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HW1, ECE 405

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Cal Poly Pomona, ECE 405, first session, summer 2010.

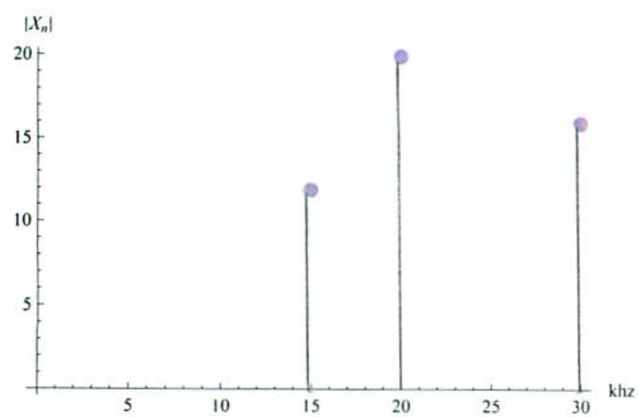
Problem 1

part(a)

```
x[t_] := 12 Cos[2 π 15 000 t - 60/180 π] - 20 Cos[2 π 20 000 t + 30/180 π] - 16 Cos[2 π 30 000 t - 70/180 π];
```

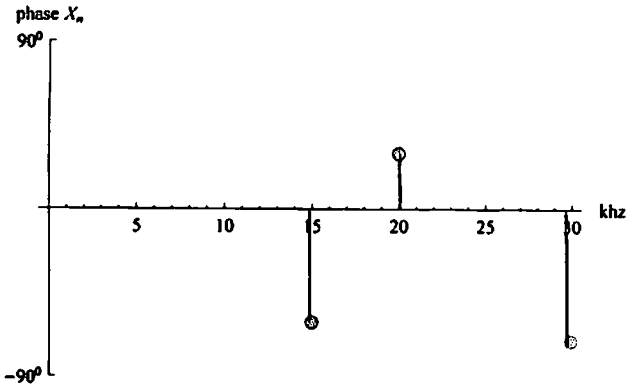
one sided magnitude spectrum

```
data = {{15, 12}, {20, -20}, {30, 16}};
ListPlot[Abs[data], Filling → Axis, AxesOrigin → {0, 0},
 PlotMarkers → {Automatic, 12}, AxesLabel → {"khz", "|Xn|"}]
```



- **part(b)**

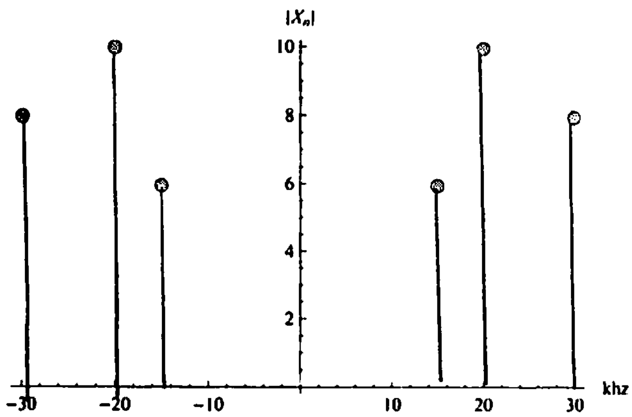
```
data = {{15, -60}, {20, 30}, {30, -70}};
ListPlot[data, Filling → Axis, AxesOrigin → {0, 0},
 Ticks → {Automatic, {{-90, "-90°"}, {90, "90°"}}}, PlotMarkers → {Automatic, 12},
 AxesLabel → {"khz", "phase Xn"}, PlotRange → {Automatic, {-90, 90}}]
```



question: As Dr Kang why key solution has angles summed in different way

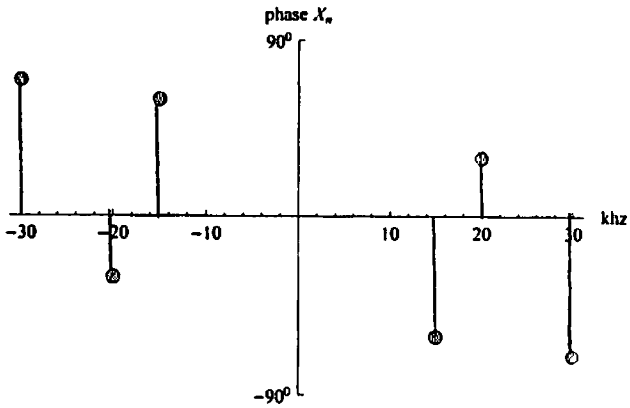
- **part(c)**

```
data = {{15, 6}, {20, 10}, {30, 8}, {-15, 6}, {-20, 10}, {-30, 8}};
ListPlot[data, Filling → Axis, AxesOrigin → {0, 0},
 PlotMarkers → {Automatic, 12}, AxesLabel → {"khz", "|Xn|"}]
```



- **part(d)**

```
data = {{15, -60}, {20, 30}, {30, -70}, {-15, 60}, {-20, -30}, {-30, 70}};
ListPlot[data, Filling → Axis, AxesOrigin → {0, 0},
 Ticks → {Automatic, {{-90, "-90°"}, {90, "90°"}}}, PlotMarkers → {Automatic, 12},
 AxesLabel → {"khz", "phase Xn"}, PlotRange → {Automatic, {-90, 90}}]
```



Problem 2

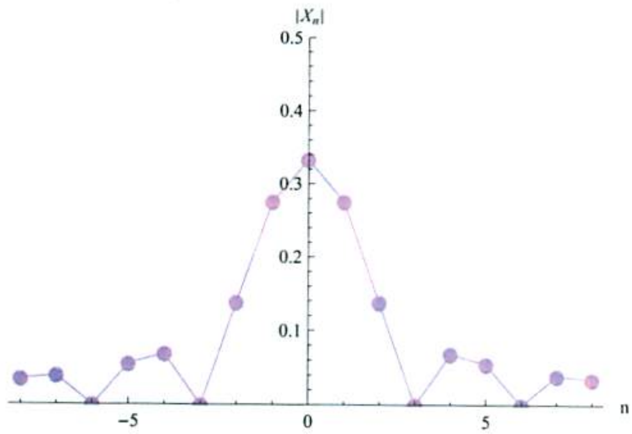
- **part(a)**

```
ClearAll["Global`*"];
xn[n_] := h d Sinc[Pi n d] Exp[-I 2 Pi f0 n t0];
parameters = {h → 1, d → 1 / 3, f0 -> 1 / (3 * 10^-3), t0 → 10^-3};
xn[n] /. parameters
```

$$\frac{1}{3} e^{-\frac{2}{3} i n \pi} \text{Sinc}\left[\frac{n \pi}{3}\right]$$

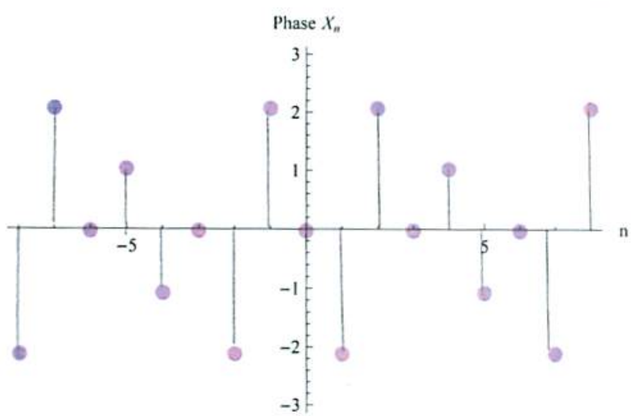
- **part(b)**

```
data = Table[{n, Abs[xn[n] /. parameters]}, {n, -8, 8}];
Show[ListPlot[data, Filling → Axis, PlotRange → {Automatic, {0, .5}},
  PlotMarkers → {Automatic, 12}, AxesLabel → {"n", "|Xn|"}], ListPlot[data, Joined → True]]
```



- **part(c)**

```
data = Table[{n, Arg[xn[n] /. parameters]}, {n, -8, 8}];
ListPlot[data, Filling → Axis, PlotRange → {Automatic, {-Pi, Pi}},
 PlotMarkers → {Automatic, 12}, AxesLabel → {"n", "Phase Xn"}]
```



- **part(d)**

Power in the n^{th} harmonic is $2\,|\,X_n\,|^2$ where we multiply by 2 to take care of both sides of the spectrum. Hence for $n = 2$

```
Abs[xn[2] /. parameters];
Row[{N[2 * %^2], " watt"}]
```

```
0.0379954 watt
```

- **part(e)**

Fourier serices of x(t) is $\sum_{n=-\infty}^{\infty} x_n$ `Exp[I 2 π f0 n t]`
at n=0

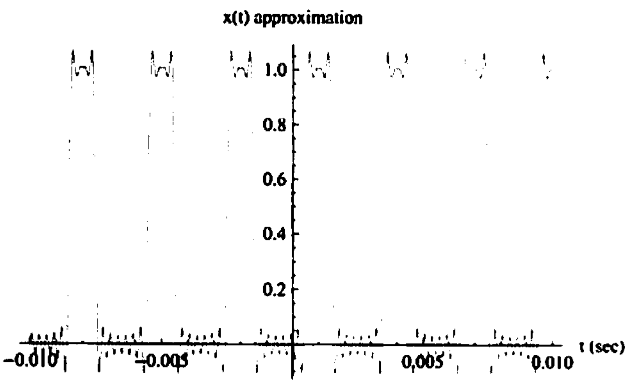
```
xn[0] /. parameters
```

$$\frac{1}{3}$$

substituting values, we obtain $x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{3} e^{-\frac{2}{3} i n \pi + \frac{2000}{3} i n \pi t} \mathrm{Sinc}\left[\frac{n\pi}{3}\right]$

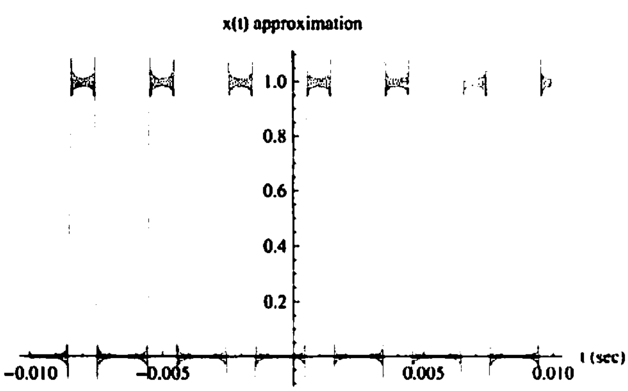
To verify, here is a plot of x(t) for n=10 terms for t=-10 ms to t=10 ms. Notice the delay which is 1 ms

```
fourier = Sum[xn[n] Exp[I 2 π f0 n t], {n, -10, 10}];
Plot[fourier /. parameters, {t, -0.01, .01}, AxesLabel -> {"t (sec)", "x(t) approximation"}]
```



Changine for numbers for terms to say n=30, we obtain

```
fourier = Sum[xn[n] Exp[I 2 π f0 n t], {n, -30, 30}];
Plot[fourier /. parameters, {t, -0.01, .01}, AxesLabel -> {"t (sec)", "x(t) approximation"}]
```



The approximation is better. But notice Gibbs phenomena at the corners.

part(f)

y(t)=x(t)*cos(2π 30000t). To find y(t), convolve the fourier transform of x(t) with the fourier transform of cos(2π 30000t). The fourier transform of cos(2π 30000t) is $\frac{1}{2}$ times impulse at frequency -30khz and at frequency 30khz. The effect of convolving the fourier transform of x(t) with these 2 impluse is to shift the fourier transform of x(t) and center it over the impulses. Hence $Y_n = \frac{1}{2} X_{n-m} + \frac{1}{2} X_{n+m}$ where m is amount of shift needed to center X_n over 30khz and -30khz

The amount of shift is given by $m = \frac{30\text{ khz}}{\frac{1}{3}\text{ khz}} = 90$, hence 90 spectral lines are needed to shift X_n, hence

$Y_n = \frac{1}{2} X_{n-90} + \frac{1}{2} X_{n+90}$

$Y_n = \frac{1}{2} h\, d\, \text{Sinc}[(n-90)\, d]\, \text{Exp}[-I\, 2\, \text{Pi}\, \text{f0}\, (n-90)\ \text{t0}] + \frac{1}{2} h\, d\, \text{Sinc}[(n+90)\, d]\, \text{Exp}[-I\, 2\, \text{Pi}\, \text{f0}\, (n+90)\ \text{t0}]$

simplify, the above becomes

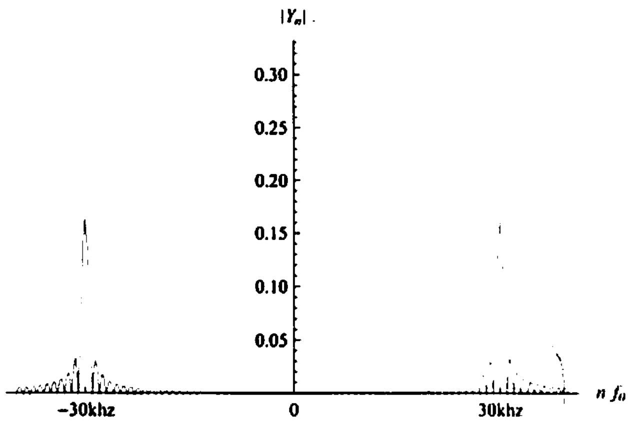
$$\text{yn[n\_] := } \frac{1}{6} e^{-\frac{2}{3} i (-90+n) \pi} \text{Sinc}\left[\text{Pi } \frac{1}{3} (-90+n)\right] + \frac{1}{6} e^{-\frac{2}{3} i (90+n) \pi} \text{Sinc}\left[\text{Pi } \frac{90+n}{3}\right]$$

```
Simplify[yn[n]]
```

$$\frac{1}{6} e^{-\frac{2}{3} i n \pi} \left(\text{Sinc}\left[\frac{1}{3} (-90+n)\, \pi\right] + \text{Sinc}\left[\frac{1}{3} (90+n)\, \pi\right]\right)$$

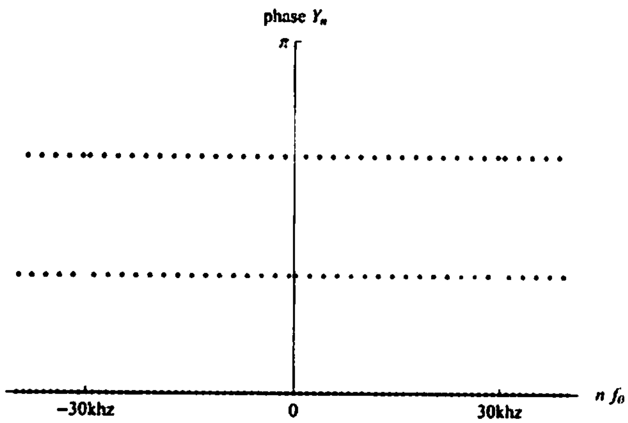
Here is a plot of the magnitude and phase of Y_n

```
data = Table[{n, Abs[yn[n]]}, {n, -120, 120, 1}];
ListPlot[data, Joined → True, AxesLabel → {Style["n f0", Italic], "|Yn|"},
 PlotRange → {Automatic, {0, 2 / 6}}, Ticks → {{{-90, "-30khz"}, 0, {90, "30khz"}}, Automatic}]
```



Now plot the phase

```
data = Table[{n, Arg[yn[n]]}, {n, -120, 120, 1}];
ListPlot[data, Joined → False, Filling → Axis,
 AxesLabel → {Style["n f0", Italic], "phase Yn"}, PlotRange → {Automatic, {0, Pi}},
 Ticks → {{{-90, "-30khz"}, 0, {90, "30khz"}}, {0, {Pi, Pi}}}]
```



- **part(g)**

Applying the filter to y(t) in the frequency domain: The filter has width of 5khz, hence 2.5khz on each side of the center of the filter. The filter is centered at 30khz, hence frequencies of 30+2.5 khz and 30-2.5 kz will be allowed through. Since each f_0 is $\frac{1}{3}$ khz, then the number of spectral lines that will be allowed through is

```
5 / (1 / 3)
15
```

Hence there will be 7 spectral lines on each side of the center of the filter. Hence n will run from 90 to 97, and also run from 83 to 89. Let $w(t)$ be the signal whose spectrum is those spectral lines obtained from the filter. Hence we write

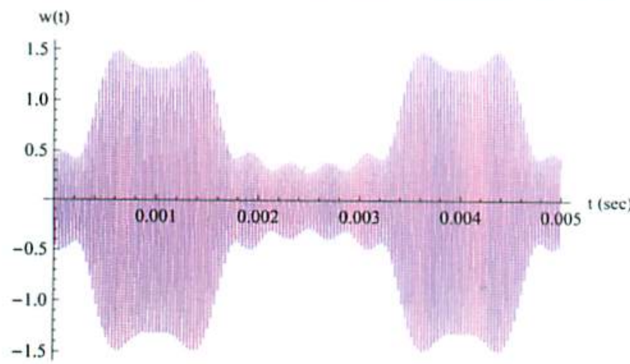
$w(t) = \sum_{n=83}^{97} Y_n\, e^{-i2\pi f_0 t}$ where $f_0 = \frac{1}{3}$ khz and $Y_n = \frac{1}{6}\, e^{-\frac{2}{3} i (-90+n)\pi}\, \text{Sinc}\left[\text{Pi}\, \frac{1}{3}(-90+n)\right] + \frac{1}{6}\, e^{-\frac{2}{3} i (90+n)\pi}\, \text{Sinc}\left[\text{Pi}\, \frac{90+n}{3}\right]$

Using the second form of the fourier series, compute to obtain

```
w1[n_] := 1/3 Sinc[Pi n/3] Cos[2 π (30 000 + n 333) t - n 2/3 π]
w2[n_] := 1/3 Sinc[Pi n/3] Cos[2 π (30 000 - n 333) t + n 2/3 π]
filterOutput = (Sum[w1[n], {n, 1, 7}] + Sum[w2[n], {n, -7, -1}] + w1[0] + w2[0]) // N
0.666667 Cos[188 496. t] - 0.0787613 Sin[0.523599 - 203 142. t] +
 0.137832 Sin[0.523599 - 196 865. t] - 0.551329 Sin[0.523599 - 190 588. t] -
 0.275664 Sin[0.523599 + 192 680. t] + 0.110266 Sin[0.523599 + 198 957. t]
```

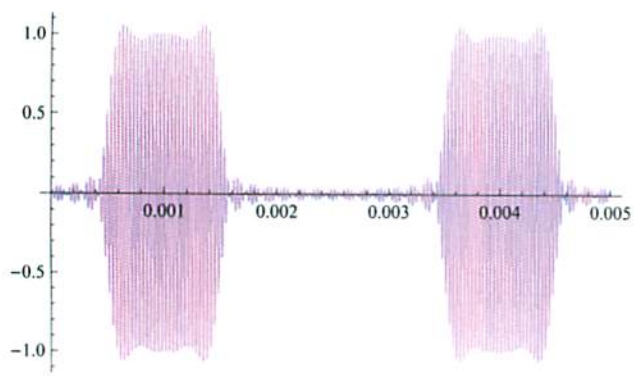
plot w(t)

```
Plot[filterOutput, {t, 0, .005}, PlotRange → All, AxesLabel → {"t (sec)", "w(t)"}]
```



compare w(t), the signal from the bandpass filter, with the original signal y(t) to see the effect of the filtering

```
Plot[(fourier /. parameters) * Cos[2 π * 30 000 t], {t, 0, .005}, PlotRange → All]
```



- **part(h)**

using the shifting property, $Z_n = Y_n\, e^{-12\pi f_0\, t_0\, n}$ where f_0 is the fundamental frequency of y(t) and t_0 is the delay amout

```
parameters = {h → 1, d → 1 / 3, f0 -> 1 / (3 * 10^-3), t0 → 10^-3 / 4};
zn[n_] := yn[n] Exp[-I 2 π f0 t0 n] /. parameters
zn[n] // Simplify
```

$$\frac{1}{6}\, e^{-\frac{5}{6} i n \pi} \left(\text{Sinc}\left[\frac{1}{3}(-90+n)\,\pi\right] + \text{Sinc}\left[\frac{1}{3}(90+n)\,\pi\right]\right)$$

Problem 3

- **part(a)**

```
width = 0.25 * 10^-3; period = 10^-3; h = 1; f0 = 1000;
```

X_n for triangular pulse the width term used to find the duty cycle is taken as 1/2 of the width of the base of the triangle.

```
d = width / period
0.25
```

Hence X_n =h d (Sinc[Pi n d])^2=$\frac{1}{2}$ (Sinc[Pi $\frac{n}{2}$])^2

$$\text{xn[n\_]} := \frac{1}{2}\left(\text{Sinc}\left[\text{Pi}\,\frac{\text{n}}{2}\right]\right)^2$$

- **part(b)**

The fourier series approximation is given by $\sum_{n=-\infty}^{\infty} X_n\, e^{-I\,2\,\pi\, f_0\, n\, t}$ where $f_0 = 1$ khz in this example. Hence

$x(t) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(\text{Sinc}\left[\text{Pi}\,\frac{n}{2}\right]\right)^2 e^{-I\,2\,\pi\,1000\,n\,t}$

- **part(c)**

$H(s) = \frac{\frac{1}{RC}}{S+\frac{1}{RC}} = \frac{1}{RCS+1}$ where R=1000 ohm, $C = 10^{-6}$, hence RC $= 10^{-3}$, then H(s)=$\frac{1}{10^{-3} S+1} = \frac{1000}{S+1000}$,

hence H(j ω) = $\frac{1000}{j\omega+1000}$

$H(j\omega) = |H(j\omega)|$ Arg($H(j\omega)$).

Now, y(t)=H(jω) x(t) , hence in terms of the fourier coefficients, we write

$Y_n = H(j\omega_0 n) X_n$=(| H(jω) | Arg(H(jω))* (| X_n | Arg(X_n))

hence

Y_n = | $H(j\omega_0 n)$ | * | $\mathrm{X_n}$ | (Arg($H(j\omega_0 n)$) + (Arg(X_n)))

Hence | Y_n | = | $H(j\omega_0 n)$ |* | $\mathrm{X_n}$ |
and
Arg(Y_n) = Arg($H(j\omega_0 n)$) + Arg(X_n)

But | $H(j\omega_0 n)$ | = $\frac{1000}{\sqrt{\omega^2{}_0 n^2+1000^2}}$
and
Arg($H(j\,\omega_0\, n)$)) = $-\arctan\left(\frac{n\,\omega_0}{1000}\right)$
Now, | X_n | = $\frac{1}{2}$ $\left(\mathtt{Sinc}\left[\mathtt{Pi}\ \frac{n}{2}\ \right]\right)^2$ and Arg[X_n] = 0 since there is no delay term.

Hence $Y_n = \frac{1000}{\sqrt{\omega^2{}_0 n^2+1000^2}} \frac{1}{2} \left(\mathtt{Sinc}\left[\mathtt{Pi}\ \frac{n}{2}\ \right]\right)^2$ Exp[-j $\mathtt{arctan}$ $\left(\frac{n\,\omega_0}{1000}\right)$]
Now, $\omega_0 = 2\pi f_0 = 2\pi$ 1000, hence
$Y_n = \frac{1000}{\sqrt{(2\pi\ 1000\ n)^2+1000^2}} \frac{1}{2} \left(\mathtt{Sinc}\left[\mathtt{Pi}\ \frac{n}{2}\ \right]\right)^2$ Exp[-j $\mathtt{arctan}$ (n 2 π)]

```
yn[n_] := 1/2 (Sinc[Pi n/2])^2 Exp[-I ArcTan[n 2 π]] 1000/Sqrt[(2 π 1000 n)^2 + 1000^2]
```

Now that Y_n is found, we can find

y(t)=$\sum_{n=-\infty}^{\infty} Y_n$ Exp[$I\ 2\pi\ f_0\ n\ t$]

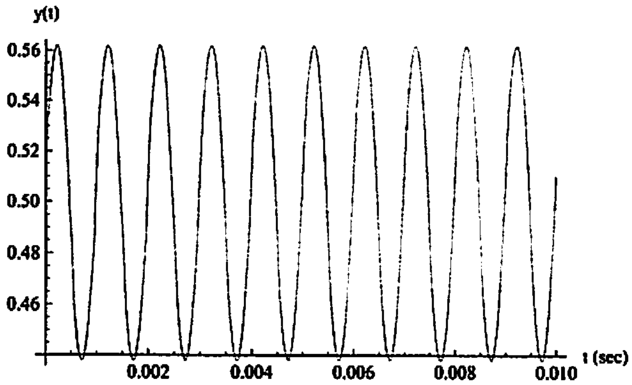
where f_0 = 1 khz, hence to plot y(t) using say 10 terms in foruier series and compare to x(t)

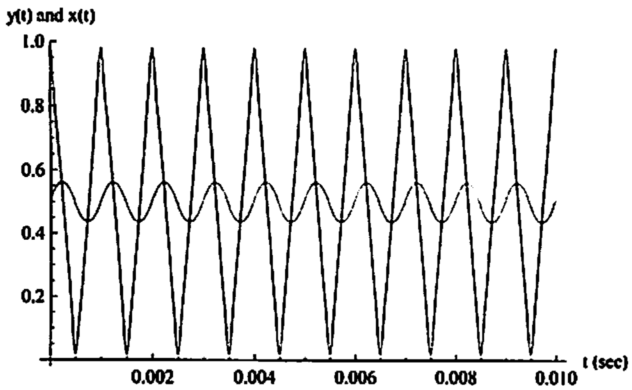
```
y[t_] := Sum[yn[n] Exp[I 2 π 1000 n t], {n, -10, 10}]
```

$$y[t\_] := \sum_{n=-10}^{10} yn[n]\, Exp[I\, 2\, \pi\, 1000\, n\, t]$$

$$x[t\_] := \frac{1}{2} \sum_{n=-10}^{10} \left(Sinc\left[Pi\, \frac{n}{2}\right]\right)^2 Exp[-I\, 2\, \pi\, 1000\, n\, t]$$

```
Plot[y[t], {t, 0, .01}, AxesLabel → {"t (sec)", "y(t)"}]

Plot[{y[t], x[t]}, {t, 0, .01}, AxesLabel → {"t (sec)", "y(t) and x(t)"}]
```



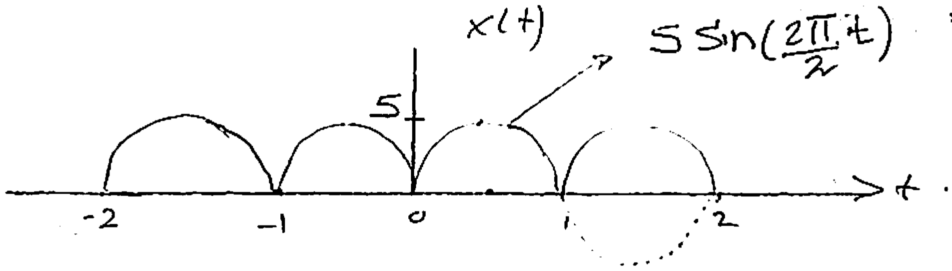


problem 4

$$5 \int_0^1 Sin[Pi\, t]\, Exp[I\, 2\, Pi\, n\, t]\, dt$$

$$\frac{5\left(1 + e^{2 i n \pi}\right)}{\pi - 4 n^2 \pi}$$

#4



$= 5\sin(\pi t)$

$T_0 = 1$ sec, $f_0 = 1$ Hz, $h = 5$, $\omega_0 = 2\pi$

Since $x(t)$ is even, it will only have a_n terms. so using

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\omega_0 n t)$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{1}\int_0^1 5\sin(\pi t)\,dt = 5\left[\frac{-\cos(\pi t)}{\pi}\right]_0^1$$

$$= -\frac{5}{\pi}\left[\cos\pi - \cos 0\right] = -\frac{5}{\pi}\left[-1-1\right] = \boxed{\frac{10}{\pi}}$$

$$a_n = \frac{2}{T}\int_T x(t)\, e^{j\frac{2\pi}{1}nt}\,dt = 2\int_0^1 5\sin(\pi t)\, e^{j2\pi nt}\,dt$$

$$= 10\int_0^1 \sin(\pi t)\, e^{j2\pi nt}\,dt$$ integration by parts gives

$$= \frac{10\cdot(1+e^{j2n\pi})}{\pi - 4n^2\pi}$$

for n integer, $e^{j2n\pi} = \cos 2n\pi + j\,\underbrace{\sin 2n\pi}_{=0} = 1.$

so $$a_n = \frac{10(2)}{\pi(1-4n^2)} = \frac{20}{\pi(1-4n^2)}$$

So $$\boxed{x(t) = \frac{10}{\pi} + \sum_{n=1}^{\infty} \frac{20}{\pi(1-4n^2)} \cos(2\pi n t)}$$

$n=-\infty$ $\qquad$ $n=-\infty$ $\quad \pi(1-4n)$

(5) $F[x(t)] = X(\omega)$. (3)

(a) $2\,x(t+5) \longrightarrow 2\,F[x(t+5)] = 2\,X(\omega)e^{j\omega 5}$ delay

(b) $10\,x[(t-7)/3] \rightarrow 10\,F[\,x(\frac{t-7}{3})\,] = 30\,X(3\omega)e^{-j\omega 7}$

Using property that $F[X(\frac{t}{a})] = aX(a\omega)$

(c) $t\,x\left(\frac{t+2}{5}\right) \otimes \frac{d}{dt}x(t)$ Conv.

using property that $F[\frac{d}{dt}x(t)] = j\omega X(\omega)$

and $F\left[x\left(\frac{t+2}{5}\right)\right] = 5X(5\omega)\,e^{j2\omega}$

and $F[t\,x(t)] = j\frac{d}{d\omega}X(\omega)$

Then $F\left[\,t\,x\left(\frac{t+2}{5}\right)\right] = j\frac{d}{d\omega}\left(5X(5\omega)\,e^{j2\omega}\right)$

so $F\left[\,(\quad)\otimes\frac{d}{dt}x(t)\right] = F[(\quad)]\;F[\frac{d}{dt}x(t)]$

i.e $F[\text{convolution}] \Rightarrow$ multiplication.

