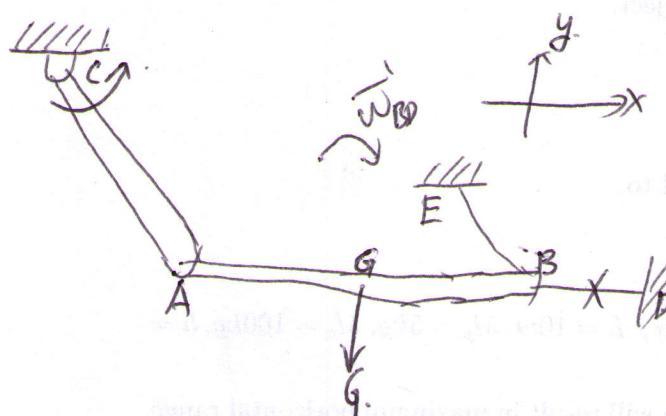


Sample problem #3:

An 8-lb uniform slender rod AB is held in position by two ropes and the link CA which has a negligible weight. After rope BD is cut, the assembly rotates in a vertical plane under the combined effect of gravity and a 4-lb-ft couple M. Determine immediately after rope BD has been cut,

- the acceleration of rod AB.
- the tension in rope EB

Solution:



Kinematic analysis

when BD is broken:

$$\omega_{BD} = 0, \frac{d\omega_{BD}}{dt} \neq 0$$

$$\vec{a}_A = \vec{a}_G + \frac{d}{dt} \vec{\omega}_{BD} \times \vec{r}_{GA}$$

$$= \vec{a}_G + \frac{d\omega_{BD}}{dt} \times \vec{r}_{GA}$$

$$\vec{a}_B = \vec{a}_G + \frac{d\omega_{BD}}{dt} \times \vec{r}_{GB}$$

$$= \vec{a}_G + \frac{d\omega_{BD}}{dt} \times \vec{r}_{GA}$$

$$\therefore \vec{a}_A + \vec{a}_B = 2\vec{a}_G \Rightarrow$$

$$\vec{a}_A + \vec{a}_B = 2 \frac{d\omega_{BD}}{dt} \times \vec{r}_{GA} \rightarrow \text{only has } y \text{ component!}$$

$$\Rightarrow \vec{a}_{Ax} = \vec{a}_{Bx} \text{ and } \vec{a}_{Ax} + \vec{a}_{Bx} = 2\vec{a}_{Ax}$$

$$\Rightarrow \vec{a}_{Ax} = \vec{a}_{Bx} = \vec{a}_{Ax}$$

Since A is on the link AC.

$$\vec{a}_A = \vec{\omega}_{AC} \times \vec{\omega}_{AC} \times \vec{r}_{AC} + \frac{d\vec{\omega}_{AC}}{dt} \times \vec{r}_{AC}$$

Initially $\vec{\omega}_{AC} = 0$

$$\Rightarrow \vec{a}_A = \frac{d\vec{\omega}_{AC}}{dt} \times \vec{r}_{AC}$$

30°

Since B is on the rope EB

$$\Rightarrow \vec{a}_B = \frac{d\vec{\omega}_{EB}}{dt} \times \vec{r}_{EB}$$

730°

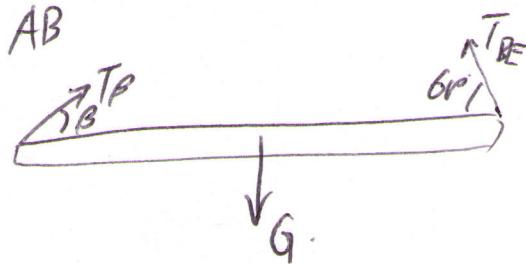
(3)

because \vec{a}_A is parallel to \vec{a}_B by Θ and β . Meanwhile, $\vec{a}_{Ax} = \vec{a}_{Bx} = \vec{a}_{Gx}$
 Thus, we get $\vec{a}_{Ay} = \vec{a}_{By}$. ($\vec{a}_{Ay} = \vec{a}_{Ax} \tan 30^\circ$; $\vec{a}_{By} = \vec{a}_{Bx} \tan 30^\circ$).

