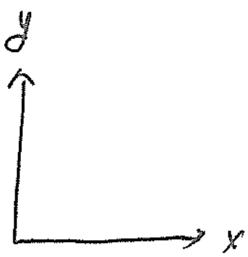
Problem 1.

Two items. the oscillating block and the moving wedge/

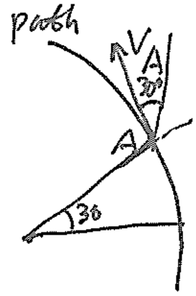
Problem #2.



step 1: 2 particles

step 2: global is rectangular coordinate.

For A: velocity $\begin{cases} \text{speed: } 180\ km/h \\ \text{direction: tangent to the path.} \end{cases}$



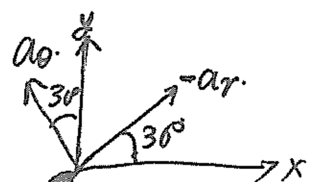
$\therefore \begin{cases} V_{Ax} = -V_A \sin 30° = -180 \cdot \frac{1}{2} = -90\ km/h = -25\ m/s \\ V_{Ay} = V_A \cos 30° = +180 \cdot \frac{\sqrt{3}}{2} = +90\sqrt{3}\ km/h = 43.3\ m/s \end{cases}$

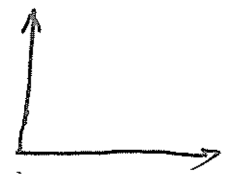
acceleration.

$a_r = \frac{V^2}{r} = \frac{(180\ km/h)^2}{400\ m.} = \frac{(180 \times 1000/3600)^2}{400} = 6.25\ m/s^2$

$a_\theta = \frac{dv}{dt} = -8\ m/s^2.$

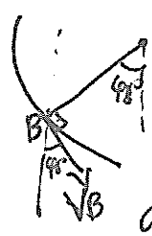
~~Frame~~ The acceleration vector.





$\therefore \begin{cases} a_{Ax} = -a_r \cos 30° - a_\theta \sin 30° \\ \quad = -1.4125\ m/s^2 \\ a_{Ay} = -a_r \sin 30° + a_\theta \cos 30° \\ \quad = -10.053\ m/s^2 \end{cases}$

For B: velocity $\begin{cases} \text{Speed: } 162\ km/h = 45\ m/s \\ \text{direction: tangent to the path.} \end{cases}$

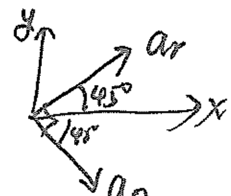


$\begin{cases} V_{Bx} = V_B \cdot \sin 45° = 45 \cdot \frac{\sqrt{2}}{2}\ m/s = 31.815 \\ V_{By} = -V_B \cdot \cos 45° = -45 \cdot \frac{\sqrt{2}}{2}\ m/s = -31.815 \end{cases}$

acceleration:

$a_r = \frac{V^2}{r} = \frac{45^2}{300} = 6.75\ m/s^2$

$a_\theta = \frac{dv}{dt} = 3\ m/s^2$



$\begin{cases} a_{Bx} = a_r \cos 45° + a_\theta \cos 45° = 6.8932\ m/s \\ a_{By} = a_r \sin 45° - a_\theta \sin 45° = 2.6512\ m/s \end{cases}$

Step 3: $\vec{V}_{B/A} = (V_{Bx} - V_{Ax})\hat{x} + (V_{By} - V_{Ay})\hat{y} = 56.815\hat{x} - 75.115\hat{y}$

~~$\vec{a}_{B/A} = (a_{Bx} - a_{Ax})\hat{x} + (a_{By} - a_{Ay})\hat{y}$~~

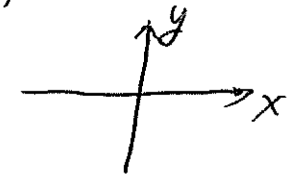
$\vec{a}_{B/A} = (a_{Bx} - a_{Ax})\hat{x} + (a_{By} - a_{Ay})\hat{y}$

$= 8.3057\hat{x} + 12.7042\hat{y}$

Problem #3:

Step 1: Two particles. B and A

Step 2: rectangular Coord.



