HW2, MAE 200A. Fall 2005. UCI

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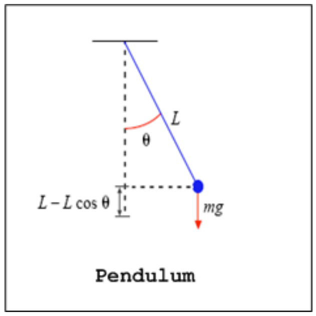
1 Problem 1

Given this simple pendulum, plot the constant energy curves in $\theta, \dot{\theta}$



Answer
The constant energy curves are curves in the Y-X plane where energy is constant. The Y-axis represents $\dot{\theta}$, and the X-axis represents θ

We are told to assume there is no damping.

We assume the pendulum is given an initial force when in the initial position ($\theta = 0^0$) that will cause it to swing anticlock wise. The pendulum will from this point obtain an energy which will remain constant. The higher the energy the pendulum posses, the larger the angle θ it will swing by will be. If the energy is large enough to cause the pendulum to reach $\theta = \pi$ (i.e. upright position) with an non zero angular velocity, then the pendulum will swing continue to rotate in the same direction and will not swing back and forth.

To be able to solve this, I therefore need to find the range of angle the pendulum will swing for a given energy level. And for this range of angle determine the angular speed, this will give me the required phase plot.

I will first derive the expression for the energy E for the pendulum above, which will be a function of $\theta, \dot{\theta}$.

$$\begin{aligned} E &= PE + KE \\ &= mgL\left(1 - \cos\theta\right) + \frac{1}{2} mL^2 \dot{\theta}^2 \end{aligned}$$

Hence

$$\dot{\theta} = \pm\sqrt{\frac{2}{mL^2}\left(E - mgL(1 - \cos\theta)\right)} \tag{1}$$