$$\text{Since: } \vec{a}_A - \vec{a}_B = 2 \frac{d\vec{u}_{BD}}{dt} \times \vec{r}_{GA} \quad \text{and} \quad \begin{cases} \vec{a}_{Ax} = \vec{a}_{Bx} = \vec{a}_{Gx} \\ \vec{a}_{Ay} = \vec{a}_{By} = \vec{a}_{Gy} \end{cases} \quad (4)$$

$$\Rightarrow \frac{d\vec{u}_{BD}}{dt} = 0$$

Translation.



$$\begin{cases} T_B \sin \beta + T_E \sin 60^\circ - G = m a_{Gy} \\ T_B \cos \beta - T_E \cos 60^\circ = m a_{Gx} \end{cases} \quad (5) \quad (6)$$

Rotation.

$$T_B \cdot \sin \beta \cdot \frac{l}{2} - T_E \cdot \sin 60^\circ \cdot \frac{l}{2} = I \cdot \frac{du_{BD}}{dt} = 0$$

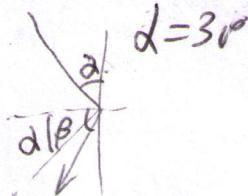
$$\Rightarrow T_B \sin \beta = T_E \sin 60^\circ \quad (7)$$

A.C.



$$-M + T_B \cdot \sin(\beta - 2) = 0$$

$$\Rightarrow T_B \sin(\beta - 30^\circ) = M. \quad (8)$$



(5) (6) (7) (8) compose four equations but there are five variables: $a_{Gx}, a_{Gy}, T_B, \beta, T_E$.

However; by (4), we have $\vec{a}_{Gy} = \vec{a}_{Gx} \cdot \tan 30^\circ \quad (9)$

So, we get

$$\left\{ \begin{array}{l} T_B S m \beta + T_{BE} S m 60^\circ - G = m a_{Gy} = m a_{Gx} \operatorname{tg} 30^\circ \dots \textcircled{⑩} \\ T_B C s \beta - T_{BE} C s 60^\circ = m a_{Gx} \dots \textcircled{⑪} \\ T_B S m (\theta - 30^\circ) = M / |Ae| \dots \textcircled{⑫} \\ T_B S m \beta = T_{BE} S m 60^\circ \dots \textcircled{⑬} \end{array} \right.$$

By ⑫ and ⑬

$$\begin{aligned} T_B S m \beta C s 30^\circ - T_B C s \beta S m 30^\circ &= M / |Ae| \\ \Rightarrow T_{BE} S m 60^\circ C s 30^\circ - T_B C s \beta S m 30^\circ &= M / |Ae| \\ \Rightarrow T_B C s \beta &= \frac{1}{S m 30^\circ} (T_{BE} S m 60^\circ C s 30^\circ - M) \\ &= 2 \cdot (T_{BE} \cdot \frac{3}{4} - M) = 2 \left(\frac{3}{4} T_{BE} - M \right) / |Ae|. \end{aligned} \quad \textcircled{⑭}$$

By ⑩, ⑪, ⑬

$$\frac{2 T_{BE} S m 60^\circ - G}{T_B C s \beta - T_{BE} C s 60^\circ} \cdot \frac{m a_{Gx} \operatorname{tg} 30^\circ}{m a_{Gx}} \Rightarrow 2 T_{BE} S m 60^\circ - G = \operatorname{tg} 30^\circ (T_B C s \beta - T_{BE} C s 60^\circ)$$

By ⑭

$$\sqrt{3} T_{BE} - G = \frac{\sqrt{3}}{3} \left(2 \left(\frac{3}{4} T_{BE} - M \right) - T_{BE} \cdot \frac{1}{2} \right)$$

$$3 T_{BE} - \sqrt{3} G = \frac{6}{4} T_{BE} - 2M - \frac{1}{2} T_{BE} = T_{BE} - 2M / |Ae|$$

$$T_{BE} = \frac{1}{2} (\sqrt{3} G - 2M) = \frac{\sqrt{3}}{2} G - \frac{M}{|Ae|} = \frac{\sqrt{3}}{2} G - \frac{4}{1.5} = \frac{\sqrt{3}}{2} \cdot 8 - \frac{4}{1.5}$$

$$\boxed{T_{BE} = 4.258 \text{ lb}}$$

$$\boxed{T_B S m \beta = T_{BE} S m 60^\circ = 3,687428 \text{ lb}}$$

$$\boxed{T_B C s \beta = 2 \left(\frac{3}{4} T_{BE} - \frac{4}{1.5} \right) = 2(3,1935 - 2.67) = 1.047}$$

$$\boxed{T_B = 3.836 \text{ lb}} \quad \boxed{\beta = 74^\circ}$$

$$\boxed{a_{Gx} = -4.32 \text{ ft/s}^2}$$

$$\boxed{a_{Gy} = -7.482 \text{ ft/s}^2}$$

§17. Plane motion of rigid body: Energy and Momentum. Method

§17.1 Principle of Work and Energy.

