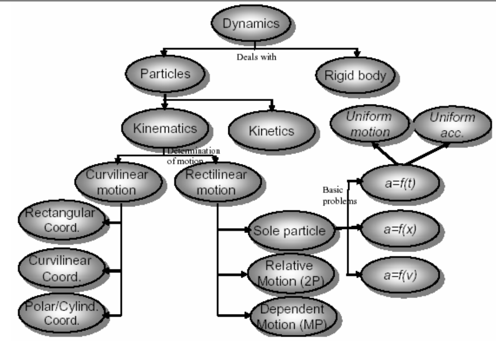
Chapter 11 Kinematics of Particles

Basic Concepts

- **Particle:** A moving body, on which any two arbitrary points keep fixed relative position.
- **Rigid body:** A moving body, on which any two arbitrary points *don't* keep fixed relative position.
- **Rectilinear motion:** The path is a straight line
- **Curvilinear motion:** The path is a curve
- **Position vector:** The vector staring from the origin of a fixed from to the instant position of the particle.
- **Average velocity vector:** The ratio of the change of two *position vectors* at two different instances to the time interval.
- **Velocity vector:** The limitation of the *average velocity vector* when the time interval approaches zero
- **Average acceleration vector:** The ratio of the change of two *velocity vectors* at two different instances to the time interval.
- **Acceleration vector:** The limitation of the *average acceleration vector* when the time interval approaches zero
- **Uniform motion:** The motion of a particle when the modulus of its acceleration vector is *zero* all the time.
- **Uniformly accelerated motion:** The motion of a particle when the modulus of its acceleration vector is a *constant*.
- **Dependent motion:** The motions of multi-particles, which are governed by geometrical constrains.
- **Rectangular coordinate:** A *fixed* frame with three fixed perpendicular axis.
- **Curvilinear coordinate:** A moving frame on the path of a particle, with one axis tangent to the path.
- **Tangential unit vector:** The unit vector of the axis of a curvilinear coordinate which is *tangent* to the particle path.
- **Radius of curvature:** The limitation of the traveled distance on the path over the direction change of the two Tangential unit vectors at the ends of the traveled distance.
- **Osculating plane:** The plane spanned by two tangential unit vectors at adjacent instances (time interval approaches 0).
- **Principle normal unit vector:** In the osculating the plane, the unit vector perpendicular to the tangential unit vector and follows right-hand trial.
- **Binormal unit vector:** The unit vector to compose a right hand coordinate system with the tangential unit vector and the principle normal unit vector.
- **Polar coordinate:** A fixed 2-D coordinate, where the point's position is defined by distance and azimuth angle to the origin
- **Cylindrical coordinate:** A fixed 3-D coordinate, spanned by a polar coordinate and a vertical axis to it

Maps of the chapter



Useful principles

(1) Velocity vector is *parallel* to tangential unit vector.
(2) *No* acceleration component along binormal unit vector.
(3) Relative motion is the subtract of motions of the particles in the same frame

Problem solving procedure

For rectilinear motion:

Step 1: The number of particles (with independent motion or dependent motion)
Step 2: Set coordinate and find the type of acceleration functions for each particle with independent motion.
Step 3: Determination of motion of the particles with independent motion by table 1
Step 4: Determination of motion of the particles with dependent motion
Step 5: Look up the solutions of the problem from the determined motions of particles.

For curvilinear motion:

Step 1: The number of particles (with independent motion or dependent motion)
Step 2: The coordinate used (type and origin for fixed frames)
Step 3: Determination of each component of motion of the particles with independent motion by table 2 (speed is given then curvilinear coord; radius is given then polar coord; x,y,z components are given then rectangular).
Step 4: Determination of each component of motion of the particles with dependent motion.
Step 5: Look up the solutions of the problem from the determined motions of particles.

Important tables

Table 1 Determination of rectilinear motion

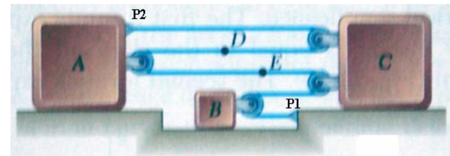
	v (velocity)	x (displacement)
$a = f(t)$	$v(t) = v(0) + \int_0^t f(\tau)d\tau$	$x(t) = x(0) + \int_0^t v(\tau)d\tau$
$a = f(x)$	$[v(x)]^2 = [v(x_0)]^2 + 2\int_0^x f(\tilde{x})d\tilde{x}$	
$a = f(v)$	$\int_{v(0)}^{v(t)} \frac{dv}{f(v)} = t$	$x(t) = x(0) + \int_{v(0)}^{x(t)} \frac{vdv}{f(v)}$

Table 2 Curvilinear motion in different coordinates

	Rectangular	Curvilinear		Polar/Cylindrical	
		Plane motion	Space motion	Plane motion	Space motion
Position vector	$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$			$\mathbf{r} = r\mathbf{e}_r$	$\mathbf{R} = r\mathbf{e}_R + z\mathbf{k}$
Velocity vector	$\mathbf{v} = \frac{dx}{dt}\mathbf{i} + \frac{dx}{dt}\mathbf{j} + \frac{dx}{dt}\mathbf{k}$	$\mathbf{v} = v\mathbf{e}_t$		$\mathbf{v} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta$	$\mathbf{v} = \frac{dR}{dt}\mathbf{e}_R + R\frac{d\theta}{dt}\mathbf{e}_\theta + \frac{dz}{dt}\mathbf{k}$

Sample Problem #1.

