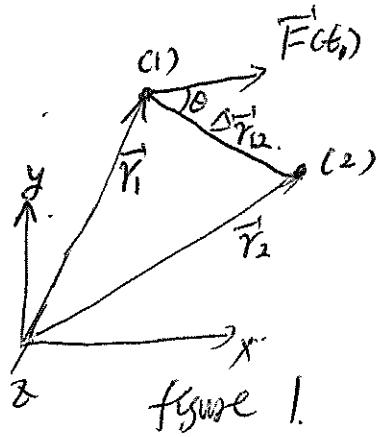


§13.1 work.

The work of the force \vec{F} is defined by the scalar product of \vec{F} and the displacement vector $\Delta \vec{r}_{12}$, as shown in figure 1.



at state 1. position vector \vec{r}_1

force \vec{F}_1

at state 2. position vector \vec{r}_2

force \vec{F}_2

$$W_{1 \rightarrow 2} = \vec{F}_1 \cdot (\vec{r}_2 - \vec{r}_1) = \vec{F}_1 \cdot \Delta \vec{r}_{12}$$

$$= |\vec{F}_1| \cos \theta \cdot |\Delta \vec{r}_{12}|$$

the projection of \vec{F} on $\Delta \vec{r}_{12}$.

* work equals to (the projection of external forces on the displacement vector.)
X(displacement)

§13.2 calculation of work in rectangular Coord / Polar. Coord.

$$\text{In Coord. } \vec{F} = F_x \vec{x} + F_y \vec{y} + F_z \vec{z}$$

$$\Delta \vec{r}_{12} = \Delta x \vec{x} + \Delta y \vec{y} + \Delta z \vec{z}$$

$$\Rightarrow W_{12} = \vec{F} \cdot \Delta \vec{r}_{12} = F_x \Delta x + F_y \Delta y + F_z \Delta z. \quad \text{--- ---} \quad \textcircled{1}$$

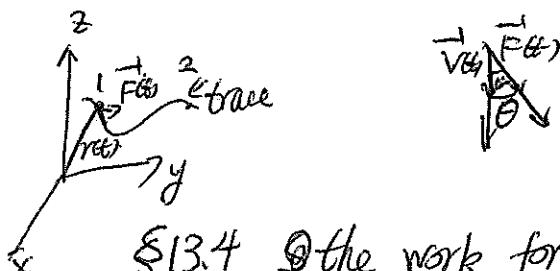
$$\text{In. Polar. Coord. } \vec{F} = F_r \vec{e}_r + F_\theta \vec{e}_\theta$$

$$\vec{r} = r \vec{e}_r \Rightarrow d\vec{r}_{12} = r_1 d\theta \vec{e}_\theta + dr \vec{e}_r.$$

$$\Rightarrow W = F_\theta r_1 d\theta + F_r dr. \quad \text{--- ---} \quad \textcircled{2}$$

§13.3 the work done by arbitrary forces and arbitrary traces.

$$\begin{aligned} U_{12} &= \int_1^2 \vec{F}(t) \cdot d\vec{r} \\ &= \int_1^2 \vec{F}(t) \cdot \sqrt{dt} \quad \xrightarrow{\text{velocity vector}} \\ &= \int_1^2 \vec{F}(t) \vec{v}(t) \cdot dt \\ &= \int_1^2 F(t) v(t) \cos \theta \cdot dt. \end{aligned}$$



§13.4 @ the work for some special cases.

