

iterative solution to differential equations diagrams

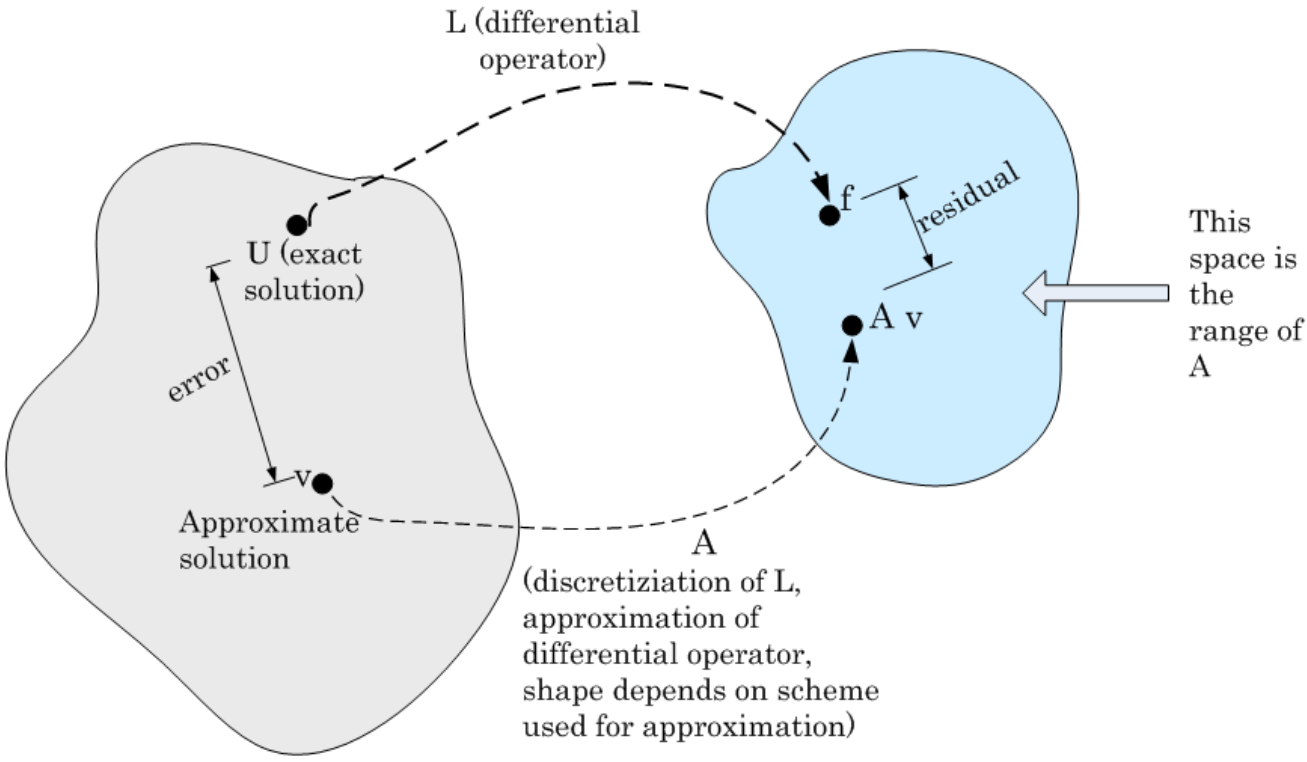
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Diagrams to help illustrate some concepts in iterative solution to differential equations.

$$Lu=f$$

Analog problem (hard)



