9

HW#6

Nasser Abbasi

MAE 200A.

Nov 1, 2005

1

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$$

let $x_1 = x$

$x_2 = \dot{x}$

hence we get $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Case 1 $\omega = 1$, $\zeta = 5$

$A = \begin{pmatrix} 0 & 1 \\ -1 & -10 \end{pmatrix}$ hence Eigenvalues $|A - \lambda I| = 0 \Rightarrow$

$\begin{vmatrix} -\lambda & 1 \\ -1 & -10-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + 10\lambda + 1 = 0 \Rightarrow \boxed{\begin{matrix} \lambda_1 = -5 + 2\sqrt{6} \\ \lambda_2 = -5 - 2\sqrt{6} \end{matrix}}$

to find eigenvectors we can solve $Av = \lambda v$.

but since λ's are distinct, then Jordan form is

$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}.$$

hence an alternative ODE form is similarity transformation:

$$\dot{\bar{z}} = (P^{-1}AP)\, z.$$

but $P^{-1}AP = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{pmatrix}$

$\bar{z} = P^{-1} X$

hence $\dot{z} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} z.$

-1

i.e. $\boxed{\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -5+2\sqrt{6} & 0 \\ 0 & -5-2\sqrt{6} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}$

Case 2 $\quad \omega_n = 1 \quad , \quad \zeta = 1$

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} , \Rightarrow |A - \lambda I| = 0 .$$

$$\begin{vmatrix} -\lambda & 1 \\ -1 & -2-\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(-2-\lambda) + 1 = 0 \Rightarrow \lambda = -1, -1 .$$

since duplicate eigenvalue, then possible Form for Jordan form is $\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$.

generalized eigenvector?

so $$\dot{z} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} z \Rightarrow \boxed{\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}$$

$\bar{z} = ?\ X$

Case 3 $\quad \omega_n = 1, \ \zeta = 0.5$

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \Rightarrow \quad A - \lambda I = \det \begin{pmatrix} -\lambda & 1 \\ -1 & -1-\lambda \end{pmatrix} = 0$$

$$\Rightarrow -\lambda(-1-\lambda) + 1 = 0 \Rightarrow \lambda^2 + \lambda + 1 = 0 \Rightarrow \lambda = \frac{-1 \pm \sqrt{1-4}}{2}$$

$\lambda = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ $\quad$ i.e. $\lambda = \sigma \pm j\omega$ where $\boxed{\alpha = -\frac{1}{2}, \ \omega = \frac{\sqrt{3}}{2}}$

~~then Jordan Form is~~ $\cancel{\begin{pmatrix} \sigma & -\omega \\ \omega & \sigma \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}}$

~~so~~ $\cancel{\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}$

$\longrightarrow$

2 distinct eigenvalues $\lambda_1 = -1 + j\frac{\sqrt{3}}{2}$, $\lambda_2 = -1 - j\frac{\sqrt{3}}{2}$.

so Jordan Form $\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \Rightarrow$

$$\boxed{\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -1 + j\frac{\sqrt{3}}{2} & 0 \\ 0 & -1 - j\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}$$

~~use~~

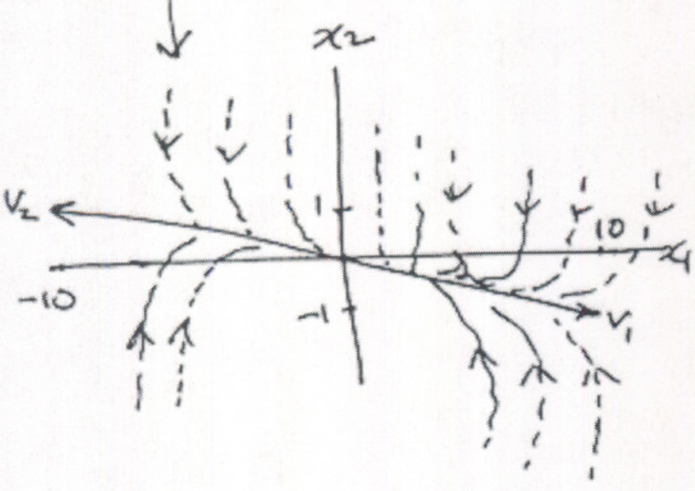
how about real matrix $\begin{bmatrix} \sigma & w \\ w & \sigma \end{bmatrix}$

