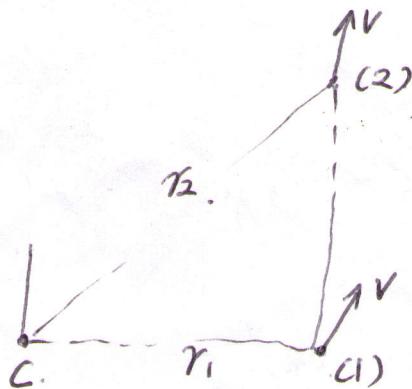


§ 13.7. motion under Gravitation force.



external forces =

$$\underline{\text{force}} \quad \underline{\text{resultant}} = -G \frac{Mm}{r^2} \hat{e}_r$$

(polar co-ord.

origin at C?)

$$U_{1 \rightarrow 2} = \frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2 \quad \leftarrow \text{principle of work and energy.}$$

$$U_{1 \rightarrow 2} = \int_{r_1}^{r_2} -G \frac{Mm}{r^2} \cdot r dr.$$

$$= G \frac{Mm}{r_2} - G \frac{Mm}{r_1}$$

(2)

By (1) and (2)

$$G \frac{Mm}{r_2} - G \frac{Mm}{r_1} = \frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2$$

$$\frac{1}{2} m V_1^2 - G \frac{Mm}{r_1} = \frac{1}{2} m V_2^2 - G \frac{Mm}{r_2}$$

determined by state (1)

determined by state (2)

So, for such a motion.

$$\frac{1}{2} m V^2 - G \frac{Mm}{r} = \text{const.}$$

Now, we calculate the minimum speed for a body to escape from earth.

when escape $\frac{GM}{r} = 0$ and $\frac{1}{2} m V^2 = 0$

\therefore escape

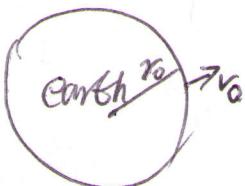
$$\Rightarrow \frac{1}{2} m V_0^2 - G \frac{Mm}{r_0} = 0 + 0$$

$$\Rightarrow \frac{1}{2} m V_0^2 = G \frac{Mm}{r_0}$$

$$V_0 = \sqrt{2G \frac{M}{r_0}} = \sqrt{2 \cdot g \cdot r_0} = \underline{\underline{11.14 \text{ km/s}}} \quad \downarrow \quad \downarrow$$

$$9.8 \text{ m/s}^2 \quad 6340 \text{ km}$$

Second
Cosmic
Velocity.



§ 13.8. principle of impulse and moment.

By Newton's 2nd Law

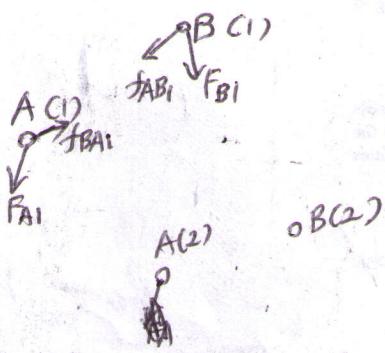
$$F = \frac{d}{dt}(m\vec{v})$$

$$\Rightarrow \vec{F} \cdot dt = \cancel{m\vec{v}} \cdot d(m\vec{v})$$

$$\Rightarrow \underbrace{\int_{t_1}^{t_2} \vec{F} \cdot dt}_{\text{impulse}} = \underbrace{\int_{t_1}^{t_2} d(m\vec{v})}_{\text{momentum}} = (\underline{m\vec{v}})_2 - (\underline{m\vec{v}})_1 \quad (8)$$

The impulse of resultant force equals to the momentum change.

