Computer Algebra
Independent Integration Tests

January 2024 special build with Reduce

0-Independent-test-suites/5-Hearn-Problems

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January 8, 2024 Compiled on January 8, 2024 at 9:10pm

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**284**]. This is test number [5].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev. 6657. December 10, 2023. On Linux.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was run directly.

1.2 Reduce test script

The following is the Reduce script used to run Reduce test for this file on my Linux
`reduce_script.red`

1.3 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (284)	0.00 (0)
Maple	99.30 (282)	0.70 (2)
Fricas	98.94 (281)	1.06 (3)
Rubi	98.24 (279)	1.76 (5)
Mupad	95.07 (270)	4.93 (14)
Giac	94.72 (269)	5.28 (15)
Reduce	92.61 (263)	7.39 (21)
Sympy	89.44 (254)	10.56 (30)
Maxima	88.38 (251)	11.62 (33)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

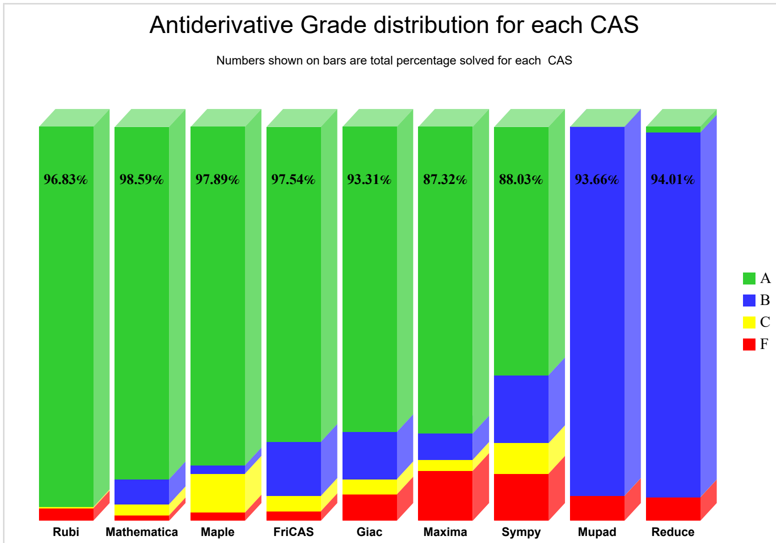
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

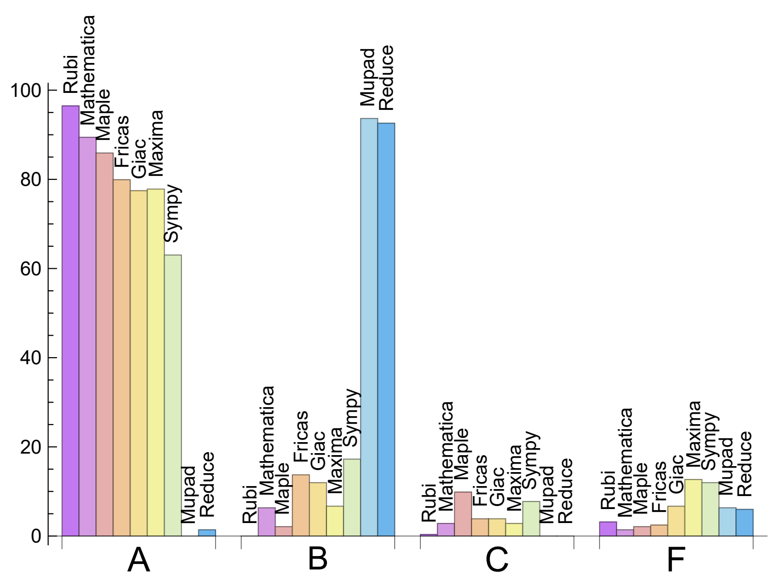
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.479	0.000	0.352	3.169
Mathematica	89.437	6.338	2.817	1.408
Maple	85.915	2.113	9.859	2.113
Fricas	79.930	13.732	3.873	2.465
Maxima	77.817	6.690	2.817	12.676
Giac	77.465	11.972	3.873	6.690
Sympy	63.028	17.254	7.746	11.972
Reduce	1.408	92.606	0.000	5.986
Mupad	0.000	93.662	0.000	6.338

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to

FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	2	50.00	50.00	0.00
Fricas	3	100.00	0.00	0.00
Rubi	5	100.00	0.00	0.00
Reduce	21	100.00	0.00	0.00
Mupad	14	0.00	100.00	0.00
Giac	15	100.00	0.00	0.00
Sympy	30	86.67	10.00	3.33
Maxima	33	66.67	0.00	33.33

Table 1.4: Failure statistics for each CAS

1.4 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Reduce	0.00
Mupad	0.17
Maple	0.18
Rubi	0.20
Maxima	0.22
Fricas	0.25
Giac	0.29
Mathematica	0.34
Sympy	2.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	35.80	1.05	18.00	0.86
Maxima	35.94	1.30	19.00	0.88
Rubi	41.24	1.03	22.00	1.00
Fricas	44.57	1.36	22.00	1.00
Reduce	45.43	1.25	22.00	1.00
Maple	51.18	0.99	20.00	0.87
Mathematica	65.69	1.31	23.00	1.00
Sympy	88.22	1.99	22.00	1.00
Giac	105.03	1.78	23.00	1.00

Table 1.6: Leaf size performance for each CAS

1.5 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

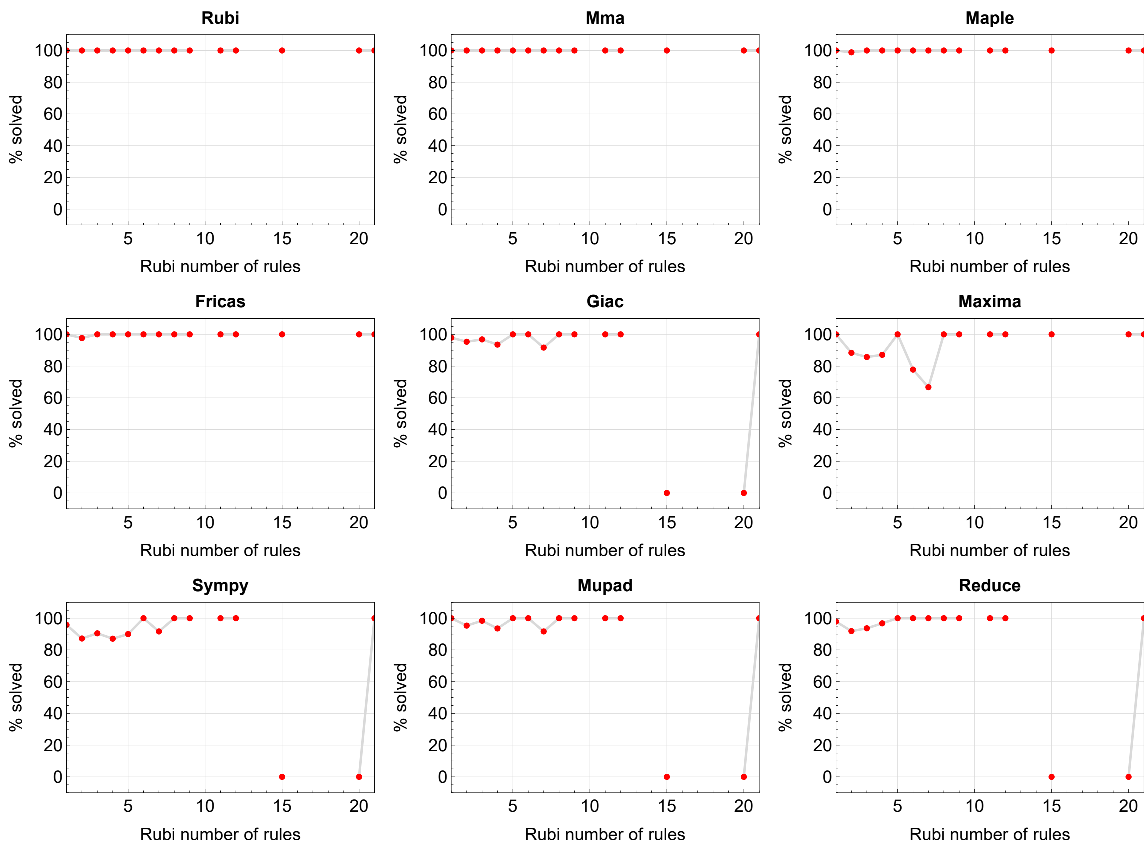


Figure 1.1: Solving statistics per number of Rubi rules used

1.6 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

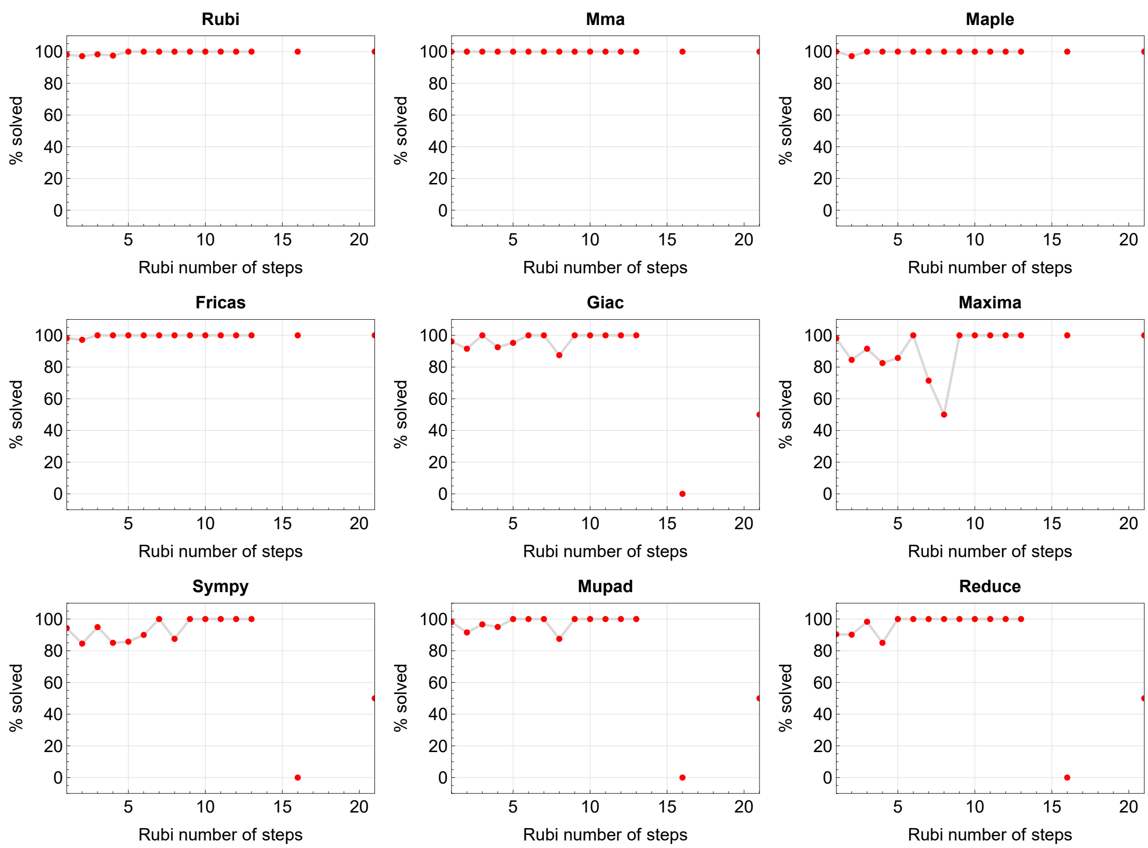


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.7 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

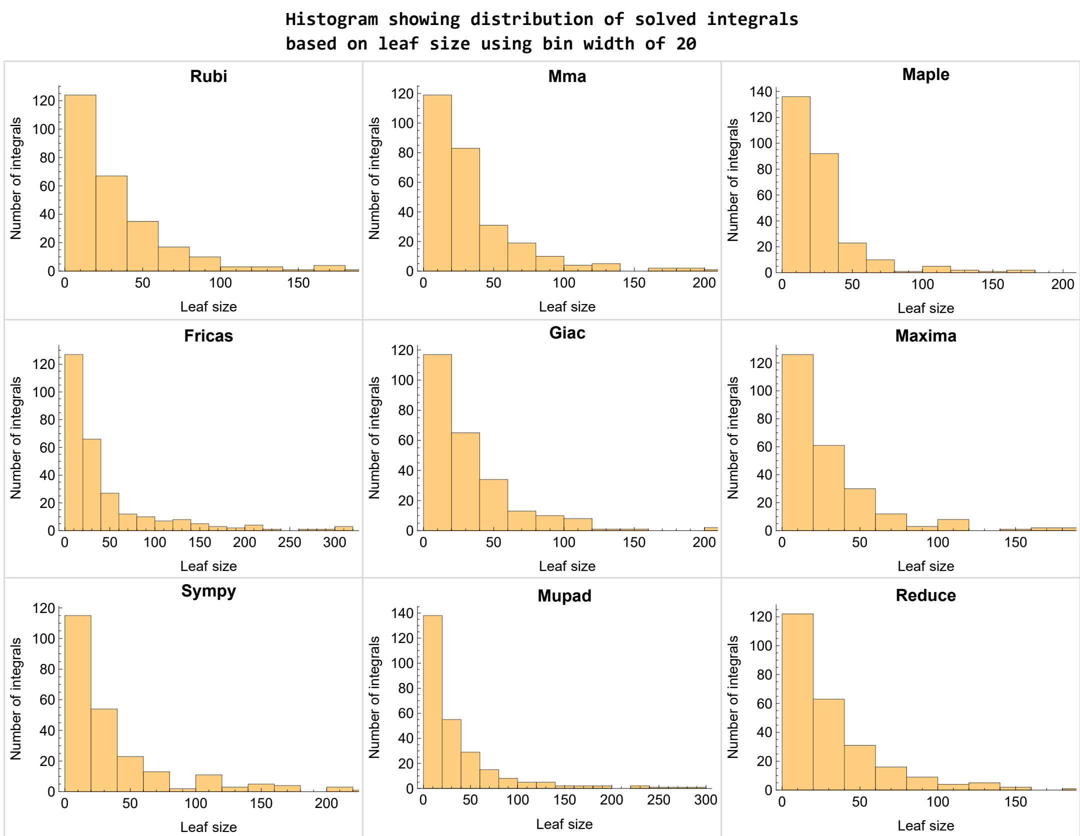


Figure 1.3: Solved integrals based on leaf size distribution

1.8 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

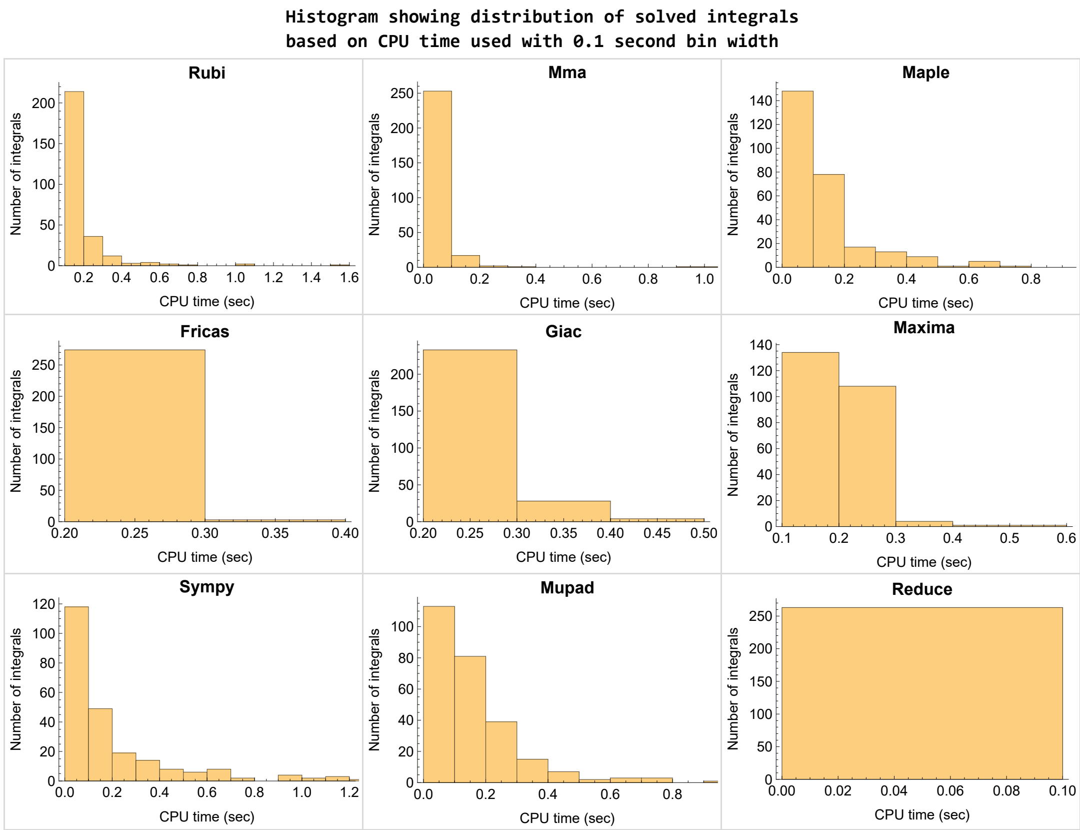


Figure 1.4: Solved integrals histogram based on CPU time used

1.9 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

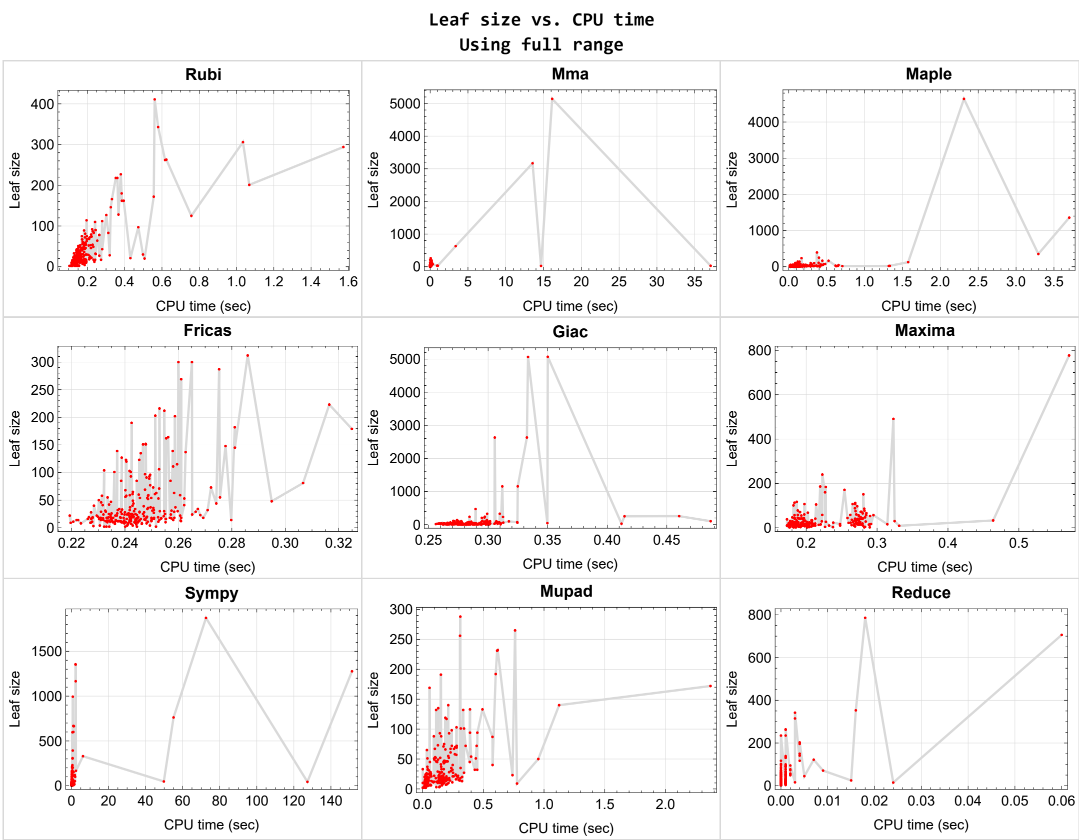


Figure 1.5: Leaf size vs. CPU time. Full range

1.10 list of integrals with no known antiderivative

$\{75, 145, 170, 273\}$

1.11 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {145}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.12 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {240}

Mathematica {278, 279, 281}

Maple {281, 282}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.13 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

For Reduce CAS, since it has no support for timelimit, there was no time limit used. But the time used was still recorded.

1.14 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica` and `Maple`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.15 Important notes about some of the results

1.15.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.15.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.15.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.15.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

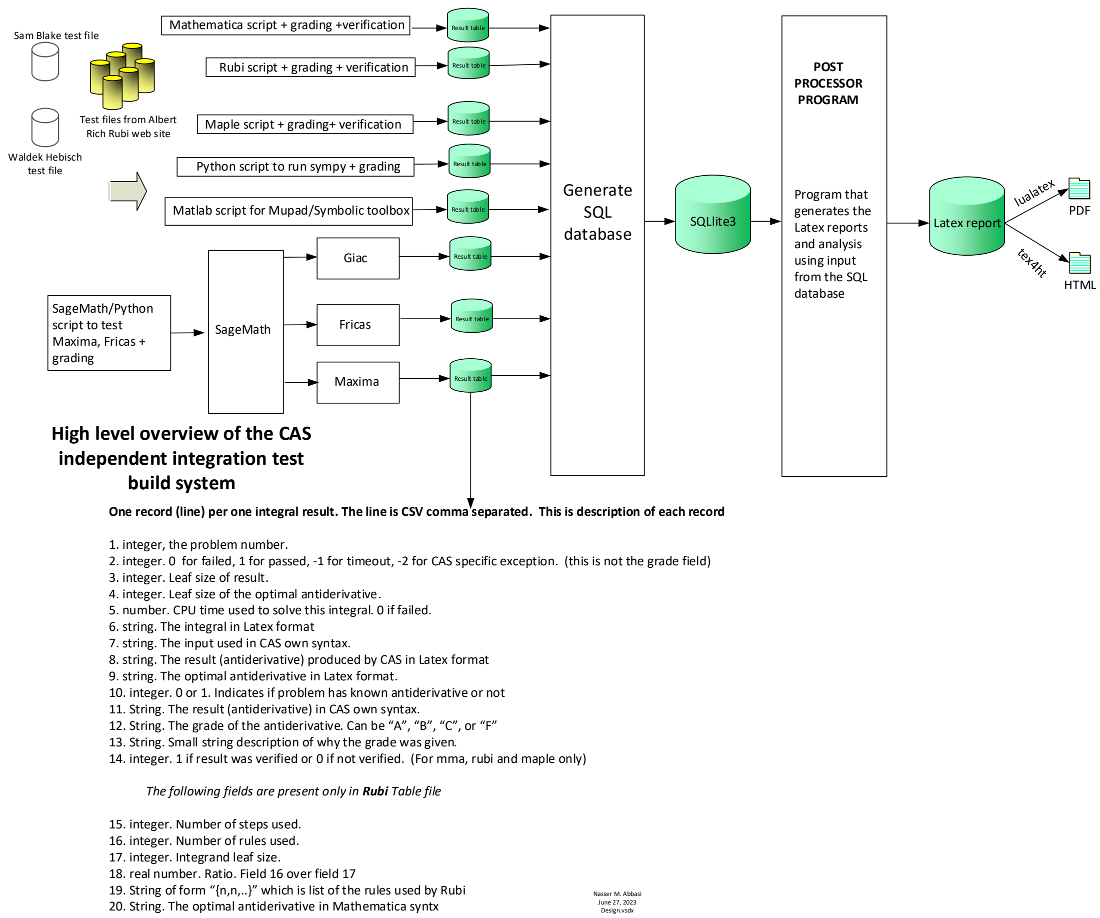
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
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2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi 22
2.1.2 Mma 23
2.1.3 Maple 23
2.1.4 Fricas 24
2.1.5 Maxima 24
2.1.6 Giac 25
2.1.7 Mupad 26
2.1.8 Sympy 26
2.1.9 Reduce 27

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283 }

B grade { }

C grade { 225 }

F normal fail { 169, 278, 279, 281, 284 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 246, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

B grade { 52, 79, 82, 108, 110, 111, 120, 121, 190, 199, 202, 235, 236, 244, 247, 256, 260, 269 }

C grade { 20, 44, 45, 51, 245, 278, 279, 281 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 41, 46, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 281, 283, 284 }

B grade { 60, 61, 110, 237, 257, 280 }

C grade { 18, 19, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 47, 48, 49, 50, 51, 95, 138, 139, 140, 141, 203, 259, 279, 282 }

F normal fail { 86 }

F(-1) timedout fail { 278 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 198, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 274, 275, 276, 277, 279, 282, 283 }

B grade { 39, 40, 41, 42, 43, 78, 79, 80, 81, 82, 90, 110, 111, 112, 127, 128, 133, 194, 195, 199, 204, 220, 222, 227, 228, 235, 236, 244, 245, 246, 247, 254, 256, 260, 268, 269, 278, 280, 284 }

C grade { 18, 19, 36, 37, 38, 44, 45, 49, 50, 51, 197 }

F normal fail { 86, 257, 281 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 164, 165, 167, 169, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 219, 220, 221, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 283 }

B grade { 32, 81, 82, 111, 112, 128, 129, 130, 146, 191, 194, 195, 204, 222, 225, 226, 241, 256, 280 }

C grade { 102, 103, 104, 105, 145, 166, 168, 197 }

F normal fail { 39, 40, 42, 43, 44, 45, 49, 51, 86, 163, 196, 203, 235, 236, 239, 251, 257, 278, 279, 281, 282, 284 }

F(-1) timedout fail { }

F(-2) exception fail { 8, 122, 123, 147, 160, 161, 176, 211, 212, 213, 214 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 165, 167, 168, 169, 171, 172, 173, 174, 175, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 196, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 274, 275, 276, 277, 282, 283, 284 }

B grade { 11, 23, 32, 67, 79, 81, 82, 110, 111, 112, 118, 127, 164, 176, 178, 179, 190, 195, 198, 199, 202, 220, 222, 228, 235, 236, 245, 246, 247, 256, 260, 268, 269, 280 }

C grade { 134, 135, 136, 137, 138, 139, 140, 141, 160, 161, 166 }

F normal fail { 56, 63, 86, 128, 129, 162, 163, 197, 200, 201, 203, 257, 278, 279, 281 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

C grade { }

F normal fail { }

F(-1) timedout fail { 102, 103, 104, 105, 128, 129, 163, 169, 203, 249, 257, 278, 279, 281 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 40, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 74, 76, 77, 78, 79, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 124, 125, 126, 127, 130, 132, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 177, 181, 182, 185, 186, 187, 190, 192, 193, 195, 199, 200, 201, 202, 206, 209, 210, 215, 216, 217, 218, 219, 221, 223, 225, 226, 227, 230, 231, 232, 240, 241, 243, 244, 245, 248, 250, 252, 253, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 282, 283, 284 }

B grade { 7, 8, 13, 22, 23, 32, 39, 41, 44, 56, 80, 81, 82, 90, 103, 104, 110, 112, 113, 115, 117, 122, 123, 131, 133, 142, 143, 161, 178, 179, 180, 183, 184, 188, 189, 204, 212, 222, 228, 229, 233, 234, 237, 239, 246, 247, 254, 256, 272 }

C grade { 9, 14, 31, 50, 70, 72, 73, 134, 135, 136, 137, 138, 139, 140, 141, 175, 191, 194, 203, 205, 220, 224 }

F normal fail { 86, 128, 129, 146, 162, 163, 176, 196, 197, 198, 207, 208, 211, 213, 214, 235, 236, 238, 242, 249, 251, 257, 265, 278, 279, 281 }

F(-1) timedout fail { 12, 147, 280 }

F(-2) exception fail { 160 }

2.1.9 Reduce

A grade { 75, 145, 170, 273 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 282 }

C grade { }

F normal fail { 86, 128, 129, 163, 193, 197, 211, 213, 214, 235, 236, 237, 238, 257, 281, 283, 284 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	0.81
time (sec)	N/A	0.119	0.000	0.014	0.188	0.238	0.015	0.276	0.001	0.021

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	16	16	17	16	15	15
N.S.	1	1.00	1.00	0.73	0.73	0.73	0.77	0.73	0.68	0.68
time (sec)	N/A	0.132	0.001	0.046	0.197	0.227	0.015	0.268	0.000	0.038

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	16	16	15	16	15	15
N.S.	1	1.00	1.00	0.73	0.73	0.73	0.68	0.73	0.68	0.68
time (sec)	N/A	0.134	0.001	0.047	0.186	0.230	0.015	0.278	0.000	0.030

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00	1.00
time (sec)	N/A	0.103	0.000	0.013	0.183	0.232	0.044	0.277	0.000	0.009

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	22	32	46	29	23	56	22
N.S.	1	1.00	0.67	0.61	0.89	1.28	0.81	0.64	1.56	0.61
time (sec)	N/A	0.147	0.015	0.086	0.199	0.231	0.051	0.266	0.000	0.133

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	33	24	34	44	26
N.S.	1	1.00	0.88	0.78	0.75	1.03	0.75	1.06	1.38	0.81
time (sec)	N/A	0.150	0.011	0.079	0.189	0.239	0.065	0.282	0.000	0.034

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	41	40	34	144	42	45	40
N.S.	1	1.00	0.85	1.02	1.00	0.85	3.60	1.05	1.12	1.00
time (sec)	N/A	0.172	0.016	0.117	0.184	0.238	0.542	0.271	0.000	0.248

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35	1.35
time (sec)	N/A	0.154	0.011	0.365	0.000	0.241	0.105	0.299	0.001	0.196

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	26	14	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.62	0.88	0.88	0.88
time (sec)	N/A	0.132	0.006	0.072	0.273	0.253	0.084	0.291	0.000	0.037

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	21	16	15	14	14	22	14	13	14
N.S.	1	1.11	0.84	0.79	0.74	0.74	1.16	0.74	0.68	0.74
time (sec)	N/A	0.137	0.004	0.624	0.277	0.237	0.044	0.270	0.000	0.029

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	35	36	39	71	41	80	117	49
N.S.	1	1.06	0.71	0.73	0.80	1.45	0.84	1.63	2.39	1.00
time (sec)	N/A	0.183	0.023	0.112	0.272	0.253	0.082	0.278	0.000	0.128

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	69	78	81	0	81	95	87
N.S.	1	1.00	0.91	1.01	1.15	1.19	0.00	1.19	1.40	1.28
time (sec)	N/A	0.209	0.026	0.156	0.188	0.307	0.000	0.277	0.001	0.576

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	34	33	43	32	121	43	33	256
N.S.	1	0.98	0.72	0.70	0.91	0.68	2.57	0.91	0.70	5.45
time (sec)	N/A	0.151	0.010	0.145	0.197	0.242	0.350	0.266	0.000	0.309

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	41	40	30	393	40	30	191
N.S.	1	1.00	0.75	1.02	1.00	0.75	9.82	1.00	0.75	4.78
time (sec)	N/A	0.155	0.014	0.195	0.272	0.253	0.658	0.270	0.000	0.151

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	20	19	19	19	20	19	25
N.S.	1	1.04	1.00	0.74	0.70	0.70	0.70	0.74	0.70	0.93
time (sec)	N/A	0.144	0.004	0.095	0.280	0.234	0.057	0.274	0.001	0.045

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	45	40	33	34	34	41	35	33	46
N.S.	1	1.10	0.98	0.80	0.83	0.83	1.00	0.85	0.80	1.12
time (sec)	N/A	0.180	0.009	0.073	0.273	0.267	0.056	0.279	0.000	0.204

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	31	40	31	36	59	33
N.S.	1	1.00	0.79	0.74	0.72	0.93	0.72	0.84	1.37	0.77
time (sec)	N/A	0.280	0.025	0.089	0.274	0.228	0.057	0.302	0.000	0.043

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	64	22	72	61	73	72	56	33
N.S.	1	1.06	0.75	0.26	0.85	0.72	0.86	0.85	0.66	0.39
time (sec)	N/A	0.243	0.013	0.061	0.277	0.251	0.094	0.278	0.001	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	64	22	72	61	73	72	56	33
N.S.	1	1.06	0.75	0.26	0.85	0.72	0.86	0.85	0.66	0.39
time (sec)	N/A	0.244	0.011	0.076	0.278	0.243	0.067	0.276	0.000	0.175

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	73	54	53	53	70	53	51	47
N.S.	1	1.06	1.09	0.81	0.79	0.79	1.04	0.79	0.76	0.70
time (sec)	N/A	0.211	0.041	0.050	0.290	0.253	0.094	0.284	0.000	0.182

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	20	18	21	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.11	1.00	1.17	1.00
time (sec)	N/A	0.127	0.005	0.076	0.203	0.242	0.017	0.277	0.000	0.220

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	76	47	94
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	1.95	1.21	2.41
time (sec)	N/A	0.159	0.083	0.076	0.194	0.262	0.299	0.276	0.001	0.391

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	140	87	192
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.33	1.45	3.20
time (sec)	N/A	0.177	0.056	0.086	0.202	0.249	0.479	0.274	0.000	0.603

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	1.00
time (sec)	N/A	0.119	0.000	0.069	0.193	0.244	0.017	0.285	0.000	0.027

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.126	0.002	0.064	0.186	0.242	0.056	0.271	0.001	0.116

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	17	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	0.94	1.00
time (sec)	N/A	0.159	0.004	0.070	0.183	0.231	0.044	0.273	0.000	0.036

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94	0.94
time (sec)	N/A	0.159	0.003	0.068	0.222	0.231	0.048	0.262	0.000	0.125

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83	0.83
time (sec)	N/A	0.131	0.004	0.074	0.194	0.255	0.094	0.268	0.000	0.134

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	26	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.93	0.89
time (sec)	N/A	0.153	0.004	0.074	0.181	0.241	0.079	0.275	0.000	0.053

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	70	45
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.67	1.07
time (sec)	N/A	0.167	0.030	0.080	0.184	0.252	0.122	0.283	0.000	0.188

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00	1.00
time (sec)	N/A	0.125	0.002	0.112	0.267	0.245	0.045	0.274	0.001	0.041

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	19	21	18	15	23	22	10
N.S.	1	1.00	1.00	1.90	2.10	1.80	1.50	2.30	2.20	1.00
time (sec)	N/A	0.121	0.002	0.076	0.184	0.238	0.053	0.295	0.000	0.155

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	87	66	24	66	63	78	57	51	72
N.S.	1	1.12	0.85	0.31	0.85	0.81	1.00	0.73	0.65	0.92
time (sec)	N/A	0.229	0.015	0.076	0.285	0.258	0.177	0.283	0.000	0.282

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	73	65	22	56	68	71	57	52	72
N.S.	1	0.99	0.88	0.30	0.76	0.92	0.96	0.77	0.70	0.97
time (sec)	N/A	0.216	0.013	0.073	0.274	0.240	0.140	0.298	0.001	0.355

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	110	89	29	97	300	20	104	68	101
N.S.	1	0.96	0.77	0.25	0.84	2.61	0.17	0.90	0.59	0.88
time (sec)	N/A	0.241	0.021	0.080	0.272	0.265	0.068	0.272	0.001	0.315

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	22	34	59	46	39	31	20
N.S.	1	1.00	1.23	0.63	0.97	1.69	1.31	1.11	0.89	0.57
time (sec)	N/A	0.140	0.013	0.076	0.270	0.261	0.183	0.288	0.000	0.162

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	24	41	59	48	39	37	18
N.S.	1	1.00	1.23	0.69	1.17	1.69	1.37	1.11	1.06	0.51
time (sec)	N/A	0.144	0.011	0.082	0.269	0.242	0.153	0.301	0.001	0.174

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	166	120	24	151	85	151	95	93	45
N.S.	1	0.97	0.70	0.14	0.88	0.50	0.88	0.56	0.54	0.26
time (sec)	N/A	0.332	0.036	0.075	0.281	0.257	0.234	0.300	0.001	0.121

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	68	35	0	151	146	74	131	93
N.S.	1	1.03	0.93	0.48	0.00	2.07	2.00	1.01	1.79	1.27
time (sec)	N/A	0.197	0.038	0.079	0.000	0.247	0.258	0.297	0.001	0.219

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	68	35	0	151	24	74	131	93
N.S.	1	1.03	0.93	0.48	0.00	2.07	0.33	1.01	1.79	1.27
time (sec)	N/A	0.170	0.020	0.070	0.000	0.248	0.214	0.324	0.001	0.143

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	54	75	91	158	81	97	67
N.S.	1	1.00	1.15	0.75	1.04	1.26	2.19	1.12	1.35	0.93
time (sec)	N/A	0.186	0.024	0.062	0.270	0.248	0.184	0.285	0.000	0.102

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	33	0	123	24	101	92	98
N.S.	1	1.00	1.00	0.49	0.00	1.84	0.36	1.51	1.37	1.46
time (sec)	N/A	0.171	0.028	0.077	0.000	0.240	0.194	0.307	0.001	0.240

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	33	0	123	92	51	100	117
N.S.	1	1.00	1.00	0.49	0.00	1.84	1.37	0.76	1.49	1.75
time (sec)	N/A	0.173	0.015	0.072	0.000	0.245	0.137	0.297	0.001	0.198

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	218	91	31	0	139	994	252	235	61
N.S.	1	1.11	0.46	0.16	0.00	0.71	5.07	1.29	1.20	0.31
time (sec)	N/A	0.361	0.041	0.139	0.000	0.258	0.681	0.414	0.001	0.231

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	218	91	35	0	139	24	256	235	132
N.S.	1	1.11	0.46	0.18	0.00	0.71	0.12	1.31	1.20	0.67
time (sec)	N/A	0.352	0.053	0.131	0.000	0.237	0.343	0.460	0.000	0.112

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	75	66	65	65	83	67	63	88
N.S.	1	1.08	1.03	0.90	0.89	0.89	1.14	0.92	0.86	1.21
time (sec)	N/A	0.220	0.011	0.106	0.280	0.243	0.132	0.309	0.000	0.093

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	127	122	22	112	127	14	114	95	140
N.S.	1	0.92	0.88	0.16	0.81	0.92	0.10	0.83	0.69	1.01
time (sec)	N/A	0.301	0.026	0.178	0.269	0.239	0.318	0.295	0.002	0.212

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	146	115	22	107	164	14	107	89	135
N.S.	1	1.06	0.83	0.16	0.78	1.19	0.10	0.78	0.64	0.98
time (sec)	N/A	0.325	0.019	0.218	0.266	0.256	0.138	0.277	0.001	0.130

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	343	209	22	0	135	14	239	203	288
N.S.	1	1.01	0.62	0.06	0.00	0.40	0.04	0.71	0.60	0.85
time (sec)	N/A	0.579	0.007	0.085	0.000	0.246	1.340	0.297	0.004	0.311

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	112	98	37	88	77	44	90	78	45
N.S.	1	1.15	1.01	0.38	0.91	0.79	0.45	0.93	0.80	0.46
time (sec)	N/A	0.279	0.025	0.125	0.269	0.249	127.316	0.286	0.001	0.173

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	411	42	30	0	101	165	205	315	53
N.S.	1	1.49	0.15	0.11	0.00	0.37	0.60	0.75	1.15	0.19
time (sec)	N/A	0.561	0.010	0.073	0.000	0.242	0.179	0.295	0.003	0.104

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	55	260	41	40	40	46	40	353	52
N.S.	1	1.12	5.31	0.84	0.82	0.82	0.94	0.82	7.20	1.06
time (sec)	N/A	0.192	0.071	0.116	0.276	0.243	0.109	0.289	0.016	0.217

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6	6
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75	0.75
time (sec)	N/A	0.131	0.002	0.017	0.177	0.240	0.046	0.301	0.000	0.023

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	11	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.65	0.53
time (sec)	N/A	0.135	0.000	0.017	0.179	0.237	0.061	0.285	0.000	0.033

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	11	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.65	0.53
time (sec)	N/A	0.136	0.002	0.024	0.184	0.250	0.055	0.283	0.000	0.032

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	26	25	56	0	23	32
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	0.88	1.23
time (sec)	N/A	0.148	0.006	0.045	0.183	0.255	0.391	0.000	0.000	0.247

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	15	16	12	15	15	15	12	12
N.S.	1	1.13	1.00	1.07	0.80	1.00	1.00	1.00	0.80	0.80
time (sec)	N/A	0.150	0.000	0.017	0.189	0.242	0.038	0.289	0.000	0.032

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	172	127	104	71	103	133	103	71	71
N.S.	1	1.35	1.00	0.82	0.56	0.81	1.05	0.81	0.56	0.56
time (sec)	N/A	0.556	0.004	0.086	0.191	0.242	0.166	0.302	0.000	0.173

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75	0.75
time (sec)	N/A	0.135	0.003	0.027	0.191	0.244	0.048	0.298	0.001	0.057

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.50	1.00
time (sec)	N/A	0.120	0.004	0.019	0.204	0.234	0.204	0.283	0.000	0.004

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	11	5	4	3	5	5	4
N.S.	1	1.00	1.00	2.75	1.25	1.00	0.75	1.25	1.25	1.00
time (sec)	N/A	0.139	0.004	0.029	0.205	0.241	0.207	0.281	0.000	0.014

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00	1.00
time (sec)	N/A	0.133	0.001	0.021	0.182	0.228	0.040	0.278	0.000	0.114

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	6	19	14	0	20	17
N.S.	1	1.00	1.00	0.88	0.35	1.12	0.82	0.00	1.18	1.00
time (sec)	N/A	0.193	0.023	0.033	0.207	0.238	0.255	0.000	0.000	0.031

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	12	22
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.00	1.83
time (sec)	N/A	0.143	0.003	0.053	0.177	0.261	0.420	0.279	0.000	0.161

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	46	28	25	25	25	22	24	22	21
N.S.	1	1.64	1.00	0.89	0.89	0.89	0.79	0.86	0.79	0.75
time (sec)	N/A	0.174	0.003	0.027	0.186	0.234	0.052	0.276	0.000	0.143

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	53	47	47	47	44	47	46	47
N.S.	1	1.09	0.98	0.87	0.87	0.87	0.81	0.87	0.85	0.87
time (sec)	N/A	0.190	0.009	0.074	0.180	0.235	0.060	0.285	0.000	0.178

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	138	39	35
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	4.76	1.34	1.21
time (sec)	N/A	0.154	0.010	0.096	0.173	0.248	0.133	0.299	0.000	0.237

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	46	39	44	39	42	58	44	66
N.S.	1	1.09	1.00	0.85	0.96	0.85	0.91	1.26	0.96	1.43
time (sec)	N/A	0.181	0.012	0.050	0.183	0.247	0.115	0.299	0.001	0.188

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	59	50	57	49	54	94	55	75
N.S.	1	1.07	1.00	0.85	0.97	0.83	0.92	1.59	0.93	1.27
time (sec)	N/A	0.196	0.015	0.059	0.184	0.243	0.101	0.275	0.000	0.178

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	24	23	24	23	23	36	23	23	23
N.S.	1	1.04	1.00	1.04	1.00	1.00	1.57	1.00	1.00	1.00
time (sec)	N/A	0.148	0.003	0.079	0.263	0.243	0.073	0.282	0.000	0.062

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	31	26	29	28	23	31	28	32	51
N.S.	1	1.15	0.96	1.07	1.04	0.85	1.15	1.04	1.19	1.89
time (sec)	N/A	0.176	0.003	0.040	0.180	0.235	0.076	0.412	0.000	0.041

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	36	36	53	36	36	65
N.S.	1	1.00	1.00	0.84	0.82	0.82	1.20	0.82	0.82	1.48
time (sec)	N/A	0.187	0.003	0.102	0.264	0.251	0.102	0.311	0.001	0.036

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	63	44	44	73
N.S.	1	1.00	1.00	0.83	0.81	0.81	1.17	0.81	0.81	1.35
time (sec)	N/A	0.192	0.005	0.153	0.258	0.243	0.119	0.350	0.000	0.127

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	27	25	32	31	31	29	33	40	25
N.S.	1	1.08	1.00	1.28	1.24	1.24	1.16	1.32	1.60	1.00
time (sec)	N/A	0.156	0.003	0.043	0.177	0.250	0.079	0.270	0.000	0.069

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	7	5	28	7	27	7	28	7
N.S.	1	1.00	1.40	1.00	5.60	1.40	5.40	1.40	5.60	1.40
time (sec)	N/A	0.155	0.039	0.050	0.263	0.252	3.797	0.275	0.003	0.584

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00	1.00
time (sec)	N/A	0.138	0.001	0.056	0.190	0.243	0.036	0.279	0.001	0.020

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.134	0.004	0.043	0.185	0.252	0.043	0.278	0.000	0.027

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	6	9	5
N.S.	1	1.00	1.00	1.20	0.60	2.20	1.00	1.20	1.80	1.00
time (sec)	N/A	0.138	0.003	0.030	0.177	0.249	0.036	0.265	0.000	0.029

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	16	3	17	13	13
N.S.	1	1.00	2.33	1.33	1.00	5.33	1.00	5.67	4.33	4.33
time (sec)	N/A	0.142	0.000	0.036	0.185	0.251	0.058	0.276	0.000	0.192

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	25	37	75	26	50	27
N.S.	1	1.00	1.00	1.24	1.19	1.76	3.57	1.24	2.38	1.29
time (sec)	N/A	0.218	0.073	0.079	0.266	0.259	0.170	0.296	0.001	0.251

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	7	15	17	15	17	17	11
N.S.	1	1.00	1.00	2.33	5.00	5.67	5.00	5.67	5.67	3.67
time (sec)	N/A	0.140	0.002	0.116	0.175	0.254	0.066	0.278	0.000	0.129

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	6	15	19	15	17	5	5
N.S.	1	1.00	3.40	1.20	3.00	3.80	3.00	3.40	1.00	1.00
time (sec)	N/A	0.142	0.006	0.063	0.178	0.252	0.061	0.273	0.000	0.044

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.145	0.001	0.078	0.204	0.239	0.019	0.276	0.001	0.029

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	18	20	17	16	16	15	16	16	16
N.S.	1	0.90	1.00	0.85	0.80	0.80	0.75	0.80	0.80	0.80
time (sec)	N/A	0.210	0.003	0.327	0.194	0.246	0.184	0.283	0.000	0.164

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	14	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	1.08	0.77
time (sec)	N/A	0.162	0.001	0.151	0.173	0.238	0.020	0.271	0.001	0.034

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	6	35
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.14	0.80
time (sec)	N/A	0.168	0.018	0.000	0.000	0.000	0.000	0.000	0.001	0.313

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	18	17	15	18	15
N.S.	1	1.00	1.00	0.84	0.79	0.95	0.89	0.79	0.95	0.79
time (sec)	N/A	0.183	0.004	0.154	0.179	0.258	0.204	0.280	0.000	0.046

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.155	0.001	0.079	0.173	0.242	0.018	0.294	0.000	0.026

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	10	8	9	12	9
N.S.	1	1.00	1.00	1.00	0.82	0.91	0.73	0.82	1.09	0.82
time (sec)	N/A	0.168	0.000	0.143	0.190	0.244	0.022	0.289	0.000	0.030

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	2	7	2
N.S.	1	1.00	1.00	1.50	1.00	3.50	2.50	1.00	3.50	1.00
time (sec)	N/A	0.155	0.001	0.132	0.176	0.242	0.038	0.282	0.001	0.023

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	10	20	6	17	6
N.S.	1	1.00	1.00	0.80	0.73	0.67	1.33	0.40	1.13	0.40
time (sec)	N/A	0.160	0.018	0.167	0.182	0.244	0.149	0.300	0.000	0.028

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00	1.00
time (sec)	N/A	0.179	0.002	0.050	0.173	0.246	0.068	0.300	0.000	0.021

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	17	15	15	17	15	17	15
N.S.	1	1.00	0.88	1.00	0.88	0.88	1.00	0.88	1.00	0.88
time (sec)	N/A	0.242	0.005	0.086	0.184	0.247	0.098	0.279	0.001	0.027

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	36	19	20	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	1.44	0.76	0.80	0.76
time (sec)	N/A	0.164	0.002	0.100	0.208	0.243	0.097	0.283	0.001	0.056

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	29	19	26	29	56	26	31	28
N.S.	1	1.12	0.71	0.46	0.63	0.71	1.37	0.63	0.76	0.68
time (sec)	N/A	0.219	0.017	0.181	0.184	0.258	0.132	0.264	0.001	0.160

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	32	31	23	23	23	39	23	25	25
N.S.	1	0.97	0.94	0.70	0.70	0.70	1.18	0.70	0.76	0.76
time (sec)	N/A	0.243	0.003	0.223	0.182	0.268	0.145	0.266	0.001	0.068

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.176	0.007	0.078	0.193	0.246	0.064	0.298	0.000	0.020

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	14	15	14	14	17	14	16	14
N.S.	1	1.06	0.88	0.94	0.88	0.88	1.06	0.88	1.00	0.88
time (sec)	N/A	0.233	0.011	0.097	0.178	0.248	0.124	0.288	0.000	0.025

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	36	19	20	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	1.44	0.76	0.80	0.76
time (sec)	N/A	0.159	0.010	0.104	0.213	0.245	0.090	0.268	0.001	0.145

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	29	27	26	29	56	26	31	28
N.S.	1	1.12	0.71	0.66	0.63	0.71	1.37	0.63	0.76	0.68
time (sec)	N/A	0.219	0.027	0.184	0.202	0.245	0.146	0.266	0.001	0.066

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	31	23	23	25	39	23	25	25
N.S.	1	0.94	0.94	0.70	0.70	0.76	1.18	0.70	0.76	0.76
time (sec)	N/A	0.245	0.007	0.231	0.183	0.242	0.133	0.265	0.001	0.177

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	2	0
N.S.	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.149	0.003	0.060	0.212	0.244	0.392	0.294	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	12	2	2	0
N.S.	1	1.00	1.00	1.50	6.50	1.00	6.00	1.00	1.00	0.00
time (sec)	N/A	0.144	0.004	0.130	0.218	0.240	0.632	0.277	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	15	13	17	13	13	0
N.S.	1	1.00	1.00	1.10	1.50	1.30	1.70	1.30	1.30	0.00
time (sec)	N/A	0.193	0.003	0.142	0.220	0.242	0.905	0.290	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	17	11	10	11	11	0
N.S.	1	1.00	1.00	0.80	1.13	0.73	0.67	0.73	0.73	0.00
time (sec)	N/A	0.177	0.011	0.174	0.248	0.240	0.663	0.281	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	20	18	12	16	16	16
N.S.	1	1.00	1.00	1.42	1.67	1.50	1.00	1.33	1.33	1.33
time (sec)	N/A	0.177	0.006	0.049	0.201	0.261	0.047	0.298	0.000	0.022

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	11	11	11
N.S.	1	1.00	2.00	1.09	1.00	1.00	1.27	1.00	1.00	1.00
time (sec)	N/A	0.136	0.009	0.084	0.184	0.234	0.055	0.284	0.000	0.022

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	10	10	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	1.00	1.00	1.00
time (sec)	N/A	0.141	0.009	0.073	0.184	0.235	0.073	0.266	0.000	0.022

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16	16
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33	1.33
time (sec)	N/A	0.143	0.009	0.036	0.193	0.251	0.058	0.277	0.000	0.197

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	13	23	26	11	27	29	56	28	28
N.S.	1	1.18	2.09	2.36	1.00	2.45	2.64	5.09	2.55	2.55
time (sec)	N/A	0.150	0.007	0.041	0.188	0.251	0.069	0.277	0.000	0.141

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	15	26	30	17	51	14	12
N.S.	1	1.00	3.17	1.25	2.17	2.50	1.42	4.25	1.17	1.00
time (sec)	N/A	0.145	0.016	0.119	0.187	0.255	0.294	0.271	0.001	0.095

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	26	28	34	28	31	11
N.S.	1	1.00	1.00	1.73	2.36	2.55	3.09	2.55	2.82	1.00
time (sec)	N/A	0.157	0.002	0.164	0.201	0.245	0.334	0.291	0.001	0.023

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	24	23	46	18	23	18
N.S.	1	1.00	0.92	0.76	0.96	0.92	1.84	0.72	0.92	0.72
time (sec)	N/A	0.159	0.022	0.141	0.187	0.244	0.091	0.269	0.000	0.185

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	24	29	22	22	22	37	25	31	24
N.S.	1	0.89	1.07	0.81	0.81	0.81	1.37	0.93	1.15	0.89
time (sec)	N/A	0.168	0.011	0.204	0.176	0.242	0.132	0.284	0.001	0.139

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	22	22	46	18	22	18
N.S.	1	1.00	0.92	0.76	0.88	0.88	1.84	0.72	0.88	0.72
time (sec)	N/A	0.160	0.016	0.143	0.185	0.261	0.091	0.295	0.001	0.159

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	22	21	36	22	23	24
N.S.	1	1.00	1.00	0.85	0.85	0.81	1.38	0.85	0.88	0.92
time (sec)	N/A	0.166	0.006	0.200	0.181	0.247	0.119	0.276	0.001	0.025

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	18	58	10	18	10
N.S.	1	1.00	1.00	1.10	1.00	1.80	5.80	1.00	1.80	1.00
time (sec)	N/A	0.160	0.003	0.177	0.210	0.248	0.620	0.285	0.001	0.126

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44	0.44
time (sec)	N/A	0.146	0.004	0.056	0.183	0.244	0.114	0.284	0.001	0.143

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.67	0.50
time (sec)	N/A	0.151	0.035	0.068	0.181	0.245	0.178	0.294	0.000	0.160

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	14	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.40	1.00
time (sec)	N/A	0.144	0.010	0.073	0.185	0.242	0.201	0.282	0.001	0.023

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	14	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	1.27	0.91
time (sec)	N/A	0.171	0.013	0.102	0.178	0.244	0.212	0.284	0.000	0.023

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	40	39	0	148	100	48	45	45
N.S.	1	1.15	1.00	0.98	0.00	3.70	2.50	1.20	1.12	1.12
time (sec)	N/A	0.191	0.033	0.152	0.000	0.278	1.976	0.269	0.001	0.389

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	44	43	0	287	1872	60	54	58
N.S.	1	1.11	0.94	0.91	0.00	6.11	39.83	1.28	1.15	1.23
time (sec)	N/A	0.207	0.073	0.300	0.000	0.275	72.566	0.282	0.002	0.266

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	47	46	117	54	105	45	67	52
N.S.	1	1.07	0.64	0.63	1.60	0.74	1.44	0.62	0.92	0.71
time (sec)	N/A	0.264	0.106	0.237	0.187	0.250	0.359	0.267	0.001	0.146

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	17	9
N.S.	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	1.13	0.60
time (sec)	N/A	0.159	0.003	0.180	0.183	0.250	0.142	0.291	0.000	0.027

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	47	46	113	54	105	45	64	52
N.S.	1	1.07	0.64	0.63	1.55	0.74	1.44	0.62	0.88	0.71
time (sec)	N/A	0.264	0.085	0.276	0.183	0.246	0.246	0.270	0.001	0.109

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	19	22	14	31	14	29	29	21
N.S.	1	1.00	1.36	1.57	1.00	2.21	1.00	2.07	2.07	1.50
time (sec)	N/A	0.188	0.006	0.051	0.185	0.257	0.039	0.281	0.000	0.172

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	125	101	138	491	182	0	0	59	0
N.S.	1	1.20	0.97	1.33	4.72	1.75	0.00	0.00	0.57	0.00
time (sec)	N/A	0.758	0.110	0.142	0.323	0.281	0.000	0.000	0.001	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	294	133	237	777	212	0	0	88	0
N.S.	1	1.92	0.87	1.55	5.08	1.39	0.00	0.00	0.58	0.00
time (sec)	N/A	1.573	0.228	0.168	0.571	0.255	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	23	19	13
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.53	1.27	0.87
time (sec)	N/A	0.194	0.011	0.053	0.269	0.255	0.079	0.275	0.000	0.022

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	13	26	11	21	13
N.S.	1	1.00	1.00	0.80	0.73	0.87	1.73	0.73	1.40	0.87
time (sec)	N/A	0.161	0.008	0.204	0.181	0.245	0.140	0.278	0.000	0.044

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	29	14	11	10	19	14	10	18	18
N.S.	1	1.21	0.58	0.46	0.42	0.79	0.58	0.42	0.75	0.75
time (sec)	N/A	0.206	0.018	0.112	0.186	0.245	0.023	0.291	0.001	0.044

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	11	9	18	12	9	17	6
N.S.	1	1.00	0.86	1.57	1.29	2.57	1.71	1.29	2.43	0.86
time (sec)	N/A	0.174	0.023	0.208	0.197	0.234	0.025	0.290	0.001	0.064

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	23	25	22	104	328	22	22
N.S.	1	1.00	0.69	0.72	0.78	0.69	3.25	10.25	0.69	0.69
time (sec)	N/A	0.156	0.016	0.130	0.196	0.243	0.307	0.299	0.001	0.025

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	21	24	20	107	329	20	20
N.S.	1	1.00	0.65	0.68	0.77	0.65	3.45	10.61	0.65	0.65
time (sec)	N/A	0.156	0.014	0.141	0.186	0.248	0.290	0.310	0.000	0.017

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	50	56	60	60	308	1156	63	57
N.S.	1	1.00	0.60	0.67	0.71	0.71	3.67	13.76	0.75	0.68
time (sec)	N/A	0.234	0.037	0.191	0.197	0.245	0.555	0.325	0.000	0.278

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	54	58	58	304	1155	61	55
N.S.	1	1.00	0.59	0.65	0.70	0.70	3.66	13.92	0.73	0.66
time (sec)	N/A	0.234	0.037	0.217	0.188	0.249	0.546	0.312	0.001	0.212

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	102	107	115	665	2631	139	133
N.S.	1	1.00	0.58	0.63	0.66	0.71	4.10	16.24	0.86	0.82
time (sec)	N/A	0.395	0.060	0.289	0.198	0.260	1.284	0.333	0.001	0.391

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	93	100	105	111	668	2631	137	132
N.S.	1	1.01	0.58	0.62	0.65	0.69	4.15	16.34	0.85	0.82
time (sec)	N/A	0.385	0.052	0.335	0.214	0.258	1.011	0.306	0.001	0.335

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	263	169	166	186	203	1355	5069	264	231
N.S.	1	1.01	0.65	0.64	0.71	0.78	5.19	19.42	1.01	0.89
time (sec)	N/A	0.624	0.101	0.443	0.220	0.251	2.140	0.350	0.001	0.614

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	262	168	164	184	202	1352	5065	262	232
N.S.	1	1.01	0.65	0.63	0.71	0.78	5.20	19.48	1.01	0.89
time (sec)	N/A	0.617	0.083	0.525	0.228	0.259	2.215	0.334	0.001	0.619

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	13	89	14
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.56	0.52	3.56	0.56
time (sec)	N/A	0.214	0.051	1.320	0.196	0.257	1.136	0.288	0.001	0.166

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	89	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	2.97	0.73
time (sec)	N/A	0.197	0.034	1.333	0.193	0.247	1.142	0.279	0.001	0.280

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	97	55	61	55	59	100	60	73	67
N.S.	1	1.14	0.65	0.72	0.65	0.69	1.18	0.71	0.86	0.79
time (sec)	N/A	0.473	0.054	0.305	0.193	0.249	0.340	0.313	0.002	0.231

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	C	N/A	N/A	N/A	F	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	15	240	22	17	17	17	17
N.S.	1	1.00	1.18	1.36	21.82	2.00	1.55	1.55	1.55	1.55
time (sec)	N/A	0.466	0.590	0.230	0.223	0.264	53.385	0.301	0.002	0.384

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	18	12	11	41	16	0	18	16	10
N.S.	1	1.50	1.00	0.92	3.42	1.33	0.00	1.50	1.33	0.83
time (sec)	N/A	0.188	0.017	0.375	0.213	0.250	0.000	0.299	0.000	0.177

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	83	76	107	0	312	0	98	150	133
N.S.	1	1.08	0.99	1.39	0.00	4.05	0.00	1.27	1.95	1.73
time (sec)	N/A	0.312	0.349	0.394	0.000	0.286	0.000	0.317	0.004	0.494

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	12	13	15	13	12	13
N.S.	1	1.00	1.00	0.76	0.71	0.76	0.88	0.76	0.71	0.76
time (sec)	N/A	0.142	0.005	0.079	0.189	0.245	0.137	0.284	0.000	0.129

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	10	13	15	13	10	13
N.S.	1	1.00	1.00	0.65	0.59	0.76	0.88	0.76	0.59	0.76
time (sec)	N/A	0.146	0.005	0.078	0.180	0.261	0.128	0.266	0.000	0.162

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	3	2
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	1.00	0.67
time (sec)	N/A	0.122	0.000	0.023	0.201	0.233	0.030	0.281	0.000	0.008

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.130	0.000	0.036	0.186	0.241	0.044	0.310	0.000	0.164

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	9	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	1.00	0.89
time (sec)	N/A	0.129	0.006	0.029	0.183	0.224	0.041	0.292	0.000	0.027

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4	4
N.S.	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00	1.00
time (sec)	N/A	0.135	0.010	0.053	0.227	0.231	0.470	0.287	0.000	0.015

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	38	23	23	22	15	26	23	22
N.S.	1	1.25	1.58	0.96	0.96	0.92	0.62	1.08	0.96	0.92
time (sec)	N/A	0.157	0.017	0.040	0.238	0.237	0.050	0.308	0.000	0.096

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.159	0.015	0.038	0.173	0.239	0.035	0.290	0.000	0.046

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	14	12	14	14	15	14
N.S.	1	1.00	1.00	1.00	1.08	0.92	1.08	1.08	1.15	1.08
time (sec)	N/A	0.137	0.009	0.035	0.192	0.237	0.058	0.276	0.000	0.055

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	23	85	24	21	30	21
N.S.	1	1.00	1.00	0.71	0.74	2.74	0.77	0.68	0.97	0.68
time (sec)	N/A	0.165	0.027	0.066	0.270	0.244	0.097	0.281	0.001	0.249

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	14	13	13	19	13	14	13
N.S.	1	1.00	0.67	0.67	0.62	0.62	0.90	0.62	0.67	0.62
time (sec)	N/A	0.158	0.010	0.039	0.187	0.240	0.039	0.276	0.000	0.027

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	201	102	102	101	101	102	101	102	101
N.S.	1	1.23	0.63	0.63	0.62	0.62	0.63	0.62	0.63	0.62
time (sec)	N/A	1.068	0.074	0.126	0.182	0.236	0.060	0.295	0.000	0.342

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	18	0	216	18	18
N.S.	1	1.00	1.00	1.06	0.00	1.00	0.00	12.00	1.00	1.00
time (sec)	N/A	0.168	0.016	0.078	0.000	0.257	0.000	0.297	0.001	0.200

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	14	31	237	14	14
N.S.	1	1.00	1.00	1.07	0.00	1.00	2.21	16.93	1.00	1.00
time (sec)	N/A	0.157	0.013	0.076	0.000	0.244	0.304	0.284	0.000	0.174

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	10	19	0	0	19	19
N.S.	1	1.00	1.00	1.24	0.59	1.12	0.00	0.00	1.12	1.12
time (sec)	N/A	0.165	0.014	0.076	0.232	0.228	0.000	0.000	0.000	0.132

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	43	79	0	54	0	0	51	0
N.S.	1	1.00	0.67	1.23	0.00	0.84	0.00	0.00	0.80	0.00
time (sec)	N/A	0.253	0.212	0.146	0.000	0.248	0.000	0.000	0.002	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	16	16	12	45	16	15
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.75	2.81	1.00	0.94
time (sec)	N/A	0.167	0.106	0.086	0.182	0.241	0.049	0.306	0.000	0.205

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.85	0.85	0.85
time (sec)	N/A	0.140	0.011	0.044	0.197	0.245	0.039	0.283	0.000	0.027

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	9	7
N.S.	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.82	0.64
time (sec)	N/A	0.133	0.012	0.022	0.174	0.236	0.083	0.284	0.000	0.018

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	7	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.78	0.67
time (sec)	N/A	0.134	0.009	0.033	0.185	0.241	0.034	0.279	0.001	0.019

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	17	18	17	17	14	24	17	17
N.S.	1	1.00	0.63	0.67	0.63	0.63	0.52	0.89	0.63	0.63
time (sec)	N/A	0.264	0.009	0.056	0.211	0.235	0.048	0.277	0.000	0.160

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	25	23	22	36	31	36	25	0
N.S.	1	0.00	1.00	0.92	0.88	1.44	1.24	1.44	1.00	0.00
time (sec)	N/A	0.000	0.101	0.137	0.260	0.249	0.145	0.284	0.015	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	5	7	7	8	7	11	7
N.S.	1	1.00	1.22	0.56	0.78	0.78	0.89	0.78	1.22	0.78
time (sec)	N/A	0.211	0.015	0.019	0.227	0.234	0.522	0.284	0.002	0.164

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	10	10	8	10	11	10
N.S.	1	1.00	1.00	1.09	0.91	0.91	0.73	0.91	1.00	0.91
time (sec)	N/A	0.162	0.006	0.049	0.213	0.233	1.134	0.277	0.001	0.028

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	21	15	15	17	15	22	28
N.S.	1	1.00	0.77	0.95	0.68	0.68	0.77	0.68	1.00	1.27
time (sec)	N/A	0.222	0.010	0.065	0.197	0.242	1.555	0.274	0.000	0.219

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	17	17	11	11	10	11	12	11
N.S.	1	1.00	0.74	0.74	0.48	0.48	0.43	0.48	0.52	0.48
time (sec)	N/A	0.167	0.005	0.036	0.187	0.228	0.043	0.274	0.000	0.044

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	11	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.69	0.75
time (sec)	N/A	0.132	0.000	0.046	0.264	0.242	0.016	0.283	0.001	0.032

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	43	58	89	1166	48	44	49
N.S.	1	1.00	0.75	0.75	1.02	1.56	20.46	0.84	0.77	0.86
time (sec)	N/A	0.185	0.018	0.106	0.262	0.249	2.242	0.287	0.000	0.097

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	114	95	140	0	300	0	232	196	88
N.S.	1	0.98	0.82	1.21	0.00	2.59	0.00	2.00	1.69	0.76
time (sec)	N/A	0.195	0.175	0.118	0.000	0.260	0.000	0.312	0.004	0.154

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	16	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	1.00	0.75
time (sec)	N/A	0.122	0.002	0.090	0.196	0.233	0.016	0.284	0.000	0.026

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	21	26	30	202	66	29	25
N.S.	1	1.00	1.00	0.62	0.76	0.88	5.94	1.94	0.85	0.74
time (sec)	N/A	0.145	0.015	0.127	0.181	0.243	0.667	0.268	0.000	0.031

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	42	666	93	41	37
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.75	0.77	0.70
time (sec)	N/A	0.162	0.019	0.133	0.190	0.234	0.998	0.274	0.000	0.154

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	73	68	32	38	27
N.S.	1	1.00	1.00	0.80	1.20	2.09	1.94	0.91	1.09	0.77
time (sec)	N/A	0.145	0.022	0.101	0.280	0.272	0.792	0.271	0.001	0.149

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	47	93	44	41	51	31
N.S.	1	1.00	1.00	0.82	1.21	2.38	1.13	1.05	1.31	0.79
time (sec)	N/A	0.142	0.051	0.125	0.277	0.249	0.967	0.294	0.000	0.053

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	11	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.79	0.86
time (sec)	N/A	0.120	0.001	0.089	0.182	0.237	0.016	0.290	0.000	0.017

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	26	19	162	23	18	25
N.S.	1	1.00	0.72	0.66	0.81	0.59	5.06	0.72	0.56	0.78
time (sec)	N/A	0.145	0.013	0.090	0.188	0.235	0.613	0.281	0.000	0.029

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	41	31	600	37	30	37
N.S.	1	1.00	0.69	0.63	0.80	0.61	11.76	0.73	0.59	0.73
time (sec)	N/A	0.151	0.019	0.099	0.192	0.233	0.987	0.276	0.000	0.040

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	31	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	1.35	0.74
time (sec)	N/A	0.127	0.016	0.093	0.268	0.240	0.523	0.288	0.000	0.140

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	60	93	44	47	51	33
N.S.	1	1.00	1.00	0.83	1.46	2.27	1.07	1.15	1.24	0.80
time (sec)	N/A	0.141	0.046	0.122	0.271	0.242	1.339	0.297	0.001	0.156

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	25	21	25	24	21	24	21
N.S.	1	1.00	1.13	1.09	0.91	1.09	1.04	0.91	1.04	0.91
time (sec)	N/A	0.124	0.004	0.085	0.185	0.243	0.019	0.290	0.000	0.257

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	45	58	216	86	51	94
N.S.	1	1.00	0.79	0.90	0.94	1.21	4.50	1.79	1.06	1.96
time (sec)	N/A	0.162	0.035	0.083	0.188	0.231	0.320	0.307	0.000	0.452

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	48	37	52	37	230	52	36	43
N.S.	1	0.98	0.87	0.67	0.95	0.67	4.18	0.95	0.65	0.78
time (sec)	N/A	0.228	0.026	0.153	0.275	0.250	0.480	0.278	0.001	0.176

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	14	14	10	26	9	10
N.S.	1	1.00	3.17	0.92	1.17	1.17	0.83	2.17	0.75	0.83
time (sec)	N/A	0.125	0.003	0.135	0.190	0.244	0.065	0.257	0.000	0.172

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	34	71	34	119	39	28	30
N.S.	1	1.00	0.91	0.79	1.65	0.79	2.77	0.91	0.65	0.70
time (sec)	N/A	0.143	0.041	0.096	0.191	0.244	1.782	0.276	0.001	0.235

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	13	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.59	0.73
time (sec)	N/A	0.211	0.012	0.064	0.200	0.235	0.102	0.272	0.000	0.217

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	18	8	11	33	11
N.S.	1	1.00	1.00	0.92	0.85	1.38	0.62	0.85	2.54	0.85
time (sec)	N/A	0.123	0.002	0.146	0.200	0.231	0.261	0.277	0.008	0.246

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	16	18	19	6	6	16
N.S.	1	1.00	1.00	0.88	2.00	2.25	2.38	0.75	0.75	2.00
time (sec)	N/A	0.127	0.092	0.287	0.314	0.236	0.500	0.288	0.000	0.275

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	8	25	29	10
N.S.	1	1.00	1.00	0.79	1.79	1.79	0.57	1.79	2.07	0.71
time (sec)	N/A	0.140	0.017	0.230	0.208	0.239	0.485	0.305	0.001	0.162

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	26	14	0	22	0	22	26	18
N.S.	1	1.00	1.44	0.78	0.00	1.22	0.00	1.22	1.44	1.00
time (sec)	N/A	0.153	0.064	0.206	0.000	0.229	0.000	0.266	0.001	0.311

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	23	17	55	0	0	19	32
N.S.	1	1.00	0.97	0.77	0.57	1.83	0.00	0.00	0.63	1.07
time (sec)	N/A	0.160	0.102	0.648	0.292	0.233	0.000	0.000	0.003	0.431

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	14	17	0	35	13	14
N.S.	1	1.00	1.00	0.88	0.82	1.00	0.00	2.06	0.76	0.82
time (sec)	N/A	0.131	0.117	0.115	0.275	0.232	0.000	0.290	0.001	0.143

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	9	2
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	4.50	1.00
time (sec)	N/A	0.112	0.012	0.118	0.282	0.222	0.061	0.278	0.000	0.026

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	15	14	14	17	0	11	14
N.S.	1	1.00	1.50	0.75	0.70	0.70	0.85	0.00	0.55	0.70
time (sec)	N/A	0.507	0.011	0.131	0.186	0.222	0.227	0.000	0.000	0.306

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	24	21	30	20	19	0	18	20
N.S.	1	1.25	1.00	0.88	1.25	0.83	0.79	0.00	0.75	0.83
time (sec)	N/A	0.498	14.666	0.195	0.325	0.230	0.280	0.000	0.001	0.327

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	71	24	31	29	24	51	25	23
N.S.	1	1.00	2.63	0.89	1.15	1.07	0.89	1.89	0.93	0.85
time (sec)	N/A	0.150	0.005	0.138	0.197	0.233	0.081	0.285	0.000	0.741

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	92	82	25	0	216	51	0	117	0
N.S.	1	1.12	1.00	0.30	0.00	2.63	0.62	0.00	1.43	0.00
time (sec)	N/A	0.226	0.070	0.072	0.000	0.253	1.829	0.000	0.004	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	40	29	48	10	16	169
N.S.	1	1.00	1.00	1.67	3.33	2.42	4.00	0.83	1.33	14.08
time (sec)	N/A	0.238	0.011	0.326	0.277	0.238	49.819	0.270	0.003	0.058

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	36	85	66	55	34	32
N.S.	1	1.00	1.00	0.82	0.90	2.12	1.65	1.38	0.85	0.80
time (sec)	N/A	0.144	0.008	0.142	0.196	0.239	0.556	0.266	0.000	0.450

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	57	41	42	40	56	40
N.S.	1	1.00	1.00	0.89	1.24	0.89	0.91	0.87	1.22	0.87
time (sec)	N/A	0.172	0.028	0.431	0.295	0.232	0.617	0.261	0.001	0.577

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	52	40	52	0	40	45	51
N.S.	1	1.00	1.22	1.41	1.08	1.41	0.00	1.08	1.22	1.38
time (sec)	N/A	0.158	0.075	0.451	0.279	0.249	0.000	0.294	0.005	0.096

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	66	74	77	97	0	57	71	72
N.S.	1	1.00	1.08	1.21	1.26	1.59	0.00	0.93	1.16	1.18
time (sec)	N/A	0.174	0.101	0.480	0.282	0.248	0.000	0.325	0.009	0.242

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	29	19	18	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	1.26	0.83	0.78	0.83
time (sec)	N/A	0.131	0.003	0.112	0.194	0.242	0.155	0.281	0.001	0.289

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	36	24	23	24
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.29	0.86	0.82	0.86
time (sec)	N/A	0.133	0.003	0.137	0.205	0.240	0.157	0.262	0.001	0.273

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	63	46	0	152	0	58	25	50
N.S.	1	1.00	1.12	0.82	0.00	2.71	0.00	1.04	0.45	0.89
time (sec)	N/A	0.174	0.111	0.657	0.000	0.248	0.000	0.286	0.004	0.953

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	70	0	190	172	75	77	67
N.S.	1	1.00	1.10	0.86	0.00	2.35	2.12	0.93	0.95	0.83
time (sec)	N/A	0.188	0.188	0.463	0.000	0.242	0.415	0.308	0.002	0.259

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	56	0	58	0	45	27	54
N.S.	1	1.00	1.20	1.27	0.00	1.32	0.00	1.02	0.61	1.23
time (sec)	N/A	0.179	0.080	0.134	0.000	0.241	0.000	0.271	0.010	0.400

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	72	78	0	106	0	62	32	72
N.S.	1	1.00	1.06	1.15	0.00	1.56	0.00	0.91	0.47	1.06
time (sec)	N/A	0.197	0.107	0.132	0.000	0.253	0.000	0.294	0.013	0.446

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	22	10	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.69	0.77	0.85	0.85	0.85
time (sec)	N/A	0.172	0.005	0.125	0.197	0.236	0.092	0.276	0.000	0.090

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	15	11	17	15	13	11
N.S.	1	1.00	1.00	0.63	0.79	0.58	0.89	0.79	0.68	0.58
time (sec)	N/A	0.142	0.002	0.029	0.200	0.221	0.040	0.263	0.000	0.168

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	20	12	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.67	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.189	0.020	0.138	0.200	0.238	0.068	0.279	0.000	0.086

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	10	10	7	10	11	10
N.S.	1	1.00	1.00	1.00	0.91	0.91	0.64	0.91	1.00	0.91
time (sec)	N/A	0.157	0.007	0.043	0.248	0.240	0.066	0.265	0.000	0.214

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	22	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	0.81	0.81
time (sec)	N/A	0.153	0.004	0.095	0.194	0.219	0.063	0.292	0.000	0.168

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	16	20	50	167	66	53	15
N.S.	1	1.00	0.64	0.48	0.61	1.52	5.06	2.00	1.61	0.45
time (sec)	N/A	0.128	0.046	0.089	0.205	0.230	2.378	0.299	0.002	0.297

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	11	9	8
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.22	1.00	0.89
time (sec)	N/A	0.120	0.002	0.089	0.191	0.227	0.035	0.270	0.000	0.203

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	28	28	39	32	25	13
N.S.	1	1.00	1.00	0.74	1.47	1.47	2.05	1.68	1.32	0.68
time (sec)	N/A	0.129	0.021	0.151	0.284	0.227	0.111	0.259	0.000	0.074

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	12	11	14	15	11	12	12
N.S.	1	1.00	0.84	0.63	0.58	0.74	0.79	0.58	0.63	0.63
time (sec)	N/A	0.128	0.008	0.039	0.188	0.232	0.308	0.267	0.000	0.023

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	112	89	86	104	153	103	141	119
N.S.	1	1.01	1.27	1.01	0.98	1.18	1.74	1.17	1.60	1.35
time (sec)	N/A	0.182	0.065	0.103	0.282	0.232	1.096	0.487	0.004	0.188

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	18	9	10	34	9	7	17	9	9
N.S.	1	2.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00	1.00
time (sec)	N/A	0.189	0.007	0.085	0.208	0.220	0.077	0.275	0.000	0.021

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9	9
N.S.	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00	1.00
time (sec)	N/A	0.185	0.006	0.077	0.208	0.227	0.067	0.267	0.000	0.023

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	26	7	13	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.89	0.78	1.44	0.78	0.78
time (sec)	N/A	0.145	0.005	0.063	0.190	0.235	0.055	0.286	0.001	0.193

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	13	26	22	23	13	13
N.S.	1	1.00	1.00	1.33	1.08	2.17	1.83	1.92	1.08	1.08
time (sec)	N/A	0.182	0.096	0.707	0.193	0.241	0.175	0.290	0.000	0.326

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	32	75	22	57	22
N.S.	1	1.00	1.00	0.82	0.79	1.14	2.68	0.79	2.04	0.79
time (sec)	N/A	0.183	0.015	0.395	0.196	0.237	0.324	0.264	0.002	0.292

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	20	16	16	15	13	15	15	15	14
N.S.	1	0.95	0.76	0.76	0.71	0.62	0.71	0.71	0.71	0.67
time (sec)	N/A	0.157	0.001	0.024	0.274	0.233	0.085	0.270	0.000	0.019

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	20	16	16	15	13	15	19	15	15
N.S.	1	0.95	0.76	0.76	0.71	0.62	0.71	0.90	0.71	0.71
time (sec)	N/A	0.156	0.003	0.078	0.290	0.235	0.085	0.263	0.001	0.048

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	25	22	23	22	19	26	22	26	41
N.S.	1	1.09	0.96	1.00	0.96	0.83	1.13	0.96	1.13	1.78
time (sec)	N/A	0.179	0.003	0.044	0.194	0.226	0.065	0.258	0.000	0.151

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	14	27	14
N.S.	1	1.00	1.00	0.83	0.78	1.56	1.78	0.78	1.50	0.78
time (sec)	N/A	0.169	0.015	0.286	0.209	0.247	0.128	0.260	0.000	0.026

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	14	27	14
N.S.	1	1.00	1.00	0.83	0.78	1.56	1.78	0.78	1.50	0.78
time (sec)	N/A	0.162	0.008	0.251	0.197	0.240	0.130	0.301	0.001	0.026

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	24	0	22	7	16
N.S.	1	1.00	3.33	1.42	0.00	2.00	0.00	1.83	0.58	1.33
time (sec)	N/A	0.150	0.008	0.309	0.000	0.232	0.000	0.273	0.000	0.164

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	23	0	26	0	35	9	18
N.S.	1	1.00	3.00	1.64	0.00	1.86	0.00	2.50	0.64	1.29
time (sec)	N/A	0.157	0.008	0.408	0.000	0.237	0.000	0.289	0.001	0.136

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	16	22	9	10	36	14	7	10
N.S.	1	1.00	1.33	1.83	0.75	0.83	3.00	1.17	0.58	0.83
time (sec)	N/A	0.151	0.024	0.263	0.331	0.236	0.390	0.279	0.000	0.154

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	20	18	0	23	9	12
N.S.	1	1.00	1.29	1.57	1.43	1.29	0.00	1.64	0.64	0.86
time (sec)	N/A	0.158	0.026	0.342	0.276	0.233	0.000	0.274	0.001	0.032

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	0	13	63	13	19	21
N.S.	1	1.00	0.81	0.67	0.00	0.62	3.00	0.62	0.90	1.00
time (sec)	N/A	0.143	0.064	0.083	0.000	0.234	0.204	0.276	0.000	0.197

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	13	22	19	18	18	20	19	16	18
N.S.	1	0.54	0.92	0.79	0.75	0.75	0.83	0.79	0.67	0.75
time (sec)	N/A	0.151	0.010	0.073	0.194	0.228	0.055	0.278	0.000	0.110

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	9	33	16	7	16	17	8
N.S.	1	1.00	1.50	0.75	2.75	1.33	0.58	1.33	1.42	0.67
time (sec)	N/A	0.129	0.119	0.185	0.187	0.244	0.492	0.256	0.000	0.037

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	36	33	25	24	24	0	24	23	24
N.S.	1	1.12	1.03	0.78	0.75	0.75	0.00	0.75	0.72	0.75
time (sec)	N/A	0.161	0.020	0.089	0.178	0.241	0.000	0.260	0.001	0.032

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	40	33	27	26	32	31	26	26	26
N.S.	1	1.21	1.00	0.82	0.79	0.97	0.94	0.79	0.79	0.79
time (sec)	N/A	0.155	0.010	0.053	0.276	0.251	0.055	0.284	0.000	0.060

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	11	3	4	17	3
N.S.	1	1.00	2.33	1.33	1.00	3.67	1.00	1.33	5.67	1.00
time (sec)	N/A	0.143	0.000	0.050	0.192	0.252	0.038	0.276	0.000	0.024

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	22	11	16	48	19	34	10	10
N.S.	1	1.00	1.83	0.92	1.33	4.00	1.58	2.83	0.83	0.83
time (sec)	N/A	0.195	0.001	0.065	0.268	0.295	0.026	0.264	0.001	0.024

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	12	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	4.00	1.00
time (sec)	N/A	0.154	0.004	0.055	0.186	0.269	0.062	0.275	0.000	0.012

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	18	12	12	16	3
N.S.	1	1.00	2.33	1.33	1.00	6.00	4.00	4.00	5.33	1.00
time (sec)	N/A	0.145	0.005	0.103	0.238	0.250	0.161	0.267	0.000	0.138

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.128	0.000	0.033	0.194	0.248	0.034	0.270	0.000	0.165

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	36	29	32	10	10	0	11	10	0
N.S.	1	0.73	0.59	0.65	0.20	0.20	0.00	0.22	0.20	0.00
time (sec)	N/A	0.183	1.008	0.064	0.280	0.239	0.000	0.275	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	11	12	12	12	13	12	10
N.S.	1	1.00	1.06	0.69	0.75	0.75	0.75	0.81	0.75	0.62
time (sec)	N/A	0.139	0.003	0.088	0.201	0.237	0.028	0.281	0.000	0.145

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	36	53	0	30	0	40	35	32
N.S.	1	1.12	0.90	1.32	0.00	0.75	0.00	1.00	0.88	0.80
time (sec)	N/A	0.170	0.008	0.024	0.000	0.260	0.000	0.276	0.001	0.043

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	67	63	58	55	109	57	77	69
N.S.	1	1.00	0.52	0.49	0.45	0.43	0.85	0.45	0.60	0.54
time (sec)	N/A	0.367	0.108	0.422	0.223	0.276	0.577	0.273	0.001	0.280

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	16	17	17	27	15	18	17
N.S.	1	1.00	0.60	0.53	0.57	0.57	0.90	0.50	0.60	0.57
time (sec)	N/A	0.192	0.020	0.127	0.195	0.245	0.168	0.264	0.001	0.187

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	7	17	17	7	19	7
N.S.	1	1.00	0.82	0.73	0.64	1.55	1.55	0.64	1.73	0.64
time (sec)	N/A	0.121	0.001	0.087	0.198	0.227	0.036	0.274	0.000	0.065

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	31	32	32	41	33	31	46
N.S.	1	1.12	1.00	0.78	0.80	0.80	1.02	0.82	0.78	1.15
time (sec)	N/A	0.181	0.006	0.106	0.270	0.271	0.053	0.259	0.000	0.112

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	7	17	17	15	18	21	6
N.S.	1	1.00	2.88	0.88	2.12	2.12	1.88	2.25	2.62	0.75
time (sec)	N/A	0.124	0.003	0.095	0.198	0.254	0.038	0.271	0.000	0.070

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	194	394	0	0	0	0	35	0
N.S.	1	1.00	0.85	1.74	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.379	0.188	0.372	0.000	0.000	0.000	0.000	0.003	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	25	24	37	25	30	51
N.S.	1	1.00	1.00	1.08	1.04	1.00	1.54	1.04	1.25	2.12
time (sec)	N/A	0.148	0.005	0.106	0.187	0.265	0.704	0.282	0.000	0.439

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	71	22	57	69	73	58	51	70
N.S.	1	0.95	0.91	0.28	0.73	0.88	0.94	0.74	0.65	0.90
time (sec)	N/A	0.223	0.016	0.093	0.285	0.248	0.147	0.277	0.001	0.269

