Application of Digital Signal Processing in Computed tomography (CT)

EE 518 project slides

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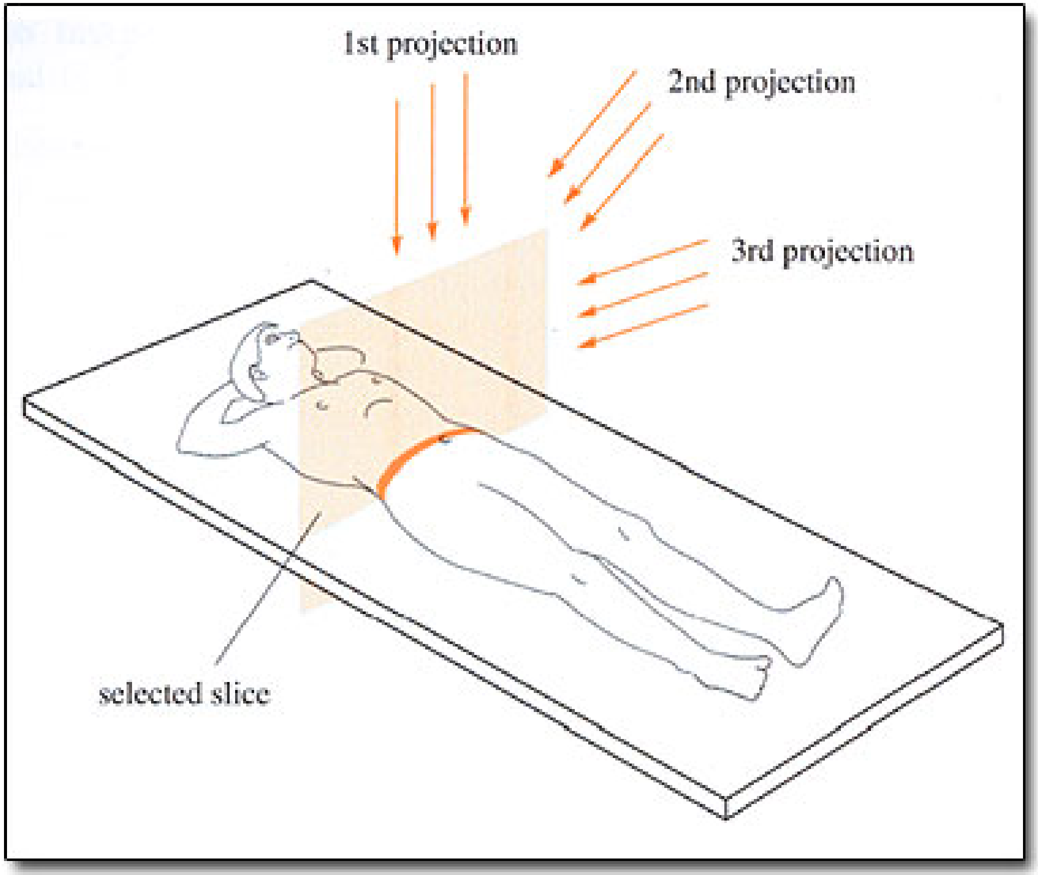
CT Overview

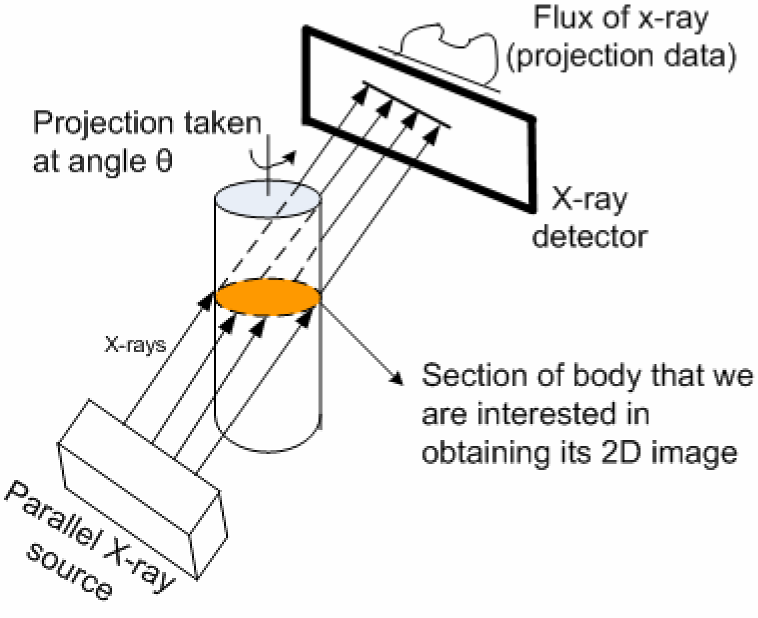
Uses X-ray to obtain multiple projections at different angles of the same cross section



Projection data processed using signal processing software to reconstruct 2D image of the cross section

