MAE 200A Engineering Analysis I

Assignment #8

Due: Tuesday, November 22, 2005

Objective: In this assignment you will apply the theory and methods for the local analysis of the near-equilibrium motion of a nonlinear dynamical system to the circular, restricted 3-body problem.

Assumptions:

(i) 3 bodies: a primary body of mass m_1, a second body of mass m_2, and a spacecraft of mass m_3

(ii) the primary body and secondary body are each in a circular orbit about their combined center of mass. It follows that the orbits have radii μ and $1-\mu$ respectively. From basic orbit mechanics, it also follows that the primary and secondary bodies are always on opposite sides of their combined center of mass, separated by 180 degrees.

(iii) the spacecraft has no influence on the orbits of the primary and secondary bodies

(iv) the spacecraft trajectory is determined only by the gravitational fields of the primary and secondary bodies

Equations of motion in the frame that rotates with the two primaries:

$$\ddot{x} - 2\dot{y} - x = -(1-\mu)\left(\frac{x - x_1}{r_1^3}\right) - \mu\left(\frac{x - x_2}{r_2^3}\right)$$

$$\ddot{y} + 2\dot{x} - y = -\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right)y$$

$$\ddot{z} = -\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right)z$$

where $r_1 = \left[(x - x_1)^2 + y^2 + z^2\right]^{1/2}$, $r_2 = \left[(x - x_2)^2 + y^2 + z^2\right]^{1/2}$, $\mu = m_1/(m_1 + m_2)$, $x_1 = -\mu$, and $x_2 = 1-\mu$.

1) Compute the 5 relative equilibria, known as the Lagrange or libration points, denoted L1, L2, L3, L4, and L5, when the primary is the Earth and the secondary is the Moon and thus $\mu = 0.012$.

2) The locations of the L1 and L4 Lagrange points are indicated approximately in the picture below. Characterize the local behavior in the neighborhood of L1 and L4 using linear analysis.