For $\dot{\theta}$ to be real, $E - mgL(1 - \cos\theta) \geq 0$, hence

$$E \geq mLg(1 - \cos\theta)$$

So, for $\theta = 0$, $E \geq 0$

For $\theta = \frac{\pi}{4}, E \geq (0.707\,11)\, mLg$

For $\theta = \frac{\pi}{2}, E \geq mLg$

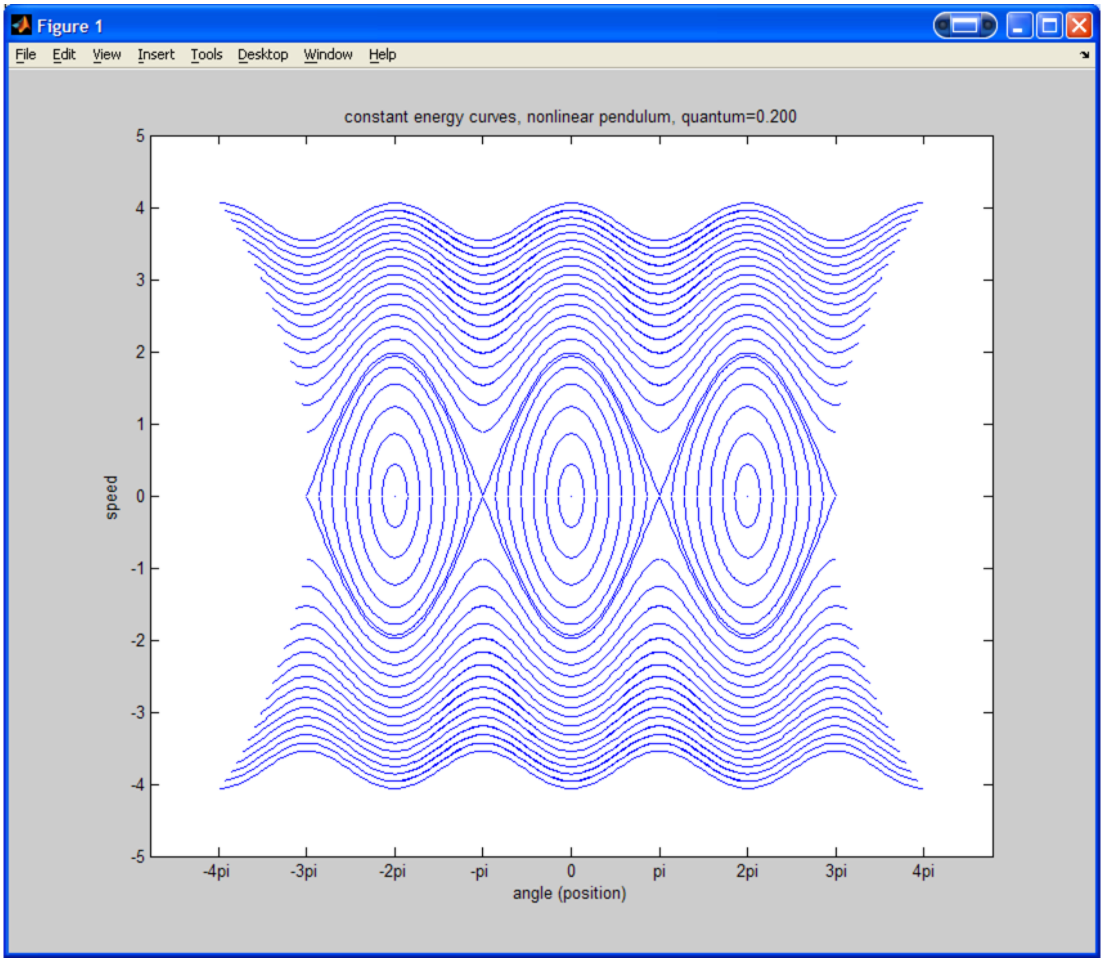
For $\theta = \frac{3\pi}{4}, E \geq (1.707\,11)\, mLg$

For $\theta = \pi, E \geq 2mLg$

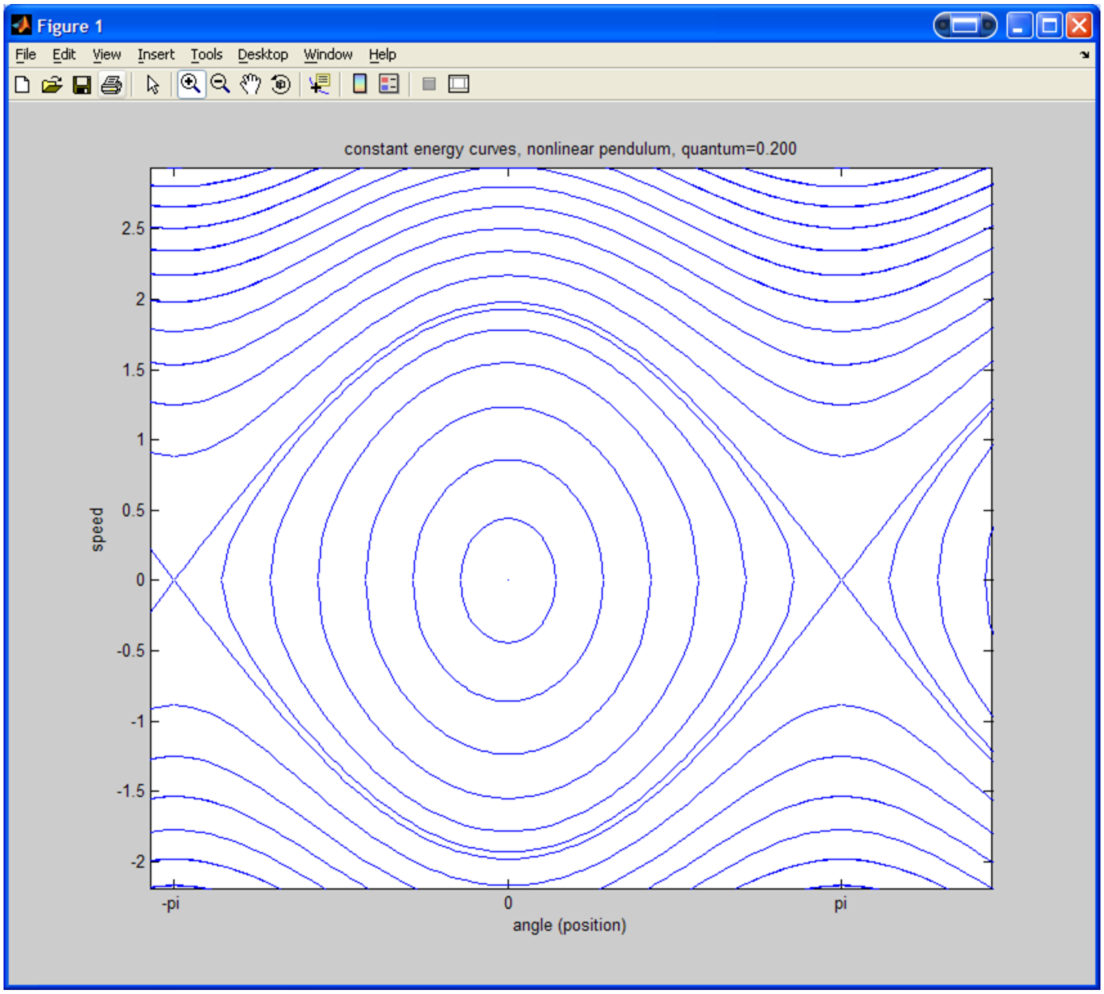
The above means that for energy level mLg for example, the angle range to plot the constant energy curve for will be between $\theta = 0..\frac{\pi}{2}$