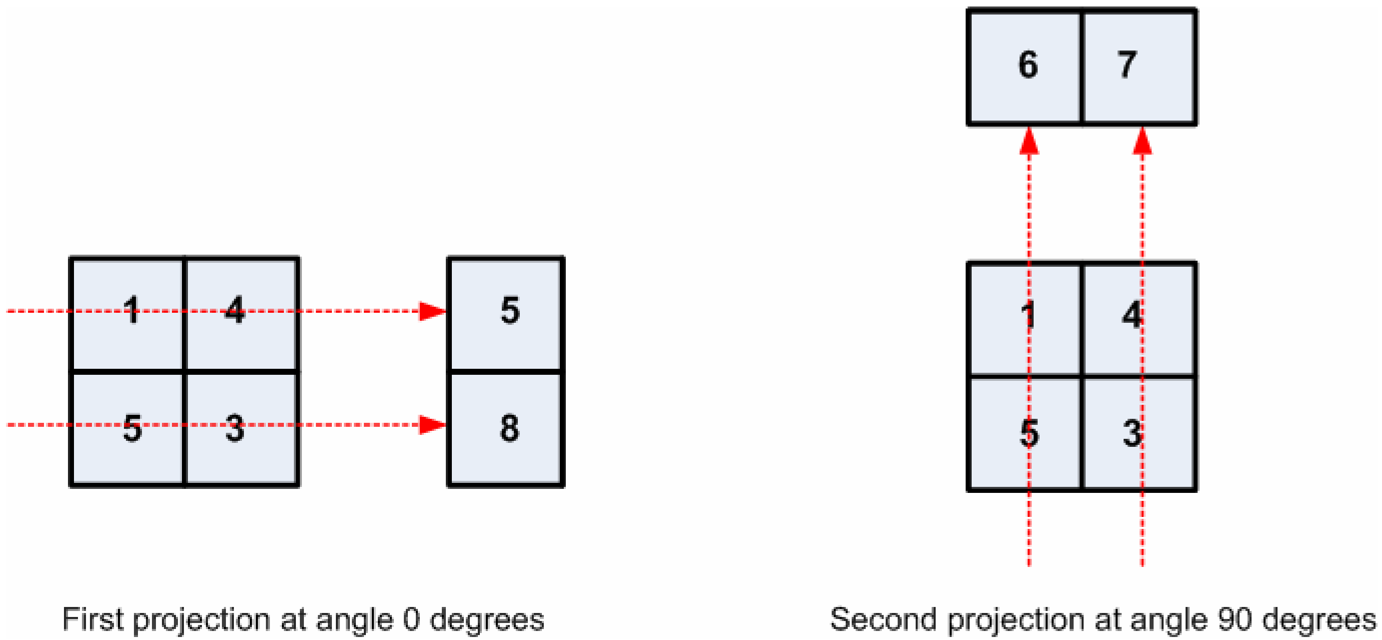
DSP medical imaging software converts projection data to 2D section images.
More projections leads to better images, but more x-ray exposure

Simplified view of CT with parallel X-ray showing projection data capture



Components of parallel X-ray CT system. Many projections are obtained at different angles. Projections are combined in software using filtered backprojection to obtain an accurate 2D image of the section of the body shown above.

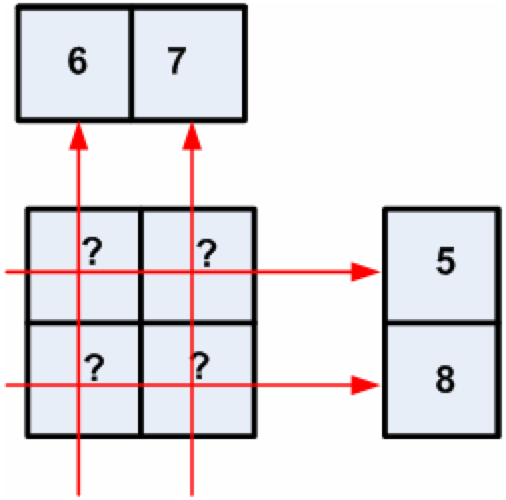
Illustrating the problem of image reconstruction on a simple 4 pixels image with 2 projections



A projection is an integral operation along the path of the ray. In other words, it sums the pixel values along its path, generating a vector of projection values.

