Name: ______________________________

EMA 545 – Practice Exam #2
Spring 2013
Prof. M. S. Allen

Honor Pledge: On my honor, I pledge that I have neither given nor received inappropriate aid in the preparation of this exam.

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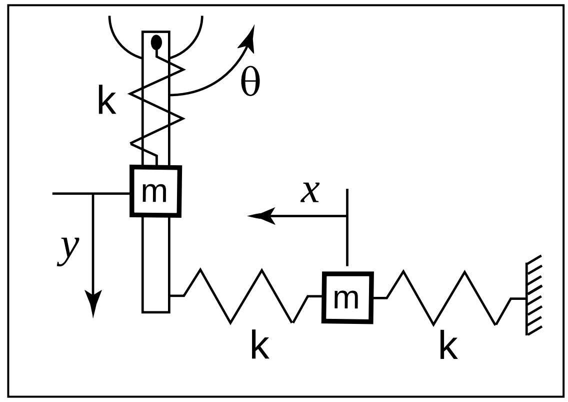
Signature

One (1) 8.5x11" double-sided sheet of notes allowed and must be turned in with your exam.

Problem #1 (10 pts)

a.) A colleague asserts that the linearized equations of motion for this system are as given below, where $\times$'s denote terms that are not given to you, which may be zero or constant. $[\,]|_{springs}$ denotes the portion of the stiffness matrix due to the springs and $[\,]|_{gravity}$ denotes that portion due to gravity. Check the ***units*** and the ***sign*** on the $K_{12}|_{springs}$ term. If incorrect, please provide the corrected term and ***explain your reasoning***. (The left mass is constrained so that it slides along the bar as the bar rotates.)



$$\begin{bmatrix} I & 0 & \times \\ 0 & m & \times \\ \times & \times & \times \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} kL^2 & k & \times \\ k & \times & \times \\ \times & \times & \times \end{bmatrix}_{springs} \begin{Bmatrix} \theta \\ x \\ y \end{Bmatrix} + \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}_{gravity} \begin{Bmatrix} \theta \\ x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

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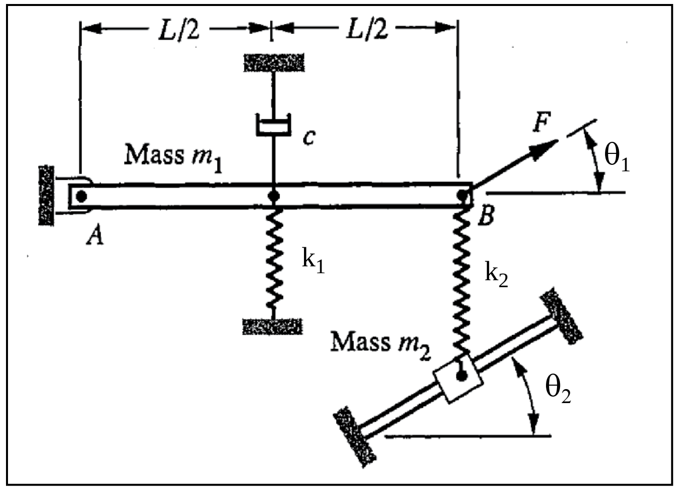
Problem #2 (45 pts)

Gravity acts downward (along dashpot c), and the initial lengths of the springs are such that the position shown corresponds to the static equilibrium when the applied dynamic force $F(t)$ is not present. The moment of inertia of a rod about its mass center is $I_g = (1/12)mL^2$ and about its end is $I_{end} = (1/3)mL^2$.



a.) Identify generalized coordinates and derive the corresponding equations of motion. Employ the stiff-spring approximation to simplify your analysis. Friction is negligible in the pin joint A and the friction force between the guide and m_2 is equal to $f=c_2v$, where v is the speed of the mass. (30 pts)

b.) Check that your answers make sense. Explain each check that you perform and why it shows that your EOM are/are not correct. (15 pts)

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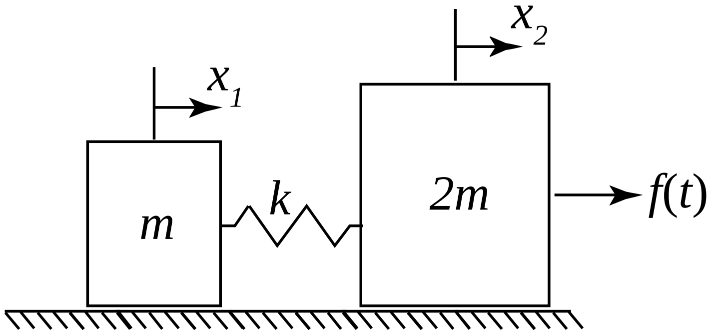
Problem #3 (45 pts)

The system pictured is initially at rest when an impulsive force $f(t) = F_0\delta(t-T)$ is applied to the mass on the right. The masses are constrained so that they only translate in the horizontal direction, and there is no friction between the masses and ground.



