Math 320 Exam Crib Sheet

1. Integration by Parts Formula

$$\int u \, dv = uv - \int v \, du$$

Example:

$$\int x \exp(x) dx = x \exp(x) - \int \exp(x) dx + C = x \exp(x) - \exp(x) + C$$

with $u = x$, $dv = \exp(x) \, dx$, $du = dx$, and $v = \exp(x)$.

2. Example of Partial Fractions

$$\int \frac{5}{(x^2 - 5x + 6)} dx = \int \frac{5}{(x-2)(x-3)} dx$$

Let

$$\frac{5}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

$$= \frac{A(x-3) + B(x-2)}{(x-2)(x-3)}$$

Therefore

$$(A + B)x = 0 \quad \text{and} \quad -3A - 2B = 5.$$

Solving $A + B = 0$ and $-3A - 2B = 5$ gives $A = -5$ and $B = 5$. So finally

$$\int \frac{5}{(x^2 - 5x + 6)} dx = \int \frac{-5}{(x-2)} dx + \int \frac{+5}{(x-3)} dx = -5 \ln |x-2| + 5 \ln |x-3| + C.$$

3. Exponentials and the Natural Logarithm: All arguments of ln are assumed greater than zero.

$$\ln(1) = 0$$

$$\ln(a/b) = \ln(a) - \ln(b)$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln(a^r) = r \ln(a)$$

$$\int \frac{1}{u} du = \ln|u| + C, \quad u \neq 0$$

$$\exp(\ln(x)) = x$$

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$$\exp(a + b) = \exp(a)\exp(b)$$

$$\exp(a - b) = \frac{\exp(a)}{\exp(b)}$$

$$\exp(ab) = (\exp(a))^b = (\exp(b))^a$$

4. **Taylor Series for $f(x)$ about the point $x = x_o$:**

$$f(x) = \sum_{n=0}^{\infty} \frac{d^n}{dx^n} f(x)|_{x=x_o} \frac{(x - x_o)^n}{n!}$$