

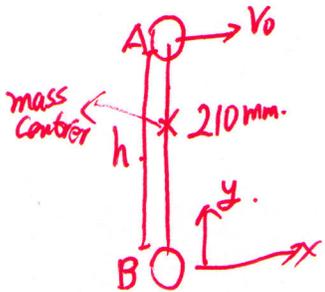
Sample problem #2.

Two small spheres A and B, with masses of 2.5kg and 1kg, respectively, are connected by a rigid rod of negligible mass. The two spheres are resting on a horizontal, frictionless surface when A is suddenly given the velocity $V_0 = (3.5\text{m/s})\hat{i}$. Determine (a) the linear momentum of the system and its angular momentum about its mass center G. (b) the velocity of A and B after the rod AB has rotated through 180° (14.45 76h.)

Step 1: 2 particle. \rightarrow 1 system

Step 2: $\begin{matrix} y \\ \uparrow \\ x \end{matrix}$

Step 3: 3.1 mass center



$$x=0$$

$$y = \frac{m_B \cdot 0 + m_A \cdot \frac{h}{2}}{m_A + m_B} = \frac{m_A}{m_A + m_B} h = \frac{2.5}{3.5} \cdot 210 = 150\text{mm.}$$

3.2. translation of mass center.

$$\sum_{i=1}^2 \vec{F}_i^e = 0 \Rightarrow a = 0\text{ m/s}^2.$$

Initial velocity of mass center

$$V = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{m_A v_0}{m_A + m_B} = \frac{5}{7} \cdot 3.5 \hat{i} = (2.5\text{m/s})\hat{i}.$$

\therefore the mass center will move along \hat{x} direction at $(2.5\text{m/s})\hat{i}$.

3.3 rotation.

$$\sum_{i=1}^2 \vec{F}_i^e \times \vec{r}_i = 0 \quad \text{and geometry is given.}$$

$$\Rightarrow \vec{M}_{AC} + \vec{M}_{BC}$$

$$\vec{r}_{AC} \times m_A \vec{v}_{AC} + \vec{r}_{BC} \times m_B \vec{v}_{BC} = \text{const.}$$

Initially: $\vec{v}_{AC} = \vec{v}_A - \vec{v} = 3.5\text{m/s} - 2.5\text{m/s} = (1\text{m/s})\hat{i}$

$$\vec{v}_{BC} = \vec{v}_B - \vec{v} = 0 - 2.5\text{m/s} = (-2.5\text{m/s})\hat{i}.$$

$$\vec{r}_{AC} = (210 - 150)\hat{j} = 60\hat{j} = 0.06\text{m}\hat{j}$$

$$\vec{r}_{BC} = (0 - 150)\hat{j} = -150\hat{j} = -0.15\text{m}\hat{j}$$

$$\Rightarrow \vec{r}_{AC} \times m_A \vec{v}_{AC} + \vec{r}_{BC} \times m_B \vec{v}_{BC} = 60 \cdot 2.5 \cdot 1 \times 10^{-3} + (-150) \cdot 1 \cdot (-2.5) \times 10^{-3}$$

$$= 150 \times 10^{-3} + 375 \times 10^{-3} = 475 \times 10^{-3}$$

$$= 0.475 \text{ kg} \cdot \text{m}^2/\text{s}$$

rotates around the mass center
angular velocity is ω

$$\vec{v}_{AC} = \vec{\omega} \times \vec{r}_{AC} = \omega \hat{k} \times r_{AC} \hat{j}$$

$$\vec{v}_{BC} = \vec{\omega} \times \vec{r}_{BC} = \omega \hat{k} \times r_{BC} \hat{j}$$

$$\Rightarrow m_A \omega r_{AC}^2 + m_B \omega r_{BC}^2 = 0.475$$

$$\omega = \frac{0.475}{2.5 \cdot (0.06)^2 + 1 \cdot (0.15)^2} = \frac{0.475}{0.009 + 0.0225} = 15.08 \text{ rad/s.}$$

$$\vec{\omega} = \omega \cdot (-\hat{k}).$$

Step 4. Solution.

