up

A small note on knot generation
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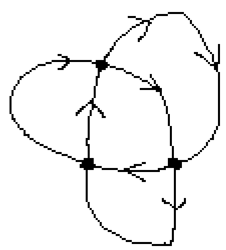
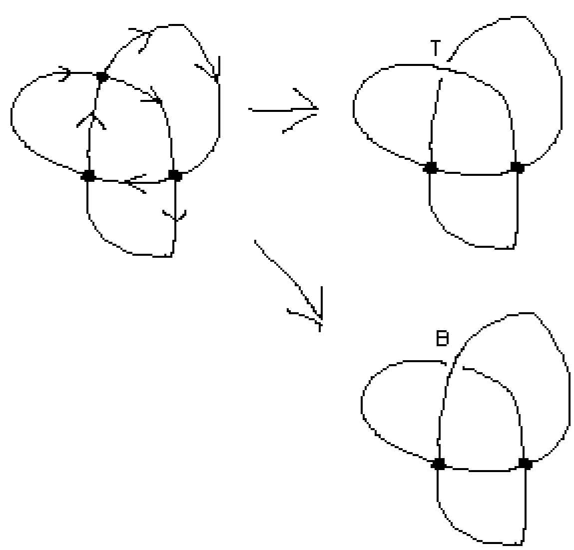
I want to generate all the knot diagrams from the planer graph generated from projecting a knot diagram into a plane.



The above knot diagram has 3 vertices. So total knots that can be generated from it as 2^n or 8 diagram. This a simple way to generate those diagram. Assign a direction from any point on the curve and follow that all the way around until you get back to the starting point. At each crossing decide if the line will cross on TOP or BOTTOM of the other curve. Assign the letter T when going on TOP of the other curve, and assign the letter B when going below the other curve.

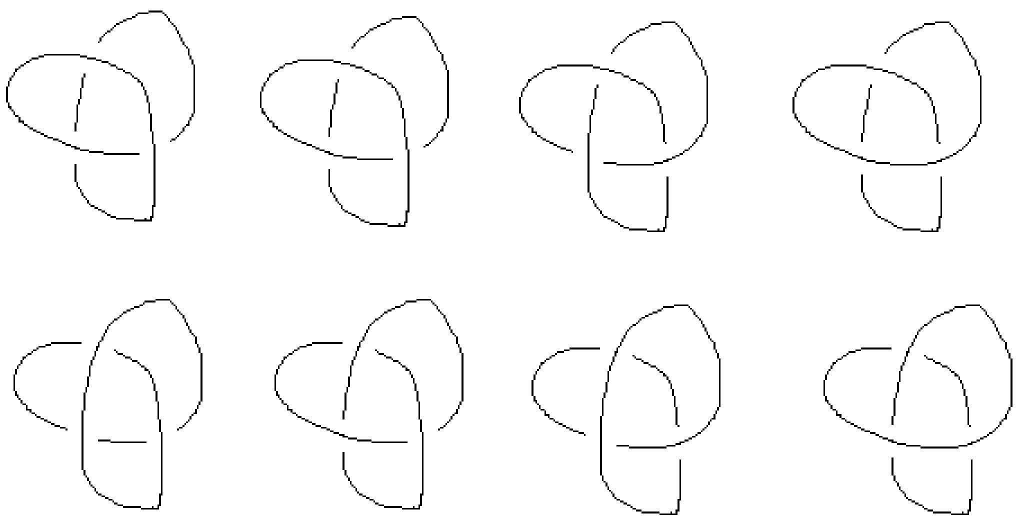
So, at each vertix, we have a choice of a T or a B :



So, at the end we will have these permutations: TTT, TTB, TBT, TBB, BTT, BTB, BBT, BBB

draw all the above 8 combinations, we get:

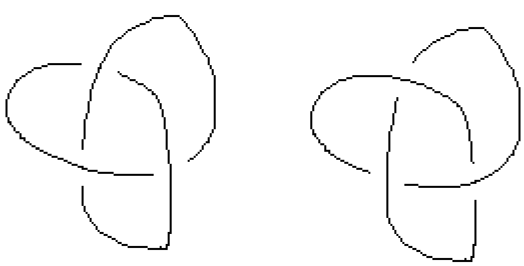
TTT TTB TBT TBB



BTT BTB BBT BBB

The interesting thing, is that only TBT and BTB are knots. The rest are unknot. So, when we have the same letter following each others, we know right away that this is a unknot.

These are the only 2 unknots out of the 8 knots:

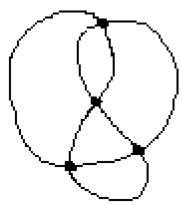


BTB TBT

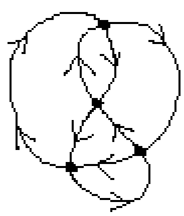
These are the left and right handed trfoil knots.

The question now is, can this be generalized? I.e., for any M vertices knot graphs, is it true that if we get a sequences of T’s and B’s with more than T or B following each other, then we have an unknot? Also, what is the ratio of the unknot to the knot being generated? In this case, we have 2/8 knots and 6/8 are unknot.

