

### Sample Problem #1. Lecture notes on Jul. 17th. 2008

A ball, B rotates around the center of a disk at a constant angular velocity  $\omega_0$  and with a constant radius  $r_0$ . A rope is bonded on B and a block A. The rope is clamped. Suddenly, the clamp is released.

Please calculate  $\frac{d^2r}{dt^2}$  and the radial velocity component of B.

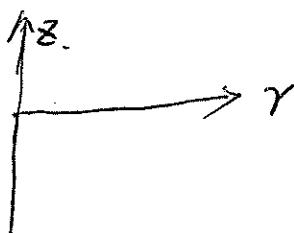
( $r$  is the instant radius of B)

Step 1: How many particles?

2. A and B

Step 2: Coordinate.

Cylindrical.



Step 3: Determination of Motion.

3.1 Kinematics.

The length of the rope is constant.

$$\text{Rope} = r - z_A$$

$$\Rightarrow \frac{dr}{dt} = \frac{dz_A}{dt} \quad \text{and} \quad \frac{d^2r}{dt^2} = \frac{d^2z_A}{dt^2} = a_{Az} \quad \dots \quad \textcircled{1}$$

The ~~free~~ resultant force of B passes through the center.

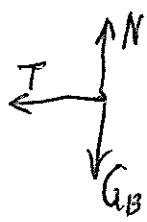
By conservation Law of moment of momentum.

$$m_B \vec{V}_B \times \vec{r}_B \equiv \text{const} \Rightarrow \frac{d}{dt} r^2 = \omega_0 r_0^2 \quad \text{--- \textcircled{2}}$$

$$\vec{V}_B \times \vec{r}_B = V_\theta \cdot r = \omega r^2 = \frac{d\theta}{dt} r^2$$

### 3.2. ~~kinetics~~-force analysis

For B:



$$\sum F_B = -T$$

$$\sum F_{B\theta} = 0$$

$$\sum F_B^z = N - G_B$$

For A:

$$\sum F_A r = 0$$

~~$\sum F_{A\theta}$~~   $\sum F_{A\theta} = 0$

$$\sum F_A^z = T - G_A$$



### 3.3 dynamic equations

$$\sum F_B r = -T = m_B a_r \quad \dots \quad (3)$$

$$\sum F_A^z = T - G_A = m_A a_z \quad \dots \quad (4)$$

in cylindrical coord.

$$a_r = \frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \quad \dots \quad (5)$$

$$v_r = \frac{dr}{dt} \quad \dots \quad (6)$$

Step 4. provide solution.

By (3) and (4)

~~$G_A = m_A a_z + m_B a_r$~~

$$-G_A = m_A a_z + m_B a_r$$

By (5)

$$-G_A = m_A a_z + m_B \left( \frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \right)$$

By (1) and (2)

$$-G_A = m_A \frac{d^2 r}{dt^2} + m_B \frac{d^2 r}{dt^2} - m_B r \left( \omega_0 \frac{r_0^2}{r^2} \right)^2$$

$$\Rightarrow \frac{d^2 r}{dt^2} = -\frac{1}{m_A + m_B} \left( -G_A + m_B \frac{r_0^4}{r^3} \omega_0^2 \right) \quad \dots \quad (7)$$

Radial velocity:

$$\frac{d^2r}{dt^2} = \frac{d}{dt} \left( \frac{dr}{dt} \right) = \frac{d}{dr} \left( \frac{dr}{dt} \right) \frac{dr}{dt} = \frac{d}{dr} (V_r) V_r.$$

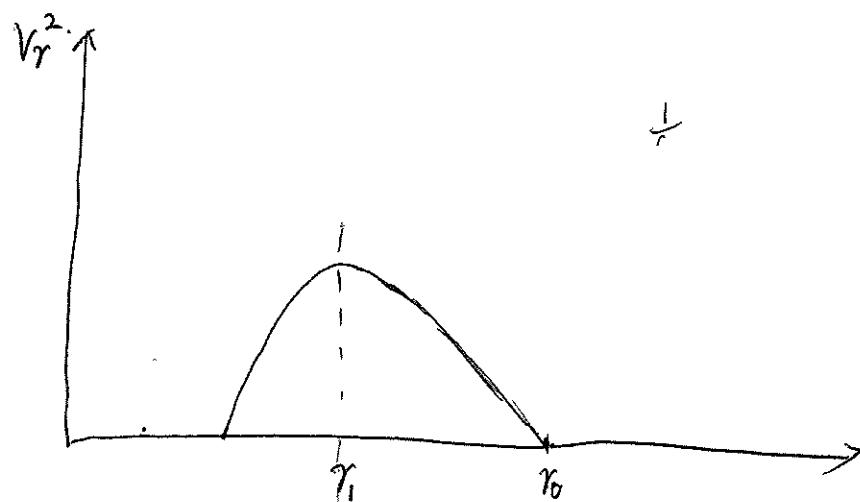
$$\Rightarrow \frac{1}{2} V_r^2 \Big|_{r_0}^r = \frac{1}{m_A + m_B} \left[ -G_A r + m_B \omega_0^2 r_0^4 \left( \frac{1}{r_0^2} - \frac{1}{r^2} \right) \right] \Big|_{r_0}^r$$

$$\frac{1}{2} V_r^2 = \frac{1}{2} V_{r_0}^2 (r=r_0) + \frac{1}{m_A + m_B} \left[ -G_A (r-r_0) - \frac{m_B \omega_0^2 r_0^4}{2} \left( \frac{1}{r_0^2} - \frac{1}{r_0^2} \right) \right]$$

$\therefore V_r^2 (r=r_0)$  is the initial radial velocity.

$$\therefore V_r^2 (r=r_0) = 0$$

$$\Rightarrow V_r^2 = \frac{-2G_A(r-r_0)}{m_A + m_B} - \cancel{\frac{m_B \omega_0^2 r_0^4}{m_A + m_B}} \left( \frac{1}{r_0^2} - \frac{1}{r^2} \right)$$



At  $r=r_1$ ,

$$\frac{d(V_r)^2}{dt} = 0 \Rightarrow 2V_r \frac{dV_r}{dt} = 0$$

$$\Rightarrow \frac{d^2r}{dt^2} \Big|_{r=r_1} = 0$$

$$\therefore r_1 = \sqrt[3]{\frac{m_B r_0^4 \omega_0^2}{G_A}}$$

The ball B would oscillate around  $r=r_1$ .