HW 2, CEE 744, Structural dynamics, Spring 2013

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0.1 Problem description

Using different shape functions an estimate of the natural frequency for the wind tower was found using the method of generalized single degree of freedom for each method.

The following table summarizes the results obtained. For each shape function the following items are calculated: Effective mass M_e, effective stiffness $K_e = K_{fe} + K_{ge}$,effective flexural stiffness K_{fe}, effective geometric stiffness K_{ge}, The ratio $\frac{M}{M_e}$and the natural frequency f in Hz.

The rows of the table below are listed from the lowest to the largest natural frequency found.

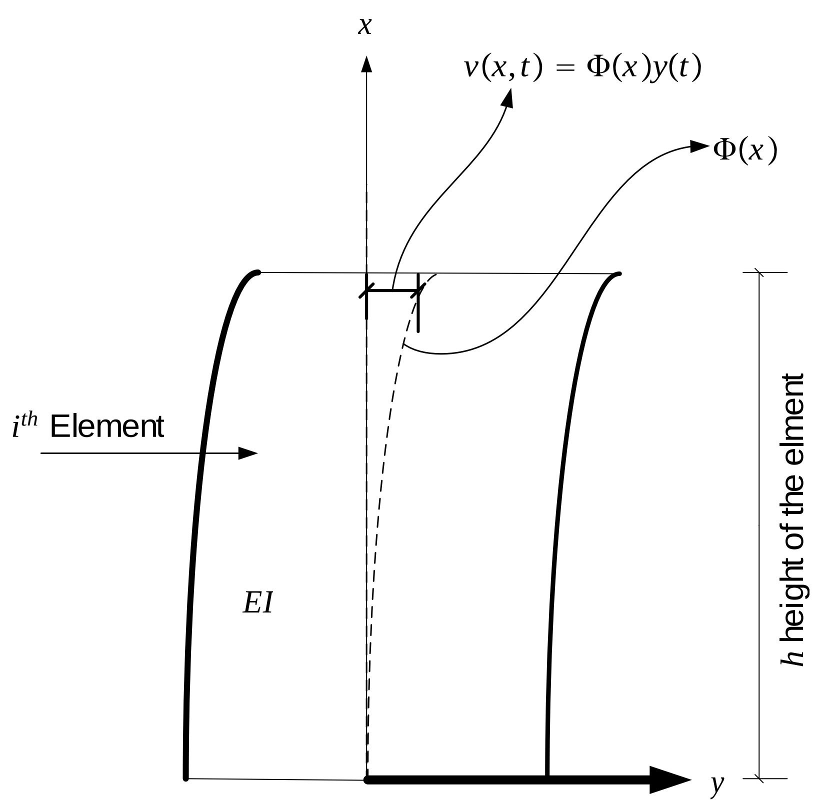
The shape function that produces the lowest natural frequency will be the one to select as the closest approximation to the real solution. The actual mass is 404171 Kg.

An Excel worksheet is also available on my web page for this HW for the lowest natural frequency case.

shape function $\Phi(x)$	M_e kg)	K_e	Flexural K_e (N/m)	Geometric K_{ge} (N/m)	$\frac{M_e}{M}$	f (Hz)
$\frac{x^2}{L^2}$	$159,636$	$383,031$	$393,520$	-10489	39.49%	0.2465
$1-\cos\left(\frac{\pi x}{2L}\right)$	$164,157$	$431,388$	$441,587$	-10198	40.62	0.2580
$\frac{2Lx^2-x^3}{2L^3}$	$165,830$	$472,453$	$482,548$	-10095	41.03	0.2686
first mode	$168,445$	$543,282$	$553,333$	-10051	41.68	0.2858
$\frac{6L^2x^2-4Lx^3+x^4}{3L^4}$	$169,764$	$595,562$	$605,586$	-10024	42	0.2981
2nd mode	$185,852$	$14,443,032$	$14,509,551$	-66519	45.98	1.403
3rd mode	$192,575$	$100,304,976$	$100,475,002$	-170026	47.65	3.6323
4th mode	$195,562$	$371,956,138$	$372,284,973$	-328835	48.386	6.941

The shape functions above indicated by the mode, are the mode shape function for a beam with fixed-free boundary conditions obtained from table 8.1 from reference [1].