§ 13.9. particle system pairs



For A:

$$\Rightarrow \int_{t_1}^{t_2} (\vec{F}_A + \vec{f}_{BA}) dt = (m_A \vec{v}_A)_2 - (m_A \vec{v}_A)_1,$$

$$\Rightarrow \int_{t_1}^{t_2} \vec{F}_A dt + \int_{t_1}^{t_2} \vec{f}_{BA} dt = (m_A \vec{v}_A)_2 - (m_A \vec{v}_A)_1, \quad (9)$$

For B:

$$\int_{t_1}^{t_2} (\vec{F}_B + \vec{f}_{AB}) dt = (m_B \vec{v}_B)_2 - (m_B \vec{v}_B)_1,$$

$$\Rightarrow \int_{t_1}^{t_2} \vec{F}_B dt + \int_{t_1}^{t_2} \vec{f}_{AB} dt = (m_B \vec{v}_B)_2 - (m_B \vec{v}_B)_1, \quad (10)$$

$$(9) + (10) \text{ and } \vec{f}_{AB} = -\vec{f}_{BA}$$

$$\Rightarrow \int_{t_1}^{t_2} \vec{F}_A dt + \int_{t_1}^{t_2} \vec{F}_B dt = [m_A \vec{v}_A + m_B \vec{v}_B]_2 - [m_A \vec{v}_A + m_B \vec{v}_B]_1,$$

$m_A \vec{v}_A + m_B \vec{v}_B$ is called as the momentum of the system. (6)

§ 13.10. particle systems.

Similarly, we have

$$\sum_{k=1}^N \int_{t_1}^{t_2} \vec{F}_k dt = (\sum_{k=1}^N m_k \vec{v}_k)_2 - (\sum_{k=1}^N m_k \vec{v}_k)_1,$$

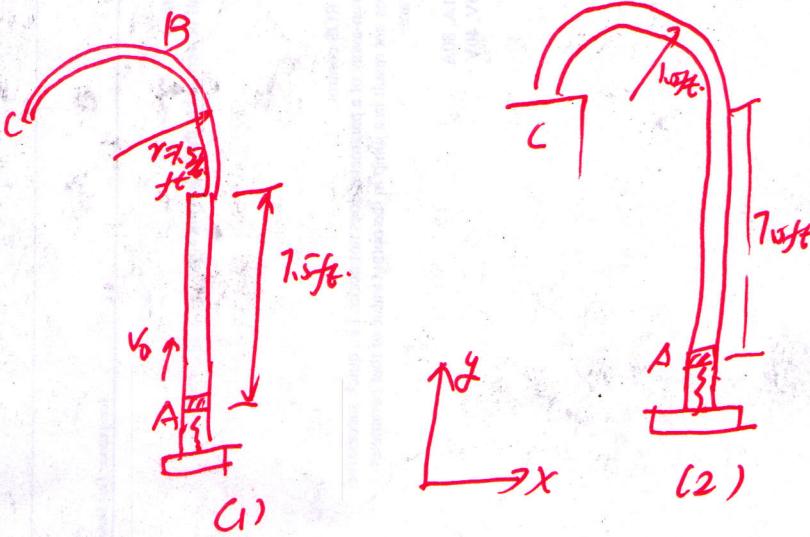
external forces to particle k
(Not resultant force!)

momentum of the system

Sample Problem #1

H.W. Project.

13. 27.5 An 8 oz package is projected upward with a velocity v_0 by a spring at A. It moves around a frictionless loop and is deposited at C. For each of the two loops shown, determine (a) the smallest velocity v_0 for which the package will reach C. (b) the corresponding force exerted by the package on the loop just before the package leaves the loop C.



Step 1: one particle.

Step 2: coordinate, rectangular.

Step 3: st.

3.1 states:

$$\text{Starting: } A \left\{ \begin{array}{l} y=0 \\ V=v_0 \text{ (unknown)} \end{array} \right.$$

$$\text{ending: } C \left\{ \begin{array}{l} y=7.5 \text{ ft} \\ V=V \text{ (unknown)} \end{array} \right.$$

~~intermediate:~~

$$\text{interim state: } B \left\{ \begin{array}{l} y=8.5 \\ V = \begin{cases} (1) \sqrt{gr} \\ (2) 0 \end{cases} \end{array} \right.$$

but
 (1) package reaches C.
 at B the normal force should be zero
 $\Rightarrow F_N = m v_B^2 / r$ the minimum resultant force is G_B it should be balanced by centrifugal forces
 $m v_B^2 / r = m g$
 (2) package reaches C. $V_B^2 = gr$.
 at B, the minimum resultant force can be zero
 $\Rightarrow V_{B \min} = 0$.