So $F\left[\,t\,x\left(\frac{t+2}{5}\right)\otimes\frac{d}{dt}x(t)\right] = j\frac{d}{d\omega}\left(5X(5\omega)e^{j2\omega}\right)\,j\omega X(\omega)$

$$= \boxed{-\frac{d}{d\omega}\left(5X(5\omega)e^{j2\omega}\right)\omega X(\omega)}$$

(d) $t\, x^*(8t)$ → complex conjugate

$$F[t\, x^*(8t)]$$

Using property $F[x^*(t)] = X^*(-\omega)$.

Then $F[x^*(8t)] = \frac{1}{8} X^*\left(-\frac{\omega}{8}\right)$.

Using property $F[t\, x(t)] = j \frac{d}{d\omega} X(\omega)$, then

$$F[t\, x^*(8t)] = j \frac{d}{d\omega}\left(\frac{1}{8} X^*\left(\frac{-\omega}{8}\right)\right) = \boxed{\frac{j}{8}\frac{d}{d\omega}\left(X^*\left(\frac{-\omega}{8}\right)\right)}$$

(e) Find Fourier transform of

$$-x\left(-\frac{(t+20)}{12}\right) e^{j1000t}$$

Using property that $F[x(t) e^{j\alpha t}] = X(\omega - \alpha)$

Then $F\left[-x\left(-\frac{t+20}{12}\right) e^{j1000t}\right]$

$$= +12\, X(-12\omega)\, e^{j\omega 20} \Big|_{\omega = \omega - 1000}$$

$$= \boxed{12 X\left(-12(\omega-1000)\right) e^{j(\omega-1000)20}}$$

Problem 5

Part (f). $\frac{d}{dt}\left(x^*(-2t)\right)\cos 500t$.

Use Property $F(x(at)) = \frac{1}{|a|}X\left(\frac{\omega}{a}\right)$ —— (1)

and Property $F\left(\frac{d}{dt}x(t)\right) = j\omega X(\omega)$ —— (2)

and Property $F(x(t)\cos(bt)) = \frac{1}{2}\left[X(\omega-a) + X(\omega+a)\right]$ —— (3)

Use property $F(x^*(t)) = X^*(-\omega)$ —— (4)

Then we obtain:

$$F[x(-2t)] = \frac{1}{|2|}X\left(\frac{\omega}{-2}\right) \quad \text{Rule (1)}$$

$$F[x^*(-2t)] = \frac{1}{2}X^*\left(\frac{\omega}{2}\right) \quad \text{rule (4)}$$

$$F\left[\frac{d}{dt}x^*(-2t)\right] = \frac{j\omega}{2}X^*\left(\frac{\omega}{2}\right) \quad \text{rule (2)}$$

↓ rule (3)

$$\frac{j(\omega-500)}{(2)(2)}\left[X^*\left(\frac{\omega-500}{2}\right) + X^*\left(\frac{\omega+500}{2}\right)\right]$$

$$= \frac{j(\omega-500)}{4}\left[X^*\left(\frac{\omega-500}{2}\right) + X^*\left(\frac{\omega+500}{2}\right)\right]$$

Problem 5

Part (g)

using rule $F[x_1(t)\, x_2(t)] = [X_1(\omega) \circledast X_2(\omega)]\frac{1}{2\pi}$

Then

$F[x^2(t)] = F[x(t)\, x(t)] = \frac{1}{2\pi} X(\omega) \circledast X(\omega)$

$F[x^3(t)] = F[x(t)\, x(t)\, x(t)] = \frac{1}{(2\pi)^3} X(\omega) \circledast X(\omega) \circledast X(\omega).$

$F[\text{constant}] = C \cdot \delta(\omega)$.

So the result is

$$\frac{1}{(2\pi)^3} X(\omega) \circledast X(\omega) \circledast X(\omega) + \frac{3}{2\pi} X(\omega) \circledast X(\omega) - 9\, X(\omega) + 15\, \delta(\omega)$$

Part h

$(t-5)\, x(3-t)$.

Using rule $F[x(-t)] = X(-\omega)$.

using rule $F[t x(t)] = j \frac{d}{d\omega} X(\omega)$

using rule $F[x(t-a)] = X(\omega)\, e^{-j\omega a}$

Then we obtain

$$F[(t-5)\, x(3-t)] = F[t x(-(t+3)) - 5 x(-(t-3))]$$

$$= F[t x(-(t-3))] - 5\, F[x(-(t-3))]$$

$$= j \frac{d}{d\omega}[X(-\omega) e^{-j\omega 3}] - 5\, X(-\omega) e^{-j3\omega}$$

Problem 5, Part (i)

$X(\frac{t}{2}) \circledast \mathbb{X}(-5t)$

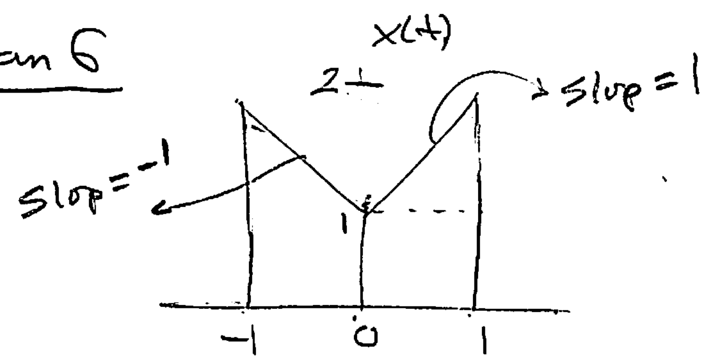
using property $F(x_1(t) \circledast x_2(t)] = \mathbb{X}_1(\omega)\,\mathbb{X}_2(\omega)$

and using property $F(x(\frac{t}{a})) = F(x(\frac{1}{a}t)) = |a|\,\mathbb{X}(a\omega)$

and property $F(\mathbb{X}(t)) = 2\pi x(-\omega)$

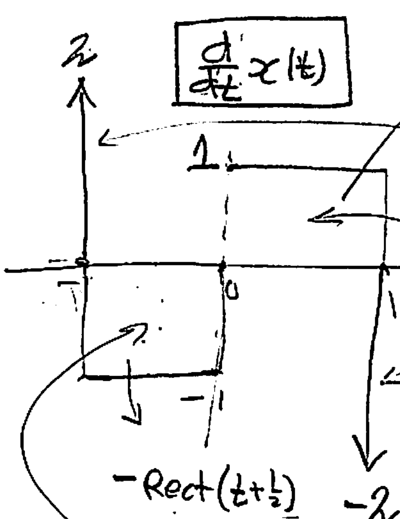
$$\left(2\mathbb{X}(2\omega)\right)\left(\frac{2\pi}{5}\,x(\tfrac{\omega}{5})\right) = \frac{4}{5}\pi\,\mathbb{X}(2\omega)\,x(\tfrac{\omega}{5})$$

Problem 6



The time differentiation property is $F[\frac{d}{dt}x(t)] = j\omega\,\mathbb{X}(\omega)$

make a function whose is the derivation of the above:



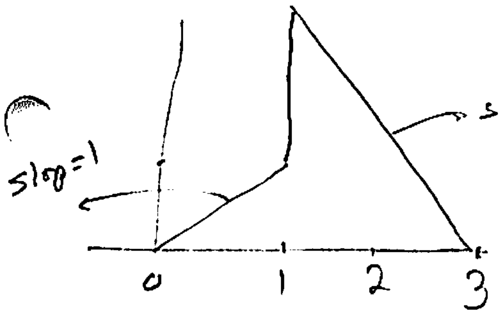
rect(t - ½)

but this has Fourier Transform of

$$2e^{j\omega} - 2e^{-j\omega} - \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)e^{j\omega\frac{1}{2}} + \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)e^{j\omega\frac{1}{2}}$$

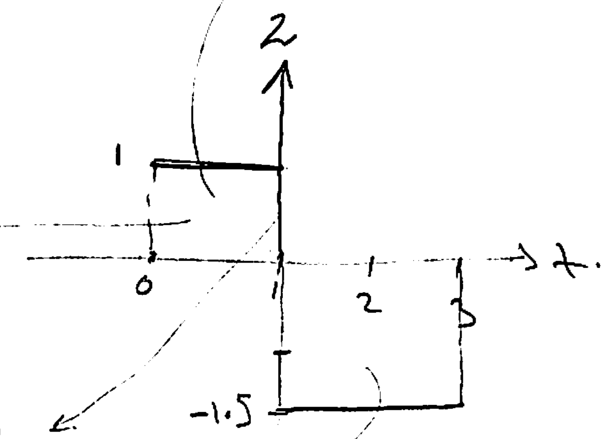
$j\omega\,\mathbb{X}(\omega) = [\text{the above}] \Rightarrow \mathbb{X}(\omega) = \frac{1}{j\omega}[\ \ ]$

Problem 6 Part (b)



$\Longrightarrow$ $x'(t)$

$\text{Rect}(t+\frac{1}{2})$



$-1.5\ \text{rect}\left(\frac{t-2}{2}\right)$

$$\text{so } F(x'(t)) = \text{Sinc}\left(\frac{\omega}{2\pi}\right)e^{-j\frac{\omega}{2}} + 2e^{-j\omega}$$

$$-1.5(2)\,\text{Sinc}\left(\frac{\omega}{2\pi}\right)e^{-2j\omega}$$

$$= \text{sinc}\left(\frac{\omega}{2\pi}\right)e^{-j\frac{\omega}{2}} - 3\,\text{Sinc}\left(\frac{\omega}{2\pi}\right)e^{-2j\omega} + 2e^{-j\omega}$$

problem 7

(a) $x(t) = t e^{-at} u(t)$

use rule $F[t\, x(t)] = j \frac{d}{dw} X(w)$.

but $F[e^{-at} u(t)] = \frac{1}{a+jw}$

so $j \frac{d}{dw}\left(\frac{1}{a+jw}\right) = j \frac{d}{dw}\left((a+jw)^{-1}\right) = j\left[-(a+jw)^{-2} \cdot j\right]$

$$= j\left(\frac{-j}{(a+jw)^2}\right) = \boxed{\frac{1}{(a+jw)^2}}$$

(b) $x(t) = t e^{-at} u(t) \cos(w_c t)$.

$$= \frac{1}{(a+jw)^2} \circledast \frac{1}{2}\left[\delta(w-w_c) + \delta(w+w_c)\right]$$

$$= \frac{1}{2} \frac{1}{(a+j(w-w_c))^2} + \frac{1}{2} \frac{1}{(a+j(w+w_c))^2}$$

(c) $10 e^{-10|t|} \cos(200t)$.

Use rule $F[e^{-a|t|}] = \frac{2a}{a^2+w^2}$, $\Rightarrow 10 F[e^{-10|t|}] = \frac{200}{100+w^2}$

Then $10 \frac{200}{(10)^2+w^2} \circledast \frac{1}{2}\left[\delta(w-200) + \delta(w+200)\right]$

$$= \frac{1}{2} \frac{200}{100+(w-200)^2} + \frac{1}{2} \frac{200}{100+(w+200)^2} = \boxed{\frac{100}{100+(w-200)^2} + \frac{100}{100+(w+200)^2}}$$

Problem 7. , part (d)

$x(t) = \text{rect}\left(\frac{t}{12}\right) e^{j200t}$.

Use property $F[x(t)\, e^{jat}] = X(\omega - a)$

Use property $F[\text{rect}\left(\frac{t}{a}\right)] = a\, \text{sinc}\left(\frac{a\omega}{2\pi}\right)$

so $12\, \text{sinc}\left(\frac{12\omega}{2\pi}\right)\Big|_{\omega = \omega - 200}$

$$= \boxed{12\, \text{sinc}\left(\frac{6\,(\omega - 200)}{\pi}\right)}$$

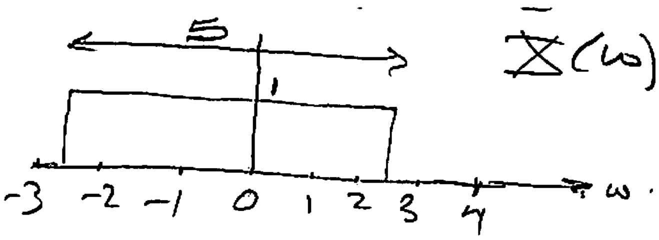
Part e $\quad x(t) = \text{sgn}(t)\, \cos(100t)$.

use property $F[\text{sgn}(t)] = \frac{2}{j\omega}$.

Then $\frac{2}{j\omega} \circledast \frac{1}{2}\left[\delta(\omega - 100) + \delta(\omega + 100)\right]$

$$= \boxed{\frac{1}{j(\omega - 100)} + \frac{1}{j(\omega + 100)}}$$

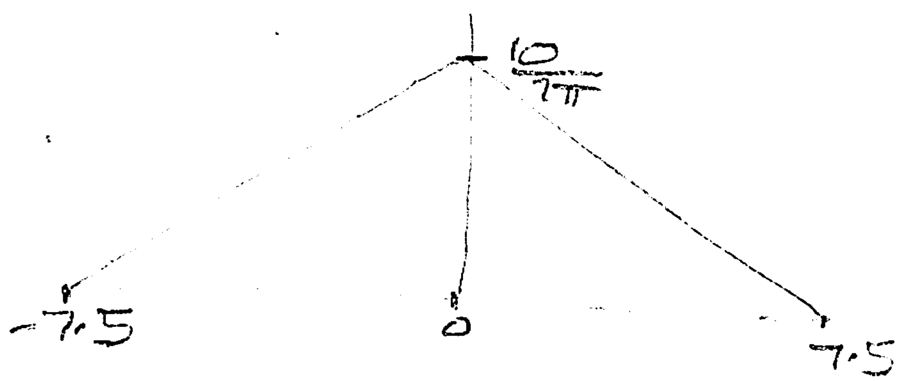
Problem 8



$F[x(t)] = \text{rect}\left(\frac{\omega}{5}\right)$

$F[x^2(t)] = \frac{1}{2\pi} X(\omega) \circledast X(\omega)$.

So need to convolve 2 rect. This gives a <u>triangle</u>. extent is from -7.5 to $+7.5$
max is at $w=0$, hight is $\frac{10}{2\pi}$ (total area when both rects are on top of each other.).



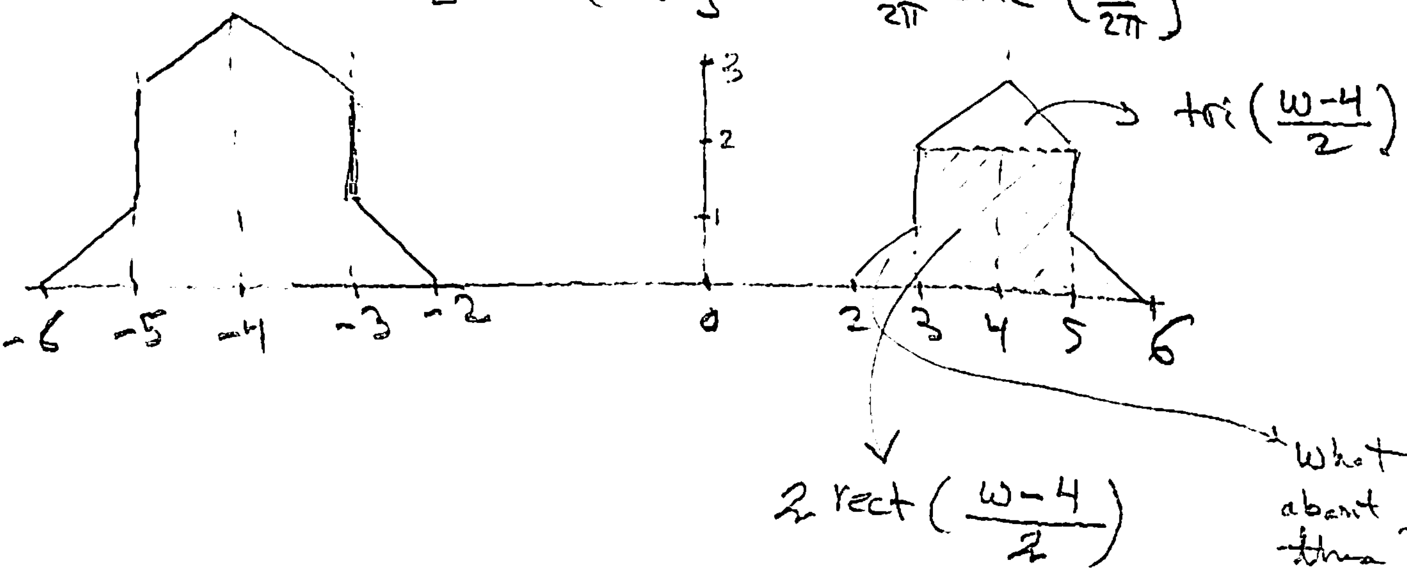
Problem 9

Use property $\mathcal{F}^{-1}[\text{tri}(\frac{\omega}{2\pi})] = \text{sinc}^2(t)$.

and $\mathcal{F}^{-1}[\text{rect}(\frac{\omega}{2\pi})] = \text{sinc}(t)$

then $\mathcal{F}^{-1}[\text{rect}(\omega)] = \frac{1}{2\pi}\text{sinc}(\frac{t}{2\pi})$

and. $\mathcal{F}^{-1}[\text{tri}(\omega)] = \frac{1}{2\pi}\text{sinc}^2(\frac{t}{2\pi})$



need to check this more.

Problem 10

(a) $\int_{-\infty}^{\infty} \frac{\sin\left(\frac{\pi t}{2}\right)}{5t} e^{-2t}\, \delta(t-1)\, dt.$ $\leftarrow$ at $\underline{\underline{t=1}}$ $\delta(t-1) = 1.$

$= \int_{-\infty}^{\infty} \frac{\sin\left(\frac{\pi}{2}\right)}{5} e^{-2} \delta(t)\, dt = \frac{\sin\left(\frac{\pi}{2}\right)}{5} e^{-2} \int_{-\infty}^{\infty} \delta(t)\, dt. = 1$

$= \frac{\sin\left(\frac{\pi}{2}\right)}{5} e^{-2} = \frac{1}{5} e^{-2} = 0.027067.$

(b) $\int_{-\infty}^{\infty} e^{j2\pi t}\, \delta\left(t-\frac{1}{8}\right) dt.$

note at $\boxed{t=\frac{1}{8}}$ $\delta\left(t-\frac{1}{8}\right)$ is not zero. hence

$\int_{-\infty}^{\infty} e^{j2\pi\frac{1}{8}}\, \delta(t)\, dt = e^{j\frac{\pi}{4}}$

$= \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = 0.707 + j\, 0.707$

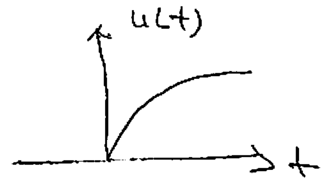
Problem 11

$$\delta(t) \rightarrow \boxed{LTI} \rightarrow u(t)$$

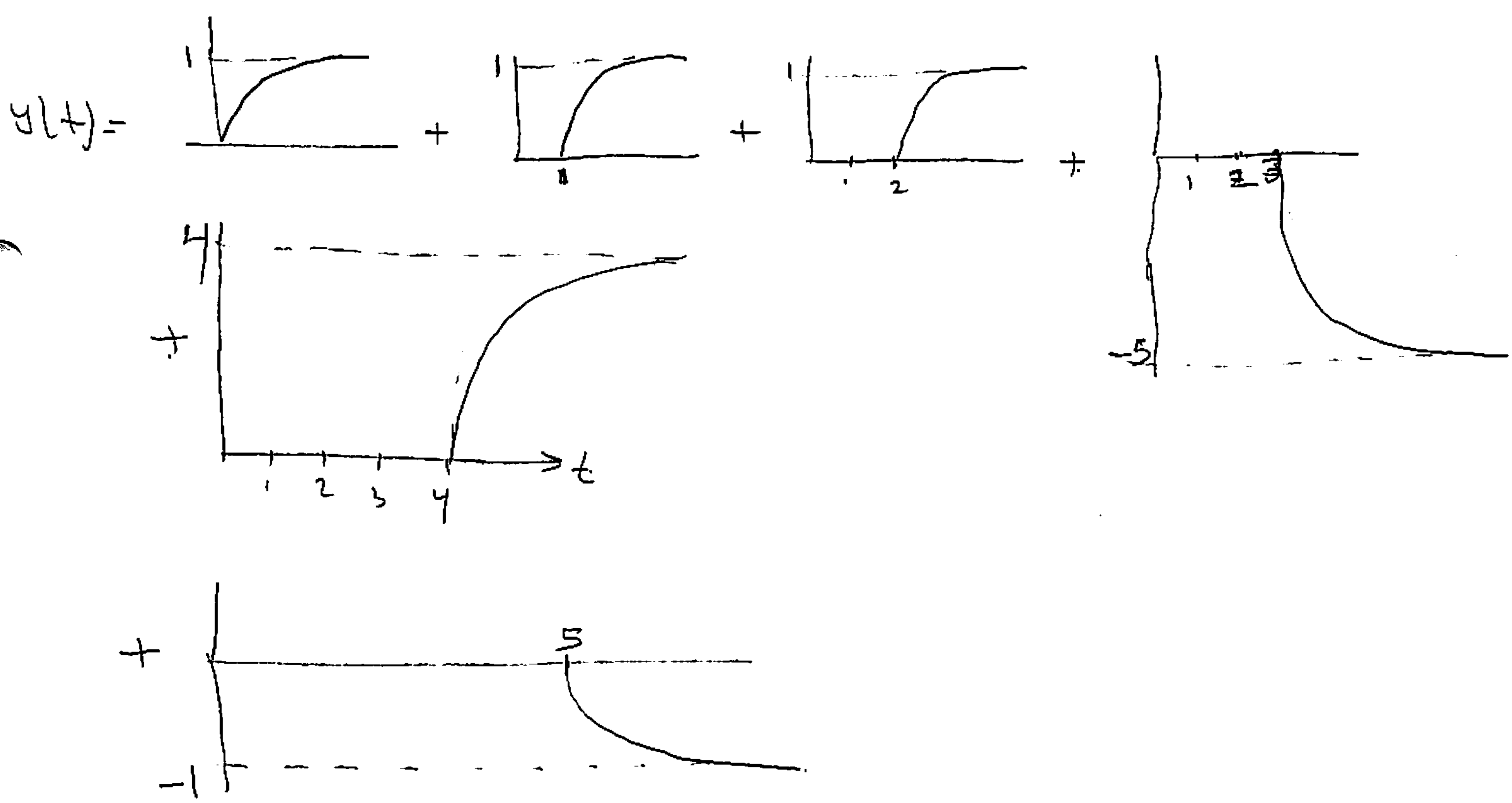
scaling the input causes scaling in the output.
delay in the input causes delay in the output.

so $y(t) = u(t) + u(t-1) + u(t-2) - 5u(t-3) + 3u(t-4) - u(t-5)$

so, assuming $u(t) =$



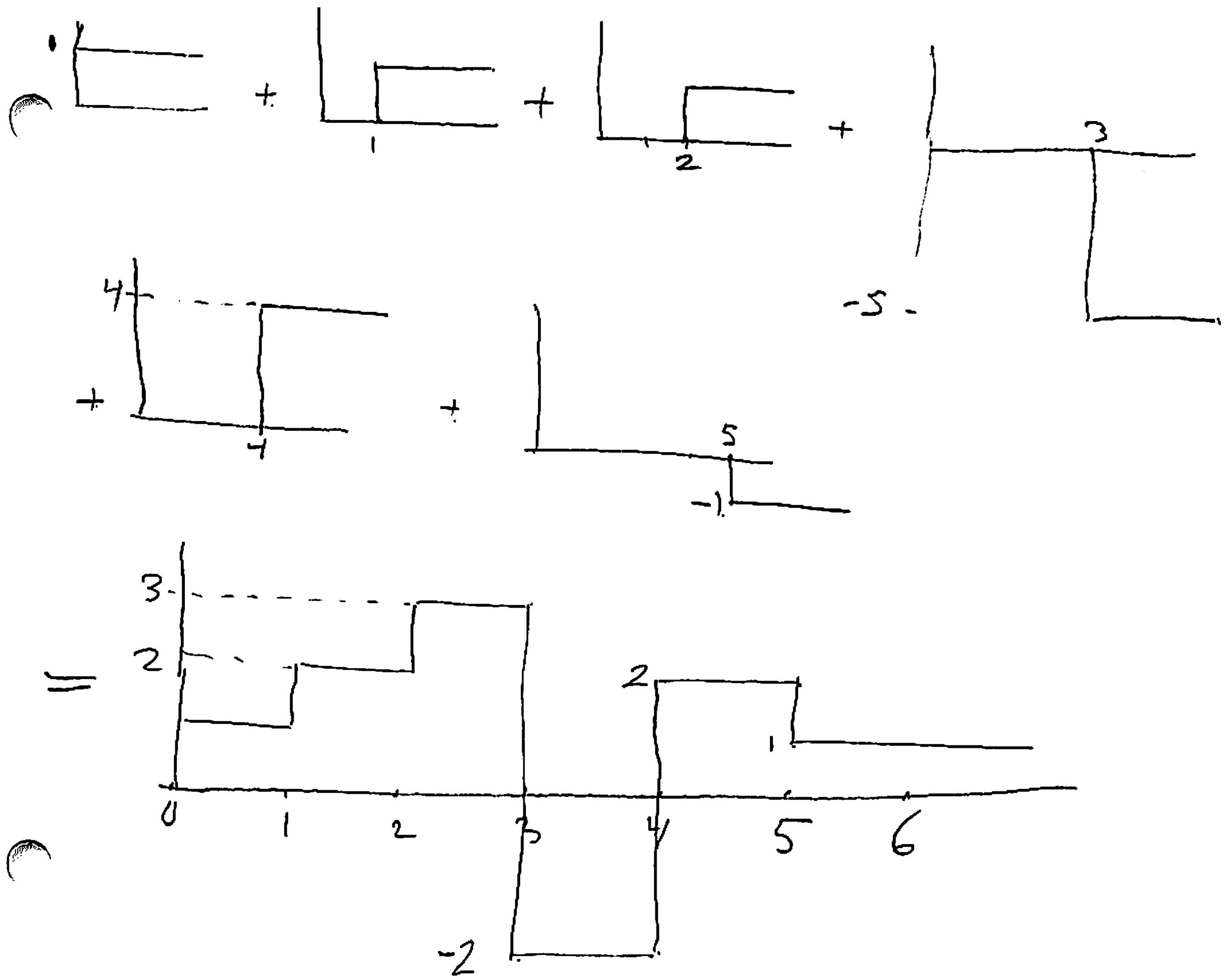
then.



so need to add all the above together to see the final result.

if $u(t)$ is the unit step, then see next page →

u(t)



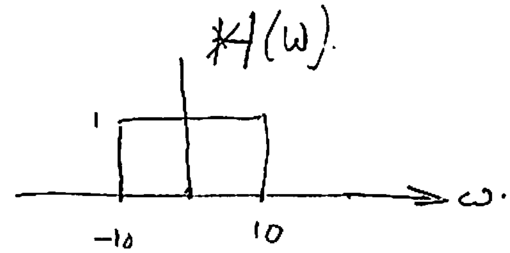
(13) $x(t)$ is band limited to B Hz.

