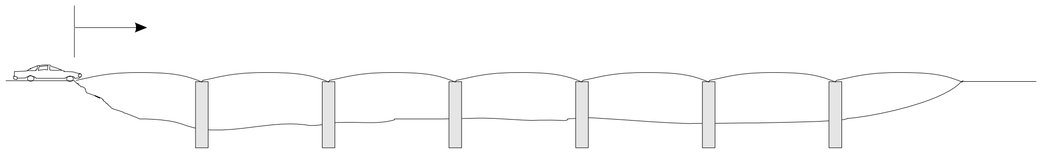
Analysis of motion in a car crossing south beltline bridge

A vehicle crossing the south beltline bridges experiences vertical dynamic vibration due to the residual camber in the bridge. Determine the extent of vertical motion that will occur. (Insert your values in the highlighted regions.)

x(t) = location of the car



Bridge data: span length = 70 ft.
upward camber = 2.5 inches

$\lambda := 70 \cdot ft$

$\Delta := \frac{2.5}{12} \cdot ft$

Car data:

$weight := 1800lb$

$speed := 60mph$

$\xi := 0.75$

$k := 5000 \frac{lb}{ft}$

$m := \frac{weight}{32.2} \cdot \frac{sec^2}{ft}$ $\quad m = 55.901 \frac{lb \cdot s^2}{ft}$

$\omega_n := \sqrt{\frac{k}{m}}$ $\quad \omega_n = 9.458 \cdot \frac{rad}{sec}$

$f_n := \frac{\omega_n}{2 \cdot \pi}$ $\quad f_n = 1.505 \frac{1}{s}$

$T_n := \frac{1}{f_n}$ $\quad T_n = 0.664\, s$

$\omega_d := \omega_n \cdot \sqrt{1 - \xi^2}$ $\quad \omega_d = 6.256 \cdot \frac{rad}{sec}$

$sp := speed$ $\quad sp = 88 \frac{ft}{s}$

PART #1: Define load and convert to a series form

Loading is as if ground is moving up and down under car. This is like an EQ load. Define the ground movement and acceleration.

-the car travels through 1/2 cycle in one span:

$$y_g(t) := \Delta \cdot \sin\left(\pi \cdot \frac{x}{\lambda}\right)$$ for 0<x<span

-the car location "x" is dependent on speed and time:

$$x := sp \cdot t$$

$$y_g(t) := \Delta \cdot \sin\left(\pi \cdot \frac{sp}{\lambda} \cdot t\right)$$

$$acc_g(t) := -\Delta \cdot \left(\frac{\pi \cdot sp}{\lambda}\right)^2 \cdot \sin\left(\pi \cdot \frac{sp}{\lambda} \cdot t\right)$$

-the span length/speed = time to cross one span,

$$T_p := \frac{\lambda}{sp} \qquad T_p = 0.795\,s \qquad \beta\epsilon\tau\alpha := \frac{T_n}{T_p} \qquad \beta\epsilon\tau\alpha = 0.835$$

$$acc_g(t) := -\Delta \cdot \left(\frac{\pi}{T_p}\right)^2 \cdot \sin\left(\frac{\pi}{T_p} \cdot t\right)$$

rounded time, more than one period:

$$T_a := 1.2s$$

Then the load in one span (0<t<Tp):

$$P_a(t) := m \cdot \Delta \cdot \left(\frac{\pi}{T_p}\right)^2 \cdot \sin\left(\frac{\pi}{T_p} \cdot t\right)$$

steps in analysis:

$$st := \frac{T_a}{.01s} \qquad st = 120$$

We need to convert this load to a periodic form that works for any point in time, until the vehicle is off of the bridge.

end of load:

$$T_{max} := 7 \cdot T_p \qquad T_{max} = 5.568\,s$$

$T_{max} =$!

<u>Convert the load to a series - Fourier Transform:</u>

$$P_a(t) := m \cdot \Delta \cdot \left(\frac{\pi}{T_p}\right)^2 \cdot \sin\left(\frac{\pi}{T_p} \cdot t\right)$$ with a period of Tp, $$P_o := m \cdot \Delta \cdot \left(\frac{\pi}{T_p}\right)^2$$

$P_o = 181.654\ \text{lb}$

$$P_f(t) := a_o + \sum_{n=1}^{\infty}\left(a_n\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T_p}\right)\right) + \sum_{n=1}^{\infty}\left(b_n\cdot\sin\left(2\cdot\pi\cdot n\cdot\frac{t}{T_p}\right)\right)$$

where:

$$a_o := \frac{1}{T_p}\cdot\int_0^{T_p} P_o\cdot\sin\left(\pi\cdot\frac{t}{T_p}\right)dt$$

$a_o = 115.644\ \text{lb}$

and: $n := 1, 2 .. 10$

$$a_n := 2\cdot\frac{P_o}{T_p}\cdot\int_0^{T_p}\sin\left(\pi\cdot\frac{t}{T_p}\right)\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T_p}\right)dt$$

$a_n =$

-77.096	lb
-15.419	
-6.608	
-3.671	
-2.336	
-1.617	
-1.186	
-0.907	
-0.716	
-0.58	

theoretically
all "b" = zero,

$a_0 := a_o$ $a_0 = 115.644\ \text{lb}$