

my Maple cheat sheet

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Contents

1	OOO in Maple	9
1.1	How to use new object method calling in Maple 2021?	9
1.2	How to make a constructor for an Object?	10
1.3	How to make different constructors for an Object?	11
1.4	How to do OOP inheritance?	13
1.5	How to extend a base class and override its method with different one?	15
1.6	How to extend a class and call base class function from the extended class??	16
1.7	How to use object as user defined record inside a proc?	17
1.8	How to make copy of list of objects?	18
1.9	How to use OOP to implement ode solver?	19
1.10	How to make a complete OOP ode solver in Maple?	26
2	Differential equations	34
2.1	How to force dsolve to use specific method for solving?	34
2.2	How to find a particular solution to ODE?	41
2.3	How to find basis solutions for homogeneous ode?	41
2.4	How to solve a differential equation with initial conditions?	41
2.5	How to verify that the ODE solution given is correct?	42
2.6	How to know the type of ODE?	42
2.7	What packages to load for differential equations?	43
2.8	How to plot solution of differential equations?	43
2.9	On High precision. Using taylor to solve ODE	44
2.10	Obtain ODE in canonical coordinates (Lie symmetry)	45
2.11	How to use hint with symgen?	46
2.12	How to parse a single ode?	47
2.13	How to check if single ODE is valid?	51
2.14	How to check if an ode has y' in it?	53
2.15	How to check if an ode is linear ode?	53
2.16	How to find the order of an ode?	54
2.17	How to find the coefficients of a linear ode?	54
2.18	find order and degree of highest derivative	55
2.19	How to obtains list of all derivatives in expression?	57
2.20	How to invert roles of dependent variable and independent variable in an ode?	59
2.21	How to find the indicial equation for an ODE?	60
2.22	How to write derivative	60

2.23	How to solve heat PDE in 1D in Maple 2017?	60
2.24	How to make Maple display $\text{diff}(y(x),x)$ as $y'(x)$ or as y' ?	62
2.25	How to set boundary conditions for <code>dsolve</code> or <code>pdsolve</code> ?	63
2.26	How force <code>dsolve</code> to use Lie?	64
2.27	How to do change of variables on the dependent variable for an ODE? .	64
2.28	How to do change of variable on the independent variable for an ODE?	64
2.29	ODE change of variable on both dependent and independent variable?	65
2.30	How to make phase plot of second order ODE?	67
2.31	How to move all terms with y to one side?	68
2.32	How to find all constants of integrations in an expression?	69
2.33	How to find if an ode is missing the dependent variable?	70
3	How to convert Mathematica expression to Maple?	71
4	How to debug internal procedures, such as <code>dsolve</code>?	71
5	How to display source code of a function?	72
6	How to display trace of a function as it runs in maple?	74
7	How to display a build in function code?	75
8	How to build a LIST or a SET on the fly?	76
9	make function display more information	78
10	How to plot a function?	79
11	How to run maple from command line?	80
12	How to use matrices in maple?	80
13	return more than value from a procedure	82
14	How does maple handle procedure arguments?	82
15	How to define your own data types?	83
16	find max element and position in matrix	83
17	How to create a package?	84
18	How to convert from floating point to Hex?	86

19 How to find taylor series expansion of functions?	87
20 How to print elements of a matrix?	87
21 How to find determinant of matrix?	87
22 How to generate Hilber matrix?	88
23 How to plot matrix data?	88
24 How to catch an error from a proc()?	89
25 How to convert 3456 to 3,456 ?	89
26 How to use units ?	90
27 How to evaluate catlan number and other sums?	90
28 write a text file that contains a package, and load it	90
29 How to find what packages are included in maple	91
30 How to plot the gradient vector field?	92
31 How to put the digits of Pi into a list?	92
32 Digits of PI in maple and mma	93
33 How to find where functions are?	95
34 on maple data types	96
35 how to extract elements from a list based on some selection?	96
36 how to test if all elements of a matrix are integers?	97
37 how to use laplace transform?	97
38 questions I have	98
39 3D plotting	98
40 How to raise each element in a list to a power?	99

41 How to generate a sequence with any increment?	99
42 What shortcuts are there for matrix manipulation?	99
43 How to solve a set of equations for the derivative?	99
44 How to solve a set of equations for differentials?	100
45 How to plot binary tree	101
46 Problem 12.4 chapter 4, Boas book	102
47 example of doing convergence test in maple	102
48 Problem ch 14, 3.18, Boas book. contour integration	103
49 How to find multiple roots to an equation such as $\sin(x) = 0$	103
50 Dr Basti Associated Legendre	104
51 Understanding conformal mapping in maple	107
52 Hide tilde character when using assumption	109
53 Fourier series in maple	109
54 How to plot graphs next to each others in a grid like fashion	112
55 How to generate Pi on X-axis	112
56 How to make output from FunctionAdvisor look better?	113
57 How to do partial fractions?	113
58 How to generate sequence sum symbolically	114
59 Nice plot from Maple	114
60 How to check if 2 expressions are the same?	115
61 converting series to factorials	115
62 How to find what new additions made to Maple?	116

63 Maple can't solve laplace equation and numerically	116
64 Some Maple Matrix operations	117
65 How set diagonal elements to some value, say 1?	119
66 How to multiply roots of a polynomial?	120
67 How to plot a surface in 3D?	120
68 How to convert trigs to sinc function in an expression	122
69 How to find NullSpace and ColumnSpace of a matrix?	123
70 How to fix the interface to using Maple notation for input?	123
71 How to find all solutions using allvalues ?	124
72 Adding only to diagonal of a matrix	124
73 How to search help for updates on some package	124
74 How to work with groups in worksheet	125
75 How to read code into worksheet?	125
76 Code editors for Maple	125
77 How to find if package is module or table?	125
78 How to replace a string?	126
79 How to use geometry and plottools ?	126
80 How to simplify log expressions ?	126
81 How to simplify hyperbolic expression ?	127
82 How to create text file and append string to it?	127
83 How to search packages and libraries?	128
84 How to numerically solve a BVP ode and plot the solution?	130

85	How to display on screen for specific width?	131
86	Maple IDE links	132
87	loading, remove and finding what packages loaded	132
88	some rules of thumbs when using Maple	133
89	How to make multiple assumptions on a symbol?	133
90	How to check if expression is an equation?	133
91	How to check if expression is a set?	133
92	How to export a plot to PDF?	134
93	How to find all roots of complex number	135
94	How to convert matrix of matrices to a matrix?	135
95	How to do pattern matching in Maple?	136
95.1	Example 1	136
95.2	Example 2	137
96	How to find trig identities?	137
97	How to find directional derivative of scalar function?	137
98	How to check if name is assigned a value?	138
99	How to select terms with sqrt or radical	139
100	How to simplify $e^{\ln(x)+\ln(y)}$	139
101	How to find all csgn() and replace them by 1	140
102	How to find symbols inside csgn() in an expression?	141
103	How to replace all abs(expr) by expr	142
104	Basis for Null space, Row space and column space of matrix	142
105	How to do Gaussian elimination on a Matrix?	143

106	How to find Reduced Echelon form of a Matrix?	143
107	How add a new row to bottom of matrix?	144
108	How to obtain list of all occurrences of some function in an expression?	145
109	How to replace $\ln(x)$ with $\ln(x)$ in an expression?	146
110	How to find all signum functions in expression and simplify it?	146
111	How to replace all signum functions in expression by 1?	147
112	How to find the cofactor matrix of a matrix?	147
113	How to normalize eigenvectors?	148
114	How to find if some function is present in an expression	149
115	How to find all functions in an expression?	149
116	find all functions except builtin math functions	150
117	How to obtain a list of all arguments of function?	150
118	Find functions whose first argument is z	150
119	Find functions whose second argument is t ?	151
120	How to typeset \hbar ?	151
121	How to find the Curl of a vector?	152
122	See all steps in RREF of an augmented matrix	153
123	How to find column space of matrix?	153
124	How to use select with own type to find subexpressions?	154
125	How to write structured types to match some expressions?	154
	125.1 type for $\sin^m(x) \cos^n(x)$	154
126	select only indexed variables from an expression	155
127	Show step by step. Calculus problem and differential equations.	156

128	How to obtain list of files with some extension in folder?	157
129	How to delete lines from text file that contains some string?	157
130	Given an expression, how to find all variables and functions in it?	158
131	How to check if an expression is integer, when it has symbols in it?	158
132	How to truncate a polynomial?	159
133	How to make a local declare like block inside a proc?	159
134	Using short name for a proc inside nested modules instead of long name	160
135	Remove duplicates objects in a list based on condition on a field	161
136	How to remove duplicates Vectors from a list?	162
137	How to find parameters such as π in an expression?	163
138	How to find all derivatives $y'(x)$ in an expression?	163
139	How combine log terms?	165
140	How to find all poles and their order of a rational function?	166
141	find series of function with specific number of terms	167
142	How to call sibling's proc without making the sibling module exported?	168
143	Find position in a list of items that are not numeric	169
144	Convert time to use seconds instead of milliseconds	170
145	How to change $\arctan(y, x)$ to $\arctan\left(\frac{y}{x}\right)$ in an expression	171
146	How to find all symbols that represent variables in an expression?	171
147	How to change first argument of function?	172
148	How to change last argument of function?	172

149	How to remove last argument of function?	173
150	How to find a pattern inside an expression	173
151	How to find parts of a Sum?	174
152	find if sequence or list is inside another and the indecies	175
153	find if some type inside some expression and its location	175
154	Change the summation index letter	176
155	How to replace generic function inside derivative?	177
156	Examples how to match types	178
157	On the order of terms when using indents	182

1 OOP in Maple

1.1 How to use new object method calling in Maple 2021?

In Maple 2021, it is now possible to use `object:-method(arg)` notation. This makes is easier to use OOP in maple. To do this, use `_self` as follows

```
restart;

module person()
  option object;
  local name::string:="";
  local age::integer:=0;

  export get_name::static:=proc(_self,$)
    return _self:-name;
  end proc;

  export set_name::static:=proc(_self,name::string,$)
    _self:-name:=name;
  end proc;

  export get_age::static:=proc(_self,$)
```

```

    return _self:-age;
end proc;

export set_age::static:=proc(_self,age::integer,$)
    _self:-age:=age;
end proc;

end module;

```

And now make an object and use it as follows

```

o:=Object(person)
           o := Object<<1846759887808>>

o:-get_name();
           ""

o:-get_age();
           0

o:-set_name("joe doe");
o:-get_name();
           "joe doe"

```

1.2 How to make a constructor for an Object?

Add ModuleCopy proc in the class. This will automatically be called to initialize the object.

Here is an example

```

restart;

module ODE()
    option object;
    local ode:=NULL;
    local y::symbol;
    local x::symbol;
    local sol;

```

```

export ModuleCopy::static := proc( _self::ODE, proto::ODE, ode, func, $ )
    print("Initilizing object with with args: ", [args]);
    _self:-ode:= ode;
    _self:-y:=op(0,func);
    _self:-x:=op(1,func);
end proc;

export dsolve::static:=proc(_self,$)
    _self:-sol := :-dsolve(ode,y(x));
end proc;

export get_sol::static:=proc(_self,$)
    return sol;
end proc;

end module;

```

And now make an object and use it as follows

```

o:=Object(ODE, diff(y(x),x)+y(x)=sin(x), y(x));
o:-dsolve();
o:-get_sol();

#y(x) = -1/2*cos(x) + 1/2*sin(x) + exp(-x)*_C1

```

So a constructor just makes it easier to initialize the object without having to make a number of set() calls to initialize each memeber data.

1.3 How to make different constructors for an Object?

This is done using overload with different ModuleCopy proc in the class.

Here is an example. Lets make a constructor that takes an ode and initial conditions, and one that only takes an ode with no initial conditions.

```

restart;

module ODE()
    option object;
    local ode:=NULL;

```

```

local y::symbol;
local x::symbol;
local ic=NULL;
local sol;

export ModuleCopy::static:= overload(
  [
    proc( _self::ODE, proto::ODE, ode, func, $ ) option overload;
      _self:-ode:= ode;
      _self:-y:=op(0,func);
      _self:-x:=op(1,func);
    end proc,

    proc( _self::ODE, proto::ODE, ode, func, ic, $ ) option overload;
      _self:-ode:= ode;
      _self:-y:=op(0,func);
      _self:-x:=op(1,func);
      _self:-ic :=ic;
    end proc
  ]
);

export dsolve::static:=proc(_self,$)
  if evalb(ic=NULL) then
    sol := :-dsolve(ode,y(x));
  else
    sol := :-dsolve([ode,ic],y(x));
  fi;
end proc;

export get_sol::static:=proc(_self,$)
  return sol;
end proc;

end module;

```

And now use it as follows

```
o:=Object(ODE, diff(y(x),x)+y(x)=sin(x), y(x), y(0)=0);
```

```

o:-dsolve():
o:-get_sol();

      #y(x) = -1/2*cos(x) + 1/2*sin(x) + 1/2*exp(-x)

o:=Object(ODE, diff(y(x),x)+y(x)=sin(x), y(x));
o:-dsolve():
o:-get_sol();

      #y(x) = -1/2*cos(x) + 1/2*sin(x) + exp(-x)*_C1

```

1.4 How to do OOP inheritance?

In the child class you want to extend from the parent class, add option `object(ParentName)`;

Here is an example

```

restart;
module ODE()
  option object;
  local ode;

  export set_ode::static:=proc(_self,ode,$)
    _self:-ode :=ode;
  end proc;

  export get_ode::static:=proc(_self,$)
    return _self:-ode;
  end proc;
end module;

#create class/module which extends the above
module second_order_ode()
  option object(ODE);
  export get_ode_order::static:=proc(_self,$)
    return 2;
  end proc;
end module;

```

In the above `second_order_ode` inherits all local variables and functions in the ODE class and adds new proc. Use as follows

```
o:=Object(second_order_ode); #create an object instance
o:-set_ode(diff(y(x),x)=sin(x));
o:-get_ode();
o:-get_ode_order();
```

Note that the child class can not have its own variable with the same name as the parent class. This is limitation. in C++ for example, local variables in extended class overrides the same named variable in the parent class.

Even if the variable have different type, Maple will not allow overriding. For example, this will fail

```
restart;
module ODE()
  option object;
  local ode;
  local id::integer;

  export set_ode::static:=proc(_self,ode,$)
    print("Enter ode::set_ode");
    _self:-ode :=ode;
  end proc;

  export get_ode::static:=proc(_self,$)
    return _self:-ode;
  end proc;
end module;

module second_order_ode()
  option object(ODE);
  local id::string;
  export get_ode_order::static:=proc(_self,$)
    return 2;
  end proc;
end module;

Error, (in second_order_ode) local `id` is declared more than once
```

There might be a way to handle this, i.e. to somehow explicitly tell Maple to override parent class proc or variable name in the child. I do not know now. The above is using Maple 2021.1

1.5 How to extend a base class and override its method with different one?

This is called polymorphism in OOP. This is a base class `animal_class` which has `make_sound` method. This method acts just as a place holder (interface), which the extending class must extend (override) with an actual implementation.

The class is extended to make cat class and implementation is made.

```
module animal_class()
  option object;

  export make_sound::static:=proc(_self,$)
    error("Not implemented, must be overridden");
  end proc;
end module;

%-----
module cat_class()
  option object(animal_class);

  make_sound::static:=proc(_self,$) #note. DO NOT USE export
    print("mewooo");
  end proc;

end module;
```

And now

```
o:=Object(animal_class);
o:-make_sound();

Error, (in anonymous procedure) Not implemented, must be overridden
```

The above is by design. As the `animal_class` is meant to be extended to be usable.

```
my_cat:=Object(cat_class);  
my_cat:-make_sound();
```

"mewooo"

So the base class can have number of methods, which are all meant to be have its implementation provided by an extending class. Each class which extends the base class must provide implementation.

1.6 How to extend a class and call base class function from the extended class??

Once a base class is extended, all methods in the base class become part of the extending class. So to call a base class method just use same way as if calling any other method in the extending class itself.

Here is an example.

```
module person()  
  option object;  
  local age::integer:=100;  
  
  export ModuleCopy::static:= proc(_self,proto::person, age::integer,$)  
    _self:-age := age;  
  end proc;  
  
  local base_class_method::static:=proc(_self,$)  
    print("In base class method...");  
  end proc;  
  
end module;  
  
#---- extend the above class  
module young_person()  
  option object(person);  
  export process::static:=proc(_self,$)  
    print("In young_person process");  
    _self:-base_class_method();  
  end proc;
```

```
end module;
```

Here is an example of usage

```
o:=Object(young_person,20);
o:-process()

      "In young_person process"
      "In base class method..."
```

The above in Maple 2023.1

1.7 How to use object as user defined record inside a proc?

A Maple Object can be used a record type in other languages, such as Ada or Pascal. This example shows how to define a local type inside a proc and use it as record.

```
restart;
foo:=proc(the_name::string,the_age::integer)::person_type;
    local module person_type() #this acts as a record type
        option object;
        export the_name::string;
        export the_age::integer;
    end module;

    local person::person_type:=Object(person_type);

    person:-the_name:=the_name;
    person:-the_age:=the_age;
    return person;
end proc;

o:=foo("joe doe",100);

o:-the_name;
           "joe doe"
o:-the_age;
           100
```

In the above person is local variable of type person_type. In the above example, the

local variable was returned back to user. But this is just an example. One can declare such variables and just use them internally inside the proc only. This method helps one organize related variables into one record/struct like type. The type can also be made global if needed.

1.8 How to make copy of list of objects?

Suppose we have list L1 of objects and we want to copy this list to another list, say L2. If we just do `L2 = L1` then this will not make an actual copy as any changes to L1 are still reflected in L2. The above only copies the reference to the same list.

To make a physical copy, need to use the `copy` command as follows

```
restart;

module person_type()
  option object;
  export the_name::string:="doe joe";
  export the_age::integer:=100;
end module;

L1:=[];
for N from 1 to 5 do
  o:=Object(person_type);
  o:-the_name:=convert(N,string);
  L1:= [ op(L1), o];
od:

L2:=map(Z->copy(Z),L1):
```

Now making any changes to L1 will not affect L2. If we just did `L2 = L1` then both will share same content which is not what we wanted.

1.9 How to use OOP to implement ode solver?

This is a basic example of using OOP in Maple to implement an ode solver. There is a base module called `ode_base_class` (I will be using class instead of module, as this is more common in OOP)

This program will for now support first order and second order ode's only.

The base ode class will contain the basic operations and data which is common to both first order and second order ode's.

Next, we will make a first order ode class, and second order ode class. Both of these will extend the base ode class.

Next, we will have more ode classes. For example, for first order ode, there will be linear first order ode class, and separable first order ode class, and Bernoulli ode class and so on. Each one these classess will extend the first order ode class which in turn extends the base ode class.

Same for second order ode's. There will be second order constant coefficients ode class, and second order variable coefficient ode class and so on. Each one of these classes will extend the second order ode class which in turn extends the base ode class.

Let `base_ode_class` be the base of an ode class which will contain all the necessary methods that are generic and applicable to an ode of any order and type.

These can be the ode expression itself, its order and any other data and operations which applicable to any ode type.

Now we want to create a first order class. This will have its own operations and private variables that are specific and make sense only to any first order ode.

Then these will be the first order separable ode class, which has methods that implement solving the ode using separable method and has other methods which makes sense only for first order separable ode. The following diagram is partial illustration of the is-A relation among possible classes.

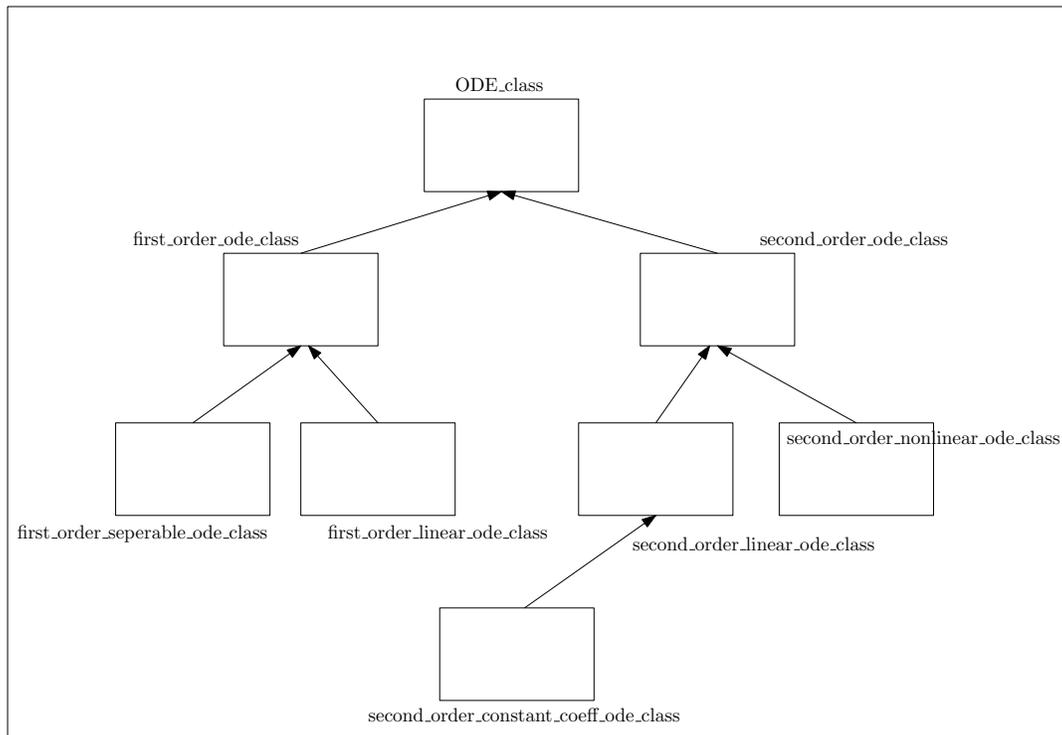


Figure 1: The is-A relation between class for ODE solver

First we define the base ode class and define private variables and method that are common to any ode type.

```

restart;

module base_ode_class()
  option object;
  local the_ode;
  local the_order::integer;

  export get_ode::static:=proc(_self,$)
    RETURN(_self:-the_ode);
  end proc;

  export get_order::static:=proc(_self,$)
    RETURN(_self:-the_order);
  end proc;
end module;

```

```
end module;
```

Note that the base ode class does not have constructor. Since it is meant to be extended only.

The following is the first order ode class.

```
module first_order_ode_class()
  option object(base_ode_class);

  local initial_conditions;
  local solution_found;

  export get_IC::static:=proc(_self,$)
    RETURN(_self:-initial_conditions);
  end proc;

  export get_solution::static:=proc(_self,$)
    RETURN(_self:-solution_found);
  end proc;

  export verify_solution::static:=proc(_self,$)::truefalse;
    #code to verify if the solution found is valid or not
    #using odetest()
  end proc;

end module;
```

The following is the first order separable ode class which extends the above first order ode class.

```
module first_order_separable_ode_class()
  option object(first_order_ode_class);
  local f,g; #ode of form  $y'=f(x)*g(y)$ 

  export ModuleCopy::static:= proc(_self,proto::first_order_separable_ode_class,
    ode, IC,$)
    _self:-the_ode := ode;
    _self:-initial_conditions := IC;
  end proc;
```

```

export dsolve::static:=proc(_self,$)
    #solve the ode for now we will use Maple but in my code
    #I have my own solver ofcourse.
    _self:-solution_found:= :-dsolve([_self:-the_ode, _self:-initial_conditions]
end proc;
end module;

```

In the above, when we create a instance of `first_order_separable_ode_class` then it now have the whole chain of classes into one. i.e. first order separable class extending the first order class which in turn extends the base ode class. For example

```

o:=Object(first_order_separable_ode_class,diff(y(x),x)=3*sin(x)*y(x),y(0)=1)
o:-dsolve();
o:-get_solution()
    #y(x) = exp(3)*exp(-3*cos(x))
o:-get_ode()
    #diff(y(x), x) = 3*sin(x)*y(x)

```

The above calls will all work, even though the first order separable class has no method in it called `get_ode` but it extends a class which does, hence it works as it.

Now we will do the same for second order ode's.

Advantage of this design, is that methods in base classes that are being extended can be reused by any class which extends them. Only methods that applies and specific to the lower level classes need to be implemented.

As we add more specific solvers, we just have to extend the base classes and the new solvers just need to implement its own specific `dsolve` and any specific methods and data that it needs itself.

Ofcourse in practice the above design is not complete as is. The user should not have to specify which class to instantiate, as user does not care what the ode type or class it is. They just want to to do

```

o:=Object(ode_class,diff(y(x),x)=3*sin(x)*y(x),y(0)=1)
o:-dsolve();
o:-get_solution()

```

To solve this problem we have to make a factory method which is called to make the correct instance of the class and return that to the user. The factory method figures

out the type of ode and it creates the correct instance of the correct class and returns that. So the call will become

```
o := make_ode_object( diff(y(x),x)=3*sin(x)*y(x), y(0)=1)
o:-dsolve();
o:-get_solution()
```

The function `make_ode_object` above is the main interface the user will call to make an ode object.

This will be explained next with examples. One possibility is to make the factory function a global function or better, a public method in a utility module. For now, it is given here as a standalone function for illustration. The user calls this method to make an object of the correct instance of the ode. Here is complete implementation of all the above including the factory method.

```
#factory method. Makes objects for users
make_ode_object:=proc(ode::`=`,func::function(name))
  local x,y,the_order;
  y:=op(0,func);
  x:=op(1,func);
  the_order := PDEtools:-difforder(ode,x);
  if the_order=1 then
    RETURN(first_order_ode_class:-make_ode_object(ode,func));
  elif the_order=2 then
    #RETURN(second_order_ode_class:-make_ode_object(ode,func));
    #implement later
    NULL;
  else
    error "Only first and second order ode's are currently supported";
  fi;
end proc;

#####
module base_ode_class()
  option object;
  local the_ode;
  local the_order::integer;
```

```

#methods bound to the object
export get_ode::static:=proc(_self,$)
    RETURN(_self:-the_ode);
end proc;

export get_order::static:=proc(_self,$)
    RETURN(_self:-the_order);
end proc;

end module:
#####
module first_order_ode_class()
    option object(base_ode_class);

    local initial_conditions;
    local solution_found;

    #public factory method not bound to the object.
export make_ode_object:=proc(ode::`=`,func::function(name))
    local x,y,ode_type::string;
    y:=op(0,func);
    x:=op(1,func);
    ode_type:="separable"; #code here which determined first order ode type
    if ode_type="separable" then
        RETURN( Object(first_order_separable_ode_class,ode,func));
    elif ode_type="linear" then
        RETURN( Object(first_order_linear_ode_class,ode,func));
    fi;
    #more ode types added here
end proc;

#methods bound to the object
export get_IC::static:=proc(_self,$)
    RETURN(_self:-initial_conditions);
end proc;

export get_solution::static:=proc(_self,$)
    RETURN(_self:-solution_found);
end proc;

```

```

export verify_solution::static:=proc(_self,$)::truefalse;
    #code to verify if the solution found is valid or not
    #using odetest()
end proc;

end module:

#####
module first_order_separable_ode_class()
    option object(first_order_ode_class);
    local f,g; #ode of form y'=f(x)*g(y)

    export ModuleCopy::static:= proc(_self,proto::first_order_separable_ode_class,ode,fu
        _self:-the_ode := ode;
    end proc;

    export dsolve::static:=proc(_self,$)
        #print("Enter first_order_separable_ode_class:-dsolve");
        #solve the ode for now we will use Maple but in my code
        #I have my own solver ofcourse.
        _self:-solution_found:= :-dsolve(_self:-the_ode);
        NULL;
    end proc;
end module:

```

It is used as follows

```

o:=make_ode_object(diff(y(x),x)=sin(x)*y(x),y(x));
o:-dsolve();
o:-get_solution();
          y(x) = c__1 exp(-cos(x))

```

1.10 How to make a complete OOP ode solver in Maple?

Here, I will start making a complete small OOP ode solver in Maple.

