Some discussion on the coefficient of restitution

(1) Objective

How to calculate coefficient restitution when the impulse of external force can **not** be dropped.

(2) Definition of coefficient of restitution

$$e = \frac{\text{Impulse during restitution period}}{\text{Impulse during deformation period}}$$

(3) Calculating formula for direct central impact

Consider direct central impact. Two balls are subjected to external forces during collision and they are parallel to the line of impact. Meanwhile, their impulse can not be neglected during collision. We denote the forces applied to the two particles are F_A and F_B respectively. The impulse during deformation period and restitution period are denoted by $\left(\int F_A dt\right)_D$, $\left(\int F_B dt\right)_D$, $\left(\int F_A dt\right)_R$, $\left(\int F_B dt\right)_R$ respectively. Thus, we have

Deformation period:

$$-\int Pdt + \left(\int F_A dt\right)_D = m_A u - m_A v_A \tag{1}$$

$$\int Pdt + \left(\int F_B dt\right)_D = m_B u - m_B v_B \tag{2}$$

Restitution period:

$$-\int Rdt + \left(\int F_A dt\right)_R = m_A v'_A - m_A u \tag{3}$$

$$\int Rdt + \left(\int F_B dt\right)_R = m_B v'_B - m_B u \tag{4}$$

By the definition of e, we have

$$e = \frac{\int Rdt}{\int Pdt} = \frac{\left(\int F_A dt\right)_R - m_A v'_A + m_A u}{\left(\int F_A dt\right)_D - m_A u + m_A v_A} \tag{5}$$

or

$$e = \frac{\int Rdt}{\int Pdt} = \frac{-\left(\int F_B dt\right)_R + m_B v'_B - m_B u}{-\left(\int F_B dt\right)_D + m_B u - m_B v_B} \tag{6}$$

So, we get:

$$e = \frac{\dfrac{\left(\int F_A dt\right)_R}{m_A} + \dfrac{-\left(\int F_B dt\right)_R}{m_B} + v'_B - v'_A}{\dfrac{\left(\int F_A dt\right)_D}{m_A} + \dfrac{-\left(\int F_B dt\right)_D}{m_B} - v_B + v_A} \tag{7}$$

(4) Conclusion:

When the impulse of external forces during collision period can be dropped, it yields

$$e = \frac{v'_B - v'_A}{-v_B + v_A}$$

when impulse of external forces are with same direction and are proportional to the mass of the particles, it yields:

$$e = \frac{v'_B - v'_A}{-v_B + v_A}$$

Otherwise, e should be only calculated by knowing the impulse of external forces and

$$e \neq \frac{v'_B - v'_A}{-v_B + v_A}$$