So, For each energy level, I will generate a plot for all the angles up to the angle limit allowed by that energy level. For each angle, I will find $\dot{\theta}$ from equation (1)

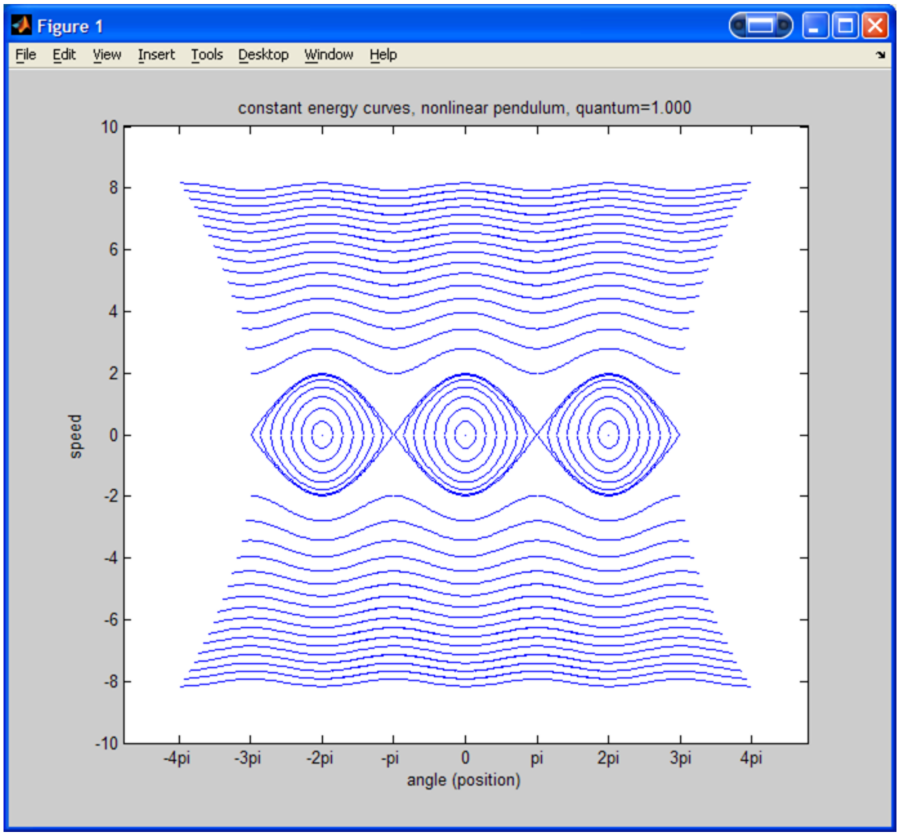
The following is the resulting plot and below that is the program I wrote to generate this plot. This was done for $m = 1, g = 9.8m/s^2, L = 10m$
I show different plots for different granuality in the energy levels. Different quantum is used for different plots.
By the quantum I mean that the energy levels for the escape energy will change from its lower level by this quantum factor. So, for a quantum of 1/2 for example, this means the energy is being incremented for each level by an amount which 1/2 the previuse energy level.