At each step more classes are added and enhanced until we get a fully working small ode solver based on OOP design that solves a first and second order ode, this is to show how it all works. Another solvers can be added later by simply extending the base class.

The base class is called `Base_ode_class`. There will be `Second_order_ode_class` and `First_order_ode_class` and these classes extend the base ode class. We can later add `higher_order_ode_class`.

Next, there are different classes which extend these. There is `First_order_linear_ode_class` and `First_order_separable_ode_class` and so on, and these extend the `First_order_ode_class`.

For example, if a user wanted to solve a first order ode which happen to be say separable, then object of class `First_order_separable_ode_class` will be created and used.

Since the user does not know and should not know what object to create, then the factory class will be used. The factory class is what the user initially calls to make the ode object.

It is the factory class which determines which type of class to instantiate based on the ode given.

The factory class is singleton (standard module in Maple, not of type object), which has the `make_ode` method which is called by the user. This method parses the ode and determines its order and then based on the order determine which subclass to use, and then instantiate this and returns the correct object to the user to use.

This object will have the `dsolve` method and other methods the user can use on the object.

The `make_ode` method in the factory module accepts only the ode itself the function such as $y(x)$. A typical use is given below

```
ode := ODE_factory_class:-make_ode( diff(y(x),x)=sin(x), y(x) );
ode:-set_IC(...);
ode:-set_hint("the hint");
.....

ode:-dsolve(); #solves the ode
ode:-is_solved(); #checks if ode was successfully solved
```

```

ode:-verify_sol(); #verifies the solution using maple odetest()
ode:-is_sol_verified(); #asks if solution is verified
ode:-get_number_of_sol(); #returns number of solutions, 1 or 2 etc...
ode:-get_sol(); #returns the solutions found in list
#and more method
...

```

Examples at the end will show how all the above works on actual odes's.

The initial call to make an ode does not have initial conditions, or hint and any other parameters. This is so to keep the call simple. As the factory method only makes the concrete ode object.

Additional methods are then used to add more information if needed by using the returned object itself, such as initial conditions, and hint and so on before calling the dsolve method on the object.

Here is a very basic setup which include the base ode class and extended to first order two subclasses for now.

```

restart;
ODE_factory_class :=module()
  #notice, normal module. No option object.

  export make_ode:=proc(ode::`=`,func::function(name),$)
    local dep_variables_found::list,item;
    local y::symbol;
    local x::symbol;
    local ode_order::integer;

    if nops(func)<>1 then
      error("Parsing error, dependent variable must contain one argument, found '
    fi;
    y:=op(0,func);
    x:=op(1,func);
    if not has(ode,y) then
      error ("Supplied ode ",ode," has no ",y);
    fi;

    if not has(ode,x) then
      error ("Supplied ode ",ode," has no ",x);

```

```

fi;

if not has(ode,func) then
    error ("Supplied ode ",ode," has no ",func);
fi;

ode_order := PDEtools:-difforder(ode,x);

#this will check that the dependent variable will show with
#SAME argument in the ode. i.e. if y(x) and y(t) show up in same ode, it
#will throw exception, which is what we want.

try
    dep_variables_found := PDEtools:-Library:-GetDepVars([y],ode);
catch:
    error lastexception;
end try;

#now go over dep_variables_found and check the independent
#variable is same as x i.e. ode can be y'(z)+y(z)=0 but function is y(x).
for item in dep_variables_found do
    if not type(item,function) then
        error("Parsing error. Expected ",func," found ",item," in ode");
    else
        if op(1,item) <> x then
            error("Parsing error. Expected ",func," found ",item," in ode");
        fi;
    fi;
od;

#now go over all indents in ode and check that y only shows as y(x) and not as
#as the PDEtools:-Library:-GetDepVars([_self:-y],ode) code above does not dete
#i.e. it does not check y'(x)+y=0

if numelems(indets(ode,identical(y))) > 0 then
    error("Parsing error, Can not have ",y," with no argument inside ",ode);
fi;

if ode_order=1 then

```

```

        RETURN(make_first_order_ode(ode,y,x));
    elif ode_order=2 then
        RETURN(make_second_order_ode(ode,y,x));
    else
        RETURN(make_higher_order_ode(ode,y,x));
    fi;
end proc;

local make_first_order_ode:=proc(ode::`=`,y::symbol,x::symbol)
    #decide on what specific type the ode is, and make instant of it

    if first_order_ode_quadrature_class:-is_quadrature(ode,y,x) then
        RETURN(Object(first_order_ode_quadrature_class,ode,y,x));
    elif first_order_ode_linear_class:-is_linear(ode,y,x) then
        RETURN(Object(first_order_ode_linear_class,ode,y,x));
    fi; #and so on
end proc;

local make_second_order_ode:=proc(ode::`=`,y::symbol,x::symbol)
    #decide on what specific type the ode is, and make instant of it
    #same as for first order
end proc;
end module;
#-----
module solution_class()
    option object;
    local the_solution;
    local is_verified_solution::truefalse:=false;
    local is_implicit_solution::truefalse:=false;

    export ModuleCopy::static:= proc(_self,proto::solution_class,the_solution::`=`,is_i
        _self:-the_solution:=the_solution;
        _self:-is_implicit_solution:=is_implicit_solution;
    end proc;

    export get_solution::static:=proc(_self,$)
        RETURN(_self:-the_solution);
    end proc;

```

```

export is_verified::static:=proc(_self,$)
    RETURN(_self:-is_verified_solution);
end proc;

export is_implicit::static:=proc(_self,$)
    RETURN(_self:-is_implicit_solution);
end proc;

export is_explicit::static:=proc(_self,$)
    RETURN(not(_self:-is_implicit_solution));
end proc;

export verify_solution::static:= overload(
[
    proc(_self, ode::`=`, $) option overload;
        local stat;
        stat:= odetest(_self:-the_solution,ode);
        if stat=0 then
            _self:-is_verified_solution:=true;
        else
            if simplify(stat)=0 then
                _self:-is_verified_solution:=true;
            else
                _self:-is_verified_solution:=false;
            fi;
        fi;
    end,

    proc(_self, ode::`=`, IC::list, $) option overload;
        local stat;
        stat:= odetest([_self:-the_solution,IC],ode);
        if stat=[0,0] then
            _self:-is_verified_solution:=true;
        else
            if simplify(stat)=[0,0] then
                _self:-is_verified_solution:=true;
            else
                _self:-is_verified_solution:=false;
            fi;
        fi;
    end,

```

```

        fi;
    end
]);
end module:
#-----

module ODE_base_class()
    option object;
    local y::symbol;
    local x::symbol;
    local func::function(name); #y(x)
    local ode::`= `;
    local ode_order::posint;
    local IC::list:=[];
    local parsed_IC::list:=[];
    local the_hint::string="";
    local solutions_found::list(solution_class):=[];

    #exported getters methods
    export get_ode::static:=proc(_self,$)
        RETURN(_self:-ode);
    end proc;

    export get_x::static:=proc(_self,$)
        RETURN(_self:-x);
    end proc;

    export get_y::static:=proc(_self,$)
        RETURN(_self:-y);
    end proc;

    export get_ode_order::static:=proc(_self,$)
        RETURN(_self:-ode_order);
    end proc;

    export get_IC::static:=proc(_self,$)
        RETURN(_self:-IC);
    end proc;

```

```

export get_parsed_IC::static:=proc(_self,$)
    RETURN(_self:-parsed_IC);
end proc;

export get_sol::static:=proc(_self,$)
    local L:=Array(1..0):
    local sol;
    for sol in _self:-solutions_found do
        L ,= sol:-get_solution();
    od;
    RETURN(convert(L,list));
end proc;

#exported setters methods
export set_hint::static:=proc(_self, hint::string,$)
    #add code to check if hint is valid
    _self:-the_hint:=hint;
end proc;
end module;

#-----
module first_order_ode_quadrature_class()
    option object(ODE_base_class);
    local f,g; #ode of form y'=f(x)*g(y)

    #this method is not an object method. It is part of the module but does
    #not have _self. It is called by the factory class to find if the ode
    #is of this type first
    export is_quadrature:=proc(ode::`=`,y::symbol,x::symbol)::truefalse;
        RETURN(true); #for now
    end proc;

    export ModuleCopy::static:= proc(_self,proto::first_order_ode_quadrature_class,ode:
        _self:-ode := ode;
        _self:-y := y;
        _self:-x := x;
        _self:-func := _self:-y(_self:-x);
        _self:-ode_order :=1;
    end proc;

```

```

export dsolve::static:=proc(_self,$)
    local sol,o;
    #print("Enter first_order_ode_quadrature_class:-dsolve");
    #solve the ode for now we will use Maple but in my code
    #I have my own solver ofcourse.
    sol:= :-dsolve(_self:-ode,_self:-func);
    o:=Object(solution_class,sol,false);
    _self:-solutions_found:= [o];
    NULL;
end proc;
end module:

#-----
module first_order_ode_linear_class()
    option object(ODE_base_class);
    local f,g; #ode of form  $y'=f(x)*g(y)$ 

    #this method is not an object method. It is part of the module but does
    #not have _self. It is called by the factory class to find if the ode
    #is of this type first
    export is_linear:=proc(ode::`=`,y::symbol,x::symbol)::truefalse;
        RETURN(true); #for now
    end proc;

    export ModuleCopy::static:= proc(_self,proto::first_order_ode_linear_class,ode::`=`,
        _self:-ode := ode;
        _self:-y := y;
        _self:-x := x;
        _self:-func := _self:-y(_self:-x);
        _self:-ode_order :=1;
    end proc;

    export dsolve::static:=proc(_self,$)
        local sol,o;
        sol:= :-dsolve(_self:-ode,_self:-func);
        o:=Object(solution_class,sol,false);
        _self:-solutions_found[1]:= [o]:
    end proc:

```

```
end module:
```

Example usage is

```
o:=ODE_factory_class:-make_ode(diff(y(x),x)=x,y(x))
o:-get_ode()
```

$$\frac{d}{dx} y(x) = x$$

```
o:-dsolve();
o:-get_sol()
```

$$\begin{bmatrix} 1 & 2 & \\ y(x) = -x^2 + c_{-1} \\ 2 & & \end{bmatrix}$$

2 Differential equations

2.1 How to force dsolve to use specific method for solving?

To find what methods dsolve uses do

```
`dsolve/methods` [1]
```

```
[quadrature, linear, Bernoulli, separable, inverse_linear,
homogeneous, Chini, lin_sym, exact, Abel, pot_sym]
```

The above gives methods for first order ODEs. More are given by

```
`dsolve/methods` [1, 'semiclass']
```

```
[Riccati, inverse_Riccati, equivalent_to_Abel,
linearizable, linearizable_by_differentiation]
```

```
`dsolve/methods` [1, 'high_degree']
```

```
[WeierstrassP, WeierstrassPPrime, JacobiSN, linearizable_by_differentiation,
missing, dAlembert, homogeneous_B, sym_implicit]
```

```
`dsolve/methods` [1,"development"]
```

```
[linearizable_by_differentiation, linearizable, con_sym,
WeierstrassP, WeierstrassPPrime, equivalent_to_Abel,
Abel_AIR, special, Riccati_symmetries]
```

```
`dsolve/methods` [1,extra]
```

```
[inverse_Riccati, Abel_AIL, `sym_pat/[F(x)*G(y),0]`, `sym_pat/[F(x),G(x)]`,
`sym_pat/[F(x),G(y)]`, `sym_pat/[F(x)+G(y),0]`,
`sym_pat/[F(x),G(x)*y+H(x)]`, sym_pat, exp_sym]
```

For example given a first order ode, we can ask dsolve to solve it using one of these methods as follows

```
restart;
```

```
ode:=diff(y(x),x)=sqrt( sqrt( y(x) ) * sin(x))/sqrt(y(x))
```

```
dsolve(ode,y(x),[`exact`])
```

$$4/5*y(x)^{(5/4)} + \text{Intat}(-\text{sqrt}(\text{sqrt}(y(x))*\sin(a))/y(x)^{(1/4)}, _a = x) + c_1 = 0$$

We can ask it to use symmetry with specific pattern as follows

```
restart;
```

```
ode:=diff(y(x),x)=sqrt( sqrt( y(x) ) * sin(x))/sqrt(y(x))
```

```
dsolve(ode,y(x),[`sym_pat/[F(x),G(x)*y+H(x)]`])
```

$$c_1 + 4/5*y(x)^{(3/2)*\text{sqrt}(\sin(x))/\text{sqrt}(\text{sqrt}(y(x))*\sin(x))} - \text{Int}(\text{sqrt}(\sin(x)), x)$$

#or using short cut, same thing can be done as follows

```
dsolve(ode,y(x),Lie)
```

$$c_1 + 4/5*y(x)^{(3/2)*\text{sqrt}(\sin(x))/\text{sqrt}(\text{sqrt}(y(x))*\sin(x))} - \text{Int}(\text{sqrt}(\sin(x)), x)$$

#another short cut for Lie is

```
dsolve(ode,[`sym_pat`])
```

$$c_1 + 4/5*y(x)^{(3/2)*\text{sqrt}(\sin(x))/\text{sqrt}(\text{sqrt}(y(x))*\sin(x))} - \text{Int}(\text{sqrt}(\sin(x)), x)$$

To find all methods, and for higher order ode, we first use indices as follows

```

indices(`dsolve/methods`)

[high, linear_nonhomogeneous],
[1],
[1, high_degree],
[3, linear_homogeneous],
[2, "linear_homogeneous all methods"],
[2, "linear_homogeneous other"],
[2, linear_homogeneous],
[2, "special_functions"],
[3, linear_nonhomogeneous],
[2, "linear_homogeneous in Normal Form"],
[2, "development"], [2, "hypergeometric"],
[1, extra],
[high, linear_homogeneous],
[high, nonlinear],
[1, "special"],
[1, "development"],
[2, nonlinear],
[high, "development"],
[1, semiclass],
[3, nonlinear],
[2, linear_nonhomogeneous],
[2, "linear_homogeneous as given"],
[3, "development"]

```

Using the above, to find all methods for second order linear_homogeneous ode, the command is

```

`dsolve/methods` [2, "linear_homogeneous all methods"]

[quadrature, const_coeffs, Euler, linear_1,
`linear/missing_y`, Kovacic, special_functions, to_const_coeffs,
exact_linear, sym_1, Mathieu, MeijerG, Heun, HeunG, HeunC, HeunB, HeunD,
HeunT, mu_xy, equivalent_to_Bessel, to_Riccati, Bessel, elliptic,
Legendre, Whittaker, Kummer, cylindrical, hypergeometric, hypergeom1,
hypergeom2, Riemann, RNF, hypergeometricsols, rationalize_lode, with_periodic_fu

```

```
`dsolve/methods` [2, "linear_homogeneous other"]
```

```
[exact_linear, sym_1, to_const_coeffs, mu_xy, equivalent_to_Bessel,  
to_Riccati, with_periodic_functions]
```

For example, given the ode $y'' + 3y' + y = 0$, we can now do

```
dsolve(ode,y(x))  
y(x) = c__1*exp(1/2*(sqrt(5) - 3)*x) + c__2*exp(-1/2*(3 + sqrt(5))*x)  
  
dsolve(ode,y(x),[`const_coeffs`])  
y(x) = c__1*exp(1/2*(sqrt(5) - 3)*x) + c__2*exp(-1/2*(3 + sqrt(5))*x)  
  
dsolve(ode,y(x),[`Kovacic`])  
y(x) = c__1*exp(1/2*(sqrt(5) - 3)*x) + c__2*exp(-1/2*(3 + sqrt(5))*x)
```

Not all methods ofcourse will work, as it depends on the ode type.

This function below lists all methods

```
ind:=indices(`dsolve/methods`);  
for item in ind do  
  cat("`dsolve/methods`",String(item));  
  eval(parse(%))  
od;
```

Which gives

```
"`dsolve/methods` [high, linear_nonhomogeneous]"  
[quadrature, fully_exact_linear, linear_nonhomogeneous_[0,1],  
exact_linear_nonhomogeneous, linear, exp_reduce]  
  
"`dsolve/methods` [1]"  
[quadrature, linear, Bernoulli, separable, inverse_linear,  
homogeneous, Chini, lin_sym, exact, Abel, pot_sym]  
  
"`dsolve/methods` [1, high_degree]"  
[WeierstrassP, WeierstrassPPrime, JacobiSN,  
linearizable_by_differentiation, missing, dAlembert,
```

```

homogeneous_B, sym_implicit]

"`dsolve/methods`[3, linear_homogeneous]"
[quadrature, const_coeffs, Euler, fully_exact_linear,
 to_const_coeffs, linear, exp_reduce, exact_linear,
 with_periodic_functions]

"`dsolve/methods`[2, "linear_homogeneous all methods]"
[quadrature, const_coeffs, Euler, linear_1, linear/missing_y,
 Kovacic, special_functions, to_const_coeffs, exact_linear,
 sym_1, Mathieu, MeijerG, Heun, HeunG, HeunC, HeunB, HeunD,
 HeunT, mu_xy, equivalent_to_Bessel, to_Riccati, Bessel,
 elliptic, Legendre, Whittaker, Kummer, cylindrical,
 hypergeometric, hypergeom1, hypergeom2, Riemann, RNF,
 hypergeometric_sols, rationalize_lode, with_periodic_functions]

"`dsolve/methods`[2, "linear_homogeneous other]"
[exact_linear, sym_1, to_const_coeffs, mu_xy,
 equivalent_to_Bessel, to_Riccati, with_periodic_functions]

"`dsolve/methods`[2, linear_homogeneous]"
[linear_homogeneous]

"`dsolve/methods`[2, "special_functions]"
[Bessel, elliptic, Legendre, Kummer, Whittaker, hypergeometric, Mathieu]

"`dsolve/methods`[3, linear_nonhomogeneous]"
[quadrature, fully_exact_linear, linear_nonhomogeneous_[0,1],
 exact_linear_nonhomogeneous, linear, exp_reduce]

"`dsolve/methods`[2, "linear_homogeneous in Normal Form]"
[linear_1]

"`dsolve/methods`[2, "development]"

```

```

[mu\_xyp, mu\_xyp2, mu\_formal, mu\_heuristic, exp\_reduce, linear,
  Bessel2, Whittaker\_old, nonlinear\_homogeneous,
  exact\_linear\_nonhomogeneous, mu\_y1, mu\_x\_y1, mu\_y\_y1,
  mu\_poly\_yn, exp\_sym, sym\_pat, sym\_8]

"`dsolve/methods`[2, "hypergeometric"]"
  [hypergeom1, hyper3]

"`dsolve/methods`[1, extra]"
  [inverse\_Riccati, Abel\_AIL, sym\_pat/\[F\(x\)\*G\(y\),0\],
  sym\_pat/\[F\(x\),G\(x\)\], sym\_pat/\[F\(x\),G\(y\)\],
  sym\_pat/\[F\(x\)+G\(y\),0\], sym\_pat/\[F\(x\),G\(x\)\*y+H\(x\)\], sym\_pat,
  exp\_sym]

"`dsolve/methods`[high, linear_homogeneous]"
  [quadrature, const\_coeffs, Euler, fully\_exact\_linear,
  to\_const\_coeffs, linear, exp\_reduce, exact\_linear,
  with\_periodic\_functions]

"`dsolve/methods`[high, nonlinear]"
  [linearizable\_by\_differentiation, linearizable, reducible,
  exact\_nonlinear, missing, mu\_formal, lin\_sym]

"`dsolve/methods`[1, "special"]"
  [80, 81]

"`dsolve/methods`[1, "development"]"
  [linearizable\_by\_differentiation, linearizable, con\_sym,
  WeierstrassP, WeierstrassPPrime, equivalent\_to\_Abel, Abel\_AIR,
  special, Riccati\_symmetries]

"`dsolve/methods`[2, nonlinear]"
  [Liouville, WeierstrassP, JacobiSN, linearizable,
  linearizable\_by\_differentiation, mu\_xy\_2, missing,
  mu\_xyp2\_dynamical\_symmetries\_fully\_reducible,
  mu\_xyp2\_singularcases, sym\_1, exact\_nonlinear, reducible,

```

```
lin_sym, S-function, mu_xyp_generalcase,  
mu_xyp2_dynamical_symmetries_not_fully_reducible]
```

```
"`dsolve/methods`[high, "development"]"
```

```
[k25, RNF, mu_heuristic, MeijerG, nonlinear_homogeneous,  
mu_poly_yn, exp_sym]
```

```
"`dsolve/methods`[1, semiclass]"
```

```
[Riccati, inverse_Riccati, equivalent_to_Abel, linearizable,  
linearizable_by_differentiation]
```

```
"`dsolve/methods`[3, nonlinear]"
```

```
[linearizable_by_differentiation, linearizable, missing,  
exact_nonlinear, reducible, mu_formal, lin_sym]
```

```
"`dsolve/methods`[2, linear_nonhomogeneous]"
```

```
[quadrature, fully_exact_linear, linear_nonhomogeneous_[0,1],  
linear_nonhomogeneous_[0,F(x)], linear_nonhomogeneous]
```

```
"`dsolve/methods`[2, "linear_homogeneous as given"]"
```

```
[quadrature, const_coeffs, Euler, linear_1, linear/missing_y,  
Kovacic, RNF, special_functions, MeijerG, Heun,  
hypergeometricsols, rationalize_lode]
```

```
"`dsolve/methods`[3, "development"]"
```

```
[k25, RNF, mu_heuristic, linear_patterns, MeijerG,  
nonlinear_homogeneous, mu_y2, mu_poly_yn, exp_sym, pFq, 3F2,  
2F2, 1F2, 0F2]
```

2.2 How to find a particular solution to ODE?

```
restart;  
ode:=diff(y(x),x)+y(x)^2*sin(x)-2*sin(x)/cos(x)^2 = 0;  
yp:=DETools:-particularsol(ode);
```

To step into the code, do

```
restart;  
ode:=diff(y(x),x)+y(x)^2*sin(x)-2*sin(x)/cos(x)^2 = 0;  
stopat(`DEtools/particularsol`);  
DETools:-particularsol(ode);
```

To print it do

```
print(`DEtools/particularsol`);
```

2.3 How to find basis solutions for homogeneous ode?

Use the output=basis option

```
ode:=diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)=0;  
dsolve(ode,output=basis);
```

2.4 How to solve a differential equation with initial conditions?

To solve

$$y'' - 3y' + 2y = 10e^{5x}$$

with $y(0) = 1, y'(0) = 5$ do

```
eq1:= diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=10*exp(5*x);  
dsolve({eq1,y(0)=1,D(y)(0)=5},y(x));
```

Methods for second order ODEs:

Trying to isolate the derivative d^2y/dx^2 ...

Successful isolation of d^2y/dx^2

--- Trying classification methods ---

```

trying a quadrature
trying high order exact linear fully integrable
trying differential order: 2; linear nonhomogeneous with symmetry [0,1]
trying a double symmetry of the form [xi=0, eta=F(x)]
<- double symmetry of the form [xi=0, eta=F(x)] successful
....

```

The above can also be written using D@@ notation, like this

```

eq:= (D@@2)(y)(x) - 3*D(y)(x) +2*y(x) = 10*exp(5*x);
IC := y(0)=1,D(y)(0)=5;
dsolve({eq,IC},y(x));

```

2.5 How to verify that the ODE solution given is correct?

use odetest and check if it gives zero.

```

eq1:= diff(diff(y(x),x),x)-3*diff(y(x),x)+2*y(x)=10*exp(5*x);
ans:=dsolve({eq1,IC},y(x));
odetest(ans,eq1);

```

0

2.6 How to know the type of ODE?

Maple can classify the ODE.

```

eq1:= diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=10*exp(5*x);
R0 := DEtools['odeadvisor'](eq1,y(x));

```

```

R0 := [[_2nd_order, _with_linear_symmetries]]

```

To get help on this type of ODE, do

```

DEtools['odeadvisor'](eq1,'help');

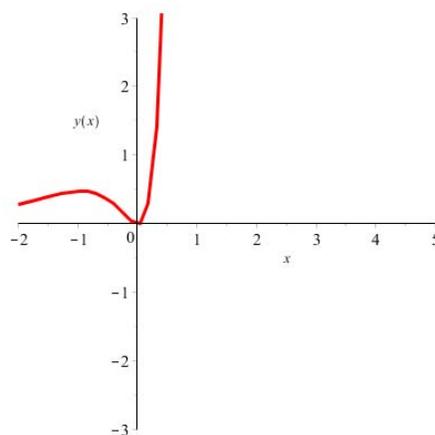
```

2.7 What packages to load for differential equations?

Use `with(DEtools);`

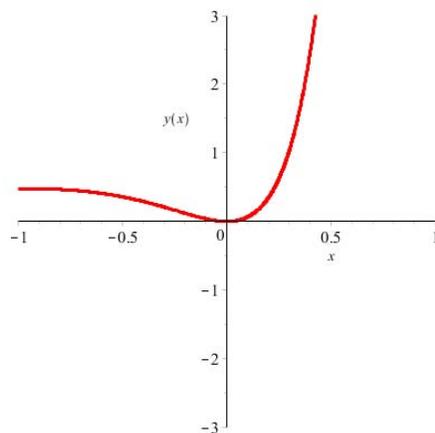
2.8 How to plot solution of differential equations?

```
restart;  
eq1:= diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=10*exp(5*x);  
DEtools[DEplot](eq1,y(x),x=-2..5, [[y(0)=0, D(y)(0)=0]], y=-3..3, linecolor=red);
```



To get a better plot, change the stepsize and independent variable range

```
restart;  
eq1:= diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=10*exp(5*x);  
DEtools[DEplot](eq1,y(x),x=-1..1, [[y(0)=0, D(y)(0)=0]], y=-3..3, stepsize=0.001, linecolor=red);
```



2.10 Obtain ODE in canonical coordinates (Lie symmetry)

How to obtain the ODE in canonical coordinates $S(R)$ for an ode using Lie symmetry?

Here is a function I wrote with the help of Maple docs, that does that. For now, I am using first order ode. It takes the ode and the variables (coordinates) to use (R, S) is what normally used, and returns back the ODE in canonical coordinates. The ode should always be a quadrature for first order.

```
get_ODE_in_canonical:=proc(ode::`=`,y::symbol,x::symbol,S::symbol,R::symbol)
  local infinitesimals,tr,itr,ODE;
  infinitesimals := [DEtools:-symgen(ode)];
  if nops(infinitesimals)=0 then
    RETURN(FAIL);
  fi;
  infinitesimals:=infinitesimals[1];
  tr := DEtools:-canoni(infinitesimals,y(x),S(R));
  itr := op(1,[solve(tr,{x,y(x)})]);
  ODE:= PDEtools:-dchange(itr,ode,[R,S(R)],simplify);
  ODE:= op(solve(ODE,{diff(S(R),R)}));
  RETURN(ODE);
end proc;
```

Example

```
ODE := diff(y(x),x) = 1/x^2+y(x);
get_ODE_in_canonical(ODE,y,x,S,R)

#diff(S(R), R) = 1/(exp(R)*R^2)

ODE := diff(y(x),x) = x^2-2*x*y(x)+y(x)^2;
get_ODE_in_canonical(ODE,y,x,S,R)
#diff(S(R), R) = 1
```

The following function is small change of the above. It accepts your choice of ξ, η to use instead of the first one obtained by Maple. This makes it possible to experiment with different infinitesimals. Note that different infinitesimals will/can result in different canonical ODE, but the final solution for $y(x)$ of course will always be the same after applying the transformation back to x, y coordinates.

```
restart;
```

```

get_ODE_in_canonical_v2:=proc(ode::`=`,y::symbol,x::symbol,S::symbol,R::symbol,xi,eta)
  local infinitesimals:=[xi,eta],tr,itr,ODE;
  tr := DEtools:-canoni(infinitesimals,y(x),S(R));
  itr := op(1,[solve(tr,{x,y(x)})]);
  ODE:= PDEtools:-dchange(itr,ode,[R,S(R)],simplify);
  ODE:= op(solve(ODE,{diff(S(R),R)}));
  RETURN(simplify(ODE,symbolic));
end proc:

```

And now

```

ODE:=diff(y(x),x) = x^2-2*x*y(x)+y(x)^2;
get_ODE_in_canonical_v2(ODE,y,x,S,R,1,1)

#diff(S(R), R) = 1/(R^2 - 1)

```

2.11 How to use hint with symgen?

Here is an example.

```

ode:=diff(y(x),x)=-y(x)^2/(exp(x)-y(x))
DEtools:-symgen(ode,y(x),HINT=[c1*x+c2*y+c3,c4*x+c5*y+c6]);

[_xi = 1, _eta = y]

```

Or

```

ode:=diff(y(x),x)=-y(x)^2/(exp(x)-y(x))
DEtools:-symgen(ode,y(x),HINT=[g(x),f(x)*y]);

[_xi = 1, _eta = y]

```

So not use $y(x)$ but only y in the hint.

