

# Chapt 13. Kinetics of Particles: Energy and Momentum Methods.

Lecture note: 07/24/2006

~~Simple~~ Procedures for using the Principle of Work and Energy.

Step 1: How many particles ~~are~~ in the system.

Step 2: Coordinate.

Step 3: Expression of Work and Energy.

3.1 determine the starting state and ending state.

3.2. the energy at all states for each particle

3.3. the work between adjacent states for each particle

3.4 governing equation by the Principle of Work and Energy

Step 4: Solutions.

## Sample Problem #1.

A 2-kg stone is dropped from a height  $h$  and strikes the ground with a velocity of ~~is~~ 24 m/s. (a) Find the kinematic energy of the stone as it strikes the ground and the height  $h$  from which it was dropped. (b) solve part a, assuming that the same stone is dropped on the moon. (Acceleration of gravity on the moon =  $1.62 \text{ m/s}^2$ ) 13.4.

Step 1 : particles --- (1)

Step 2 : coordinate.

rectangular



B ————— ground

Step 3 :

3.1. starting state A

ending state B

3.2. energy .

Kinematics  $\frac{1}{2} mv^2$

Potential energy of gravity:  $mgh$ .  
Spray = 0

So we get,

$$\text{At state A: } E_A = \frac{1}{2}mv_A^2 + mgh_A \quad \dots \quad \textcircled{1}$$

$$\text{At State B: } E_B = \frac{1}{2}mv_B^2 + mgh_B \quad \dots \quad \textcircled{2}$$

3.3 work for ~~other~~ external forces.

$$W=0 \quad \dots \quad \textcircled{3}$$

3.4 governing Eq.

$$E_B - E_A = W$$

$$\Rightarrow \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}mv_A^2 + mgh_A \quad \dots \quad \textcircled{4}$$

Step 4: Solutions.

(a) Kinematic energy at B

$$E_{kB} = \frac{1}{2}mv_B^2 = \frac{1}{2} \cdot 2 \cdot 24^2 = 576 \text{ (J)}$$

$$\text{at A: } v_A = 0$$

$$\xrightarrow{\text{by } \textcircled{4}} mg(h_A - h_B) = \frac{1}{2}mv_B^2$$

$$h_A - h_B = \frac{v_B^2}{2g} = \frac{576}{2 \cdot 9.8} = 29.39 \text{ (m)}.$$

(b) Kinematic energy at B

$$E_{kB} = \frac{1}{2}mv_B^2 = 576 \text{ (J)}$$

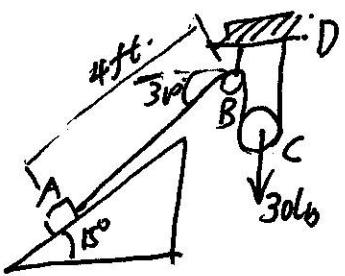
$$\text{at A: } v_A = 0$$

$$\xrightarrow{\text{by } \textcircled{4}} mg(h_A - h_B) = \frac{1}{2}mv_B^2$$

$$h_A - h_B = \frac{v_B^2}{2g} = \frac{576}{2 \cdot 9.8} = 177.78 \text{ (m)}$$

Sample Problem 11-2:

The 16-lb block A is released from rest in the position shown. Neglecting the effect of friction and the masses of pulleys. Determine the velocity of the block after it has moved 2ft up the incline. (13.9)



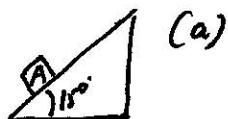
Step 1: Particles  
one

Step 2: Coordinate  
rectangular  $\rightarrow$  origin at D

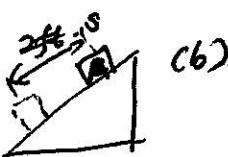
Step 3:

3.1 states

Starting state:



ending state:



geometry:  $s = 2\text{ft}$

$$\begin{cases} y_b - y_a = s \cdot \sin 15^\circ \\ \frac{ds}{dt} = v \end{cases} \quad \dots \dots \quad (1)$$

3.2. energy

$$\begin{cases} \text{kinetic energy: } \frac{1}{2}mv^2 \\ \text{potential energy: } mgh \\ \text{spring: } 0 \end{cases}$$

So we get:

$$\text{at state "a": } F_A = \frac{1}{2}mv_A^2 + mgh_A \quad \dots \quad (2)$$

$$\text{at state "b": } F_B = \frac{1}{2}mv_B^2 + mgh_B \quad \dots \quad (3)$$

3.3 work for other external forces

$$W = \bar{F} \cdot (\vec{r}_{cb} - \vec{r}_{ca}) \hat{y} = 30(y_{ca} - y_{cb}) \quad \dots \quad (4)$$

geometry:

The length of the rope is constant.