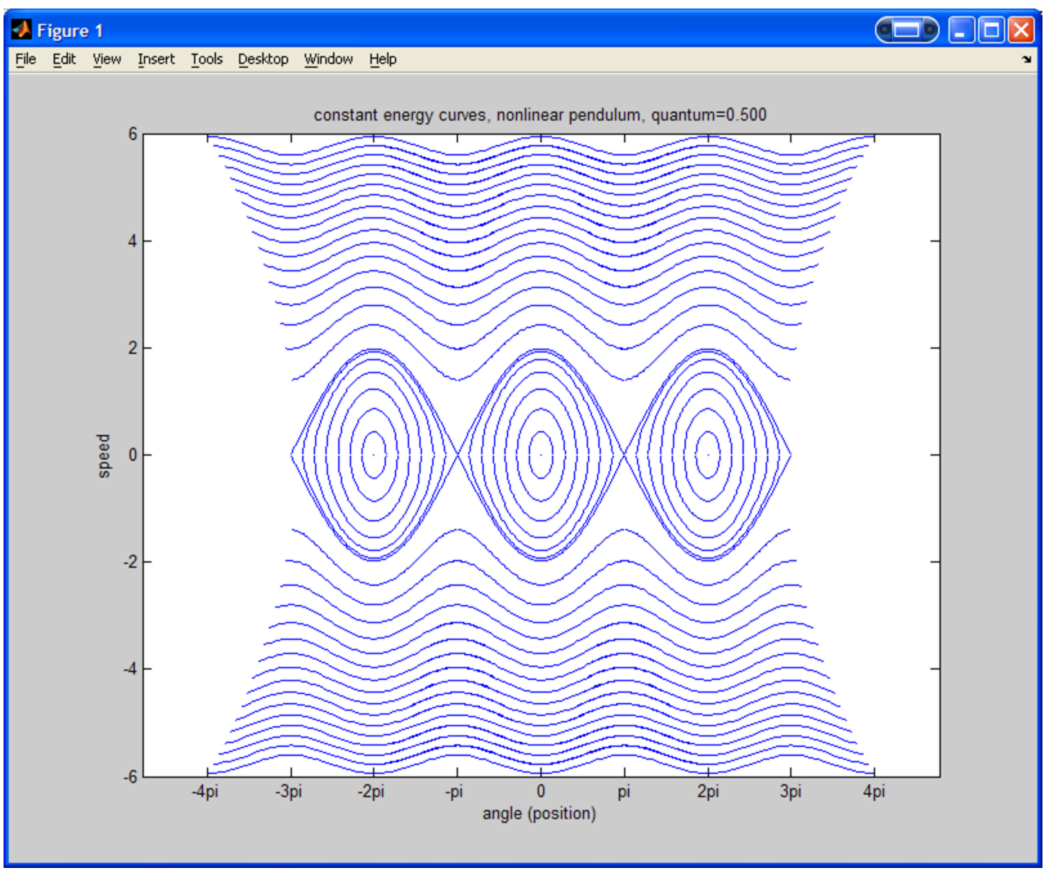


This is a zoom to a smaller area to better show the energy lines



This below is a plot using different quantum





If we need to calculate the equilibrium points, we can do find the rate of change of the energy, and set that to zero as follows