2.12 How to parse a single ode?

The following function takes in a single ode and parses it to verify it is valid. It returns back 3 values. The dependent variable, the independent variable and the ode order.

```
#added 12/8/2022.
interface(warnlevel=4);
kernelopts('assertlevel'=2):

parse_single_ode:=proc(ode::`= `)::symbol,symbol,integer;
#parses single ode. returns back dep_var,indep_var,ode_order
#throws exception when it detects parsing errors

local func;
local y,x;
local dep_variables_found::list,item;
local the_order;

func:=PDEtools:-Library:-GetDepVars("not given",ode,onlydifferentiatedfunctions=true);
if nops(func)=0 then
    error ("not differential equation ",ode);
fi;

func := func[1];

if nops(func)<>1 then
    error("Parsing error, dependent variable must contain one argument, found ", func);
fi;

y:=op(0,func);
x:=op(1,func);

#basic verification
if not has(ode,y) then
    error ("Supplied ode ",ode," has no ",y);
fi;

if not has(ode,x) then
    error ("Supplied ode ",ode," has no ",x);
```

```

fi;

if not has(ode,func) then
    error ("Supplied ode ",ode," has no ",func);
fi;

the_order := PDEtools:-difforder(ode,x);
#if the_order=0 then
#    error ("No derivative found in ",ode,". Input is not differential equation");
#fi;

#note that the following call will also return y(x) if the input is not an ode
#this will check that the dependent variable will show with SAME argument in the ode
#i.e. if y(x) and y(t) show up in same ode, it will throw exception, which is what
#we want.
try
    dep_variables_found := PDEtools:-Library:-GetDepVars([y],ode);
catch:
    error lastexception;
end try;

#now go over dep_variables_found and check the independent variable is same as x
#i.e. ode can be y'(z)+y(z)=0 but function is y(x).
for item in dep_variables_found do
    if not type(item,function) then
        error("Parsing error. Expected ",func," found ",item," in ode");
    else
        if op(1,item) <> x then
            error("Parsing error. Expected ",func," found ",item," in ode");
        fi;
    fi;
od;

#now go over all indets in ode and check that y only shows as y(x) and not as just
#as the PDEtools:-Library:-GetDepVars([_self:-y],ode) code above does not detect this
#i.e. it does not check y'(x)+y=0

if numelems(indets(ode,identical(y))) > 0 then
    error("Parsing error, Can not have ",y," with no argument inside ",ode);

```

```

fi;

return y,x,the_order;
end proc:

```

To use do

```

y,x,the_order := parse_single_ode(diff(y(x),x)+y(x)=0);

```

An alternative to the above is to pass the dependent function itself as well as the ode. This is what I do myself in my ode solver. Like this

```

parse_single_ode:=proc(ode::`=`,func::function(name))::symbol,symbol,integer;
#parses single ode. returns back dep_var,indep_var,ode_order
#throws exception when it detects parsing errors

local y,x;
local dep_variables_found::list,item;
local the_order;

    if nops(func)<>1 then
        error("Parsing error, dependent variable must contain one argument, found ", func);
    fi;

    y:=op(0,func);
    x:=op(1,func);

    #basic verification
    if not has(ode,y) then
        error ("Supplied ode ",ode," has no ",y);
    fi;

    if not has(ode,x) then
        error ("Supplied ode ",ode," has no ",x);
    fi;

    if not has(ode,func) then
        error ("Supplied ode ",ode," has no ",func);
    fi;

```

```

the_order := PDEtools:-difforder(ode,x);

#note that the following call will also return y(x) if the input is not an ode
#this will check that the dependent variable will show with SAME argument in the
#i.e. if y(x) and y(t) show up in same ode, it will throw exception, which is wh
#we want.
try
  dep_variables_found := PDEtools:-Library:-GetDepVars([y],ode);
  #print("dep_variables_found=",dep_variables_found);
catch:
  error lastexception;
end try;

#now go over dep_variables_found and check the independent variable is same as x
#i.e. ode can be y'(z)+y(z)=0 but function is y(x).
for item in dep_variables_found do
  if not type(item,function) then
    error("Parsing error. Expected ",func," found ",item," in ode");
  else
    if op(1,item) <> x then
      error("Parsing error. Expected ",func," found ",item," in ode");
    fi;
  fi;
od;

#now go over all indents in ode and check that y only shows as y(x) and not as y
#as the PDEtools:-Library:-GetDepVars([_self:-y],ode) code above does not detect
#i.e. it does not check y'(x)+y=0

if numelems(indets(ode,identical(y))) > 0 then
  error("Parsing error, Can not have ",y," with no argument inside ",ode);
fi;

return y,x,the_order;
end proc:

```

To use do the same as before, but need to add $y(x)$ as second argument. Like this

```
y,x,the_order := parse_single_ode(diff(y(x),x)+y(x)=0, y(x));
```

2.13 How to check if single ODE is valid?

This is very much the same as above, but this function returns True if the ode is syntactically valid, else false. Can be used just to check if the ode is valid before using it.

It takes in the ode and the function $y(x)$ and returns either true or false.

```
#March 15, 2024
```

```
export is_valid_single_ode :=proc(ode_in::`=`,func::function(name))::truefalse;  
  local x:=op(1,func),y:=op(0,func);  
  local ode:=ode_in;  
  local dep_variables_found::list;  
  local item;  
  
  if nops(func)<>1 then RETURN(false); fi;  
  
  if not has(ode,diff) then ode:=convert(ode,diff); fi;  
  
  if `or`(not has(ode,diff), not has(ode,x), not has(ode,func)) then  
    RETURN(false);  
  fi;  
  
  try  
    dep_variables_found := PDEtools:-Library:-GetDepVars([y],ode);  
  catch:  
    RETURN(false);  
  end try;  
  
  map( X-> `if`( `or`(not type(X,function), op(1,X) <> x) ,RETURN(false),NULL),  
    dep_variables_found  
  );  
  
  #check there is no y on its own. Should always be y(x)  
  if nops(indets(ode,identical(y))) <> 0 then  
    RETURN(false);  
  fi;
```

```

    RETURN(true);
end proc:

```

Example usages

```

is_valid_single_ode(diff(y(x),x)+sin(x)=0, y(x) )

```

```

    true

```

To test, do

```

L:=[[3*(D@@2)(y)(x)+diff(y(x),x)+1=sin(x),y(x),true],
     [diff(y(x),x)^2=z,y(x),true],
     [diff(y(x),x)^2=z,y(x),true],
     [diff(y(x),x)^2=y(x)^2,y(x),true],
     [diff(y(x),x)=y(z)^2,y(x),false],
     [1/diff(y(x),x)=y(x)^2,y(x),true],
     [y(x)=1,y(x),false],
     [diff(y(x),z)=1,y(x),false],
     [D(y)(x)=1,y(x),true],
     [(D@@2)(y)(x)+diff(y(x),x)=sin(x),y(x),true],
     [diff(y(x),x)+diff(y(z),z)=1,y(x),false],
     [diff(y(x),x)+diff(g(z),z)=1,y(x),true]
]:

map(X->evalb(is_valid_single_ode(X[1],X[2])=X[3]),L);
if andmap(x->evalb(x=true),%) then
    print("all tests passed");
else
    print("WARNING, not all tests passed");
fi;

#gives

[true, true, true, true, true, true, true, true, true, true, true]
    "all tests passed"

```

2.14 How to check if an ode has y' in it?

Suppose to want to check if an ode has $y'(x)$ in it. We can not write `has(ode, diff(y(x),x))` and see if this gives true or not, because this will also match y'' and y''' and any higher order.

One way is to first convert the ode to D form and then use `has` on `D(y)(x)`. This will not match y'' anymore which is what we wanted, because y'' becomes `(D@@2)(y)(x)` and so the check for first order diff will work as expected.

```
ode:=diff(y(x),x$2)+3*y(x)=0;
ode:=convert(ode,D);
has(ode,(D)(y)(x))

#false
```

Another example

```
ode:=diff(y(x),x$2)+1/diff(y(x),x)=0;
ode:=convert(ode,D);
has(ode,(D)(y)(x))

#true
```

The same thing if we wanted to check if the ode has y'' or not.

```
ode:=diff(y(x),x$3)+diff(y(x),x)=0;
ode:=convert(ode,D);
has(ode,(D@@2)(y)(x))

#false
```

2.15 How to check if an ode is linear ode?

```
ode:=diff(y(x),x$3)+ x*diff(y(x),x)+sin(x)=0;
if has(DEtools:-odeadvisor(ode,y(x),['linear']),_linear) then
    print("linear");
else
    print("not linear ode");
fi
```

The above prints "linear"

```
ode:=y(x)*diff(y(x),x$3)+ x*diff(y(x),x)+sin(x)=0;
if has(DEtools:-odeadvisor(ode,y(x),['linear']),_linear) then
  print("linear");
else
  print("not linear ode");
fi
```

The above prints "not linear ode". This works for any ode order.

2.16 How to find the order of an ode?

```
ode:=diff(y(x),x$3)+ x*diff(y(x),x)+sin(x)=0;
PDEtools:-difforder(ode)
```

Gives 3.

2.17 How to find the coefficients of a linear ode?

Given a linear ode of the form $Ay'' + By' + Cy = f(x)$ how to find $A, B, C, f(x)$?

```
ode:=diff(y(x),x$3)+ x*diff(y(x),x)+99*y(x)=sin(x);
L:=DEtools:-convertAlg(ode,y(x)); #this only works on linear ode's
```

Gives $L := [[99, x, 0, 1], \sin(x)]$.

L is a list. The second entry is $f(x)$ and the first entry of L is a list which gives the coefficients of the ode. Notice they are ordered from lowest order to highest order of the ode. Since this is third order ode, then there are 4 entries. The first is the coefficient of $y(x)$, the second is the coefficient of y' and the third is the coefficient of y'' and the fourth entry is the coefficient of y''' . Notice that coefficient of y'' is zero since y'' is missing from the ode.

This function `DEtools:-convertAlg` only works on linear ode's. Therefore we need to check if the ode is linear first before using. How to check for linear ode is given above.

2.18 find order and degree of highest derivative

I need to find the degree of the highest derivative in an ode.

For example, if the input is

$$x \left(\frac{d^2}{dx^2} y(x) \right)^5 + x \left(\frac{d^2}{dx^2} y(x) \right)^2 + \left(\frac{d}{dx} y(x) \right) y(x) + \left(\frac{d^3}{dx^3} y(x) \right)^4 \sin(x) + 5y(x) + \sin(x) + \frac{d^6}{dx^6} r(x) + \left(\frac{d^3}{dx^3} y(x) \right)^4 \cos(x) = 0$$

Then the degree for highest derivative of y w.r.t. x is 4. For

$$y''(x)^2 + y'(x) + y(x) = 0$$

It will be 2.

This function returns the order and degree of such term.

```
find_all_derivatives_of_specific_order:=proc(expr,y::symbol,x::symbol,N::posint)::set
local t1,t2;

if not has(expr,diff(y(x),x$N)) then
return {};
fi;

t1 := identical(diff(y(x),x$N))^anything;
t2 := identical(diff(y(x),x$N));
return indets[flat](expr,{t1,t2}); #MUST use flat

end proc:

get_order_and_degree_of_largest_derivative:=proc(expr,y::symbol,x::symbol)::integer,an
local the_order,the_degree;
local cand::set;
local the_exponent;
local item;

if not has(expr,diff(y(x),x)) then return 0,0; fi;
the_order := PDEtools:-difforder(expr,x);
cand := find_all_derivatives_of_specific_order(expr,y,x,the_order);
```

```

if nops(cand)=0 then
    the_degree := 0;
else
    the_degree:=1;
    for item in cand do
        if type(item, ``) then
            the_exponent := op(2,item);
            if type( the_exponent,symbol) or not type( the_exponent,numeric) then
                the_degree :=the_exponent;
            else
                if type(the_degree,symbol) or not type( the_exponent,numeric) then
                    next;
                else
                    if op(2,item)>the_degree then
                        the_degree := op(2,item);
                    fi;
                fi;
            fi;
        else
            if type(the_degree,symbol) then
                next;
            else
                if the_degree=0 then
                    the_degree := 1;
                fi;
            fi;
        fi;
    od;
fi;

return the_order,the_degree;
end proc:

```

And now it can be called as follows

```

ode:=diff(y(x),x$2)+diff(y(x),x)+y(x)=0;
get_order_and_degree_of_largest_derivative(lhs(ode),y,x)

```

```
#2,1
```

```
ode:=diff(y(x),x$2)*diff(y(x),x$2)^4+diff(y(x),x)+y(x)=0;  
get_order_and_degree_of_largest_derivative(lhs(ode),y,x)
```

```
#2,5
```

```
ode:=y(x)=0;  
get_order_and_degree_of_largest_derivative(lhs(ode),y,x)
```

```
#0,0
```

```
ode:=diff(y(x),x)^n=0;  
get_order_and_degree_of_largest_derivative(lhs(ode),y,x)
```

```
#1,n
```

2.19 How to obtains list of all derivatives in expression?

I had a need to find all derivatives of form `diff(y(anything), anything)` in an ode so to check that y argument is not different among them.

For an example, given

$$\begin{aligned} & a \left(\frac{d^2}{dx^2} y(x) \right) \left(\frac{d^3}{dx^3} y(x) \right) - \sqrt{1 + \left(\frac{d^2}{dx^2} y(x) \right)^2} + \frac{1}{\sin \left(\frac{d^5}{dx^5} y(x) \right)} \\ & = \frac{d}{dz} y(z) + e^{y(x) + \frac{d}{dx} y(x)} + \frac{d}{dx} r(x) \end{aligned}$$

the result should be

$$\begin{bmatrix} \frac{d^5}{dx^5} y(x) \\ \frac{d^4}{dx^4} y(x) \\ \frac{d^3}{dx^3} y(x) \\ \frac{d^2}{dx^2} y(x) \\ \frac{d}{dx} y(x) \\ \frac{d}{dz} y(z) \end{bmatrix}$$

One issue is how to check for diff and also check that the dependent variable is y so

as not to pick other dependent variables such as z in this example. This was done by converting `diff` to `D` otherwise it will not work.

```
restart;
expr:=a*diff(y(x),x$2)*diff(y(x),x$3)-sqrt(1+ diff(y(x),x$2)^2)+1/sin(diff(y(x),x$5))=

#this finds all derivatives
list_of_diffs:=indets(expr,'satisfies'(s->op(0,s)='diff' and op([0,1],convert(s,D))=y)

#This finds all dependent variables
list_of_diffs:=convert(list_of_diffs,list);
list_of_indep_variables := map(x->PDEtools:-Library:-GetIndepVars(x)[-1],list_of_diffs)

#this converts it to set. If the ODE is valid, then the list_of_indep_variables should
#have one entry $x$ in it and nothing else.
list_of_indep_variables := convert(ListTools:-Flatten(list_of_indep_variables),set);

if nops(list_of_indep_variables)>1 then
  error( cat("Only one independent variable expected in differential form, found ",
            convert(list_of_indep_variables,string)) );
fi;

if list_of_indep_variables[1]<>x then
  error( cat("Independent variable expected in differential form not same as independent
            convert(list_of_indep_variables,string)) );
fi;
```

Another option instead of doing all the above is to do this

```
expr:=a*diff(y(x),x$2)*diff(y(x),x$3)-sqrt(1+ diff(y(x),x$2)^2)+1/sin(diff(y(x),x$5))=
try
  PDEtools:-Library:-GetDepVars([y(x)],expr);
catch
  error "functions with name [y] having different dependency: [[y(x), y(z)]]"
end try;
```

The function `PDEtools:-Library:-GetDepVars([y(x)],expr)` checks that only $y(x)$ dependency shows up. It throws an error otherwise. So if an error is thrown, then this means y shows up with different independent variables.

2.20 How to invert roles of dependent variable and independent variable in an ode?

Sometimes it is useful to invert an ode. i.e. make the independent variable the dependent variable, and the dependent variable the independent. For example, given

$$1 + \left(\frac{x}{y(x)} - \sin(y(x)) \right) \left(\frac{d}{dx} y(x) \right) = 0$$

We want the ode to become

$$-\sin(y)y + y \left(\frac{d}{dy} x(y) \right) + x(y) = 0$$

This can be done as follows

```
restart;  
  
ode:=1+ (x/y(x)-sin(y(x) ))*diff(y(x),x)=0;  
tr:={x=u(t),y(x)=t};  
ode:=PDEtools:-dchange(tr,ode);  
ode:=eval(ode,[t=y,u=x]);  
ode:=simplify(ode);
```

$$\frac{-\sin(y)y + y \left(\frac{d}{dy} x(y) \right) + x(y)}{y \left(\frac{d}{dy} x(y) \right)} = 0$$

In this case, we can get rid of the denominator, but this is a manual step for now.

```
ode:=numer(lhs(ode))=0;
```

$$-\sin(y)y + y \left(\frac{d}{dy} x(y) \right) + x(y) = 0$$

The above can now be solved more easily for $x(y)$ than solving the original ode for $y(x)$.

2.21 How to find the indicial equation for an ODE?

For say Bessel ode of order zero:

```
eq:= x^2*diff(y(x),x$2)+x*diff(y(x),x)+x^2*y(x)=0;
DEtools[indicialeq](eq,x,0,y(x));
#x^2 = 0
```

The third argument above is the singularity point of interest. So we have two roots, both zero. These are now used for finding the power series solution $y(x)$ if needed.

Another example, is Bessel of order 1

```
eq:= x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0;
DEtools[indicialeq](eq,x,0,y(x));
#x^2-1 = 0
```

2.22 How to write derivative

To write $y'(x) = x$, one way is `diff(y(x),x)=x` and another is `D(y)(x)=x`. To write $y''(x) = x$, one way is `diff(y(x),x$2)=x` and another is `(D@@2)(y)(x)=x`.

To convert from one form to another use `convert(eq,diff)` or `convert(eq,D)`

2.23 How to solve heat PDE in 1D in Maple 2017?

to solve $\frac{\partial u(x,t)}{\partial t} = k \frac{\partial^2 u(x,t)}{\partial x^2}$ with homogeneous dirichlet boundary conditions $u(0,t) = 0, u(L,t) = 0$ the commands are

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
sol:=pdsolve([pde,bc]) assuming 0<L:
```

Which gives

$$u(x,t) = \sum_{n=1}^{\infty} C1(n) \sin\left(\frac{\pi n x}{L}\right) e^{-\frac{k\pi^2 n^2 t}{L^2}}$$

Which can be made more readable as follows

```
sol:=algsubs(_Z1=n,sol):
sol:=algsubs(Pi*n/L=lambda(n),sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} C1(n) \sin(x\lambda(n)) e^{-kt(\lambda(n))^2}$$

For homogeneous Neumann B.C., at $x = 0$, let $\frac{\partial u}{\partial x} = 0$ and at $x = L$ let $u(L,t) = 0$, the solution it gives looks different than my hand solution

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
pdsolve([pde,bc]) assuming 0<L;
```

It gives

$$u(x,t) = C3 C2 \left(e^{1/4 \frac{2i\pi xL - k\pi^2 t}{L^2}} + e^{-1/4 \frac{\pi(2ixL + k\pi t)}{L^2}} \right)$$

I need to look more into the above and see if this comes out to be the same as my hand solution.

Another example, with initial conditions now given

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L;
sol1:=algsubs(_Z2=n,sol);
```

The result is

$$u(x,t) = \sum_{n=1}^{\infty} \left(2 \frac{1}{L} e^{-1/4 \frac{k\pi^2 t(1+2n)^2}{L^2}} \cos\left(1/2 \frac{\pi x(1+2n)}{L}\right) \int_0^L f(x) \cos\left(1/2 \frac{\pi x(1+2n)}{L}\right) dx \right)$$

Another example

```

restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
ic:=u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L;
sol1:=algsubs(_Z2=n,sol);

```

$$u(x,t) = \sum_{n=1}^{\infty} 768 \frac{1}{\pi (16n^4 + 32n^3 - 136n^2 - 152n + 105)} e^{-1/4 \frac{k\pi^2 t(1+2n)^2}{L^2}} \cos\left(\frac{1}{2} \frac{\pi x(1+2n)}{L}\right)$$

Another example

```

restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L;

```

$$u(x,t) = \sin\left(\frac{\pi x}{L}\right) e^{-9 \frac{\pi^2 kt}{L^2}} \left(-2 \cos\left(2 \frac{\pi x}{L}\right) + 3 e^{8 \frac{\pi^2 kt}{L^2}} - 1\right)$$

The above answer seems wrong. There is not even a summation in it. It is different from my hand solution. Look more into it.

2.24 How to make Maple display $\text{diff}(y(x),x)$ as $y'(x)$ or as y' ?

Add this

```

expr:=diff(y(x),x);
Typesetting:-Settings(typesetprime=true, prime=x);

```

The above will display the expression as $y'(x)$. To make it now show the x do

```

expr:=diff(y(x),x);
Typesetting:-Settings(typesetprime=true, prime=x);
Typesetting:-Suppress(y(x));

```

Now it will show the expression as just y' . For all the above to work, make sure you have `Typesetting` level set to `Extended` in the GUI.

This is done inside `Tools->Options->Display` menu.

To clear all the above `Typesetting`, do `restart` or do `Typesetting:-Unsuppress(y(x))`

2.25 How to set boundary conditions for `dsolve` or `pdsolve`?

The Maple syntax for setting initial and boundary conditions is very confusing, as compared to Mathematica, which seems to me to be simpler. So I wrote this to remind me of the syntax each time.

For PDE, assuming dependent variable is $u(x, t)$ then

Conditions	Maple code
$u(0, t) = 0$	<code>u(0, t)=0</code>
$\frac{\partial u}{\partial x} = 0$ at $x = 0$	<code>D[1](u)(0, t)=0</code>
$\frac{\partial^2 u}{\partial x^2} = 0$ at $x = 0$	<code>D[1, 1](u)(0, t)=0</code>
$\frac{\partial^3 u}{\partial x^3} = 0$ at $x = 0$	<code>D[1, 1, 1](u)(0, t)=0</code>
$\frac{\partial u}{\partial t} = 0$ at $t = 0$	<code>D[2](u)(x, 0)=0</code>
$\frac{\partial^2 u}{\partial t^2} = 0$ at $t = 0$	<code>D[2, 2](u)(x, 0)=0</code>
$\frac{\partial^3 u}{\partial t^3} = 0$ at $t = 0$	<code>D[2, 2, 2](u)(x, 0)=0</code>

Notice the syntax for the last one above. It is `(D[1]@@2)(u)(0, t)=0` and not `(D@@2)[1](u)(0, t)=0`

For an ODE, assuming dependent variable is $y(x)$ then the syntax is

Conditions	Maple code
$y(0) = 0$	<code>y(0)=0</code>
$\frac{dy}{dx} = 0$ at $x = 0$	<code>D(y)(0)=0</code>
$\frac{d^2 y}{dx^2} = 0$ at $x = 0$	<code>(D@@2)(y)(0)=0</code>

2.26 How force dsolve to use Lie?

Use `dsolve(ode,Lie)`

To find symmetries, do

```
DEtools:-symgen(ode,y(x),HINT=[c__1+c__2*x+c__3*y,c__4+c__5*x+c__6*y])
```

or just

```
DEtools:-symgen(ode,y(x))
```

To debug it do

```
stopat('ODEtools/symgen'); before calling dsolve or DEtools:-symgen
```

2.27 How to do change of variables on the dependent variable for an ODE?

given an ode

$$y(x) = \left(\frac{d}{dx} y(x) \right)^3 y(x)^2 + 2x \left(\frac{d}{dx} y(x) \right)$$

do change of variable $u(x) = y(x)^2$

```
restart;  
ode:=y(x)=diff(y(x),x)^3*y(x)^2+2*x*diff(y(x),x);  
new_ode:=PDEtools:-dchange({y(x)=sqrt(u(x))},ode,{u});
```

$$\sqrt{u(x)} = \frac{\left(\frac{d}{dx} u(x) \right)^3}{8\sqrt{u(x)}} + \frac{x \left(\frac{d}{dx} u(x) \right)}{\sqrt{u(x)}}$$

2.28 How to do change of variable on the independent variable for an ODE?

given an ode

$$\frac{d^2}{dt^2} y(t) + y(t) = 2t$$

do change of variable $t = \tau + \pi$

```
restart;
ode:=diff(y(t),t$2)+y(t)=2*t;
PDEtools:-dchange({t=tau+Pi},ode,known={t},unknown={tau},params=Pi)
```

$$\frac{d^2}{d\tau^2}y(\tau) + y(\tau) = 2\tau + 2\pi$$

it is important to use `params=Pi`. Watch what happens if we do not do that

```
restart;
ode:=diff(y(t),t$2)+y(t)=2*t;
PDEtools:-dchange({t=tau+Pi},ode,known={t},unknown={tau});
```

$$\frac{d^2}{d\tau^2}y(\tau, \pi) + y(\tau, \pi) = 2\tau + 2\pi$$

Which is not what we want.