(a) Linear momentum of the system

$$\sum_{i=1}^2 m_i \vec{v}_i = m_A \vec{v}_A + m_B \vec{v}_B = m_A \cdot \vec{v}_A^0 = 2.5 \cdot 3.5 = 8.75 \text{ kg m/s } \vec{i}$$

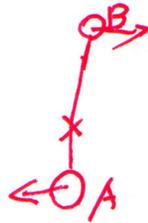
angular momentum about mass center.

$$\sum_{i=1}^2 \vec{r}_{ic} \times m_i \vec{v}_i = \vec{r}_{Ac} \times m_A \vec{v}_A + \vec{r}_{Bc} \times m_B \vec{v}_B = 0.525 \text{ kg} \cdot \text{m}^2/\text{s} (-\vec{k})$$

c) ~~$0.525 \text{ kg} \cdot \text{m}^2/\text{s} (-\vec{k})$~~

$$\begin{cases} m_A v_A + m_B v_B = 3.5 \cdot 2.5 \\ \vec{v}_{Ac} = \vec{\omega} \times \vec{r}_{Ac} \\ \vec{v}_{Bc} = \vec{\omega} \times \vec{r}_{Bc} \end{cases}$$

after rotate 180°



$$\vec{v}_{Ac} = \vec{\omega} \times \vec{r}_{Ac} = -1 \text{ m/s } \vec{i}$$

$$\vec{v}_{Bc} = \vec{\omega} \times \vec{r}_{Bc} = 2.5 \text{ m/s } \vec{i}$$

$$\vec{v}_A = \vec{v}_{Ac} + \vec{v} = -1 + 2.5 = 1.5 \text{ m/s } \vec{i}$$

$$\vec{v}_B = \vec{v}_{Bc} + \vec{v} = 2.5 + 2.5 = 5 \text{ m/s } \vec{i}$$

Test Description

Hardware Model

Date:

5/30/04

Input Frequency:

20Hz

Input Voltage:

380VAC

Serial #:

00012

Model #:

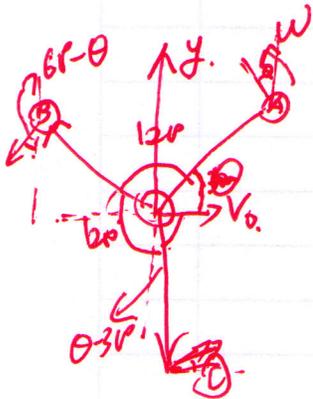
42070

Producer:

000-V.A

Sample Problem # 3.

Three small identical spheres A, B, C, which can slide on a horizontal, frictionless surface, are attached to three strings of length l which are tied to a ring G. Initially the spheres rotate about the ring, which moves along the x axis with a velocity V_0 . Suddenly the ring breaks and the three spheres move freely in the xy plane. Knowing that $V_A = 8.66 \text{ ft/s } \hat{j}$, $V_C = 15 \text{ ft/s } \hat{i}$, $a = 0.866 \text{ ft}$ and $d = 0.5 \text{ ft}$, determine. (a) the initial velocity of the ring, (b) the length l of the strings (c) the rate in rad/s at which the spheres were rotating about G.



Step 1: Particles \rightarrow 3.

Step 2: Coord. \rightarrow x, y

Step 3:

3.1. mass center: at origin.

3.2. translation of mass center.

$$\sum \vec{F}_i^e = 0 \Rightarrow a = 0$$

$$m_A \vec{V}_A + m_B \vec{V}_B + m_C \vec{V}_C = (m_A + m_B + m_C) \vec{V} = \text{constant}$$

before break: $\omega, V_0 \rightarrow$ ring velocity
 \rightarrow angular velocity

$$V_{Ax} = V_0 - \omega l \sin \theta$$

$$V_{Ay} = \omega l \cos \theta$$

$$V_{Bx} = V_0 - \omega l \sin(60^\circ - \theta)$$

$$V_{By} = -\omega l \cos(60^\circ - \theta)$$

$$V_{Cx} = V_0 + \omega l \cos(\theta - 30^\circ) = V_0 + \omega l \sin(60^\circ + \theta)$$