The equations of motion of this system are:

$$\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}$$

Find the response of the first mass, $x_1(t)$, as a function of time.

APPENDIX B

TRANSIENT RESPONSES FOR UNDERDAMPED ONE-DEGREE-OF-FREEDOM SYSTEMS

$$\ddot{q} + 2\zeta\omega_{\text{nat}}\dot{q} + \omega_{\text{nat}}^2 q = \frac{F(t)}{M} \quad \zeta < 1, \quad \omega_{\text{d}} = \omega_{\text{nat}}\sqrt{1-\zeta^2}$$

- Free vibration: $F(t) = 0$

$$q = \exp(-\zeta\omega_{\text{nat}}t)\left[q(0)\cos(\omega_{\text{d}}t) \quad \frac{\dot{q}(0) + \zeta\omega_{\text{nat}}q(0)}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]$$

- Impulse excitation: $F(t) = \delta(t)$

$$q = \frac{1}{M\omega_{\text{d}}}\exp(-\zeta\omega_{\text{nat}}t)\sin(\omega_{\text{d}}t)h(t)$$

- Step excitation: $F(t) = h(t)$

$$q = \frac{1}{M\omega_{\text{nat}}^2}\left\{1 - \exp(-\zeta\omega_{\text{nat}}t)\left[\cos(\omega_{\text{d}}t) + \frac{\zeta\omega_{\text{nat}}}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]\right\}h(t)$$

- Ramp excitation: $F(t) = th(t)$

$$q = \frac{1}{M\omega_{\text{nat}}^3}\left\{(\omega_{\text{nat}}t) - 2\zeta + \exp(-\zeta\omega_{\text{nat}}t)\left[2\zeta\cos(\omega_{\text{d}}t) - (1-2\zeta^2)\frac{\omega_{\text{nat}}}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]\right\}h(t)$$

- Quadratic excitation: $F(t) = t^2h(t)$

$$q = \frac{1}{M\omega_{\text{nat}}^4}\left\{(\omega_{\text{nat}}t)^2 - 4\zeta(\omega_{\text{nat}}t) - 2(1-4\zeta^2) + \exp(-\zeta\omega_{\text{nat}}t) \times \left[2(1-4\zeta^2)\cos(\omega_{\text{d}}t) + (6\zeta - 8\zeta^3\frac{\omega_{\text{nat}}}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]\right\}h(t)$$

- Exponential excitation:

$$F(t) = \exp(-\beta t)\,h(t)$$

$$q = \frac{1}{M(\omega_{\text{nat}}^2 - 2\zeta\omega_{\text{nat}}\beta + \beta^2)}\left\{\exp(-\beta t) - \exp(-\zeta\omega_{\text{nat}}t)\left[\cos(\omega_{\text{d}}t) + \frac{\zeta\omega_{\text{nat}} - \beta}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]\right\}h(t)$$

- Transient sinusoidal excitation:

$$F(t) = \sin(\omega t)h(t), \quad \omega \neq \omega_{\text{nat}} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{\text{nat}}^2 - \omega^2)^2 + 4\zeta^2\omega_{\text{nat}}^2\omega^2]}$$
$$\times\left\{(\omega_{\text{nat}}^2 - \omega^2)\sin(\omega t) - 2\zeta\omega_{\text{nat}}\omega\cos(\omega t) + \omega\exp(-\zeta\omega_{\text{nat}}t)\left[2\zeta\omega_{\text{nat}}\cos(\omega_{\text{d}}t) - \frac{(1-2\zeta^2)\omega_{\text{nat}}^2 - \omega^2}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]\right\}h(t)$$

- Transient co-sinusoidal excitation:

$$F(t) = \cos(\omega t)h(t), \quad \omega \neq \omega_{\text{nat}} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{\text{nat}}^2 - \omega^2)^2 + 4\zeta^2\omega_{\text{nat}}^2\omega^2]}$$
$$\times\left\{(\omega_{\text{nat}}^2 - \omega^2)\cos(\omega t) + 2\zeta\omega_{\text{nat}}\omega\sin(\omega t) - \exp(-\zeta\omega_{\text{nat}}t)\left[(\omega_{\text{nat}}^2 - \omega^2)\cos\omega_{\text{d}}t + \frac{\zeta\omega_{\text{nat}}(\omega_{\text{nat}} + \omega^2)}{\omega_{\text{d}}}\sin(\omega_{\text{d}}t)\right]\right\}h(t)$$