The CT Inverse problem

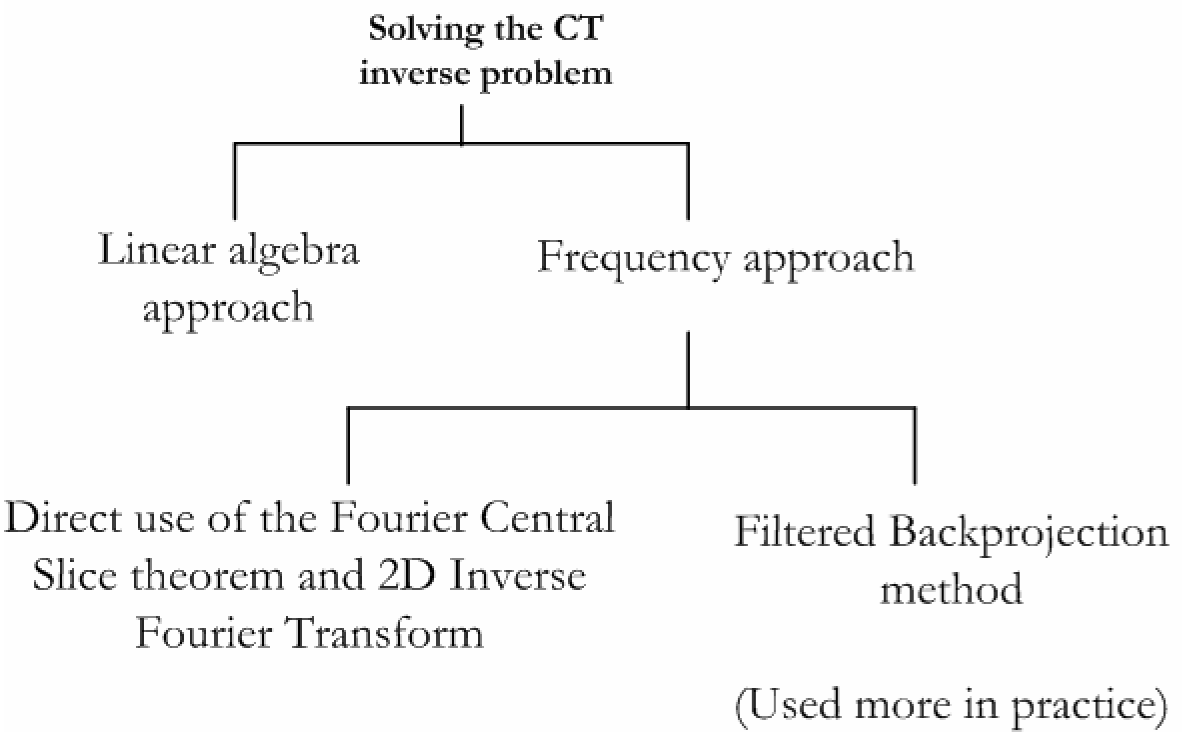
Determine the original image from the projection data only



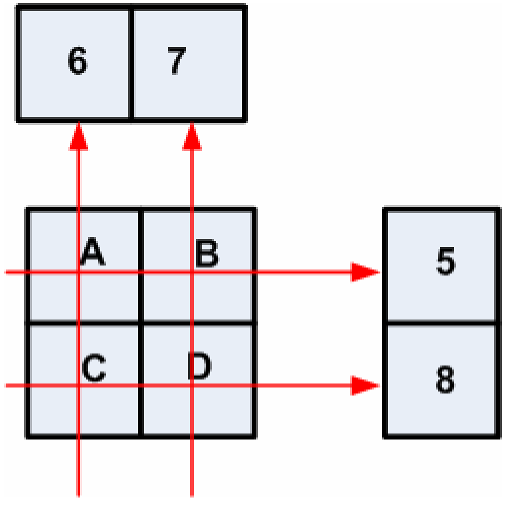
The problem of image reconstruction from projections and possible solutions

The problem is the following:

Given a set of projections with corresponding angles that these projections obtained at, determined the original image Method of solution



Solving the problem using linear algebra



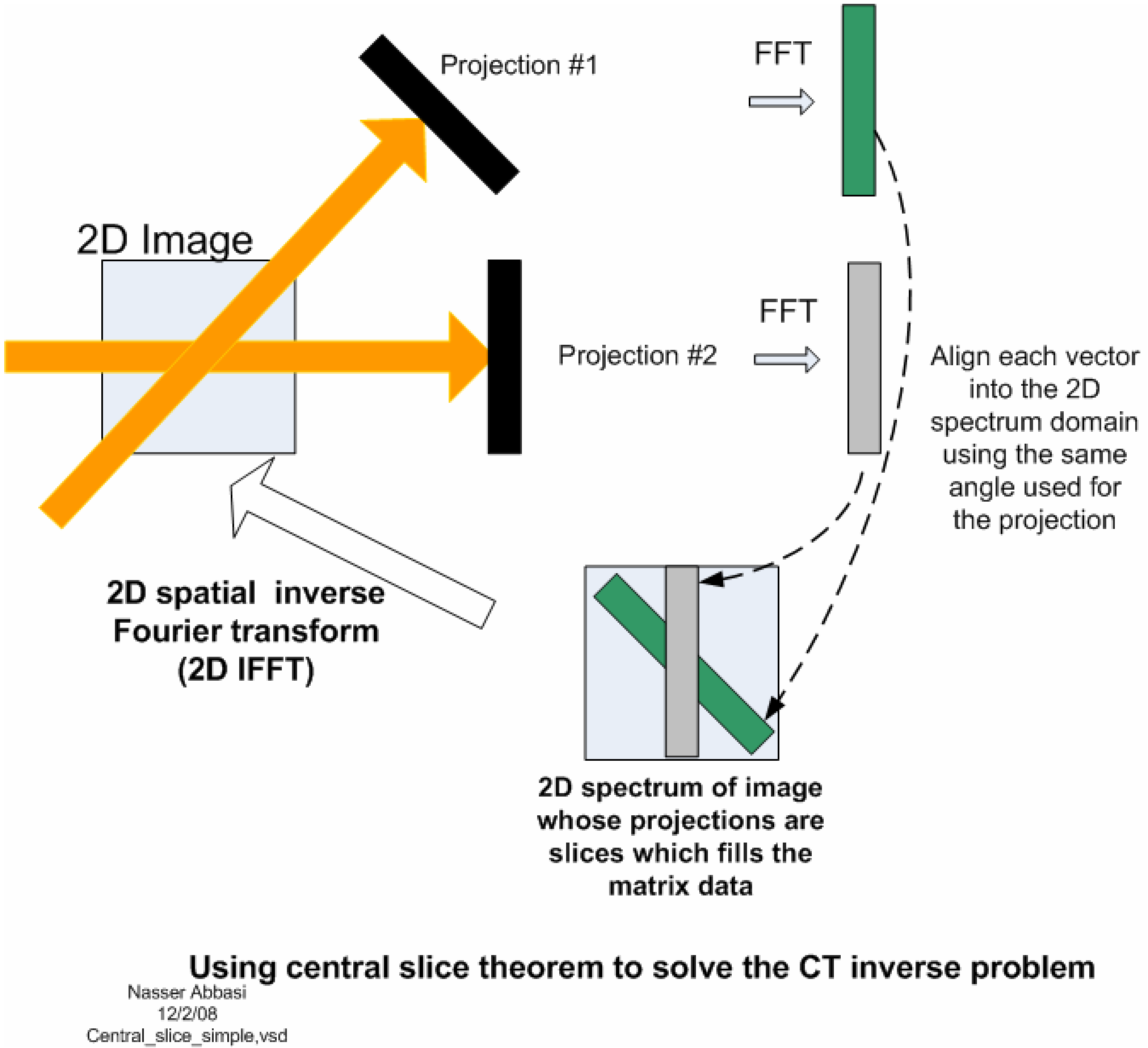
$$A + B = 5$$
$$C + D = 8$$
$$C + A = 6$$
$$D + B = 7$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 6 \\ 7 \end{pmatrix}$$