2.29 ODE change of variable on both dependent and independent variable?

This verifies solution given in <https://math.stackexchange.com/questions/3477732/can-t-see-that-an-ode-is-equivalent-to-a-bessel-equation> Where a change of variables on

$$\xi^2 \frac{d^2 \eta}{d\xi^2} + \xi \frac{d\eta}{d\xi} - (\xi^2 + n^2)\eta = 0$$

Was made using

$$\eta = \frac{y}{x^\alpha}, \quad \xi = \beta x^\gamma,$$

To produce the ode

$$\frac{d^2 y}{dx^2} - \frac{(2\alpha - 1)}{x} \frac{dy}{dx} - (\beta^2 \gamma^2 x^{2\gamma-2} + \frac{n^2 \gamma^2 - \alpha^2}{x^2}) y = 0.$$

In Maple the above is done using

```
restart;
ode := zeta^2*diff(eta(zeta),zeta$2) + zeta*diff(eta(zeta),zeta) - (zeta^2 + n^2)*eta
the_tr:={zeta=beta*x^gamma,eta(zeta)=y(x)/x^alpha};
PDEtools:-dchange(the_tr,ode,{y(x),x},'known'={eta(zeta)},'uknown'={y(x)},'params'={alpha,
simplify(%);
```

```

numer(lhs(%))=0;
simplify(numer(lhs(%))/(x^(1-alpha)))=0;
numer(lhs(%))=0;
collect(%, [y(x), diff(y(x), x), diff(y(x), x$2)]);

```

Which gives

$$(-\gamma^2 x^{-1+2\gamma} \beta^2 x - \gamma^2 n^2 + \alpha^2) y(x) + (-2\alpha x + x) \left(\frac{d}{dx} y(x) \right) + x^2 \left(\frac{d^2}{dx^2} y(x) \right) = 0$$

Here is another example. Here to make change of variables to polar coordinates by making $x = r \cos \theta$ and $y = r \sin \theta$ The ode is

$$\frac{y - xy'}{\sqrt{1 + (y')^2}} = x^2 + y^2$$

In Maple

```

restart;
ode := (y(x)-x*diff(y(x),x))/sqrt(1+ diff(y(x),x)^2) = x^2+y(x)^2;
the_tr:={x=r(t)*cos(t),y(x)=r(t)*sin(t)};
PDEtools:-dchange(the_tr,ode,{r(t),t}, 'known'={y(x)}, 'unknown'={r(t)});

```

Which gives

$$\frac{r(t) \sin(t) - \frac{r(t) \cos(t) \left(\left(\frac{d}{dt} r(t) \right) \sin(t) + r(t) \cos(t) \right)}{\left(\frac{d}{dt} r(t) \right) \cos(t) - r(t) \sin(t)}}{\sqrt{1 + \frac{\left(\left(\frac{d}{dt} r(t) \right) \sin(t) + r(t) \cos(t) \right)^2}{\left(\left(\frac{d}{dt} r(t) \right) \cos(t) - r(t) \sin(t) \right)^2}}} = r(t)^2 (\cos^2(t)) + r(t)^2 (\sin^2(t))$$

Here is another example. Where we want to change $R(r)$ to $y(x)$ everywhere

$$\frac{d^2}{dr^2} R(r) + \frac{d}{dr} R(r) + R(r) = 0$$

```

restart;
ode:=diff(R(r),r$2)+diff(R(r),r)+R(r)=0;
the_tr:={r=x,R(r)=y(x)};
PDEtools:-dchange(the_tr,ode,{y(x),x}, 'known'={R(r),r}, 'unknown'={y(x),x});

```

$$\frac{d^2}{dx^2}y(x) + \frac{d}{dx}y(x) + y(x) = 0$$

The format of the transformation is `old_independent_variable=new_independent_variable` and `old_dependent_variable=new_dependent_variable`

2.30 How to make phase plot of second order ODE?

Make a phase plot of

$$\frac{d^2}{dt^2}x(t) + \frac{d}{dt}x(t) + x(t) = u(t)$$

By plotting $x(t)$ vs $x'(t)$ without solving the ODE.

```
restart;
alias(DS=DynamicSystems):
ode := diff(x(t),t$2) +1/2*diff(x(t),t)+ x(t) = u(t);
sys:=DS:-DiffEquation(ode, 'outputvariable'=[x(t)], 'inputvariable'=[u(t)]);
sys0:=DS:-StateSpace(sys);
eq1:=diff(x1(t),t)=sys0:-a[1,..].Vector([x1(t),x2(t)]);
eq2:=diff(x2(t),t)=sys0:-a[2,..].Vector([x1(t),x2(t)]);
DEtools:-DEplot([eq1,eq2], [x1(t),x2(t)], t=0..35, [[x1(0)=1,x2(0)=1]], x1=-2..2,x2=-2..2,
numpoints=200, linecolor=black, axes=boxed);
```

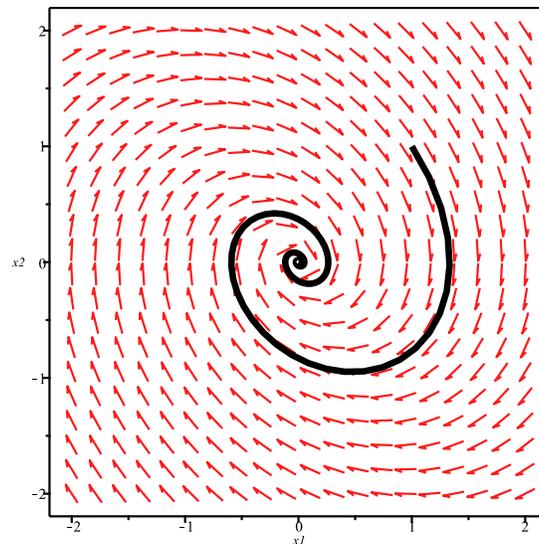


Figure 2: Phase Plot

2.31 How to move all terms with y to one side?

Given ode where y is dependent variable and x is independent variable, how to normalize the ode so that all terms with $y(x)$ are on left side and everything is on the RHS? THIS makes it easier to see what the forcing function is. i.e. we want to write the ode as

$$ay'' + by' + cy = f(x)$$

Here is function to do this. This works for any ode or any expression.

```
export move_y_x_to_each_side:=proc(expr_in::`=`,y::symbol,x::symbol)::`=`;
  local expr:=expr_in;
  local stuff_with_y,stuff_not_with_y;
  expr := lhs(expr)-rhs(expr);
  if not expr::`+` then
    RETURN(expr_in);
  fi;
  expr := numer(normal(expr));
  stuff_with_y,stuff_not_with_y := selectremove(has,expr,y);
  stuff_not_with_y := convert(stuff_not_with_y,diff);

  RETURN(stuff_with_y = -stuff_not_with_y);
end proc;
```

Examples

```
ode:=diff(y(x),x$2)+x*y(x)+sin(x)-3=tan(x);
expr:=move_y_x_to_each_side(ode,y,x);
```

$$\# \text{diff}(y(x), x, x) + x*y(x) = 3 - \sin(x) + \tan(x)$$

And

```
ode:=diff(y(x),x$2)+x*y(x)+sin(x)-3=tan(x)+y(x)+Pi;
expr:=move_y_x_to_each_side(ode,y,x);
```

$$\# \text{diff}(y(x), x, x) + x*y(x) - y(x) = 3 - \sin(x) + \tan(x) + Pi$$

And

```
ode:=move_y_x_to_each_side(diff(x(t),t$2)/x(t) + diff(x(t),t)/x(t) - F(t)/x(t) =1,x,t)
expr:=move_y_x_to_each_side(ode,x,t);
```

```
# diff(x(t), t, t) + diff(x(t), t) - x(t) = F(t)
```

2.32 How to find all constants of integrations in an expression?

Sometimes I make solution for an ode with constants of integrations. These must be of form $_Cn$ where n is positive integer, such as $_C1$ or $_C5$. Now to integrate this solution once more, need to come up with new constant that does not already show in the solution.

To find current constants in an expression use this

```
sol:=x-_C1*x+_C3*x^2;
indets(sol,And(symbol, suffixed(_C, nonnegint)));
{ _C1, _C3 }
```

Therefore, one way to make a new constant of integration, is to find the largest numbered one and increase by one. Like this

```
restart;
sol:=x-_C1*x+_C3*x^2;
myconstants:=indets(sol,And(symbol, suffixed(_C, nonnegint)));
map(X->String(X),myconstants);
map(X->X[3..],%);
map(X->:-parse(X),%);
n:=max(%);
new_constant:=_C|| (n+1);
```

2.33 How to find if an ode is missing the dependent variable?

Sometimes it is useful to know if an ode is missing $y(x)$ as this allows one to do the substitution $y' = u$ and reduce the order of the ode.

This is a function which takes in an ode and returns true if it is missing $y(x)$ or false otherwise.

```
#added 2/18/2024. checks if an ode is missing y
export is_ode_missing_y:=proc(ode_in::`=`,y::symbol,x::symbol)::truefalse;
local ode:=ode_in;
local ode_order::posint := PDEtools:-difforder(ode_in);
local N::posint;

ode:=lhs(ode)-rhs(ode);
ode:=convert(ode,D);
for N from 1 to ode_order do
    ode:=eval(ode,[(D@@N)(y)(x)=Y||N]);
od;

RETURN(not has(ode,y(x)));

end proc;
```

To use do

```
restart;
ode:=diff(y(x),x$2)+diff(y(x),x$4)+x=0;
is_ode_missing_y(ode,y,x);

    #true

ode:=diff(y(x),x$2)+diff(y(x),x$4)+x=sin(x)+y(x);
is_ode_missing_y(ode,y,x);

    #false
```

3 How to convert Mathematica expression to Maple?

```
restart;  
with(MmaTranslator); #load the package  
FromMma(`Integrate[Cos[x],x]`);
```

Or

```
restart;  
with(MmaTranslator); #load the package  
convert(`Integrate[Cos[x],x]`, FromMma);
```

4 How to debug internal procedures, such as dsolve?

```
f:=proc()  
eq:=x*diff(y(x),x)+y(x)=exp(2*x);  
dsolve(eq,y(x));  
end proc;
```

Then used the command `stopat(f)`; then called the procedure `f()`; and now the debugger comes up. Did `step` command and now it steps inside `dsolve`

Some examples

```
stopat(`ODEtools/symtest`);  
stopat(`ODEtools/test`);  
stopat(`ODEtools/normal/expanded`);  
stopat(`ODEtools/odepde`);  
  
DEtools:-symtest([-3,y],ode,y(x));
```

‘ODEtools/normal/expanded’

5 How to display source code of a function?

For integration use

```
infolevel[`evalf/int`]:=5;infolevel[int]:=5;
```

Another option

```
restart;  
interface(verboseproc=3)  #(try 2 also)
```

then `print(procedure)`; or `eval(procedure_name)`; for example

```
restart:  
interface(verboseproc=3):  
print(LinearAlgebra:-GramSchmidt);  
print(lcm);
```

Also can use `showstat`, in this case `interface(verboseproc=3)` is not needed. Also `showstat` gives line numbers and I think it is easier to read. Some examples

```
showstat(`odsolve/2nd_order`)  
showstat(`evalf/hypergeom`);  
showstat(`evalf/exp/general`);  
showstat(`evalf/Psi`);  
showstat(`evalf/int`);  
showstat(`dsolve/SERIES`);  
  
#these 3 shows the main 3 functions by each solver  
showstat(`odeadv/dAlembert`); #used by advisor  
showstat(`odsolve/dAlembert`); # main API.  
showstat(`odsolve/dAlembert/integrate`); #used to integrate the ode  
  
showstat(`ODEtools/odeadv`);  
showstat(DEtools:-odeadvisor);  
showstat(`dsolve/series/froben/inhom`)  
showstat(`dsolve/series/froben`)
```

To stop at anyone of these functions in debugger do

```
stopat(`dsolve/series/froben/inhom`)
#code here, say dsolve command.
```

The above will stop in the debugger in the above function.

There is also a function by Joe Riel here here is the post by Joe Riel:

"A disadvantage of showstat, particularly if you want to cut and paste the output, is that it includes line numbers. Here is a simple procedure I threw together to remove the line numbers."

```
PrintProc := proc(p::name,lines::{posint,posint..posint})
local width;
option `Copyright (C) 2004 by Joseph S. Riel. All rights reserved.`;
description "Print like showstat, but without line numbers";
width := interface('screenwidth'=200);
try
printf("%s",
StringTools:-RegSubs(
"\n ...." = "\n"
,debugopts('procdump'=
`if`(nargs=1,p,[args]))))
catch "procedure name expected":
error "%1 is not a procedure name",p
finally interface('screenwidth'=width)
end try;
NULL
end:
```

To print source code to file using the above, do the following

```
currentdir("C:\\data");
interface('prettyprint'=1):
interface('verboseproc'=3):
writeto("listing.txt")
PrintProc('singular');
writeto('terminal');
```

Now the output will show up in the file "listing.txt" and also no line wrapping. The above I found is the best solution so far to do this.

6 How to display trace of a function as it runs in maple?

```
trace(foo);  
untrace(foo);
```

also see `debug(foo);`

Also

```
infolevel[all]:=5:  
printlevel:=10:
```

See <http://www.mapleprimes.com/questions/35951-How-To-Debugtrace-Things-In-Maple>

Also look at `kernelopts(opaquemodules=true)`

Here is a useful post by Carl Love from Maple prime forum that summarizes all of these

Here are four things that you can do to get more information. I have listed them in order by how structured the information is, with the most structured first.

1. Set

```
infolevel[all] := 5;
```

That will cause programs to print out additional information of the programmers' choosing. You can use higher or lower numbers for more or less information. Most programs don't use levels higher than 5.

2. Print the code of procedures with showstat:

```
showstat(int);  
showstat(sin);  
showstat(cos);
```

3. Trace the execution of particular procedures with trace:

```
trace(int);  
trace(sin);
```

4. Trace the execution of everything with printlevel:

```
printlevel := 10000;
```

You can use higher or lower numbers for more or less information.

7 How to display a build in function code?

Some examples

```
interface(verboseproc=3);  
print(DEtools)  
print(`ODEtools/symgen`);  
print(`symgen/methods`);  
print(`symgen/do`);
```

To stop the debugger at symgen do

```
stopat(`ODEtools/symgen`);
```

To get infolevel on symgen do

```
infolevel[`symgen`]:=5;
```

Or to see line numbers

```
interface(verboseproc=3);  
showstat(dsolve)
```

Or can use the Browse(); command

```
with(LibraryTools);  
Browse();
```

Another option I found is

```
s:=debugopts(procdump=`showstat`);
```

Then the above produces listing that can be copied as string with line wrapping ok.

8 How to build a LIST or a SET on the fly?

One way

```
L:=[]:  
for i from 1 to 3 do :  
    L:=[op(L),i];  
end do;
```

But a better way is to use seq if one knows the length

```
L:=[seq(i,i=1..3)];
```

```
L := [1, 2, 3]
```

Since list is unmutable, a more efficient method, for long lists, is to use Array, and then convert the result back to list at the end since Array can grow dynamically without preallocation each time something is inserted as follows

```
L:=Array():
for i from 1 to 3 do :
  L(i):=i;
end do;

for i from 1 to numelems(L) do :
  print(L[i]);
end do;

L := convert(L,list)
```

Which wil print

```
      L := [1]

      L := [1, 2]

      L := [1, 2, 3]

      1

      2

      3

      L := [1, 2, 3]
```

Notice that to add to an Array, () is used. But to access an entry in an array [] is used.

And finally, using Array also, it can be done without using any indexing as follows

```
L:=Array(1..0):
for i from 1 to 3 do :
  L ,= i;
end do;
```

```
L := convert(L,list)
```

For the above to work, the array must be declared using `Array(1..0)`. The new syntax `A ,= i` will append to the array, and there is no need to write `A(i) := i`

9 make function display more information

By Carol Devore on the net:

```
Use infolevel.
```

```
For example, to show what logic dsolve uses, do this:
```

```
First try
```

```
> infolevel[all]:= 5;
```

```
That will probably give more information than you want, but if not,  
then try
```

```
> printlevel:= 1000;
```

```
If you want information about a specific procedure, you can use debug.
```

```
For example,
```

```
restart;
```

```
debug('int/int');
```

```
int(p, x= 0..1);
```

```
To find out what procedures are being called without getting too much  
extra information, use excallgraph.
```

Trying on dsolve

```
infolevel[dsolve]:= 3;
```

```
dsolve({eq1},y(x));
```

```
Methods for second order ODEs:
```

```
Trying to isolate the derivative  $d^2y/dx^2$ ...
```

```
Successful isolation of  $d^2y/dx^2$ 
```

```
--- Trying classification methods ---
```

```
trying a quadrature
```

```
trying high order exact linear fully integrable
```

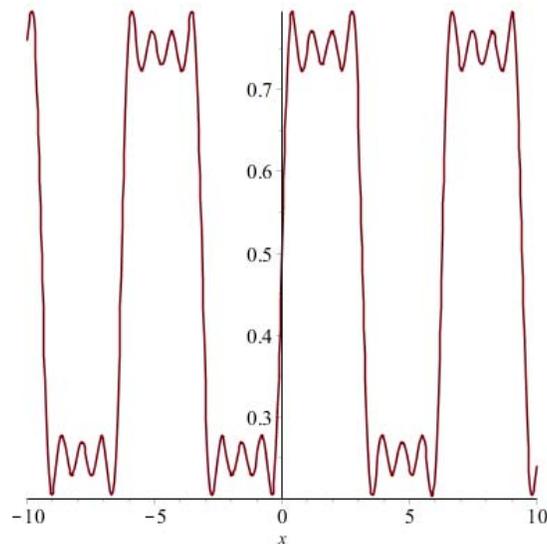
```
trying differential order: 2; linear nonhomogeneous with symmetry [0,1]
trying a double symmetry of the form [xi=0, eta=F(x)]
<- double symmetry of the form [xi=0, eta=F(x)] successful
```

10 How to plot a function?

Here, I am looking at fouries series expansion of $f(x) = 0$ between π and 0 , and $f(x) = 1$ between 0 and π .

The Fouries series expansion is worked out to be as below. This shows that the series approximate the above $f(x)$ as more terms are added

```
restart;
f:=(x)-> 1/2 + (1/Pi)*(sin(x)+sin(3*x)/3+sin(5*x)/5+sin(7*x)/7);
plot(f(x),x=-10..10);
```



11 How to run maple from command line?

From DOS, point to where your cmaple is

```
>"C:\Program Files\Maple 7\BIN.WNT\"cmaple
```

To make it execute maple commands use the < foo.txt to pipe maple commands in the file to it.

12 How to use matrices in maple?

```
A:= Matrix( [ [1, 2, 3] ,
               [3, 6, 7] ,
               [5, 6, 9] ,
               [7, 7, 7]
             ]);
whattype(A);
      Matrix
size:=LinearAlgebra:-Dimension(A);
      size := 4, 3
row:=size[1];
      row := 4
col:=size[2];
      col := 3
```

You can extract any part of the matrix like this:

```
B:=A[1..3,2..2];
```

$$\begin{bmatrix} 2 \\ 6 \\ 6 \end{bmatrix}$$

By Carl Devore <http://mathforum.org/kb/message.jspa?messageID=1570678>

Maple **list and** sequence structures are more flexible than Matrices, which are highly structured. A Maple **list** of lists (called a **listlist** in Maplese) **is** akin to a **matrix** in some other languages. Many **matrix** operations can be performed directly on the **listlist** form, but **to do**

serious linear algebra, you should convert to a Matrix. Of course, it is trivial to convert a listlist to Matrix:

```
LL:= [[1,2], [3,4]];
M:= Matrix(LL);
```

So here is another solution in line with your original wishes. This is "index free", but the table-based solution I gave earlier should be faster. (It is usually considered bad form to repeatedly append to a list or sequence

```
L:= [] []; # Create a NULL sequence
do
  line:= readline(file);
  if line::string then
    if line contains valid data then
      Z:= a list of that data;
      L:= L, Z
    fi
  else
    break
  fi
od

A:= Matrix([L]); # Note []: seq -> list.
```

To move a column into a matrix: Here, I want to copy 2nd column to the 3rd column:

A;

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 7 \\ 5 & 6 & 9 \\ 7 & 7 & 7 \end{bmatrix}$$

B:=A[1..row,2];

$$\begin{bmatrix} 2 \\ 6 \\ 6 \\ 7 \end{bmatrix}$$

A[1..row,3]:=B: A;

$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 6 & 6 \\ 5 & 6 & 6 \\ 7 & 7 & 7 \end{bmatrix}$$

13 return more than value from a procedure

Maple can return multiple values. Make sure to use the comma "," in the body of the procedure to separate each return value. Example:

```
size_matrix:=proc(x) 3*x, 4*x; end proc;
row,col :=size_matrix(5);
```

14 How does maple handle procedure arguments?

When passing a variable to maple procedure, the variable VALUE is passed to the procedure (This is different from say Fortran where the default is pass by reference). But this is the same as with Mathematica.

For example, if a variable X had value 10, then you call a procedure FOO passing it X, then inside FOO, X will be the number 10, not the argument variable X. So, this means one can not have X on the left hand side inside FOO. Like this `x:=1`

The only way to assign new value to the input and return new value, is to use a local variable, like this:

```
one:= proc(x)
    local y;
    print(x);
    y:=x+ 1;
    print(x);
    y;
end proc;

z:='z';
z:=5;
f:=one(z);
```

```
f := 6
```

15 How to define your own data types?

Use 'type/name' to define new type name.

```
`type/char` := x-> x::string and length(x)=1;
```

```
P:= proc(c::char) print(c) end proc:
```

```
P("x");
```

```
      "x"
```

```
P("xy");
```

Error, invalid input: P expects its 1st argument, c, to be of type char, but received

```
> `type/byte` := x-> x::integer and (x>= 0 and x<256);
```

```
#will define a byte (unsigned integer)
```

16 find max element and position in matrix

Code from net by Carl Devore:

```
MMax:= proc(M::{Matrix,matrix})  
  local C,r,c,mx,L,p;  
  C:= op(`if`(M::Matrix, [1,2], [2,2,2]), eval(M));  
  L:= map(op, convert(M, listlist));  
  mx:= max(L[]);  
  member(mx,L,'p');  
  r:= iquo(p, C, 'c');  
  mx, `if`(c=0, [r,C], [r+1,c])  
end;
```

Code below from C W

```
A:=matrix(12,12,rand(100));  
Ao:=array((proc(E)  
  local i; [seq(i=(rhs=lhs)(E[i]),i=1..nops(E))]end)
```

```

        (sort(op(3,eval(A)),proc(E1,E2) if rhs(E1)>rhs(E2)
        then
            true
        else
            false
        fi
        end)))));
Ao[1];

```

17 How to create a package?

First create the module:

```

restart;

nma:= module()
  option package;
  export getMaxMatrix;
  getMaxMatrix := proc (M::{matrix, Matrix})
    local C, r, c, mx, L, p;
    C := op(`if`(M::Matrix,[1, 2],[2,2,2]),eval(M));
    L := map(op,convert(M,listlist));
    mx := max(L[]); member(mx,L,'p');
    r := iquo(p,C,'c');
    mx, `if`(c = 0,[r, C],[r+1, c])
  end proc;
end module;

A:= Matrix( [ [1, 2, 3] ,
               [3, 6, 7] ,
               [5, 6, 9] ,
               [7, 7, 7]
             ]);

nma[getMaxMatrix](A);|

```

Gives 9, [3, 3]. Now save the module.

```
savelibname := "C:/MAPLE_PACKAES";
march('create', savelibname, 20);
```

now save the library to disk. `savelib(nma);`

Now we can test everything by reinitialize everything and reload the library.

```
>restart
#Add my library to LIBNAME
>libname:="C:/MAPLE_PACKAGES",libname;
> A:=matrix( [ [1,2,3],[4,6,9] ] );
>with(nma);
>nma[getMaxMatrix](A);
```

Now to print a `proc()` in the package, do

```
>interface(verboseproc=3);
> print(nma[getMaxMatrix]);
```

Now you can list what packages exist in the archive:

```
march('list',savelibname);
march('extract',savelibname,":-1.m","C:MAPLE_PACKAGES/t.m")
```

Some notes. need to clean later

```
> module1lib:=`module1\\lib`;
> system("md "||module1lib);
> march('create',module1lib,100);
> makehelp(module1,`module1/module1.mws`,module1lib):
> makehelp(`module1/export1`,`module1/export1.mws`,module1lib):
> savelibname:=module1lib: ### doesn't affect current libname
> savelib(module1); ### no error message
> restart;
> module1lib:="module1\\lib":
> libname:=module1lib,libname; ### now Maple will find module1
> with(module1);
> ?module1
```

Also there is a long thread here on Maple prime on making personal packages in Maple [How-To-Create-A-Personal-Package](#)

18 How to convert from floating point to Hex?

From: Robert Israel (israel@math.ubc.ca)
Subject: Re: Getting non-integral results in hex
Newsgroups: comp.soft-sys.math.maple
Date: 2003-06-13 00:07:37 PST

I assume you mean floating-point numbers. Note that Maple floats (as opposed to "hardware floats") are in fact stored in base 10. To convert a float to hex with n digits after the ".", you can use this:

```
> `convert/hexfloat` := proc(x::numeric, n::nonnegint)
  local A,B,ax,R;
  if nargs = 1 then return procname(x,round(Digits*log[16](10))) fi;
  if x = 0 then return cat(`0.` , `0`$n) fi;
  ax:= abs(x);
  A:= floor(ax);
  B:= round(frac(ax)*16^n);
  if B = 16^n then A:= A+1; B:= 0 fi;
  R:= cat(convert(A,hex),`.`);
  if x < 0 then R:= cat(`-`,R) fi;
  cat(R,substring(convert(16^n+B,hex),2..-1));
end;
```

And then, e.g.:

```
> convert(1234.5678, hexfloat, 4);
```

4D2.915B

19 How to find taylor series expansion of functions?

```
mtaylor(sin(x),[x],10);
```

$$x - 1/6 x^3 + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}$$

20 How to print elements of a matrix?

```
restart;
a:=Matrix([ [2,3,4],[4,5,6] ]);
nRow,nCol :=LinearAlgebra[Dimension](a);
for i from 1 to nRow do
  for j from 1 to nCol do
    printf("a(%d,%d)=%d\n",i,j,a[i,j]);
  end do;
end do;

a(1,1)=2
a(1,2)=3
a(1,3)=4
a(2,1)=4
a(2,2)=5
a(2,3)=6
```

21 How to find determinant of matrix?

```
restart;
a:=Matrix([ [2,4],[5,7] ]);
LinearAlgebra:-Determinant(a);
-6
```

22 How to generate Hilber matrix?

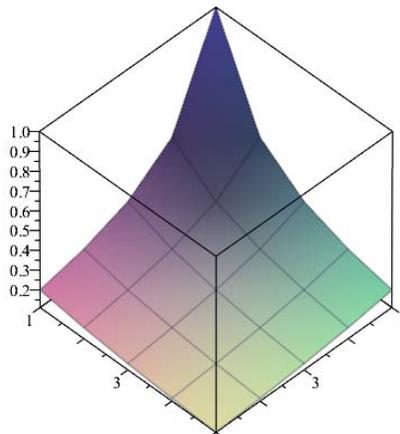
```
H := LinearAlgebra:-HilbertMatrix(5);
```

$$\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 \\ 1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\ 1/5 & 1/6 & 1/7 & 1/8 & 1/9 \end{bmatrix}$$

23 How to plot matrix data?

Matlab is much easier here. In maple, need to covert the matrix to a list of list of points first.

```
restart;  
H := LinearAlgebra:-HilbertMatrix(5):  
nRow,nCol :=LinearAlgebra[Dimension](H):  
L:=[seq([seq( [i,j,H[i,j]], i=1..nRow) ], j=1..nCol)]:  
plots:-surfdata(L);
```



24 How to catch an error from a proc()?

An error in maple raises an exception. So, use try catch to trap it as follows:

```
try
  v,pos:=MMax(4);
catch:
  printf("an error is caught\n");
end try;
```

25 How to convert 3456 to 3,456 ?

From the net, by Carl Devor:

```
`print/commas` := proc(N::integer)
  local n,s,i,b;
  n:= ListTools:-Reverse(convert(abs(N), base, 1000));
  if N<0 then n:= subsop(1= -n[1], n) fi;
  nprintf("%s", sprintf(cat("%d", "%03d" $ nops(n)-1), n[]))
end proc:

commas(456554);

      456,554
```

To convert a string to array of chars use `array(StringTools:-Explode(S))`

```
s:="Nasser M. Abbasi":
r:=array(StringTools:-Explode(s));
      r:=["N" "a" "s" .....]
```

Now can use the string as normal array

```
r[4];
      "s"
```

26 How to use units ?

```
Units[GetDimensions](base);  
amount_of_information, amount_of_substance, currency, electric_current, length,  
logarithmic_gain, luminous_intensity, mass, thermodynamic_temperature, time
```

27 How to evaluate catlan number and other sums?

Use the Sum command.

```
restart;  
expr:= (-1)^i/(2*i+1)^2;  
Sum(expr,i=0..infinity);  
evalf(%,50);  
0.91596559417721901505460351493238411077414937428167
```

Notice, if I used the sum command instead of the Sum command I get this result:

```
sum(expr,i=0..infinity);  
Catalan
```

28 write a text file that contains a package, and load it

This shows how to do a simple package and use it without building a library. Just using a plain text file.

