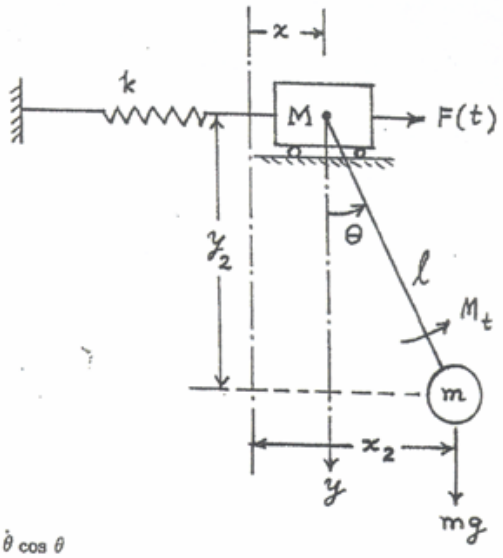
possible error in key solution:

2.



$x_2 = x + \ell \sin\theta \;;\; \dot{x}_2 = \dot{x} + \ell\,\dot{\theta}\cos\theta$

$y_2 = \ell\cos\theta \;;\; \dot{y}_2 = -\ell\,\dot{\theta}\sin\theta$

$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m (\dot{x}_2^2 + \dot{y}_2^2)$$

$$= \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \left[(\dot{x} + \ell\,\dot{\theta}\cos\theta)^2 + (-\ell\,\dot{\theta}\sin\theta)^2\right]$$

$$= \frac{1}{2} (M+m)\,\dot{x}^2 + \frac{1}{2} m \ell^2 \dot{\theta}^2 + m \ell \dot{x}\,\dot{\theta}\cos\theta$$

$$V = \frac{1}{2} k x^2 + m g \ell (1 - \cos\theta)$$

$Q_x = F(t) \;;\; Q_\theta = M_t(t)$

Equations of motion:

$$(M+m)\,\ddot{x} + m\ell\,\ddot{\theta}\cos\theta - m\ell\,\dot{\theta}^2\sin\theta + k x = F(t) \qquad (1)$$

$$m\ell^2\,\ddot{\theta} + m\ell\,\ddot{x}\cos\theta - m\ell\,\dot{x}\,\dot{\theta}\sin\theta + m g \ell \sin\theta = M_t(t) \qquad (2)$$

P2

$$T = \frac{1}{2}(M+m)\,\dot{x}^2 + \frac{1}{2} m L^2 \dot{\theta}^2 + m L \dot{x}\dot{\theta}\cos\theta$$

$$V = \frac{1}{2} k x^2 + m g l (1 - \cos\theta)$$

Now

$$
\begin{aligned}
L &= T - V \\
&= \frac{1}{2}(M+m)\,\dot{x}^2 + \frac{1}{2}mL^2\dot{\theta}^2 + mL\dot{x}\dot{\theta}\cos\theta - \left(\frac{1}{2}kx^2 + mgl(1-\cos\theta)\right) \\
&= \frac{1}{2}(M+m)\,\dot{x}^2 + \frac{1}{2}mL^2\dot{\theta}^2 + mL\dot{x}\dot{\theta}\cos\theta - \frac{1}{2}kx^2 - mgl(1-\cos\theta)
\end{aligned}
$$

EQM for θ show is WRONG. Proof:

$$
\begin{aligned}
\frac{\partial L}{\partial \dot{\theta}} &= mL^2\dot{\theta} + mL\dot{x}\cos\theta \\
\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} &= mL^2\ddot{\theta} + mL\ddot{x}\cos\theta - mL\dot{x}\dot{\theta}\sin\theta
\end{aligned}
$$

and

$$
\frac{\partial L}{\partial \theta} = -mL\dot{x}\dot{\theta}\sin\theta - mgl(\sin\theta)
$$

Hence, EQM is

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} &= M_t \\
mL^2\ddot{\theta} + mL\ddot{x}\cos\theta - mL\dot{x}\dot{\theta}\sin\theta - \left(-mL\dot{x}\dot{\theta}\sin\theta - mgl(\sin\theta)\right) &= M_t \\
mL^2\ddot{\theta} + mL\ddot{x}\cos\theta - mL\dot{x}\dot{\theta}\sin\theta + mL\dot{x}\dot{\theta}\sin\theta + mgl(\sin\theta) &= M_t \\
mL^2\ddot{\theta} + mL\ddot{x}\cos\theta + mgl(\sin\theta) &= M_t
\end{aligned}
$$

Which is NOT the same as shown in the key solution