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	9	2
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	4.50	1.00
time (sec)	N/A	0.115	0.000	0.098	0.290	0.231	0.056	0.277	0.000	0.002

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	21	20	25	24	25	26	20
N.S.	1	1.00	1.22	0.78	0.74	0.93	0.89	0.93	0.96	0.74
time (sec)	N/A	0.131	0.020	0.186	0.273	0.244	0.079	0.269	0.001	0.042

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	20	10
N.S.	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	2.00	1.00
time (sec)	N/A	0.132	0.002	0.077	0.210	0.226	0.030	0.280	0.000	0.033

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	14	13	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	0.88	0.81	0.88
time (sec)	N/A	0.147	0.002	0.013	0.288	0.280	0.051	0.292	0.000	0.227

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	29	34	33	24	32	38	31	24
N.S.	1	1.08	0.72	0.85	0.82	0.60	0.80	0.95	0.78	0.60
time (sec)	N/A	0.181	0.010	0.014	0.464	0.257	0.094	0.293	0.000	0.030

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	17	29	14	13	29	0	15	49	9
N.S.	1	0.81	1.38	0.67	0.62	1.38	0.00	0.71	2.33	0.43
time (sec)	N/A	0.273	0.081	0.401	0.200	0.266	0.000	0.282	0.002	0.778

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71	0.71
time (sec)	N/A	0.154	0.001	0.062	0.185	0.255	0.020	0.283	0.001	0.002

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	18	14	18	18	16
N.S.	1	1.00	1.00	0.94	0.89	1.00	0.78	1.00	1.00	0.89
time (sec)	N/A	0.155	0.003	0.119	0.189	0.239	0.056	0.266	0.000	0.153

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	7	6	16	7	29	17	6
N.S.	1	1.00	2.00	0.70	0.60	1.60	0.70	2.90	1.70	0.60
time (sec)	N/A	0.115	0.012	0.155	0.269	0.250	0.076	0.260	0.000	0.041

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	16	5	4	14	3	25	13	4
N.S.	1	1.00	2.67	0.83	0.67	2.33	0.50	4.17	2.17	0.67
time (sec)	N/A	0.115	0.012	0.126	0.283	0.258	0.071	0.278	0.001	0.030

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	21	17	16	16	22	16	15	16
N.S.	1	1.14	1.00	0.81	0.76	0.76	1.05	0.76	0.71	0.76
time (sec)	N/A	0.137	0.008	0.632	0.272	0.255	0.049	0.269	0.000	0.143

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	47	73	46	46	79	45
N.S.	1	1.00	0.96	0.79	0.89	1.38	0.87	0.87	1.49	0.85
time (sec)	N/A	0.218	0.014	0.055	0.283	0.251	0.077	0.267	0.000	0.061

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	41	41	762	44	65	46
N.S.	1	1.00	1.00	0.94	0.84	0.84	15.55	0.90	1.33	0.94
time (sec)	N/A	0.232	0.018	0.123	0.191	0.262	55.063	0.268	0.001	0.207

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	207	54	270	14	54	14
N.S.	1	1.00	1.17	1.00	17.25	4.50	22.50	1.17	4.50	1.17
time (sec)	N/A	0.144	2.289	0.102	0.306	0.252	7.443	0.259	0.002	0.286

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	27	44	26	27	54	27
N.S.	1	1.00	1.00	1.00	0.96	1.57	0.93	0.96	1.93	0.96
time (sec)	N/A	0.320	37.104	0.617	0.228	0.274	0.166	0.278	0.001	0.342

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	180	136	125	160	145	1277	114	122	140
N.S.	1	0.90	0.68	0.63	0.80	0.73	6.42	0.57	0.61	0.70
time (sec)	N/A	0.384	0.148	1.576	0.227	0.281	151.384	0.271	0.007	1.124

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	20	15	15	25	15
N.S.	1	1.00	1.00	0.89	0.83	1.11	0.83	0.83	1.39	0.83
time (sec)	N/A	0.153	0.007	0.049	0.193	0.243	0.154	0.287	0.000	0.165

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	21	37	36	37	64	21
N.S.	1	1.00	1.00	0.92	0.88	1.54	1.50	1.54	2.67	0.88
time (sec)	N/A	0.163	0.006	0.065	0.205	0.242	0.213	0.263	0.000	0.073

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F(-1)	F	B	F	F	B	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	5141	0	0	179	0	0	786	0
N.S.	1	0.00	54.69	0.00	0.00	1.90	0.00	0.00	8.36	0.00
time (sec)	N/A	0.000	16.142	180.000	0.000	0.325	0.000	0.000	0.018	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	F	F	B	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	630	352	0	223	0	0	706	0
N.S.	1	0.00	4.44	2.48	0.00	1.57	0.00	0.00	4.97	0.00
time (sec)	N/A	0.000	3.375	3.294	0.000	0.316	0.000	0.000	0.060	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	250	171	162	0	76	15	172
N.S.	1	1.00	1.48	11.90	8.14	7.71	0.00	3.62	0.71	8.19
time (sec)	N/A	0.431	0.923	0.398	0.254	0.255	0.000	0.284	0.024	2.370

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	4030	0	3168	4640	0	0	0	0	907	0
N.S.	1	0.00	0.79	1.15	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	13.546	2.312	0.000	0.000	0.000	0.000	0.188	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	330	306	181	1356	0	269	330	472	342	265
N.S.	1	0.93	0.55	4.11	0.00	0.82	1.00	1.43	1.04	0.80
time (sec)	N/A	1.035	0.157	3.702	0.000	0.261	6.170	0.289	0.003	0.762

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	5	4	11	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	1.25	1.00	2.75	1.00
time (sec)	N/A	0.185	0.002	0.411	0.202	0.249	0.273	0.262	0.001	0.021

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	0	71	61	0	137	76	94	339	103
N.S.	1	0.00	1.00	0.86	0.00	1.93	1.07	1.32	4.77	1.45
time (sec)	N/A	0.000	0.066	0.094	0.000	0.263	0.106	0.297	0.006	0.280

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [159] had the largest ratio of [3]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	3	3	1.00	13	0.231
3	A	3	3	1.00	10	0.300
4	A	1	1	1.00	3	0.333
5	A	2	2	1.00	11	0.182
6	A	2	2	1.00	14	0.143
7	A	2	2	1.00	20	0.100
8	A	3	2	1.00	12	0.167
9	A	3	3	1.00	13	0.231
10	A	3	2	1.11	10	0.200
11	A	4	4	1.06	13	0.308
12	A	2	2	1.00	23	0.087
13	A	4	3	0.98	20	0.150
14	A	2	2	1.00	22	0.091
15	A	5	5	1.04	14	0.357
16	A	8	7	1.10	9	0.778
17	A	2	2	1.00	16	0.125
18	A	9	8	1.06	7	1.143
19	A	9	8	1.06	11	0.727
20	A	7	6	1.06	10	0.600
21	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	9	0.222
23	A	2	2	1.00	11	0.182
24	A	1	1	1.00	7	0.143
25	A	1	1	1.00	7	0.143
26	A	2	2	1.00	9	0.222
27	A	2	2	1.00	11	0.182
28	A	3	3	1.00	11	0.273
29	A	2	2	1.00	11	0.182
30	A	2	2	1.00	11	0.182
31	A	1	1	1.00	9	0.111
32	A	1	1	1.00	11	0.091
33	A	10	9	1.12	9	1.000
34	A	9	8	0.99	7	1.143
35	A	9	8	0.96	11	0.727
36	A	3	3	1.00	7	0.429
37	A	3	3	1.00	9	0.333
38	A	8	7	0.97	9	0.778
39	A	3	3	1.03	12	0.250
40	A	3	3	1.03	12	0.250
41	A	2	2	1.00	12	0.167
42	A	2	2	1.00	12	0.167
43	A	2	2	1.00	12	0.167
44	A	7	6	1.11	10	0.600
45	A	7	6	1.11	12	0.500
46	A	9	8	1.08	7	1.143
47	A	10	9	0.92	7	1.286
48	A	10	9	1.06	7	1.286
49	A	8	7	1.01	7	1.000
50	A	13	12	1.15	7	1.714
51	A	8	7	1.49	12	0.583
52	A	9	8	1.12	11	0.727
53	A	1	1	1.00	2	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	1	1	1.00	4	0.250
55	A	1	1	1.00	6	0.167
56	A	1	1	1.00	6	0.167
57	A	2	2	1.13	4	0.500
58	A	11	11	1.35	8	1.375
59	A	3	2	1.00	8	0.250
60	A	1	1	1.00	4	0.250
61	A	3	2	1.00	6	0.333
62	A	3	2	1.00	8	0.250
63	A	4	3	1.00	8	0.375
64	A	3	2	1.00	8	0.250
65	A	4	4	1.64	8	0.500
66	A	4	4	1.09	10	0.400
67	A	2	2	1.00	10	0.200
68	A	3	3	1.09	8	0.375
69	A	3	3	1.07	10	0.300
70	A	3	3	1.04	8	0.375
71	A	4	3	1.15	10	0.300
72	A	3	3	1.00	12	0.250
73	A	3	3	1.00	12	0.250
74	A	4	4	1.08	10	0.400
75	N/A	1	0	1.00	5	0.000
76	A	2	2	1.00	2	1.000
77	A	2	2	1.00	2	1.000
78	A	2	2	1.00	2	1.000
79	A	3	3	1.00	2	1.500
80	A	4	4	1.00	6	0.667
81	A	2	2	1.00	2	1.000
82	A	2	2	1.00	2	1.000
83	A	3	3	1.00	4	0.750
84	A	6	5	0.90	8	0.625
85	A	4	3	1.00	4	0.750

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	4	0.500
87	A	5	4	1.00	11	0.364
88	A	3	3	1.00	4	0.750
89	A	4	3	1.00	4	0.750
90	A	4	3	1.00	4	0.750
91	A	2	2	1.00	7	0.286
92	A	4	4	1.00	4	1.000
93	A	7	7	1.00	6	1.167
94	A	3	3	1.00	6	0.500
95	A	6	6	1.12	8	0.750
96	A	6	6	0.97	6	1.000
97	A	5	5	1.00	4	1.250
98	A	7	7	1.06	6	1.167
99	A	3	3	1.00	6	0.500
100	A	6	6	1.12	8	0.750
101	A	7	7	0.94	6	1.167
102	A	2	2	1.00	6	0.333
103	A	2	2	1.00	6	0.333
104	A	4	4	1.00	6	0.667
105	A	3	3	1.00	8	0.375
106	A	4	4	1.00	4	1.000
107	A	2	2	1.00	6	0.333
108	A	2	2	1.00	6	0.333
109	A	2	2	1.00	6	0.333
110	A	3	3	1.18	6	0.500
111	A	2	2	1.00	6	0.333
112	A	2	2	1.00	6	0.333
113	A	3	3	1.00	8	0.375
114	A	4	3	0.89	8	0.375
115	A	3	3	1.00	8	0.375
116	A	4	3	1.00	8	0.375
117	A	4	3	1.00	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	6	0.333
119	A	2	2	1.00	8	0.250
120	A	2	2	1.00	6	0.333
121	A	2	2	1.00	8	0.250
122	A	5	4	1.15	8	0.500
123	A	5	4	1.11	10	0.400
124	A	6	6	1.07	12	0.500
125	A	2	2	1.00	7	0.286
126	A	6	6	1.07	12	0.500
127	A	7	7	1.00	4	1.750
128	A	16	15	1.20	8	1.875
129	A	21	20	1.92	8	2.500
130	A	5	5	1.00	6	0.833
131	A	2	2	1.00	9	0.222
132	A	5	5	1.21	9	0.556
133	A	5	4	1.00	9	0.444
134	A	1	1	1.00	6	0.167
135	A	1	1	1.00	6	0.167
136	A	2	2	1.00	7	0.286
137	A	2	2	1.00	7	0.286
138	A	4	4	1.00	9	0.444
139	A	3	3	1.01	9	0.333
140	A	4	4	1.01	9	0.444
141	A	3	3	1.01	9	0.333
142	A	3	3	1.00	11	0.273
143	A	3	3	1.00	11	0.273
144	A	12	11	1.14	10	1.100
145	N/A	1	0	1.00	11	0.000
146	A	6	5	1.50	9	0.556
147	A	8	7	1.08	15	0.467
148	A	1	1	1.00	3	0.333
149	A	1	1	1.00	3	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	1	1	1.00	3	0.333
151	A	1	1	1.00	3	0.333
152	A	1	1	1.00	5	0.200
153	A	1	1	1.00	9	0.111
154	A	5	4	1.25	11	0.364
155	A	4	3	1.00	13	0.231
156	A	1	1	1.00	9	0.111
157	A	3	2	1.00	18	0.111
158	A	2	2	1.00	7	0.286
159	A	21	21	1.23	7	3.000
160	A	2	2	1.00	9	0.222
161	A	2	2	1.00	7	0.286
162	A	2	2	1.00	7	0.286
163	A	2	2	1.00	12	0.167
164	A	1	1	1.00	14	0.071
165	A	1	1	1.00	7	0.143
166	A	1	1	1.00	5	0.200
167	A	1	1	1.00	7	0.143
168	A	2	2	1.00	12	0.167
169	F	0	0	N/A	0.000	N/A
170	N/A	3	0	1.00	9	0.000
171	A	2	2	1.00	6	0.333
172	A	3	3	1.00	7	0.429
173	A	3	3	1.00	10	0.300
174	A	1	1	1.00	13	0.077
175	A	5	4	1.00	12	0.333
176	A	5	4	0.98	19	0.211
177	A	1	1	1.00	9	0.111
178	A	2	2	1.00	11	0.182
179	A	2	2	1.00	13	0.154
180	A	4	3	1.00	13	0.231
181	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	1	1	1.00	9	0.111
183	A	2	2	1.00	11	0.182
184	A	2	2	1.00	13	0.154
185	A	3	2	1.00	13	0.154
186	A	4	3	1.00	13	0.231
187	A	1	1	1.00	11	0.091
188	A	2	2	1.00	13	0.154
189	A	9	8	0.98	18	0.444
190	A	3	2	1.00	9	0.222
191	A	5	4	1.00	13	0.308
192	A	6	5	1.00	6	0.833
193	A	1	1	1.00	13	0.077
194	A	3	2	1.00	13	0.154
195	A	4	3	1.00	13	0.231
196	A	4	3	1.00	14	0.214
197	A	4	3	1.00	18	0.167
198	A	1	1	1.00	20	0.050
199	A	1	1	1.00	9	0.111
200	A	5	4	1.00	65	0.062
201	A	4	4	1.25	68	0.059
202	A	1	1	1.00	21	0.048
203	A	8	7	1.12	21	0.333
204	A	5	4	1.00	23	0.174
205	A	3	2	1.00	16	0.125
206	A	4	3	1.00	25	0.120
207	A	3	2	1.00	24	0.083
208	A	3	2	1.00	29	0.069
209	A	1	1	1.00	18	0.056
210	A	1	1	1.00	23	0.043
211	A	4	3	1.00	24	0.125
212	A	4	3	1.00	22	0.136
213	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	4	3	1.00	31	0.097
215	A	5	4	1.00	16	0.250
216	A	1	1	1.00	8	0.125
217	A	4	4	1.00	6	0.667
218	A	1	1	1.00	20	0.050
219	A	4	3	0.96	13	0.231
220	A	2	2	1.00	13	0.154
221	A	3	3	1.00	9	0.333
222	A	3	2	1.00	13	0.154
223	A	2	2	1.00	11	0.182
224	A	6	5	1.01	13	0.385
225	C	5	5	2.00	4	1.250
226	A	6	6	1.00	4	1.500
227	A	3	3	1.00	4	0.750
228	A	1	1	1.00	28	0.036
229	A	2	2	1.00	11	0.182
230	A	3	3	0.95	4	0.750
231	A	3	3	0.95	4	0.750
232	A	4	3	1.09	8	0.375
233	A	2	2	1.00	7	0.286
234	A	2	2	1.00	7	0.286
235	A	2	2	1.00	8	0.250
236	A	2	2	1.00	10	0.200
237	A	2	2	1.00	8	0.250
238	A	2	2	1.00	10	0.200
239	A	3	3	1.00	17	0.176
240	A	5	4	0.54	13	0.308
241	A	3	2	1.00	11	0.182
242	A	5	4	1.12	13	0.308
243	A	3	3	1.21	8	0.375
244	A	3	3	1.00	2	1.500
245	A	5	5	1.00	4	1.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	3	3	1.00	2	1.500
247	A	3	3	1.00	2	1.500
248	A	1	1	1.00	3	0.333
249	A	4	4	0.73	12	0.333
250	A	2	2	1.00	13	0.154
251	A	3	3	1.12	14	0.214
252	A	2	2	1.00	12	0.167
253	A	2	2	1.00	7	0.286
254	A	1	1	1.00	5	0.200
255	A	8	7	1.12	9	0.778
256	A	3	2	1.00	9	0.222
257	A	2	2	1.00	15	0.133
258	A	1	1	1.00	11	0.091
259	A	8	7	0.95	7	1.000
260	A	1	1	1.00	9	0.111
261	A	2	2	1.00	9	0.222
262	A	2	2	1.00	7	0.286
263	A	2	2	1.00	2	1.000
264	A	5	4	1.08	6	0.667
265	A	5	4	0.81	17	0.235
266	A	3	3	1.00	4	0.750
267	A	2	2	1.00	19	0.105
268	A	1	1	1.00	11	0.091
269	A	1	1	1.00	9	0.111
270	A	3	2	1.14	12	0.167
271	A	3	3	1.00	24	0.125
272	A	2	2	1.00	29	0.069
273	N/A	1	0	1.00	12	0.000
274	A	1	1	1.00	54	0.019
275	A	4	4	0.90	21	0.190
276	A	1	1	1.00	2	0.500
277	A	1	1	1.00	4	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	F	0	0	N/A	0.000	N/A
279	F	0	0	N/A	0.000	N/A
280	A	1	1	1.00	85	0.012
281	F	0	0	N/A	0.000	N/A
282	A	4	4	0.93	107	0.037
283	A	4	4	1.00	7	0.571
284	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

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3.25 $\int \frac{1}{(a+bx)^2}\, dx$ . . . 246
3.26 $\int \frac{x}{a+bx}\, dx$ . . . 250

3.27 $\int \frac{x^2}{a+bx}\,dx$ . . . 255
3.28 $\int \frac{1}{x(a+bx)}\,dx$ . . . 260
3.29 $\int \frac{1}{x^2(a+bx)}\,dx$ . . . 265
3.30 $\int \frac{1}{x^2(a+bx)^2}\,dx$ . . . 270
3.31 $\int \frac{1}{c^2+x^2}\,dx$ . . . 275
3.32 $\int \frac{1}{c^2-x^2}\,dx$ . . . 280
3.33 $\int \frac{1}{-1+2x^3}\,dx$ . . . 285
3.34 $\int \frac{1}{-2+x^3}\,dx$ . . . 293
3.35 $\int \frac{1}{-b+ax^3}\,dx$ . . . 300
3.36 $\int \frac{1}{-2+x^4}\,dx$ . . . 309
3.37 $\int \frac{1}{-1+5x^4}\,dx$ . . . 315
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3.39 $\int \frac{1}{-1+3x^2+x^4}\,dx$ . . . 328
3.40 $\int \frac{1}{-1-3x^2+x^4}\,dx$ . . . 335
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3.43 $\int \frac{1}{1+4x^2+x^4}\,dx$ . . . 353
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3.47 $\int \frac{1}{-2+x^6}\,dx$ . . . 383
3.48 $\int \frac{1}{2+x^6}\,dx$ . . . 391
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3.54 $\int x\log(x)\,dx$ . . . 440
3.55 $\int x^2\log(x)\,dx$ . . . 445
3.56 $\int x^p\log(x)\,dx$ . . . 450
3.57 $\int \log^2(x)\,dx$ . . . 455
3.58 $\int x^9\log^{11}(x)\,dx$ . . . 460
3.59 $\int \frac{\log^2(x)}{x}\,dx$ . . . 467
3.60 $\int \frac{1}{\log(x)}\,dx$ . . . 472
3.61 $\int \frac{1}{\log(1+x)}\,dx$ . . . 476
3.62 $\int \frac{1}{x\log(x)}\,dx$ . . . 481
3.63 $\int \frac{1}{x^2\log^2(x)}\,dx$ . . . 486
3.64 $\int \frac{\log^p(x)}{x}\,dx$ . . . 491

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3.77 $\int \cos(x)\,dx$ . . . 556
3.78 $\int \tan(x)\,dx$ . . . 561
3.79 $\int \cot(x)\,dx$ . . . 566
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3.81 $\int \sec(x)\,dx$ . . . 577
3.82 $\int \csc(x)\,dx$ . . . 582
3.83 $\int \sin^2(x)\,dx$ . . . 587
3.84 $\int x^3\sin\left(x^2\right)\,dx$ . . . 592
3.85 $\int \sin^3(x)\,dx$ . . . 597
3.86 $\int \sin^p(x)\,dx$ . . . 602
3.87 $\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx$ . . . 607
3.88 $\int \cos^2(x)\,dx$ . . . 612
3.89 $\int \cos^3(x)\,dx$ . . . 617
3.90 $\int \sec^2(x)\,dx$ . . . 622
3.91 $\int \sin(x)\sin(2x)\,dx$ . . . 627
3.92 $\int x\sin(x)\,dx$ . . . 632
3.93 $\int x^2\sin(x)\,dx$ . . . 637
3.94 $\int x\sin^2(x)\,dx$ . . . 642
3.95 $\int x^2\sin^2(x)\,dx$ . . . 647
3.96 $\int x\sin^3(x)\,dx$ . . . 653
3.97 $\int x\cos(x)\,dx$ . . . 658
3.98 $\int x^2\cos(x)\,dx$ . . . 663
3.99 $\int x\cos^2(x)\,dx$ . . . 668
3.100 $\int x^2\cos^2(x)\,dx$ . . . 673
3.101 $\int x\cos^3(x)\,dx$ . . . 679
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3.104 $\int \frac{\sin(x)}{x^2}\,dx$ 694
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3.109 $\int \tan(a+bx)\,dx$ 719
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3.111 $\int \csc(a+bx)\,dx$ 729
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3.113 $\int \sin^2(a+bx)\,dx$ 739
3.114 $\int \sin^3(a+bx)\,dx$ 744
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3.120 $\int \frac{1}{1+\sin(x)}\,dx$ 774
3.121 $\int \frac{1}{1-\sin(x)}\,dx$ 779
3.122 $\int \frac{1}{a+b\sin(x)}\,dx$ 784
3.123 $\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$ 790
3.124 $\int x^2\sin^2(a+bx)\,dx$ 796
3.125 $\int \cos(x)\cos(2x)\,dx$ 802
3.126 $\int x^2\cos^2(a+bx)\,dx$ 807
3.127 $\int \cot^3(x)\,dx$ 813
3.128 $\int x^3\tan^4(x)\,dx$ 818
3.129 $\int x^3\tan^6(x)\,dx$ 827
3.130 $\int x\tan^2(x)\,dx$ 838
3.131 $\int \cos(3x)\sin(2x)\,dx$ 843
3.132 $\int \cos^2(x)\sin^2(x)\,dx$ 848
3.133 $\int \csc^2(x)\sec^2(x)\,dx$ 853
3.134 $\int d^x\sin(x)\,dx$ 858
3.135 $\int d^x\cos(x)\,dx$ 863
3.136 $\int d^x x\sin(x)\,dx$ 868
3.137 $\int d^x x\cos(x)\,dx$ 875
3.138 $\int d^x x^2\sin(x)\,dx$ 882
3.139 $\int d^x x^2\cos(x)\,dx$ 889
3.140 $\int d^x x^3\sin(x)\,dx$ 896
3.141 $\int d^x x^3\cos(x)\,dx$ 904

3.142 $\int \sin(x)\sin(2x)\sin(3x)\,dx$ . . . 912
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3.144 $\int x^2\sin^3(kx)\,dx$ . . . 924
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3.147 $\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$ . . . 943
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3.149 $\int \cos(\log(x))\,dx$ . . . 955
3.150 $\int e^x\,dx$ . . . 959
3.151 $\int a^x\,dx$ . . . 964
3.152 $\int e^{ax}\,dx$ . . . 969
3.153 $\int \frac{e^{ax}}{x}\,dx$ . . . 974
3.154 $\int \frac{1}{a+be^{mx}}\,dx$ . . . 978
3.155 $\int \frac{e^{2x}}{1+e^x}\,dx$ . . . 983
3.156 $\int e^{2x+ax}\,dx$ . . . 988
3.157 $\int \frac{1}{be^{-mx}+ae^{mx}}\,dx$ . . . 993
3.158 $\int e^{ax}x\,dx$ . . . 998
3.159 $\int e^x x^{20}\,dx$ . . . 1003
3.160 $\int a^x b^{-x}\,dx$ . . . 1012
3.161 $\int a^x b^x\,dx$ . . . 1017
3.162 $\int \frac{a^x}{x^2}\,dx$ . . . 1023
3.163 $\int \frac{a^x x}{(1+bx)^2}\,dx$ . . . 1028
3.164 $\int \frac{e^{ax}x}{(1+ax)^2}\,dx$ . . . 1033
3.165 $\int k^{x^2}x\,dx$ . . . 1038
3.166 $\int e^{x^2}\,dx$ . . . 1043
3.167 $\int e^{x^2}x\,dx$ . . . 1048
3.168 $\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx$ . . . 1053
3.169 $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx$ . . . 1058
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3.171 $\int e^x\log(x)\,dx$ . . . 1068
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3.1 $\int (1 + x + x^2) \, dx$

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3.1.1 Optimal result

Integrand size = 6, antiderivative size = 16

$$\int (1 + x + x^2) \, dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

output
```
x+1/2*x^2+1/3*x^3
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 + x + x^2) \, dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

input
```
Integrate[1 + x + x^2,x]
```

output
```
x + x^2/2 + x^3/3
```

3.1.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2 + x + 1)\, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

input
```
Int[1 + x + x^2,x]
```

output
```
x + x^2/2 + x^3/3
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
default	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
norman	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
risch	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
parallelrisch	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
parts	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13

input
```
int(x^2+x+1,x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+1/3*x^3
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1+x+x^2)\, dx = \frac{1}{3}\, x^3 + \frac{1}{2}\, x^2 + x$$

input
```
integrate(x^2+x+1,x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int (1+x+x^2)\, dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

input
```
integrate(x**2+x+1,x)
```

output
```
x**3/3 + x**2/2 + x
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1+x+x^2)\, dx = \frac{1}{3}\, x^3 + \frac{1}{2}\, x^2 + x$$

input
```
integrate(x^2+x+1,x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1 + x + x^2) \, dx = \frac{1}{3} x^3 + \frac{1}{2} x^2 + x$$

input
```
integrate(x^2+x+1,x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int (1 + x + x^2) \, dx = \frac{x (2 x^2 + 3 x + 6)}{6}$$

input
```
int(x + x^2 + 1,x)
```

output
```
(x*(3*x + 2*x^2 + 6))/6
```

3.1.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int (1 + x + x^2) \, dx = \frac{x(2x^2 + 3x + 6)}{6}$$

input
```
int(x**2 + x + 1,x)
```

output
```
(x*(2*x**2 + 3*x + 6))/6
```

3.2 $\int x^2(x+2x^2)^2\,dx$

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3.2.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int x^2(x+2x^2)^2\,dx=\frac{x^5}{5}+\frac{2x^6}{3}+\frac{4x^7}{7}$$

output
```
1/5*x^5+2/3*x^6+4/7*x^7
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(x+2x^2)^2\,dx=\frac{x^5}{5}+\frac{2x^6}{3}+\frac{4x^7}{7}$$

input
```
Integrate[x^2*(x + 2*x^2)^2,x]
```

output
```
x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

3.2.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(2x^2+x\right)^2 dx$$

$$\Big\downarrow \; 9$$

$$\int x^4(2x+1)^2dx$$

$$\Big\downarrow \; 49$$

$$\int \left(4x^6+4x^5+x^4\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

input
```
Int[x^2*(x + 2*x^2)^2,x]
```

output
```
x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

3.2.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x^{5}\left(60 x^{2}+70 x+21\right)}{105}$	16
default	$\frac{1}{5} x^{5}+\frac{2}{3} x^{6}+\frac{4}{7} x^{7}$	17
norman	$\frac{1}{5} x^{5}+\frac{2}{3} x^{6}+\frac{4}{7} x^{7}$	17
risch	$\frac{1}{5} x^{5}+\frac{2}{3} x^{6}+\frac{4}{7} x^{7}$	17
parallelrisch	$\frac{1}{5} x^{5}+\frac{2}{3} x^{6}+\frac{4}{7} x^{7}$	17

input
```
int(x^2*(2*x^2+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^5*(60*x^2+70*x+21)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2\left(x+2 x^2\right)^2 \, dx=\frac{4}{7} \, x^{7} + \frac{2}{3} \, x^{6} + \frac{1}{5} \, x^{5}$$

input
```
integrate(x^2*(2*x^2+x)^2,x, algorithm="fricas")
```

output
```
4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2(x+2x^2)^2\,dx = \frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

input
```
integrate(x**2*(2*x**2+x)**2,x)
```

output
```
4*x**7/7 + 2*x**6/3 + x**5/5
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(x+2x^2)^2\,dx = \frac{4}{7}\,x^7 + \frac{2}{3}\,x^6 + \frac{1}{5}\,x^5$$

input
```
integrate(x^2*(2*x^2+x)^2,x, algorithm="maxima")
```

output
```
4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(x+2x^2)^2\,dx = \frac{4}{7}\,x^7 + \frac{2}{3}\,x^6 + \frac{1}{5}\,x^5$$

input
```
integrate(x^2*(2*x^2+x)^2,x, algorithm="giac")
```

output
```
4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x^2 (x + 2x^2)^2 \, dx = \frac{x^5 (60 \, x^2 + 70 \, x + 21)}{105}$$

input
```
int(x^2*(x + 2*x^2)^2,x)
```

output
```
(x^5*(70*x + 60*x^2 + 21))/105
```

3.2.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x^2 (x + 2x^2)^2 \, dx = \frac{x^5(60x^2 + 70x + 21)}{105}$$

input
```
int(x**4*(4*x**2 + 4*x + 1),x)
```

output
```
(x**5*(60*x**2 + 70*x + 21))/105
```

3.3 $\int x(1+2x+x^2)\,dx$

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3.3.1 Optimal result

Integrand size = 10, antiderivative size = 22

$$\int x\left(1+2x+x^2\right)\,dx=\frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}$$

output
```
1/2*x^2+2/3*x^3+1/4*x^4
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(1+2x+x^2\right)\,dx=\frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}$$

input
```
Integrate[x*(1 + 2*x + x^2),x]
```

output
```
x^2/2 + (2*x^3)/3 + x^4/4
```

3.3.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x^2+2x+1)\,dx$$

$$\Big\downarrow \text{1098}$$

$$\int x(x+1)^2dx$$

$$\Big\downarrow \text{49}$$

$$\int (x^3+2x^2+x)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

input
```
Int[x*(1 + 2*x + x^2),x]
```

output
```
x^2/2 + (2*x^3)/3 + x^4/4
```

3.3.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x^2(3x^2+8x+6)}{12}$	16
default	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17
norman	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17
risch	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17
parallelrisch	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17

input
```
int(x*(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*x^2+8*x+6)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1+2x+x^2)\,dx=\frac{1}{4}\,x^4+\frac{2}{3}\,x^3+\frac{1}{2}\,x^2$$

input
```
integrate(x*(x^2+2*x+1),x, algorithm="fricas")
```

output
```
1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1+2x+x^2)\,dx=\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

input
```
integrate(x*(x**2+2*x+1),x)
```

output
```
x**4/4 + 2*x**3/3 + x**2/2
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1+2x+x^2)\,dx=\frac{1}{4}\,x^4+\frac{2}{3}\,x^3+\frac{1}{2}\,x^2$$

input
```
integrate(x*(x^2+2*x+1),x, algorithm="maxima")
```

output
```
1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1+2x+x^2)\,dx=\frac{1}{4}\,x^4+\frac{2}{3}\,x^3+\frac{1}{2}\,x^2$$

input
```
integrate(x*(x^2+2*x+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1+2x+x^2)\,dx = \frac{x^2\,(3\,x^2+8\,x+6)}{12}$$

input
```
int(x*(2*x + x^2 + 1),x)
```

output
```
(x^2*(8*x + 3*x^2 + 6))/12
```

3.3.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1+2x+x^2)\,dx = \frac{x^2(3x^2+8x+6)}{12}$$

input
```
int(x*(x**2 + 2*x + 1),x)
```

output
```
(x**2*(3*x**2 + 8*x + 6))/12
```

3.4 $\int \frac{1}{x} dx$

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3.4.10 Reduce [B] (verification not implemented) . . . 135

3.4.1 Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} dx = \log(x)$$

output
```
ln(x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input
```
Integrate[x^(-1),x]
```

output
```
Log[x]
```

3.4.3 Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{14\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x} \, dx$$

$$\Big\downarrow \; 14$$

$$\log(x)$$

input
```
Int[x^(-1),x]
```

output
```
Log[x]
```

3.4.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

3.4.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3
parallelrisch	$\ln(x)$	3

input
```
int(1/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log (x)$$

input
```
integrate(1/x,x, algorithm="fricas")
```

output
```
log(x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log (x)$$

input
```
integrate(1/x,x)
```

output
```
log(x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log (x)$$

input
```
integrate(1/x,x, algorithm="maxima")
```

output
```
log(x)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x} \, dx = \log (|x|)$$

input
```
integrate(1/x,x, algorithm="giac")
```

output
```
log(abs(x))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \ln (x)$$

input
```
int(1/x,x)
```

output
```
log(x)
```

3.4.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

input
```
int(1/x,x)
```

output
```
log(x)
```

3.5 $\int \frac{(1+x)^3}{(-1+x)^4} \, dx$

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3.5.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(1+x)^3}{(-1+x)^4} \, dx = \frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x)$$

output
```
8/3/(1-x)^3-6/(1-x)^2+6/(1-x)+ln(1-x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(1+x)^3}{(-1+x)^4} \, dx = -\frac{2(4-9x+9x^2)}{3(-1+x)^3} + \log(-1+x)$$

input
```
Integrate[(1 + x)^3/(-1 + x)^4,x]
```

output
```
(-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]
```

3.5.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^3}{(x-1)^4}\,dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{x-1}+\frac{6}{(x-1)^2}+\frac{12}{(x-1)^3}+\frac{8}{(x-1)^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{6}{1-x}-\frac{6}{(1-x)^2}+\frac{8}{3(1-x)^3}+\log(1-x)$$

input
```
Int[(1 + x)^3/(-1 + x)^4,x]
```

output
```
8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

3.5.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

method	result	size
norman	$\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3}+\ln(-1+x)$	22
risch	$\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3}+\ln(-1+x)$	22
default	$\ln(-1+x)-\frac{8}{3(-1+x)^3}-\frac{6}{-1+x}-\frac{6}{(-1+x)^2}$	27
parallelrisch	$\frac{3\ln(-1+x)x^3-8-9\ln(-1+x)x^2+9\ln(-1+x)x-18x^2-3\ln(-1+x)+18x}{3(-1+x)^3}$	49
meijerg	$\frac{x(x^2-3x+3)}{3(1-x)^3}+\frac{x^2(3-x)}{2(1-x)^3}+\frac{x^3}{(1-x)^3}+\frac{x(22x^2-30x+12)}{12(1-x)^3}+\ln(1-x)$	74

input
```
int((1+x)^3/(-1+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(-6*x^2+6*x-8/3)/(-1+x)^3+ln(-1+x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{(1+x)^3}{(-1+x)^4}\,dx = -\frac{18\,x^2-3\,(x^3-3\,x^2+3\,x-1)\log(x-1)-18\,x+8}{3\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((1+x)^3/(-1+x)^4,x, algorithm="fricas")
```

output
```
-1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*log(x - 1) - 18*x + 8)/(x^3 - 3*x
^2 + 3*x - 1)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)^3}{(-1+x)^4}\,dx = \frac{-18x^2+18x-8}{3x^3-9x^2+9x-3} + \log{(x-1)}$$

input
```
integrate((1+x)**3/(-1+x)**4,x)
```

output
```
(-18*x**2 + 18*x - 8)/(3*x**3 - 9*x**2 + 9*x - 3) + log(x - 1)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)^3}{(-1+x)^4}\,dx = -\frac{2\,(9\,x^2-9\,x+4)}{3\,(x^3-3\,x^2+3\,x-1)} + \log{(x-1)}$$

input
```
integrate((1+x)^3/(-1+x)^4,x, algorithm="maxima")
```

output
```
-2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{(1+x)^3}{(-1+x)^4}\,dx = -\frac{2\,(9\,x^2-9\,x+4)}{3\,(x-1)^3} + \log{(|x-1|)}$$

input
```
integrate((1+x)^3/(-1+x)^4,x, algorithm="giac")
```

output
```
-2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + log(abs(x - 1))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{(1+x)^3}{(-1+x)^4}\,dx = \ln(x-1) - \frac{6\,x^2 - 6\,x + \frac{8}{3}}{(x-1)^3}$$

input
```
int((x + 1)^3/(x - 1)^4,x)
```

output
```
log(x - 1) - (6*x^2 - 6*x + 8/3)/(x - 1)^3
```

3.5.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{(1+x)^3}{(-1+x)^4}\,dx = \frac{3\log(x-1)\,x^3 - 9\log(x-1)\,x^2 + 9\log(x-1)\,x - 3\log(x-1) - 6x^3 - 2}{3x^3 - 9x^2 + 9x - 3}$$

input
```
int((x**3 + 3*x**2 + 3*x + 1)/(x**4 - 4*x**3 + 6*x**2 - 4*x + 1),x)
```

output
```
(3*log(x - 1)*x**3 - 9*log(x - 1)*x**2 + 9*log(x - 1)*x - 3*log(x - 1) - 6
*x**3 - 2)/(3*(x**3 - 3*x**2 + 3*x - 1))
```

3.6 $\int \frac{1}{(-1+x)x(1+x)^2}\,dx$

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3.6.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = -\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(1+x)$$

output
```
-1/2/(1+x)+1/4*ln(1-x)-ln(x)+3/4*ln(1+x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = \frac{1}{4}\left(-\frac{2}{1+x}+\log(1-x)-4\log(x)+3\log(1+x)\right)$$

input
```
Integrate[1/((-1 + x)*x*(1 + x)^2),x]
```

output
```
(-2/(1 + x) + Log[1 - x] - 4*Log[x] + 3*Log[1 + x])/4
```

3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)x(x+1)^2} \, dx$$

$$\Big\downarrow \text{93}$$

$$\int \left(-\frac{1}{x}+\frac{3}{4(x+1)}+\frac{1}{2(x+1)^2}+\frac{1}{4(x-1)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

input
```
Int[1/((-1 + x)*x*(1 + x)^2),x]
```

output
```
-1/2*1/(1 + x) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

3.6.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln(-1+x)}{4}-\ln(x)-\frac{1}{2(1+x)}+\frac{3\ln(1+x)}{4}$	25
norman	$\frac{\ln(-1+x)}{4}-\ln(x)-\frac{1}{2(1+x)}+\frac{3\ln(1+x)}{4}$	25
risch	$\frac{\ln(-1+x)}{4}-\ln(x)-\frac{1}{2(1+x)}+\frac{3\ln(1+x)}{4}$	25
parallelrisch	$-\frac{4x\ln(x)-\ln(-1+x)x-3\ln(1+x)x+2+4\ln(x)-\ln(-1+x)-3\ln(1+x)}{4(1+x)}$	45

input
```
int(1/(-1+x)/x/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-1+x)-ln(x)-1/2/(1+x)+3/4*ln(1+x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = \frac{3\,(x+1)\log{(x+1)}+(x+1)\log{(x-1)}-4\,(x+1)\log{(x)}-2}{4\,(x+1)}$$

input
```
integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="fricas")
```

output
```
1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) - 4*(x + 1)*log(x) - 2)/(x
+ 1)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = -\log(x) + \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} - \frac{1}{2x+2}$$

input
```
integrate(1/(-1+x)/x/(1+x)**2,x)
```

output
```
-log(x) + log(x - 1)/4 + 3*log(x + 1)/4 - 1/(2*x + 2)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = -\frac{1}{2(x+1)} + \frac{3}{4}\log(x+1) + \frac{1}{4}\log(x-1) - \log(x)$$

input
```
integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="maxima")
```

output
```
-1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1) - log(x)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = -\frac{1}{2(x+1)} - \log\left(\left|-\frac{1}{x+1}+1\right|\right) + \frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

input
```
integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="giac")
```

output
```
-1/2/(x + 1) - log(abs(-1/(x + 1) + 1)) + 1/4*log(abs(-2/(x + 1) + 1))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx = \frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} - \ln(x) - \frac{1}{2(x+1)}$$

input
```
int(1/(x*(x - 1)*(x + 1)^2),x)
```

output
```
log(x - 1)/4 + (3*log(x + 1))/4 - log(x) - 1/(2*(x + 1))
```

3.6.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{1}{(-1+x)x(1+x)^2}\,dx$$
$$= \frac{\log(x-1)\,x + \log(x-1) + 3\log(x+1)\,x + 3\log(x+1) - 4\log(x)\,x - 4\log(x) + 2x}{4x+4}$$

input
```
int(1/(x*(x**3 + x**2 - x - 1)),x)
```

output
```
(log(x - 1)*x + log(x - 1) + 3*log(x + 1)*x + 3*log(x + 1) - 4*log(x)*x -
4*log(x) + 2*x)/(4*(x + 1))
```

3.7 $\int \frac{b+ax}{(-p+x)(-q+x)}\,dx$

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3.7.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx = \frac{(b+ap)\log(p-x)}{p-q} - \frac{(b+aq)\log(q-x)}{p-q}$$

output
```
(a*p+b)*ln(p-x)/(p-q)-(a*q+b)*ln(q-x)/(p-q)
```

3.7.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx = \frac{(b+ap)\log(-p+x)-(b+aq)\log(-q+x)}{p-q}$$

input
```
Integrate[(b + a*x)/((-p + x)*(-q + x)),x]
```

output
```
((b + a*p)*Log[-p + x] - (b + a*q)*Log[-q + x])/(p - q)
```

3.7.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+b}{(x-p)(x-q)} \, dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{-ap-b}{(p-q)(p-x)}+\frac{aq+b}{(p-q)(q-x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ap+b)\log(p-x)}{p-q}-\frac{(aq+b)\log(q-x)}{p-q}$$

input
```
Int[(b + a*x)/((-p + x)*(-q + x)),x]
```

output
```
((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

3.7.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result	size
norman	$\frac{(ap+b)\ln(p-x)}{p-q}-\frac{(aq+b)\ln(q-x)}{p-q}$	41
default	$\frac{(-aq-b)\ln(q-x)}{p-q}+\frac{(ap+b)\ln(p-x)}{p-q}$	43
parallelrisch	$\frac{\ln(-p+x)ap-\ln(-q+x)aq+\ln(-p+x)b-\ln(-q+x)b}{p-q}$	46
risch	$-\frac{\ln(-q+x)aq}{p-q}-\frac{\ln(-q+x)b}{p-q}+\frac{\ln(p-x)ap}{p-q}+\frac{\ln(p-x)b}{p-q}$	66

input
```
int((a*x+b)/(-p+x)/(-q+x),x,method=_RETURNVERBOSE)
```

output
```
(a*p+b)*ln(p-x)/(p-q)-(a*q+b)*ln(q-x)/(p-q)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx = \frac{(ap+b)\log(-p+x)-(aq+b)\log(-q+x)}{p-q}$$

input
```
integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="fricas")
```

output
```
((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(26) = 52.

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.60

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx = \frac{(ap+b)\log\left(x+\frac{-2apq-bp-bq-\frac{p^2(ap+b)}{p-q}+\frac{2pq(ap+b)}{p-q}-\frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p-q} - \frac{(aq+b)\log\left(x+\frac{-2apq-bp-bq+\frac{p^2(aq+b)}{p-q}-\frac{2pq(aq+b)}{p-q}+\frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p-q}$$

input
```
integrate((a*x+b)/(-p+x)/(-q+x),x)
```

output
```
(a*p + b)*log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q) + 2*p*q*(
a*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q + 2*b))/(p - q) - (a
*q + b)*log(x + (-2*a*p*q - b*p - b*q + p**2*(a*q + b)/(p - q) - 2*p*q*(a*
q + b)/(p - q) + q**2*(a*q + b)/(p - q))/(a*p + a*q + 2*b))/(p - q)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx = \frac{(ap+b)\log(-p+x)}{p-q} - \frac{(aq+b)\log(-q+x)}{p-q}$$

input
```
integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="maxima")
```

output
```
(a*p + b)*log(-p + x)/(p - q) - (a*q + b)*log(-q + x)/(p - q)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx = \frac{(ap+b)\log(|-p+x|)}{p-q} - \frac{(aq+b)\log(|-q+x|)}{p-q}$$

input
```
integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="giac")
```

output
```
(a*p + b)*log(abs(-p + x))/(p - q) - (a*q + b)*log(abs(-q + x))/(p - q)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx=\frac{\ln\left(x-p\right)\,\left(b+a\,p\right)}{p-q}-\frac{\ln\left(x-q\right)\,\left(b+a\,q\right)}{p-q}$$

input
```
int((b + a*x)/((p - x)*(q - x)),x)
```

output
```
(log(x - p)*(b + a*p))/(p - q) - (log(x - q)*(b + a*q))/(p - q)
```

3.7.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{b+ax}{(-p+x)(-q+x)}\,dx=\frac{\log(p-x)\,ap+\log(p-x)\,b-\log(q-x)\,aq-\log(q-x)\,b}{p-q}$$

input
```
int((a*x + b)/(p*q - p*x - q*x + x**2),x)
```

output
```
(log(p - x)*a*p + log(p - x)*b - log(q - x)*a*q - log(q - x)*b)/(p - q)
```

3.8 $\int \frac{1}{c+bx+ax^2} dx$

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3.8.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{1}{c+bx+ax^2} dx = -\frac{2\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{c+bx+ax^2} dx = \frac{2\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[(c + b*x + a*x^2)^(-1),x]
```

output
```
(2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

3.8.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx+c}\,dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{b^2-(b+2ax)^2-4ac}d(b+2ax)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(c + b*x + a*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

3.8.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.8.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2ax+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2ax+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

input
```
int(1/(a*x^2+b*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{1}{c+bx+ax^2}\,dx = \left[\frac{\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(1/(a*x^2+b*x+c),x, algorithm="fricas")
```

output
```
[log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(
a*x^2 + b*x + c))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{1}{c+bx+ax^2}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)$$

input
```
integrate(1/(a*x**2+b*x+c),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a))
```

3.8.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{c+bx+ax^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*x^2+b*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{c+bx+ax^2}\,dx = \frac{2 \arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(a*x^2+b*x+c),x, algorithm="giac")
```

output
```
2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{c+bx+ax^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,a\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(c + b*x + a*x^2),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*a*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

3.8.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{c+bx+ax^2}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{4ac-b^2}$$

input
```
int(1/(a*x**2 + b*x + c),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((2*a*x + b)/sqrt(4*a*c - b**2)))/(4*a*c - b**2)
```

3.9 $\int \frac{b+ax}{1+x^2}\,dx$

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3.9.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{b+ax}{1+x^2}\,dx=b\arctan(x)+\frac{1}{2}a\log\left(1+x^2\right)$$

output
```
b*arctan(x)+1/2*a*ln(x^2+1)
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+ax}{1+x^2}\,dx=b\arctan(x)+\frac{1}{2}a\log\left(1+x^2\right)$$

input
```
Integrate[(b + a*x)/(1 + x^2),x]
```

output
```
b*ArcTan[x] + (a*Log[1 + x^2])/2
```

3.9.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+b}{x^2+1}dx$$

$$\Big\downarrow \text{ 452}$$

$$a\int \frac{x}{x^2+1}dx+b\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{ 216}$$

$$a\int \frac{x}{x^2+1}dx+b\arctan(x)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{2}a\log\left(x^2+1\right)+b\arctan(x)$$

input
```
Int[(b + a*x)/(1 + x^2),x]
```

output
```
b*ArcTan[x] + (a*Log[1 + x^2])/2
```

3.9.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$b \arctan\left(x\right)+\frac{a \ln\left(x^{2}+1\right)}{2}$	15
meijerg	$b \arctan\left(x\right)+\frac{a \ln\left(x^{2}+1\right)}{2}$	15
risch	$b \arctan\left(x\right)+\frac{a \ln\left(x^{2}+1\right)}{2}$	15
parallelrisch	$\frac{\ln\left(x-i\right) a}{2}-\frac{i \ln\left(x-i\right) b}{2}+\frac{\ln\left(x+i\right) a}{2}+\frac{i \ln\left(x+i\right) b}{2}$	36

input
```
int((a*x+b)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
b*arctan(x)+1/2*a*ln(x^2+1)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+ax}{1+x^2}\,dx = b\arctan\left(x\right)+\frac{1}{2}\,a\log\left(x^2+1\right)$$

input
```
integrate((a*x+b)/(x^2+1),x, algorithm="fricas")
```

output
```
b*arctan(x) + 1/2*a*log(x^2 + 1)
```

3.9.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{b+ax}{1+x^2}\,dx = \left(\frac{a}{2}-\frac{ib}{2}\right)\log(x-i)+\left(\frac{a}{2}+\frac{ib}{2}\right)\log(x+i)$$

input
```
integrate((a*x+b)/(x**2+1),x)
```

output
```
(a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+ax}{1+x^2}\,dx = b\arctan(x)+\frac{1}{2}a\log\left(x^2+1\right)$$

input
```
integrate((a*x+b)/(x^2+1),x, algorithm="maxima")
```

output
```
b*arctan(x) + 1/2*a*log(x^2 + 1)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+ax}{1+x^2}\,dx = b\arctan(x)+\frac{1}{2}a\log\left(x^2+1\right)$$

input
```
integrate((a*x+b)/(x^2+1),x, algorithm="giac")
```

output
```
b*arctan(x) + 1/2*a*log(x^2 + 1)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+ax}{1+x^2}\,dx = \frac{a\ln(x^2+1)}{2} + b\,\mathrm{atan}(x)$$

input
```
int((b + a*x)/(x^2 + 1),x)
```

output
```
(a*log(x^2 + 1))/2 + b*atan(x)
```

3.9.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+ax}{1+x^2}\,dx = atan(x)\,b + \frac{\log(x^2+1)\,a}{2}$$

input
```
int((a*x + b)/(x**2 + 1),x)
```

output
```
(2*atan(x)*b + log(x**2 + 1)*a)/2
```

3.10 $\int \frac{1}{3-2x+x^2}\,dx$

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3.10.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{3-2x+x^2}\,dx = -\frac{\arctan\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(1/2*(1-x)*2^(1/2))*2^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{\arctan\left(\frac{-1+x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(3 - 2*x + x^2)^(-1),x]
```

output
```
ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]
```

3.10.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-2x+3} \, dx$$
$$\Big\downarrow \text{1083}$$
$$-2\int \frac{1}{-(2x-2)^2-8}d(2x-2)$$
$$\Big\downarrow \text{217}$$
$$\frac{\arctan\left(\frac{2x-2}{2\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[(3 - 2*x + x^2)^(-1),x]
```

output
```
ArcTan[(-2 + 2*x)/(2*Sqrt[2])]/Sqrt[2]
```

3.10.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.10.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{\sqrt{2}\,\arctan\left(\frac{(-1+x)\sqrt{2}}{2}\right)}{2}$	15
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(-2+2x)\sqrt{2}}{4}\right)}{2}$	17

input
```
int(1/(x^2-2*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/2*(-1+x)*2^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

input
```
integrate(1/(x^2-2*x+3),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}-\frac{\sqrt{2}}{2}\right)}{2}$$

input
```
integrate(1/(x**2-2*x+3),x)
```

output
```
sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

input
```
integrate(1/(x^2-2*x+3),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

input
```
integrate(1/(x^2-2*x+3),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(x-1)}{2}\right)}{2}$$

input
```
int(1/(x^2 - 2*x + 3),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(x - 1))/2))/2
```

3.10.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{3-2x+x^2}\,dx = \frac{\sqrt{2}\,atan\left(\frac{x-1}{\sqrt{2}}\right)}{2}$$

input
```
int(1/(x**2 - 2*x + 3),x)
```

output
```
(sqrt(2)*atan((x - 1)/sqrt(2)))/2
```

3.11 $\int \frac{1}{(-1+x)^2(1+x^2)^2} dx$

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3.11.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{1}{4(1-x)} - \frac{1}{4(1+x^2)} + \frac{\arctan(x)}{4} - \frac{1}{2}\log(1-x) + \frac{1}{4}\log(1+x^2)$$

output
```
1/4/(1-x)-1/4/(x^2+1)+1/4*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{1}{4}\left(\frac{1}{1-x} - \frac{1}{1+x^2} + \arctan(x) - 2\log(-1+x) + \log(1+x^2)\right)$$

input
```
Integrate[1/((-1 + x)^2*(1 + x^2)^2),x]
```

output
```
((1 - x)^(-1) - (1 + x^2)^(-1) + ArcTan[x] - 2*Log[-1 + x] + Log[1 + x^2])
/4
```

3.11.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {496, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)^2 \left(x^2+1\right)^2}dx$$
$$\Big\downarrow \ 496$$
$$-\frac{1}{4} \int -\frac{2(2-x)}{(1-x)^2 \left(x^2+1\right)}dx-\frac{1}{4 \left(x^2+1\right)}$$
$$\Big\downarrow \ 27$$
$$\frac{1}{2} \int \frac{2-x}{(1-x)^2 \left(x^2+1\right)}dx-\frac{1}{4 \left(x^2+1\right)}$$
$$\Big\downarrow \ 657$$
$$\frac{1}{2} \int \left(\frac{2 x+1}{2 \left(x^2+1\right)}+\frac{1}{1-x}+\frac{1}{2(x-1)^2}\right)dx-\frac{1}{4 \left(x^2+1\right)}$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2} \left(\frac{\arctan (x)}{2}+\frac{1}{2} \log \left(x^2+1\right)+\frac{1}{2(1-x)}-\log (1-x)\right)-\frac{1}{4 \left(x^2+1\right)}$$

input
```
Int[1/((-1 + x)^2*(1 + x^2)^2),x]
```

output
```
-1/4*1/(1 + x^2) + (1/(2*(1 - x)) + ArcTan[x]/2 - Log[1 - x] + Log[1 + x^2
]/2)/2
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{2}-\frac{1}{4(x^2+1)}+\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{4}$
risch	$\frac{-\frac{1}{4}x^2-\frac{1}{4}x}{(x^2+1)(-1+x)}-\frac{\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{4}$
parallelrisch	$-\frac{2x+2x^2+2\ln(x-i)+2\ln(x+i)-i\ln(x-i)x^2+i\ln(x-i)x^3+i\ln(x+i)-i\ln(x+i)x^3-i\ln(x+i)x+4\ln(-1+x)x^3-i\ln(x-i)-}{}$

input
```
int(1/(-1+x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-1+x)-1/2*ln(-1+x)-1/4/(x^2+1)+1/4*ln(x^2+1)+1/4*arctan(x)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{1}{(-1+x)^2 (1+x^2)^2} \, dx = -\frac{x^2 - (x^3 - x^2 + x - 1) \arctan(x) - (x^3 - x^2 + x - 1) \log(x^2 + 1) + 2 (x^3 - x^2 + x - 1) \log(x - 1) -}{4 (x^3 - x^2 + x - 1)}$$

input
```
integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/4*(x^2 - (x^3 - x^2 + x - 1)*arctan(x) - (x^3 - x^2 + x - 1)*log(x^2 +
1) + 2*(x^3 - x^2 + x - 1)*log(x - 1) + x)/(x^3 - x^2 + x - 1)
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-1+x)^2 (1+x^2)^2} \, dx = \frac{-x^2 - x}{4x^3 - 4x^2 + 4x - 4} - \frac{\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{4}$$

input
```
integrate(1/(-1+x)**2/(x**2+1)**2,x)
```

output
```
(-x**2 - x)/(4*x**3 - 4*x**2 + 4*x - 4) - log(x - 1)/2 + log(x**2 + 1)/4 +
 atan(x)/4
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-1+x)^2 (1+x^2)^2} \, dx = -\frac{x^2 + x}{4 (x^3 - x^2 + x - 1)} + \frac{1}{4} \arctan(x) + \frac{1}{4} \log\left(x^2 + 1\right) - \frac{1}{2} \log(x - 1)$$

input
```
integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*arctan(x) + 1/4*log(x^2 + 1) - 1/
2*log(x - 1)
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \frac{1}{(-1+x)^2 (1+x^2)^2} \, dx = \frac{1}{16}\pi - \frac{1}{4}\pi \left\lfloor \frac{\pi + 4 \arctan(x)}{4\pi} + \frac{1}{2} \right\rfloor + \frac{\frac{2}{x-1} + 1}{8\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)} - \frac{1}{4(x-1)} + \frac{1}{4}\arctan(x) + \frac{1}{4}\log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)$$

input
```
integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/16*pi - 1/4*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/8*(2/(x - 1) +
 1)/(2/(x - 1) + 2/(x - 1)^2 + 1) - 1/4/(x - 1) + 1/4*arctan(x) + 1/4*log(
2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-1+x)^2 (1+x^2)^2} \, dx = -\frac{\ln(x-1)}{2} - \frac{\frac{x^2}{4} + \frac{x}{4}}{x^3 - x^2 + x - 1} + \ln(x - \mathrm{i})\left(\frac{1}{4} - \frac{1}{8}\mathrm{i}\right) + \ln(x + 1\mathrm{i})\left(\frac{1}{4} + \frac{1}{8}\mathrm{i}\right)$$

input
```
int(1/((x^2 + 1)^2*(x - 1)^2),x)
```

output
```
log(x - 1i)*(1/4 - 1i/8) - log(x - 1)/2 + log(x + 1i)*(1/4 + 1i/8) - (x/4
+ x^2/4)/(x - x^2 + x^3 - 1)
```

3.11.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.39

$$\int \frac{1}{(-1+x)^2 (1+x^2)^2} dx$$
$$= \frac{atan(x)\, x^3 - atan(x)\, x^2 + atan(x)\, x - atan(x) + \log(x^2+1)\, x^3 - \log(x^2+1)\, x^2 + \log(x^2+1)\, x - \log(}{4x^3 - 4x^2 + 4x -}$$

input
```
int(1/(x**6 - 2*x**5 + 3*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)
```

output
```
(atan(x)*x**3 - atan(x)*x**2 + atan(x)*x - atan(x) + log(x**2 + 1)*x**3 -
log(x**2 + 1)*x**2 + log(x**2 + 1)*x - log(x**2 + 1) - 2*log(x - 1)*x**3 +
 2*log(x - 1)*x**2 - 2*log(x - 1)*x + 2*log(x - 1) - x**3 - 2*x + 1)/(4*(x
**3 - x**2 + x - 1))
```

3.12 $\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx$

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3.12.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \frac{a\log(a-x)}{(a-b)(a-c)} - \frac{b\log(b-x)}{(a-b)(b-c)} + \frac{c\log(c-x)}{(a-c)(b-c)}$$

output
```
a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)
```

3.12.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \frac{a(b-c)\log(-a+x)+b(-a+c)\log(-b+x)+(a-b)c\log(-c+x)}{(a-b)(a-c)(b-c)}$$

input
```
Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]
```

output
```
(a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-c + x])/(
(a - b)*(a - c)*(b - c))
```

3.12.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x-a)(x-b)(x-c)} \, dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(-\frac{a}{(a-b)(a-c)(a-x)}+\frac{b}{(a-b)(b-c)(b-x)}+\frac{c}{(a-c)(c-b)(c-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

input
```
Int[x/((-a + x)*(-b + x)*(-c + x)),x]
```

output
```
(a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*L
og[c - x])/((a - c)*(b - c))
```

3.12.3.1 Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{a\ln(a-x)}{(a-b)(a-c)}-\frac{b\ln(b-x)}{(a-b)(b-c)}+\frac{c\ln(c-x)}{(a-c)(b-c)}$	69
norman	$\frac{c\ln(c-x)}{ab-ac-bc+c^2}+\frac{a\ln(a-x)}{(a-b)(a-c)}-\frac{b\ln(b-x)}{(a-b)(b-c)}$	72
risch	$-\frac{b\ln(-b+x)}{ab-ac-b^2+bc}+\frac{a\ln(-a+x)}{a^2-ab-ac+bc}+\frac{c\ln(c-x)}{ab-ac-bc+c^2}$	79
parallelrisch	$\frac{\ln(-a+x)ab-\ln(-a+x)ac-\ln(-b+x)ab+\ln(-b+x)bc+\ln(-c+x)ac-\ln(-c+x)bc}{(ab-ac-bc+c^2)(a-b)}$	84

input
```
int(x/(-a+x)/(-b+x)/(-c+x),x,method=_RETURNVERBOSE)
```

output
```
a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \frac{(a-b)c\log(-c+x)+(ab-ac)\log(-a+x)-(ab-bc)\log(-b+x)}{a^2b-ab^2+(a-b)c^2-(a^2-b^2)c}$$

input
```
integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="fricas")
```

output
```
((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*log(-b + x)
)/(a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \text{Timed out}$$

input
```
integrate(x/(-a+x)/(-b+x)/(-c+x),x)
```

output
```
Timed out
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \frac{a\log(-a+x)}{a^2-ab-(a-b)c} - \frac{b\log(-b+x)}{ab-b^2-(a-b)c} + \frac{c\log(-c+x)}{ab-(a+b)c+c^2}$$

input
```
integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="maxima")
```

output
```
a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2 - (a - b)
*c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \frac{a\log(|-a+x|)}{a^2-ab-ac+bc} - \frac{b\log(|-b+x|)}{ab-b^2-ac+bc} + \frac{c\log(|-c+x|)}{ab-ac-bc+c^2}$$

input
```
integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="giac")
```

output
```
a*log(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*log(abs(-b + x))/(a*b - b^2
 - a*c + b*c) + c*log(abs(-c + x))/(a*b - a*c - b*c + c^2)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \ln(x-a)\left(\frac{b}{(a-b)\,(b-c)} - \frac{c}{(a-c)\,(b-c)}\right) - \frac{b\ln(x-b)}{(a-b)\,(b-c)} + \frac{c\ln(x-c)}{(a-c)\,(b-c)}$$

input
```
int(-x/((a - x)*(b - x)*(c - x)),x)
```

output
```
log(x - a)*(b/((a - b)*(b - c)) - c/((a - c)*(b - c))) - (b*log(x - b))/((
a - b)*(b - c)) + (c*log(x - c))/((a - c)*(b - c))
```

3.12.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.40

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx = \frac{\log(a-x)\,ab - \log(a-x)\,ac - \log(b-x)\,ab + \log(b-x)\,bc + \log(c-x)\,ac - \log(c-x)\,bc}{a^2b - a^2c - a\,b^2 + a\,c^2 + b^2c - b\,c^2}$$

input
```
int(( - x)/(a*b*c - a*b*x - a*c*x + a*x**2 - b*c*x + b*x**2 + c*x**2 - x**
3),x)
```

output
```
(log(a - x)*a*b - log(a - x)*a*c - log(b - x)*a*b + log(b - x)*b*c + log(c
 - x)*a*c - log(c - x)*b*c)/(a**2*b - a**2*c - a*b**2 + a*c**2 + b**2*c -
b*c**2)
```

3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)} \, dx$

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3.13.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)} \, dx = -\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

output
```
-1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)} \, dx = \frac{-\log(a^2+x^2)+\log(b^2+x^2)}{2(a^2-b^2)}$$

input
```
Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

output
```
(-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))
```

3.13.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{1}{(a^2+x^2)(b^2+x^2)}dx^2$$

$$\Big\downarrow \; 47$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{b^2+x^2}dx^2}{a^2-b^2}-\frac{\int \frac{1}{a^2+x^2}dx^2}{a^2-b^2}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(\frac{\log(b^2+x^2)}{a^2-b^2}-\frac{\log(a^2+x^2)}{a^2-b^2}\right)$$

input
```
Int[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

output
```
(-(Log[a^2 + x^2]/(a^2 - b^2)) + Log[b^2 + x^2]/(a^2 - b^2))/2
```

3.13.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$-\frac{\ln(a^2+x^2)-\ln(b^2+x^2)}{2(a^2-b^2)}$	33
default	$-\frac{\ln(a^2+x^2)}{2(a^2-b^2)}+\frac{\ln(b^2+x^2)}{2a^2-2b^2}$	44
norman	$-\frac{\ln(a^2+x^2)}{2(a^2-b^2)}+\frac{\ln(b^2+x^2)}{2a^2-2b^2}$	44
risch	$-\frac{\ln(-a^2-x^2)}{2(a^2-b^2)}+\frac{\ln(b^2+x^2)}{2a^2-2b^2}$	48

input
```
int(x/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(a^2+x^2)-ln(b^2+x^2))/(a^2-b^2)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = -\frac{\log(a^2+x^2)-\log(b^2+x^2)}{2(a^2-b^2)}$$

input
```
integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

output
```
-1/2*(log(a^2 + x^2) - log(b^2 + x^2))/(a^2 - b^2)
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(36) = 72.

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.57

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{\log\left(-\frac{a^4}{2(a-b)(a+b)}+\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}-\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2(a-b)(a+b)} - \frac{\log\left(\frac{a^4}{2(a-b)(a+b)}-\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}+\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2(a-b)(a+b)}$$

input
```
integrate(x/(a**2+x**2)/(b**2+x**2),x)
```

output
```
log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**2/2 - b**
4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b)) - log(a**4/(2*(
a - b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) + a**2/2 + b**4/(2*(a - b)*(
a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = -\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

input
```
integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

output
```
-1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = -\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

input
```
integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

output
```
-1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.45

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}-4x^2\right)1\mathrm{i}}{2(a^2-b^2)}-\frac{\left(\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}+4x^2\right)1\mathrm{i}}{2(a^2-b^2)}}{\frac{\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}-4x^2}{2(a^2-b^2)}+\frac{\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}+4x^2}{2(a^2-b^2)}}\right)1\mathrm{i}}{a^2-b^2}$$

input
```
int(x/((a^2 + x^2)*(b^2 + x^2)),x)
```

output
```
(atan(((((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)*1i)/(
2*(a^2 - b^2)) - (((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*
x^2)*1i)/(2*(a^2 - b^2)))/(((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b
^2)) - 4*x^2)/(2*(a^2 - b^2)) + ((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^
2 - b^2)) + 4*x^2)/(2*(a^2 - b^2))))*1i)/(a^2 - b^2)
```

3.13.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{-\log(a^2+x^2)+\log(b^2+x^2)}{2a^2-2b^2}$$

input
```
int(x/(a**2*b**2 + a**2*x**2 + b**2*x**2 + x**4),x)
```

output
```
( - log(a**2 + x**2) + log(b**2 + x**2))/(2*(a**2 - b**2))
```

3.14 $\int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx$

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3.14.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

output
```
a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right) - b \arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

input
```
Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

output
```
(a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)
```

3.14.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}dx$$

$$\Big\downarrow \text{383}$$

$$\frac{a^2\int \frac{1}{a^2+x^2}dx}{a^2-b^2}-\frac{b^2\int \frac{1}{b^2+x^2}dx}{a^2-b^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

input
```
Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

output
```
(a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

3.14.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

3.14.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$	41
risch	$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$	41
parallelrisch	$-\frac{ia\ln(-ia+x)-ib\ln(-ib+x)-ia\ln(ia+x)+ib\ln(ib+x)}{2(a^2-b^2)}$	59

input
```
int(x^2/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx=\frac{a\arctan\left(\frac{x}{a}\right)-b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

input
```
integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

output
```
(a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)
```

3.14.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 393, normalized size of antiderivative = 9.82

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx$$
$$= -\frac{ia\log\left(-\frac{2ia^7}{(a-b)^3(a+b)^3}+\frac{4ia^5b^2}{(a-b)^3(a+b)^3}-\frac{2ia^3b^4}{(a-b)^3(a+b)^3}+\frac{ia^3}{(a-b)(a+b)}+\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}$$
$$+\frac{ia\log\left(\frac{2ia^7}{(a-b)^3(a+b)^3}-\frac{4ia^5b^2}{(a-b)^3(a+b)^3}+\frac{2ia^3b^4}{(a-b)^3(a+b)^3}-\frac{ia^3}{(a-b)(a+b)}-\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}$$
$$-\frac{ib\log\left(-\frac{2ia^4b^3}{(a-b)^3(a+b)^3}+\frac{4ia^2b^5}{(a-b)^3(a+b)^3}+\frac{ia^2b}{(a-b)(a+b)}-\frac{2ib^7}{(a-b)^3(a+b)^3}+\frac{ib^3}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}$$
$$+\frac{ib\log\left(\frac{2ia^4b^3}{(a-b)^3(a+b)^3}-\frac{4ia^2b^5}{(a-b)^3(a+b)^3}-\frac{ia^2b}{(a-b)(a+b)}+\frac{2ib^7}{(a-b)^3(a+b)^3}-\frac{ib^3}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}$$

input
```
integrate(x**2/(a**2+x**2)/(b**2+x**2),x)
```

output
```
-I*a*log(-2*I*a**7/((a - b)**3*(a + b)**3) + 4*I*a**5*b**2/((a - b)**3*(a
+ b)**3) - 2*I*a**3*b**4/((a - b)**3*(a + b)**3) + I*a**3/((a - b)*(a + b)
) + I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) + I*a*log(2*I*a**7
/((a - b)**3*(a + b)**3) - 4*I*a**5*b**2/((a - b)**3*(a + b)**3) + 2*I*a**
3*b**4/((a - b)**3*(a + b)**3) - I*a**3/((a - b)*(a + b)) - I*a*b**2/((a -
 b)*(a + b)) + x)/(2*(a - b)*(a + b)) - I*b*log(-2*I*a**4*b**3/((a - b)**3
*(a + b)**3) + 4*I*a**2*b**5/((a - b)**3*(a + b)**3) + I*a**2*b/((a - b)*(
a + b)) - 2*I*b**7/((a - b)**3*(a + b)**3) + I*b**3/((a - b)*(a + b)) + x)
/(2*(a - b)*(a + b)) + I*b*log(2*I*a**4*b**3/((a - b)**3*(a + b)**3) - 4*I
*a**2*b**5/((a - b)**3*(a + b)**3) - I*a**2*b/((a - b)*(a + b)) + 2*I*b**7
/((a - b)**3*(a + b)**3) - I*b**3/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b
))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

input
```
integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

output
```
a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

input
```
integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

output
```
a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 4.78

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx = -\frac{a\,\mathrm{atan}\left(\frac{x\,(2\,a^4+2\,b^4)-\frac{a^2\,x\,(8\,a^6-8\,a^4\,b^2-8\,a^2\,b^4+8\,b^6)}{(2\,a^2-2\,b^2)^2}}{a\,b^2\,(2\,a^2-2\,b^2)}\right)}{a^2-b^2} - \frac{b\,\mathrm{atan}\left(\frac{x\,(2\,a^4+2\,b^4)-\frac{b^2\,x\,(8\,a^6-8\,a^4\,b^2-8\,a^2\,b^4+8\,b^6)}{(2\,a^2-2\,b^2)^2}}{a^2\,b\,(2\,a^2-2\,b^2)}\right)}{a^2-b^2}$$

input
```
int(x^2/((a^2 + x^2)*(b^2 + x^2)),x)
```

output
```
- (a*atan((x*(2*a^4 + 2*b^4) - (a^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b
^2))/(2*a^2 - 2*b^2)^2)/(a*b^2*(2*a^2 - 2*b^2))))/(a^2 - b^2) - (b*atan((x
*(2*a^4 + 2*b^4) - (b^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^2))/(2*a^2
- 2*b^2)^2)/(a^2*b*(2*a^2 - 2*b^2))))/(a^2 - b^2)
```

3.14.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{atan\left(\frac{x}{a}\right)a - atan\left(\frac{x}{b}\right)b}{a^2-b^2}$$

input
```
int(x**2/(a**2*b**2 + a**2*x**2 + b**2*x**2 + x**4),x)
```

output
```
(atan(x/a)*a - atan(x/b)*b)/(a**2 - b**2)
```

3.15 $\int \frac{x}{(-1+x)(1+x^2)}\,dx$

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3.15.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{x}{(-1+x)\,(1+x^2)}\,dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1-x) - \frac{1}{4}\log\left(1+x^2\right)$$

output
```
1/2*arctan(x)+1/2*ln(1-x)-1/4*ln(x^2+1)
```

3.15.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-1+x)\,(1+x^2)}\,dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1-x) - \frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[x/((-1 + x)*(1 + x^2)),x]
```

output
```
ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

3.15.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x-1)(x^2+1)} \, dx$$

$$\Big\downarrow \; 587$$

$$\frac{1}{2}\int \frac{1-x}{x^2+1}dx+\frac{1}{2}\int \frac{1}{x-1}dx$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\int \frac{1-x}{x^2+1}dx+\frac{1}{2}\log(1-x)$$

$$\Big\downarrow \; 452$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx-\int \frac{x}{x^2+1}dx\right)+\frac{1}{2}\log(1-x)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\arctan(x)-\int \frac{x}{x^2+1}dx\right)+\frac{1}{2}\log(1-x)$$

$$\Big\downarrow \; 240$$

$$\frac{1}{2}\left(\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{2}\log(1-x)$$

input
```
Int[x/((-1 + x)*(1 + x^2)),x]
```

output
```
Log[1 - x]/2 + (ArcTan[x] - Log[1 + x^2]/2)/2
```

3.15.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

3.15.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(-1+x)}{2}-\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{2}$	20
risch	$\frac{\ln(-1+x)}{2}-\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{2}$	20
parallelrisch	$\frac{\ln(-1+x)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

input
```
int(x/(-1+x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-1+x)-1/4*ln(x^2+1)+1/2*arctan(x)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(x-1)$$

input
```
integrate(x/(-1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)}\,dx = \frac{\log(x-1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x/(-1+x)/(x**2+1),x)
```

output
```
log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(x-1)$$

input
```
integrate(x/(-1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x}{(-1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x/(-1+x)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x - 1))
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x}{(-1+x)(1+x^2)}\,dx = \frac{\ln(x-1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x/((x^2 + 1)*(x - 1)),x)
```

output
```
log(x - 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.15.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)}\,dx = \frac{atan(x)}{2} - \frac{\log(x^2+1)}{4} + \frac{\log(x-1)}{2}$$

input
```
int(x/(x**3 - x**2 + x - 1),x)
```

output
```
(2*atan(x) - log(x**2 + 1) + 2*log(x - 1))/4
```

3.16 $\int \frac{x}{1+x^3} \, dx$

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3.16.1 Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \frac{x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x}{1+x^3} \, dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

input
```
Integrate[x/(1 + x^3),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

3.16.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^3+1} \, dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)$$

input
```
Int[x/(1 + x^3),x]
```

output
```
-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2]/2
)/3
```

3.16.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.16.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

input
```
int(x/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3}\,dx=\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

input
```
integrate(x/(x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log
(x + 1)
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^3}\,dx = -\frac{\log (x+1)}{3} + \frac{\log (x^2-x+1)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**3+1),x)
```

output
```
-log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)
/3)/3
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1)$$

input
```
integrate(x/(x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log
(x + 1)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(|x+1|)$$

input
```
integrate(x/(x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log
(abs(x + 1))
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{1+x^3}\,dx = -\frac{\ln(x+1)}{3} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(x/(x^3 + 1),x)
```

output
```
log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

3.16.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+x^3}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x+1)}{3}$$

input
```
int(x/(x**3 + 1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + log(x**2 - x + 1) - 2*log(x + 1))/6
```

3.17 $\int \frac{x^3}{(-1+x)^2(1+x^3)} \, dx$

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3.17.1 Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} \, dx = \frac{1}{2(1-x)} + \frac{3}{4}\log(1-x) - \frac{1}{12}\log(1+x) - \frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
1/2/(1-x)+3/4*ln(1-x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)
```

3.17.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} \, dx = \frac{1}{12}\left(-\frac{6}{-1+x} + 9\log(-1+x) - \log(1+x) - 4\log\left((-1+x)^2+x\right)\right)$$

input
```
Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]
```

output
```
(-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x])/12
```

3.17.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x-1)^2 (x^3+1)} \, dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{1-2x}{3 (x^2-x+1)}+\frac{3}{4(x-1)}-\frac{1}{12(x+1)}+\frac{1}{2(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(x+1)$$

input
```
Int[x^3/((-1 + x)^2*(1 + x^3)),x]
```

output
```
1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^2]/3
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{2(-1+x)}+\frac{3\ln(-1+x)}{4}-\frac{\ln(x^2-x+1)}{3}-\frac{\ln(1+x)}{12}$	32
norman	$-\frac{1}{2(-1+x)}+\frac{3\ln(-1+x)}{4}-\frac{\ln(x^2-x+1)}{3}-\frac{\ln(1+x)}{12}$	32
risch	$-\frac{1}{2(-1+x)}+\frac{3\ln(-1+x)}{4}-\frac{\ln(x^2-x+1)}{3}-\frac{\ln(1+x)}{12}$	32
parallelrisch	$\frac{9\ln(-1+x)x-\ln(1+x)x-4\ln(x^2-x+1)x-6-9\ln(-1+x)+\ln(1+x)+4\ln(x^2-x+1)}{-12+12x}$	57

input
```
int(x^3/(-1+x)^2/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+x)+3/4*ln(-1+x)-1/3*ln(x^2-x+1)-1/12*ln(1+x)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(-1+x)^2(1+x^3)}\,dx = -\frac{4(x-1)\log(x^2-x+1)+(x-1)\log(x+1)-9(x-1)\log(x-1)+6}{12(x-1)}$$

input
```
integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="fricas")
```

output
```
-1/12*(4*(x - 1)*log(x^2 - x + 1) + (x - 1)*log(x + 1) - 9*(x - 1)*log(x -
 1) + 6)/(x - 1)
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(-1+x)^2(1+x^3)}\,dx = \frac{3\log(x-1)}{4} - \frac{\log(x+1)}{12} - \frac{\log(x^2-x+1)}{3} - \frac{1}{2x-2}$$

input
```
integrate(x**3/(-1+x)**2/(x**3+1),x)
```

output
```
3*log(x - 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 - 1/(2*x - 2)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(-1+x)^2(1+x^3)}\,dx = -\frac{1}{2(x-1)} - \frac{1}{3}\log(x^2-x+1) - \frac{1}{12}\log(x+1) + \frac{3}{4}\log(x-1)$$

input
```
integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="maxima")
```

output
```
-1/2/(x - 1) - 1/3*log(x^2 - x + 1) - 1/12*log(x + 1) + 3/4*log(x - 1)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(-1+x)^2(1+x^3)}\,dx = -\frac{1}{2(x-1)} - \frac{1}{3}\log\left(\frac{1}{x-1} + \frac{1}{(x-1)^2} + 1\right) - \frac{1}{12}\log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

input
```
integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="giac")
```

output
```
-1/2/(x - 1) - 1/3*log(1/(x - 1) + 1/(x - 1)^2 + 1) - 1/12*log(abs(-2/(x -
 1) - 1))
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(-1+x)^2\,(1+x^3)}\,dx = \frac{3\,\ln(x-1)}{4} - \frac{\ln(x+1)}{12} - \frac{\ln(x^2-x+1)}{3} - \frac{1}{2\,(x-1)}$$

input
```
int(x^3/((x^3 + 1)*(x - 1)^2),x)
```

output
```
(3*log(x - 1))/4 - log(x + 1)/12 - log(x^2 - x + 1)/3 - 1/(2*(x - 1))
```

3.17.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(-1+x)^2\,(1+x^3)}\,dx$$
$$= \frac{-4\log(x^2-x+1)\,x + 4\log(x^2-x+1) + 9\log(x-1)\,x - 9\log(x-1) - \log(x+1)\,x + \log(x+1) -}{12x-12}$$

input
```
int(x**3/(x**5 - 2*x**4 + x**3 + x**2 - 2*x + 1),x)
```

output
```
( - 4*log(x**2 - x + 1)*x + 4*log(x**2 - x + 1) + 9*log(x - 1)*x - 9*log(x
 - 1) - log(x + 1)*x + log(x + 1) - 6*x)/(12*(x - 1))
```

3.18 $\int \frac{1}{1+x^4} \, dx$

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3.18.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \frac{1}{1+x^4} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+
x^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^4} \, dx = \frac{-2\arctan\left(1-\sqrt{2}x\right)+2\arctan\left(1+\sqrt{2}x\right)-\log\left(1-\sqrt{2}x+x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

input
```
Integrate[(1 + x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x +
x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

3.18.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+1} \, dx$$

$$\Big\downarrow \text{ 755}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[(1 + x^4)^(-1),x]
```

output
```
(-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1
/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2])
)/2
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$-\frac{x\sqrt{2}\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{2}\left(x^4\right)}\right)}{4(x^4)^{\frac{1}{4}}}$

input
```
int(1/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.18.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{1}{1+x^4}\,dx = \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right)-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)$$

input
```
integrate(1/(x^4+1),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*l
og(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2
)) - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

input
```
integrate(1/(x**4+1),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(1/(x^4+1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(1/(x^4+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{1}{1+x^4}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x^4 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x
*(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.18.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \frac{1}{1+x^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)+2atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)-\log\left(-\sqrt{2}\,x+x^2+1\right)+\log\left(\sqrt{2}\,x+x^2+1\right)\right)}{8}$$

input
```
int(1/(x**4 + 1),x)
```

output
```
(sqrt(2)*( - 2*atan((sqrt(2) - 2*x)/sqrt(2)) + 2*atan((sqrt(2) + 2*x)/sqrt
(2)) - log( - sqrt(2)*x + x**2 + 1) + log(sqrt(2)*x + x**2 + 1)))/8
```

3.19 $\int \frac{x^2}{1+x^4} dx$

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3.19.1 Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{x^2}{1+x^4} dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+
x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{1+x^4} dx = \frac{-2\arctan\left(1-\sqrt{2}x\right)+2\arctan\left(1+\sqrt{2}x\right)+\log\left(1-\sqrt{2}x+x^2\right)-\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

input
```
Integrate[x^2/(1 + x^4),x]
```

output
```
(-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] + Log[1 - Sqrt[2]*x +
x^2] - Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

3.19.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+1}dx$$

$$\Big\downarrow \ 826$$

$$\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \ 1476$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \ 1479$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[x^2/(1 + x^4),x]
```

output
```
(-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (Lo
g[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))
/2
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^{2}-x\sqrt{2}}{1+x^{2}+x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$\frac{x^{3}\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}+\sqrt{x^{4}}\right)}{8\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{4\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}+\sqrt{x^{4}}\right)}{8\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{4}\right.}{2+\sqrt{2}\left(x^{4}\right.}\right.}{4\left(x^{4}\right)^{\frac{3}{4}}}$

input
```
int(x^2/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.19.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{1+x^4}\,dx = \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right)-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right) + \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right) - \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)$$

input
```
integrate(x^2/(x^4+1),x, algorithm="fricas")
```

output
```
(1/8*I - 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I + 1/8)*sqrt(2)*l
og(2*x - (I - 1)*sqrt(2)) + (1/8*I + 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2
)) - (1/8*I - 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{1+x^4}\,dx = \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

input
```
integrate(x**2/(x**4+1),x)
```

output
```
sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^2/(x^4+1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^2/(x^4+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{1+x^4}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^2/(x^4 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*atan(2^(1/2)*x
*(1/2 + 1i/2))*(1/4 + 1i/4)
```

3.19.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{1+x^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)+2atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)+\log\left(-\sqrt{2}\,x+x^2+1\right)-\log\left(\sqrt{2}\,x+x^2+1\right)\right)}{8}$$

input
```
int(x**2/(x**4 + 1),x)
```

output
```
(sqrt(2)*( - 2*atan((sqrt(2) - 2*x)/sqrt(2)) + 2*atan((sqrt(2) + 2*x)/sqrt
(2)) + log( - sqrt(2)*x + x**2 + 1) - log(sqrt(2)*x + x**2 + 1)))/8
```

3.20 $\int \frac{1}{1+x^2+x^4}\,dx$

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3.20.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right)$$

output
```
-1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1
/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.20.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right) - \sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

input
```
Integrate[(1 + x^2 + x^4)^(-1),x]
```

output
```
(I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]
*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

3.20.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[(1 + x^2 + x^4)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x + x^2]/2)/2 + (ArcTan[(1 +
 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

3.20.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	54
risch	$\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(4x^2+4x+4)}{4}-\frac{\ln(4x^2-4x+4)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	60

input
```
int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1
/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**4+x**2+1),x)
```

output
```
-log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \operatorname{atanh}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^2 + x^4 + 1),x)
```

output
```
atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

3.20.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4}$$

input
```
int(1/(x**4 + x**2 + 1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3
*log(x**2 - x + 1) + 3*log(x**2 + x + 1))/12
```

