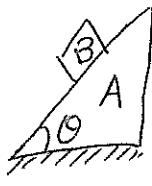


Sample Problem #1: Lecture notes on Jul. 13rd, 2006

Block B is placed on a wedge. The wedge A is fixed and the friction coefficient between B and A is μ .



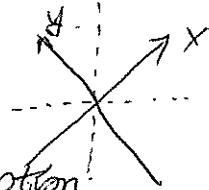
B will slide down when θ exceeds a critical value. Please find this critical value.

Step 1: How many particles?

2: A and B

Step 2: Coordinate.

rectangular.



Step 3: Determine of motion

3.1 Kinematics

$$a_{Ay} = 0 \quad \dots \quad (1)$$

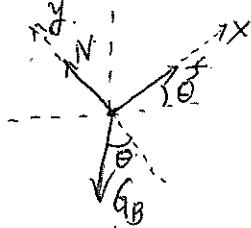
$$a_{Ax} = 0 \quad \dots \quad (2)$$

$$\begin{cases} \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = \vec{a}_B \\ (\vec{a}_{B/A})_y = 0 \end{cases} \Rightarrow a_{By} = 0 \quad \dots \quad (3)$$

3.2 force Analysis

$$\text{For B: } f - G_B \sin \theta = \sum F_x \quad \dots \quad (4)$$

$$N - G_B \cos \theta = \sum F_y \quad \dots \quad (5)$$



3.3 Dynamic Eq.

$$\sum F_x = m_B a_{Bx} = f - G_B \sin \theta \quad \dots \quad (6)$$

$$\sum F_y = m_B a_{By} = N - G_B \cos \theta \quad \dots \quad (7)$$

By (3) and (7), we get.

$$N = G_B \cos \theta \quad \dots \quad (8)$$

By (8), we have

$$a_{Bx} = \frac{f - G_B \sin \theta}{m_B} \quad \dots \quad (9)$$

Step 4: Go back to the question and provide solution.

When B moves, we have

$$a_{Bx} \leq 0 \quad \text{and} \quad f = \mu N$$

By equation ⑨

$$\frac{f - G_B \sin \theta}{m_B} \leq 0$$

$$\Rightarrow f \leq G_B \sin \theta$$

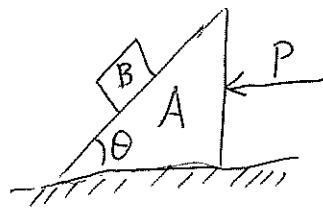
$$\Rightarrow \mu N \leq G_B \sin \theta$$

$$\Rightarrow \cancel{\mu} \leq \tan \theta.$$

The critical value for θ is $\theta^c = \arctan(\mu)$.

When $\theta > \theta^c$, B slips down automatically!

Sample problem #2.:



The wedge is placed on a frictionless plane, and a block is placed on it. The friction coefficient between B and A is μ . A is subjected to a horizontal force P .

The friction coefficient μ follows $\mu < \tan \theta$. So the block B will slip down if P is zero.

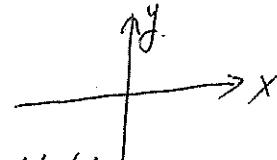
Please find (a) the range for P when B moves with A together
 (b) calculate the accelerations of B and A when B elevates relative to A.

Step 1: How many particles?

Two: A and B

Step 2: Coordinate

rectangular.



Step 3: Determination of Motion

3.1 Kinematics

$$a_{Ay} = 0 \quad \text{--- (1)}$$

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$ should be parallel to the inclined surface.

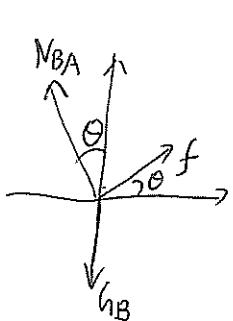
$$\frac{(\vec{a}_{B/A})_y}{(\vec{a}_{B/A})_x} = \tan \theta \Rightarrow \frac{a_{By} - a_{Ax}}{a_{Bx} - a_{Ax}} = \tan \theta.$$

By (1), we get

$$a_{By} = \tan \theta (a_{Bx} - a_{Ax}) \quad \text{--- (2)}$$

3.2 Force analysis

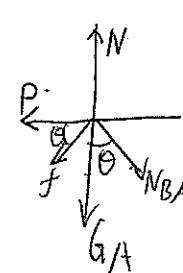
For B.



$$\sum F_{Bx} = f \cos \theta - N_B \sin \theta \quad \text{--- (3)}$$

$$\sum F_{By} = N_B \cos \theta + f \sin \theta - G_B \quad \text{--- (4)}$$

For A.



