Summary

Chapter 13 Kinetics of particles: Energy and Momentum Methods

Basic Concepts

- **Work of a force:** Work is the scale product between the external force and the displacement vector. (a process parameter and it is a scalar)
- **Power:** the scale product between the external force and the velocity vector. (an instant parameter and it is a scalar)
- **Efficiency of a power system:** the ratio of the output mechanical energy to the input mechanical energy (capability of energy transmission)
- **Impulse:** the product between force and time (a process parameter and a vector)
- **Momentum:** the product between mass and velocity (an instant parameter and a vector)
- **Line of impact:** The line normal to the contacting surface
- **Direct central impact:** the impact when the initial velocities are all parallel to the line of impact (The momentum along the line of impact is conserved during collision when the impulse of external forces are **negligible**)
- **Oblique central impact:** the impact when the initial velocity are NOT parallel to the line of impact (The momentum along the line of impact is conserved during collision and the velocities normal to the line of impact are unchanged when the impulse of external forces are **negligible**)
- **Coefficient of Restitution:** the ratio of the impulse of interacting forces during restitution period to that during deformation period (**Only** when the impulse of external forces are negligible, $e = \frac{v_B' - v_A'}{v_A - v_B}$).
- **Perfect plastic impact:** the impact for $e = 0$ (energy lost is maximum)
- **Perfect elastic impact:** the impact for $e = 1$ (no energy lost)

Useful formulas

- **Work in rectangular coordinate**

$$dU = F_x dx + F_y dy + F_z dz$$

- **Work in polar coordinate**

$$dU = F_\theta r d\theta + F_r dr$$

- **Work of an arbitrary force and an arbitrary trace**

$$U = \int_1^2 \mathbf{F} \cdot \mathbf{v} dt$$

- **Work of gravity**

$$U = -mg(y_2 - y_1)$$

- **Work of spring forces**

$$U = -1/2\, kx_2^{\,2} + 1/2\, kx_1^{\,2}$$

- **Work of gravitational force**

$$U = -G\frac{Mm}{r}$$

- **Potential of gravity**

$$U = mgy$$

- **Potential of spring forces**

$$U = 1/2\, kx^2$$

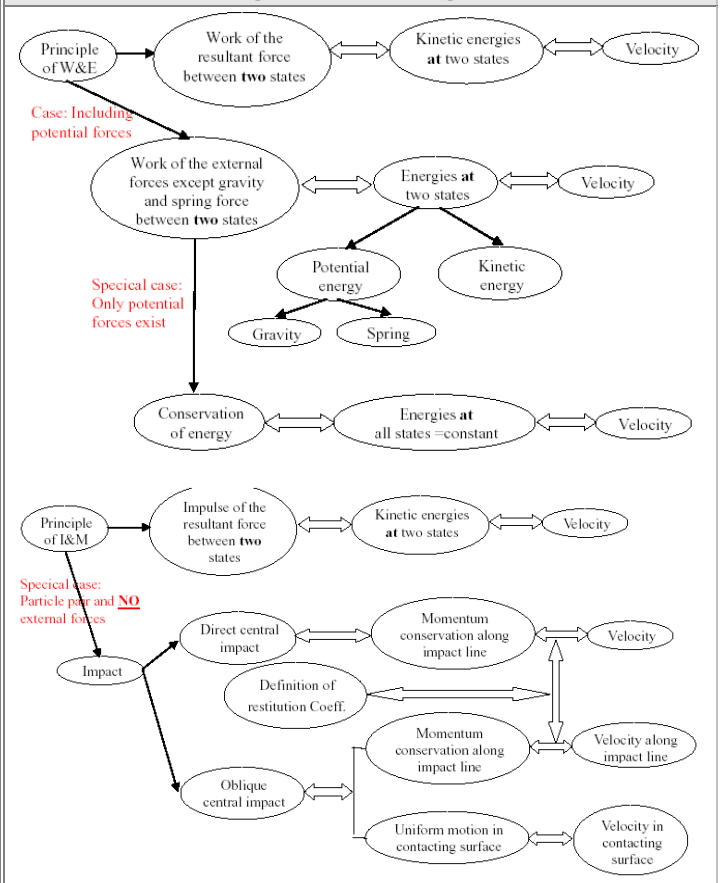
- **Coefficients of restitution (no impulse of external forces)**

$$e = \frac{v_B' - v_A'}{v_A - v_B}$$

Tips

When the impulse of external forces are negligible, the **momentum conservation** along the impact line and the **coefficient of restitution** represent the two governing equations for impact problem.

Maps of the chapter



Useful principles

(1) Principle of work and Energy: The work of resultant forces equals to the change of kinetic energy of the particle.
(2) Principle of Impulse of Momentum: The impulse of resultant forces equals to the change of momentum of the particle.

Step-by-step procedure

For using the principle of work and energy
Step 1: How many particles in the system
Step 2: Setup a coordinate (Newton frame)
Step 3: Expression of work and energy
- 3.1 Determine the staring start and ending state
- 3.2 The energy at all states for each particle (kinetic energy, potentials [gravity and spring])
- 3.3 The work of forces except gravity and spring forces between the two state
- 3.4 Link the work and energy to setup governing equations

Step 4: Solutions

For using the principle of Impulse and Momentum
Step 1: How many particles in the system
Step 2: Setup a coordinate (Newton Frame)
Step 3: Expression of Impulse and Momentum
- 3.1 Determine the staring start and ending state (before and after collision)
- 3.2 The Momentum at all states for each particle
- 3.3 The Impulse of resultant forces.
- 3.4 Link the Impulse and Momentum to setup governing equations

Step 4: Solutions