What is bandwidth of $x^3(t)$?

from convolution property: $F(x^3(t)) = \frac{1}{2\pi} X(\omega) \circledast X(\omega) \circledast X(\omega)$.

when convolving, the bandwidth become to sum of the bandwidth of the $X(\omega)$. hence final bandwidth is

$\boxed{3B}$



(14) $H(\omega) = rect\left(\frac{\omega}{40\pi}\right)$

$= rect\left(\frac{\omega}{20(2\pi)}\right)$

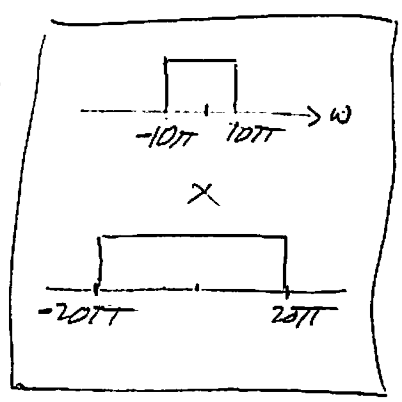
when $\delta(t)$ is applied, the the output is the $F^{-1}[H(\omega)]$

Which is $\boxed{20 \operatorname{sinc}(20t)}$

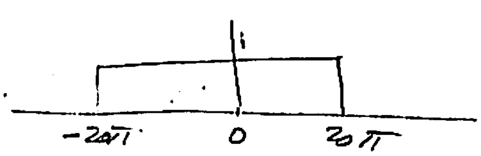
(b) if input is $10 \operatorname{sinc}(10t)$, then output is

$F[10 \operatorname{sinc}(10t)] \quad H(\omega)$

$= rect\left(\frac{\omega}{10(2\pi)}\right) \quad rect\left(\frac{\omega}{20(2\pi)}\right)$



so result is (in ω domain) :



so the $F^{-1}[\quad]$ is a sinc.

so $\boxed{y(t) = 20 \operatorname{sinc}(20t)}$

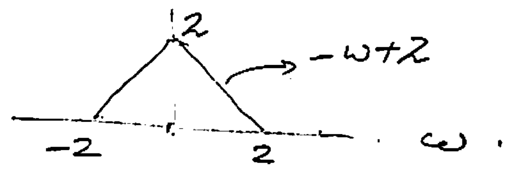
problems 15

$|X(\omega)| = 2\,tri\left(\frac{\omega}{2}\right)$. Find energy of $x(t)$.

from parsevall, energy in ω domain = energy in time domain.

so energy of $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$

the triangle is



so $2 \cdot \frac{1}{2\pi}\int_0^2 (-\omega+2)^2\, d\omega$.

this 2 since adding both sides.

so $\frac{1}{\pi}\int_0^2 \omega^2+4-4\omega\, d\omega = \frac{1}{\pi}\left(\left[\frac{\omega^3}{3}\right]_0^2 + 4\left[\omega\right]_0^2 - 4\left[\frac{\omega^2}{2}\right]_0^2\right)$

$= \frac{1}{\pi}\left(\frac{1}{3}(8) + 4(2) - 2(4)\right)$

$= \frac{1}{\pi}\left(\frac{8}{3}+8-8\right) = \boxed{\frac{1}{\pi}\left(\frac{8}{3}\right)}$

HW # 2

Nasser M. Abbasi

Problem (1)

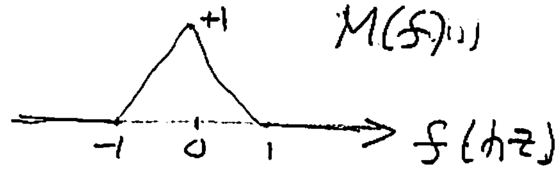
$m(t) = \text{sinc}^2(t)$.

$c(t) = 2\cos(2\pi 10 t)$.

(a) $F(m(t)) = F(\text{sinc}(t)) \circledast F(\text{sinc}(t))$

$= \text{rect}(f) \circledast \text{rect}(f)$

$= \text{tri}(f)$

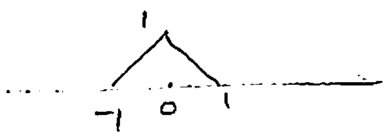


M(f)

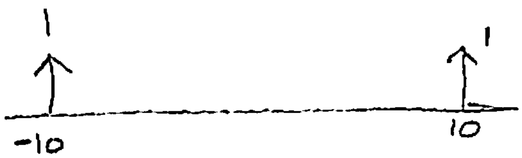
(b) AM:

$s(t) = (m(t) \cdot 2\cos(2\pi 10 t)) + 2\cos(2\pi 10 t)$

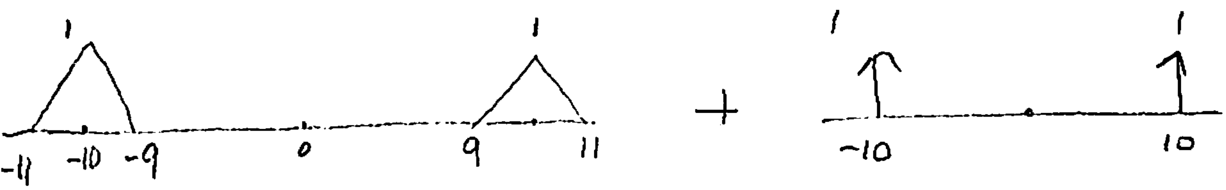
$F(2\cos(2\pi f_c t))$



$\circledast$

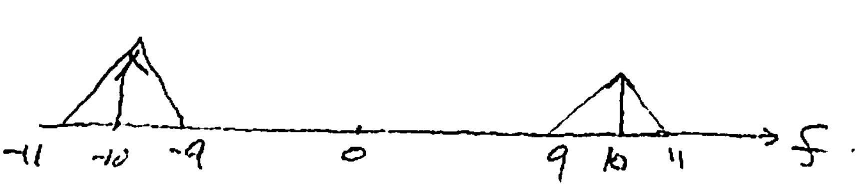


$\Downarrow$



$\Downarrow$

S(f)



Problem (1).

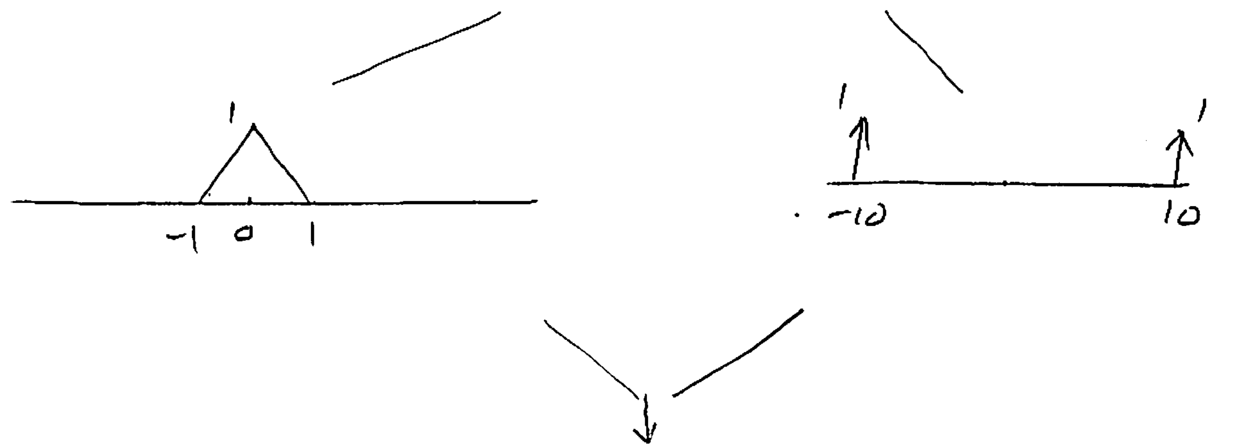
(b) (2) DSB-SC.

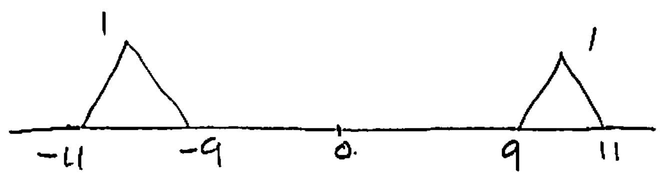
for DSB-SC, the modulated carrier is given by:

$$s(t) = m(t) \cdot c(t) = \text{sinc}^2(t) \cdot 2\cos(2\pi 10 t).$$

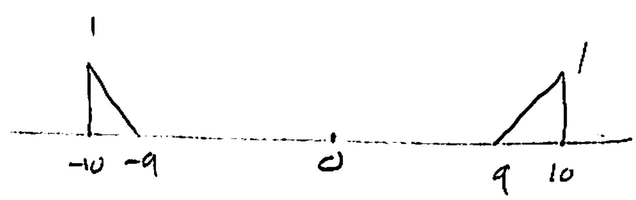
so

$$F(s(t)) = F(\text{sinc}^2(t)) \circledast F(2\cos(2\pi 10 t))$$

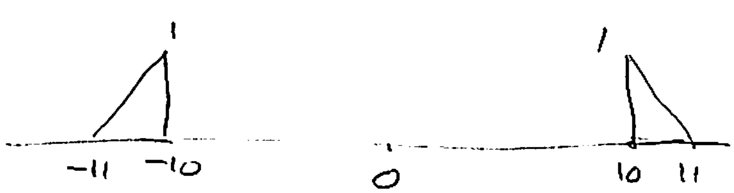




(3) LSB. here, we remove USB from DSB-SC to obtain:



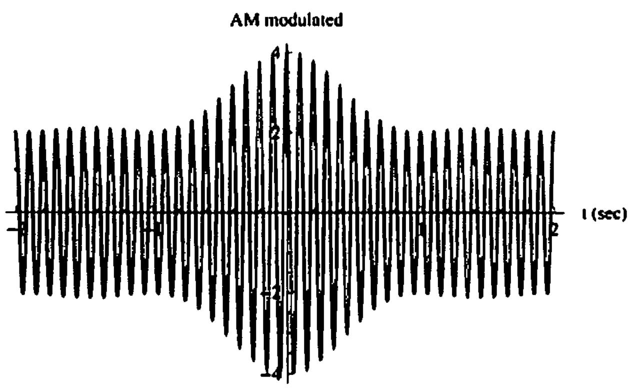
(3) VSB. remove LSB to obtain:



Problem (1). Part (c)

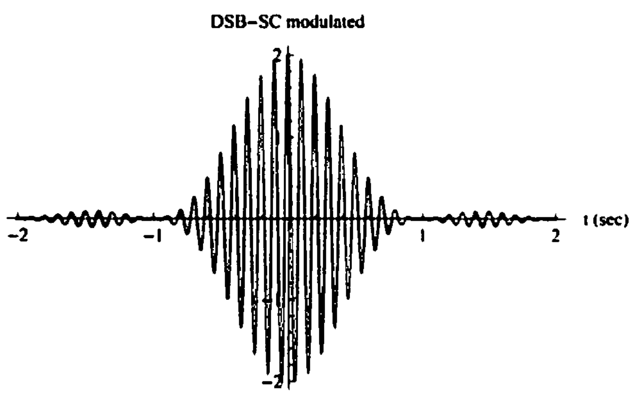
```
In[18]:= s = Sinc[Pi t]^2 2 Cos[2 π 10 t] + 2 Cos[2 π 10 t]
         Plot[s, {t, -2, 2}, PlotRange → All,
          AxesLabel → {"t (sec)", "AM modulated"}, PlotStyle → Thick]

Out[18]= 2 Cos[20 π t] + 2 Cos[20 π t] Sinc[π t]^2
```

Out[19]=



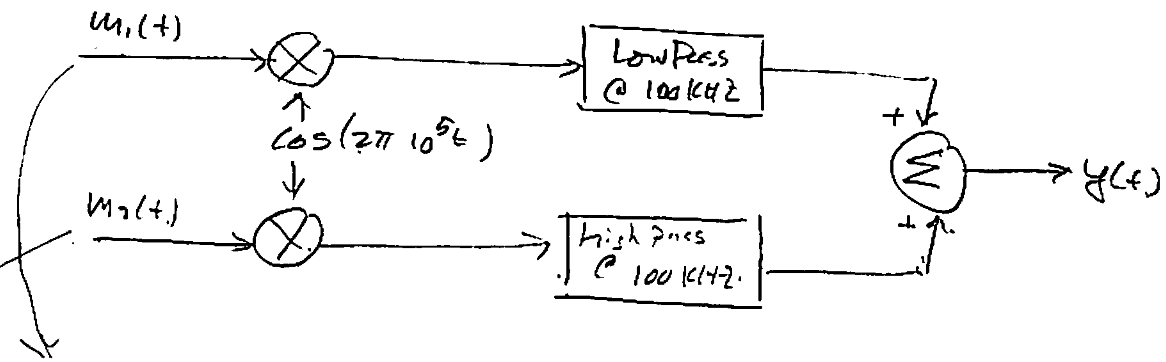

```
In[20]:= s = Sinc[Pi t]^2 2 Cos[2 π 10 t]
         Plot[s, {t, -2, 2}, PlotRange → All,
          AxesLabel → {"t (sec)", "DSB-SC modulated"}, PlotStyle → Thick]

Out[20]= 2 Cos[20 π t] Sinc[π t]^2
```

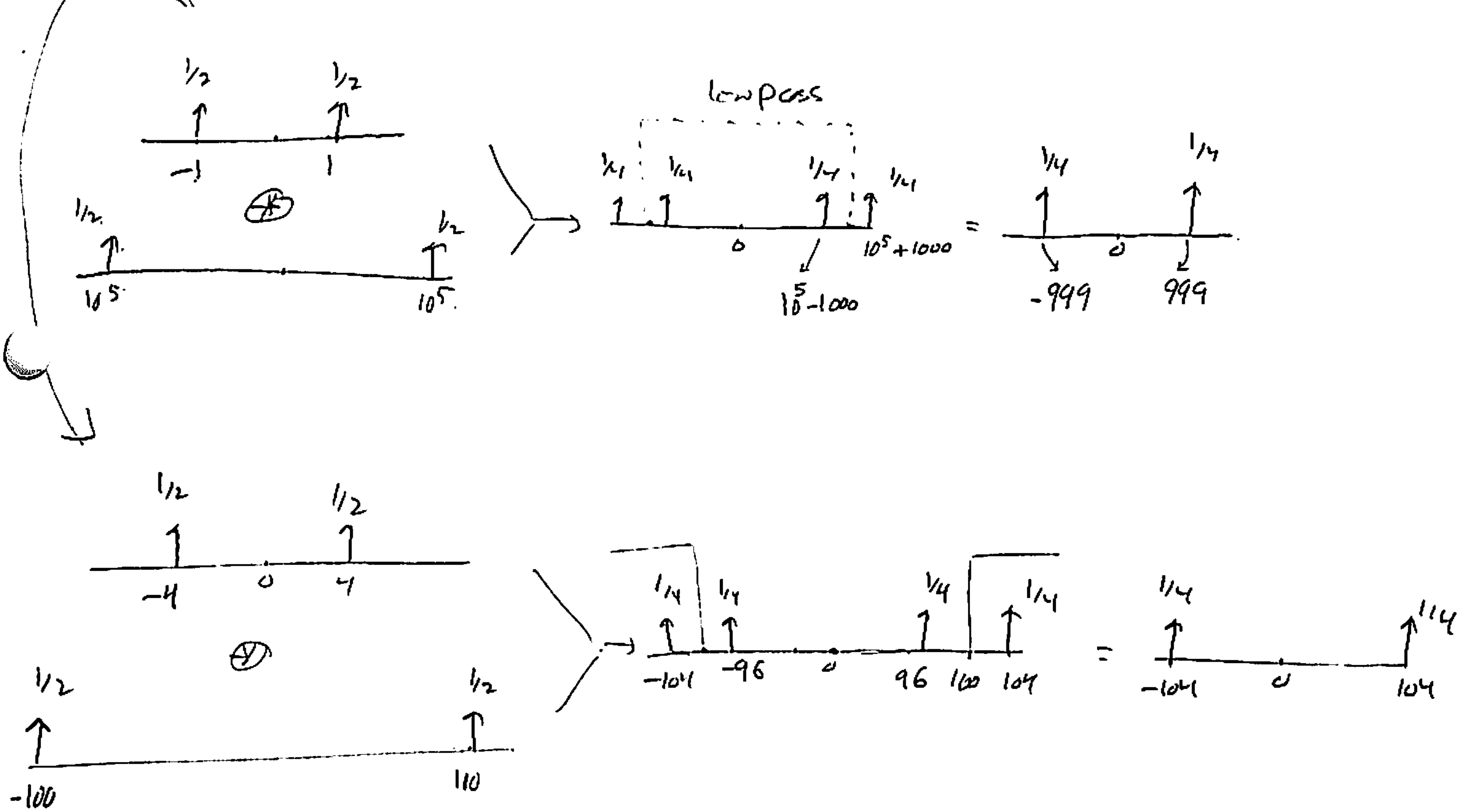
Out[21]=



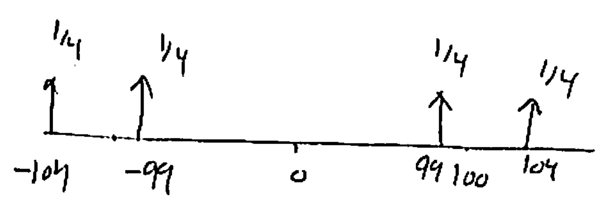
Problem 6

(a) $m_1(t) = \cos(2\pi\, 1000t)$, $m_2(t) = \cos(2\pi\, 4000t)$.
Find $y(t)$.





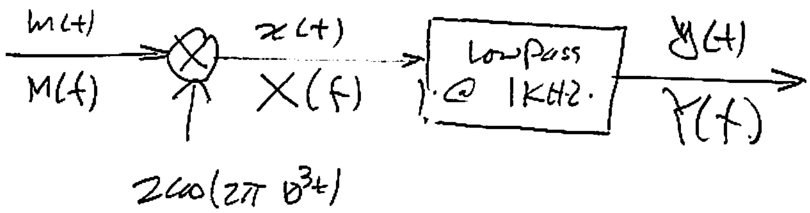
So in frequency, $Y(f) =$

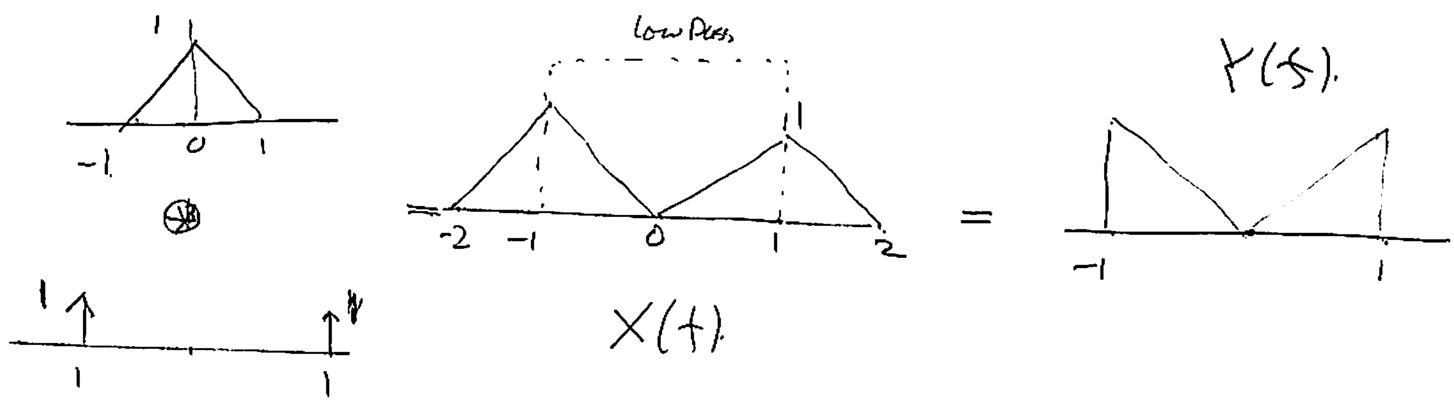


So in time domain

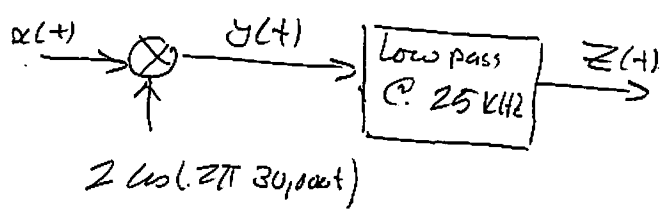
$$\boxed{y(t) = \frac{1}{2}\cos(2\pi\, 99000t) + \frac{1}{2}\cos(2\pi\, 10400t)}$$

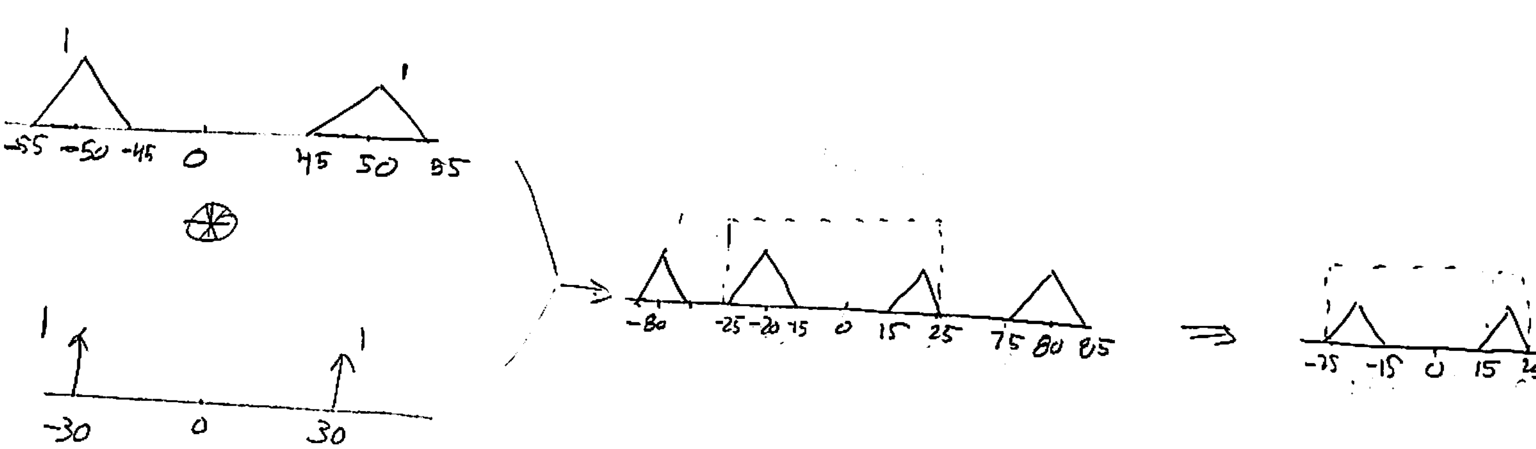
3 Sketch the spectrum X(f) and Y(f). in the system below



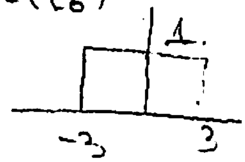


4 Sketch the pectrum





rect$\left(\frac{t}{6}\right)$



HW # 3

problem ①

$m(t) = 10\, \text{rect}\left(\frac{t}{6}\right)$.

ⓐ Find $\hat{m}(t)$

$$\hat{m}(t) = \frac{10}{\pi}\int_{-\infty}^{\infty} m(\tau)\,\frac{1}{(t-\tau)}\,d\tau = \frac{10}{\pi}\int_{-\infty}^{\infty} \frac{\text{rect}\left(\frac{\tau}{6}\right)}{t-\tau}\,d\tau$$

$$= \frac{10}{\pi}\left[\int_{-3}^{3} \frac{1}{t-\tau}\,d\tau\right] = \frac{10}{\pi}\ \ln(t-\tau)\Big|_{-3}^{3} = \frac{10}{\pi}\left[\ln(t-3) - \ln(t+3)\right]$$

$$= \boxed{\frac{10}{\pi}\ln\left(\frac{t-3}{t+3}\right)}$$

ⓑ $m_{USSB}(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$.

$$= \boxed{10\,\text{rect}\left(\frac{t}{6}\right)\cos(2\pi\, 1000t) - \frac{10}{\pi}\ln\left(\frac{t-3}{t+3}\right)\sin(2\pi\, 1000t)}$$

② $m(t) = 4\sin(12t)\cos(7t)\cos(3t)$

$$= 4\left[\tfrac{1}{2}\left(\sin 5t + \sin 19t\right)\right]\cos(3t)$$

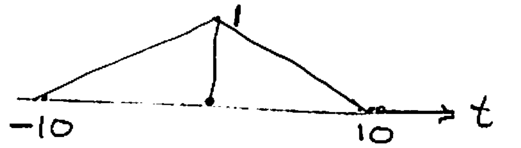
$$= 2\left[\sin 5t\cos 3t + \sin 19t\cos 3t\right]$$

$$= \sin 2t + \sin 8t + \sin 16t + \sin 22t.$$

$$\hat{m}(t) = \sin\left(2t - \tfrac{\pi}{2}\right) + \sin\left(8t - \tfrac{\pi}{2}\right) + \sin\left(16t - \tfrac{\pi}{2}\right) + \sin\left(22t - \tfrac{\pi}{2}\right)$$

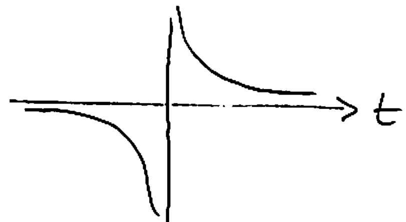
$$= -\cos 2t - \cos 8t - \cos 16t - \cos 22t.$$

③ $m(t) = tri\left(\frac{t}{10}\right)$.



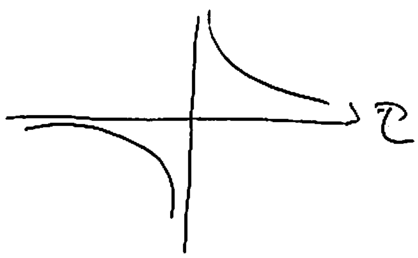
$\hat{m}(t) = m(t) \circledast \frac{1}{\pi t}$.

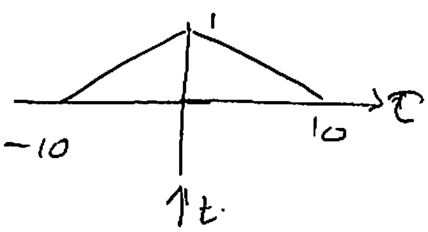
$\frac{1}{\pi t}$ =



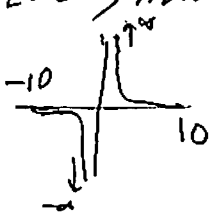
. so need to convolve with.

flip the tri function (easier). stay the same;

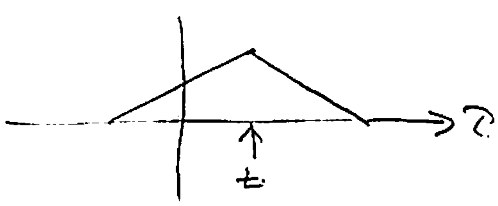




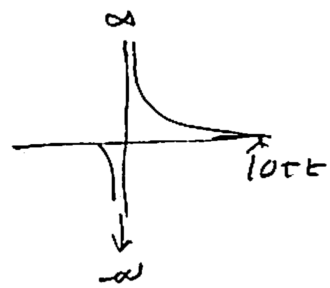
at $t=0$, multiply and integrate



so area = 0 (areas cancel) so at $t=0 \Rightarrow 0$

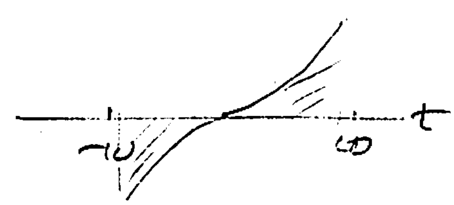


at $t>0$,

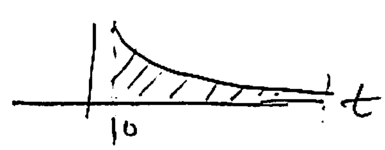


$\Rightarrow$

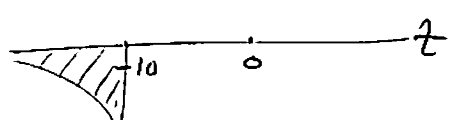
so for $-10 \le t \le 10$ we have



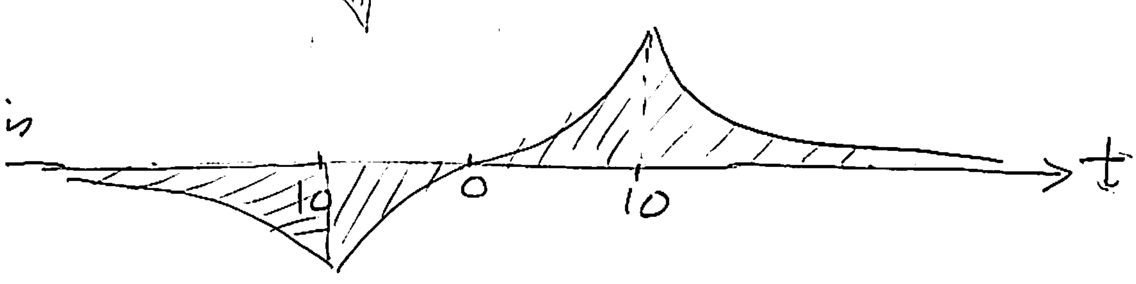
for $t > 10$, we get



for $t < 10$, we get



so final answer is



⑤ $m(t) = 2\sin(5000t)\sin(7000t)$.