The following diagram describes the computation done at each element of the wind tower



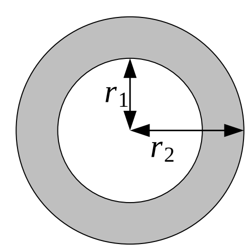
The effective flexural stiffness is $K_{fe} = \sum_i EI_i M_i \theta_i$

Where θ_i angle increment given by $\Phi_i'' h$

M_i is the bending moment given by $\Phi_i'' y_i$

Where $I_i = \frac{\pi}{4}(r_2^4 - r_1^4)$

Hence $K_{fe} = \sum_i EI_i (\Phi_i'')^2 h$



Effective mass $M_e = \sum_i m_i \Phi_i^2$

And effective geometric stiffness is $K_{ge} = \sum_i (\Phi_i')^2 \bar{m}_i g h$

where $\bar{m}$ is the accumlative mass from all the top elements
and g is 9.81 meter/sec^2

0.2 Conclusions

The lowest approximate natural frequency found is $\boxed{0.2465 \text{ Hz}}$ for the shape function $\frac{x^2}{L^2}$. The effective mass to actual mass ratio for this case was 39.49%

The higher the natural frequency became as the shape function is changed, this ratio also increased. At $f = 6.941$ Hz, this ratio became almost 50%.

An applet was written to simulate the result allowing one to select different shape functions and observe the result.

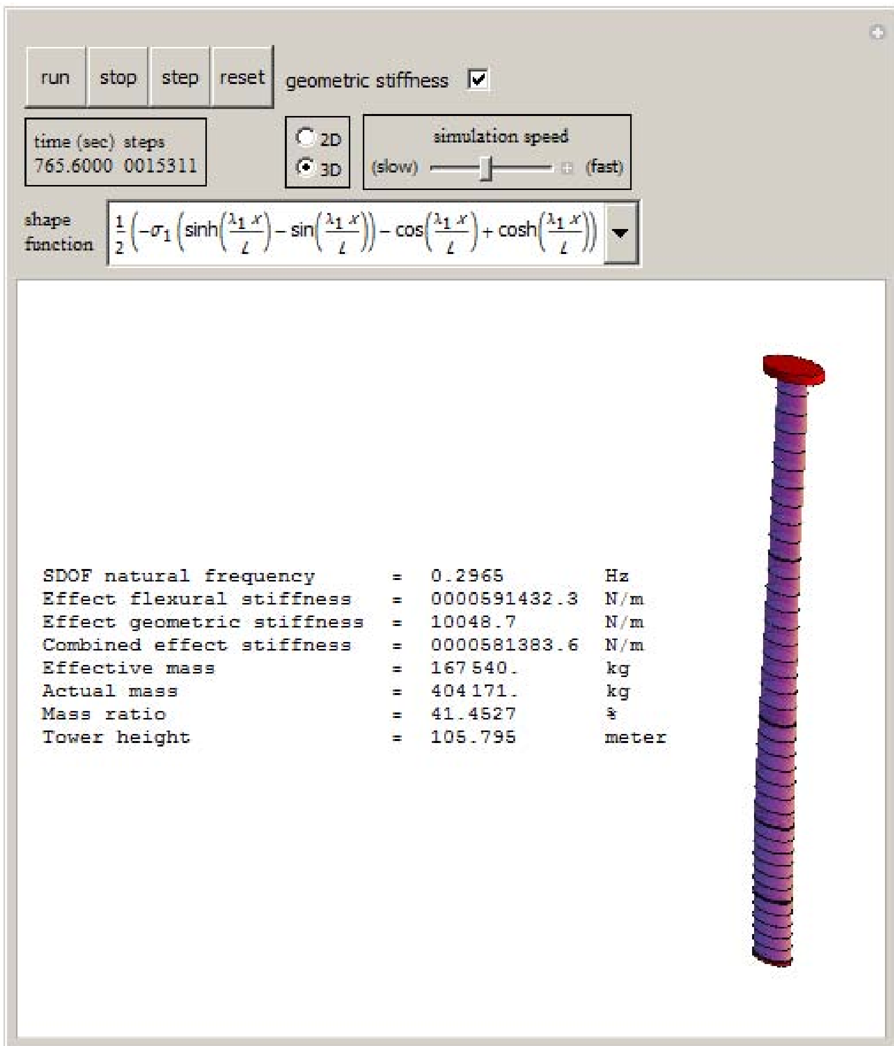


Figure 1: Mathematica demonstration

This table shows the final computation result for the case that gave the lowest natural frequency