$$U_{1 \rightarrow 2} = T_2 - T_1$$

↓ ↓ ↓

The work of the resultant force $\Rightarrow \{ \begin{array}{l} \int \vec{F} \cdot d\vec{R} \\ \int M \cdot d\theta \end{array}$

kinetic energy at state 1 kinetic energy at state 2.

The kinetic energy of a rigid body in plane motion.

$$\vec{v} = \vec{v}_c + \omega \times \vec{r}_{cx} = \vec{v}_c + \omega r_{cx} \vec{k}$$

position vector relative to the mass center.
velocity at mass center

$$\begin{aligned} T &= \frac{1}{2} \iint p (\vec{v} \cdot \vec{v}) ds \\ &= \frac{1}{2} \iint p (\vec{v}_c \cdot \vec{v}_c + \vec{v}_c \cdot \overset{\rightarrow 0}{\omega r_{cx} \vec{k}} + \omega r_{cx} \vec{k} \cdot \vec{v}_c + \omega^2 r_{cx}^2) ds \\ &= \frac{1}{2} (\iint p ds) v_c^2 + \frac{\omega^2}{2} \iint p r_{cx}^2 ds \\ &= \frac{1}{2} m v_c^2 + \frac{I_c}{2} \omega^2. \quad \text{moment of inertia} \end{aligned}$$

Expression 1:

$$U_{1 \rightarrow 2} = W_{1 \rightarrow 2} + (V_1 - V_2)$$

↓ ↓ ↓

the work done by forces excluding potential forces.

If $W_{1 \rightarrow 2} = 0$

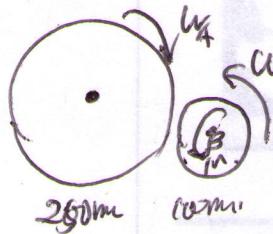
$$\Rightarrow V_1 - V_2 = T_2 - T_1 \Rightarrow T_1 + V_1 = T_2 + V_2$$

↓ ↓

energy conservation.

Sample problem #1.

Gear A has a mass of 10kg. and a radius of gyration of 80mm. The system is at rest when a couple M of magnitude 6 N·m is applied to gear B. Neglecting friction, find (a) the number of revolutions executed by gear B before its angular velocity reaches 600 rpm (b) the tangential force which gear B exerts on gear A.



Solution: System

$$\text{Energy} \quad \underbrace{\frac{1}{2} I_A \omega_A^2 + \frac{1}{2} I_B \omega_B^2}$$

$$\text{Work} \quad U_{1 \rightarrow 2} = \int_0^{M \cdot d\theta} M \cdot d\theta = M \cdot \Theta_{\text{all}} = M \cdot N \cdot 2\pi$$

State 2

$$\underbrace{\frac{1}{2} I_A \omega_A'^2 + \frac{1}{2} I_B \omega_B'^2}$$

number of revolutions

~~$I_A = \int r^2 dm = \int r^2 \rho \cdot 2\pi r dr \cdot d\theta$~~

~~$= \int r^2 dm = \rho \cdot \frac{4}{3} \pi \cdot r^4 \cdot 2\pi = \rho \cdot \frac{\pi}{2} R^4$~~

~~$\Rightarrow I_A = \frac{\rho \cdot \pi R^4}{2} \cdot m_A = m_A (\bar{R}_A)^2 = m_A (0.2)^2 = 0.4 \text{ kg} \cdot \text{m}^2$~~

$$I_B = m_B (\bar{R}_B)^2 = m_B (0.08)^2 = 0.0192 \text{ kg} \cdot \text{m}^2$$

Motion Dependence:

$$\omega_B \cdot r_B = \omega_A - \omega_A \Rightarrow \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{100}{250} = 0.4$$

$$U_{1 \rightarrow 2} = T_2 - T_1$$

$$\Rightarrow M \cdot N \cdot 2\pi = \frac{1}{2} \cdot 0.4 \cdot \omega_A'^2 + \frac{1}{2} \cdot 0.0192 \cdot \omega_B'^2$$

$$\omega_B' = 600 \text{ rpm.} = \frac{600 \times 2\pi}{60} \text{ rad/s} = 20\pi \text{ rad/s}$$

$$\omega_A' = 0.4 \cdot \omega_B'$$

$$\Rightarrow N = \frac{M}{2\pi} \left(\frac{1}{2} \cdot 0.4 \cdot 0.16 \omega_B'^2 + \frac{1}{2} \cdot 0.0192 \cdot \omega_B'^2 \right)$$

$$= 4.35 \text{ rounds.}$$

Force: for A.



