

§17.2 Principle of impulse and momentum for the plane motion of rigid body.

For a particle system:

$$\sum_k \int_1^2 \vec{F}_k^e dt = \left(\sum_k m_k \vec{v} \right)_2 - \left(\sum_k m_k \vec{v} \right)_1$$

velocity at mass center

$$= \left(\sum_k m_k \vec{v}_k \right)_2 - \left(\sum_k m_k \vec{v}_k \right)_1 \quad (1)$$

$$\sum_k \vec{r}_k \times \vec{F}_k^e = \frac{d}{dt} (H_0) = \frac{d}{dt} \left(\vec{r} \times m_0 \vec{v} + \sum_{k=1}^N m_k (\vec{r}_{kc} \times \vec{v}_{kc}) \right) \quad (2)$$

For a rigid body in plane motion

$$\sum_k \int_1^2 \vec{F}_k^e dt = \int_1^2 (m \vec{v})_2 - (m \vec{v})_1$$

total mass

velocity at mass center

(3)

$$\sum_k \vec{r}_k \times \vec{F}_k^e = \frac{d}{dt} \left(\vec{r} \times m \vec{v} + \int_S \rho ds (\vec{r}_{kc} \times \omega \times \vec{r}_{kc}) \right)$$

$$= \frac{d}{dt} \left(\vec{r} \times m \vec{v} + \omega \int_S \rho r_{kc}^2 ds \right)$$

$$= \frac{d}{dt} \left(\vec{r} \times m \vec{v} + I \omega \right) \quad (4)$$

$$\sum_k \int_1^2 (\vec{r}_{kc} \times \vec{F}_k^e) dt = (I \omega)_2 - (I \omega)_1$$

Sample Problem #1.

A uniform sphere of mass m and radius r is projected along a rough horizontal surface with a linear velocity \vec{v}_1 and no angular velocity. Denoting by μ_k the coefficient of kinetic friction between the sphere and the surface, determine (a) the time t_2 at which the sphere and the surface will start rolling without sliding, (b) the linear and angular velocity of the sphere at time t_2 .

$$\Rightarrow \sum_k \int_1^2 \vec{r}_k \times \vec{F}_k^e dt = (\vec{r} \times m \vec{v} + I \omega)_2 - (\vec{r} \times m \vec{v} + I \omega)_1$$

Solution:

	State 1 <i>initial</i>	State 2. $t=t_2$
Momentum	$m\bar{v}_1$	$m\bar{v}_2$
angular momentum <i>about mass center</i>	$I\omega = 0$	$I\omega'$
Impulse		$F \cdot t_2$
Moment Impulse <i>About mass Center</i>		$-F \cdot r \cdot t_2$



Motion Dependence.

at $t=t_2$. $v_A = 0$

$$\Rightarrow v_A = v_C + \omega r = 0$$

$$\Rightarrow v_C = -\omega r = \bar{v}_2$$

$$\Rightarrow \int F t_2 = m\bar{v}_2 - m\bar{v}_1 = m\omega r - m\bar{v}_1$$

$$-F r t_2 = I\omega'$$

$$-F t_2 = -m \frac{F r t_2}{I} r - m\bar{v}_1$$

$$\Rightarrow \left(F + m \frac{F r^2}{I} \right) t_2 = + m\bar{v}_1$$

$$t_2 = \frac{m\bar{v}_1}{\left(m \frac{r^2}{I} + 1 \right) F} = \frac{\bar{v}_1}{\mu_k g \left(m \frac{r^2}{I} + 1 \right)}$$

$$= \frac{\bar{v}_1}{\mu_k g \left(m \frac{r^2}{\frac{5}{2} m r^2} + 1 \right)} = \frac{\bar{v}_1}{\mu_k g \left(\frac{5}{2} + 1 \right)} = \frac{2\bar{v}_1}{7\mu_k g}$$

$$I = \int \rho \cdot dv \cdot (r \sin \theta)^2 \cdot r^2 \sin \theta dr d\theta d\phi$$

$$= \int \rho dr \int r^4 \sin^3 \theta d\theta \int d\phi$$

$$I = \int \rho dr r^2 \int \sin^2 \theta d\theta \int d\phi$$

$$= \frac{2}{3} \pi \rho r^5 \int_0^\pi \sin^2 \theta d\theta$$

$$= \frac{2}{3} \pi \rho r^5 \int_0^\pi (1 - \cos^2 \theta) d\theta$$

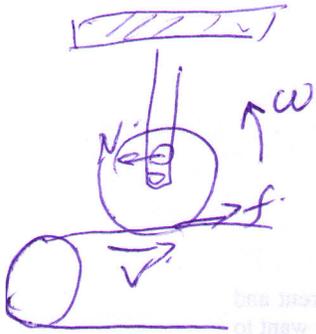
$$= \frac{2}{3} \pi \rho r^5 \left(\theta - \frac{\sin^3 \theta}{3} \right) \Big|_0^\pi$$

$$= \frac{2}{3} \pi \rho r^5 \left(\pi - \frac{1}{3} - \left(-\pi + \frac{1}{3} \right) \right)$$

$$= \frac{2}{3} \pi \rho r^5 \left(2 - \frac{2}{3} \right) = \frac{8}{15} \pi \rho r^5$$

Sample Problem #2.