$\alpha \qquad \beta$

$c(t) = \cos(2\pi 10^7 t)$

ⓓ Find $m_{LSSB}(t)$

$$m_{LSSB}(t) = m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t.$$

$$m(t) = 2\left[\tfrac{1}{2}\cos(-2000t) - \tfrac{1}{2}\cos(12000t)\right]$$

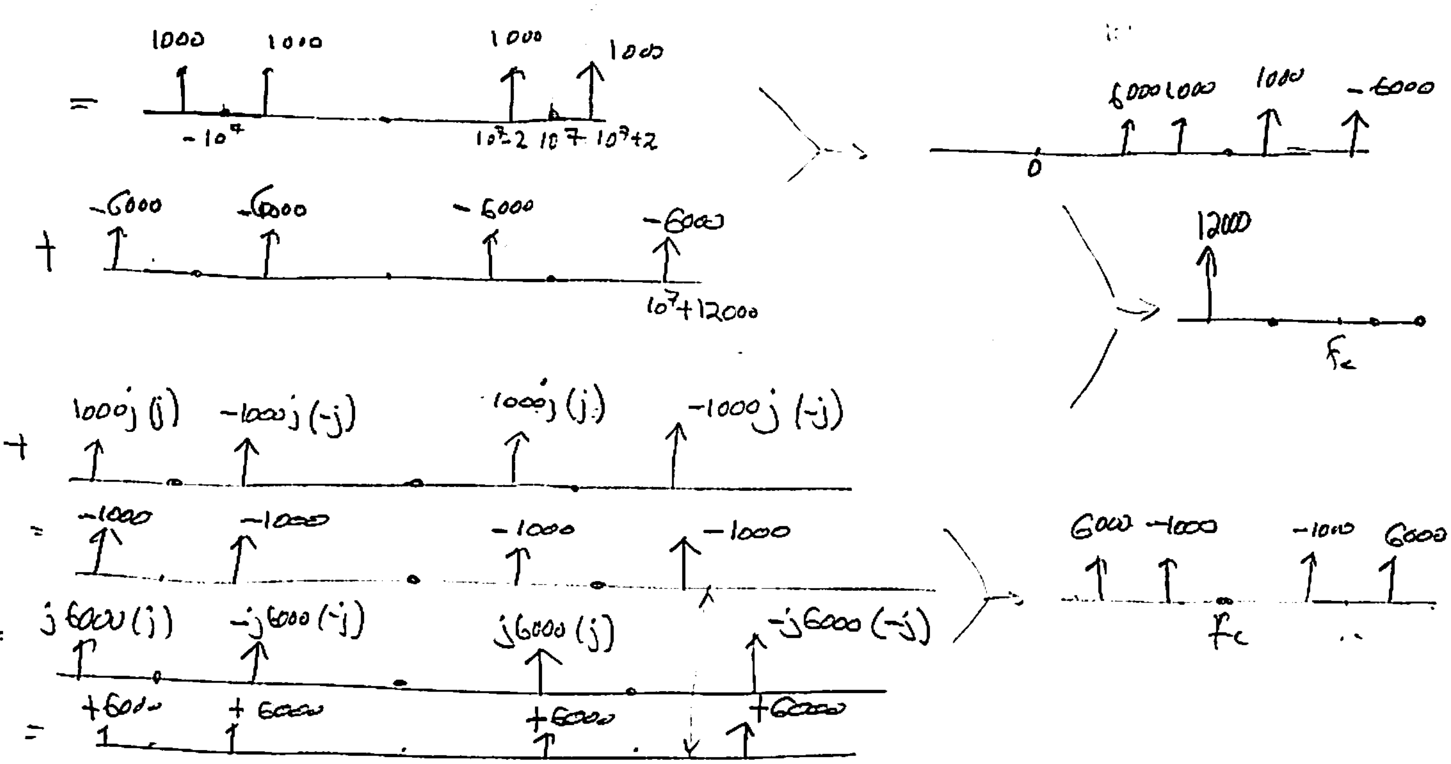
$$= \boxed{\cos(2000t) - \cos(12000t)}$$

so $\hat{m}(t) = \cos(2000t - \frac{\pi}{2}) - \cos(12000t - \frac{\pi}{2})$

$$= \boxed{\sin(2000t) - \sin(12000t)}$$

so $m_{LSSB}(t) = [\cos(2000t) - \cos(12000t)]\cos\omega_c t$
$+ [\sin(2000t) - \sin(12000t)]\sin\omega_c t$

$= \cos 2000t\cos\omega_c t - \cos 12000t\cos\omega_c t + \sin 2000t\sin\omega_c t -$
$\sin(12000t)\sin\omega_c t$.



(6) $m(t) = 24\cos(2\pi 100t) + 16\cos(2\pi 200t)$

$c(t) = 2\cos(2\pi 1000t)$.

let $m(t) = 24\cos(\alpha t) + 16\cos(\beta t)$

$c(t) = 2\cos(W_c t)$.

so $x(t) = 48\cdot\cos(\alpha t)\cos W_c t + 32\cos(\beta t)\cos W_c t$

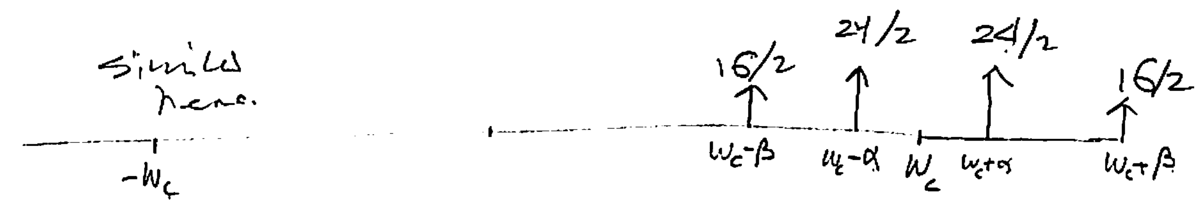
$= 48\left[\frac{1}{2}\cos((W_c-\alpha)t) + \frac{1}{2}\cos(W_c+\alpha)t\right]$

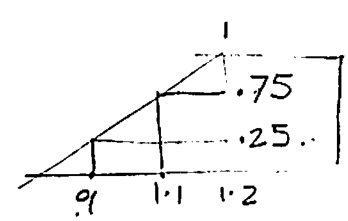
$+ 32\left[\frac{1}{2}\cos(W_c-\beta)t + \frac{1}{2}\cos(W_c+\beta)t\right]$

$= 24\cos(W_c-\alpha)t + 24\cos(W_c+\alpha)t$

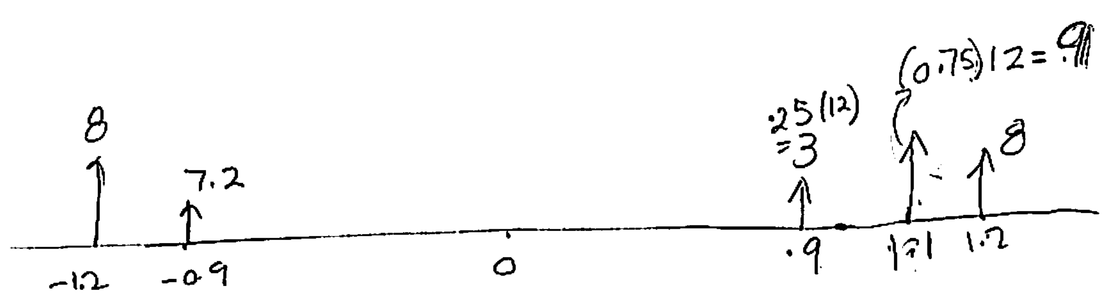
$+ 16\cos(W_c-\beta)t + 16\cos(W_c+\beta)t$.

so $X(f)$:



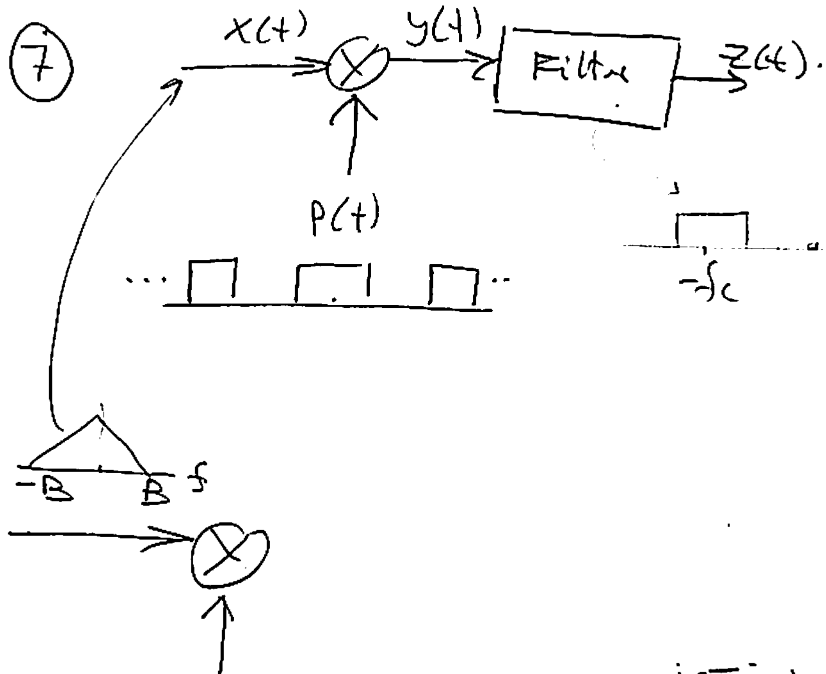


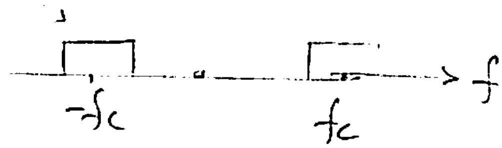
So $Y(f)$.



so $\boxed{y(t) = 16\cos(1200t) + 18\cos(1100t) + 6\cos(900t)}$

(7)



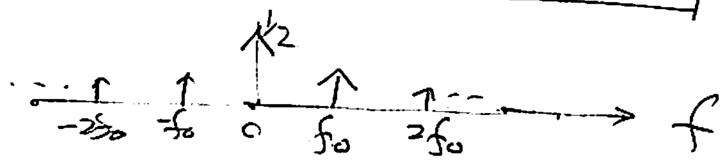


$$P(t) = \sum \tfrac{1}{2}\,\mathrm{sinc}\left(\tfrac{n}{2}\right) e^{j\frac{2\pi}{T_c}nt}.$$

$n = 1$
$d = \frac{1}{2}$

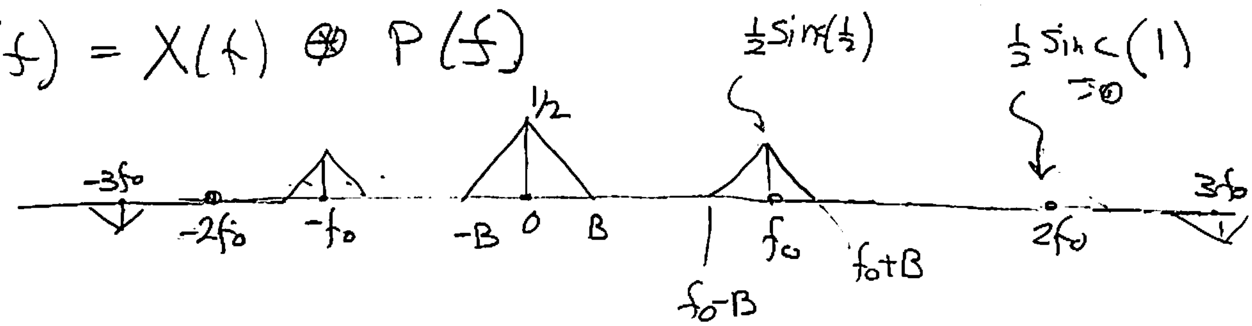
so $\boxed{P(f) = \tfrac{1}{2}\,\mathrm{sinc}\left(\tfrac{n}{2}\right) \sum \delta(f - nf_0)}$

so $P(f) =$

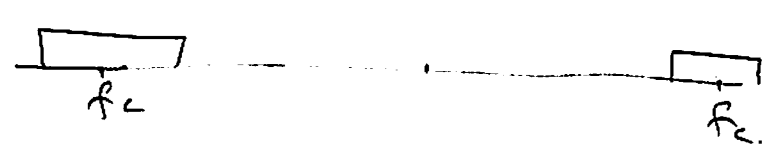


So $y(t) = x(t) \cdot p(t)$

or $Y(f) = X(f) \circledast P(f)$



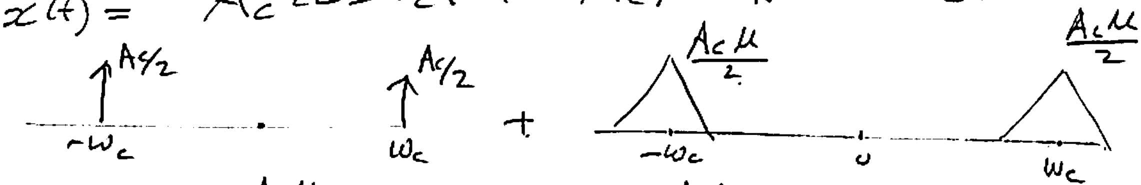
so when we multiply the above with $H(f)$ which is

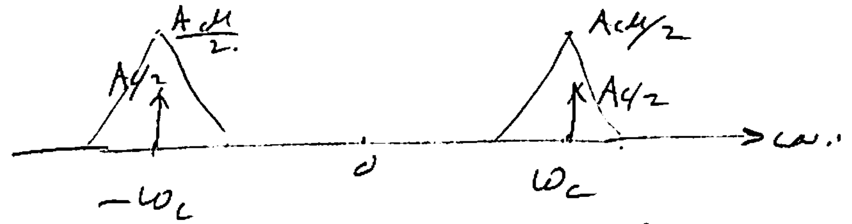


we will get, in the frequency domain $X(f_c - f) \times \underbrace{\tfrac{1}{2}\,\mathrm{sinc}(f - f_c)}_{\text{scale factor}}$

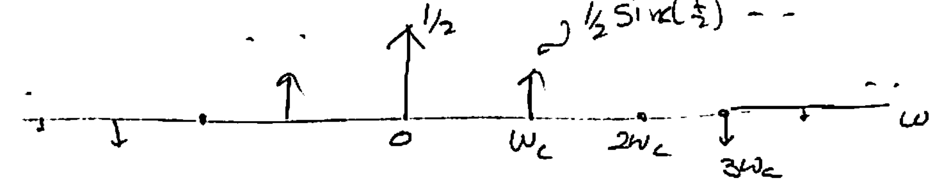
(b) if $x(t) = A_c[1+\mu m_n(t)] \cos(\omega_c t)$.

Then $x(t) = A_c \cos\omega_c t + A_c \mu\, m_N(t) \cos\omega_c t$.

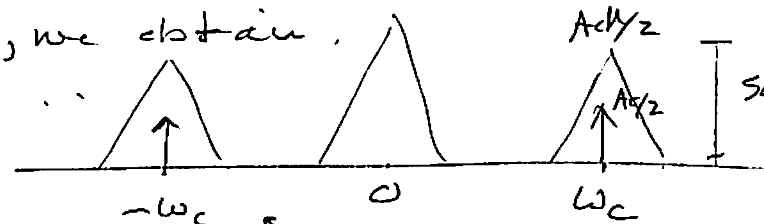
$X(f) =$ 

$=$ 

Convolve the above with $P(f)$, which is a puls train.

$P(f) =$ 

so, at ω_c, we obtain.

$Y(f) =$ 

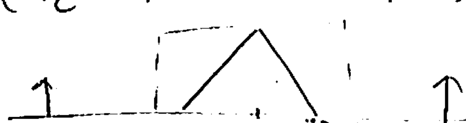
scaled by ½ sinc(½)

so $Z(f)$ will only contain these bandpass regions for bandpass.

hence $Z(f) = \frac{1}{2}\text{sinc}(\frac{1}{2})\left[\frac{A_c}{2}(f-f_c) + \frac{A_c \mu}{2} X(f-f_c)\right]$

$+ \frac{1}{2}\text{sinc}(\frac{1}{2})\left[\frac{A_c}{2}(f+f_c) + \frac{A_c \mu}{2} X(f+f_c)\right]$

so $z(t) = \frac{1}{2}\text{sinc}(\frac{1}{2}) \left[A_c \cos(\omega_c t) + A_c \mu.\ m_N(t)\right]$

If low pass, then 

here only this will be used.

$Z(f) = \frac{1}{2}\text{sinc}(\frac{1}{2}) \left[X(f)\right] = \frac{1}{2}\text{sinc}(\frac{1}{2}) \left[A_c \mu M(f)\right]$

so $z(t) = \frac{1}{2}\text{sinc}(\frac{1}{2}) A_c \mu\, m_N(t)$

$= \frac{1}{2} \frac{\sin(\pi/2)}{\pi/2} A_c \mu\, m_N(t) = \boxed{\frac{A_c}{\pi} \mu\, m_N(t)}$

(8)

$$v_i(t) = \frac{5}{1000}\cos(2\pi\, 600t) + \frac{1}{1000}\cos(2\pi\, 1000t).$$

$$v_o(t) = a_1 v_1(t) + a_2 v_2^2(t)$$

$$V_o(t) = a_1\left[\frac{5}{1000}\cos(2\pi 600t) + \frac{1}{1000}\cos(2\pi\, 1000t)\right]$$
$$+ a_2\left[\frac{5}{1000}\cos(2\pi\, 600t) + \frac{1}{1000}\cos(2\pi 1000t)\right]^2$$

$$= a_1 \frac{5}{1000}\cos(2\pi 600t) + a_1\frac{1}{1000}\cos(2\pi 1000t)$$
$$+ a_2\left[\left(\frac{5}{1000}\right)^2\cos^2(2\pi\, 600t) + \left(\frac{1}{1000}\right)^2\cos^2(2\pi\, 1000t)\right.$$
$$\left. + 2\left(\frac{5}{1000}\right)\left(\frac{1}{1000}\right)\cos(2\pi\, 600t)\cos(2\pi\, 1000t)\right]$$

expand, use trig identity, look for coeff. of $\cos(2\pi\, 1000t)$ and $\cos(2\pi\, 400t)$. equate to 1 and 0.001 to solve for a_1, a_2.

(9.) $$m_{AM}(t) = 10\left[1 + 0.8\cos(2\pi\, 2000t)\right]\cos(2\pi 10^6 t)$$

(a) Find modulation index. μ.

~~the~~ rewrite as $m_{AM}(t) = A_c[1 + \mu\, m_N(t)]\cos(2\pi \omega_c t)$

so $\boxed{\mu = 0.8}$

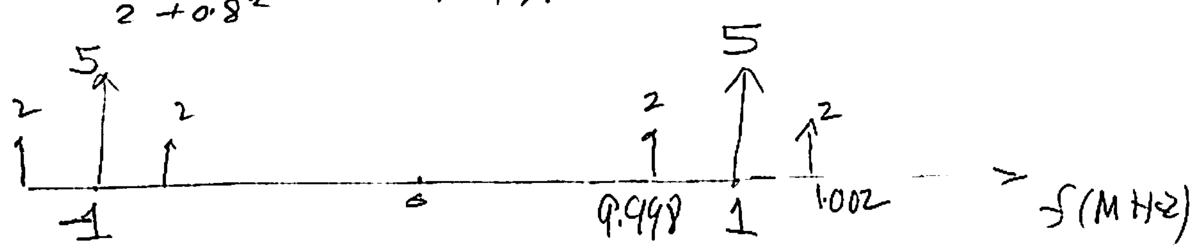
(b) transmission bandwidth. $2B$. when $B = 2$ kHz.

hence $B_T = \boxed{4\,KHZ}$

(c) efficiency = $$\frac{\text{Power in signal}}{\text{Total power}} = \frac{\frac{A_c^2\mu^2}{2}}{A_c^2 + \frac{A_c^2\mu^2}{2}} = \frac{\mu^2}{2+\mu^2}$$

$$= \frac{0.8^2}{2+0.8^2} = 24.24\%$$

(d)



(9) $m(t) = -6\cos(2\pi 10t) - 4\cos(2\pi 30t)$.

$c(t) = A_c \cos(2\pi 100t)$.

$\mu = 0.8$.

(a). $AM(t) = m(t)\cos W_c t + A_c \cos W_c t$

$= [A_c + m(t)] \cos W_c t$

$= A_c \left[1 + \frac{1}{A_c} m(t)\right] \cos W_c t$.

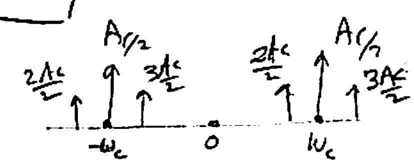
$= A_c \left[1 + \frac{1}{A_c}\left(-6\cos(2\pi 10t) - 4\cos(2\pi 30t)\right)\right] \cos W_c t$.

$= A_c \left[1 + \underbrace{\frac{\max(m(t))}{A_c}}_{\mu = 0.8} \cdot \frac{m(t)}{\max(m(t))}\right] \cos W_c t$

$$AM(t) = \boxed{A_c\left[1 + \mu\, m_N(t)\right] \cos W_c t.}$$

So $\boxed{m_N(t) = \frac{-6}{10}\cos(2\pi 10t) - \frac{4}{10}\cos(2\pi 30t)}$

(b) $\overline{m_N^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} m_N^2(t)\, dt$



Can also Find power from $|M(t)|$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{36}{100}\cos^2(2\pi 10t) + \frac{16}{100}\cos^2(2\pi 30t)$

$+ \frac{80}{100}\cos(2\pi 10t)\cos(2\pi 30t)\; dt$.

$= \frac{36}{100}\left(\frac{1}{2}\right) + \frac{16}{100}\left(\frac{1}{2}\right) + \frac{80}{100}\int \underbrace{\quad}_{=0} dt$

$= \boxed{0.26}$

(c) $Eff = \frac{\text{Av. Power in signal}}{\text{Total av. power}} = \frac{\frac{1}{2}A_c^2\mu^2\overline{m_N^2(t)}}{\frac{A_c^2}{2} + \frac{1}{2}A_c^2\mu^2\overline{m_N^2(t)}}$

$= \frac{0.8^2(0.26)}{1 + 0.8^2(0.26)} = 14.27\%$

HW#4

① $M_{EM}(t) = 50 \cos\left[2\pi\, 10^7 t + 6 \sin(2\pi\, 5000 t)\right]$

ⓐ For PM, $M_{EM}(t) = 50 \cos\left[2\pi f_c t + k_p\, m(t)\right]$.

$k_p = 3$, hence $M_{EM}(t) = 50 \cos\left[2\pi f_c t + 3\, m(t)\right]$.

hence $3\, m(t) = 6 \sin(2\pi\, 5000 t)$

hence $\boxed{m(t) = 2 \sin(2\pi\, 5000 t)}$

ⓑ for FM, $M_{EM}(t) = 50 \cos\left[2\pi f_c t + k_f \int m(\lambda)\, d\lambda\right]$

for $k_f = 2\pi\, 3000$ rad/s/V, then

$M_{EM}(t) = 50 \cos\left[2\pi f_c t + 2\pi\, 3000 \int m(\lambda)\, d\lambda\right]$.

hence. $2\pi\, 3000 \int m(\lambda)\, d\lambda = 6 \sin(2\pi\, 5000 t)$

or $\int m(\lambda)\, d\lambda = \frac{6}{2\pi\, 3000} \sin(2\pi\, 5000 t)$

or $m(t) = \frac{6}{2\pi\, 3000} \frac{d}{dt} \sin(2\pi\, 5000 t)$

$= \frac{6}{\cancel{2\pi}\, 3000} \cdot 2\cancel{\pi}\, 5000 \cdot \cos(2\pi\, 5000 t)$

$\boxed{m(t) = 10 \cos(2\pi\, 5000 t)}$

HW 4

② BW for FM = 220 KHZ.
Using Carson Rule.

ⓐ $\Delta f = 80$ KHZ. (max freq. deviation)

$$(B_T)_{carson} = 2(B + \Delta f) = 2(B + 80 \text{ KHZ}).$$

So $220 \text{ KHZ} = 2B + 160 \text{ KHZ}$.

So $B = \frac{60}{2} = \boxed{30 \text{ KHZ}}$

ⓑ $(B_T)_{carson} = 2(B + \Delta f)$.

Given $B = 20$ KHZ, then

$$220 \text{ KHZ} = 2(20 + \Delta f)$$

$$\frac{220 - 40}{2} = \Delta f$$

So $\boxed{\Delta f = 90 \text{ KHZ}}$

③

$$FM(t) = \cos\left(2\pi \cdot 30 \cdot 10^6 t + k_f \int m(\lambda)\, d\lambda\right)$$

$$w_i(t) = 2\pi\, 30 \cdot 10^6 + k_f\, m(t)$$

$$\Delta w = k_f \max|m(t)|$$

$$\boxed{\Delta w = k_f\,(6)} \Rightarrow \Delta f = 6\,(2000) = 12 \text{ KHZ}.$$

So Bandwidth is $(B_T)_{carson} = 2(f_m + \Delta f) = 2(6 + 12) = 36$ KHZ

HW 4

(4) $m(t) = 6\cos(2\pi\, 1000\, t)$

$f_c = 50\ \text{kHz}$.

$\beta = 9$.

Unmodulated Carrier Power $= 32$ Watt.

(a) Find frequency deviation Constant K_f.

$A_m = 6$

$f_m = 1000$ hz.

$$\beta = \frac{K_f A_m}{\omega_m}$$

so $$K_f = \frac{\beta \omega_m}{A_m} = \frac{(9)(2\pi\, 1000)}{6} = \pi\, 3000 \ \text{rad/s/V}.$$

(b) 60 kHz is 10 kHz away from Carrier

so

$n=0$ $n=1$ ... $n=10$

f_c f_c+f_m ... f_c+f_m

So for one side band, power is $\frac{A_c^2}{2} J_{10}^2(\beta=9)$

$= \frac{A_c^2}{2}(0.1247)^2$ Watt..

so 2 sided power is $\boxed{0.1247}^2 A_c^2$.

but $\frac{A_c^2}{2} = 32 \Rightarrow A_c^2 = 64$.

So 2 sided power $= (0.1247)^2\, 64$

50   51   52   53   ...   59   60

$n=0$ $n=1$ $n=2$ $n=3$ ... $n=9$ $n=10$

HW 4

5. $m_c(t) = 50 \cos[2\pi 10^6 t + 20t + 5\sin(2\pi 10t)]$

$\omega_i(t) = \frac{d}{dt}\theta_i(t) = 2\pi 10^6 + 20 + 5(2\pi \times 10)\cos(2\pi 10t)$

6. $m_{FM}(t) = 50 \cos[2\pi 10^7 t + 8\cos(2\pi 2000t)]$

what is max freq. deviation?

frequency deviation $= \frac{d}{dt}\phi(t) = \frac{d}{dt}[8\cos(2\pi 2000t)]$

$= -8\sin(2\pi 2000t)\ (2\pi 2000)$

so max is $8(2\pi 2000) = 2\pi 16000$

$= \boxed{16\ \text{KHz}}$

7. 30 MHz carrier.

single tone. $f_m = 11$ KHz.

max freq. deviation = 99 KHz.

a. 1% side bandwidth.

$\cos(\omega_c t + \beta \sin\omega_m t)$

$\beta = \frac{\Delta f}{f_m} = \frac{99}{11} = \boxed{9}$

from table, we find $\boxed{n = 13}$

so $(B_T)_{1\%} = 2 f_m n_{max} = 2(11)(13) = 286$ KHz.

b. $(B_T)_{Carson} = 2 f_m (1+\beta) = 2(11)(1+9) = 220$ KHz.

c. $(B_T)_{Carson} = 2(f_m + \Delta f)$

so $(B_T)^c_{Carson} \rightarrow 2(3 f_m + 3\Delta f) = 2(3\times 11 + 3\times 99) =$ 660 KHz

HW 4

(a) $m(t) = 60\cos(2\pi 1000t) + 20\cos(2\pi 3000t)$.

$c(t) = \cos(2\pi 10^5 t)$

$K_f = 2\pi 100$ rad/s/V.

(a) $M_{FM}(t) = \cos\left(2\pi 10^5 t + K_f \int 60\cos(2\pi 1000\lambda) + 20\cos(2\pi 3000\lambda)\, d\lambda\right)$

$= \cos\left(2\pi 10^5 t + K_f\left[\frac{60\sin(2\pi 1000t)}{2\pi 1000} + \frac{20\sin(2\pi 3000t)}{2\pi 3000}\right]\right)$

$= \cos\left(2\pi 10^5 t + 6\sin(2\pi 1000t) + \frac{2}{3}\sin(2\pi 3000t)\right)$

so $A_{m_1} = 6$, $f_{m_1} = 1000$ hz

$A_{m_2} = \frac{2}{3}$, $f_{m_2} = 3000$ hz.

$(B_T)_{Carson} = 2(f_m + \Delta f)$

$\Delta f = K_f \max|(m(t))|$.

but $\max|m(t)| = 60 + 20 = 80$.

so $\Delta f = 2\pi 100\,(80) = 2\pi\, 8000$ rad/sec. $= \boxed{8\,KHz}$

so $(B_T)_{Carson} = 2(3 + 8) = 22$ KHz

we take the larger of the frequencies

HW# 4

(12) FM transmitter A_m, f_m

$m(t) = 8\cos(2\pi\, 200t)$, $\beta = 6$.

unmodulated power is 12 Watt. across 50Ω.