$$V_{Cy} = \omega l \sin(\theta - 30^\circ) = -\omega l \cos(60^\circ + \theta)$$

$$\begin{aligned} \sum V_{Ax} + V_{Bx} + V_{Cx} &= 3V_0 - \omega l \sin \theta + \omega l \sin(60^\circ + \theta) - \omega l \sin(60^\circ - \theta) \\ &= 3V_0 - \omega l \sin \theta + \omega l [2 \sin 60^\circ \cos \theta] \\ &= 3V_0 \end{aligned}$$

$$\sum V_{Ay} = \omega l \cos \theta - \omega l \cos(60^\circ - \theta) - \omega l \cos(60^\circ + \theta) = 0$$

after break:

$$m_A \vec{V}_A + m_B \vec{V}_B + m_C \vec{V}_C = m \vec{V}_C \Rightarrow \vec{V}_{Ax} = 0, \vec{V}_{Ay} = V_A$$

$$\vec{V}_{Bx} = 0, \vec{V}_{By} = -V_B$$

$$\vec{V}_{Cx} = V_C, \vec{V}_{Cy} = 0$$

$$\Rightarrow 3mV_0 = mV_C \Rightarrow V_0 = \frac{V_C}{3} = 5 \text{ ft/s}$$

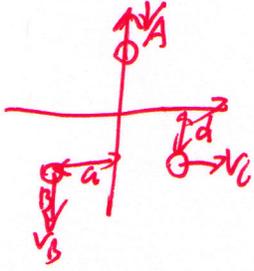
$$mV_A + mV_B = 0$$

After the strings are ^{broken} ~~broken~~, the motions of the particles are uniform motion.

So: when the strings are broken.

$$\begin{cases} V_{Ax} = V_0 - \omega L \sin \theta = 0 & \Rightarrow \omega L = 1 \text{ ft/s} \\ V_{Ay} = \omega L \cos \theta = 2.66 \text{ ft/s} & \Rightarrow \omega L = 2.66 \text{ ft/s} \end{cases}$$

$$\begin{cases} V_{Cx} = V_0 \omega L \sin(60^\circ + \theta) = 1 \text{ ft/s} & \Rightarrow \omega L = 1 \text{ ft/s} \\ V_{Cy} = -\omega L \cos(60^\circ + \theta) = 0 & \Rightarrow \theta = 30^\circ \end{cases}$$



3.3 rotation.

Because $\sum_{i=1}^N \vec{F}_i \times \vec{r}_i = \frac{d}{dt} \left[\left(\sum_{i=1}^N m_i \right) \vec{v} \times \vec{r} \right] + \frac{d}{dt} \left[\sum_{i=1}^N m_i \vec{v}_i \times \vec{r}_i \right]$

In this problem, \vec{v} parallel to \vec{r} axis $\Rightarrow \vec{v} \times \vec{r} = 0$

$$\Rightarrow \sum_{i=1}^N \vec{F}_i \times \vec{r}_i = \frac{d}{dt} \left[\sum_{i=1}^N m_i \vec{v}_i \times \vec{r}_i \right] = 0$$

$$\Rightarrow \text{before: } \sum_{i=1}^N m_i \vec{v}_i \times \vec{r}_i = \sum_{i=1}^N m_i \vec{v}_i \times \vec{r}_i = 3 \omega L^2$$

$$\text{after: } \sum_{i=1}^N m_i \vec{v}_i \times \vec{r}_i = \vec{v}_c \cdot d + V_A \cdot a + V_B \cdot b = V_c \cdot d + V_A \cdot a$$

$$\Rightarrow V_c \cdot d + V_A \cdot a = 3 \omega L^2 = 15 \cdot 0.5 + 2.66 \cdot 0.866$$

$$\omega = 2 \text{ rad/s}$$

$$L = 0.5 \text{ ft}$$

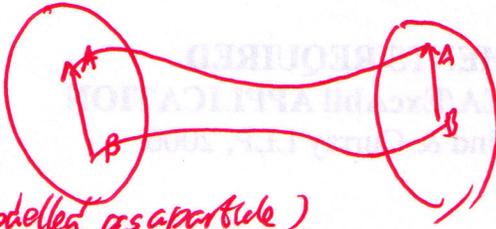
Chapter 15. Kinematics of Rigid Bodies

§1. Motion of Rigid Body.

