Correct 2

Using x and θ as the coordinates, the kinetic and potential energies of the system can be expressed as

$$T = \frac{1}{2}\, m\, \dot{x}^2 + \frac{1}{2}\, J_0\, \dot{\theta}^2 \tag{1}$$

where $J_0 = m\,(\ell + x)^2$ and

$$V = \frac{1}{2}\, k\,(x + \delta_{st})^2 - m\, g\,(\ell + x)\cos\theta \tag{2}$$

where $\delta_{st} = \frac{mg}{k}$. Equations (1) and (2) give

$$\frac{\partial T}{\partial \dot{x}} = m\,\dot{x} \quad ; \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\,\ddot{x}$$

$$\frac{\partial T}{\partial \dot{\theta}} = J_0\,\dot{\theta} \quad ; \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \dot{J}_0\,\dot{\theta} + J_0\,\ddot{\theta} = 2\, m\,(\ell + x)\,\dot{x}\,\dot{\theta} + J_0\,\ddot{\theta}$$

$$\frac{\partial T}{\partial x} = m\,(\ell + x)\,\dot{\theta}^2 \quad ; \quad \frac{\partial T}{\partial \theta} = 0$$

$$\frac{\partial V}{\partial x} = k\,(x + \delta_{st}) - m\, g\cos\theta \quad ; \quad \frac{\partial V}{\partial \theta} = m\, g\,(\ell + x)\sin\theta$$

The equations of motion can be derived using Lagrange's equations, Eq. (6.44), as:

$$m\,\ddot{x} - m\,(\ell + x)\,\dot{\theta}^2 + k\,x + m\,g - m\,g\cos\theta = 0 \tag{3}$$

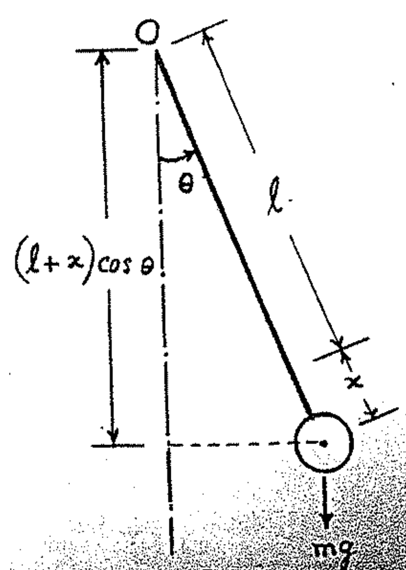
$$m\,(\ell + x)^2\,\ddot{\theta} + 2\,m\,(\ell + x)\,\dot{x}\,\dot{\theta} + m\,g\,(\ell + x)\sin\theta = 0 \tag{4}$$

Using $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and neglecting nonlinear terms involving $x^2\,\ddot{\theta}$, $\dot{\theta}^2$, $x\,\theta$, and $\dot{x}\,\dot{\theta}$, Eqs. (3) and (4) can be reduced (linearized) to obtain:

$$m\,\ddot{x} + k\,x = 0 \tag{5}$$

$$m\,\ell^2\,\ddot{\theta} + m\,g\,\ell\,\theta = 0 \tag{6}$$

Equations (5) and (6) correspond to the natural frequencies:



$$\omega_{n1} = \sqrt{\frac{k}{m}} \tag{7}$$

$$\omega_{n2} = \sqrt{\frac{g}{\ell}} \tag{8}$$