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Generating the four Kharitonov polynomials and displaying corresponding Hurwitz stability matrix

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Introduction

Software written in Mathematica to generate the four Kharitonov's polynomials from the interval polynomial specification and construct the four Hurwitz stability matrices to test for stability of each polynomial. Examples from chapter 5, "New tools for robustness of linear systems" by Professor B. Ross Barmish are used for illustration.

- **Example 5.5.2**

This function takes interval polynomal and generates the 4 Kharitonov's polynomials

In[141]:=
```
(p = kharitonovPoly[{{11, 12}, {9, 10}, {7, 8}, {5, 6}, {3, 4}, {1, 2}}, s]) // TableForm
```

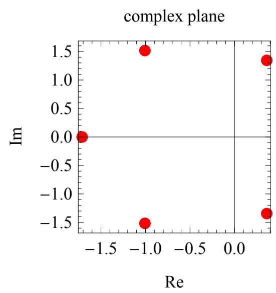
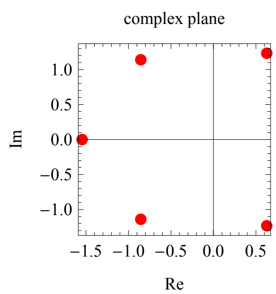
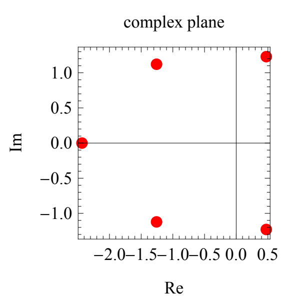
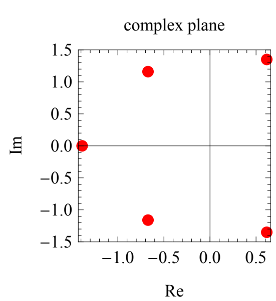
Out[141]//TableForm=

$11 + 9 s + 8 s^2 + 6 s^3 + 3 s^4 + s^5$
$12 + 10 s + 7 s^2 + 5 s^3 + 4 s^4 + 2 s^5$
$12 + 9 s + 7 s^2 + 6 s^3 + 4 s^4 + s^5$
$11 + 10 s + 8 s^2 + 5 s^3 + 3 s^4 + 2 s^5$

This function takes the result and generate the Hurwitz matrix and root locations. The polynomial is stable when all leading minors are positive.

In[178]:= `displayHurwitz[p, s]`

Out[178]=

Hurwitz Matrix	Δ_i	root locations	Real part of roots
$s^5 + 3 s^4 + 6 s^3 + 8 s^2 + 9 s + 11$ $\begin{pmatrix} 9 & 6 & 1 & 0 & 0 \\ 11 & 8 & 3 & 0 & 0 \\ 0 & 9 & 6 & 1 & 0 \\ 0 & 11 & 8 & 3 & 0 \\ 0 & 0 & 9 & 6 & 1 \end{pmatrix}$	Δ_1 = 9 Δ_2 = 6 Δ_3 = -108 Δ_4 = -196 Δ_5 = -196		-1.71185 -1.00624 -1.00624 0.362161 0.362161
$2 s^5 + 4 s^4 + 5 s^3 + 7 s^2 + 10 s + 12$ $\begin{pmatrix} 10 & 5 & 2 & 0 & 0 \\ 12 & 7 & 4 & 0 & 0 \\ 0 & 10 & 5 & 2 & 0 \\ 0 & 12 & 7 & 4 & 0 \\ 0 & 0 & 10 & 5 & 2 \end{pmatrix}$	Δ_1 = 10 Δ_2 = 10 Δ_3 = -110 Δ_4 = -196 Δ_5 = -392		-1.5452 -0.854858 -0.854858 0.627459 0.627459
$s^5 + 4 s^4 + 6 s^3 + 7 s^2 + 9 s + 12$ $\begin{pmatrix} 9 & 6 & 1 & 0 & 0 \\ 12 & 7 & 4 & 0 & 0 \\ 0 & 9 & 6 & 1 & 0 \\ 0 & 12 & 7 & 4 & 0 \\ 0 & 0 & 9 & 6 & 1 \end{pmatrix}$	Δ_1 = 9 Δ_2 = -9 Δ_3 = -270 Δ_4 = -729 Δ_5 = -729		-2.43433 -1.25737 -1.25737 0.474539 0.474539
$2 s^5 + 3 s^4 + 5 s^3 + 8 s^2 + 10 s + 11$ $\begin{pmatrix} 10 & 5 & 2 & 0 & 0 \\ 11 & 8 & 3 & 0 & 0 \\ 0 & 10 & 5 & 2 & 0 \\ 0 & 11 & 8 & 3 & 0 \\ 0 & 0 & 10 & 5 & 2 \end{pmatrix}$	Δ_1 = 10 Δ_2 = 25 Δ_3 = 45 Δ_4 = -89 Δ_5 = -178		-1.3877 -0.672514 -0.672514 0.616365 0.616365