(1) Translation:

Any straight line in the body keeps the same direction.

(The body can be modelled as a particle.)

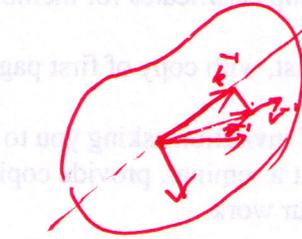


$$\vec{a}'(x, y, z) = \vec{a}$$

$$\vec{v}'(x, y, z) = \vec{v}$$

(2) Rotation about a fixed axis

all the ~~points~~ points move on a circular path centered at the axis



$$\vec{v}'(x, y, z) = \vec{\omega} \times \vec{r}'$$

$$\vec{a}'(x, y, z) = \frac{d\vec{v}'}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r}' + \vec{\omega} \times \vec{v}'$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}')$$

→ angular acceleration.

$$\vec{\omega} = \frac{d\theta}{dt} \vec{k} \Rightarrow \theta \text{ is the only variable for such a motion}$$

$$\frac{d\vec{\omega}}{dt} = \frac{d^2\theta}{dt^2} \vec{k}$$

$\theta = \text{const}$ uniform rotation
 $\frac{d\theta}{dt} = \text{const}$ uniform angular rotation

Special case: (Two dimensional condition) → slabs

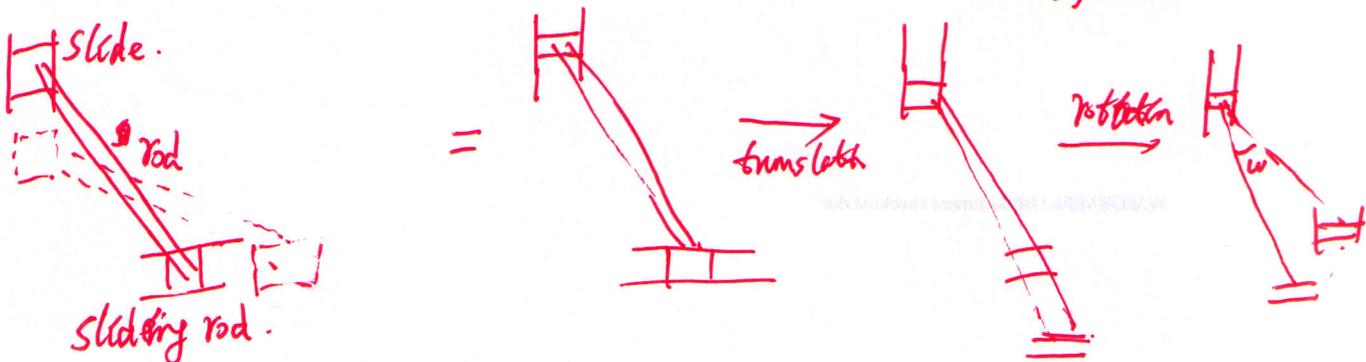
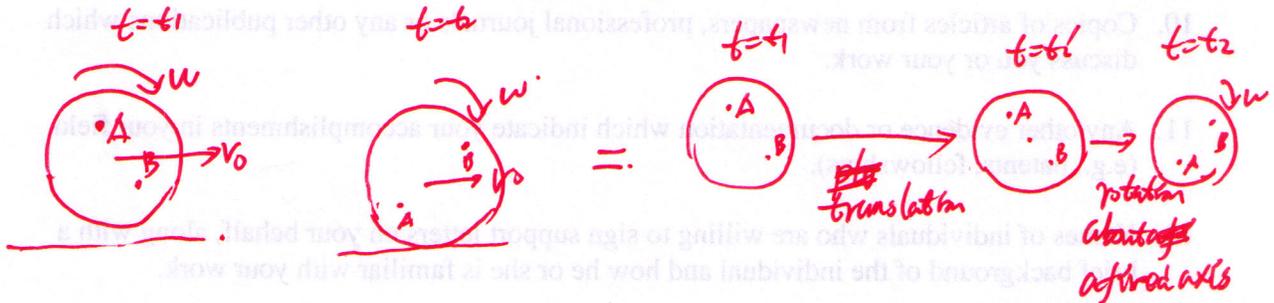
$$\vec{v}(x, y) = \vec{\omega} \times \vec{r} = r\omega \vec{\theta}$$

