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HW # 9

MAE 200 A

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1. plot the zero velocity curves for different values of the constant α (or C). Hint $C=-1.7$ should produce roughly the first zero velocity curves presented in the lecture.

Answer

Curves of zero velocity are given as the (x,y) points from the solution of the equation

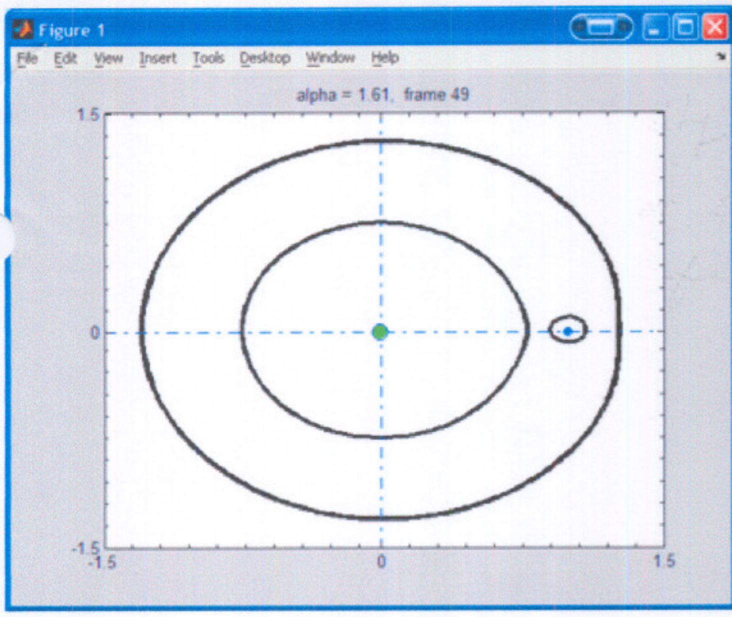
$$U = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} = \alpha$$

the above represents the P.E $=\alpha$. i.e when K.E. $=0$ because we assume that velocity is zero.

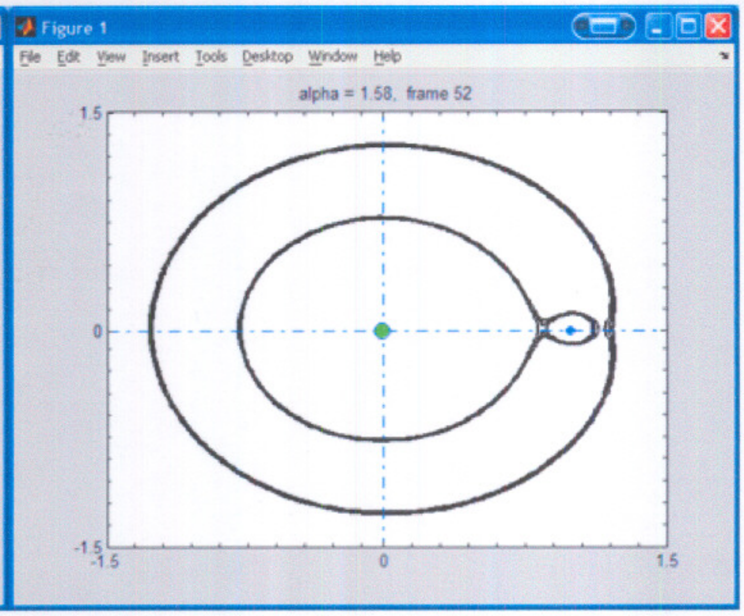
$\mu = 0.012$
$x_1 = -0.012$
$x_2 = 0.988$

I have written a matlab animation to draw the zero velocity curves for $\alpha=2$ down to $\alpha=1.5$. I have emailed the script. please run to see result.