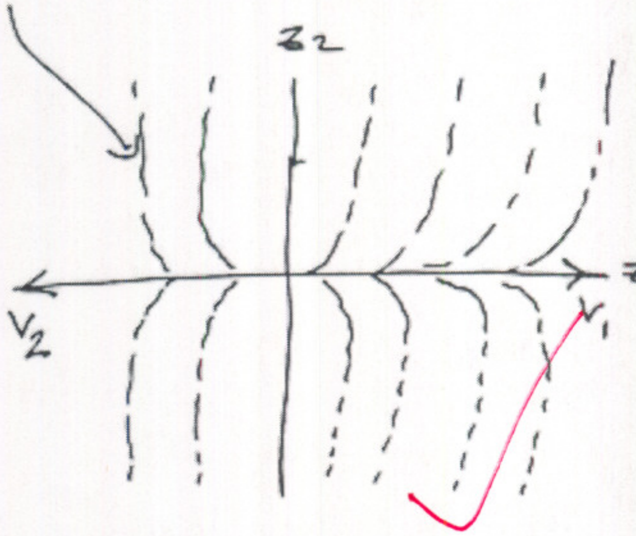
sketch of state portrait

(p.s. For this part I used matlab to help find eigenvectors [v,d] = eig(A))

Case 1 original rep. $\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -10 \end{pmatrix} x$.

modified rep. $\dot{z} = \begin{pmatrix} -5+2\sqrt{6} & 0 \\ 0 & -5-2\sqrt{6} \end{pmatrix} z$

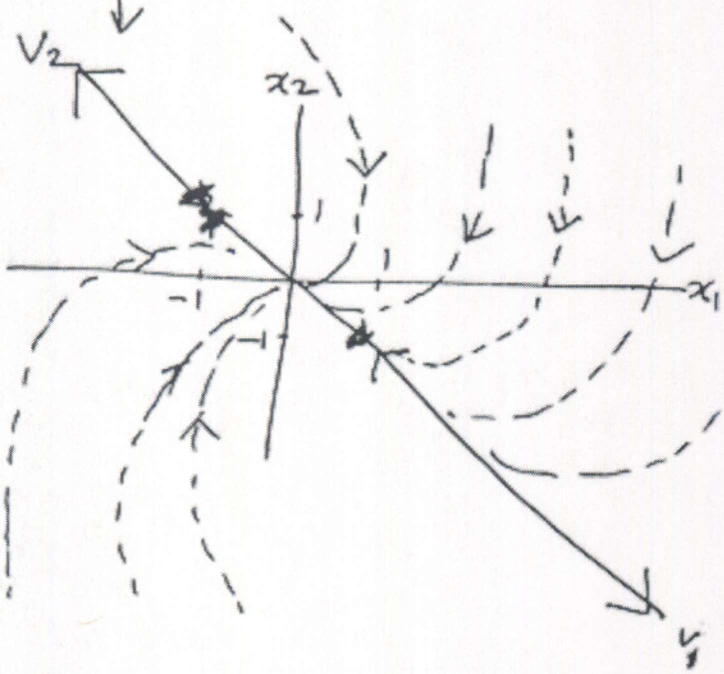


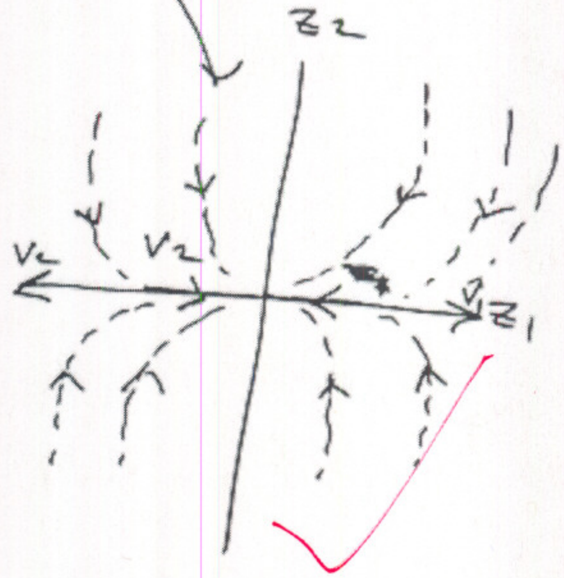


Case 2

original $\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x$

transformed $\dot{z} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} z$.





Case 3

original $\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} x$

modified $\dot{z} = \begin{pmatrix} -1+j\frac{\sqrt{3}}{2} & 0 \\ 0 & -1-j\frac{\sqrt{3}}{2} \end{pmatrix} z$