Create this nma_pkg1.txt file:

```
nma_pkg1 := module()  
  export f1;  
  option package;  
  
  f1:= proc()  
    print("in package nma_pkg1");
```

```
end proc;  
  
end module;
```

now save it, and from maple do

```
>read("c:\\nma_pkg1.txt");
```

now execute f1() as this:

```
>nma_pkg1[f1]();  
      "in pakcage nma_pkg1"
```

now put it in a library (so that we can use with, instead of read)

```
> savelibname:=("c:/maple");  
> march('create', savelibname, 20);  
> savelib(nma_pkg1);  
>restart;  
> libname := "c:/maple",libname;  
> with(nma_pkg1);  
> f1();  
      "in pakcage nma_pkg1"
```

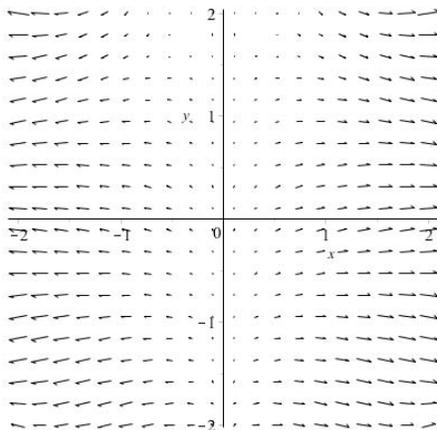
now make changes to the nma_pkg1.txt file and updated again as above.

29 How to find what packages are included in maple

?index,package

30 How to plot the gradient vector field?

```
restart;
f:=3*x^2 + y* cos(x*y);
the_grad :=linalg[grad](f,[x,y]);
plots[fieldplot](the_grad,x=-2..2,y=-2..2);
```



or

or can do it in just one command: `plots[gradplot](f,x=-2..2,y=-2..2);`

31 How to put the digits of Pi into a list?

Suppose you want the 100 digits of Pi put in a list. This is one way to do it:

```
restart;
L:=evalf(Pi,100);
S:=convert(L,string);
the_list:=[seq(parse(S[i]),i=3..length(S))];

the_list := [1, 4, 1, 5, 9, 2, 6, 5, 3, ..
```

This below now tells how many times each digits occurs.

```
>stats[transform,tally](the_list);

[Weight(0, 8), Weight(1, 8), Weight(2, 12), Weight(3, 11),
```

```
Weight(4, 10), Weight(5, 8), Weight(6, 9), Weight(7, 7),  
Weight(8, 13), Weight(9, 13)]
```

32 Digits of PI in maple and mma

Written sometime in 2005? I should really record the time when I write something.

I just run these now, Auust 2014, and now Maple 18 as very fast. So this all below is no longer valid. I will leave it here for now for reference until I update it all later

I have written a few lines of code, which counts how many times each digit occurs after the decimal points of π

Written this in maple first. Then did similar thin in mma 5.0. Both are run on the same PC. No other applications are running at the time when I run the code.

The basic idea of the algorithm is to use `evalf(Pi,digits)` in maple to find π for any number of decimal digits, and to use `N[Pi,digits]` in mma for doing the same. (Where the variable digits above is the number of digits)

Then in maple convert the above π to a string, and generate a sequence of the characters to right of decimal point, then use `stats[transform,tally]` to do the actual counting.

In mma, I use `RealDigits[]` to get a list of the digits, and then use `Count[]` to do the counting.

This is result of some of the runs to find Pi to some digits, and the total time (to find Pi and do the counting)

All times are in cpu seconds, machine is P4, 2.8 Ghz, 500 MB of RAM, single CPU, hyperthreading enabled, running XP home edition. Maple 9.03 student version, and mma 5.0 student version.

Below is the result, and below that I show the maple code and the mma code.

Because of this, before each run in mma, I exited the application and started it fresh. In maple, it does not matter for the above reason.

```
100,000 digits:  
          Find_Pi      Total  
Maple 9.0      55         84  
Mma 5.0        0.9        1.54
```

Mma is 60 times faster in finding pi and about 56 times faster overall

300,000 digits:

	Find_Pi	Total
Maple 9.0	309	781
Mma 5.0	3.7	6

Mma is 300 times faster in finding Pi, and 130 times faster overall.

3,000,000 digits

	Find_Pi	Total
Maple 9.0		
Mma 5.0	85	118

Maple **time in hours** ! Still running.

Maple code

```
> restart;
startingTime :=time();
L:=evalf(Pi,100000):
timeToFindPiInSecs:=time()-startingTime;
S:=convert(L,string):
the_list:=seq(parse(S[i]),i=3..length(S)):
stats[transform,tally](the_list);
endingTime :=time():
cpuTimeInSecs := endingTime - startingTime;
```

mma code

```
Clear[]
startingTime=TimeUsed[]
t1=N[Pi,100000];
timeToFindPiInSecs=TimeUsed[]-startingTime
{c,d}=RealDigits[t1];
theList=c[[Range[2,Length[c]]]];
f[digit_]:=Count[theList,digit];
r=Range[0,9];
Map[f,r]
cpuTimeInSecs=TimeUsed[]-startingTime
```

update 12/25/03 Changed maple code on how to do the counting : To use

```
StringTools[CharacterFrequencies](S)
```

Now the counting in maple is much faster. It is always hard to know which is the best function to use.

```
restart;
startingTime :=time();
L:=evalf(Pi,300000):
timeToFindPiInSecs:=time()-startingTime;
S:=convert(L,string):
StringTools[CharacterFrequencies](S);
endingTime :=time():
cpuTimeInSecs := endingTime - startingTime;
```

33 How to find where functions are?

```
From: Ken Lin (maplemath@tp.edu.tw)
Subject: Re: how to find which package a function belongs to?
Newsgroups: comp.soft-sys.math.maple
Date: 2003-12-04 03:49:26 PST
```

When Maple first loaded, There are only two kinds of "internal" commands which can be called directly. One is the "kernal" commands coded in C, and the other includes many "internal" prodecures programmed by the kernal commands which lies in the "Main Library", There are also many other "external" procedures which were categorized into so called "packages", plots[display](...) for example, plots[] is a package(Library), and display() is the prodecure inside plots[]. All the packages can be loaded by with() command, like

```
> with(plots);
```

Because Different Packages include user library might have the same procedure name, Maple doesn't realize the "procedure_name" you type in, it took it for a "symbol". If you really want to know which packages provided by Maple the external procedure lies in, just mark

the `procedure_name` and press F1 key, the Maple Help Browser will show you the `packages` you might be interested.

By the way, `plot3d()` is a "internal" procedure lies in the Main Library. You can confirm that by:

```
> op(0, eval(plot3d));  
  
                                procedure  
or in Maple 9  
> type( plot3d, 'std' );          #Is it internal?                true  
> type( plot3d, 'stdlib' );      #Does is lie in "Standard(Main) Library"?  
                                true
```

If you are interested the codes inside `plot3d()`...

```
> interface(verboseproc=2):      #Turn on verboseproc  
> print(plot3d);                 #eval() also works  
> interface(verboseproc=1):      #Turn off verboseproc
```

I hope this will give you some `help`. Have fun with Maple.

Ken Lin

34 on maple data types

See <http://www.maplesoft.com/applications/view.aspx?SID=1533&view=html&L=G>

35 how to extract elements from a list based on some selection?

use `select`. For example

```
>restart;  
>my_list:=[1,3.4,3+I,5];  
>select(x->evalb(Im(x)=0),my_list);  
      [1, 3.4, 5]
```

36 how to test if all elements of a matrix are integers?

```
restart;
m:=Matrix( [[1.3,2,3],[3,4,4] ]);
matrixTestQ := proc(m::Matrix)

    local r,c,i,j;

    (r,c):=LinearAlgebra[Dimensions](m);
    for i from 1 to r do
        for j from 1 to c do
            if( not evalb( whattype(m[i,j]) = integer) ) then
                return(false);
            end if;
        end do;
    end do;

    return true;
end proc;

>matrixTestQ(m);

false
```

I am sure there is a better way than the above. Need to find out.

37 how to use laplace transform?

```
restart;
f:= t->sin(omega*t) ;
L:=convert(inttrans[laplace](f(t),t,s),int);
```

$$\frac{\omega}{\omega^2 + s^2}$$

To find the inverse, do:

```
inttrans[invlaplace](L,s,t);
```

$\sin(\omega t)$

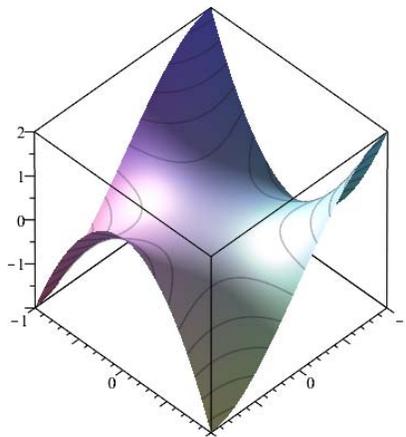
38 questions I have

Any difference between using

```
`diffalg/Rosenfeld_Groebner`(args)  
or  
diffalg[Rosenfeld_Groebner](args)
```

39 3D plotting

```
restart;  
f:= (x,y)->x^3-3*x*y^2;  
plot3d(f,-1..1,-1..1,numpoints=2500,style=patchcontour);
```



40 How to raise each element in a list to a power?

Use map

```
map(`^`, {1,2,3}, 3);  
{1, 8, 27}
```

41 How to generate a sequence with any increment?

```
incr:=.25; start:=0; last:=3;  
seq(start+i*incr, i=1..(last/incr));
```

42 What shortcuts are there for matrix manipulation?

read ?MVshortcut, ?MVassignment, and ?Mvextract and Transpose(R) can be shortened to R^%T

43 How to solve a set of equations for the derivative?

Written feb 20, 2004

This is problem 7.4 chapter 4, in the Mary Boas book. Given

$$\begin{aligned}xs^2 + yt^2 &= 1 \\x^2s + y^2t &= xy - 4\end{aligned}$$

Find $\frac{dx}{dt}$, $\frac{dx}{ds}$, $\frac{dy}{dt}$, $\frac{dy}{ds}$ at $x = 1, y = -3, s = 2, t = -1$

This is how I did it in maple:

```
restart;  
alias(x=x(s,t));  
alias(y=y(s,t));
```

```

alias(Xt= diff(x(s,t), t));
alias(Xs= diff(x(s,t), s));
alias(Yt= diff(y(s,t), t));
alias(Ys= diff(y(s,t), s));

eq1:= x*s^2+y*t^2=1;
eq2:= x^2*s+y^2*t=x*y-4;

r1:=diff(eq1,t);
r2:=diff(eq1,s);
r3:=diff(eq2,t);
r4:=diff(eq2,s);

sol:=solve({r1,r2,r3,r4},{Xt,Xs,Yt,Ys});

```

$$\frac{\partial}{\partial s}x(s,t) = -\frac{x(s,t)(x(s,t)t^2 - 4y(s,t)st + 2x(s,t)s)}{2x(s,t)st^2 - 2y(s,t)ts^2 + x(s,t)s^2 - y(s,t)t^2}$$

$$\frac{\partial}{\partial t}x(s,t) = -\frac{y(s,t)t(-3y(s,t)t + 2x(s,t))}{2x(s,t)st^2 - 2y(s,t)ts^2 + x(s,t)s^2 - y(s,t)t^2}$$

$$\frac{\partial}{\partial s}y(s,t) = -\frac{x(s,t)(3x(s,t)s - 2y(s,t))s}{2x(s,t)st^2 - 2y(s,t)ts^2 + x(s,t)s^2 - y(s,t)t^2}$$

$$\frac{\partial}{\partial t}y(s,t) = -\frac{y(s,t)(4x(s,t)st - y(s,t)s^2 - 2y(s,t)t)}{2x(s,t)st^2 - 2y(s,t)ts^2 + x(s,t)s^2 - y(s,t)t^2}$$

```

points:= {x=1,y=-3,s=2,t=-1};
subs(points,sol);

```

44 How to solve a set of equations for differentials?

This is problem 7.15 chapter 4 in Boas:

Given $x^2u - y^2v = 1$ and $x + y = uv$ Find $\frac{dx}{du}, v$ and $\frac{dy}{du}, y$

This is the maple code to solve this:

```

restart;
eq1:=x^2*u-y^2*v=1;
eq2:=x+y=u*v;

```

```

r1:=D(eq1);
r2:=D(eq2);
r1_:=subs(D(v)=0,r1);
r2_:=subs(D(v)=0,r2);
sol:=solve({r1_,r2_},{D(x),D(u)});
print("dx/du,v=");
rhs(sol[1])/rhs(sol[2]);

r1_:=subs(D(y)=0,r1);
r2_:=subs(D(y)=0,r2);
sol:=solve({r1_,r2_},{D(x),D(u)});
print("dx/du,y=");

rhs(sol[1])/rhs(sol[2]);

```

$$\begin{aligned}
eq1 &:= u x^2 - v y^2 = 1 \\
eq2 &:= x + y = u v \\
r1 &:= 2 D(x) x u + x^2 D(u) - 2 D(y) y v - y^2 D(v) = 0 \\
r2 &:= D(x) + D(y) = D(u) v + u D(v) \\
r1_ &:= 2 D(x) x u + x^2 D(u) - 2 D(y) y v = 0 \\
r2_ &:= D(x) + D(y) = D(u) v \\
sol &:= \left\{ D(u) = \frac{2 D(y) (u x + v y)}{x (2 u v + x)}, D(x) = \frac{D(y) (2 v^2 y - x^2)}{x (2 u v + x)} \right\} \\
&\quad \text{"dx/du,v="} \\
&\quad \frac{2 (u x + v y)}{2 v^2 y - x^2} \\
r1_ &:= 2 D(x) x u + x^2 D(u) - y^2 D(v) = 0 \\
r2_ &:= D(x) = D(u) v + u D(v) \\
sol &:= \left\{ D(u) = -\frac{D(y) (2 u^2 x - y^2)}{x (2 u v + x)}, D(x) = \frac{D(y) (u x^2 + v y^2)}{x (2 u v + x)} \right\} \\
&\quad \text{"dx/du,y="} \\
&\quad -\frac{2 u^2 x - y^2}{u x^2 + v y^2}
\end{aligned}$$

45 How to plot binary tree

by <http://www.math.fsu.edu/~bellenot>

```

restart;
t2 := proc(i, x, y)
    if i < 2 then [[x, y], [x, y - 1]], [[x, y], [x + 2^i, y - 1]]
    else [[x, y], [x, y - 1]], [[x, y], [x + 2^i, y - 1]],

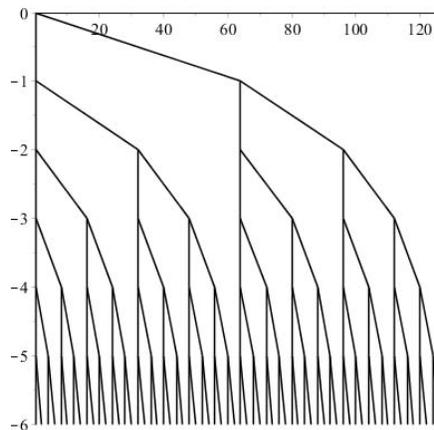
```

```

    t2(i - 1, x, y - 1), t2(i - 1, x + 2^i, y - 1)
  end if
end proc;

PLOT(CURVES(t2(6,0,0)));

```



46 Problem 12.4 chapter 4, Boas book

```

restart;
z:= Int( sin(t)/t, t=sin(x)..cos(x));
diff(z,x);

```

$$\frac{\sin(x) \sin(\cos(x))}{\cos(x)} - \frac{\cos(x) \sin(\sin(x))}{\sin(x)}$$

47 example of doing convergence test in maple

```

restart;
c:='c': C:='C': n:='n': P:='P':
C := n -> ((n+2)/(3*n+1))^n;
### WARNING: calls to `C` for generating C code should be replaced by codegen[C]
`The general term is ` , c[n]= C(n); ` `;
`The n-th root is:`;
### WARNING: calls to `C` for generating C code should be replaced by codegen[C]

```

```
P := C(n)^(1/n):
abs(c[n])^(1/n) = P;
P := simplify(P, assume=positive):
abs(c[n])^(1/n) = P;
```

The general term is, $c_n = \left(\frac{n+2}{3n+1} \right)^n$

The n -th root is:

$$|c_n|^{\frac{1}{n}} = \left(\left(\frac{n+2}{3n+1} \right)^n \right)^{\frac{1}{n}}$$

$$|c_n|^{\frac{1}{n}} = \frac{n+2}{3n+1}$$

48 Problem ch 14, 3.18, Boas book. contour integration

```
restart;
f:= 1/( (1-2*z)*(5*z-4) );
residue(f,z=4/5);
```

$$\frac{-1}{3}$$

49 How to find multiple roots to an equation such as $\sin(x) = 0$

```
_EnvAllSolutions:=true;
solve(sin(x)=0);
```

Pi _Z1~

50 Dr Basti Associated Legendre

Subject: Associated Legendre
Author: Mehran Basti <Basti@worldnet.att.net>
Organization: AT&T Worldnet
Date: Mon, 25 Nov 2002 02:48:15 GMT

Dear newsgroup:

I had mentioned that my methods will `solve` classical equations without the `use` of infinite `series`.

The following `is` a Maple code of my old files. Those days I had Maple2 but the general idea `is` the same `in` the process `and` you see that we can also `solve` the integrals involved.

It does `not` make sense how are the theory behind it but eventually it will come into 1

Just `read` the procedures `and` you can see the solution of associated legendre AL at the

```
> s1:=-diff(p(t),t)+p(t)^2;
>
> s2:=exp(2*int(p(t),t))*T(t);
> s3:=s1+s2;
> s4:=diff(T(t),t)/T(t);
> s5:=-1/2*(diff(s4,t))+(1/4)*s4^2;
> s6:=s5+s2;
> p(t):=-1/t+(1)/(2-t);
> s1:=simplify(s1);
> s1:=collect(%,t);
> s2:=simplify(s2);
> s1+s2=(2*t^2-4*t+m^2-1)/(t*(-2+t))^2;
> solve(%,T(t));
> T(t):=simplify(%);
> s2:=simplify(s2);
> s2+s1;
> s3:=simplify(%);
```

```

>
> s6:=simplify(s6);
> t*(-2+t);
> simplify(%);
> z:=(r3*t^3+r2*t^2+r1*t+r0)/(%);
>
> simplify(diff(z,t)+z^2-s6);
> s7:=collect(numer(%),t);
>
> coeff(%,t,0);
> solve(%,r0);
> r0:=op(1,{%});
> coeff(s7,t,1);
> solve(%,r1);
> r1:=simplify(%);
> coeff(s7,t,2);
> solve(%,r2);
> r2:=simplify(%);
> coeff(s7,t,3);
> solve(%,r3);
> r3:=simplify(%);
> simplify(s7);
> s3:=simplify(s3);
> s4:=simplify(s4);
> s6:=simplify(s6);
> T(t):=simplify(T(t));
> z:=simplify(z);
> 1/2*s4+2*p(t)+z;
> s8:=simplify(%);
> exp(int(%,t));
> expand(%);
> g:=(%);
> simplify(g,power);
> g:=%;
> Int(%,t);
> Integralg:=(%);
> int(g1(t),t);
> x1:=-p(t)+g1(t)/(%);
> diff(x1,t)+x1^2-s3;

```

```

> simplify(%);
> s10:=numer(%);
> solve(%,int(g1(t),t));
> Ing:=(%);
> simplify(subs(g1(t)=g,%));
>
> Ing:=(%);
> expand(%);
> Ing:=simplify(%);
> simplify(diff(%,t)-g);
> expand(%);
> simplify(%);
> x:=-p(t)+g/Ing;
> simplify(diff(x,t)+x^2-s3);
> int(x,t);
> exp(%);
> expand(%);
> s11:=simplify(%);
> ALT:=t*(2-t)*diff(u(t),t$2)+2*(1-t)*diff(u(t),t)+(2-m^2/(1-(1-t)^2))*u(t);
> -2*(1-t)/(2*t*(2-t));
> int(%,t);
> exp(%);
> s12:=simplify(%,power);
>
> u1:=s12*s11;
> u1:=simplify(%,power);
> simplify(subs(u(t)=u1,ALT));
> AL:=(1-nu^2)*diff(u(nu),nu$2)-2*nu*diff(u(nu),nu)+(2-m^2/(1-nu^2))*u(nu);
>
> u2:=subs(t=1-nu,u1);
> simplify(subs(u(nu)=u2,AL));
>

```

The advantage of these methods are that there are ample rooms for advances.

Today my skills for solving classical equations such as Riccati is much advanced.

Highly complicated and more general Riccati equations in its billions now possible.

Sincerely

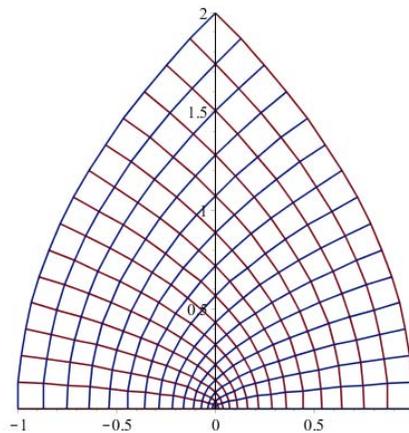
Dr.M.Basti

51 Understanding conformal mapping in maple

To plot mapping of complex function in maple, use `[plots]conformal`. The trick is to how to specify the quadrant in the x-y plane. This example shows how.

Suppose we want to map the first quadrant. Then we specify the `DIAGONAL` points in the range, from the lower left corner to the upper right corner, which then should be `0..1+I`. Because 0 is the lower left corner, and $(1, i)$ is the upper right corner. Example:

```
restart;
assume(y,real);
assume(x,real);
#f:= z->I+z*exp(I*Pi/4);
f:= z->z^2;
w:=f(x+I*y);
u:=Re(w);
v:=Im(w);
plots:-conformal(f(z),z=0..1+I,grid=[16,16],numxy=[16,16],scaling=constrained);
```



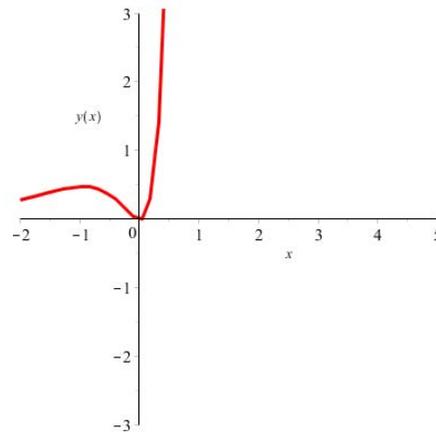
This below uses the first TWO quadrants, i.e. the upper half of the x-y plane

```
restart;
assume(y,real);
```

```

assume(x,real);
#f:= z->I+z*exp(I*Pi/4);
f:= z->z^2;
w:=f(x+I*y);
u:=Re(w);
v:=Im(w);
plots:-conformal(f(z),z=-1-I..1+I,grid=[16,16],numxy=[16,16],scaling=constrained);

```

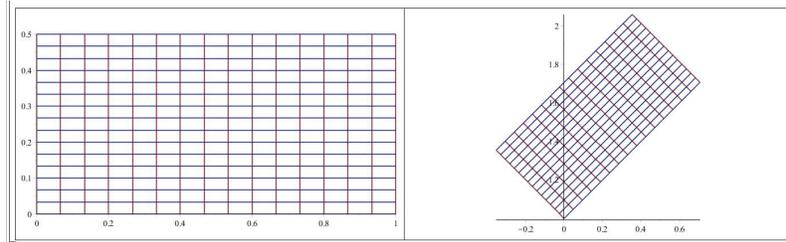


This below puts the plots next to each others so to see them

```

restart;
assume(y,real);
assume(x,real);
f:= z->I+z*exp(I*Pi/4);
#f:= z->z^2;
w:=f(x+I*y);
u:=Re(w);
v:=Im(w);
A := array(1..2):
A[1]:=plots:-conformal(z,z=0..1+I/2,grid=[16,16],numxy=[16,16],scaling=constrained):
A[2]:=plots:-conformal(f(z),z=0..1+I/2,grid=[16,16],numxy=[16,16],scaling=constrained);
plots:-display(A);

```



52 Hide tilde character when using assumption

`interface(showassumed=0)` removes all tildas and `interface(showassumed=1)` adds the tildas.