$$\sum F_{Ax} = N_B \sin \theta - P - f \cos \theta \quad \text{--- (5)}$$

$$\sum F_{Ay} = N - f \sin \theta - N_B \cos \theta - G_A \quad \text{--- (6)}$$

3.3 Dynamic Equations.

$$\sum F_{Bx} = m_B a_{Bx} = f \cos \theta - N_B S \sin \theta \quad \dots \dots \quad (7)$$

$$\sum F_{By} = m_B a_{By} = N_B A \cos \theta + f S \sin \theta - G_B \quad \dots \dots \quad (8)$$

$$\sum F_{Ax} = m_A a_{Ax} = N_B S \sin \theta - P - f \cos \theta \quad \dots \dots \quad (9)$$

$$\sum F_{Ay} = m_A a_{Ay} = N - f S \sin \theta - N_B A \cos \theta - G_A \quad \dots \dots \quad (10)$$

together with

$$a_{Ay} = 0 \quad \dots \dots \quad (11)$$

$$a_{By} = f \sin \theta (a_{Bx} - a_{Ax}) \quad \dots \dots \quad (12)$$

7 unknowns. but 6 equations.

The rest is. $\left\{ \begin{array}{l} (7)(8) \text{ A moves with B : } a_{Bx} = a_{Ax} \text{ and } a_{By} = a_{Ay} \dots \dots (11) \\ (6) \text{ B moves relative to A : } f = \pm \mu N_{BA} \end{array} \right.$

(Sign is determined by the moving direction)
 "+" B moves downwards
 "-" B moves upwards.

Step 4: provide solution.

(a) B moves with A together.

$$\text{By (11) we have. } \left\{ \begin{array}{l} a_{By} = a_{Ay} \\ a_{Bx} = a_{Ax} \\ |f| \leq \mu N_{BA} \end{array} \right.$$

By Eq. (7) and (9)

$$m_A a_{Ax} + m_B a_{Bx} = -P \Rightarrow a_{Ax} = a_{Bx} = \frac{-P}{m_A + m_B} \quad \dots \dots \quad (13)$$

By Eq (8) and $a_{By} = a_{Ay} = 0$

$$N_B A \cos \theta + f S \sin \theta - G_B = 0 \quad \dots \dots \quad (14)$$

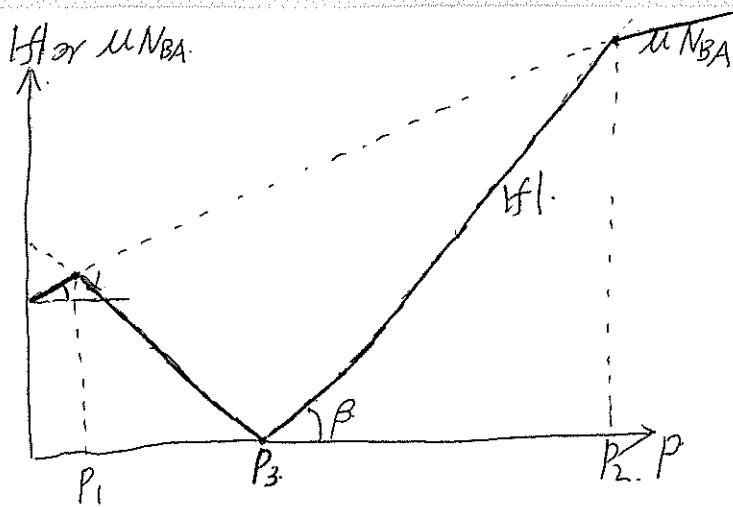
By Eq (13) and (7)

$$f \cos \theta - N_B S \sin \theta = -P \frac{m_B}{m_A + m_B} \quad \dots \dots \quad (15)$$

By (14) and (15), we have

$$f = G_B S \sin \theta - \frac{m_B}{m_A + m_B} P \cos \theta$$

$$N_{AB} = G_B C \cos \theta + \frac{m_B}{m_A + m_B} P S \sin \theta \quad \dots \dots \quad (16)$$



Case 1

$$\text{By Eq. (B)} \quad \begin{cases} \operatorname{tg} \alpha = \mu \frac{m_B}{m_A + m_B} \operatorname{tg} \theta \\ \operatorname{tg} \beta = \frac{m_B}{m_A + m_B} \operatorname{tg} \theta \end{cases}$$

$$\text{So when } \operatorname{tg} \alpha < \operatorname{tg} \beta \Rightarrow \mu \operatorname{tg} \theta < \operatorname{tg} \theta \Rightarrow \mu < 1$$

{ μ_{NBA} curve will cross with f_f curve at P_2 . — key ①

Because $\mu < \operatorname{tg} \theta$, The two curves cross at P_1 too. — key ②.

