Summary

Chapter 15 Kinematics of Rigid Bodies

Basic Concepts

- **Translation:** Any straight line in the body keeps the same direction during motion (The body can be modeled by a particle)
- **Rotation about a fixed axis:** All the points on the body move on circular paths centered at the same axis
- **General plane motion and motion decomposition:** Arbitrary plane motion and the motion can be decomposed by two steps: translation of an arbitrary point on the body and the rotation about the axis passing through the point
- **Instantaneous center:** The point, as if all the points on the body rotate about it[**It may not on the body**]. (The directions of velocity vectors of any two arbitrary points on the body can determine the instantaneous center. Simply drawing the normal lines to the two velocity vectors and their cross point is the instantaneous center)
- **Coriolis acceleration:** A virtual acceleration, which presents the effect of frame rotation on the acceleration of a moving particle in the rotating frame.
- **Coincide points:** The point in a rotating frame. It occupies the instant position of a moving point but has no relative motion

Useful formulas

- **Velocity distribution for a rigid body in translation:**

$$\mathbf{v} = \mathbf{v}(t)$$

- **Acceleration distribution for a rigid body in translation:**

$$\mathbf{a} = \mathbf{a}(t)$$

- **Velocity distribution for a rigid body in rotation about a fixed axis:**

$$\mathbf{v}(x, y, z) = \boldsymbol{\omega}(t) \times \mathbf{r}(x, y, z)$$

- **Acceleration distribution for a rigid body in rotation about a fixed axis:**

$$\mathbf{a} = \frac{d\boldsymbol{\omega}(t)}{dt} \times \mathbf{r}(x, y, z) + \boldsymbol{\omega}(t) \times [\boldsymbol{\omega}(t) \times \mathbf{r}(x, y, z)]$$

- **Velocity distribution for a rigid body in general plane motion:**

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}(t) \times \mathbf{r}_{B/A}$$

- **Acceleration distribution for a rigid body in general plane motion:**

$$\mathbf{a}_B = \mathbf{a}_A + \frac{d\boldsymbol{\omega}(t)}{dt} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}(t) \times [\boldsymbol{\omega}(t) \times \mathbf{r}_{B/A}]$$

- **Velocity distribution about the instantaneous point:**

$$\mathbf{v}_B = \boldsymbol{\omega}(t) \times \mathbf{r}_{B/(I.C)}$$

- **The relationship between the absolute velocity (in a lab frame) and the relative velocity in a rotating frame[The two frames have the same origin]:**

$$\mathbf{v} = \mathbf{v}_{p'} + \mathbf{v}_r$$

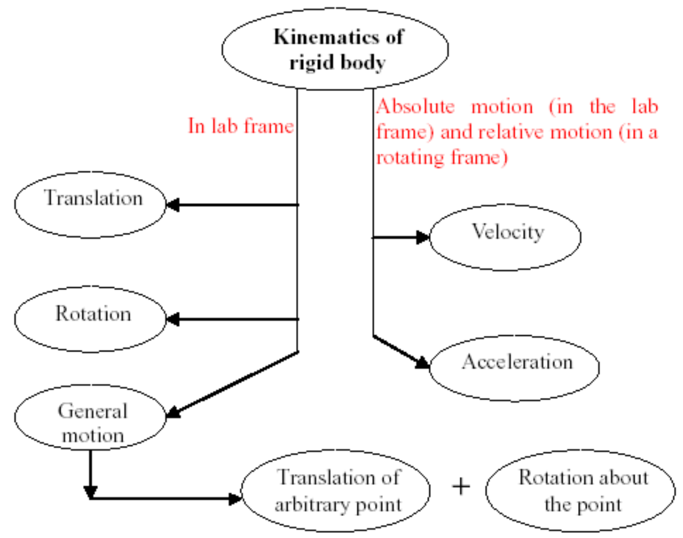
where $\mathbf{v}$ is the absolute velocity, $\mathbf{v}_{p'}$ is the absolute velocity at the coincide point($\mathbf{v}_{p'} = \omega X \mathbf{r}$), and $\mathbf{v}_r$ is the relative velocity

- **The relationship between the absolute acceleration (in a lab frame) and the relative acceleration in a rotating frame[The two frames have the same origin]:**

$$\mathbf{a} = \mathbf{a}_{p'} + \mathbf{a}_r + \mathbf{a}_c$$

where $\mathbf{a}$ is the absolute acceleration, $\mathbf{a}_{p'}$ is the absolute acceleration at the coincide point ($\mathbf{a}_{p'} = (d\omega/dt) X \mathbf{r} + \omega X \omega X \mathbf{r}$), $\mathbf{a}_r$ is the relative acceleration, and $\mathbf{a}_c$ is the acceleration at the Coriolis acceleration ($\mathbf{a}_c = 2\omega X v_r$)

