collection of animations of search path in optimization

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Contents

These are animations[1] showing the search path u^k on top of a contour plot from different starting point u^0 toward a minimizer u^* for different objective functions using different algorithms. More animations will be added.

The animation run once and stop. To re-start the animation, please force reload the HTML page (using shift-reload).

1 First example $f(u) = (11 - u_1 - u_2)^2 + (1 + u_1 + 10u_2 - u_1u_2)^2$

1.1 compare steepest descent optimal step with conjugate gradient

The objective function is $f(u) = (11-u_1-u_2)^2+(1+u_1+10u_2-u_1u_2)^2$ Starting from $u^0 = \{14; 23.59\}$.

steepest descent, optimal step size, 76 iterations	conjugate gradient, Polak-Ribiere formula, 14 iteratio Completes much faster with less itreations.

1.2 compare steepest descent optimal step with conjugate gradient, larger range

The objective function is $f(u) = (11 - u_1 - u_2)^2 + (1 + u_1 + 10u_2 - u_1u_2)^2$

This is the same as the earlier animation but uses larger range. Starting from $u^0 = \{14; 23.59\}$.

steepest descent, optimal step size, 76 iterations	conjugate gradient, Polak-Ribiere formula, 14 iteration

1.3 compare steepest descent optimal step with fixed step h=0.25

The objective function is $f(u) = (11 - u_1 - u_2)^2 + (1 + u_1 + 10u_2 - u_1u_2)^2$

Starting from $u^0 = \{14; 23.59\}$.

steepest descent, optimal step size, 76 iterations	steepest descent, fixed step size $h = 0.25$, 150 iteration

[1] Made during taking course ECE 719 optimal systems at University Wisconsin, Madison. Course given by Professor B Ross Barmish in spring 2016

THe search using optimal step size is slower (due to performing line search at each step) and consumes more CPU time, but it does converge. While the search using fixed step is faster (since it does not perform line search) but it failed to converge when it u^k was very close to u^* due to oscillation around u^* as the step size was relatively large.

1.4 compare steepest descent optimal step with fixed step h=0.1

The objective function is $f(u) = (11 - u_1 - u_2)^2 + (1 + u_1 + 10u_2 - u_1u_2)^2$

Starting from $u^0 = \{0.4; 4.3\}$.

steepest descent, optimal step size, 47 iterations	steepest descent, fixed step size $h = 0.1$, 114 iterations

2 second example, Rosenbrock's banana function $f(u) = 100 * (u_2 - u_1^2)^2 + (1 - u_1)^2$

2.1 compare steepest descent optimal step with fixed step h=0.1

The objective function is $f(u) = 100 * (u_2 - u_1^2)^2 + (1 - u_1)^2$

Starting from $u^0 = \{1.828, -1.878\}$.

steepest descent, optimal step size, Very slow convergence near u^* but converged in 2079 steps. (Animation stops at step 300 to reduce size).	steepest descent, fixed step size $h = 0.1$, 300 iteratio Stopped due to oscillation. Do not converge.