$$
\begin{aligned}
\dot{E} &= mgL\left(\sin\theta\ \dot{\theta}\right) + \frac{1}{2}mL^2 2\dot{\theta}\ \ddot{\theta} \\
&= mgL\left(\sin\theta\ \dot{\theta}\right) + mL^2\dot{\theta}\ \ddot{\theta}
\end{aligned}
$$

substitue into the above the following system dynaics equation

$$
mL^2\ddot{\theta} = -c\dot{\theta} - mgL\sin\theta
$$

We are told to set $c = 0$, hence the equation becomes

$$
mL^2\ddot{\theta} = -mgL\sin\theta \tag{2}
$$

so we get

$$
\begin{aligned}
\dot{E} &= mgL\sin\theta\ \dot{\theta} - mgL\sin\theta\ \dot{\theta} \\
&= -2mgL\sin\theta\ \dot{\theta}
\end{aligned}
$$

For $\dot{E} = 0$ means $\theta = \pm n\pi$ or $\theta = 0, \pm\pi, \pm 2\pi, \cdots$

```
function HW2
%HW2 problem. MAE 200A. UCI, Fall 2005
%by Nasser Abbasi

%This MATLAB function generate the constant energy level
%curves for a nonlinear pendulum with no damping

close all; clear all;
m=1; g=9.8; L=10;

nPoints=40;  %number of data points (positio vs speed) to find per curve

nEnergyLevels = 8; %number of energy levels
lowAnglesToVisit = linspace(0,pi,nEnergyLevels);
lowEnergyLevels(1:nEnergyLevels) = m*g*L*(1-cos(lowAnglesToVisit));
highAnglesToVisit = linspace(pi,2*pi,2*nEnergyLevels);

initialHighEnergy=2*m*g*L;
Q=0.2;
for i=1:2*nEnergyLevels
    highEnergyLevels(i) = initialHighEnergy+(Q*i*initialHighEnergy);
end

A = zeros(length(lowAnglesToVisit)+length(highAnglesToVisit),2);
A(:,1) = [lowAnglesToVisit highAnglesToVisit];
A(:,2) = [lowEnergyLevels highEnergyLevels];

[nAngles,nCol]=size(A);
data=zeros(nPoints,2);

for j=1:nAngles

    currentAngle=A(j,1);
    currentEnergyLevel =A(j,2) ;
    angles=linspace(0,currentAngle,nPoints);
    data(1:nPoints,1)=angles(:);

    for m=1:nPoints
        data(m,2)=speed(currentEnergyLevel,angles(m));
    end
    doPlots(data);
end

title(sprintf('constant energy curves, nonlinear pendulum, quantum=%1.3f',Q));
xlabel('angle (position)');
ylabel('speed');
set(gca,'xtick',[-4*pi,-3*pi,-2*pi,-pi,0,pi,2*pi,3*pi,4*pi]);
set(gca,'XTickLabel',{'-4pi','-3pi','-2pi','-pi','0','pi','2pi','3pi','4pi'})

function s=speed(energy,angle)
m=1; g=9.8; L=10;
if angle<pi
    s=sqrt(2/(m*L^2)*(energy-m*g*L*(1-cos(angle))));
else
    s=sqrt(2/(m*L^2)*(energy-m*g*L*(1+cos(angle-pi))));
end

function doPlots(data)
plotCurves(data,0);
plotCurves(data,2*pi);
plotCurves(data,-2*pi);

function plotCurves(data,k)
plot(data(:,1)+k,data(:,2));
hold on;
plot(data(:,1)+k,-data(:,2));
plot(-data(:,1)+k,-data(:,2));
plot(-data(:,1)+k,data(:,2));
```