Because, μ_{NBA} is the upper bound of friction forces.

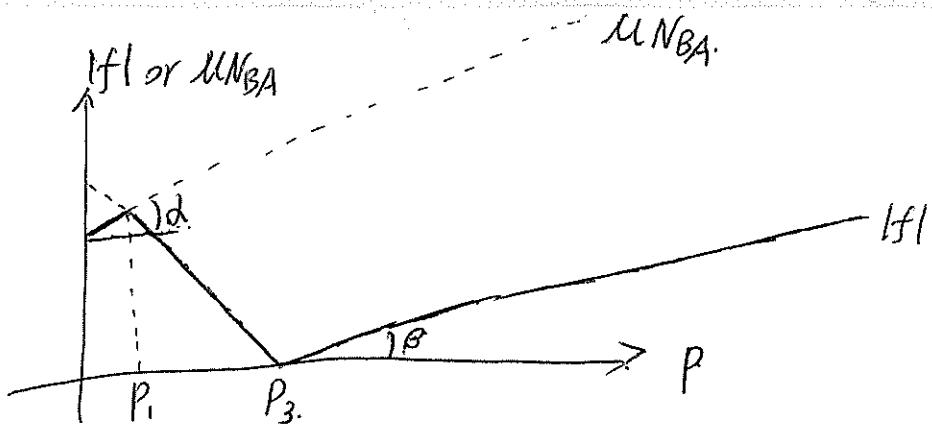
The real friction forces ~~can~~ can be illustrated by the ~~solid~~ curve with solid line. — key ③.

When $P < P_1$; B slips down ~~and~~ ^{by gravity} friction can not hold it.

When $P_1 < P < P_3$; B intends to slip down but friction holds it.

When $P_3 < P < P_2$; B intends to move up but friction drags it back.

When $P > P_2$; B moves up.



Case 2.

$$\text{By Eq. ⑯ } \mu \tan \alpha = \mu \frac{m_B}{m_A + m_B} \sin \theta$$

$$\tan \beta = \frac{m_B}{m_A + m_B} \cos \theta$$

So; when $\tan \alpha > \tan \beta \Rightarrow \mu > \tan \theta$.

μ_{NBA} curve will not cross with $|f|$ curve when $P > P_3$. --- key ①

Because $\mu < \tan \theta$, the two curves cross at P_1 . --- key ②

Because, μ_{NBA} is the upper bound of friction forces.

The real friction forces can be illustrated by the ~~solid~~ curve with solid line.

when $P < P_1$; B slips down by gravity and friction can not hold it.

when $P < P \cancel{<} P_3$; B intends to slip down but friction holds it.

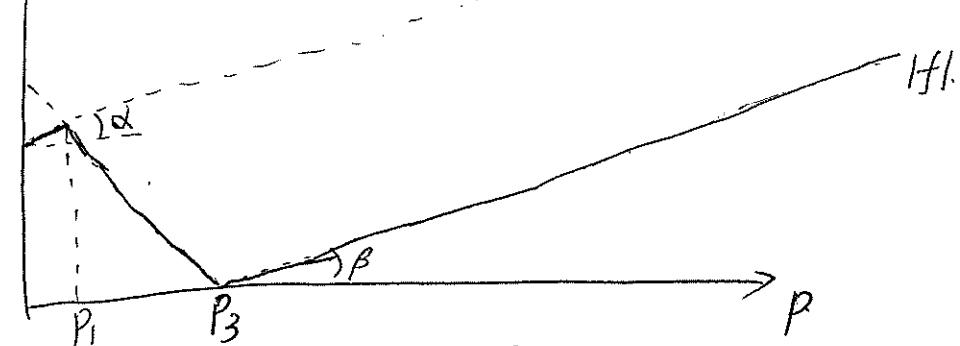
when $P > P_3$; B intends to move up but friction draws it back.

No matter how high P is, B will never move.

μ_{f1} or μ_{NBA}

μ_{NBA}

Case 3.



By Eq. ⑥ $f_g d = \mu \frac{m_B}{m_A + m_B} \sin \theta$.

$$f_g \beta = \frac{m_B}{m_A + m_B} \cos \theta.$$

So when $f_g d = f_g \beta \Rightarrow \mu = \cot \theta$.