3.2 energy.

A energy } kinetic $\frac{1}{2}mv^2$

potential. } gravity mgh

$$E_A = mgh + \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2$$

$$E_B = ngh + \frac{1}{2}mv^2 = \begin{cases} mg \cdot 8.5 + \frac{1}{2}mvr \\ mg \cdot 8.5 + 0 \end{cases}$$

$$E_C = mgh + \frac{1}{2}mv^2 = mg \cdot 7.5 + \frac{1}{2}mv^2$$

3.3 work

$$W=0 \quad (\text{no other forces})$$

3.4 G.E.

$$E_A = E_B = E_C$$

$$\Rightarrow (1) mg \cdot 8.5 + \frac{1}{2}mg \cdot 1.5 = mg \cdot 7.5 + \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{3.5mg}{m}$$

$$(2) mg \cdot 8.5 + 0 = mg \cdot 7.5 + \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2mg}{m}$$

Step 4: Solution.

① minimum velocity:

$$v_c = \sqrt{3.5g} \quad v_c = \sqrt{2g}$$

② at C



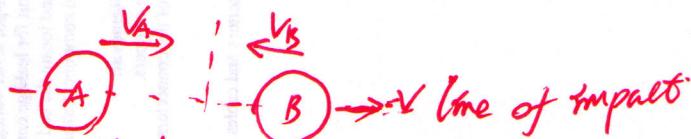
$$N = \frac{mv^2}{r}$$

$$\begin{cases} (1) N = \frac{3.5g \cdot m}{r} = \frac{7}{3}mg \\ (2) N = \frac{m \cdot 2g}{r} = \frac{4}{3}mg \end{cases}$$

§ 13. Impact.

Concept: ① Line of impact.

The line normal to the contacting surface



② Direct central impact.

Velocities are parallel to the line of impact

③ Oblique central impact

Velocities are not in the line of impact!

§ 13.

~~the procedure of collision. Direct central impact?~~ Velocity is equal.

~~contracted.~~
A touch B.

A, B are
deformed.

maximum
deformation
occurs

restitution.

v_A' v_B'
separate.

§ 13

(A) mathematical description for direct central impact

(B) compose a particle system.

 $t_1 \rightarrow t_5$ (there is no external forces)

⇒ momentum conservation.

$$\Rightarrow m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}_A' + m_B \vec{v}_B'$$

(1 eq. 2 unknowns)

(1)

A new terminology: coefficient of restitution

$$e = \frac{\text{Impulse during restitution} \rightarrow t_3 - t_2}{\text{Impulse during deformation.}}$$

 $\rightarrow t_1 \rightarrow t_1$

$$= \frac{m_A v_A' - m_A u_A}{m_A u_A - m_A v_A} = \frac{v_A' - u_A}{u_A - v_A} \dots \quad (2)$$

$$\underline{\text{Newton 3rd.}} \quad \frac{m_B v_B' - m_B u_B}{m_B u_B - m_B v_B} = \frac{v_B' - u_B}{u_B - v_B} \dots \quad (3)$$

$$\therefore u_A = u_B$$

$$\Rightarrow e = \frac{v_B' - v_A'}{v_A - v_B} \dots$$

(4) the ratio of relative velocities before and after

▷ collision

Two special cases.