■ Example 5.6.2

In[179]:=
```
(p = kharitonovPoly[{{0.25, 1.25}, {0.75, 1.25}, {2.75, 3.25}, {0.25, 1.25}}, s]) // TableForm
```

Out[179]//TableForm=

$0.25 + 0.75\, s + 3.25\, s^2 + 1.25\, s^3$

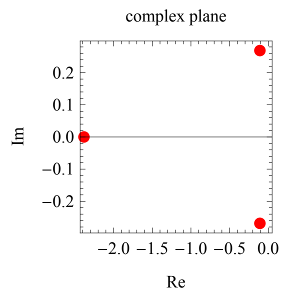
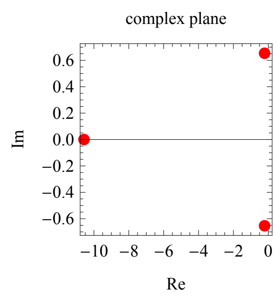
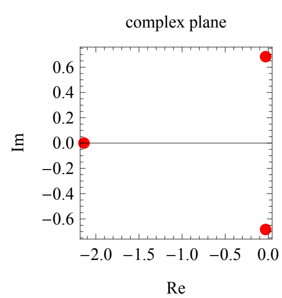
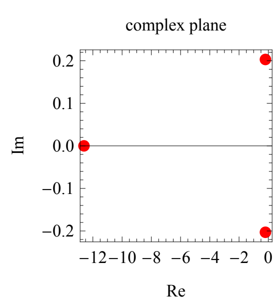
$1.25 + 1.25\, s + 2.75\, s^2 + 0.25\, s^3$

$1.25 + 0.75\, s + 2.75\, s^2 + 1.25\, s^3$

$0.25 + 1.25\, s + 3.25\, s^2 + 0.25\, s^3$

In[180]:= **displayHurwitz[p, s]**

Out[180]=

Hurwitz Matrix	Δ_i	root locations	Real part of roots
$1.25\, s^3 + 3.25\, s^2 + 0.75\, s + 0.25$ $\begin{pmatrix} 0.75 & 1.25 & 0 \\ 0.25 & 3.25 & 0 \\ 0 & 0.75 & 1.25 \end{pmatrix}$	Δ_1 0.75 Δ_2 2.125 Δ_3 2.65625		-2.38347 -0.108264 -0.108264
$0.25\, s^3 + 2.75\, s^2 + 1.25\, s + 1.25$ $\begin{pmatrix} 1.25 & 0.25 & 0 \\ 1.25 & 2.75 & 0 \\ 0 & 1.25 & 0.25 \end{pmatrix}$	Δ_1 1.25 Δ_2 3.125 Δ_3 0.78125		-10.5718 -0.21411 -0.21411
$1.25\, s^3 + 2.75\, s^2 + 0.75\, s + 1.25$ $\begin{pmatrix} 0.75 & 1.25 & 0 \\ 1.25 & 2.75 & 0 \\ 0 & 0.75 & 1.25 \end{pmatrix}$	Δ_1 0.75 Δ_2 0.5 Δ_3 0.625		-2.13812 -0.0309384 -0.0309384
$0.25\, s^3 + 3.25\, s^2 + 1.25\, s + 0.25$ $\begin{pmatrix} 1.25 & 0.25 & 0 \\ 0.25 & 3.25 & 0 \\ 0 & 1.25 & 0.25 \end{pmatrix}$	Δ_1 1.25 Δ_2 4. Δ_3 1.		-12.6098 -0.195114 -0.195114