State 1

$$\frac{1}{2} I_A \omega_A^2 = 0$$

State 2

$$\frac{1}{2} I_A \omega_A'^2$$

Work.

$$U_{12} = \int \mathbf{F} \cdot d\mathbf{s} = F \cdot \text{Path} \cdot r_A$$

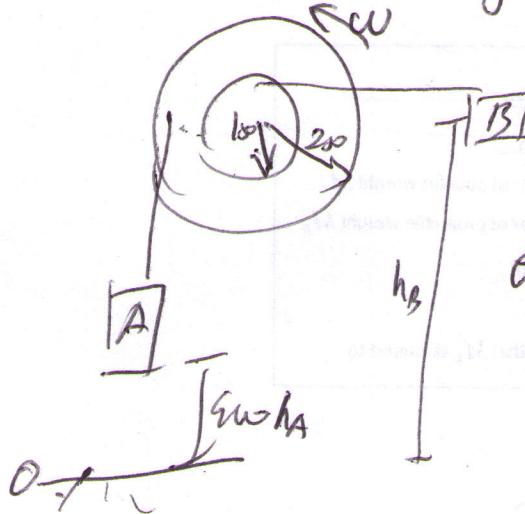
$$= F \cdot N \cdot 2\pi \cdot r_A$$

$$\Rightarrow F \cdot N \cdot 2\pi \cdot r_A = \frac{1}{2} I_A \omega_A'^2$$

$$\Rightarrow F = 46.2 \text{ N.}$$

Sample problem #2.

The double pulley shown has a mass of 14kg and a centroidal radius of gyration of 165mm. Cylinder A and block B are attached to cords that are wrapped on the pulley as shown. The coefficient of kinetic friction between block B and the surface is 0.25. knowing that the system is released from rest in the position shown. find (a) the velocity of A as it strikes the ground (b) the total distance that B moves before coming to rest.



~~start solution:~~
System

State 1

$$\frac{1}{2} m_B v_0^2 + \frac{1}{2} I \omega_0^2 + \frac{1}{2} m_A v_0^2 = 0.$$

State 2.

$$\frac{1}{2} m_B v_2^2 + \frac{1}{2} I \omega_2^2 + \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_A V_A^2$$

$$m_A g h_A + m_B g h_B$$

$$m_B g h_B$$

work done

$$\text{by non-potential forces} + \cancel{U_{12}} = \cancel{\bar{m}_B g \cdot \mu_k \cdot s}$$

$$I = m_A (\cancel{R})^2 = 14 \cdot (0.165)^2 = 0.38115 \text{ kg m}^2.$$

motion dependence:

$$\begin{cases} \mathbf{c} \cdot \mathbf{v}_0 = V_A \\ \mathbf{c} \cdot \mathbf{v}_2 = V_B \end{cases}$$

~~$$S = \frac{V_2}{V_0} \cdot S_0$$~~

~~$$V_B = \frac{r_c}{r_o} \cdot V_A$$~~

$$\Rightarrow S = \frac{r_c}{r_o} \cdot h_A = \frac{r_c}{r_o} \cdot 0.1$$

G.E

$$W_{1 \rightarrow 2} = -m_B g h_B + \frac{1}{2} m_B V_B'^2 + \frac{1}{2} I w'^2 + \frac{1}{2} m_A V_A'^2$$

$$-m_A g h_A - m_B g h_B = 0$$

$$-m_B g M_k S = -m_A g h_A + \frac{1}{2} m_B w'^2 r_i^2 + \frac{1}{2} 0.38115 w'^2 + \frac{1}{2} m_A w'^2 r_o^2$$

$$\Rightarrow \cancel{m_A g h_A} = \frac{-m_A g h_A - m_B g M_k \cdot \frac{r_i}{r_o} h_A}{0.6012} = \frac{11.5 \cdot 9.8 \cdot 0.9 - 9 \cdot 9.8 \cdot 0.25 \cdot \frac{150}{250} \cdot 0.9}{0.6012}$$

$$\cancel{\frac{101.43 - 11.907}{0.0012}}$$

$$w'^2 = \frac{89.523}{0.6012} \Rightarrow w' = \cancel{11.72} \text{ rad/s.}$$

$$V_A = 2.93 \text{ m/s}$$

(b) For B and ~~wheel~~ pulley.