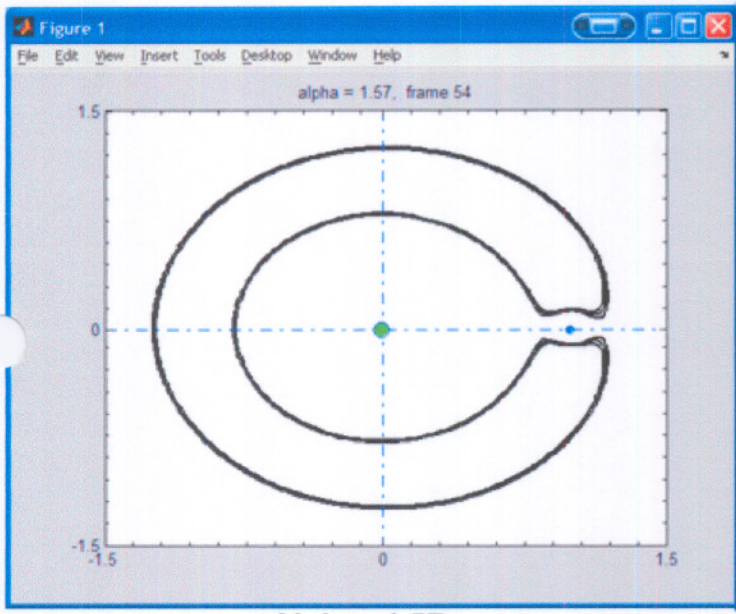
next page I show few frames from the animation for different α.