(a) find frequency deviation k_f.

$$FM(t) = A_c \cos\left(\omega_c t + k_f \int m(\lambda)\,d\lambda\right)$$

$$= A_c \cos\left(\omega_c t + 8 k_f \frac{\sin(2\pi\, 200t)}{2\pi\, 200}\right)$$

Compare to canonical form

$$FM(t) = \cos(\omega_c t + \beta \sin(\omega_m t))$$

so $\boxed{\beta = \frac{8 k_f}{2\pi\, 200}}$

so $k_f = \frac{(6)(2\pi\, 200)}{8} = \boxed{942.477}$ rad/s/Volt

(b) to Find A_c, use Power specifications

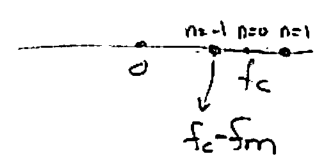
since $\text{Power} = \frac{1}{\Omega} \frac{A_c^2}{2}$

Then $12 = \frac{1}{50} \frac{A_c^2}{2}$. solve for $A_c = \sqrt{(12)(50)(2)} = 34.64$ Volt

Peak amplitude at $f_c - 200$ hence $\boxed{n = +1}$ since $f_m = 200$.

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

From tables $\boxed{J_1(6) = -0.2767}$

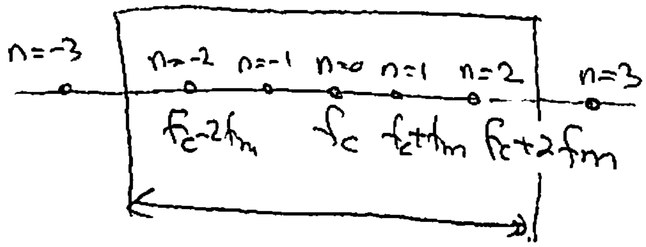


so for $n = +1$, we have

$$\text{Amplitude} = \underbrace{A_c\,(J_1(6))}\cos(\omega_c + \omega_m)t$$

$$= (34.64)(-0.2767) = \boxed{9.584 \text{ Volt}}$$

(10) $\beta = 1$,



$J_0(1) = 0.7652$

$J_1(1) = 0.4401$

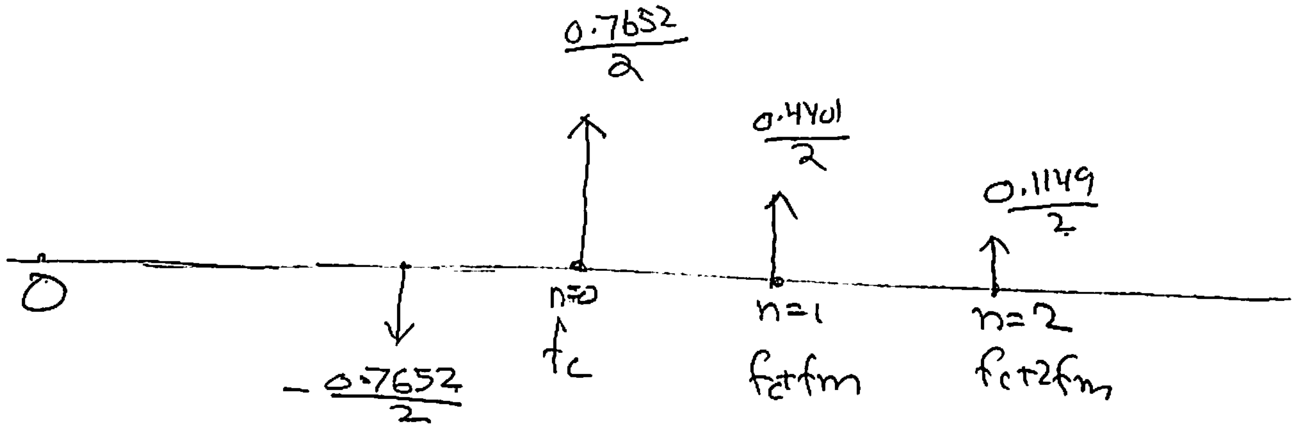
$J_2(2) = 0.1149$

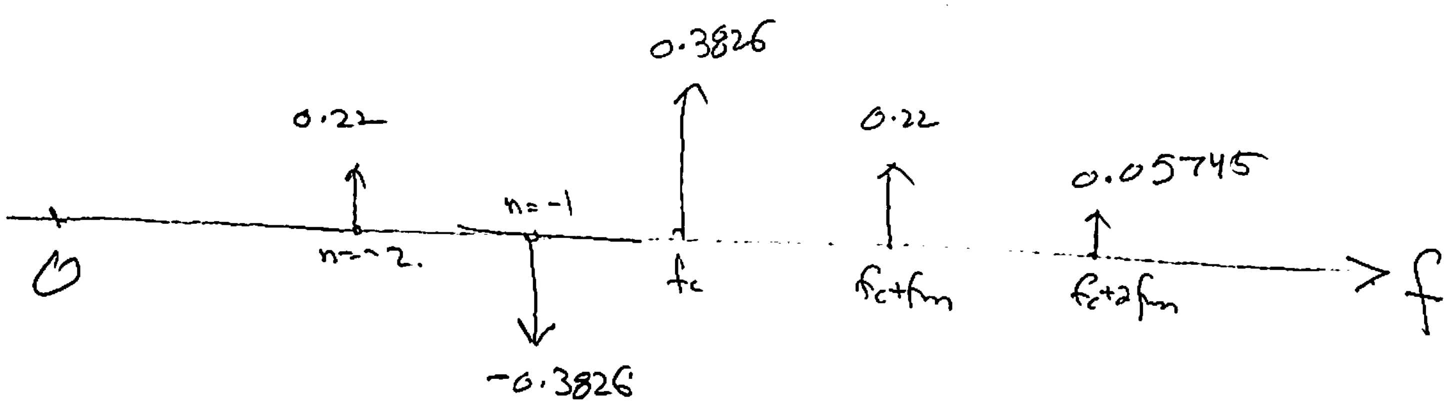
$$FM(t) = \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right).$$

so output $FM(f)$ is

$$1 = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta\left(f - (f_c + n f_m)\right) + \delta\left(f + (f_c + n f_m)\right)\right]$$

need to go to $n=2$ on right side.





HW#4

#11

$m_{FM}(t) = 25\cos(2\pi 10^6 t + 8\sin(2\pi 3000t)]$

(a) Total average power $= \left(\frac{25^2}{2}\right)/50 = 6.25$ Watt.

(b) The Fourier Series of the above is

$$\hat{m}_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m t)$$

where $\omega_c = 2\pi 10^6$ rad/sec $\quad \boxed{\beta = 8}$
$\omega_m = 2\pi 3000$ rad/sec.

so at $n=0$, we have $A_c J_0(8) \cos(2\pi 10^6 t)$

so the power of this is $\frac{[A_c J_0(8)]^2}{2} = \frac{[(25)(0.1717)]^2}{2}$

$= 9.212$

so over $50\Omega \longrightarrow \frac{9.212}{50} = 0.18425$ Watt.

so % is $\frac{0.18425}{6.25} \times 100 = 2.9\%$.

(c) to Find peak freq. deviation:

Frequency deviation $= \frac{d}{dt}$ Phase deviation

$= \frac{d}{dt}\left(8\sin(2\pi 3000t)\right)$

$= 8 \cos(2\pi 3000t)\ (2\pi 3000)$

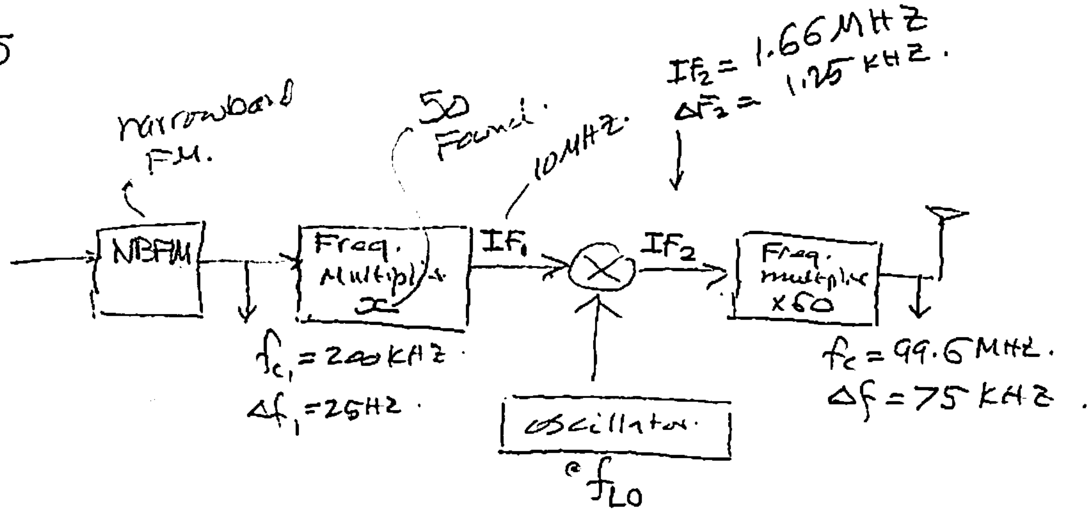
so max of the above is $(8)(2\pi 3000) = 150{,}796$ rad/sec
$= \boxed{24\ \text{KHZ}}$

(d) $n_{max} = 11$

so $(B_T)_{1\%} = 2 f_m n_{max} = 2(3\text{KHZ})(11) = 66$ KHZ

HW 5

(1)



we need $IF_1 = \dfrac{99.6\text{ MHz}}{60} = 1.66\text{ MHz}$.

Oscillator do not affect Δf.

hence $(\Delta f_1) x (60) = \Delta f$

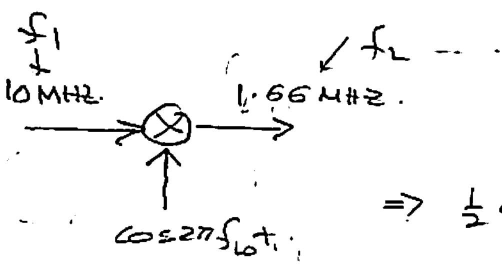
so $(25) x (60) = 75{,}000$.

hence $x = \dfrac{75{,}000}{(25)(60)} = \boxed{50}$ so multiplication factor = 50

now we use this to find the rest.

$IF_1 = (200\text{ kHz})(50) = 10\text{ MHz}$.

$IF_2 = \dfrac{99.6\text{ MHz}}{60} = 1.66\text{ MHz}$.



$\Rightarrow \frac{1}{2}\cos(2\pi(f_{LO}+f_1)t + \frac{1}{2}\cos(2\pi(f_{LO}-f_1)t)$.

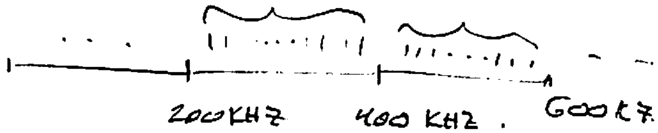
need $f_{LO}+f_1 = 1.66\text{ MHz}$.

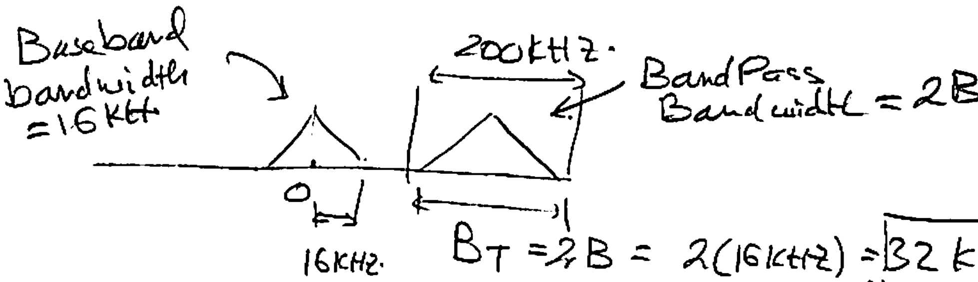
or $f_{LO}-f_1 = 1.66\text{ MHz}$.

so $f_{LO} = 10\text{ MHz} - 1.66\text{ MHz} = \boxed{8.34\text{ MHz}}$

HW5
3

12 audio @ 16KHZ.





$B_T = 2B = 2(16\,\text{KHZ}) = \boxed{32\,\text{KHZ}}$

So $200\,\text{KHZ} \geq 32\,\text{KHZ}\,(1+\beta)$ ← ask about this.

$= \beta \leq 5.25$

(b) $(B_T) = 2(B + \Delta f) = 2(16\,\text{kHZ} + 2\,\text{MHZ})$

B_T: Carson.

B: 16kHZ.

$= 4.032\ \text{MHZ}.$

(Why answer 9MHZ)?

HW #5

9

FM

$$\longrightarrow \boxed{\quad} \longrightarrow (SNR)_o = \frac{3}{4}\frac{A_c^2 \beta^2}{N_o f_m} = \frac{3}{4}\frac{A_{FM}^2 \beta^2}{N_o f_m}$$

for AM

$$AM(t) = A_c \cos w_c t + m(t) \cos w_c t$$

$$= A_c (1 + \mu\, m(t)) \cos w_c t.$$

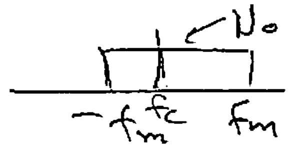
let $\mu = 1$

$$= A_c (1 + m(t)) \cos w_c t.$$

$$= A_c (1 + A_m \cos(2\pi f_m t)) \cos w_c t.$$

$$(S_o)_{Am} = \frac{A_m^2}{2}.$$

$$(P_o)_{Am} = 2 N_o f_m$$



so $(SNR)_{o,Am} = \boxed{\dfrac{A_m^2}{4 N_o f_m}}$

so need.

$$\frac{A_m^2}{4 N_o f_m} = \frac{3}{4}\frac{A_{FM}^2 \beta^2}{N_o f_m}$$

so $\dfrac{A_{Am}}{A_{FM}} = \sqrt{3\beta^2}$ but $\beta = 9$.

$$= \sqrt{3\, 9^2} = \sqrt{3}\, 9 = \boxed{15.58}$$

HW#5

10

$m(t) = 8\cos(2\pi 5000t)$

$K_f = 2\pi 4000$ rad/s/V.

$A_c = 0.1V$.

$$(SNR)_o = \frac{3}{4}\frac{A_c^2 \beta^2}{N_o f_m} = \frac{3}{4}\frac{(0.1)^2 \beta^2}{(10^{-6})(5000)}. \quad \text{—— } (1)$$

$$\text{but } \beta = \frac{K_f A_m}{2\pi f_m} = \frac{(2\pi 4000)(8)}{2\pi(5000)}$$

Plug all this into (1) and calculate

$$(SNR)_o = 614.4$$

$$\text{so in db} = 10\log_{10}(614.4) = 27.88 \text{ db}$$

HW5
11

FM, $(SNR)_i = 30$ db.

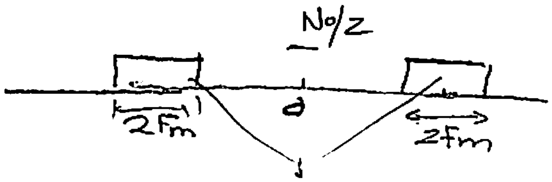
$(SNR)_o = 48$ db.

$$(SNR)_i \xrightarrow[P_i]{S_i} \boxed{\text{FM receiver}} \longrightarrow (SNR)_o$$

$$(SNR)_o \to \frac{3}{4}\frac{A_c^2 \beta^2}{N_o f_m}$$

$S_i = \dfrac{A_c^2}{2}$.

$P_i = 2 N_o f_m$



area $= 2N_o f_m$

here. $$10 \log\left(\frac{A_c^2}{2N_o f_m}\right) = 30$$

$$10 \log\left(\frac{3}{4}\frac{A_c^2 \beta^2}{N_o f_m}\right) = 48$$

so $$\frac{A_c^2}{2N_o f_m} = 10^3$$

so $$10 \log\left(\frac{3}{2}\beta^2 (10^3)\right) = 48.$$

so $$\frac{3}{2}\beta^2 (10^3) = 10^{4.8}$$

so $$\beta^2 = \frac{\left(\frac{2}{3} 10^{4.8}\right)}{10^3} \Rightarrow \boxed{\beta = 6.485}$$

HW#5

7

Power in signal after filter =

$$\int_{-2,000}^{2,000} S_m(f)\,df = (2\text{ kHz})(10^{-2}) + (1\text{ kHz})(10^{-2})$$

$$= (3,000)(10^{-2}) = 30 \text{ Watt}$$

Power in noise after filter $= \int_{-2,000}^{2,000} \frac{1}{2}\times 10^{-6}\,df$

$$= \left(\tfrac{1}{2}\times 10^{-6}\right)(4,000) = (10^{-6})(2,000) = 2\times 10^{-3} \text{ watt}$$

so $SNR = \dfrac{30}{2\times 10^{-3}} = 1.5\times 10^{4}$

so in db,

$$(SNR)_{db} = 10\log_{10}(1.5\times 10^{4})$$

$$= 10\left[\log_{10}1.5 + \log_{10}10^{4}\right]$$

$$= 10\left(\log_{10}1.5 + 4\right)$$

$$= 40 + 10\log_{10}1.5$$

$$= \boxed{41.76}$$

HW#5

8) $m(t) = 2\cos(2\pi\, 5000t)$

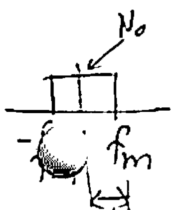
$c(t) \Rightarrow \omega_c = 2\pi\, 30\times 10^3$

$k_f = 2\pi\, 15000$ rad/sec per Volt.

$SNR_i \longrightarrow \boxed{} \longrightarrow SNR_o = 40$ db.

$S_i = \dfrac{A_c^2}{2}$ $\qquad\qquad$ $S_o = k_f^2 \dfrac{A_m^2}{2}$

$\longrightarrow$ demodulator FM $\longrightarrow$



$P_i = f_m N_o$ $\qquad\qquad$ $P_o = \dfrac{8\pi^2 N_o f_m^3}{3A_c^2}$

Bandwidth at Baseband = fm

so $(SNR)_i = \dfrac{A_c^2}{2 f_m N_o}$ $\qquad\qquad$ $(SNR)_o = \dfrac{3}{4}\dfrac{A_c^2 \beta^2}{N_o f_m}$

hence we see that $\boxed{(SNR_o) = \frac{3}{2}\beta^2 (SNR)_i}$

so in db, $40 = 10\log_{10}\left(\frac{3}{2}\beta^2\right) + (SNR)_{i,db}$

so $(SNR)_{i,db} = 40 - 10\log_{10}\left(\frac{3}{2}\beta^2\right)$.

But $\beta = \dfrac{k_f A_m}{2\pi f_m} = \dfrac{2\pi(15000)(2)}{2\pi(5000)} = \dfrac{30{,}000}{5{,}000} = \boxed{6}$

so $(SNR_i)_{db} = 40 - 10\log_{10}\left(\frac{3}{2}6^2\right) = \boxed{22.6761}$

HW 5

12. AM: $A_c \cos \omega_c t + m(t) \cos \omega_c t$.

FM: $A_c \cos(\omega_c t + k_f \int m(\lambda) d\lambda)$

(a) B_T for AM $= 2 f_m$.

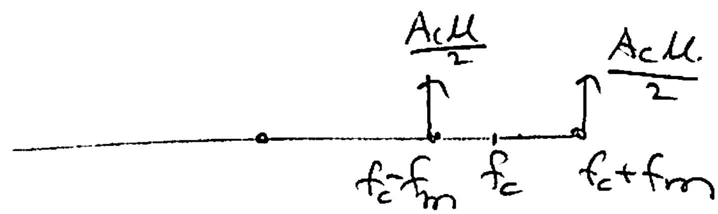
Frequency deviation for FM is $\frac{d}{dt}\left(k_f \int m(\lambda) d\lambda\right) = k_f\, m(t)$.

so max of this $= k_f |m(t)|_{max} = k_f A_m \equiv \Delta f$

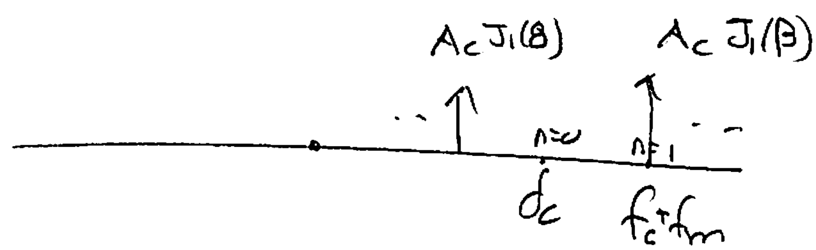
so $\overset{\Delta f}{(k_f A_m)} = 4(2 f_m)$

$\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \frac{8 f_m}{f_m} = \boxed{8}$

(b) For AM



For FM



So

$$A_c J_1(8) = \frac{A_c \mu}{2}$$

so

$$\mu = 2 J_1(8) = \boxed{0.469}$$

HW 6, Problem 1

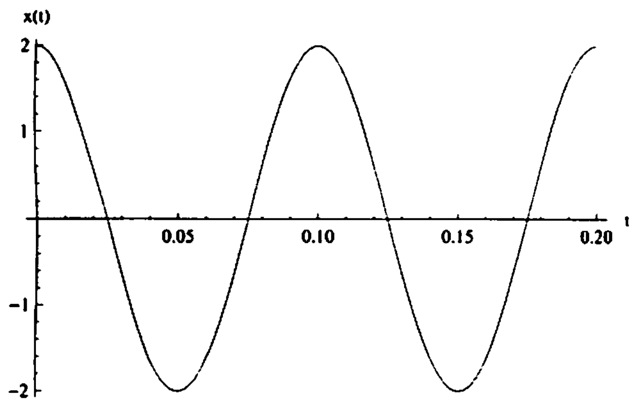
by Nasser M. Abbasi

In[281] := << dsp`

- **part (a)**

In[282] :=
```
Clear[w, t];
fm = 10;
period = 1 / fm;
x[t_] := 2 Cos[2 Pi fm t]
Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}]
```

Out[286]=



- **part(b)**

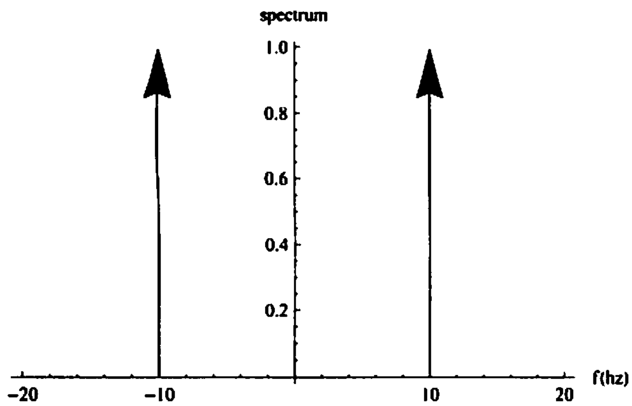
In[287] :=
```
ft = FourierTransform[x[t], t, f, FourierParameters → {0, -2 Pi}];
Print["Fourier Transform of x(t) is"];
ft
```

Fourier Transform of x(t) is

Out[289]= DiracDelta[-10 + f] + DiracDelta[10 + f]

In[290]:=
```
dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

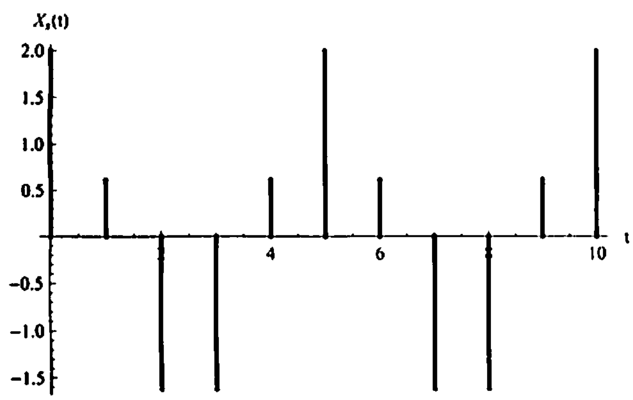
Out[291]=



- **part(c)**

In[292]:=
```
Ts = 0.02;
nSamples = 2 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling → Axis, FillingStyle → Thick, AxesLabel → {"t", "Xs(t)"}]
```

Out[295]=



- **part(d)**

In[296]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

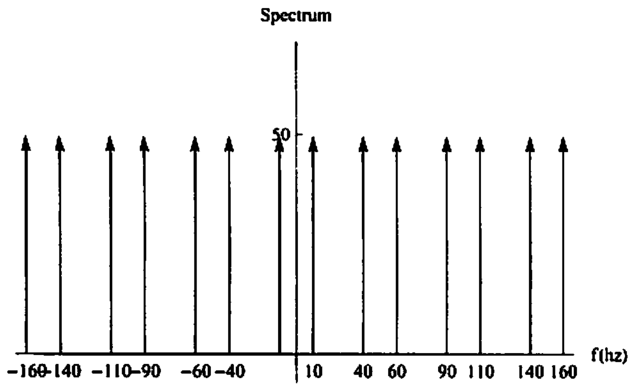
```
Sampling frequency = 50. hz
```

In[390]:=
```
spectrum = Expand[fs Sum[ft /. f → (f - n * fs), {n, -3, 3}]];
Print["spectrum=", spectrum];
```

```
spectrum=50. DiracDelta[-160. + f] + 50. DiracDelta[-140. + f] + 50. DiracDelta[-110. + f] +
 50. DiracDelta[-90. + f] + 50. DiracDelta[-60. + f] + 50. DiracDelta[-40. + f] +
 50. DiracDelta[-10 + f] + 50. DiracDelta[10 + f] + 50. DiracDelta[40. + f] + 50. DiracDelta[60. + f] +
 50. DiracDelta[90. + f] + 50. DiracDelta[110. + f] + 50. DiracDelta[140. + f] + 50. DiracDelta[160. + f]
```

In[352]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -3 * fs, 3 * fs, -.1 fs, 1.4 fs, Small],
  AxesLabel → {"f(hz)", "Spectrum"},
  Ticks → {{-160, -140, -110, -90, -60, -40, -10, 10, 40, 60, 90, 110, 140, 160}, {50}}]
```

Out[352]=



- **part(e)**

In[374]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

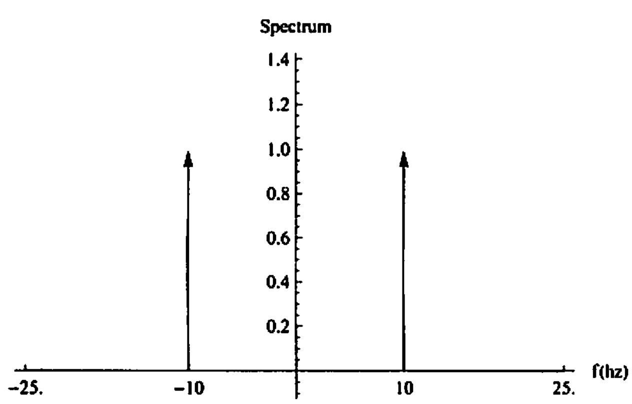
```
bandwidth = 25. hz
```

In[345]:=
```
gain = Ts;
Print["Gain=", gain];
```

```
Gain=0.02
```

In[396]:=
```
spectrum = fs DiracDelta[-10 + f] + fs DiracDelta[10 + f];
Show[First@dsp`plotFourierTransform[
   Expand[gain * spectrum], f, -bandwidth, bandwidth, -.1 , 1.4 , Small],
  AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-bandwidth, -10, 0, 10, bandwidth}}]
```

Out[397]=



- **part(f)**

From the output above, we conclude that $y(t) = 2 \text{Cos}[2 \pi\, 10\, t]$

HW 6, Problem 2

by Nasser M. Abbasi

In[96] := << dsp`

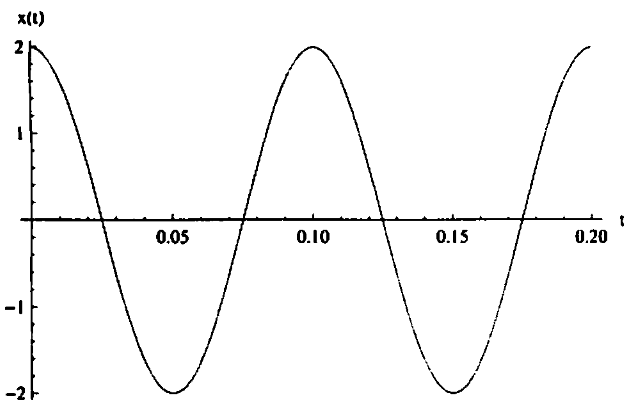
- **part (a)**

In[144] :=
```
Clear[w, t, t, f];
fm = 10;
period = 1 / fm;
Print["Period of message = ", N@period , " seconds"];
```

```
Period of message = 0.1 seconds
```

In[148] :=
```
x[t_] := 2 Cos[2 Pi fm t]
Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}]
```

