HW5, Problem 25.5.

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Problem: Use Runga-Kutta classical 4th order method with h=0.5 to solve $\frac{dy}{dx} = yx^2 - 1.2y$ over the interval $x = 0$ to 2 with $y(0) = 1$.

Solution:

in the equation $y' = f(x, y)$, when using RK classical 4th order, future value of y are found using the equation:

$y_{i+1} = y_i + h\left(\frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\right)$

where

$k_1 = f(x_i, y_i)$

$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_1\right)$

$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_2\right)$

$k_4 = f(x_i + h, y_i + hk_3)$

I illustrate the method for one step, then write a matlab function to implement RK 4th order to fully solve this problem.

Let $h = 0.5$

First step, Calculate the k_1, k_2, k_3, k_4 :

$k_1 = f(x_0, y_0) = y_0\left(x_0^2 - 1.2\right) = [1]\left(0^2 - 1.2\right) = -1.2$

$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_1\right) = \left[1 + \frac{1}{2}(0.5)(-1.2)\right]\left(\left(0 + \frac{1}{2}(0.5)\right)^2 - 1.2\right) = -0.7525$

$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_2\right) = \left[1 + \frac{1}{2}(0.5)(-0.7525)\right]\left(\left(0 + \frac{1}{2}(0.5)\right)^2 - 1.2\right) = -0.872765625$

$k_4 = f(x_i + h, y_i + hk_3) = [1 + (0.5)(-0.872765625)]\left((0 + (0.5))^2 - 1.2\right) = -0.535436328125$

$y_{i+1} = y_i + h\left(\frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\right) \Longrightarrow$

$y_1 = 1 + (0.5)\left(\frac{1}{6}(-1.2 + 2(-0.7525) + 2(-0.872765625) - 0.535436328125)\right) = 0.584502701822917$

the above completes one step. Repeate the above for the number of steps.

Results: The following results shows that RK4 classical method produced an excellent result compared to the true solution. Even at the last troublesome point, RK4 did much better than midpoint and heun methods.