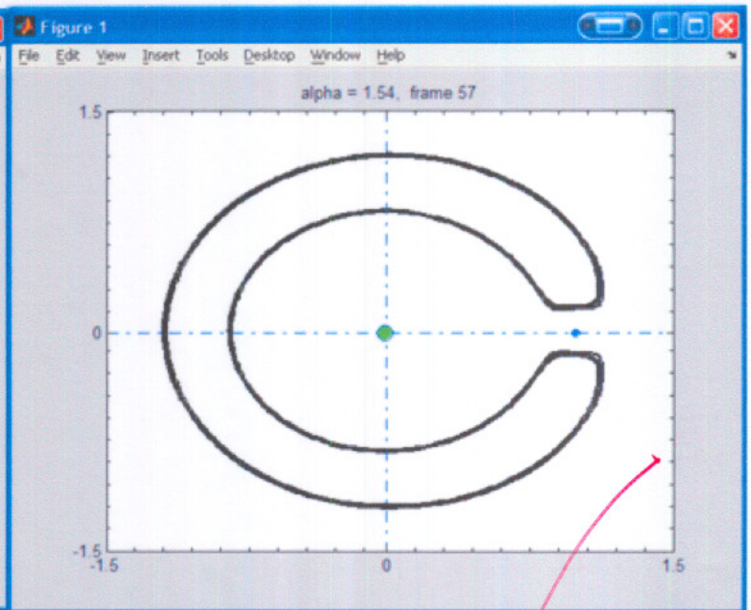
Alpha= 1.61



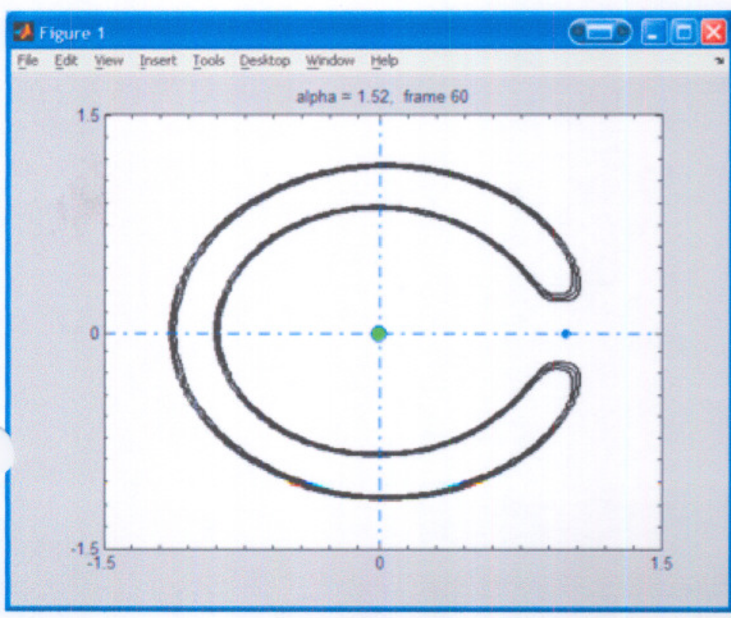
Alpha= 1.58



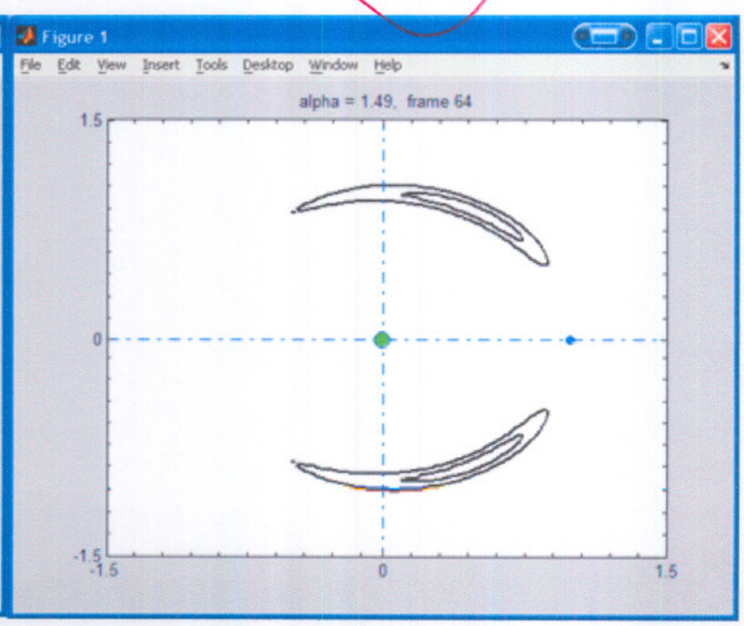
Alpha= 1.57



Alpha= 1.54

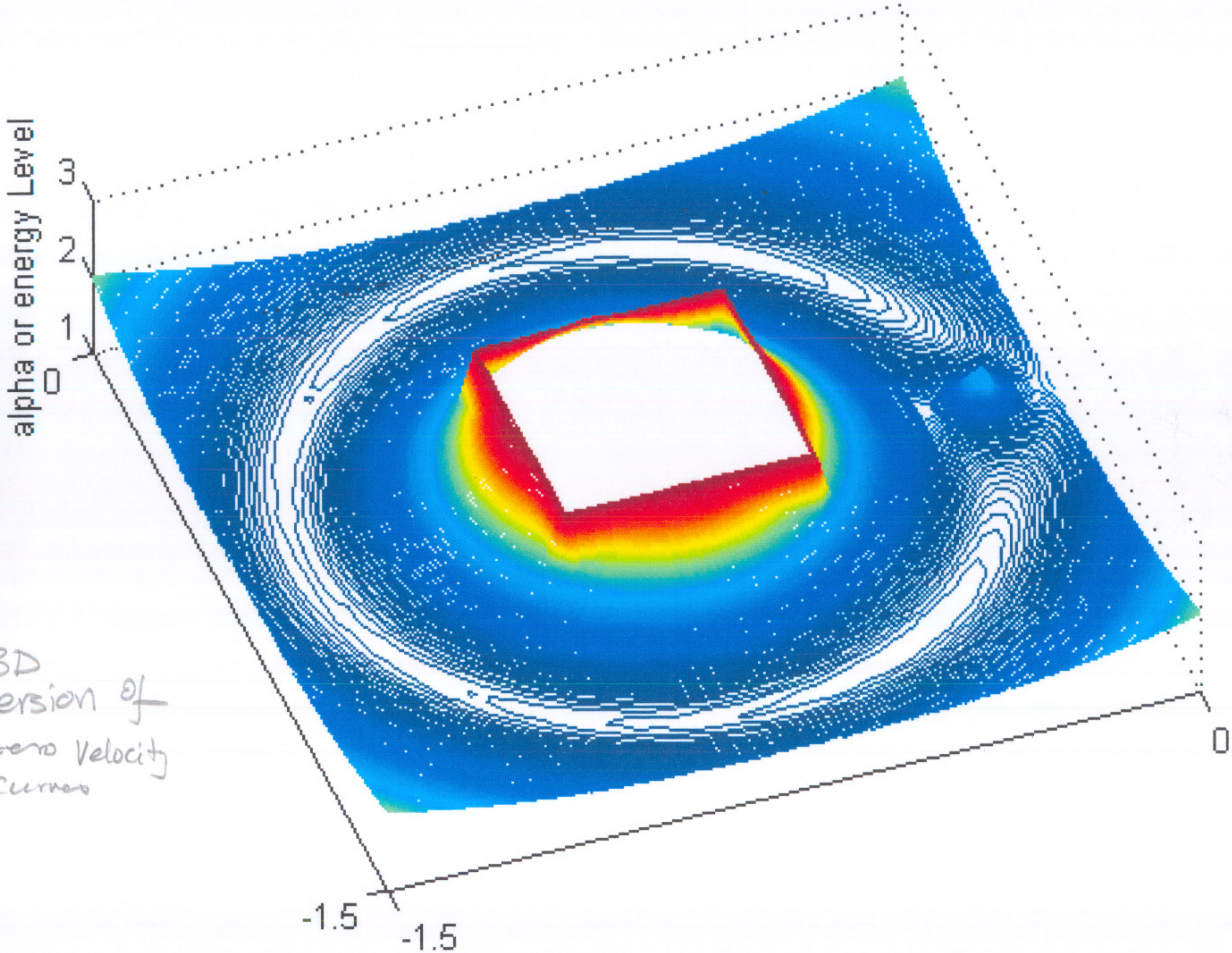


Alpha= 1.52