#	height	T (m)	D (m)	mass (kg)	E (GPa)	geometric Stiffness (N/m)	flexural Stiffness (N/m)	effective stiffness (N/m)	current height (m)	shape Function	curveture	angle	I (m^4)	effective mass (kg)
1	1.34	0.	0.	130000.	2.1×10^{11}	0	0	0	106.815	1.	0	0	0	130000.
2	0.295	0.121	2.8	2374.83	2.1×10^{11}	65.4782	1742.69	1677.21	105.475	0.975067	0.000175293	0.0000517115	0.915479	2257.88
3	2.3	0.015	2.8	2386.14	2.1×10^{11}	516.807	1888.5	1371.7	105.18	0.969621	0.000175293	0.000403175	0.127245	2243.36
4	2.94	0.015	2.822	3062.18	2.1×10^{11}	646.4	2471.66	1825.26	102.88	0.927678	0.000175293	0.000515363	0.130284	2635.27
5	2.94	0.015	2.844	3086.28	2.1×10^{11}	623.643	2530.23	1906.59	99.94	0.875415	0.000175293	0.000515363	0.133371	2365.18
6	2.935	0.015	2.868	3106.17	2.1×10^{11}	599.42	2590.76	1991.34	97.	0.824668	0.000175293	0.000514486	0.136794	2112.43
7	2.935	0.015	2.89	3131.32	2.1×10^{11}	575.951	2651.15	2075.2	94.065	0.775518	0.000175293	0.000514486	0.139983	1883.26
8	2.935	0.015	2.912	3155.37	2.1×10^{11}	552.162	2712.48	2160.32	91.13	0.727877	0.000175293	0.000514486	0.143222	1671.73
9	2.935	0.016	2.934	3390.22	2.1×10^{11}	528.833	2956.7	2427.87	88.195	0.681747	0.000175293	0.000514486	0.156116	1575.7
10	2.93	0.017	2.956	3621.95	2.1×10^{11}	505.006	3204.35	2699.34	85.26	0.637127	0.000175293	0.00051361	0.169481	1470.26
11	2.93	0.018	2.978	3862.51	2.1×10^{11}	482.455	3466.1	2983.65	82.33	0.594089	0.000175293	0.00051361	0.183326	1363.24
12	2.925	0.019	3.	4099.09	2.1×10^{11}	459.353	3730.72	3271.37	79.4	0.552556	0.000175293	0.000512733	0.197659	1251.53
13	0.28	0.18	3.	3529.65	2.1×10^{11}	41.6633	2875.77	2834.1	76.475	0.512595	0.000175293	0.0000490821	1.59164	927.428
14	2.885	0.02	3.052	4307.75	2.1×10^{11}	437.018	4075.61	3638.59	76.195	0.508848	0.000175293	0.000505721	0.218926	1115.39
15	2.885	0.02	3.124	4396.6	2.1×10^{11}	414.825	4372.9	3958.07	73.31	0.471045	0.000175293	0.000505721	0.234895	975.529
16	2.88	0.021	3.196	4715.07	2.1×10^{11}	392.306	4905.37	4513.07	70.425	0.4347	0.000175293	0.000504845	0.263955	890.978
17	2.88	0.021	3.268	4823.23	2.1×10^{11}	370.427	5246.71	4876.28	67.545	0.399873	0.000175293	0.000504845	0.282322	771.226
18	2.88	0.022	3.34	5164.63	2.1×10^{11}	348.886	5865.08	5516.19	64.665	0.3665	0.000175293	0.000504845	0.315596	693.725
19	2.875	0.022	3.412	5268.77	2.1×10^{11}	326.664	6244.36	5917.7	61.785	0.334581	0.000175293	0.000503968	0.336589	589.81
20	2.875	0.022	3.484	5381.87	2.1×10^{11}	305.063	6650.73	6345.67	58.91	0.304168	0.000175293	0.000503968	0.358494	497.921
21	2.87	0.023	3.556	5733.12	2.1×10^{11}	283.321	7376.82	7093.5	56.035	0.275204	0.000175293	0.000503092	0.398325	434.21
22	2.87	0.023	3.628	5851.16	2.1×10^{11}	262.196	7837.06	7574.87	53.165	0.247735	0.000175293	0.000503092	0.423176	359.101
23	2.86	0.023	3.7	5947.94	2.1×10^{11}	240.32	8287.09	8046.77	50.295	0.22171	0.000175293	0.000501339	0.449041	292.373
24	0.33	0.23	3.7	6540.68	2.1×10^{11}	25.3972	8071.31	8045.92	47.435	0.197212	0.000175293	0.0000578468	3.79036	254.384