$$\vec{a}(x, y) = \frac{d\vec{v}}{dt} = \frac{d\omega}{dt} \cdot r \vec{\theta} + \omega^2 \cdot r (-\vec{r})$$

$$= \frac{d^2\theta}{dt^2} \cdot r \vec{\theta} + \omega^2 \cdot r (-\vec{r})$$

the acceleration in polar coord.

(3) General plane motion (motion de composition)



So General Plane motion = Translation + Rotation.

(3.1.) velocity distribution.



translation \rightarrow rotation about A

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$= \vec{v}_A + \omega \vec{k} \times \vec{r}_{B/A}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \vec{v}_B + \omega \vec{k} \times \vec{r}_{A/B}$$

$$\Rightarrow \vec{v}_B = \vec{v}_A - \omega \vec{k} \times \vec{r}_{A/B}$$

we get Since $\vec{r}_{A/B} = -\vec{r}_{B/A}$, we get

$$\boxed{\omega_A = \omega_B}$$

angular velocity on the rigid body are same anywhere.

(3.2) Instantaneous center.

The geometry point; as if all the points on the body rotate about it.



Instantaneous center

We first prove such a point exist.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \omega \vec{k} \times \vec{r}_{B/A}$$

$$\vec{v}_C = \vec{v}_A + \omega \vec{k} \times \vec{r}_{C/A}$$

$$\vec{v}_B = \vec{\omega}' \times \vec{r}_B \quad \vec{v}_A = \vec{\omega}' \times \vec{r}_A$$

$$\vec{\omega}' \times \vec{r}_B = \vec{v}_B = \vec{v}_A + \omega \vec{k} \times \vec{r}_{B/A} = \vec{\omega}' \times \vec{r}_A + \omega \vec{k} \times \vec{r}_{B/A}$$

$$\Rightarrow \vec{\omega}' \times (\vec{r}_B - \vec{r}_A) = \omega \vec{k} \times \vec{r}_{B/A}$$

$$\Rightarrow \boxed{\vec{\omega}' = \omega \vec{k}} \quad \text{--- (1)}$$

$$\vec{v}_C = \vec{v}_A + \omega \vec{k} \times \vec{r}_{C/A} = \vec{\omega}' \times \vec{r}_A + \omega \vec{k} \times \vec{r}_{C/A}$$

$$= \omega \vec{k} \times \vec{r}_A + \omega \vec{k} \times \vec{r}_{C/A}$$

$$= \omega \vec{k} \times (\vec{r}_A + \vec{r}_{C/A}) = \omega \vec{k} \times \vec{r}_C$$

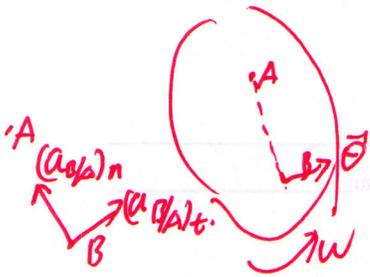
proved!

The Instantaneous center exist and the angular velocity is $\omega \vec{k}$.

~~Different for same of the body h.~~

(3.3) acceleration distribution.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$



$$\vec{a}_{B/A} = \frac{d\vec{v}_{B/A}}{dt} = \frac{d(\vec{\omega} \times \vec{r}_{B/A})}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r}_{B/A} + \vec{\omega} \times \vec{v}_{B/A}$$

$$= \frac{d\omega}{dt} (\vec{k} \times \vec{r}_{B/A}) + \omega \vec{k} \times (\omega \vec{k} \times \vec{r}_{B/A})$$

$$= \frac{d\omega}{dt} \vec{\theta} - \omega^2 \vec{r}_{B/A}$$

$$\Rightarrow \vec{a}_B = \vec{a}_A + (\underbrace{a_{B/A}}_t)_t + (\underbrace{a_{B/A}}_n)_n$$

Sample problem #1.

