Summary

# Chapter 14 System of particles

## Basic Concepts

- **Mass center:** A virtual point defined by the weighted average of particles' position vector. (It is **completely** determined by the distribution of particles and does not depend on the selected coordinate)
- **Momentum of a particle system:** The summation of all particles' momentum. (*It is a vector and depends on the selected Newton frame*)
- **Angular momentum of a particle system:** The summation of moment of momentum of all particles about the origin of the selected Newton frame(*depending on the selected frame*)
- **Resultant force:** The summation of all external forces applied to the particles (*independent to the selected frame*)
- **Force moment:** The summation of the moment of external forces on the particles about the origin of the selected Newton frame (*depending on the selected frame*)
- **Impulse:** The summation of the impulse of external forces on the particles. (*independent to the selected frame*)

## Useful formulas

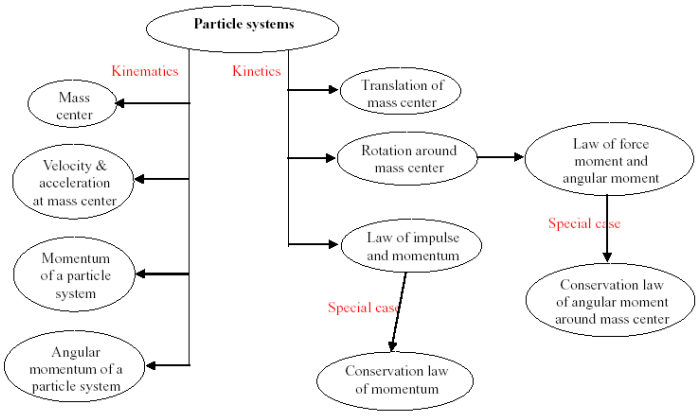
- **Position vector of mass center:** $\mathbf{r} = \sum_{i=1}^{N} m_i \mathbf{r}_i \Big/ \sum_{i=1}^{N} m_i$
- **Velocity vector of mass center:** $\mathbf{v} = \sum_{i=1}^{N} m_i \mathbf{v}_i \Big/ \sum_{i=1}^{N} m_i$
- **Acceleration vector of mass center:**
- $\mathbf{a} = \sum_{i=1}^{N} m_i \mathbf{a}_i \Big/ \sum_{i=1}^{N} m_i$
- **Angular momentum (moment of momentum):**
- $\sum_{i=1}^{N} (\mathbf{r}_i \times m_i \mathbf{v}_i) = \sum_{i=1}^{N} (\mathbf{r}_{ic} \times m_i \mathbf{v}_{ic}) + \left( \sum_{i=1}^{N} m_i \right) \mathbf{r} \times \mathbf{v}$
- **Translation of mass center:**
- $\sum_{i=1}^{N} \mathbf{F}_i^e = \left( \sum_{i=1}^{N} m_i \right) d^2\mathbf{r}/dt^2$
- **Rotation around mass center:**
- $\sum_{i=1}^{N} \mathbf{F}_i^e \times \mathbf{r}_{ic} = \frac{d}{dt} \left[ \sum_{i=1}^{N} m_i (\mathbf{v}_{ic} \times \mathbf{r}_{ic}) \right]$
- **Principle of impulse and moment:**

$$\sum_{i=1}^{N} \mathbf{F}_i^e t = \left( \sum_{i=1}^{N} m_i \mathbf{v}_i \right)_2 - \left( \sum_{i=1}^{N} m_i \mathbf{v}_i \right)_1$$

- **Kinetic energy of particle systems:**
- $T = \frac{1}{2} \left( \sum_{i=1}^{N} m_i \right) \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \sum_{i=1}^{N} (m_i \mathbf{v}_{ic} \cdot \mathbf{v}_{ic})$
- **Principle of work and energy for particle systems**
- $\left( \sum U \right)_{1 \to 2} = T_2 - T_1$

Work done by external forces and internal forces

## Maps of the chapter




## Useful principles

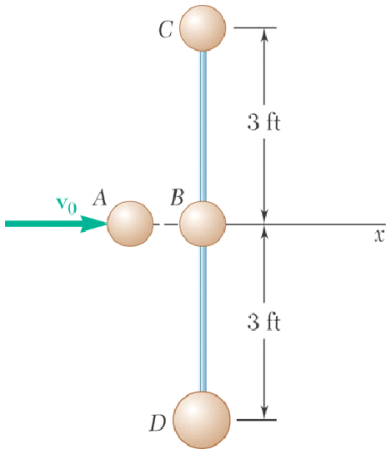
(1) Conservation law of momentum: If the impulse of external forces of a particle system is zero, the momentum of the system is conserved.
(2) Conservation law of angular momentum: If the moment of external forces about the mass center is zero, the relative angular moment about the mass center (using both the relative velocities and position vectors) is conserved.
(3) Conservation law of energy: If there is no external forces and the relative distances between the particles are kept unchanged, the energy of the system is conserved.