Out[149]=



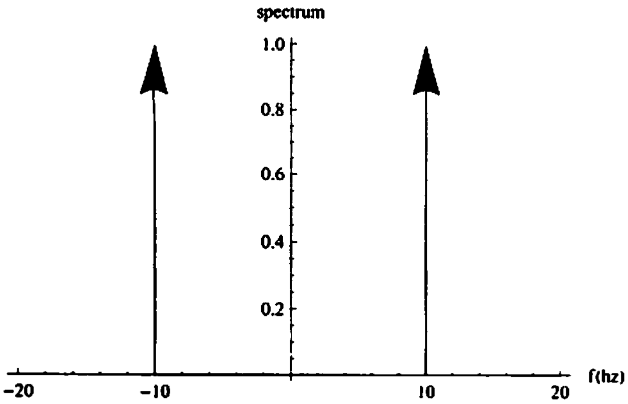
- **part(b)**

In[150] :=
```
ft = FourierTransform[x[t], t, f, FourierParameters → {0, -2 Pi}];
Print["Fourier Transform of x(t) is", ft];
```

```
Fourier Transform of x(t) isDiracDelta[-10 + f] + DiracDelta[10 + f]
```

In[152]:=
```
dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[153]=



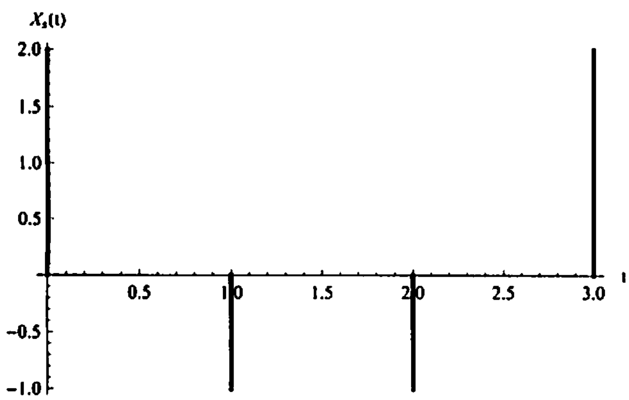
- **part(c)**

In[156]:=
```
Ts = 1 / 15;
Print[" samping period = ", N@Ts, " seconds"];
```

```
 samping period = 0.0666667 seconds
```

In[158]:=
```
nSamples = 2 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling → Axis, FillingStyle → Thick, AxesLabel → {"t", "Xs(t)"}]
```

Out[160]=



- **part(d)**

In[161]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

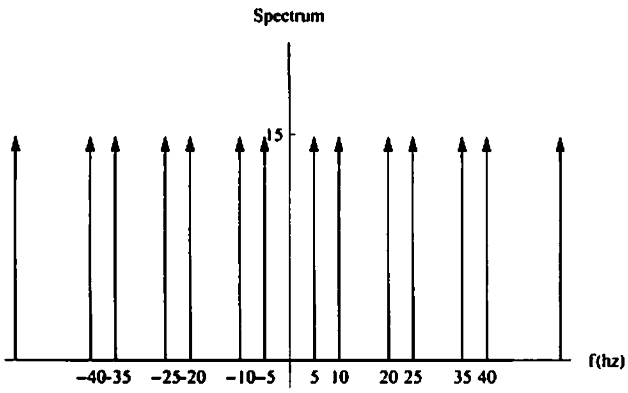
```
Sampling frequency = 15 hz
```

In[164]:=
```
spectrum = Expand[fs Sum[ft /. f → (f - n * fs), {n, -3, 3}]];
Print["spectrum=", spectrum];
```

```
spectrum=15 DiracDelta[-55 + f] + 15 DiracDelta[-40 + f] + 15 DiracDelta[-35 + f] +
  15 DiracDelta[-25 + f] + 15 DiracDelta[-20 + f] + 15 DiracDelta[-10 + f] +
  15 DiracDelta[-5 + f] + 15 DiracDelta[5 + f] + 15 DiracDelta[10 + f] + 15 DiracDelta[20 + f] +
  15 DiracDelta[25 + f] + 15 DiracDelta[35 + f] + 15 DiracDelta[40 + f] + 15 DiracDelta[55 + f]
```

In[166]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -3 * fs, 3 * fs, -.1 fs, 1.4 fs, Small],
  AxesLabel → {"f(hz)", "Spectrum"},
  Ticks → {{-40, -35, -25, -20, -10, -5, 5, 10, 20, 25, 35, 40}, {fs}}]
```

Out[166]=



- **part(e)**

In[167]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

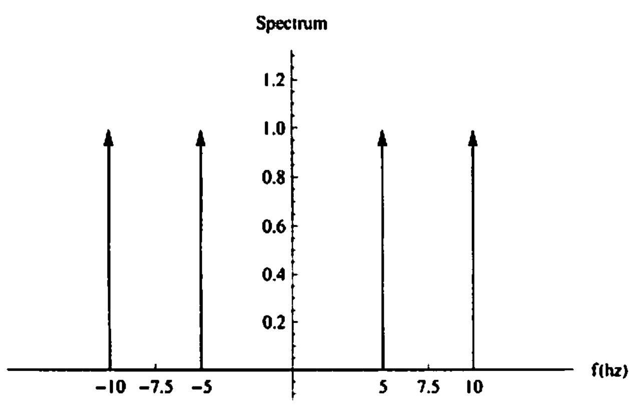
```
bandwidth = 7.5 hz
```

In[169]:=
```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.0666667
```

In[173]:=
```
spectrum =
  (15 DiracDelta[-10 + f] + 15 DiracDelta[-5 + f] + 15 DiracDelta[5 + f] + 15 DiracDelta[10 + f]);
Show[First@dsp`plotFourierTransform[Expand[gain * spectrum],
   f, -2 * bandwidth, 2 * bandwidth, -.1 , 1.3 , Small],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-bandwidth, -10, -5, 0, 5, 10, bandwidth}}]
```

Out[174]=

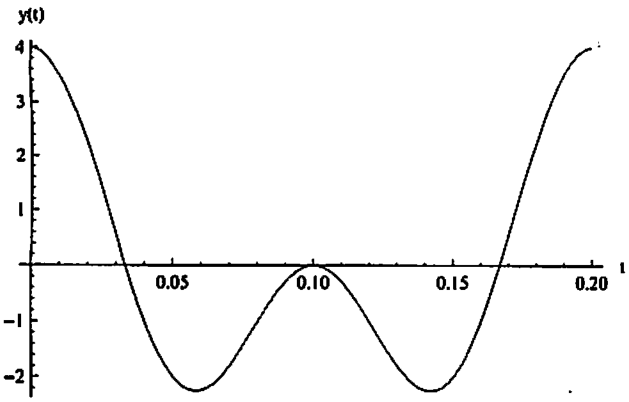


- **part(f)**

From the output above, we conclude that $y(t) = 2\,\mathrm{Cos}[2\,\pi\,10\,t]+2\mathrm{Cos}[2\,\pi\,5\,\mathrm{t}]$

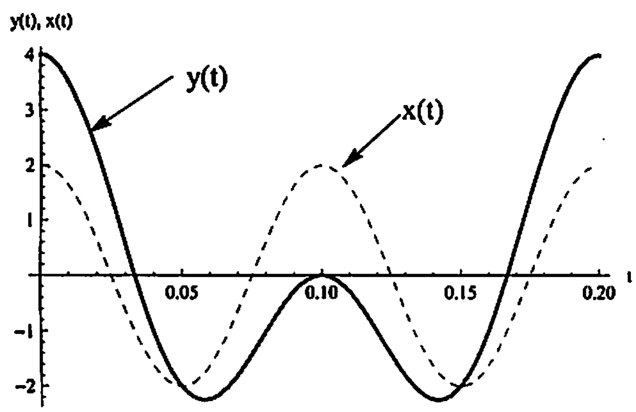
In[175]:=
```
y[t_] := 2 Cos[2 Pi 10 t] + 2 Cos[2 Pi 5 t];
Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```

Out[176]=



Compare this with the original signal x(t) to see aliasing

In[178]:=
```
Plot[{x[t], y[t]}, {t, 0, 2 period},
 AxesLabel → {"t", "y(t), x(t)"}, PlotStyle → {Dashed, Thick}]
```



HW 6, Problem 3

by Nasser M. Abbasi

```
<< dsp`
```

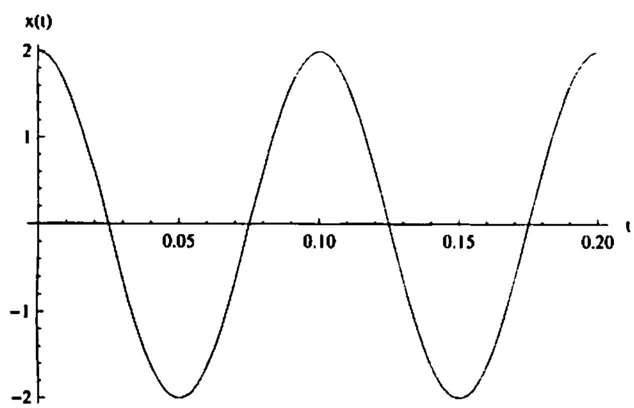
- **part (a)**

```
In[468]:= Clear[w, t, t, f];
          fm = 10;
          period = 1 / fm;
          Print["Period of message = ", N@period , " seconds"];
```

```
Period of message = 0.1 seconds
```

```
In[472]:= x[t_] := 2 Cos[2 Pi fm t]
          Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```

Out[473]=



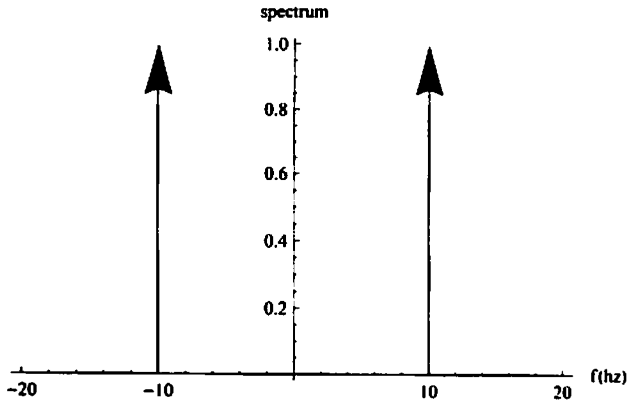
- **part(b)**

```
In[485]:= ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
          Print["Fourier Transform of x(t) = ", ft];
```

Fourier Transform of x(t) = $\delta(f-10)+\delta(f+10)$

In[476]:=
```
dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[477]=



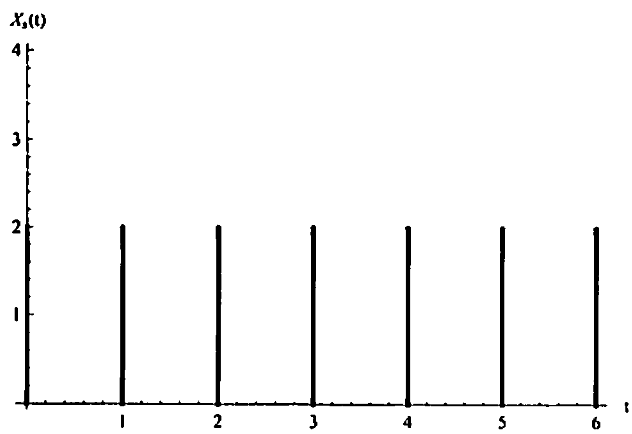
- **part(c)**

In[478]:=
```
Ts = 1 / 10;
Print[" samping period = ", N@Ts, " seconds"];
```

```
samping period = 0.1 seconds
```

In[480]:=
```
nSamples = 6 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling → Axis, FillingStyle → Thick, AxesLabel → {"t", "Xs(t)"}]
```

Out[482]=

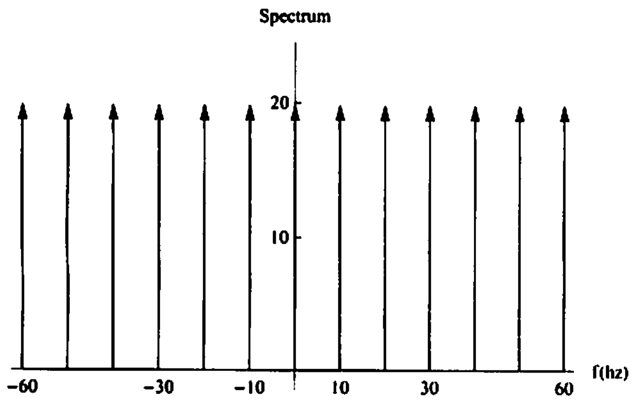


- **part(d)**

```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

```
Sampling frequency = 10 hz
```

```
spectrum = fs Sum[ft /. f → (f - n * fs), {n, -7, 7}];
spectrum = Expand[spectrum];
spectrum = 20 DiracDelta[-60 + f] + 20 DiracDelta[-50 + f] + 20 DiracDelta[-40 + f] +
    20 DiracDelta[-30 + f] + 20 DiracDelta[-20 + f] + 20 DiracDelta[-10 + f] +
    20 DiracDelta[f] + 20 DiracDelta[10 + f] + 20 DiracDelta[20 + f] + 20 DiracDelta[30 + f] +
    20 DiracDelta[40 + f] + 20 DiracDelta[50 + f] + 20 DiracDelta[60 + f];

Show[First@dsp`plotFourierTransform[spectrum, f, -6 * fs, 6 * fs, -.1 fs, 2.4 fs, Small],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-60, -30, -10, 10, 30, 60}, {fs, 2 * fs}}]
```



- **part(e)**

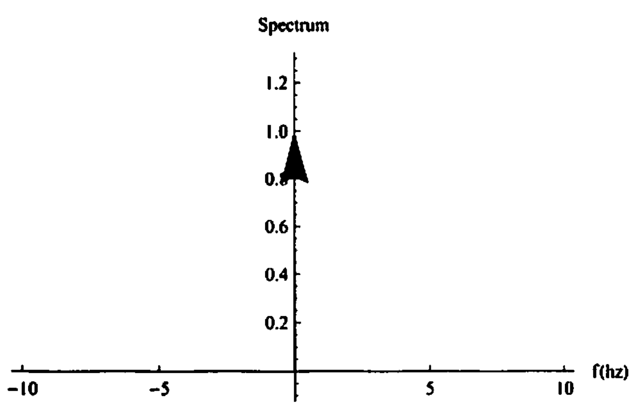
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

```
bandwidth = 5. hz
```

```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.1
```

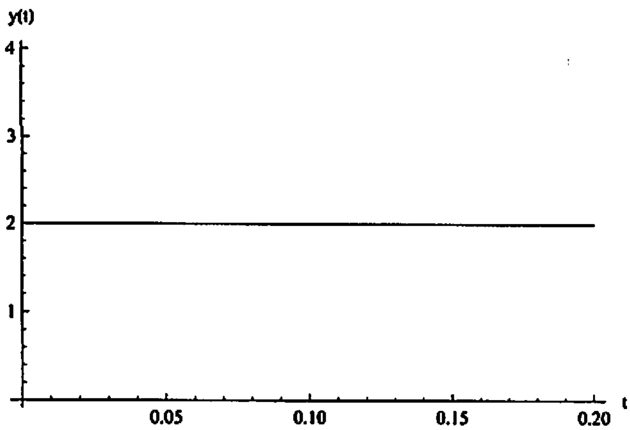
```
spectrum = ( 20 DiracDelta[f]);
Show[First@dsp`plotFourierTransform[
    Expand[gain * spectrum], f, -2 * bandwidth, 2 * bandwidth, -.1 , 1.3 , Large],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-10, -5, 0, 5, 10}}]
```



- **part(f)**

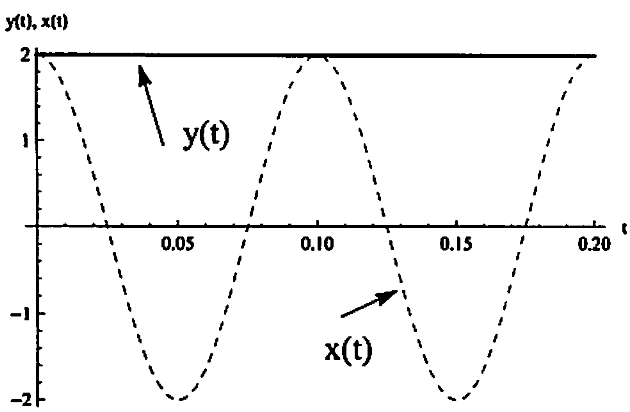
From the output above, we conclude that $y(t) = 2 \text{Cos}[2\pi\, 0\, t]$=2

```
y[t_] := 2 ;
Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```



Compare this with the original signal x(t)

```
Plot[{x[t], y[t]}, {t, 0, 2 period},
 AxesLabel → {"t", "y(t), x(t)"}, PlotStyle → {Dashed, Thick}]
```



HW 6, Problem 4

by Nasser M. Abbasi

In[266] := `<< dsp``

- **part (a)**

In[269] :=
```
Clear[w, t, t, f];
f1 = 20;
f2 = 10;
period1 = 1 / f1;
period2 = 1 / f2;

Print["Period of message 1 = ", N@period1 , " seconds"];
```

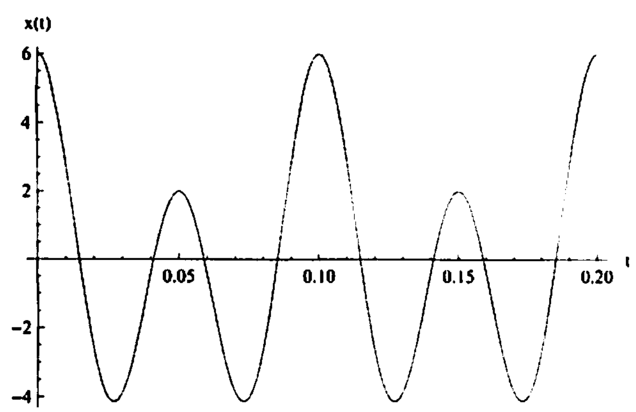
```
Period of message 1 = 0.05 seconds
```

In[275] :=
```
Print["Period of message 2 = ", N@period2 , " seconds"];
```

```
Period of message 2 = 0.1 seconds
```

In[302] :=
```
x[t_] := 4 Cos[2 Pi f1 t] + 2 Cos[2 Pi f2 t];
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}]
```

Out[303]=



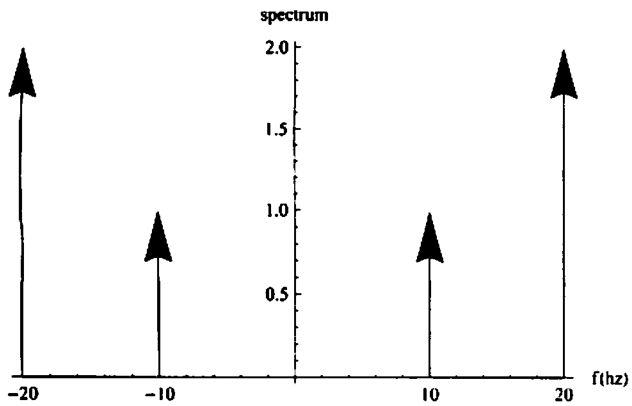
- **part(b)**

In[338] :=
```
ft = FourierTransform[x[t], t, f, FourierParameters → {0, -2 Pi}];
Print["Fourier Transform of x(t) = ", ft];
```

```
Fourier Transform of x(t) =
 2 DiracDelta[-20 + f] + DiracDelta[-10 + f] + DiracDelta[10 + f] + 2 DiracDelta[20 + f]
```

In[340]:=
```
dsp`plotFourierTransform[ft, f, -2 f2, 2 f2, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[341]=



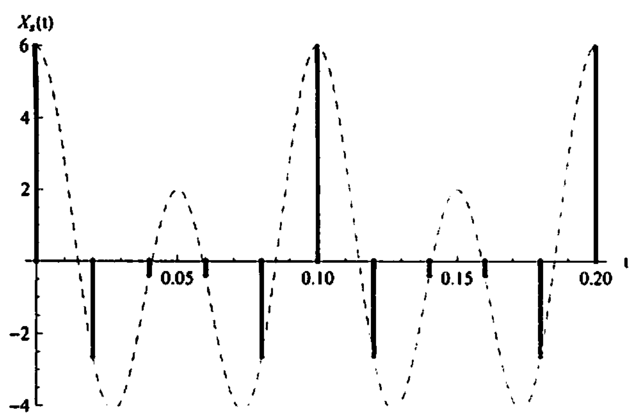
- **part(c)**

In[282]:=
```
Ts = 1 / 50;
Print[" samping period = ", N@Ts, " seconds"];
```

```
samping period = 0.02 seconds
```

In[322]:=
```
nSamples = 2 * period2 / Ts;
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}, PlotStyle → Dashed];
data = Table[{n * Ts, x[n Ts]}, {n, 0, nSamples}];
Show[{ListPlot[data, Filling → Axis, FillingStyle → Thick,
   AxesLabel → {"t", "Xs(t)"}, PlotRange → {Automatic, {-4, 6}}], p1}]
```

Out[325]=



- **part(d)**

In[326]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

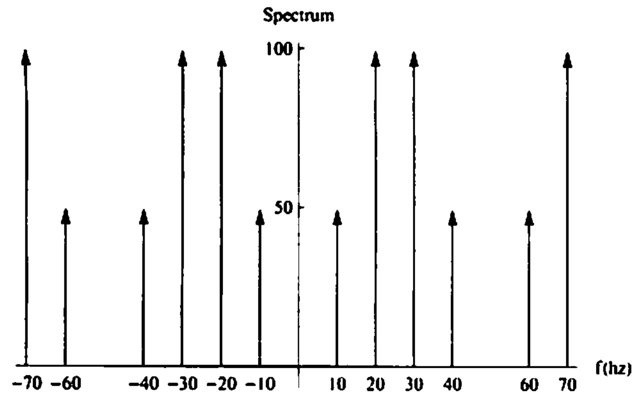
```
Sampling frequency = 50 hz
```

In[365]:=
```
spectrum = fs Sum[ft /. f → (f - n * fs), {n, -1, 1}];
spectrum = Expand[spectrum]
```

Out[366]=
```
100 DiracDelta[-70 + f] + 50 DiracDelta[-60 + f] + 50 DiracDelta[-40 + f] +
 100 DiracDelta[-30 + f] + 100 DiracDelta[-20 + f] + 50 DiracDelta[-10 + f] +
 50 DiracDelta[10 + f] + 100 DiracDelta[20 + f] + 100 DiracDelta[30 + f] +
 50 DiracDelta[40 + f] + 50 DiracDelta[60 + f] + 100 DiracDelta[70 + f]
```

In[370]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -fs, fs, -.1 fs, 2 fs, Small],
 AxesLabel → {"f(hz)", "Spectrum"},
 Ticks → {{-70, -60, -40, -30, -20, -10, 10, 20, 30, 40, 60, 70}, {fs, 2 fs}}]
```

Out[370]=



- **part(e)**

In[371]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

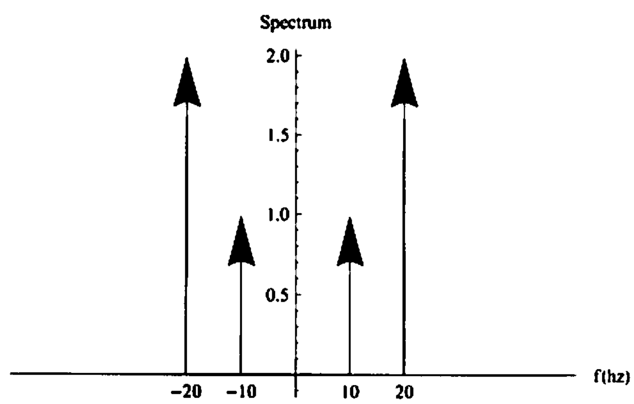
```
bandwidth = 25. hz
```

In[373]:=
```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.02
```

In[381]:=
```
spectrum = ( 100 DiracDelta[-20 + f] +
     50 DiracDelta[-10 + f] + 50 DiracDelta[10 + f] + 100 DiracDelta[20 + f]);
Show[First@dsp`plotFourierTransform[Expand[gain * spectrum], f,
   -2 * bandwidth, 2 * bandwidth, -.1 , 2 , Large],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-20, -10, 0, 10, 20}}]
```

Out[382]=

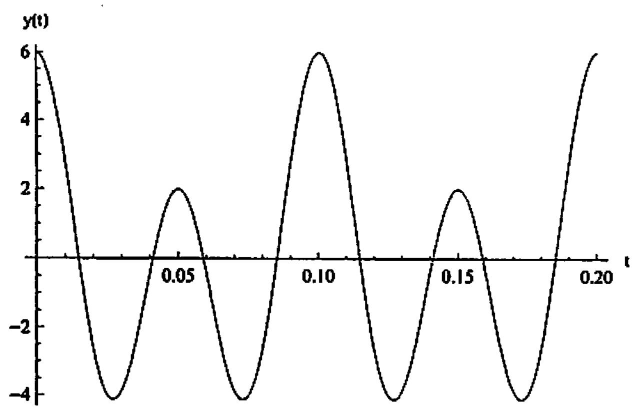


part(f)

From the output above, we conclude that $y(t)$ = 2 Cos[2 π 10 t]+4Cos[2π 20t]

```
In[383]:= y[t_] := 2 Cos[2 Pi 10 t] + 4 Cos[2 π 20 t];
          Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```

Out[384]=



which is the same as orginal x(t)

HW 6, Problem 5

by Nasser M. Abbasi

In[385]:=
```
<< dsp`
```

- **part (a)**

In[498]:=
```
Clear[w, t, t, f];
f1 = 20;
f2 = 10;
period1 = 1/f1;
period2 = 1/f2;
Print["Period of message 1 = ", N[period1], " seconds"];
```

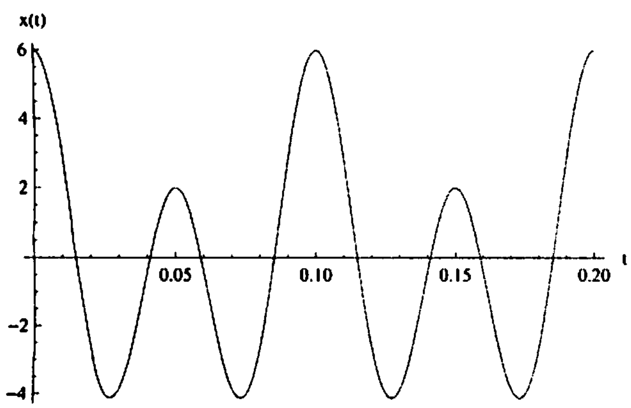
Period of message 1 = 0.05 seconds

In[504]:=
```
Print["Period of message 2 = ", N@period2 , " seconds"];
```

Period of message 2 = 0.1 seconds

In[505]:=
```
x[t_] := 4 Cos[2 Pi f1 t] + 2 Cos[2 Pi f2 t];
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```

Out[506]=



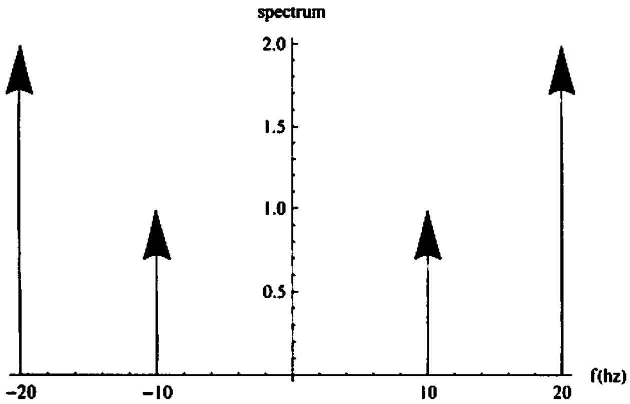
- **part(b)**

In[507]:=
```
ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
Print["Fourier Transform of x(t) = ", ft];
```

Fourier Transform of x(t) = $2\,\delta(f-20) + \delta(f-10) + \delta(f+10) + 2\,\delta(f+20)$

In[397]:=
```
dsp`plotFourierTransform[ft, f, -2 f2, 2 f2, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[398]=



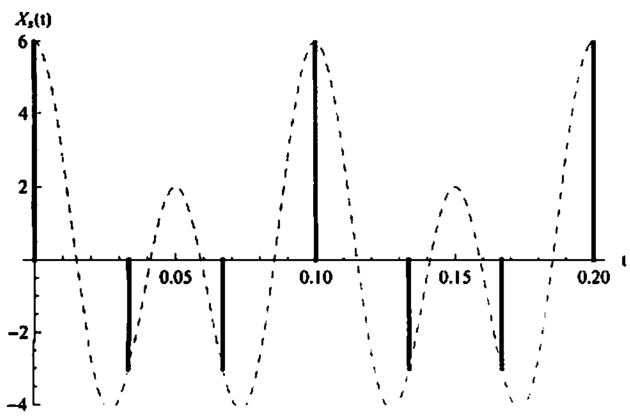
- **part(c)**

In[431]:=
```
Ts = 1 / 30;
Print[" samping period = ", N@Ts, " seconds"];
```

```
samping period = 0.0333333 seconds
```

In[433]:=
```
nSamples = 2 * period2 / Ts;
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}, PlotStyle → Dashed];
data = Table[{n * Ts, x[n Ts]}, {n, 0, nSamples}];
Show[{ListPlot[data, Filling → Axis, FillingStyle → Thick,
   AxesLabel → {"t", "Xs(t)"}, PlotRange → {Automatic, {-4, 6}}], p1}]
```