3.21 $\int (a+bx)^p \, dx$

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3.21.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

output
```
(b*x+a)^(p+1)/b/(p+1)
```

3.21.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

input
```
Integrate[(a + b*x)^p,x]
```

output
```
(a + b*x)^(1 + p)/(b*(1 + p))
```

3.21.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^p\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

input
```
Int[(a + b*x)^p,x]
```

output
```
(a + b*x)^(1 + p)/(b*(1 + p))
```

3.21.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+p}}{b(1+p)}$	19
default	$\frac{(bx+a)^{1+p}}{b(1+p)}$	19
risch	$\frac{(bx+a)(bx+a)^{p}}{b(1+p)}$	22
parallelrisch	$\frac{x(bx+a)^{p}ab+(bx+a)^{p}a^{2}}{(1+p)ab}$	36
norman	$\frac{x\,\mathrm{e}^{p\ln(bx+a)}}{1+p}+\frac{a\,\mathrm{e}^{p\ln(bx+a)}}{b(1+p)}$	37

input
```
int((b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(1+p)/b/(1+p)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^p\,dx=\frac{(bx+a)(bx+a)^p}{bp+b}$$

input
```
integrate((b*x+a)^p,x, algorithm="fricas")
```

output
```
(b*x + a)*(b*x + a)^p/(b*p + b)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^p\,dx=\frac{\begin{cases}\frac{(a+bx)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx) & \text{otherwise}\end{cases}}{b}$$

input
```
integrate((b*x+a)**p,x)
```

output
```
Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p\,dx = \frac{(bx+a)^{p+1}}{b(p+1)}$$

input
```
integrate((b*x+a)^p,x, algorithm="maxima")
```

output
```
(b*x + a)^(p + 1)/(b*(p + 1))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p\,dx = \frac{(bx+a)^{p+1}}{b(p+1)}$$

input
```
integrate((b*x+a)^p,x, algorithm="giac")
```

output
```
(b*x + a)^(p + 1)/(b*(p + 1))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p\,dx = \frac{(a+b\,x)^{p+1}}{b\,(p+1)}$$

input
```
int((a + b*x)^p,x)
```

output
```
(a + b*x)^(p + 1)/(b*(p + 1))
```

3.21.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int (a+bx)^p\,dx = \frac{(bx+a)^p\,(bx+a)}{b\,(p+1)}$$

input
```
int((a + b*x)**p,x)
```

output
```
((a + b*x)**p*(a + b*x))/(b*(p + 1))
```

3.22 $\int x(a+bx)^p\,dx$

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3.22.1 Optimal result

Integrand size = 9, antiderivative size = 39

$$\int x(a+bx)^p\,dx=-\frac{a(a+bx)^{1+p}}{b^2(1+p)}+\frac{(a+bx)^{2+p}}{b^2(2+p)}$$

output
```
-a*(b*x+a)^(p+1)/b^2/(p+1)+(b*x+a)^(2+p)/b^2/(2+p)
```

3.22.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int x(a+bx)^p\,dx=\frac{(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)}$$

input
```
Integrate[x*(a + b*x)^p,x]
```

output
```
((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))
```

3.22.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^p\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{(a+bx)^{p+1}}{b}-\frac{a(a+bx)^p}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

input
```
Int[x*(a + b*x)^p,x]
```

output
```
-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))
```

3.22.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{(bx+a)^{1+p}(-xpb-bx+a)}{b^2(p^2+3p+2)}$	36
risch	$-\frac{(-x^2b^2p-xapb-x^2b^2+a^2)(bx+a)^p}{b^2(2+p)(1+p)}$	50
parallelrisch	$\frac{x^2(bx+a)^p b^2 p+x^2(bx+a)^p b^2+x(bx+a)^p abp-(bx+a)^p a^2}{b^2(p^2+3p+2)}$	69
norman	$\frac{x^2 e^{p\ln(bx+a)}}{2+p}+\frac{pax\, e^{p\ln(bx+a)}}{b(p^2+3p+2)}-\frac{a^2 e^{p\ln(bx+a)}}{b^2(p^2+3p+2)}$	73

input
```
int(x*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(b*x+a)^(1+p)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int x(a+bx)^p\,dx = \frac{(abpx+(b^2p+b^2)x^2-a^2)(bx+a)^p}{b^2p^2+3\,b^2p+2\,b^2}$$

input
```
integrate(x*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2
)
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(31) = 62.

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 5.15

$$\int x(a+bx)^p\,dx = \begin{cases} \frac{a^p x^2}{2} & \text{for } b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p=-2 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p=-1 \\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(
a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log
(a/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*
p + 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*
p*x**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)*
*p/(b**2*p**2 + 3*b**2*p + 2*b**2), True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int x(a+bx)^p\,dx = \frac{(b^2(p+1)x^2 + abpx - a^2)(bx+a)^p}{(p^2+3\,p+2)b^2}$$

input
```
integrate(x*(b*x+a)^p,x, algorithm="maxima")
```

output
```
(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

$$\int x(a+bx)^p\,dx = \frac{(bx+a)^p b^2 p x^2 + (bx+a)^p abpx + (bx+a)^p b^2 x^2 - (bx+a)^p a^2}{b^2p^2 + 3\,b^2p + 2\,b^2}$$

input
```
integrate(x*(b*x+a)^p,x, algorithm="giac")
```

output
```
((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x
+ a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.41

$$\int x(a+bx)^p\,dx = \begin{cases} -\frac{a\ln(a+b\,x)-b\,x}{b^2} & \text{if } p=-1 \\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } p=-2 \\ \frac{2\left(\frac{(a+b\,x)^{p+2}}{2\,p+4}-\frac{a\,(a+b\,x)^{p+1}}{2\,p+2}\right)}{b^2} & \text{if } p\neq -1\wedge p\neq -2 \end{cases}$$

input
```
int(x*(a + b*x)^p,x)
```

output
```
piecewise(p == -1, -(a*log(a + b*x) - b*x)/b^2, p == -2, (log(a + b*x) + a
/(a + b*x))/b^2, p ~= -1 & p ~= -2, (2*((a + b*x)^(p + 2)/(2*p + 4) - (a*(
a + b*x)^(p + 1))/(2*p + 2)))/b^2)
```

3.22.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int x(a+bx)^p\,dx = \frac{(bx+a)^p\,(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2\,(p^2+3p+2)}$$

input
```
int((a + b*x)**p*x,x)
```

output
```
((a + b*x)**p*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b**2*(p**2 +
 3*p + 2))
```

3.22. $\int x(a+bx)^p\,dx$

3.23 $\int x^2(a+bx)^p\,dx$

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3.23.1 Optimal result

Integrand size = 11, antiderivative size = 60

$$\int x^2(a+bx)^p\,dx=\frac{a^2(a+bx)^{1+p}}{b^3(1+p)}-\frac{2a(a+bx)^{2+p}}{b^3(2+p)}+\frac{(a+bx)^{3+p}}{b^3(3+p)}$$

output
```
a^2*(b*x+a)^(p+1)/b^3/(p+1)-2*a*(b*x+a)^(2+p)/b^3/(2+p)+(b*x+a)^(3+p)/b^3/
(3+p)
```

3.23.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x^2(a+bx)^p\,dx=\frac{(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2\left(2+3p+p^2\right)x^2\right)}{b^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^2*(a + b*x)^p,x]
```

output
```
((a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b
^3*(1 + p)*(2 + p)*(3 + p))
```

3.23.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^p\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{p+1}}{b^2}+\frac{(a+bx)^{p+2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

input
```
Int[x^2*(a + b*x)^p,x]
```

output
```
(a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 +
p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

3.23.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
gosper	$\frac{(bx+a)^{1+p}\left(b^{2}p^{2}x^{2}+3x^{2}b^{2}p-2xapb+2x^{2}b^{2}-2axb+2a^{2}\right)}{b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$
risch	$\frac{\left(b^{3}p^{2}x^{3}+a\,b^{2}p^{2}x^{2}+3b^{3}p\,x^{3}+x^{2}ap\,b^{2}+2x^{3}b^{3}-2a^{2}pxb+2a^{3}\right)(bx+a)^{p}}{(2+p)(3+p)(1+p)b^{3}}$
norman	$\frac{x^{3}{\mathrm e}^{p\ln(bx+a)}}{3+p}+\frac{ap\,x^{2}{\mathrm e}^{p\ln(bx+a)}}{b(p^{2}+5p+6)}+\frac{2a^{3}{\mathrm e}^{p\ln(bx+a)}}{b^{3}(p^{3}+6p^{2}+11p+6)}-\frac{2p\,a^{2}x\,{\mathrm e}^{p\ln(bx+a)}}{b^{2}(p^{3}+6p^{2}+11p+6)}$
parallelrisch	$\frac{x^{3}(bx+a)^{p}a\,b^{3}p^{2}+3x^{3}(bx+a)^{p}a\,b^{3}p+x^{2}(bx+a)^{p}a^{2}b^{2}p^{2}+2x^{3}(bx+a)^{p}a\,b^{3}+x^{2}(bx+a)^{p}a^{2}b^{2}p-2x(bx+a)^{p}a^{3}bp+2a^{4}(bx+a)^{p}}{a\,b^{3}(p^{2}+5p+6)(1+p)}$

input
```
int(x^2*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(b*x+a)^(1+p)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+
2*b^2*x^2-2*a*b*x+2*a^2)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

$$\int x^2(a+bx)^p\,dx = -\frac{(2\,a^2bpx-(b^3p^2+3\,b^3p+2\,b^3)x^3-2\,a^3-(ab^2p^2+ab^2p)x^2)(bx+a)^p}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

input
```
integrate(x^2*(b*x+a)^p,x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b
^2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(51) = 102$.

Time = 0.48 (sec) , antiderivative size = 597, normalized size of antiderivative = 9.95

$$\int x^2(a+bx)^p\,dx = \begin{cases} \frac{a^p x^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} - \frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a
*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) +
 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b*
*3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b
**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3
- a*x/b**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*
b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3
 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p*
*3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3
*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b*
*3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b*
*3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3), True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x^2(a+bx)^p\,dx = \frac{((p^2+3\,p+2)b^3x^3+(p^2+p)ab^2x^2-2\,a^2bpx+2\,a^3)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3}$$

input
```
integrate(x^2*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x
 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.33

$$\int x^2(a+bx)^p\,dx$$
$$= \frac{(bx+a)^pb^3p^2x^3+(bx+a)^pab^2p^2x^2+3\,(bx+a)^pb^3px^3+(bx+a)^pab^2px^2+2\,(bx+a)^pb^3x^3-2\,(bx+a}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

input
```
integrate(x^2*(b*x+a)^p,x, algorithm="giac")
```

output
```
((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p
*x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2
*b*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int x^2(a+bx)^p\,dx$$
$$= \begin{cases} \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3} & \text{if } p=-1 \\ \frac{x}{b^2}-\frac{a^2}{b^3\,(a+b\,x)}-\frac{2\,a\,\ln(a+b\,x)}{b^3} & \text{if } p=-2 \\ \frac{\ln(a+b\,x)+\frac{2\,a}{a+b\,x}-\frac{a^2}{2\,(a+b\,x)^2}}{b^3} & \text{if } p=-3 \\ \frac{2\,(a+b\,x)^{p+1}\,\left(8\,a^2-8\,a\,b\,p\,x-8\,a\,b\,x+4\,b^2\,p^2\,x^2+12\,b^2\,p\,x^2+8\,b^2\,x^2\right)}{b^3\,(8\,p^3+48\,p^2+88\,p+48)} & \text{if } p\neq -1\land p\neq -2\land p\neq -3 \end{cases}$$

input
```
int(x^2*(a + b*x)^p,x)
```

output
```
piecewise(p == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), p ==
-2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, p == -3, (log(a
+ b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, p ~= -1 & p ~= -2 & p
 ~= -3, (2*(a + b*x)^(p + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*p*x^2 - 8*a*b*x +
 4*b^2*p^2*x^2 - 8*a*b*p*x))/(b^3*(88*p + 48*p^2 + 8*p^3 + 48)))
```

3.23.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int x^2(a+bx)^p\,dx$$
$$= \frac{(bx+a)^p\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3\,(p^3+6p^2+11p+6)}$$

input
```
int((a + b*x)**p*x**2,x)
```

output
```
((a + b*x)**p*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2*p*x**2 +
b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*(p**3 + 6*p**2 + 11*p
 + 6))
```

3.24 $\int \frac{1}{a+bx} \, dx$

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3.24.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

output
```
ln(b*x+a)/b
```

3.24.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

input
```
Integrate[(a + b*x)^(-1),x]
```

output
```
Log[a + b*x]/b
```

3.24.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{16\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx}\,dx$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+bx)}{b}$$

input
```
Int[(a + b*x)^(-1),x]
```

output
```
Log[a + b*x]/b
```

3.24.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.24.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

input
```
int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log{(bx+a)}}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+bx}\,dx = \frac{\log{(a+bx)}}{b}$$

input
```
integrate(1/(b*x+a),x)
```

output
```
log(a + b*x)/b
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log{(bx+a)}}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="maxima")
```

output
```
log(b*x + a)/b
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+bx}\,dx = \frac{\log\left(|bx+a|\right)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\ln\left(a+b\,x\right)}{b}$$

input
```
int(1/(a + b*x),x)
```

output
```
log(a + b*x)/b
```

3.24.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log(bx+a)}{b}$$

input
```
int(1/(a + b*x),x)
```

output
```
log(a + b*x)/b
```

3.25 $\int \frac{1}{(a+bx)^2} \, dx$

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3.25.1 Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

3.25.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[(a + b*x)^(-2),x]
```

output
```
-(1/(b*(a + b*x)))
```

3.25.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{b(a+bx)}$$

input
```
Int[(a + b*x)^(-2),x]
```

output
```
-(1/(b*(a + b*x)))
```

3.25.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.25.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13

input
```
int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{ab+b^2x}$$

input
```
integrate(1/(b*x+a)**2,x)
```

output
```
-1/(a*b + b**2*x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/((b*x + a)*b)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b\,(a+b\,x)}$$

input
```
int(1/(a + b*x)^2,x)
```

output
```
-1/(b*(a + b*x))
```

3.25.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = \frac{x}{a\,(bx+a)}$$

input
```
int(1/(a**2 + 2*a*b*x + b**2*x**2),x)
```

output
```
x/(a*(a + b*x))
```

3.26 $\int \frac{x}{a+bx} \, dx$

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3.26.1 Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{x}{a+bx} \, dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

output
```
x/b-a*ln(b*x+a)/b^2
```

3.26.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx} \, dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

input
```
Integrate[x/(a + b*x),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

3.26.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx}\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

input
```
Int[x/(a + b*x),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

3.26.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
parallelrisch	$-\frac{a\ln(bx+a)-bx}{b^2}$	19

input
```
int(x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x/b-a*ln(b*x+a)/b^2
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+bx}\,dx = \frac{bx-a\log\left(bx+a\right)}{b^2}$$

input
```
integrate(x/(b*x+a),x, algorithm="fricas")
```

output
```
(b*x - a*log(b*x + a))/b^2
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{a+bx}\,dx = -\frac{a\log\left(a+bx\right)}{b^2}+\frac{x}{b}$$

input
```
integrate(x/(b*x+a),x)
```

output
```
-a*log(a + b*x)/b**2 + x/b
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(bx+a)}{b^2}$$

input
```
integrate(x/(b*x+a),x, algorithm="maxima")
```

output
```
x/b - a*log(b*x + a)/b^2
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(|bx+a|)}{b^2}$$

input
```
integrate(x/(b*x+a),x, algorithm="giac")
```

output
```
x/b - a*log(abs(b*x + a))/b^2
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx}\,dx = -\frac{a\,\ln(a+b\,x) - b\,x}{b^2}$$

input
```
int(x/(a + b*x),x)
```

output
```
-(a*log(a + b*x) - b*x)/b^2
```

3.26.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+bx}\,dx = \frac{-\log(bx+a)\,a+bx}{b^2}$$

input
```
int(x/(a + b*x),x)
```

output
```
( - log(a + b*x)*a + b*x)/b**2
```

3.27 $\int \frac{x^2}{a+bx}\,dx$

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3.27.10 Reduce [B] (verification not implemented) 259

3.27.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x^2}{a+bx}\,dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2\log(a+bx)}{b^3}$$

output
```
-a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

3.27.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx}\,dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2\log(a+bx)}{b^3}$$

input
```
Integrate[x^2/(a + b*x),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

3.27.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

input
```
Int[x^2/(a + b*x),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

3.27.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}x^2b+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
norman	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
parallelrisch	$\frac{x^2b^2+2a^2\ln(bx+a)-2axb}{2b^3}$	30

input
```
int(x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*x^2*b+a*x)+a^2*ln(b*x+a)/b^3
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{b^2x^2-2\,abx+2\,a^2\log(bx+a)}{2\,b^3}$$

input
```
integrate(x^2/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

input
```
integrate(x**2/(b*x+a),x)
```

output
```
a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log(bx+a)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^2/(b*x+a),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log(|bx+a|)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^2/(b*x+a),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

input
```
int(x^2/(a + b*x),x)
```

output
```
(2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.27.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{2\log(bx+a)\,a^2 - 2abx + b^2x^2}{2b^3}$$

input
```
int(x**2/(a + b*x),x)
```

output
```
(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2)/(2*b**3)
```

3.28 $\int \frac{1}{x(a+bx)}\,dx$

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3.28.10 Reduce [B] (verification not implemented) 264

3.28.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{x(a+bx)}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

output
```
ln(x)/a-ln(b*x+a)/a
```

3.28.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

input
```
Integrate[1/(x*(a + b*x)),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

3.28.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)}dx$$

$$\Big\downarrow \; 47$$

$$\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 14$$

$$\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

input
```
Int[1/(x*(a + b*x)),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

3.28.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

3.28.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(bx+a)}{a}$	16
default	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
risch	$-\frac{\ln(bx+a)}{a}+\frac{\ln(-x)}{a}$	21

input
```
int(1/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-ln(b*x+a))/a
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log(bx+a)-\log(x)}{a}$$

input
```
integrate(1/x/(b*x+a),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{x(a+bx)}\,dx = \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

input
```
integrate(1/x/(b*x+a),x)
```

output
```
(log(x) - log(a/b + x))/a
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

input
```
integrate(1/x/(b*x+a),x, algorithm="maxima")
```

output
```
-log(b*x + a)/a + log(x)/a
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log(|bx+a|)}{a} + \frac{\log(|x|)}{a}$$

input
```
integrate(1/x/(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a + log(abs(x))/a
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{2\,\operatorname{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

input
```
int(1/(x*(a + b*x)),x)
```

output
```
-(2*atanh((2*b*x)/a + 1))/a
```

3.28.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)}\,dx = \frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int(1/(x*(a + b*x)),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.29 $\int \frac{1}{x^2(a+bx)}\,dx$

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3.29.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

3.29.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[1/(x^2*(a + b*x)),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

3.29.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)}\,dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x)),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

3.29.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.29.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{bx\ln(x)-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

input
```
int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

input
```
integrate(1/x**2/(b*x+a),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2} - \frac{1}{a\,x}$$

input
```
int(1/(x^2*(a + b*x)),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.29.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{\log(bx+a)\,bx - \log(x)\,bx - a}{a^2x}$$

input
```
int(1/(x**2*(a + b*x)),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.30 $\int \frac{1}{x^2(a+bx)^2}\,dx$

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3.30.1 Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{x^2(a+bx)^2}\,dx=-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(a+bx)^2}\,dx=-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

input
```
Integrate[1/(x^2*(a + b*x)^2),x]
```

output
```
-((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

3.30.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{2b^2}{a^3(a+bx)}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{1}{a^2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

input
```
Int[1/(x^2*(a + b*x)^2),x]
```

output
```
-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a
^3
```

3.30.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.30.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{a^2x}-\frac{b}{a^2(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	43
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{x(bx+a)}+\frac{2b\ln(-bx-a)}{a^3}-\frac{2b\ln(x)}{a^3}$	49
norman	$\frac{\frac{2b^2x^2}{a^3}-\frac{1}{a}}{x(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	50
parallelrisch	$-\frac{2\ln(x)x^2b^2-2\ln(bx+a)x^2b^2+2\ln(x)xab-2\ln(bx+a)xab-2x^2b^2+a^2}{a^3x(bx+a)}$	70

input
```
int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2(a+bx)^2}\,dx=-\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log{(bx+a)}+2\,(b^2x^2+abx)\log{(x)}}{a^3bx^2+a^4x}$$

input
```
integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*l
og(x))/(a^3*b*x^2 + a^4*x)
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(a+bx)^2}\,dx=\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^3}$$

input
```
integrate(1/x**2/(b*x+a)**2,x)
```

output
```
(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)^2}\,dx = -\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

input
```
integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+bx)^2}\,dx = -\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

input
```
integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
 a) - 1))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)^2}\,dx = \frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3} - \frac{1}{a\,x\,\left(a+b\,x\right)} - \frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

input
```
int(1/(x^2*(a + b*x)^2),x)
```

output
```
(2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

3.30.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^2(a+bx)^2}\,dx$$
$$= \frac{2\log(bx+a)\,abx + 2\log(bx+a)\,b^2x^2 - 2\log(x)\,abx - 2\log(x)\,b^2x^2 - a^2 + 2b^2x^2}{a^3x\,(bx+a)}$$

input
```
int(1/(x**2*(a**2 + 2*a*b*x + b**2*x**2)),x)
```

output
```
(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x - 2*log(
x)*b**2*x**2 - a**2 + 2*b**2*x**2)/(a**3*x*(a + b*x))
```

3.31 $\int \frac{1}{c^2+x^2} dx$

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3.31.10 Reduce [B] (verification not implemented) 279

3.31.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

output
```
arctan(x/c)/c
```

3.31.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input
```
Integrate[(c^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/c]/c
```

3.31.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^2+x^2}\,dx$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input
```
Int[(c^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/c]/c
```

3.31.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.31.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{c}\right)}{c}$	11
risch	$\frac{\arctan\left(\frac{x}{c}\right)}{c}$	11
parallelrisch	$-\frac{i\ln(-ic+x)-i\ln(ic+x)}{2c}$	27

input
```
int(1/(c^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/c)/c
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} \, dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input
```
integrate(1/(c^2+x^2),x, algorithm="fricas")
```

output
```
arctan(x/c)/c
```

3.31.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{c^2 + x^2} \, dx = \frac{-\frac{i\log\left(-ic+x\right)}{2} + \frac{i\log\left(ic+x\right)}{2}}{c}$$

input
```
integrate(1/(c**2+x**2),x)
```

output
```
(-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input
```
integrate(1/(c^2+x^2),x, algorithm="maxima")
```

output
```
arctan(x/c)/c
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input
```
integrate(1/(c^2+x^2),x, algorithm="giac")
```

output
```
arctan(x/c)/c
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2+x^2}\,dx=\frac{\mathrm{atan}\left(\frac{x}{c}\right)}{c}$$

input
```
int(1/(c^2 + x^2),x)
```

output
```
atan(x/c)/c
```

3.31.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} \, dx = \frac{atan\left(\frac{x}{c}\right)}{c}$$

input
```
int(1/(c**2 + x**2),x)
```

output
```
atan(x/c)/c
```

3.32 $\int \frac{1}{c^2-x^2}\,dx$

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3.32.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{c^2-x^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{x}{c}\right)}{c}$$

output
```
arctanh(x/c)/c
```

3.32.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2-x^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{x}{c}\right)}{c}$$

input
```
Integrate[(c^2 - x^2)^(-1),x]
```

output
```
ArcTanh[x/c]/c
```

3.32.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^2-x^2}\,dx$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{x}{c}\right)}{c}$$

input
```
Int[(c^2 - x^2)^(-1),x]
```

output
```
ArcTanh[x/c]/c
```

3.32.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.32.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

method	result	size
parallelrisch	$-\frac{\ln(-c+x)-\ln(c+x)}{2c}$	19
default	$-\frac{\ln(c-x)}{2c}+\frac{\ln(c+x)}{2c}$	22
norman	$-\frac{\ln(c-x)}{2c}+\frac{\ln(c+x)}{2c}$	22
risch	$\frac{\ln(c+x)}{2c}-\frac{\ln(-c+x)}{2c}$	22

input
```
int(1/(c^2-x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(-c+x)-ln(c+x))/c
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{c^2 - x^2} \, dx = \frac{\log{(c + x)} - \log{(-c + x)}}{2 \, c}$$

input
```
integrate(1/(c^2-x^2),x, algorithm="fricas")
```

output
```
1/2*(log(c + x) - log(-c + x))/c
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{c^2 - x^2} \, dx = -\frac{\frac{\log{(-c+x)}}{2} - \frac{\log{(c+x)}}{2}}{c}$$

input
```
integrate(1/(c**2-x**2),x)
```

output
```
-(log(-c + x)/2 - log(c + x)/2)/c
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{1}{c^2 - x^2}\,dx = \frac{\log(c + x)}{2\,c} - \frac{\log(-c + x)}{2\,c}$$

input
```
integrate(1/(c^2-x^2),x, algorithm="maxima")
```

output
```
1/2*log(c + x)/c - 1/2*log(-c + x)/c
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{c^2 - x^2}\,dx = \frac{\log(|c + x|)}{2\,c} - \frac{\log(|-c + x|)}{2\,c}$$

input
```
integrate(1/(c^2-x^2),x, algorithm="giac")
```

output
```
1/2*log(abs(c + x))/c - 1/2*log(abs(-c + x))/c
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 - x^2}\,dx = \frac{\operatorname{atanh}\left(\frac{x}{c}\right)}{c}$$

input
```
int(1/(c^2 - x^2),x)
```

output
```
atanh(x/c)/c
```

3.32.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{1}{c^2 - x^2} \, dx = \frac{\log(-c - x) - \log(c - x)}{2c}$$

input
```
int(1/(c**2 - x**2),x)
```

output
```
(log( - c - x) - log(c - x))/(2*c)
```

3.33 $\int \frac{1}{-1+2x^3}\,dx$

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3.33.2 Mathematica [A] (verified) 285
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3.33.1 Optimal result

Integrand size = 9, antiderivative size = 78

$$\int \frac{1}{-1+2x^3}\,dx=-\frac{\arctan\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

output
```
1/6*ln(1-2^(1/3)*x)*2^(2/3)-1/12*ln(1+2^(1/3)*x+2^(2/3)*x^2)*2^(2/3)-1/6*a
rctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{1}{-1+2x^3}\,dx=-\frac{2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right)-2\log\left(1-\sqrt[3]{2}x\right)+\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

input
```
Integrate[(-1 + 2*x^3)^(-1),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]] - 2*Log[1 - 2^(1/3)*x] +
 Log[1 + 2^(1/3)*x + 2^(2/3)*x^2])/2^(1/3)
```

3.33.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {750, 16, 25, 27, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2x^3-1}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\int -\frac{\sqrt[3]{2}x+2}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{2}x-1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int -\frac{\sqrt[3]{2}x+2}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(x+2^{2/3}\right)}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\sqrt[3]{2}\int \frac{x+2^{2/3}}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(2\sqrt[3]{2}x+1\right)}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx}{2\sqrt[3]{2}}+\frac{\int\frac{2\sqrt[3]{2}x+1}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int\frac{2\sqrt[3]{2}x+1}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx}{2\sqrt[3]{2}}-\frac{3\int\frac{1}{-\left(2\sqrt[3]{2}x+1\right)^2-3}d\left(2\sqrt[3]{2}x+1\right)}{2^{2/3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int\frac{2\sqrt[3]{2}x+1}{2^{2/3}x^2+\sqrt[3]{2}x+1}dx}{2\sqrt[3]{2}}+\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{2^{2/3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{3}\sqrt[3]{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[(-1 + 2*x^3)^(-1),x]
```

output
```
Log[1 - 2^(1/3)*x]/(3*2^(1/3)) - (2^(1/3)*((Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*
x)/Sqrt[3]])/2^(2/3) + Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(2*2^(2/3))))/3
```

3.33.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.33.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^3-1\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{6}$	24
default	$\frac{2^{\frac{2}{3}}\ln\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\ln\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{2^{\frac{1}{3}}}{2}\right)}{12}-\frac{\arctan\left(\frac{\left(1+2\,2^{\frac{1}{3}}x\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}}{6}$	58
meijerg	$\frac{2^{\frac{2}{3}}x\left(\ln\left(1-2^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+2^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}}{2+2^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{6\left(x^3\right)^{\frac{1}{3}}}$	82

input
```
int(1/(2*x^3-1),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(1/_R^2*ln(x-_R),_R=RootOf(2*_Z^3-1))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int\frac{1}{-1+2x^3}\,dx=-\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2\,x^2+2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(2\,x-2^{\frac{2}{3}}\right)$$

input
```
integrate(1/(2*x^3-1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(6)*2^(1/6)*arctan(1/6*sqrt(6)*2^(1/6)*(2*2^(2/3)*x + 2^(1/3))) -
 1/12*2^(2/3)*log(2*x^2 + 2^(2/3)*x + 2^(1/3)) + 1/6*2^(2/3)*log(2*x - 2^(
2/3))
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+2x^3}\,dx = \frac{2^{\frac{2}{3}}\log\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6} - \frac{2^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)}{12} - \frac{2^{\frac{2}{3}}\sqrt{3}\,\mathrm{atan}\left(\frac{2\cdot\sqrt[3]{2}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(2*x**3-1),x)
```

output
```
2**(2/3)*log(x - 2**(2/3)/2)/6 - 2**(2/3)*log(x**2 + 2**(2/3)*x/2 + 2**(1/
3)/2)/12 - 2**(2/3)*sqrt(3)*atan(2*2**(1/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{1}{-1+2x^3}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x+1\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{1}{2}\cdot 2^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)\right)$$

input
```
integrate(1/(2*x^3-1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*2^(2/3)*x + 2^(1/3))) -
 1/12*2^(2/3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) + 1/6*2^(2/3)*log(1/2*2^(2/
3)*(2^(1/3)*x - 1))
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{1}{-1+2x^3}\,dx = -\frac{1}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(2x+\left(\frac{1}{2}\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 4^{\frac{1}{3}}\log\left(x^2+\left(\frac{1}{2}\right)^{\frac{1}{3}}x+\left(\frac{1}{2}\right)^{\frac{2}{3}}\right) + \frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{1}{2}\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(1/(2*x^3-1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(1/2)^(1/3)*arctan(2/3*sqrt(3)*(1/2)^(2/3)*(2*x + (1/2)^(1/3)
)) - 1/12*4^(1/3)*log(x^2 + (1/2)^(1/3)*x + (1/2)^(2/3)) + 1/3*(1/2)^(1/3)
*log(abs(x - (1/2)^(1/3)))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{1}{-1+2x^3}\,dx = \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}}{2}\right)}{6} + \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(1/(2*x^3 - 1),x)
```

output
```
(2^(2/3)*log(x - 2^(2/3)/2))/6 + (2^(2/3)*log(x - (2^(2/3)*(3^(1/2)*1i - 1
))/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 1))/4
)*(3^(1/2)*1i + 1))/12
```

3.33.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{1}{-1+2x^3}\,dx = \frac{2^{\frac{2}{3}}\left(-2\sqrt{3}\,atan\left(\frac{2\,2^{\frac{1}{3}}x+1}{\sqrt{3}}\right)-\log\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x+1\right)+2\log\left(2^{\frac{1}{3}}x-1\right)\right)}{12}$$

input
```
int(1/(2*x**3 - 1),x)
```

output
```
(2**(2/3)*( - 2*sqrt(3)*atan((2*2**(1/3)*x + 1)/sqrt(3)) - log(2**(2/3)*x*
*2 + 2**(1/3)*x + 1) + 2*log(2**(1/3)*x - 1)))/12
```

3.34 $\int \frac{1}{-2+x^3} \, dx$

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3.34.1 Optimal result

Integrand size = 7, antiderivative size = 74

$$\int \frac{1}{-2+x^3} \, dx = -\frac{\arctan\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}} - \frac{\log\left(2^{2/3}+\sqrt[3]{2}x+x^2\right)}{6\ 2^{2/3}}$$

output
```
1/6*ln(2^(1/3)-x)*2^(1/3)-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arcta
n(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{-2+x^3} \, dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right) - 2\log\left(2-2^{2/3}x\right) + \log\left(2+2^{2/3}x+\sqrt[3]{2}x^2\right)}{6\ 2^{2/3}}$$

input
```
Integrate[(-2 + x^3)^(-1),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] - 2*Log[2 - 2^(2/3)*x] + L
og[2 + 2^(2/3)*x + 2^(1/3)*x^2])/2^(2/3)
```

3.34.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3-2}dx$$

$$\Big\downarrow \ 750$$

$$\frac{\int -\frac{x+2\sqrt[3]{2}}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{3\ 2^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{2}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 16$$

$$\frac{\int -\frac{x+2\sqrt[3]{2}}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 25$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\int \frac{x+2\sqrt[3]{2}}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\frac{3\int \frac{1}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{2^{2/3}}+\frac{1}{2}\int \frac{\sqrt[3]{2}(2^{2/3}x+1)}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\frac{3\int \frac{1}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{2^{2/3}}+\frac{\int \frac{2^{2/3}x+1}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{2^{2/3}}}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 1082$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\frac{\int \frac{2^{2/3}x+1}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{2^{2/3}}-3\int \frac{1}{-(2^{2/3}x+1)^2-3}d(2^{2/3}x+1)}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 217$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\frac{\int \frac{2^{2/3}x+1}{x^2+\sqrt[3]{2}x+2^{2/3}}dx}{2^{2/3}}+\sqrt{3}\arctan\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{3\ 2^{2/3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{2}x+2^{2/3}\right)}{3\ 2^{2/3}}$$

input
```
Int[(-2 + x^3)^(-1),x]
```

output
```
Log[2^(1/3) - x]/(3*2^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] +
Log[2^(2/3) + 2^(1/3)*x + x^2]/2)/(3*2^(2/3))
```

3.34.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.34.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}-2\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}}{3}$	22
default	$\frac{2^{\frac{1}{3}}\ln\left(x-2^{\frac{1}{3}}\right)}{6}-\frac{\ln\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}x+x^{2}\right)2^{\frac{1}{3}}}{12}-\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}}x\right)\sqrt{3}}{3}\right)2^{\frac{1}{3}}\sqrt{3}}{6}$	54
meijerg	$\frac{2^{\frac{1}{3}}x\left(\ln\left(1-\frac{2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{2}\right)-\frac{\ln\left(1+\frac{2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{2}+\frac{2^{\frac{1}{3}}\left(x^{3}\right)^{\frac{2}{3}}}{2}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{4+2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{6\left(x^{3}\right)^{\frac{1}{3}}}$	84

input
```
int(1/(x^3-2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3-2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{1}{-2+x^3}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}x+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(2\,x^2+4^{\frac{2}{3}}x+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(2\,x-4^{\frac{2}{3}}\right)$$

input
```
integrate(1/(x^3-2),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*x + 4^(1/3)*sqrt(
3))) - 1/24*4^(2/3)*log(2*x^2 + 4^(2/3)*x + 2*4^(1/3)) + 1/12*4^(2/3)*log(
2*x - 4^(2/3))
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{-2+x^3}\,dx = \frac{\sqrt[3]{2}\log\left(x-\sqrt[3]{2}\right)}{6}-\frac{\sqrt[3]{2}\log\left(x^2+\sqrt[3]{2}x+2^{\frac{2}{3}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**3-2),x)
```

output
```
2**(1/3)*log(x - 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x + 2**(2/3))/
12 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{1}{-2+x^3}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}\right)\right) - \frac{1}{12} \cdot 2^{\frac{1}{3}}\log\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(x-2^{\frac{1}{3}}\right)$$

input
```
integrate(1/(x^3-2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^
(1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(x - 2^(1/3))
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{1}{-2+x^3}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}\right)\right) - \frac{1}{12} \cdot 2^{\frac{1}{3}}\log\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|x-2^{\frac{1}{3}}\right|\right)$$

input
```
integrate(1/(x^3-2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^
(1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(abs(x - 2^(1/3)))
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{-2+x^3}\,dx = \frac{2^{1/3}\ln\left(x-2^{1/3}\right)}{6} + \frac{2^{1/3}\ln\left(x-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{1/3}\ln\left(x+\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(1/(x^3 - 2),x)
```

output
```
(2^(1/3)*log(x - 2^(1/3)))/6 + (2^(1/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 1))
/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(x + (2^(1/3)*(3^(1/2)*1i + 1))/2)*
(3^(1/2)*1i + 1))/12
```

3.34.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{1}{-2+x^3}\,dx = \frac{2^{\frac{1}{3}}\left(-2\sqrt{3}\,atan\left(\frac{\left(2^{\frac{1}{3}}+2x\right)2^{\frac{2}{3}}}{2\sqrt{3}}\right)-\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}x+x^2\right)+2\log\left(-2^{\frac{1}{3}}+x\right)\right)}{12}$$

input
```
int(1/(x**3 - 2),x)
```

output
```
(2**(1/3)*( - 2*sqrt(3)*atan((2**(1/3) + 2*x)/(2**(1/3)*sqrt(3))) - log(2*
*(2/3) + 2**(1/3)*x + x**2) + 2*log( - 2**(1/3) + x)))/12
```

3.35 $\int \frac{1}{-b+ax^3}\,dx$

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3.35.1 Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{1}{-b+ax^3}\,dx=-\frac{\arctan\left(\frac{\sqrt[3]{b}+2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
1/3*ln(b^(1/3)-a^(1/3)*x)/a^(1/3)/b^(2/3)-1/6*ln(b^(2/3)+a^(1/3)*b^(1/3)*x
+a^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*a^(1/3)*x)/b^(1/3)
*3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{-b+ax^3}\,dx=-\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)+\log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[(-b + a*x^3)^(-1),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3
) - a^(1/3)*x] + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(a^(1/3)*
b^(2/3))
```

3.35.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^3-b}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{\int -\frac{\sqrt[3]{a}x+2\sqrt[3]{b}}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x-\sqrt[3]{b}}dx}{3b^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int -\frac{\sqrt[3]{a}x+2\sqrt[3]{b}}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\int \frac{\sqrt[3]{a}x+2\sqrt[3]{b}}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{a}x+\sqrt[3]{b}\right)}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\frac{1}{2}\int \frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}+1\right)}{\sqrt[3]{a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\frac{1}{2}\int \frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}$$

input
```
Int[(-b + a*x^3)^(-1),x]
```

output
```
Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2*a^
(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3) + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^
(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3))
```

3.35. $\int \frac{1}{-b+ax^3}\,dx$

3.35.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.35.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^3-b\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{3a}$	29
default	$\frac{\ln\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2+\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	92

input
```
int(1/(a*x^3-b),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a-b))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.61

$$\int \frac{1}{-b+ax^3}\,dx$$
$$=\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abx^3-3\,(ab^2)^{\frac{1}{3}}bx+b^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2-(ab^2)^{\frac{2}{3}}x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ax^3-b}\right)-(ab^2)^{\frac{2}{3}}\log\left(abx^2+\right.}{6\,ab^2}\right.$$
$$-\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(ab^2)^{\frac{2}{3}}x+(ab^2)^{\frac{1}{3}}b\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b^2}\right)+(ab^2)^{\frac{2}{3}}\log\left(abx^2+(ab^2)^{\frac{2}{3}}x+(ab^2)^{\frac{1}{3}}b\right)-}{6\,ab^2}$$

input
```
integrate(1/(a*x^3-b),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1
/3)*b*x + b^2 - 3*sqrt(1/3)*(2*a*b*x^2 - (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b
)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 - b)) - (a*b^2)^(2/3)*log(a*b*x^2 + (a*b^
2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x - (a*b^2)^(2/3))
)/(a*b^2), -1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*(a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) + (a*b^2)^(
2/3)*log(a*b*x^2 + (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) - 2*(a*b^2)^(2/3)*lo
g(a*b*x - (a*b^2)^(2/3)))/(a*b^2)]
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{-b+ax^3}\,dx = \operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**3-b),x)
```

output
```
RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{1}{-b+ax^3}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(a*x^3-b),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^
(2/3)) - 1/6*log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(2/3)) + 1/3*
log(x - (b/a)^(1/3))/(a*(b/a)^(2/3))
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{1}{-b+ax^3}\,dx = \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b} - \frac{\sqrt{3}(a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab} - \frac{(a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

input
```
integrate(1/(a*x^3-b),x, algorithm="giac")
```

output
```
1/3*(b/a)^(1/3)*log(abs(x - (b/a)^(1/3)))/b - 1/3*sqrt(3)*(a^2*b)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*b) - 1/6*(a^2*b)^(1/3
)*log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*b)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int \frac{1}{-b+ax^3}\,dx = \frac{\ln\left(a^{1/3}\,x-b^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}} + \frac{\ln\left(3\,a^2\,x-\frac{3\,a^{5/3}\,b^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}} - \frac{\ln\left(3\,a^2\,x+\frac{3\,a^{5/3}\,b^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

input
```
int(-1/(b - a*x^3),x)
```

output
```
log(a^(1/3)*x - b^(1/3))/(3*a^(1/3)*b^(2/3)) + (log(3*a^2*x - (3*a^(5/3)*b
^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)) - (log(3
*a^2*x + (3*a^(5/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(1
/3)*b^(2/3))
```

3.35.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

$$\int \frac{1}{-b+ax^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x+b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}x^2+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)+2\log\left(a^{\frac{1}{3}}x-b^{\frac{1}{3}}\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(1/(a*x**3 - b),x)
```

output
```
(b**(1/3)*( - 2*sqrt(3)*atan((2*a**(1/3)*x + b**(1/3))/(b**(1/3)*sqrt(3)))
 - log(a**(2/3)*x**2 + b**(1/3)*a**(1/3)*x + b**(2/3)) + 2*log(a**(1/3)*x
- b**(1/3))))/(6*a**(1/3)*b)
```

3.36 $\int \frac{1}{-2+x^4} \, dx$

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3.36.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \frac{1}{-2+x^4} \, dx = -\frac{\arctan\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

output
```
-1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/4*arctanh(1/2*x*2^(3/4))*2^(1/4)
```

3.36.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{-2+x^4} \, dx = -\frac{2\arctan\left(\frac{x}{\sqrt[4]{2}}\right) - \log\left(2-2^{3/4}x\right) + \log\left(2+2^{3/4}x\right)}{4\ 2^{3/4}}$$

input
```
Integrate[(-2 + x^4)^(-1),x]
```

output
```
-1/4*(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/2^(3/
4)
```

3.36.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-2}dx$$

$$\Big\downarrow \text{756}$$

$$-\frac{\int \frac{1}{\sqrt{2}-x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{x^2+\sqrt{2}}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\int \frac{1}{\sqrt{2}-x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

input
```
Int[(-2 + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x/2^(1/4)]/2^(3/4) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

3.36.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.36.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}-2\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4}$	22
default	$-\frac{2^{\frac{1}{4}}\left(\ln\left(\frac{x+2^{\frac{1}{4}}}{x-2^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x\,2^{\frac{3}{4}}}{2}\right)\right)}{8}$	32
meijerg	$\frac{2^{\frac{1}{4}}x\left(\ln\left(1-\frac{2^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{2^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{2^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)\right)}{8\left(x^{4}\right)^{\frac{1}{4}}}$	54

input
```
int(1/(x^4-2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4-2))
```

3.36.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{-2+x^4}\,dx = -\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4\,x+8^{\frac{3}{4}}\right) - \frac{1}{32}i\cdot 8^{\frac{3}{4}}\log\left(4\,x+i\cdot 8^{\frac{3}{4}}\right) + \frac{1}{32}i\cdot 8^{\frac{3}{4}}\log\left(4\,x-i\cdot 8^{\frac{3}{4}}\right) + \frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4\,x-8^{\frac{3}{4}}\right)$$

input
```
integrate(1/(x^4-2),x, algorithm="fricas")
```

output
```
-1/32*8^(3/4)*log(4*x + 8^(3/4)) - 1/32*I*8^(3/4)*log(4*x + I*8^(3/4)) + 1
/32*I*8^(3/4)*log(4*x - I*8^(3/4)) + 1/32*8^(3/4)*log(4*x - 8^(3/4))
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{1}{-2+x^4}\,dx = \frac{\sqrt[4]{2}\log\left(x-\sqrt[4]{2}\right)}{8} - \frac{\sqrt[4]{2}\log\left(x+\sqrt[4]{2}\right)}{8} - \frac{\sqrt[4]{2}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

input
```
integrate(1/(x**4-2),x)
```

output
```
2**(1/4)*log(x - 2**(1/4))/8 - 2**(1/4)*log(x + 2**(1/4))/8 - 2**(1/4)*ata
n(2**(3/4)*x/2)/4
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{-2+x^4}\,dx = -\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right) + \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{x-2^{\frac{1}{4}}}{x+2^{\frac{1}{4}}}\right)$$

input
```
integrate(1/(x^4-2),x, algorithm="maxima")
```

output
```
-1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) + 1/8*2^(1/4)*log((x - 2^(1/4))/(x + 2^
(1/4)))
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{-2+x^4}\,dx = -\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right) - \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x+2^{\frac{1}{4}}\right|\right) + \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x-2^{\frac{1}{4}}\right|\right)$$

input
```
integrate(1/(x^4-2),x, algorithm="giac")
```

output
```
-1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) - 1/8*2^(1/4)*log(abs(x + 2^(1/4))) + 1
/8*2^(1/4)*log(abs(x - 2^(1/4)))
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{1}{-2+x^4}\,dx = -\frac{2^{1/4}\left(\operatorname{atan}\left(\frac{2^{3/4}x}{2}\right)+\operatorname{atanh}\left(\frac{2^{3/4}x}{2}\right)\right)}{4}$$

input
```
int(1/(x^4 - 2),x)
```

output
```
-(2^(1/4)*(atan((2^(3/4)*x)/2) + atanh((2^(3/4)*x)/2)))/4
```

3.36.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{-2+x^4}\,dx = \frac{2^{\frac{1}{4}}\left(-2atan\left(\frac{x2^{\frac{3}{4}}}{2}\right)-\log\left(2^{\frac{1}{4}}+x\right)+\log\left(-2^{\frac{1}{4}}+x\right)\right)}{8}$$

input
```
int(1/(x**4 - 2),x)
```

output
```
(2**(1/4)*( - 2*atan(x/2**(1/4)) - log(2**(1/4) + x) + log( - 2**(1/4) + x
)))/8
```

3.37 $\int \frac{1}{-1+5x^4}\,dx$

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3.37.1 Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \frac{1}{-1+5x^4}\,dx = -\frac{\arctan\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

output
```
-1/10*arctan(5^(1/4)*x)*5^(3/4)-1/10*arctanh(5^(1/4)*x)*5^(3/4)
```

3.37.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{-1+5x^4}\,dx = -\frac{2\arctan\left(\sqrt[4]{5}x\right) - \log\left(1-\sqrt[4]{5}x\right) + \log\left(1+\sqrt[4]{5}x\right)}{4\sqrt[4]{5}}$$

input
```
Integrate[(-1 + 5*x^4)^(-1),x]
```

output
```
-1/4*(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/5^(1/
4)
```

3.37.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5x^4-1}dx$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{2}\int \frac{1}{1-\sqrt{5}x^2}dx-\frac{1}{2}\int \frac{1}{\sqrt{5}x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{1-\sqrt{5}x^2}dx-\frac{\arctan\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\text{arctanh}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

input
```
Int[(-1 + 5*x^4)^(-1),x]
```

output
```
-1/2*ArcTan[5^(1/4)*x]/5^(1/4) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

3.37.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.37.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(5\_Z^4-1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{20}$	24
default	$-\frac{5^{\frac{3}{4}}\left(\ln\left(\frac{x+\frac{5^{\frac{3}{4}}}{5}}{x-\frac{5^{\frac{3}{4}}}{5}}\right)+2\arctan\left(5^{\frac{1}{4}}x\right)\right)}{20}$	33
meijerg	$\frac{5^{\frac{3}{4}}x\left(\ln\left(1-5^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+5^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(5^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right)\right)}{20\left(x^4\right)^{\frac{1}{4}}}$	52

input
```
int(1/(5*x^4-1),x,method=_RETURNVERBOSE)
```

output
```
1/20*sum(1/_R^3*ln(x-_R),_R=RootOf(5*_Z^4-1))
```

3.37.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{-1+5x^4}\,dx = -\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(5\,x+5^{\frac{3}{4}}\right)-\frac{1}{20}i\cdot 5^{\frac{3}{4}}\log\left(5\,x+i\cdot 5^{\frac{3}{4}}\right)+\frac{1}{20}i\cdot 5^{\frac{3}{4}}\log\left(5\,x-i\cdot 5^{\frac{3}{4}}\right)+\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(5\,x-5^{\frac{3}{4}}\right)$$

input
```
integrate(1/(5*x^4-1),x, algorithm="fricas")
```

output
```
-1/20*5^(3/4)*log(5*x + 5^(3/4)) - 1/20*I*5^(3/4)*log(5*x + I*5^(3/4)) + 1
/20*I*5^(3/4)*log(5*x - I*5^(3/4)) + 1/20*5^(3/4)*log(5*x - 5^(3/4))
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{-1+5x^4}\,dx = \frac{5^{\frac{3}{4}}\log\left(x-\frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}}\log\left(x+\frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{5}x\right)}{10}$$

input
```
integrate(1/(5*x**4-1),x)
```

output
```
5**(3/4)*log(x - 5**(3/4)/5)/20 - 5**(3/4)*log(x + 5**(3/4)/5)/20 - 5**(3/
4)*atan(5**(1/4)*x)/10
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{1}{-1+5x^4}\,dx = -\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5^{\frac{1}{4}}x\right) + \frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\frac{\sqrt{5}x-5^{\frac{1}{4}}}{\sqrt{5}x+5^{\frac{1}{4}}}\right)$$

input
```
integrate(1/(5*x^4-1),x, algorithm="maxima")
```

output
```
-1/10*5^(3/4)*arctan(5^(1/4)*x) + 1/20*5^(3/4)*log((sqrt(5)*x - 5^(1/4))/(
sqrt(5)*x + 5^(1/4)))
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{-1+5x^4}\,dx = -\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5\left(\frac{1}{5}\right)^{\frac{3}{4}}x\right) - \frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\left|x+\left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right) + \frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\left|x-\left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)$$

input
```
integrate(1/(5*x^4-1),x, algorithm="giac")
```

output
```
-1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*log(abs(x + (1/5)^(1/
4))) + 1/20*5^(3/4)*log(abs(x - (1/5)^(1/4)))
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{1}{-1+5x^4}\,dx = -\frac{5^{3/4}\left(\operatorname{atan}\left(5^{1/4}\,x\right)+\operatorname{atanh}\left(5^{1/4}\,x\right)\right)}{10}$$

input
```
int(1/(5*x^4 - 1),x)
```

output
```
-(5^(3/4)*(atan(5^(1/4)*x) + atanh(5^(1/4)*x)))/10
```

3.37.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{-1+5x^4}\,dx = \frac{\sqrt{5}\,5^{\frac{1}{4}}\left(-2atan\left(\frac{\sqrt{5}\,x5^{\frac{3}{4}}}{5}\right)+\log\left(5^{\frac{1}{4}}x-1\right)-\log\left(5^{\frac{1}{4}}x+1\right)\right)}{20}$$

input
```
int(1/(5*x**4 - 1),x)
```

output
```
(sqrt(5)*5**(1/4)*( - 2*atan((sqrt(5)*x)/5**(1/4)) + log(5**(1/4)*x - 1) -
 log(5**(1/4)*x + 1)))/20
```

3.38 $\int \frac{1}{7+3x^4} dx$

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3.38.1 Optimal result

Integrand size = 9, antiderivative size = 171

$$\int \frac{1}{7+3x^4}\,dx = -\frac{\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\arctan\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} - \frac{\log\left(\sqrt{21}-\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\log\left(\sqrt{21}+\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

output
```
1/84*arctan(-1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/84
*arctan(1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)-1/168*ln(
3*x^2-3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/168*ln
(3*x^2+3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{1}{7+3x^4}\,dx = \frac{-2\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)+2\arctan\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)-\log\left(7-\sqrt{2}\sqrt[4]{3}7^{3/4}x+\sqrt{21}x^2\right)+\log\left(7+\sqrt{}\right.}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

input
```
Integrate[(7 + 3*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x
] - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] + Log[7 + Sqrt[2]*3^
(1/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

3.38.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {755, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^4+7}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7}dx}{2\sqrt{7}}+\frac{\int \frac{\sqrt{3}x^2+\sqrt{7}}{3x^4+7}dx}{2\sqrt{7}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\sqrt{2}\sqrt[4]{\frac{7}{3}}x+\sqrt{\frac{7}{3}}}dx}{2\sqrt{3}}+\frac{\int \frac{1}{x^2+\sqrt{2}\sqrt[4]{\frac{7}{3}}x+\sqrt{\frac{7}{3}}}dx}{2\sqrt{3}}}{2\sqrt{7}}+\frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7}dx}{2\sqrt{7}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7}dx}{2\sqrt{7}}+\frac{\frac{\int \frac{1}{-\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)^2-1}d\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{\sqrt{2}\sqrt[4]{21}}-\frac{\int \frac{1}{-\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)^2-1}d\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7}dx}{2\sqrt{7}}+\frac{\frac{\arctan\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{\sqrt{2}\sqrt[4]{21}}-\frac{\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}$$

$\Big\downarrow$ 1479

$$\frac{-\frac{\int -\frac{\sqrt{2}3^{3/4}\sqrt[4]{7}-6x}{3x^2-\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}}dx}{2\sqrt{2}\sqrt[4]{21}}-\frac{\int -\frac{6x+\sqrt{2}3^{3/4}\sqrt[4]{7}}{3x^2+\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}}dx}{2\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}+\frac{\frac{\arctan\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{\sqrt{2}\sqrt[4]{21}}-\frac{\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\sqrt{2}3^{3/4}\sqrt[4]{7}-6x}{3x^2-\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}}dx}{2\sqrt{2}\sqrt[4]{21}}+\frac{\int \frac{6x+\sqrt{2}3^{3/4}\sqrt[4]{7}}{3x^2+\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}}dx}{2\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}+\frac{\frac{\arctan\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{\sqrt{2}\sqrt[4]{21}}-\frac{\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\arctan\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{\sqrt{2}\sqrt[4]{21}}-\frac{\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}+\frac{\frac{\log\left(3x^2+\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{2\sqrt{2}\sqrt[4]{21}}-\frac{\log\left(3x^2-\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{2\sqrt{2}\sqrt[4]{21}}}{2\sqrt{7}}$$

input
```
Int[(7 + 3*x^4)^(-1),x]
```

output
```
(-(ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(Sqrt[2]*21^(1/4))) + ArcTan[1 + (3/7
)^(1/4)*Sqrt[2]*x]/(Sqrt[2]*21^(1/4)))/(2*Sqrt[7]) + (-1/2*Log[Sqrt[21] -
Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(Sqrt[2]*21^(1/4)) + Log[Sqrt[21] + Sqr
t[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(2*Sqrt[2]*21^(1/4)))/(2*Sqrt[7])
```

3.38. $\int \frac{1}{7+3x^4}\,dx$

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.38.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+7\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,21^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{21}}{3}}{x^{2}-\frac{\sqrt{3}\,21^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{21}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,21^{\frac{3}{4}}x}{21}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,21^{\frac{3}{4}}x}{21}-1\right)\right)}{168}$
meijerg	$\frac{1029^{\frac{3}{4}}\Bigg(-\frac{x\sqrt{2}\,\ln\left(1-\frac{\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{7}+\frac{\sqrt{3}\sqrt{7}\sqrt{x^{4}}}{7}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{14-\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\frac{\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{7}+\frac{\sqrt{3}\sqrt{7}\sqrt{x^{4}}}{7}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}}{4116}$

input
```
int(1/(3*x^4+7),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum(1/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+7))
```

3.38.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.50

$$\begin{aligned}\int \frac{1}{7+3x^4}\,dx = &\left(\frac{1}{8232}i+\frac{1}{8232}\right)\cdot 1029^{\frac{3}{4}}\sqrt{2}\log\left((i+1)\cdot 1029^{\frac{3}{4}}\sqrt{2}+294\,x\right)\\ &-\left(\frac{1}{8232}i-\frac{1}{8232}\right)\cdot 1029^{\frac{3}{4}}\sqrt{2}\log\left(-(i-1)\cdot 1029^{\frac{3}{4}}\sqrt{2}+294\,x\right)\\ &+\left(\frac{1}{8232}i-\frac{1}{8232}\right)\cdot 1029^{\frac{3}{4}}\sqrt{2}\log\left((i-1)\cdot 1029^{\frac{3}{4}}\sqrt{2}+294\,x\right)\\ &-\left(\frac{1}{8232}i+\frac{1}{8232}\right)\cdot 1029^{\frac{3}{4}}\sqrt{2}\log\left(-(i+1)\cdot 1029^{\frac{3}{4}}\sqrt{2}+294\,x\right)\end{aligned}$$

input
```
integrate(1/(3*x^4+7),x, algorithm="fricas")
```

output
```
(1/8232*I + 1/8232)*1029^(3/4)*sqrt(2)*log((I + 1)*1029^(3/4)*sqrt(2) + 29
4*x) - (1/8232*I - 1/8232)*1029^(3/4)*sqrt(2)*log(-(I - 1)*1029^(3/4)*sqrt
(2) + 294*x) + (1/8232*I - 1/8232)*1029^(3/4)*sqrt(2)*log((I - 1)*1029^(3/
4)*sqrt(2) + 294*x) - (1/8232*I + 1/8232)*1029^(3/4)*sqrt(2)*log(-(I + 1)*
1029^(3/4)*sqrt(2) + 294*x)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{1}{7+3x^4}\,dx = -\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2-\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{168} + \frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2+\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{168} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{84} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{84}$$

input
```
integrate(1/(3*x**4+7),x)
```

output
```
-189**(1/4)*sqrt(2)*log(x**2 - 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 +
189**(1/4)*sqrt(2)*log(x**2 + 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + s
qrt(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt
(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{1}{7+3x^4}\,dx = {} & \frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}x+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right) \\
& + \frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}x-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right) \\
& + \frac{1}{168}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\log\left(\sqrt{3}x^2+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) \\
& - \frac{1}{168}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\log\left(\sqrt{3}x^2-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)
\end{aligned}
$$

input
```
integrate(1/(3*x^4+7),x, algorithm="maxima")
```

output
```
1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3
)*x + 7^(1/4)*3^(1/4)*sqrt(2))) + 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42
*7^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)*x - 7^(1/4)*3^(1/4)*sqrt(2))) + 1/168*
7^(1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 + 7^(1/4)*3^(1/4)*sqrt(2)*x + sqrt
(7)) - 1/168*7^(1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 - 7^(1/4)*3^(1/4)*sqr
t(2)*x + sqrt(7))
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$
\begin{aligned}
\int \frac{1}{7+3x^4}\,dx = {} & \frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2\,x+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right) \\
& + \frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2\,x-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right) \\
& + \frac{1}{168}\cdot 756^{\frac{1}{4}}\log\left(x^2+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{7}{3}}\right) \\
& - \frac{1}{168}\cdot 756^{\frac{1}{4}}\log\left(x^2-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{7}{3}}\right)
\end{aligned}
$$

input
```
integrate(1/(3*x^4+7),x, algorithm="giac")
```

output
```
1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x + (7/3)^(1/4)*sqrt(2))
) + 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x - (7/3)^(1/4)*sqrt
(2))) + 1/168*756^(1/4)*log(x^2 + (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3)) - 1/1
68*756^(1/4)*log(x^2 - (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3))
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

$$\int \frac{1}{7+3x^4}\,dx = \sqrt{2}\,189^{1/4}\,\mathrm{atan}\left(\sqrt{2}\,189^{3/4}\,x\left(\frac{1}{126}-\frac{1}{126}\mathrm{i}\right)\right)\left(\frac{1}{84}+\frac{1}{84}\mathrm{i}\right) + \sqrt{2}\,189^{1/4}\,\mathrm{atan}\left(\sqrt{2}\,189^{3/4}\,x\left(\frac{1}{126}+\frac{1}{126}\mathrm{i}\right)\right)\left(\frac{1}{84}-\frac{1}{84}\mathrm{i}\right)$$

input
```
int(1/(3*x^4 + 7),x)
```

output
```
2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 - 1i/126))*(1/84 + 1i/84
) + 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 + 1i/126))*(1/84 - 1
i/84)
```

3.38.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.54

$$\int \frac{1}{7+3x^4}\,dx = \frac{\sqrt{6}\,21^{\frac{1}{4}}\left(-2atan\left(\frac{\left(\sqrt{2}\,21^{\frac{1}{4}}-2\sqrt{3}\,x\right)21^{\frac{3}{4}}}{21\sqrt{2}}\right)+2atan\left(\frac{\left(\sqrt{2}\,21^{\frac{1}{4}}+2\sqrt{3}\,x\right)21^{\frac{3}{4}}}{21\sqrt{2}}\right)-\log\left(-\sqrt{2}\,21^{\frac{1}{4}}x+\sqrt{7}+\sqrt{3}\,x^{2}\right)+}{168}$$

input
```
int(1/(3*x**4 + 7),x)
```

output
```
(sqrt(6)*21**(1/4)*( - 2*atan((sqrt(2)*21**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*2
1**(1/4))) + 2*atan((sqrt(2)*21**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*21**(1/4)))
 - log( - sqrt(2)*21**(1/4)*x + sqrt(7) + sqrt(3)*x**2) + log(sqrt(2)*21**
(1/4)*x + sqrt(7) + sqrt(3)*x**2)))/168
```

3.39 $\int \frac{1}{-1+3x^2+x^4}\,dx$

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3.39.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{-1+3x^2+x^4}\,dx = -\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right) - \sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right)$$

output
```
-1/13*arctan(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/2
6*arctanh(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1+3x^2+x^4}\,dx = -\frac{\sqrt{-3+\sqrt{13}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)+\sqrt{3+\sqrt{13}}\operatorname{arctanh}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right)}{\sqrt{26}}$$

input
```
Integrate[(-1 + 3*x^2 + x^4)^(-1),x]
```

output
```
-((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x] + Sqrt[3 + Sqrt[13
]]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x])/Sqrt[26])
```

3.39.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+3x^2-1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{x^2+\frac{1}{2}\left(3-\sqrt{13}\right)}dx}{\sqrt{13}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(3+\sqrt{13}\right)}dx}{\sqrt{13}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{x^2+\frac{1}{2}\left(3-\sqrt{13}\right)}dx}{\sqrt{13}}-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

$$\Big\downarrow \text{220}$$

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)-\sqrt{\frac{2}{13\left(\sqrt{13}-3\right)}}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)$$

input
```
Int[(-1 + 3*x^2 + x^4)^(-1),x]
```

output
```
-(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) - Sqrt[2/(
13*(-3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

3.39.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.39.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+3\_Z^{2}-1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{3}+3\_R}\right)}{2}$	35
default	$-\frac{2\sqrt{13}\,\arctan\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$	56

input
```
int(1/(x^4+3*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3+3*_R)*ln(x-_R),_R=RootOf(_Z^4+3*_Z^2-1))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.07

$$\int \frac{1}{-1+3x^2+x^4}\,dx = \frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}+3}\log\left(\sqrt{26}\left(3\sqrt{13}-13\right)\sqrt{\sqrt{13}+3}+52x\right) - \frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}+3}\log\left(-\sqrt{26}\left(3\sqrt{13}-13\right)\sqrt{\sqrt{13}+3}+52x\right) - \frac{1}{52}\sqrt{26}\sqrt{-\sqrt{13}+3}\log\left(\sqrt{26}\left(3\sqrt{13}+13\right)\sqrt{-\sqrt{13}+3}+52x\right) + \frac{1}{52}\sqrt{26}\sqrt{-\sqrt{13}+3}\log\left(-\sqrt{26}\left(3\sqrt{13}+13\right)\sqrt{-\sqrt{13}+3}+52x\right)$$

input
```
integrate(1/(x^4+3*x^2-1),x, algorithm="fricas")
```

output
```
1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(sqrt(26)*(3*sqrt(13) - 13)*sqrt(sqrt(
13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(-sqrt(26)*(3*sqrt(
13) - 13)*sqrt(sqrt(13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(-sqrt(13) + 3)*l
og(sqrt(26)*(3*sqrt(13) + 13)*sqrt(-sqrt(13) + 3) + 52*x) + 1/52*sqrt(26)*
sqrt(-sqrt(13) + 3)*log(-sqrt(26)*(3*sqrt(13) + 13)*sqrt(-sqrt(13) + 3) +
52*x)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.00

$$\int \frac{1}{-1+3x^2+x^4}\,dx = \sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\log\left(x-22\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}+312\left(\frac{3}{104}+\frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}\right) - \sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\log\left(x-312\left(\frac{3}{104}+\frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}+22\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\right) - 2\sqrt{-\frac{3}{104}+\frac{\sqrt{13}}{104}}\operatorname{atan}\left(\frac{2\sqrt{2}x}{3\sqrt{-3+\sqrt{13}}+\sqrt{13}\sqrt{-3+\sqrt{13}}}\right)$$

input
```
integrate(1/(x**4+3*x**2-1),x)
```

output
```
sqrt(3/104 + sqrt(13)/104)*log(x - 22*sqrt(3/104 + sqrt(13)/104) + 312*(3/
104 + sqrt(13)/104)**(3/2)) - sqrt(3/104 + sqrt(13)/104)*log(x - 312*(3/10
4 + sqrt(13)/104)**(3/2) + 22*sqrt(3/104 + sqrt(13)/104)) - 2*sqrt(-3/104
+ sqrt(13)/104)*atan(2*sqrt(2)*x/(3*sqrt(-3 + sqrt(13)) + sqrt(13)*sqrt(-3
 + sqrt(13))))
```

3.39.7 Maxima [F]

$$\int \frac{1}{-1+3x^2+x^4}\,dx = \int \frac{1}{x^4+3\,x^2-1}\,dx$$

input
```
integrate(1/(x^4+3*x^2-1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + 3*x^2 - 1), x)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{1}{-1+3x^2+x^4}\,dx = -\frac{1}{26}\sqrt{26\sqrt{13}-78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}}\right) - \frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right) + \frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)$$

input
```
integrate(1/(x^4+3*x^2-1),x, algorithm="giac")
```

output
```
-1/26*sqrt(26*sqrt(13) - 78)*arctan(x/sqrt(1/2*sqrt(13) + 3/2)) - 1/52*sqr
t(26*sqrt(13) + 78)*log(abs(x + sqrt(1/2*sqrt(13) - 3/2))) + 1/52*sqrt(26*
sqrt(13) + 78)*log(abs(x - sqrt(1/2*sqrt(13) - 3/2)))
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{1}{-1+3x^2+x^4}\,dx = -\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{\sqrt{13}+3}}+\frac{3\sqrt{13}\sqrt{26}\,x}{26\sqrt{\sqrt{13}+3}}\right)\sqrt{\sqrt{13}+3}}{26} - \frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{3-\sqrt{13}}}-\frac{3\sqrt{13}\sqrt{26}\,x}{26\sqrt{3-\sqrt{13}}}\right)\sqrt{3-\sqrt{13}}}{26}$$

input
```
int(1/(3*x^2 + x^4 - 1),x)
```

output
```
- (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) + 3)^(1/2)) + (3*13^(1/2)*26^(
1/2)*x)/(26*(13^(1/2) + 3)^(1/2)))*(13^(1/2) + 3)^(1/2))/26 - (26^(1/2)*at
anh((26^(1/2)*x)/(2*(3 - 13^(1/2))^(1/2)) - (3*13^(1/2)*26^(1/2)*x)/(26*(3
 - 13^(1/2))^(1/2)))*(3 - 13^(1/2))^(1/2))/26
```

3.39.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{1}{-1+3x^2+x^4}\,dx$$
$$= \frac{\sqrt{2}\left(6\sqrt{\sqrt{13}+3}\,\sqrt{13}\,atan\left(\frac{2x}{\sqrt{\sqrt{13}+3}\,\sqrt{2}}\right)-26\sqrt{\sqrt{13}+3}\,atan\left(\frac{2x}{\sqrt{\sqrt{13}+3}\,\sqrt{2}}\right)+3\sqrt{\sqrt{13}-3}\,\sqrt{13}\log\left(-\sqrt{\right.\right.}{}$$

input
```
int(1/(x**4 + 3*x**2 - 1),x)
```

output
```
(sqrt(2)*(6*sqrt(sqrt(13) + 3)*sqrt(13)*atan((2*x)/(sqrt(sqrt(13) + 3)*sqr
t(2))) - 26*sqrt(sqrt(13) + 3)*atan((2*x)/(sqrt(sqrt(13) + 3)*sqrt(2))) +
3*sqrt(sqrt(13) - 3)*sqrt(13)*log( - sqrt(sqrt(13) - 3) + sqrt(2)*x) - 3*s
qrt(sqrt(13) - 3)*sqrt(13)*log(sqrt(sqrt(13) - 3) + sqrt(2)*x) + 13*sqrt(s
qrt(13) - 3)*log( - sqrt(sqrt(13) - 3) + sqrt(2)*x) - 13*sqrt(sqrt(13) - 3
)*log(sqrt(sqrt(13) - 3) + sqrt(2)*x)))/104
```

3.40 $\int \frac{1}{-1-3x^2+x^4} \, dx$

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3.40.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{-1-3x^2+x^4} \, dx = -\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)} \arctan\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right) - \sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}} \operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

output
```
-1/13*arctanh(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/
26*arctan(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1-3x^2+x^4} \, dx = -\frac{\sqrt{3+\sqrt{13}} \arctan\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right) + \sqrt{-3+\sqrt{13}} \operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)}{\sqrt{26}}$$

input
```
Integrate[(-1 - 3*x^2 + x^4)^(-1),x]
```

output
```
-((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x] + Sqrt[-3 + Sqrt[1
3]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x])/Sqrt[26])
```

3.40.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-3x^2-1}dx$$

$$\Big\downarrow \; 1406$$

$$\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-3-\sqrt{13}\right)}dx}{\sqrt{13}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-3+\sqrt{13}\right)}dx}{\sqrt{13}}$$

$$\Big\downarrow \; 216$$

$$\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-3-\sqrt{13}\right)}dx}{\sqrt{13}}-\sqrt{\frac{2}{13\left(\sqrt{13}-3\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)$$

$$\Big\downarrow \; 220$$

$$-\sqrt{\frac{2}{13\left(\sqrt{13}-3\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

input
```
Int[(-1 - 3*x^2 + x^4)^(-1),x]
```

output
```
-(Sqrt[2/(13*(-3 + Sqrt[13]))]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sqrt[2
/(13*(3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

3.40.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.40.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-3\_Z^{2}-1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{3}-3\_R}\right)}{2}$	35
default	$-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}-\frac{2\sqrt{13}\,\arctan\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$	56

input
```
int(1/(x^4-3*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3-3*_R)*ln(x-_R),_R=RootOf(_Z^4-3*_Z^2-1))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.07

$$\int \frac{1}{-1-3x^2+x^4}\,dx = -\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\log\left(\sqrt{26}\left(3\sqrt{13}+13\right)\sqrt{\sqrt{13}-3}+52\,x\right) + \frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\log\left(-\sqrt{26}\left(3\sqrt{13}+13\right)\sqrt{\sqrt{13}-3}+52\,x\right) + \frac{1}{52}\sqrt{26}\sqrt{-\sqrt{13}-3}\log\left(\sqrt{26}\left(3\sqrt{13}-13\right)\sqrt{-\sqrt{13}-3}+52\,x\right) - \frac{1}{52}\sqrt{26}\sqrt{-\sqrt{13}-3}\log\left(-\sqrt{26}\left(3\sqrt{13}-13\right)\sqrt{-\sqrt{13}-3}+52\,x\right)$$

input
```
integrate(1/(x^4-3*x^2-1),x, algorithm="fricas")
```

output
```
-1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(sqrt(26)*(3*sqrt(13) + 13)*sqrt(sqrt
(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(-sqrt(26)*(3*sqrt
(13) + 13)*sqrt(sqrt(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(-sqrt(13) - 3)*
log(sqrt(26)*(3*sqrt(13) - 13)*sqrt(-sqrt(13) - 3) + 52*x) - 1/52*sqrt(26)
*sqrt(-sqrt(13) - 3)*log(-sqrt(26)*(3*sqrt(13) - 13)*sqrt(-sqrt(13) - 3) +
 52*x)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{1}{-1-3x^2+x^4}\,dx = \operatorname{RootSum}\left(2704t^4+156t^2-1,\left(t\mapsto t\log\left(-312t^3-22t+x\right)\right)\right)$$

input
```
integrate(1/(x**4-3*x**2-1),x)
```

output
```
RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3 - 22*_t +
 x)))
```

3.40.7 Maxima [F]

$$\int \frac{1}{-1-3x^2+x^4}\,dx = \int \frac{1}{x^4-3\,x^2-1}\,dx$$

input
```
integrate(1/(x^4-3*x^2-1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 - 3*x^2 - 1), x)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{1}{-1-3x^2+x^4}\,dx = -\frac{1}{26}\,\sqrt{26\,\sqrt{13}+78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\,\sqrt{13}-\frac{3}{2}}}\right) - \frac{1}{52}\,\sqrt{26\,\sqrt{13}-78}\log\left(\left|x+\sqrt{\frac{1}{2}\,\sqrt{13}+\frac{3}{2}}\right|\right) + \frac{1}{52}\,\sqrt{26\,\sqrt{13}-78}\log\left(\left|x-\sqrt{\frac{1}{2}\,\sqrt{13}+\frac{3}{2}}\right|\right)$$

input
```
integrate(1/(x^4-3*x^2-1),x, algorithm="giac")
```

output
```
-1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) - 1/52*sqr
t(26*sqrt(13) - 78)*log(abs(x + sqrt(1/2*sqrt(13) + 3/2))) + 1/52*sqrt(26*
sqrt(13) - 78)*log(abs(x - sqrt(1/2*sqrt(13) + 3/2)))
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{1}{-1-3x^2+x^4}\,dx = -\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{\sqrt{13}-3}}-\frac{3\sqrt{13}\sqrt{26}\,x}{26\sqrt{\sqrt{13}-3}}\right)\sqrt{\sqrt{13}-3}}{26} - \frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{-\sqrt{13}-3}}+\frac{3\sqrt{13}\sqrt{26}\,x}{26\sqrt{-\sqrt{13}-3}}\right)\sqrt{-\sqrt{13}-3}}{26}$$

input
```
int(-1/(3*x^2 - x^4 + 1),x)
```

output
```
- (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) - 3)^(1/2)) - (3*13^(1/2)*26^(
1/2)*x)/(26*(13^(1/2) - 3)^(1/2)))*(13^(1/2) - 3)^(1/2))/26 - (26^(1/2)*at
anh((26^(1/2)*x)/(2*(- 13^(1/2) - 3)^(1/2)) + (3*13^(1/2)*26^(1/2)*x)/(26*
(- 13^(1/2) - 3)^(1/2)))*(- 13^(1/2) - 3)^(1/2))/26
```

3.40.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{1}{-1-3x^2+x^4}\,dx = \frac{\sqrt{2}\left(-6\sqrt{\sqrt{13}-3}\,\sqrt{13}\,atan\left(\frac{2x}{\sqrt{\sqrt{13}-3}\,\sqrt{2}}\right)-26\sqrt{\sqrt{13}-3}\,atan\left(\frac{2x}{\sqrt{\sqrt{13}-3}\,\sqrt{2}}\right)-3\sqrt{\sqrt{13}+3}\,\sqrt{13}\log\left(-\sqrt{}\right.\right.}{}$$

input
```
int(1/(x**4 - 3*x**2 - 1),x)
```

output
```
(sqrt(2)*( - 6*sqrt(sqrt(13) - 3)*sqrt(13)*atan((2*x)/(sqrt(sqrt(13) - 3)*
sqrt(2))) - 26*sqrt(sqrt(13) - 3)*atan((2*x)/(sqrt(sqrt(13) - 3)*sqrt(2)))
 - 3*sqrt(sqrt(13) + 3)*sqrt(13)*log( - sqrt(sqrt(13) + 3) + sqrt(2)*x) +
3*sqrt(sqrt(13) + 3)*sqrt(13)*log(sqrt(sqrt(13) + 3) + sqrt(2)*x) + 13*sqr
t(sqrt(13) + 3)*log( - sqrt(sqrt(13) + 3) + sqrt(2)*x) - 13*sqrt(sqrt(13)
+ 3)*log(sqrt(sqrt(13) + 3) + sqrt(2)*x)))/104
```

3.41 $\int \frac{1}{1-3x^2+x^4}\,dx$

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3.41.1 Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{1-3x^2+x^4}\,dx = -\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

output
```
-1/5*arctanh(x*2^(1/2)/(3+5^(1/2))^(1/2))*10^(1/2)/(3+5^(1/2))^(1/2)+arcta
nh(x*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))
```

3.41.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{1}{1-3x^2+x^4}\,dx = \frac{1}{20}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x\right)\right) - \left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x\right) + \left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x\right) + \left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\right)$$

input
```
Integrate[(1 - 3*x^2 + x^4)^(-1),x]
```

output
```
(-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5]
 - 2*x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x] + (-5 + Sqrt[5])*Log[1 + S
qrt[5] + 2*x])/20
```

3.41.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-3x^2+1}\,dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx}{\sqrt{5}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{220}$$

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

input
```
Int[(1 - 3*x^2 + x^4)^(-1),x]
```

output
```
-(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + Sqrt[(3 +
Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

3.41.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.41.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result
default	$\frac{\ln(x^2-x-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{10}-\frac{\ln(x^2+x-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)}{10}$
risch	$\frac{\ln\left(2x-1+\sqrt{5}\right)}{4}+\frac{\ln\left(2x-1+\sqrt{5}\right)\sqrt{5}}{20}+\frac{\ln\left(2x-1-\sqrt{5}\right)}{4}-\frac{\ln\left(2x-1-\sqrt{5}\right)\sqrt{5}}{20}-\frac{\ln\left(2x+\sqrt{5}+1\right)}{4}+\frac{\ln\left(2x+\sqrt{5}+1\right)\sqrt{5}}{20}-$

input
```
int(1/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2-x-1)+1/10*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/4*ln(x^2+x-1)+
1/10*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{1}{1-3x^2+x^4}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2\,x^2+\sqrt{5}(2\,x+1)+2\,x+3}{x^2+x-1}\right) + \frac{1}{20}\sqrt{5}\log\left(\frac{2\,x^2+\sqrt{5}(2\,x-1)-2\,x+3}{x^2-x-1}\right) - \frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-x-1\right)$$

input
```
integrate(1/(x^4-3*x^2+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/
20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x - 1) - 2*x + 3)/(x^2 - x - 1)) - 1/4*
log(x^2 + x - 1) + 1/4*log(x^2 - x - 1)
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(58) = 116$.

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \frac{1}{1-3x^2+x^4}\,dx = \left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x-\frac{7}{2}-\frac{7\sqrt{5}}{10}+120\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)^3\right) + \left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7}{2}+120\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^3+\frac{7\sqrt{5}}{10}\right) + \left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7\sqrt{5}}{10}+120\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)^3+\frac{7}{2}\right) + \left(-\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x+120\left(-\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

input
```
integrate(1/(x**4-3*x**2+1),x)
```

output
```
(sqrt(5)/20 + 1/4)*log(x - 7/2 - 7*sqrt(5)/10 + 120*(sqrt(5)/20 + 1/4)**3)
 + (1/4 - sqrt(5)/20)*log(x - 7/2 + 120*(1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/
10) + (-1/4 + sqrt(5)/20)*log(x - 7*sqrt(5)/10 + 120*(-1/4 + sqrt(5)/20)**
3 + 7/2) + (-1/4 - sqrt(5)/20)*log(x + 120*(-1/4 - sqrt(5)/20)**3 + 7*sqrt
(5)/10 + 7/2)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{1}{1-3x^2+x^4}\,dx = -\frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right) -\frac{1}{4}\,\log\left(x^2+x-1\right)+\frac{1}{4}\,\log\left(x^2-x-1\right)$$

input
```
integrate(1/(x^4-3*x^2+1),x, algorithm="maxima")
```

output
```
-1/20*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/20*sqrt(5)*
log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) - 1/4*log(x^2 + x - 1) + 1/4*
log(x^2 - x - 1)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int \frac{1}{1-3x^2+x^4}\,dx = -\frac{1}{20}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)-\frac{1}{20}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}-1\right|}{\left|2\,x+\sqrt{5}-1\right|}\right) -\frac{1}{4}\,\log\left(\left|x^2+x-1\right|\right)+\frac{1}{4}\,\log\left(\left|x^2-x-1\right|\right)$$

input
```
integrate(1/(x^4-3*x^2+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/20*sq
rt(5)*log(abs(2*x - sqrt(5) - 1)/abs(2*x + sqrt(5) - 1)) - 1/4*log(abs(x^2
 + x - 1)) + 1/4*log(abs(x^2 - x - 1))
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{1}{1-3x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{4\,x}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{10}-\frac{1}{2}\right) + \operatorname{atanh}\left(\frac{4\,x}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)$$

input
```
int(1/(x^4 - 3*x^2 + 1),x)
```

output
```
atanh((4*x)/(5^(1/2) - 3) - (2*5^(1/2)*x)/(5^(1/2) - 3))*(5^(1/2)/10 - 1/2
) + atanh((4*x)/(5^(1/2) + 3) + (2*5^(1/2)*x)/(5^(1/2) + 3))*(5^(1/2)/10 +
 1/2)
```

3.41.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{1}{1-3x^2+x^4}\,dx = -\frac{\sqrt{5}\log\left(-\sqrt{5}+2x-1\right)}{20} - \frac{\sqrt{5}\log\left(-\sqrt{5}+2x+1\right)}{20} + \frac{\sqrt{5}\log\left(\sqrt{5}+2x-1\right)}{20} + \frac{\sqrt{5}\log\left(\sqrt{5}+2x+1\right)}{20} + \frac{\log\left(-\sqrt{5}+2x-1\right)}{4} - \frac{\log\left(-\sqrt{5}+2x+1\right)}{4} + \frac{\log\left(\sqrt{5}+2x-1\right)}{4} - \frac{\log\left(\sqrt{5}+2x+1\right)}{4}$$

input
```
int(1/(x**4 - 3*x**2 + 1),x)
```

output
```
( - sqrt(5)*log( - sqrt(5) + 2*x - 1) - sqrt(5)*log( - sqrt(5) + 2*x + 1)
+ sqrt(5)*log(sqrt(5) + 2*x - 1) + sqrt(5)*log(sqrt(5) + 2*x + 1) + 5*log(
 - sqrt(5) + 2*x - 1) - 5*log( - sqrt(5) + 2*x + 1) + 5*log(sqrt(5) + 2*x
- 1) - 5*log(sqrt(5) + 2*x + 1))/20
```

3.42 $\int \frac{1}{1-4x^2+x^4} dx$

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3.42.10 Reduce [B] (verification not implemented) . . . 351

3.42.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \frac{1}{1-4x^2+x^4} \, dx = \frac{\text{arctanh}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\text{arctanh}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
1/2*arctanh(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arc
tanh(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.42.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-4x^2+x^4} \, dx = \frac{\text{arctanh}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\text{arctanh}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

input
```
Integrate[(1 - 4*x^2 + x^4)^(-1),x]
```

output
```
ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2
+ Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

3.42.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-4x^2+1} \, dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{x^2-\sqrt{3}-2}dx}{2\sqrt{3}}-\frac{\int \frac{1}{x^2+\sqrt{3}-2}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\text{arctanh}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

input
```
Int[(1 - 4*x^2 + x^4)^(-1),x]
```

output
```
ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2
+ Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

3.42.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.42.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^2+1\right)} \frac{\ln\left(x-\_R\right)}{\_R^3-2\_R}\right)}{4}$	33
default	$-\frac{\sqrt{3}\,\text{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}+\frac{\sqrt{3}\,\text{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	60

input
```
int(1/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3-2*_R)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^2+1))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(55) = 110.

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \frac{1}{1-4x^2+x^4}\,dx = & -\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right) \\ & +\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right) \\ & -\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+x\right) \\ & +\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log\left(-\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+x\right)\end{aligned}$$

input
```
integrate(1/(x^4-4*x^2+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) +
 1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(-sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x)
- 1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x
) + 1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log(-(sqrt(3) + 2)*sqrt(-sqrt(3) + 2)
+ x)
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.36

$$\int \frac{1}{1-4x^2+x^4}\,dx = \text{RootSum}\left(2304t^4-192t^2+1,\left(t\mapsto t\log\left(384t^3-28t+x\right)\right)\right)$$

input
```
integrate(1/(x**4-4*x**2+1),x)
```

output
```
RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 - 28*_t +
x)))
```

3.42.7 Maxima [F]

$$\int \frac{1}{1-4x^2+x^4}\,dx = \int \frac{1}{x^4-4\,x^2+1}\,dx$$

input
```
integrate(1/(x^4-4*x^2+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 - 4*x^2 + 1), x)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{1}{1-4x^2+x^4}\,dx =& \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)\\ &+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)\\ &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)\\ &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(\left|x-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)\end{aligned}$$

input
```
integrate(1/(x^4-4*x^2+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) - 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) + 1/2*sqrt(2))) + 1/24*
(sqrt(6) + 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) - 1/2*sqrt(2))) - 1/24*(sqrt
(6) + 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) + 1/2*sqrt(2))) - 1/24*(sqrt(6) -
 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) - 1/2*sqrt(2)))
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.46

$$\int \frac{1}{1-4x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{5\sqrt{2}\,x}{\sqrt{2}\,\sqrt{6}+4}+\frac{3\sqrt{6}\,x}{\sqrt{2}\,\sqrt{6}+4}\right)\left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\,x}{\sqrt{2}\,\sqrt{6}-4}-\frac{3\sqrt{6}\,x}{\sqrt{2}\,\sqrt{6}-4}\right)\left(\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{12}\right)$$

input
```
int(1/(x^4 - 4*x^2 + 1),x)
```

output
```
atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) + 4) + (3*6^(1/2)*x)/(2^(1/2)*6^(1/2)
 + 4))*(2^(1/2)/4 + 6^(1/2)/12) - atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) - 4
) - (3*6^(1/2)*x)/(2^(1/2)*6^(1/2) - 4))*(2^(1/2)/4 - 6^(1/2)/12)
```