## Problem solving procedure

Step 1: How many particles and what is the particle system
Step 2: Coordinate (*You can only set up one coordinate to easily describe the motion of all particles*)
Step 3: Determination of motion
3.1 Calculate the position of mass center
3.2 Determination the motion of mass center (*By the principle of* ***Translation of mass center***)
3.3 Determination of the motion of all particles relative to mass center
*Case 1: If the moment of external forces is known and the geometry of the particle system is known, use the principle of* ***Rotation around mass center***
*Case 2:If the relative distance between particles has no change, use the principle of* ***work and energy****.*
Step 4: Solutions

**Sample Problem #1.**

**Four small disks A,B,C, and D can slide freely on a frictionless horizontal surface. Disk B,C and D are connected by light rods and are at rest in the position shown when disk B is struck squarely by disk A which is moving to the right with a velocity $v_0$=(38.5ft/s)i. The weights of disks are $W_A=W_B=W_C$=15lb, and $W_D$=30lb. Knowing that the velocities of the disks immediately after the impact are $V_A=V_B$=(8.25ft/s)I, $v_C=V_C$i and $v_D=V_D$i, determine (a) the speeds $v_C$ and $v_D$, (b) the fraction of the initial kinetic energy of the system which is dissipated during the collision.**

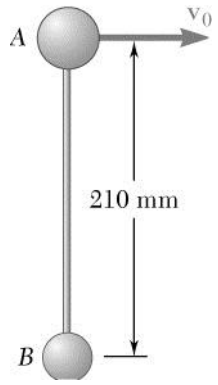



**Step-by-Step Solution:**

| # Step | Attacking Strategy | Your Solution |
|---|---|---|
| **#1** | How many particles and what the particle system is | 4 particles compose the particle system. Before collision, A moves along x direction, B,C,D are at rest. After collision, all the particles move along x direction. Velocities of A,B are known, while, velocity of C,D are known. The motions of B,C, D are dependent. |
| **#2** | Setup a coordinate to describe the motion of all particles | A,B,C,D moves along x direction. We setup rectangular coordinate with x axis pointing rightwards and with origin at B. |
| **#3.1** | Determine mass center | Before collision: x=0<br>and $y=(m_C*y_C+m_D*y_D)/(m_A+m_B+m_C+m_D)$=-0.6ft<br>After collision: x=0 and y=-0.6ft |
| **#3.2** | Translation of mass center | $\Sigma \mathbf{F}_i^e=0$; ∴a=0 and v=constant<br>$\therefore(m_A*v_A+m_B*v_B+m_C*v_C+m_D*v_D)$=constant<br>before collision, $m_A*v_A+m_B*v_B+m_C*v_C+m_D*v_D=m_A*v_0=330$<br>$\therefore 15*8.25+15*8.25+15*v_C+30*v_D=330$<br>$\therefore v_C+2*v_D=22$ (1) |
| **#3.3** | Rotation of the particle system relative to mass center | $\Sigma \mathbf{r}_{kc}X\mathbf{F}_i^e=0$; ∴ moment of momentum relative to the mass center is conserved.<br>$\therefore m_Av_0*0.6=(m_A+m_B)v_A'0.6+m_C*v_C'(3+0.6)-m_Dv_D'(3-0.6)$<br>$\therefore 3v_C-4*v_D=11$ (2) |
| **#4** | Solutions | By (1) and (2), **$v_C$=11ft/s; $v_D$=5.5ft/s**<br>(Energy lost)/(initial energy)=$(T_2-T_1)/T_1$=**78.57%** |
| **After Thought** | The keys for the problems of a particle system are the determination of translation of mass center and the determination of rotation around the mass center. | |

**Sample Problem #2.**

**Two small spheres A and B, with masses of 2.5kg and 1kg, respectively, are connected by a rigid rod of negligible mass. The two spheres are resting on a horizontal; frictionless surface when A is suddenly given the velocity $v_0$=3.5m/si, Determine (a) the linear momentum of the system and its angular momentum about its mass center G. (b) the velocity of A and B after the rod AB has rotated through 180°**




**Step-by-Step Solution:**

| # Step | Attacking Strategy | Your Solution |
|---|---|---|
| **#1** | How many particles and what the particle system is | Two particles compose a particle system. Initially, A is given a velocity. Then, A and B will move with a fixed relative position vector. |
| **#2** | Setup a coordinate to describe the motion of all particles | The velocity of A is given along horizontal direction. So, we setup rectangular coordinate with x axis pointing rightwards and with origin at B. |
| **#3.1** | Determine mass center | Initially:<br>x=0 and $y=(m_A*y_A+m_B*y_B)/(m_A+m_B)$=150mm |
| **#3.2** | Translation of mass center | $\Sigma \mathbf{F}_i^e=0$; ∴a=0 and v=constant<br>$\therefore(m_A*v_A+m_B*v_B)$=constant<br>before collision, $m_A*v_A+m_B*v_B=m_A*v_0=2.5*3.5$(kgm/s)<br>∴v=2.5m/s and $m_A*v_A+m_B*v_B$=8.75(kgm/s) (1) |
| **#3.3** | Rotation of the particle system relative to mass center | $\Sigma \mathbf{r}_{kc} X \mathbf{F}_i^e=0$; ∴ moment of momentum relative to the mass center is conserved.<br>∴ $m_A v_A' 0.06-m_B v_B' 0.15= m_A*3.5*0.06=0.525 kgm^2/s$ (2) |
| **#4** | Solutions | The angular moment about mass center equals to **0.525kgm²/s.**<br>Initially $v_A'=v+\omega*0.06$; $v_B'=v-\omega*0.21$<br>By (2) and (1), $\omega=15.08 rad/s$<br>Since there is no force moment, ω will be kept at constant.<br>∴, after rotating 180°,<br>$v_A'=v-\omega*0.06$=**1.5m/s**; $v_B'=v+\omega*0.21$=**5m/s** |
| **After Thought** | The keys for the problems of a particle system are the determination of translation of mass center and the determination of rotation around the mass center. | |