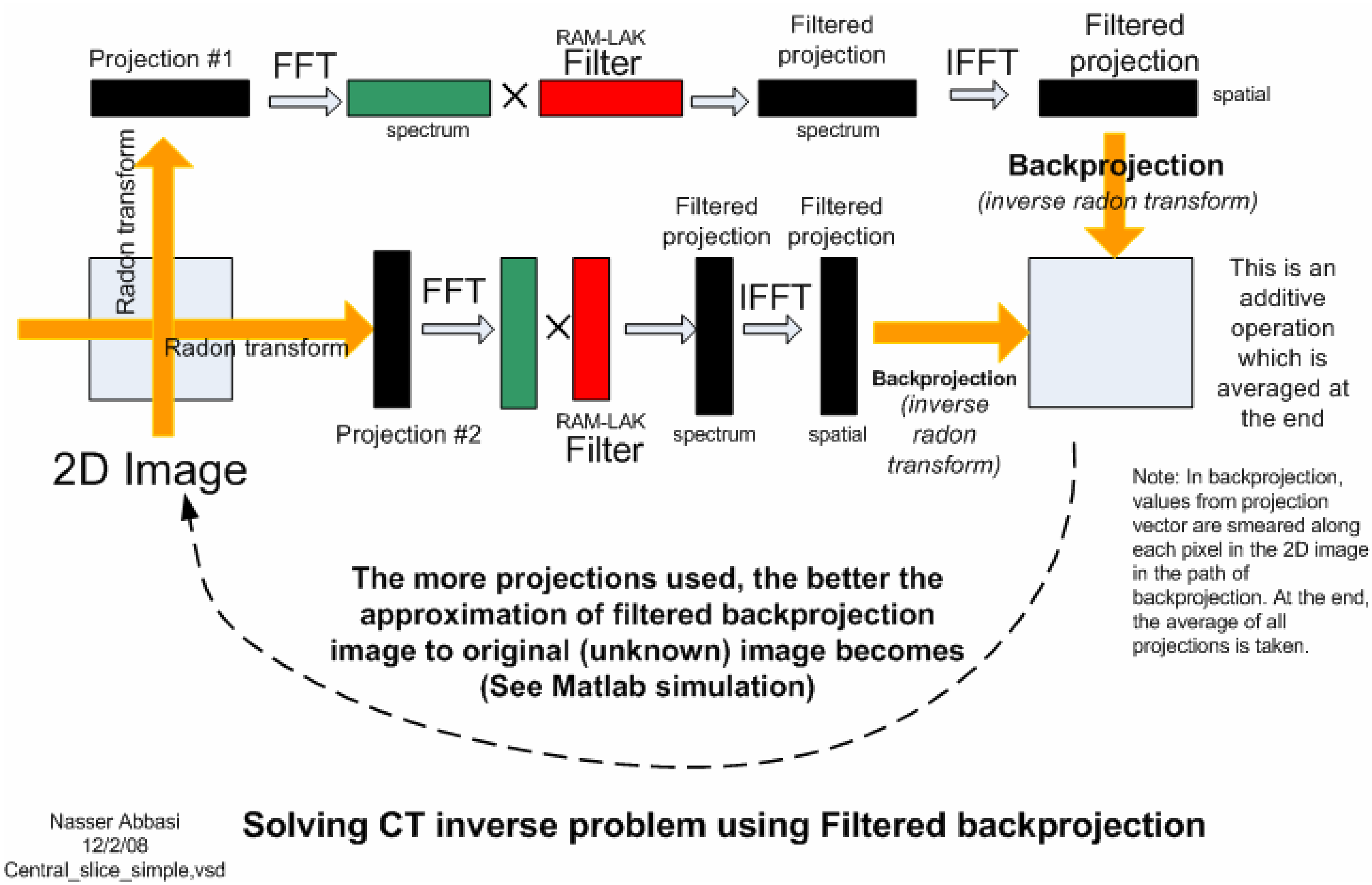
A x b

Determinant of A is zero. No unique solution exist. Infinite number of solutions. Use least square approach.