$$\left. \begin{array}{l} \text{kinetic energy} \\ E \\ \text{P.E} \end{array} \right\}$$

State 1

$$\frac{1}{2} m_B V_B'^2 + \frac{1}{2} I w'^2$$

$$m_B g h_B$$

State 2.

$$\frac{1}{2} m_B V_B'^2 + \frac{1}{2} I w'^2$$

$$m_B g h_B$$

W.

$$W_{1 \rightarrow 2} = -m_B g M_k S'$$

$$-\frac{1}{2} I w'^2 - \frac{1}{2} m_B V_B'^2 = -m_B g M_k S'$$

$$-\frac{1}{2} I w'^2 - \frac{1}{2} m_B w'^2 r_i^2 = -m_B g M_k S'$$

$$26.177 / m_B$$

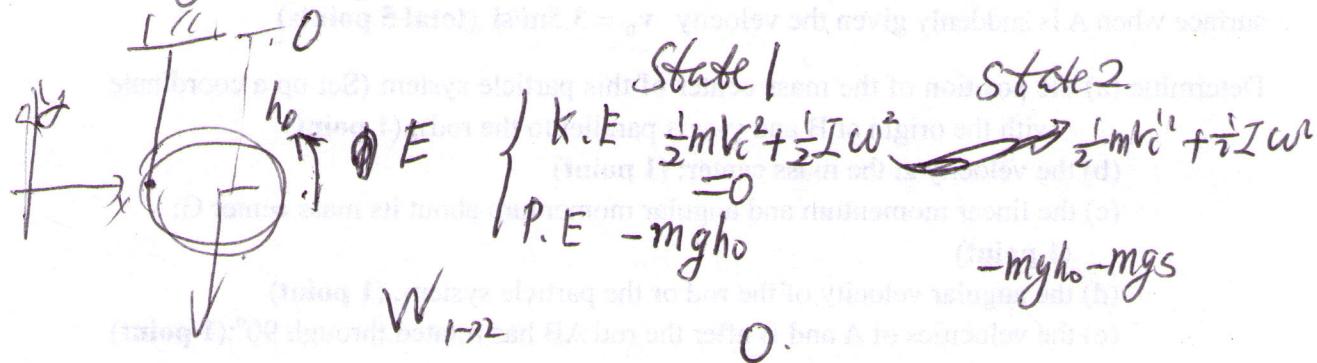
$$S' = \frac{\frac{1}{2} I w'^2 + \frac{1}{2} w'^2 r_i^2}{g M_k} = \frac{\frac{1}{2} \cdot 11.72^2 \cdot 0.25^2 + \frac{1}{2} \cdot 0.38115 \cdot 11.72^2}{9.8 \cdot 0.25} = \cancel{0.63} \text{ m}$$

$$S_{\text{tot}} = S + S' = \frac{0.15}{0.25} \cdot 0.7 + 0.63 = \frac{2.9 + 1.5452}{2.28} = \cancel{2.483}$$

$$= 1.83 \text{ m} + \frac{0.9}{2.28} = 2.483 \text{ m}$$

Sample problem #3 (17.21)

A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward distance s .



$$V_c \cancel{\oplus} w r = 0$$

$$V_c = w \cdot r$$

$$\Rightarrow 0 = \frac{1}{2}mv_c'^2 + \frac{1}{2}Iw^2 - mgs$$

$$I = \frac{1}{2}m r^2$$

$$\cancel{0} mgs = \frac{1}{2}m w^2 r^2 + \frac{1}{2}\frac{1}{2}m r^2 \cdot w^2$$

$$\cancel{mgs} = \left(\frac{1}{2}r^2 + \frac{1}{4}r^2 \right) w^2$$

$$= \frac{3}{4}r^2 w^2$$

$$|w| = \sqrt{\frac{4gs}{3r^2}} = \frac{2\sqrt{gs}}{\sqrt{3}r}$$

$$|V| = |w| \cdot |r| = \frac{2\sqrt{gs}}{\sqrt{3}} = \sqrt{\frac{4gs}{3}}$$