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■ Example 5.10.1

In[181]:=
```
(p = kharitonovPoly[{{0.45, 0.55}, {1.95, 2.05}, {2.95, 3.05},
     {5.95, 6.05}, {3.95, 4.05}, {3.95, 4.05}, {1, 1}}, s]) // TableForm
```

Out[181]//TableForm=

$0.45 + 1.95\, s + 3.05\, s^2 + 6.05\, s^3 + 3.95\, s^4 + 3.95\, s^5 + s^6$

$0.55 + 2.05\, s + 2.95\, s^2 + 5.95\, s^3 + 4.05\, s^4 + 4.05\, s^5 + s^6$

$0.55 + 1.95\, s + 2.95\, s^2 + 6.05\, s^3 + 4.05\, s^4 + 3.95\, s^5 + s^6$

$0.45 + 2.05\, s + 3.05\, s^2 + 5.95\, s^3 + 3.95\, s^4 + 4.05\, s^5 + s^6$

In[182]:=
```
displayHurwitz[p, s]
```

Out[182]=

<table>
<tr><th>Hurwitz Matrix</th><th>Δ_i</th><th>root locations</th><th>Real part of root</th></tr>
<tr>
<td>

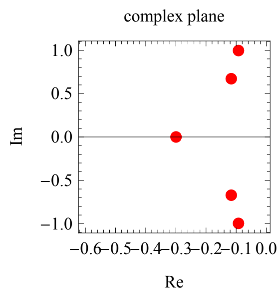
$s^6 + 3.95\, s^5 + 3.95\, s^4 + 6.05\, s^3 + 3.05\, s^2 + 1.95\, s + 0.45$

$$\begin{pmatrix} 1.95 & 6.05 & 3.95 & 0 & 0 & 0 \\ 0.45 & 3.05 & 3.95 & 1 & 0 & 0 \\ 0 & 1.95 & 6.05 & 3.95 & 0 & 0 \\ 0 & 0.45 & 3.05 & 3.95 & 1 & 0 \\ 0 & 0 & 1.95 & 6.05 & 3.95 & 0 \\ 0 & 0 & 0.45 & 3.05 & 3.95 & 1 \end{pmatrix}$$

</td>
<td>

Δ_1 1.95
Δ_2 3.225
Δ_3 7.9575
Δ_4 9.39937
Δ_5 6.41034
Δ_6 6.41034

</td>
<td>



</td>
<td>-3.2334
-0.299508
-0.116271
-0.116271
-0.0922772
-0.0922772</td>
</tr>
<tr>
<td>

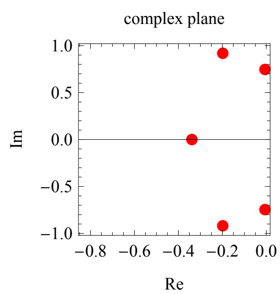
$s^6 + 4.05\, s^5 + 4.05\, s^4 + 5.95\, s^3 + 2.95\, s^2 + 2.05\, s + 0.55$

$$\begin{pmatrix} 2.05 & 5.95 & 4.05 & 0 & 0 & 0 \\ 0.55 & 2.95 & 4.05 & 1 & 0 & 0 \\ 0 & 2.05 & 5.95 & 4.05 & 0 & 0 \\ 0 & 0.55 & 2.95 & 4.05 & 1 & 0 \\ 0 & 0 & 2.05 & 5.95 & 4.05 & 0 \\ 0 & 0 & 0.55 & 2.95 & 4.05 & 1 \end{pmatrix}$$