$$L = -y_c - y_c + \left(\frac{-y_A}{\sin 30}\right) \quad \dots \quad (5)$$

$$-\gamma_{ca} - \gamma_{ca} + \left( \frac{-\gamma_{Aa}}{\sin 30^\circ} \right) = -2\gamma_{cb} - \frac{\gamma_{Ab}}{\sin 30^\circ}$$

$$\Rightarrow -2(\gamma_{cb} - \gamma_{ca}) = \frac{\gamma_{Ab} - \gamma_{Aa}}{\sin 30^\circ}$$

$$\xrightarrow{\text{by (4)}} W = 30 \cdot \left( \frac{\gamma_{Ab} - \gamma_{Aa}}{2 \sin 30^\circ} \right) = -30(\gamma_{Aa} - \gamma_{Ab}) \quad (J)$$

3.4 governing E.8.

$$E_b - E_a = W$$

$$\frac{1}{2}mV_b^2 + mgY_b - \frac{1}{2}mV_a^2 - mgY_{Aa} = 30(\gamma_{Aa} - \gamma_{Ab})$$

$$\text{at (a)} \quad V_{Aa} = 0$$

by (6)

$$\Rightarrow \frac{1}{2}mV_{Ab}^2 = 30(\gamma_{Aa} - \gamma_{Ab}) + mg(\gamma_{Aa} - \gamma_{Ab}) \\ = (mg - 30)(\gamma_{Aa} - \gamma_{Ab}) \quad \text{--- (6)}$$

~~geometry~~

Step 4: solutions.

$$\text{geometry: } \gamma_{Ab} - \gamma_{Aa} = 16 \cdot \sin 15^\circ \quad \text{---}$$

by (6)

$$\Rightarrow \frac{1}{2}mV_{Ab}^2 = (16 - 30) \left( -2 \cdot \frac{1}{2} \right) \\ = 14$$

$$V_{Ab} = \sqrt{\frac{2 \times 14}{16/322}} = 7.5 \text{ ft/s}$$

Sample #3.

A 20 lb block is attached to spring A and connected to spring B by a cord and pulley. The block is held in the position shown with both springs unstretched. When the support is removed and the block is released with no initial velocity, knowing  $k_A, k_B = 12 \text{ lb/in}$ .

(a) Velocity of the block after it has moved down 2 in

(b) the maximum velocity.

Step 1: one particle

Step 2: rectangular cord.

Step 3: 3/1 starting state. ("a")

$y_a = y_{A0}$  spring at rest

$v = 0$

ending state. ("b")

$y_b = y_{Aa}$

3.2. energy.  $\left\{ \begin{array}{l} \text{kinetic} : \frac{1}{2} m V^2 \\ \text{potential} : \text{gravity } m g y_{Aa} \\ \text{spring. } \frac{1}{2} k (y_{Aa} - y_{A0})^2 \\ + \frac{1}{2} k (y_p - y_{p0})^2 \end{array} \right.$

So : We get :

at state "a":  $E_a = \frac{1}{2} m V_a^2 + \frac{1}{2} k (y_{Aa} - y_{A0})^2 + \frac{1}{2} k (y_{pa} - y_{p0})^2 + m g y_{Aa}$

state "b":  $E_b = \frac{1}{2} m V_b^2 + \frac{1}{2} k (y_{Ab} - y_{A0})^2 + \frac{1}{2} k (y_{pb} - y_{p0})^2 + m g y_{Ab}$

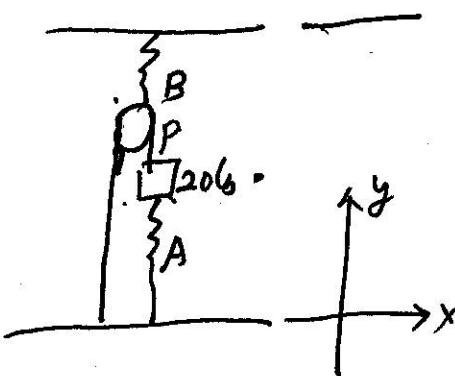
3.3. Work for other external forces.

$$W = 0$$

3.4 governing Eq.

$$\begin{aligned} E_b - E_a &= 0 = \frac{1}{2} m V_b^2 + \frac{1}{2} k (y_{Ab} - y_{A0})^2 + \frac{1}{2} k (y_{Ab} - y_{p0})^2 + m g y_{Ab} \\ &\quad + \frac{1}{2} m V_a^2 + \frac{1}{2} k (y_{Aa} - y_{A0})^2 + \frac{1}{2} k (y_{pa} - y_{p0})^2 + m g y_{Aa} \\ &= \frac{1}{2} m V_b^2 + \frac{1}{2} k (y_{Ab} - y_{A0})^2 + \frac{1}{2} k (y_{pb} - y_{p0})^2 + m g y_{Ab} \\ &\quad - 0 - 0 - 0 + m g y_{Aa}. \end{aligned}$$