Alpha= 1.49

frame 35



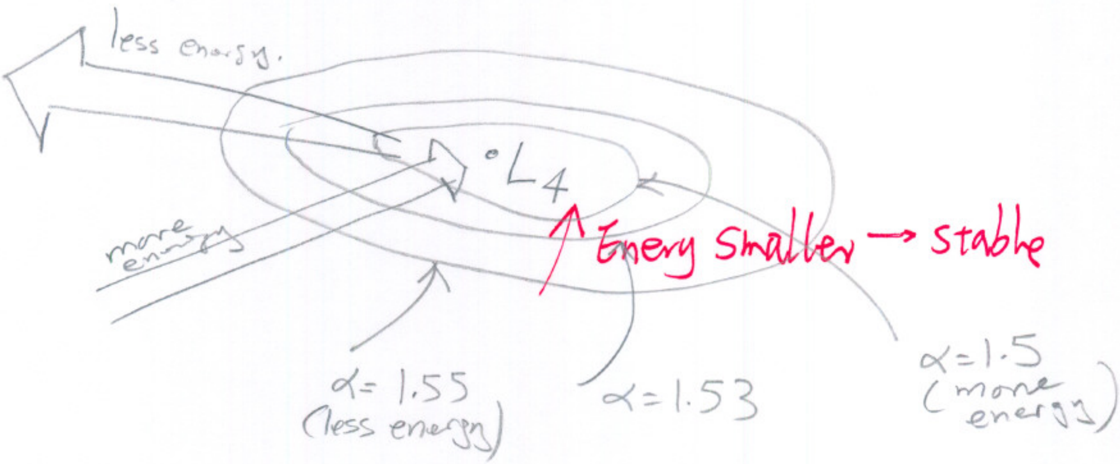
3D
Version of
Zero Velocity
Curves

part 2

in HW#8 we did Linear Analysis. based on Linear analysis It was found that L_1 was definitly an unstable equilibrium.