3.42.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{1}{1-4x^2+x^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{3}\,atanh\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)+6atanh\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)-\sqrt{3}\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+x\right)+\sqrt{3}\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+x\right)+3\log\right.}{24}$$

input
```
int(1/(x**4 - 4*x**2 + 1),x)
```

output

```
(sqrt(2)*(2*sqrt(3)*atanh((2*x)/(sqrt(6) - sqrt(2))) + 6*atanh((2*x)/(sqrt
(6) - sqrt(2))) - sqrt(3)*log(( - sqrt(6) - sqrt(2) + 2*x)/2) + sqrt(3)*lo
g((sqrt(6) + sqrt(2) + 2*x)/2) + 3*log(( - sqrt(6) - sqrt(2) + 2*x)/2) - 3
*log((sqrt(6) + sqrt(2) + 2*x)/2)))/24
```

3.42. $\int \frac{1}{1-4x^2+x^4}\,dx$

3.43 $\int \frac{1}{1+4x^2+x^4}\,dx$

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3.43.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \frac{1}{1+4x^2+x^4}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arct
an(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+4x^2+x^4}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

input
```
Integrate[(1 + 4*x^2 + x^4)^(-1),x]
```

output
```
ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 +
Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

3.43.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+4x^2+1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{x^2-\sqrt{3}+2}dx}{2\sqrt{3}}-\frac{\int \frac{1}{x^2+\sqrt{3}+2}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

input
```
Int[(1 + 4*x^2 + x^4)^(-1),x]
```

output
```
ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 +
Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

3.43.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+4\_Z^{2}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}+2\_R}\right)}{4}$	33
default	$\frac{\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}-\frac{\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}$	60

input
```
int(1/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3+2*_R)*ln(x-_R),_R=RootOf(_Z^4+4*_Z^2+1))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \frac{1}{1+4x^2+x^4}\,dx = &-\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}-2}\log\left(\left(\sqrt{3}+2\right)\sqrt{\sqrt{3}-2}+x\right)\\&+\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}-2}\log\left(-\left(\sqrt{3}+2\right)\sqrt{\sqrt{3}-2}+x\right)\\&-\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}-2}\log\left(\left(\sqrt{3}-2\right)\sqrt{-\sqrt{3}-2}+x\right)\\&+\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}-2}\log\left(-\left(\sqrt{3}-2\right)\sqrt{-\sqrt{3}-2}+x\right)\end{aligned}$$

input
```
integrate(1/(x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*sqrt(sqrt(3) - 2)*log((sqrt(3) + 2)*sqrt(sqrt(3) - 2) + x) +
 1/12*sqrt(3)*sqrt(sqrt(3) - 2)*log(-(sqrt(3) + 2)*sqrt(sqrt(3) - 2) + x)
- 1/12*sqrt(3)*sqrt(-sqrt(3) - 2)*log((sqrt(3) - 2)*sqrt(-sqrt(3) - 2) + x
) + 1/12*sqrt(3)*sqrt(-sqrt(3) - 2)*log(-(sqrt(3) - 2)*sqrt(-sqrt(3) - 2)
+ x)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{1}{1+4x^2+x^4}\,dx = -2\sqrt{\frac{1}{24}-\frac{\sqrt{3}}{48}}\,\operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2-\sqrt{3}}+2\sqrt{2-\sqrt{3}}}\right) - 2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\,\operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$$

input
```
integrate(1/(x**4+4*x**2+1),x)
```

output
```
-2*sqrt(1/24 - sqrt(3)/48)*atan(x/(sqrt(3)*sqrt(2 - sqrt(3)) + 2*sqrt(2 -
sqrt(3)))) - 2*sqrt(sqrt(3)/48 + 1/24)*atan(x/(-2*sqrt(sqrt(3) + 2) + sqrt
(3)*sqrt(sqrt(3) + 2)))
```

3.43.7 Maxima [F]

$$\int \frac{1}{1+4x^2+x^4}\,dx = \int \frac{1}{x^4+4\,x^2+1}\,dx$$

input
```
integrate(1/(x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + 4*x^2 + 1), x)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+4x^2+x^4}\,dx = \frac{1}{12}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{2\,x}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{12}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{2\,x}{\sqrt{6}-\sqrt{2}}\right)$$

input
```
integrate(1/(x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/12*(sqrt(6) - 3*sqrt(2))*arctan(2*x/(sqrt(6) + sqrt(2))) + 1/12*(sqrt(6)
 + 3*sqrt(2))*arctan(2*x/(sqrt(6) - sqrt(2)))
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.75

$$\int \frac{1}{1+4x^2+x^4}\,dx = 2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}-4}-\frac{16\,\sqrt{3}\,x\,\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}-4}\right)\,\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}} - 2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}+4} + \frac{16\,\sqrt{3}\,x\,\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}+4}\right)\,\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}$$

input
```
int(1/(4*x^2 + x^4 + 1),x)
```

output
```
2*atanh((24*x*(3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4) - (16*3^(1/2)*x*(
3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4))*(3^(1/2)/48 - 1/24)^(1/2) - 2*a
tanh((24*x*(- 3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4) + (16*3^(1/2)*x*(-
 3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4))*(- 3^(1/2)/48 - 1/24)^(1/2)
```

3.43.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.49

$$\int \frac{1}{1+4x^2+x^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{3}\,atan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)-6atan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)-\sqrt{3}\log\left(-\frac{\sqrt{6}\,i}{2}+\frac{\sqrt{2}\,i}{2}+x\right)i+\sqrt{3}\log\left(\frac{\sqrt{6}\,i}{2}-\frac{\sqrt{2}\,i}{2}+x\right)i-3}{24}$$

input
```
int(1/(x**4 + 4*x**2 + 1),x)
```

output
```
(sqrt(2)*(2*sqrt(3)*atan((2*x)/(sqrt(6) + sqrt(2))) - 6*atan((2*x)/(sqrt(6
) + sqrt(2))) - sqrt(3)*log(( - sqrt(6)*i + sqrt(2)*i + 2*x)/2)*i + sqrt(3
)*log((sqrt(6)*i - sqrt(2)*i + 2*x)/2)*i - 3*log(( - sqrt(6)*i + sqrt(2)*i
 + 2*x)/2)*i + 3*log((sqrt(6)*i - sqrt(2)*i + 2*x)/2)*i))/24
```

3.43. $\int \frac{1}{1+4x^2+x^4}\,dx$

3.44 $\int \frac{1}{2+x^2+x^4}\,dx$

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3.44.10 Reduce [B] (verification not implemented) 366

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 196

$$\int \frac{1}{2+x^2+x^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)-\frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

output
```
-1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-14+28*2^(1
/2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1
4+28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))/(-2+4*2^(1/
2))^(1/2)+1/4*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))/(-2+4*2^(1/2))^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+x^2+x^4}\,dx = -\frac{i\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1-i\sqrt{7}\right)}}+\frac{i\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1+i\sqrt{7}\right)}}$$

input
```
Integrate[(2 + x^2 + x^4)^(-1),x]
```

output
```
((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))/2] + (I*
ArcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7]))/2]
```

3.44.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+x^2+2}dx$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{\int \frac{\sqrt{-1+2\sqrt{2}}-x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\int \frac{x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\frac{1}{2}\int -\frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\int \frac{1}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\int \frac{1}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{2\sqrt{2}-1}\int \frac{1}{-\left(2x-\sqrt{-1+2\sqrt{2}}\right)^2-2\sqrt{2}-1}d\left(2x-\sqrt{-1+2\sqrt{2}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{2\sqrt{2}-1}\int \frac{1}{-\left(2x+\sqrt{-1+2\sqrt{2}}\right)^2-2\sqrt{2}-1}d\left(2x+\sqrt{-1+2\sqrt{2}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 1103$$

$$\frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

3.44. $\int \frac{1}{2+x^2+x^4}\,dx$

input
```
Int[(2 + x^2 + x^4)^(-1),x]
```

output
```
(Sqrt[(-1 + 2*Sqrt[2])/(1 + 2*Sqrt[2])]*ArcTan[(-Sqrt[-1 + 2*Sqrt[2]] + 2*
x)/Sqrt[1 + 2*Sqrt[2]]] - Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/2)/(
2*Sqrt[2*(-1 + 2*Sqrt[2])]) + (Sqrt[(-1 + 2*Sqrt[2])/(1 + 2*Sqrt[2])]*ArcT
an[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]] + Log[Sqrt[2] + Sqrt[
-1 + 2*Sqrt[2]]*x + x^2]/2)/(2*Sqrt[2*(-1 + 2*Sqrt[2])])
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

3.44.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+\_Z^{2}+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{3}+\_R}\right)}{2}$
default	$\frac{\left(-\sqrt{-1+2\sqrt{2}}\,\sqrt{2}-4\sqrt{-1+2\sqrt{2}}\right)\ln\left(x^{2}+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)}{56}+\frac{\left(7\sqrt{2}+\frac{\left(-\sqrt{-1+2\sqrt{2}}\,\sqrt{2}-4\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}}{2}\right)\arctan\left(\frac{2x-\sqrt{}}{\sqrt{1}}\right.}{14\sqrt{1+2\sqrt{2}}}$

input
```
int(1/(x^4+x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+_Z^2+2))
```

3.44.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+x^2+x^4}\,dx = \frac{1}{28}\sqrt{7}\sqrt{i\sqrt{7}+1}\log\left(\left(\sqrt{7}+i\right)\sqrt{i\sqrt{7}+1}+4x\right) - \frac{1}{28}\sqrt{7}\sqrt{i\sqrt{7}+1}\log\left(-\left(\sqrt{7}+i\right)\sqrt{i\sqrt{7}+1}+4x\right) + \frac{1}{28}\sqrt{7}\sqrt{-i\sqrt{7}+1}\log\left(\left(\sqrt{7}-i\right)\sqrt{-i\sqrt{7}+1}+4x\right) - \frac{1}{28}\sqrt{7}\sqrt{-i\sqrt{7}+1}\log\left(-\left(\sqrt{7}-i\right)\sqrt{-i\sqrt{7}+1}+4x\right)$$

input
```
integrate(1/(x^4+x^2+2),x, algorithm="fricas")
```

output
```
1/28*sqrt(7)*sqrt(I*sqrt(7) + 1)*log((sqrt(7) + I)*sqrt(I*sqrt(7) + 1) + 4
*x) - 1/28*sqrt(7)*sqrt(I*sqrt(7) + 1)*log(-(sqrt(7) + I)*sqrt(I*sqrt(7) +
 1) + 4*x) + 1/28*sqrt(7)*sqrt(-I*sqrt(7) + 1)*log((sqrt(7) - I)*sqrt(-I*s
qrt(7) + 1) + 4*x) - 1/28*sqrt(7)*sqrt(-I*sqrt(7) + 1)*log(-(sqrt(7) - I)*
sqrt(-I*sqrt(7) + 1) + 4*x)
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(158) = 316.

Time = 0.68 (sec) , antiderivative size = 994, normalized size of antiderivative = 5.07

$$\int \frac{1}{2+x^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**4+x**2+2),x)
```

output
```
sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7 +
 5*sqrt(14)*sqrt(1 + 2*sqrt(2))/28 + 3*sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4
*sqrt(2) + 9)/28) - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*
sqrt(2) + 9)/28 + 83*sqrt(2)/28) - sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*
(-3*sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/28 - 5*sqrt(14)*sqrt(
1 + 2*sqrt(2))/28 + 4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7) - 33*sqrt(4*sqrt(2) +
 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28) + 2*sq
rt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sq
rt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2
*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) - 8*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sq
rt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2
*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqr
t(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*
sqrt(2) + 9) + 1 + 6*sqrt(2))) + 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)
/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqr
t(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)))) + 2*sqrt(-sqrt(4*sqrt(2) + 9)/
112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-
2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1
 + 6*sqrt(2))) - 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2)
 + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*s...
```

3.44.7 Maxima [F]

$$\int \frac{1}{2+x^2+x^4}\,dx = \int \frac{1}{x^4+x^2+2}\,dx$$

input
```
integrate(1/(x^4+x^2+2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + x^2 + 2), x)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.29

$$\int \frac{1}{2+x^2+x^4}\,dx$$

$$= \frac{1}{112}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}-2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}\right)\arctan\left(\frac{2\cdot 2^{\frac{3}{4}}\sqrt{\frac{1}{2}}\left(x+2^{\frac{1}{4}}\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}\right)}{\sqrt{\sqrt{2}+4}}\right)$$
$$+\frac{1}{112}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}-2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}\right)\arctan\left(\frac{2\cdot 2^{\frac{3}{4}}\sqrt{\frac{1}{2}}\left(x-2^{\frac{1}{4}}\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}\right)}{\sqrt{\sqrt{2}+4}}\right)$$
$$+\frac{1}{224}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}+2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}\right)\log\left(x^2+2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}\right.$$
$$\left.+\sqrt{2}\right)-\frac{1}{224}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}+2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}\right)\log\left(x^2-2\right.$$
$$\left.\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)$$

input
```
integrate(1/(x^4+x^2+2),x, algorithm="giac")
```

output
```
1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(
2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))
/sqrt(sqrt(2) + 4)) + 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(2*sqrt(2) + 8) -
 2^(1/4)*sqrt(-2*sqrt(2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqr
t(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/224*sqrt(7)*(sqrt(7)*2^(1/4)
*sqrt(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*log(x^2 + 2*2^(1/4)*x
*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/224*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt
(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*log(x^2 - 2*2^(1/4)*x*sqrt
(-1/8*sqrt(2) + 1/2) + sqrt(2))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

$$\int \frac{1}{2+x^2+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{7}\,x\,\sqrt{7-\sqrt{7}\,7\mathrm{i}}}{14}\right)\,\sqrt{7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{14} - \frac{\sqrt{7}\operatorname{atan}\left(\frac{x\,\sqrt{1+\sqrt{7}\,1\mathrm{i}}}{2}\right)\,\sqrt{1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{14}$$

input
```
int(1/(x^2 + x^4 + 2),x)
```

output
```
(atan((7^(1/2)*x*(7 - 7^(1/2)*7i)^(1/2))/14)*(7 - 7^(1/2)*7i)^(1/2)*1i)/14
 - (7^(1/2)*atan((x*(7^(1/2)*1i + 1)^(1/2))/2)*(7^(1/2)*1i + 1)^(1/2)*1i)/
14
```

3.44.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.20

$$\int \frac{1}{2+x^2+x^4}\,dx = \frac{\sqrt{2\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{2\sqrt{2}+1}}\right)}{28} - \frac{\sqrt{2\sqrt{2}+1}\,atan\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{2\sqrt{2}+1}}\right)}{7} - \frac{\sqrt{2\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{2\sqrt{2}-1}+2x}{\sqrt{2\sqrt{2}+1}}\right)}{28} + \frac{\sqrt{2\sqrt{2}+1}\,atan\left(\frac{\sqrt{2\sqrt{2}-1}+2x}{\sqrt{2\sqrt{2}+1}}\right)}{7} - \frac{\sqrt{2\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}+x^2\right)}{56} + \frac{\sqrt{2\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}+x^2\right)}{56} - \frac{\sqrt{2\sqrt{2}-1}\log\left(-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}+x^2\right)}{14} + \frac{\sqrt{2\sqrt{2}-1}\log\left(\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}+x^2\right)}{14}$$

input
```
int(1/(x**4 + x**2 + 2),x)
```

output
```
(2*sqrt(2*sqrt(2) + 1)*sqrt(2)*atan((sqrt(2*sqrt(2) - 1) - 2*x)/sqrt(2*sqr
t(2) + 1)) - 8*sqrt(2*sqrt(2) + 1)*atan((sqrt(2*sqrt(2) - 1) - 2*x)/sqrt(2
*sqrt(2) + 1)) - 2*sqrt(2*sqrt(2) + 1)*sqrt(2)*atan((sqrt(2*sqrt(2) - 1) +
 2*x)/sqrt(2*sqrt(2) + 1)) + 8*sqrt(2*sqrt(2) + 1)*atan((sqrt(2*sqrt(2) -
1) + 2*x)/sqrt(2*sqrt(2) + 1)) - sqrt(2*sqrt(2) - 1)*sqrt(2)*log( - sqrt(2
*sqrt(2) - 1)*x + sqrt(2) + x**2) + sqrt(2*sqrt(2) - 1)*sqrt(2)*log(sqrt(2
*sqrt(2) - 1)*x + sqrt(2) + x**2) - 4*sqrt(2*sqrt(2) - 1)*log( - sqrt(2*sq
rt(2) - 1)*x + sqrt(2) + x**2) + 4*sqrt(2*sqrt(2) - 1)*log(sqrt(2*sqrt(2)
- 1)*x + sqrt(2) + x**2))/56
```

3.44. $\int \frac{1}{2+x^2+x^4}\,dx$

3.45 $\int \frac{1}{2-x^2+x^4} dx$

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3.45.1 Optimal result

Integrand size = 12, antiderivative size = 196

$$\begin{aligned}\int \frac{1}{2-x^2+x^4} dx =& -\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\arctan\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right) \\ &+\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\arctan\left(\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right) \\ &-\frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}\end{aligned}$$

output
```
-1/28*arctan((-2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28*2^(1/
2))^(1/2)+1/28*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+
28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^
(1/2)+1/4*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{1}{2-x^2+x^4}\,dx = -\frac{i\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1-i\sqrt{7}\right)}}+\frac{i\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1+i\sqrt{7}\right)}}$$

input
```
Integrate[(2 - x^2 + x^4)^(-1),x]
```

output
```
((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]))/2] + (
I*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sqrt[7]))/2]
```

3.45.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-x^2+2}\,dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{1+2\sqrt{2}}-x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\int \frac{x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{1+2\sqrt{2}}\int \frac{1}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx-\frac{1}{2}\int -\frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\frac{1}{2}\sqrt{1+2\sqrt{2}}\int \frac{1}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\sqrt{1+2\sqrt{2}}\int \frac{1}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\frac{1}{2}\sqrt{1+2\sqrt{2}}\int \frac{1}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{1+2\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{1+2\sqrt{2}}\right)^2-2\sqrt{2}+1}d\left(2x-\sqrt{1+2\sqrt{2}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{1+2\sqrt{2}}\int \frac{1}{-\left(2x+\sqrt{1+2\sqrt{2}}\right)^2-2\sqrt{2}+1}d\left(2x+\sqrt{1+2\sqrt{2}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}}\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}}\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}}\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}}\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

3.45. $\int \frac{1}{2-x^2+x^4}\,dx$

input
```
Int[(2 - x^2 + x^4)^(-1),x]
```

output
```
(Sqrt[(1 + 2*Sqrt[2])/(-1 + 2*Sqrt[2])]*ArcTan[(-Sqrt[1 + 2*Sqrt[2]] + 2*x
)/Sqrt[-1 + 2*Sqrt[2]]] - Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[2]]*x + x^2]/2)/(2
*Sqrt[2*(1 + 2*Sqrt[2])]) + (Sqrt[(1 + 2*Sqrt[2])/(-1 + 2*Sqrt[2])]*ArcTan
[(Sqrt[1 + 2*Sqrt[2]] + 2*x)/Sqrt[-1 + 2*Sqrt[2]]] + Log[Sqrt[2] + Sqrt[1
+ 2*Sqrt[2]]*x + x^2]/2)/(2*Sqrt[2*(1 + 2*Sqrt[2])])
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

3.45.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-\_Z^2+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$
default	$\frac{\left(\sqrt{1+2\sqrt{2}}\,\sqrt{2}-4\sqrt{1+2\sqrt{2}}\right)\ln\left(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}}\right)}{56}+\frac{\left(7\sqrt{2}+\frac{\left(\sqrt{1+2\sqrt{2}}\,\sqrt{2}-4\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}}{2}\right)\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{14\sqrt{-1+2\sqrt{2}}}+$

input
```
int(1/(x^4-x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+2))
```

3.45.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-x^2+x^4}\,dx = \frac{1}{28}\sqrt{7}\sqrt{i\sqrt{7}-1}\log\left(\left(\sqrt{7}-i\right)\sqrt{i\sqrt{7}-1}+4x\right) - \frac{1}{28}\sqrt{7}\sqrt{i\sqrt{7}-1}\log\left(-\left(\sqrt{7}-i\right)\sqrt{i\sqrt{7}-1}+4x\right) + \frac{1}{28}\sqrt{7}\sqrt{-i\sqrt{7}-1}\log\left(\left(\sqrt{7}+i\right)\sqrt{-i\sqrt{7}-1}+4x\right) - \frac{1}{28}\sqrt{7}\sqrt{-i\sqrt{7}-1}\log\left(-\left(\sqrt{7}+i\right)\sqrt{-i\sqrt{7}-1}+4x\right)$$

input
```
integrate(1/(x^4-x^2+2),x, algorithm="fricas")
```

output
```
1/28*sqrt(7)*sqrt(I*sqrt(7) - 1)*log((sqrt(7) - I)*sqrt(I*sqrt(7) - 1) + 4
*x) - 1/28*sqrt(7)*sqrt(I*sqrt(7) - 1)*log(-(sqrt(7) - I)*sqrt(I*sqrt(7) -
 1) + 4*x) + 1/28*sqrt(7)*sqrt(-I*sqrt(7) - 1)*log((sqrt(7) + I)*sqrt(-I*s
qrt(7) - 1) + 4*x) - 1/28*sqrt(7)*sqrt(-I*sqrt(7) - 1)*log(-(sqrt(7) + I)*
sqrt(-I*sqrt(7) - 1) + 4*x)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.12

$$\int \frac{1}{2-x^2+x^4}\,dx = \text{RootSum}\left(1568t^4+28t^2+1,\left(t\mapsto t\log\left(-112t^3+6t+x\right)\right)\right)$$

input
```
integrate(1/(x**4-x**2+2),x)
```

output
```
RootSum(1568*_t**4 + 28*_t**2 + 1, Lambda(_t, _t*log(-112*_t**3 + 6*_t + x
)))
```

3.45.7 Maxima [F]

$$\int \frac{1}{2-x^2+x^4}\,dx = \int \frac{1}{x^4-x^2+2}\,dx$$

input
```
integrate(1/(x^4-x^2+2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 - x^2 + 2), x)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{1}{2-x^2+x^4}\,dx &= \frac{1}{112}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}+2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}+4}+2x\right)}{4\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}}\right)\\ &+\frac{1}{112}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}+2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}+4}-2x\right)}{4\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}}\right)\\ &+\frac{1}{224}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}-2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}\right)\log\left(2^{\frac{1}{4}}\sqrt{\frac{1}{2}}x\sqrt{\sqrt{2}+4}+x^2+\sqrt{2}\right)\\ &-\frac{1}{224}\sqrt{7}\left(\sqrt{7}2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}-2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}\right)\log\left(-2^{\frac{1}{4}}\sqrt{\frac{1}{2}}x\sqrt{\sqrt{2}+4}+x^2+\sqrt{2}\right)\end{aligned}$$

input
```
integrate(1/(x^4-x^2+2),x, algorithm="giac")
```

output
```
1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(
2) + 8))*arctan(1/4*2^(3/4)*(2^(1/4)*sqrt(1/2)*sqrt(sqrt(2) + 4) + 2*x)/sq
rt(-1/8*sqrt(2) + 1/2)) + 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(-2*sqrt(2) +
 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*arctan(-1/4*2^(3/4)*(2^(1/4)*sqrt(1/2)*
sqrt(sqrt(2) + 4) - 2*x)/sqrt(-1/8*sqrt(2) + 1/2)) + 1/224*sqrt(7)*(sqrt(7
)*2^(1/4)*sqrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(2) + 8))*log(2^(1/4)*
sqrt(1/2)*x*sqrt(sqrt(2) + 4) + x^2 + sqrt(2)) - 1/224*sqrt(7)*(sqrt(7)*2^
(1/4)*sqrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(2) + 8))*log(-2^(1/4)*sqr
t(1/2)*x*sqrt(sqrt(2) + 4) + x^2 + sqrt(2))
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.67

$$\int \frac{1}{2-x^2+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x\,\sqrt{-7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{4\,\left(\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}+\frac{\sqrt{7}\,x\,\sqrt{-7-\sqrt{7}\,7\mathrm{i}}}{28\,\left(\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}\right)\,\sqrt{-7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{14} + \frac{\sqrt{7}\operatorname{atan}\left(\frac{x\,\sqrt{-1+\sqrt{7}\,1\mathrm{i}}}{4\,\left(-\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}-\frac{\sqrt{7}\,x\,\sqrt{-1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{4\,\left(-\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}\right)\,\sqrt{-1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{14}$$

input
```
int(1/(x^4 - x^2 + 2),x)
```

output
```
(atan((x*(- 7^(1/2)*7i - 7)^(1/2)*1i)/(4*((7^(1/2)*1i)/2 + 1/2)) + (7^(1/2
)*x*(- 7^(1/2)*7i - 7)^(1/2))/(28*((7^(1/2)*1i)/2 + 1/2)))*(- 7^(1/2)*7i -
 7)^(1/2)*1i)/14 + (7^(1/2)*atan((x*(7^(1/2)*1i - 1)^(1/2))/(4*((7^(1/2)*1
i)/2 - 1/2)) - (7^(1/2)*x*(7^(1/2)*1i - 1)^(1/2)*1i)/(4*((7^(1/2)*1i)/2 -
1/2)))*(7^(1/2)*1i - 1)^(1/2)*1i)/14
```

3.45.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.20

$$\int \frac{1}{2-x^2+x^4}\,dx = -\frac{\sqrt{2\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{2\sqrt{2}+1}-2x}{\sqrt{2\sqrt{2}-1}}\right)}{28} - \frac{\sqrt{2\sqrt{2}-1}\,atan\left(\frac{\sqrt{2\sqrt{2}+1}-2x}{\sqrt{2\sqrt{2}-1}}\right)}{7} + \frac{\sqrt{2\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{2\sqrt{2}+1}+2x}{\sqrt{2\sqrt{2}-1}}\right)}{28} + \frac{\sqrt{2\sqrt{2}-1}\,atan\left(\frac{\sqrt{2\sqrt{2}+1}+2x}{\sqrt{2\sqrt{2}-1}}\right)}{7} + \frac{\sqrt{2\sqrt{2}+1}\,\sqrt{2}\log\left(-\sqrt{2\sqrt{2}+1}\,x+\sqrt{2}+x^2\right)}{56} - \frac{\sqrt{2\sqrt{2}+1}\,\sqrt{2}\log\left(\sqrt{2\sqrt{2}+1}\,x+\sqrt{2}+x^2\right)}{56} - \frac{\sqrt{2\sqrt{2}+1}\log\left(-\sqrt{2\sqrt{2}+1}\,x+\sqrt{2}+x^2\right)}{14} + \frac{\sqrt{2\sqrt{2}+1}\log\left(\sqrt{2\sqrt{2}+1}\,x+\sqrt{2}+x^2\right)}{14}$$

input
```
int(1/(x**4 - x**2 + 2),x)
```

output
```
( - 2*sqrt(2*sqrt(2) - 1)*sqrt(2)*atan((sqrt(2*sqrt(2) + 1) - 2*x)/sqrt(2*
sqrt(2) - 1)) - 8*sqrt(2*sqrt(2) - 1)*atan((sqrt(2*sqrt(2) + 1) - 2*x)/sqr
t(2*sqrt(2) - 1)) + 2*sqrt(2*sqrt(2) - 1)*sqrt(2)*atan((sqrt(2*sqrt(2) + 1
) + 2*x)/sqrt(2*sqrt(2) - 1)) + 8*sqrt(2*sqrt(2) - 1)*atan((sqrt(2*sqrt(2)
 + 1) + 2*x)/sqrt(2*sqrt(2) - 1)) + sqrt(2*sqrt(2) + 1)*sqrt(2)*log( - sqr
t(2*sqrt(2) + 1)*x + sqrt(2) + x**2) - sqrt(2*sqrt(2) + 1)*sqrt(2)*log(sqr
t(2*sqrt(2) + 1)*x + sqrt(2) + x**2) - 4*sqrt(2*sqrt(2) + 1)*log( - sqrt(2
*sqrt(2) + 1)*x + sqrt(2) + x**2) + 4*sqrt(2*sqrt(2) + 1)*log(sqrt(2*sqrt(
2) + 1)*x + sqrt(2) + x**2))/56
```

3.46 $\int \frac{1}{-1+x^6}\,dx$

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3.46.1 Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \frac{1}{-1+x^6}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{3} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{1}{12}\log\left(1+x+x^2\right)$$

output
```
-1/3*arctanh(x)+1/12*ln(x^2-x+1)-1/12*ln(x^2+x+1)+1/6*arctan(1/3*(1-2*x)*3
^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{1}{-1+x^6}\,dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 2\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) - \log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(-1 + x^6)^(-1),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3
]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

3.46.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-1}\,dx$$

$$\Big\downarrow \ 754$$

$$-\frac{1}{3}\int \frac{1}{1-x^2}dx-\frac{1}{3}\int \frac{2-x}{2\left(x^2-x+1\right)}dx-\frac{1}{3}\int \frac{x+2}{2\left(x^2+x+1\right)}dx$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{3}\int \frac{1}{1-x^2}dx-\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx$$

$$\Big\downarrow \ 219$$

$$-\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{6}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+$$
$$\frac{1}{6}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\operatorname{arctanh}(x)}{3}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{6}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{\operatorname{arctanh}(x)}{3}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{6}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)-\frac{\operatorname{arctanh}(x)}{3}$$

input
```
Int[(-1 + x^6)^(-1),x]
```

output
```
-1/3*ArcTanh[x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 - x + x^2
]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 + x + x^2]/2)/6
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.46.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result
default	$\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}$
risch	$\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{6}+\frac{\ln(-1+x)}{6}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x\cdots}{2+(x\cdots}\right.\cdots\right.}{6(x^6)^{\frac{1}{6}}}$

input
```
int(1/(x^6-1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/12
*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(1+x)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\,\log\left(x^2+x+1\right)+\frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)+\frac{1}{6}\,\log\left(x-1\right)$$

input
```
integrate(1/(x^6-1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x +
 1) + 1/6*log(x - 1)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{1}{-1+x^6}\,dx = \frac{\log\left(x-1\right)}{6} - \frac{\log\left(x+1\right)}{6} + \frac{\log\left(x^2-x+1\right)}{12} - \frac{\log\left(x^2+x+1\right)}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**6-1),x)
```

output
```
log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12
- sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/6
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right) + \frac{1}{6}\log\left(x-1\right)$$

input
```
integrate(1/(x^6-1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x +
 1) + 1/6*log(x - 1)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(|x+1|\right) + \frac{1}{6}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^6-1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs
(x + 1)) + 1/6*log(abs(x - 1))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \frac{1}{-1+x^6}\,dx = -\frac{\operatorname{atanh}(x)}{3} - \operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}} + \frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}} - \frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right)$$

input
```
int(1/(x^6 - 1),x)
```

output
```
- atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1)
)*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)
*1i - 1))*(3^(1/2)/6 - 1i/6)
```

3.46.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{1}{-1+x^6}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x^2+x+1)}{12} + \frac{\log(x-1)}{6} - \frac{\log(x+1)}{6}$$

input
```
int(1/(x**6 - 1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ log(x**2 - x + 1) - log(x**2 + x + 1) + 2*log(x - 1) - 2*log(x + 1))/12
```

3.47 $\int \frac{1}{-2+x^6}\,dx$

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3.47.1 Optimal result

Integrand size = 7, antiderivative size = 138

$$\int \frac{1}{-2+x^6}\,dx=\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}}-\frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}}$$

output
```
-1/6*arctanh(1/2*x*2^(5/6))*2^(1/6)+1/24*ln(2^(1/3)-2^(1/6)*x+x^2)*2^(1/6)
-1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/
6)*x*3^(1/2))*2^(1/6)*3^(1/2)-1/12*arctan(1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2
))*2^(1/6)*3^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \frac{1}{-2+x^6}\,dx= -\frac{2\sqrt{3}\arctan\left(\frac{-1+2^{5/6}x}{\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right)-2\log\left(2-2^{5/6}x\right)+2\log\left(2+2^{5/6}x\right)-\log\left(2-2^{5/6}\right.}{12\ 2^{5/6}}$$

input
```
Integrate[(-2 + x^6)^(-1),x]
```

output
```
-1/12*(2*Sqrt[3]*ArcTan[(-1 + 2^(5/6)*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 +
2^(5/6)*x)/Sqrt[3]] - 2*Log[2 - 2^(5/6)*x] + 2*Log[2 + 2^(5/6)*x] - Log[2
- 2^(5/6)*x + 2^(2/3)*x^2] + Log[2 + 2^(5/6)*x + 2^(2/3)*x^2])/2^(5/6)
```

3.47.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {754, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-2}dx$$

$$\Big\downarrow \text{754}$$

$$-\frac{\int \frac{1}{\sqrt[3]{2}-x^2}dx}{3\ 2^{2/3}}-\frac{\int \frac{2\sqrt[6]{2}-x}{2\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}dx}{3\ 2^{5/6}}-\frac{\int \frac{x+2\sqrt[6]{2}}{2\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}dx}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\sqrt[3]{2}-x^2}dx}{3\ 2^{2/3}}-\frac{\int \frac{2\sqrt[6]{2}-x}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\int \frac{x+2\sqrt[6]{2}}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\int \frac{2\sqrt[6]{2}-x}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\int \frac{x+2\sqrt[6]{2}}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{3\int \frac{1}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}-\frac{1}{2}\int -\frac{\sqrt[6]{2}\left(1-2^{5/6}x\right)}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\frac{3\int \frac{1}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{1}{2}\int \frac{\sqrt[6]{2}\left(2^{5/6}x+1\right)}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{3\int \frac{1}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{1}{2}\int \frac{\sqrt[6]{2}\left(1-2^{5/6}x\right)}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\frac{3\int \frac{1}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{1}{2}\int \frac{\sqrt[6]{2}\left(2^{5/6}x+1\right)}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{1}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{\int \frac{1-2^{5/6}x}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}}{6\ 2^{5/6}}-\frac{\frac{3\int \frac{1}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{\int \frac{2^{5/6}x+1}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}}{6\ 2^{5/6}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{\int \frac{1-2^{5/6}x}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+3\int \frac{1}{-\left(1-2^{5/6}x\right)^2-3}d\left(1-2^{5/6}x\right)}{6\ 2^{5/6}}-\frac{\frac{\int \frac{2^{5/6}x+1}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}-3\int \frac{1}{-\left(2^{5/6}x+1\right)^2-3}d\left(2^{5/6}x+1\right)}{6\ 2^{5/6}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{\int \frac{1-2^{5/6}x}{x^2-\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}-\sqrt{3}\arctan\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right)}{6\ 2^{5/6}}-\frac{\frac{\int \frac{2^{5/6}x+1}{x^2+\sqrt[6]{2}x+\sqrt[3]{2}}dx}{2^{5/6}}+\sqrt{3}\arctan\left(\frac{2^{5/6}x+1}{\sqrt{3}}\right)}{6\ 2^{5/6}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{-\sqrt{3}\arctan\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}{6\ 2^{5/6}}-\frac{\sqrt{3}\arctan\left(\frac{2^{5/6}x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}{6\ 2^{5/6}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

input
```
Int[(-2 + x^6)^(-1),x]
```

3.47. $\int \frac{1}{-2+x^6}\,dx$

output
```
-1/3*ArcTanh[x/2^(1/6)]/2^(5/6) - (-(Sqrt[3]*ArcTan[(1 - 2^(5/6)*x)/Sqrt[3
]]) - Log[2^(1/3) - 2^(1/6)*x + x^2]/2)/(6*2^(5/6)) - (Sqrt[3]*ArcTan[(1 +
 2^(5/6)*x)/Sqrt[3]] + Log[2^(1/3) + 2^(1/6)*x + x^2]/2)/(6*2^(5/6))
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.47.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}}\right)}{6}$
default	$\frac{\ln\left(2^{\frac{1}{3}}-2^{\frac{1}{6}}x+x^{2}\right)2^{\frac{1}{6}}}{24}-\frac{\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2^{\frac{1}{6}}\sqrt{3}}{12}+\frac{2^{\frac{1}{6}}\ln\left(x-2^{\frac{1}{6}}\right)}{12}-\frac{\ln\left(2^{\frac{1}{3}}+2^{\frac{1}{6}}x+x^{2}\right)2^{\frac{1}{6}}}{24}-\frac{\arctan\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2}{12}$
meijerg	$\frac{2^{\frac{1}{6}}x\left(\ln\left(1-\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}\right)-\ln\left(1+\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}\right)+\frac{\ln\left(1-\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}\left(x^{6}\right)^{\frac{1}{3}}}{2}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{4-2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}}{}\right)}{2}\right.}{12\left(x^{6}\right)^{\frac{1}{6}}}$

input
```
int(1/(x^6-2),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(1/_R^5*ln(x-_R),_R=RootOf(_Z^6-2))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{1}{-2+x^6}\,dx = -\frac{1}{384}\cdot 32^{\frac{5}{6}}\left(\sqrt{-3}+1\right)\log\left(32^{\frac{5}{6}}\left(\sqrt{-3}+1\right)+32\,x\right)+\frac{1}{384}\cdot 32^{\frac{5}{6}}\left(\sqrt{-3}+1\right)\log\left(-32^{\frac{5}{6}}\left(\sqrt{-3}+1\right)+32\,x\right)-\frac{1}{384}\cdot 32^{\frac{5}{6}}\left(\sqrt{-3}-1\right)\log\left(32^{\frac{5}{6}}\left(\sqrt{-3}-1\right)+32\,x\right)+\frac{1}{384}\cdot 32^{\frac{5}{6}}\left(\sqrt{-3}-1\right)\log\left(-32^{\frac{5}{6}}\left(\sqrt{-3}-1\right)+32\,x\right)-\frac{1}{192}\cdot 32^{\frac{5}{6}}\log\left(16\,x+32^{\frac{5}{6}}\right)+\frac{1}{192}\cdot 32^{\frac{5}{6}}\log\left(16\,x-32^{\frac{5}{6}}\right)$$

input
```
integrate(1/(x^6-2),x, algorithm="fricas")
```

output
```
-1/384*32^(5/6)*(sqrt(-3) + 1)*log(32^(5/6)*(sqrt(-3) + 1) + 32*x) + 1/384
*32^(5/6)*(sqrt(-3) + 1)*log(-32^(5/6)*(sqrt(-3) + 1) + 32*x) - 1/384*32^(
5/6)*(sqrt(-3) - 1)*log(32^(5/6)*(sqrt(-3) - 1) + 32*x) + 1/384*32^(5/6)*(
sqrt(-3) - 1)*log(-32^(5/6)*(sqrt(-3) - 1) + 32*x) - 1/192*32^(5/6)*log(16
*x + 32^(5/6)) + 1/192*32^(5/6)*log(16*x - 32^(5/6))
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.10

$$\int \frac{1}{-2+x^6}\,dx = \operatorname{RootSum}\left(1492992t^6-1,\left(t\mapsto t\log\left(-12t+x\right)\right)\right)$$

input
```
integrate(1/(x**6-2),x)
```

output
```
RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \frac{1}{-2+x^6}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right) - \frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right) + \frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2-2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(x+2^{\frac{1}{6}}\right) + \frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(x-2^{\frac{1}{6}}\right)$$

input
```
integrate(1/(x^6-2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*s
qrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*
log(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3
)) - 1/12*2^(1/6)*log(x + 2^(1/6)) + 1/12*2^(1/6)*log(x - 2^(1/6))
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{1}{-2+x^6}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right) - \frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right) + \frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2-2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(\left|x+2^{\frac{1}{6}}\right|\right) + \frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(\left|x-2^{\frac{1}{6}}\right|\right)$$

input
```
integrate(1/(x^6-2),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*s
qrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*
log(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3
)) - 1/12*2^(1/6)*log(abs(x + 2^(1/6))) + 1/12*2^(1/6)*log(abs(x - 2^(1/6)
))
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{1}{-2+x^6}\,dx = -\frac{2^{1/6}\,\mathrm{atanh}\left(\frac{2^{5/6}\,x}{2}\right)}{6} + \frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x\,1\mathrm{i}}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)} - \frac{2^{1/6}\sqrt{3}\,x}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{12} + \frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x\,1\mathrm{i}}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)} + \frac{2^{1/6}\sqrt{3}\,x}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{12}$$

input
```
int(1/(x^6 - 2),x)
```

output
```
(2^(1/6)*atan((2^(1/6)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)) - (2
^(1/6)*3^(1/2)*x)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)))*(3^(1/2)*1i +
1)*1i)/12 - (2^(1/6)*atanh((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x*1i
)/(2*((2^(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x)/(2*((2^(1
/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2)*1i - 1)*1i)/12
```

3.47.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69

$$\int \frac{1}{-2+x^6}\,dx = \frac{2^{\frac{1}{6}}\left(2\sqrt{3}\,atan\left(\frac{\left(2^{\frac{1}{6}}-2x\right)2^{\frac{5}{6}}}{2\sqrt{3}}\right) - 2\sqrt{3}\,atan\left(\frac{\left(2^{\frac{1}{6}}+2x\right)2^{\frac{5}{6}}}{2\sqrt{3}}\right) - 2\log\left(2^{\frac{1}{6}}+x\right) + 2\log\left(-2^{\frac{1}{6}}+x\right) + \log\left(-2^{\frac{1}{6}}x\right.\right.}{24}$$

input
```
int(1/(x**6 - 2),x)
```

output
```
(2**(1/6)*(2*sqrt(3)*atan((2**(1/6) - 2*x)/(2**(1/6)*sqrt(3))) - 2*sqrt(3)
*atan((2**(1/6) + 2*x)/(2**(1/6)*sqrt(3))) - 2*log(2**(1/6) + x) + 2*log(
- 2**(1/6) + x) + log( - 2**(1/6)*x + 2**(1/3) + x**2) - log(2**(1/6)*x +
2**(1/3) + x**2)))/24
```

3.48 $\int \frac{1}{2+x^6}\,dx$

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3.48.1 Optimal result

Integrand size = 7, antiderivative size = 138

$$\int \frac{1}{2+x^6}\,dx = \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} - \frac{\arctan\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}} + \frac{\arctan\left(\sqrt{3}+2^{5/6}x\right)}{6\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}}$$

output
```
1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1
/12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2
))*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{1}{2+x^6}\,dx = \frac{4\arctan\left(\frac{x}{\sqrt[6]{2}}\right) - 2\arctan\left(\sqrt{3}-2^{5/6}x\right) + 2\arctan\left(\sqrt{3}+2^{5/6}x\right) - \sqrt{3}\log\left(2-2^{5/6}\sqrt{3}x+2^{2/3}x^2\right) + \cdots}{12\ 2^{5/6}}$$

input
```
Integrate[(2 + x^6)^(-1),x]
```

output
```
(4*ArcTan[x/2^(1/6)] - 2*ArcTan[Sqrt[3] - 2^(5/6)*x] + 2*ArcTan[Sqrt[3] +
2^(5/6)*x] - Sqrt[3]*Log[2 - 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2] + Sqrt[3]*Lo
g[2 + 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2])/(12*2^(5/6))
```

3.48.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {753, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+2}dx$$

$$\Big\downarrow \text{753}$$

$$\frac{\int \frac{1}{x^2+\sqrt[3]{2}}dx}{3\ 2^{2/3}}+\frac{\int \frac{2\sqrt[6]{2}-\sqrt{3}x}{2\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}dx}{3\ 2^{5/6}}+\frac{\int \frac{\sqrt{3}x+2\sqrt[6]{2}}{2\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}dx}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2+\sqrt[3]{2}}dx}{3\ 2^{2/3}}+\frac{\int \frac{2\sqrt[6]{2}-\sqrt{3}x}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\int \frac{\sqrt{3}x+2\sqrt[6]{2}}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{2\sqrt[6]{2}-\sqrt{3}x}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\int \frac{\sqrt{3}x+2\sqrt[6]{2}}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\int \frac{1}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt[6]{2}\left(\sqrt{3}-2^{5/6}x\right)}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\frac{\int \frac{1}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt[6]{2}\left(2^{5/6}x+\sqrt{3}\right)}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{1}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt[6]{2}\left(\sqrt{3}-2^{5/6}x\right)}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\frac{\int \frac{1}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt[6]{2}\left(2^{5/6}x+\sqrt{3}\right)}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{1}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{\sqrt{3}\int \frac{\sqrt{3}-2^{5/6}x}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}}{6\ 2^{5/6}}+\frac{\frac{\int \frac{1}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{\sqrt{3}\int \frac{2^{5/6}x+\sqrt{3}}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\sqrt{3}\int \frac{\sqrt{3}-2^{5/6}x}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\frac{\int \frac{1}{-\left(1-\frac{2^{5/6}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2^{5/6}x}{\sqrt{3}}\right)}{\sqrt{3}}}{6\ 2^{5/6}}+\frac{\frac{\sqrt{3}\int \frac{2^{5/6}x+\sqrt{3}}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}-\frac{\int \frac{1}{-\left(\frac{2^{5/6}x}{\sqrt{3}}+1\right)^2-\frac{1}{3}}d\left(\frac{2^{5/6}x}{\sqrt{3}}+1\right)}{\sqrt{3}}}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\sqrt{3}\int \frac{\sqrt{3}-2^{5/6}x}{x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}-\arctan\left(\sqrt{3}\left(1-\frac{2^{5/6}x}{\sqrt{3}}\right)\right)}{6\ 2^{5/6}}+\frac{\frac{\sqrt{3}\int \frac{2^{5/6}x+\sqrt{3}}{x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}}dx}{2^{5/6}}+\arctan\left(\sqrt{3}\left(\frac{2^{5/6}x}{\sqrt{3}}+1\right)\right)}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{-\arctan\left(\sqrt{3}\left(1-\frac{2^{5/6}x}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{6\ 2^{5/6}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2^{5/6}x}{\sqrt{3}}+1\right)\right)+\frac{1}{2}\sqrt{3}\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{6\ 2^{5/6}}+\frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

3.48. $\int \frac{1}{2+x^6}\,dx$

input
```
Int[(2 + x^6)^(-1),x]
```

output
```
ArcTan[x/2^(1/6)]/(3*2^(5/6)) + (-ArcTan[Sqrt[3]*(1 - (2^(5/6)*x)/Sqrt[3])
] - (Sqrt[3]*Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x + x^2])/2)/(6*2^(5/6)) + (Arc
Tan[Sqrt[3]*(1 + (2^(5/6)*x)/Sqrt[3])] + (Sqrt[3]*Log[2^(1/3) + 2^(1/6)*Sq
rt[3]*x + x^2])/2)/(6*2^(5/6))
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.48.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}}\right)}{6}$
default	$\frac{\arctan\left(\frac{x 2^{\frac{5}{6}}}{2}\right) 2^{\frac{1}{6}}}{6}+\frac{\arctan\left(x 2^{\frac{5}{6}}-\sqrt{3}\right) 2^{\frac{1}{6}}}{12}+\frac{\arctan\left(x 2^{\frac{5}{6}}+\sqrt{3}\right) 2^{\frac{1}{6}}}{12}-\frac{\ln\left(2^{\frac{1}{3}}+x^{2}-2^{\frac{1}{6}} x \sqrt{3}\right) 2^{\frac{1}{6}} \sqrt{3}}{24}+\frac{\ln\left(2^{\frac{1}{3}}+x^{2}+2^{\frac{1}{6}} x \sqrt{3}\right) 2}{24}$
meijerg	$\frac{2^{\frac{1}{6}}\left(-\frac{x\sqrt{3}\,\ln\left(1-\frac{\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}\left(x^{6}\right)^{\frac{1}{3}}}{2}\right)}{2\left(x^{6}\right)^{\frac{1}{6}}}+\frac{x\arctan\left(\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{4-\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}\right)}{\left(x^{6}\right)^{\frac{1}{6}}}+\frac{2x\arctan\left(\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}\right)}{\left(x^{6}\right)^{\frac{1}{6}}}+\frac{x\sqrt{3}\,\ln\left(1+\frac{\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}\left(x^{6}\right.}{2}\right.}{2\left(x^{6}\right)^{\frac{1}{6}}}\right.}{12}$

input
```
int(1/(x^6+2),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(1/_R^5*ln(x-_R),_R=RootOf(_Z^6+2))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.19

$$
\begin{aligned}
\int \frac{1}{2+x^6}\,dx = {} & \frac{1}{384}\cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)\log\left(32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)+32\,x\right) \\
& - \frac{1}{384}\cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)\log\left(-32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)+32\,x\right) \\
& + \frac{1}{384}\cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)\log\left(32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)+32\,x\right) \\
& - \frac{1}{384}\cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)\log\left(-32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)+32\,x\right) \\
& + \frac{1}{192}\cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\log\left(32^{\frac{5}{6}}(-1)^{\frac{1}{6}}+16\,x\right) \\
& - \frac{1}{192}\cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}\log\left(-32^{\frac{5}{6}}(-1)^{\frac{1}{6}}+16\,x\right)
\end{aligned}
$$

input
```
integrate(1/(x^6+2),x, algorithm="fricas")
```

output
```
1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(32^(5/6)*(-1)^(1/6)*(sqrt(-3)
 + 1) + 32*x) - 1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-32^(5/6)*(-1
)^(1/6)*(sqrt(-3) + 1) + 32*x) + 1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*
log(32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1) + 32*x) - 1/384*32^(5/6)*(-1)^(1/6)
*(sqrt(-3) - 1)*log(-32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1) + 32*x) + 1/192*32
^(5/6)*(-1)^(1/6)*log(32^(5/6)*(-1)^(1/6) + 16*x) - 1/192*32^(5/6)*(-1)^(1
/6)*log(-32^(5/6)*(-1)^(1/6) + 16*x)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.10

$$
\int \frac{1}{2+x^6}\,dx = \operatorname{RootSum}\left(1492992t^6+1,\left(t\mapsto t\log\left(12t+x\right)\right)\right)
$$

input
```
integrate(1/(x**6+2),x)
```

output
```
RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+x^6}\,dx = \frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2+\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2-\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x+\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{12} \cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x-\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{6}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}x\right)$$

input
```
integrate(1/(x^6+2),x, algorithm="maxima")
```

output
```
1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)
*2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*
2^(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x -
sqrt(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+x^6}\,dx = \frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2+\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2-\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x+\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{12} \cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x-\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{6}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}x\right)$$

input
```
integrate(1/(x^6+2),x, algorithm="giac")
```

output
```
1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)
*2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*
2^(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x -
sqrt(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{1}{2+x^6}\,dx = \frac{2^{1/6}\operatorname{atan}\left(\frac{2^{5/6}x}{2}\right)}{6} + \frac{2^{1/6}\operatorname{atan}\left(\frac{2^{1/6}x}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}+\frac{2^{1/6}\sqrt{3}\,x\,1\mathrm{i}}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(\sqrt{3}-\mathrm{i}\right)1\mathrm{i}}{12} + \frac{2^{1/6}\operatorname{atan}\left(\frac{2^{1/6}x}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}-\frac{2^{1/6}\sqrt{3}\,x\,1\mathrm{i}}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(\sqrt{3}+1\mathrm{i}\right)1\mathrm{i}}{12}$$

input
```
int(1/(x^6 + 2),x)
```

output
```
(2^(1/6)*atan((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^(1/3)*3
^(1/2)*1i)/2 - 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(1/2)*1
i)/2 - 2^(1/3)/2)))*(3^(1/2) - 1i)*1i)/12 + (2^(1/6)*atan((2^(1/6)*x)/(2*(
(2^(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) - (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)
*3^(1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2) + 1i)*1i)/12
```

3.48.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{1}{2+x^6}\,dx = \frac{2^{\frac{1}{6}}\left(-2atan\left(\frac{\left(2^{\frac{1}{6}}\sqrt{3}-2x\right)2^{\frac{5}{6}}}{2}\right)+2atan\left(\frac{\left(2^{\frac{1}{6}}\sqrt{3}+2x\right)2^{\frac{5}{6}}}{2}\right)+4atan\left(\frac{x2^{\frac{5}{6}}}{2}\right)-\sqrt{3}\log\left(-2^{\frac{1}{6}}\sqrt{3}\,x+2^{\frac{1}{3}}+x^2\right)+\right.}{24}$$

input
```
int(1/(x**6 + 2),x)
```

output
```
(2**(1/6)*( - 2*atan((2**(1/6)*sqrt(3) - 2*x)/2**(1/6)) + 2*atan((2**(1/6)
*sqrt(3) + 2*x)/2**(1/6)) + 4*atan(x/2**(1/6)) - sqrt(3)*log( - 2**(1/6)*s
qrt(3)*x + 2**(1/3) + x**2) + sqrt(3)*log(2**(1/6)*sqrt(3)*x + 2**(1/3) +
x**2)))/24
```

3.49 $\int \frac{1}{1+x^8} dx$

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3.49.1 Optimal result

Integrand size = 7, antiderivative size = 339

$$\begin{aligned}\int \frac{1}{1+x^8} dx = &-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}\\
&+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)\\
&+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)\\
&-\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)\\
&+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)\end{aligned}$$

output
```
-1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2-2
^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^
(1/2))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/4*arct
an((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arc
tan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.62

$$\begin{aligned}\int \frac{1}{1+x^8}\,dx = &\frac{1}{4}\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x-\sin\left(\frac{\pi}{8}\right)\right)\right)\cos\left(\frac{\pi}{8}\right)\\ &+\frac{1}{4}\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\cos\left(\frac{\pi}{8}\right)\\ &-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)\\ &+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)\\ &+\frac{1}{4}\arctan\left(\left(x-\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)\\ &+\frac{1}{4}\arctan\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)\\ &-\frac{1}{8}\log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)\\ &+\frac{1}{8}\log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)\end{aligned}$$

input
```
Integrate[(1 + x^8)^(-1),x]
```

output
```
(ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + S
in[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Co
s[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8
]]*Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1
 + x^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[P
i/8])/8
```

3.49.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {757, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+1}\,dx$$

$$\Big\downarrow \; 757$$

$$\frac{\int \frac{\sqrt{2}-x^2}{x^4-\sqrt{2}x^2+1}dx}{2\sqrt{2}}+\frac{\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}dx}{2\sqrt{2}}$$

$$\Big\downarrow \; 1483$$

$$\frac{\frac{\int \frac{\left(1-\sqrt{2}\right)x+\sqrt{2\left(2-\sqrt{2}\right)}}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(1-\sqrt{2}\right)x}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\left(1+\sqrt{2}\right)x+\sqrt{2\left(2+\sqrt{2}\right)}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\left(1-\sqrt{2}\right)\int -\frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{2}}\right)^2-\sqrt{2}-2}d\left(2x-\sqrt{2-\sqrt{2}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{2}}\int \frac{\;}{-\left(2x+\sqrt{\;\;}\right.}}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\sqrt{2-\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{2}}\right)^2+\sqrt{2}-2}d\left(2x-\sqrt{2+\sqrt{2}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx-\sqrt{2-\sqrt{2}}\int \frac{1}{-\left(2x+\sqrt{2+\right.}}}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

$$\Big\downarrow \; 217$$

3.49. $\int \frac{1}{1+x^8}\,dx$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int\frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int\frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

input
```
Int[(1 + x^8)^(-1),x]
```

output
```
((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Lo
g[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]) + (ArcTan[(Sqrt
[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 + Sqrt[2 -
Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]))/(2*Sqrt[2]) + ((ArcTan[(-Sqrt
[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - Sqrt[2 +
Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[2]]) + (ArcTan[(Sqrt[2 + Sqrt[2]] +
 2*x)/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^
2])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2])
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 757
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Simp[r/(2*Sqrt[2]*a)   Int[(Sqrt[2]*r -
 s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Simp[r/(2*S
qrt[2]*a)   Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^
(n/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.49.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.06

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$
meijerg	$-\frac{x\cos\left(\frac{\pi}{8}\right)\ln\left(1-2\cos\left(\frac{\pi}{8}\right)\left(x^{8}\right)^{\frac{1}{8}}+\left(x^{8}\right)^{\frac{1}{4}}\right)}{8\left(x^{8}\right)^{\frac{1}{8}}}+\frac{x\sin\left(\frac{\pi}{8}\right)\arctan\left(\frac{\sin\left(\frac{\pi}{8}\right)\left(x^{8}\right)^{\frac{1}{8}}}{1-\cos\left(\frac{\pi}{8}\right)\left(x^{8}\right)^{\frac{1}{8}}}\right)}{4\left(x^{8}\right)^{\frac{1}{8}}}-\frac{x\cos\left(\frac{3\pi}{8}\right)\ln\left(1-2\cos\left(\frac{3\pi}{8}\right)\left(x^{8}\right)^{\frac{1}{8}}+\left(x^{8}\right)\right.}{8\left(x^{8}\right)^{\frac{1}{8}}}$

input
```
int(1/(x^8+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(1/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

3.49.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.40

$$\begin{aligned}\int\frac{1}{1+x^{8}}\,dx=&\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(2x+(i+1)\sqrt{2}(-1)^{\frac{1}{8}}\right)\\&-\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(2x-(i-1)\sqrt{2}(-1)^{\frac{1}{8}}\right)\\&+\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(2x+(i-1)\sqrt{2}(-1)^{\frac{1}{8}}\right)\\&-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(2x-(i+1)\sqrt{2}(-1)^{\frac{1}{8}}\right)\\&+\frac{1}{8}(-1)^{\frac{1}{8}}\log\left(x+(-1)^{\frac{1}{8}}\right)+\frac{1}{8}i(-1)^{\frac{1}{8}}\log\left(x+i(-1)^{\frac{1}{8}}\right)\\&-\frac{1}{8}i(-1)^{\frac{1}{8}}\log\left(x-i(-1)^{\frac{1}{8}}\right)-\frac{1}{8}(-1)^{\frac{1}{8}}\log\left(x-(-1)^{\frac{1}{8}}\right)\end{aligned}$$

input
```
integrate(1/(x^8+1),x, algorithm="fricas")
```

output
```
(1/16*I + 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x + (I + 1)*sqrt(2)*(-1)^(1/8)) -
 (1/16*I - 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x - (I - 1)*sqrt(2)*(-1)^(1/8))
+ (1/16*I - 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x + (I - 1)*sqrt(2)*(-1)^(1/8))
 - (1/16*I + 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x - (I + 1)*sqrt(2)*(-1)^(1/8)
) + 1/8*(-1)^(1/8)*log(x + (-1)^(1/8)) + 1/8*I*(-1)^(1/8)*log(x + I*(-1)^(
1/8)) - 1/8*I*(-1)^(1/8)*log(x - I*(-1)^(1/8)) - 1/8*(-1)^(1/8)*log(x - (-
1)^(1/8))
```

3.49.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.04

$$\int \frac{1}{1+x^8}\,dx = \mathrm{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(8t+x\right)\right)\right)$$

input
```
integrate(1/(x**8+1),x)
```

output
```
RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))
```

3.49.7 Maxima [F]

$$\int \frac{1}{1+x^8}\,dx = \int \frac{1}{x^8+1}\,dx$$

input
```
integrate(1/(x^8+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 + 1), x)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+x^8}\,dx = \frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{16}\sqrt{\sqrt{2}+2}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) - \frac{1}{16}\sqrt{\sqrt{2}+2}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right) + \frac{1}{16}\sqrt{-\sqrt{2}+2}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) - \frac{1}{16}\sqrt{-\sqrt{2}+2}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

input
```
integrate(1/(x^8+1),x, algorithm="giac")
```

output
```
1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) -
1/16*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqr
t(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*lo
g(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^8}\,dx = \operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}} - \frac{x\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right) - \operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2}+\sqrt{\sqrt{2}-2}\,\sqrt{\sqrt{2}+2}} + \frac{x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{\sqrt{2}+\sqrt{\sqrt{2}-2}\,\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right) + \operatorname{atan}\left(-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}}{2}+x\sqrt{\sqrt{2}+2}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{16}-\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,2\mathrm{i} - \operatorname{atan}\left(x\sqrt{\sqrt{2}+2}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{2}}{16}-\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,2\mathrm{i}$$

input
```
int(1/(x^8 + 1),x)
```

output
```
atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/
2) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2)
 - 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/
2)*1i)/8) - atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)
*(2^(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2
)^(1/2)*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)
^(1/2)*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1
/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*
2i - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1
/2)*1i)/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

3.49.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.60

$$\int \frac{1}{1+x^8}\,dx = -\frac{\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2x}{\sqrt{\sqrt{2}+2}}\right)}{8} + \frac{\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}+2x}{\sqrt{\sqrt{2}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}-2x}{\sqrt{-\sqrt{2}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}+2x}{\sqrt{-\sqrt{2}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}\,x+x^2+1\right)}{16} + \frac{\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}\,x+x^2+1\right)}{16} - \frac{\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}\,x+x^2+1\right)}{16} + \frac{\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\,x+x^2+1\right)}{16}$$

input
```
int(1/(x**8 + 1),x)
```

output
```
( - 2*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) - 2*x)/sqrt(sqrt(2) + 2
)) + 2*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) + 2*x)/sqrt(sqrt(2) +
2)) - 2*sqrt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) - 2*x)/sqrt( - sqrt(2
) + 2)) + 2*sqrt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) + 2*x)/sqrt( - sq
rt(2) + 2)) - sqrt( - sqrt(2) + 2)*log( - sqrt( - sqrt(2) + 2)*x + x**2 +
1) + sqrt( - sqrt(2) + 2)*log(sqrt( - sqrt(2) + 2)*x + x**2 + 1) - sqrt(sq
rt(2) + 2)*log( - sqrt(sqrt(2) + 2)*x + x**2 + 1) + sqrt(sqrt(2) + 2)*log(
sqrt(sqrt(2) + 2)*x + x**2 + 1))/16
```

3.50 $\int \frac{1}{-1+x^8} dx$

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3.50.1 Optimal result

Integrand size = 7, antiderivative size = 97

$$\int \frac{1}{-1+x^8} dx = -\frac{\arctan(x)}{4} + \frac{\arctan\left(1-\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\operatorname{arctanh}(x)}{4} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}}$$

output
```
-1/4*arctan(x)-1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(
1+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1
/2))*2^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{1}{-1+x^8} dx = \frac{1}{16}\Big(-4\arctan(x) + 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) - 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + 2\log(1-x) - 2\log(1+x) + \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\Big)$$

input
```
Integrate[(-1 + x^8)^(-1),x]
```

output
```
(-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqr
t[2]*x] + 2*Log[1 - x] - 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] -
 Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

3.50.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.714$, Rules used = {758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-1} \, dx$$

$$\Big\downarrow \text{ 758}$$

$$-\frac{1}{2}\int \frac{1}{1-x^4}dx-\frac{1}{2}\int \frac{1}{x^4+1}dx$$

$$\Big\downarrow \text{ 755}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)-\frac{1}{2}\int \frac{1}{1-x^4}dx$$

$$\Big\downarrow \text{ 756}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}\right)$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}\right)$$

3.50. $\int \frac{1}{-1+x^8}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[(-1 + x^8)^(-1),x]
```

output
```
(-1/2*ArcTan[x] - ArcTanh[x]/2)/2 + ((ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcT
an[1 + Sqrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log
[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2)/2
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.50. $\int \frac{1}{-1+x^8} \, dx$

3.50.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\_R\ln\left(x-\_R\right)\right)}{8}-\frac{\arctan(x)}{4}-\frac{\ln(1+x)}{8}+\frac{\ln(-1+x)}{8}$
default	$-\frac{\arctan(x)}{4}-\frac{\operatorname{arctanh}(x)}{4}-\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{16}$
meijerg	$\frac{x\left(\ln\left(1-\left(x^8\right)^{\frac{1}{8}}\right)-\ln\left(1+\left(x^8\right)^{\frac{1}{8}}\right)+\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^8\right)^{\frac{1}{8}}+\left(x^8\right)^{\frac{1}{4}}\right)}{2}-\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^8\right)^{\frac{1}{8}}}{2-\sqrt{2}\left(x^8\right)^{\frac{1}{8}}}\right)-2\arctan\left(\left(x^8\right)^{\frac{1}{8}}\right)-\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(\right.\right.}{}\right.}{8\left(x^8\right)^{\frac{1}{8}}}$

input
```
int(1/(x^8-1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R*ln(x-_R),_R=RootOf(_Z^4+1))-1/4*arctan(x)-1/8*ln(1+x)+1/8*ln(-1
+x)
```

3.50.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{1}{-1+x^8}\,dx = &-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right)\\ &+\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right)\\ &-\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right)\\ &+\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)\\ &-\frac{1}{4}\arctan\left(x\right)-\frac{1}{8}\log\left(x+1\right)+\frac{1}{8}\log\left(x-1\right)\end{aligned}$$

input
```
integrate(1/(x^8-1),x, algorithm="fricas")
```

output
```
-(1/16*I + 1/16)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) + (1/16*I - 1/16)*sqrt
(2)*log(2*x - (I - 1)*sqrt(2)) - (1/16*I - 1/16)*sqrt(2)*log(2*x + (I - 1)
*sqrt(2)) + (1/16*I + 1/16)*sqrt(2)*log(2*x - (I + 1)*sqrt(2)) - 1/4*arcta
n(x) - 1/8*log(x + 1) + 1/8*log(x - 1)
```

3.50.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 127.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{-1+x^8}\,dx = \frac{\log(x-1)}{8} - \frac{\log(x+1)}{8} + \frac{i\log(x-i)}{8} - \frac{i\log(x+i)}{8} + \operatorname{RootSum}\left(4096t^4+1, (t \mapsto t\log(-8t+x))\right)$$

input
```
integrate(1/(x**8-1),x)
```

output
```
log(x - 1)/8 - log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 + RootSum(40
96*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{1}{-1+x^8}\,dx = -\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{1}{4}\arctan(x) - \frac{1}{8}\log(x+1) + \frac{1}{8}\log(x-1)$$

input
```
integrate(1/(x^8-1),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(
x - 1)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1+x^8}\,dx = -\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) -\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) -\frac{1}{4}\arctan(x)-\frac{1}{8}\log(|x+1|)+\frac{1}{8}\log(|x-1|)$$

input
```
integrate(1/(x^8-1),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(abs(x + 1)) + 1/8
*log(abs(x - 1))
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{1}{-1+x^8}\,dx = \frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\mathrm{atan}(x)}{4}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right) +\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

input
```
int(1/(x^8 - 1),x)
```

output
```
(atan(x*1i)*1i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8
+ 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)
```

3.50.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{1}{-1+x^8}\,dx = \frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)}{8} - \frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)}{8} - \frac{atan(x)}{4} + \frac{\sqrt{2}\log\left(-\sqrt{2}\,x+x^2+1\right)}{16} - \frac{\sqrt{2}\log\left(\sqrt{2}\,x+x^2+1\right)}{16} + \frac{\log(x-1)}{8} - \frac{\log(x+1)}{8}$$

input
```
int(1/(x**8 - 1),x)
```

output
```
(2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2)) - 2*sqrt(2)*atan((sqrt(2) + 2*x)/
sqrt(2)) - 4*atan(x) + sqrt(2)*log( - sqrt(2)*x + x**2 + 1) - sqrt(2)*log(
sqrt(2)*x + x**2 + 1) + 2*log(x - 1) - 2*log(x + 1))/16
```

3.51 $\int \frac{1}{1-x^4+x^8}\,dx$

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3.51.1 Optimal result

Integrand size = 12, antiderivative size = 275

$$\begin{aligned}\int \frac{1}{1-x^4+x^8}\,dx = &-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}\\ &+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}\\ &-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\\ &-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\end{aligned}$$

output
```
-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(
1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2
)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/
2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{1}{1-x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4 + x^8)^(-1),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

3.51.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1684, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{1684}$$

$$\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1483}$$

$$\frac{\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(1-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \ 1103$$

3.51. $\int \frac{1}{1-x^4+x^8}\,dx$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[(1 - x^4 + x^8)^(-1),x]
```

output
```
((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]
]] + ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqr
t[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[
2 - Sqrt[3]]))/(2*Sqrt[3]) + ((Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + Sqr
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[3]]
*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2/(2 - Sqrt[3])]*ArcTan[(Sqrt[
2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((1 + Sqrt[3])*Log[1 + Sqrt[2 + S
qrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30

input
```
int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

3.51.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{1-x^4+x^8}\,dx = \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right) - \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) + \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) - \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
(1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 +
6*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x +
6*x^2 - 6*I) + (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(
2)*x + 6*x^2 - 6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(
3)*sqrt(2)*x + 6*x^2 + 6*I)
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(1/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*
x + 1)/24
```

3.51.7 Maxima [F]

$$\int \frac{1}{1-x^4+x^8}\,dx = \int \frac{1}{x^8-x^4+1}\,dx$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 - x^4 + 1), x)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{1}{1-x^4+x^8}\,dx &= \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{1}{1-x^4+x^8}\,dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(1/(x^8 - x^4 + 1),x)
```

output
```
- 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
 - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12
)
```

3.51.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.15

$$\int \frac{1}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{12} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} + \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24}$$

input
```
int(1/(x**8 - x**4 + 1),x)
```

output
```
( - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 6*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)/
(2*sqrt( - sqrt(3) + 2))) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) +
 sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 6*sqrt( - sqrt(3) + 2)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sqrt
( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - sqrt( - sqrt(3) + 2)*sqrt(3)*log
( - sqrt( - sqrt(3) + 2)*x + x**2 + 1) + sqrt( - sqrt(3) + 2)*sqrt(3)*log(
sqrt( - sqrt(3) + 2)*x + x**2 + 1) - 3*sqrt( - sqrt(3) + 2)*log( - sqrt( -
 sqrt(3) + 2)*x + x**2 + 1) + 3*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) +
 2)*x + x**2 + 1) - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)
 + sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2))/24
```

3.52 $\int \frac{x^7}{1+x^{12}} \, dx$

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3.52.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{x^7}{1+x^{12}} \, dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\log\left(1-x^4+x^8\right)$$

output
```
-1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(
1/2)
```

3.52.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 260 vs. $2(49) = 98$.

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.31

$$\int \frac{x^7}{1+x^{12}} \, dx = \frac{1}{24}\left(2\sqrt{3}\arctan\left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right)-2\log\left(1-\sqrt{2}x+x^2\right)-2\log\left(1+\sqrt{2}x+x^2\right)+\log\left(2+\sqrt{2}x-\sqrt{6}x+2x^2\right)+\log\left(2+\sqrt{2}\left(-1+\sqrt{3}\right)x+2x^2\right)+\log\left(2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)+\log\left(2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)\right)$$

input
```
Integrate[x^7/(1 + x^12),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*A
rcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 +
Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*
Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2
]*x + x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1
+ Sqrt[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (
Sqrt[2] + Sqrt[6])*x + 2*x^2])/24
```

3.52.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^{12}+1} \, dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{4}\int \frac{x^4}{x^{12}+1}dx^4$$

$$\Big\downarrow \; 821$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{x^4+1}{x^8-x^4+1}dx^4-\frac{1}{3}\int \frac{1}{x^4+1}dx^4\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{x^4+1}{x^8-x^4+1}dx^4-\frac{1}{3}\log \left(x^4+1\right)\right)$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log \left(x^4+1\right)\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log \left(x^4+1\right)\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{4}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^8-3}d\left(2x^4-1\right)-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log \left(x^4+1\right)\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\sqrt{3}\arctan \left(\frac{2x^4-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log \left(x^4+1\right)\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\sqrt{3}\arctan \left(\frac{2x^4-1}{\sqrt{3}}\right)+\frac{1}{2}\log \left(x^8-x^4+1\right)\right)-\frac{1}{3}\log \left(x^4+1\right)\right)$$

input
```
Int[x^7/(1 + x^12),x]
```

output
```
(-1/3*Log[1 + x^4] + (Sqrt[3]*ArcTan[(-1 + 2*x^4)/Sqrt[3]] + Log[1 - x^4 +
 x^8]/2)/3)/4
```

3.52. $\int \frac{x^7}{1+x^{12}}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.52.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(x^8-x^4+1)}{24}+\frac{\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x^4+1)}{12}$	41
risch	$\frac{\ln(4x^8-4x^4+4)}{24}+\frac{\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x^4+1)}{12}$	43
meijerg	$-\frac{x^8\ln\left(1+(x^{12})^{\frac{1}{3}}\right)}{12(x^{12})^{\frac{2}{3}}}+\frac{x^8\ln\left(1-(x^{12})^{\frac{1}{3}}+(x^{12})^{\frac{2}{3}}\right)}{24(x^{12})^{\frac{2}{3}}}+\frac{x^8\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^{12})^{\frac{1}{3}}}{2-(x^{12})^{\frac{1}{3}}}\right)}{12(x^{12})^{\frac{2}{3}}}$	80

input
```
int(x^7/(x^12+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/2)-1/12*ln(x^4+
1)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx=\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^12+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1
/12*log(x^4 + 1)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{1+x^{12}}\,dx = -\frac{\log\left(x^4+1\right)}{12} + \frac{\log\left(x^8-x^4+1\right)}{24} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**7/(x**12+1),x)
```

output
```
-log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/
3 - sqrt(3)/3)/12
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{24}\,\log\left(x^8-x^4+1\right) - \frac{1}{12}\,\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^12+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1
/12*log(x^4 + 1)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{24}\,\log\left(x^8-x^4+1\right) - \frac{1}{12}\,\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^12+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1
/12*log(x^4 + 1)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{1+x^{12}}\,dx = -\frac{\ln\left(x^4+1\right)}{12} - \ln\left(x^4 - \frac{\sqrt{3}\,1\mathrm{i}}{2} - \frac{1}{2}\right)\left(-\frac{1}{24} + \frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x^4 + \frac{\sqrt{3}\,1\mathrm{i}}{2} - \frac{1}{2}\right)\left(\frac{1}{24} + \frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

input
```
int(x^7/(x^12 + 1),x)
```

output
```
log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

3.52.10 Reduce [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 353, normalized size of antiderivative = 7.20

$$\int \frac{x^7}{1+x^{12}}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\log\left(-\sqrt{2}\,x+x^2+1\right)}{12} + \frac{\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\log\left(\sqrt{2}\,x+x^2+1\right)}{12} + \frac{\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} + \frac{\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24}$$

input
```
int(x**7/(x**12 + 1),x)
```

output
```
( - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
 sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) -
 4*x)/(2*sqrt( - sqrt(3) + 2))) - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(
6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqr
t(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + sqrt( - sq
rt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))
) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sq
rt(6) + sqrt(2))) + sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*s
qrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + log( - sqrt( - sqrt(3) +
 2)*x + x**2 + 1) - 2*log( - sqrt(2)*x + x**2 + 1) + log(sqrt( - sqrt(3) +
 2)*x + x**2 + 1) - 2*log(sqrt(2)*x + x**2 + 1) + log(( - sqrt(6)*x - sqrt
(2)*x + 2*x**2 + 2)/2) + log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2))/24
```

3.52. $\int \frac{x^7}{1+x^{12}}\,dx$

3.53 $\int \log(x)\,dx$

3.53.1 Optimal result 436
3.53.2 Mathematica [A] (verified) 436
3.53.3 Rubi [A] (verified) 437
3.53.4 Maple [A] (verified) 437
3.53.5 Fricas [A] (verification not implemented) 438
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3.53.7 Maxima [A] (verification not implemented) 438
3.53.8 Giac [A] (verification not implemented) 439
3.53.9 Mupad [B] (verification not implemented) 439
3.53.10 Reduce [B] (verification not implemented) 439

3.53.1 Optimal result

Integrand size = 2, antiderivative size = 8

$$\int \log(x)\,dx = -x + x\log(x)$$

output
```
-x+x*ln(x)
```

3.53.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = -x + x\log(x)$$

input
```
Integrate[Log[x],x]
```

output
```
-x + x*Log[x]
```

3.53.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\,dx$$

$$\Big\downarrow \; 2732$$

$$x\log(x) - x$$

input
```
Int[Log[x],x]
```

output
```
-x + x*Log[x]
```

3.53.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

3.53.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
lookup	$-x+x\ln\left(x\right)$	9
default	$-x+x\ln\left(x\right)$	9
norman	$-x+x\ln\left(x\right)$	9
risch	$-x+x\ln\left(x\right)$	9
parallelrisch	$-x+x\ln\left(x\right)$	9
parts	$-x+x\ln\left(x\right)$	9

input
```
int(ln(x),x,method=_RETURNVERBOSE)
```

output
```
-x+x*ln(x)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(log(x),x, algorithm="fricas")
```

output
```
x*log(x) - x
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(ln(x),x)
```

output
```
x*log(x) - x
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(log(x),x, algorithm="maxima")
```

output
```
x*log(x) - x
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log{(x)} - x$$

input
```
integrate(log(x),x, algorithm="giac")
```

output
```
x*log(x) - x
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x)\,dx = x\,(\ln{(x)} - 1)$$

input
```
int(log(x),x)
```

output
```
x*(log(x) - 1)
```

3.53.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x)\,dx = x(\log(x) - 1)$$

input
```
int(log(x),x)
```

output
```
x*(log(x) - 1)
```

3.54 $\int x \log(x)\, dx$

3.54.1 Optimal result 440
3.54.2 Mathematica [A] (verified) 440
3.54.3 Rubi [A] (verified) 441
3.54.4 Maple [A] (verified) 442
3.54.5 Fricas [A] (verification not implemented) 442
3.54.6 Sympy [A] (verification not implemented) 442
3.54.7 Maxima [A] (verification not implemented) 443
3.54.8 Giac [A] (verification not implemented) 443
3.54.9 Mupad [B] (verification not implemented) 443
3.54.10 Reduce [B] (verification not implemented) 444

3.54.1 Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.54.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

input
```
Integrate[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.54.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log(x)\,dx$$

$$\Big\downarrow \; 2741$$

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

input
```
Int[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.54.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

input
```
int(x*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="fricas")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log\left(x\right)}{2} - \frac{x^2}{4}$$

input
```
integrate(x*ln(x),x)
```

output
```
x**2*log(x)/2 - x**2/4
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="maxima")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="giac")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\left(\ln{(x)} - \frac{1}{2}\right)}{2}$$

input
```
int(x*log(x),x)
```

output
```
(x^2*(log(x) - 1/2))/2
```

3.54.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int x\log(x)\,dx = \frac{x^2(2\log(x)-1)}{4}$$

input
```
int(log(x)*x,x)
```

output
```
(x**2*(2*log(x) - 1))/4
```

3.55 $\int x^2 \log(x)\,dx$

3.55.1 Optimal result 445
3.55.2 Mathematica [A] (verified) 445
3.55.3 Rubi [A] (verified) 446
3.55.4 Maple [A] (verified) 447
3.55.5 Fricas [A] (verification not implemented) 447
3.55.6 Sympy [A] (verification not implemented) 447
3.55.7 Maxima [A] (verification not implemented) 448
3.55.8 Giac [A] (verification not implemented) 448
3.55.9 Mupad [B] (verification not implemented) 448
3.55.10 Reduce [B] (verification not implemented) 449

3.55.1 Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^2 \log(x)\,dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

output
```
-1/9*x^3+1/3*x^3*ln(x)
```

3.55.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \log(x)\,dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

input
```
Integrate[x^2*Log[x],x]
```

output
```
-1/9*x^3 + (x^3*Log[x])/3
```

3.55.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(x) \, dx$$

$$\Big\downarrow \; 2741$$

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

input
```
Int[x^2*Log[x],x]
```

output
```
-1/9*x^3 + (x^3*Log[x])/3
```

3.55.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.55.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14
norman	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14
risch	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14
parallelrisch	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14
parts	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14

input
```
int(x^2*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/9*x^3+1/3*x^3*ln(x)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\log(x)\,dx=\frac{1}{3}\,x^3\log\left(x\right)-\frac{1}{9}\,x^3$$

input
```
integrate(x^2*log(x),x, algorithm="fricas")
```

output
```
1/3*x^3*log(x) - 1/9*x^3
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2\log(x)\,dx=\frac{x^3\log\left(x\right)}{3}-\frac{x^3}{9}$$

input
```
integrate(x**2*ln(x),x)
```

output
```
x**3*log(x)/3 - x**3/9
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \log(x)\, dx = \frac{1}{3}\, x^3 \log\left(x\right) - \frac{1}{9}\, x^3$$

input
```
integrate(x^2*log(x),x, algorithm="maxima")
```

output
```
1/3*x^3*log(x) - 1/9*x^3
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \log(x)\, dx = \frac{1}{3}\, x^3 \log\left(x\right) - \frac{1}{9}\, x^3$$

input
```
integrate(x^2*log(x),x, algorithm="giac")
```

output
```
1/3*x^3*log(x) - 1/9*x^3
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x^2 \log(x)\, dx = \frac{x^3 \left(\ln\left(x\right) - \frac{1}{3}\right)}{3}$$

input
```
int(x^2*log(x),x)
```

output
```
(x^3*(log(x) - 1/3))/3
```

3.55.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int x^2 \log(x)\,dx = \frac{x^3(3\log(x)-1)}{9}$$

input
```
int(log(x)*x**2,x)
```

output
```
(x**3*(3*log(x) - 1))/9
```

3.56 $\int x^p \log(x)\,dx$

3.56.1 Optimal result . . . 450
3.56.2 Mathematica [A] (verified) . . . 450
3.56.3 Rubi [A] (verified) . . . 451
3.56.4 Maple [A] (verified) . . . 451
3.56.5 Fricas [A] (verification not implemented) . . . 452
3.56.6 Sympy [B] (verification not implemented) . . . 452
3.56.7 Maxima [A] (verification not implemented) . . . 453
3.56.8 Giac [F] . . . 453
3.56.9 Mupad [B] (verification not implemented) . . . 453
3.56.10 Reduce [B] (verification not implemented) . . . 454

3.56.1 Optimal result

Integrand size = 6, antiderivative size = 26

$$\int x^p \log(x)\,dx = -\frac{x^{1+p}}{(1+p)^2} + \frac{x^{1+p}\log(x)}{1+p}$$

output
```
-x^(p+1)/(p+1)^2+x^(p+1)*ln(x)/(p+1)
```

3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int x^p \log(x)\,dx = \frac{x^{1+p}(-1+(1+p)\log(x))}{(1+p)^2}$$

input
```
Integrate[x^p*Log[x],x]
```

output
```
(x^(1 + p)*(-1 + (1 + p)*Log[x]))/(1 + p)^2
```

3.56.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p \log(x)\,dx$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{x^{p+1}\log(x)}{p+1}-\frac{x^{p+1}}{(p+1)^2}$$

input
```
Int[x^p*Log[x],x]
```

output
```
-(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

3.56.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.56.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{x(\ln(x)p+\ln(x)-1)x^{p}}{(1+p)^{2}}$	19
norman	$\frac{x\ln(x){\mathrm e}^{\ln(x)p}}{1+p}-\frac{x\,{\mathrm e}^{\ln(x)p}}{p^{2}+2p+1}$	34
parallelrisch	$\frac{x\,x^{p}\ln(x)p+x^{p}\ln(x)x-x\,x^{p}}{p^{2}+2p+1}$	34

input
```
int(x^p*ln(x),x,method=_RETURNVERBOSE)
```

output
```
x*(ln(x)*p+ln(x)-1)/(1+p)^2*x^p
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^p \log(x)\, dx = \frac{((p+1)x\log{(x)} - x)x^p}{p^2 + 2\,p + 1}$$

input
```
integrate(x^p*log(x),x, algorithm="fricas")
```

output
```
((p + 1)*x*log(x) - x)*x^p/(p^2 + 2*p + 1)
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(20) = 40.

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int x^p \log(x)\, dx = \begin{cases} \frac{pxx^p\log{(x)}}{p^2+2p+1} + \frac{xx^p\log{(x)}}{p^2+2p+1} - \frac{xx^p}{p^2+2p+1} & \text{for}\ p \neq -1 \\ \frac{\log{(x)}^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**p*ln(x),x)
```

output
```
Piecewise((p*x*x**p*log(x)/(p**2 + 2*p + 1) + x*x**p*log(x)/(p**2 + 2*p +
1) - x*x**p/(p**2 + 2*p + 1), Ne(p, -1)), (log(x)**2/2, True))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^p \log(x)\,dx = \frac{x^{p+1}\log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

input
```
integrate(x^p*log(x),x, algorithm="maxima")
```

output
```
x^(p + 1)*log(x)/(p + 1) - x^(p + 1)/(p + 1)^2
```

3.56.8 Giac [F]

$$\int x^p \log(x)\,dx = \int x^p \log(x)\,dx$$

input
```
integrate(x^p*log(x),x, algorithm="giac")
```

output
```
integrate(x^p*log(x), x)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int x^p \log(x)\,dx = \begin{cases} \frac{\ln(x)^2}{2} & \text{if } p = -1 \\ \frac{x^{p+1}(\ln(x)(p+1)-1)}{(p+1)^2} & \text{if } p \neq -1 \end{cases}$$

input
```
int(x^p*log(x),x)
```

output
```
piecewise(p == -1, log(x)^2/2, p ~= -1, (x^(p + 1)*(log(x)*(p + 1) - 1))/(
p + 1)^2)
```

3.56.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int x^p \log(x) \, dx = \frac{x^p x(\log(x) \, p + \log(x) - 1)}{p^2 + 2p + 1}$$

input
```
int(x**p*log(x),x)
```

output
```
(x**p*x*(log(x)*p + log(x) - 1))/(p**2 + 2*p + 1)
```

3.57 $\int \log^2(x)\,dx$

3.57.1 Optimal result 455
3.57.2 Mathematica [A] (verified) 455
3.57.3 Rubi [A] (verified) 456
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3.57.1 Optimal result