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~~Step 4: Solution~~  
the length of the rope

$$L = \cancel{\frac{1}{2}k} \cdot (y_p - y_A) + y_p = \text{const.}$$

(a) when  $y_{Ab} - y_{Aa} = 2 \text{ inch}$ .

$$\left. \begin{aligned} \frac{1}{2}mV_b^2 + \frac{1}{2}k \cdot \cancel{(y_{Aa})^2} + \frac{1}{2}k(y_{pb} - y_{pa})^2 + mg(y_{Ab} - y_{Aa}) &= 0 \\ y_{Ab} - y_{Aa} &= -2 \text{ inch} = -\frac{1}{12} \text{ feet} \end{aligned} \right\}$$

$$y_{pb} - y_{pa} = \frac{y_{Aa} + \text{const.}}{2} - \frac{y_{Aa} + \text{const.}}{2} = \frac{y_{Ab} - y_{Aa}}{2}$$

~~$\frac{1}{2}mV_b^2 + \frac{1}{2}k(y_{pb} - y_{pa})^2 + mg(y_{Ab} - y_{Aa}) = \frac{1}{2} \cdot 12 \cdot \left(\frac{-2}{2}\right)^2$~~

~~$= \frac{1}{2} \cdot 12 \cdot 40 = 6$~~

~~$= 10$~~

~~$V_b = \sqrt{\frac{10}{20}} = 1 \text{ m/s}$~~

$$\frac{1}{2}mV_b^2 = -\frac{1}{2}k(y_{Ab} - y_{Aa})^2 - \frac{1}{2}k(y_{pb} - y_{pa})^2 + mg(y_{Ab} - y_{Aa})$$

$$k = 12 \text{ lb/in} = 12 \text{ lb/}\frac{1}{12} \text{ feet} = 144 \text{ lb/feet.}$$

$$y_{Ab} - y_{Aa} = -\frac{1}{12} \text{ feet.}$$

$$y_{pb} - y_{pa} = -\frac{1}{12} \text{ feet}$$

$$\frac{1}{2}mV_b^2 = -\frac{1}{2} \cdot 144 \cdot \frac{1}{36} - \frac{1}{2} \cdot 144 \cdot \frac{1}{144} + 20 \cdot \frac{1}{6}$$

$$= -2 - \frac{1}{2} + \frac{20}{6} = \frac{20}{6} - \frac{12}{6} - \frac{3}{6} = \frac{5}{6}$$

$$V_b^2 = \frac{5}{36} = \frac{5 \times 32.2}{3 \times 20}$$

$$V_b = 1.6381 \text{ feet/s.}$$

~~(b)~~

$$(b) \quad \begin{cases} \frac{1}{2}mV_b^2 + \frac{1}{2}k(y_{Ab} - y_{Aa})^2 + \frac{1}{2}k(y_{Pb} - y_{Pa})^2 + mg(y_{Ab} - y_{Aa}) = 0 \\ y_{Pb} - y_{Pa} = \frac{y_{Ab} - y_{Aa}}{2} \end{cases}$$

$$\Rightarrow \frac{1}{2}mV_b^2 + \frac{1}{2}k(y_{Ab} - y_{Aa})^2 + \frac{1}{2}k \cdot \frac{(y_{Ab} - y_{Aa})^2}{4} + mg(y_{Ab} - y_{Aa}) = 0$$

$$\Rightarrow V_b(y_{Ab}) \rightarrow V_{b\max} \Rightarrow \frac{dV_b}{dy_{Ab}} = 0$$

~~$\frac{1}{2}m \cdot 2V_b \frac{dV_b}{dy_{Ab}}$~~

~~$\frac{1}{2}m \cdot 2V_b \frac{dV_b}{dy_{Ab}} + \frac{1}{2}k \cdot 2(y_{Ab} - y_{Aa}) + \frac{1}{8}k \cdot 2(y_{Ab} - y_{Aa}) + mg = 0$~~

$$\Rightarrow k(y_{Ab} - y_{Aa}) + \frac{1}{4}k(y_{Ab} - y_{Aa}) + mg = 0$$

$$\frac{5}{4}k(y_{Ab} - y_{Aa}) = -mg$$

$$y_{Ab} - y_{Aa} = -\frac{4mg}{5k} = -\frac{4 \cdot 20}{5 \cdot 144} = -0.111 \text{ feet}$$

$$\frac{1}{2}mV_b^2 = -20 \quad \text{---} \quad -mg(y_{Ab} - y_{Aa}) - \frac{1}{2}k(y_{Ab} - y_{Aa})^2 - \frac{1}{2}k \frac{(y_{Ab} - y_{Aa})^2}{4}$$

$$= -20 \cdot (-0.111) - \frac{1}{2} \cdot 144 \cdot 0.111^2 - \frac{1}{2} \cdot 144 \frac{0.111^2}{4}$$

$$= 2.222 - 0.8887 - 0.2222$$

$$= 1.8915 \text{ feet/s}$$