① $e = 0 \rightarrow$ Perfectly Plastic Impact.

$$V_B' = V_A' \Rightarrow V_B' = V_A' = \frac{m_A}{m_A + m_B} V_A + \frac{m_B}{m_A + m_B} V_B$$

② $e = 1 \rightarrow$ Perfectly elastic Impact.

$$\left\{ \begin{array}{l} V_B' - V_A' = V_A - V_B \\ m_A V_A + m_B V_B = m_A V_A' + m_B V_B' \end{array} \right.$$

$$\Rightarrow \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = \frac{1}{2} m_A (V_A')^2 + \frac{1}{2} m_B (V_B')^2$$

no energy lost during collision,

§13. OBLIQUE central impact.

neglect friction.

so we get

$$\left\{ \begin{array}{l} \text{along line of impact: } m_A V_{At} + m_B V_{Bn} = m_A V_{Atn}' + m_B V_{Bn}' \\ \text{e. } e = \frac{V_{Atn}' - V_{At}}{V_{Bn}' - V_{Bn}} \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{normal to the line of impact: } V_{At} = V_{Atn}' \\ V_{Bn} = V_{Bn}' \end{array} \right.$$

Summary on the content

Impact.

direct central impact

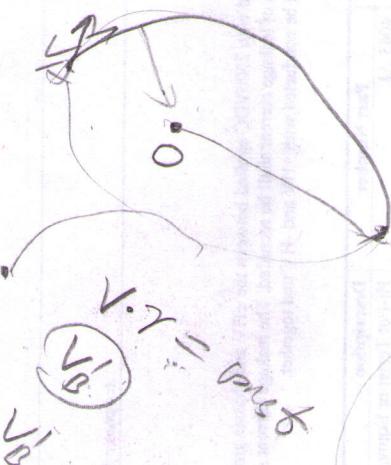
oblique central impact.

$$\left\{ \begin{array}{l} \text{momentum conservation: } m_A V_A' + m_B V_B' = m_A V_A + m_B V_B \\ \text{coefficient of restitution: ratio of relative velocity} \end{array} \right.$$

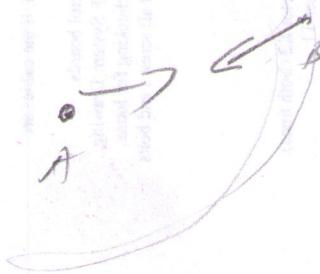
e. $\frac{V_{Atn}' - V_{At}}{V_{Bn}' - V_{Bn}}$. impulse.

along line of impact: direct central impact

normal to the line of impact: velocity conservation.



$$F = G \frac{m_A m_B}{r^2}$$



$$m_A v_A = m_A \frac{V^2}{2}$$

$$G \frac{m_A m_B}{r^2} = m_A \frac{V^2}{2}$$

~~Step~~-by-step procedure for using the principle of impulse and momentum.

Step 1: How many particles and what is the particle system

Step 2: Coordinate. ~~frame~~ (Newton frame).

Step 3:

3.1 states and category of collision

3.2. momentum at each state.

3.3 Impulse of ~~force~~ resultant force

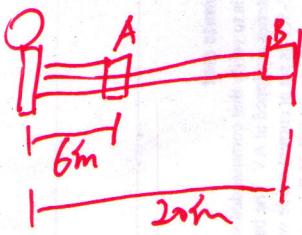
3.4 G.E. (Eqn. 32 and 33)

Step 4: Solution.

Sample problem #2.

A 3-lb collar can slide on a horizontal rod which is free to rotate about a vertical shaft. The collar is initially held at A by a cord attached to the shaft. As the rod rotates at a rate $\dot{\theta} = 18 \text{ rad/s}$, the cord is cut and the collar moves out along the rod and strikes the stop at B without rebounding. Neglecting friction and mass of the rod. determine the magnitude of the impulse of the force exerted by the stop on the collar.

(13.142)



5 Impulse problem.

Step 1: one particle.

Step 2: polar origin at O. (Newton).