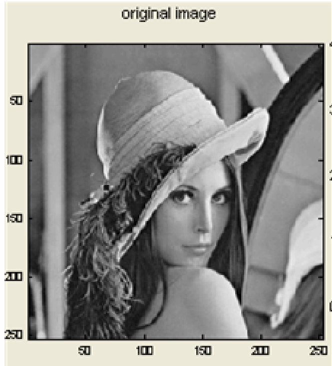
Solving the problem using frequency domain with Fourier Central Slice Theorem



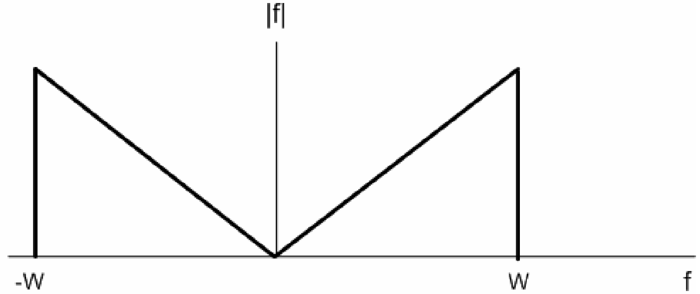
Solving the inverse CT problem using Filtered back projection



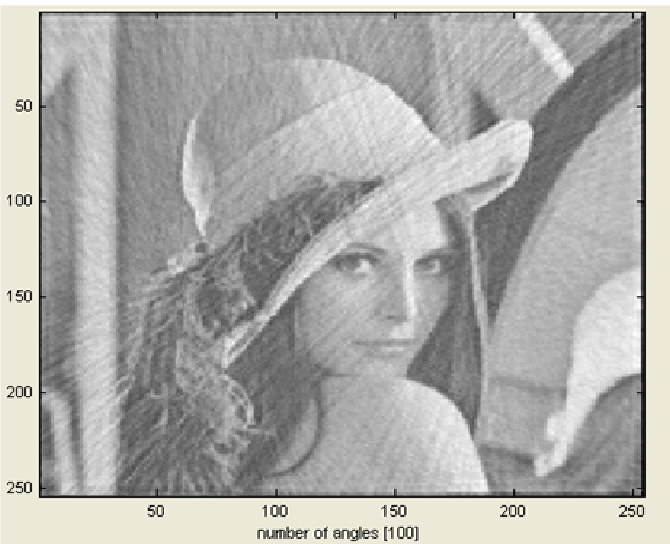
Affect of Filtering using RAM-LAK on quality of back projection image, result found by simulation using Matlab radon/inverse radon functions



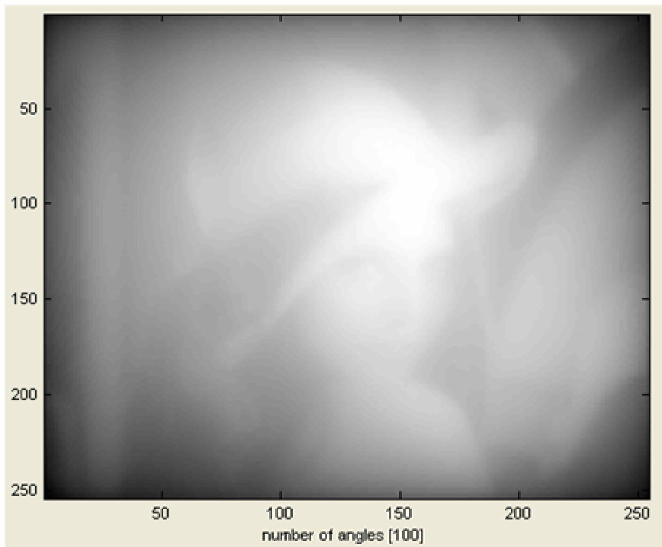
Original Image



RAM-LAK spectrum magnitude



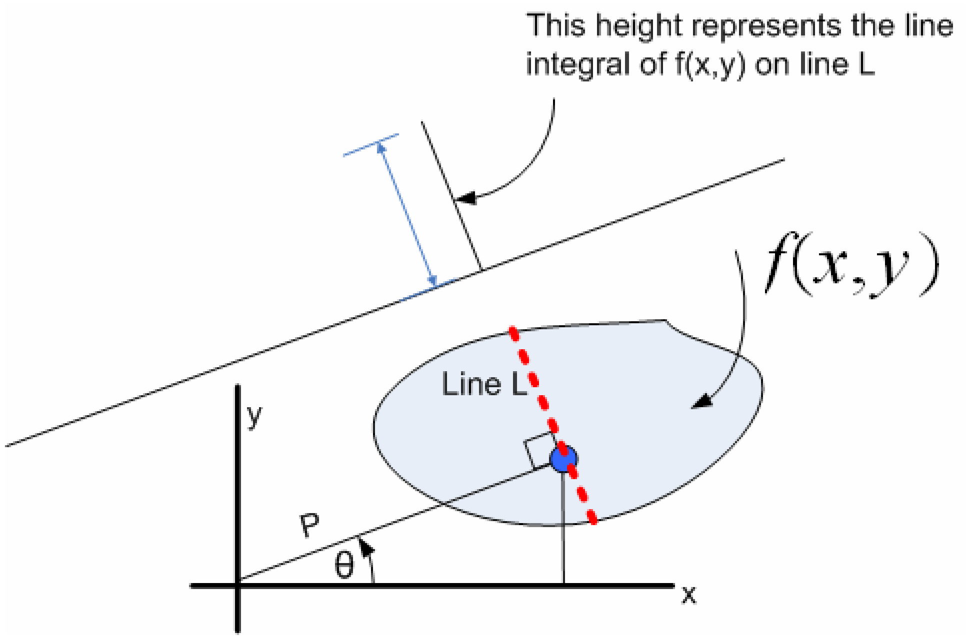
Reconstruction, 100 projections, Filtered backprojection (RAM-LAK filter)