However, for L4 it was unconclusive since all eigenvalues were on the imaginary axis.

but from plotting zero velocity curves, we see that around L4 we get a contour plot as:



this means an object (such as spacecraft) will posses more Potential energy (U), the closer it is to L4, since smaller α means larger P.E.

i.e trajectories closer to L4 tend to say close to L4. trajectories in the outer zero velocity curves can't move inside without external energy added.

hence this means spacecraft is harder to perturbate away from L4 the closer it is to L4. i.e. we need more energy to perturbate object away from L4 the closer the object is to L4.

because more external work needed to overcome the higher energy levels

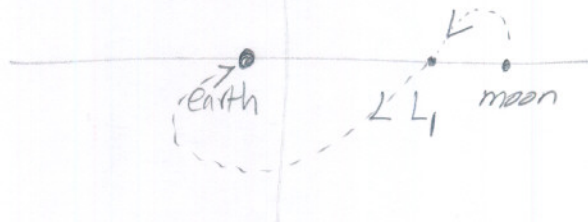
Wrong conclusion ⟹ L4 is stable.

hence using zero velocity curves helps in determing stability of L4.

part 3

PS. I First generate the equation of solution of motion near L4 using eigenvalues and eigenvectors.

After that, I will solve this using ODE45.

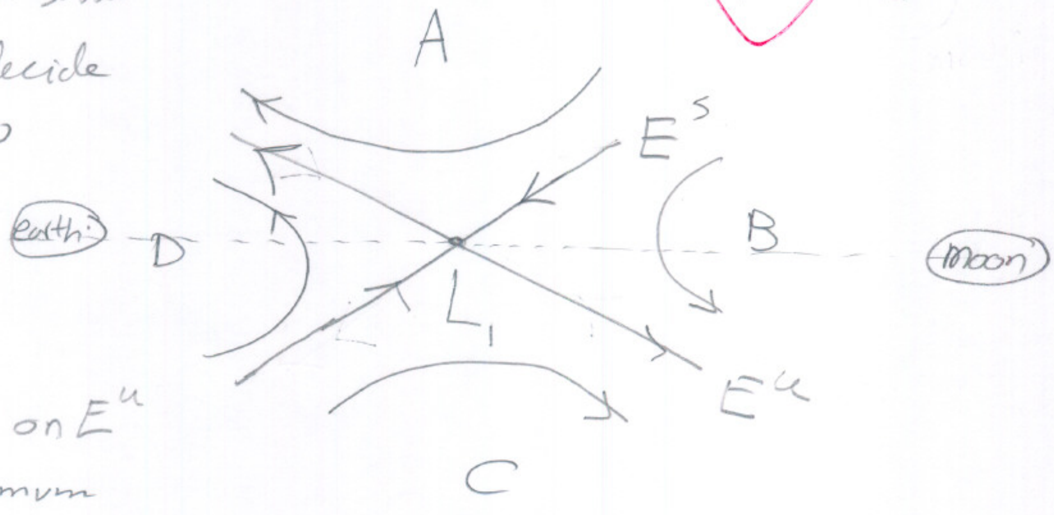


From L_1, if we integrate backwards to get to the moon orbit.

from L_1, we integrate forward to get to the earth orbit. L_1 is a saddle point (from HW # 8)

state portrait near L1

There are 4 regions in this mainfold, need to decide on which trajectory to pick. we need to pick A trajectory That is closest to L_1 but Not on E^s nor on E^u to given us the minumum energy need to go from moon to earth. trajectories Further away means more energy wasted.



Jacobi integral is

$$C = -2E = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - \dot{x}^2 - \dot{y}^2$$

total Energy of space craft.