</td>
<td>

Δ_1 2.05
Δ_2 2.775
Δ_3 4.0575
Δ_4 2.49938
Δ_5 0.404656
Δ_6 0.404656

</td>
<td>



</td>
<td>-3.3032
-0.338496
-0.1981
-0.1981
-0.00605111
-0.00605111</td>
</tr>
<tr>
<td>

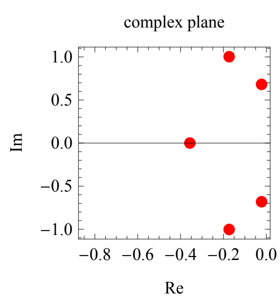
$s^6 + 3.95\, s^5 + 4.05\, s^4 + 6.05\, s^3 + 2.95\, s^2 + 1.95\, s + 0.55$

$$\begin{pmatrix} 1.95 & 6.05 & 3.95 & 0 & 0 & 0 \\ 0.55 & 2.95 & 4.05 & 1 & 0 & 0 \\ 0 & 1.95 & 6.05 & 3.95 & 0 & 0 \\ 0 & 0.55 & 2.95 & 4.05 & 1 & 0 \\ 0 & 0 & 1.95 & 6.05 & 3.95 & 0 \\ 0 & 0 & 0.55 & 2.95 & 4.05 & 1 \end{pmatrix}$$

</td>
<td>

Δ_1 1.95
Δ_2 2.425
Δ_3 3.5075
Δ_4 3.11438
Δ_5 2.34509
Δ_6 2.34509

</td>
<td>



</td>
<td>-3.20234
-0.356709
-0.173359
-0.173359
-0.0221182
-0.0221182</td>
</tr>
<tr>
<td>

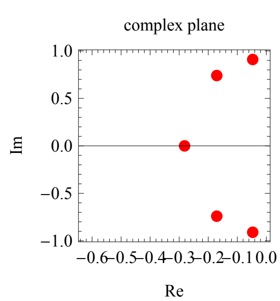
$s^6 + 4.05\, s^5 + 3.95\, s^4 + 5.95\, s^3 + 3.05\, s^2 + 2.05\, s + 0.45$

$$\begin{pmatrix} 2.05 & 5.95 & 4.05 & 0 & 0 & 0 \\ 0.45 & 3.05 & 3.95 & 1 & 0 & 0 \\ 0 & 2.05 & 5.95 & 4.05 & 0 & 0 \\ 0 & 0.45 & 3.05 & 3.95 & 1 & 0 \\ 0 & 0 & 2.05 & 5.95 & 4.05 & 0 \\ 0 & 0 & 0.45 & 3.05 & 3.95 & 1 \end{pmatrix}$$

</td>
<td>

Δ_1 2.05
Δ_2 3.575
Δ_3 8.4075
Δ_4 7.81438
Δ_5 2.68991
Δ_6 2.68991

</td>
<td>



</td>
<td>-3.33369
-0.281521
-0.17061
-0.17061
-0.0467823
-0.0467823</td>
</tr>
</table>

▪ page 71 example, in conlcusion

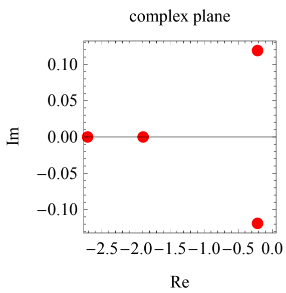
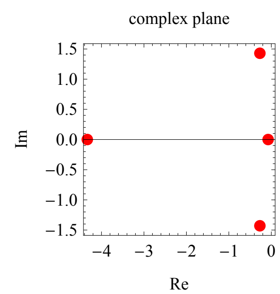
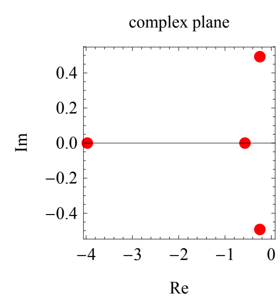
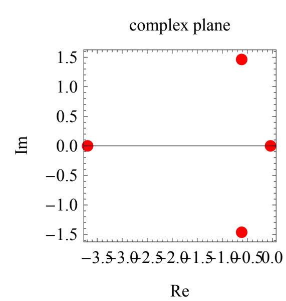
In[183]:=
```
(p = kharitonovPoly[{{0.3125, 0.6875}, {2.5, 9.5},
      {4.8125, 7.1875}, {4.9475, 5.0375}, {1, 1}}, s]) // TableForm
```