53 Fourier series in maple

I wrote this to generate FS in Maple for some HW I was doing. I think this was for Math 121A at UC Berkeley in 2003

```
restart;
f:=x->piecewise(-Pi<x and x<Pi/2,-1,
                Pi/2<x and x<1,0,1);

assume(n, integer);

nmaFourier2:=proc(f,freq,from_,to_,maxN)
  local n::integer,denomC,denomS,a,b;
  denomC:=( to_ - from_ ) / 2;
  denomS:=( to_ - from_ ) / 2;

  a:=proc(n)
    int(f(x)*cos(n*freq*x),x=from_..to_) /denomC;
  end proc;

  b:=proc(n)
    int(f(x)*sin(n*freq*x),x=from_..to_) / denomS;
  end proc;

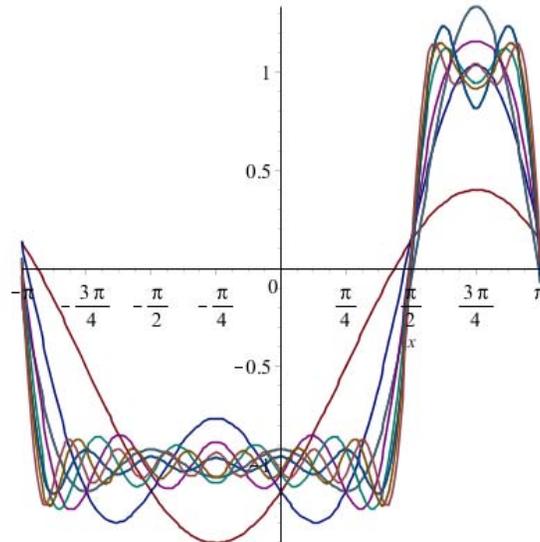
  evalf(denomC);
```

```

1/2*a(0) + sum( a(n) * cos(n*freq*x) ,n=1..maxN)
           + sum( b(n) * sin(n*freq*x) ,n=1..maxN)
end proc;

r:= [seq(nmaFourier2(f,1,-Pi,Pi,nIter),nIter=1..10)];
plot(r,x=-Pi..Pi);

```



To animate do

```

g:=n->plot(nmaFourier2(f,1,-Pi,Pi,n),x=-2*Pi..2*Pi);
plots:-animate(g,[n],n=1..40);

```

Another version

```

restart;
f:=x->piecewise(-Pi<x and x<Pi/2,-1,
                 Pi/2<x and x<1,0,1);

assume(n,integer);
nmaFourier2:=proc(f,freq,from_,to_,maxN::integer)
  local n::integer,denomC,denomS,a,b;

  denomC:=( to_ - from_ ) / 2;
  denomS:=( to_ - from_ ) / 2;

```

```

a:=proc(n)
  int(f(x)*cos(n*freq*x),x=from_.to_) /denomC;
end proc;

b:=proc(n)
  int(f(x)*sin(n*freq*x),x=from_.to_) / denomS;
end proc;

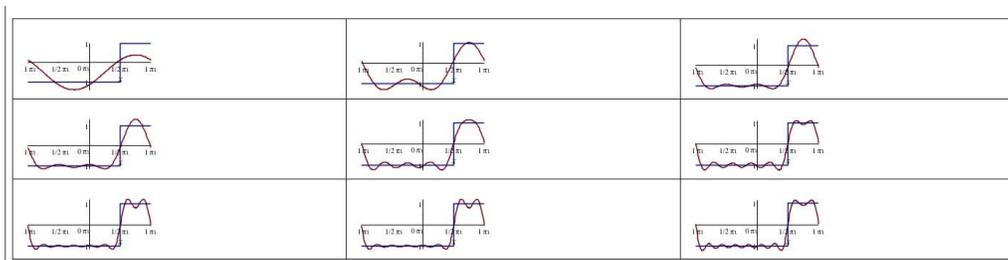
1/2*a(0) + sum( a(n) * cos(n*freq*x) ,n=1..maxN)
          + sum( b(n) * sin(n*freq*x) ,n=1..maxN)
end proc;

plots[setoptions](title=` ` , axesfont=[SYMBOL,8] ,font=[COURIER,1],
  xtickmarks=[seq(evalf(k*Pi/2)=sprintf("%a %s" , k/2 , "pi" ),k= -3..3)],
  ytickmarks=[-1.0="-1" ,-0.5="" ,0.0="0" ,0.5="" ,1.0="1"]);

B:=array(1..3,1..3);
k:=0;
for i from 1 to 3 do
  for j from 1 to 3 do
    k:=k+1;
    B[i,j]:=plot({f(x),nmaFourier2(f,1,-Pi,Pi,k)},x=-Pi..Pi,size=[200,100]);
  end do;
end do;

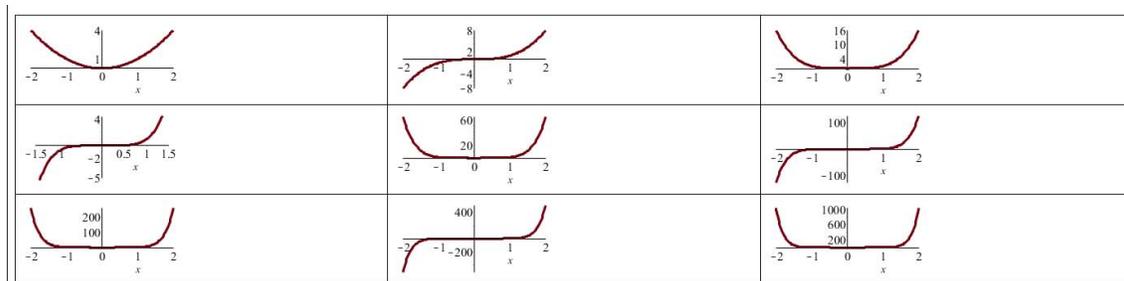
plots:-display( B);

```



54 How to plot graphs next to each others in a grid like fashion

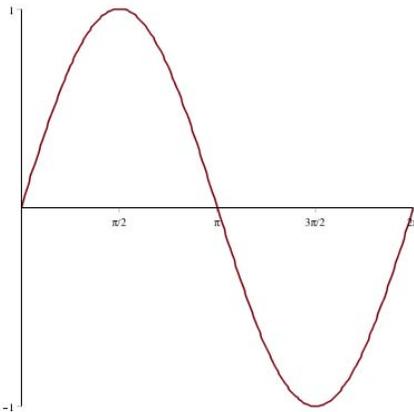
```
restart;
v:=1;
B:=Matrix(3,3);
for i from 1 to 3 do
  for j from 1 to 3 do
    v:=v+1;
    B[i,j]:= plot(x^v,x=-2..2,thickness=3,size=[200,100] );
  end do;
end do;
plots:-display(B);
```



55 How to generate Pi on X-axis

From book Maple animation by John Putz

```
plot( sin(x), x=0..2*Pi, xtickmarks=evalf([Pi/2="p/2", Pi="p",
3*Pi/2="3p/2", 2*Pi="2p"]), ytickmarks=[-1,1], axesfont=[SYMBOL,16], labels=["", ""] );
```



56 How to make output from FunctionAdvisor look better?

From Preben Alsholm

```
res:=FunctionAdvisor(sin):
res2:=op(2,eval(res)):
map(print,res2);
```

or answer by Thomas Richard

```
> FunctionAdvisor( display, sin );
```

57 How to do partial fractions?

Use `convert(expr,parfrac)` or `convert(f,fullparfrac)`

58 How to generate sequence sum symbolically

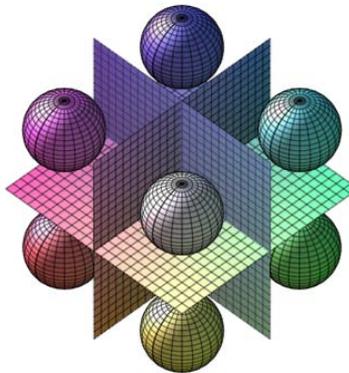
```
n := 7;  
f:=sum('a[k]*b[k]', 'k'=1..n);
```

$$a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 + a_5b_5 + a_6b_6 + a_7b_7$$

59 Nice plot from Maple

from Serge from the net:

```
restart;  
with(geom3d):  
plane(OYZ,x=0,[x,y,z]):  
plane(OXZ,y=0,[x,y,z]):  
plane(OXY,z=0,[x,y,z]):  
c:=1/2:r:=1/4:  
L:=combinat[permute]([-c$3,c$3],3):  
S:=seq(sphere(s||i,[point(A||i,op(op(i,L))),r]),i=1..8):  
draw([OYZ,OXZ,OXY,S]);
```



60 How to check if 2 expressions are the same?

Use `evalb()`. For example `evalb(I*sinh(x)=sin(I*x))`; gives true

The above does not always work. Only sure way is to do this

```
> m1 := exp(I*n*x);
m2 := (cos(n*x)+I*sin(n*x));
simplify(m1-m2);
simplify(m1-convert(m2,exp));
```

61 converting series to factorials

Function by Robert Israel from the net:

```
restart;

thefacts:= [seq(i!,i=2..20)]:
getfacts:= proc(x::{algebraic,series})
  local i;
  if type(x, {`+`,`*`,series}) then
    map(getfacts,x)
  elif type(x, fraction) then
    getfacts(numer(x))/getfacts(denom(x))
  elif type(x, ``) then
    getfacts(op(1,x))^op(2,x)
  elif type(x,negint) then
    -getfacts(-x)
  elif type(x,posint) then
    for i from 1 to 19 while irem(x, thefacts[i]) = 0 do od:
    if i = 1 then x
    elif thefacts[i-1] = x then ``(i)!
    else ``(i-1)!*getfacts(x/thefacts[i])
    fi
  else x
  fi
end;

getfacts(series(sin(x),x));
```

$$\text{series}\left(x - \frac{x^3}{((3))!} + \frac{x^5}{((5))!} + O(x^7), x, 7\right)$$

62 How to find what new additions made to Maple?

?updates,maple10

63 Maple can't solve laplace equation and numerically

Maple 2020.

```
restart;
PDE := diff(u(x,y), y$2 ) + diff(u(x,y), x$2) = 0;
BC:= u(x,0)=0, u(x,100)=100, u(0,y)=0, u(10,y)=0;
sol:=pdsolve(PDE,[BC] ,numeric);
```

Error, (in pdsolve/numeric) unable to handle elliptic PDEs

Compare to

```
restart;
PDE := diff(u(x,y), y$2 ) + diff(u(x,y), x$2) = 0;
BC:= u(x,0)=0, u(x,100)=100, u(0,y)=0, u(10,y)=0;
sol:=pdsolve([PDE,BC]);
```

$$u(x, y) = \sum_{n=1}^{\infty} -200 \frac{((-1)^n - 1) e^{10\pi n} \sin(1/10 n\pi x) (e^{1/10 n\pi y} - e^{-1/10 n\pi y})}{\pi n (e^{20\pi n} - 1)}$$

64 Some Maple Matrix operations

Create a new matrix, by appending some rows of one matrix to rows from another matrix:

```
restart; with(LinearAlgebra):  
A:=< <1|2|3> , <4|5|6> >;
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

```
B:=< <7|8|10> , <11|12|13> , <14|15|16> >;
```

$$\begin{bmatrix} 7 & 8 & 10 \\ 11 & 12 & 13 \\ 14 & 15 & 16 \end{bmatrix}$$

Now append first row of A to last 2 rows of B

```
C:=< A[1,1..-1] , B[2..-1,1..-1] >;
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 11 & 12 & 13 \\ 14 & 15 & 16 \end{bmatrix}$$

```
# Now append first column of A to first 2 rows of B
```

```
A[1..-1,1];
```

```
B[1..2,1..-1];
```

```
C:=< A[1..-1,1] | B[1..2,1..-1] >;
```

$$\begin{bmatrix} 1 & 7 & 8 & 10 \\ 4 & 11 & 12 & 13 \end{bmatrix}$$

```
#Now remove the middle row of B
```

```
B;
```

```
B:=<B[1,1..-1] , B[-1,1..-1] >;
```

$$\begin{bmatrix} 7 & 8 & 10 \\ 14 & 15 & 16 \end{bmatrix}$$

```
#now set the diagonal elements of B to be 0  
B:=RandomMatrix(3);  
for i from 1 to 3 do  
  B[i,i]:=0;  
end do:  
B;
```

$$B := \begin{bmatrix} 0 & 99 & 92 \\ 8 & 0 & -31 \\ 69 & 44 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 99 & 92 \\ 8 & 0 & -31 \\ 69 & 44 & 0 \end{bmatrix}$$

To find inverse.

```
restart;  
with(LinearAlgebra):  
A:=Matrix( [ [2,0],[4,2] ] );  
MatrixInverse(A);
```

$$\begin{bmatrix} 1/2 & 0 \\ -1 & 1/2 \end{bmatrix}$$

To check that for any matrix A, then $A \cdot \text{transpose}(A)$ is always a matrix which is symmetrical

```
A:=RandomMatrix(2,3);  
A.Transpose(A);
```

$$A := \begin{bmatrix} 99 & 44 & -31 \\ 29 & 92 & 67 \end{bmatrix}$$

$$\begin{bmatrix} 99 & 44 & -31 \\ 29 & 92 & 67 \end{bmatrix}$$

how to create a random lower triangular matrix?

```
restart;
with(LinearAlgebra);
A:=RandomMatrix(4,4,outputoptions=[shape=triangular[lower]]);
```

$$\begin{bmatrix} 67 & 0 & 0 & 0 \\ -31 & 92 & 0 & 0 \\ 44 & 29 & 99 & 0 \\ 69 & 8 & 27 & -4 \end{bmatrix}$$

65 How set diagonal elements to some value, say 1?

```
restart;
with(LinearAlgebra);
A:=RandomMatrix(5);
LinearAlgebra:-Map[(i,j)->evalb(i=j)](x->1,A);
```

$$A := \begin{bmatrix} 1 & -98 & -76 & -4 & 29 \\ -38 & 1 & -72 & 27 & 44 \\ -18 & 57 & 1 & 8 & 92 \\ 87 & 27 & -32 & 1 & -31 \\ 33 & -93 & -74 & 99 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -98 & -76 & -4 & 29 \\ -38 & 1 & -72 & 27 & 44 \\ -18 & 57 & 1 & 8 & 92 \\ 87 & 27 & -32 & 1 & -31 \\ 33 & -93 & -74 & 99 & 1 \end{bmatrix}$$

66 How to multiply roots of a polynomial?

```
eq:=3*x^3+2*x^2+x+5=0;
s:=[evalf(solve(eq,x))];
mul(s[i],i=1..nops(s));
```

Gives

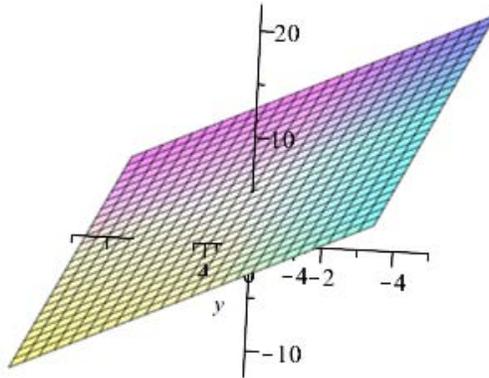
$$eq := 3x^3 + 2x^2 + x + 5 = 0$$

$$s := [-1.342780428, 0.3380568807 - 1.061566392 I, 0.3380568807 + 1.061566392 I]$$

$$-1.666666666 - 1.157693742 \cdot 10^{-10} I$$

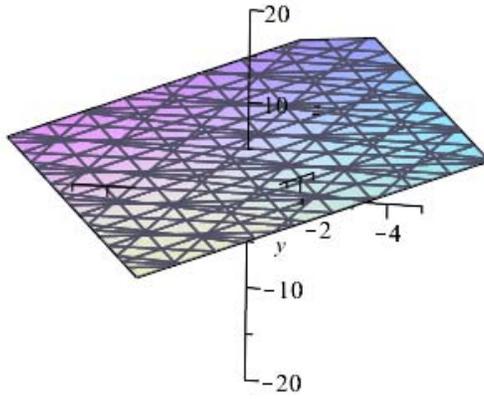
67 How to plot a surface in 3D?

```
restart;
eq:=3*x+4*y+2*z=10;
plot3d(solve(eq,z),x=-5..5,y=-5..5,axes=normal);
```



One can also use `implicitplot3d`

```
restart;  
with(plots):  
implicitplot3d(3*x+4*y+2*z=10, x=-5..5, y=-5..5, z=-20..20, axes=normal);
```



68 How to convert trigs to sinc function in an expression

From <http://www.mapleprimes.com/questions/40470-Trigonometric-Function-To-Sinc-Function>

Maple doesn't have a sinc function. If you mean the function $\text{sinc}(x) = \sin(x)/x$, you could say something like

```
> eval(expr, {sin = (x -> x*sinc(x)),
              cos = (x -> (x+Pi/2)*sinc(x+Pi/2)),
              tan = (x -> x*sinc(x)/(x+Pi/2)/sinc(x+Pi/2))});
```

69 How to find NullSpace and ColumnSpace of a matrix?

```
restart;  
with(LinearAlgebra):  
A:=Matrix([[1,0,1,0,1],[0,1,0,1,0]]);  
NullSpace(A);  
ColumnSpace(A);
```

$$A := \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$
$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$
$$\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]$$

70 How to fix the interface to using Maple notation for input?

Go to tools->options, and Display, and select Maple notation for input display.

71 How to find all solutions using allvalues ?

```
solve(x^2-sin(x),x);  
RootOf(-sin(_Z)+_Z^2)  
  
allvalues(%);  
RootOf(-sin(_Z)+_Z^2, 0.), RootOf(-sin(_Z)+_Z^2, .8767262154)  
  
evalf(%);  
0., .8767262154
```

72 Adding only to diagonal of a matrix

Use Map with filter

```
A:=< 1,2,3;4,5,6;7,8,9>;  
LinearAlgebra:-Map[(i,j)->evalb(i=j)](x->x+1,A);
```

73 How to search help for updates on some package

Go to <http://www.maplesoft.com/support/help/search.aspx>
and type say updates,Maple17,DE in the small box there.

74 How to work with groups in worksheet

From <http://www.mapleprimes.com/questions/201092-How-To-Insert-New-Paragraph-On-Its-Own> by Carl Love:

I use these special keystrokes constantly in my Maple worksheet typing:

Ctrl-J: Insert execution group below cursor.

Ctrl-K: Insert execution group above cursor.

Ctrl-T: Switch from executable code mode to text mode (for entering extended formatted comments).

Ctrl-M: Switch from text mode to executable code mode.

Shift-Enter (or Shift-Return): Begin a new line in the same execution group.

Func-3: Split execution group into two (at cursor).

Func-4: Join cursor execution group with execution group below.

75 How to read code into worksheet?

Use the read command, as in `read "mycode.mpl"` where `mycode.mpl` is plain text file that contains maple code

76 Code editors for Maple

1. <http://www.mapleprimes.com/forum/codeeditormaple>
2. <http://www.mapleprimes.com/blog/joe-riel/emacs-mode-for-maple>
3. <http://www.mapleprimes.com/blog/jacquesc/vim-mode-for-maple>
4. <http://www.maplesoft.com/products/toolboxes/IDE/index.aspx>

77 How to find if package is module or table?

New packages are module, which allows using `packageName:-function()` since it is easier. Old packages use tables which needs `packageName[function]()` which is not common.

To find if package is based on module or not, use the command

```
type(combstruct, `module`);
```

This will return true or false. To know if name is package use the command

```
type(combstruct, 'package');
```

78 How to replace a string?

```
file_name :=StringTools:-SubstituteAll(file_name,":","-");
```

79 How to use geometry and plottools ?

```
restart;  
c:= i->([i/(1+i),0],1/(1+i));  
d:= i->([1,1/i],1/i):  
geometry:-circle(c1,[geometry:-point(o,2/3,0),1/3],[x,y]):  
geometry:-circle(c2,[geometry:-point(o,1,1),1],[x,y]):  
geometry:-intersection(o,c1,c2,[u,v]):  
plots:-display(plottools:-circle(c(2)),plottools:-circle(d(1)),geometry:-draw(o));
```

To know more about the intersection, use this:

```
geometry:-detail(o);
```

80 How to simplify log expressions ?

Use symbolic option

```
restart;  
simplify(ln(3^x/2^y) =ln(n),symbolic);
```

81 How to simplify hyperbolic expression ?

How to convert

$$\frac{3 + 2 \sinh(x)^2}{\sinh(x)^2 \tanh(x)}$$

to

$$3 \coth^3(x) - \coth(x)$$

```
restart;
e := (3+2*sinh(x)^2)/(sinh(x)^2*tanh(x));
expand(student[changevar](sinh(x)^2=tanh(x)^2/(1-tanh(x)^2),e));
```

82 How to create text file and append string to it?

```
restart;
try
  fd :=-1;
  fd := fopen("C:\\output3.txt",APPEND,TEXT);
catch:
  print(`Unable to open file, error is`);
  print(StringTools:-FormatMessage(lastexception[2]));
end try:

if not(evalb(fd=-1)) then #file open ok
  str:="hello world";
  try
    fprintf(fd,"%s\n",str);
  catch:
    print(`failed to append to file, error is`);
    print(StringTools:-FormatMessage(lastexception[2]));
  finally:
    close(fd);
  end try;
fi:
```

83 How to search packages and libraries?

To find in which library a command is do

```
with(LibraryTools);
FindLibrary('int',all); #find which library command int is in

"C:\Program Files\Maple 18\lib\update.mla",
"C:\Program Files\Maple 18\lib\DEsAndMathematicalFunctions18.mla",
"C:\Program Files\Maple 18\lib\maple.mla"
```

To get content of library do

```
restart;
with(LibraryTools):
LibLocation:=cat(kernelopts(mapledir),"/lib/maple.mla");
c:=ShowContents(LibLocation);
```

Then can use this to print the name of each symbol/command, and then use whattype command to find its type

```
seq(c[i,1],i=1..20);
```

To get list of Maple kernel builtin commands and symbols, use this. Written by Acer from Maple prime site:

```
restart:
interface(warnlevel=0):
started := false:
T := 'T':
for i from 1 to 1000 do
  f := eval(parse(cat("proc() option builtin=",i,"; end proc")));
  p := (s->StringTools:-Take(s,StringTools:-Search(";",s)-1))(convert(eval(f),string))
  if not type(parse(p),posint) then
    T[i] := p;
    started := true;
  else
    if started then i:=1000; next; end if;
  end if;
end do:
```

```
i;
[ entries(T,nolist) ];
nops(%);
```

The above gives on Maple 18.02 the following

```
["crinterp", "equation", "{", "even", "debugopts",
  "embedded_imaginary", "define_external", "embedded_real",
  "coeff", "cx_zero", "coeffs", "embedded_axis", "conjugate",
  "constant", "convert", "cx_infinity", "dlclose", "identical",
  "divide", "hfloat", "done", "function", "$", "fraction",
  "denom", "float", "degree", "finite", "disassemble",
  "extended_rational", "diff", "extended_numeric", "frem",
  "union", "frontend", "upperbound", "exports", "writeto",
  "factorial", "xor", "evalgf1", "type", "expand", "typematch",
  "entries", "unames", "evalb", "unbind",
  "evalf/hypergeom/kernel", "atomic", "hfarray", "anything",
  "hastype", "complex", "has", "boolean", "goto", ":-",
  "gmp_isprime", "!", "genpoly", "anyfunc", "gc", "algebraic",
  "SFloatMantissa", "ssystem", "Scale10", "stop", "Scale2",
  "sort", "SearchText", "[", "~", "subset", "~Array",
  "subsindets", "~Matrix", "streamcall", "~Vector", "subs",
  "Unordered", "table", "ToInert", "system",
  "_hackwareToPointer", "substring", "UpdateSource", "subsop",
  "_maplet", "trunc", "_jvm", "kernel/transpose", "_treeMatch",
  "tcoeff", "_savelib", "taylor", "abs", "rtable_num_dims",
  "addressof", "rtable_num_elems", "_unify", "rtable_options",
  "_xml", "rtable_redim", "and", "rtable_scale", "andmap",
  "rtable_scanblock", "alias", "rtable_size", "anames",
  "rtable_sort_indices", "assign", "savelib", "assemble",
  "rtable_zip", "array", "select", "appendto", "searchtext",
  "cat", "series", "callback", "selectremove", "bind", "sign",
  "attributes", "setattribute", "ormap", "ArrayOptions", "order",
  "Array", "parse", "**", "overload", "*", ":", "numer",
  "CopySign", "numelems", "^", "or", "||", "op", "nops",
  "seq", "normal", "time", "not", "piecewise", "numboccur",
  "?[", "userinfo", "modp2", "inner", "mods", "timelimit",
  "mvMultiply", "traperror", "negate", "rtable_normalize_index",
  "call_external", "rtable_is_zero", "assigned", "rtable_indfns",
  "evalf", "rtable_histogram", "eval", "evaln", "rtable_eval",
  "truefalse", "evalhf", "rtable_convolution", "tabular", "mul",
  "rtableInfo", "zppoly", "if", "rtable", "uneval", "remove",
```

```

"sfloat", "rhs", "specfunc", "readlib", "string", "reduce_opr",
"symbol", "ASSERT", "'?()'", "realcons", "TRACE", "'quit'",
"relation", "_local", "pointto", "sequential", "add", "print",
"set", "SFloatExponent", "iolib", "radical", "SDMPolynom",
"'int/series'", "protected", "Record", "irem", "procedure",
"Re", "iquo", "poszero", "isqrt", "real_infinity", "RETURN",
"is_gmp", "ratpoly", "'+'", "lcoeff", "rational", "OrderedNE",
"kernelopts", "range", "Object", "NumericEventHandler",
"icontent", "numeric", "NumericStatus", "igcd", "odd",
"NumericClass", "ilog10", "nonpositive", "NumericEvent",
"ilog2", "nonreal", "'implies'", "posint", "NameSpace",
"indets", "positive", "NextAfter", "indices", "polynom",
"MPFloat", "'intersect'", "pos_infinity", "MorrBrilCull",
"'<'", "member", "neg_infinity", "Im", "maxnorm", "name",
"'>'", "max", "negint", "'<='", "map2", "negative", "modp1",
"nonnegative", "FromInert", "modp", "negzero",
"EqualStructure", "'minus'", "nonposint", "'>='", "min",
"nonnegint", "'>'", "DefaultUnderflow", "lexorder",
"imaginary", "'='", "lhs", "indexable", "ERROR", "ldegree",
"indexed", "EqualEntries", "length", "integer", "macro",
"list", "DEBUG", "map", "literal", "'..''", "lowerbound",
"'module'", "Default0", "lprint", "moduledefinition",
"DefaultOverflow"]

```

296

84 How to numerically solve a BVP ode and plot the solution?

This one has one solution

```

eq:=diff(u(z),z$2)+(k-1)*diff(u(z),z)/z+lambda*exp(u(z))=0;
sol:=dsolve({subs({k=1,lambda=2},eq),u(0)=1,u(1)=0},numeric,u(z),
            method=bvp[midrich],'abserr'=0.001);
plots[odeplot](sol);

```

This solved coupled ODE's, so there are 2 solutions. Say $x_1(t)$ and $x_2(r)$, It is a little tricky to plot all solutions generated, but here is an example

```
restart;
```

```

R := 0.4; px := 32000; Mm := 0.1; Ds := 9; D02 := 7.2; YXS := 0.3; KS := 10;
Sp := 30; Cb := 8; K02 := 0.2; R0 := 0.000001; YX0 := 0.42857;
Vs := px*1/YXS*(Mm*x2(r))/(KS + x2(r))*x1(r)/(K02 + x1(r));
Vo := px*1/YX0*(Mm*x2(r))/(KS + x2(r))*x1(r)/(K02 + x1(r));

eqs := diff(x1(r),r$2) + 2/r*diff(x1(r),r)= Vo/D02,
diff(x2(r),r$2) + 2/r* diff(x2(r),r)= Vs/Ds;
ic:=D(x1)(R0)=0,x1(R) = Cb,D(x2)(R0)= 0, x2(R) = Sp;
sol:=dsolve({eqs,ic},numeric,{x1(r),x2(r)},'abserr'=.52,'maxmesh'=1000,output=listproc

```

And now to plot do

```

x1Sol:=rhs(sol[2]);
plot(x1Sol(r),r=0..0.4);

x2Sol:=rhs(sol[4]);
plot(x2Sol(r),r=0..0.4);

```

85 How to display on screen for specific width?

This below by Axel Vogt posted on sci.math.symbolic which does a nice job of formatting output to specific width.

```

split_for_print:=proc(expr, len)
  # expr = some Maple expression
  # len = length to split with line breaks
  local L,s,tmp,j;
  s:=convert(expr, string);
  L:=[StringTools:-LengthSplit(s, len)];
  for j from 1 to nops(L) do
    # if j = nops(L) then printf("%s ;", L[-1])
    if j = nops(L) then printf("%s", L[-1])
    else printf("%s\\n", L[j]);
    end if;
  end do;
end proc;

evalf[100](Pi);

```

```
split_for_print(%, 40);

3.14159265358979323846264338327950288419\
7169399375105820974944592307816406286208\
998628034825342117068
```

86 Maple IDE links

for VIM

1. <https://code.google.com/p/maplevim/source/browse/trunk/syntax/maple.vim>

in vim, type `set syntax=maple` after putting the file `maple.vim` in `~/.vim/syntax/maple.vim`.
I found `maple.vim` in above link.

For Maple IDE

MapleIDE18

87 loading, remove and finding what packages loaded

use `packages()`; to find what packages loaded. use `unwith` to remove package

```
packages();
                                []

with(DynamicSystems):
packages();
                                [DynamicSystems]

unwith(DynamicSystems);
packages();
                                []
```

88 some rules of thumbs when using Maple

1. put `restart` in separate execution group
2. do not use `with` inside `proc()`. Use `uses` instead.

89 How to make multiple assumptions on a symbol?

```
assume( A::AndProp(NonZero, constant) );
```

Now can use `is(A, constant)`;

90 How to check if expression is an equation?

check for `'='` as follows

```
eq:= x=1;
whattype(eq); # `='`

if whattype(eq) = `=` then
    print("yes");
else
    print("no");
fi;

"yes"
```

91 How to check if expression is a set?

check for `'set'` as follows

```
eq:= {diff(y(x),x)=1,x(0)=1};

if whattype(eq) = `set` then
    print("yes");
else
```

```
print("no");
fi;
"yes"
```

92 How to export a plot to PDF?

I could only find a way to export to eps

```
plotsetup(default):
plotsetup(postscript, plotoutput=`t.eps`, plotoptions=`color,portrait,height=300`);
plot(sin(x),x=-Pi..Pi,'gridlines');
plotsetup(default):
```

Make sure not to put `:` at the end of the plot command! else it will not be exported. It has to end with `;`

This will save it to `t.eps` in the `currentdir()` location. Then use `ps2pdf t.eps t.pdf` to convert it to PDF. Or just `ps2pdf t.eps` it will automatically create `t.pdf`

Or `ps2pdf -dCompatibilityLevel=1.4 t.eps` but maybe it is best to do

```
ps2pdf -dCompatibilityLevel=1.4 -dEmbedAllFonts=true t.eps
```

Also try adding

```
-dPDFSETTINGS=/printer
```

to the above. This tells it to optimize it for printing.