Out[436]=



- **part(d)**

In[509]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

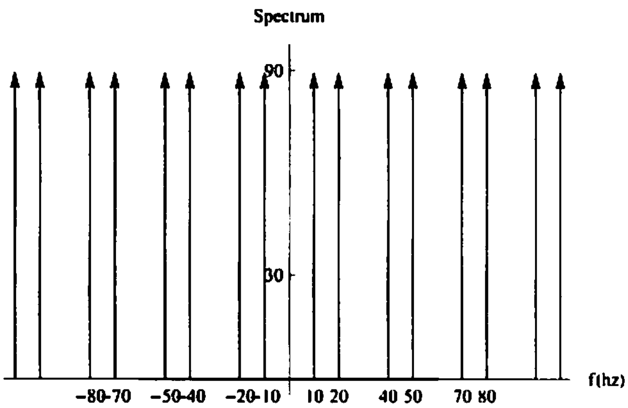
Sampling frequency = 10 hz

In[515]:=
```
spectrum = fs Sum[ft /. f → (f - n * fs), {n, -4, 4}];
spectrum = Expand[spectrum];
spectrum = 90 DiracDelta[-110 + f] + 90 DiracDelta[-100 + f] + 90 DiracDelta[-80 + f] +
   90 DiracDelta[-70 + f] + 90 DiracDelta[-50 + f] + 90 DiracDelta[-40 + f] +
   90 DiracDelta[-20 + f] + 90 DiracDelta[-10 + f] + 90 DiracDelta[10 + f] +
   90 DiracDelta[20 + f] + 90 DiracDelta[40 + f] + 90 DiracDelta[50 + f] + 90 DiracDelta[70 + f] +
   90 DiracDelta[80 + f] + 90 DiracDelta[100 + f] + 90 DiracDelta[110 + f]
```

Out[517]= $90\delta(f-110)+90\delta(f-100)+90\delta(f-80)+90\delta(f-70)+90\delta(f-50)+90\delta(f-40)+90\delta(f-20)+90\delta(f-10)+90\delta(f+10)+90\delta(f+20)+90\delta(f+40)+90\delta(f+50)+90\delta(f+70)+90\delta(f+80)+90\delta(f+100)+90\delta(f+110)$

In[454]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -2 fs, 2 fs, -.1 fs, 3.2 fs, Small],
 AxesLabel → {"f(hz)", "Spectrum"},
 Ticks → {{-80, -70, -50, -40, -20, -10, 10, 20, 40, 50, 70, 80}, {fs, 3 fs}}]
```

Out[454]=



- **part(e)**

In[455]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

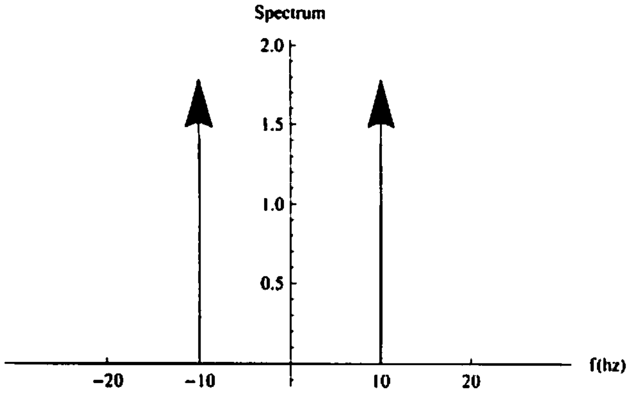
```
bandwidth = 15. hz
```

In[373]:=
```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.02
```

In[457]:=
```
spectrum = (90 DiracDelta[-10 + f] + 90 DiracDelta[10 + f]);
Show[First@dsp`plotFourierTransform[
   Expand[gain * spectrum], f, -2 * bandwidth, 2 * bandwidth, -.1 , 2 , Large],
  AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-20, -10, 0, 10, 20}}]
```

Out[458]=



In[460]:=
```
h = 0.02 * 90;
v = 2 * 1.8
```

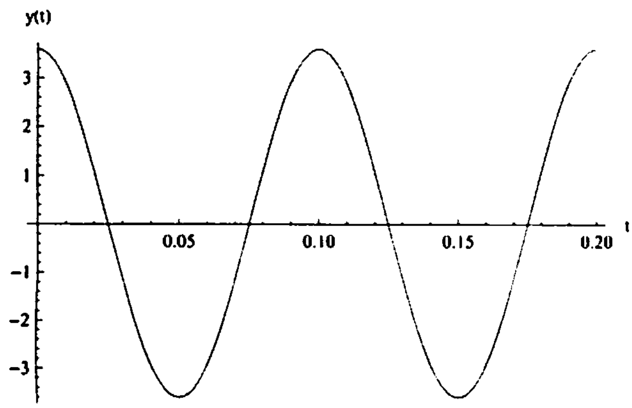
Out[461]= 3.6

- **part(f)**

From the output above, we conclude that $y(t) = 3.6\,\mathrm{Cos}[2\,\pi\,10\,t]$

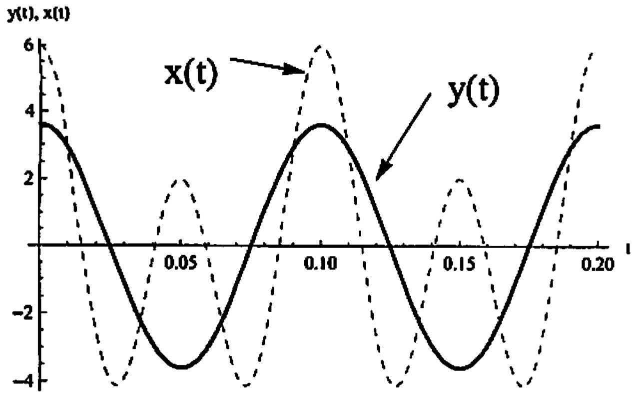
In[462]:=
```
y[t_] := 3.6 Cos[2 Pi 10 t];
Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```

Out[463]=



compare to orginal x(t)

In[464]:=
```
Plot[{x[t], y[t]}, {t, 0, 2 period},
  AxesLabel → {"t", "y(t), x(t)"}, PlotStyle → {Dashed, Thick}]
```


y(t), x(t)
6
4
2
-2
-4
0.05
0.10
0.15
0.20
t
x(t)
y(t)

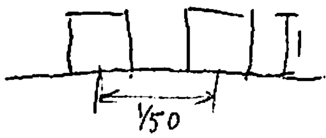
HW#6

6

$T = 0.1\,\text{sec.}$

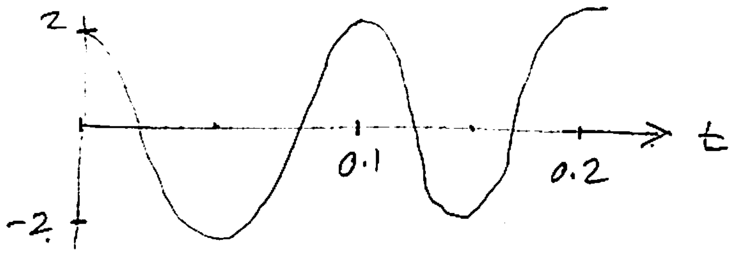
$\frac{\tau}{T_s} = \frac{1}{4}$ so $\boxed{\tau = \frac{T_s}{4}}$

$x(t) = 2\cos(2\pi 10 t)$.

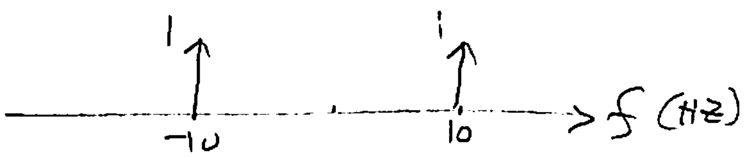
sampled uniformly by  dutycycle $= \frac{1}{4}$, $T_s = \frac{1}{50}$.

$= \frac{1}{5}\left(\frac{1}{10}\right)$ sec

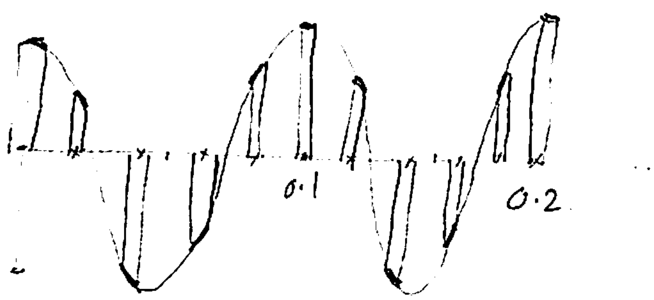
(a)



(b)

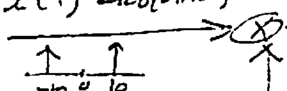


(c)



(d)

$x(t) = 2\cos(2\pi 10 t)$



$x_s(t) = x(t)g(t) = x(t) \sum h\,\text{rect}\left(\frac{t-nT_s}{\tau}\right)$

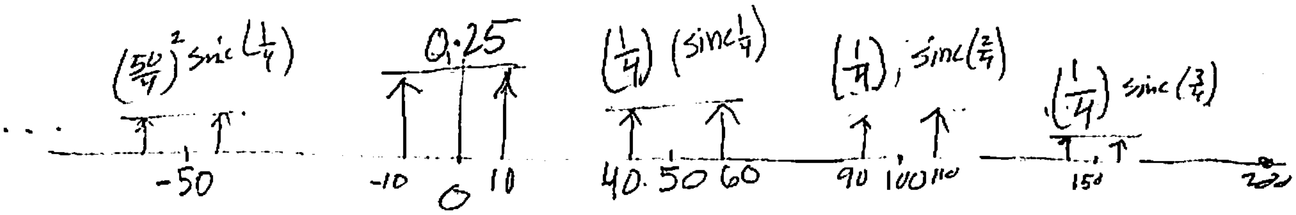
$\downarrow$ F.T

$X_s(f) = X(f) \circledast f_s h \sum \tau\,\text{sinc}\left(\frac{n\tau}{T_s}\right)\delta(f - nf_s)$

$g(t) = \sum h\,\text{rect}\left(\frac{t-nT_s}{\tau}\right)$

$= f_s h \tau \sum \text{sinc}\left(\frac{n\tau}{T_s}\right) X(f - nf_s)$ 50

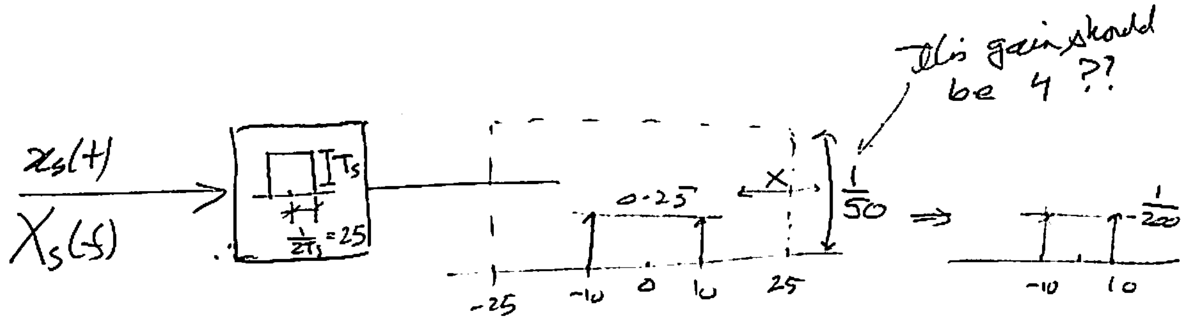
$G(f) = f_s \sum h\tau\,\text{sinc}\left(n\frac{\tau}{T_s}\right)\delta(f - nf_s)$ $= (50)\left(\frac{1}{4}\cdot\frac{1}{50}\right) \sum \text{sinc}\left(\frac{n}{4}\right) X(f - nf_s)$



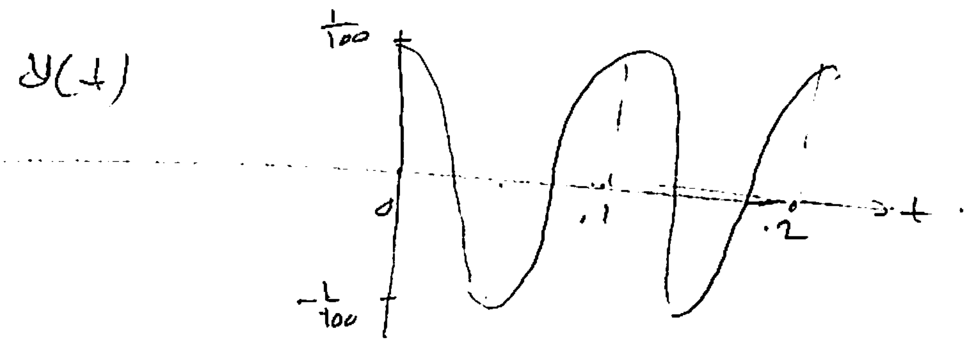
HW6

6.

(e)



(f) we see that $\boxed{y(t) = \frac{1}{100} \cos(2\pi 10 t)}$

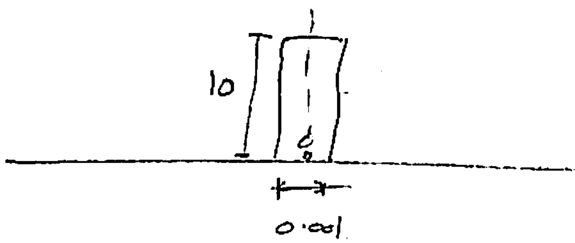


note amplitude is different, because gain we are asked to use is T_s which is not what normally used for pulse rect train which should have been (d).

HW6

(7) determine Nyquist rate for

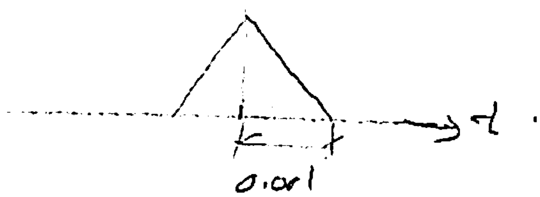
(a) $x(t) = 10 \text{ rect}\left(\frac{t}{0.001}\right)$.



so. $\tau = 0.001$

this signal is not periodic. hence its bandwidth is ∞. hence require ∞ sampling frequency.

(b) $x(t) = \text{tri}\left(\frac{t}{0.001}\right)$.



for similar reasoning as (a). this is not periodic, hence ∞ bandwidth $\Rightarrow$ ∞ sampling freq.

(c) $\text{sinc}(1000t) = \frac{\sin(\pi 1000 t)}{\pi 1000}$

so $f_m = 500$ Hz.

hence Nyquist $= 1000$ Hz.

(d) $\text{sinc}(2000t) + \text{sinc}^2(1100t)$

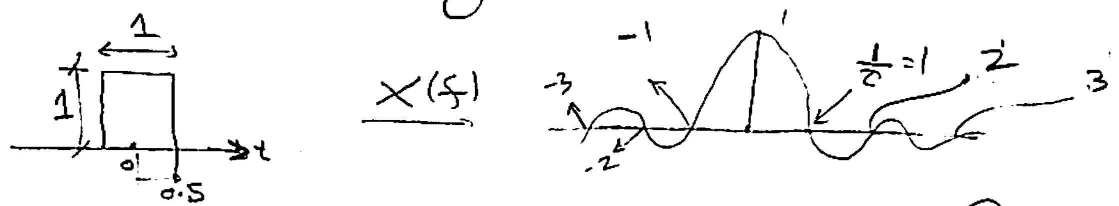
$= \frac{\sin(2\pi 1000 t)}{\pi 2000} + \left[\frac{\sin(2\pi 550 t)}{2\pi 550}\right]^2$

$= \frac{\sin 2\pi 1000 t}{\pi 2000} + \left(\frac{1}{2\pi 550}\right)^2 \left[\frac{1}{2} - \frac{1}{2}\cos(2\pi\, 1100 t)\right] \Rightarrow$ Nyquist $=$ 2200 H

HW 6

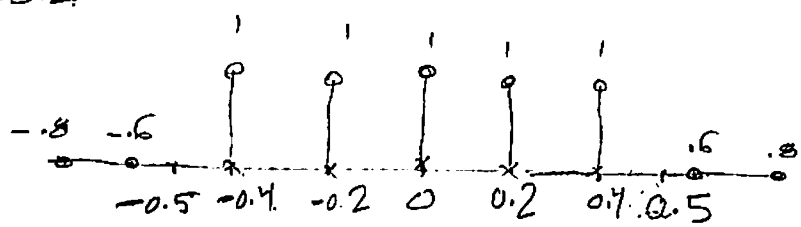
8. $x(t) = \text{rect}(t)$ is ideally sampled at rate 5 samples/s.

(a)



Nyquist rate $= \infty$. see problem 7, part (a).

(b) $T_s = \frac{1}{5} = 0.2$



(c) need to print spectrum of $\hat{x}$.

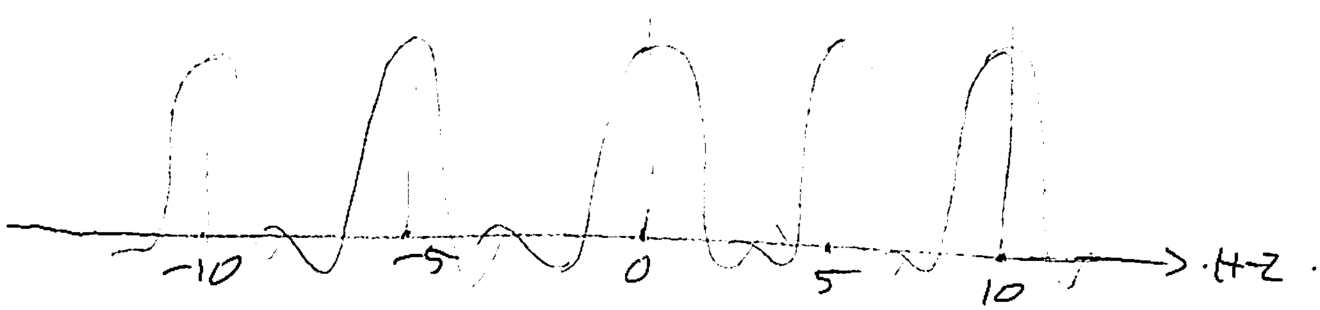
$$x_s(t) = \delta(t) + \delta(t-0.2) + \delta(t-0.4) + \delta(t+0.2) + \delta(t+0.4).$$

$$\text{so } F[x_s(t)] = X_s(f) = f_s\left[X(f) + X(f-f_s) + X(f-2f_s) + X(f+f_s) + X(f+2f_s)\right]$$

but $X(f) = 1 \cdot \text{sinc}(f)$

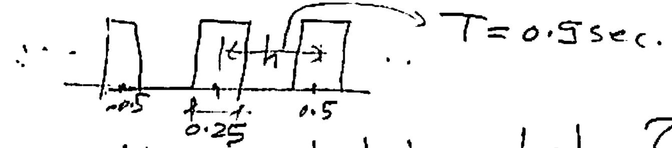
$$\text{so } X_s(f) = 5\left[\text{sinc}(f) + \text{sinc}(f-5) + \text{sinc}(f-10) + \text{sinc}(f+5) + \text{sinc}(f+10)\right]$$

so, place a sinc function of height $= 5$ at $f = 0, \pm 5, \pm 10$.



(d) it is not possible to recover, since sinc function extent for $\pm\infty$. so there will always be a loss.

HW6



9.

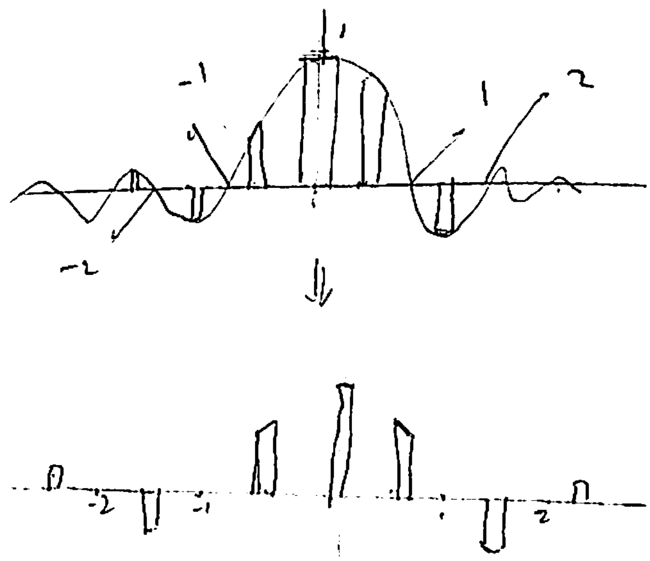
$x(t) = \text{sinc}(t)$ sampled by rect train, $h=1$, $\tau = 0.25$ s,

$T = 0.5$ sec.

(a) Nyquist rate for $x(t)$ is ∞ since it is a sinc function with ∞ bandwidth? but answer says 1 Hz? I do not understand.

if we apply this to the rect pulse, then I get $(2)(f_m) = 2\left(\frac{1}{T}\right) = 2\left(\frac{1}{0.5}\right) = 2(2) = 4$ Hz.

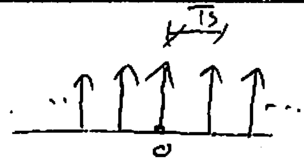
(b)



HW6

(10) $x(t) = 2\cos(2\pi\, 10t) + \cos(2\pi\, 20t)$.

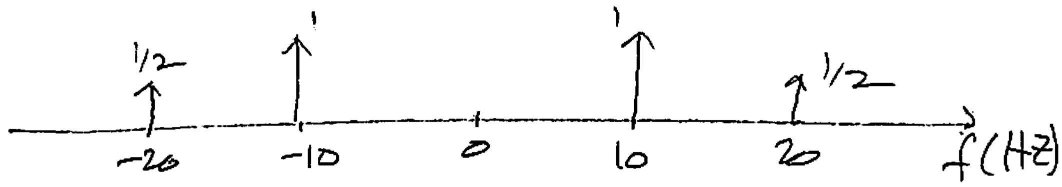
$f_s = 40\text{ Hz}$.



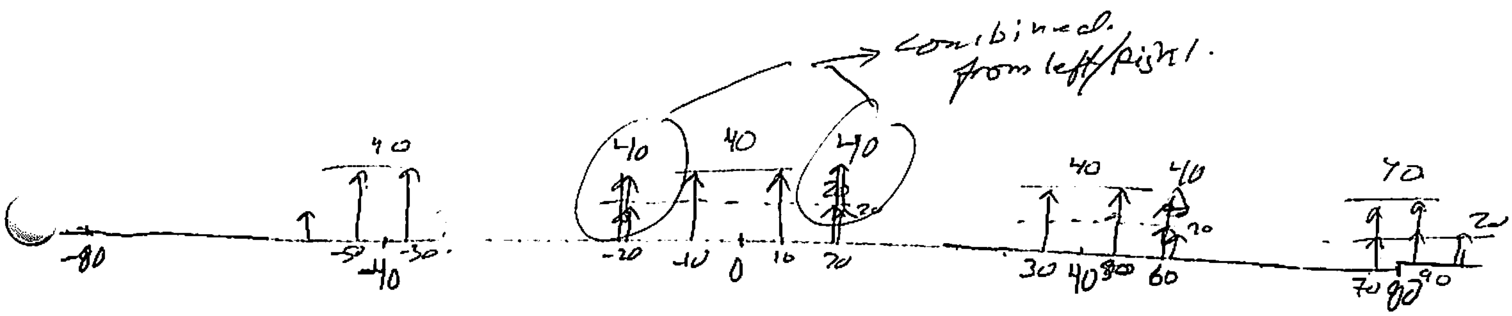
$T_s = 0.025$ sec

(a) see plot next page.

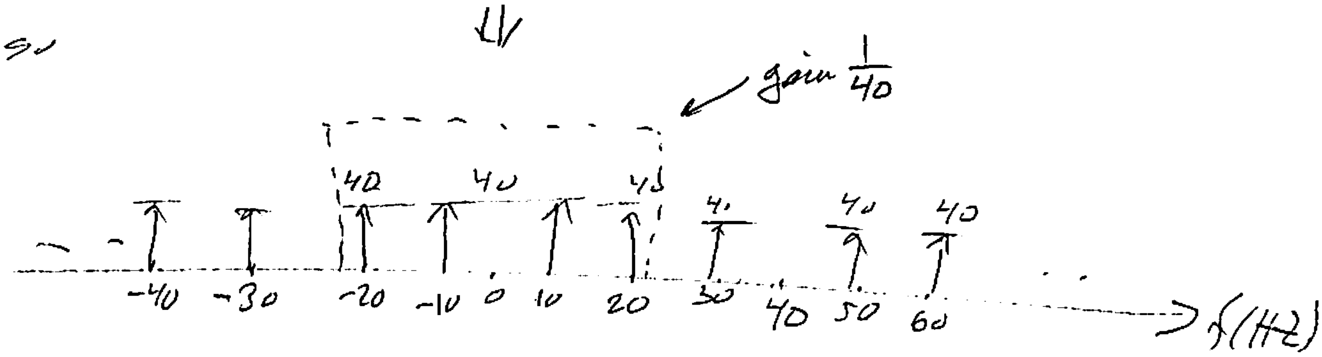
X(f).



(b) $X_s(f) = f_s \sum X(f - nf_s)$.

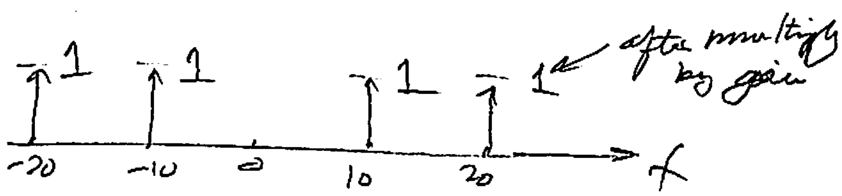


so



d)

after Sampling:



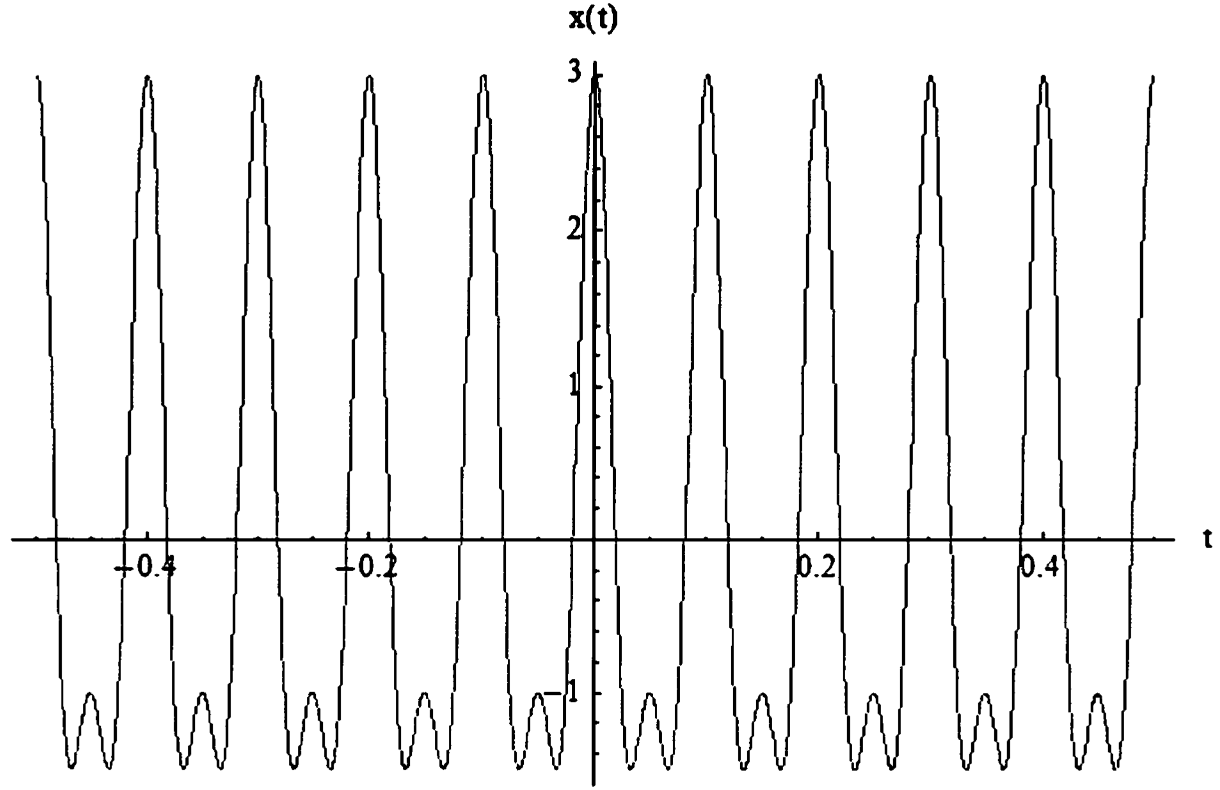
so time domain is

$$\boxed{2\cos(2\pi\, 10t) + 2\cos(2\pi\, 20t)}$$

(e) $\boxed{2\cos(2\pi\, 10\, t)}$

In[15]:= `Plot[2 Cos[2 Pi 10 t] + Cos[2 Pi 20 t], {t, -.5, .5}, AxesLabel → {"t", "x(t)"}]`

Out[15]=

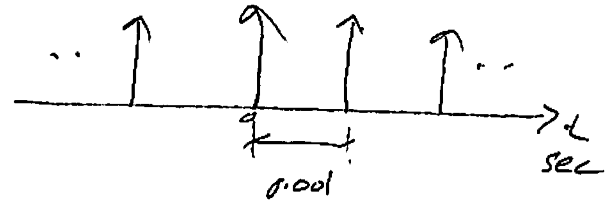


HW6

12

$m_1(t) = 500\ \text{sinc}^2(500t)$.

