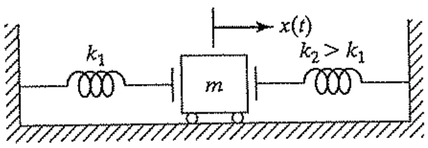
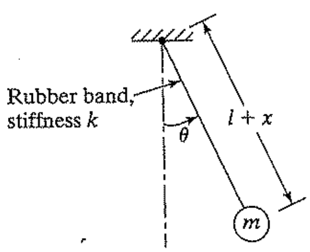
EGME 511 HW #3

1 Two springs, having different stiffnesses k_1 and k_2 with $k_2 > k_1$, are placed on either side of a mass m, as shown When the mass is in its equilibrium position, no spring is in contact with the mass. However, when the mass is displaced from its equilibrium position, only one spring will be compressed. If the mass is given an initial velocity $\dot{x}_0$ at $t = 0$, determine (a) the maximum deflection and (b) the period of vibration of the mass.



2 A mass m, connected to an elastic rubber band of unstretched length l and stiffness k, is permitted to swing as a pendulum bob, as shown. Derive the nonlinear equations of motion of the system using x and θ as coordinates. Linearize the equations of motion and determine the natural frequencies of vibration of the system.



3 Find the exact solution of the nonlinear pendulum equation

$$\ddot{\theta} + \omega_0^2\left(\theta - \frac{\theta^3}{6}\right) = 0$$

with $\dot{\theta} = 0$ when $\theta = \theta_0$, where θ_0 denotes the maximum angular displacement.

4 Find the equilibrium position and plot the trajectories in the neighborhood of the equilibrium position corresponding to the following equation:

$$\ddot{x} + 0.1(x^2 - 1)\dot{x} + x = 0$$

5 The equation of motion of a simple pendulum, subjected to external force, is given by

$$\ddot{\theta} + 0.5\,\dot{\theta} + \sin\theta = 0.8$$

Find the nature of singularity at $\theta = \sin^{-1}(0.8)$.

6 The equation of motion of a simple pendulum subject to viscous damping can be expressed as

$$\ddot{\theta} + c\dot{\theta} + \sin\theta = 0$$

If the initial conditions are $\theta(0) = \theta_0$ and $\dot{\theta}(0) = 0$, show that the origin in the phase plane diagram represents (a) a stable focus for $c > 0$ and (b) an unstable focus for $c < 0$.

7 A single degree of freedom system is subjected to Coulomb friction so that the equation of motion is given by

$$\ddot{x} + f\frac{\dot{x}}{|\dot{x}|} + \omega_n^2 x = 0$$

Construct the phase plane trajectories of the system using the initial conditions $x(0) = 10(f/\omega_n^2)$ and $\dot{x}(0) = 0$.

8 The phase plane equation of a single degree of freedom system is given by

$$\frac{dy}{dx} = \frac{-cy - (x - 0.1x^3)}{y}$$

Investigate the nature of singularity at $(x, y) = (0, 0)$ for $c > 0$.

9 Using perturbation method, find the solution of the van der Pol's equation.

$$\ddot{x} - \alpha(1 - x^2)\dot{x} + x = 0 \qquad \alpha > 0$$