Step 3: Motion

3.1 Kinematics

$a_{Ay} = 0$ ...... ①

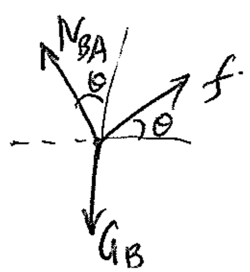
$\vec{a}_{B/A}$ is along the inclined surface of A

$\Rightarrow \dfrac{a_{By} - a_{Ay}}{a_{Bx} - a_{Ax}} = tg\theta = 1.$

$\Rightarrow a_{By} = a_{Bx} - a_{Ax}$ ...... ②

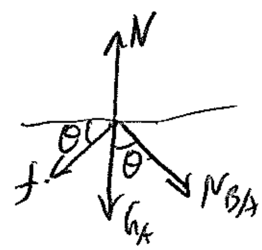
3.2 force analysis.

For B



$$\begin{cases} \Sigma F_{Bx} = f\cos\theta - N_{BA}\sin\theta \quad \cdots\cdots ③ \\ \Sigma F_{By} = f\sin\theta + N_{BA}\cos\theta - G_B \cdots ④ \\ f = \mu N_{BA} \end{cases}$$

For A



$$\begin{cases} \Sigma F_{Ax} = N_{BA}\sin\theta - f\cos\theta \quad \cdots ⑤ \\ \Sigma F_{Ay} = N - f\sin\theta - G_A - N_{BA}\cos\theta \cdots ⑥ \\ f = \mu N_{BA} \end{cases}$$

③ + ⑤ $\Rightarrow \Sigma F_{Bx} + \Sigma F_{Ax} = 0$

3.3 dynamic Eq.

$\Sigma F_{Bx} + \Sigma F_{Ax} = 0 = m_B a_{Bx} + m_A a_{Ax} = 0$ .... ⑦

By ④

$m_B a_{By} = \mu N_{BA} \sin\theta + N_{BA}\cos\theta - G_B$

By ②

$m_B(a_{Bx} - a_{Ax}) = \mu N_{BA}\sin\theta + N_{BA}\cos\theta - G_B$ ...... ⑧

By ⑧

$$m_B a_{BX} = \mu N_{BA}\cos\theta - N_{BA}\sin\theta \quad \text{----} \quad (9)$$

By ⑦ and ⑧ ~~and ⑨~~

$$m_B\left(a_{BX} + \frac{m_B a_{BX}}{m_A}\right) = \mu N_{BA}\sin\theta + N_{BA}\cos\theta - G_B$$

$$\Rightarrow m_B\left(1+\frac{m_B}{m_A}\right)a_{BX} = \mu N_{BA}\sin\theta + N_{BA}\cos\theta - G_B \quad \text{——} \quad (10)$$

By ⑨ and ⑩

$$m_B\left(1+\frac{m_B}{m_A}\right)a_{BX} = (\mu\sin\theta + \cos\theta)\frac{m_B a_{BX}}{\mu\cos\theta - \sin\theta} - G_B$$

$$\boxed{a_{BX} = \frac{G_B}{\frac{\mu\sin\theta+\cos\theta}{\mu\cos\theta-\sin\theta}m_B - m_B\left(1+\frac{m_B}{m_A}\right)} = \frac{g}{\frac{\mu\sin\theta+\cos\theta}{\mu\cos\theta-\sin\theta} - \left(1+\frac{m_B}{m_A}\right)} = \frac{g}{\frac{\mu+1}{\mu-1} - \left(1+\frac{1}{2}\right)} = \frac{g}{-\frac{1.3}{0.7} - 1.5} = -0.29g}$$

By eq ⑦

$$\boxed{a_{AX} = -\frac{m_B}{m_A}a_{BX} = -\frac{1}{2}\cdot a_{BX} = 0.145g}$$

By ②

$$\boxed{a_{By} = a_{BX} - a_{AX} = (-0.29 - 0.145)g = -0.435g}$$

By ⑥

$$N = f\sin\theta + G_A + N_{BA}\cos\theta = \mu N_{BA}\overset{\sin\theta}{} + G_A + N_{BA}\cos\theta$$