Another example of a direction field for an ODE

```
plotsetup(postscript, plotoutput=`t0.eps`, plotoptions=`color,portrait,height=300`);
ode:= diff(y(x),x) = 3*x^2 - 1;
DEtools:-DEplot( ode, y(x), x=-2..2, [y(0) = 0], y=-2..2,
                linecolour=red, color = blue, stepsize=.05,arrows=MEDIUM );
plotsetup(default);
```

93 How to find all roots of complex number

To find roots of $(3 + 4i)^{1/3}$, do

```
fsolve(z^3=(3+4*I),z);
```

#gives

```
-1.26495290635775+1.15061369838445*I,  
-.363984239564424-1.67078820068900*I,  
1.62893714592218+.520174502304545*I
```

94 How to convert matrix of matrices to a matrix?

```
A:= Matrix(2, 2, {(1, 1) = 0, (1, 2) = 0, (2, 1) = 0, (2, 2) = 2});  
f:=x->`if`(x<>0,x*LinearAlgebra:-IdentityMatrix(2),0*Matrix(2));  
B:=map(f,A);
```

Which gives

$$\begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \end{bmatrix}$$

now

```
r:=Matrix(convert(B,listlist))
```

Gives

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

95 How to do pattern matching in Maple?

Maple has a simple but easy to use pattern matching, which works well. Here are some examples. For each case, will show what pattern to detect and how to do it. I am still not very good at pattern matching in Maple and will need to make improvements in this with time.

95.1 Example 1

Detect \sqrt{xy} in expression.

```
restart;
expr:= sin(x)*sqrt(x*y);
if patmatch(expr,a::anything*(b::anything*x*y)^(c::anything),'la') then
  assign(la);
  if c =1/2 or c=-1/2 then
    print("found sqrt(x*y)");
  else
    print("did not find sqrt(x*y)");
  fi;
fi;
```

But if the expression was $\sin(x)\sqrt{xy} + 3$ then the above would fail, because there are a term after \sqrt{xy} , so the pattern has to change to

```
restart;
expr:= sin(x)*sqrt(x*y)+3;
if patmatch(expr,a::anything*(b::anything*x*y)^(c::anything)+d::anything,'la') then
  assign(la);
  if c =1/2 or c=-1/2 then
    print("found sqrt(x*y)");
  else
    print("did not find sqrt(x*y)");
  fi;
fi;
```

95.2 Example 2

There was a case where I wanted to detect form $f(x)g(\frac{y}{x})$, i.e. $f(x)$ can be any expression which is function of x only (it can be constant also) multiplied by a function whose argument must be $\frac{y}{x}$ or a constant multiplied by $\frac{y}{x}$.

This means something like $xg(\frac{y}{x})$ or $x^2e^{3\frac{y}{x}}$ or $f(x)\sin\frac{y}{x}$ or $\cos\frac{y}{x}$ where in this last case $f(x) = 1$ which is allowed.

TO FINISH.

96 How to find trig identities?

use `trigsubs`, very useful command. For example

```
trigsubs(cos(theta)^3)
```

Gives

$$[1/2 \cos(\theta) + 1/2 \cos(2\theta) \cos(\theta), 1/4 \cos(3\theta) + 3/4 \cos(\theta)]$$

97 How to find directional derivative of scalar function?

Given $f(x, y, z) = x^2z + y^3z^2 - xyz$ we want to find its directional derivative along the vector n .

One way

```
n:=<-1,0,3>;  
g:=VectorCalculus[Gradient](x^2*z+y^3*z^2-x*y*z, [x,y,z]);  
Student[VectorCalculus][DotProduct](g,n/LinearAlgebra[Norm](n,2))
```

Gives

$$-\frac{(2xz - yz)\sqrt{10}}{10} + \frac{(6y^3z + 3x^2 - 3xy)\sqrt{10}}{10}$$

Another is

```
Student[MultivariateCalculus][DirectionalDerivative](x^2*z+y^3*z^2-x*y*z, [x,y,z], [-1
```

Gives the same result.

98 How to check if name is assigned a value?

For simple variable, use `assigned`

```
restart;
x:=10:
assigned(x)
                                true
assigned(y)
                                false
```

For a field in table do

```
restart;
A:=table(["x"]=10, "y"]=20):
assigned(A["x"])
                                true
assigned(A["z"])
                                false
```

For field in Record, I do not know how yet, other than using try catch, as `assigned` does not seem to work for Record fields.

```
restart;
A:=Record('x'=10, 'y'=20);
try
  assigned(A:-x)
catch:
  print("no such field in record")
end try;

                                true

try
  assigned(A:-z)
catch:
```

```
print("no such field in record")
end try;

      "no such field in record"
```

99 How to select terms with sqrt or radical

Given

$$3 + x + \sqrt{-4ac + b^2} + \sin(y) + x^3\sqrt{39} + \sqrt{\cos x}$$

Find terms that are sqrt. Use `indets`

```
restart;
expr_with_radical:= 3+x+sqrt(b^2-4*a*c)+sin(y)+x^3*sqrt(39)+sqrt(cos(x));
indets(expr_with_radical, algebraic^fraction)
```

$$\{\sqrt{39}, \sqrt{-4ac + b^2}, \sqrt{\cos x}\}$$

Alternative is to use type `radical`

```
restart;
expr_with_radical:= 3+x+sqrt(b^2-4*a*c)+sin(y)+x^3*sqrt(39)+sqrt(cos(x));
indets(expr_with_radical, radical)
```

$$\{\sqrt{39}, \sqrt{-4ac + b^2}, \sqrt{\cos x}\}$$

100 How to simplify $e^{\ln(x)+\ln(y)}$

given

$$e^{\frac{2 \ln(\sqrt{p^2+1+p})+2 \ln(a)+\ln(p^2+1)a}{2a}} + e^{3x}$$

`simplify(expr)` does not work. So tried `subsindets`

```
restart;
expr := exp((2*ln(sqrt(p^2 + 1) + p) + 2*ln(a) + ln(p^2 + 1)*a)/(2*a))+ exp(3*x);
subsindets(expr, 'specfunc( anything, exp )', f->(if (has(op(1,f), 'ln'), expand(f), f)))
```

$$\left(\sqrt{p^2+1}+p\right)^{\frac{1}{a}} a^{\frac{1}{a}} \sqrt{p^2+1}+e^{3x}$$

It is possible to also try `simplify(expr,exp)` in some cases, but for the above example, this did not work, i.e. it did not simplify it.

Update december 2023. Trying Maple 2023.2.1, it simplifies the above using `simplify(expr,exp)`

```
restart;
expr := exp((2*ln(sqrt(p^2 + 1) + p) + 2*ln(a) + ln(p^2 + 1)*a)/(2*a)) + exp(3*x);
simplify(expr,exp)
```

$$\sqrt{p^2+1} \left(\sqrt{p^2+1}+p\right)^{\frac{1}{a}} a^{\frac{1}{a}} + e^{3x}$$

And

```
restart;
expr:=exp(ln(x)+ln(y));
simplify(expr)
```

$$xy$$

101 How to find all `csgn()` and replace them by 1

I wanted to simplify an expression which could have `csgn()` in it, and find all the arguments.

$$\frac{1 + \text{csgn}(a) a}{3 \text{csgn}(b) b}$$

One way is

```
restart;
expr:=(1+csgn(a)*a)/(3*csgn(b)*b);
expr:=subsindets(expr,'specfunc( anything, csgn )',f->1);
```

$$\frac{1+a}{3b}$$

102 How to find symbols inside `csgn()` in an expression?

Given `sol:=1/2*2^(1/2)*csgn(x)*x*csgn(y)`; how to find all symbols inside `csgn` which will be x, y in this case?

```
restart;
sol:=1/2*2^(1/2)*csgn(x)*x*csgn(y);
indets(sol,'specfunc( anything, csgn )');
vars:=subsindets(%, 'specfunc( anything, csgn )', f->op(f))
```

Gives $\{x, y\}$

Now if we want to simplify the above solution by assuming that all variables inside `vars` are positive, how to do that?

```
restart;
sol:=1/2*2^(1/2)*csgn(x)*x*csgn(y);
indets(sol,'specfunc( anything, csgn )');
vars:=subsindets(%, 'specfunc( anything, csgn )', f->op(f));
simplify(sol) assuming op(map2('<',0,vars))
```

Gives $\frac{\sqrt{2}x}{2}$. Notice in the above the use of `op(map2('<',0,vars))`, this will generate the sequence $0 < x, 0 < y$ automatically. `op` is needed otherwise the result will be $\{0 < x, 0 < y\}$ which will give syntax error when passed to `assuming`

Ofcourse, it would have been also possible to just write

```
simplify(sol) assuming positive;
```

And get the same result. But sometimes we might want to specify which variables are to be assumed positive and not all of them at once in the expression.

103 How to replace all abs(expr) by expr

I wanted to replace $|\text{expr}|$ by (expr)

One way is

```
restart;  
expr:=u(x) = _C1*exp(-3*x^(1/3)*sqrt(c))*(3*x^(1/3)*sqrt(c) + 1) + _C2*exp(3*x^(1/3)*sqrt(c))*
```

$$u(x) = {}_C1 e^{-3x^{\frac{1}{3}}\sqrt{c}} \left(3x^{\frac{1}{3}}\sqrt{c} + 1 \right) + {}_C2 e^{3x^{\frac{1}{3}}\sqrt{c}} \left| -1 + 3x^{\frac{1}{3}}\sqrt{c} \right|$$

```
expr:=subsindets(expr,'specfunc( anything, abs )',f->op(f));
```

$$u(x) = {}_C1 e^{-3x^{\frac{1}{3}}\sqrt{c}} \left(3x^{\frac{1}{3}}\sqrt{c} + 1 \right) + {}_C2 e^{3x^{\frac{1}{3}}\sqrt{c}} \left(-1 + 3x^{\frac{1}{3}}\sqrt{c} \right)$$

104 Basis for Null space, Row space and column space of matrix

Given

$$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 2 & -2 \\ 0 & 2 & 3 & -1 \end{bmatrix}$$

Find its Null, Row and Column space basis vectors.

```
restart;  
A:=Matrix([[1,-1,0,2],[1,2,2,-2],[0,2,3,-1]]);  
LinearAlgebra:-NullSpace(A)
```

$$\left\{ \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} \right\}$$

```
restart;  
A:=Matrix([[1,-1,0,2],[1,2,2,-2],[0,2,3,-1]]);  
LinearAlgebra:-RowSpace(A)
```

$$[[1, 0, 0, 0], [0, 1, 0, -2], [0, 0, 1, 1]]$$

```
restart;
A:=Matrix([[1,-1,0,2],[1,2,2,-2],[0,2,3,-1]]);
LinearAlgebra:-ColumnSpace(A)
```

$$\left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right]$$

105 How to do Gaussian elimination on a Matrix?

Given

$$\begin{bmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{bmatrix}$$

Find the new form after Gaussian elimination

```
restart;
A:=Matrix([[1,-4,-3,-7],[2,-1,1,7],[1,2,3,11]]);
LinearAlgebra:-GaussianElimination(A);
```

$$\begin{bmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{bmatrix}$$

106 How to find Reduced Echelon form of a Matrix?

Given matrix

$$\begin{bmatrix} 5 & 2 & 18 \\ 0 & 1 & 4 \\ 4 & 1 & 12 \end{bmatrix}$$

Find its Reduced Echelon form.

```
restart;
A:=Matrix([[5,2,18],[0,1,4],[4,1,12]]);
Student:-LinearAlgebra:-ReducedRowEchelonForm(A)
```

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

Another option is

```
restart;
A:=Matrix([[5,2,18],[0,1,4],[4,1,12]]);
MTM:-rref(A)
```

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

107 How add a new row to bottom of matrix?

Given matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}$$

How to add row

$$[a, b]$$

to end of the matrix?

```
restart;
A:=Matrix([[1,1],[2,3],[4,5]]);
the_row:=convert([a,b],Vector['row']);
ArrayTools:-Concatenate(1,A,the_row);
```

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 5 \\ a & b \end{bmatrix}$$

108 How to obtain list of all occurrences of some function in an expression?

For an example, How to find list of all ln functions in this expression?

$$\ln(|x + 1|) + 2x \ln(x) + \sin(x)$$

```
restart;
expr:=ln(abs(x+1))+2*x*ln(x)+sin(x);
tmp := indets(expr, 'specfunc(anything, ln)');

# tmp := {ln(x), ln(abs(x + 1))}
```

To pick only ln functions which has *abs* inside them anywhere, replace the above with

```
restart;
expr:=ln(abs(x+1))+2*x*ln(x)+sin(x);
lis:=indets(expr, 'specfunc(anything, ln)');
select(Z->has(Z, abs), lis)

# tmp := {ln(abs(x + 1))}
```

Or, better alternative to the above is

```
restart;
expr:=ln(abs(x+1))+2*x*ln(x)+sin(x);
indets(expr, 'specfunc( satisfies(u->has(u, abs)) , ln )');

# tmp := {ln(abs(x + 1))}
```

109 How to replace $\ln(|x|)$ with $\ln(x)$ in an expression?

Given

$$\sin(x) + \ln(|x|) + \ln\left(x + \frac{|y|}{\sqrt{|x+3|}}\right) + \ln(x^3) + \cos(|x|)$$

How to remove the absolute, the ones only inside each \ln in the above expression?

```
restart;
expr:=sin(x)+ln(abs(x))+ln(x+abs(y)/sqrt(abs(x+3)))+ln(x^3)+cos(abs(x));
expr:=evalindets(expr,'specfunc(ln)',f->evalindets(f,'specfunc(abs)',f->op(1,f)))
# sin(x) + ln(x) + ln(x + y/sqrt(x + 3)) + ln(x^3) + cos(abs(x))
```

$$\sin(x) + \ln(x) + \ln\left(x + \frac{y}{\sqrt{x+3}}\right) + \ln(x^3) + \cos(|x|)$$

110 How to find all signum functions in expression and simplify it?

Given

$$\frac{\left(\ln\left(\frac{(b + \sqrt{b^2 + y(x)^2} \operatorname{signum}(b))b}{y(x)}\right) + \ln(2)\right) \operatorname{signum}(b)}{b} = _C1 + \frac{-\ln(a) + \ln(x) - \ln(a + \sqrt{a^2 + x^2} \operatorname{signum}(a))}{|a|}$$

How to find all arguments of `signum` and simplify the above by assuming they are all positive?

```
restart;
expr:=-ln((b + sqrt(b^2 + y(x)^2)*signum(b))*b/y(x)) + ln(2))*signum(b)/b = _C1 + (-1
lis:=indets(expr,'specfunc(anything,signum)');
assum:=convert(map(x->op(1,x)>0,lis),list);
simplify(expr,assume=assum);
```

$$\frac{-\ln(b) - \ln\left(\frac{b + \sqrt{b^2 + y(x)^2}}{y(x)}\right) - \ln(2)}{b} = \frac{-C1 a - \ln(a) - \ln(a + \sqrt{a^2 + x^2}) + \ln(x) - \ln(2)}{a}$$

111 How to replace all signum functions in expression by 1?

Given

$$\frac{\left(\ln\left(\frac{(b + \sqrt{b^2 + y(x)^2} \operatorname{signum}(b))b}{y(x)}\right) + \ln(2)\right) \operatorname{signum}(b)}{b} = -C1 + \frac{-\ln(a) + \ln(x) - \ln(a + \sqrt{a^2 + x^2} \operatorname{signum}(a))}{|a|}$$

How to replace all `signum` by 1?

```
restart;
expr:=- (ln((b + sqrt(b^2 + y(x)^2)*signum(b))*b/y(x)) + ln(2))*signum(b)/b = _C1 + (-1
evalindets(expr, 'specfunc(anything,signum)', f -> 1);
```

$$\frac{\ln\left(\frac{(b + \sqrt{b^2 + y(x)^2})b}{y(x)}\right) + \ln(2)}{b} = -C1 + \frac{-\ln(a) + \ln(x) - \ln(a + \sqrt{a^2 + x^2}) - \ln(2)}{|a|}$$

112 How to find the cofactor matrix of a matrix?

Use `LinearAlgebra:-Adjoint` and then transpose the result. Since the Adjoint is the transpose of the cofactor.

Given

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

then

```
restart;
A:=Matrix([[1,2,3],[4,5,6],[7,8,10]]);
LinearAlgebra:-Transpose(LinearAlgebra:-Adjoint(A))
```

$$\begin{bmatrix} 2 & 2 & -3 \\ 4 & -11 & 6 \\ -3 & 6 & -3 \end{bmatrix}$$

113 How to normalize eigenvectors?

When finding eigenvectors of matrix, using `LinearAlgebra`, the vectors are not normalized. How to normalized them so the length is one?

One way is

```
restart;
LA:=LinearAlgebra;
Sx:=Matrix([[0,1,0],[1,0,1],[0,1,0]]);

#this finds eigenvectors in v
lam,v:=LA:-Eigenvectors(Sx);

#this normalize it
B:=map(n -> v[.., n]/norm(v[.., n], 2), [$1..LA:-RowDimension(v)]):
B:=`<|>`(op(B)); #this converts the list back to matrix.
```

$$v = \begin{bmatrix} -1 & 1 & 1 \\ 0 & \sqrt{2} & -\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

114 How to find if some function is present in an expression

Given expression $3 \sin(x) + t + 3f(x,t)t + g(x,t)$ find if it contains function $f()$.

Use `indets` with `specfunc(f)`

```
restart;
expr := 3*sin(x)+t+3*f(x,t)*t+g(x,t);
res := indets(expr, specfunc(f));
if numelems(res)<>0 then
  print("Found f(x,t)");
else
  print("could not find f(x,t)");
fi;
```

"Found f(x,t)"

115 How to find all functions in an expression?

Given expression $3 \sin(x) + t + 3f(x,t)t + g(x,t)$ find all functions, if any, in the expression.

Use `indets` with `function`

```
restart;
expr := 3*sin(x)+t+3*f(x,t)*t+g(x,t);
res := indets(expr,function);
if numelems(res)<>0 then
  print("Found these functions",res);
else
  print("could not find any function");
fi;
```

"Found these functions", f(x, t), g(x, t), sin(x)

116 find all functions except builtin math functions

Given expression $3 \sin(x) + t + 3f(x,t)t + g(x,t)$ find all functions, if any, in the expression but exclude the math functions such as sin in the above.

```
restart;
expr := 3*sin(x)+t+3*f(x,t)*t+g(x,t);
res := indets(expr, And( function, Not(typefunc(mathfunc))));
if numelems(res)<>0 then
  print("Found these functions",res);
else
  print("could not find any function");
fi;
```

"Found these functions", f(x, t), g(x, t)

117 How to obtain a list of all arguments of function?

use op

```
restart;
op(1..,f(x,t))

x, t
```

Note that `op(0,f(x,t))` finds the function name.

118 Find functions whose first argument is z

```
restart;
expr := 3*sin(z)+t+3*f(z,t,y)*t+g(x,t);
res := indets(expr, patfunc(identical(z), anything));
if numelems(res)<>0 then
  print("Found these functions",res);
else
```

```
print("could not find any function");  
fi;
```

gives

"Found these functions", f(z, t, y), sin(z)

119 Find functions whose second argument is t ?

```
expr := 3*sin(z)+t+3*f(z,t,y)*t+g(x,t);  
res := indets(expr, patfunc(anything, identical(t), anything));  
if numelems(res)<>0 then  
  print("Found these functions",res);  
else  
  print("could not find any function");  
fi;
```

gives

"Found these functions", f(z, t, y), g(x, t)

120 How to typeset \hbar ?

```
expr := `&hbar;`*x
```

gives

$\hbar x$

Notice, the ; is needed. This ‘&hbar’*x will not work. It must be ‘ℏ’*x

121 How to find the Curl of a vector?

First example

```
restart;  
VectorCalculus:-SetCoordinates( 'cartesian' [x,y,z] );  
F:=VectorCalculus:-VectorField(<y,-x,0>);
```

$$F = y\bar{e}_x - x\bar{e}_y$$

And now

```
VectorCalculus:-Curl(F);
```

$$-2\bar{e}_z$$

Second example

```
restart;  
VectorCalculus:-SetCoordinates( 'cartesian' [x,y,z] );  
F:=VectorCalculus:-VectorField(<y*z^2,x*z^2+2,2*x*y*z-1>);
```

$$F = yz^2\bar{e}_x + (xz^2 + 2)\bar{e}_y + (2xyz - 1)\bar{e}_z$$

And now

```
VectorCalculus:-Curl(F);
```

$$0$$

Since Curl is zero, field is conservative.

Third example, in cylindrical coordinates

```
restart;  
VectorCalculus:-SetCoordinates( 'cylindrical' [rho,phi,z] );  
F:=VectorCalculus:-VectorField(<0,-rho,2>);
```

$$F = -\rho \bar{e}_{\mathbb{E}} + 2\bar{e}_z$$

And now

```
VectorCalculus:-Curl(F);
```

$$2\bar{e}_z$$

122 See all steps in RREF of an augmented matrix

Use `Student:-LinearAlgebra:-GaussJordanEliminationTutor(A, output=steps)` Where A is your augmented matrix.

123 How to find column space of matrix?

Do not use the Maple command `LinearAlgebra:-ColumnSpace` for this. it gives the columns in the RREF. The correct way is to obtain the corresponding columns of the pivot columns in the original matrix A . Hence use the command `Basis` like this

```
A:=Matrix([[1,0,0],[1,1,1]]);
LinearAlgebra:-Basis([seq(A[..,i],i=1..LinearAlgebra:-ColumnDimension(A))]);
```

Which gives

$$\left[\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]$$

If you use `ColumnSpace` command you'll get this

```
A:=Matrix([[1,0,0],[1,1,1]]);
LinearAlgebra:-ColumnSpace(A);
```

$$\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]$$

These are different. `Basis` is the correct command to use, which matches the standard definition in textbooks.

124 How to use select with own type to find subexpressions?

Given expression such as $3 + (1 + x) \sin x$ or $3 + (1 + x) \sin^2 x$ use select to find any polynomial * sin^n subexpressions.

```
restart;
mytype_1 := ``*``( {polynom(And(algebraic, satisfies(u -> not has(u, I))),x),
                    Or( 'specfunc(sin)'^integer, 'specfunc(sin)')
                  }
                );
select(type, 3+(1+x)*sin(x),mytype_1);
select(type, 3+(1+x)*sin(x)^2,mytype_1);
```

Gives

```
(1 + x) sin(x)
                2
(1 + x) sin(x)
```

125 How to write structured types to match some expressions?

125.1 type for $\sin^m(x) \cos^n(x)$

```
restart;
my_type:=``*``( { Or('specfunc'(sin),'specfunc'(sin)^Or(integer,rational)),
                  Or('specfunc'(cos),'specfunc'(cos)^Or(integer,rational))} );
type(sin(x)^2*cos(x)^3,my_type);
type(sin(x)^2*cos(x),my_type);
type(sin(x)*cos(x),my_type);
type(cos(x)*sin(x)^(1/2),my_type);

true
```

```
true
true
true
```

I could not find a way to avoid writing `Or('specfunc'(sin), 'specfunc'(sin)^Or(integer, rational))` in order to match both $\sin x$ and $\sin^2 x$. For these things, I find Mathematica patterns more flexible. The above can be done as follows in Mathematica

```
ClearAll[x,n,m,any]
patt=any_.*Sin[_]^n_.*Cos[_]^m_
MatchQ[Sin[x]^2*Cos[2*x]^3,patt]
MatchQ[Sin[x]^2*Cos[x],patt]
MatchQ[Sin[x]*Cos[x],patt]
MatchQ[Cos[x]*Sin[x],patt]
```

```
True
True
True
True
```

In Mathematica `n_.` says basically to match $\sin x$ or $\sin^2 x$ since the dot says to match zero or more. So no need to duplicate things as I did above in Maple. There might be a way to do the same in Maple using structured type, but I could not find it. In General, I find patterns in Mathematica more flexible and easier to use for this sort of thing. Maple has `patmatch` command, but not as easy to use as `Patterns` in Mathematica.

126 select only indexed variables from an expression

use `indets` with type 'indexed'

```
expr:=16*a[3]+6*a[1];
terms:=indets(expr,'indexed');

terms := {a[1], a[3]}
```

#to find maximum index, then do

```
map(x->op(x), terms)
```

```
{1, 3}
```

127 Show step by step. Calculus problem and differential equations.

For integration do

```
Student:-Calculus1:-ShowSolution(Int(x*sin(x),x));
```

The steps are displayed. This does not work all the time. For example

```
integrand:=x*y(x)*diff(y(x),x$2)+x*(diff(y(x),x))^2-y(x)*diff(y(x),x);  
Student:-Calculus1:-ShowSolution(Int(integrand,x));
```

gives

Error, (in Student:-Calculus1:-ShowSolution) unable to determine which calculus operation is being applied in this problem; you can provide this information as the 2nd argument on your call to Rule or Hint

For differential equations, support is limited but these are the steps

```
restart;  
ode:=diff(y(x),x)=sin(x);  
Student:-ODEs:-ODESteps(ode)
```

Prints the steps. If IC is there, then

```
restart;  
ode:=diff(y(x),x)=sin(x);  
ic:=y(0)=1;  
Student:-ODEs:-ODESteps([ode,ic])
```

128 How to obtain list of files with some extension in folder?

Use FileTools:-ListDirectory

```
dir_name:="C:/tmp";
currentdir(dir_name); #cd to directory
files_to_process := FileTools:-ListDirectory(dir_name,'all','returnonly'="*.tex"):
numelems(files_to_process)
100
```

In the above, `files_to_process` is a list of the files in the current folder with extension `.tex`

129 How to delete lines from text file that contains some string?

There was a case when I needed to delete lines from text file that contains a say "foo" as an example.

This is what I did. use `readline` to read the lines, check, and if the line contains "foo" skip, else write the line to a temporary file. At the line, use `Rename` to rename the temporary file to the file being read.

```
dir_name:="C:/tmp";
currentdir(dir_name);

tmp_file_name      := "TMP.txt";
source_file_name   := "source.txt";
file_id            := fopen(tmp_file_name,WRITE):
line               := readline(source_file_name):

while line<>0 do

    if not StringTools:-Has(line,"foo") then
        fprintf(file_id,"%s\n",line);
    fi;

    line := readline(source_file_name):
```

```
od:
```

```
fclose(file_id);  
FileTools:-Rename(tmp_file_name,source_file_name,force=true);
```

130 Given an expression, how to find all variables and functions in it?

Given say $\frac{d^2}{dx^2}y(x) + n\left(\frac{d}{dx}y(x)\right) + 3 = \sin(x)$ how to find all variables and functions in it, not including math functions such as $\sin x$?

So the result should be $n, x, y(x)$.

```
ode:=diff(y(x),x$2)+n*diff(y(x),x)+3=sin(x);  
vars:=indets(ode, Or( And(symbol,Not(constant)), And(function,Not(typefunc(mathfunc))));  
  
#gives  
# vars := {n, x, diff(y(x), x), diff(y(x), x, x), y(x)}
```

I still need to work on excluding derivatives from the search.

131 How to check if an expression is integer, when it has symbols in it?

I had case where I needed to check if something is integer or not. The problem is that the result had a symbol n in it. I need a way to tell Maple that to check if the result can be an integer given that n is also an integer.

Using `type` does not work, since can't use assumptions. One way is to use `coulditbe` as follows

```
restart;  
expr:=n-1+2*m;  
vars:=indets(expr,And(symbol,Not(constant)));  
coulditbe(expr,integer) assuming op(map(Z->Z::integer,vars))  
  
# true
```

In the above `indets(expr, And(symbol, Not(constant)))` picks all variables in the expression, and `assuming op(map(Z->Z::integer, vars))` makes assumption that each is integer.

132 How to truncate a polynomial?

Given $9x^5 + 4x^4 + 3x^3 + x^2 + x + 1$ how to truncate it, so that all terms of x^3 and higher are removed?