Integrand size = 4, antiderivative size = 15

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

output
```
2*x-2*x*ln(x)+x*ln(x)^2
```

3.57.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

input
```
Integrate[Log[x]^2,x]
```

output
```
2*x - 2*x*Log[x] + x*Log[x]^2
```

3.57.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2(x)\,dx$$

$$\Big\downarrow \ 2733$$

$$x\log^2(x) - 2\int \log(x)dx$$

$$\Big\downarrow \ 2732$$

$$x\log^2(x) - 2(x\log(x) - x)$$

input
```
Int[Log[x]^2,x]
```

output
```
x*Log[x]^2 - 2*(-x + x*Log[x])
```

3.57.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.57.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$2x-2x\ln\left(x\right)+x\ln\left(x\right)^{2}$	16
norman	$2x-2x\ln\left(x\right)+x\ln\left(x\right)^{2}$	16
risch	$2x-2x\ln\left(x\right)+x\ln\left(x\right)^{2}$	16
parallelrisch	$2x-2x\ln\left(x\right)+x\ln\left(x\right)^{2}$	16

input
```
int(ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x-2*x*ln(x)+x*ln(x)^2
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log\left(x\right)^{2} - 2\,x\log\left(x\right) + 2\,x$$

input
```
integrate(log(x)^2,x, algorithm="fricas")
```

output
```
x*log(x)^2 - 2*x*log(x) + 2*x
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log\left(x\right)^{2} - 2x\log\left(x\right) + 2x$$

input
```
integrate(ln(x)**2,x)
```

output
```
x*log(x)**2 - 2*x*log(x) + 2*x
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = \left(\log\left(x\right)^2 - 2\,\log\left(x\right) + 2\right)x$$

input
```
integrate(log(x)^2,x, algorithm="maxima")
```

output
```
(log(x)^2 - 2*log(x) + 2)*x
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log\left(x\right)^2 - 2\,x\log\left(x\right) + 2\,x$$

input
```
integrate(log(x)^2,x, algorithm="giac")
```

output
```
x*log(x)^2 - 2*x*log(x) + 2*x
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = x\left(\ln\left(x\right)^2 - 2\,\ln\left(x\right) + 2\right)$$

input
```
int(log(x)^2,x)
```

output
```
x*(log(x)^2 - 2*log(x) + 2)
```

3.57.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = x\big(\log(x)^2 - 2\log(x) + 2\big)$$

input
```
int(log(x)**2,x)
```

output
```
x*(log(x)**2 - 2*log(x) + 2)
```

3.58 $\int x^9 \log^{11}(x)\,dx$

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3.58.1 Optimal result

Integrand size = 8, antiderivative size = 127

$$\int x^9 \log^{11}(x)\,dx = -\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\log(x)}{15625000}-\frac{6237x^{10}\log^2(x)}{3125000}+\frac{2079x^{10}\log^3(x)}{312500}-\frac{2079x^{10}\log^4(x)}{125000}+\frac{2079x^{10}\log^5(x)}{62500}-\frac{693x^{10}\log^6(x)}{12500}+\frac{99x^{10}\log^7(x)}{1250}-\frac{99x^{10}\log^8(x)}{1000}+\frac{11}{100}x^{10}\log^9(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{1}{10}x^{10}\log^{11}(x)$$

output
```
-6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+20
79/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-69
3/12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10
*ln(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

3.58.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int x^9 \log^{11}(x)\,dx = -\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\log(x)}{15625000}-\frac{6237x^{10}\log^2(x)}{3125000}+\frac{2079x^{10}\log^3(x)}{312500}-\frac{2079x^{10}\log^4(x)}{125000}+\frac{2079x^{10}\log^5(x)}{62500}-\frac{693x^{10}\log^6(x)}{12500}+\frac{99x^{10}\log^7(x)}{1250}-\frac{99x^{10}\log^8(x)}{1000}+\frac{11}{100}x^{10}\log^9(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{1}{10}x^{10}\log^{11}(x)$$

input
```
Integrate[x^9*Log[x]^11,x]
```

output
```
(-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2
)/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (2
079*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/
1250 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^
10)/100 + (x^10*Log[x]^11)/10
```

3.58.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.375$, Rules used = {2742, 2742, 2742, 2742, 2742, 2742, 2742, 2742, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \log^{11}(x) \, dx$$

$$\Big\downarrow \; 2742$$

$$\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{10}\int x^9\log^{10}(x)dx$$

$$\Big\downarrow \; 2742$$

$$\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{10}\left(\frac{1}{10}x^{10}\log^{10}(x)-\int x^9\log^9(x)dx\right)$$

$$\Big\downarrow \; 2742$$

$$\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{10}\left(\frac{9}{10}\int x^9\log^8(x)dx+\frac{1}{10}x^{10}\log^{10}(x)-\frac{1}{10}x^{10}\log^9(x)\right)$$

$$\Big\downarrow \; 2742$$

$$\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\int x^9\log^7(x)dx\right)+\frac{1}{10}x^{10}\log^{10}(x)-\frac{1}{10}x^{10}\log^9(x)\right)$$

$$\Big\downarrow \; 2742$$

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\int x^9\log^6(x)dx\right)\right)+\frac{1}{10}x^{10}\log^{10}(x)-\frac{1}{10}x^{10}\log^9(x)\right)$$

$\downarrow$ 2742

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x)-\frac{3}{5}\int x^9\log^5(x)dx\right)\right)\right)+\frac{1}{10}x^{10}\log^{10}(x)-\frac{1}{10}\right.$$

$\downarrow$ 2742

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x)-\frac{3}{5}\left(\frac{1}{10}x^{10}\log^5(x)-\frac{1}{2}\int x^9\log^4(x)dx\right)\right)\right)\right)\right.$$

$\downarrow$ 2742

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x)-\frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{5}\int x^9\log^3(x)dx-\frac{1}{10}x^{10}\log^4(x)\right)+\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2742

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x)-\frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{10}x^{10}\log^3(x)-\frac{3}{10}\int x^9\log^2(x)dx\right.\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2742

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x)-\frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{10}x^{10}\log^3(x)-\frac{3}{10}\left(\frac{1}{10}x^{10}\log^2(x)\right.\right.\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2741

$$\frac{1}{10}x^{10}\log^{11}(x)-$$
$$\frac{11}{10}\left(\frac{1}{10}x^{10}\log^{10}(x)-\frac{1}{10}x^{10}\log^9(x)+\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x)-\frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x)-\frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x)-\frac{3}{5}\left(\frac{1}{10}x^{10}\right.\right.\right.\right.\right.\right.$$

input
```
Int[x^9*Log[x]^11,x]
```

3.58. $\int x^9\log^{11}(x)\,dx$

output
```
(x^10*Log[x]^11)/10 - (11*(-1/10*(x^10*Log[x]^9) + (x^10*Log[x]^10)/10 + (
9*((x^10*Log[x]^8)/10 - (4*((x^10*Log[x]^7)/10 - (7*((x^10*Log[x]^6)/10 -
(3*((x^10*Log[x]^5)/10 + (-1/10*(x^10*Log[x]^4) + (2*((x^10*Log[x]^3)/10 -
 (3*((x^10*Log[x]^2)/10 + (x^10/100 - (x^10*Log[x])/10)/5))/10))/5)/2))/5)
)/10))/5))/10))/10
```

3.58.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

method	result
default	$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\ln(x)}{15625000}-\frac{6237x^{10}\ln(x)^2}{3125000}+\frac{2079x^{10}\ln(x)^3}{312500}-\frac{2079x^{10}\ln(x)^4}{125000}+\frac{2079x^{10}\ln(x)^5}{62500}-\frac{693x^{10}\ln}{1250}$
risch	$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\ln(x)}{15625000}-\frac{6237x^{10}\ln(x)^2}{3125000}+\frac{2079x^{10}\ln(x)^3}{312500}-\frac{2079x^{10}\ln(x)^4}{125000}+\frac{2079x^{10}\ln(x)^5}{62500}-\frac{693x^{10}\ln}{1250}$
parallelrisch	$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\ln(x)}{15625000}-\frac{6237x^{10}\ln(x)^2}{3125000}+\frac{2079x^{10}\ln(x)^3}{312500}-\frac{2079x^{10}\ln(x)^4}{125000}+\frac{2079x^{10}\ln(x)^5}{62500}-\frac{693x^{10}\ln}{1250}$
parts	$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\ln(x)}{15625000}-\frac{6237x^{10}\ln(x)^2}{3125000}+\frac{2079x^{10}\ln(x)^3}{312500}-\frac{2079x^{10}\ln(x)^4}{125000}+\frac{2079x^{10}\ln(x)^5}{62500}-\frac{693x^{10}\ln}{1250}$

input
```
int(x^9*ln(x)^11,x,method=_RETURNVERBOSE)
```

output
```
-6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+20
79/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-69
3/12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10
*ln(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\int x^9\log^{11}(x)\,dx = \frac{1}{10}\,x^{10}\log\left(x\right)^{11} - \frac{11}{100}\,x^{10}\log\left(x\right)^{10} + \frac{11}{100}\,x^{10}\log\left(x\right)^{9} - \frac{99}{1000}\,x^{10}\log\left(x\right)^{8} + \frac{99}{1250}\,x^{10}\log\left(x\right)^{7} - \frac{693}{12500}\,x^{10}\log\left(x\right)^{6} + \frac{2079}{62500}\,x^{10}\log\left(x\right)^{5} - \frac{2079}{125000}\,x^{10}\log\left(x\right)^{4} + \frac{2079}{312500}\,x^{10}\log\left(x\right)^{3} - \frac{6237}{3125000}\,x^{10}\log\left(x\right)^{2} + \frac{6237}{15625000}\,x^{10}\log\left(x\right) - \frac{6237}{156250000}\,x^{10}$$

input
```
integrate(x^9*log(x)^11,x, algorithm="fricas")
```

output
```
1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/10
00*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/
62500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^
3 - 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/15625000
0*x^10
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int x^9\log^{11}(x)\,dx = \frac{x^{10}\log\left(x\right)^{11}}{10} - \frac{11x^{10}\log\left(x\right)^{10}}{100} + \frac{11x^{10}\log\left(x\right)^{9}}{100} - \frac{99x^{10}\log\left(x\right)^{8}}{1000} + \frac{99x^{10}\log\left(x\right)^{7}}{1250} - \frac{693x^{10}\log\left(x\right)^{6}}{12500} + \frac{2079x^{10}\log\left(x\right)^{5}}{62500} - \frac{2079x^{10}\log\left(x\right)^{4}}{125000} + \frac{2079x^{10}\log\left(x\right)^{3}}{312500} - \frac{6237x^{10}\log\left(x\right)^{2}}{3125000} + \frac{6237x^{10}\log\left(x\right)}{15625000} - \frac{6237x^{10}}{156250000}$$

input
```
integrate(x**9*ln(x)**11,x)
```

output
```
x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**9/100 - 9
9*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x**10*log(x)**6/125
00 + 2079*x**10*log(x)**5/62500 - 2079*x**10*log(x)**4/125000 + 2079*x**10
*log(x)**3/312500 - 6237*x**10*log(x)**2/3125000 + 6237*x**10*log(x)/15625
000 - 6237*x**10/156250000
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^9 \log^{11}(x)\,dx$$
$$= \frac{1}{156250000} \left(15625000 \log(x)^{11} - 17187500 \log(x)^{10} + 17187500 \log(x)^9 - 15468750 \log(x)^8 + 123750\right.$$

input
```
integrate(x^9*log(x)^11,x, algorithm="maxima")
```

output
```
1/156250000*(15625000*log(x)^11 - 17187500*log(x)^10 + 17187500*log(x)^9 -
 15468750*log(x)^8 + 12375000*log(x)^7 - 8662500*log(x)^6 + 5197500*log(x)
^5 - 2598750*log(x)^4 + 1039500*log(x)^3 - 311850*log(x)^2 + 62370*log(x)
- 6237)*x^10
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\begin{aligned}\int x^9 \log^{11}(x)\,dx = {} & \frac{1}{10}x^{10}\log(x)^{11} - \frac{11}{100}x^{10}\log(x)^{10} + \frac{11}{100}x^{10}\log(x)^9 \\ & - \frac{99}{1000}x^{10}\log(x)^8 + \frac{99}{1250}x^{10}\log(x)^7 - \frac{693}{12500}x^{10}\log(x)^6 \\ & + \frac{2079}{62500}x^{10}\log(x)^5 - \frac{2079}{125000}x^{10}\log(x)^4 + \frac{2079}{312500}x^{10}\log(x)^3 \\ & - \frac{6237}{3125000}x^{10}\log(x)^2 + \frac{6237}{15625000}x^{10}\log(x) - \frac{6237}{156250000}x^{10}\end{aligned}$$

input
```
integrate(x^9*log(x)^11,x, algorithm="giac")
```

output
```
1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/10
00*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/
62500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^
3 - 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/15625000
0*x^10
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^9 \log^{11}(x) \, dx$$
$$= \frac{6237\,x^{10}\left(\frac{15625000\ln(x)^{11}}{6237} - \frac{1562500\ln(x)^{10}}{567} + \frac{1562500\ln(x)^9}{567} - \frac{156250\ln(x)^8}{63} + \frac{125000\ln(x)^7}{63} - \frac{12500\ln(x)^6}{9} + \frac{2500\ln(x)^5}{3}\right.}{156250000}$$

input
```
int(x^9*log(x)^11,x)
```

output
```
(6237*x^10*(10*log(x) - 50*log(x)^2 + (500*log(x)^3)/3 - (1250*log(x)^4)/3
 + (2500*log(x)^5)/3 - (12500*log(x)^6)/9 + (125000*log(x)^7)/63 - (156250
*log(x)^8)/63 + (1562500*log(x)^9)/567 - (1562500*log(x)^10)/567 + (156250
00*log(x)^11)/6237 - 1))/156250000
```

3.58.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^9 \log^{11}(x) \, dx$$
$$= \frac{x^{10}(15625000\log(x)^{11} - 17187500\log(x)^{10} + 17187500\log(x)^9 - 15468750\log(x)^8 + 12375000\log(x)^7 -}{1}$$

input
```
int(log(x)**11*x**9,x)
```

output
```
(x**10*(15625000*log(x)**11 - 17187500*log(x)**10 + 17187500*log(x)**9 - 1
5468750*log(x)**8 + 12375000*log(x)**7 - 8662500*log(x)**6 + 5197500*log(x
)**5 - 2598750*log(x)**4 + 1039500*log(x)**3 - 311850*log(x)**2 + 62370*lo
g(x) - 6237))/156250000
```

3.59 $\int \frac{\log^2(x)}{x} dx$

3.59.1 Optimal result 467
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3.59.8 Giac [A] (verification not implemented) 470
3.59.9 Mupad [B] (verification not implemented) 470
3.59.10 Reduce [B] (verification not implemented) 471

3.59.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\log^2(x)}{x} dx = \frac{\log^3(x)}{3}$$

output
```
1/3*ln(x)^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(x)}{x} dx = \frac{\log^3(x)}{3}$$

input
```
Integrate[Log[x]^2/x,x]
```

output
```
Log[x]^3/3
```

3.59.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)}{x} \, dx$$

$$\Big\downarrow \; 2739$$

$$\int \log^2(x) d\log(x)$$

$$\Big\downarrow \; 15$$

$$\frac{\log^3(x)}{3}$$

input
```
Int[Log[x]^2/x,x]
```

output
```
Log[x]^3/3
```

3.59.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.59.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln(x)^3}{3}$	7
default	$\frac{\ln(x)^3}{3}$	7
norman	$\frac{\ln(x)^3}{3}$	7
risch	$\frac{\ln(x)^3}{3}$	7
parts	$\frac{\ln(x)^3}{3}$	7

input
```
int(ln(x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x)^3
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x}\,dx = \frac{1}{3}\log(x)^3$$

input
```
integrate(log(x)^2/x,x, algorithm="fricas")
```

output
```
1/3*log(x)^3
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)}{x}\,dx = \frac{\log(x)^3}{3}$$

input
```
integrate(ln(x)**2/x,x)
```

output
```
log(x)**3/3
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x}\,dx = \frac{1}{3}\log(x)^3$$

input
```
integrate(log(x)^2/x,x, algorithm="maxima")
```

output
```
1/3*log(x)^3
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x}\,dx = \frac{1}{3}\log(x)^3$$

input
```
integrate(log(x)^2/x,x, algorithm="giac")
```

output
```
1/3*log(x)^3
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x}\,dx = \frac{\ln(x)^3}{3}$$

input
```
int(log(x)^2/x,x)
```

output
```
log(x)^3/3
```

3.59.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x}\,dx = \frac{\log(x)^3}{3}$$

input
```
int(log(x)**2/x,x)
```

output
```
log(x)**3/3
```

3.60 $\int \frac{1}{\log(x)} dx$

3.60.1 Optimal result 472
3.60.2 Mathematica [A] (verified) 472
3.60.3 Rubi [A] (verified) 473
3.60.4 Maple [B] (verified) 473
3.60.5 Fricas [A] (verification not implemented) 474
3.60.6 Sympy [A] (verification not implemented) 474
3.60.7 Maxima [A] (verification not implemented) 474
3.60.8 Giac [A] (verification not implemented) 475
3.60.9 Mupad [B] (verification not implemented) 475
3.60.10 Reduce [B] (verification not implemented) 475

3.60.1 Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \frac{1}{\log(x)} dx = \text{LogIntegral}(x)$$

output
```
Li(x)
```

3.60.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)} dx = \text{LogIntegral}(x)$$

input
```
Integrate[Log[x]^(-1),x]
```

output
```
LogIntegral[x]
```

3.60.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log(x)} \, dx$$

$$\Big\downarrow \text{ 2735}$$

$$\text{LogIntegral}(x)$$

input
```
Int[Log[x]^(-1),x]
```

output
```
LogIntegral[x]
```

3.60.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

method	result	size
default	$-\operatorname{Ei}_1(-\ln(x))$	9
risch	$-\operatorname{Ei}_1(-\ln(x))$	9

input
```
int(1/ln(x),x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-ln(x))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{log\_integral}\,(x)$$

input
```
integrate(1/log(x),x, algorithm="fricas")
```

output
```
log_integral(x)
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{li}\,(x)$$

input
```
integrate(1/ln(x),x)
```

output
```
li(x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\,dx = \text{Ei}(\log\,(x))$$

input
```
integrate(1/log(x),x, algorithm="maxima")
```

output
```
Ei(log(x))
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\, dx = \mathrm{Ei}(\log(x))$$

input
```
integrate(1/log(x),x, algorithm="giac")
```

output
```
Ei(log(x))
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\, dx = \mathrm{logint}(x)$$

input
```
int(1/log(x),x)
```

output
```
logint(x)
```

3.60.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\, dx = ei(\log(x))$$

input
```
int(1/log(x),x)
```

output
```
ei(log(x))
```

3.61 $\int \frac{1}{\log(1+x)}\,dx$

3.61.1 Optimal result . . . 476
3.61.2 Mathematica [A] (verified) . . . 476
3.61.3 Rubi [A] (verified) . . . 477
3.61.4 Maple [B] (verified) . . . 478
3.61.5 Fricas [A] (verification not implemented) . . . 478
3.61.6 Sympy [A] (verification not implemented) . . . 478
3.61.7 Maxima [A] (verification not implemented) . . . 479
3.61.8 Giac [A] (verification not implemented) . . . 479
3.61.9 Mupad [B] (verification not implemented) . . . 479
3.61.10 Reduce [B] (verification not implemented) . . . 480

3.61.1 Optimal result

Integrand size = 6, antiderivative size = 4

$$\int \frac{1}{\log(1+x)}\,dx = \text{LogIntegral}(1+x)$$

output
```
Li(1+x)
```

3.61.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(1+x)}\,dx = \text{LogIntegral}(1+x)$$

input
```
Integrate[Log[1 + x]^(-1),x]
```

output
```
LogIntegral[1 + x]
```

3.61.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2836, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log(x+1)}dx$$

$$\Big\downarrow \; 2836$$

$$\int \frac{1}{\log(x+1)}d(x+1)$$

$$\Big\downarrow \; 2735$$

$$\text{LogIntegral}(x+1)$$

input
```
Int[Log[1 + x]^(-1),x]
```

output
```
LogIntegral[1 + x]
```

3.61.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(4) = 8.

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

method	result	size
derivativedivides	$-\operatorname{Ei}_{1}\left(-\ln\left(1+x\right)\right)$	11
default	$-\operatorname{Ei}_{1}\left(-\ln\left(1+x\right)\right)$	11
risch	$-\operatorname{Ei}_{1}\left(-\ln\left(1+x\right)\right)$	11

input
```
int(1/ln(1+x),x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-ln(1+x))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(1+x)}\,dx = \text{log\_integral}\left(x+1\right)$$

input
```
integrate(1/log(1+x),x, algorithm="fricas")
```

output
```
log_integral(x + 1)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{\log(1+x)}\,dx = \operatorname{li}\left(x+1\right)$$

input
```
integrate(1/ln(1+x),x)
```

output
```
li(x + 1)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{\log(1+x)}\,dx = \mathrm{Ei}(\log{(x+1)})$$

input
```
integrate(1/log(1+x),x, algorithm="maxima")
```

output
```
Ei(log(x + 1))
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{\log(1+x)}\,dx = \mathrm{Ei}(\log{(x+1)})$$

input
```
integrate(1/log(1+x),x, algorithm="giac")
```

output
```
Ei(log(x + 1))
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(1+x)}\,dx = \mathrm{logint}(x+1)$$

input
```
int(1/log(x + 1),x)
```

output
```
logint(x + 1)
```

3.61.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{\log(1+x)} \, dx = ei(\log(x+1))$$

input
```
int(1/log(x + 1),x)
```

output
```
ei(log(x + 1))
```

3.62 $\int \frac{1}{x \log(x)} \, dx$

3.62.1 Optimal result 481
3.62.2 Mathematica [A] (verified) 481
3.62.3 Rubi [A] (verified) 482
3.62.4 Maple [A] (verified) 483
3.62.5 Fricas [A] (verification not implemented) 483
3.62.6 Sympy [A] (verification not implemented) 483
3.62.7 Maxima [A] (verification not implemented) 484
3.62.8 Giac [A] (verification not implemented) 484
3.62.9 Mupad [B] (verification not implemented) 484
3.62.10 Reduce [B] (verification not implemented) 485

3.62.1 Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

output
```
ln(ln(x))
```

3.62.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

input
```
Integrate[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

3.62.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)}\,dx$$
$$\Big\downarrow \; 2739$$
$$\int \frac{1}{\log(x)}d\log(x)$$
$$\Big\downarrow \; 14$$
$$\log(\log(x))$$

input
```
Int[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

3.62.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.62.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4
parallelrisch	$\ln(\ln(x))$	4

input
```
int(1/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="fricas")
```

output
```
log(log(x))
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/ln(x),x)
```

output
```
log(log(x))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="maxima")
```

output
```
log(log(x))
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\log(x)}\,dx = \log(|\log(x)|)$$

input
```
integrate(1/x/log(x),x, algorithm="giac")
```

output
```
log(abs(log(x)))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \ln(\ln(x))$$

input
```
int(1/(x*log(x)),x)
```

output
```
log(log(x))
```

3.62.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
int(1/(log(x)*x),x)
```

output
```
log(log(x))
```

3.63 $\int \frac{1}{x^2 \log^2(x)} \, dx$

3.63.1 Optimal result 486
3.63.2 Mathematica [A] (verified) 486
3.63.3 Rubi [A] (verified) 487
3.63.4 Maple [A] (verified) 488
3.63.5 Fricas [A] (verification not implemented) 488
3.63.6 Sympy [A] (verification not implemented) 489
3.63.7 Maxima [A] (verification not implemented) 489
3.63.8 Giac [F] 489
3.63.9 Mupad [B] (verification not implemented) 490
3.63.10 Reduce [B] (verification not implemented) 490

3.63.1 Optimal result

Integrand size = 8, antiderivative size = 17

$$\int \frac{1}{x^2 \log^2(x)} \, dx = -\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

output
```
-Ei(-ln(x))-1/x/ln(x)
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \log^2(x)} \, dx = -\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

input
```
Integrate[1/(x^2*Log[x]^2),x]
```

output
```
-ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

3.63.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2743, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\log^2(x)}\,dx$$
$$\Big\downarrow \text{2743}$$
$$-\int \frac{1}{x^2\log(x)}dx-\frac{1}{x\log(x)}$$
$$\Big\downarrow \text{2746}$$
$$-\int \frac{1}{x\log(x)}d\log(x)-\frac{1}{x\log(x)}$$
$$\Big\downarrow \text{2609}$$
$$-\text{ExpIntegralEi}(-\log(x))-\frac{1}{x\log(x)}$$

input
```
Int[1/(x^2*Log[x]^2),x]
```

output
```
-ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

3.63.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

3.63.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{x \ln(x)} + \mathrm{Ei}_1(\ln(x))$	15
risch	$-\frac{1}{x \ln(x)} + \mathrm{Ei}_1(\ln(x))$	15

input
```
int(1/x^2/ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x/ln(x)+Ei(1,ln(x))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2 \log^2(x)}\, dx = -\frac{x \log(x) \operatorname{log\_integral}\left(\frac{1}{x}\right) + 1}{x \log(x)}$$

input
```
integrate(1/x^2/log(x)^2,x, algorithm="fricas")
```

output
```
-(x*log(x)*log_integral(1/x) + 1)/(x*log(x))
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \log^2(x)}\,dx = -\operatorname{Ei}\left(-\log\left(x\right)\right) - \frac{1}{x\log\left(x\right)}$$

input
```
integrate(1/x**2/ln(x)**2,x)
```

output
```
-Ei(-log(x)) - 1/(x*log(x))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2 \log^2(x)}\,dx = -\Gamma\left(-1, \log\left(x\right)\right)$$

input
```
integrate(1/x^2/log(x)^2,x, algorithm="maxima")
```

output
```
-gamma(-1, log(x))
```

3.63.8 Giac [F]

$$\int \frac{1}{x^2 \log^2(x)}\,dx = \int \frac{1}{x^2 \log\left(x\right)^2}\,dx$$

input
```
integrate(1/x^2/log(x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^2*log(x)^2), x)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \log^2(x)} \, dx = -\mathrm{ei}(-\ln(x)) - \frac{1}{x \ln(x)}$$

input
```
int(1/(x^2*log(x)^2),x)
```

output
```
- ei(-log(x)) - 1/(x*log(x))
```

3.63.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2 \log^2(x)} \, dx = \frac{-ei(-\log(x)) \log(x) \, x - 1}{\log(x) \, x}$$

input
```
int(1/(log(x)**2*x**2),x)
```

output
```
( - (ei( - log(x))*log(x)*x + 1))/(log(x)*x)
```

3.64 $\int \frac{\log^p(x)}{x} \, dx$

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3.64.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{\log^p(x)}{x} \, dx = \frac{\log^{1+p}(x)}{1+p}$$

output
```
ln(x)^(p+1)/(p+1)
```

3.64.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x} \, dx = \frac{\log^{1+p}(x)}{1+p}$$

input
```
Integrate[Log[x]^p/x,x]
```

output
```
Log[x]^(1 + p)/(1 + p)
```

3.64.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^p(x)}{x} \, dx$$

$$\Big\downarrow \text{2739}$$

$$\int \log^p(x) d \log(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\log^{p+1}(x)}{p+1}$$

input
```
Int[Log[x]^p/x,x]
```

output
```
Log[x]^(1 + p)/(1 + p)
```

3.64.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.64.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+p}}{1+p}$	13
default	$\frac{\ln(x)^{1+p}}{1+p}$	13
risch	$\frac{\ln(x)\ln(x)^{p}}{1+p}$	13
norman	$\frac{\ln(x)\,\mathrm{e}^{p\ln(\ln(x))}}{1+p}$	15

input
```
int(ln(x)^p/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)^(1+p)/(1+p)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x}\,dx = \frac{\log(x)^p \log(x)}{p+1}$$

input
```
integrate(log(x)^p/x,x, algorithm="fricas")
```

output
```
log(x)^p*log(x)/(p + 1)
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log^p(x)}{x}\,dx = \begin{cases} \frac{\log(x)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(x)**p/x,x)
```

output
```
Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x}\,dx = \frac{\log{(x)}^{p+1}}{p+1}$$

input
```
integrate(log(x)^p/x,x, algorithm="maxima")
```

output
```
log(x)^(p + 1)/(p + 1)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x}\,dx = \frac{\log{(x)}^{p+1}}{p+1}$$

input
```
integrate(log(x)^p/x,x, algorithm="giac")
```

output
```
log(x)^(p + 1)/(p + 1)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\log^p(x)}{x}\,dx = \begin{cases} \ln{(\ln{(x)})} & \text{if } p = -1 \\ \frac{\ln(x)^{p+1}}{p+1} & \text{if } p \neq -1 \end{cases}$$

input
```
int(log(x)^p/x,x)
```

output
```
piecewise(p == -1, log(log(x)), p ~= -1, log(x)^(p + 1)/(p + 1))
```

3.64.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x}\,dx = \frac{\log(x)^p \log(x)}{p+1}$$

input
```
int(log(x)**p/x,x)
```

output
```
(log(x)**p*log(x))/(p + 1)
```

3.65 $\int (b+ax)\log(x)\,dx$

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3.65.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int (b+ax)\log(x)\,dx = -bx-\frac{ax^2}{4}+bx\log(x)+\frac{1}{2}ax^2\log(x)$$

output
```
-b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)
```

3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (b+ax)\log(x)\,dx = -bx-\frac{ax^2}{4}+bx\log(x)+\frac{1}{2}ax^2\log(x)$$

input
```
Integrate[(b + a*x)*Log[x],x]
```

output
```
-(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

3.65.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)(ax+b)\,dx$$

$$\Big\downarrow \text{ 2750}$$

$$\frac{\log(x)(ax+b)^2}{2a}-\int \frac{(b+ax)^2}{2ax}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\log(x)(ax+b)^2}{2a}-\frac{\int \frac{(b+ax)^2}{x}dx}{2a}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\log(x)(ax+b)^2}{2a}-\frac{\int \left(xa^2+2ba+\frac{b^2}{x}\right)dx}{2a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(x)(ax+b)^2}{2a}-\frac{\frac{a^2x^2}{2}+2abx+b^2\log(x)}{2a}$$

input
```
Int[(b + a*x)*Log[x],x]
```

output
```
((b + a*x)^2*Log[x])/(2*a) - (2*a*b*x + (a^2*x^2)/2 + b^2*Log[x])/(2*a)
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2750
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n])
  u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b,
 c, d, e, n, r}, x] && IGtQ[q, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
norman	$-bx-\frac{a\,x^{2}}{4}+bx\ln\left(x\right)+\frac{a\,x^{2}\ln\left(x\right)}{2}$	25
risch	$\left(\frac{1}{2}a\,x^{2}+bx\right)\ln\left(x\right)-\frac{a\,x^{2}}{4}-bx$	25
parallelrisch	$-bx-\frac{a\,x^{2}}{4}+bx\ln\left(x\right)+\frac{a\,x^{2}\ln\left(x\right)}{2}$	25
parts	$-bx-\frac{a\,x^{2}}{4}+bx\ln\left(x\right)+\frac{a\,x^{2}\ln\left(x\right)}{2}$	25
default	$a\left(-\frac{x^{2}}{4}+\frac{x^{2}\ln\left(x\right)}{2}\right)+b\left(-x+x\ln\left(x\right)\right)$	27

input
```
int((a*x+b)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (b+ax)\log(x)\,dx = -\frac{1}{4}\,ax^2 - bx + \frac{1}{2}\left(ax^2 + 2\,bx\right)\log\left(x\right)$$

input
```
integrate((a*x+b)*log(x),x, algorithm="fricas")
```

output
```
-1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (b+ax)\log(x)\,dx = -\frac{ax^2}{4} - bx + \left(\frac{ax^2}{2} + bx\right)\log\left(x\right)$$

input
```
integrate((a*x+b)*ln(x),x)
```

output
```
-a*x**2/4 - b*x + (a*x**2/2 + b*x)*log(x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (b+ax)\log(x)\,dx = -\frac{1}{4}\,ax^2 - bx + \frac{1}{2}\left(ax^2 + 2\,bx\right)\log\left(x\right)$$

input
```
integrate((a*x+b)*log(x),x, algorithm="maxima")
```

output
```
-1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (b+ax)\log(x)\,dx = \frac{1}{2}\,ax^2\log{(x)} - \frac{1}{4}\,ax^2 + bx\log{(x)} - bx$$

input
```
integrate((a*x+b)*log(x),x, algorithm="giac")
```

output
```
1/2*a*x^2*log(x) - 1/4*a*x^2 + b*x*log(x) - b*x
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (b+ax)\log(x)\,dx = -\frac{x\,(4\,b + a\,x - 4\,b\,\ln{(x)} - 2\,a\,x\,\ln{(x)})}{4}$$

input
```
int(log(x)*(b + a*x),x)
```

output
```
-(x*(4*b + a*x - 4*b*log(x) - 2*a*x*log(x)))/4
```

3.65.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (b+ax)\log(x)\,dx = \frac{x(2\log(x)\,ax + 4\log(x)\,b - ax - 4b)}{4}$$

input
```
int(log(x)*(a*x + b),x)
```

output
```
(x*(2*log(x)*a*x + 4*log(x)*b - a*x - 4*b))/4
```

3.66 $\int (b+ax)^2 \log(x)\,dx$

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3.66.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int (b+ax)^2 \log(x)\,dx = -b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} - \frac{b^3\log(x)}{3a} + \frac{(b+ax)^3\log(x)}{3a}$$

output
```
-b^2*x-1/2*a*b*x^2-1/9*a^2*x^3-1/3*b^3*ln(x)/a+1/3*(a*x+b)^3*ln(x)/a
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int (b+ax)^2 \log(x)\,dx = -b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} + b^2x\log(x) + abx^2\log(x) + \frac{1}{3}a^2x^3\log(x)$$

input
```
Integrate[(b + a*x)^2*Log[x],x]
```

output
```
-(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 + b^2*x*Log[x] + a*b*x^2*Log[x] + (a^
2*x^3*Log[x])/3
```

3.66.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)(ax+b)^2\,dx$$

$$\Big\downarrow \text{2750}$$

$$\frac{\log(x)(ax+b)^3}{3a}-\int \frac{(b+ax)^3}{3ax}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\log(x)(ax+b)^3}{3a}-\frac{\int \frac{(b+ax)^3}{x}dx}{3a}$$

$$\Big\downarrow \text{49}$$

$$\frac{\log(x)(ax+b)^3}{3a}-\frac{\int \left(x^2a^3+3bxa^2+3b^2a+\frac{b^3}{x}\right)dx}{3a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(x)(ax+b)^3}{3a}-\frac{\frac{a^3x^3}{3}+\frac{3}{2}a^2bx^2+3ab^2x+b^3\log(x)}{3a}$$

input
```
Int[(b + a*x)^2*Log[x],x]
```

output
```
((b + a*x)^3*Log[x])/(3*a) - (3*a*b^2*x + (3*a^2*b*x^2)/2 + (a^3*x^3)/3 +
b^3*Log[x])/(3*a)
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2750
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n])
  u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b,
 c, d, e, n, r}, x] && IGtQ[q, 0]
```

3.66.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result	size
risch	$-b^2x-\frac{ab\,x^2}{2}-\frac{a^2x^3}{9}-\frac{b^3\ln(x)}{3a}+\frac{(ax+b)^3\ln(x)}{3a}$	47
default	$a^2\left(-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}\right)+2ab\left(-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}\right)+b^2(-x+x\ln(x))$	48
norman	$b^2x\ln(x)+ab\,x^2\ln(x)-\frac{a^2x^3}{9}-b^2x-\frac{ab\,x^2}{2}+\frac{a^2x^3\ln(x)}{3}$	48
parallelrisch	$b^2x\ln(x)+ab\,x^2\ln(x)-\frac{a^2x^3}{9}-b^2x-\frac{ab\,x^2}{2}+\frac{a^2x^3\ln(x)}{3}$	48
parts	$\frac{a^2x^3\ln(x)}{3}+ab\,x^2\ln(x)+b^2x\ln(x)+\frac{b^3\ln(x)}{3a}-\frac{\frac{a^3x^3}{3}+\frac{3a^2b\,x^2}{2}+3a\,b^2x+b^3\ln(x)}{3a}$	73

input
```
int((a*x+b)^2*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-b^2*x-1/2*a*b*x^2-1/9*a^2*x^3-1/3*b^3*ln(x)/a+1/3*(a*x+b)^3*ln(x)/a
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b+ax)^2 \log(x)\,dx = -\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\,\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log\left(x\right)$$

input
```
integrate((a*x+b)^2*log(x),x, algorithm="fricas")
```

output
```
-1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*l
og(x)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (b+ax)^2 \log(x)\,dx = -\frac{a^2x^3}{9} - \frac{abx^2}{2} - b^2x + \left(\frac{a^2x^3}{3} + abx^2 + b^2x\right)\log\left(x\right)$$

input
```
integrate((a*x+b)**2*ln(x),x)
```

output
```
-a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b**2*x)*log
(x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b+ax)^2 \log(x)\,dx = -\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\,\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log\left(x\right)$$

input
```
integrate((a*x+b)^2*log(x),x, algorithm="maxima")
```

output
```
-1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*l
og(x)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b+ax)^2 \log(x)\,dx = \frac{1}{3}\,a^2x^3\log{(x)} - \frac{1}{9}\,a^2x^3 + abx^2\log{(x)} - \frac{1}{2}\,abx^2 + b^2x\log{(x)} - b^2x$$

input
```
integrate((a*x+b)^2*log(x),x, algorithm="giac")
```

output
```
1/3*a^2*x^3*log(x) - 1/9*a^2*x^3 + a*b*x^2*log(x) - 1/2*a*b*x^2 + b^2*x*lo
g(x) - b^2*x
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b+ax)^2 \log(x)\,dx = b^2\,x\,\ln{(x)} - \frac{a^2\,x^3}{9} - b^2\,x + \frac{a^2\,x^3\,\ln{(x)}}{3} - \frac{a\,b\,x^2}{2} + a\,b\,x^2\,\ln{(x)}$$

input
```
int(log(x)*(b + a*x)^2,x)
```

output
```
b^2*x*log(x) - (a^2*x^3)/9 - b^2*x + (a^2*x^3*log(x))/3 - (a*b*x^2)/2 + a*
b*x^2*log(x)
```

3.66.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int (b+ax)^2 \log(x)\,dx = \frac{x(6\log(x)\,a^2x^2 + 18\log(x)\,abx + 18\log(x)\,b^2 - 2a^2x^2 - 9abx - 18b^2)}{18}$$

input
```
int(log(x)*(a**2*x**2 + 2*a*b*x + b**2),x)
```

output
```
(x*(6*log(x)*a**2*x**2 + 18*log(x)*a*b*x + 18*log(x)*b**2 - 2*a**2*x**2 -
9*a*b*x - 18*b**2))/18
```

3.67 $\int \frac{\log(x)}{(b+ax)^2} dx$

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3.67.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{(b+ax)^2} dx = \frac{x\log(x)}{b(b+ax)} - \frac{\log(b+ax)}{ab}$$

output
```
x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\log(x)}{(b+ax)^2} dx = \frac{\frac{x\log(x)}{b+ax} - \frac{\log(b+ax)}{a}}{b}$$

input
```
Integrate[Log[x]/(b + a*x)^2,x]
```

output
```
((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b
```

3.67.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{(ax+b)^2}\,dx$$

$$\Big\downarrow \text{ 2751}$$

$$\frac{x\log(x)}{b(ax+b)} - \frac{\int \frac{1}{b+ax}dx}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x\log(x)}{b(ax+b)} - \frac{\log(ax+b)}{ab}$$

input
```
Int[Log[x]/(b + a*x)^2,x]
```

output
```
(x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

3.67.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

3.67.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x\ln(x)}{b(ax+b)}-\frac{\ln(ax+b)}{ab}$	30
norman	$\frac{x\ln(x)}{b(ax+b)}-\frac{\ln(ax+b)}{ab}$	30
parts	$-\frac{\ln(x)}{a(ax+b)}+\frac{\frac{\ln(x)}{b}-\frac{\ln(ax+b)}{b}}{a}$	38
parallelrisch	$\frac{-\ln(ax+b)xa+\ln(x)ax-\ln(ax+b)b}{ab(ax+b)}$	40
risch	$-\frac{\ln(x)}{a(ax+b)}+\frac{\ln(-x)}{ba}-\frac{\ln(ax+b)}{ab}$	41

input
```
int(ln(x)/(a*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\log(x)}{(b+ax)^2}\,dx = \frac{ax\log(x)-(ax+b)\log(ax+b)}{a^2bx+ab^2}$$

input
```
integrate(log(x)/(a*x+b)^2,x, algorithm="fricas")
```

output
```
(a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\log(x)}{(b+ax)^2}\,dx = -\frac{\log(x)}{a^2x+ab}+\frac{\log(x)-\log\left(x+\frac{b}{a}\right)}{ab}$$

input
```
integrate(ln(x)/(a*x+b)**2,x)
```

output
```
-log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\log(x)}{(b+ax)^2}\,dx = -\frac{\frac{\log(ax+b)}{b}-\frac{\log(x)}{b}}{a}-\frac{\log(x)}{(ax+b)a}$$

input
```
integrate(log(x)/(a*x+b)^2,x, algorithm="maxima")
```

output
```
-(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(29) = 58.

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.76

$$\int \frac{\log(x)}{(b+ax)^2}\,dx = a^2\left(\frac{\log\left(\frac{(ax+b)^2|a|\left|\frac{b}{ax+b}-1\right|}{a^2|ax+b|}\right)}{a^3b}+\frac{\log\left(-\frac{b+\frac{(ax+b)a\left(\frac{b}{ax+b}-1\right)-ab}{a}}{a}\right)}{\left((ax+b)\left(\frac{b}{ax+b}-1\right)-b\right)a^3}-\frac{\log\left(\left|-(ax+b)\left(\frac{b}{ax+b}-1\right)+b\right|\right)}{a^3b}\right)$$

input
```
integrate(log(x)/(a*x+b)^2,x, algorithm="giac")
```

output
```
a^2*(log((a*x + b)^2*abs(a)*abs(b/(a*x + b) - 1)/(a^2*abs(a*x + b)))/(a^3*
b) + log(-(b + ((a*x + b)*a*(b/(a*x + b) - 1) - a*b)/a)/a)/(((a*x + b)*(b/
(a*x + b) - 1) - b)*a^3) - log(abs(-(a*x + b)*(b/(a*x + b) - 1) + b))/(a^3
*b))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\log(x)}{(b+ax)^2} \, dx = \frac{x^2 \ln(x)}{b\,(a\,x^2+b\,x)} - \frac{\ln(b+a\,x)}{a\,b}$$

input
```
int(log(x)/(b + a*x)^2,x)
```

output
```
(x^2*log(x))/(b*(b*x + a*x^2)) - log(b + a*x)/(a*b)
```

3.67.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\log(x)}{(b+ax)^2} \, dx = \frac{-\log(ax+b)\,ax - \log(ax+b)\,b + \log(x)\,ax}{ab\,(ax+b)}$$

input
```
int(log(x)/(a**2*x**2 + 2*a*b*x + b**2),x)
```

output
```
( - log(a*x + b)*a*x - log(a*x + b)*b + log(x)*a*x)/(a*b*(a*x + b))
```

3.68 $\int x \log(b + ax) \, dx$

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3.68.1 Optimal result

Integrand size = 8, antiderivative size = 46

$$\int x \log(b + ax) \, dx = \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b + ax)}{2a^2} + \frac{1}{2}x^2 \log(b + ax)$$

output
```
1/2*b*x/a-1/4*x^2-1/2*b^2*ln(a*x+b)/a^2+1/2*x^2*ln(a*x+b)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int x \log(b + ax) \, dx = \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b + ax)}{2a^2} + \frac{1}{2}x^2 \log(b + ax)$$

input
```
Integrate[x*Log[b + a*x],x]
```

output
```
(b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

3.68.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log(ax+b)\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}x^2\log(ax+b)-\frac{1}{2}a\int \frac{x^2}{b+ax}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}x^2\log(ax+b)-\frac{1}{2}a\int \left(\frac{b^2}{a^2(b+ax)}-\frac{b}{a^2}+\frac{x}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2\log(ax+b)-\frac{1}{2}a\left(\frac{b^2\log(ax+b)}{a^3}-\frac{bx}{a^2}+\frac{x^2}{2a}\right)$$

input
```
Int[x*Log[b + a*x],x]
```

output
```
(x^2*Log[b + a*x])/2 - (a*(-((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b + a*x])/a
^3))/2
```

3.68.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.68.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\ln(ax+b)}{2a^2}+\frac{x^2\ln(ax+b)}{2}$	39
risch	$\frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\ln(ax+b)}{2a^2}+\frac{x^2\ln(ax+b)}{2}$	39
parts	$\frac{x^2\ln(ax+b)}{2}-\frac{a\left(\frac{\frac{1}{2}a\,x^2-bx}{a^2}+\frac{b^2\ln(ax+b)}{a^3}\right)}{2}$	45
parallelrisch	$\frac{2x^2\ln(ax+b)a^2-a^2x^2+2axb-2b^2\ln(ax+b)-2b^2}{4a^2}$	50
derivativedivides	$\frac{-b(\ln(ax+b)(ax+b)-ax-b)+\frac{(ax+b)^2\ln(ax+b)}{2}-\frac{(ax+b)^2}{4}}{a^2}$	53
default	$\frac{-b(\ln(ax+b)(ax+b)-ax-b)+\frac{(ax+b)^2\ln(ax+b)}{2}-\frac{(ax+b)^2}{4}}{a^2}$	53

input
```
int(x*ln(a*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x/a-1/4*x^2-1/2*b^2*ln(a*x+b)/a^2+1/2*x^2*ln(a*x+b)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int x\log(b+ax)\,dx=-\frac{a^2x^2-2\,abx-2\,(a^2x^2-b^2)\log{(ax+b)}}{4\,a^2}$$

input
```
integrate(x*log(a*x+b),x, algorithm="fricas")
```

output
```
-1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x\log(b+ax)\,dx = -a\left(\frac{x^2}{4a}-\frac{bx}{2a^2}+\frac{b^2\log{(ax+b)}}{2a^3}\right)+\frac{x^2\log{(ax+b)}}{2}$$

input
```
integrate(x*ln(a*x+b),x)
```

output
```
-a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**2*log(a*x
 + b)/2
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x\log(b+ax)\,dx = \frac{1}{2}\,x^2\log{(ax+b)} - \frac{1}{4}\,a\left(\frac{2\,b^2\log{(ax+b)}}{a^3}+\frac{ax^2-2\,bx}{a^2}\right)$$

input
```
integrate(x*log(a*x+b),x, algorithm="maxima")
```

output
```
1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b*x)/a^2
)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int x\log(b+ax)\,dx = \frac{(ax+b)^2\log{(ax+b)}}{2\,a^2} - \frac{(ax+b)b\log{(ax+b)}}{a^2} - \frac{(ax+b)^2}{4\,a^2}+\frac{(ax+b)b}{a^2}$$

input
```
integrate(x*log(a*x+b),x, algorithm="giac")
```

output
```
1/2*(a*x + b)^2*log(a*x + b)/a^2 - (a*x + b)*b*log(a*x + b)/a^2 - 1/4*(a*x
 + b)^2/a^2 + (a*x + b)*b/a^2
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int x\log(b+ax)\,dx = \begin{cases} \frac{x^2\left(\ln(a\,x)-\frac{1}{2}\right)}{2} & \text{if } b=0 \\ \frac{\ln(b+a\,x)\left(x^2-\frac{b^2}{a^2}\right)}{2}-\frac{b^2\left(\frac{a^2\,x^2}{2\,b^2}-\frac{a\,x}{b}\right)}{2\,a^2} & \text{if } b\neq 0 \end{cases}$$

input
```
int(x*log(b + a*x),x)
```

output
```
piecewise(b == 0, (x^2*(log(a*x) - 1/2))/2, b ~= 0, (log(b + a*x)*(x^2 - b
^2/a^2))/2 - (b^2*((a^2*x^2)/(2*b^2) - (a*x)/b))/(2*a^2))
```

3.68.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x\log(b+ax)\,dx = \frac{2\log(ax+b)\,a^2x^2-2\log(ax+b)\,b^2-a^2x^2+2abx}{4a^2}$$

input
```
int(log(a*x + b)*x,x)
```

output
```
(2*log(a*x + b)*a**2*x**2 - 2*log(a*x + b)*b**2 - a**2*x**2 + 2*a*b*x)/(4*
a**2)
```

3.69 $\int x^2 \log(b+ax)\,dx$

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3.69.10 Reduce [B] (verification not implemented) 520

3.69.1 Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x^2 \log(b+ax)\,dx = -\frac{b^2x}{3a^2}+\frac{bx^2}{6a}-\frac{x^3}{9}+\frac{b^3\log(b+ax)}{3a^3}+\frac{1}{3}x^3\log(b+ax)$$

output
```
-1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*ln(a*x+b)/a^3+1/3*x^3*ln(a*x+b)
```

3.69.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^2 \log(b+ax)\,dx = -\frac{b^2x}{3a^2}+\frac{bx^2}{6a}-\frac{x^3}{9}+\frac{b^3\log(b+ax)}{3a^3}+\frac{1}{3}x^3\log(b+ax)$$

input
```
Integrate[x^2*Log[b + a*x],x]
```

output
```
-1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x
^3*Log[b + a*x])/3
```

3.69.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(ax+b) \, dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}x^3 \log(ax+b) - \frac{1}{3}a \int \frac{x^3}{b+ax} dx$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}x^3 \log(ax+b) - \frac{1}{3}a \int \left(-\frac{b^3}{a^3(b+ax)} + \frac{b^2}{a^3} - \frac{xb}{a^2} + \frac{x^2}{a}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \log(ax+b) - \frac{1}{3}a\left(-\frac{b^3 \log(ax+b)}{a^4} + \frac{b^2 x}{a^3} - \frac{bx^2}{2a^2} + \frac{x^3}{3a}\right)$$

input
```
Int[x^2*Log[b + a*x],x]
```

output
```
(x^3*Log[b + a*x])/3 - (a*((b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*a) - (b^
3*Log[b + a*x])/a^4))/3
```

3.69.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.69.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

method	result	size
norman	$-\frac{b^{2} x}{3 a^{2}}+\frac{b\,x^{2}}{6 a}-\frac{x^{3}}{9}+\frac{b^{3} \ln \left(a x +b \right)}{3 a^{3}}+\frac{x^{3} \ln \left(a x +b \right)}{3}$	50
risch	$-\frac{b^{2} x}{3 a^{2}}+\frac{b\,x^{2}}{6 a}-\frac{x^{3}}{9}+\frac{b^{3} \ln \left(a x +b \right)}{3 a^{3}}+\frac{x^{3} \ln \left(a x +b \right)}{3}$	50
parts	$\frac{x^{3} \ln \left(a x +b \right)}{3}-\frac{a \left(\frac{\frac{1}{3} a^{2} x^{3}-\frac{1}{2} a b\,x^{2}+b^{2} x}{a^{3}}-\frac{b^{3} \ln \left(a x +b \right)}{a^{4}}\right)}{3}$	56
parallelrisch	$\frac{6 x^{3} \ln \left(a x +b \right) a^{3}-2 a^{3} x^{3}+3 a^{2} b\,x^{2}-6 a\,b^{2} x +6 b^{3} \ln \left(a x +b \right)+6 b^{3}}{18 a^{3}}$	61
derivativedivides	$\frac{b^{2} \left(\ln \left(a x +b \right) \left(a x +b \right)-a x -b \right)-2 b \left(\frac{\left(a x +b \right)^{2} \ln \left(a x +b \right)}{2}-\frac{\left(a x +b \right)^{2}}{4}\right)+\frac{\left(a x +b \right)^{3} \ln \left(a x +b \right)}{3}-\frac{\left(a x +b \right)^{3}}{9}}{a^{3}}$	82
default	$\frac{b^{2} \left(\ln \left(a x +b \right) \left(a x +b \right)-a x -b \right)-2 b \left(\frac{\left(a x +b \right)^{2} \ln \left(a x +b \right)}{2}-\frac{\left(a x +b \right)^{2}}{4}\right)+\frac{\left(a x +b \right)^{3} \ln \left(a x +b \right)}{3}-\frac{\left(a x +b \right)^{3}}{9}}{a^{3}}$	82

input
```
int(x^2*ln(a*x+b),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*ln(a*x+b)/a^3+1/3*x^3*ln(a*x+b)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \log(b+ax)\,dx = -\frac{2\,a^3x^3 - 3\,a^2bx^2 + 6\,ab^2x - 6\,(a^3x^3 + b^3)\log{(ax+b)}}{18\,a^3}$$

input
```
integrate(x^2*log(a*x+b),x, algorithm="fricas")
```

output
```
-1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*log(a*x + b
))/a^3
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^2 \log(b+ax)\, dx = -a\left(\frac{x^3}{9a} - \frac{bx^2}{6a^2} + \frac{b^2x}{3a^3} - \frac{b^3 \log{(ax+b)}}{3a^4}\right) + \frac{x^3 \log{(ax+b)}}{3}$$

input
```
integrate(x**2*ln(a*x+b),x)
```

output
```
-a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x + b)/(3*
a**4)) + x**3*log(a*x + b)/3
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^2 \log(b+ax)\, dx = \frac{1}{3}\, x^3 \log{(ax+b)} + \frac{1}{18}\, a\left(\frac{6\, b^3 \log{(ax+b)}}{a^4} - \frac{2\, a^2x^3 - 3\, abx^2 + 6\, b^2x}{a^3}\right)$$

input
```
integrate(x^2*log(a*x+b),x, algorithm="maxima")
```

output
```
1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3 - 3*a*b
*x^2 + 6*b^2*x)/a^3)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int x^2 \log(b+ax)\, dx = \frac{(ax+b)^3 \log{(ax+b)}}{3\, a^3} - \frac{(ax+b)^2 b \log{(ax+b)}}{a^3} + \frac{(ax+b)b^2 \log{(ax+b)}}{a^3} - \frac{(ax+b)^3}{9\, a^3} + \frac{(ax+b)^2 b}{2\, a^3} - \frac{(ax+b)b^2}{a^3}$$

input
```
integrate(x^2*log(a*x+b),x, algorithm="giac")
```

output
```
1/3*(a*x + b)^3*log(a*x + b)/a^3 - (a*x + b)^2*b*log(a*x + b)/a^3 + (a*x +
 b)*b^2*log(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x + b)^2*b/a^3 - (
a*x + b)*b^2/a^3
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x^2 \log(b+ax)\,dx = \begin{cases} \frac{x^3\left(\ln(a\,x)-\frac{1}{3}\right)}{3} & \text{if } b=0 \\ \frac{\ln(b+a\,x)\left(x^3+\frac{b^3}{a^3}\right)}{3} - \frac{b^3\left(\frac{a^3\,x^3}{3\,b^3}-\frac{a^2\,x^2}{2\,b^2}+\frac{a\,x}{b}\right)}{3\,a^3} & \text{if } b\neq 0 \end{cases}$$

input
```
int(x^2*log(b + a*x),x)
```

output
```
piecewise(b == 0, (x^3*(log(a*x) - 1/3))/3, b ~= 0, (log(b + a*x)*(x^3 + b
^3/a^3))/3 - (b^3*(- (a^2*x^2)/(2*b^2) + (a^3*x^3)/(3*b^3) + (a*x)/b))/(3*
a^3))
```

3.69.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^2 \log(b+ax)\,dx = \frac{6\log(ax+b)\,a^3x^3+6\log(ax+b)\,b^3-2a^3x^3+3a^2b\,x^2-6a\,b^2x}{18a^3}$$

input
```
int(log(a*x + b)*x**2,x)
```

output
```
(6*log(a*x + b)*a**3*x**3 + 6*log(a*x + b)*b**3 - 2*a**3*x**3 + 3*a**2*b*x
**2 - 6*a*b**2*x)/(18*a**3)
```

3.70 $\int \log \left(a^2+x^2\right) dx$

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3.70.1 Optimal result

Integrand size = 8, antiderivative size = 23

$$\int \log \left(a^2+x^2\right) dx=-2x+2a\arctan\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)$$

output
```
-2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log \left(a^2+x^2\right) dx=-2x+2a\arctan\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)$$

input
```
Integrate[Log[a^2 + x^2],x]
```

output
```
-2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

3.70.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2898, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a^2+x^2\right) \, dx$$

$$\Big\downarrow \text{ 2898}$$

$$x \log \left(a^2+x^2\right)-2 \int \frac{x^2}{a^2+x^2}dx$$

$$\Big\downarrow \text{ 262}$$

$$x \log \left(a^2+x^2\right)-2 \left(x-a^2 \int \frac{1}{a^2+x^2}dx\right)$$

$$\Big\downarrow \text{ 216}$$

$$x \log \left(a^2+x^2\right)-2 \left(x-a \arctan \left(\frac{x}{a}\right)\right)$$

input
```
Int[Log[a^2 + x^2],x]
```

output
```
-2*(x - a*ArcTan[x/a]) + x*Log[a^2 + x^2]
```

3.70.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

3.70.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$-2x+2a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^{2}+x^{2}\right)$	24
risch	$-2x+2a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^{2}+x^{2}\right)$	24
parts	$-2x+2a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^{2}+x^{2}\right)$	24
parallelrisch	$-2i\ln\left(-ia+x\right)a+ia\ln\left(a^{2}+x^{2}\right)+x\ln\left(a^{2}+x^{2}\right)-2x$	38

input
```
int(ln(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log \left(a^2 + x^2\right) \, dx = 2\, a \arctan \left(\frac{x}{a}\right) + x \log \left(a^2 + x^2\right) - 2\, x$$

input
```
integrate(log(a^2+x^2),x, algorithm="fricas")
```

output
```
2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

3.70.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \log \left(a^2 + x^2\right) \, dx = -2a \left(\frac{i \log \left(-ia + x\right)}{2} - \frac{i \log \left(ia + x\right)}{2}\right) + x \log \left(a^2 + x^2\right) - 2x$$

input
```
integrate(ln(a**2+x**2),x)
```

output
```
-2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2) - 2*x
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log \left(a^2 + x^2\right) \, dx = 2\, a \arctan \left(\frac{x}{a}\right) + x \log \left(a^2 + x^2\right) - 2\, x$$

input
```
integrate(log(a^2+x^2),x, algorithm="maxima")
```

output
```
2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log \left(a^2+x^2\right) \, dx = 2\, a \arctan \left(\frac{x}{a}\right) + x \log \left(a^2+x^2\right) - 2\, x$$

input
```
integrate(log(a^2+x^2),x, algorithm="giac")
```

output
```
2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log \left(a^2+x^2\right) \, dx = x\, \ln \left(a^2+x^2\right) - 2\, x + 2\, a\, \mathrm{atan}\left(\frac{x}{a}\right)$$

input
```
int(log(a^2 + x^2),x)
```

output
```
x*log(a^2 + x^2) - 2*x + 2*a*atan(x/a)
```

3.70.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log \left(a^2+x^2\right) \, dx = 2 atan\left(\frac{x}{a}\right) a + \log\left(a^2+x^2\right) x - 2x$$

input
```
int(log(a**2 + x**2),x)
```

output
```
2*atan(x/a)*a + log(a**2 + x**2)*x - 2*x
```

3.71 $\int x \log(a^2+x^2)\,dx$

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3.71.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int x \log(a^2+x^2)\,dx = -\frac{x^2}{2}+\frac{1}{2}(a^2+x^2)\log(a^2+x^2)$$

output
```
-1/2*x^2+1/2*(a^2+x^2)*ln(a^2+x^2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x \log(a^2+x^2)\,dx = \frac{1}{2}\left(-x^2+(a^2+x^2)\log(a^2+x^2)\right)$$

input
```
Integrate[x*Log[a^2 + x^2],x]
```

output
```
(-x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2
```

3.71.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(a^2+x^2\right) \, dx$$

$$\Big\downarrow \; 2904$$

$$\frac{1}{2} \int \log \left(a^2+x^2\right) dx^2$$

$$\Big\downarrow \; 2836$$

$$\frac{1}{2} \int \log \left(a^2+x^2\right) d\left(a^2+x^2\right)$$

$$\Big\downarrow \; 2732$$

$$\frac{1}{2}\left(\left(a^2+x^2\right) \log \left(a^2+x^2\right)-a^2-x^2\right)$$

input
```
Int[x*Log[a^2 + x^2],x]
```

output
```
(-a^2 - x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2
```

3.71.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

3.71.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{(a^2+x^2)\ln(a^2+x^2)}{2}-\frac{a^2}{2}-\frac{x^2}{2}$	29
default	$\frac{(a^2+x^2)\ln(a^2+x^2)}{2}-\frac{a^2}{2}-\frac{x^2}{2}$	29
norman	$-\frac{x^2}{2}+\frac{x^2\ln(a^2+x^2)}{2}+\frac{\ln(a^2+x^2)a^2}{2}$	33
risch	$-\frac{x^2}{2}+\frac{x^2\ln(a^2+x^2)}{2}+\frac{\ln(a^2+x^2)a^2}{2}$	33
parts	$-\frac{x^2}{2}+\frac{x^2\ln(a^2+x^2)}{2}+\frac{\ln(a^2+x^2)a^2}{2}$	33
parallelrisch	$\frac{x^2\ln(a^2+x^2)}{2}+\frac{\ln(a^2+x^2)a^2}{2}-\frac{x^2}{2}+\frac{a^2}{2}$	38

input
```
int(x*ln(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2+x^2)*ln(a^2+x^2)-1/2*a^2-1/2*x^2
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x\log\left(a^2+x^2\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(a^2+x^2\right)\log\left(a^2+x^2\right)$$

input
```
integrate(x*log(a^2+x^2),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x\log\left(a^2+x^2\right)\,dx=\frac{a^2\log\left(a^2+x^2\right)}{2}+\frac{x^2\log\left(a^2+x^2\right)}{2}-\frac{x^2}{2}$$

input
```
integrate(x*ln(a**2+x**2),x)
```

output
```
a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int x\log\left(a^2+x^2\right)\,dx=-\frac{1}{2}\,a^2-\frac{1}{2}\,x^2+\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)$$

input
```
integrate(x*log(a^2+x^2),x, algorithm="maxima")
```

output
```
-1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int x\log\left(a^2+x^2\right)\,dx=-\frac{1}{2}\,a^2-\frac{1}{2}\,x^2+\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)$$

input
```
integrate(x*log(a^2+x^2),x, algorithm="giac")
```

output
```
-1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int x\log\left(a^2+x^2\right)\,dx=\frac{a^2\,\ln\left(x-\sqrt{-a^2}\right)}{2}+\frac{x^2\,\ln\left(a^2+x^2\right)}{2}-\frac{x^2}{2}+\frac{a^2\,\ln\left(x+\sqrt{-a^2}\right)}{2}$$

input
```
int(x*log(a^2 + x^2),x)
```

output
```
(a^2*log(x - (-a^2)^(1/2)))/2 + (x^2*log(a^2 + x^2))/2 - x^2/2 + (a^2*log(
x + (-a^2)^(1/2)))/2
```

3.71.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x\log\left(a^2+x^2\right)\,dx=\frac{\log(a^2+x^2)\,a^2}{2}+\frac{\log(a^2+x^2)\,x^2}{2}-\frac{x^2}{2}$$

input
```
int(log(a**2 + x**2)*x,x)
```

output
```
(log(a**2 + x**2)*a**2 + log(a**2 + x**2)*x**2 - x**2)/2
```

3.72 $\int x^2 \log(a^2+x^2)\,dx$

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3.72.1 Optimal result

Integrand size = 12, antiderivative size = 44

$$\int x^2 \log(a^2+x^2)\,dx = \frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2+x^2)$$

output
```
2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x^2 \log(a^2+x^2)\,dx = \frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2+x^2)$$

input
```
Integrate[x^2*Log[a^2 + x^2],x]
```

output
```
(2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

3.72.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(a^2+x^2\right) \, dx$$

$$\Big\downarrow \; 2905$$

$$\frac{1}{3} x^3 \log \left(a^2+x^2\right)-\frac{2}{3} \int \frac{x^4}{a^2+x^2}dx$$

$$\Big\downarrow \; 254$$

$$\frac{1}{3} x^3 \log \left(a^2+x^2\right)-\frac{2}{3} \int \left(\frac{a^4}{a^2+x^2}-a^2+x^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} x^3 \log \left(a^2+x^2\right)-\frac{2}{3} \left(a^3 \arctan \left(\frac{x}{a}\right)-a^2 x+\frac{x^3}{3}\right)$$

input
```
Int[x^2*Log[a^2 + x^2],x]
```

output
```
(-2*(-(a^2*x) + x^3/3 + a^3*ArcTan[x/a]))/3 + (x^3*Log[a^2 + x^2])/3
```

3.72.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

3.72.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2a^3\arctan\left(\frac{x}{a}\right)}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}$	37
risch	$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2a^3\arctan\left(\frac{x}{a}\right)}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}$	37
parts	$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2a^3\arctan\left(\frac{x}{a}\right)}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}$	37
parallelrisch	$\frac{2i\ln(-ia+x)a^3}{3}-\frac{i\ln\left(a^2+x^2\right)a^3}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}-\frac{2x^3}{9}+\frac{2a^2x}{3}$	53

input
```
int(x^2*ln(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2\log\left(a^2+x^2\right)\,dx=-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

input
```
integrate(x^2*log(a^2+x^2),x, algorithm="fricas")
```

output
```
-2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int x^2 \log \left(a^2+x^2\right) \, dx = -2a^3\left(-\frac{i\log \left(-ia+x\right)}{6}+\frac{i\log \left(ia+x\right)}{6}\right) + \frac{2a^2x}{3}+\frac{x^3\log \left(a^2+x^2\right)}{3}-\frac{2x^3}{9}$$

input
```
integrate(x**2*ln(a**2+x**2),x)
```

output
```
-2*a**3*(-I*log(-I*a + x)/6 + I*log(I*a + x)/6) + 2*a**2*x/3 + x**3*log(a*
*2 + x**2)/3 - 2*x**3/9
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \log \left(a^2+x^2\right) \, dx = -\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

input
```
integrate(x^2*log(a^2+x^2),x, algorithm="maxima")
```

output
```
-2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \log \left(a^2+x^2\right) \, dx = -\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

input
```
integrate(x^2*log(a^2+x^2),x, algorithm="giac")
```

output
```
-2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int x^2 \log \left(a^2+x^2\right) \, dx = \frac{2 a^2 x}{3} - \frac{\ln \left(x-\sqrt{-a^2}\right) \left(-a^2\right)^{3/2}}{3} + \frac{x^3 \ln \left(a^2+x^2\right)}{3} + \frac{\ln \left(x+\sqrt{-a^2}\right) \left(-a^2\right)^{3/2}}{3} - \frac{2 x^3}{9}$$

input
```
int(x^2*log(a^2 + x^2),x)
```

output
```
(2*a^2*x)/3 - (log(x - (-a^2)^(1/2))*(-a^2)^(3/2))/3 + (x^3*log(a^2 + x^2)
)/3 + (log(x + (-a^2)^(1/2))*(-a^2)^(3/2))/3 - (2*x^3)/9
```

3.72.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \log \left(a^2+x^2\right) \, dx = -\frac{2atan\left(\frac{x}{a}\right) a^3}{3} + \frac{\log(a^2+x^2)\, x^3}{3} + \frac{2a^2x}{3} - \frac{2x^3}{9}$$

input
```
int(log(a**2 + x**2)*x**2,x)
```

output
```
( - 6*atan(x/a)*a**3 + 3*log(a**2 + x**2)*x**3 + 6*a**2*x - 2*x**3)/9
```

3.73 $\int x^4 \log (a^2 + x^2)\, dx$

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3.73.1 Optimal result

Integrand size = 12, antiderivative size = 54

$$\int x^4 \log \left(a^2+x^2\right)\, dx=-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)$$

output
```
-2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^4 \log \left(a^2+x^2\right)\, dx=-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)$$

input
```
Integrate[x^4*Log[a^2 + x^2],x]
```

output
```
(-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*
Log[a^2 + x^2])/5
```

3.73.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log \left(a^2+x^2\right) \, dx$$

$$\Big\downarrow \ 2905$$

$$\frac{1}{5} x^5 \log \left(a^2+x^2\right)-\frac{2}{5} \int \frac{x^6}{a^2+x^2}dx$$

$$\Big\downarrow \ 254$$

$$\frac{1}{5} x^5 \log \left(a^2+x^2\right)-\frac{2}{5} \int \left(-\frac{a^6}{a^2+x^2}+a^4-x^2 a^2+x^4\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5} x^5 \log \left(a^2+x^2\right)-\frac{2}{5} \left(a^5 \left(-\arctan \left(\frac{x}{a}\right)\right)+a^4 x-\frac{a^2 x^3}{3}+\frac{x^5}{5}\right)$$

input
```
Int[x^4*Log[a^2 + x^2],x]
```

output
```
(-2*(a^4*x - (a^2*x^3)/3 + x^5/5 - a^5*ArcTan[x/a]))/5 + (x^5*Log[a^2 + x^
2])/5
```

3.73.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

3.73.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2a^5\arctan\left(\frac{x}{a}\right)}{5}+\frac{x^5\ln\left(a^2+x^2\right)}{5}$	45
risch	$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2a^5\arctan\left(\frac{x}{a}\right)}{5}+\frac{x^5\ln\left(a^2+x^2\right)}{5}$	45
parts	$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2a^5\arctan\left(\frac{x}{a}\right)}{5}+\frac{x^5\ln\left(a^2+x^2\right)}{5}$	45
parallelrisch	$-\frac{2i\ln\left(-ia+x\right)a^5}{5}+\frac{i\ln\left(a^2+x^2\right)a^5}{5}+\frac{x^5\ln\left(a^2+x^2\right)}{5}-\frac{2x^5}{25}+\frac{2a^2x^3}{15}-\frac{2a^4x}{5}$	61

input
```
int(x^4*ln(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4\log\left(a^2+x^2\right)\,dx=\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

input
```
integrate(x^4*log(a^2+x^2),x, algorithm="fricas")
```

output
```
2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 -
2/25*x^5
```

3.73.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int x^4 \log \left(a^2+x^2\right) \, dx = -2a^5 \left(\frac{i \log \left(-ia+x\right)}{10} - \frac{i \log \left(ia+x\right)}{10}\right) - \frac{2a^4x}{5} + \frac{2a^2x^3}{15} + \frac{x^5 \log \left(a^2+x^2\right)}{5} - \frac{2x^5}{25}$$

input
```
integrate(x**4*ln(a**2+x**2),x)
```

output
```
-2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2*a**2*x**
3/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4 \log \left(a^2+x^2\right) \, dx = \frac{2}{5} \, a^5 \arctan \left(\frac{x}{a}\right) + \frac{1}{5} \, x^5 \log \left(a^2+x^2\right) - \frac{2}{5} \, a^4 x + \frac{2}{15} \, a^2 x^3 - \frac{2}{25} \, x^5$$

input
```
integrate(x^4*log(a^2+x^2),x, algorithm="maxima")
```

output
```
2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 -
2/25*x^5
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4 \log \left(a^2+x^2\right) \, dx = \frac{2}{5} \, a^5 \arctan \left(\frac{x}{a}\right) + \frac{1}{5} \, x^5 \log \left(a^2+x^2\right) - \frac{2}{5} \, a^4 x + \frac{2}{15} \, a^2 x^3 - \frac{2}{25} \, x^5$$

input
```
integrate(x^4*log(a^2+x^2),x, algorithm="giac")
```

output
```
2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 -
2/25*x^5
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int x^4 \log \left(a^2+x^2\right) \, dx = \frac{x^5 \ln \left(a^2+x^2\right)}{5} - \frac{2\,a^4\,x}{5} - \frac{\ln \left(x-\sqrt{-a^2}\right) \left(-a^2\right)^{5/2}}{5} + \frac{\ln \left(x+\sqrt{-a^2}\right) \left(-a^2\right)^{5/2}}{5} - \frac{2\,x^5}{25} + \frac{2\,a^2\,x^3}{15}$$

input
```
int(x^4*log(a^2 + x^2),x)
```

output
```
(x^5*log(a^2 + x^2))/5 - (2*a^4*x)/5 - (log(x - (-a^2)^(1/2))*(-a^2)^(5/2)
)/5 + (log(x + (-a^2)^(1/2))*(-a^2)^(5/2))/5 - (2*x^5)/25 + (2*a^2*x^3)/15
```

3.73.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4 \log \left(a^2+x^2\right) \, dx = \frac{2atan\left(\frac{x}{a}\right) a^5}{5} + \frac{\log(a^2+x^2)\,x^5}{5} - \frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25}$$

input
```
int(log(a**2 + x**2)*x***4,x)
```

output
```
(30*atan(x/a)*a**5 + 15*log(a**2 + x**2)*x***5 - 30*a**4*x + 10*a**2*x**3 -
 6*x**5)/75
```

3.74 $\int \log(-a^2+x^2)\,dx$

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3.74.9 Mupad [B] (verification not implemented) 545
3.74.10 Reduce [B] (verification not implemented) 545

3.74.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \log(-a^2+x^2)\,dx = -2x + 2a\operatorname{arctanh}\left(\frac{x}{a}\right) + x\log(-a^2+x^2)$$

output
```
-2*x+2*a*arctanh(x/a)+x*ln(-a^2+x^2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log(-a^2+x^2)\,dx = -2x + 2a\operatorname{arctanh}\left(\frac{x}{a}\right) + x\log(-a^2+x^2)$$

input
```
Integrate[Log[-a^2 + x^2],x]
```

output
```
-2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

3.74.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2898, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x^2-a^2\right) \, dx$$

$$\Big\downarrow \ 2898$$

$$x \log \left(x^2-a^2\right)-2 \int -\frac{x^2}{a^2-x^2}dx$$

$$\Big\downarrow \ 25$$

$$2 \int \frac{x^2}{a^2-x^2}dx+x \log \left(x^2-a^2\right)$$

$$\Big\downarrow \ 262$$

$$2 \left(a^2 \int \frac{1}{a^2-x^2}dx-x\right)+x \log \left(x^2-a^2\right)$$

$$\Big\downarrow \ 219$$

$$x \log \left(x^2-a^2\right)+2 \left(a \text{arctanh}\left(\frac{x}{a}\right)-x\right)$$

input
```
Int[Log[-a^2 + x^2],x]
```

output
```
2*(-x + a*ArcTanh[x/a]) + x*Log[-a^2 + x^2]
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

3.74.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
default	$x\ln\left(-a^{2}+x^{2}\right)-2x-a\ln\left(a-x\right)+a\ln\left(a+x\right)$	32
risch	$x\ln\left(-a^{2}+x^{2}\right)-2x+a\ln\left(a+x\right)-a\ln\left(-a+x\right)$	32
parts	$x\ln\left(-a^{2}+x^{2}\right)-2x-a\ln\left(a-x\right)+a\ln\left(a+x\right)$	32
parallelrisch	$-2a\ln\left(-a+x\right)+x\ln\left(-a^{2}+x^{2}\right)+a\ln\left(-a^{2}+x^{2}\right)-2x$	38

input
```
int(ln(-a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
x*ln(-a^2+x^2)-2*x-a*ln(a-x)+a*ln(a+x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \log \left(-a^2+x^2\right) \, dx = x \log \left(-a^2+x^2\right)+a \log (a+x)-a \log (-a+x)-2 \, x$$

input
```
integrate(log(-a^2+x^2),x, algorithm="fricas")
```

output
```
x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \log \left(-a^2+x^2\right) \, dx = -2a\left(\frac{\log (-a+x)}{2}-\frac{\log (a+x)}{2}\right)+x \log \left(-a^2+x^2\right)-2x$$

input
```
integrate(ln(-a**2+x**2),x)
```

output
```
-2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \log \left(-a^2+x^2\right) \, dx = x \log \left(-a^2+x^2\right)+a \log (a+x)-a \log (-a+x)-2 \, x$$

input
```
integrate(log(-a^2+x^2),x, algorithm="maxima")
```

output
```
x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \log \left(-a^2+x^2\right)\, dx = x \log \left(-a^2+x^2\right)+a \log \left(|a+x|\right)-a \log \left(|-a+x|\right)-2\, x$$

input
```
integrate(log(-a^2+x^2),x, algorithm="giac")
```

output
```
x*log(-a^2 + x^2) + a*log(abs(a + x)) - a*log(abs(-a + x)) - 2*x
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(-a^2+x^2\right)\, dx = x\, \ln \left(x^2-a^2\right)-2\, x+2\, a\, \mathrm{atanh}\left(\frac{x}{a}\right)$$

input
```
int(log(x^2 - a^2),x)
```

output
```
x*log(x^2 - a^2) - 2*x + 2*a*atanh(x/a)
```

3.74.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \log \left(-a^2+x^2\right)\, dx = -\log\left(-a^2+x^2\right) a+\log\left(-a^2+x^2\right) x+2 \log(-a-x) a-2x$$

input
```
int(log( - a**2 + x**2),x)
```

output
```
- log( - a**2 + x**2)*a + log( - a**2 + x**2)*x + 2*log( - a - x)*a - 2*x
```

3.75 $\int \log(\log(\log(\log(x)))) \, dx$

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3.75.8 Giac [N/A] 549
3.75.9 Mupad [N/A] 549
3.75.10 Reduce [F] 550

3.75.1 Optimal result

Integrand size = 5, antiderivative size = 5

$$\int \log(\log(\log(\log(x)))) \, dx = \mathrm{Int}(\log(\log(\log(\log(x)))), x)$$

output
```
CannotIntegrate(ln(ln(ln(ln(x)))),x)
```

3.75.2 Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x)))) \, dx = \int \log(\log(\log(\log(x)))) \, dx$$

input
```
Integrate[Log[Log[Log[Log[x]]]],x]
```

output
```
Integrate[Log[Log[Log[Log[x]]]], x]
```

3.75.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(\log(\log(\log(x)))) \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \log(\log(\log(\log(x))))dx$$

input
```
Int[Log[Log[Log[Log[x]]]],x]
```

output
```
$Aborted
```

3.75.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.75.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \ln \left(\ln \left(\ln \left(\ln \left(x\right)\right)\right)\right) dx$$

input
```
int(ln(ln(ln(ln(x)))),x)
```

output
```
int(ln(ln(ln(ln(x)))),x)
```

3.75.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x))))\,dx = \int \log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right)\,dx$$

input
```
integrate(log(log(log(log(x)))),x, algorithm="fricas")
```

output
```
integral(log(log(log(log(x)))), x)
```

3.75.6 Sympy [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 5.40

$$\int \log(\log(\log(\log(x))))\,dx = x\log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right) - \int \frac{1}{\log\left(x\right)\log\left(\log\left(x\right)\right)\log\left(\log\left(\log\left(x\right)\right)\right)}\,dx$$

input
```
integrate(ln(ln(ln(ln(x)))),x)
```

output
```
x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(log(x))))
, x)
```

3.75.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 5.60

$$\int \log(\log(\log(\log(x))))\,dx = \int \log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right)\,dx$$

input
```
integrate(log(log(log(log(x)))),x, algorithm="maxima")
```

output
```
x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log(log(x)))
), x)
```

3.75.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x))))\,dx = \int \log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right)\,dx$$

input
```
integrate(log(log(log(log(x)))),x, algorithm="giac")
```

output
```
integrate(log(log(log(log(x)))), x)
```

3.75.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x))))\,dx = \int \ln\left(\ln\left(\ln\left(\ln\left(x\right)\right)\right)\right)\,dx$$

input
```
int(log(log(log(log(x)))),x)
```

output
```
int(log(log(log(log(x)))), x)
```

3.75.10 Reduce [F]

$$\int \log(\log(\log(\log(x))))\,dx = -\left(\int \frac{1}{\log(\log(\log(x)))\log(\log(x))\log(x)}dx\right) + \log(\log(\log(\log(x))))\,x$$

input
```
int(log(log(log(log(x)))),x)
```

output
```
 - int(1/(log(log(log(x)))*log(log(x))*log(x)),x) + log(log(log(log(x))))*
x
```

3.76 $\int \sin(x)\,dx$

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3.76.5 Fricas [A] (verification not implemented) . . . 553
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3.76.7 Maxima [A] (verification not implemented) . . . 554
3.76.8 Giac [A] (verification not implemented) . . . 554
3.76.9 Mupad [B] (verification not implemented) . . . 554
3.76.10 Reduce [B] (verification not implemented) . . . 555

3.76.1 Optimal result

Integrand size = 2, antiderivative size = 4

$$\int \sin(x)\,dx = -\cos(x)$$

output
```
-cos(x)
```

3.76.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

input
```
Integrate[Sin[x],x]
```

output
```
-Cos[x]
```

3.76.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)dx$$

$$\Big\downarrow \; 3118$$

$$-\cos(x)$$

input
```
Int[Sin[x],x]
```

output
```
-Cos[x]
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.76.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
lookup	$-\cos \left(x \right)$	5
default	$-\cos \left(x \right)$	5
risch	$-\cos \left(x \right)$	5
parallelrisch	$-\cos \left(x \right)-1$	7
norman	$-\frac{2}{1+\tan^{2}\left(\frac{x}{2}\right)}$	13
meijerg	$\sqrt{\pi}\, \left(\frac{1}{\sqrt{\pi}}-\frac{\cos \left(x \right)}{\sqrt{\pi}}\right)$	16

input
```
int(sin(x),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos\left(x\right)$$

input
```
integrate(sin(x),x, algorithm="fricas")
```

output
```
-cos(x)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(x)\,dx = -\cos\left(x\right)$$

input
```
integrate(sin(x),x)
```

output
```
-cos(x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\, dx = -\cos(x)$$

input
```
integrate(sin(x),x, algorithm="maxima")
```

output
```
-cos(x)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\, dx = -\cos(x)$$

input
```
integrate(sin(x),x, algorithm="giac")
```

output
```
-cos(x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\, dx = -\cos(x)$$

input
```
int(sin(x),x)
```

output
```
-cos(x)
```

3.76.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

input
```
int(sin(x),x)
```

output
```
 - cos(x)
```

3.77 $\int \cos(x)\,dx$

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3.77.8 Giac [A] (verification not implemented) 559
3.77.9 Mupad [B] (verification not implemented) 559
3.77.10 Reduce [B] (verification not implemented) 560

3.77.1 Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cos(x)\,dx = \sin(x)$$

output
```
sin(x)
```

3.77.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
Integrate[Cos[x],x]
```

output
```
Sin[x]
```

3.77.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(x+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{3117}$$
$$\sin(x)$$

input
```
Int[Cos[x],x]
```

output
```
Sin[x]
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.77.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sin(x)$	3
default	$\sin(x)$	3
meijerg	$\sin(x)$	3
risch	$\sin(x)$	3
parallelrisch	$\sin(x)$	3
norman	$\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}$	17

input
```
int(cos(x),x,method=_RETURNVERBOSE)
```

output
```
sin(x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
integrate(cos(x),x, algorithm="fricas")
```

output
```
sin(x)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
integrate(cos(x),x)
```

output
```
sin(x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin(x)$$

input
```
integrate(cos(x),x, algorithm="maxima")
```

output
```
sin(x)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin(x)$$

input
```
integrate(cos(x),x, algorithm="giac")
```

output
```
sin(x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin(x)$$

input
```
int(cos(x),x)
```

output
```
sin(x)
```

3.77.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
int(cos(x),x)
```

output
```
sin(x)
```

3.78 $\int \tan(x)\,dx$

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3.78.1 Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \tan(x)\,dx = -\log(\cos(x))$$

output
```
-ln(cos(x))
```

3.78.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\log(\cos(x))$$

input
```
Integrate[Tan[x],x]
```

output
```
-Log[Cos[x]]
```

3.78.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan(x)dx$$
$$\Big\downarrow \text{ 3956}$$
$$-\log(\cos(x))$$

input
```
Int[Tan[x],x]
```

output
```
-Log[Cos[x]]
```

3.78.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.78.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
lookup	$-\ln(\cos(x))$	6
default	$-\ln(\cos(x))$	6
derivativedivides	$\frac{\ln(1+\tan^2(x))}{2}$	10
norman	$\frac{\ln(1+\tan^2(x))}{2}$	10
parallelrisch	$\frac{\ln(1+\tan^2(x))}{2}$	10
risch	$ix - \ln(e^{2ix} + 1)$	16

input
```
int(tan(x),x,method=_RETURNVERBOSE)
```

output
```
-ln(cos(x))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \tan(x)\,dx = -\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

input
```
integrate(tan(x),x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(x)^2 + 1))
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\log\left(\cos\left(x\right)\right)$$

input
```
integrate(tan(x),x)
```

output
```
-log(cos(x))
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \tan(x)\,dx = \log\left(\sec\left(x\right)\right)$$

input
```
integrate(tan(x),x, algorithm="maxima")
```

output
```
log(sec(x))
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \tan(x)\,dx = -\log\left(\left|\cos\left(x\right)\right|\right)$$

input
```
integrate(tan(x),x, algorithm="giac")
```

output
```
-log(abs(cos(x)))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\ln\left(\cos\left(x\right)\right)$$

input
```
int(tan(x),x)
```

output
```
-log(cos(x))
```

3.78.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \tan(x)\,dx = \frac{\log\left(\tan\left(x\right)^2+1\right)}{2}$$

input
```
int(tan(x),x)
```

output
```
log(tan(x)**2 + 1)/2
```

3.79 $\int \cot(x)\,dx$

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3.79.10 Reduce [B] (verification not implemented) 570

3.79.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x)\,dx = \log(\sin(x))$$

output
```
ln(sin(x))
```

3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x)\,dx = \log(\cos(x)) + \log(\tan(x))$$

input
```
Integrate[Cot[x],x]
```

output
```
Log[Cos[x]] + Log[Tan[x]]
```

3.79.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\log(\sin(x))$$

input
```
Int[Cot[x],x]
```

output
```
Log[Sin[x]]
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.79.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln (\sin (x))$	4
default	$\ln (\sin (x))$	4
derivativedivides	$-\frac{\ln(1+\tan^2(x))}{2}+\ln (\tan (x))$	14
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln (\tan (x))$	14
risch	$-ix+\ln (e^{2ix}-1)$	14
parallelrisch	$-\frac{\ln(1+\tan^2(x))}{2}+\ln (\tan (x))$	14

input
```
int(1/tan(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \cot(x)\,dx = \frac{1}{2}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)$$

input
```
integrate(1/tan(x),x, algorithm="fricas")
```

output
```
1/2*log(tan(x)^2/(tan(x)^2 + 1))
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log(\sin(x))$$

input
```
integrate(1/tan(x),x)
```

output
```
log(sin(x))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log(\sin(x))$$

input
```
integrate(1/tan(x),x, algorithm="maxima")
```

output
```
log(sin(x))
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \cot(x)\,dx = -\frac{1}{2}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log\left(\tan(x)^2\right)$$

input
```
integrate(1/tan(x),x, algorithm="giac")
```

output
```
-1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 4.33

$$\int \cot(x)\,dx = \ln(\tan(x)) - \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

input
```
int(1/tan(x),x)
```

output
```
log(tan(x)) - log(tan(x)^2 + 1)/2
```

3.79.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 4.33

$$\int \cot(x)\,dx = -\frac{\log\left(\tan(x)^2 + 1\right)}{2} + \log(\tan(x))$$

input
```
int(1/tan(x),x)
```

output
```
( - log(tan(x)**2 + 1) + 2*log(tan(x)))/2
```

3.80 $\int \frac{1}{(1+\tan(x))^2} \, dx$

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3.80.9 Mupad [B] (verification not implemented) 575
3.80.10 Reduce [B] (verification not implemented) 576

3.80.1 Optimal result

Integrand size = 6, antiderivative size = 21

$$\int \frac{1}{(1+\tan(x))^2} \, dx = \frac{1}{2}\log(\cos(x)+\sin(x)) - \frac{1}{2(1+\tan(x))}$$

output
```
1/2*ln(cos(x)+sin(x))-1/2/(1+tan(x))
```

3.80.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+\tan(x))^2} \, dx = \frac{1}{2}\left(\log(\cos(x)) + \log(1+\tan(x)) - \frac{1}{1+\tan(x)}\right)$$

input
```
Integrate[(1 + Tan[x])^(-2),x]
```

output
```
(Log[Cos[x]] + Log[1 + Tan[x]] - (1 + Tan[x])^(-1))/2
```

3.80.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3964, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\tan(x)+1)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(\tan(x)+1)^2}dx$$
$$\Big\downarrow \text{ 3964}$$
$$\frac{1}{2}\int \frac{1-\tan(x)}{\tan(x)+1}dx-\frac{1}{2(\tan(x)+1)}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{2}\int \frac{1-\tan(x)}{\tan(x)+1}dx-\frac{1}{2(\tan(x)+1)}$$
$$\Big\downarrow \text{ 4013}$$
$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

input
```
Int[(1 + Tan[x])^(-2),x]
```

output
```
Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

3.80.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.80.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$-\frac{1}{2(\tan(x)+1)}+\frac{\ln(\tan(x)+1)}{2}-\frac{\ln(1+\tan^2(x))}{4}$	26
default	$-\frac{1}{2(\tan(x)+1)}+\frac{\ln(\tan(x)+1)}{2}-\frac{\ln(1+\tan^2(x))}{4}$	26
norman	$-\frac{1}{2(\tan(x)+1)}+\frac{\ln(\tan(x)+1)}{2}-\frac{\ln(1+\tan^2(x))}{4}$	26
risch	$-\frac{ix}{2}-\frac{1}{2(e^{2ix}+i)}+\frac{\ln(e^{2ix}+i)}{2}$	29
parallelrisch	$\frac{2\ln(\tan(x)+1)\tan(x)-\ln(1+\tan^2(x))\tan(x)-2+2\ln(\tan(x)+1)-\ln(1+\tan^2(x))}{4\tan(x)+4}$	47

input
```
int(1/(tan(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(tan(x)+1)+1/2*ln(tan(x)+1)-1/4*ln(1+tan(x)^2)
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{(1+\tan(x))^2}\,dx = \frac{(\tan(x)+1)\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)+\tan(x)-1}{4(\tan(x)+1)}$$

input
```
integrate(1/(1+tan(x))^2,x, algorithm="fricas")
```

output
```
1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) + tan(x) -
 1)/(tan(x) + 1)
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(17) = 34$.