25	2.71	0.024	3.76	5986.44	2.1×10^{11}	211.099	8594.67	8383.57	47.105	0.194478	0.000175293	0.000475045	0.491484	226.416
26	2.71	0.024	3.825	6087.4	2.1×10^{11}	192.408	9051.11	8858.7	44.395	0.172744	0.000175293	0.000475045	0.517585	181.652
27	2.71	0.024	3.89	6192.4	2.1×10^{11}	174.03	9523.42	9349.39	41.685	0.152298	0.000175293	0.000475045	0.544595	143.631
28	2.705	0.025	3.955	6546.	2.1×10^{11}	155.911	10401.9	10246.	38.975	0.13314	0.000175293	0.000474169	0.59593	116.036
29	2.705	0.025	4.02	6655.17	2.1×10^{11}	138.59	10926.6	10788.	36.27	0.1153	0.000175293	0.000474169	0.62599	88.475
30	2.705	0.025	4.085	6764.34	2.1×10^{11}	121.796	11468.6	11346.8	33.565	0.0987436	0.000175293	0.000474169	0.657044	65.9543
31	2.685	0.026	4.15	7093.36	2.1×10^{11}	104.928	12408.	12303.1	30.86	0.0834694	0.000175293	0.000470663	0.716154	49.4204
32	0.36	0.24	4.15	8389.62	2.1×10^{11}	12.0884	13136.7	13124.6	28.175	0.0695766	0.000175293	0.0000631056	5.65503	40.6134
33	2.41	0.026	4.15	6417.42	2.1×10^{11}	80.6735	11137.1	11056.5	27.815	0.06781	0.000175293	0.000422457	0.716154	29.5086
34	2.41	0.027	4.15	6662.63	2.1×10^{11}	68.8614	11557.1	11488.3	25.405	0.0565684	0.000175293	0.000422457	0.74316	21.3203
35	2.41	0.028	4.15	6907.72	2.1×10^{11}	57.743	11976.5	11918.7	22.995	0.0463449	0.000175293	0.000422457	0.770126	14.8368
36	2.41	0.029	4.15	7152.69	2.1×10^{11}	47.3746	12395.2	12347.9	20.585	0.0371396	0.000175293	0.000422457	0.797053	9.86606
37	2.405	0.029	4.15	7137.85	2.1×10^{11}	37.7092	12369.5	12331.8	18.175	0.0289524	0.000175293	0.000421581	0.797053	5.98324
38	2.405	0.03	4.15	7382.19	2.1×10^{11}	29.0553	12786.8	12757.7	15.77	0.0217971	0.000175293	0.000421581	0.82394	3.50738
39	0.44	0.39	4.15	16023.5	2.1×10^{11}	4.00782	23363.5	23359.5	13.365	0.0156557	0.000175293	0.0000771291	8.22878	3.92739
40	2.4	0.031	4.15	7610.56	2.1×10^{11}	20.905	13176.	13155.1	12.925	0.0146419	0.000175293	0.000420704	0.850788	1.63158
41	2.4	0.032	4.15	7854.15	2.1×10^{11}	14.177	13591.2	13577.	10.525	0.00970912	0.000175293	0.000420704	0.877597	0.740387
42	2.395	0.034	4.15	8323.61	2.1×10^{11}	8.62937	14389.7	14381.	8.125	0.00578605	0.000175293	0.000419828	0.931096	0.278661
43	2.395	0.06	4.15	14595.9	2.1×10^{11}	4.46481	24919.	24914.5	5.73	0.00287769	0.000175293	0.000419828	1.61241	0.120871
44	2.395	0.06	4.15	14595.9	2.1×10^{11}	1.57107	24919.	24917.4	3.335	0.000974826	0.000175293	0.000419828	1.61241	0.0138703
45	0.24	0.4	4.15	8940.34	2.1×10^{11}	0.0127931	12974.4	12974.4	0.94	0.0000774446	0.000175293	0.0000420704	8.37774	0.000053621
46	0.7	0.055	4.15	3915.31	2.1×10^{11}	0.0208944	6700.56	6700.54	0.7	0.0000429469	0.000175293	0.000122705	1.48341	7.22154×10

Figure 2: Final table

0.3 References

1. Formulas for Natural Frequency and Mode Shape, Robert D. Blevins
2. Dynamics of structures by Ray W. Clough and Joseph Penzien.
3. Structural Dynamics, 5th edition by Mario Paz and William Leigh.
4. Professor Oliva class lecture notes, CEE 744, structural dynamics, spring 2013, University of Wisconsin, Madison.
5. http://en.wikipedia.org/wiki/List_of_moment_of_areas