The motion of an oscillatory flywheel is defined by the relation $\theta = \theta_0 e^{-7\pi t/6}$ where θ is expressed in radians and t in second. knowing the $\theta_0 = 0.5$ rad.

determine: angular velocity and angular acceleration when (a) $t=0$ (b) $t=0.125$

angular velocity: $\omega = \frac{d\theta}{dt} \vec{k} = \theta_0 \left(-\frac{7\pi}{6}\right) \exp\left(-\frac{7\pi t}{6}\right) \cos 4\pi t$ 15.3

$\vec{k} \theta_0 \left(-\frac{7\pi}{6}\right) \exp\left(-\frac{7\pi t}{6}\right) \sin 4\pi t$

$\omega(0) = \cancel{\theta_0 \left(-\frac{7\pi}{6}\right)} \cdot \cancel{\theta_0} = 0.5 \cdot \left(-\frac{7\pi}{6}\right) = -4.77 \text{ rad/s}$

$\omega(0.125) = -\theta_0 4\pi \left(-\frac{7\pi}{6} - \frac{1}{8}\right) = -1.934 \text{ rad/s}$

angular acceleration: $\alpha = \frac{d^2\theta}{dt^2} \vec{k}$

$= \theta_0 \left(-\frac{7\pi}{6}\right)^2 \exp\left(-\frac{7\pi t}{6}\right) \cos 4\pi t$

$\vec{k} \theta_0 \left(-\frac{7\pi}{6}\right) 4\pi \exp\left(-\frac{7\pi t}{6}\right) \sin 4\pi t$

$- \theta_0 (4\pi) \exp\left(-\frac{7\pi t}{6}\right) \sin 4\pi t \left(-\frac{7\pi}{6}\right)$

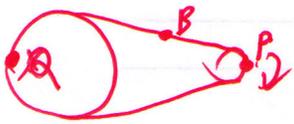
$- \theta_0 (4\pi)^2 \exp\left(-\frac{7\pi t}{6}\right) \cos 4\pi t$

$\alpha(0) = -34.5 \text{ rad/s}^2$

$\alpha(0.125) = 36.5 \text{ rad/s}^2$

Sample Problem #2. (15/17 ZSL)

The belt shown moves over two pulleys without slipping. At the instant shown, the pulleys are rotating clockwise and the speed of point B on the belt is 4 m/s increasing at the rate of 32 m/s².



Determine: (a) angular velocity and angular acceleration of each pulley
(b) the acceleration on P.

$$(a) \begin{cases} v_B = 4 \text{ m/s} & \frac{dv_B}{dt} = 32 \text{ m/s}^2 \\ v_A = \omega_A \cdot r_A & v_P = \omega_B r_B \\ v_P = v_A = v_B & (\text{same belt}) \end{cases}$$

$$\omega_A r_A = \omega_B r_B = 4 \text{ m/s}$$

$$\omega_A = \frac{4}{0.16}$$

$$\omega_B = \frac{4}{0.1}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\omega}{dt} \cdot r \cdot \hat{\theta} - \omega^2 r \cdot \hat{r}$$

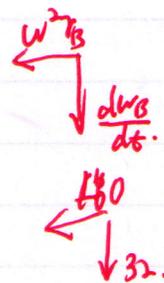
$$\text{at edge: } a_{\theta} = \frac{d\omega}{dt} \cdot r = \frac{dv_B}{dt}$$

$$\Rightarrow \frac{d\omega_A}{dt} \cdot r_A = \frac{dv_B}{dt} = \frac{d\omega_B}{dt} \cdot r_B = 32$$