$\rightarrow$

hence $C = \underbrace{x^2+y^2+\frac{2(1-\mu)}{r_1}+\frac{2\mu}{r_2}} - (\dot{x}^2+\dot{y}^2)$

near L_1, minimum energy
for this term is the
P.E. which is α found
in part 1. i.e it is α when the
zero velocity curves just meet at L_1.

from part 1, $\alpha = 1.58$ was the energy level (P.E.)
when zero level curve meet at L_1.

hence $C = 1.58 - (\dot{x}^2+\dot{y}^2)$

so need to pick initial speed $\dot{x}_0$ and $\dot{y}_0$ s.t. C is
just over zero. for example $\boxed{\dot{x}_0 = 0.88 \quad , \dot{y}_0 = 0.88}$
will statisfy this.

next need to determine the eigenvectors at L_1.
from problem HW 8. the linearized equations at L_1
are

$$\dot{\Delta s} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 11.27 & 0 & 0 & 2 \\ 0 & -4.14 & -2 & 0 \end{bmatrix} \Delta s$$

and eigen values found to be
$\boxed{\lambda = -2.9, -2.3j, +2.3j, 2.9}$

for $\lambda=-2.9, \lambda=+2.9$ we need to find v s.t.

$(A - \lambda I)v = 0$

$\longrightarrow$

using Matlab, these are the eigenvectors

$\lambda_1 = 2.93$, $v_1 = [0.2936 \quad -0.1352 \quad 0.8595 \quad -0.3959]$

$\lambda_2 = -2.93$, $v_2 = [0.2936 \quad 0.1352 \quad -0.8595 \quad -0.3959]$

$\lambda_3 = 2.33j$, $v_3 = [0.11 \quad 0.38j \quad 0.25j \quad -0.89]$

$\lambda_4 = -2.33j$, $v_4 = [0.11 \quad -0.38j \quad -0.25j \quad -0.89]$

hence the Linear solution of the equation of motion near L_1 is

$$\begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix} = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2 + c_3 e^{\lambda_3 t} v_3 + c_4 e^{\lambda_4 t} v_4$$

was $x_0 = 0.8376$
$y_0 = 0$

at $t=0$, $x_0^+ = 0.83761$ (from HW#8, coordinate of L_1), $y_0^+ = 0.0001$

$\dot{x}_0 = 0.88$ (from making Jacobi integral min.)

$\dot{y}_0 = 0.88$. (note: I made x_0^+, y_0^+ slightly different from L_1 to perturbe it)

$$\begin{pmatrix} 0.83761 \\ 0.0001 \\ 0.88 \\ 0.88 \end{pmatrix} = c_1 \begin{pmatrix} 0.294 \\ -0.135 \\ 0.859 \\ -0.396 \end{pmatrix} + c_2 \begin{pmatrix} 0.294 \\ 0.135 \\ -0.859 \\ -0.396 \end{pmatrix} + c_3 \begin{pmatrix} 0.11 \\ 0.38j \\ 0.25j \\ -0.89 \end{pmatrix} + c_4 \begin{pmatrix} 0.11 \\ -0.38j \\ -0.25j \\ -0.89 \end{pmatrix}$$

Solve for c_1, c_2, c_3, c_4 $\longrightarrow$

$$\begin{pmatrix} 0.8376 \\ 0.0001 \\ 0.88 \\ 0.88 \end{pmatrix} = \begin{pmatrix} 0.294 & 0.294 & 0.11 & 0.11 \\ -0.135 & 0.135 & 0.38j & -0.38j \\ 0.859 & -0.859 & 0.25j & -0.25j \\ -0.396 & -0.396 & -0.89 & -0.89 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \cong \begin{pmatrix} 2.395 \\ 1.4667 \\ -1.354 - 0.165j \\ -1.354 + 0.165j \end{pmatrix}$$

