HW8

1) Definition of relative equilibria:

$\dot{x} = \ddot{x} = \dot{y} = \ddot{y} = \dot{z} = \ddot{z} = 0$

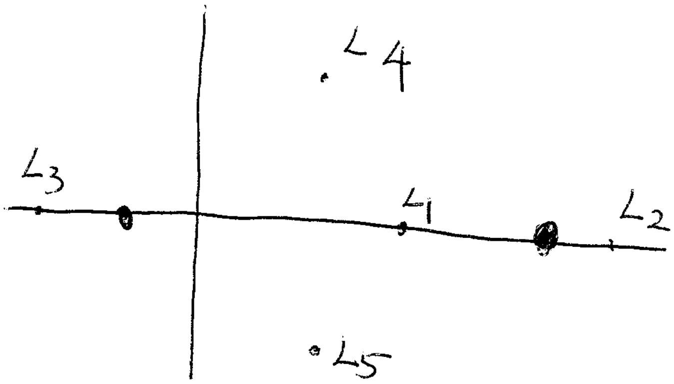
substitute these into eqns of motion:

$$x - (1-\mu)\left(\frac{x - x_1}{r_1^3}\right) - \mu\left(\frac{x - x_2}{r_2^3}\right) = 0$$

$$y - \left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right) y = 0$$

$$-\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right) z = 0$$

These are non-linear equations, use matlab command fsolve (fun, x0) = x, try several initial conditions x_0 to get all the libration points.



$L_1 = (0.837, 0)$

$L_2 = (1.155 \quad 0)$

$L_3 = (-1.005 \quad 0)$

$L_4 = (0.488 \quad 0.866)$

$L_5 = (0.488 \quad -0.866)$

2) We consider this in $x-y$ plane, $z=0$

convert eqns of motion to four-dimensional first order ODEs.

$x = x_1 \qquad \dot{x} = x_2 \qquad y = x_3 \qquad \dot{y} = x_4$

then we have

$$\dot{x}_1 = x_2$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_2 = 2x_4 + x_1 - (1-\mu)\left(\frac{x_1 - x_1'}{r_1^3}\right) + \mu\left(\frac{x_1 - x_2'}{r_2^3}\right)$$
$$\dot{x}_4 = -2x_2 + x_2 - \left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right)x_2$$

Note: $x_1' = -\mu \qquad x_2' = 1-\mu$

First use the "Jacobian" command in Matlab to get the Jacobian matrix symbolically. Then compute the eigenvalues and eigenvectors at points $L1$ and $L4$.

$$J_1 = \frac{\partial f}{\partial \mathbf{x}}\bigg|_{\mathbf{x}_{eq} = L_1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 11.284 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & -4.142 & 0 \end{pmatrix}$$

Eigen values are $\lambda_1 = 2.9302$, $\lambda_2 = -2.9302$

$\lambda_3 = +2.3332i$, $\lambda_4 = -2.3332i$

Because $\lambda_1 > 0$, L_1 is Un-Stable.

Eigenvectors associated with J_1 are

$$V_1 = \begin{pmatrix} 0.293 \\ 0.859 \\ -0.135 \\ -0.395 \end{pmatrix} \quad V_2 = \begin{pmatrix} -0.2934 \\ 0.8597 \\ -0.1351 \\ 0.3958 \end{pmatrix} \quad V_3 = \begin{pmatrix} 0.1059 \\ 0 \\ 0 \\ -0.885 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.247 \\ 0.3795 \\ 0 \end{pmatrix} i$$

$$V_4 = \begin{pmatrix} 0.1059 \\ 0 \\ 0 \\ -0.885 \end{pmatrix} - \begin{pmatrix} 0 \\ 0.247 \\ 0.3795 \\ 0 \end{pmatrix} i \qquad V_{3,4} = u \pm w i$$

So L_1 is a saddle point with $E_u = \text{span}\{V_1\}$

$E_s = \text{span}\{V_2\}$, $E_c = \text{span}\{u, w\}$

Linear behavior is superposition of 3 modes: one decaying exponential, one growing exponential, and one oscillatory.

For L_4, Jacobian matrix is

$$J_2 = \left.\frac{\partial f}{\partial \mathbb{X}}\right|_{x_{eq}=L_4} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0.75 & 0 & 1.262 \\ 0 & 0 & 0 & 1 \\ 1.26 & -2 & 2.25 & 0 \end{pmatrix} \qquad x_{eq} = L_2 = \begin{pmatrix} 0.488 \\ 0 \\ 0.866 \\ 0 \end{pmatrix}$$

Eigenvalues are

$\lambda_1 = 0.9551i$, $\lambda_2 = -0.9551i$, $\lambda_3 = 0.2962i$, $\lambda_4 = -0.2962i$

with associated eigenvectors

~~$v_1 = u_1 \pm j.$~~ $v_1 = u_1 + w_1 i$, $v_2 = u_1 - w_1 i$.

$v_3 = u_2 + w_2 i$, $v_4 = u_2 - w_2 i$

center with $E_c = \text{span}\{u_1, w_1, u_2, w_2\}$

linear behavior is superposition of 2 undamped, oscillatory modes.

No real parts for λ_i. These are non-hyperbolic points. Linearization analysis is not definitive.