Out[183]//TableForm=

$0.3125 + 2.5\, s + 7.1875\, s^2 + 5.0375\, s^3 + s^4$
$0.6875 + 9.5\, s + 4.8125\, s^2 + 4.9475\, s^3 + s^4$
$0.6875 + 2.5\, s + 4.8125\, s^2 + 5.0375\, s^3 + s^4$
$0.3125 + 9.5\, s + 7.1875\, s^2 + 4.9475\, s^3 + s^4$

In[184]:= `displayHurwitz[p, s]`

Out[184]=

<table>
<tr><th>Hurwitz Matrix</th><th>Δ_i</th><th>root locations</th><th>Real part of roots</th></tr>
<tr>
<td>$s^4 + 5.0375\, s^3 + 7.1875\, s^2 + 2.5\, s + 0.3125$
$\begin{pmatrix} 2.5 & 5.0375 & 0 & 0 \\ 0.3125 & 7.1875 & 1 & 0 \\ 0 & 2.5 & 5.0375 & 0 \\ 0 & 0.3125 & 7.1875 & 1 \end{pmatrix}$</td>
<td>Δ_1 2.5
Δ_2 16.3945
Δ_3 76.3375
Δ_4 76.3375</td>
<td>
</td>
<td>-2.70998
-1.89535
-0.216088
-0.216088</td>
</tr>
<tr>
<td>$s^4 + 4.9475\, s^3 + 4.8125\, s^2 + 9.5\, s + 0.6875$
$\begin{pmatrix} 9.5 & 4.9475 & 0 & 0 \\ 0.6875 & 4.8125 & 1 & 0 \\ 0 & 9.5 & 4.9475 & 0 \\ 0 & 0.6875 & 4.8125 & 1 \end{pmatrix}$</td>
<td>Δ_1 9.5
Δ_2 42.3173
Δ_3 119.115
Δ_4 119.115</td>
<td>
</td>
<td>-4.33442
-0.269038
-0.269038
-0.0750017</td>
</tr>
<tr>
<td>$s^4 + 5.0375\, s^3 + 4.8125\, s^2 + 2.5\, s + 0.6875$
$\begin{pmatrix} 2.5 & 5.0375 & 0 & 0 \\ 0.6875 & 4.8125 & 1 & 0 \\ 0 & 2.5 & 5.0375 & 0 \\ 0 & 0.6875 & 4.8125 & 1 \end{pmatrix}$</td>
<td>Δ_1 2.5
Δ_2 8.56797
Δ_3 36.9111
Δ_4 36.9111</td>
<td>
</td>
<td>-3.9738
-0.568926
-0.247385
-0.247385</td>
</tr>
<tr>
<td>$s^4 + 4.9475\, s^3 + 7.1875\, s^2 + 9.5\, s + 0.3125$
$\begin{pmatrix} 9.5 & 4.9475 & 0 & 0 \\ 0.3125 & 7.1875 & 1 & 0 \\ 0 & 9.5 & 4.9475 & 0 \\ 0 & 0.3125 & 7.1875 & 1 \end{pmatrix}$</td>
<td>Δ_1 9.5
Δ_2 66.7352
Δ_3 239.922
Δ_4 239.922</td>
<td>
</td>
<td>-3.69136
-0.611201
-0.611201
-0.033736</td>
</tr>
</table>