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.57

$$\int \frac{1}{(1+\tan(x))^2}\,dx = \frac{2\log(\tan(x)+1)\tan(x)}{4\tan(x)+4} + \frac{2\log(\tan(x)+1)}{4\tan(x)+4} - \frac{\log(\tan^2(x)+1)\tan(x)}{4\tan(x)+4} - \frac{\log(\tan^2(x)+1)}{4\tan(x)+4} - \frac{2}{4\tan(x)+4}$$

input
```
integrate(1/(1+tan(x))**2,x)
```

output
```
2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*tan(x) + 4)
 - log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)**2 + 1)/(4*tan(x)
 + 4) - 2/(4*tan(x) + 4)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1+\tan(x))^2}\,dx = -\frac{1}{2(\tan(x)+1)} - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \frac{1}{2}\log(\tan(x)+1)$$

input
```
integrate(1/(1+tan(x))^2,x, algorithm="maxima")
```

output
```
-1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1+\tan(x))^2}\,dx = -\frac{1}{2(\tan(x)+1)} - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \frac{1}{2}\log(|\tan(x)+1|)$$

input
```
integrate(1/(1+tan(x))^2,x, algorithm="giac")
```

output
```
-1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(abs(tan(x) + 1))
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(1+\tan(x))^2}\,dx = \frac{\ln(\tan(x)+1)}{2} - \frac{\ln\left(\tan(x)^2+1\right)}{4} - \frac{1}{2\,(\tan(x)+1)}$$

input
```
int(1/(tan(x) + 1)^2,x)
```

output
```
log(tan(x) + 1)/2 - log(tan(x)^2 + 1)/4 - 1/(2*(tan(x) + 1))
```

3.80.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{1}{(1+\tan(x))^2}\,dx$$
$$= \frac{-\log\left(\tan(x)^2+1\right)\tan(x) - \log\left(\tan(x)^2+1\right) + 2\log(\tan(x)+1)\tan(x) + 2\log(\tan(x)+1) + 2\tan}{4\tan(x)+4}$$

input
```
int(1/(tan(x)**2 + 2*tan(x) + 1),x)
```

output
```
( - log(tan(x)**2 + 1)*tan(x) - log(tan(x)**2 + 1) + 2*log(tan(x) + 1)*tan
(x) + 2*log(tan(x) + 1) + 2*tan(x))/(4*(tan(x) + 1))
```

3.81 $\int \sec(x)\,dx$

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3.81.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \sec(x)\,dx = \operatorname{arctanh}(\sin(x))$$

output
```
arctanh(sin(x))
```

3.81.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sec(x)\,dx = \operatorname{arctanh}(\sin(x))$$

input
```
Integrate[Sec[x],x]
```

output
```
ArcTanh[Sin[x]]
```

3.81.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4257}$$

$$\operatorname{arctanh}(\sin(x))$$

input
```
Int[Sec[x],x]
```

output
```
ArcTanh[Sin[x]]
```

3.81.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.81.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

method	result	size
default	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)$	7
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
risch	$\ln\left(i+\mathrm{e}^{ix}\right)-\ln\left(\mathrm{e}^{ix}-i\right)$	22

input
```
int(1/cos(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x)+tan(x))
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\,dx = \frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\,\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(1/cos(x),x, algorithm="fricas")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.81.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sec(x)\,dx = -\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}$$

input
```
integrate(1/cos(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sec(x)\, dx = \frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(\sin(x) - 1)$$

input
```
integrate(1/cos(x),x, algorithm="maxima")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\, dx = \frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(-\sin(x) + 1)$$

input
```
integrate(1/cos(x),x, algorithm="giac")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \sec(x)\,dx = \ln\left(\frac{1}{\cos(x)}\right) + \ln(\sin(x) + 1)$$

input
```
int(1/cos(x),x)
```

output
```
log(1/cos(x)) + log(sin(x) + 1)
```

3.81.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\,dx = -\log\left(\tan\left(\frac{x}{2}\right) - 1\right) + \log\left(\tan\left(\frac{x}{2}\right) + 1\right)$$

input
```
int(1/cos(x),x)
```

output
```
 - log(tan(x/2) - 1) + log(tan(x/2) + 1)
```

3.82 $\int \csc(x)\,dx$

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3.82.1 Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \csc(x)\,dx = -\text{arctanh}(\cos(x))$$

output
```
-arctanh(cos(x))
```

3.82.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \csc(x)\,dx = -\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csc[x],x]
```

output
```
-Log[Cos[x/2]] + Log[Sin[x/2]]
```

3.82.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)dx$$

$$\Big\downarrow \text{4257}$$

$$-\text{arctanh}(\cos(x))$$

input
```
Int[Csc[x],x]
```

output
```
-ArcTanh[Cos[x]]
```

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.82.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
parallelrisch	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)$	9
risch	$\ln\left({\mathrm e}^{i x}-1\right)-\ln\left({\mathrm e}^{i x}+1\right)$	20

input
```
int(1/sin(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*x))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int \csc(x)\,dx = -\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

input
```
integrate(1/sin(x),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

3.82.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.00

$$\int \csc(x)\,dx = \frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}$$

input
```
integrate(1/sin(x),x)
```

output
```
log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.00

$$\int \csc(x)\,dx = -\frac{1}{2}\log\left(\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)-1\right)$$

input
```
integrate(1/sin(x),x, algorithm="maxima")
```

output
```
-1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \csc(x)\,dx = -\frac{1}{2}\log\left(\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(-\cos\left(x\right)+1\right)$$

input
```
integrate(1/sin(x),x, algorithm="giac")
```

output
```
-1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \csc(x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(1/sin(x),x)
```

output
```
log(tan(x/2))
```

3.82.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \csc(x)\,dx = \log\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(1/sin(x),x)
```

output
```
log(tan(x/2))
```

3.83 $\int \sin^2(x)\,dx$

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3.83.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.83.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

input
```
Integrate[Sin[x]^2,x]
```

output
```
x/2 - Sin[2*x]/4
```

3.83.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)^2dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \; 24$$

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Sin[x]^2,x]
```

output
```
x/2 - (Cos[x]*Sin[x])/2
```

3.83.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.83.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^{3}\left(\frac{x}{2}\right)+x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	45

input
```
int(sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

input
```
integrate(sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*cos(x)*sin(x) + 1/2*x
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(sin(x)**2,x)
```

output
```
x/2 - sin(x)*cos(x)/2
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2\,x)}{4}$$

input
```
int(sin(x)^2,x)
```

output
```
x/2 - sin(2*x)/4
```

3.83.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

input
```
int(sin(x)**2,x)
```

output
```
( - cos(x)*sin(x) + x)/2
```

3.84 $\int x^3 \sin(x^2)\,dx$

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3.84.1 Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3 \sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

input
```
Integrate[x^3*Sin[x^2],x]
```

output
```
-1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

3.84.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin\left(x^2\right) dx$$
$$\Big\downarrow \text{3860}$$
$$\frac{1}{2} \int x^2 \sin\left(x^2\right) dx^2$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2} \int x^2 \sin\left(x^2\right) dx^2$$
$$\Big\downarrow \text{3777}$$
$$\frac{1}{2} \left(\int \cos\left(x^2\right) dx^2 - x^2 \cos\left(x^2\right)\right)$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2} \left(\int \sin\left(x^2 + \frac{\pi}{2}\right) dx^2 - x^2 \cos\left(x^2\right)\right)$$
$$\Big\downarrow \text{3117}$$
$$\frac{1}{2} \left(\sin\left(x^2\right) - x^2 \cos\left(x^2\right)\right)$$

input
```
Int[x^3*Sin[x^2],x]
```

output
```
(-(x^2*Cos[x^2]) + Sin[x^2])/2
```

3.84.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.84.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
default	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
risch	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
parallelrisch	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
meijerg	$\sqrt{\pi}\left(-\frac{x^2\cos(x^2)}{2\sqrt{\pi}}+\frac{\sin(x^2)}{2\sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^2}{2}+\frac{x^2\left(\tan^2\left(\frac{x^2}{2}\right)\right)}{2}+\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	39
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x^3}{2}-\frac{3\pi^2\left(\frac{2\,\mathrm{S}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x^3}{3\pi^{\frac{3}{2}}}+\frac{2x^2\cos\left(x^2\right)}{3\pi^2}-\frac{2\sin\left(x^2\right)}{3\pi^2}\right)}{4}$	69

input
```
int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\, dx = -\frac{1}{2}\, x^2 \cos\left(x^2\right) + \frac{1}{2}\, \sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="fricas")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \sin\left(x^2\right)\, dx = -\frac{x^2 \cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

input
```
integrate(x**3*sin(x**2),x)
```

output
```
-x**2*cos(x**2)/2 + sin(x**2)/2
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\, dx = -\frac{1}{2}\, x^2 \cos\left(x^2\right) + \frac{1}{2}\, \sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="maxima")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="giac")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = \frac{\sin\left(x^2\right)}{2} - \frac{x^2\,\cos\left(x^2\right)}{2}$$

input
```
int(x^3*sin(x^2),x)
```

output
```
sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.84.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{\cos\left(x^2\right)x^2}{2} + \frac{\sin\left(x^2\right)}{2}$$

input
```
int(sin(x**2)*x**3,x)
```

output
```
( - cos(x**2)*x**2 + sin(x**2))/2
```

3.85 $\int \sin^3(x)\,dx$

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3.85.8 Giac [A] (verification not implemented) 600
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3.85.10 Reduce [B] (verification not implemented) 601

3.85.1 Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x)\,dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

output
```
-cos(x)+1/3*cos(x)^3
```

3.85.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x)\,dx = -\frac{3\cos(x)}{4} + \frac{1}{12}\cos(3x)$$

input
```
Integrate[Sin[x]^3,x]
```

output
```
(-3*Cos[x])/4 + Cos[3*x]/12
```

3.85.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(x)^3 dx$$
$$\Big\downarrow \text{3113}$$
$$-\int \left(1-\cos^2(x)\right) d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\cos^3(x)}{3}-\cos(x)$$

input
```
Int[Sin[x]^3,x]
```

output
```
-Cos[x] + Cos[x]^3/3
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
parallelrisch	$-\frac{2}{3}-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	13
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

input
```
int(sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2+sin(x)^2)*cos(x)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="fricas")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

input
```
integrate(sin(x)**3,x)
```

output
```
cos(x)**3/3 - cos(x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="maxima")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="giac")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x)\,dx = \frac{\cos(x)\,\left(\cos(x)^2-3\right)}{3}$$

input
```
int(sin(x)^3,x)
```

output
```
(cos(x)*(cos(x)^2 - 3))/3
```

3.85.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sin^3(x)\,dx = -\frac{\cos(x)\sin(x)^2}{3} - \frac{2\cos(x)}{3} + \frac{2}{3}$$

input
```
int(sin(x)**3,x)
```

output
```
( - cos(x)*sin(x)**2 - 2*cos(x) + 2)/3
```

3.86 $\int \sin^p(x)\,dx$

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3.86.1 Optimal result

Integrand size = 4, antiderivative size = 44

$$\int \sin^p(x)\,dx=\frac{\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(x)\right)\sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

output
```
cos(x)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],sin(x)^2)*sin(x)^(p+1)/(p+1)
/(cos(x)^2)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sin^p(x)\,dx=\frac{\sqrt{\cos^2(x)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(x)\right)\sec(x)\sin^{1+p}(x)}{1+p}$$

input
```
Integrate[Sin[x]^p,x]
```

output
```
(Sqrt[Cos[x]^2]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sec
[x]*Sin[x]^(1 + p))/(1 + p)
```

3.86.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^p(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)^p dx$$

$$\Big\downarrow \; 3122$$

$$\frac{\cos(x)\sin^{p+1}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

input
```
Int[Sin[x]^p,x]
```

output
```
(Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sin[x]^(1 +
 p))/((1 + p)*Sqrt[Cos[x]^2])
```

3.86.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.86.4 Maple [F]

$$\int (\sin^p(x))\,dx$$

input
```
int(sin(x)^p,x)
```

output
```
int(sin(x)^p,x)
```

3.86.5 Fricas [F]

$$\int \sin^p(x)\,dx = \int \sin(x)^p\,dx$$

input
```
integrate(sin(x)^p,x, algorithm="fricas")
```

output
```
integral(sin(x)^p, x)
```

3.86.6 Sympy [F]

$$\int \sin^p(x)\,dx = \int \sin^p(x)\,dx$$

input
```
integrate(sin(x)**p,x)
```

output
```
Integral(sin(x)**p, x)
```

3.86.7 Maxima [F]

$$\int \sin^p(x)\,dx = \int \sin(x)^p\,dx$$

input
```
integrate(sin(x)^p,x, algorithm="maxima")
```

output
```
integrate(sin(x)^p, x)
```

3.86.8 Giac [F]

$$\int \sin^p(x)\,dx = \int \sin(x)^p\,dx$$

input
```
integrate(sin(x)^p,x, algorithm="giac")
```

output
```
integrate(sin(x)^p, x)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \sin^p(x)\,dx = -\frac{\cos(x)\,\sin(x)^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-\frac{p}{2};\ \frac{3}{2};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{\frac{p}{2}+\frac{1}{2}}}$$

input
```
int(sin(x)^p,x)
```

output
```
-(cos(x)*sin(x)^(p + 1)*hypergeom([1/2, 1/2 - p/2], 3/2, cos(x)^2))/(sin(x
)^2)^(p/2 + 1/2)
```

3.86.10 Reduce [F]

$$\int \sin^p(x)\,dx = \int \sin{(x)}^p\,dx$$

input
```
int(sin(x)**p,x)
```

output
```
int(sin(x)**p,x)
```

3.87 $\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx$

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3.87.10 Reduce [B] (verification not implemented) 611

3.87.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx = \sin(x)+\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}$$

output
```
sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

3.87.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx = \sin(x)+\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}$$

input
```
Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]
```

output
```
Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

3.87.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3669, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^2(x)+1\right)^2 \cos(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sin(x)^2+1\right)^2 \cos(x)dx$$

$$\Big\downarrow \text{ 3669}$$

$$\int \left(\sin^2(x)+1\right)^2 d\sin(x)$$

$$\Big\downarrow \text{ 210}$$

$$\int \left(\sin^4(x)+2\sin^2(x)+1\right) d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

input
```
Int[Cos[x]*(1 + Sin[x]^2)^2,x]
```

output
```
Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

3.87.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.87.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\sin(x)+\frac{2(\sin^3(x))}{3}+\frac{(\sin^5(x))}{5}$	16
default	$\sin(x)+\frac{2(\sin^3(x))}{3}+\frac{(\sin^5(x))}{5}$	16
risch	$\frac{13\sin(x)}{8}+\frac{\sin(5x)}{80}-\frac{11\sin(3x)}{48}$	18
parallelrisch	$\frac{13\sin(x)}{8}+\frac{\sin(5x)}{80}-\frac{11\sin(3x)}{48}$	18

input
```
int(cos(x)*(1+sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx = \frac{1}{15} \left(3 \cos(x)^4 - 16 \cos(x)^2 + 28\right) \sin(x)$$

input
```
integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx = \frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)$$

input
```
integrate(cos(x)*(1+sin(x)**2)**2,x)
```

output
```
sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx = \frac{1}{5} \sin(x)^5 + \frac{2}{3} \sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx = \frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx = \frac{\sin(x)^5}{5} + \frac{2\sin(x)^3}{3} + \sin(x)$$

input
```
int(cos(x)*(sin(x)^2 + 1)^2,x)
```

output
```
sin(x) + (2*sin(x)^3)/3 + sin(x)^5/5
```

3.87.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx = \frac{\sin(x)\left(3\sin(x)^4 + 10\sin(x)^2 + 15\right)}{15}$$

input
```
int(cos(x)*(sin(x)**4 + 2*sin(x)**2 + 1),x)
```

output
```
(sin(x)*(3*sin(x)**4 + 10*sin(x)**2 + 15))/15
```

3.88 $\int \cos^2(x)\,dx$

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3.88.9 Mupad [B] (verification not implemented) 616
3.88.10 Reduce [B] (verification not implemented) 616

3.88.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.88.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

3.88.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.88.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.88.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

input
```
integrate(cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(cos(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

3.88.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

input
```
int(cos(x)**2,x)
```

output
```
(cos(x)*sin(x) + x)/2
```

3.89 $\int \cos^3(x)\,dx$

3.89.1 Optimal result 617
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3.89.1 Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

output
```
sin(x)-1/3*sin(x)^3
```

3.89.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

input
```
Integrate[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

3.89.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$-\int \left(1-\sin^2(x)\right)d(-\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\sin(x)-\frac{\sin^3(x)}{3}$$

input
```
Int[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.89.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\left(2+\cos^{2}\left(x\right)\right)\sin\left(x\right)}{3}$	11
risch	$\frac{3\sin\left(x\right)}{4}+\frac{\sin\left(3x\right)}{12}$	12
parallelrisch	$\frac{3\sin\left(x\right)}{4}+\frac{\sin\left(3x\right)}{12}$	12

input
```
int(cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2+cos(x)^2)*sin(x)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x)\,dx=\frac{1}{3}\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*sin(x)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x)\,dx = -\frac{\sin^3(x)}{3} + \sin(x)$$

input
```
integrate(cos(x)**3,x)
```

output
```
-sin(x)**3/3 + sin(x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="maxima")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="giac")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin(x)^3}{3}$$

input
```
int(cos(x)^3,x)
```

output
```
sin(x) - sin(x)^3/3
```

3.89.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \cos^3(x)\,dx = \frac{\sin(x)\left(-\sin(x)^2+3\right)}{3}$$

input
```
int(cos(x)**3,x)
```

output
```
(sin(x)*( - sin(x)**2 + 3))/3
```

3.90 $\int \sec^2(x)\,dx$

3.90.1 Optimal result 622
3.90.2 Mathematica [A] (verified) 622
3.90.3 Rubi [A] (verified) 623
3.90.4 Maple [A] (verified) 624
3.90.5 Fricas [B] (verification not implemented) 624
3.90.6 Sympy [B] (verification not implemented) 624
3.90.7 Maxima [A] (verification not implemented) 625
3.90.8 Giac [A] (verification not implemented) 625
3.90.9 Mupad [B] (verification not implemented) 625
3.90.10 Reduce [B] (verification not implemented) 626

3.90.1 Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \sec^2(x)\,dx = \tan(x)$$

output
```
tan(x)
```

3.90.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

input
```
Integrate[Sec[x]^2,x]
```

output
```
Tan[x]
```

3.90.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{4254}$$

$$-\int 1d(-\tan(x))$$

$$\Big\downarrow \text{24}$$

$$\tan(x)$$

input
```
Int[Sec[x]^2,x]
```

output
```
Tan[x]
```

3.90.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.90.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\tan(x)$	3
parallelrisch	$\tan(x)$	3
risch	$\frac{2i}{e^{2ix}+1}$	13
norman	$-\frac{2\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}$	17

input
```
int(1/cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
tan(x)
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(2) = 4$.

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 3.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(1/cos(x)^2,x, algorithm="fricas")
```

output
```
sin(x)/cos(x)
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5 vs. $2(2) = 4$.

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 2.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(1/cos(x)**2,x)
```

output
```
sin(x)/cos(x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan{(x)}$$

input
```
integrate(1/cos(x)^2,x, algorithm="maxima")
```

output
```
tan(x)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan{(x)}$$

input
```
integrate(1/cos(x)^2,x, algorithm="giac")
```

output
```
tan(x)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

input
```
int(1/cos(x)^2,x)
```

output
```
tan(x)
```

3.90.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 3.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

input
```
int(1/cos(x)**2,x)
```

output
```
sin(x)/cos(x)
```

3.91 $\int \sin(x)\sin(2x)\,dx$

3.91.1 Optimal result . . . 627
3.91.2 Mathematica [A] (verified) . . . 627
3.91.3 Rubi [A] (verified) . . . 628
3.91.4 Maple [A] (verified) . . . 629
3.91.5 Fricas [A] (verification not implemented) . . . 629
3.91.6 Sympy [A] (verification not implemented) . . . 629
3.91.7 Maxima [A] (verification not implemented) . . . 630
3.91.8 Giac [A] (verification not implemented) . . . 630
3.91.9 Mupad [B] (verification not implemented) . . . 630
3.91.10 Reduce [B] (verification not implemented) . . . 631

3.91.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sin(x)\sin(2x)\,dx=\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

output
```
1/2*sin(x)-1/6*sin(3*x)
```

3.91.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\,dx=\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

input
```
Integrate[Sin[x]*Sin[2*x],x]
```

output
```
Sin[x]/2 - Sin[3*x]/6
```

3.91.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\sin(2x)dx$$

$$\Big\downarrow \text{ 4770}$$

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

input
```
Int[Sin[x]*Sin[2*x],x]
```

output
```
Sin[x]/2 - Sin[3*x]/6
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.91.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
parallelrisch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{2\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{4\left(\tan^2(x)\right)\tan\left(\frac{x}{2}\right)}{3}+\frac{2\tan(x)}{3}-\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(x)\right)}$	51

input
```
int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)-1/6*sin(3*x)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \sin(x)\sin(2x)\,dx=-\frac{2}{3}\left(\cos{(x)}^2-1\right)\sin{(x)}$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

output
```
-2/3*(cos(x)^2 - 1)*sin(x)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \sin(x)\sin(2x)\,dx=-\frac{2\sin{(x)}\cos{(2x)}}{3}+\frac{\sin{(2x)}\cos{(x)}}{3}$$

input
```
integrate(sin(x)*sin(2*x),x)
```

output
```
-2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \sin(x)\sin(2x)\,dx = -\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/6*sin(3*x) + 1/2*sin(x)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sin(2x)\,dx = \frac{2}{3}\sin(x)^3$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

output
```
2/3*sin(x)^3
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sin(2x)\,dx = \frac{2\sin(x)^3}{3}$$

input
```
int(sin(2*x)*sin(x),x)
```

output
```
(2*sin(x)^3)/3
```

3.91.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sin(x)\sin(2x)\,dx = -\frac{2\cos(2x)\sin(x)}{3} + \frac{\cos(x)\sin(2x)}{3}$$

input
```
int(sin(2*x)*sin(x),x)
```

output
```
( - 2*cos(2*x)*sin(x) + cos(x)*sin(2*x))/3
```

3.92 $\int x \sin(x)\, dx$

3.92.1 Optimal result 632
3.92.2 Mathematica [A] (verified) 632
3.92.3 Rubi [A] (verified) 633
3.92.4 Maple [A] (verified) 634
3.92.5 Fricas [A] (verification not implemented) 634
3.92.6 Sympy [A] (verification not implemented) 635
3.92.7 Maxima [A] (verification not implemented) 635
3.92.8 Giac [A] (verification not implemented) 635
3.92.9 Mupad [B] (verification not implemented) 636
3.92.10 Reduce [B] (verification not implemented) 636

3.92.1 Optimal result

Integrand size = 4, antiderivative size = 8

$$\int x \sin(x)\, dx = -x \cos(x) + \sin(x)$$

output
```
-x*cos(x)+sin(x)
```

3.92.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x)\, dx = -x \cos(x) + \sin(x)$$

input
```
Integrate[x*Sin[x],x]
```

output
```
-(x*Cos[x]) + Sin[x]
```

3.92.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int x\sin(x)dx$$
$$\Big\downarrow \; 3777$$
$$\int \cos(x)dx - x\cos(x)$$
$$\Big\downarrow \; 3042$$
$$\int \sin\left(x+\frac{\pi}{2}\right)dx - x\cos(x)$$
$$\Big\downarrow \; 3117$$
$$\sin(x) - x\cos(x)$$

input
```
Int[x*Sin[x],x]
```

output
```
-(x*Cos[x]) + Sin[x]
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.92.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$-x\cos(x)+\sin(x)$	9
risch	$-x\cos(x)+\sin(x)$	9
parallelrisch	$-x\cos(x)+\sin(x)$	9
parts	$-x\cos(x)+\sin(x)$	9
meijerg	$2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	22
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	30

input
```
int(x*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-x*cos(x)+sin(x)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x)+\sin(x)$$

input
```
integrate(x*sin(x),x, algorithm="fricas")
```

output
```
-x*cos(x) + sin(x)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

input
```
integrate(x*sin(x),x)
```

output
```
-x*cos(x) + sin(x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

input
```
integrate(x*sin(x),x, algorithm="maxima")
```

output
```
-x*cos(x) + sin(x)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

input
```
integrate(x*sin(x),x, algorithm="giac")
```

output
```
-x*cos(x) + sin(x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = \sin(x) - x\cos(x)$$

input
```
int(x*sin(x),x)
```

output
```
sin(x) - x*cos(x)
```

3.92.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -\cos(x)\,x + \sin(x)$$

input
```
int(sin(x)*x,x)
```

output
```
 - cos(x)*x + sin(x)
```

3.93 $\int x^2 \sin(x) \, dx$

3.93.1 Optimal result 637
3.93.2 Mathematica [A] (verified) 637
3.93.3 Rubi [A] (verified) 638
3.93.4 Maple [A] (verified) 639
3.93.5 Fricas [A] (verification not implemented) 640
3.93.6 Sympy [A] (verification not implemented) 640
3.93.7 Maxima [A] (verification not implemented) 640
3.93.8 Giac [A] (verification not implemented) 641
3.93.9 Mupad [B] (verification not implemented) 641
3.93.10 Reduce [B] (verification not implemented) 641

3.93.1 Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^2 \sin(x) \, dx = 2\cos(x) - x^2 \cos(x) + 2x \sin(x)$$

output
```
2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) \, dx = -\left(\left(-2 + x^2\right) \cos(x)\right) + 2x \sin(x)$$

input
```
Integrate[x^2*Sin[x],x]
```

output
```
-((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

3.93.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin(x)dx$$

$$\Big\downarrow \text{3777}$$

$$2\int x\cos(x)dx - x^2\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$2\int x\sin\left(x+\frac{\pi}{2}\right)dx - x^2\cos(x)$$

$$\Big\downarrow \text{3777}$$

$$2(\int -\sin(x)dx + x\sin(x)) - x^2\cos(x)$$

$$\Big\downarrow \text{25}$$

$$2(x\sin(x) - \int \sin(x)dx) - x^2\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$2(x\sin(x) - \int \sin(x)dx) - x^2\cos(x)$$

$$\Big\downarrow \text{3118}$$

$$2(x\sin(x) + \cos(x)) - x^2\cos(x)$$

input
```
Int[x^2*Sin[x],x]
```

output
```
-(x^2*Cos[x]) + 2*(Cos[x] + x*Sin[x])
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
risch	$(-x^2+2)\cos(x)+2x\sin(x)$	17
default	$2\cos(x)-x^2\cos(x)+2x\sin(x)$	18
parts	$2\cos(x)-x^2\cos(x)+2x\sin(x)$	18
parallelrisch	$-x^2\cos(x)+2x\sin(x)+2\cos(x)+2$	19
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2}{2}+1\right)\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)$	34
norman	$\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)-x^2+4x\tan\left(\frac{x}{2}\right)+4}{1+\tan^2\left(\frac{x}{2}\right)}$	36

input
```
int(x^2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
(-x^2+2)*cos(x)+2*x*sin(x)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\,dx = -\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

input
```
integrate(x^2*sin(x),x, algorithm="fricas")
```

output
```
-(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \sin(x)\,dx = -x^2\cos\left(x\right)+2x\sin\left(x\right)+2\cos\left(x\right)$$

input
```
integrate(x**2*sin(x),x)
```

output
```
-x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\,dx = -\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

input
```
integrate(x^2*sin(x),x, algorithm="maxima")
```

output
```
-(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\,dx = -\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

input
```
integrate(x^2*sin(x),x, algorithm="giac")
```

output
```
-(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\,dx = 2\,x\,\sin\left(x\right)-\cos\left(x\right)\,\left(x^2-2\right)$$

input
```
int(x^2*sin(x),x)
```

output
```
2*x*sin(x) - cos(x)*(x^2 - 2)
```

3.93.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \sin(x)\,dx = -\cos\left(x\right)x^2+2\cos\left(x\right)+2\sin\left(x\right)x$$

input
```
int(sin(x)*x**2,x)
```

output
```
- cos(x)*x**2 + 2*cos(x) + 2*sin(x)*x
```

3.94 $\int x \sin^2(x)\, dx$

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3.94.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int x \sin^2(x)\, dx = \frac{x^2}{4} - \frac{1}{2} x \cos(x) \sin(x) + \frac{\sin^2(x)}{4}$$

output
```
1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

3.94.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x \sin^2(x)\, dx = \frac{x^2}{4} - \frac{1}{8} \cos(2x) - \frac{1}{4} x \sin(2x)$$

input
```
Integrate[x*Sin[x]^2,x]
```

output
```
x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

3.94.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^2(x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int x\sin(x)^2dx$$
$$\Big\downarrow \text{ 3791}$$
$$\frac{\int xdx}{2}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$
$$\Big\downarrow \text{ 15}$$
$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x*Sin[x]^2,x]
```

output
```
x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

3.94.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.94.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^2}{4}-\frac{\cos(2x)}{8}-\frac{x\sin(2x)}{4}$	20
default	$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin^2(x))}{4}$	25
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x^2+1}{2\sqrt{\pi}}-\frac{\cos(2x)}{2\sqrt{\pi}}-\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{4}$	38
norman	$\frac{\tan^2\left(\frac{x}{2}\right)+\left(\tan^3\left(\frac{x}{2}\right)\right)x+\frac{x^2}{4}-x\tan\left(\frac{x}{2}\right)+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{x^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

input
```
int(x*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/8*cos(2*x)-1/4*x*sin(2*x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx=-\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x^2-\frac{1}{4}\,\cos\left(x\right)^2$$

input
```
integrate(x*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x\sin^2(x)\,dx = \frac{x^2\sin^2(x)}{4} + \frac{x^2\cos^2(x)}{4} - \frac{x\sin(x)\cos(x)}{2} - \frac{\cos^2(x)}{4}$$

input
```
integrate(x*sin(x)**2,x)
```

output
```
x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 - cos(x)**2/4
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x)$$

input
```
integrate(x*sin(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x)$$

input
```
integrate(x*sin(x)^2,x, algorithm="giac")
```

output
```
1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{\sin(x)^2}{4} - \frac{x\,\sin(2\,x)}{4} + \frac{x^2}{4}$$

input
```
int(x*sin(x)^2,x)
```

output
```
sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

3.94.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x\sin^2(x)\,dx = -\frac{\cos(x)\sin(x)\,x}{2} + \frac{\sin(x)^2}{4} + \frac{x^2}{4} - \frac{1}{2}$$

input
```
int(sin(x)**2*x,x)
```

output
```
( - 2*cos(x)*sin(x)*x + sin(x)**2 + x**2 - 2)/4
```

3.95 $\int x^2 \sin^2(x)\,dx$

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3.95.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^2 \sin^2(x)\,dx = -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

output
```
-1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(x)\,dx = \frac{1}{24}\left(4x^3-6x\cos(2x)+\left(3-6x^2\right)\sin(2x)\right)$$

input
```
Integrate[x^2*Sin[x]^2,x]
```

output
```
(4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

3.95.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2(x) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^2 \sin(x)^2 dx$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\int x^2dx}{2} - \frac{1}{2}\int \sin^2(x)dx - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{2}\int \sin^2(x)dx + \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}\int \sin(x)^2dx + \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x) - \frac{\int 1dx}{2}\right) + \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x) + \frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x) - \frac{x}{2}\right)$$

input
```
Int[x^2*Sin[x]^2,x]
```

output
```
x^3/6 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2 + (-1/2*x + (Cos[x]*Sin[x])
/2)/2
```

3.95.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.95.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

method	result	size
meijerg	$\frac{x^{5}{}_{2}F_{3}\left(1,\frac{5}{2};\frac{3}{2},2,\frac{7}{2};-x^{2}\right)}{5}$	19
risch	$\frac{x^{3}}{6}-\frac{x\cos\left(2x\right)}{4}-\frac{\left(2x^{2}-1\right)\sin\left(2x\right)}{8}$	27
default	$x^{2}\left(\frac{x}{2}-\frac{\cos\left(x\right)\sin\left(x\right)}{2}\right)-\frac{\left(\cos^{2}\left(x\right)\right)x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{4}+\frac{x}{4}-\frac{x^{3}}{3}$	37
norman	$\frac{x^{2}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-\frac{x}{4}+\frac{x^{3}}{6}-\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}-x^{2}\tan\left(\frac{x}{2}\right)+\frac{x^{3}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{x^{3}\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	94

input
```
int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1,5/2],[3/2,2,7/2],-x^2)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(x) \, dx = \frac{1}{6} \, x^3 - \frac{1}{2} \, x \cos \left(x\right)^2 - \frac{1}{4} \, \left(2 \, x^2 - 1\right) \cos \left(x\right) \sin \left(x\right) + \frac{1}{4} \, x$$

input
```
integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

output
```
1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int x^2 \sin^2(x) \, dx = \frac{x^3 \sin^2 \left(x\right)}{6} + \frac{x^3 \cos^2 \left(x\right)}{6} - \frac{x^2 \sin \left(x\right) \cos \left(x\right)}{2} + \frac{x \sin^2 \left(x\right)}{4} - \frac{x \cos^2 \left(x\right)}{4} + \frac{\sin \left(x\right) \cos \left(x\right)}{4}$$

input
```
integrate(x**2*sin(x)**2,x)
```

output
```
x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(x)\, dx = \frac{1}{6}\, x^3 - \frac{1}{4}\, x \cos{(2\, x)} - \frac{1}{8}\, \left(2\, x^2 - 1\right) \sin{(2\, x)}$$

input
```
integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

output
```
1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(x)\, dx = \frac{1}{6}\, x^3 - \frac{1}{4}\, x \cos{(2\, x)} - \frac{1}{8}\, \left(2\, x^2 - 1\right) \sin{(2\, x)}$$

input
```
integrate(x^2*sin(x)^2,x, algorithm="giac")
```

output
```
1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2 \sin^2(x)\, dx = \frac{\sin{(2\, x)}}{8} - \frac{x\, \cos{(2\, x)}}{4} - \frac{x^2\, \sin{(2\, x)}}{4} + \frac{x^3}{6}$$

input
```
int(x^2*sin(x)^2,x)
```

output
```
sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

3.95.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int x^2 \sin^2(x)\, dx = -\frac{\cos(x)\sin(x)\, x^2}{2} + \frac{\cos(x)\sin(x)}{4} + \frac{\sin(x)^2\, x}{2} + \frac{x^3}{6} - \frac{x}{4}$$

input
```
int(sin(x)**2*x**2,x)
```

output
```
( - 6*cos(x)*sin(x)*x**2 + 3*cos(x)*sin(x) + 6*sin(x)**2*x + 2*x**3 - 3*x)
/12
```

3.96 $\int x \sin^3(x)\,dx$

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3.96.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int x \sin^3(x)\,dx = -\frac{2}{3}x\cos(x) + \frac{2\sin(x)}{3} - \frac{1}{3}x\cos(x)\sin^2(x) + \frac{\sin^3(x)}{9}$$

output
```
-2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

3.96.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int x \sin^3(x)\,dx = -\frac{3}{4}x\cos(x) + \frac{1}{12}x\cos(3x) + \frac{3\sin(x)}{4} - \frac{1}{36}\sin(3x)$$

input
```
Integrate[x*Sin[x]^3,x]
```

output
```
(-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin(x)^3dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{2}{3}\int x\sin(x)dx+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int x\sin(x)dx+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}(\int\cos(x)dx-x\cos(x))+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\int\sin\left(x+\frac{\pi}{2}\right)dx-x\cos(x)\right)+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)+\frac{2}{3}(\sin(x)-x\cos(x))$$

input
```
Int[x*Sin[x]^3,x]
```

output
```
-1/3*(x*Cos[x]*Sin[x]^2) + Sin[x]^3/9 + (2*(-(x*Cos[x]) + Sin[x]))/3
```

3.96.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.96.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{x(2+\sin^2(x))\cos(x)}{3}+\frac{(\sin^3(x))}{9}+\frac{2\sin(x)}{3}$	23
risch	$-\frac{3x\cos(x)}{4}+\frac{3\sin(x)}{4}+\frac{x\cos(3x)}{12}-\frac{\sin(3x)}{36}$	24
parallelrisch	$-\frac{3x\cos(x)}{4}+\frac{3\sin(x)}{4}+\frac{x\cos(3x)}{12}-\frac{\sin(3x)}{36}$	24
norman	$\frac{-\frac{2x}{3}+\frac{32\left(\tan^3\left(\frac{x}{2}\right)\right)}{9}+\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)+2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	65

input
```
int(x*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{3}\,x\cos\left(x\right)^{3} - x\cos\left(x\right) - \frac{1}{9}\,\left(\cos\left(x\right)^{2} - 7\right)\sin\left(x\right)$$

input
```
integrate(x*sin(x)^3,x, algorithm="fricas")
```

output
```
1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x\sin^3(x)\,dx = -x\sin^2\left(x\right)\cos\left(x\right) - \frac{2x\cos^3\left(x\right)}{3} + \frac{7\sin^3\left(x\right)}{9} + \frac{2\sin\left(x\right)\cos^2\left(x\right)}{3}$$

input
```
integrate(x*sin(x)**3,x)
```

output
```
-x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2
/3
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos\left(x\right) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin\left(x\right)$$

input
```
integrate(x*sin(x)^3,x, algorithm="maxima")
```

output
```
1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{12}\,x\cos(3\,x) - \frac{3}{4}\,x\cos(x) - \frac{1}{36}\,\sin(3\,x) + \frac{3}{4}\,\sin(x)$$

input
```
integrate(x*sin(x)^3,x, algorithm="giac")
```

output
```
1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\sin^3(x)\,dx = \frac{x\cos(x)^3}{3} - \frac{\sin(x)\,\cos(x)^2}{9} - x\,\cos(x) + \frac{7\,\sin(x)}{9}$$

input
```
int(x*sin(x)^3,x)
```

output
```
(7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

3.96.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\sin^3(x)\,dx = -\frac{\cos(x)\sin(x)^2\,x}{3} - \frac{2\cos(x)\,x}{3} + \frac{\sin(x)^3}{9} + \frac{2\sin(x)}{3}$$

input
```
int(sin(x)**3*x,x)
```

output
```
( - 3*cos(x)*sin(x)**2*x - 6*cos(x)*x + sin(x)**3 + 6*sin(x))/9
```

3.97 $\int x\cos(x)\,dx$

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3.97.1 Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

output
```
cos(x)+x*sin(x)
```

3.97.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

input
```
Integrate[x*Cos[x],x]
```

output
```
Cos[x] + x*Sin[x]
```

3.97.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x\sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\int -\sin(x)dx + x\sin(x)$$

$$\Big\downarrow \text{ 25}$$

$$x\sin(x) - \int \sin(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$x\sin(x) - \int \sin(x)dx$$

$$\Big\downarrow \text{ 3118}$$

$$x\sin(x) + \cos(x)$$

input
```
Int[x*Cos[x],x]
```

output
```
Cos[x] + x*Sin[x]
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\cos\left(x\right)+x\sin\left(x\right)$	8
risch	$\cos\left(x\right)+x\sin\left(x\right)$	8
parts	$\cos\left(x\right)+x\sin\left(x\right)$	8
parallelrisch	$x\sin\left(x\right)+\cos\left(x\right)+1$	9
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2}{1+\tan^{2}\left(\frac{x}{2}\right)}$	21
meijerg	$2\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(x\right)}{2\sqrt{\pi}}+\frac{x\sin\left(x\right)}{2\sqrt{\pi}}\right)$	27

input
```
int(x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+x*sin(x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="fricas")
```

output
```
x*sin(x) + cos(x)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x)
```

output
```
x*sin(x) + cos(x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="maxima")
```

output
```
x*sin(x) + cos(x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="giac")
```

output
```
x*sin(x) + cos(x)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\,\sin(x)$$

input
```
int(x*cos(x),x)
```

output
```
cos(x) + x*sin(x)
```

3.97.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + \sin(x)\,x$$

input
```
int(cos(x)*x,x)
```

output
```
cos(x) + sin(x)*x
```

3.98 $\int x^2 \cos(x)\,dx$

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3.98.1 Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x^2 \cos(x)\,dx = 2x\cos(x) - 2\sin(x) + x^2\sin(x)$$

output
```
2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x)\,dx = 2x\cos(x) + \left(-2 + x^2\right)\sin(x)$$

input
```
Integrate[x^2*Cos[x],x]
```

output
```
2*x*Cos[x] + (-2 + x^2)*Sin[x]
```

3.98.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin\left(x+\frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{3777}$$

$$2\int -x\sin(x)dx + x^2\sin(x)$$

$$\Big\downarrow \text{25}$$

$$x^2\sin(x) - 2\int x\sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x^2\sin(x) - 2\int x\sin(x)dx$$

$$\Big\downarrow \text{3777}$$

$$x^2\sin(x) - 2(\int \cos(x)dx - x\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$x^2\sin(x) - 2\left(\int \sin\left(x+\frac{\pi}{2}\right) dx - x\cos(x)\right)$$

$$\Big\downarrow \text{3117}$$

$$x^2\sin(x) - 2(\sin(x) - x\cos(x))$$

input
```
Int[x^2*Cos[x],x]
```

output
```
x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x])
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
risch	$2x\cos(x)+(x^2-2)\sin(x)$	15
default	$2x\cos(x)-2\sin(x)+x^2\sin(x)$	17
parallelrisch	$2x\cos(x)-2\sin(x)+x^2\sin(x)$	17
parts	$2x\cos(x)-2\sin(x)+x^2\sin(x)$	17
meijerg	$4\sqrt{\pi}\left(\frac{x\cos(x)}{2\sqrt{\pi}}-\frac{\left(-\frac{3x^2}{2}+3\right)\sin(x)}{6\sqrt{\pi}}\right)$	29
norman	$\frac{2x-2x\left(\tan^2\left(\frac{x}{2}\right)\right)+2x^2\tan\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	40

input
```
int(x^2*cos(x),x,method=_RETURNVERBOSE)
```

output
```
2*x*cos(x)+(x^2-2)*sin(x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x)\,dx = 2\,x\cos(x) + (x^2 - 2)\sin(x)$$

input
```
integrate(x^2*cos(x),x, algorithm="fricas")
```

output
```
2*x*cos(x) + (x^2 - 2)*sin(x)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^2 \cos(x)\,dx = x^2\sin(x) + 2x\cos(x) - 2\sin(x)$$

input
```
integrate(x**2*cos(x),x)
```

output
```
x**2*sin(x) + 2*x*cos(x) - 2*sin(x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x)\,dx = 2\,x\cos(x) + (x^2 - 2)\sin(x)$$

input
```
integrate(x^2*cos(x),x, algorithm="maxima")
```

output
```
2*x*cos(x) + (x^2 - 2)*sin(x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) \, dx = 2 x \cos(x) + (x^2 - 2) \sin(x)$$

input
```
integrate(x^2*cos(x),x, algorithm="giac")
```

output
```
2*x*cos(x) + (x^2 - 2)*sin(x)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) \, dx = \sin(x) \, (x^2 - 2) + 2 x \cos(x)$$

input
```
int(x^2*cos(x),x)
```

output
```
sin(x)*(x^2 - 2) + 2*x*cos(x)
```

3.98.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2 \cos(x) \, dx = 2 \cos(x) x + \sin(x) x^2 - 2 \sin(x)$$

input
```
int(cos(x)*x**2,x)
```

output
```
2*cos(x)*x + sin(x)*x**2 - 2*sin(x)
```

3.99 $\int x \cos^2(x) \, dx$

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3.99.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int x \cos^2(x) \, dx = \frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2} x \cos(x) \sin(x)$$

output
```
1/4*x^2+1/4*cos(x)^2+1/2*x*cos(x)*sin(x)
```

3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x \cos^2(x) \, dx = \frac{x^2}{4} + \frac{1}{8} \cos(2x) + \frac{1}{4} x \sin(2x)$$

input
```
Integrate[x*Cos[x]^2,x]
```

output
```
x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4
```

3.99.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{\int xdx}{2}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x*Cos[x]^2,x]
```

output
```
x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

3.99.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.99.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^{2}}{4}+\frac{\cos\left(2 x\right)}{8}+\frac{x \sin\left(2 x\right)}{4}$	20
parallelrisch	$\frac{x^{2}}{4}-\frac{1}{8}+\frac{x \sin\left(2 x\right)}{4}+\frac{\cos\left(2 x\right)}{8}$	21
default	$x \left(\frac{x}{2}+\frac{\cos\left(x\right) \sin\left(x\right)}{2}\right)-\frac{x^{2}}{4}-\frac{\left(\sin^{2}\left(x\right)\right)}{4}$	25
norman	$\frac{x \tan\left(\frac{x}{2}\right)-\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{x^{2}}{4}+\frac{x^{2} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{x^{2} \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}-\left(\tan^{3}\left(\frac{x}{2}\right)\right) x}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	63

input
```
int(cos(x)^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2+1/8*cos(2*x)+1/4*x*sin(2*x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \cos^2(x)\,dx=\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x^{2}+\frac{1}{4}\,\cos\left(x\right)^{2}$$

input
```
integrate(x*cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x\cos^2(x)\,dx = \frac{x^2\sin^2(x)}{4} + \frac{x^2\cos^2(x)}{4} + \frac{x\sin(x)\cos(x)}{2} + \frac{\cos^2(x)}{4}$$

input
```
integrate(x*cos(x)**2,x)
```

output
```
x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 + cos(x)**2/4
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\cos^2(x)\,dx = \frac{1}{4}x^2 + \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x)$$

input
```
integrate(x*cos(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\cos^2(x)\,dx = \frac{1}{4}x^2 + \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x)$$

input
```
integrate(x*cos(x)^2,x, algorithm="giac")
```

output
```
1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\cos^2(x)\,dx = \frac{x\,\sin(2\,x)}{4} - \frac{\sin(x)^2}{4} + \frac{x^2}{4}$$

input
```
int(x*cos(x)^2,x)
```

output
```
(x*sin(2*x))/4 - sin(x)^2/4 + x^2/4
```

3.99.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x\cos^2(x)\,dx = \frac{\cos(x)\sin(x)\,x}{2} - \frac{\sin(x)^2}{4} + \frac{x^2}{4} + \frac{1}{2}$$

input
```
int(cos(x)**2*x,x)
```

output
```
(2*cos(x)*sin(x)*x - sin(x)**2 + x**2 + 2)/4
```

3.100 $\int x^2 \cos^2(x)\,dx$

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3.100.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^2 \cos^2(x)\,dx = -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)$$

output
```
-1/4*x+1/6*x^3+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)+1/2*x^2*cos(x)*sin(x)
```

3.100.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \cos^2(x)\,dx = \frac{1}{24}\left(4x^3+6x\cos(2x)+\left(-3+6x^2\right)\sin(2x)\right)$$

input
```
Integrate[x^2*Cos[x]^2,x]
```

output
```
(4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24
```

3.100.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{\int x^2dx}{2}-\frac{1}{2}\int \cos^2(x)dx+\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}\int \cos^2(x)dx+\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(-\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)+\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(-\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[x^2*Cos[x]^2,x]
```

output
```
x^3/6 + (x*Cos[x]^2)/2 + (x^2*Cos[x]*Sin[x])/2 + (-1/2*x - (Cos[x]*Sin[x])
/2)/2
```

3.100.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.100.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

method	result	siz
risch	$\frac{x^3}{6}+\frac{x\cos(2x)}{4}+\frac{(2x^2-1)\sin(2x)}{8}$	27
parallelrisch	$\frac{x^3}{6}+\frac{x\cos(2x)}{4}+\frac{x^2\sin(2x)}{4}-\frac{\sin(2x)}{8}$	29
default	$x^2\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)+\frac{(\cos^2(x))x}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}$	37
norman	$\frac{x^2\tan\left(\frac{x}{2}\right)+\frac{x}{4}+\frac{x^3}{6}+\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}-\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}-x^2\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{x^3\left(\tan^4\left(\frac{x}{2}\right)\right)}{6}-\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	94

input
```
int(x^2*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3+1/4*x*cos(2*x)+1/8*(2*x^2-1)*sin(2*x)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \cos^2(x)\, dx = \frac{1}{6}\, x^3 + \frac{1}{2}\, x \cos{(x)}^2 + \frac{1}{4}\, \left(2\, x^2 - 1\right) \cos{(x)} \sin{(x)} - \frac{1}{4}\, x$$

input
```
integrate(x^2*cos(x)^2,x, algorithm="fricas")
```

output
```
1/6*x^3 + 1/2*x*cos(x)^2 + 1/4*(2*x^2 - 1)*cos(x)*sin(x) - 1/4*x
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int x^2 \cos^2(x)\, dx = \frac{x^3 \sin^2{(x)}}{6} + \frac{x^3 \cos^2{(x)}}{6} + \frac{x^2 \sin{(x)} \cos{(x)}}{2} - \frac{x \sin^2{(x)}}{4} + \frac{x \cos^2{(x)}}{4} - \frac{\sin{(x)} \cos{(x)}}{4}$$

input
```
integrate(x**2*cos(x)**2,x)
```

output
```
x**3*sin(x)**2/6 + x**3*cos(x)**2/6 + x**2*sin(x)*cos(x)/2 - x*sin(x)**2/4
 + x*cos(x)**2/4 - sin(x)*cos(x)/4
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \cos^2(x)\,dx = \frac{1}{6}\,x^3 + \frac{1}{4}\,x\cos\left(2\,x\right) + \frac{1}{8}\,\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

input
```
integrate(x^2*cos(x)^2,x, algorithm="maxima")
```

output
```
1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \cos^2(x)\,dx = \frac{1}{6}\,x^3 + \frac{1}{4}\,x\cos\left(2\,x\right) + \frac{1}{8}\,\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

input
```
integrate(x^2*cos(x)^2,x, algorithm="giac")
```

output
```
1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2 \cos^2(x)\,dx = \frac{x\,\cos\left(2\,x\right)}{4} - \frac{\sin\left(2\,x\right)}{8} + \frac{x^2\,\sin\left(2\,x\right)}{4} + \frac{x^3}{6}$$

input
```
int(x^2*cos(x)^2,x)
```

output
```
(x*cos(2*x))/4 - sin(2*x)/8 + (x^2*sin(2*x))/4 + x^3/6
```

3.100.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int x^2\cos^2(x)\,dx = \frac{\cos(x)\sin(x)\,x^2}{2} - \frac{\cos(x)\sin(x)}{4} - \frac{\sin(x)^2\,x}{2} + \frac{x^3}{6} + \frac{x}{4}$$

input
```
int(cos(x)**2*x**2,x)
```

output
```
(6*cos(x)*sin(x)*x**2 - 3*cos(x)*sin(x) - 6*sin(x)**2*x + 2*x**3 + 3*x)/12
```

3.101 $\int x \cos^3(x) \, dx$

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3.101.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int x \cos^3(x) \, dx = \frac{2\cos(x)}{3} + \frac{\cos^3(x)}{9} + \frac{2}{3}x\sin(x) + \frac{1}{3}x\cos^2(x)\sin(x)$$

output
```
2/3*cos(x)+1/9*cos(x)^3+2/3*x*sin(x)+1/3*x*cos(x)^2*sin(x)
```

3.101.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int x \cos^3(x) \, dx = \frac{3\cos(x)}{4} + \frac{1}{36}\cos(3x) + \frac{3}{4}x\sin(x) + \frac{1}{12}x\sin(3x)$$

input
```
Integrate[x*Cos[x]^3,x]
```

output
```
(3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12
```

3.101.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{2}{3}\int x\cos(x)dx+\frac{\cos^3(x)}{9}+\frac{1}{3}x\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int x\sin\left(x+\frac{\pi}{2}\right)dx+\frac{\cos^3(x)}{9}+\frac{1}{3}x\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}(\int -\sin(x)dx+x\sin(x))+\frac{\cos^3(x)}{9}+\frac{1}{3}x\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}(x\sin(x)-\int \sin(x)dx)+\frac{\cos^3(x)}{9}+\frac{1}{3}x\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}(x\sin(x)-\int \sin(x)dx)+\frac{\cos^3(x)}{9}+\frac{1}{3}x\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{3118}$$

$$\frac{\cos^3(x)}{9}+\frac{1}{3}x\sin(x)\cos^2(x)+\frac{2}{3}(x\sin(x)+\cos(x))$$

input
```
Int[x*Cos[x]^3,x]
```

output
```
Cos[x]^3/9 + (x*Cos[x]^2*Sin[x])/3 + (2*(Cos[x] + x*Sin[x]))/3
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.101.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{x\left(2+\cos^{2}\left(x\right)\right)\sin\left(x\right)}{3}+\frac{\left(\cos^{3}\left(x\right)\right)}{9}+\frac{2\cos\left(x\right)}{3}$	23
risch	$\frac{3\cos\left(x\right)}{4}+\frac{3x\sin\left(x\right)}{4}+\frac{\cos\left(3x\right)}{36}+\frac{x\sin\left(3x\right)}{12}$	24
parallelrisch	$\frac{7}{9}+\frac{3\cos\left(x\right)}{4}+\frac{3x\sin\left(x\right)}{4}+\frac{\cos\left(3x\right)}{36}+\frac{x\sin\left(3x\right)}{12}$	25
norman	$\frac{2\left(\tan^{4}\left(\frac{x}{2}\right)\right)+\frac{8\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+2x\tan\left(\frac{x}{2}\right)+2x\left(\tan^{5}\left(\frac{x}{2}\right)\right)+\frac{4\left(\tan^{3}\left(\frac{x}{2}\right)\right)x}{3}+\frac{14}{9}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}$	55

input
```
int(x*cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2+cos(x)^2)*sin(x)+1/9*cos(x)^3+2/3*cos(x)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\cos^3(x)\,dx = \frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(x\cos(x)^2 + 2\,x\right)\sin(x) + \frac{2}{3}\cos(x)$$

input
```
integrate(x*cos(x)^3,x, algorithm="fricas")
```

output
```
1/9*cos(x)^3 + 1/3*(x*cos(x)^2 + 2*x)*sin(x) + 2/3*cos(x)
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x\cos^3(x)\,dx = \frac{2x\sin^3(x)}{3} + x\sin(x)\cos^2(x) + \frac{2\sin^2(x)\cos(x)}{3} + \frac{7\cos^3(x)}{9}$$

input
```
integrate(x*cos(x)**3,x)
```

output
```
2*x*sin(x)**3/3 + x*sin(x)*cos(x)**2 + 2*sin(x)**2*cos(x)/3 + 7*cos(x)**3/
9
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\cos^3(x)\,dx = \frac{1}{12}\,x\sin(3\,x) + \frac{3}{4}\,x\sin(x) + \frac{1}{36}\cos(3\,x) + \frac{3}{4}\cos(x)$$

input
```
integrate(x*cos(x)^3,x, algorithm="maxima")
```

output
```
1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\cos^3(x)\,dx = \frac{1}{12}\,x\sin(3\,x) + \frac{3}{4}\,x\sin(x) + \frac{1}{36}\,\cos(3\,x) + \frac{3}{4}\,\cos(x)$$

input
```
integrate(x*cos(x)^3,x, algorithm="giac")
```

output
```
1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\cos^3(x)\,dx = \frac{\cos(x)^3}{9} + \frac{x\,\sin(x)\,\cos(x)^2}{3} + \frac{2\,\cos(x)}{3} + \frac{2\,x\,\sin(x)}{3}$$

input
```
int(x*cos(x)^3,x)
```

output
```
(2*cos(x))/3 + cos(x)^3/9 + (2*x*sin(x))/3 + (x*cos(x)^2*sin(x))/3
```

3.101.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\cos^3(x)\,dx = -\frac{\cos(x)\sin(x)^2}{9} + \frac{7\cos(x)}{9} - \frac{\sin(x)^3 x}{3} + \sin(x)\,x + \frac{1}{9}$$

input
```
int(cos(x)**3*x,x)
```

output
```
( - cos(x)*sin(x)**2 + 7*cos(x) - 3*sin(x)**3*x + 9*sin(x)*x + 1)/9
```

3.102 $\int \frac{\sin(x)}{x} dx$

3.102.1 Optimal result 684
3.102.2 Mathematica [A] (verified) 684
3.102.3 Rubi [A] (verified) 685
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3.102.5 Fricas [A] (verification not implemented) 686
3.102.6 Sympy [A] (verification not implemented) 686
3.102.7 Maxima [C] (verification not implemented) 687
3.102.8 Giac [A] (verification not implemented) 687
3.102.9 Mupad [F(-1)] 687
3.102.10Reduce [B] (verification not implemented) 688

3.102.1 Optimal result

Integrand size = 6, antiderivative size = 2

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

output
```
Si(x)
```

3.102.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

input
```
Integrate[Sin[x]/x,x]
```

output
```
SinIntegral[x]
```

3.102.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{x} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{x} dx$$

$$\Big\downarrow \text{ 3780}$$

$$\text{Si}(x)$$

input
```
Int[Sin[x]/x,x]
```

output
```
SinIntegral[x]
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.102.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\mathrm{Si}\left(x\right)$	3
meijerg	$\mathrm{Si}\left(x\right)$	3
risch	$-\frac{\pi \,\mathrm{csgn}\left(x\right)}{2}+\mathrm{Si}\left(x\right)$	9

input
```
int(sin(x)/x,x,method=_RETURNVERBOSE)
```

output
```
Si(x)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x}\,dx = \mathrm{Si}\left(x\right)$$

input
```
integrate(sin(x)/x,x, algorithm="fricas")
```

output
```
sin_integral(x)
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x}\,dx = \mathrm{Si}\left(x\right)$$

input
```
integrate(sin(x)/x,x)
```

output
```
Si(x)
```

3.102.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{\sin(x)}{x}\,dx = -\frac{1}{2}i\,\mathrm{Ei}(i\,x) + \frac{1}{2}i\,\mathrm{Ei}(-i\,x)$$

input
```
integrate(sin(x)/x,x, algorithm="maxima")
```

output
```
-1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x}\,dx = \mathrm{Si}\,(x)$$

input
```
integrate(sin(x)/x,x, algorithm="giac")
```

output
```
sin_integral(x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{x}\,dx = \mathrm{sinint}(x)$$

input
```
int(sin(x)/x,x)
```

output
```
sinint(x)
```

3.102.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x}\,dx = si(x)$$

input
```
int(sin(x)/x,x)
```

output
```
si(x)
```

3.103 $\int \frac{\cos(x)}{x} \, dx$

3.103.1 Optimal result 689
3.103.2 Mathematica [A] (verified) 689
3.103.3 Rubi [A] (verified) 690
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3.103.5 Fricas [A] (verification not implemented) 691
3.103.6 Sympy [B] (verification not implemented) 691
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3.103.8 Giac [A] (verification not implemented) 692
3.103.9 Mupad [F(-1)] 692
3.103.10Reduce [B] (verification not implemented) 693

3.103.1 Optimal result

Integrand size = 6, antiderivative size = 2

$$\int \frac{\cos(x)}{x} \, dx = \text{CosIntegral}(x)$$

output
```
Ci(x)
```

3.103.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x} \, dx = \text{CosIntegral}(x)$$

input
```
Integrate[Cos[x]/x,x]
```

output
```
CosIntegral[x]
```

3.103.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{x} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{ 3783}$$

$$\text{CosIntegral}(x)$$

input
```
Int[Cos[x]/x,x]
```

output
```
CosIntegral[x]
```

3.103.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.103.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{Ci}(x)$	3
risch	$\operatorname{Ci}(x)-\frac{i\pi\,\operatorname{csgn}(ix)\operatorname{csgn}(x)}{2}+\frac{i\pi\,\operatorname{csgn}(ix)}{2}$	24
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}}+\frac{2\operatorname{Ci}(x)}{\sqrt{\pi}}\right)}{2}$	48

input
```
int(cos(x)/x,x,method=_RETURNVERBOSE)
```

output
```
Ci(x)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x}\,dx = \operatorname{Ci}(x)$$

input
```
integrate(cos(x)/x,x, algorithm="fricas")
```

output
```
cos_integral(x)
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4.

Time = 0.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{\cos(x)}{x}\,dx = -\log(x)+\frac{\log(x^2)}{2}+\operatorname{Ci}(x)$$

input
```
integrate(cos(x)/x,x)
```

output
```
-log(x) + log(x**2)/2 + Ci(x)
```

3.103.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{\cos(x)}{x}\,dx = \frac{1}{2}\,\mathrm{Ei}(i\,x) + \frac{1}{2}\,\mathrm{Ei}(-i\,x)$$

input
```
integrate(cos(x)/x,x, algorithm="maxima")
```

output
```
1/2*Ei(I*x) + 1/2*Ei(-I*x)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x}\,dx = \mathrm{Ci}\,(x)$$

input
```
integrate(cos(x)/x,x, algorithm="giac")
```

output
```
cos_integral(x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{x}\,dx = \mathrm{cosint}(x)$$

input
```
int(cos(x)/x,x)
```

output
```
cosint(x)
```

3.103.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x}\,dx = ci(x)$$

input
```
int(cos(x)/x,x)
```

output
```
ci(x)
```

3.104 $\int \frac{\sin(x)}{x^2} \, dx$

3.104.1 Optimal result 694
3.104.2 Mathematica [A] (verified) 694
3.104.3 Rubi [A] (verified) 695
3.104.4 Maple [A] (verified) 696
3.104.5 Fricas [A] (verification not implemented) 697
3.104.6 Sympy [B] (verification not implemented) 697
3.104.7 Maxima [C] (verification not implemented) 697
3.104.8 Giac [A] (verification not implemented) 698
3.104.9 Mupad [F(-1)] 698
3.104.10 Reduce [B] (verification not implemented) 698

3.104.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{\sin(x)}{x^2} \, dx = \text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

output
```
Ci(x)-sin(x)/x
```

3.104.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x^2} \, dx = \text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

input
```
Integrate[Sin[x]/x^2,x]
```

output
```
CosIntegral[x] - Sin[x]/x
```

3.104.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{x^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\int \frac{\cos(x)}{x}dx - \frac{\sin(x)}{x}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{x}dx - \frac{\sin(x)}{x}$$

$$\Big\downarrow \text{3783}$$

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

input
```
Int[Sin[x]/x^2,x]
```

output
```
CosIntegral[x] - Sin[x]/x
```

3.104.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\operatorname{Ci}(x)-\frac{\sin(x)}{x}$	11
risch	$\operatorname{Ci}(x)-\frac{i\pi\,\mathrm{csgn}(ix)\,\mathrm{csgn}(x)}{2}+\frac{i\pi\,\mathrm{csgn}(ix)}{2}-\frac{\sin(x)}{x}$	31
meijerg	$\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}}-\frac{4\sin(x)}{\sqrt{\pi}\,x}+\frac{4\operatorname{Ci}(x)}{\sqrt{\pi}}\right)}{4}$	65

input
```
int(sin(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
Ci(x)-sin(x)/x
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin(x)}{x^2}\,dx = \frac{x\,\mathrm{Ci}\,(x) - \sin\,(x)}{x}$$

input
```
integrate(sin(x)/x^2,x, algorithm="fricas")
```

output
```
(x*cos_integral(x) - sin(x))/x
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\sin(x)}{x^2}\,dx = -\log\,(x) + \frac{\log\,(x^2)}{2} + \mathrm{Ci}\,(x) - \frac{\sin\,(x)}{x}$$

input
```
integrate(sin(x)/x**2,x)
```

output
```
-log(x) + log(x**2)/2 + Ci(x) - sin(x)/x
```

3.104.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\sin(x)}{x^2}\,dx = \frac{1}{2}\,\Gamma(-1, i\,x) + \frac{1}{2}\,\Gamma(-1, -i\,x)$$

input
```
integrate(sin(x)/x^2,x, algorithm="maxima")
```

output
```
1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin(x)}{x^2} \, dx = \frac{x \operatorname{Ci}(x) - \sin(x)}{x}$$

input
```
integrate(sin(x)/x^2,x, algorithm="giac")
```

output
```
(x*cos_integral(x) - sin(x))/x
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{x^2} \, dx = \operatorname{cosint}(x) - \frac{\sin(x)}{x}$$

input
```
int(sin(x)/x^2,x)
```

output
```
cosint(x) - sin(x)/x
```

3.104.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin(x)}{x^2} \, dx = \frac{ci(x)\, x - \sin(x)}{x}$$

input
```
int(sin(x)/x**2,x)
```

output
```
(ci(x)*x - sin(x))/x
```

3.105 $\int \frac{\sin^2(x)}{x}\,dx$

3.105.1 Optimal result . . . 699
3.105.2 Mathematica [A] (verified) . . . 699
3.105.3 Rubi [A] (verified) . . . 700
3.105.4 Maple [A] (verified) . . . 701
3.105.5 Fricas [A] (verification not implemented) . . . 701
3.105.6 Sympy [A] (verification not implemented) . . . 702
3.105.7 Maxima [C] (verification not implemented) . . . 702
3.105.8 Giac [A] (verification not implemented) . . . 702
3.105.9 Mupad [F(-1)] . . . 703
3.105.10 Reduce [B] (verification not implemented) . . . 703

3.105.1 Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \frac{\sin^2(x)}{x}\,dx = -\frac{\text{CosIntegral}(2x)}{2} + \frac{\log(x)}{2}$$

output
```
-1/2*Ci(2*x)+1/2*ln(x)
```

3.105.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{x}\,dx = -\frac{\text{CosIntegral}(2x)}{2} + \frac{\log(x)}{2}$$

input
```
Integrate[Sin[x]^2/x,x]
```

output
```
-1/2*CosIntegral[2*x] + Log[x]/2
```

3.105.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{x} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(x)^2}{x} dx$$
$$\Big\downarrow \; 3793$$
$$\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\log(x)}{2} - \frac{\text{CosIntegral}(2x)}{2}$$

input
```
Int[Sin[x]^2/x,x]
```

output
```
-1/2*CosIntegral[2*x] + Log[x]/2
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.105.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\operatorname{Ci}(2x)}{2}+\frac{\ln(x)}{2}$	12
risch	$-\frac{\operatorname{Ci}(2x)}{2}+\frac{i\pi\ \operatorname{csgn}(ix)\operatorname{csgn}(x)}{4}-\frac{i\pi\ \operatorname{csgn}(ix)}{4}+\frac{\ln(x)}{2}$	32
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma}{\sqrt{\pi}}+\frac{2\ln(2)}{\sqrt{\pi}}+\frac{2\ln(x)}{\sqrt{\pi}}-\frac{2\ \operatorname{Ci}(2x)}{\sqrt{\pi}}\right)}{4}$	36

input
```
int(sin(x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Ci(2*x)+1/2*ln(x)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin^2(x)}{x}\,dx=-\frac{1}{2}\operatorname{Ci}(2x)+\frac{1}{2}\log(x)$$

input
```
integrate(sin(x)^2/x,x, algorithm="fricas")
```

output
```
-1/2*cos_integral(2*x) + 1/2*log(x)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\sin^2(x)}{x}\,dx = \frac{\log(x)}{2} - \frac{\mathrm{Ci}(2x)}{2}$$

input
```
integrate(sin(x)**2/x,x)
```

output
```
log(x)/2 - Ci(2*x)/2
```

3.105.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(x)}{x}\,dx = -\frac{1}{4}\,\mathrm{Ei}(2i\,x) - \frac{1}{4}\,\mathrm{Ei}(-2i\,x) + \frac{1}{2}\,\log(x)$$

input
```
integrate(sin(x)^2/x,x, algorithm="maxima")
```

output
```
-1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin^2(x)}{x}\,dx = -\frac{1}{2}\,\mathrm{Ci}(2\,x) + \frac{1}{2}\,\log(x)$$

input
```
integrate(sin(x)^2/x,x, algorithm="giac")
```

output
```
-1/2*cos_integral(2*x) + 1/2*log(x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)}{x}\,dx = \frac{\ln(x)}{2} - \frac{\text{cosint}(2\,x)}{2}$$

input
```
int(sin(x)^2/x,x)
```

output
```
log(x)/2 - cosint(2*x)/2
```

3.105.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin^2(x)}{x}\,dx = -\frac{ci(2x)}{2} + \frac{\log(x)}{2}$$

input
```
int(sin(x)**2/x,x)
```

output
```
( - ci(2*x) + log(x))/2
```

3.106 $\int \tan^3(x)\,dx$

3.106.1 Optimal result . . . 704
3.106.2 Mathematica [A] (verified) . . . 704
3.106.3 Rubi [A] (verified) . . . 705
3.106.4 Maple [A] (verified) . . . 706
3.106.5 Fricas [A] (verification not implemented) . . . 707
3.106.6 Sympy [A] (verification not implemented) . . . 707
3.106.7 Maxima [A] (verification not implemented) . . . 707
3.106.8 Giac [A] (verification not implemented) . . . 708
3.106.9 Mupad [B] (verification not implemented) . . . 708
3.106.10Reduce [B] (verification not implemented) . . . 708

3.106.1 Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \tan^3(x)\,dx = \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

output
```
ln(cos(x))+1/2*tan(x)^2
```

3.106.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^3(x)\,dx = \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

input
```
Integrate[Tan[x]^3,x]
```

output
```
Log[Cos[x]] + Tan[x]^2/2
```

3.106.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(x)^3 dx$$

$$\Big\downarrow \; 3954$$

$$\frac{\tan^2(x)}{2} - \int \tan(x)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan^2(x)}{2} - \int \tan(x)dx$$

$$\Big\downarrow \; 3956$$

$$\frac{\tan^2(x)}{2} + \log(\cos(x))$$

input
```
Int[Tan[x]^3,x]
```

output
```
Log[Cos[x]] + Tan[x]^2/2
```

3.106.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.106.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
default	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
norman	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
parallelrisch	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
risch	$-ix + \frac{2\,e^{2ix}}{(e^{2ix}+1)^2} + \ln(e^{2ix}+1)$	30

input
```
int(tan(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)^2-1/2*ln(1+tan(x)^2)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan^3(x)\,dx = \frac{1}{2}\tan(x)^2 + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

input
```
integrate(tan(x)^3,x, algorithm="fricas")
```

output
```
1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^3(x)\,dx = \log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

input
```
integrate(tan(x)**3,x)
```

output
```
log(cos(x)) + 1/(2*cos(x)**2)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \tan^3(x)\,dx = -\frac{1}{2\left(\sin(x)^2-1\right)} + \frac{1}{2}\log\left(\sin(x)^2-1\right)$$

input
```
integrate(tan(x)^3,x, algorithm="maxima")
```

output
```
-1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x)\,dx = \frac{1}{2}\tan(x)^2 - \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

input
```
integrate(tan(x)^3,x, algorithm="giac")
```

output
```
1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x)\,dx = \ln(\cos(x)) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

input
```
int(tan(x)^3,x)
```

output
```
log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)
```

3.106.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x)\,dx = -\frac{\log(\tan(x)^2 + 1)}{2} + \frac{\tan(x)^2}{2}$$

input
```
int(tan(x)**3,x)
```

output
```
( - log(tan(x)**2 + 1) + tan(x)**2)/2
```

3.107 $\int \sin(a+bx)\,dx$

3.107.1 Optimal result 709
3.107.2 Mathematica [A] (verified) 709
3.107.3 Rubi [A] (verified) 710
3.107.4 Maple [A] (verified) 711
3.107.5 Fricas [A] (verification not implemented) 711
3.107.6 Sympy [A] (verification not implemented) 712
3.107.7 Maxima [A] (verification not implemented) 712
3.107.8 Giac [A] (verification not implemented) 712
3.107.9 Mupad [B] (verification not implemented) 713
3.107.10 Reduce [B] (verification not implemented) 713

3.107.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

output
```
-cos(b*x+a)/b
```

3.107.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(a)\cos(bx)}{b} + \frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Sin[a + b*x],x]
```

output
```
-((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

3.107.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)dx$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\cos(a+bx)}{b}$$

input
```
Int[Sin[a + b*x],x]
```

output
```
-(Cos[a + b*x]/b)
```

3.107.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.107.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\cos(bx+a)}{b}$	12
default	$-\frac{\cos(bx+a)}{b}$	12
risch	$-\frac{\cos(bx+a)}{b}$	12
parallelrisch	$\frac{-\cos(bx+a)-1}{b}$	15
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	32
meijerg	$\frac{\sin(a)\sin(bx)}{b}+\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	34

input
```
int(sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-cos(b*x+a)/b
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a),x, algorithm="fricas")
```

output
```
-cos(b*x + a)/b
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \sin(a+bx)\,dx = \begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x\sin(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a),x)
```

output
```
Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a),x, algorithm="maxima")
```

output
```
-cos(b*x + a)/b
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a),x, algorithm="giac")
```

output
```
-cos(b*x + a)/b
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+b\,x)}{b}$$

input
```
int(sin(a + b*x),x)
```

output
```
-cos(a + b*x)/b
```

3.107.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

input
```
int(sin(a + b*x),x)
```

output
```
( - cos(a + b*x))/b
```

3.108 $\int \cos(a+bx)\,dx$

3.108.1 Optimal result 714
3.108.2 Mathematica [B] (verified) 714
3.108.3 Rubi [A] (verified) 715
3.108.4 Maple [A] (verified) 716
3.108.5 Fricas [A] (verification not implemented) 716
3.108.6 Sympy [A] (verification not implemented) 717
3.108.7 Maxima [A] (verification not implemented) 717
3.108.8 Giac [A] (verification not implemented) 717
3.108.9 Mupad [B] (verification not implemented) 718
3.108.10Reduce [B] (verification not implemented) 718

3.108.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cos(a+bx)\,dx = \frac{\sin(a+bx)}{b}$$

output
```
sin(b*x+a)/b
```

3.108.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(a+bx)\,dx = \frac{\cos(bx)\sin(a)}{b} + \frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x],x]
```

output
```
(Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

3.108.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{3117}$$
$$\frac{\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x],x]
```

output
```
Sin[a + b*x]/b
```

3.108.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.108.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\sin(bx+a)}{b}$	11
default	$\frac{\sin(bx+a)}{b}$	11
risch	$\frac{\sin(bx+a)}{b}$	11
parallelrisch	$\frac{\sin(bx+a)}{b}$	11
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	30
meijerg	$\frac{\cos(a)\sin(bx)}{b}-\frac{\sin(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	35

input
```
int(cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
sin(b*x+a)/b
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx = \frac{\sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a),x, algorithm="fricas")
```

output
```
sin(b*x + a)/b
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\,dx = \begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a),x)
```

output
```
Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx = \frac{\sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a),x, algorithm="maxima")
```

output
```
sin(b*x + a)/b
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx = \frac{\sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a),x, algorithm="giac")
```

output
```
sin(b*x + a)/b
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx=\frac{\sin\left(a+b\,x\right)}{b}$$

input
```
int(cos(a + b*x),x)
```

output
```
sin(a + b*x)/b
```

3.108.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx=\frac{\sin\left(bx+a\right)}{b}$$

input
```
int(cos(a + b*x),x)
```

output
```
sin(a + b*x)/b
```

3.109 $\int \tan(a+bx)\,dx$

3.109.1 Optimal result 719
3.109.2 Mathematica [A] (verified) 719
3.109.3 Rubi [A] (verified) 720
3.109.4 Maple [A] (verified) 721
3.109.5 Fricas [A] (verification not implemented) 721
3.109.6 Sympy [A] (verification not implemented) 722
3.109.7 Maxima [A] (verification not implemented) 722
3.109.8 Giac [A] (verification not implemented) 722
3.109.9 Mupad [B] (verification not implemented) 723
3.109.10Reduce [B] (verification not implemented) 723

3.109.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}$$

output
```
-ln(cos(b*x+a))/b
```

3.109.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}$$

input
```
Integrate[Tan[a + b*x],x]
```

output
```
-(Log[Cos[a + b*x]]/b)
```

3.109.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\log(\cos(a+bx))}{b}$$

input
```
Int[Tan[a + b*x],x]
```

output
```
-(Log[Cos[a + b*x]]/b)
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.109.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
default	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
norman	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
parallelrisch	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
risch	$ix+\frac{2ia}{b}-\frac{\ln(e^{2i(bx+a)}+1)}{b}$	30

input
```
int(tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(1+tan(b*x+a)^2)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(a+bx)\,dx = -\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2\,b}$$

input
```
integrate(tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(b*x + a)^2 + 1))/b
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \tan(a+bx)\,dx = \begin{cases} \frac{\log(\tan^2(a+bx)+1)}{2b} & \text{for } b \neq 0 \\ x\tan(a) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(b*x+a),x)
```

output
```
Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \tan(a+bx)\,dx = \frac{\log(\sec(bx+a))}{b}$$

input
```
integrate(tan(b*x+a),x, algorithm="maxima")
```

output
```
log(sec(b*x + a))/b
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \tan(a+bx)\,dx = -\frac{\log(|\cos(bx+a)|)}{b}$$

input
```
integrate(tan(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(cos(b*x + a)))/b
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

input
```
int(tan(a + b*x),x)
```

output
```
log(tan(a + b*x)^2 + 1)/(2*b)
```

3.109.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(a+bx)\,dx = \frac{\log\left(\tan\left(bx+a\right)^2+1\right)}{2b}$$

input
```
int(tan(a + b*x),x)
```

output
```
log(tan(a + b*x)**2 + 1)/(2*b)
```

3.110 $\int \cot(a+bx)\,dx$

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3.110.2 Mathematica [B] (verified) . . . 724
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3.110.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

output
```
ln(sin(b*x+a))/b
```

3.110.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \cot(a+bx)\,dx = \frac{\log(\cos(a+bx))}{b} + \frac{\log(\tan(a+bx))}{b}$$

input
```
Integrate[Cot[a + b*x],x]
```

output
```
Log[Cos[a + b*x]]/b + Log[Tan[a + b*x]]/b
```

3.110.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\log(-\sin(a+bx))}{b}$$

input
```
Int[Cot[a + b*x],x]
```

output
```
Log[-Sin[a + b*x]]/b
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

method	result	size
derivativedivides	$\frac{\ln(\tan(bx+a))-\frac{\ln\left(1+\tan^2(bx+a)\right)}{2}}{b}$	26
default	$\frac{\ln(\tan(bx+a))-\frac{\ln\left(1+\tan^2(bx+a)\right)}{2}}{b}$	26
norman	$\frac{\ln(\tan(bx+a))}{b}-\frac{\ln(1+\tan^2(bx+a))}{2b}$	29
risch	$-ix-\frac{2ia}{b}+\frac{\ln(e^{2i(bx+a)}-1)}{b}$	29
parallelrisch	$\frac{2\ln(\tan(bx+a))-\ln(1+\tan^2(bx+a))}{2b}$	29

input
```
int(1/tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(tan(b*x+a))-1/2*ln(1+tan(b*x+a)^2))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \cot(a+bx)\,dx = \frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2\,b}$$

input
```
integrate(1/tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \cot(a+bx)\,dx = \begin{cases} -\frac{\log(\tan^2(a+bx)+1)}{2b} + \frac{\log(\tan(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\tan(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/tan(b*x+a),x)
```

output
```
Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0)
), (x/tan(a), True))
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(bx+a))}{b}$$

input
```
integrate(1/tan(b*x+a),x, algorithm="maxima")
```

output
```
log(sin(b*x + a))/b
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \cot(a+bx)\,dx = \frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{2b}$$

input
```
integrate(1/tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b
*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \cot(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

input
```
int(1/tan(a + b*x),x)
```

output
```
log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b)
```

3.110.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \cot(a+bx)\,dx = \frac{-\log\left(\tan\left(bx+a\right)^2+1\right)+2\log\left(\tan\left(bx+a\right)\right)}{2b}$$

input
```
int(1/tan(a + b*x),x)
```

output
```
( - log(tan(a + b*x)**2 + 1) + 2*log(tan(a + b*x)))/(2*b)
```

3.111 $\int \csc(a+bx)\,dx$

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3.111.9 Mupad [B] (verification not implemented) 733
3.111.10 Reduce [B] (verification not implemented) 733