Reconstruction, 100 projections, No filtering used before backprojection

Radon Transform

- Mathematically, a projection is performed using radon transform



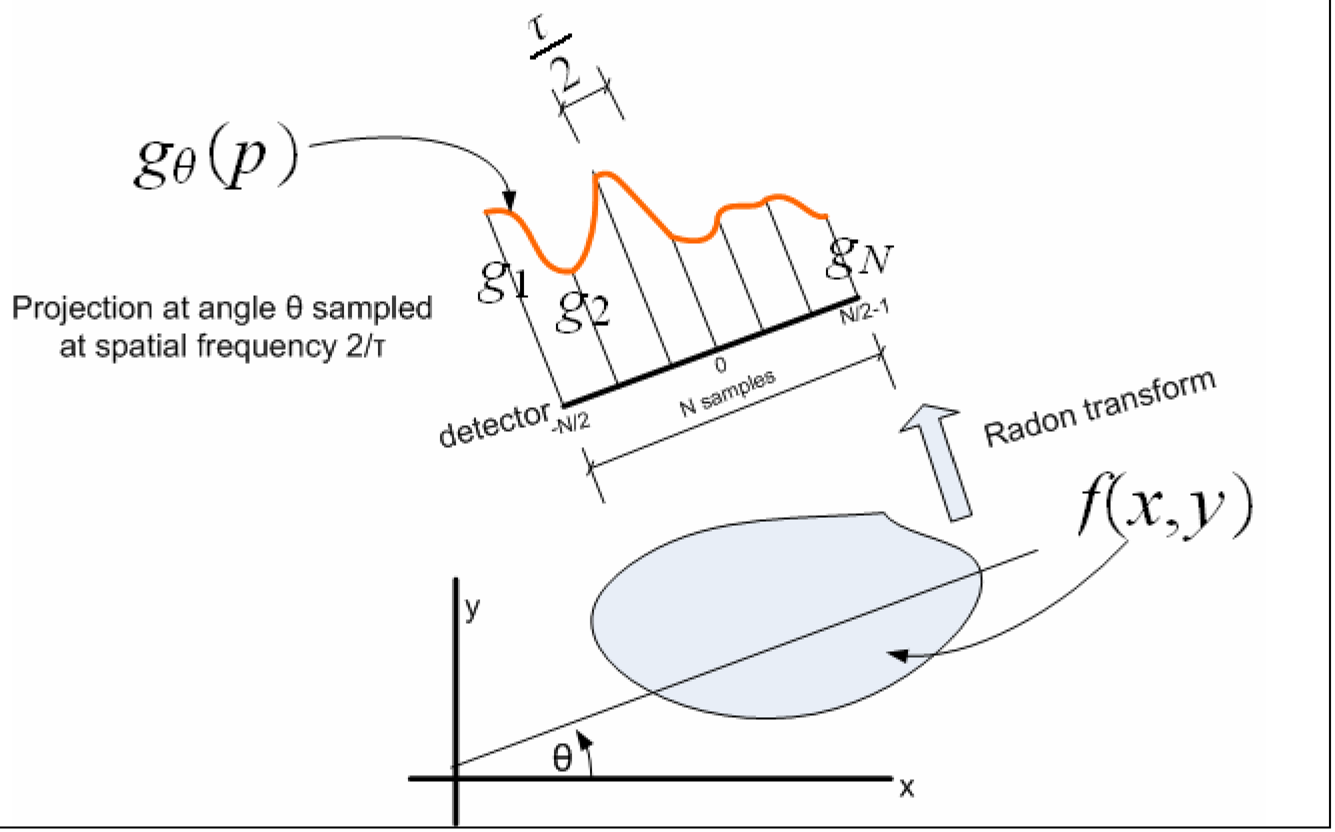
$$g(p,\theta) = \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} f(x,y)\, \delta(x\cos\theta + y\sin\theta - p)\, dxdy$$