17.50. Disk A, of mass 2kg and radius $r=60\text{mm}$, is at rest when it is placed in contact with a belt which moves at a constant speed $v=15\text{m/s}$ knowing that $\mu_k=0.2$, determine the time required for the disk to reach a constant angular velocity.



disk	State 1	State 2
L. M.	0	0
A. M.	0	$I\omega$
I. P.	$+M_k m g \cdot t - N t = 0$	
I. P. M.	$+M_k m g \cdot r t$	

$$M_k m g r t = I \omega \quad I = \int \rho \cdot r^3 dr d\theta = \frac{R^4}{4} \rho 2\pi = \frac{m}{2} R^2$$

$$M_k m g r t = \frac{m}{2} R^2 \omega$$

$$\omega = \frac{2 M_k g}{R} t$$

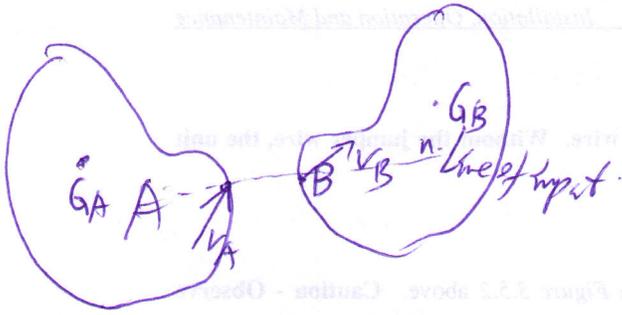
4.3 Second Boundary

If a second boundary is encountered, the DC Converter will again override the control mode and use the second boundary, encountered at the internal control mode. If necessary, to stay within the operating space, the DC Converter is unable to maintain the limits set by the user. If with abnormality, discover the output remains to prevent the system another load from a damaging failure. In summary, the DC Converter will follow the limits set by the user in all cases.

4.4 The Load

The load must exist in the operating space defined by the limits. A simple check to determine the desired operating space will be useful and may avoid frustration. Finally, there are a few basic rules that must be followed in setting the operating limits. The DC Converter will enforce each of these rules:

§ 17.3. Impact of rigid bodies.



~~when it~~
The deflection of the line of impact.

The coefficient of restitution

- ① no external forces
- ② the moment of external forces about a point on the ~~body~~ body is zero

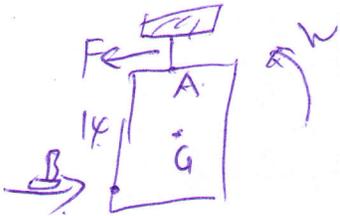
$$e = \frac{\int R dt}{\int P dt} = \frac{(v_B)_n (v_A)_n}{(v_A)_n - (v_B)_n}$$

↓
deflection

Sample Problem #3.

A 0.05-lb bullet B is fired with a horizontal velocity of 1000 ft/s into the side of a 20-lb square panel suspended from a hinge at A.

Find (a) the angular velocity after impact (b) impulse reaction at A. $T = 0.006$ s.



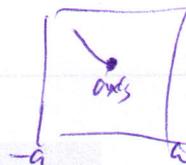
system	state 1 ^b	state 2 ^a
L.M.	$m_B v_B$	$(m_B + m_G) v_G$
A.M.	$m_B v_B \cdot \frac{l_A}{12}$	$(m_G + m_B) \cdot \left(\frac{9}{12}\right) l_A \omega$
I.P	$F_A \cdot l_A$	
I.P.M A:	0	

$$I_p = \int_S \rho(x^2 + y^2) \cdot ds$$

$$= \int_{-a}^a \int_{-a}^a \rho(x^2 + y^2) dx dy$$

$$= \rho \int_{-a}^a \left[\int_{-a}^a (x^2 + y^2) dx \right] dy$$

$$= \rho \int_{-a}^a \left(\frac{2a^3}{3} + 2ay^2 \right) dy = \rho \left(\frac{4}{3}a^4 + \frac{4}{3}a^4 \right) = \rho \cdot \frac{8}{3}a^4 = \rho \cdot \frac{1}{8}(2a)^4 = \frac{ma'^2}{6}$$



$$\Rightarrow \cancel{m_B V_B} (m_A + m_B) V_G = W \cdot \frac{g}{12}$$

$$\Rightarrow m_B V_B \frac{1}{12} = (m_A + m_B) \frac{g}{12} V_G + I_O \omega$$

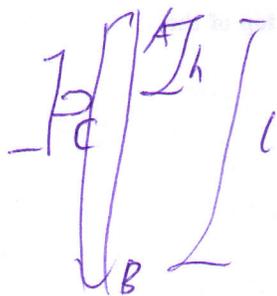
$$\Rightarrow \omega = 4.67 \text{ rad/s}$$

$$V_2 = 3.57 \text{ m/s}$$

$$F_A \cdot \Delta t = (m_B + m_G) V_G - m_B V_B = -259 \text{ lb}$$

Sample Problem #4 (17.88)

A. 40g bullet is fired with a horizontal velocity of 600 m/s into the lower end of a slender 7-kg bar of length $L = 600 \text{ mm}$ knowing that $h = 240 \text{ mm}$



and the bar is initially at rest.

(a) angular velocity after impact.

(b) reaction at C.

System	state 1 (s)	state 2 (s)
LM	$-m_B V_0$	$-(m_B + m_r) V_G$
A.M _c	$-m_0 V_0 \cdot \left(\frac{L}{2} - h\right)$	$-(m_B + m_r) V_0 \left(\frac{L}{2} - h\right) + I_G \cdot \omega$
IP	$F \cdot dt$	
I.P.M. C		

0