$$Av=f$$

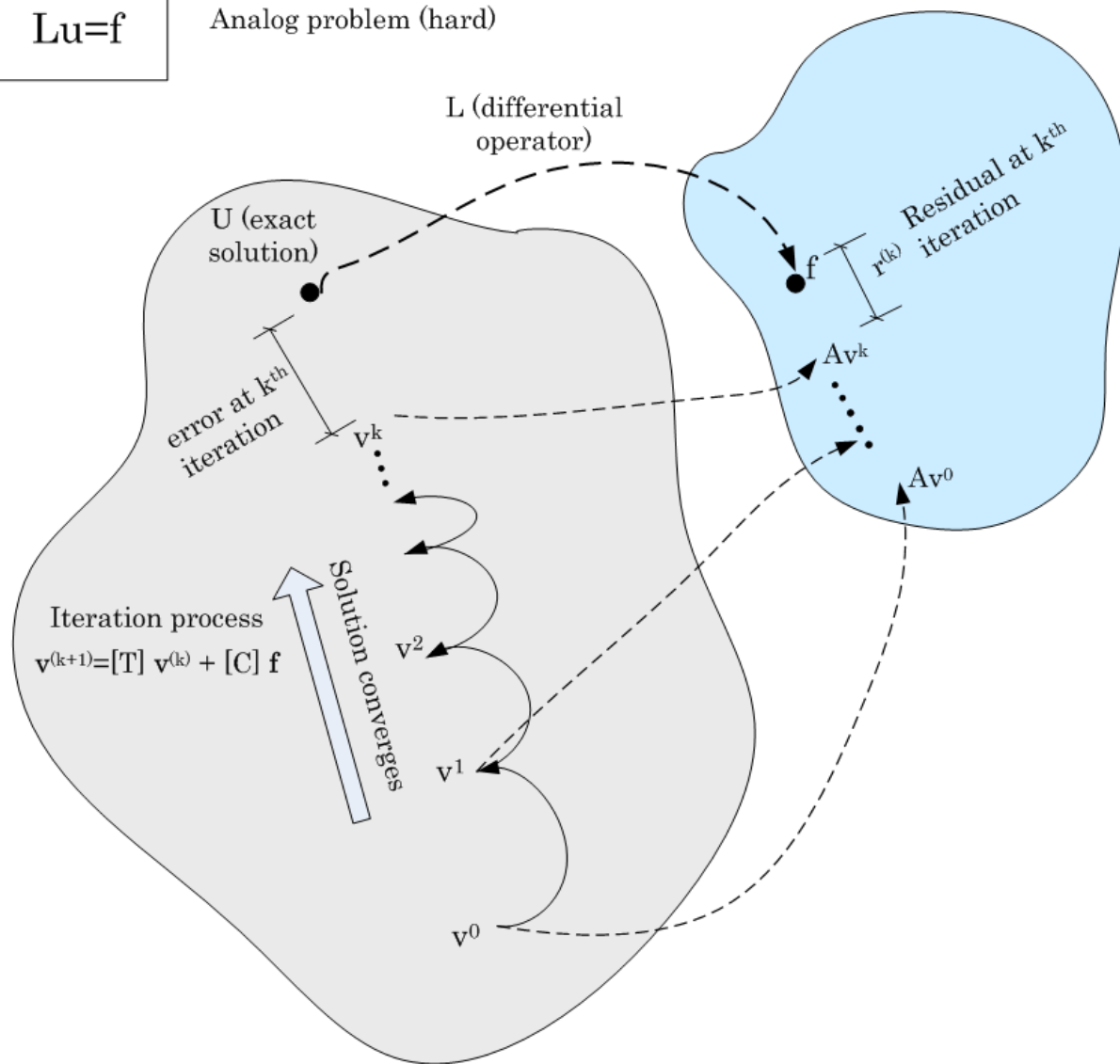
Discrete problem (easier)

It is required to solve $Lu=f$.
 In the above problem, the following are known (given): f, L
 Algorithm:

1. A is constructed by approximating L using appropriate difference scheme.
2. The problem thus has been translated from $Lu=f$ to an algebraic problem $Av=f$, Now it is solved numerically for v .
3. $f-Av$ is the residual r , $u-v$ is the error e , and $(f-Av)/f$ is the relative residual
4. Note that $Ae=r$

$$Lu=f$$

Analog problem (hard)



It is required to solve $Lu=f$.

In the above problem, the following are known (given): f, L

Algorithm for iterative solution (also called relaxation)

A is not used at all. Only the point by point discretization approximation is used.

LOOP until convergence

 LOOP over all grid points in solution space

 Update each point using point discretization scheme using Jacobian,

 Gauss-Seidel, or SOR etc... method

 END LOOP

END LOOP

Good convergence test to use is to check that relative residual is less than some tolerance.