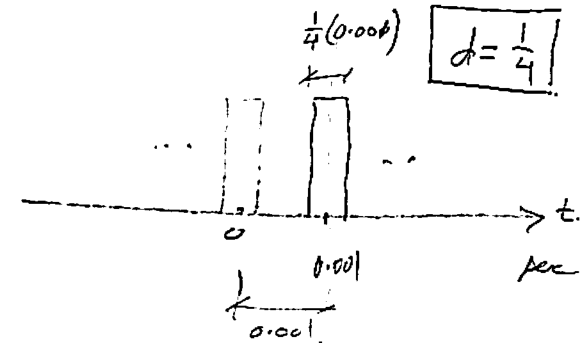
$P_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 0.001k)$.



$P_2(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t - 0.001k}{0.00025}\right)$

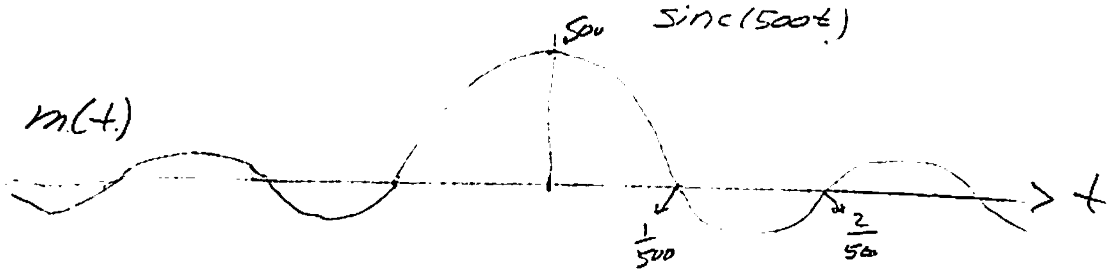
delay

width.

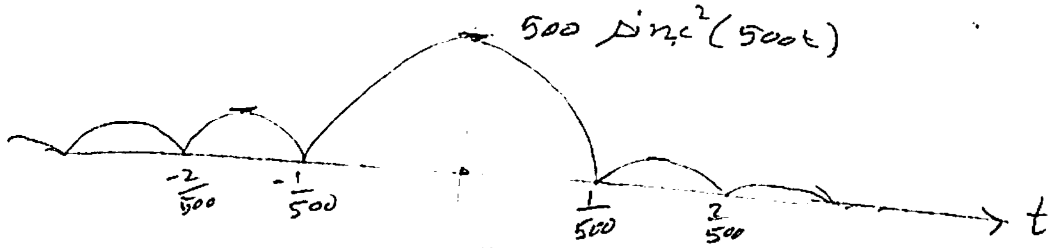


$\boxed{d = \frac{1}{4}}$

ⓐ $x_1(t) = m_1(t)\, P_1(t)$.



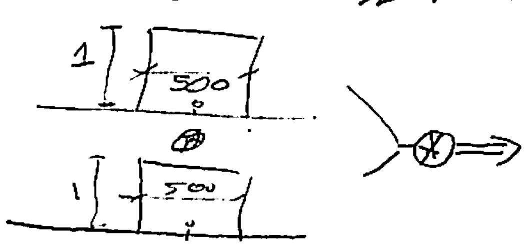
$\text{sinc}^2(500t)$



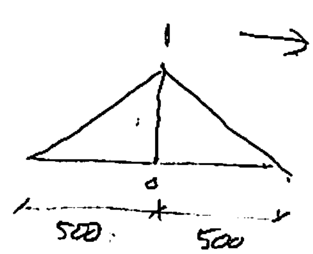
to find spectrum of ↑ we can use relation.

$$F[\text{sinc}^2] = F[(\text{sinc})(\text{sinc})]$$
$$= F(\text{sinc}) \circledast F(\text{sinc})$$

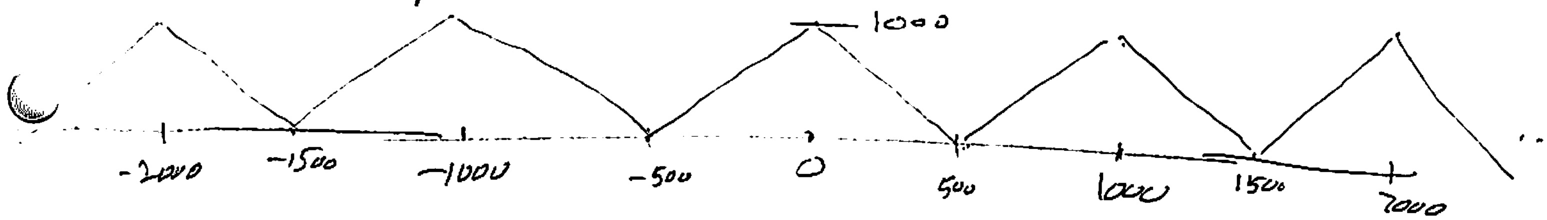
so need to convolve 2 rect.



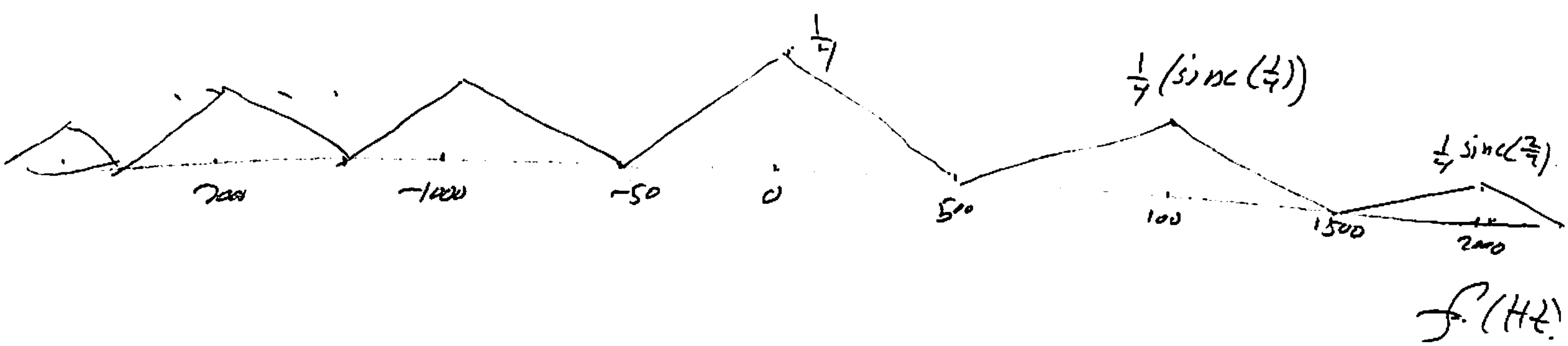
$\text{tri}\left(\frac{f}{500}\right)$



so ④ has spectrum (note $f_s = \frac{1}{0.001} = 1000$.

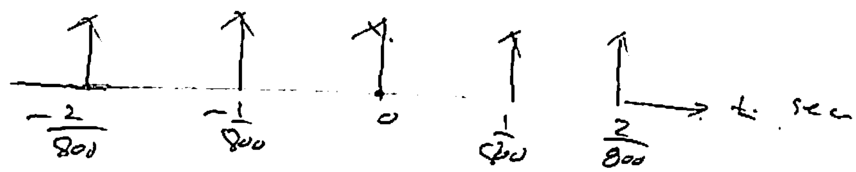


⑤. note that $d = \frac{1}{4} \cdot 50$,

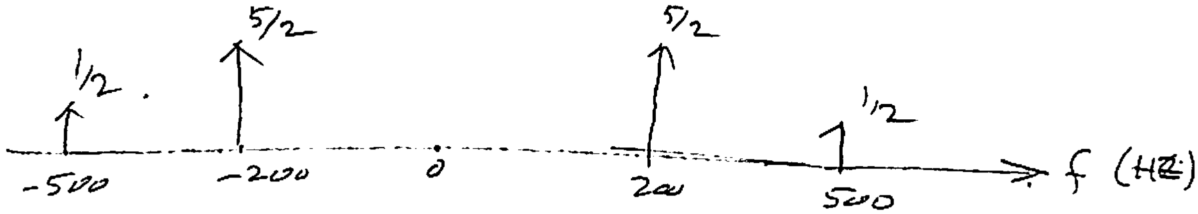


HW 6

13 $\quad m(t) = 5\cos(2\pi\,\overset{f_1}{200}\,t) + \cos(2\pi\,\overset{f_2}{500}\,t).$ $\quad \Rightarrow f_s = \frac{1}{T_s} = 800$ Hz

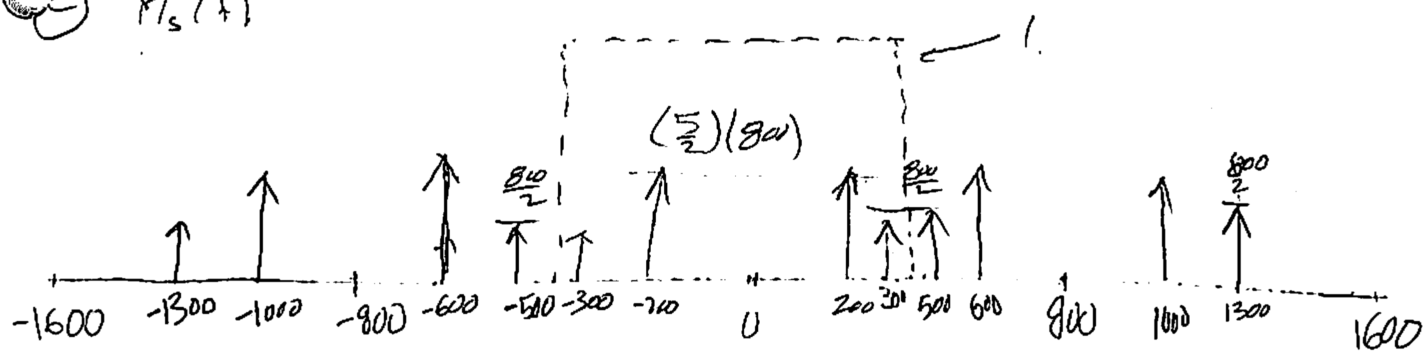


(a) $\mathcal{F}(m(t)) = \frac{5}{2}\left(\delta(f-f_1)+\delta(f+f_1)\right)$
$+ \frac{1}{2}\left(\delta(f-f_2)+\delta(f+f_2)\right).$

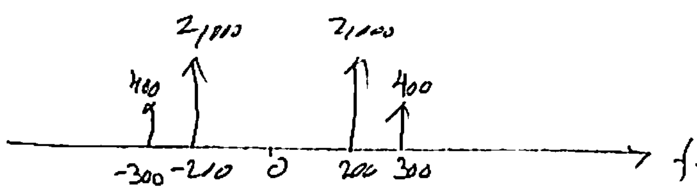


(b) $B = 500$, hence $2B = 1000$ Hz $\Rightarrow \boxed{1\,\text{KHz}}$

(c) $M_s(f)$



(d) so $Y(f)$



so $y(t) = 4000\cos(2\pi 200t) + 800\cos(2\pi 300t).$

HW7

(1) $m_p = 16V$.
$x = -8.7V$.

(a) $N = 8$ bits. Find ~~t~~offset binary code.

$\Delta = \frac{16}{2^7} = \boxed{0.125}$

quantization level $= \left(\frac{Abs(x)}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since x is negative, then Code $= 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary this is $\boxed{0011\ 1010}$

(b) sign/magnitude..

since $x < 0$ Then Code $= 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{1100\ 0110}$

(c) 2's Complement.

since $x < -\frac{\Delta}{2}$ then Code. $= 2^8 - 70 = 256 - 70$
$= 186$

which in binary is $\boxed{1011\ 1010}$

(d) 1's complement.

since $x < 0$ Then
Code $(2^8 - 1) - 70 = 255 - 70 = 185$

which in binary is $\boxed{1011\ 1001}$

extra: to illustrate this more, this is the Calculations assuming $x = +8.7V$.

offset binary

$\Delta = \frac{16}{2^7} = 0.125$

Level $=$ round $\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

since $x > 0$ then code $(70)_2 = 0100\ 0110$

Sign magnitude

since $x > 0$ then code $(70)_2 = 0100\ 0110$

2's Complement

since $x > 0$ then code $= (70)_2 = 0100\ 0110$

1's complement

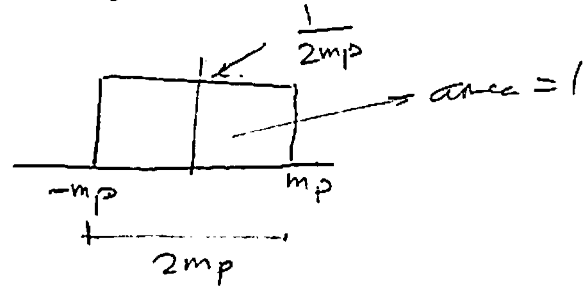
since $x > 0$, then code $= (70)_2 = 0100\ 0110$.

HW 7

(3) $m_p = 16$.

$N = 11$ bits.

$m(t)$ Pdf



(a) $$SNR = \frac{\text{signal power}}{\text{noise power}} = \frac{\overline{m^2(t)}}{\frac{1}{12}S^2}$$

when $S = \frac{m_p}{2^{N-1}}$ hence Noise $\boxed{P_{av} = \frac{1}{12}\left(\frac{m_p}{2^{10}}\right)^2}$

and since this is a Random message, then

$$\overline{m^2(t)} = E(m^2(t)) = \int x^2 f_X \, dx$$

$$= \int_{-m_p}^{m_p} \left(\frac{1}{2m_p}\right) x^2 dx$$

$$= \frac{1}{2m_p}\left[\frac{x^3}{3}\right]_{-m_p}^{m_p} = \frac{1}{6m_p}\left[m_p^3 - (-m_p)^3\right]$$

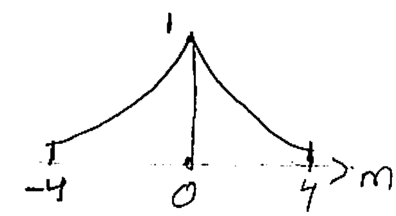
$$= \frac{1}{6m_p}\left[m_p^3 + m_p^3\right] = \boxed{\frac{1}{3}m_p^2}$$

So $$SNR = \frac{\frac{1}{3}m_p^2}{\frac{1}{12}\left(\frac{m_p}{2^{10}}\right)^2} = \frac{(12)(2^{20})}{3} = (4)(2)^{20};$$

$$= \boxed{66.226 \text{ db}}$$

HW 7

(6) $f(m) = \begin{cases} k e^{-|m|} & -4 < m < 4 \\ 0 & \text{o.w.} \end{cases}$



(a) Find k.

$$\int_{-4}^{4} k e^{-|m|}\,dm = 1 \Rightarrow \int_{-4}^{0} k e^{m}\,dm + \int_{0}^{4} k e^{-m}\,dm$$

$$= k\left[\left[e^{m}\right]_{-4}^{0} + \frac{\left[e^{-m}\right]_{0}^{4}}{-1} \right]$$

$$= k\left[e^{0} - e^{-4} + \frac{e^{-4} - e^{0}}{-1} \right] = k\left[1 - e^{-4} + \frac{e^{-4} - 1}{-1} \right]$$

$$= k\left[(1 - e^{-4}) + (1 - e^{-4}) \right] = k\left(2 - 2e^{-4}\right)$$

$$= \boxed{2k\left(1 - e^{-4}\right) = 1}$$

so $k = \frac{1}{2(1-e^{-4})} = \boxed{0.50932}$

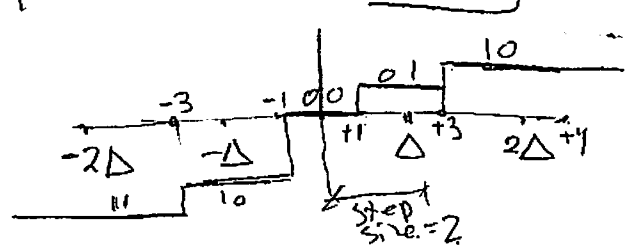
(b) $m_p = 4$. so $\Delta = \frac{m_p}{2^{N-1}}$ where N is number of bits.

or $\Delta = \frac{8}{2^N}$

but 2^N = number of levels.

but we are told to use 4 levels. hence $\boxed{N=2}$

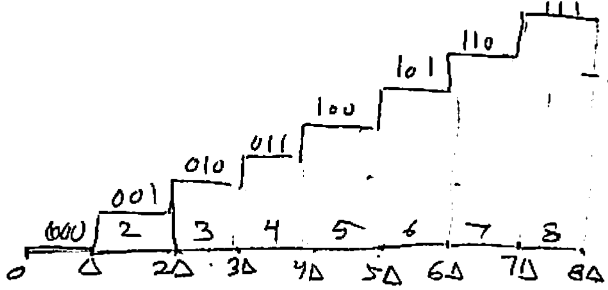
so $\Delta = \frac{8}{2^2} = \frac{8}{4} = \boxed{2}$.



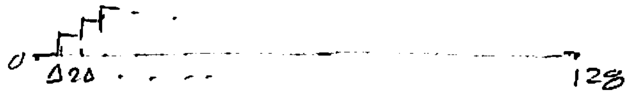
HW 7

$\boxed{7}$

1st channel:



2nd channel:



$m_p = 2$ Volt.

noise power $= \frac{1}{12} S^2$ where S is step size.

for channel 1, $S_1 = \frac{m_p}{8} = \frac{2}{8}$.

for channel 2, $S_2 = \frac{m_p}{128} = \frac{2}{128}$.

so for channel 1, $\overline{e_1^2} = \frac{1}{12}\left(\frac{2}{8}\right)^2 = \frac{4}{(12)(64)}$.

for channel 2, $\overline{e_2^2} = \frac{1}{12}\left(\frac{2}{128}\right)^2$

so $\left(\frac{\overline{e_1^2}}{\overline{e_2^2}}\right)_{db} = 10 \log_{10} \frac{\overline{e_1^2}}{\overline{e_2^2}} = 10 \left[\log_{10} \overline{e_1^2} - \log \overline{e_2^2}\right]$

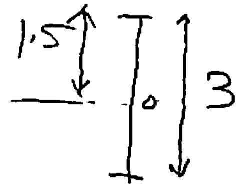
$= 10 \left[\log_{10} \frac{4}{(12)(64)} - \log_{10} \frac{1}{12} \frac{4}{128^2}\right] =$

$= 10 \left[-2.2833 + 4.6915\right] = \boxed{24.082 \text{ db}}$

HW 7.

$\boxed{8.}$ $m_p = 3V.$

levels = 64.



RMS of noise = $\sqrt{\overline{e^2}} = \sqrt{\frac{1}{12}s^2}$

but $s = \frac{3}{64}$, so $RMS = \sqrt{\frac{1}{12}\left(\frac{3}{64}\right)^2} = 0.01353$

so peak signal to RMS ratio is

$$\frac{1.5}{0.01353} = 110.85$$

$\boxed{9.}$

number of Levels = 512

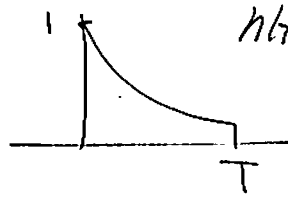
$B = 4.2\ MHz.$ so $2B = 8.4 \times 10^6\ Hz.$

hence $N = 9.$ $(2^9 = 512)$.

so binary pulses $= \underbrace{(9)}_{\text{bits per Sample}}\underbrace{(8.4 \times 10^6)}_{\text{Samples/sec.}} = \boxed{75600}$ binary pulses/sec.

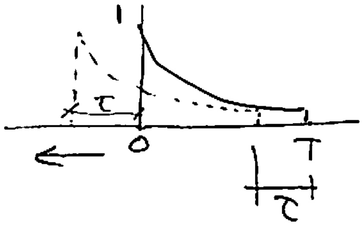
HW 8

#1



$h(t) = e^{-at}u(t)$.

Case 1



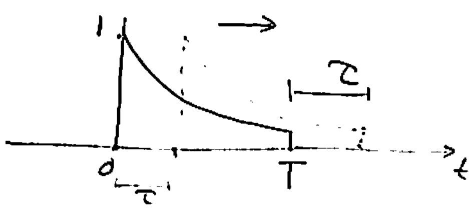
$0 \leq \tau \leq T$.

$$R(\tau) = \int_0^{T-\tau} h(t)\,h(t+\tau)\,dt = \int_0^{T-\tau} e^{-at} e^{-a(t+\tau)}\,dt$$

$$= e^{-a\tau}\int_0^{T-\tau} e^{-2at}\,dt = \frac{e^{-a\tau}}{-2a}\Big[e^{-2at}\Big]_0^{T-\tau}$$

$$= \frac{e^{-a\tau}}{-2a}\Big[e^{-2a(T-\tau)} - 1\Big] = \boxed{\frac{e^{-a\tau}}{2a}\left(1 - e^{-2a(T-\tau)}\right)}$$

Case 2



$0 \leq \tau \leq T$

$$R(\tau) = \int_\tau^{T} h(t)\,h(t-\tau)\,dt = \int_\tau^{T} e^{-at} e^{-a(t-\tau)}\,dt$$

$$= e^{a\tau}\int_\tau^{T} e^{-2at}\,dt = \frac{e^{a\tau}}{-2a}\Big[e^{-2at}\Big]_\tau^{T}$$

$$= \frac{e^{a\tau}}{-2a}\Big[e^{-2aT} - e^{-2a\tau}\Big] = \frac{e^{a\tau}}{2a}\Big[e^{-2a\tau} - e^{-2aT}\Big]$$

$$= \frac{1}{2a}\Big[e^{-a\tau} - e^{-2aT+a\tau}\Big] = \frac{e^{-a\tau}}{2a}\Big[1 - e^{-2aT+2a\tau}\Big]$$

$$= \boxed{\frac{e^{-a\tau}}{2a}\Big[1 - e^{-2a(T-\tau)}\Big]}$$

We see that $R_-(\tau) = R_+(\tau)$ as expected.

so $R(\tau) = \begin{cases} \dfrac{e^{-a\tau}}{2a}\left[1 - e^{-2a(T-\tau)}\right] & |\tau| \le T \\ 0 & \text{o.w.} \end{cases}$

so $R_X(\tau) = \frac{1}{T} R(\tau)$

now $S_X(f) = F.T\left[R_X(\tau)\right] \cdot F.T\left[R_X(\tau)\right]^*$

or $S_X(f) = \left| F.T\left[R_X(\tau)\right] \right|^2$

so need to find F.T. of $R_X(\tau)$ first.

$$F.T\left(R_X(\tau)\right) = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$= \int_{-T}^{T} \frac{e^{-a|\tau|}}{2a}\left(1 - e^{-2a(T-|\tau|)}\right) e^{-j2\pi f\tau}\, d\tau$$

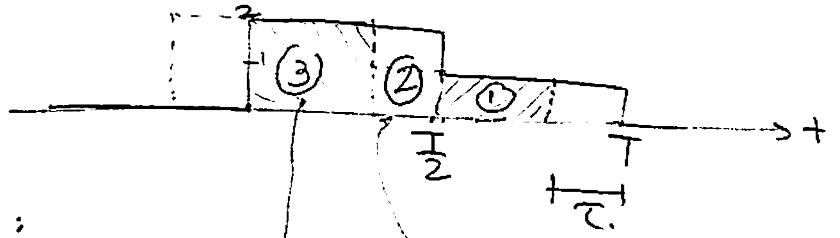
$$H(\omega) \longrightarrow 1 - e^{-aT}\frac{\left[\cos(2\pi f T) - j\sin(2\pi f T)\right]}{a + j\omega}$$

so $S_X(f) = H(\omega)\, H^*(\omega)$

$$\Rightarrow \frac{1}{T}\frac{1}{a^2 + (2\pi f)^2}\left(1 - 2e^{-aT}\cos(\omega T) + e^{-2aT}\right)$$

HW 8

(2)



(a) $0 < \tau < \frac{T}{2}$:

3 regions as above.

$$R_p(\tau) = \int_0^{\frac{T}{2}-\tau} (2)(2)\, d\lambda + \int_{\frac{T}{2}-\tau}^{\frac{T}{2}} (1)(2)\, d\lambda + \int_{\frac{T}{2}}^{T-\tau} (1)(1)\, d\lambda$$

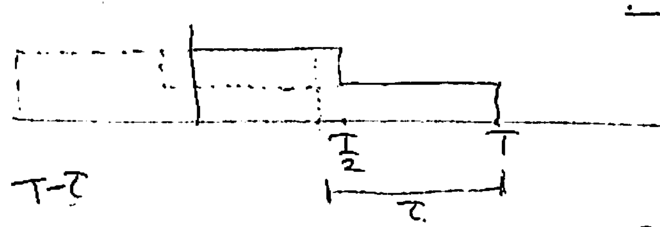
$$= 4\,[\lambda]_0^{\frac{T}{2}-\tau} + 2\,[\lambda]_{\frac{T}{2}-\tau}^{\frac{T}{2}} + 1\,[\lambda]_{\frac{T}{2}}^{T-\tau}$$

$$= 4\left(\frac{T}{2}-\tau\right) + 2\left(\frac{T}{2}-\frac{T}{2}+\tau\right) + \left(T-\tau-\frac{T}{2}\right)$$

$$= 4\frac{T}{2} - 4\tau + 2\tau + \frac{T}{2} - \tau = \frac{5}{2}T - 3\tau$$

$$= \boxed{\frac{5}{2}T\left(1-\frac{6}{5}\tau\right)}$$

$\frac{T}{2} < \tau \leq T$

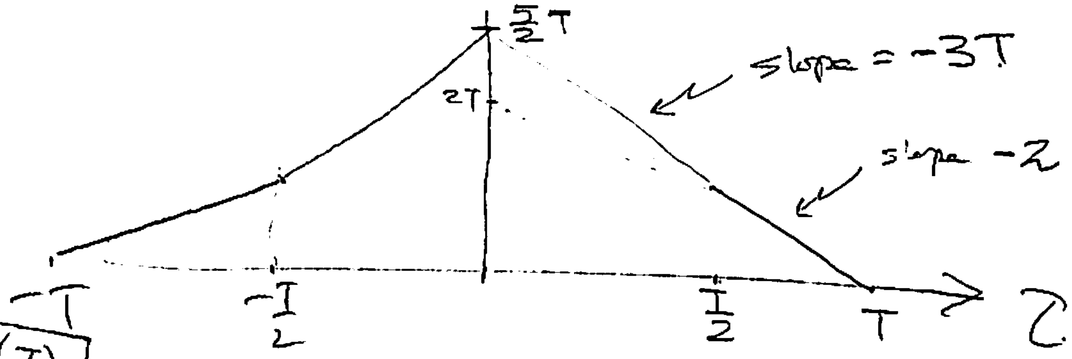


$$R_p(\tau) = \int_0^{T-\tau} (1)(2)\, d\lambda = 2\,[\lambda]_0^{T-\tau} = \boxed{2(T-\tau)}$$

hence
$$R_p(\tau) = \begin{cases} \frac{5}{2}T\left(1-\frac{6}{5}|\tau|\right) & 0 < |\tau| < \frac{T}{2} \\ 2(T-|\tau|) & \frac{T}{2} \leq |\tau| \leq T \\ 0 & o.w. \end{cases}$$

$\longrightarrow$

B. plot



so $\boxed{R_X = \frac{1}{T} R_p(\tau)}$

to find F.T. of this, use tri function.

so $H(f) \longrightarrow$ sinc.

and $S_X(f) = |H(f)|^2$

HW7

(1) $m_p = 16V$.
$x = -8.7V$.

(a) $N = 8$ bits. Find ~~offset~~ offset binary code.

$\Delta = \frac{16}{2^7} = \boxed{0.125}$

quantization level $= \left(\frac{Abs(x)}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since x is negative, then Code $= 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary this is $\boxed{0011\ 1010}$

(b) sign/magnitude..

since $x < 0$ Then Code $= 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{1100\ 0110}$

(c) 2's Complement.

since $x < -\frac{\Delta}{2}$ then Code. $= 2^8 - 70 = 256 - 70$
$= 186$

which in binary is $\boxed{1011\ 1010}$

(d) 1's complement.

since $x < 0$ Then

Code $(2^8 - 1) - 70 = 255 - 70 = 185$

which in binary is $\boxed{1011\ 1001}$

extra: to illustrate this more, this is the Calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{16}{2^7} = 0.125$$

$$\text{Level} = \text{round}\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$$

Since $x > 0$ Then code $(70)_2 = 0100\ 0110$

Sign magnitude

Since $x > 0$ Then code $(70)_2 = 0100\ 0110$

2's Complement

Since $x > 0$ then code $= (70)_2 = 0100\ 0110$

1's Complement

Since $x > 0$, then code $= (70)_2 = 0100\ 0110$.