Maps of the chapter



Step-by-step procedure

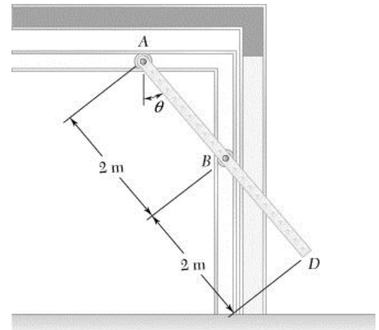
Step 1: How many rigid bodies in the problem
Step 2: Lab frames (*the lab frames you selected can only have fixed offsets*) and, if there are rotating bodies, set up rotating frames fixed on them
Step 3: Determination of motion (Use instantaneous centers if directions of velocity vectors at two points are given; use motion decomposition method if the components of velocity vectors are given; use motion transformation between lab frames and rotating frames if there is any rotational part)
Step 4: Solutions

Some tips

- When will you use instantaneous center?
  Directions of velocities of two points are given and the amplitude of one of them is given. Under such a case, **Use instantaneous center to determine motion of the rigid body.**
- When you determine the motion of rotational machines, be sure to know the following three points: (a) which is the rotating frame; (b) which is the lab frame; and (c) the rotating frame and the lab frame should have the same origin.

Sample Problem #1.

An overhead door is guided by wheels at A and B that roll in horizontal and vertical tracks, Knowing that when θ=40° the velocity of B is 0.6m/s upward, determine (a) the angular velocity of the door, (b) the velocity of end D of the door.

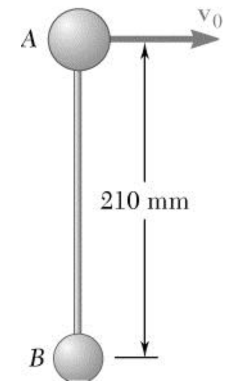


Step-by-Step Solution:

# Step	Attacking Strategy	Your Solution
#1	How many particles and what the particle system is	
#2	Setup a coordinate to describe the motion of all particles	
#3.1	Determine mass center	
#3.2	Translation of mass center	
#3.3	Rotation of the particle system relative to mass center	
#4	Solutions	
After Thought	The keys for the problems of a particle system are the determination of translation of mass center and the determination of rotation around the mass center.	

Sample Problem #2.

Two small spheres A and B, with masses of 2.5kg and 1kg, respectively, are connected by a rigid rod of negligible mass. The two spheres are resting on a horizontal; frictionless surface when A is suddenly given the velocity v_0=3.5m/si, Determine (a) the linear momentum of the system and its angular momentum about its mass center G. (b) the velocity of A and B after the rod AB has rotated through 180°



Step-by-Step Solution:

# Step	Attacking Strategy	Your Solution
#1	How many particles and what the particle system is	Two particles compose a particle system. Initially, A is given a velocity. Then, A and B will move with a fixed relative position vector.
#2	Setup a coordinate to describe the motion of all particles	The velocity of A is given along horizontal direction. So, we setup rectangular coordinate with x axis pointing rightwards and with origin at B.
#3.1	Determine mass center	Initially: x=0 and y=(m_A*y_A+m_B*y_B)/ (m_A+m_B)=150mm
#3.2	Translation of mass center	$\Sigma \mathbf{F}_i^e=0$; ∴a=0 and v=constant ∴(m_A*v_A+m_B*v_B)=constant before collision, m_A*v_A+m_B*v_B =m_A*v_0=2.5*3.5(kgm/s) ∴v=2.5m/s and m_A*v_A+m_B*v_B=8.75(kgm/s) (1)
#3.3	Rotation of the particle system relative to mass center	$\Sigma \mathbf{r}_{kc} X \mathbf{F}_i^e=0$; ∴ moment of momentum relative to the mass center is conserved. ∴ $m_A v_A$'0.06-$m_B v_B$'0.15= m_A*3.5*0.06=0.525kgm^2/s (2)
#4	Solutions	The angular moment about mass center equals to **0.525kgm^2/s.** Initially v_A'=v+ω*0.06; v_B'=v-ω*0.21 By (2) and (1), ω=15.08rad/s Since there is no force moment, ω will be kept at constant. ∴, after rotating 180°, v_A'=v-ω*0.06=**1.5m/s**; v_B'=v+ω*0.21=**5m/s**
After Thought	The keys for the problems of a particle system are the determination of translation of mass center and the determination of rotation around the mass center.	