Step 3:

3.1 states.

before collision $\begin{cases} m \\ V_A = ? \end{cases}$

after collision $\begin{cases} V_A = 0 \\ V_A \theta = W \cdot r_B = \dot{\theta} \cdot r_B \end{cases}$

$V_A \text{ const} = W r_B \theta = \dot{\theta} \cdot r_B = \dot{\theta} \cdot 6 \dot{\theta}$.

force resultant force on A.

$$\therefore a_{Ar} = 0$$

relative velocity

$$\vec{a}_{Ar, \text{rel}} = a_{Ar} - \omega^2 r_{\text{rod/r}} = +W^2 r$$



relative displacement.

$$\Delta r = r_B - r_A$$

~~$$\frac{d}{dt} \Delta r_{\text{rod}} = \frac{d \Delta r}{dt}$$~~

$$\left. \begin{aligned} \vec{a}_{\text{Arod}} &= \frac{d \vec{r}_{\text{rod}}}{dt} \\ \vec{v}_r &= \frac{d \vec{r}}{dt} \end{aligned} \right\} \rightarrow \text{relative velocity } A \rightarrow \text{rod}$$

$$V_r = \frac{d \vec{r}}{dt}$$

$$\Rightarrow \frac{d V_r}{d r} = V_r = +\omega^2 \cdot r$$

$$\Rightarrow \frac{1}{2} V_r^2(r_2) - \frac{1}{2} V_r^2(r_1) = \omega^2 \cdot \cancel{r^2} \cdot \cancel{r^2}$$

$$\Rightarrow V_r^2(r_2) = 2\omega^2 r^2 - 2\omega^2 r_1^2$$

$$\Rightarrow \vec{V}_{A/r}(\text{before collision}) = \sqrt{2(r^2 - r_1^2)} \omega$$

$$3.2. \text{ momentum } \vec{V}_{A/r} = \vec{V}_{\text{rod}} + \vec{V}_{A/r} = \omega \cdot r_B \hat{\theta} + \sqrt{2(r^2 - r_1^2)} \omega \hat{r}$$

before collision: $m \vec{V}_{A/r}^b$

after collision: $m(\vec{V}_{A/r}^a) = 0$

$$m(\vec{V}_{A/r}^a)_\theta = m \dot{\theta} r_B$$

3.3 force impulse

$$\vec{I} = I_r \vec{e}_r + I_\theta \vec{e}_\theta = I_r \hat{r} + I_\theta \hat{\theta}$$

3.4 G. E.

~~$$m \vec{V}_{A/r}^a - m \vec{V}_{A/r}^b = I$$~~

$$0 + m \dot{\theta} r_B \hat{\theta} - m \dot{\theta} r_B \hat{\theta} - \cancel{m \dot{\theta} r_B \hat{\theta}} - \cancel{\sqrt{2(r^2 - r_1^2)} \omega \hat{r}} = I_r \hat{r} + I_\theta \hat{\theta}$$

$$\Rightarrow I_r = -m \sqrt{2(r^2 - r_1^2)} \cdot \omega = \cancel{-\frac{3}{32.2} \sqrt{[(\frac{12}{12})^2 - (\frac{1}{12})^2] - 18}} \quad a,$$

$$= \cancel{0.816}$$

Sample problem #2

13.15) A 75-g ball is projected from a height of 1.6m with a horizontal velocity of 2m/s and bounces from a 400-g smooth plate. knowing that the height of the rebound is 0.6m.

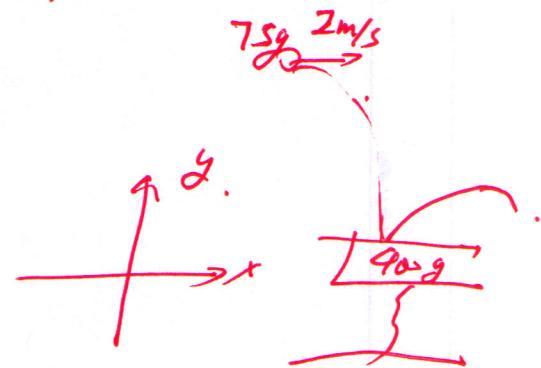
- the velocity of the plate after the impact
- energy lost.