Sampling the projection data

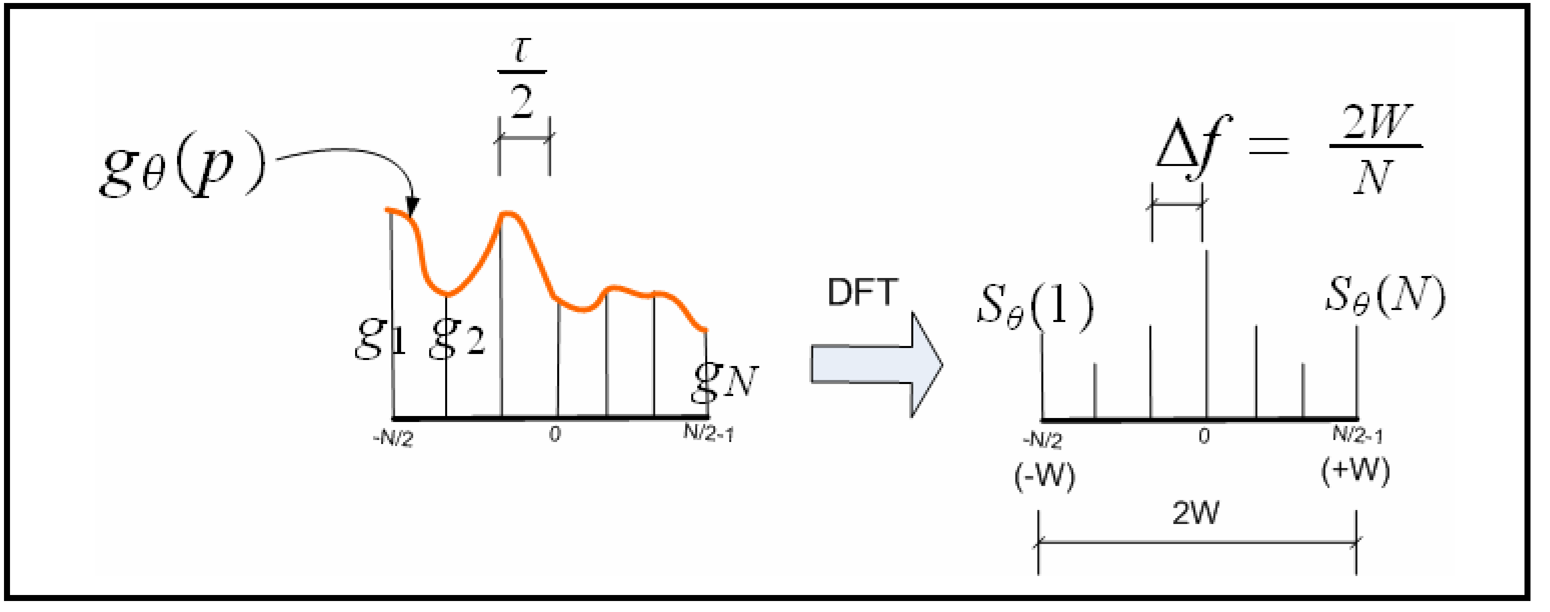
Once a complete projection is obtained, it is sampled at frequency larger than its Nyquist spatial frequency

$g_\theta[p]$ sampled to sequence of samples

$g_\theta\left(\frac{n}{2W}\right)$ where $f_s = 2W$, where $W = \frac{1}{\tau}$



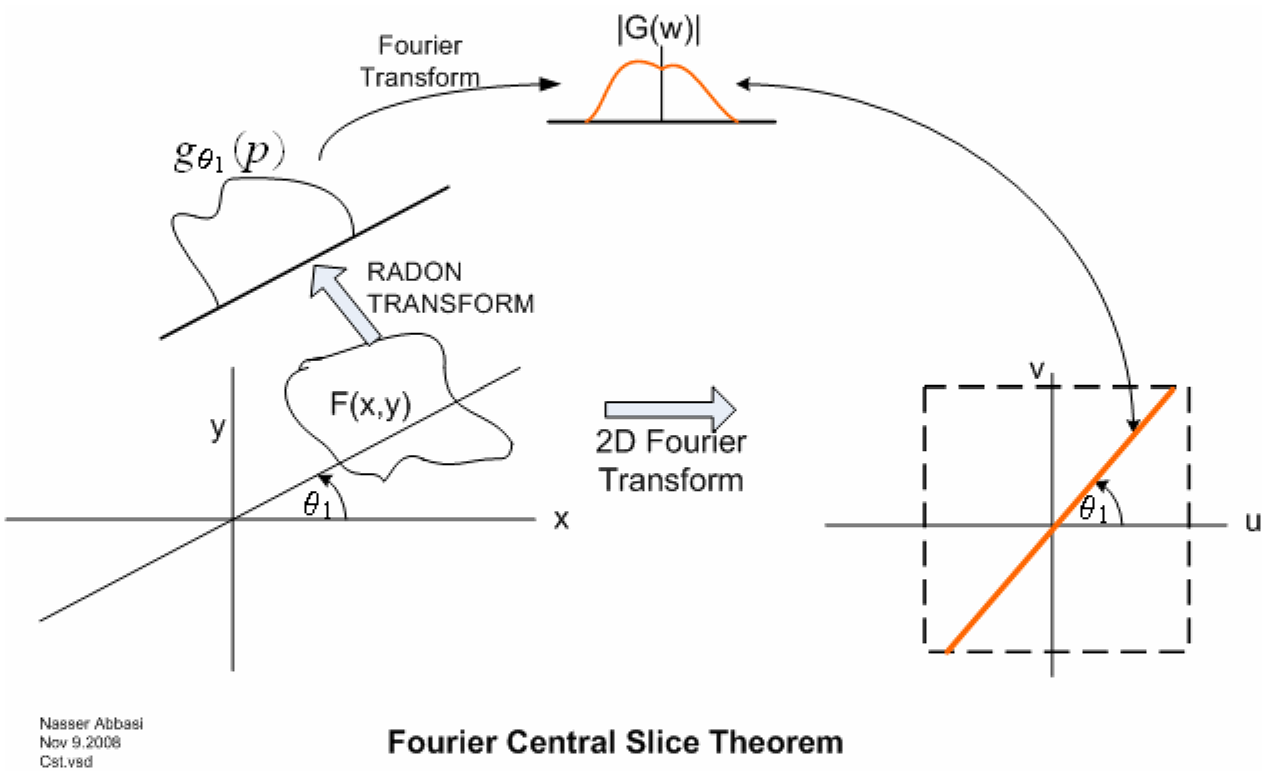
Obtain Fourier transform of each sampled projection (using FFT)



$$S(k,\theta_m) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} g\left(\frac{n}{2W},\theta_m\right)e^{-j\frac{2\pi}{N}kn} \qquad k = 0, 1, \cdots, N-1$$

