my Quantum Mechanics cheat sheet

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Table 1: QM cheat sheet

	Position Operator X	Momentum operator P	Hamilitonian operator H
Eigenvalue eigenvector relation	$X\|x\rangle = x\|x\rangle$ where x is the eigenvalue (size) of the $\|x\rangle$ which is the position vector associated with x measured.	$P\|\phi_p\rangle = p\|\phi_p\rangle$ where p is the momentum of the particle.	$H\|\Psi_{E_i}\rangle = E_i\|\Psi_{E_i}\rangle$ where E_i is the energy level of the particle.
Normalization relation	$\int_{-\infty}^{\infty} \|x\rangle \langle x\| \ dx = 1$	$\int_{-\infty}^{\infty} \|\phi_p\rangle \langle \phi_p\| \ dp = 1$	$\int_{-\infty}^{\infty} \|\Psi_{E_i}\rangle \langle \Psi_{E_i}\| \ dE = 1$
orthogonality	$\langle x\|x'\rangle = \delta(x-x')$	$\langle \phi_p\|\phi_{p'}\rangle = \delta(p-p')$	$\langle \Psi_{E_i}\|\Psi_{E_j}\rangle = \delta(E_i-E_j)$
Matrix element of operator	$\langle x \mid X \mid x'\rangle = x'\delta(x-x')$. Operator X is diagonal matrix.	$\langle x\|P\|x'\rangle = -i\hbar\delta(x-x')\frac{d}{dx'}$ where momentum operator P is expressed in position operator $\|x\rangle$ basis. Note that operator P is not a diagonal matrix.	$\langle x \mid H \mid x'\rangle =?$

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	Position Operator X	Momentum operator Φ_p	Hamiltonian operator
Function form of the state function $\|\Psi\rangle$	N/A ?	$P\|\phi_p\rangle = p\|\phi_p\rangle$ $\int P\|x'\rangle \langle x'\|\phi_p\rangle \; dx = p\int \|x'\rangle \langle x'\|\phi_p\rangle \; dx$ $\int \langle x\|P\|x'\rangle \langle x'\|\phi_p\rangle \; dx = p\int \langle x\|x'\rangle \langle x'\|\phi_p\rangle \; dx$ $\int -i\hbar\delta(x-x')\frac{d}{dx'}\phi_p(x')\,dx = p\int \delta(x-x')\phi_p(x')\,dx$ $= \frac{2}{L}\frac{L}{2}$ $= 1$	$\langle\Psi\|\Psi\rangle = \int_{-\infty}^{\infty} \langle\Psi\|x\rangle \langle x\|\Psi\rangle \; dx$ $= \int_{-\infty}^{\infty} \langle x\|\Psi\rangle \langle x\|\Psi\rangle \; dx$ $= \int_{-\infty}^{\infty} \Psi^*(x)\Psi(x)\,dx$ $= \int_0^L \left(\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\right)^2 \; dx$ $= \frac{2}{L}\frac{L}{2}$ $= 1$
Vector form to function form	$\langle x \mid \Psi\rangle = \Psi(x)$	$\langle x \mid \phi_p\rangle = \phi_p(x)$	$\langle x \mid \Psi_E\rangle = \Psi_E(x)$
Expansion of state vector $\|\Psi\rangle$	$\|\Psi\rangle = \int_{-\infty}^{\infty} \|x\rangle \langle x\|\Psi\rangle \; dx$	$\|\Psi\rangle = \int_{-\infty}^{\infty} \|\phi_p\rangle \langle \phi_p\|\Psi\rangle \; dp$	$\|\Psi\rangle = \int_{-\infty}^{\infty} \|E_i\rangle \langle E_i\|\Psi\rangle \; di$
State function $\|\Psi\rangle$ For infinite potential deep well of width $x < 0 < L$	todo	todo	todo

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Table1 – continued from previous page

	Position Operator X	Momentum operator Φ_p	Hamilitonian operator
Probability of measurement	1. Probability of measuring x given system is in state $\|\Psi\rangle$ is $\|\langle\Psi\|\Psi\rangle\|^2$. For infinite potential deep well of width $x < 0 < L$ this becomes $\langle\Psi\|\Psi\rangle = \int_{-\infty}^{\infty} \langle\Psi\|x\rangle \langle x\|\Psi\rangle \, dx$ $= \int_{-\infty}^{\infty} \langle x\|\Psi\rangle \langle x\|\Psi\rangle \, dx$ $= \int_{-\infty}^{\infty} \Psi^*(x)\Psi(x) \, dx$ $= \int_0^L \left(\sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}\right)^2 dx$ $= \frac{2}{L}\frac{L}{2}$ $= 1$ 2. Probability of measuring x given system is in state ϕ_p	$\|\Psi\rangle = \int_{-\infty}^{\infty} \|\phi_p\rangle \langle\phi_p\|\Psi\rangle \, dp$	$\|\Psi\rangle = \int_{-\infty}^{\infty} \|E_i\rangle \langle E_i\|\Psi\rangle \, di$