$$\frac{d\omega_A}{dt} = \frac{32}{0.16}$$

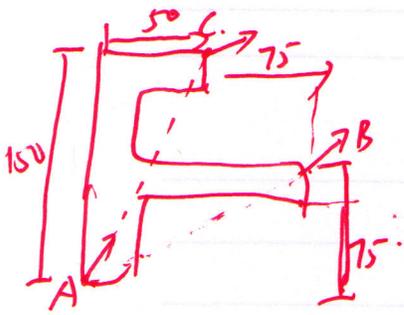
$$\frac{d\omega_B}{dt} = \frac{32}{0.1}$$

$$(b) \vec{a}_P = \frac{d\omega_B}{dt} \cdot r_B \hat{\theta} - \omega_B^2 \cdot r_B \hat{r} \\ = 32 \hat{\theta} - \frac{16}{0.1} \hat{r}$$



~~15.45~~ Sample Problem #3 (15.45 7th)

The sheet metal form shown moves in the x,y plane. knowly that $V_{Ax} = 100 \text{ mm/s}$
 $V_{By} = -75 \text{ mm/s}$ $V_{Cx} = 40 \text{ mm/s}$. determine (a) angular velocity (b) velocity at A



$$\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{V}_C = \vec{V}_A + \vec{\omega} \times \vec{r}_{C/A}$$

$$V_{Cx} = V_{Ax} - \omega r_{CA} \sin \theta$$

$$= 100 - \omega \cdot 150 \cdot \frac{1}{\sqrt{2}}$$

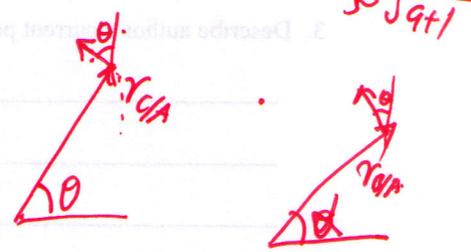
$$= 100 - \omega \cdot 106.066$$

$$\Rightarrow 40 = 100 - \omega \cdot 106.066 \Rightarrow \omega = -0.943 \text{ rad/s}$$

$$V_{By} = V_{Ay} + \omega r_{BA} \cos \theta = V_{Ay} + \omega \cdot 125$$

$$\Rightarrow V_{By} = -75 + 0.943 \cdot 125 = 17.875 \text{ mm/s}$$

$$\vec{V}_A = (100 \text{ mm/s}) \hat{i} + (17.875 \text{ mm/s}) \hat{j}$$



Sample problem #4

(15.93)

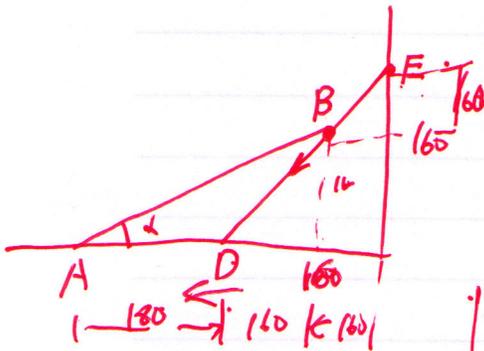
$V_D = 800 \text{ mm/s}$ ←

(a) determine angular velocity of each rod.

(b) the velocity of point A.

Solution

Instantaneous center of DE.



$$V_D = \omega_{DE} \cdot 320 \text{ mm}$$

$$\omega_{DE} = \frac{800}{320} = \frac{10}{4} = 2.5 \text{ rad/s}$$

$$V_B = \omega_{DE} \cdot 160\sqrt{2} = 2.5 \cdot 160 \cdot 1.414 = 565.6 \text{ mm/s}$$

$$\frac{V_B}{\sin(90^\circ - \alpha)} = \frac{x}{\sin 45^\circ}$$

$$x = (160 + 180) / \cos \alpha$$

$$r_B = \frac{x \cos \alpha}{\sin 45^\circ} = \frac{160 + 180}{\sqrt{2}}$$

$$\omega_{AB} \cdot r_B = V_B$$

$$\omega_{AB} = \frac{\omega_{DE} \cdot 160\sqrt{2}}{(160 + 180)}$$

$$= \frac{2.5 \cdot 160}{160 + 180} = 1.176 \text{ rad/s}$$