3.111.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \csc(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}$$

output
```
-arctanh(cos(b*x+a))/b
```

3.111.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \csc(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

input
```
Integrate[Csc[a + b*x],x]
```

output
```
-(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b
```

3.111.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(a+bx)dx$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}$$

input
```
Int[Csc[a + b*x],x]
```

output
```
-(ArcTanh[Cos[a + b*x]]/b)
```

3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.111.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
parallelrisch	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
derivativedivides	$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$	21
default	$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$	21
risch	$-\frac{\ln(e^{i(bx+a)}+1)}{b}+\frac{\ln(e^{i(bx+a)}-1)}{b}$	35

input
```
int(1/sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(tan(1/2*b*x+1/2*a))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \csc(a+bx)\,dx = -\frac{\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)}{2\,b}$$

input
```
integrate(1/sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \csc(a+bx)\,dx = \begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\sin(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/sin(b*x+a),x)
```

output
```
Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \csc(a+bx)\,dx = -\frac{\log(\cos(bx+a)+1) - \log(\cos(bx+a)-1)}{2\,b}$$

input
```
integrate(1/sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.25

$$\int \csc(a+bx)\,dx = \frac{\log\left(\left|-\frac{\cos(bx+a)}{b}+\frac{1}{|b|}\right|\right)}{2\,|b|} - \frac{\log\left(\left|-\frac{\cos(bx+a)}{b}-\frac{1}{|b|}\right|\right)}{2\,|b|}$$

input
```
integrate(1/sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(abs(-cos(b*x + a)/b + 1/abs(b)))/abs(b) - 1/2*log(abs(-cos(b*x + a
)/b - 1/abs(b)))/abs(b)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\,dx = -\frac{\operatorname{atanh}(\cos{(a+b\,x)})}{b}$$

input
```
int(1/sin(a + b*x),x)
```

output
```
-atanh(cos(a + b*x))/b
```

3.111.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \csc(a+bx)\,dx = \frac{\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$$

input
```
int(1/sin(a + b*x),x)
```

output
```
log(tan((a + b*x)/2))/b
```

3.112 $\int \sec(a+bx)\,dx$

3.112.1 Optimal result 734
3.112.2 Mathematica [A] (verified) 734
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3.112.9 Mupad [B] (verification not implemented) 738
3.112.10Reduce [B] (verification not implemented) 738

3.112.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sec(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b}$$

output
```
arctanh(sin(b*x+a))/b
```

3.112.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b}$$

input
```
Integrate[Sec[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/b
```

3.112.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4257$$

$$\frac{\text{arctanh}(\sin(a+bx))}{b}$$

input
```
Int[Sec[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/b
```

3.112.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.112.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

method	result	size
derivativedivides	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
default	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	32
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	35
risch	$\frac{\ln(e^{i(bx+a)}+i)}{b}-\frac{\ln(e^{i(bx+a)}-i)}{b}$	37

input
```
int(1/cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(sec(b*x+a)+tan(b*x+a))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \sec(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)}{2\,b}$$

input
```
integrate(1/cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

3.112.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \sec(a+bx)\,dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{b} + \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\cos(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/cos(b*x+a),x)
```

output
```
Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) + 1)/b, Ne(
b, 0)), (x/cos(a), True))
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \sec(a+bx)\,dx = \frac{\log(\sin(bx+a)+1) - \log(\sin(bx+a)-1)}{2b}$$

input
```
integrate(1/cos(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \sec(a+bx)\,dx = \frac{\log(|\sin(bx+a)+1|) - \log(|\sin(bx+a)-1|)}{2b}$$

input
```
integrate(1/cos(b*x+a),x, algorithm="giac")
```

output
```
1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)))/b
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))}{b}$$

input
```
int(1/cos(a + b*x),x)
```

output
```
atanh(sin(a + b*x))/b
```

3.112.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \sec(a+bx)\,dx = \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$$

input
```
int(1/cos(a + b*x),x)
```

output
```
( - log(tan((a + b*x)/2) - 1) + log(tan((a + b*x)/2) + 1))/b
```

3.113 $\int \sin^2(a+bx)\,dx$

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3.113.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

3.113.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = -\frac{-2(a+bx)+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Sin[a + b*x]^2,x]
```

output
```
-1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

3.113.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]^2,x]
```

output
```
x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

3.113.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.113.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}-\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx-\sin(2bx+2a)}{4b}$	22
derivativedivides	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

input
```
int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/4/b*sin(2*b*x+2*a)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx=\frac{bx-\cos(bx+a)\sin(bx+a)}{2\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

3.113.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sin^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a
+ b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\,dx = \frac{2\,bx + 2\,a - \sin(2\,bx + 2\,a)}{4\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{1}{2}\,x - \frac{\sin(2\,bx + 2\,a)}{4\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\sin(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(sin(a + b*x)^2,x)
```

output
```
x/2 - sin(2*a + 2*b*x)/(4*b)
```

3.113.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = \frac{-\cos(bx+a)\sin(bx+a)+bx}{2b}$$

input
```
int(sin(a + b*x)**2,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x) + b*x)/(2*b)
```

3.114 $\int \sin^3(a+bx)\,dx$

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3.114.9 Mupad [B] (verification not implemented) 748
3.114.10Reduce [B] (verification not implemented) 748

3.114.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sin^3(a+bx)\,dx=-\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}$$

output
```
-cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sin^3(a+bx)\,dx=-\frac{3\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b}$$

input
```
Integrate[Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

3.114.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^3dx$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x] - Cos[a + b*x]^3/3)/b)
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.114.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\left(2+\sin^{2}\left(b x +a \right)\right) \cos \left(b x +a \right)}{3 b}$	22
default	$-\frac{\left(2+\sin^{2}\left(b x +a \right)\right) \cos \left(b x +a \right)}{3 b}$	22
parallelrisch	$\frac{-8-9 \cos \left(b x +a \right)+\cos \left(3 b x +3 a \right)}{12 b}$	25
risch	$-\frac{3 \cos \left(b x +a \right)}{4 b}+\frac{\cos \left(3 b x +3 a \right)}{12 b}$	27
norman	$\frac{-\frac{4 \left(\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4}{3 b}}{\left(1+\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)^{3}}$	39

input
```
int(sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\,dx = \frac{\cos\left(bx+a\right)^3 - 3\,\cos\left(bx+a\right)}{3\,b}$$

input
```
integrate(sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**3,x)
```

output
```
Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b
, 0)), (x*sin(a)**3, True))
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\,dx = \frac{\cos(bx+a)^3 - 3\cos(bx+a)}{3b}$$

input
```
integrate(sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sin^3(a+bx)\,dx = \frac{\cos(bx+a)^3}{3b} - \frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sin^3(a+bx)\,dx = -\frac{3\,\cos(a+b\,x)-\cos(a+b\,x)^3}{3\,b}$$

input
```
int(sin(a + b*x)^3,x)
```

output
```
-(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)
```

3.114.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \sin^3(a+bx)\,dx = \frac{-\cos(bx+a)\sin(bx+a)^2-2\cos(bx+a)+2}{3b}$$

input
```
int(sin(a + b*x)**3,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x)**2 - 2*cos(a + b*x) + 2)/(3*b)
```

3.115 $\int \cos^2(a+bx)\,dx$

3.115.1 Optimal result 749
3.115.2 Mathematica [A] (verified) 749
3.115.3 Rubi [A] (verified) 750
3.115.4 Maple [A] (verified) 751
3.115.5 Fricas [A] (verification not implemented) 751
3.115.6 Sympy [B] (verification not implemented) 752
3.115.7 Maxima [A] (verification not implemented) 752
3.115.8 Giac [A] (verification not implemented) 752
3.115.9 Mupad [B] (verification not implemented) 753
3.115.10Reduce [B] (verification not implemented) 753

3.115.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \cos^2(a+bx)\,dx=\frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b
```

3.115.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \cos^2(a+bx)\,dx=\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Cos[a + b*x]^2,x]
```

output
```
(2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

3.115.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

input
```
Int[Cos[a + b*x]^2,x]
```

output
```
x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

3.115.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.115.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx+\sin(2bx+2a)}{4b}$	20
derivativedivides	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

input
```
int(cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4/b*sin(2*b*x+2*a)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\,dx=\frac{bx+\cos(bx+a)\sin(bx+a)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

3.115.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \cos^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a
+ b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\,dx = \frac{2\,bx + 2\,a + \sin(2\,bx + 2\,a)}{4\,b}$$

input
```
integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(a+bx)\,dx = \frac{1}{2}\,x + \frac{\sin(2\,bx + 2\,a)}{4\,b}$$

input
```
integrate(cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(a+bx)\,dx = \frac{x}{2} + \frac{\sin(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(cos(a + b*x)^2,x)
```

output
```
x/2 + sin(2*a + 2*b*x)/(4*b)
```

3.115.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\,dx = \frac{\cos(bx+a)\sin(bx+a)+bx}{2b}$$

input
```
int(cos(a + b*x)**2,x)
```

output
```
(cos(a + b*x)*sin(a + b*x) + b*x)/(2*b)
```

3.116 $\int \cos^3(a+bx)\,dx$

3.116.1 Optimal result 754
3.116.2 Mathematica [A] (verified) 754
3.116.3 Rubi [A] (verified) 755
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3.116.5 Fricas [A] (verification not implemented) 756
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3.116.9 Mupad [B] (verification not implemented) 758
3.116.10 Reduce [B] (verification not implemented) 758

3.116.1 Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \cos^3(a+bx)\,dx = \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

output
```
sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

3.116.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\,dx = \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]^3,x]
```

output
```
Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

3.116.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx$$
$$\Big\downarrow \text{3113}$$
$$-\frac{\int \left(1-\sin^2(a+bx)\right) d(-\sin(a+bx))}{b}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3,x]
```

output
```
-((-Sin[a + b*x] + Sin[a + b*x]^3/3)/b)
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
default	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
parallelrisch	$\frac{9\sin(bx+a)+\sin(3bx+3a)}{12b}$	24
risch	$\frac{3\sin(bx+a)}{4b}+\frac{\sin(3bx+3a)}{12b}$	27

input
```
int(cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\,dx = \frac{(\cos(bx+a)^2+2)\sin(bx+a)}{3\,b}$$

input
```
integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \cos^3(a+bx)\,dx = \begin{cases} \frac{2\sin^3(a+bx)}{3b} + \frac{\sin(a+bx)\cos^2(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**3,x)
```

output
```
Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
 0)), (x*cos(a)**3, True))
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\,dx = -\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\,dx = -\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)\,dx = \frac{3\,\sin\left(a+b\,x\right)-{\sin\left(a+b\,x\right)}^3}{3\,b}$$

input
```
int(cos(a + b*x)^3,x)
```

output
```
(3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)
```

3.116.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \cos^3(a+bx)\,dx = \frac{\sin\left(bx+a\right)\left(-\sin\left(bx+a\right)^2+3\right)}{3b}$$

input
```
int(cos(a + b*x)**3,x)
```

output
```
(sin(a + b*x)*( - sin(a + b*x)**2 + 3))/(3*b)
```

3.117 $\int \sec^2(a+bx)\,dx$

3.117.1 Optimal result 759
3.117.2 Mathematica [A] (verified) 759
3.117.3 Rubi [A] (verified) 760
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3.117.5 Fricas [A] (verification not implemented) 761
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3.117.8 Giac [A] (verification not implemented) 763
3.117.9 Mupad [B] (verification not implemented) 763
3.117.10Reduce [B] (verification not implemented) 763

3.117.1 Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \sec^2(a+bx)\,dx = \frac{\tan(a+bx)}{b}$$

output
```
tan(b*x+a)/b
```

3.117.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/b
```

3.117.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \ 4254$$

$$-\frac{\int 1d(-\tan(a+bx))}{b}$$

$$\Big\downarrow \ 24$$

$$\frac{\tan(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/b
```

3.117.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.117.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tan(bx+a)}{b}$	11
default	$\frac{\tan(bx+a)}{b}$	11
risch	$\frac{2i}{b\left(e^{2i(bx+a)}+1\right)}$	20
norman	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	30
parallelrisch	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	30

input
```
int(1/cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
tan(b*x+a)/b
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \sec^2(a+bx)\,dx = \frac{\sin(bx+a)}{b\cos(bx+a)}$$

input
```
integrate(1/cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
sin(b*x + a)/(b*cos(b*x + a))
```

3.117.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(7) = 14$.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \sec^2(a+bx)\,dx = \begin{cases} \tilde{\infty}x & \text{for } \left(a=-\frac{\pi}{2}\vee a=-bx-\frac{\pi}{2}\right)\wedge\left(a=-bx-\frac{\pi}{2}\vee b=0\right) \\ \frac{x}{\cos^2(a)} & \text{for } b=0 \\ -\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)-b} & \text{otherwise} \end{cases}$$

input
```
integrate(1/cos(b*x+a)**2,x)
```

output
```
Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2) | Eq(a,
-b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b*x/2)/(b*tan(a/2 +
 b*x/2)**2 - b), True))
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan(bx+a)}{b}$$

input
```
integrate(1/cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
tan(b*x + a)/b
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan(bx+a)}{b}$$

input
```
integrate(1/cos(b*x+a)^2,x, algorithm="giac")
```

output
```
tan(b*x + a)/b
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{b}$$

input
```
int(1/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)/b
```

3.117.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \sec^2(a+bx)\,dx = \frac{\sin(bx+a)}{\cos(bx+a)\,b}$$

input
```
int(1/cos(a + b*x)**2,x)
```

output
```
sin(a + b*x)/(cos(a + b*x)*b)
```

3.118 $\int \frac{1}{1+\cos(x)}\,dx$

3.118.1 Optimal result 764
3.118.2 Mathematica [A] (verified) 764
3.118.3 Rubi [A] (verified) 765
3.118.4 Maple [A] (verified) 766
3.118.5 Fricas [A] (verification not implemented) 766
3.118.6 Sympy [A] (verification not implemented) 766
3.118.7 Maxima [A] (verification not implemented) 767
3.118.8 Giac [B] (verification not implemented) 767
3.118.9 Mupad [B] (verification not implemented) 767
3.118.10 Reduce [B] (verification not implemented) 768

3.118.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)}\,dx=\frac{\sin(x)}{1+\cos(x)}$$

output
```
sin(x)/(1+cos(x))
```

3.118.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)}\,dx=\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x])^(-1),x]
```

output
```
Tan[x/2]
```

3.118.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+1}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{ 3127}$$
$$\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[(1 + Cos[x])^(-1),x]
```

output
```
Sin[x]/(1 + Cos[x])
```

3.118.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.118.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
parallelrisch	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

input
```
int(1/(cos(x)+1),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(cos(x) + 1)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
integrate(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(cos(x) + 1)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(cos(x) + 1),x)
```

output
```
tan(x/2)
```

3.118.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(cos(x) + 1),x)
```

output
```
tan(x/2)
```

3.119 $\int \frac{1}{1-\cos(x)}\,dx$

3.119.1 Optimal result . . . 769
3.119.2 Mathematica [A] (verified) . . . 769
3.119.3 Rubi [A] (verified) . . . 770
3.119.4 Maple [A] (verified) . . . 771
3.119.5 Fricas [A] (verification not implemented) . . . 771
3.119.6 Sympy [A] (verification not implemented) . . . 771
3.119.7 Maxima [A] (verification not implemented) . . . 772
3.119.8 Giac [A] (verification not implemented) . . . 772
3.119.9 Mupad [B] (verification not implemented) . . . 772
3.119.10Reduce [B] (verification not implemented) . . . 773

3.119.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

output
```
-sin(x)/(1-cos(x))
```

3.119.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 - Cos[x])^(-1),x]
```

output
```
-Cot[x/2]
```

3.119.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3127}$$
$$-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[(1 - Cos[x])^(-1),x]
```

output
```
-(Sin[x]/(1 - Cos[x]))
```

3.119.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.119.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

input
```
int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(1/2*x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)),x)
```

output
```
-1/tan(x/2)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int(-1/(cos(x) - 1),x)
```

output
```
-cot(x/2)
```

3.119.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(( - 1)/(cos(x) - 1),x)
```

output
```
( - 1)/tan(x/2)
```

3.120 $\int \frac{1}{1+\sin(x)}\,dx$

3.120.1 Optimal result 774
3.120.2 Mathematica [B] (verified) 774
3.120.3 Rubi [A] (verified) 775
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3.120.5 Fricas [A] (verification not implemented) 776
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3.120.7 Maxima [A] (verification not implemented) 777
3.120.8 Giac [A] (verification not implemented) 777
3.120.9 Mupad [B] (verification not implemented) 777
3.120.10Reduce [B] (verification not implemented) 778

3.120.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{\cos(x)}{1+\sin(x)}$$

output
```
-cos(x)/(1+sin(x))
```

3.120.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{1+\sin(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(1 + Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

3.120.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3127}$$
$$-\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[(1 + Sin[x])^(-1),x]
```

output
```
-(Cos[x]/(1 + Sin[x]))
```

3.120.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.120.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
parallelrisch	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
risch	$-\frac{2}{i+e^{ix}}$	13

input
```
int(1/(sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-2/(1+tan(1/2*x))
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="fricas")
```

output
```
-(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
integrate(1/(1+sin(x)),x)
```

output
```
-2/(tan(x/2) + 1)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) + 1)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) + 1)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(sin(x) + 1),x)
```

output
```
-2/(tan(x/2) + 1)
```

3.120.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{1+\sin(x)}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(sin(x) + 1),x)
```

output
```
(2*tan(x/2))/(tan(x/2) + 1)
```

3.121 $\int \frac{1}{1-\sin(x)}\,dx$

3.121.1 Optimal result 779
3.121.2 Mathematica [B] (verified) 779
3.121.3 Rubi [A] (verified) 780
3.121.4 Maple [A] (verified) 781
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3.121.8 Giac [A] (verification not implemented) 782
3.121.9 Mupad [B] (verification not implemented) 782
3.121.10 Reduce [B] (verification not implemented) 783

3.121.1 Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)}{1-\sin(x)}$$

output
```
cos(x)/(1-sin(x))
```

3.121.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(1 - Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

3.121.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sin(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin(x)}dx$$
$$\Big\downarrow \text{3127}$$
$$\frac{\cos(x)}{1-\sin(x)}$$

input
```
Int[(1 - Sin[x])^(-1),x]
```

output
```
Cos[x]/(1 - Sin[x])
```

3.121.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.121.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
parallelrisch	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

input
```
int(1/(-sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-2/(tan(1/2*x)-1)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

input
```
integrate(1/(1-sin(x)),x, algorithm="fricas")
```

output
```
(cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
integrate(1/(1-sin(x)),x)
```

output
```
-2/(tan(x/2) - 1)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

input
```
integrate(1/(1-sin(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) - 1)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

input
```
integrate(1/(1-sin(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) - 1)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(-1/(sin(x) - 1),x)
```

output
```
-2/(tan(x/2) - 1)
```

3.121.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(( - 1)/(sin(x) - 1),x)
```

output
```
( - 2*tan(x/2))/(tan(x/2) - 1)
```

3.122 $\int \frac{1}{a+b\sin(x)}\,dx$

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3.122.2 Mathematica [A] (verified) . . . 784
3.122.3 Rubi [A] (verified) . . . 785
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3.122.10Reduce [B] (verification not implemented) . . . 789

3.122.1 Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \frac{1}{a+b\sin(x)}\,dx=\frac{2\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin(x)}\,dx=\frac{2\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Integrate[(a + b*Sin[x])^(-1),x]
```

output
```
(2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

3.122.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\sin(x)}dx$$

$$\Big\downarrow \; 3139$$

$$2\int \frac{1}{a\tan^2\left(\frac{x}{2}\right)+2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 1083$$

$$-4\int \frac{1}{-\left(2b+2a\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 217$$

$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Int[(a + b*Sin[x])^(-1),x]
```

output
```
(2*ArcTan[(2*b + 2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2]
```

3.122.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.122.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2 \arctan\left(\frac{2 a \tan\left(\frac{x}{2}\right)+2 b}{2 \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}$	39
risch	$-\frac{\ln\left({\mathrm e}^{i x}+\frac{i a \sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{b \sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left({\mathrm e}^{i x}+\frac{i a \sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{b \sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}$	119

input
```
int(1/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.70

$$\int \frac{1}{a+b\sin(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)}{2(a^2-b^2)}, -\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 -
b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a
*b*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b
^2)*cos(x)))/sqrt(a^2 - b^2)]
```

3.122.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(31) = 62.

Time = 1.98 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.50

$$\int \frac{1}{a+b\sin(x)}\,dx = \begin{cases} \tilde{\infty}\log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ \frac{2}{b\tan\left(\frac{x}{2}\right)-b} & \text{for } a=-b \\ -\frac{2}{b\tan\left(\frac{x}{2}\right)+b} & \text{for } a=b \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*sin(x)),x)
```

output
```
Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a
, 0)), (2/(b*tan(x/2) - b), Eq(a, -b)), (-2/(b*tan(x/2) + b), Eq(a, b)), (
log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/
2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))
```

3.122.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))/sqrt(a^2 - b^2)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b*sin(x)),x)
```

output
```
(2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)
^(1/2)
```

3.122.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+b\sin(x)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

input
```
int(1/(sin(x)*b + a),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan(x/2)*a + b)/sqrt(a**2 - b**2)))/(a**2 - b**
2)
```

3.123 $\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$

3.123.1 Optimal result . . . 790
3.123.2 Mathematica [A] (verified) . . . 790
3.123.3 Rubi [A] (verified) . . . 791
3.123.4 Maple [A] (verified) . . . 792
3.123.5 Fricas [A] (verification not implemented) . . . 793
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3.123.8 Giac [A] (verification not implemented) . . . 795
3.123.9 Mupad [B] (verification not implemented) . . . 795
3.123.10 Reduce [B] (verification not implemented) . . . 795

3.123.1 Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{1-a^2+b^2}}\right)}{\sqrt{1-a^2+b^2}}$$

output
```
-2*arctanh((b-(1-a)*tan(1/2*x))/(-a^2+b^2+1)^(1/2))/(-a^2+b^2+1)^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx=\frac{2\arctan\left(\frac{b+(-1+a)\tan\left(\frac{x}{2}\right)}{\sqrt{-1+a^2-b^2}}\right)}{\sqrt{-1+a^2-b^2}}$$

input
```
Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]
```

output
```
(2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1 + a^2 - b
^2]
```

3.123.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin(x)+\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a+b\sin(x)+\cos(x)}dx$$

$$\Big\downarrow \text{ 3603}$$

$$2\int \frac{1}{-\left((1-a)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+1}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$-4\int \frac{1}{4\left(-a^2+b^2+1\right)-\left(2b-2(1-a)\tan\left(\frac{x}{2}\right)\right)^2}d\left(2b-2(1-a)\tan\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{2b-2(1-a)\tan\left(\frac{x}{2}\right)}{2\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

input
```
Int[(a + Cos[x] + b*Sin[x])^(-1),x]
```

output
```
(-2*ArcTanh[(2*b - 2*(1 - a)*Tan[x/2])/(2*Sqrt[1 - a^2 + b^2])])/Sqrt[1 -
a^2 + b^2]
```

3.123.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result
default	$\frac{2\arctan\left(\frac{2(a-1)\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$
risch	$-\frac{\ln\left(e^{ix}+\frac{iab\sqrt{-a^2+b^2+1}+ia^2-ib^2-a^2b+b^3+a\sqrt{-a^2+b^2+1}-i+b}{\left(b^2+1\right)\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}+\frac{\ln\left(e^{ix}+\frac{iab\sqrt{-a^2+b^2+1}-ia^2+ib^2+a^2b-b^3+a\sqrt{-a^2+b^2+1}+i-}{\left(b^2+1\right)\sqrt{-a^2+b^2+1}}\right.}{\sqrt{-a^2+b^2+1}}$

input
```
int(1/(a+cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2-b^2-1)^(1/2)*arctan(1/2*(2*(a-1)*tan(1/2*x)+2*b)/(a^2-b^2-1)^(1/2))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 6.11

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2+1}\log\left(-\frac{b^4+(a^2+3)b^2-(2\,a^2b^2-b^4-2\,a^2+1)\cos(x)^2-a^2+2\,(ab^2+a)\cos(x)+2\,(ab^3+ab-(b^3-(2\,a^2-1)b)\cos(x))\,\mathrm{s}}{(b^2-1)\cos(x)^2-a^2-b^2-2\,a\cos(x)-}\right.}{2\,(a^2-b^2-1)}\right.$$

input
```
integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2 + 1)*log(-(b^4 + (a^2 + 3)*b^2 - (2*a^2*b^2 - b^4 -
2*a^2 + 1)*cos(x)^2 - a^2 + 2*(a*b^2 + a)*cos(x) + 2*(a*b^3 + a*b - (b^3 -
 (2*a^2 - 1)*b)*cos(x))*sin(x) - 2*(2*a*b*cos(x)^2 - a*b + (b^3 + b)*cos(x
) - (b^2 - (a*b^2 - a)*cos(x) + 1)*sin(x))*sqrt(-a^2 + b^2 + 1) + 2)/((b^2
 - 1)*cos(x)^2 - a^2 - b^2 - 2*a*cos(x) - 2*(a*b + b*cos(x))*sin(x)))/(a^2
 - b^2 - 1), arctan(-(a*b*sin(x) + b^2 + a*cos(x) + 1)*sqrt(a^2 - b^2 - 1)
/((b^3 - (a^2 - 1)*b)*cos(x) + (a^2 - b^2 - 1)*sin(x)))/sqrt(a^2 - b^2 - 1
)]
```

3.123.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1872 vs. $2(37) = 74$.

Time = 72.57 (sec) , antiderivative size = 1872, normalized size of antiderivative = 39.83

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+cos(x)+b*sin(x)),x)
```

output
```
Piecewise((log(tan(x/2) + 1/b)/b, Eq(a, 1)), (2*b**4*sqrt(b**2 + 1)/(b**6*
tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) +
 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 +
 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**
2 + 1)*tan(x/2) + 32*tan(x/2)) + 10*b**4/(b**6*tan(x/2) - b**5*sqrt(b**2 +
 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3
*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(
x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/
2)) + 24*b**2*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5
 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 +
 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*
sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 40*b**
2/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*ta
n(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqr
t(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*
sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32*sqrt(b**2 + 1)/(b**6*tan(x/2)
- b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*
tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x
/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*ta
n(x/2) + 32*tan(x/2)) + 32/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**...
```

3.123.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + \cos(x) + b\sin(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2-a^2+1>0)', see `assume?` for
 more deta
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b-\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2-1}}\right)\right)}{\sqrt{a^2-b^2-1}}$$

input
```
integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) + b - tan(
1/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx = \begin{cases} \frac{\ln\left(b\tan\left(\frac{x}{2}\right)+1\right)}{b} & \text{if } a=1 \\ \frac{2\,\mathrm{atan}\left(\frac{b+\tan\left(\frac{x}{2}\right)(a-1)}{\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}} & \text{if } a\neq 1 \end{cases}$$

input
```
int(1/(a + cos(x) + b*sin(x)),x)
```

output
```
piecewise(a == 1, log(b*tan(x/2) + 1)/b, a ~= 1, (2*atan((b + tan(x/2)*(a
- 1))/(a^2 - b^2 - 1)^(1/2)))/(a^2 - b^2 - 1)^(1/2))
```

3.123.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx = \frac{2\sqrt{a^2-b^2-1}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-1}}\right)}{a^2-b^2-1}$$

input
```
int(1/(cos(x) + sin(x)*b + a),x)
```

output
```
(2*sqrt(a**2 - b**2 - 1)*atan((tan(x/2)*a - tan(x/2) + b)/sqrt(a**2 - b**2
 - 1)))/(a**2 - b**2 - 1)
```

3.124 $\int x^2 \sin^2(a+bx)\,dx$

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3.124.10 Reduce [B] (verification not implemented) 801

3.124.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int x^2 \sin^2(a+bx)\,dx = -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}$$

output
```
-1/4*x/b^2+1/6*x^3+1/4*cos(b*x+a)*sin(b*x+a)/b^3-1/2*x^2*cos(b*x+a)*sin(b*
x+a)/b+1/2*x*sin(b*x+a)^2/b^2
```

3.124.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int x^2 \sin^2(a+bx)\,dx = \frac{4b^3x^3-6bx\cos(2(a+bx))+(3-6b^2x^2)\sin(2(a+bx))}{24b^3}$$

input
```
Integrate[x^2*Sin[a + b*x]^2,x]
```

output
```
(4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)])/(2
4*b^3)
```

3.124.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^2 \sin(a+bx)^2dx$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{\int \sin^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\int \sin^2(a+bx)dx}{2b^2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \sin(a+bx)^2dx}{2b^2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

$$\Big\downarrow \text{ 3115}$$

$$-\frac{\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}}{2b^2}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x\sin^2(a+bx)}{2b^2}-\frac{\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

input
```
Int[x^2*Sin[a + b*x]^2,x]
```

output
```
x^3/6 - (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
 - (x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b))/(2*b^2)
```

3.124.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.124.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result
risch	$\frac{x^{3}}{6}-\frac{x\cos(2bx+2a)}{4b^{2}}-\frac{(2x^{2}b^{2}-1)\sin(2bx+2a)}{8b^{3}}$
derivativedivides	$\frac{a^{2}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-2a\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^{2}}{4}+\frac{\left(\sin^{2}(bx+a)\right)}{4}\right)+(bx+a)^{2}\left(\cdots\right.}{b^{3}}$
default	$\frac{a^{2}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-2a\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^{2}}{4}+\frac{\left(\sin^{2}(bx+a)\right)}{4}\right)+(bx+a)^{2}\left(\cdots\right.}{b^{3}}$
norman	$\frac{\frac{x^{2}\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{x^{3}}{6}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^{3}}-\frac{\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^{3}}-\frac{x}{4b^{2}}+\frac{x^{3}\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{x^{3}\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}+\frac{3x\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^{2}}-\cdots}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}$

input
```
int(x^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3-1/4/b^2*x*cos(2*b*x+2*a)-1/8*(2*b^2*x^2-1)/b^3*sin(2*b*x+2*a)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^2 \sin^2(a+bx)\,dx = \frac{2\,b^3x^3 - 6\,bx\cos\left(bx+a\right)^2 - 3\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right) + 3\,bx}{12\,b^3}$$

input
```
integrate(x^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*si
n(b*x + a) + 3*b*x)/b^3
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int x^2 \sin^2(a+bx)\,dx = \begin{cases} \frac{x^3\sin^2\left(a+bx\right)}{6} + \frac{x^3\cos^2\left(a+bx\right)}{6} - \frac{x^2\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b} + \frac{x\sin^2\left(a+bx\right)}{4b^2} - \frac{x\cos^2\left(a+bx\right)}{4b^2} + \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{4b^3} & \text{for } b \neq \\ \frac{x^3\sin^2\left(a\right)}{3} & \text{otherw} \end{cases}$$

input
```
integrate(x**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2*sin(a +
b*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x*cos(a + b*x)**2/(
4*b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*sin(a)**2/3
, True))
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int x^2 \sin^2(a+bx)\,dx$$
$$= \frac{4\,(bx+a)^3 + 6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2 - 6\,\left(2\,(bx+a)^2 - 2\,(bx+a)\sin(2\,bx+2\,a) - \cos(2\,bx\right.}{24\,b^3}$$

input
```
integrate(x^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
 a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a - 6*(b*x + a)*c
os(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^2 \sin^2(a+bx)\,dx = \frac{1}{6}\,x^3 - \frac{x\cos(2\,bx+2\,a)}{4\,b^2} - \frac{(2\,b^2x^2-1)\sin(2\,bx+2\,a)}{8\,b^3}$$

input
```
integrate(x^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a
)/b^3
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(a+bx)\,dx = \frac{x^3}{6} + \frac{\sin(2\,a+2\,b\,x)}{8\,b^3} - \frac{x\,\cos(2\,a+2\,b\,x)}{4\,b^2} - \frac{x^2\,\sin(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(x^2*sin(a + b*x)^2,x)
```

output
```
x^3/6 + sin(2*a + 2*b*x)/(8*b^3) - (x*cos(2*a + 2*b*x))/(4*b^2) - (x^2*sin
(2*a + 2*b*x))/(4*b)
```

3.124.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int x^2 \sin^2(a+bx)\,dx$$
$$= \frac{-6\cos(bx+a)\sin(bx+a)\,b^2x^2+3\cos(bx+a)\sin(bx+a)+6\sin(bx+a)^2\,bx+9a+2b^3x^3-3bx}{12b^3}$$

input
```
int(sin(a + b*x)**2*x**2,x)
```

output
```
( - 6*cos(a + b*x)*sin(a + b*x)*b**2*x**2 + 3*cos(a + b*x)*sin(a + b*x) +
6*sin(a + b*x)**2*b*x + 9*a + 2*b**3*x**3 - 3*b*x)/(12*b**3)
```

3.125 $\int \cos(x)\cos(2x)\,dx$

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3.125.10Reduce [B] (verification not implemented) . . . 806

3.125.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\cos(2x)\,dx = \frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

output
```
1/2*sin(x)+1/6*sin(3*x)
```

3.125.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\,dx = \frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

input
```
Integrate[Cos[x]*Cos[2*x],x]
```

output
```
Sin[x]/2 + Sin[3*x]/6
```

3.125.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)\cos(2x)dx$$

$$\Big\downarrow \; 4771$$

$$\frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

input
```
Int[Cos[x]*Cos[2*x],x]
```

output
```
Sin[x]/2 + Sin[3*x]/6
```

3.125.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
parallelrisch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{4\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^2(x)\right)\tan\left(\frac{x}{2}\right)}{3}+\frac{4\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(x)\right)}$	51

input
```
int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)+1/6*sin(3*x)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{3}\left(2\cos\left(x\right)^2+1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

output
```
1/3*(2*cos(x)^2 + 1)*sin(x)
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(x)\cos(2x)\,dx = -\frac{\sin\left(x\right)\cos\left(2x\right)}{3}+\frac{2\sin\left(2x\right)\cos\left(x\right)}{3}$$

input
```
integrate(cos(x)*cos(2*x),x)
```

output
```
-sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

output
```
1/6*sin(3*x) + 1/2*sin(x)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

output
```
1/6*sin(3*x) + 1/2*sin(x)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(x)\cos(2x)\,dx = \sin(x) - \frac{2\sin(x)^3}{3}$$

input
```
int(cos(2*x)*cos(x),x)
```

output
```
sin(x) - (2*sin(x)^3)/3
```

3.125.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(x)\cos(2x)\,dx = -\frac{\cos(2x)\sin(x)}{3} + \frac{2\cos(x)\sin(2x)}{3}$$

input
```
int(cos(2*x)*cos(x),x)
```

output
```
( - cos(2*x)*sin(x) + 2*cos(x)*sin(2*x))/3
```

3.126 $\int x^2 \cos^2(a+bx)\,dx$

3.126.1 Optimal result 807
3.126.2 Mathematica [A] (verified) 807
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3.126.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int x^2 \cos^2(a+bx)\,dx = -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2} -\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
-1/4*x/b^2+1/6*x^3+1/2*x*cos(b*x+a)^2/b^2-1/4*cos(b*x+a)*sin(b*x+a)/b^3+1/
2*x^2*cos(b*x+a)*sin(b*x+a)/b
```

3.126.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int x^2 \cos^2(a+bx)\,dx = \frac{4b^3x^3+6bx\cos(2(a+bx))+(-3+6b^2x^2)\sin(2(a+bx))}{24b^3}$$

input
```
Integrate[x^2*Cos[a + b*x]^2,x]
```

output
```
(4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a + b*x)])/(
24*b^3)
```

3.126.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{\int \cos^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int \cos^2(a+bx)dx}{2b^2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}}{2b^2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\cos^2(a+bx)}{2b^2}-\frac{\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}$$

input
```
Int[x^2*Cos[a + b*x]^2,x]
```

output
```
x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
 - (x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b))/(2*b^2)
```

3.126.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.126.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result
risch	$\frac{x^3}{6}+\frac{x\cos(2bx+2a)}{4b^2}+\frac{(2x^2b^2-1)\sin(2bx+2a)}{8b^3}$
parallelrisch	$\frac{(6x^2b^2-3)\sin(2bx+2a)+4x^3b^3+6x\cos(2bx+2a)b}{24b^3}$
derivativedivides	$\frac{a^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-2a\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{(\sin^2(bx+a))}{4}\right)+(bx+a)^2\left(\cos\right.}{b^3}$
default	$\frac{a^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-2a\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{(\sin^2(bx+a))}{4}\right)+(bx+a)^2\left(\cos\right.}{b^3}$
norman	$\frac{\frac{x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x^3}{6}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^3}+\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^3}+\frac{x}{4b^2}+\frac{x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}-\frac{3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2}+\frac{x(}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$

input
```
int(x^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3+1/4/b^2*x*cos(2*b*x+2*a)+1/8*(2*b^2*x^2-1)/b^3*sin(2*b*x+2*a)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^2\cos^2(a+bx)\,dx = \frac{2\,b^3x^3+6\,bx\cos(bx+a)^2+3\,(2\,b^2x^2-1)\cos(bx+a)\sin(bx+a)-3\,bx}{12\,b^3}$$

input
```
integrate(x^2*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 6*b*x*cos(b*x + a)^2 + 3*(2*b^2*x^2 - 1)*cos(b*x + a)*si
n(b*x + a) - 3*b*x)/b^3
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int x^2 \cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{x^3 \sin^2(a+bx)}{6} + \frac{x^3 \cos^2(a+bx)}{6} + \frac{x^2 \sin(a+bx)\cos(a+bx)}{2b} - \frac{x \sin^2(a+bx)}{4b^2} + \frac{x \cos^2(a+bx)}{4b^2} - \frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for } b \neq \\ \frac{x^3 \cos^2(a)}{3} & \text{otherw} \end{cases}$$

input
```
integrate(x**2*cos(b*x+a)**2,x)
```

output
```
Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2*sin(a +
b*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x*cos(a + b*x)**2/(
4*b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*cos(a)**2/3
, True))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.55

$$\int x^2 \cos^2(a+bx)\,dx$$
$$= \frac{4(bx+a)^3 + 6(2bx+2a+\sin(2bx+2a))a^2 - 6\left(2(bx+a)^2 + 2(bx+a)\sin(2bx+2a) + \cos(2bx\right.}{24b^3}$$

input
```
integrate(x^2*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
 a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a + 6*(b*x + a)*c
os(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^2 \cos^2(a+bx)\,dx = \frac{1}{6}x^3 + \frac{x\cos(2bx+2a)}{4b^2} + \frac{(2b^2x^2-1)\sin(2bx+2a)}{8b^3}$$

input
```
integrate(x^2*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a
)/b^3
```

3.126.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^2 \cos^2(a+bx)\,dx = \frac{x^3}{6} - \frac{\sin(2a+2bx)}{8b^3} + \frac{x\cos(2a+2bx)}{4b^2} + \frac{x^2\sin(2a+2bx)}{4b}$$

input
```
int(x^2*cos(a + b*x)^2,x)
```

output
```
x^3/6 - sin(2*a + 2*b*x)/(8*b^3) + (x*cos(2*a + 2*b*x))/(4*b^2) + (x^2*sin
(2*a + 2*b*x))/(4*b)
```

3.126.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^2 \cos^2(a+bx)\,dx$$
$$= \frac{6\cos(bx+a)\sin(bx+a)b^2x^2 - 3\cos(bx+a)\sin(bx+a) - 6\sin(bx+a)^2 bx + 2b^3x^3 + 3bx}{12b^3}$$

input
```
int(cos(a + b*x)**2*x**2,x)
```

output
```
(6*cos(a + b*x)*sin(a + b*x)*b**2*x**2 - 3*cos(a + b*x)*sin(a + b*x) - 6*s
in(a + b*x)**2*b*x + 2*b**3*x**3 + 3*b*x)/(12*b**3)
```

3.127 $\int \cot^3(x)\,dx$

3.127.1 Optimal result 813
3.127.2 Mathematica [A] (verified) 813
3.127.3 Rubi [A] (verified) 814
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3.127.9 Mupad [B] (verification not implemented) 817
3.127.10Reduce [B] (verification not implemented) 817

3.127.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cot^3(x)\,dx = -\frac{1}{2}\cot^2(x) - \log(\sin(x))$$

output
```
-1/2*cot(x)^2-ln(sin(x))
```

3.127.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\,dx = -\frac{1}{2}\cot^2(x) - \log(\cos(x)) - \log(\tan(x))$$

input
```
Integrate[Cot[x]^3,x]
```

output
```
-1/2*Cot[x]^2 - Log[Cos[x]] - Log[Tan[x]]
```

3.127.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3954}$$

$$\int -\cot(x)dx-\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(x)dx-\frac{1}{2}\cot^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int -\tan\left(x+\frac{\pi}{2}\right)dx-\frac{1}{2}\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)dx-\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

input
```
Int[Cot[x]^3,x]
```

output
```
-1/2*Cot[x]^2 - Log[Sin[x]]
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.127.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$-\frac{1}{2\tan\left(x\right)^{2}}-\ln\left(\tan\left(x\right)\right)+\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	22
default	$-\frac{1}{2\tan\left(x\right)^{2}}-\ln\left(\tan\left(x\right)\right)+\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	22
norman	$-\frac{1}{2\tan\left(x\right)^{2}}-\ln\left(\tan\left(x\right)\right)+\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	22
parallelrisch	$-\frac{2\ln\left(\tan\left(x\right)\right)\left(\tan^{2}\left(x\right)\right)-\ln\left(1+\tan^{2}\left(x\right)\right)\left(\tan^{2}\left(x\right)\right)+1}{2\tan\left(x\right)^{2}}$	31
risch	$ix+\frac{2\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}-1\right)^{2}}-\ln\left({\mathrm e}^{2 i x}-1\right)$	32

input
```
int(1/tan(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/tan(x)^2-ln(tan(x))+1/2*ln(1+tan(x)^2)
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \cot^3(x)\, dx = -\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^2 + \tan(x)^2 + 1}{2\tan(x)^2}$$

input
```
integrate(1/tan(x)^3,x, algorithm="fricas")
```

output
```
-1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x)^2
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\, dx = -\log(\sin(x)) - \frac{1}{2\sin^2(x)}$$

input
```
integrate(1/tan(x)**3,x)
```

output
```
-log(sin(x)) - 1/(2*sin(x)**2)
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\, dx = -\frac{1}{2\sin(x)^2} - \frac{1}{2}\log\left(\sin(x)^2\right)$$

input
```
integrate(1/tan(x)^3,x, algorithm="maxima")
```

output
```
-1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \cot^3(x)\,dx = \frac{\tan(x)^2 - 1}{2\tan(x)^2} + \frac{1}{2}\log\left(\tan(x)^2 + 1\right) - \frac{1}{2}\log\left(\tan(x)^2\right)$$

input
```
integrate(1/tan(x)^3,x, algorithm="giac")
```

output
```
1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*log(tan(x)^2 + 1) - 1/2*log(tan(x)^2)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \cot^3(x)\,dx = \frac{\ln\left(\tan(x)^2 + 1\right)}{2} - \ln\left(\tan(x)\right) - \frac{1}{2\tan(x)^2}$$

input
```
int(1/tan(x)^3,x)
```

output
```
log(tan(x)^2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)^2)
```

3.127.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \cot^3(x)\,dx = \frac{\log\left(\tan(x)^2 + 1\right)\tan(x)^2 - 2\log(\tan(x))\tan(x)^2 - 1}{2\tan(x)^2}$$

input
```
int(1/tan(x)**3,x)
```

output
```
(log(tan(x)**2 + 1)*tan(x)**2 - 2*log(tan(x))*tan(x)**2 - 1)/(2*tan(x)**2)
```

3.128 $\int x^3 \tan^4(x)\,dx$

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3.128.1 Optimal result

Integrand size = 8, antiderivative size = 104

$$\int x^3 \tan^4(x)\,dx = -\frac{x^2}{2}+\frac{4ix^3}{3}+\frac{x^4}{4}-4x^2\log\left(1+e^{2ix}\right)+\log(\cos(x)) + 4ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-2\,\text{PolyLog}\left(3,-e^{2ix}\right) + x\tan(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)+\frac{1}{3}x^3\tan^3(x)$$

output
```
-1/2*x^2+4/3*I*x^3+1/4*x^4-4*x^2*ln(1+exp(2*I*x))+ln(cos(x))+4*I*x*polylog
(2,-exp(2*I*x))-2*polylog(3,-exp(2*I*x))+x*tan(x)-x^3*tan(x)-1/2*x^2*tan(x
)^2+1/3*x^3*tan(x)^3
```

3.128.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x^3 \tan^4(x)\,dx = \frac{4ix^3}{3}+\frac{x^4}{4}-4x^2\log\left(1+e^{2ix}\right)+\log(\cos(x)) + 4ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-2\,\text{PolyLog}\left(3,-e^{2ix}\right) - \frac{1}{2}x^2\sec^2(x)+x\tan(x)-\frac{4}{3}x^3\tan(x)+\frac{1}{3}x^3\sec^2(x)\tan(x)$$

input
```
Integrate[x^3*Tan[x]^4,x]
```

output
```
((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] + (4*I)*x
*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] - (x^2*Sec[x]^2)/2
+ x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*Tan[x])/3
```

3.128.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {3042, 4203, 3042, 4203, 15, 3042, 4202, 2620, 3011, 2720, 4203, 15, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tan^4(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \tan(x)^4 dx$$

$$\Big\downarrow \text{4203}$$

$$-\int x^3 \tan^2(x)dx - \int x^2 \tan^3(x)dx + \frac{1}{3}x^3 \tan^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3 \tan(x)^2dx - \int x^2 \tan(x)^3dx + \frac{1}{3}x^3 \tan^3(x)$$

$$\Big\downarrow \text{4203}$$

$$\int x^3dx + 4\int x^2 \tan(x)dx + \int x \tan^2(x)dx + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x)$$

$$\Big\downarrow \text{15}$$

$$4\int x^2 \tan(x)dx + \int x \tan^2(x)dx + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x)$$

$$\Big\downarrow \text{3042}$$

$$4\int x^2\tan(x)dx+\int x\tan(x)^2dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)$$

$$\Big\downarrow \text{4202}$$

$$4\left(\frac{ix^3}{3}-2i\int\frac{e^{2ix}x^2}{1+e^{2ix}}dx\right)+\int x\tan(x)^2dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\int x\log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)+\int x\tan(x)^2dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)$$

$$\Big\downarrow \text{3011}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\int\text{PolyLog}\left(2,-e^{2ix}\right)dx\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)+\int x\tan(x)^2dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)$$

$$\Big\downarrow \text{2720}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)+\int x\tan(x)^2dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)$$

$$\Big\downarrow \text{4203}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-\int xdx-\int\tan(x)dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{1}{2}x^2\tan^2(x)+x\tan(x)$$

$$\Big\downarrow \text{15}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-\int\tan(x)dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{x^2}{2}-\frac{1}{2}x^2\tan^2(x)+x\tan(x)$$

$$\Big\downarrow \text{3042}$$

3.128. $\int x^3\tan^4(x)\,dx$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-\int\tan(x)dx+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{x^2}{2}-\frac{1}{2}x^2\tan^2(x)+x\tan(x)$$

$$\Big\downarrow \text{ 3956}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{x^2}{2}-\frac{1}{2}x^2\tan^2(x)+x\tan(x)+\log(\cos(x))$$

$$\Big\downarrow \text{ 7143}$$

$$4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2ix}\right)\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)+\frac{x^4}{4}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{x^2}{2}-\frac{1}{2}x^2\tan^2(x)+x\tan(x)+\log(\cos(x))$$

input
```
Int[x^3*Tan[x]^4,x]
```

output
```
-1/2*x^2 + x^4/4 + Log[Cos[x]] + 4*((I/3)*x^3 - (2*I)*((-1/2*I)*x^2*Log[1
+ E^((2*I)*x)] + I*((I/2)*x*PolyLog[2, -E^((2*I)*x)] - PolyLog[3, -E^((2*I
)*x)]/4))) + x*Tan[x] - x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^3)/3
```

3.128.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))    Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.128. $\int x^3 \tan^4(x)\, dx$

3.128.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.33

method	result
risch	$\frac{x^4}{4}-\frac{2ix(6x^2e^{4ix}+6x^2e^{2ix}-3e^{4ix}-3ix\,e^{4ix}+4x^2-6e^{2ix}-3ix\,e^{2ix}-3)}{3(e^{2ix}+1)^3}+\ln(e^{2ix}+1)-2\ln(e^{ix})+\frac{8ix^3}{3}-4x^2\ln$

input
```
int(x^3*tan(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-2/3*I*x*(6*x^2*exp(4*I*x)+6*x^2*exp(2*I*x)-3*exp(4*I*x)-3*I*x*exp(
4*I*x)+4*x^2-6*exp(2*I*x)-3*I*x*exp(2*I*x)-3)/(exp(2*I*x)+1)^3+ln(exp(2*I*
x)+1)-2*ln(exp(I*x))+8/3*I*x^3-4*x^2*ln(exp(2*I*x)+1)+4*I*x*polylog(2,-exp
(2*I*x))-2*polylog(3,-exp(2*I*x))
```

3.128.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.75

$$\begin{aligned}\int x^3\tan^4(x)\,dx = {} & \frac{1}{3}\,x^3\tan(x)^3+\frac{1}{4}\,x^4-\frac{1}{2}\,x^2\tan(x)^2-\frac{1}{2}\,x^2 \\ & -2i\,x\mathrm{Li}_2\left(\frac{2\,(i\,\tan(x)-1)}{\tan(x)^2+1}+1\right)+2i\,x\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(x)-1)}{\tan(x)^2+1}+1\right) \\ & -\frac{1}{2}\,(4\,x^2-1)\log\left(-\frac{2\,(i\,\tan(x)-1)}{\tan(x)^2+1}\right) \\ & -\frac{1}{2}\,(4\,x^2-1)\log\left(-\frac{2\,(-i\,\tan(x)-1)}{\tan(x)^2+1}\right) \\ & -(x^3-x)\tan(x)-\mathrm{polylog}\left(3,\frac{\tan(x)^2+2i\,\tan(x)-1}{\tan(x)^2+1}\right) \\ & -\mathrm{polylog}\left(3,\frac{\tan(x)^2-2i\,\tan(x)-1}{\tan(x)^2+1}\right)\end{aligned}$$

input
```
integrate(x^3*tan(x)^4,x, algorithm="fricas")
```

output
```
1/3*x^3*tan(x)^3 + 1/4*x^4 - 1/2*x^2*tan(x)^2 - 1/2*x^2 - 2*I*x*dilog(2*(I
*tan(x) - 1)/(tan(x)^2 + 1) + 1) + 2*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2
 + 1) + 1) - 1/2*(4*x^2 - 1)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*(
4*x^2 - 1)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - (x^3 - x)*tan(x) - pol
ylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) - polylog(3, (tan(x)^2
 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

3.128.6 Sympy [F]

$$\int x^3 \tan^4(x)\,dx = \int x^3 \tan^4(x)\,dx$$

input
```
integrate(x**3*tan(x)**4,x)
```

output
```
Integral(x**3*tan(x)**4, x)
```

3.128.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(80) = 160$.

Time = 0.32 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.72

$$\int x^3 \tan^4(x)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*tan(x)^4,x, algorithm="maxima")
```

output
```
-(3*I*x^4 + 12*(4*x^2 + (4*x^2 - 1)*cos(6*x) + 3*(4*x^2 - 1)*cos(4*x) + 3*
(4*x^2 - 1)*cos(2*x) - (-4*I*x^2 + I)*sin(6*x) - 3*(-4*I*x^2 + I)*sin(4*x)
 - 3*(-4*I*x^2 + I)*sin(2*x) - 1)*arctan2(sin(2*x), cos(2*x) + 1) + (3*I*x
^4 - 32*x^3 + 24*x)*cos(6*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 16*x)*cos(
4*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 8*x)*cos(2*x) - 48*(x*cos(6*x) + 3
*x*cos(4*x) + 3*x*cos(2*x) + I*x*sin(6*x) + 3*I*x*sin(4*x) + 3*I*x*sin(2*x
) + x)*dilog(-e^(2*I*x)) - 6*(4*I*x^2 + (4*I*x^2 - I)*cos(6*x) + 3*(4*I*x^
2 - I)*cos(4*x) + 3*(4*I*x^2 - I)*cos(2*x) - (4*x^2 - 1)*sin(6*x) - 3*(4*x
^2 - 1)*sin(4*x) - 3*(4*x^2 - 1)*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2
 + 2*cos(2*x) + 1) - 24*(I*cos(6*x) + 3*I*cos(4*x) + 3*I*cos(2*x) - sin(6*
x) - 3*sin(4*x) - 3*sin(2*x) + I)*polylog(3, -e^(2*I*x)) - (3*x^4 + 32*I*x
^3 - 24*I*x)*sin(6*x) - 3*(3*x^4 + 16*I*x^3 - 8*x^2 - 16*I*x)*sin(4*x) - 3
*(3*x^4 + 16*I*x^3 - 8*x^2 - 8*I*x)*sin(2*x))/(-12*I*cos(6*x) - 36*I*cos(4
*x) - 36*I*cos(2*x) + 12*sin(6*x) + 36*sin(4*x) + 36*sin(2*x) - 12*I)
```

3.128.8 Giac [F]

$$\int x^3 \tan^4(x)\,dx = \int x^3 \tan{(x)}^4\,dx$$

input
```
integrate(x^3*tan(x)^4,x, algorithm="giac")
```

output
```
integrate(x^3*tan(x)^4, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \tan^4(x)\,dx = \int x^3 \tan(x)^4\,dx$$

input
```
int(x^3*tan(x)^4,x)
```

output
```
int(x^3*tan(x)^4, x)
```

3.128.10 Reduce [F]

$$\int x^3 \tan^4(x)\,dx = 4\left(\int \tan(x)\,x^2dx\right) - \frac{\log\left(\tan(x)^2+1\right)}{2} + \frac{\tan(x)^3\,x^3}{3} - \frac{\tan(x)^2\,x^2}{2} - \tan(x)\,x^3 + \tan(x)\,x + \frac{x^4}{4} - \frac{x^2}{2}$$

input

```
int(tan(x)**4*x**3,x)
```

output

```
(48*int(tan(x)*x**2,x) - 6*log(tan(x)**2 + 1) + 4*tan(x)**3*x**3 - 6*tan(x
)**2*x**2 - 12*tan(x)*x**3 + 12*tan(x)*x + 3*x**4 - 6*x**2)/12
```

3.129 $\int x^3 \tan^6(x) \, dx$

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3.129.1 Optimal result

Integrand size = 8, antiderivative size = 153

$$\begin{aligned}\int x^3 \tan^6(x) \, dx = {} & \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log\left(1 + e^{2ix}\right) - 2\log(\cos(x)) \\ & - \frac{23}{5}ix \operatorname{PolyLog}\left(2, -e^{2ix}\right) + \frac{23}{10}\operatorname{PolyLog}\left(3, -e^{2ix}\right) \\ & - \frac{19}{10}x\tan(x) + x^3\tan(x) - \frac{\tan^2(x)}{20} + \frac{4}{5}x^2\tan^2(x) \\ & + \frac{1}{10}x\tan^3(x) - \frac{1}{3}x^3\tan^3(x) - \frac{3}{20}x^2\tan^4(x) + \frac{1}{5}x^3\tan^5(x)\end{aligned}$$

output
```
19/20*x^2-23/15*I*x^3-1/4*x^4+23/5*x^2*ln(1+exp(2*I*x))-2*ln(cos(x))-23/5*
I*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-exp(2*I*x))-19/10*x*tan(x)+x^3
*tan(x)-1/20*tan(x)^2+4/5*x^2*tan(x)^2+1/10*x*tan(x)^3-1/3*x^3*tan(x)^3-3/
20*x^2*tan(x)^4+1/5*x^3*tan(x)^5
```

3.129.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^3 \tan^6(x)\,dx = \frac{1}{60}\left(-92ix^3 - 15x^4 + 276x^2 \log\left(1+e^{2ix}\right) - 120\log(\cos(x)) - 276ix\,\text{PolyLog}\left(2,-e^{2ix}\right) + 138\,\text{PolyLog}\left(3,-e^{2ix}\right) - 3\sec^2(x) + 66x^2\sec^2(x) - 9x^2\sec^4(x) - 120x\tan(x) + 92x^3\tan(x) + 6x\sec^2(x)\tan(x) - 44x^3\sec^2(x)\tan(x) + 12x^3\sec^4(x)\tan(x)\right)$$

input

```
Integrate[x^3*Tan[x]^6,x]
```

output

```
((-92*I)*x^3 - 15*x^4 + 276*x^2*Log[1 + E^((2*I)*x)] - 120*Log[Cos[x]] - (
276*I)*x*PolyLog[2, -E^((2*I)*x)] + 138*PolyLog[3, -E^((2*I)*x)] - 3*Sec[x
]^2 + 66*x^2*Sec[x]^2 - 9*x^2*Sec[x]^4 - 120*x*Tan[x] + 92*x^3*Tan[x] + 6*
x*Sec[x]^2*Tan[x] - 44*x^3*Sec[x]^2*Tan[x] + 12*x^3*Sec[x]^4*Tan[x])/60
```

3.129.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.92, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 2.500$, Rules used = {3042, 4203, 3042, 4203, 3042, 4203, 15, 3042, 3954, 3042, 3956, 4202, 2620, 3011, 2720, 4203, 15, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tan^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \tan(x)^6 dx$$

$$\Big\downarrow \text{4203}$$

$$-\int x^3 \tan^4(x)dx - \frac{3}{5}\int x^2 \tan^5(x)dx + \frac{1}{5}x^3 \tan^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3\tan(x)^4dx-\frac{3}{5}\int x^2\tan(x)^5dx+\frac{1}{5}x^3\tan^5(x)$$

$\big\downarrow$ 4203

$$\int x^3\tan^2(x)dx+\int x^2\tan^3(x)dx-\frac{3}{5}\left(-\int x^2\tan^3(x)dx-\frac{1}{2}\int x\tan^4(x)dx+\frac{1}{4}x^2\tan^4(x)\right)+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)$$

$\big\downarrow$ 3042

$$\int x^3\tan(x)^2dx-\frac{3}{5}\left(-\int x^2\tan(x)^3dx-\frac{1}{2}\int x\tan(x)^4dx+\frac{1}{4}x^2\tan^4(x)\right)+\int x^2\tan(x)^3dx+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)$$

$\big\downarrow$ 4203

$$-\int x^3dx-\frac{3}{5}\left(\int x^2\tan(x)dx+\int x\tan^2(x)dx+\frac{1}{2}\left(\frac{1}{3}\int\tan^3(x)dx+\int x\tan^2(x)dx-\frac{1}{3}x\tan^3(x)\right)+\frac{1}{4}x^2\tan^4(x)-\frac{1}{2}x^2\right.$$
$$4\int x^2\tan(x)dx-\int x\tan^2(x)dx+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\big\downarrow$ 15

$$-\frac{3}{5}\left(\int x^2\tan(x)dx+\int x\tan^2(x)dx+\frac{1}{2}\left(\frac{1}{3}\int\tan^3(x)dx+\int x\tan^2(x)dx-\frac{1}{3}x\tan^3(x)\right)+\frac{1}{4}x^2\tan^4(x)-\frac{1}{2}x\right.$$
$$4\int x^2\tan(x)dx-\int x\tan^2(x)dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\big\downarrow$ 3042

$$-\frac{3}{5}\left(\int x^2\tan(x)dx+\frac{1}{2}\left(\int x\tan(x)^2dx+\frac{1}{3}\int\tan(x)^3dx-\frac{1}{3}x\tan^3(x)\right)+\int x\tan(x)^2dx+\frac{1}{4}x^2\tan^4(x)-\frac{1}{2}x\right.$$
$$4\int x^2\tan(x)dx-\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\big\downarrow$ 3954

$$-\frac{3}{5}\left(\int x^2\tan(x)dx+\frac{1}{2}\left(\frac{1}{3}\left(\frac{\tan^2(x)}{2}-\int\tan(x)dx\right)+\int x\tan(x)^2dx-\frac{1}{3}x\tan^3(x)\right)+\int x\tan(x)^2dx+\frac{1}{4}x^2\right.$$
$$4\int x^2\tan(x)dx-\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\big\downarrow$ 3042

3.129. $\int x^3\tan^6(x)\,dx$

$$-\frac{3}{5}\left(\int x^2\tan(x)dx+\frac{1}{2}\left(\frac{1}{3}\left(\frac{\tan^2(x)}{2}-\int\tan(x)dx\right)+\int x\tan(x)^2dx-\frac{1}{3}x\tan^3(x)\right)+\int x\tan(x)^2dx+\frac{1}{4}x^2\right.$$
$$4\int x^2\tan(x)dx-\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\downarrow$ 3956

$$-4\int x^2\tan(x)dx-$$
$$\frac{3}{5}\left(\int x^2\tan(x)dx+\int x\tan(x)^2dx+\frac{1}{2}\left(\int x\tan(x)^2dx-\frac{1}{3}x\tan^3(x)+\frac{1}{3}\left(\frac{\tan^2(x)}{2}+\log(\cos(x))\right)\right)+\frac{1}{4}x^2\tan\right.$$
$$\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\downarrow$ 4202

$$-4\left(\frac{ix^3}{3}-2i\int\frac{e^{2ix}x^2}{1+e^{2ix}}dx\right)-$$
$$\frac{3}{5}\left(-2i\int\frac{e^{2ix}x^2}{1+e^{2ix}}dx+\int x\tan(x)^2dx+\frac{1}{2}\left(\int x\tan(x)^2dx-\frac{1}{3}x\tan^3(x)+\frac{1}{3}\left(\frac{\tan^2(x)}{2}+\log(\cos(x))\right)\right)+\frac{ix^3}{3}\right.$$
$$\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\downarrow$ 2620

$$-4\left(\frac{ix^3}{3}-2i\left(i\int x\log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\int x\log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\int x\tan(x)^2dx+\frac{1}{2}\left(\int x\tan(x)^2dx-\frac{1}{3}x\tan^3(x)+\frac{1}{3}\right.\right.$$
$$\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\downarrow$ 3011

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\int\text{PolyLog}\left(2,-e^{2ix}\right)dx\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\int\text{PolyLog}\left(2,-e^{2ix}\right)dx\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\int x\tan(x)^2dx+\frac{1}{2}\right.$$
$$\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$\downarrow$ 2720

3.129. $\int x^3\tan^6(x)\,dx$

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\int x\tan(x)^2\right.$$
$$\int x\tan(x)^2dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)$$

$$\Big\downarrow \text{4203}$$

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)-\int xdx-\int\right.$$
$$\int xdx+\int\tan(x)dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{1}{2}x^2\tan^2(x)-x\tan(x)$$

$$\Big\downarrow \text{15}$$

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\frac{1}{2}\left(-\int \tan\right.\right.$$
$$\int\tan(x)dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{x^2}{2}+\frac{1}{2}x^2\tan^2(x)-x\tan(x)$$

$$\Big\downarrow \text{3042}$$

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\frac{1}{2}\left(-\int \tan\right.\right.$$
$$\int\tan(x)dx-\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{x^2}{2}+\frac{1}{2}x^2\tan^2(x)-x\tan(x)$$

$$\Big\downarrow \text{3956}$$

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-e^{2ix}\right)de^{2ix}\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\frac{ix^3}{3}-\frac{x^2}{2}+\right.$$
$$\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{x^2}{2}+\frac{1}{2}x^2\tan^2(x)-x\tan(x)-\log(\cos(x))$$

$$\Big\downarrow \text{7143}$$

3.129. $\int x^3\tan^6(x)\,dx$

$$-4\left(\frac{ix^3}{3}-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2ix}\right)\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)\right)-$$
$$\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2ix}\right)\right)-\frac{1}{2}ix^2\log\left(1+e^{2ix}\right)\right)+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{4}x^2\tan^4(x)-\right.$$
$$\frac{x^4}{4}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{x^2}{2}+\frac{1}{2}x^2\tan^2(x)-x\tan(x)-\log(\cos(x))$$

input
```
Int[x^3*Tan[x]^6,x]
```

output
```
x^2/2 - x^4/4 - Log[Cos[x]] - 4*((I/3)*x^3 - (2*I)*((-1/2*I)*x^2*Log[1 + E
^((2*I)*x)] + I*((I/2)*x*PolyLog[2, -E^((2*I)*x)] - PolyLog[3, -E^((2*I)*x
)]/4))) - x*Tan[x] + x^3*Tan[x] + (x^2*Tan[x]^2)/2 - (x^3*Tan[x]^3)/3 + (x
^3*Tan[x]^5)/5 - (3*(-1/2*x^2 + (I/3)*x^3 + Log[Cos[x]] - (2*I)*((-1/2*I)*
x^2*Log[1 + E^((2*I)*x)] + I*((I/2)*x*PolyLog[2, -E^((2*I)*x)] - PolyLog[3
, -E^((2*I)*x)]/4)) + x*Tan[x] - (x^2*Tan[x]^2)/2 + (x^2*Tan[x]^4)/4 + (-1
/2*x^2 + Log[Cos[x]] + x*Tan[x] - (x*Tan[x]^3)/3 + (Log[Cos[x]] + Tan[x]^2
/2)/3)/2))/5
```

3.129.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.129. $\int x^3 \tan^6(x)\,dx$

3.129.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{x^4}{4}+\frac{i\left(9i\mathrm{e}^{6ix}+90x^3\mathrm{e}^{8ix}-162ix^2\mathrm{e}^{4ix}-66ix^2\mathrm{e}^{2ix}+180x^3\mathrm{e}^{6ix}-66x\,\mathrm{e}^{8ix}+3i\mathrm{e}^{8ix}+9i\mathrm{e}^{4ix}+280\,\mathrm{e}^{4ix}x^3-246x\,\mathrm{e}^{6ix}-162ix^2\mathrm{e}^{6ix}+3\right.}{15\left(\mathrm{e}^{2ix}+1\right)^5}$

input
```
int(x^3*tan(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+1/15*I*(9*I*exp(6*I*x)+90*x^3*exp(8*I*x)-162*I*x^2*exp(4*I*x)-66*
I*x^2*exp(2*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)+3*I*exp(8*I*x)+9*I*exp
(4*I*x)+280*exp(4*I*x)*x^3-246*x*exp(6*I*x)-162*I*x^2*exp(6*I*x)+3*I*exp(2
*I*x)+140*exp(2*I*x)*x^3-354*x*exp(4*I*x)-66*I*x^2*exp(8*I*x)+46*x^3-234*x
*exp(2*I*x)-60*x)/(exp(2*I*x)+1)^5-2*ln(exp(2*I*x)+1)+4*ln(exp(I*x))-46/15
*I*x^3+23/5*x^2*ln(exp(2*I*x)+1)-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*pol
ylog(3,-exp(2*I*x))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

$$\begin{aligned}\int x^3\tan^6(x)\,dx &= \frac{1}{5}\,x^3\tan{(x)}^5-\frac{3}{20}\,x^2\tan{(x)}^4-\frac{1}{4}\,x^4-\frac{1}{30}\,\left(10\,x^3-3\,x\right)\tan{(x)}^3\\ &\quad+\frac{1}{20}\,\left(16\,x^2-1\right)\tan{(x)}^2+\frac{19}{20}\,x^2+\frac{23}{10}i\,x\mathrm{Li}_2\left(\frac{2\left(i\,\tan{(x)}-1\right)}{\tan{(x)}^2+1}+1\right)\\ &\quad-\frac{23}{10}i\,x\mathrm{Li}_2\left(\frac{2\left(-i\,\tan{(x)}-1\right)}{\tan{(x)}^2+1}+1\right)\\ &\quad+\frac{1}{10}\,\left(23\,x^2-10\right)\log\left(-\frac{2\left(i\,\tan{(x)}-1\right)}{\tan{(x)}^2+1}\right)\\ &\quad+\frac{1}{10}\,\left(23\,x^2-10\right)\log\left(-\frac{2\left(-i\,\tan{(x)}-1\right)}{\tan{(x)}^2+1}\right)\\ &\quad+\frac{1}{10}\,\left(10\,x^3-19\,x\right)\tan{(x)}\\ &\quad+\frac{23}{20}\,\mathrm{polylog}\left(3,\,\frac{\tan{(x)}^2+2i\,\tan{(x)}-1}{\tan{(x)}^2+1}\right)\\ &\quad+\frac{23}{20}\,\mathrm{polylog}\left(3,\,\frac{\tan{(x)}^2-2i\,\tan{(x)}-1}{\tan{(x)}^2+1}\right)\end{aligned}$$

input
```
integrate(x^3*tan(x)^6,x, algorithm="fricas")
```

output
```
1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3*x)*tan(x
)^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 + 23/10*I*x*dilog(2*(I*tan(x)
 - 1)/(tan(x)^2 + 1) + 1) - 23/10*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 +
1) + 1) + 1/10*(23*x^2 - 10)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*
(23*x^2 - 10)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(10*x^3 - 19*x
)*tan(x) + 23/20*polylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) +
23/20*polylog(3, (tan(x)^2 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

3.129.6 Sympy [F]

$$\int x^3 \tan^6(x)\,dx = \int x^3 \tan^6{(x)}\,dx$$

input
```
integrate(x**3*tan(x)**6,x)
```

output
```
Integral(x**3*tan(x)**6, x)
```

3.129.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(113) = 226$.

Time = 0.57 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.08

$$\int x^3 \tan^6(x)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*tan(x)^6,x, algorithm="maxima")
```

output
```
(15*I*x^4 + 12*(23*x^2 + (23*x^2 - 10)*cos(10*x) + 5*(23*x^2 - 10)*cos(8*x
) + 10*(23*x^2 - 10)*cos(6*x) + 10*(23*x^2 - 10)*cos(4*x) + 5*(23*x^2 - 10
)*cos(2*x) - (-23*I*x^2 + 10*I)*sin(10*x) - 5*(-23*I*x^2 + 10*I)*sin(8*x)
- 10*(-23*I*x^2 + 10*I)*sin(6*x) - 10*(-23*I*x^2 + 10*I)*sin(4*x) - 5*(-23
*I*x^2 + 10*I)*sin(2*x) - 10)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^4
- 184*x^3 + 240*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x + 1
2*I)*cos(8*x) - 2*(-75*I*x^4 + 560*x^3 + 324*I*x^2 - 708*x - 18*I)*cos(6*x
) - 6*(-25*I*x^4 + 120*x^3 + 108*I*x^2 - 164*x - 6*I)*cos(4*x) - 3*(-25*I*
x^4 + 120*x^3 + 88*I*x^2 - 88*x - 4*I)*cos(2*x) - 276*(x*cos(10*x) + 5*x*c
os(8*x) + 10*x*cos(6*x) + 10*x*cos(4*x) + 5*x*cos(2*x) + I*x*sin(10*x) + 5
*I*x*sin(8*x) + 10*I*x*sin(6*x) + 10*I*x*sin(4*x) + 5*I*x*sin(2*x) + x)*di
log(-e^(2*I*x)) - 6*(23*I*x^2 + (23*I*x^2 - 10*I)*cos(10*x) + 5*(23*I*x^2
- 10*I)*cos(8*x) + 10*(23*I*x^2 - 10*I)*cos(6*x) + 10*(23*I*x^2 - 10*I)*co
s(4*x) + 5*(23*I*x^2 - 10*I)*cos(2*x) - (23*x^2 - 10)*sin(10*x) - 5*(23*x^
2 - 10)*sin(8*x) - 10*(23*x^2 - 10)*sin(6*x) - 10*(23*x^2 - 10)*sin(4*x) -
 5*(23*x^2 - 10)*sin(2*x) - 10*I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x)
 + 1) - 138*(I*cos(10*x) + 5*I*cos(8*x) + 10*I*cos(6*x) + 10*I*cos(4*x) +
5*I*cos(2*x) - sin(10*x) - 5*sin(8*x) - 10*sin(6*x) - 10*sin(4*x) - 5*sin(
2*x) + I)*polylog(3, -e^(2*I*x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x
) - (75*x^4 + 560*I*x^3 - 264*x^2 - 936*I*x + 12)*sin(8*x) - 2*(75*x^4 ...
```

3.129.8 Giac [F]

$$\int x^3 \tan^6(x)\,dx = \int x^3 \tan(x)^6\,dx$$

input
```
integrate(x^3*tan(x)^6,x, algorithm="giac")
```

output
```
integrate(x^3*tan(x)^6, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \tan^6(x)\, dx = \int x^3 \tan(x)^6\, dx$$

input
```
int(x^3*tan(x)^6,x)
```

output
```
int(x^3*tan(x)^6, x)
```

3.129.10 Reduce [F]

$$\int x^3 \tan^6(x)\, dx = -\frac{23\left(\int \tan(x)\, x^2 dx\right)}{5} + \log\left(\tan(x)^2+1\right) + \frac{\tan(x)^5 x^3}{5} - \frac{3\tan(x)^4 x^2}{20} - \frac{\tan(x)^3 x^3}{3} + \frac{\tan(x)^3 x}{10} + \frac{4\tan(x)^2 x^2}{5} - \frac{\tan(x)^2}{20} + \tan(x)\, x^3 - \frac{19\tan(x)\, x}{10} - \frac{x^4}{4} + \frac{19x^2}{20}$$

input
```
int(tan(x)**6*x**3,x)
```

output
```
( - 276*int(tan(x)*x**2,x) + 60*log(tan(x)**2 + 1) + 12*tan(x)**5*x**3 - 9
*tan(x)**4*x**2 - 20*tan(x)**3*x**3 + 6*tan(x)**3*x + 48*tan(x)**2*x**2 -
3*tan(x)**2 + 60*tan(x)*x**3 - 114*tan(x)*x - 15*x**4 + 57*x**2)/60
```

3.130 $\int x\tan^2(x)\,dx$

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3.130.1 Optimal result

Integrand size = 6, antiderivative size = 15

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

output
```
-1/2*x^2+ln(cos(x))+x*tan(x)
```

3.130.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

input
```
Integrate[x*Tan[x]^2,x]
```

output
```
-1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

3.130.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 4203, 15, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan^2(x)\, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x \tan(x)^2 dx$$

$$\Big\downarrow \text{ 4203}$$

$$-\int x dx - \int \tan(x) dx + x \tan(x)$$

$$\Big\downarrow \text{ 15}$$

$$-\int \tan(x) dx - \frac{x^2}{2} + x \tan(x)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \tan(x) dx - \frac{x^2}{2} + x \tan(x)$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

input
```
Int[x*Tan[x]^2,x]
```

output
```
-1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

3.130.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

method	result	size
norman	$x \tan(x) - \frac{x^2}{2} - \frac{\ln(1+\tan^2(x))}{2}$	20
parallelrisch	$x \tan(x) - \frac{x^2}{2} - \frac{\ln(1+\tan^2(x))}{2}$	20
risch	$-\frac{x^2}{2} - 2ix + \frac{2ix}{e^{2ix}+1} + \ln(e^{2ix}+1)$	32

input
```
int(x*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan(x) + \frac{1}{2}\,\log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

input
```
integrate(x*tan(x)^2,x, algorithm="fricas")
```

output
```
-1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

3.130.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + x\tan(x) - \frac{\log(\tan^2(x) + 1)}{2}$$

input
```
integrate(x*tan(x)**2,x)
```

output
```
-x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.13

$$\int x\tan^2(x)\,dx =$$

$$-\frac{x^2\cos(2x)^2 + x^2\sin(2x)^2 + 2x^2\cos(2x) + x^2 - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(2\right.}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

input
```
integrate(x*tan(x)^2,x, algorithm="maxima")
```

output
```
-1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan\left(x\right) + \frac{1}{2}\,\log\left(\frac{4}{\tan\left(x\right)^2 + 1}\right)$$

input
```
integrate(x*tan(x)^2,x, algorithm="giac")
```

output
```
-1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\tan^2(x)\,dx = \ln\left(\cos\left(x\right)\right) + x\tan(x) - \frac{x^2}{2}$$

input
```
int(x*tan(x)^2,x)
```

output
```
log(cos(x)) + x*tan(x) - x^2/2
```

3.130.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\tan^2(x)\,dx = -\frac{\log\left(\tan\left(x\right)^2 + 1\right)}{2} + \tan\left(x\right)x - \frac{x^2}{2}$$

input
```
int(tan(x)**2*x,x)
```

output
```
( - log(tan(x)**2 + 1) + 2*tan(x)*x - x**2)/2
```

3.131 $\int \cos(3x)\sin(2x)\,dx$

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3.131.10Reduce [B] (verification not implemented) 847

3.131.1 Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x)\sin(2x)\,dx = \frac{\cos(x)}{2} - \frac{1}{10}\cos(5x)$$

output
```
1/2*cos(x)-1/10*cos(5*x)
```

3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x)\sin(2x)\,dx = \frac{\cos(x)}{2} - \frac{1}{10}\cos(5x)$$

input
```
Integrate[Cos[3*x]*Sin[2*x],x]
```

output
```
Cos[x]/2 - Cos[5*x]/10
```

3.131.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\cos(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(2x)\cos(3x)dx$$

$$\Big\downarrow \text{ 4772}$$

$$\frac{\cos(x)}{2} - \frac{1}{10}\cos(5x)$$

input
```
Int[Cos[3*x]*Sin[2*x],x]
```

output
```
Cos[x]/2 - Cos[5*x]/10
```

3.131.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
parallelrisch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}-\frac{2}{5}$	13
norman	$\frac{-\frac{4\left(\tan^2(x)\right)}{5}-\frac{4\left(\tan^2\left(\frac{3x}{2}\right)\right)}{5}+\frac{12\tan(x)\tan\left(\frac{3x}{2}\right)}{5}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(x)\right)}$	43

input
```
int(cos(3*x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)-1/10*cos(5*x)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(3x)\sin(2x)\,dx = -\frac{8}{5}\cos(x)^5 + 2\cos(x)^3$$

input
```
integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")
```

output
```
-8/5*cos(x)^5 + 2*cos(x)^3
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x)\sin(2x)\,dx = \frac{3\sin(2x)\sin(3x)}{5} + \frac{2\cos(2x)\cos(3x)}{5}$$

input
```
integrate(cos(3*x)*sin(2*x),x)
```

output
```
3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\sin(2x)\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) + 1/2*cos(x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\sin(2x)\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(3*x)*sin(2*x),x, algorithm="giac")
```

output
```
-1/10*cos(5*x) + 1/2*cos(x)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(3x)\sin(2x)\,dx = 2\cos(x)^3 - \frac{8\cos(x)^5}{5}$$

input
```
int(cos(3*x)*sin(2*x),x)
```

output
```
2*cos(x)^3 - (8*cos(x)^5)/5
```

3.131.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cos(3x)\sin(2x)\,dx = \frac{2\cos(3x)\cos(2x)}{5} + \frac{3\sin(3x)\sin(2x)}{5}$$

input
```
int(cos(3*x)*sin(2*x),x)
```

output
```
(2*cos(3*x)*cos(2*x) + 3*sin(3*x)*sin(2*x))/5
```

3.132 $\int \cos^2(x)\sin^2(x)\,dx$

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3.132.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)$$

output
```
1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

3.132.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8}-\frac{1}{32}\sin(4x)$$

input
```
Integrate[Cos[x]^2*Sin[x]^2,x]
```

output
```
x/8 - Sin[4*x]/32
```

3.132.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos^2(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)^2\cos(x)^2dx$$

$$\Big\downarrow \ 3048$$

$$\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \ 3115$$

$$\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \ 24$$

$$\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

input
```
Int[Cos[x]^2*Sin[x]^2,x]
```

output
```
-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4
```

3.132.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.132.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos^3(x))\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

input
```
int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x-1/32*sin(4*x)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}x$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

input
```
integrate(cos(x)**2*sin(x)**2,x)
```

output
```
x/8 - sin(2*x)*cos(2*x)/16
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}x - \frac{1}{32}\sin(4x)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
1/8*x - 1/32*sin(4*x)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin(4\,x)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
1/8*x - 1/32*sin(4*x)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos(x)\ \sin(x)^3}{4} - \frac{\cos(x)\ \sin(x)}{8} + \frac{x}{8}$$

input
```
int(cos(x)^2*sin(x)^2,x)
```

output
```
x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

3.132.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos(x)\sin(x)^3}{4} - \frac{\cos(x)\sin(x)}{8} + \frac{x}{8}$$

input
```
int(cos(x)**2*sin(x)**2,x)
```

output
```
(2*cos(x)*sin(x)**3 - cos(x)*sin(x) + x)/8
```

3.133 $\int \csc^2(x) \sec^2(x) \, dx$

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3.133.1 Optimal result

Integrand size = 9, antiderivative size = 7

$$\int \csc^2(x) \sec^2(x) \, dx = -\cot(x) + \tan(x)$$

output
```
-cot(x)+tan(x)
```

3.133.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x) \sec^2(x) \, dx = -2\cot(2x)$$

input
```
Integrate[Csc[x]^2*Sec[x]^2,x]
```

output
```
-2*Cot[2*x]
```

3.133.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(x)^2\sec(x)^2dx$$

$$\Big\downarrow \text{ 3100}$$

$$\int \left(\tan^2(x)+1\right)\cot^2(x)d\tan(x)$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\cot^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\tan(x)-\cot(x)$$

input
```
Int[Csc[x]^2*Sec[x]^2,x]
```

output
```
-Cot[x] + Tan[x]
```

3.133.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.133.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

method	result	size
parallelrisch	$-2\cot(x)+\sec(x)\csc(x)$	11
default	$\frac{1}{\cos(x)\sin(x)}-2\cot(x)$	15
risch	$-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	22
norman	$\frac{\frac{1}{2}-3\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}$	36

input
```
int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*cot(x)+sec(x)*csc(x)
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \csc^2(x)\sec^2(x)\,dx=-\frac{2\cos(x)^2-1}{\cos(x)\sin(x)}$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

output
```
-(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{2\cos(2x)}{\sin(2x)}$$

input
```
integrate(1/cos(x)**2/sin(x)**2,x)
```

output
```
-2*cos(2*x)/sin(2*x)
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
```

output
```
-1/tan(x) + tan(x)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
```

output
```
-1/tan(x) + tan(x)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2\,x)$$

input
```
int(1/(cos(x)^2*sin(x)^2),x)
```

output
```
-2*cot(2*x)
```

3.133.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc^2(x)\sec^2(x)\,dx = \frac{2\sin(x)^2-1}{\cos(x)\sin(x)}$$

input
```
int(1/(cos(x)**2*sin(x)**2),x)
```

output
```
(2*sin(x)**2 - 1)/(cos(x)*sin(x))
```

3.134 $\int d^x \sin(x) \, dx$

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3.134.1 Optimal result

Integrand size = 6, antiderivative size = 32

$$\int d^x \sin(x) \, dx = -\frac{d^x \cos(x)}{1+\log^2(d)} + \frac{d^x \log(d) \sin(x)}{1+\log^2(d)}$$

output
```
-d^x*cos(x)/(1+ln(d)^2)+d^x*ln(d)*sin(x)/(1+ln(d)^2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x \sin(x) \, dx = \frac{d^x(-\cos(x) + \log(d)\sin(x))}{1+\log^2(d)}$$

input
```
Integrate[d^x*Sin[x],x]
```

output
```
(d^x*(-Cos[x] + Log[d]*Sin[x]))/(1 + Log[d]^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4932\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int d^x \sin(x)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{d^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x \cos(x)}{\log^2(d)+1}$$

input
```
Int[d^x*Sin[x],x]
```

output
```
-((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