Step 1: ~~2 particles~~ → system

Step 2: rectangular.

Step 3:

- State



ball before: $v_{ball} = v_x \vec{i} + v_y \vec{j} = 2 \text{ m/s} \vec{i} + 5.6 \text{ m/s} \vec{j}$

after $v_{ball} = v_x \vec{i} + v_y \vec{j} = 2 \text{ m/s} \vec{i} + 3.4 \text{ m/s} \vec{j}$
 Support before v_s

3.2 moment. after $v = v_y \vec{j}$

before: $m_b \vec{v}_{ball} + m_s \vec{v}_s = m_b v_b = m_b \cdot 2 \vec{i} - m_b \cdot 5.6 \vec{j}$

after: $m_b \vec{v}_b + m_s \vec{v}_s = m_b \cdot 2 \vec{i} + m_b \cdot 3.4 \vec{j} + m_s v_s$

3.3 Impact

$I = 0.0$ → spring has no deformation at that moment

3.4. G. E.

$\Theta I = \Delta m$

Step 4:

Solution:

$$m_b (3.4 - (-5.6)) + m_s v_s = 0$$

$$v_s = - \frac{m_s}{m_b} \cdot 9 = - \frac{75}{400} \cdot 9 = -1.6875 \text{ m/s}$$

B. For A

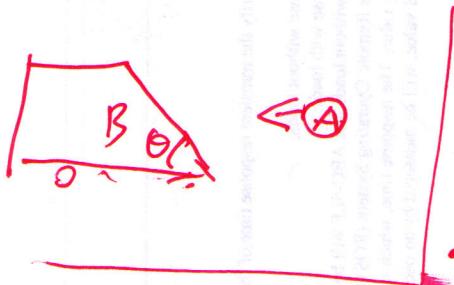
~~$I = m_b + m_s$~~

$$\Theta \Delta E = \frac{1}{2} m_s v_b^2 + \frac{1}{2} m_b v_b'^2 + \frac{1}{2} m_s v_s^2$$

$$= \cancel{\frac{1}{2} \cdot 75} \cdot 75 \cdot (3.4)^2 - \cancel{\frac{1}{2} \cdot 75 \cdot 5.6^2} + \cancel{\frac{1}{2} \cdot 400 \cdot (1.6875)^2} = 0.173,7$$

Sample problem #3

13.169. A. 3-16 state. A mass with a velocity v_0 parallel to the ground and of magnitude $v_0 = 6 \text{ ft/s}$ strikes the inclined face of a 12-6 wedge-B, which can roll freely on the ground and is initially at rest. knowing $\theta = 60^\circ$, $e = 1$. determine the velocity of the wedge immediately after impact.



2 particles \rightarrow system.

Rectangular $\xrightarrow{\text{g}, \vec{x}}$

3..1 States.

before: $v_A = -v_0$ & $v_B = 0$

after: ~~$v_B = 0$~~ oblique impact

$$\vec{v}_A' = v_A ? \quad \vec{v}_B' = v_B \vec{x}$$

$$= v_{Ax} \vec{x} + v_{Ay} \vec{y}$$

line of impact

3.2 moment.

$$\text{before: } m_A \vec{r}_A = -m_A v_0 \vec{x}$$

$$\text{after: } m_A v_{Ax} \vec{x} + m_A v_{Ay} \vec{y} + m_B v_B \vec{x}$$



3.3 Impact

$$I = 0$$

3.4. G. E.

$$\text{t direction: } v_{At}' = (v_0)_t = v_0 \cdot \sin 30^\circ \quad v_{Bt}' = 0$$

$$\begin{aligned} \text{n direction: } & m_A v_{An}' + m_B v_{Bn}' = m_A v_{An} + m_B v_{Bn} \\ & e = 1 \quad \frac{v_{An} - v_{Bn}'}{v_{An} - v_{Bn}} = 1 \end{aligned}$$