hence the solution is

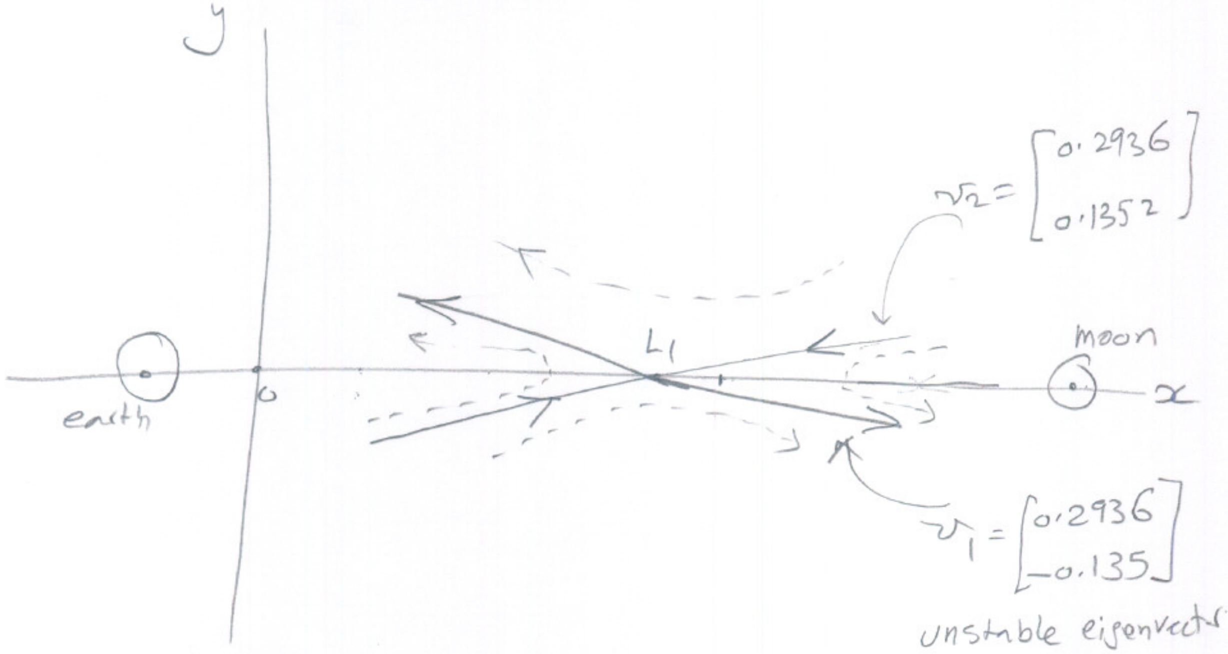
$$\begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix} = 2.395 e^{2.93t} \begin{pmatrix} 0.294 \\ -.135 \\ .859 \\ -.396 \end{pmatrix} + 1.467 e^{-2.93t} \begin{pmatrix} .294 \\ .135 \\ -.859 \\ -.396 \end{pmatrix}$$

$$+ (-1.354 - 0.165j) e^{2.33jt} \begin{pmatrix} .11 \\ .38j \\ .25j \\ -.89 \end{pmatrix} + (-1.354 + .165j) e^{-2.33jt} \begin{pmatrix} .11 \\ -.38j \\ -.25j \\ -.89 \end{pmatrix}$$