This can be done as follows

```
restart;
p:=1+x+x^2+3*x^3+4*x^4+9*x^5;
simplify(p, {x^3=0})
```

$$x^2 + x + 1$$

133 How to make a local declare like block inside a proc?

Sometimes it is useful to make a small local piece of code inside a proc, with its own local variables that do not interfere with the variables of the proc. In Ada, this is done using `declare` clause for example. In Maple one can do the same as follows

```
restart;

foo:=proc()
  local n;
  n:=10;
  proc()
    local n;
    n:=99;
    print("inside inner proc, n=",n);
  end proc();
  print("n=",n);
end proc;

foo();
```

Which prints

```
"inside inner proc, n=", 99
    "n=", 10
```

Notice the end of the inner anonymous proc above. It has `end proc()`; and not `end proc`; as normal proc. This defines the inner proc and calls it at same time. All the local variables inside the anonymous proc only exist inside that proc and go away after the call, and they do not interfere with the outer proc variables. This way we can declare temporary variables and use them where they are needed.

134 Using short name for a proc inside nested modules instead of long name

There was a case where I was making lots of calls from many places to one specific proc inside a module. I did not want to keep using the long name each time.

the command alias did not work. After some trial and error, found that using `use` works. Here is the solution. First this is the original layout

```
restart;

A:=module()
  export B:=module()
    export foo:=proc()
      print("in A:-B:-foo()");
    end proc;
  end module;

  export C:=module()
    export boo:=proc()
      print("in A:-C:-boo()");
      A:-B:-foo();
    end proc;
  end module;
end module;
```

In the above, the goal is replace `A:-B:-foo()`; with just `foo()` and have it bind to `A:-B:-foo()`; automatically.

This is done by modifying the above to

```
restart;
A:=module()
  export B:=module()
    export foo:=proc()
      print("in A:-B:-foo()");
    end proc;
  end module;

  use foo=A:-B:-foo in #add this line here
  export C:=module()
    export boo:=proc()
      print("in A:-C:-boo()");
      foo(); #now can just use the short name
    end proc;
  end module;
  end use; #add this line here.
end module;
```

Wrapping the whole module where the short name is used worked.

Any module that needs to use the short name, can do the same. This solved the problem.

135 Remove duplicates objects in a list based on condition on a field

I had case where there is list of Objects, and wanted to removed duplicate entries in the list based on if some field is the same among the objects.

This can be done using the command `ListTools:-MakeUnique` and using a proc which checks for the condition. In this example, we want to remove objects where the field `age` in each object is the same.

```
restart;

#create data type
```

```

module person_type()
  option object;
  export name::string:="me";
  export age:=25;
end module:

#make few objects
o1:=Object(person_type);
o2:=Object(person_type);
o3:=Object(person_type);
o3:-age:=46;
o4:=Object(person_type);

#make list of them
list_of_people:=[o1,o2,o3,o4];

nops(list_of_people); #this will print 4

#now delete object if age is same
list_of_people:=ListTools:-MakeUnique(
  list_of_people,
  1,
  proc(a,b)
    evalb(a:-age=b:-age);
  end proc
);

nops(list_of_people); #this will print 2

```

136 How to remove duplicates Vectors from a list?

Converting a list of Vectors to set will not remove duplicates, as each Vector occupies different memory address, even if the structure is the same. To remove duplicate vector, use `ListTools:-MakeUnique` as follows

```

restart;

my_list:=[Vector([1,0]),Vector([1,0]),Vector([2,0])];

```

```
convert(my_list,set); #this will still show the 3 vectors.
```

```
ListTools:-MakeUnique(my_list,1,proc(a,b) LinearAlgebra:-Equal(a,b) end proc)
```

```
#now only 2 vectors will remain. Duplicate one was removed
```

137 How to find parameters such as π in an expression?

Use

```
restart;  
expr:=4*Pi+sin(x);  
indets(expr,And(name,constant))  
  
{Pi}
```

138 How to find all derivatives $y'(x)$ in an expression?

Given an expression such as

$$\sin(x) + \left(\frac{d}{dx}y(x)\right)^3 + \left(\frac{d}{dx}y(x)\right) \ln\left(y(x) \left(\frac{d}{dx}y(x)\right)^2\right) + \sqrt{\frac{d}{dx}y(x)} + \frac{x}{\left(\frac{d}{dx}y(x)\right)^7}$$

Find all $y'(x)$ for any power that show up, so the result should be

$$\left\{ \frac{1}{\left(\frac{d}{dx}y(x)\right)^7}, \left(\frac{d}{dx}y(x)\right)^2, \left(\frac{d}{dx}y(x)\right)^3, \sqrt{\frac{d}{dx}y(x)}, \frac{d}{dx}y(x) \right\}$$

Use `indets` with type `identical(diff(y(x),x))~anything` is used. But must use the `flat` option to work correctly.

```
restart;
```

```

expr:= y(x)*diff(y(x),x)^(1/3)+sin(x)*diff(y(x),x)^3 + z*diff(y(x),x)*ln(y(x)*diff(y(x),x))
t1:=identical(diff(y(x),x))^anything;
t2:=identical(diff(y(x),x));
indets[flat](expr, 'Or'( t1, t2 ));

```

gives

```

{diff(y(x), x)^(1/3), 1/diff(y(x), x)^7, diff(y(x), x)^2, diff(y(x), x)^3, diff(y(x), x),

```

Without using flat it will give wrong result. For example

```

restart;
expr:=diff(y(x),x)^2;
t1:=identical(diff(y(x),x))^anything;
t2:=identical(diff(y(x),x));
indets(expr, 'Or'( t1, t2 ));

```

Gives

```

{diff(y(x), x)^2, diff(y(x), x)}

```

You see, it has extra $\text{diff}(y(x), x)$ showing up. Adding flat it gives

```

restart;
expr:=diff(y(x),x)^2;
t1:=identical(diff(y(x),x))^anything;
t2:=identical(diff(y(x),x));
indets[flat](expr, 'Or'( t1, t2 ));

```

Now it gives

```

{diff(y(x), x)^2}

```

Which is the correct result.

139 How combine log terms?

To go from $\ln(AB)$ to $\ln A + \ln B$ need to use `simplify` with `ln` option but add assumptions that one of the terms is positive. Else it will not do it

```
restart;
simplify(ln(A*B),ln); # no change
simplify(ln(A*B),ln) assuming A>0; # ln(A) + ln(B)
simplify(ln(A*B),ln) assuming B>0; # ln(A) + ln(B)
```

To go from $\ln(\frac{A}{B})$ to $\ln A - \ln B$ need to use `simplify` with `ln` option but add assumptions that $B > 0$.

```
restart;
simplify(ln(A/B),ln); # no change
simplify(ln(A/B),ln) assuming A>0; # ln(A) + ln(1/B)
simplify(ln(A/B),ln) assuming B>0; # do this: -ln(B) + ln(A)
```

To go from $\ln A + \ln B$ to $\ln(AB)$ need to use `combine` with assumptions that either A or B is positive, else it will not do it.

```
restart;
combine(ln(A) + ln(B),ln); # no change
combine(ln(A) + ln(B),ln) assuming A>0; # ln(A*B)
combine(ln(A) + ln(B),ln) assuming B>0 # ln(A*B)
```

To go from $\ln A - \ln B$ to $\ln \frac{A}{B}$ need to use `combine` with assumptions that either B is positive.

```
restart;
combine(ln(A) - ln(B),ln); #no change
combine(ln(A) - ln(B),ln) assuming A>0; # -ln(B/A)
combine(ln(A) - ln(B),ln) assuming B>0 # use this ln(A/B)
```

140 How to find all poles and their order of a rational function?

Gives a rational function in x , such as

$$\frac{1}{10(x-4)(x-5)^3}$$

How to find all its poles which are $x = 4$ and $x = 5$ and the order of each pole which will be 1 and 3 in this example?

Using `sqrfree` as follows

```
restart;
get_poles_and_order:=proc(r_in,x::symbol)::list;
  local r:=r_in,N::posint;
  local the_poles::list;
  local item;

  r:=normal(r);
  if not type(r,'ratpoly'(anything,x)) then
    error("Not be a polynomial or a rational function in ",x)
  fi;

  the_poles := sqrfree(denom(r),x);
  the_poles := the_poles[2,..]; #we do not need the overall factor
  for N,item in the_poles do
    the_poles[N]:=[solve(item[1]=0,x),item[2]];
  od;
  return the_poles;
end proc;
```

The above proc `get_poles_and_order` returns back a list of lists. Each sublist has its first entry the pole and the second entry the order.

Here are some examples

```
r:=1/(10*(x-4)*(x-5)^3);
get_poles_and_order(r,x)
```

```
#[[4, 1], [5, 3]]
```

The above says there is a pole at $x = 4$ of order 1 and pole at $x = 5$ of order 3.

141 find series of function with specific number of terms

Doing `series` in Maple with specific order value, the number of terms generated ofcourse depends on the function. I had need to have the series generated always with same number of terms. I could not find an option in Maple to do that. This function does this. It keeps finding the series for the function with increasing order until the number terms that comes out is what requested. There is an upper limit that can be changed if needed to protect against pathological cases.

```
restart;

get_series_by_terms:=proc(expr,x::symbol,at::numeric,number_terms_needed::posint)
local keep_running::boolean:=true;
local current_order::integer:=0;
local MAX_ORDER_TO_TRY::posint:=100; #change as needed
local result;

do
    current_order := current_order+1;
    result := convert(series(expr,x=at,current_order),polynom);
    if nops(result) >= number_terms_needed or current_order>MAX_ORDER_TO_TRY then
        keep_running:=false;
    fi;
until keep_running=false;

return result;
end proc;
```

And now

```
get_series_by_terms(sin(x),x,0,10)
```

returns

$$\begin{aligned} x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \\ \frac{1}{39916800}x^{11} + \frac{1}{6227020800}x^{13} - \frac{1}{1307674368000}x^{15} + \\ \frac{1}{355687428096000}x^{17} - \frac{1}{121645100408832000}x^{19} \end{aligned} \quad (1)$$

142 How to call sibling's proc without making the sibling module exported?

Given a parent module A and inside it there are two child modules (local modules) with names say B and C . To call a proc `foo` inside B from another proc inside C , the proc `foo` has to be exported. But the module B does not have to be exported, if we make sure to use `B:-foo()` call instead of full name `A:-B:-foo()` call.

So make sure to use `child:-proc()` from other siblings to avoid having to make each child exported. Making children exported means they can be seen and called directly from outside the parent which is not what we want.

Here is an example

```
restart;

A:=module() #parent

  export main:=proc()
    C:-foo();
  end proc;

  local B:=module() #child
    export foo:=proc()
      print("in A:-B:-foo() proc");
    end proc;
  end module;

  local C:=module() #child
    export foo:=proc()
      print("in A:-C:-foo(). About to call A:-B:-foo()");
      B:-foo(); #do this and NOT A:-B:-foo()
    end proc;
```

```
end module;  
end module;
```

and now

```
A:-main()  
  "in A:-C:-foo(). About to call A:-B:-foo()"  
  "in A:-B:-foo() proc"
```

If instead we have written `A:-B:-foo()` in the above call, then Maple will complain with the error `Error, (in foo) module does not export 'B'`

143 Find position in a list of items that are not numeric

Given list such as `[1,2,3,4,5,x,y,8,9,Pi]` find position of elements that are not numeric. In this case the answer should be `[6,7,10]`

```
restart;  
lis:=[1,2,3,4,5,x,y,8,9,Pi];  
lis2:=select(x->not(type(x,numeric)),lis);  
map(x->ListTools:-Search(x,lis),lis2)  
  
[6,7,10]
```

I could not find a way to do it using one command like with Mathematica. The first command above uses `select` to first find non numeric entries. The second command `ListTools:-Search` then find the index/position.

Maple's `ListTools:-Search` should really have a version that allows one to select the element directly. Something like this

```
lis:=[1,2,3,4,5,x,y,8,9,Pi];  
ListTools:-Search(x->not(type(x,'numeric')),lis,all)
```

144 Convert time to use seconds instead of milliseconds

Maple's command `Value(Time())` returns 13 digits number, which is number of milliseconds from epoch. I wanted this value to be in seconds, to match the file changed time from `FileTools[Status]("A.txt")` which uses seconds and not milliseconds. I could not find an option to tell `Date` or `Time` to do this. Here is one way to do this.

```
r:=Value(Time()); #r := 1652677498870
length(r); #13
r:=convert(r, base, 10);
r:=ListTools:-Reverse(r);
r:=r[1..-4]; #remove last 3 digits
nops(r);
r:=parse(cat(op(r))) #r := 1652677498
length(r); #10
```

This can be made into a function

```
get_time_in_seconds:=proc()::integer;
local r;
r:=Value(Time());
r:=convert(r, base, 10);
r:=ListTools:-Reverse(r);
r:=r[1..-4];
r:=parse(cat(op(r)));
return r;
end proc;

get_time_in_seconds() #1652679222
```

145 How to change $\arctan(y, x)$ to $\arctan\left(\frac{y}{x}\right)$ in an expression

I needed to do this as I was translating Maple code to Sagemath. Where sagemath supports \arctan with only one argument.

Given an expression such as

$$x + \arctan(y, x) + \sin(x)$$

Convert it to

$$x + \arctan\left(\frac{y}{x}\right) + \sin(x)$$

```
evalindets(expr, 'specfunc(arctan)', f->'if'(nops(f)=2, arctan(op(1,f)/op(2,f)), f))
```

146 How to find all symbols that represent variables in an expression?

Given an expression such as $a + \sin(x) + \pi + y + f(r)$ how to find all symbols in it, which will be a, x, y, r ?

One way

```
expr:=a+sin(x) + Pi + y+ f(r);  
indets(expr, And(symbol, Not(constant)));
```

Another is

```
expr:=a+sin(x) + Pi + y+ f(r);  
indets(expr, assignable(name));
```

Both give $\{a, r, x, y\}$

147 How to change first argument of function?

This question came up in <https://mathematica.stackexchange.com/questions/274535/replacing-only-variables-in-specific-locations-with-replace-all>

where the user wanted to I would like to replace $R0$ with r , but only in the arguments of f

The input they had is $\text{line} = R0*f[R0,x] + R0^2*42*D[g[R0,x],x]$

In Maple, this can be done as follows, which I think is a easier thanks to Maple's strong type system.

```
line := R0*f(R0,x) + R0^2*42*diff(g(R0,x),x);
evalindets(line, 'patfunc(identical(R0),anything)', Z-> subsop(1 = r, Z));
```

Which gives

$$R0 f(r, x) + 42 R0^2 \left(\frac{\partial}{\partial x} g(r, x) \right)$$

148 How to change last argument of function?

Given an expression which contains some different functions each with different number of arguments. Suppose we want to change only the last argument of each function if the last argument is x , and change it to say x^2 .

how to do that?

Hence given $R0*f(R0,x)+42*R0^2*D[2](g)(R0,x)+h(x,y,z,r,x)$ we want to change it to $R0*f(R0,x^2)+42*R0^2*D[2](g)(R0,x^2)+h(x,y,z,r,x^2)$.

```
expr:= R0*f(R0,x) + R0^2*42*diff(g(R0,x),x)+ h(x,y,z,r,x);
evalindets(expr, 'patfunc[reverse](anything,identical(x))', Z-> subsop(-1 = x^2, Z));
```

Note the use of `subsop(-1 = x^2, Z)` where `-1` means the last entry in the argument of the function. This is basically same as last example, but uses `patfunc[reverse]` instead of just `patfunc`

149 How to remove last argument of function?

This is the same example as the above, but now we want to remove the last argument instead of changing it.

Hence, given an expression which contains some different functions each with different number of arguments. Suppose we want to remove only the last argument of each function if the last argument is x .

how to do that?

Hence given $R0*f(R0,x)+h(x,y,z,r,x)$ we want to change it to $R0*f(R0)+h(x,y,z,r)$.

```
expr:=R0*f(R0,x)+h(x,y,z,r,x)
evalindets(expr, 'patfunc[reverse](anything, identical(x))', Z-> subsop(-1 = NULL, Z ))
```

Be careful using the above on expression that has $\text{diff}(f(y,x),x)$ as this will give 0, because we basically removed the variable of the differentiation.

150 How to find a pattern inside an expression

Given $3e^x + \sin(ae^x)f(e^{5x})$ how to find all terms with pattern `anything*exp`? If we do this

```
expr:=3*exp(x)+sin(a*exp(x))*f(exp(5*x));
indets(expr, `&*&`(anything, 'specfunc(exp)'));

#      {a*exp(x), 3*exp(x)}
```

We see it did not find `exp(5*x)` this is because there is nothing multiplying the `exp` function. To find this we add `Or` like this to count for both cases

```
expr:=3*exp(x)+sin(a*exp(x))*f(exp(5*x));
indets[flat](expr, Or( `&*&`(anything, 'specfunc(exp)'), 'specfunc(exp)' ))

#      {a*exp(x), 3*exp(x), exp(5*x)}
```

The `flat` option is needed, as without it this will be the result

```
expr:=3*exp(x)+sin(a*exp(x))*f(exp(5*x));
```

```
indets(expr, Or( `&*` (anything, 'specfunc(exp)') , 'specfunc(exp)' ))
#      {a*exp(x), 3*exp(x), exp(x), exp(5*x)}
```

151 How to find parts of a Sum?

Given an inert Sum such as

$$r = \sum_{n=2}^{\infty} a_{n-1} x^{n-1}$$

How to obtain the body of the sum, the index variable, the lower starting value and the upper limit?

```
r:=Sum(a[n - 1]*x^(n - 1), n = 2 .. infinity);
op(0,r) #head
                Sum
op(1,r) #body
                a[n - 1] x^(n - 1)
op(2,r) # sum specs
                n = 2 .. infinity
lhs(op(2,r)) #name of summation index
                n
rhs(op(2,r)) # lower..upper limits
                2 .. infinity
op(1,rhs(op(2,r))) #lower limit
                2
op(2,rhs(op(2,r))) #upper limit
                infinity
```

152 find if sequence or list is inside another and the indecies

Given a 1D container, such as vector or list, called A , how to find if this sequence is inside another sequence say B and the indecies in B where the sequence A is located?

Use `ArrayTools` in version 2023

```
A:=[1,3,5]; B:=[1,3,4,5];

status,indices_list := ArrayTools:-IsSubsequence( A, B , 'output' = ['check','indices']
if status then
    print("Sequence ",A," Was found in ",B," At indices ",indices_list);
else
    print("Sequence ",A," Was not found in ",B);
fi;
```

The above gives

```
"Sequence ", [1, 3, 5], " Was found in ", [1, 3, 4, 5], " At indices ", [1, 2, 4]
```

If A was not sequence inside B , then status will be false otherwise.

153 find if some type inside some expression and its location

Use `membertype`

```
restart;
expr:=cos(x)+x+9+sin(x)*(x^2+4);
membertype( polynom(anything,x), expr, 'loc' );
    true
loc;
    2
```

The above says there is a polynomial in x inside the expression at `op(2,expr)` notice that $x^2 + 4$ is not a polynomial since the expression will expand and $\sin x$ will be multiplied by it causing it not to become polynomial. So only $9 + x$ is the polynomial.

The location is where the member starts at. Notice that Maple sorts polynomial from lower to higher powers.

```
restart;
expr:=cos(x)+x+9+sin(x)*(x^2+4);
membertype( integer, expr, 'loc' );
      true
loc;
      3
```

The above says that there is an integer (which is 9 in this example) inside the expression at `op(3,expr)`

154 Change the summation index letter

I noticed that Maple returns the summation index variable using leading underscore as in `_n` or `_m` which makes the latex looks not as good. Here is an example

```
restart;
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),'formal_series');

y(x) = _C1*Sum((-1/2 - sqrt(3)*I/2)^_n*x^_n/_n!, _n = 0 .. infinity) +
      _C2*Sum((-1/2 + sqrt(3)*I/2)^_n*x^_n/_n!, _n = 0 .. infinity)
```

The latex of the above is

$$y(x) = -C1 \left(\sum_{-n=0}^{\infty} \frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^{-n} x^{-n}}{-n!} \right) + -C2 \left(\sum_{-n=0}^{\infty} \frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{-n} x^{-n}}{-n!} \right)$$

Not seeing an option to change `_n` to `n`, I wrote the following function which takes in the solution, use `subsindets` and remove the leading underscore.

This is the above example showing how to use the function

```
restart;

fix_summation_index:=proc(expr)
local fix_it:=proc(the_sum)
  local the_letter::symbol,the_new_letter::symbol,the_letter_as_string::string;
  the_letter:= op([2,1],the_sum);
```

```

the_letter_as_string:=String(the_letter);
if the_letter_as_string[1]="_" then
  the_new_letter:=parse(the_letter_as_string[2..]);
  RETURN(subs(the_letter=the_new_letter,the_sum));
else
  RETURN(the_sum);
fi;
end proc;

if not(has(expr,Sum)) then
  RETURN(expr);
else
  RETURN(subsindets( expr, 'specfunc( anything, Sum )', f->fix_it(f)));
fi;

end proc;

sol:=dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),'formal_series'):
sol:=fix_summation_index(sol);

y(x) = _C1*Sum((-1/2 - sqrt(3)*I/2)^n*x^n/n!, n = 0 .. infinity) +
      _C2*Sum((-1/2 + sqrt(3)*I/2)^n*x^n/n!, n = 0 .. infinity)

```

The latex now is

$$y(x) = -C1 \left(\sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^n x^n}{n!} \right) + -C2 \left(\sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^n x^n}{n!} \right)$$

155 How to replace generic function inside derivative?

I had case where I wanted to substitute $_F=(y-x^2)/x$ in an expression obtained which is $x*\text{diff}(F1, x) + 2*y*\text{diff}(F1, y)$

Using eval does not work

```

restart;
eval(x*diff(F1, x) + 2*y*diff(F1, y),F1=(y-x^2)/x);

```

```
0
```

Also using delayed does not work

```
restart;  
eval('x*diff(F1, x) + 2*y*diff(F1, y)', F1=(y-x^2)/x);  
value(%)  
0
```

ALso using subs does not work

```
restart;  
subs(F1=(y-x^2)/x, x*diff(F1, x) + 2*y*diff(F1, y))  
0
```

However, delayed with subs finally worked

```
restart;  
subs(F1=(y-x^2)/x, 'x*diff(F1, x) + 2*y*diff(F1, y)');  
value(%)  
  
x*(-2 - (-x^2 + y)/x^2) + 2*y/x
```

So the rule is, if you want to replace a function inside diff, use subs and not eval, and make sure to delay evaluation of the expression and then use value() to obtain the final result.

156 Examples how to match types

These are examples how to match expression types

1. expr is product $a*b$ which is matched using

```
type(a*b, `&*&`(anything, anything))  
true
```

The above can also be written using infix notation

```
type(a*b, anything &* anything)  
true
```

However, the above only match product of two terms. To match $3*a*b$ use

```
type(3*a*b, `*`)  
true
```

2. expr is division a/b which is matched using same as `*`

```
type(a/b, `&*` (anything, anything))  
true
```

3. Match type of $a*f(x)$, which is anything times a function that takes one argument x .

```
type(a*f(x), `&*` (anything, patfunc(identical(x), anything)))  
true
```

Using `patfunc` is better than using `function`

```
type(a*f(x,y), `&*` (anything, function(identical(x))))  
true
```

Because `patfunc` matches on $f(x,y,z,\dots)$ and not just function which takes only one argument $f(x)$. But if you know your function takes only one argument, then use `function`

4. To match $3*y/x$. This was tricky. Had to use

```
expr:=3*y/x;  
type(expr, `&*` (anything, identical(y), `^` (identical(x), -1)))  
or  
type(expr, `&*` (identical(y), `^` (identical(x), -1)))  
  
true
```

I used `or` to account for possible `expr:=y/x`, i.e. missing constant at front. Note that

```
expr:=3*y/x;  
type(3*y/x, `&*` (anything, identical(y/x)))  
false
```

Does not match, since $3*y/x$ is actually $3*y$ times $1/x$ internally.

5. Match on $f(b* y/x)$

```
type(f(3*y/x),function(`&*`(anything,identical(y),`~`(identical(x),-1))))  
  
true
```

6. Match on $f(b* y/x)$ or $f(y/x)$

```
expr:=f(3*y/x);  
type(expr, function(`&*`(anything,identical(y),`~`(identical(x),-1))))  
or  
type(expr, function(`&*`(identical(y),`~`(identical(x),-1))))  
  
true
```

Again, had to use `or` to account for missing constant multiplier.

7. Match on $a*f(b* y/x)$

```
expr:=a*f(3*y/x);  
type(expr, `&*`(anything,patfunc(`&*`(anything,identical(y),`~`(identical(x),-1))))  
true
```

8. Match against $9+3 y/x$

```
expr:=9+3*y/x;  
selector:=`&+`(anything,`&*`(anything,identical(y),`~`(identical(x),-1))):  
type(expr, selector);  
  
true  
  
select(type, [expr], selector);  
[9 + 3*y/x]
```

Notice in the above, when using `select` we need to put the expression inside a list, as `select` looks at each operand, This way the whole expression is taken as one. If we just used `select(type,expr,selector)`; it would not have found it.

To use `patmatch` the command becomes

```
patmatch(expr, a::anything+b::anything*y/x, 'la');
la

[a = 9, b = 3]
```

The nice thing about patmatch is that it allowed one to assign variable to parts of the expression automatically.

- Match against $9+f(3 y/x)$ where now f is function. Using patmatch. I could not do this in one command, as all my attempts failed:

```
expr:=3+4*x*f(3*y/x);
body_of_function:=C::anything*y/x;
patmatch(expr,A::anything+B::anything*F::function(C::anything*y/x),'la');
patmatch(expr,A::anything+B::anything*F::patfunc(C::anything*y/x),'la');

Error, (in PatternMatching:-AlgStruct:-Match) testing against an invalid type
Error, (in type/patfunc) testing against an invalid type
```

So I had to do it in two steps. First match on the function as whole, then use that to match on $f(3*y/x)$ in second stage, like this

```
expr:=3+4*x*f(3*y/x);
patmatch(expr,A::anything+B::anything*F::function(anything),'la');
la;

[A = 3, B = 4*x, F = f(3*y/x)]
```

And now

```
assign(la);
A:='A';
patmatch (op(1,F),A::anything*y/x,'la');
la

[A = 3]
```

Overall, I find Mathematica's pattern matching constructs much simpler and more intuitive to use and easier to learn as there are many examples and tutorials. For example, the last example above in Mathematica could be done as follows

```

expr=9+(4*x)*f[3*y/x];
Cases[{expr},any0_.*any1_.*any2_[any3_.*y/x]:>{any0,any1,any2,any3}]

{{9, 4 x, f, 3}}

```

Maple's help pages are not good at all and provide little or no examples to learn from compared to Mathematica's excellent help pages. For any serious pattern matching tasks, I would use Mathematica. Maple has a better debugger and hence easier to debug the code because of this. So it is a tradeoff between these two systems.

157 On the order of terms when using indets

I was trying to match on anyfunction that has x inside its arguments. It turned out that matching $f(ax)$ vs. $f(a+x)$ needed to have the `identical(x)` being placed first when it is a sum and last when it is a product. Very strange. Just be aware of this

```

indets(f(a+x),function(`&+`(anything,identical(x))));
#failed
{}

```

But

```

indets(f(a+x),function(`&+`(identical(x),anything)));

{f(x + a)}

```

While with product, it is the other way around

```

indets(f(a*x),function(`&*`(anything,identical(x))));

{f(a*x)}

```

But now this fail

```

indets(f(a*x),function(`&*`(identical(x),anything)));
#fail
{}

```

I have not yet figure how to tell it that the order does not matter. Maple 2023.1

A better way than the above, if I want to find any function that takes in x or y as arguments is to use `patfunc` like this

```
indets(f(a*x+b*y), patfunc(anything, `Or`({identical(x), identical(y)})));  
{f(a*b + y)}
```

```
indets(f(b+y), patfunc(anything, `Or`({identical(x), identical(y)})));  
{f(b + y)}
```

```
indets(exp(a*x), patfunc(anything, `Or`({identical(x), identical(y)})));  
{exp(a*x)}
```

```
indets(exp(a+x), patfunc(anything, `Or`({identical(x), identical(y)})));  
{exp(x + a)}
```