1. Constant force and rectilinear motion.

$$U_{12} = \int_1^2 \underbrace{F(t)}_{\substack{\text{constant}}} \underbrace{v(t) \cos \theta}_{\substack{\text{constant}}} dt = F \cos \theta \int_1^2 v(t) dt = F \cos \theta (x_2 - x_1)$$

2. the force for gravity (figure 2)

$$\begin{aligned} U_{12} &= \int_1^2 \underbrace{F(t)}_{\substack{\text{constant} \\ \rightarrow mg}} \underbrace{v(t) \cos \theta}_{\substack{\text{constant}}} dt = -mg \int_1^2 v(t) \cos \theta dt \\ &= -mg \int_1^2 v_y dt \\ &= -mg (y_2 - y_1) \end{aligned}$$

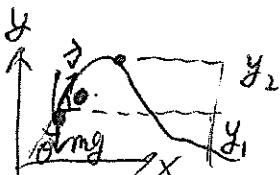
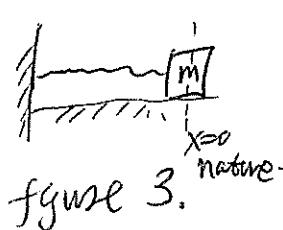
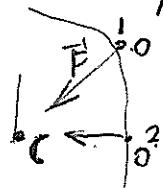


Figure 2. 3. the force ~~for~~ exerted by a spring (figure 3)



$$\begin{aligned} U_{12} &= \int_1^2 \underbrace{F(t) v(t) \cos \theta}_{\substack{\text{constant} \\ \rightarrow F = -kx \\ \rightarrow v(t) = \frac{dx}{dt}}} dt \\ &= \int_1^2 -kx \frac{dx}{dt} dt \\ &= -\frac{1}{2} kx^2 = \frac{1}{2} kx_1^2 - \frac{1}{2} kx_2^2 \end{aligned}$$

4. Gravitational force.



$$F = G \frac{Mm}{r^2} \vec{e}_r \quad (\text{polar coord, origin at 'C'})$$

by (2).

$$U_{12} = \int_1^2 (F_r r d\theta + F_\theta dr)$$

$$= \int_1^2 -G \frac{Mm}{r^2} dr = +G \frac{Mm}{r_2} - G \frac{Mm}{r_1}$$

§ 13.5 Principle of Work and Energy

By Newton's 2nd Law

$$\begin{aligned} \vec{F} &= \cancel{\frac{d\vec{m}\vec{v}}{dt}} \\ \cancel{\frac{d\vec{r}}{dt}} \quad U_{12} &= \int_1^2 \vec{F} \cdot \vec{r} dt \end{aligned} \quad \Rightarrow \quad \left. \begin{aligned} U_{12} &= \int_1^2 \vec{F} \cdot \vec{V} \cdot dt \\ &= \int_1^2 \frac{d\vec{m}\vec{v}}{dt} \cdot \vec{V} \cdot dt \\ &= \int_1^2 \frac{d}{dt} \left(\frac{1}{2} \vec{m}(\vec{v} \cdot \vec{v}) \right) dt \\ &= \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 \end{aligned} \right\}$$

$\frac{1}{2}mv^2$ is defined as kinetic energy! ③

③ means the work done by ~~the~~ resultant force equals to the change of kinetic energy. !!

If we denote the work done by gravity as $U_{G1 \rightarrow 2} = -(m g y_2 - m g y_1)$ and we denote the work done by spring as $U_{S1 \rightarrow 2} = \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2$

We get

$$U_{12}^e - mg y_2 + mgy_1 + \frac{1}{2} k X_1^2 - \frac{1}{2} k X_2^2 = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$$

$$\Rightarrow U_{12}^e = (\underbrace{\frac{1}{2} m V_2^2 + mgy_2 + \frac{1}{2} k X_2^2}_{\substack{\downarrow \\ \text{extension of the spring}}}) - (\underbrace{\frac{1}{2} m V_1^2 + mgy_1 + \frac{1}{2} k X_1^2}_{\substack{\downarrow \\ \text{height of the particle}}})$$

(4)

define : $E = \frac{1}{2} m v^2 + mgh + \frac{1}{2} k X$ (constant energy)

(4) means the work done by external forces (exclude gravity and spring)
 equals to the energy change $E_2 - E_1$. ~~✓~~

§ 13.6 power and Efficiency

$$\text{power} = \vec{F} \cdot \vec{V}$$

efficiency \longrightarrow for a mechanical system

$$\text{output energy } P_o$$

$$y = \frac{P_o}{P_i}$$