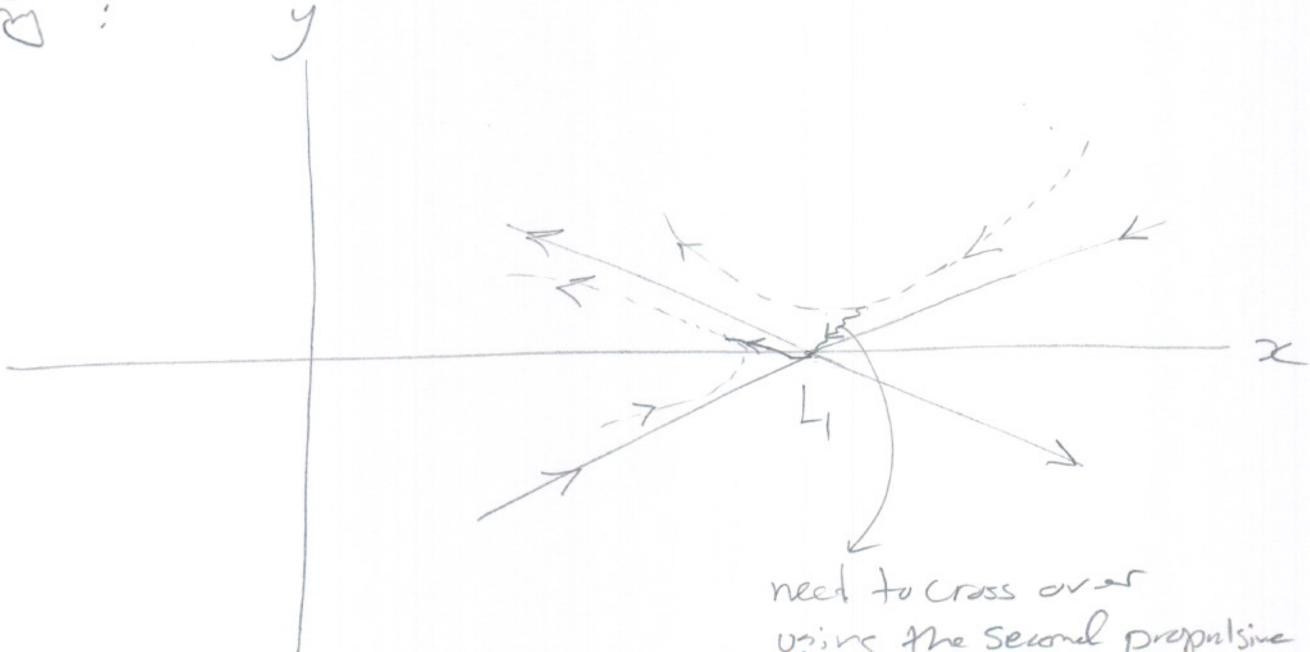
this is the analytical solution based on linear analysis near L_1.

However now I will use ode45 to integrate the motion using the same initial conditions but using the $\dot{s} = As$ equation where $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 11.27 & 0 & 0 & 2 \\ 0 & -4.14 & -2 & 0 \end{bmatrix}$ from before. $\longrightarrow$

First let me show the eigenvectors for L_1 (saddle point). I'll show only v_1 and v_2 associated with λ_1, λ_2.



so to go from moon to earth via L_1 we need this trajectory :



Now I solve this using ODE45 → code and output next

PART 3 HW9

```
function nma_HW9_part3_v3

%by nasser abbasi. user ode45 to plot moon-earth orbit via L1

clear all; close all;
mu=0.012;
t0=0;
tfinal=10;
xL1=0.8376; yL1=0.0;
x0=xL1; y0=yL1; xdot0=-0.88; ydot0=-0.88;
IC=[x0 y0 xdot0 ydot0];

figure;
%integrate forward in time from L1 to earth
[t,x]=ode45(@f,[0:0.01:0.5],IC);
plot(x(:,1),x(:,2))

hold on;
%integrate backward in time from L1 to moon
x0=xL1; y0=yL1; xdot0=-0.88; ydot0=-0.88;
IC=[x0 y0 xdot0 ydot0];
[t,x]=ode45(@f,[.1:-0.01:0],IC);
plot(x(:,1),x(:,2),'r');


xlim([-.5 2]);
ylim([-.5 .3]);
line([-.5 2],[0 0],'LineStyle','-.')
line([0 0],[-.5 .5],'LineStyle','-.')
plot(-mu,0,'o');
plot(1-mu,0,'o');
plot(xL1,0,'*');

title('part 4, HW9. forward and backward ode45 integration for L1
solution');
xlabel('x');
ylabel('y');
legend('forward integration','backward integration');


function dx=f(t,x)

dx=zeros(4,1);
mu=0.012;
a1=-mu;
a2=1-mu;
r1=sqrt((x(1)-a1)^2+x(2)^2);
r2=sqrt((x(1)-a2)^2+x(2)^2);
dx(1)=x(3);
dx(2)=x(4);
dx(3)=x(1)-(1-mu)*(x(1)-a1)/r1^3 - mu*(x(1)-a2)/r2^3 + 2*x(4);
dx(4)=x(2)*( 1 - (1-mu)/r1^3 - mu/r2^3) - 2*x(3);
```