2. (i) By making the substitution $t = 1/u$ in the integral $\int_1^x \frac{dt}{t}$, prove that, for $x > 0$, $\ln(1/x) = -\ln x$.

(ii) The function f is such that $f(x + \pi) = f(x)$ for all values of x. In the interval $0 \leqslant x < \pi$, $f(x) = x - \sin x$. Sketch the curve $y = f(x)$ for $-2\pi \leqslant x \leqslant 2\pi$, and state all the values of x for which the function f is discontinuous. Evaluate the integrals

(*a*) $\int_{-\pi/2}^{\pi/2} f(x)\,dx$, (*b*) $\int_0^{3\pi/2} f(x)\,dx$.

(16 marks)

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$

13. Of the vectors

$$\mathbf{a} = \begin{pmatrix} 5 \\ 6 \\ -11 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 9 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} -5 \\ -14 \\ 24 \end{pmatrix}$$

show that **a**, **b**, **d** form a set of basis vectors. Express **c** in terms of this basis.

If **a**, **b**, **c** and **d** are the position vectors of points A, B, C and D respectively, show that the point P (1, −2, 3) lies on AD, find the ratio $AP : AD$, and show that BP is perpendicular to PC.

(16 marks)

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14. The planes

$$\begin{aligned} 2x + y + z &= 4 \\ x + 2y + z &= 2 \\ x + y + 2z &= 6 \end{aligned}$$

meet only in the point (1, −1, 3). The x, y, z coordinate system is transformed by the linear transformation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

In the X, Y, Z system, obtain the equations of the planes and the coordinates of the point(s) in which they meet.

(16 marks)

V **15.** (i) If $I_n = \int_0^{\pi/2} x^n \cos x\,dx$, prove that for $n \geqslant 2$

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)\,I_{n-2}\,.$$

Hence find I_4.

(ii) Solve the equation

$$3\sinh^2 x - 2\cosh x - 2 = 0\,.$$

Give the values of x as natural logarithms.

(16 marks)

$x = \ln 3$ & $x = \ln \frac{1}{3}$

$\frac{13}{4}$

16. (i) Express in the form $\cos\theta + i\sin\theta$ each of the cube roots of unity. If $\alpha^3 = \beta^3 = 1$ and $\alpha \neq \beta$, use the Argand diagram to find the value of $|\alpha - \beta|$. On the same diagram plot the points representing the three possible values of $\alpha + \beta$, and evaluate $(\alpha + \beta)^3$.

(ii) Show geometrically or otherwise that for any two complex numbers z_1 and z_2

$$|z_1 + z_2| \leqslant |z_1| + |z_2|\,.$$

Prove by induction that the sum of the moduli of any finite number of complex numbers is not less than the modulus of their sum.

(16 marks)