Let me try it with the figure 8 knot. This is the planer projection, it has 4 vertices. So total number of knot diagrams is 2^4= 16

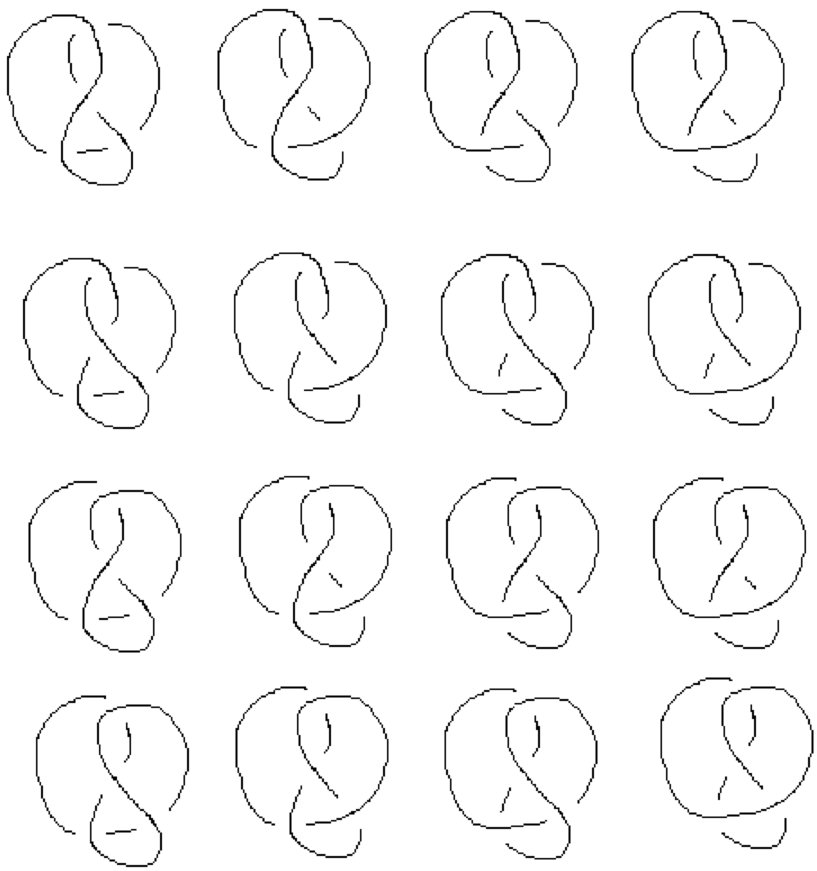


Assign a direction to follow:



Now, follow the arrow and assign a T or B according to the combination TTTT, TTTB, TTBT, TTBB, TBTT, TBTB, TBBT, TBBB, BTTT, BTTB, BTBT, BTBB, BBTT, BBTB, BBBT, BBBB

Here they draw, in the same order given above, from left to right, top to bottom:



Which are the unknot in the above? Looking at them I see numbers 1,2,3,4,5,8,9,12,13,14,15,16 are unknot.
These numbers refer to the diagram numbers above in the order left to right, top to bottom.

So, the following are the unknot and knot combinations:

TTTT, TTTB, TTBT, TTBB, TBTT (1,2,3,4,5) are unknot
TBTB, TBBT, (6,7) are knot
TBBB, BTTT, (8,9) are unknot
BTTB, BTBT, (10,11) are knot
BTBB, BBTT, BBTB, BBBT, BBBB (12,13,14,15,16) are unknot

The ratio for knot is 4/16 and for unknot is 12/16

Notice, this is double the ratio I saw above for n=3, where we had 2/8 for knot and 6/8 for unknot. Does this mean the ratio will double each time the number of vertices is increase by one?

Does this mean, for n=5 we will get 8/32 knot ratio and 24/32 unknot ratio, and so forth for larger n’s ?

Write a program to find out.

Now, let me look at the pattern of the T’s and B’s.

For the unknot, I see either a 3 or more same consecetive letters (BBB or TTT), or a 2 same consecutive letters (TT or BB). I see in the knot pattern having a TT or BB next to each others as well, however, in the knot pattern these are flip flips, while for unknot pattern those where not.

For example, looking at unknot pattern TTBB, and the knot pattern TBBT. Both have the pattern ‘BB’. But if we take 2 letters at a time from left to right, we never get the same two letters in the knot patter, while in the unknot pattern we do. How can I generalize this, so that given number of vertices, I can generate all the knot diagrams?

In the above, for n=4, we get
TBTB, TBBT, BTTB, BTBT
As the knots.
I can see the pattern here. If we assign X to TB, and Y to BT, then the pattern is

XX, XY, YX, YY

So, for n=5, will we get

XXX, XXY, XYX, XYY, YXX, YXY, YYX,YYY

Or

TBTBTB, TBTBBT, TBBTTB, TBBTBT, BTTBTB, BTTBBT, BTBTTB, BTBTBT

Let me try it and found out.