{ μ_{NBA} curve will not cross with f_1 curve when $P > P_3$ (they are parallel)

Because $\mu < \cot \theta$, the two curves cross at P_1 . key ①

Because, μ_{NBA} is the upper bound of friction forces. key ②

The ideal friction force can be illustrated by the solid line.

- * { when $P < P_1$; B slips down and friction can not hold it.
- when $P_1 < P < P_3$; B intends to slip down but friction ~~does not hold it~~ holds it
- when $P > P_3$; B intends to move up but friction draws it back

no matter how high P_C , B will never move.

Now we solve P_1 .

at P_1 ; $f = \mu N_{BA}$.

By ⑥, we have.

$$G_B S_m \theta - \frac{m_B}{m_A + m_B} P_1 C_s \theta = \mu \cdot G_B C_s \theta + \mu \frac{m_B}{m_A + m_B} P_1 S_m \theta$$

$$P_1 = \frac{G_B S_m \theta - \mu G_B C_s \theta}{C_s \theta + \cancel{\mu} S_m \theta} - \frac{m_A + m_B}{m_B} \quad \dots \quad ⑦$$

Now we solve P_3 .

at P_3 ; $f = 0$

By ⑥, we have.

$$G_B S_m \theta - \frac{m_B}{m_A + m_B} P_3 C_s \theta = 0$$

$$P_3 = \frac{G_B S_m \theta}{C_s \theta} - \frac{m_A + m_B}{m_B} \quad \dots \quad ⑧$$

Now, we solve P_2 .

at P_2 $f = -\mu N_{BA}$

$$-G_B S_m \theta + \frac{m_B}{m_A + m_B} P_2 C_s \theta = \mu G_B C_s \theta + \mu \frac{m_B}{m_A + m_B} P_2 S_m \theta$$

$$P_2 = \frac{-G_B S_m \theta - \mu G_B C_s \theta}{\mu S_m \theta - C_s \theta} - \frac{m_A + m_B}{m_B} \quad \dots \quad ⑨$$

(b) when B rotates relative to A, calculate the accelerations.

By (1) we have $\mu = -MN_{BA}$... (1)

By (1) and (2)

$$m_A a_{Ax} + m_B a_{Bx} = -P \quad \dots \dots \quad (2)$$

By (1) and (2).

$$m_B a_{Bx} = -MN_{BA} \cdot N_{BA} \sin\theta \quad \dots \dots \quad (2)$$

By (2)

$$m_B a_{By} = N_{BA} \frac{\cos\theta}{\sin\theta} - MN_{BA} \sin\theta - G_B \quad \dots \dots \quad (2)$$

~~(2)~~ $\frac{(2)}{(2)}$ yields.

$$\frac{m_B a_{By} + G_B}{m_B a_{Bx}} = \frac{\cos\theta - \mu \sin\theta}{-\mu \cos\theta - \sin\theta} = \frac{\mu \sin\theta - \cos\theta}{\sin\theta + \mu \cos\theta} \quad \dots \dots$$

$$\Rightarrow m_B a_{By} + G_B = \frac{\mu \sin\theta - \cos\theta}{\sin\theta + \mu \cos\theta} m_B a_{Bx} \quad \dots \dots \quad (2)$$

By (2), (2), and (2), we have.

$$a_{By} = \tan\theta (a_{Bx} - a_{Ax}) = \tan\theta (a_{Bx} - (-P/m_B a_{Bx})) \frac{1}{m_A}$$

$$= \tan\theta \left(1 + \frac{m_B}{m_A}\right) a_{Bx} + \tan\theta P \cdot \frac{1}{m_A}$$

$$= \frac{P}{m_A} \tan\theta + \tan\theta \left(1 + \frac{m_B}{m_A}\right) \frac{m_B a_{Bx} + G_B}{m_B} \frac{\sin\theta + \mu \cos\theta}{\mu \sin\theta - \cos\theta}$$

$$\text{So: } a_{By} = \frac{\frac{P}{m_A} \tan\theta + \tan\theta \left(\frac{\sin\theta + \mu \cos\theta}{\mu \sin\theta - \cos\theta}\right) \left(1 + \frac{m_B}{m_A}\right) \frac{G_B}{m_B}}{1 - \tan\theta \left(1 + \frac{m_B}{m_A}\right) \frac{\sin\theta + \mu \cos\theta}{\mu \sin\theta - \cos\theta}} \quad \dots \dots \quad (2)$$

By (2) and (2)

a_{Ax} and a_{Bx} can be obtained.