By ⑨

$$N_{BA} = \frac{m_B a_{BX}}{\mu\cos\theta - \sin\theta}$$

$$\Rightarrow N = \frac{\mu\sin\theta + \cos\theta}{\mu\cos\theta - \sin\theta} m_B a_{BX} + G_A$$

$$= G_A + \frac{\mu+1}{\mu-1} m_B a_{BX}$$

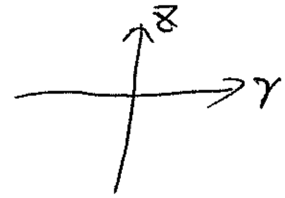
$$= 2g - \frac{1.3}{0.7}\cdot 1\cdot(-0.29g$$

$$= 2.538g$$

Problem # ~~3~~4

Step 1. 2 particles

Step 2: cylindrical coord.



Step 3: Motion

3.1 Kinematics

$L_{rope} = r - Z_B = const$

$\Rightarrow \frac{dr}{dt} = \frac{dz_B}{dt}$ and $\frac{d^2 r}{dt^2} = \frac{d^2 z_B}{dt^2}$ ..... ①

conservation of moment of Momentum. (the moment of resultant force is zero about the center of the disk)

$\Rightarrow m_A \vec{V}_A \times \vec{r}_A = const$

$\begin{cases} \vec{V}_A = V_r \vec{e}_r + V_\theta \vec{e}_\theta \\ \vec{r}_A = r \vec{e}_r \end{cases}$

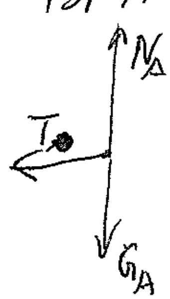
$\Rightarrow m_A V_\theta r = const$, $V_\theta = \frac{d\theta}{dt} \cdot r$ $\Rightarrow \frac{d\theta}{dt} r^2 = const$

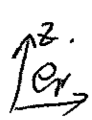
Because, $\frac{d\theta}{dt} = \omega_0$ and $r = r_0$ initially,

we get $\frac{d\theta}{dt} = \omega_0 \left(\frac{r_0}{r}\right)^2$ ..... ②

3.2 force analysis

For A





$\begin{cases} \Sigma F_{Ar} = -T \\ N_A - G_A = \Sigma F_{Az} \\ \Sigma F_{A\theta} = 0 \end{cases}$ ..... ③

For B

$\Sigma F_{Bz} = T - G_B$ ..... ④

$\Sigma F_{Br} = 0$

$\Sigma F_{B\theta} = 0$

~~By ③ and ④~~

3.3 dynamics. Eqs

$$\begin{cases} m_A a_{Ar} = \sum F_{Ar} = -T \\ m_B a_{Bz} = \sum F_{Bz} = T - G_B \end{cases}$$

$$\Rightarrow T = m_B a_{Bz} + G_B = -m_A a_{Ar} \quad \cdots\cdots \text{⑤}$$

By Kinematic relation in cylindrical-coord.

$$a_{Ar} = \frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 \quad \cdots\cdots \text{⑥}$$

By ⑤. ⑥

$$T = -m_A a_{Ar} = -m_A \left(\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right) \Leftarrow \text{by ①}$$

$$= -m_A \left(a_{Bz} - r\left(\frac{d\theta}{dt}\right)^2\right) \Leftarrow a_{Bz} = \frac{T - G_B}{m_B}$$

$$= -m_A \left(\frac{T - G_B}{m_B} - r\left(\frac{d\theta}{dt}\right)^2\right)$$

$$\Rightarrow (m_B + m_A) T = m_A G_B + m_A m_B r \left(\frac{d\theta}{dt}\right)^2 \Leftarrow \frac{d\theta}{dt} = \left(\frac{r_0}{r}\right)^2 \omega_0$$

$$\Rightarrow T = \frac{m_A}{m_B + m_A} G_B + \frac{m_A m_B}{m_A + m_B} r \frac{r_0^4}{r^4} \omega_0^2$$

$$\boxed{\begin{aligned} T(r=0.8) &= \frac{1}{3} 2 \cdot g + \frac{1 \times 2}{3} \frac{1^4}{0.8^3} \cdot 2^2 \\ &= \frac{2}{3} g + 5.2 = 11.7 \text{ N} \end{aligned}}$$