Slider block *B* moves to the left with a constant velocity of 2 in/s. At *t=0*, slider block *A* is moving to the right with a constant acceleration and a velocity of 4in/s. Knowing that at *t=2*s, slider block *C* has moved 1.5in to the right, determine (a) the velocity of slide block *C* at t=0; (b) the velocity of portion *D* of the cable at t=0, (c) the accelerations of *A* and *C*.

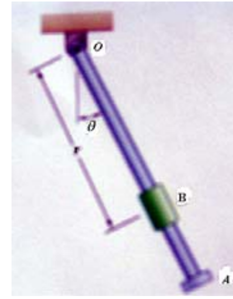


Step-by-Step Solution:

# Step	Attacking Strategy	Your Solution
#1	The number of particles	4 (A, B, C, D) and they are all in rectilinear motion. A and B are in independent motion, C and D are in dependent motion.
#2	Set Coordinate and find the type of acceleration functions for each particle with independent motion.	Coordinate is along horizontal direction and points to the right. For A: Constant acceleration; initial velocity of 4in/s For B: Constant velocity; initial velocity of -2in/s (right is positive) So, all the motions belongs to $a=f(t)$.
#3	Determination of motion of the particles with independent motion by table 1	For A: $v_A(t)=4+a_A*t$; $x_A(t)=x_A(0)+4*t+1/2\ a_A*t^2$ For B: $v_B(t)=-2$; $X_B(t)=x_B(0)-2*t$
#4	Determination of motion of the particles with dependent motion	Constrains: the length of the rope is fixed. $L_{rope}=$ P1-$[x_B(0)+x_B(t)]+[x_c(0)+x_c(t)]-[x_B(0)+x_B(t)]+3*\{[x_c(0)+x_c(t)]-[x_A(0)+x_A(t)]\}$ yields velocity dependence: $0-[0+v_B(t)]+[0+v_c(t)]-[0+v_B(t)]+3*\{[0+v_c(t)]-[0+v_A(t)]\}=0$ we get: $-2v_B(t)+4v_c(t)-3v_A(t)=0$ Thus: $v_c(t)=[8+3a_A*t]/4$, $a_c(t)=3a_A/4$ and $x_c(t)=x_c(0)+2*t+3/8\ a_A*t^2$ For D: the length P2 to D should be fixed. We have $[x_c(0)+x_c(t)]-[x_A(0)+x_A(t)]+[x_c(0)+x_c(t)]-[x_D(0)+x_D(t)]=const$ We get: $2v_c(t)-v_A(t)-v_D(t)=0$ Thus: $v_D(t)=1/2*a_A*t$ and $x_D(t)=1/4*a_A*t^2$
#5	Look up the solutions of the problem from the determined motions of particles.	$v_c(0)=v_c(t=0)=2in/s$ $v_D(0)=v_D(t=0)=0in/s$ $1.5in=x_c(t=2)-x_c(0)=2*(2)+3/8\ a_A*(2)^2$, we get $a_A=-5/3(in/s^2)$; $a_c(t)=-5/4(in/s^2)$
After Thought	Please be careful in finding constrains. Frequently ask yourself, whether they are physically reasonable.	

Sample Problem #2.

The oscillation of rod *OA* about *O*, is defined by the relation *θ=(4/π) (sinπt)*, where *θ* and t are expressed in radians and seconds, respectively. Collar *B* slides along the rod so that its distance from *O* is *r=10/(t+6)*, where r and t are expressed in mm and seconds, respectively. When t=1s, determine (a) the velocity of the collar; (b) the total acceleration of the collar; (c) the acceleration of the collar relative to the rod



Step-by-Step Solution:

# Step	Attacking Strategy	Your Solution
#1	The number of particles (with independent motion or dependent motion)	1 (Collar B)
#2	The coordinate used (type and origin for fixed frames)	Plane motion. *r* and *θ* are known. Select polar coordinate and the origin is set at *O*.
#3	Determination of each component of motion of the particles with independent motion by table 2 (speed is given then curvilinear coord; radius is given then polar coord; x,y,z components are given then rectangular).	By table 2, we get r component: $a_r(t)=\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2=\frac{20}{(t+6)^3}-\frac{10}{t+6}(4\cos\pi t)^2$ and $v_r=\frac{dr}{dt}=-\frac{10}{(t+6)^2}$ θ component: $a_\theta=r\frac{d^2\theta}{dt^2}+2\frac{dr}{dt}\frac{d\theta}{dt}=-\pi\frac{40}{t+6}\sin(\pi t)-\frac{80}{(t+6)^2}\cos(\pi t)$ and $v_\theta=\frac{40}{t+6}\cos(\pi t)$
#4	Determination of each component of motion of the particles with dependent motion.	None!
#5	Look up the solutions of the problem from the determined motions of particles.	At t=1s, we get $v_r=-\frac{10}{49}$, $v_\theta=-\frac{40}{7}$. So the velocity of the collar is $v=5.71$; $\beta=0+\mathrm{atan}\left(\frac{v_\theta}{v_r}\right)=2^o$ (the angle from vertical direction) Acceleration: $a_r(t)=\frac{20}{7^3}-\frac{160}{7}=-22.8$; $a_\theta=\frac{80}{7^2}=1.63$; So the acceleration of the collar is 22.85 at the angle of $\beta=0+\mathrm{atan}\left(\frac{a_\theta}{a_r}\right)=-4^o$ Since the relative motion between the rod and the collar is rectilinear motion, the relative velocity would be $v_{c/r}=\frac{dr}{dt}=-\frac{10}{49}$
After Thought		