First solution method
Apply the Fourier Central Slice Theorem

The Fourier transform of a projection taken at angle θ_m is equal to the values found along a slice in the 2D fourier transform of the original image itself, as long as this line goes through the origin of the 2D fourier transform plane and has the same angle θ_m



Fourier Central Slice Theorem

Second solution method: Filtered back projection

- Flow diagram shown earlier
- The mathematical expression of FBP derived from first principles in the project paper as follows

$$h(x,y) = \frac{2W\pi}{M} \sum_{m=0}^{M-1} \left[\overbrace{\frac{1}{N} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} \overbrace{S\left(n\frac{2W}{N}, m\frac{\pi}{M}\right)\left|n\frac{2W}{N}\right|}^{\text{Filtered Projection}} e^{nj\frac{4W\pi}{N}\left(x\cos\left(m\frac{\pi}{M}\right)+y\sin\left(m\frac{\pi}{M}\right)\right)}}^{\text{One filtered backprojection image}} \right]$$

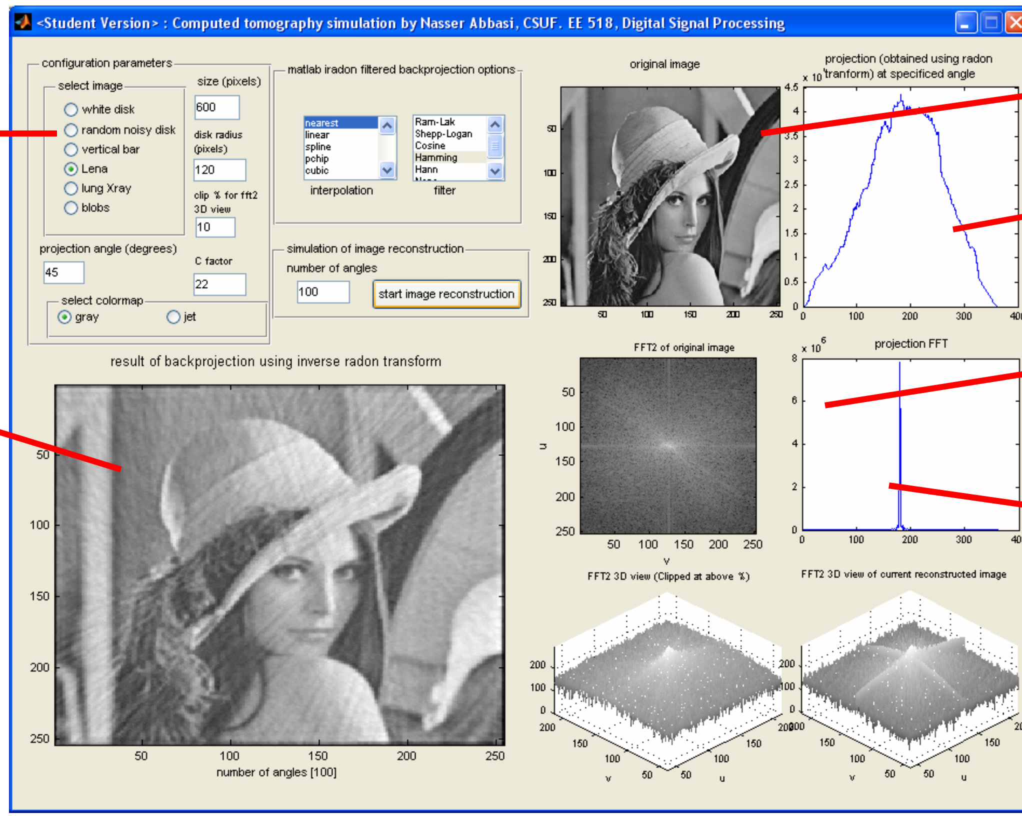
N number of samples in projection
M number of projections
W largest spatial frequency in projection

Matlab simulation

- A Matlab application is written to simulate the CT reconstructions. Matlab radon and iradon used for the implementation.

Options to select number of projections, Filter type and other parameters

Reconstructed image



Original image

Current projection

2D spectrum of original image

Fourier transform of current projection

Conclusions

1. CT image reconstruction is an inverse problem.
2. Hard problem to solve using linear algebra due to large number of equations to solve.
3. 2 methods based on frequency domain examined: Central slice theorem (SCT) and filtered back projection (FBP)
4. SCT requires gridding and interpolation of 2D spectrum to enable 2D IFFT.
5. FBP filters the projection spectrum before applying back projection. Back projection is an accumulative and averaging approach. Used more in practice than SCT.
6. Digital signal processing is critical to implement all the important current medical imaging methods such as CT, MRI, SPECT, PET and others

The END