3.134.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.134.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{d^{x}\left(\ln\left(d\right)\sin\left(x\right)-\cos\left(x\right)\right)}{1+\ln\left(d\right)^{2}}$	23
risch	$-\frac{d^{x}\cos\left(x\right)}{1+\ln\left(d\right)^{2}}+\frac{d^{x}\ln\left(d\right)\sin\left(x\right)}{1+\ln\left(d\right)^{2}}$	33
norman	$\frac{\frac{{\mathrm e}^{x\ln\left(d\right)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\ln\left(d\right)^{2}}-\frac{{\mathrm e}^{x\ln\left(d\right)}}{1+\ln\left(d\right)^{2}}+\frac{2\ln\left(d\right){\mathrm e}^{x\ln\left(d\right)}\tan\left(\frac{x}{2}\right)}{1+\ln\left(d\right)^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	69

input
```
int(d^x*sin(x),x,method=_RETURNVERBOSE)
```

output
```
d^x*(ln(d)*sin(x)-cos(x))/(1+ln(d)^2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x\sin(x)\,dx=\frac{(\log\left(d\right)\sin\left(x\right)-\cos\left(x\right))d^{x}}{\log\left(d\right)^{2}+1}$$

input
```
integrate(d^x*sin(x),x, algorithm="fricas")
```

output
```
(log(d)*sin(x) - cos(x))*d^x/(log(d)^2 + 1)
```

3.134.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int d^x\sin(x)\,dx=\begin{cases}\frac{xe^{-ix}\sin\left(x\right)}{2}-\frac{ixe^{-ix}\cos\left(x\right)}{2}-\frac{e^{-ix}\cos\left(x\right)}{2} & \text{for}\: d=e^{-i}\\ \frac{xe^{ix}\sin\left(x\right)}{2}+\frac{ixe^{ix}\cos\left(x\right)}{2}-\frac{e^{ix}\cos\left(x\right)}{2} & \text{for}\: d=e^{i}\\ \frac{d^{x}\log\left(d\right)\sin\left(x\right)}{\log\left(d\right)^{2}+1}-\frac{d^{x}\cos\left(x\right)}{\log\left(d\right)^{2}+1} & \text{otherwise}\end{cases}$$

input
```
integrate(d**x*sin(x),x)
```

output
```
Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I*x)*cos(x
)/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x)*cos(x)/2 - exp(I
*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*c
os(x)/(log(d)**2 + 1), True))
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int d^x \sin(x)\,dx = \frac{d^x \log(d)\sin(x) - d^x \cos(x)}{\log(d)^2 + 1}$$

input
```
integrate(d^x*sin(x),x, algorithm="maxima")
```

output
```
(d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)
```

3.134.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 10.25

$$\int d^x \sin(x)\,dx = \text{Too large to display}$$

input
```
integrate(d^x*sin(x),x, algorithm="giac")
```

output
```
abs(d)^x*((pi - pi*sgn(d) - 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi -
pi*sgn(d) - 2)^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) -
1/2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*((pi
- pi*sgn(d) + 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)
^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/
((pi - pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*(-I*e^(1/2*I*pi*x*s
gn(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I)
- I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*
log(abs(d)) - 4*I)) - abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)
/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) + I*e^(-1/2*I*pi*x*sgn(d)
 + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x \sin(x)\, dx = -\frac{d^x\left(\cos\left(x\right) - \ln\left(d\right)\,\sin\left(x\right)\right)}{\ln\left(d\right)^2 + 1}$$

input
```
int(d^x*sin(x),x)
```

output
```
-(d^x*(cos(x) - log(d)*sin(x)))/(log(d)^2 + 1)
```

3.134.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x \sin(x)\, dx = \frac{d^x(-\cos\left(x\right) + \log(d)\sin\left(x\right))}{\log\left(d\right)^2 + 1}$$

input
```
int(d**x*sin(x),x)
```

output
```
(d**x*( - cos(x) + log(d)*sin(x)))/(log(d)**2 + 1)
```

3.135 $\int d^x \cos(x) \, dx$

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3.135.1 Optimal result

Integrand size = 6, antiderivative size = 31

$$\int d^x \cos(x) \, dx = \frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)}$$

output
```
d^x*cos(x)*ln(d)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^x \cos(x) \, dx = \frac{d^x (\cos(x) \log(d) + \sin(x))}{1 + \log^2(d)}$$

input
```
Integrate[d^x*Cos[x],x]
```

output
```
(d^x*(Cos[x]*Log[d] + Sin[x]))/(1 + Log[d]^2)
```

3.135.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int d^x \cos(x) \, dx$$

$$\Big\downarrow \; 4933$$

$$\frac{d^x \sin(x)}{\log^2(d)+1} + \frac{d^x \log(d) \cos(x)}{\log^2(d)+1}$$

input
```
Int[d^x*Cos[x],x]
```

output
```
(d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

3.135.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.135.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{d^{x}(\ln(d)\cos(x)+\sin(x))}{1+\ln(d)^{2}}$	21
risch	$\frac{d^{x}\cos(x)\ln(d)}{1+\ln(d)^{2}}+\frac{d^{x}\sin(x)}{1+\ln(d)^{2}}$	32
norman	$\frac{\frac{\ln(d)e^{x\ln(d)}}{1+\ln(d)^{2}}+\frac{2\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^{2}}-\frac{\ln(d)e^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\ln(d)^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	71

input
```
int(d^x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
d^x*(ln(d)*cos(x)+sin(x))/(1+ln(d)^2)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^{x}\cos(x)\,dx=\frac{(\cos(x)\log(d)+\sin(x))d^{x}}{\log(d)^{2}+1}$$

input
```
integrate(d^x*cos(x),x, algorithm="fricas")
```

output
```
(cos(x)*log(d) + sin(x))*d^x/(log(d)^2 + 1)
```

3.135.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int d^{x}\cos(x)\,dx=\begin{cases}\frac{ixe^{-ix}\sin(x)}{2}+\frac{xe^{-ix}\cos(x)}{2}+\frac{ie^{-ix}\cos(x)}{2} & \text{for } d=e^{-i}\\ -\frac{ixe^{ix}\sin(x)}{2}+\frac{xe^{ix}\cos(x)}{2}-\frac{ie^{ix}\cos(x)}{2} & \text{for } d=e^{i}\\ \frac{d^{x}\log(d)\cos(x)}{\log(d)^{2}+1}+\frac{d^{x}\sin(x)}{\log(d)^{2}+1} & \text{otherwise}\end{cases}$$

input
```
integrate(d**x*cos(x),x)
```

output
```
Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos
(x)/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*
exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*cos(x)/(log(d)**2 + 1) + d
**x*sin(x)/(log(d)**2 + 1), True))
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int d^x \cos(x) \, dx = \frac{d^x \cos(x) \log(d) + d^x \sin(x)}{\log(d)^2 + 1}$$

input
```
integrate(d^x*cos(x),x, algorithm="maxima")
```

output
```
(d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)
```

3.135.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 10.61

$$\int d^x \cos(x) \, dx = \text{Too large to display}$$

input
```
integrate(d^x*cos(x),x, algorithm="giac")
```

output
```
abs(d)^x*(2*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)*log(abs(d))/((pi - pi*sgn(
d) - 2)^2 + 4*log(abs(d))^2) - (pi - pi*sgn(d) - 2)*sin(1/2*pi*x*sgn(d) -
1/2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) + abs(d)^x*(2*co
s(1/2*pi*x*sgn(d) - 1/2*pi*x - x)*log(abs(d))/((pi - pi*sgn(d) + 2)^2 + 4*
log(abs(d))^2) - (pi - pi*sgn(d) + 2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/
((pi - pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) + I*abs(d)^x*(I*e^(1/2*I*pi*x*
sgn(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I)
 - I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4
*log(abs(d)) - 4*I)) + I*abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I
*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) - I*e^(-1/2*I*pi*x*sgn
(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^x \cos(x)\,dx = \frac{d^x\,(\sin{(x)} + \ln{(d)}\,\cos{(x)})}{\ln{(d)}^2 + 1}$$

input
```
int(d^x*cos(x),x)
```

output
```
(d^x*(sin(x) + log(d)*cos(x)))/(log(d)^2 + 1)
```

3.135.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^x \cos(x)\,dx = \frac{d^x(\cos{(x)}\log(d) + \sin{(x)})}{\log{(d)}^2 + 1}$$

input
```
int(d**x*cos(x),x)
```

output
```
(d**x*(cos(x)*log(d) + sin(x)))/(log(d)**2 + 1)
```

3.136 $\int d^x x \sin(x)\,dx$

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3.136.1 Optimal result

Integrand size = 7, antiderivative size = 84

$$\int d^x x \sin(x)\,dx = \frac{2d^x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x \cos(x)}{1+\log^2(d)} + \frac{d^x \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{d^x \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x \log(d)\sin(x)}{1+\log^2(d)}$$

output
```
2*d^x*cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x*cos(x)/(1+ln(d)^2)+d^x*sin(x)/(1+ln
(d)^2)^2-d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x*ln(d)*sin(x)/(1+ln(d)^2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int d^x x \sin(x)\,dx = \frac{d^x\left(-\cos(x)\left(x-2\log(d)+x\log^2(d)\right)+\left(1+x\log(d)-\log^2(d)+x\log^3(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^2}$$

input
```
Integrate[d^x*x*Sin[x],x]
```

output
```
(d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log[d]^2 + x
*Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2
```

3.136.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4968, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xd^x \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-\int \left(\frac{d^x \log(d) \sin(x)}{\log^2(d)+1}-\frac{d^x \cos(x)}{\log^2(d)+1}\right)dx+\frac{xd^x \log(d) \sin(x)}{\log^2(d)+1}-\frac{xd^x \cos(x)}{\log^2(d)+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{xd^x \log(d) \sin(x)}{\log^2(d)+1}-\frac{d^x \log^2(d) \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{xd^x \cos(x)}{\log^2(d)+1}+\frac{2d^x \log(d) \cos(x)}{\left(\log^2(d)+1\right)^2}$$

input
```
Int[d^x*x*Sin[x],x]
```

output
```
(2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[d]^2) + (
d^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d
^x*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{d^{x} \left(\ln \left(d \right)^{3} x \sin \left(x \right)+\left(-x \cos \left(x \right)-\sin \left(x \right)\right) \ln \left(d \right)^{2}+\left(x \sin \left(x \right)+2 \cos \left(x \right)\right) \ln \left(d \right)-x \cos \left(x \right)+\sin \left(x \right)\right)}{\left(1+\ln \left(d \right)^{2}\right)^{2}}$
risch	$-\frac{i \left(-1+x \ln \left(d \right)+i x \right) d^{x} {\mathrm e}^{i x}}{2 \left(\ln \left(d \right)+i\right)^{2}}+\frac{i \left(-1+x \ln \left(d \right)-i x \right) d^{x} {\mathrm e}^{-i x}}{2 \left(\ln \left(d \right)-i\right)^{2}}$
norman	$\frac{\frac{x \,{\mathrm e}^{x \ln \left(d \right)} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\ln \left(d \right)^{2}}+\frac{2 \ln \left(d \right) {\mathrm e}^{x \ln \left(d \right)}}{\left(1+\ln \left(d \right)^{2}\right)^{2}}-\frac{x \,{\mathrm e}^{x \ln \left(d \right)}}{1+\ln \left(d \right)^{2}}-\frac{2 \ln \left(d \right) {\mathrm e}^{x \ln \left(d \right)} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1+\ln \left(d \right)^{2}\right)^{2}}-\frac{2 \left(\ln \left(d \right)^{2}-1\right) {\mathrm e}^{x \ln \left(d \right)} \tan \left(\frac{x}{2}\right)}{\left(1+\ln \left(d \right)^{2}\right)^{2}}+\frac{2 \ln \left(d \right) x \,{\mathrm e}^{x \ln \left(d \right)} \tan \left(\frac{x}{2}\right)}{1+\ln \left(d \right)^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$

input
```
int(d^x*x*sin(x),x,method=_RETURNVERBOSE)
```

output
```
d^x*(ln(d)^3*x*sin(x)+(-x*cos(x)-sin(x))*ln(d)^2+(x*sin(x)+2*cos(x))*ln(d)
-x*cos(x)+sin(x))/(1+ln(d)^2)^2
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int d^x x \sin(x) \, dx = -\frac{{\left(x \cos \left(x\right) \log \left(d\right)^{2} + x \cos \left(x\right) - 2 \, \cos \left(x\right) \log \left(d\right) - {\left(x \log \left(d\right)^{3} + x \log \left(d\right) - \log \left(d\right)^{2} + 1\right)} \sin \left(x\right)\right)} d^{x}}{\log \left(d\right)^{4} + 2 \, \log \left(d\right)^{2} + 1}$$

input
```
integrate(d^x*x*sin(x),x, algorithm="fricas")
```

output
```
-(x*cos(x)*log(d)^2 + x*cos(x) - 2*cos(x)*log(d) - (x*log(d)^3 + x*log(d)
- log(d)^2 + 1)*sin(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

3.136.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.67

$$\int d^x x \sin(x)\,dx$$
$$= \begin{cases} \frac{x^2 e^{-ix}\sin(x)}{4} - \frac{ix^2 e^{-ix}\cos(x)}{4} + \frac{ixe^{-ix}\sin(x)}{4} - \frac{xe^{-ix}\cos(x)}{4} + \frac{ie^{-ix}\cos(x)}{4} \\ \frac{x^2 e^{ix}\sin(x)}{4} + \frac{ix^2 e^{ix}\cos(x)}{4} - \frac{ixe^{ix}\sin(x)}{4} - \frac{xe^{ix}\cos(x)}{4} - \frac{ie^{ix}\cos(x)}{4} \\ \frac{d^x x\log(d)^3\sin(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x x\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\log(d)\sin(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x x\cos(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1} + \frac{2d^x \mathrm{lo}}{\log(d)^4} \end{cases}$$

input
```
integrate(d**x*x*sin(x),x)
```

output
```
Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 + I*x*exp(-
I*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I)
)), (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*cos(x)/4 - I*x*exp(I*x)*sin(
x)/4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*
log(d)**3*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(
log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)*
*2 + 1) - d**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin
(x)/(log(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4 + 2*lo
g(d)**2 + 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int d^x x \sin(x)\, dx = -\frac{\left(\left(\log(d)^2+1\right)x - 2\log(d)\right)d^x\cos(x) - \left(\left(\log(d)^3+\log(d)\right)x - \log(d)^2+1\right)d^x\sin(x)}{\log(d)^4+2\log(d)^2+1}$$

input
```
integrate(d^x*x*sin(x),x, algorithm="maxima")
```

output
```
-(((log(d)^2 + 1)*x - 2*log(d))*d^x*cos(x) - ((log(d)^3 + log(d))*x - log(
d)^2 + 1)*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

3.136.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1156, normalized size of antiderivative = 13.76

$$\int d^x x \sin(x)\, dx = \text{Too large to display}$$

input
```
integrate(d^x*x*sin(x),x, algorithm="giac")
```

output
```
1/2*(((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(pi*
x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*p
i*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)
))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))*(x*log(
abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) -
 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos
(1/2*pi*x*sgn(d) - 1/2*pi*x + x) + 2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(a
bs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^
2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*l
og(abs(d)) + 2*log(abs(d)))^2) + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2
*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(ab
s(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x +
 1/2*(((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(pi
*x*sgn(d) - pi*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*
pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d
)))^2) + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))*(x*log
(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d)
+ 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*co
s(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - 2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*l...
```

3.136.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int d^x x \sin(x)\,dx$$
$$= \frac{d^x\,(\sin(x) + 2\,\ln(d)\,\cos(x) - \ln(d)^2\,\sin(x) - x\,\cos(x) + x\,\ln(d)\,\sin(x) - x\ln(d)^2\,\cos(x) + x\ln(d)^3}{\left(\ln(d)^2 + 1\right)^2}$$

input
```
int(d^x*x*sin(x),x)
```

output
```
(d^x*(sin(x) + 2*log(d)*cos(x) - log(d)^2*sin(x) - x*cos(x) + x*log(d)*sin
(x) - x*log(d)^2*cos(x) + x*log(d)^3*sin(x)))/(log(d)^2 + 1)^2
```

3.136.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int d^x x \sin(x) \, dx$$
$$= \frac{d^x \left(-\cos(x) \log(d)^2 x + 2\cos(x) \log(d) - \cos(x) x + \log(d)^3 \sin(x) x - \log(d)^2 \sin(x) + \log(d) \sin(x) x}{\log(d)^4 + 2\log(d)^2 + 1}$$

input
```
int(d**x*sin(x)*x,x)
```

output
```
(d**x*( - cos(x)*log(d)**2*x + 2*cos(x)*log(d) - cos(x)*x + log(d)**3*sin(
x)*x - log(d)**2*sin(x) + log(d)*sin(x)*x + sin(x)))/(log(d)**4 + 2*log(d)
**2 + 1)
```

3.137 $\int d^x x \cos(x) \, dx$

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3.137.1 Optimal result

Integrand size = 7, antiderivative size = 83

$$\int d^x x \cos(x) \, dx = \frac{d^x \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{d^x \cos(x) \log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x \cos(x) \log(d)}{1+\log^2(d)} - \frac{2 d^x \log(d) \sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x \sin(x)}{1+\log^2(d)}$$

output
```
d^x*cos(x)/(1+ln(d)^2)^2-d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x*cos(x)*ln(
d)/(1+ln(d)^2)-2*d^x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x*sin(x)/(1+ln(d)^2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int d^x x \cos(x) \, dx = \frac{d^x \left(\cos(x) \left(1 + x \log(d) - \log^2(d) + x \log^3(d)\right) + \left(x - 2 \log(d) + x \log^2(d)\right) \sin(x)\right)}{\left(1+\log^2(d)\right)^2}$$

input
```
Integrate[d^x*x*Cos[x],x]
```

output
```
(d^x*(Cos[x]*(1 + x*Log[d] - Log[d]^2 + x*Log[d]^3) + (x - 2*Log[d] + x*Lo
g[d]^2)*Sin[x]))/(1 + Log[d]^2)^2
```

3.137.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xd^x \cos(x)\,dx$$

$$\Big\downarrow \text{4969}$$

$$-\int \left(\frac{\sin(x)d^x}{\log^2(d)+1}+\frac{\cos(x)\log(d)d^x}{\log^2(d)+1}\right)dx+\frac{xd^x\sin(x)}{\log^2(d)+1}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{xd^x\sin(x)}{\log^2(d)+1}-\frac{2d^x\log(d)\sin(x)}{(\log^2(d)+1)^2}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\cos(x)}{(\log^2(d)+1)^2}+\frac{d^x\cos(x)}{(\log^2(d)+1)^2}$$

input
```
Int[d^x*x*Cos[x],x]
```

output
```
(d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (
d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)
^2 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

3.137.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.137.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{d^{x}\left(\ln(d)^{3}\cos(x)x+(x\sin(x)-\cos(x))\ln(d)^{2}+(x\cos(x)-2\sin(x))\ln(d)+x\sin(x)+\cos(x)\right)}{\left(1+\ln(d)^{2}\right)^{2}}$
risch	$\frac{(-1+x\ln(d)+ix)d^{x}e^{ix}}{2(\ln(d)+i)^{2}}+\frac{(-1+x\ln(d)-ix)d^{x}e^{-ix}}{2(\ln(d)-i)^{2}}$
norman	$\frac{\frac{\left(\ln(d)^{2}-1\right)e^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^{2}\right)^{2}}+\frac{\ln(d)x\,e^{x\ln(d)}}{1+\ln(d)^{2}}-\frac{\left(\ln(d)^{2}-1\right)e^{x\ln(d)}}{\left(1+\ln(d)^{2}\right)^{2}}-\frac{4\ln(d)e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(1+\ln(d)^{2}\right)^{2}}+\frac{2x\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^{2}}-\frac{\ln(d)x\,e^{x\ln(d)}(\mathrm{ta}}{1+\ln(d)^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$

input
```
int(d^x*x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
d^x*(ln(d)^3*cos(x)*x+(x*sin(x)-cos(x))*ln(d)^2+(x*cos(x)-2*sin(x))*ln(d)+
x*sin(x)+cos(x))/(1+ln(d)^2)^2
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int d^{x}x\cos(x)\,dx$$
$$=\frac{\left(x\cos\left(x\right)\log\left(d\right)^{3}+x\cos\left(x\right)\log\left(d\right)-\cos\left(x\right)\log\left(d\right)^{2}+\left(x\log\left(d\right)^{2}+x-2\,\log\left(d\right)\right)\sin\left(x\right)+\cos\left(x\right)\right)d^{x}}{\log\left(d\right)^{4}+2\,\log\left(d\right)^{2}+1}$$

input
```
integrate(d^x*x*cos(x),x, algorithm="fricas")
```

output
```
(x*cos(x)*log(d)^3 + x*cos(x)*log(d) - cos(x)*log(d)^2 + (x*log(d)^2 + x -
 2*log(d))*sin(x) + cos(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

3.137.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.66

$$\int d^x x \cos(x)\, dx$$
$$= \begin{cases} \frac{ix^2e^{-ix}\sin(x)}{4} + \frac{x^2e^{-ix}\cos(x)}{4} + \frac{xe^{-ix}\sin(x)}{4} + \frac{ixe^{-ix}\cos(x)}{4} + \frac{e^{-ix}\cos(x)}{4} \\ -\frac{ix^2e^{ix}\sin(x)}{4} + \frac{x^2e^{ix}\cos(x)}{4} + \frac{xe^{ix}\sin(x)}{4} - \frac{ixe^{ix}\cos(x)}{4} + \frac{e^{ix}\cos(x)}{4} \\ \frac{d^x x\log(d)^3\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\log(d)\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\sin(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1} - \frac{2d^x \mathrm{lo}}{\log(d)^4\cdots} \end{cases}$$

input
```
integrate(d**x*x*cos(x),x)
```

output
```
Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*
x)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I)))
, (-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/
4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log
(d)**3*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log
(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2
+ 1) + d**x*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)
/(log(d)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 + 2*log(d
)**2 + 1) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int d^x x \cos(x)\, dx = \frac{\left(\left(\log(d)^3 + \log(d)\right)x - \log(d)^2 + 1\right)d^x \cos(x) + \left(\left(\log(d)^2 + 1\right)x - 2\log(d)\right)d^x \sin(x)}{\log(d)^4 + 2\log(d)^2 + 1}$$

input
```
integrate(d^x*x*cos(x),x, algorithm="maxima")
```

output
```
(((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*cos(x) + ((log(d)^2 + 1)*x - 2
*log(d))*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

3.137.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1155, normalized size of antiderivative = 13.92

$$\int d^x x \cos(x)\, dx = \text{Too large to display}$$

input
```
integrate(d^x*x*cos(x),x, algorithm="giac")
```

output
```
1/2*(2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d))
 + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn
(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2)
 + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(x*log(
abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) -
 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos
(1/2*pi*x*sgn(d) - 1/2*pi*x + x) - ((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs
(d))^2 - 2*pi*sgn(d) - 2)*(pi*x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d)
- pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) -
 pi*log(abs(d)) + 2*log(abs(d)))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(ab
s(d)) + 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2
*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(ab
s(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x +
 1/2*(2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)
) - 2*log(abs(d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sg
n(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2
) - (2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(x*log
(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d)
+ 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*co
s(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((2*pi - pi^2*sgn(d) + pi^2 - 2*log...
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int d^x x \cos(x)\,dx$$
$$= \frac{d^x \left(\cos(x) - 2\ln(d)\sin(x) - \ln(d)^2\cos(x) + x\sin(x) + x\ln(d)\cos(x) + x\ln(d)^3\cos(x) + x\ln(d)^2 \cdots\right.}{\left(\ln(d)^2 + 1\right)^2}$$

input
```
int(d^x*x*cos(x),x)
```

output
```
(d^x*(cos(x) - 2*log(d)*sin(x) - log(d)^2*cos(x) + x*sin(x) + x*log(d)*cos
(x) + x*log(d)^3*cos(x) + x*log(d)^2*sin(x)))/(log(d)^2 + 1)^2
```

3.137.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int d^x x\cos(x)\,dx$$
$$=\frac{d^x\left(\cos\left(x\right)\log(d)^3x-\cos\left(x\right)\log(d)^2+\cos\left(x\right)\log(d)\,x+\cos\left(x\right)+\log(d)^2\sin\left(x\right)x-2\log(d)\sin\left(x\right)+\text{s}\right.}{\log\left(d\right)^4+2\log\left(d\right)^2+1}$$

input
```
int(d**x*cos(x)*x,x)
```

output
```
(d**x*(cos(x)*log(d)**3*x - cos(x)*log(d)**2 + cos(x)*log(d)*x + cos(x) +
log(d)**2*sin(x)*x - 2*log(d)*sin(x) + sin(x)*x))/(log(d)**4 + 2*log(d)**2
 + 1)
```

3.138 $\int d^x x^2 \sin(x)\,dx$

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3.138.9 Mupad [B] (verification not implemented) 888
3.138.10 Reduce [B] (verification not implemented) 888

3.138.1 Optimal result

Integrand size = 9, antiderivative size = 162

$$\int d^x x^2 \sin(x)\,dx = \frac{2d^x\cos(x)}{(1+\log^2(d))^3} - \frac{6d^x\cos(x)\log^2(d)}{(1+\log^2(d))^3} + \frac{4d^x x\cos(x)\log(d)}{(1+\log^2(d))^2} - \frac{d^x x^2\cos(x)}{1+\log^2(d)} - \frac{6d^x\log(d)\sin(x)}{(1+\log^2(d))^3} + \frac{2d^x\log^3(d)\sin(x)}{(1+\log^2(d))^3} + \frac{2d^x x\sin(x)}{(1+\log^2(d))^2} - \frac{2d^x x\log^2(d)\sin(x)}{(1+\log^2(d))^2} + \frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}$$

output
```
2*d^x*cos(x)/(1+ln(d)^2)^3-6*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+4*d^x*x*cos(
x)*ln(d)/(1+ln(d)^2)^2-d^x*x^2*cos(x)/(1+ln(d)^2)-6*d^x*ln(d)*sin(x)/(1+ln
(d)^2)^3+2*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+2*d^x*x*sin(x)/(1+ln(d)^2)^2-2
*d^x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int d^x x^2 \sin(x)\,dx$$
$$= \frac{d^x\left(-\cos(x)\left(-2+x^2-4x\log(d)+2(3+x^2)\log^2(d)-4x\log^3(d)+x^2\log^4(d)\right)+\left(2x+(-6+x^2)\log\right.\right.}{\left(1+\log^2(d)\right)^3}$$

input
```
Integrate[d^x*x^2*Sin[x],x]
```

output
```
(d^x*(-(Cos[x]*(-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[d]^
3 + x^2*Log[d]^4)) + (2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x
*Log[d]^4 + x^2*Log[d]^5)*Sin[x]))/(1 + Log[d]^2)^3
```

3.138.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4968, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 d^x \sin(x)\,dx$$
$$\Big\downarrow \text{4968}$$
$$-2\int -x\left(\frac{d^x\cos(x)}{\log^2(d)+1}-\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}\right)dx+\frac{x^2d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2d^x\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \text{25}$$
$$2\int x\left(\frac{d^x\cos(x)}{\log^2(d)+1}-\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}\right)dx+\frac{x^2d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2d^x\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \text{2010}$$
$$2\int\left(\frac{d^xx\cos(x)}{\log^2(d)+1}-\frac{d^xx\log(d)\sin(x)}{\log^2(d)+1}\right)dx+\frac{x^2d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2d^x\cos(x)}{\log^2(d)+1}$$

$\Big\downarrow$ 2009

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{x^2 d^x \cos(x)}{\log^2(d)+1} + 2\left(-\frac{x d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2} + \frac{x d^x \sin(x)}{\left(\log^2(d)+1\right)^2} - \frac{3 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3} + \frac{d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3} + \frac{2 x d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2} - \frac{3 d^x \log\right.$$

input
```
Int[d^x*x^2*Sin[x],x]
```

output
```
-((d^x*x^2*Cos[x])/(1 + Log[d]^2)) + (d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2
) + 2*((d^x*Cos[x])/(1 + Log[d]^2)^3 - (3*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]
^2)^3 + (2*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (3*d^x*Log[d]*Sin[x])/(
1 + Log[d]^2)^3 + (d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3 + (d^x*x*Sin[x])/
(1 + Log[d]^2)^2 - (d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2)
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.138.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{i\left(2+\ln(d)^2x^2+2i\ln(d)x^2-x^2-2x\ln(d)-2ix\right)d^x e^{ix}}{2(\ln(d)+i)^3}+\frac{i\left(2-2x\ln(d)+2ix+\ln(d)^2x^2-2i\ln(d)x^2-x^2\right)d^x e^{-ix}}{2(\ln(d)-i)^3}$
parallelrisch	$\frac{d^x\left(\ln(d)^5x^2\sin(x)+(-x^2\cos(x)-2x\sin(x))\ln(d)^4+(2x^2\sin(x)+4x\cos(x)+2\sin(x))\ln(d)^3+(-2x^2-6)\cos(x)\ln(d)^2+(x^2\right.}{\left(1+\ln(d)^2\right)^3}$
norman	$\frac{\frac{x^2e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}-\frac{x^2e^{x\ln(d)}}{1+\ln(d)^2}-\frac{2\left(3\ln(d)^2-1\right)e^{x\ln(d)}}{\left(1+\ln(d)^2\right)^3}+\frac{4\ln(d)x\,e^{x\ln(d)}}{\left(1+\ln(d)^2\right)^2}+\frac{2\left(3\ln(d)^2-1\right)e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^2\right)^3}-\frac{4\ln(d)x\,e^{x\ln(d)}(\tan}{\left(1+\ln(d)^2\right)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$

input
```
int(d^x*x^2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(2+ln(d)^2*x^2+2*I*ln(d)*x^2-x^2-2*x*ln(d)-2*I*x)*d^x/(ln(d)+I)^3*e
xp(I*x)+1/2*I*(2-2*x*ln(d)+2*I*x+ln(d)^2*x^2-2*I*ln(d)*x^2-x^2)*d^x/(ln(d)
-I)^3*exp(-I*x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.71

$$\int d^x x^2\sin(x)\,dx = -\frac{\left(x^2\cos\left(x\right)\log\left(d\right)^4-4\,x\cos\left(x\right)\log\left(d\right)^3+2\,\left(x^2+3\right)\cos\left(x\right)\log\left(d\right)^2-4\,x\cos\left(x\right)\log\left(d\right)+\left(x^2-2\right)\cos\right.}{\log\left(d\right)^6+3\,\log\left(d\right)^4+3}$$

input
```
integrate(d^x*x^2*sin(x),x, algorithm="fricas")
```

output
```
-(x^2*cos(x)*log(d)^4 - 4*x*cos(x)*log(d)^3 + 2*(x^2 + 3)*cos(x)*log(d)^2
- 4*x*cos(x)*log(d) + (x^2 - 2)*cos(x) - (x^2*log(d)^5 - 2*x*log(d)^4 + 2*
(x^2 + 1)*log(d)^3 + (x^2 - 6)*log(d) + 2*x)*sin(x))*d^x/(log(d)^6 + 3*log
(d)^4 + 3*log(d)^2 + 1)
```

3.138.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.10

$$\int d^x x^2 \sin(x)\, dx = \text{Too large to display}$$

input
```
integrate(d**x*x**2*sin(x),x)
```

output
```
Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 + I*x**2*ex
p(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*ex
p(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**3*exp(I*x)*sin
(x)/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x
)*cos(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)
/4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 +
 3*log(d)**2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 +
3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 +
 3*log(d)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4
+ 3*log(d)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1) - d**x*x**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 +
 1) - 2*d**x*x*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1
) + 4*d**x*x*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1)
+ 4*d**x*x*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d
**x*x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**
3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)**2*co
s(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*sin(x)/(l
og(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos(x)/(log(d)**6 + 3*l
og(d)**4 + 3*log(d)**2 + 1), True))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$\int d^x x^2 \sin(x)\, dx = -\frac{\left(\left(\log(d)^4 + 2\log(d)^2 + 1\right)x^2 - 4\left(\log(d)^3 + \log(d)\right)x + 6\log(d)^2 - 2\right)d^x \cos(x) - \left(\left(\log(d)^5 + 2\log\right.\right.}{\log(d)^6 + 3\log(d)^4 + 3\log(d)^2}$$

input
```
integrate(d^x*x^2*sin(x),x, algorithm="maxima")
```

output
```
-(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2
- 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(
log(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^
2 + 1)
```

3.138.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 2631, normalized size of antiderivative = 16.24

$$\int d^x x^2 \sin(x)\, dx = \text{Too large to display}$$

input
```
integrate(d^x*x^2*sin(x),x, algorithm="giac")
```

output
```
-1/2*(((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi
^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^
2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2
*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d
))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)
) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(a
bs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log
(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*l
og(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d))
- 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(
d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d)
)^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(
1/2*pi*x*sgn(d) - 1/2*pi*x + x) - (2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d)
)^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(ab
s(d))^2 - 3*pi*sgn(d) - 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d))
 + 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi
^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) ...
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int d^x x^2 \sin(x)\,dx$$
$$= \frac{d^x\,(2\,\cos\left(x\right) - x^2\,\cos\left(x\right) + 2\,x\,\sin\left(x\right)) + d^x\,\ln\left(d\right)^3\,(2\,\sin\left(x\right) + 2\,x^2\,\sin\left(x\right) + 4\,x\,\cos\left(x\right)) - d^x\,\ln\left(d\right)^2\,(6}{}$$

input
```
int(d^x*x^2*sin(x),x)
```

output
```
(d^x*(2*cos(x) - x^2*cos(x) + 2*x*sin(x)) + d^x*log(d)^3*(2*sin(x) + 2*x^2
*sin(x) + 4*x*cos(x)) - d^x*log(d)^2*(6*cos(x) + 2*x^2*cos(x)) + d^x*log(d
)*(x^2*sin(x) - 6*sin(x) + 4*x*cos(x)) - d^x*log(d)^4*(x^2*cos(x) + 2*x*si
n(x)) + d^x*x^2*log(d)^5*sin(x))/(log(d)^2 + 1)^3
```

3.138.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int d^x x^2 \sin(x)\,dx$$
$$= \frac{d^x\left(-\cos\left(x\right)\log(d)^4 x^2 + 4\cos\left(x\right)\log(d)^3 x - 2\cos\left(x\right)\log(d)^2 x^2 - 6\cos\left(x\right)\log(d)^2 + 4\cos\left(x\right)\log(d)\,x\right.}{}$$

input
```
int(d**x*sin(x)*x**2,x)
```

output
```
(d**x*( - cos(x)*log(d)**4*x**2 + 4*cos(x)*log(d)**3*x - 2*cos(x)*log(d)**
2*x**2 - 6*cos(x)*log(d)**2 + 4*cos(x)*log(d)*x - cos(x)*x**2 + 2*cos(x) +
 log(d)**5*sin(x)*x**2 - 2*log(d)**4*sin(x)*x + 2*log(d)**3*sin(x)*x**2 +
2*log(d)**3*sin(x) + log(d)*sin(x)*x**2 - 6*log(d)*sin(x) + 2*sin(x)*x))/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1)
```

3.139 $\int d^x x^2 \cos(x)\,dx$

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3.139.1 Optimal result

Integrand size = 9, antiderivative size = 161

$$\int d^x x^2 \cos(x)\,dx = -\frac{6d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^3}+\frac{2d^x\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3}+\frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)}-\frac{2d^x\sin(x)}{\left(1+\log^2(d)\right)^3}+\frac{6d^x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^3}-\frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\sin(x)}{1+\log^2(d)}$$

output
```
-6*d^x*cos(x)*ln(d)/(1+ln(d)^2)^3+2*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^3+2*d^x
*x*cos(x)/(1+ln(d)^2)^2-2*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x^2*cos(x
)*ln(d)/(1+ln(d)^2)-2*d^x*sin(x)/(1+ln(d)^2)^3+6*d^x*ln(d)^2*sin(x)/(1+ln(
d)^2)^3-4*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x^2*sin(x)/(1+ln(d)^2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\int d^x x^2 \cos(x)\,dx$$
$$= \frac{d^x\left(\cos(x)\left(2x+(-6+x^2)\log(d)+2(1+x^2)\log^3(d)-2x\log^4(d)+x^2\log^5(d)\right)+\left(-2+x^2-4x\log(d\right.\right.}{\left(1+\log^2(d)\right)^3}$$

input
```
Integrate[d^x*x^2*Cos[x],x]
```

output
```
(d^x*(Cos[x]*(2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*Log[d]^
4 + x^2*Log[d]^5) + (-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Lo
g[d]^3 + x^2*Log[d]^4)*Sin[x]))/(1 + Log[d]^2)^3
```

3.139.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4969, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 d^x \cos(x)\,dx$$
$$\Big\downarrow \text{4969}$$
$$-2\int x\left(\frac{\sin(x)d^x}{\log^2(d)+1}+\frac{\cos(x)\log(d)d^x}{\log^2(d)+1}\right)dx+\frac{x^2d^x\sin(x)}{\log^2(d)+1}+\frac{x^2d^x\log(d)\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \text{2010}$$
$$-2\int\left(\frac{x\sin(x)d^x}{\log^2(d)+1}+\frac{x\cos(x)\log(d)d^x}{\log^2(d)+1}\right)dx+\frac{x^2d^x\sin(x)}{\log^2(d)+1}+\frac{x^2d^x\log(d)\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1} + \frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1} - 2\left(\frac{2xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2} - \frac{3d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3} + \frac{d^x \sin(x)}{\left(\log^2(d)+1\right)^3} + \frac{xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2} - \frac{xd^x \cos(x)}{\left(\log^2(d)+1\right)^2} + \frac{3d^x \log(d)}{\left(\log^2(d)\right.}\right.$$

input
```
Int[d^x*x^2*Cos[x],x]
```

output
```
(d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*x^2*Sin[x])/(1 + Log[d]^2) -
 2*((3*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 - (d^x*Cos[x]*Log[d]^3)/(1 + Lo
g[d]^2)^3 - (d^x*x*Cos[x])/(1 + Log[d]^2)^2 + (d^x*x*Cos[x]*Log[d]^2)/(1 +
 Log[d]^2)^2 + (d^x*Sin[x])/(1 + Log[d]^2)^3 - (3*d^x*Log[d]^2*Sin[x])/(1
+ Log[d]^2)^3 + (2*d^x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^2)
```

3.139.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.139.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\left(2+\ln(d)^2x^2+2i\ln(d)x^2-x^2-2x\ln(d)-2ix\right)d^x\mathrm{e}^{ix}}{2(\ln(d)+i)^3}+\frac{\left(2-2x\ln(d)+2ix+\ln(d)^2x^2-2i\ln(d)x^2-x^2\right)d^x\mathrm{e}^{-ix}}{2(\ln(d)-i)^3}$
parallelrisch	$\frac{d^x\left(\ln(d)^5x^2\cos(x)+x(-2\cos(x)+x\sin(x))\ln(d)^4+(2x^2\cos(x)-4x\sin(x)+2\cos(x))\ln(d)^3+(2x^2+6)\sin(x)\ln(d)^2+(x^2\cos\right.}{\left(1+\ln(d)^2\right)^3}$
norman	$\frac{\frac{\ln(d)x^2\mathrm{e}^{x\ln(d)}}{1+\ln(d)^2}+\frac{2x^2\mathrm{e}^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}-\frac{2\left(\ln(d)^2-1\right)x\,\mathrm{e}^{x\ln(d)}}{\left(1+\ln(d)^2\right)^2}+\frac{4\left(3\ln(d)^2-1\right)\mathrm{e}^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(1+\ln(d)^2\right)^3}+\frac{2\ln(d)\left(\ln(d)^2-3\right)\mathrm{e}^{x\ln(d)}}{\left(1+\ln(d)^2\right)^3}-\frac{8\ln(d)x\,\mathrm{e}^{x}}{(1+}}{1+\tan^2\left(\frac{x}{2}\right)}$

input
```
int(d^x*x^2*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2+ln(d)^2*x^2+2*I*ln(d)*x^2-x^2-2*x*ln(d)-2*I*x)*d^x/(ln(d)+I)^3*exp(
I*x)+1/2*(2-2*x*ln(d)+2*I*x+ln(d)^2*x^2-2*I*ln(d)*x^2-x^2)*d^x/(ln(d)-I)^3
*exp(-I*x)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.69

$$\int d^x x^2\cos(x)\,dx$$
$$=\frac{\left(x^2\cos\left(x\right)\log\left(d\right)^5-2\,x\cos\left(x\right)\log\left(d\right)^4+2\left(x^2+1\right)\cos\left(x\right)\log\left(d\right)^3+\left(x^2-6\right)\cos\left(x\right)\log\left(d\right)+2\,x\cos\left(x\right.\right.}{\log\left(d\right)^6+3\,\log\left(d\right)^4+3\,\log}$$

input
```
integrate(d^x*x^2*cos(x),x, algorithm="fricas")
```

output
```
(x^2*cos(x)*log(d)^5 - 2*x*cos(x)*log(d)^4 + 2*(x^2 + 1)*cos(x)*log(d)^3 +
 (x^2 - 6)*cos(x)*log(d) + 2*x*cos(x) + (x^2*log(d)^4 - 4*x*log(d)^3 + 2*(
x^2 + 3)*log(d)^2 + x^2 - 4*x*log(d) - 2)*sin(x))*d^x/(log(d)^6 + 3*log(d)
^4 + 3*log(d)^2 + 1)
```

3.139.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 668, normalized size of antiderivative = 4.15

$$\int d^x x^2 \cos(x)\, dx = \text{Too large to display}$$

input
```
integrate(d**x*x**2*cos(x),x)
```

output
```
Piecewise((I*x**3*exp(-I*x)*sin(x)/6 + x**3*exp(-I*x)*cos(x)/6 + x**2*exp(
-I*x)*sin(x)/4 + I*x**2*exp(-I*x)*cos(x)/4 - I*x*exp(-I*x)*sin(x)/4 + x*ex
p(-I*x)*cos(x)/4 - I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (-I*x**3*exp(I*x
)*sin(x)/6 + x**3*exp(I*x)*cos(x)/6 + x**2*exp(I*x)*sin(x)/4 - I*x**2*exp(
I*x)*cos(x)/4 + I*x*exp(I*x)*sin(x)/4 + x*exp(I*x)*cos(x)/4 + I*exp(I*x)*c
os(x)/4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*cos(x)/(log(d)**6 + 3*log(d)
**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)*
*4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)
**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**2*sin(x)/(log(d)**6 + 3*log(d
)**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4
 + 3*log(d)**2 + 1) + d**x*x**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)
**2 + 1) - 2*d**x*x*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**
2 + 1) - 4*d**x*x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
+ 1) - 4*d**x*x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1)
+ 2*d**x*x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log
(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 6*d**x*log(d)*
*2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*cos(
x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*sin(x)/(log(d)**6
+ 3*log(d)**4 + 3*log(d)**2 + 1), True))
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int d^x x^2 \cos(x)\, dx = \frac{\left(\left(\log(d)^5 + 2\log(d)^3 + \log(d)\right)x^2 + 2\log(d)^3 - 2\left(\log(d)^4 - 1\right)x - 6\log(d)\right)d^x\cos(x) + \left(\left(\log(d)^4 + \right.\right.}{\log(d)^6 + 3\log(d)^4 + 3\log(d)^2 +}$$

input
```
integrate(d^x*x^2*cos(x),x, algorithm="maxima")
```

output
```
(((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x -
 6*log(d))*d^x*cos(x) + ((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + l
og(d))*x + 6*log(d)^2 - 2)*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2
 + 1)
```

3.139.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 2631, normalized size of antiderivative = 16.34

$$\int d^x x^2 \cos(x)\,dx = \text{Too large to display}$$

input
```
integrate(d^x*x^2*cos(x),x, algorithm="giac")
```

output
```
1/2*((2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(
abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(p
i*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d)) - pi*x*s
gn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^
3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*s
gn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs
(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2) +
(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi
*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*lo
g(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) -
 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d
) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))
^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1
/2*pi*x*sgn(d) - 1/2*pi*x + x) + ((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2
*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d
))^2 - 3*pi*sgn(d) - 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2
- 2*pi*x^2*sgn(d) + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*
sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sg
n(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(...
```

3.139.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int d^x x^2 \cos(x)\, dx$$
$$= \frac{d^x\,(x^2\,\sin(x) - 2\,\sin(x) + 2\,x\,\cos(x)) + d^x \ln(d)^3\,(2\,\cos(x) + 2\,x^2\,\cos(x) - 4\,x\,\sin(x)) + d^x \ln(d)^2\,(6}{}$$

input
```
int(d^x*x^2*cos(x),x)
```

output
```
(d^x*(x^2*sin(x) - 2*sin(x) + 2*x*cos(x)) + d^x*log(d)^3*(2*cos(x) + 2*x^2
*cos(x) - 4*x*sin(x)) + d^x*log(d)^2*(6*sin(x) + 2*x^2*sin(x)) - d^x*log(d
)*(6*cos(x) - x^2*cos(x) + 4*x*sin(x)) + d^x*log(d)^4*(x^2*sin(x) - 2*x*co
s(x)) + d^x*x^2*log(d)^5*cos(x))/(log(d)^2 + 1)^3
```

3.139.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int d^x x^2 \cos(x)\, dx$$
$$= \frac{d^x\left(\cos(x)\log(d)^5 x^2 - 2\cos(x)\log(d)^4 x + 2\cos(x)\log(d)^3 x^2 + 2\cos(x)\log(d)^3 + \cos(x)\log(d)\,x^2 - 6\right.}{}$$

input
```
int(d**x*cos(x)*x**2,x)
```

output
```
(d**x*(cos(x)*log(d)**5*x**2 - 2*cos(x)*log(d)**4*x + 2*cos(x)*log(d)**3*x
**2 + 2*cos(x)*log(d)**3 + cos(x)*log(d)*x**2 - 6*cos(x)*log(d) + 2*cos(x)
*x + log(d)**4*sin(x)*x**2 - 4*log(d)**3*sin(x)*x + 2*log(d)**2*sin(x)*x**
2 + 6*log(d)**2*sin(x) - 4*log(d)*sin(x)*x + sin(x)*x**2 - 2*sin(x)))/(log
(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1)
```

3.140 $\int d^x x^3 \sin(x)\,dx$

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3.140.1 Optimal result

Integrand size = 9, antiderivative size = 261

$$\begin{aligned}\int d^x x^3 \sin(x)\,dx = &-\frac{24d^x\cos(x)\log(d)}{(1+\log^2(d))^4}+\frac{24d^x\cos(x)\log^3(d)}{(1+\log^2(d))^4}+\frac{6d^x x\cos(x)}{(1+\log^2(d))^3}\\&-\frac{18d^x x\cos(x)\log^2(d)}{(1+\log^2(d))^3}+\frac{6d^x x^2\cos(x)\log(d)}{(1+\log^2(d))^2}\\&-\frac{d^x x^3\cos(x)}{1+\log^2(d)}-\frac{6d^x\sin(x)}{(1+\log^2(d))^4}+\frac{36d^x\log^2(d)\sin(x)}{(1+\log^2(d))^4}\\&-\frac{6d^x\log^4(d)\sin(x)}{(1+\log^2(d))^4}-\frac{18d^x x\log(d)\sin(x)}{(1+\log^2(d))^3}+\frac{6d^x x\log^3(d)\sin(x)}{(1+\log^2(d))^3}\\&+\frac{3d^x x^2\sin(x)}{(1+\log^2(d))^2}-\frac{3d^x x^2\log^2(d)\sin(x)}{(1+\log^2(d))^2}+\frac{d^x x^3\log(d)\sin(x)}{1+\log^2(d)}\end{aligned}$$

output

```
-24*d^x*cos(x)*ln(d)/(1+ln(d)^2)^4+24*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^4+6*d
^x*x*cos(x)/(1+ln(d)^2)^3-18*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+6*d^x*x^2*
cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x^3*cos(x)/(1+ln(d)^2)-6*d^x*sin(x)/(1+ln(d
)^2)^4+36*d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^4-6*d^x*ln(d)^4*sin(x)/(1+ln(d)^2
)^4-18*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^3+6*d^x*x*ln(d)^3*sin(x)/(1+ln(d)^2)
^3+3*d^x*x^2*sin(x)/(1+ln(d)^2)^2-3*d^x*x^2*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d
^x*x^3*ln(d)*sin(x)/(1+ln(d)^2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

$$\int d^x x^3 \sin(x)\,dx$$
$$= \frac{d^x\left(-\cos(x)\left(x(-6+x^2)-6(-4+x^2)\log(d)+3x(4+x^2)\log^2(d)-12(2+x^2)\log^3(d)+3x(6+x^2)\log\right.\right.}{}$$

input
```
Integrate[d^x*x^3*Sin[x],x]
```

output
```
(d^x*(-(Cos[x]*(x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^
2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*
Log[d]^6)) + (3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2
+ 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5
- 3*x^2*Log[d]^6 + x^3*Log[d]^7)*Sin[x]))/(1 + Log[d]^2)^4
```

3.140.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4968, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 d^x \sin(x)\,dx$$
$$\Big\downarrow \text{ 4968}$$
$$-3\int -x^2\left(\frac{d^x\cos(x)}{\log^2(d)+1}-\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}\right)dx+\frac{x^3d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3d^x\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \text{ 25}$$
$$3\int x^2\left(\frac{d^x\cos(x)}{\log^2(d)+1}-\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}\right)dx+\frac{x^3d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3d^x\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \text{ 2010}$$

$$3\int\left(\frac{d^x x^2\cos(x)}{\log^2(d)+1}-\frac{d^x x^2\log(d)\sin(x)}{\log^2(d)+1}\right)dx+\frac{x^3d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3d^x\cos(x)}{\log^2(d)+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3d^x\cos(x)}{\log^2(d)+1}+$$
$$3\left(\frac{x^2d^x\sin(x)}{(\log^2(d)+1)^2}-\frac{x^2d^x\log^2(d)\sin(x)}{(\log^2(d)+1)^2}+\frac{2x^2d^x\log(d)\cos(x)}{(\log^2(d)+1)^2}-\frac{6xd^x\log(d)\sin(x)}{(\log^2(d)+1)^3}+\frac{12d^x\log^2(d)\sin(x)}{(\log^2(d)+1)^4}-\frac{2}{(\log}\right.$$

input
```
Int[d^x*x^3*Sin[x],x]
```

output
```
-((d^x*x^3*Cos[x])/(1 + Log[d]^2)) + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2
) + 3*((-8*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (8*d^x*Cos[x]*Log[d]^3)/(
1 + Log[d]^2)^4 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*x*Cos[x]*Log[
d]^2)/(1 + Log[d]^2)^3 + (2*d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (2*d
^x*Sin[x])/(1 + Log[d]^2)^4 + (12*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^4 -
(2*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Log[d]*Sin[x])/(1 + Lo
g[d]^2)^3 + (2*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3 + (d^x*x^2*Sin[x])/
(1 + Log[d]^2)^2 - (d^x*x^2*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2)
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.140.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{i\left(-6+\ln(d)^3x^3+3i\ln(d)^2x^3-3\ln(d)x^3-ix^3+6x\ln(d)+6ix-3\ln(d)^2x^2-6i\ln(d)x^2+3x^2\right)d^x e^{ix}}{2(\ln(d)+i)^4}+\frac{i\left(-6+6x\ln(d)-6ix\right.}{}$
parallelrisch	$\frac{d^x\left(\ln(d)^7x^3\sin(x)+(-x^3\cos(x)-3x^2\sin(x))\ln(d)^6+3((x^3+2x)\sin(x)+2x^2\cos(x))\ln(d)^5+3(-x^3\cos(x)-x^2\sin(x)-6x\cos\right.}{}$
norman	$\frac{\frac{x^3e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}-\frac{x^3e^{x\ln(d)}}{1+\ln(d)^2}+\frac{6\ln(d)x^2e^{x\ln(d)}}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(\ln(d)^2-1\right)x^2e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(3\ln(d)^2-1\right)xe^{x\ln(d)}}{\left(1+\ln(d)^2\right)\left(\ln(d)^4+2\ln(d)^2+1\right)}-\frac{12\left(\ln(d\right.}{\left(\ln(d)^6+\right.}}{}$

input
```
int(d^x*x^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3
*ln(d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*I*(-6+6*x*l
n(d)-6*I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3
*ln(d)*x^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.78

$$\int d^x x^3 \sin(x)\,dx =$$
$$-\frac{\left(x^3\cos\left(x\right)\log\left(d\right)^6-6\,x^2\cos\left(x\right)\log\left(d\right)^5+3\,\left(x^3+6\,x\right)\cos\left(x\right)\log\left(d\right)^4-12\,\left(x^2+2\right)\cos\left(x\right)\log\left(d\right)^3+\right.}{}$$

input
```
integrate(d^x*x^3*sin(x),x, algorithm="fricas")
```

output
```
-(x^3*cos(x)*log(d)^6 - 6*x^2*cos(x)*log(d)^5 + 3*(x^3 + 6*x)*cos(x)*log(d
)^4 - 12*(x^2 + 2)*cos(x)*log(d)^3 + 3*(x^3 + 4*x)*cos(x)*log(d)^2 - 6*(x^
2 - 4)*cos(x)*log(d) + (x^3 - 6*x)*cos(x) - (x^3*log(d)^7 - 3*x^2*log(d)^6
 + 3*(x^3 + 2*x)*log(d)^5 - 3*(x^2 + 2)*log(d)^4 + 3*(x^3 - 4*x)*log(d)^3
+ 3*(x^2 + 12)*log(d)^2 + 3*x^2 + (x^3 - 18*x)*log(d) - 6)*sin(x))*d^x/(lo
g(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

3.140.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 1355, normalized size of antiderivative = 5.19

$$\int d^x x^3 \sin(x)\,dx = \text{Too large to display}$$

input
```
integrate(d**x*x**3*sin(x),x)
```

output
```
Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 + I*x**3*ex
p(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(-I*x)*sin(x)/8 + 3
*I*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*sin(x)/8 + 3*x*exp(-I*x)*cos(
x)/8 - 3*I*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (x**4*exp(I*x)*sin(x)/8 +
I*x**4*exp(I*x)*cos(x)/8 - I*x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)
/4 + 3*x**2*exp(I*x)*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x
)*sin(x)/8 + 3*x*exp(I*x)*cos(x)/8 + 3*I*exp(I*x)*cos(x)/8, Eq(d, exp(I)))
, (d**x*x**3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*l
og(d)**2 + 1) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*lo
g(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*sin(x)/(log(d)**8 + 4*l
og(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**4*cos(x)/(
log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log
(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) -
3*d**x*x**3*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*lo
g(d)**2 + 1) + d**x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log
(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*sin(x)/(log(d)**8 + 4*lo
g(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)**5*cos(x)/(l
og(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(
d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) ...
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.71

$$\int d^x x^3 \sin(x)\,dx =$$

$$-\frac{\left(\left(\log(d)^6 + 3\log(d)^4 + 3\log(d)^2 + 1\right)x^3 - 6\left(\log(d)^5 + 2\log(d)^3 + \log(d)\right)x^2 - 24\log(d)^3 + 6\left(3\right.}{}$$

input
```
integrate(d^x*x^3*sin(x),x, algorithm="maxima")
```

output
```
-(((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3
 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^2 - 1)*x + 24*log(
d))*d^x*cos(x) - ((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*lo
g(d)^4 - 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(
d)^3 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 +
 6*log(d)^4 + 4*log(d)^2 + 1)
```

3.140.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 5069, normalized size of antiderivative = 19.42

$$\int d^x x^3 \sin(x) \, dx = \text{Too large to display}$$

input
```
integrate(d^x*x^3*sin(x),x, algorithm="giac")
```

output
```
1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*lo
g(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d)
 + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^
2 - 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi
^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*l
og(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 -
6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x
 - 12*x)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^
2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sg
n(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(
d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^
3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d)
 + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*l
og(abs(d)) + 2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2
*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*p
i*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6
*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d))
 - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d
)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2
*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs...
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.89

$$\int d^x x^3 \sin(x)\,dx =$$

$$-\frac{d^x\,(6\,\sin(x)+x^3\,\cos(x)-3\,x^2\,\sin(x)-6\,x\,\cos(x))-d^x\,\ln(d)^5\,(6\,x^2\,\cos(x)+3\,x^3\,\sin(x)+6\,x\,\sin$$

input
```
int(d^x*x^3*sin(x),x)
```

output
```
-(d^x*(6*sin(x) + x^3*cos(x) - 3*x^2*sin(x) - 6*x*cos(x)) - d^x*log(d)^5*(
6*x^2*cos(x) + 3*x^3*sin(x) + 6*x*sin(x)) + d^x*log(d)^4*(6*sin(x) + 3*x^3
*cos(x) + 3*x^2*sin(x) + 18*x*cos(x)) - d^x*log(d)^3*(24*cos(x) + 12*x^2*c
os(x) + 3*x^3*sin(x) - 12*x*sin(x)) - d^x*log(d)^2*(36*sin(x) - 3*x^3*cos(
x) + 3*x^2*sin(x) - 12*x*cos(x)) + d^x*log(d)^6*(x^3*cos(x) + 3*x^2*sin(x)
) + d^x*log(d)*(24*cos(x) - 6*x^2*cos(x) - x^3*sin(x) + 18*x*sin(x)) - d^x
*x^3*log(d)^7*sin(x))/(log(d)^2 + 1)^4
```

3.140.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.01

$$\int d^x x^3 \sin(x)\,dx$$

$$= \frac{d^x\left(-6\sin\left(x\right)+24\cos\left(x\right)\log(d)^3-6\log(d)^4\sin\left(x\right)-\cos\left(x\right)x^3-\cos\left(x\right)\log(d)^6x^3+\log(d)^7\sin\left(x\right)x^3}{}$$

input
```
int(d**x*sin(x)*x**3,x)
```

output
```
(d**x*( - cos(x)*log(d)**6*x**3 + 6*cos(x)*log(d)**5*x**2 - 3*cos(x)*log(d
)**4*x**3 - 18*cos(x)*log(d)**4*x + 12*cos(x)*log(d)**3*x**2 + 24*cos(x)*l
og(d)**3 - 3*cos(x)*log(d)**2*x**3 - 12*cos(x)*log(d)**2*x + 6*cos(x)*log(
d)*x**2 - 24*cos(x)*log(d) - cos(x)*x**3 + 6*cos(x)*x + log(d)**7*sin(x)*x
**3 - 3*log(d)**6*sin(x)*x**2 + 3*log(d)**5*sin(x)*x**3 + 6*log(d)**5*sin(
x)*x - 3*log(d)**4*sin(x)*x**2 - 6*log(d)**4*sin(x) + 3*log(d)**3*sin(x)*x
**3 - 12*log(d)**3*sin(x)*x + 3*log(d)**2*sin(x)*x**2 + 36*log(d)**2*sin(x
) + log(d)*sin(x)*x**3 - 18*log(d)*sin(x)*x + 3*sin(x)*x**2 - 6*sin(x)))/(
log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
```

3.141 $\int d^x x^3 \cos(x) \, dx$

3.141.1 Optimal result . . . 904
3.141.2 Mathematica [A] (verified) . . . 905
3.141.3 Rubi [A] (verified) . . . 905
3.141.4 Maple [C] (verified) . . . 907
3.141.5 Fricas [A] (verification not implemented) . . . 907
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3.141.7 Maxima [A] (verification not implemented) . . . 908
3.141.8 Giac [C] (verification not implemented) . . . 909
3.141.9 Mupad [B] (verification not implemented) . . . 910
3.141.10Reduce [B] (verification not implemented) . . . 911

3.141.1 Optimal result

Integrand size = 9, antiderivative size = 260

$$\begin{aligned}\int d^x x^3 \cos(x) \, dx = &-\frac{6d^x \cos(x)}{(1+\log^2(d))^4} + \frac{36d^x \cos(x) \log^2(d)}{(1+\log^2(d))^4} - \frac{6d^x \cos(x) \log^4(d)}{(1+\log^2(d))^4} \\ &- \frac{18d^x x \cos(x) \log(d)}{(1+\log^2(d))^3} + \frac{6d^x x \cos(x) \log^3(d)}{(1+\log^2(d))^3} \\ &+ \frac{3d^x x^2 \cos(x)}{(1+\log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1+\log^2(d))^2} + \frac{d^x x^3 \cos(x) \log(d)}{1+\log^2(d)} \\ &+ \frac{24d^x \log(d) \sin(x)}{(1+\log^2(d))^4} - \frac{24d^x \log^3(d) \sin(x)}{(1+\log^2(d))^4} - \frac{6d^x x \sin(x)}{(1+\log^2(d))^3} \\ &+ \frac{18d^x x \log^2(d) \sin(x)}{(1+\log^2(d))^3} - \frac{6d^x x^2 \log(d) \sin(x)}{(1+\log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)}\end{aligned}$$

output
```
-6*d^x*cos(x)/(1+ln(d)^2)^4+36*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^4-6*d^x*cos(
x)*ln(d)^4/(1+ln(d)^2)^4-18*d^x*x*cos(x)*ln(d)/(1+ln(d)^2)^3+6*d^x*x*cos(x
)*ln(d)^3/(1+ln(d)^2)^3+3*d^x*x^2*cos(x)/(1+ln(d)^2)^2-3*d^x*x^2*cos(x)*ln
(d)^2/(1+ln(d)^2)^2+d^x*x^3*cos(x)*ln(d)/(1+ln(d)^2)+24*d^x*ln(d)*sin(x)/(
1+ln(d)^2)^4-24*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^4-6*d^x*x*sin(x)/(1+ln(d)^2
)^3+18*d^x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^3-6*d^x*x^2*ln(d)*sin(x)/(1+ln(d)^
2)^2+d^x*x^3*sin(x)/(1+ln(d)^2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.65

$$\int d^x x^3 \cos(x)\,dx$$
$$= \frac{d^x \left(\cos(x) \left(3(-2+x^2) + x(-18+x^2)\log(d) + 3(12+x^2)\log^2(d) + 3x(-4+x^2)\log^3(d) - 3(2+x^2)\log\right.\right.}{}$$

input
```
Integrate[d^x*x^3*Cos[x],x]
```

output
```
(d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2
+ 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5
- 3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3
*x*(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6
*x^2*Log[d]^5 + x^3*Log[d]^6)*Sin[x]))/(1 + Log[d]^2)^4
```

3.141.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4969, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 d^x \cos(x)\,dx$$
$$\Big\downarrow \; 4969$$
$$-3\int x^2\left(\frac{\sin(x)d^x}{\log^2(d)+1} + \frac{\cos(x)\log(d)d^x}{\log^2(d)+1}\right)dx + \frac{x^3 d^x \sin(x)}{\log^2(d)+1} + \frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \; 2010$$
$$-3\int \left(\frac{x^2\sin(x)d^x}{\log^2(d)+1} + \frac{x^2\cos(x)\log(d)d^x}{\log^2(d)+1}\right)dx + \frac{x^3 d^x \sin(x)}{\log^2(d)+1} + \frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}$$
$$\Big\downarrow \; 2009$$

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1} + \frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1} -$$
$$3\left(\frac{2x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2} + \frac{x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2} - \frac{x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2} - \frac{6x d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3} + \frac{2x d^x \sin(x)}{\left(\log^2(d)+1\right)^3} - \frac{8d^x \log}{\left(\log^2(\right.}\right.$$

input
```
Int[d^x*x^3*Cos[x],x]
```

output
```
(d^x*x^3*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*x^3*Sin[x])/(1 + Log[d]^2) -
 3*((2*d^x*Cos[x])/(1 + Log[d]^2)^4 - (12*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]
^2)^4 + (2*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 + (6*d^x*x*Cos[x]*Log[d])
/(1 + Log[d]^2)^3 - (2*d^x*x*Cos[x]*Log[d]^3)/(1 + Log[d]^2)^3 - (d^x*x^2*
Cos[x])/(1 + Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 - (8
*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^4 + (8*d^x*Log[d]^3*Sin[x])/(1 + Log[d]
^2)^4 + (2*d^x*x*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x*Log[d]^2*Sin[x])/(1 +
 Log[d]^2)^3 + (2*d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)^2)
```

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.141.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\left(-6+\ln(d)^3x^3+3i\ln(d)^2x^3-3\ln(d)x^3-ix^3+6x\ln(d)+6ix-3\ln(d)^2x^2-6i\ln(d)x^2+3x^2\right)d^x e^{ix}}{2(\ln(d)+i)^4}+\frac{\left(-6+6x\ln(d)-6ix-3\ln\right.}{}$
parallelrisch	$\frac{d^x\left(\ln(d)^7x^3\cos(x)+x^2(x\sin(x)-3\cos(x))\ln(d)^6+3\left(\left(x^3+2x\right)\cos(x)-2x^2\sin(x)\right)\ln(d)^5+3\left(x^3\sin(x)-x^2\cos(x)+6x\sin(x)\right)\right.}{}$
norman	$\frac{\frac{\ln(d)x^3e^{x\ln(d)}}{1+\ln(d)^2}+\frac{2x^3e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}-\frac{3\left(\ln(d)^2-1\right)x^2e^{x\ln(d)}}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(\ln(d)^4-6\ln(d)^2+1\right)e^{x\ln(d)}}{\left(\ln(d)^6+3\ln(d)^4+3\ln(d)^2+1\right)\left(1+\ln(d)^2\right)}+\frac{3\left(\ln(d)^2-1\right)x^2e^{x\ln(d)}\left(\mathrm{ta}\right.}{\ln(d)^4+2\ln(d)^2+1}}{}$

input
```
int(d^x*x^3*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3*ln
(d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*(-6+6*x*ln(d)-
6*I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3*ln(d
)*x^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.78

$$\int d^x x^3 \cos(x)\,dx$$
$$= \frac{\left(x^3\cos\left(x\right)\log\left(d\right)^7-3\,x^2\cos\left(x\right)\log\left(d\right)^6+3\,\left(x^3+2\,x\right)\cos\left(x\right)\log\left(d\right)^5-3\,\left(x^2+2\right)\cos\left(x\right)\log\left(d\right)^4+3\,(x\right.}{}$$

input
```
integrate(d^x*x^3*cos(x),x, algorithm="fricas")
```

output
```
(x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(x)*log(d)
^5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*log(d)^3 + 3*(x^2
+ 12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log(d) + 3*(x^2 - 2)*cos(x) +
(x^3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log
(d)^3 + x^3 + 3*(x^3 + 4*x)*log(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d
^x/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

3.141.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 1352, normalized size of antiderivative = 5.20

$$\int d^x x^3 \cos(x)\,dx = \text{Too large to display}$$

input
```
integrate(d**x*x**3*cos(x),x)
```

output
```
Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 + x**3*exp(
-I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*exp(-I*x)*sin(x)/8 +
 3*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(x)/8 - 3*I*x*exp(-I*x)*cos(
x)/8 - 3*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (-I*x**4*exp(I*x)*sin(x)/8 +
 x**4*exp(I*x)*cos(x)/8 + x**3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/
4 + 3*I*x**2*exp(I*x)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*s
in(x)/8 + 3*I*x*exp(I*x)*cos(x)/8 - 3*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (
d**x*x**3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(
d)**2 + 1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d
)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*cos(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**4*sin(x)/(log
(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)
**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d
**x*x**3*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d
)**2 + 1) + d**x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
 + 4*log(d)**2 + 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*cos(x)/(log(d)**8 + 4*log(d
)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)**5*sin(x)/(log(
d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)*
*4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 1...
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.71

$$\int d^x x^3 \cos(x)\,dx$$
$$= \frac{\left(\left(\log(d)^7 + 3\log(d)^5 + 3\log(d)^3 + \log(d)\right)x^3 - 6\log(d)^4 - 3\left(\log(d)^6 + \log(d)^4 - \log(d)^2 - 1\right)x^2 +\right.}{}$$

input
```
integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

output
```
(((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(
d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))
*x + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
1)*x^3 - 6*(log(d)^5 + 2*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d
)^4 + 2*log(d)^2 - 1)*x + 24*log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 +
6*log(d)^4 + 4*log(d)^2 + 1)
```

3.141.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 5065, normalized size of antiderivative = 19.48

$$\int d^x x^3 \cos(x)\, dx = \text{Too large to display}$$

input
```
integrate(d^x*x^3*cos(x),x, algorithm="giac")
```

output
```
-1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*l
og(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d
) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))
^2 - 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(
d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs
(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(
d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi
 + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2
 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 -
 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sg
n(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3
*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(a
bs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*
log(abs(d)))^2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^
3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*lo
g(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6
*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x
- 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(
d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) -
2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(ab...
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.89

$$\int d^x x^3 \cos(x)\, dx =$$

$$-\frac{d^x\,(6\,\cos(x) - 3\,x^2\,\cos(x) - x^3\,\sin(x) + 6\,x\,\sin(x)) - d^x\ln(d)^5\,(3\,x^3\,\cos(x) - 6\,x^2\,\sin(x) + 6\,x\,\mathrm{co}}{}$$

input
```
int(d^x*x^3*cos(x),x)
```

output
```
-(d^x*(6*cos(x) - 3*x^2*cos(x) - x^3*sin(x) + 6*x*sin(x)) - d^x*log(d)^5*(
3*x^3*cos(x) - 6*x^2*sin(x) + 6*x*cos(x)) + d^x*log(d)^4*(6*cos(x) + 3*x^2
*cos(x) - 3*x^3*sin(x) - 18*x*sin(x)) + d^x*log(d)^3*(24*sin(x) - 3*x^3*co
s(x) + 12*x^2*sin(x) + 12*x*cos(x)) - d^x*log(d)^2*(36*cos(x) + 3*x^2*cos(
x) + 3*x^3*sin(x) + 12*x*sin(x)) + d^x*log(d)^6*(3*x^2*cos(x) - x^3*sin(x)
) - d^x*log(d)*(24*sin(x) + x^3*cos(x) - 6*x^2*sin(x) - 18*x*cos(x)) - d^x
*x^3*log(d)^7*cos(x))/(log(d)^2 + 1)^4
```

3.141.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01

$$\int d^x x^3 \cos(x) \, dx$$
$$= \frac{d^x \left(-6\cos(x) + 18\log(d)^4 \sin(x)\, x - 12\log(d)^3 \sin(x)\, x^2 + 3\cos(x)\log(d)^2 x^2 + \cos(x)\log(d)^7 x^3 + \cos\right.}{}$$

input
```
int(d**x*cos(x)*x**3,x)
```

output
```
(d**x*(cos(x)*log(d)**7*x**3 - 3*cos(x)*log(d)**6*x**2 + 3*cos(x)*log(d)**
5*x**3 + 6*cos(x)*log(d)**5*x - 3*cos(x)*log(d)**4*x**2 - 6*cos(x)*log(d)*
*4 + 3*cos(x)*log(d)**3*x**3 - 12*cos(x)*log(d)**3*x + 3*cos(x)*log(d)**2*
x**2 + 36*cos(x)*log(d)**2 + cos(x)*log(d)*x**3 - 18*cos(x)*log(d)*x + 3*c
os(x)*x**2 - 6*cos(x) + log(d)**6*sin(x)*x**3 - 6*log(d)**5*sin(x)*x**2 +
3*log(d)**4*sin(x)*x**3 + 18*log(d)**4*sin(x)*x - 12*log(d)**3*sin(x)*x**2
 - 24*log(d)**3*sin(x) + 3*log(d)**2*sin(x)*x**3 + 12*log(d)**2*sin(x)*x -
 6*log(d)*sin(x)*x**2 + 24*log(d)*sin(x) + sin(x)*x**3 - 6*sin(x)*x))/(log
(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
```

3.142 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

3.142.1 Optimal result 912
3.142.2 Mathematica [A] (verified) 912
3.142.3 Rubi [A] (verified) 913
3.142.4 Maple [A] (verified) 914
3.142.5 Fricas [A] (verification not implemented) 914
3.142.6 Sympy [B] (verification not implemented) 915
3.142.7 Maxima [A] (verification not implemented) 915
3.142.8 Giac [A] (verification not implemented) 916
3.142.9 Mupad [B] (verification not implemented) 916
3.142.10Reduce [B] (verification not implemented) 916

3.142.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

output
```
-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

3.142.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

input
```
Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

3.142.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(2x)\sin(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\sin(2x)\sin(3x)dx$$

$$\Big\downarrow \text{ 4855}$$

$$\int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

input
```
Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

3.142.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.142.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
parallelrisch	$-\frac{3}{16}+\frac{\cos(6x)}{24}-\frac{\cos(4x)}{16}-\frac{\cos(2x)}{8}$	21

input
```
int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

3.142.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(19) = 38$.

Time = 1.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{5\sin(x)\sin(2x)\cos(3x)}{24} - \frac{\sin(2x)\sin(3x)\cos(x)}{8} - \frac{\cos(x)\cos(2x)\cos(3x)}{6}$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

output
```
x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*c
os(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*
x)/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{4}{3}\sin{(x)}^6 + \frac{3}{2}\sin{(x)}^4$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
-4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{\sin{(x)}^4\,(8\sin{(x)}^2 - 9)}{6}$$

input
```
int(sin(2*x)*sin(3*x)*sin(x),x)
```

output
```
-(sin(x)^4*(8*sin(x)^2 - 9))/6
```

3.142.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx = &-\frac{\cos{(3x)}\cos{(2x)}\cos{(x)}}{24} + \frac{\cos{(3x)}\cos{(2x)}\sin{(x)}\,x}{4} \\ &+ \frac{\cos{(3x)}\cos{(x)}\sin{(2x)}\,x}{4} \\ &- \frac{\cos{(3x)}\sin{(2x)}\sin{(x)}}{3} - \frac{\cos{(2x)}\cos{(x)}\sin{(3x)}\,x}{4} \\ &+ \frac{\cos{(2x)}\sin{(3x)}\sin{(x)}}{8} + \frac{\sin{(3x)}\sin{(2x)}\sin{(x)}\,x}{4}\end{aligned}$$

input
```
int(sin(3*x)*sin(2*x)*sin(x),x)
```

output
```
( - cos(3*x)*cos(2*x)*cos(x) + 6*cos(3*x)*cos(2*x)*sin(x)*x + 6*cos(3*x)*c
os(x)*sin(2*x)*x - 8*cos(3*x)*sin(2*x)*sin(x) - 6*cos(2*x)*cos(x)*sin(3*x)
*x + 3*cos(2*x)*sin(3*x)*sin(x) + 6*sin(3*x)*sin(2*x)*sin(x)*x)/24
```

3.142. $\int \sin(x)\sin(2x)\sin(3x)\,dx$

3.143 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

3.143.1 Optimal result 918
3.143.2 Mathematica [A] (verified) 918
3.143.3 Rubi [A] (verified) 919
3.143.4 Maple [A] (verified) 920
3.143.5 Fricas [A] (verification not implemented) 920
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3.143.8 Giac [A] (verification not implemented) 922
3.143.9 Mupad [B] (verification not implemented) 922
3.143.10Reduce [B] (verification not implemented) 922

3.143.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sin(2x) + \frac{1}{16}\sin(4x) + \frac{1}{24}\sin(6x)$$

output
```
1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

3.143.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sin(2x) + \frac{1}{16}\sin(4x) + \frac{1}{24}\sin(6x)$$

input
```
Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

3.143.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2x)\cos(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cos(2x)\cos(3x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

input
```
Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.143.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
parallelrisch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23

input
```
int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{12}\left(16\cos(x)^5 - 10\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{4}x$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

output
```
1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

3.143.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.

Time = 1.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.80

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = -\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} - \frac{\sin(x)\cos(2x)\cos(3x)}{24} - \frac{\sin(2x)\cos(x)\cos(3x)}{6} + \frac{3\sin(3x)\cos(x)\cos(2x)}{8}$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

output
```
-x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*
sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 - sin(x)*cos(2*x)*cos(3*x
)/24 - sin(2*x)*cos(x)*cos(3*x)/6 + 3*sin(3*x)*cos(x)*cos(2*x)/8
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

output
```
1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

output
```
1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} + \frac{\sin\left(4\,x\right)}{16} + \frac{\sin\left(6\,x\right)}{24}$$

input
```
int(cos(2*x)*cos(3*x)*cos(x),x)
```

output
```
x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

3.143.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx = {} & \frac{\cos\left(3x\right)\cos\left(2x\right)\cos\left(x\right)x}{4} + \frac{\cos\left(3x\right)\cos\left(2x\right)\sin\left(x\right)}{3} \\ & + \frac{5\cos\left(3x\right)\cos\left(x\right)\sin\left(2x\right)}{24} \\ & - \frac{\cos\left(3x\right)\sin\left(2x\right)\sin\left(x\right)x}{4} + \frac{\cos\left(2x\right)\sin\left(3x\right)\sin\left(x\right)x}{4} \\ & + \frac{\cos\left(x\right)\sin\left(3x\right)\sin\left(2x\right)x}{4} + \frac{3\sin\left(3x\right)\sin\left(2x\right)\sin\left(x\right)}{8}\end{aligned}$$

input
```
int(cos(3*x)*cos(2*x)*cos(x),x)
```

output
```
(6*cos(3*x)*cos(2*x)*cos(x)*x + 8*cos(3*x)*cos(2*x)*sin(x) + 5*cos(3*x)*co
s(x)*sin(2*x) - 6*cos(3*x)*sin(2*x)*sin(x)*x + 6*cos(2*x)*sin(3*x)*sin(x)*
x + 6*cos(x)*sin(3*x)*sin(2*x)*x + 9*sin(3*x)*sin(2*x)*sin(x))/24
```

3.143. $\int \cos(x)\cos(2x)\cos(3x)\,dx$

3.144 $\int x^2 \sin^3(kx)\,dx$

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3.144.2 Mathematica [A] (verified) 924
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3.144.10 Reduce [B] (verification not implemented) 930

3.144.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int x^2 \sin^3(kx)\,dx = \frac{14\cos(kx)}{9k^3} - \frac{2x^2\cos(kx)}{3k} - \frac{2\cos^3(kx)}{27k^3} + \frac{4x\sin(kx)}{3k^2} - \frac{x^2\cos(kx)\sin^2(kx)}{3k} + \frac{2x\sin^3(kx)}{9k^2}$$

output
```
14/9*cos(k*x)/k^3-2/3*x^2*cos(k*x)/k-2/27*cos(k*x)^3/k^3+4/3*x*sin(k*x)/k^
2-1/3*x^2*cos(k*x)*sin(k*x)^2/k+2/9*x*sin(k*x)^3/k^2
```

3.144.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int x^2 \sin^3(kx)\,dx = \frac{-81(-2+k^2x^2)\cos(kx) + (-2+9k^2x^2)\cos(3kx) - 6kx(-27\sin(kx)+\sin(3kx))}{108k^3}$$

input
```
Integrate[x^2*Sin[k*x]^3,x]
```

output
```
(-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*x*(-27*Si
n[k*x] + Sin[3*k*x]))/(108*k^3)
```

3.144.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^3(kx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin(kx)^3 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{2 \int \sin^3(kx) dx}{9k^2}+\frac{2}{3} \int x^2 \sin(kx) dx+\frac{2x \sin^3(kx)}{9k^2}-\frac{x^2 \sin^2(kx) \cos(kx)}{3k}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2 \int \sin(kx)^3 dx}{9k^2}+\frac{2}{3} \int x^2 \sin(kx) dx+\frac{2x \sin^3(kx)}{9k^2}-\frac{x^2 \sin^2(kx) \cos(kx)}{3k}$$

$$\Big\downarrow \text{3113}$$

$$\frac{2 \int \left(1-\cos^2(kx)\right) d\cos(kx)}{9k^3}+\frac{2}{3} \int x^2 \sin(kx) dx+\frac{2x \sin^3(kx)}{9k^2}-\frac{x^2 \sin^2(kx) \cos(kx)}{3k}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3} \int x^2 \sin(kx) dx+\frac{2\left(\cos(kx)-\frac{1}{3} \cos^3(kx)\right)}{9k^3}+\frac{2x \sin^3(kx)}{9k^2}-\frac{x^2 \sin^2(kx) \cos(kx)}{3k}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3} \left(\frac{2 \int x \cos(kx) dx}{k}-\frac{x^2 \cos(kx)}{k}\right)+\frac{2\left(\cos(kx)-\frac{1}{3} \cos^3(kx)\right)}{9k^3}+\frac{2x \sin^3(kx)}{9k^2}-\frac{x^2 \sin^2(kx) \cos(kx)}{3k}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{2\int x\sin\left(kx+\frac{\pi}{2}\right)dx}{k}-\frac{x^2\cos(kx)}{k}\right)+\frac{2\left(\cos(kx)-\frac{1}{3}\cos^3(kx)\right)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

$\big\downarrow$ 3777

$$\frac{2}{3}\left(\frac{2\left(\frac{\int -\sin(kx)dx}{k}+\frac{x\sin(kx)}{k}\right)}{k}-\frac{x^2\cos(kx)}{k}\right)+\frac{2\left(\cos(kx)-\frac{1}{3}\cos^3(kx)\right)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

$\big\downarrow$ 25

$$\frac{2}{3}\left(\frac{2\left(\frac{x\sin(kx)}{k}-\frac{\int \sin(kx)dx}{k}\right)}{k}-\frac{x^2\cos(kx)}{k}\right)+\frac{2\left(\cos(kx)-\frac{1}{3}\cos^3(kx)\right)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

$\big\downarrow$ 3042

$$\frac{2}{3}\left(\frac{2\left(\frac{x\sin(kx)}{k}-\frac{\int \sin(kx)dx}{k}\right)}{k}-\frac{x^2\cos(kx)}{k}\right)+\frac{2\left(\cos(kx)-\frac{1}{3}\cos^3(kx)\right)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

$\big\downarrow$ 3118

$$\frac{2\left(\cos(kx)-\frac{1}{3}\cos^3(kx)\right)}{9k^3}+\frac{2}{3}\left(\frac{2\left(\frac{\cos(kx)}{k^2}+\frac{x\sin(kx)}{k}\right)}{k}-\frac{x^2\cos(kx)}{k}\right)+\frac{2x\sin^3(kx)}{9k^2}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

input
```
Int[x^2*Sin[k*x]^3,x]
```

output
```
(2*(Cos[k*x] - Cos[k*x]^3/3))/(9*k^3) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k) +
(2*x*Sin[k*x]^3)/(9*k^2) + (2*(-((x^2*Cos[k*x])/k) + (2*(Cos[k*x]/k^2 + (x
*Sin[k*x])/k))/k))/3
```

3.144. $\int x^2\sin^3(kx)\,dx$

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.144.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{3\left(x^{2} k^{2}-2\right) \cos\left(k x\right)}{4 k^{3}}+\frac{3 x \sin\left(k x\right)}{2 k^{2}}+\frac{\left(9 x^{2} k^{2}-2\right) \cos\left(3 k x\right)}{108 k^{3}}-\frac{x \sin\left(3 k x\right)}{18 k^{2}}$
derivativedivides	$\frac{-\frac{k^{2} x^{2} \left(2+\sin^{2}\left(k x\right)\right) \cos\left(k x\right)}{3}+\frac{4 \cos\left(k x\right)}{3}+\frac{4 k x \sin\left(k x\right)}{3}+\frac{2 k x \left(\sin^{3}\left(k x\right)\right)}{9}+\frac{2 \left(2+\sin^{2}\left(k x\right)\right) \cos\left(k x\right)}{27}}{k^{3}}$
default	$\frac{-\frac{k^{2} x^{2} \left(2+\sin^{2}\left(k x\right)\right) \cos\left(k x\right)}{3}+\frac{4 \cos\left(k x\right)}{3}+\frac{4 k x \sin\left(k x\right)}{3}+\frac{2 k x \left(\sin^{3}\left(k x\right)\right)}{9}+\frac{2 \left(2+\sin^{2}\left(k x\right)\right) \cos\left(k x\right)}{27}}{k^{3}}$
norman	$\frac{-\frac{2 x^{2}}{3 k}+\frac{80}{27 k^{3}}+\frac{8 x \tan\left(\frac{k x}{2}\right)}{3 k^{2}}+\frac{64 x \left(\tan^{3}\left(\frac{k x}{2}\right)\right)}{9 k^{2}}+\frac{8 x \left(\tan^{5}\left(\frac{k x}{2}\right)\right)}{3 k^{2}}-\frac{2 x^{2} \left(\tan^{2}\left(\frac{k x}{2}\right)\right)}{k}+\frac{2 x^{2} \left(\tan^{4}\left(\frac{k x}{2}\right)\right)}{k}+\frac{2 x^{2} \left(\tan^{6}\left(\frac{k x}{2}\right)\right)}{3 k}+8(t}{\left(1+\tan^{2}\left(\frac{k x}{2}\right)\right)^{3}}$

input
```
int(x^2*sin(k*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(k^2*x^2-2)/k^3*cos(k*x)+3/2*x*sin(k*x)/k^2+1/108*(9*k^2*x^2-2)/k^3*c
os(3*k*x)-1/18*x/k^2*sin(3*k*x)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int x^2 \sin^3(kx)\,dx = \frac{\left(9\,k^{2} x^{2} - 2\right) \cos\left(k x\right)^{3} - 3 \,\left(9\,k^{2} x^{2} - 14\right) \cos\left(k x\right) - 6 \,\left(k x \cos\left(k x\right)^{2} - 7\,k x\right) \sin\left(k x\right)}{27\,k^{3}}$$

input
```
integrate(x^2*sin(k*x)^3,x, algorithm="fricas")
```

output
```
1/27*((9*k^2*x^2 - 2)*cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*cos(k*x) - 6*(k*x*co
s(k*x)^2 - 7*k*x)*sin(k*x))/k^3
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int x^2 \sin^3(kx)\,dx$$
$$= \begin{cases} -\frac{x^2\sin^2(kx)\cos(kx)}{k} - \frac{2x^2\cos^3(kx)}{3k} + \frac{14x\sin^3(kx)}{9k^2} + \frac{4x\sin(kx)\cos^2(kx)}{3k^2} + \frac{14\sin^2(kx)\cos(kx)}{9k^3} + \frac{40\cos^3(kx)}{27k^3} & \text{for } k \neq 0 \\ 0 & \text{otherwis} \end{cases}$$

input
```
integrate(x**2*sin(k*x)**3,x)
```

output
```
Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k) + 14*x*
sin(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2) + 14*sin(k*x)**2*
cos(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), Ne(k, 0)), (0, True))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int x^2 \sin^3(kx)\,dx$$
$$= -\frac{6\,kx\sin(3\,kx) - 162\,kx\sin(kx) - (9\,k^2x^2 - 2)\cos(3\,kx) + 81\,(k^2x^2 - 2)\cos(kx)}{108\,k^3}$$

input
```
integrate(x^2*sin(k*x)^3,x, algorithm="maxima")
```

output
```
-1/108*(6*k*x*sin(3*k*x) - 162*k*x*sin(k*x) - (9*k^2*x^2 - 2)*cos(3*k*x) +
 81*(k^2*x^2 - 2)*cos(k*x))/k^3
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int x^2 \sin^3(kx)\,dx = -\frac{x \sin(3\,kx)}{18\,k^2} + \frac{3\,x \sin(kx)}{2\,k^2} + \frac{(9\,k^2x^2 - 2)\cos(3\,kx)}{108\,k^3} - \frac{3\,(k^2x^2 - 2)\cos(kx)}{4\,k^3}$$

input
```
integrate(x^2*sin(k*x)^3,x, algorithm="giac")
```

output
```
-1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 - 2)*cos(3*
k*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int x^2 \sin^3(kx)\,dx = \frac{\frac{14\,\cos(k\,x)}{9} - \frac{2\cos(k\,x)^3}{27} + k\left(\frac{14\,x\,\sin(k\,x)}{9} - \frac{2\,x\cos(k\,x)^2\,\sin(k\,x)}{9}\right) + k^2\left(\frac{x^2\cos(k\,x)^3}{3} - x^2\,\cos(k\,x)\right)}{k^3}$$

input
```
int(x^2*sin(k*x)^3,x)
```

output
```
((14*cos(k*x))/9 - (2*cos(k*x)^3)/27 + k*((14*x*sin(k*x))/9 - (2*x*cos(k*x
)^2*sin(k*x))/9) + k^2*((x^2*cos(k*x)^3)/3 - x^2*cos(k*x)))/k^3
```

3.144.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^2 \sin^3(kx)\,dx = \frac{-9\cos(kx)\sin(kx)^2 k^2x^2 + 2\cos(kx)\sin(kx)^2 - 18\cos(kx)\,k^2x^2 + 40\cos(kx) + 6\sin(kx)^3 kx + 36\,\mathrm{si}}{27k^3}$$

input
```
int(sin(k*x)**3*x**2,x)
```

output
```
( - 9*cos(k*x)*sin(k*x)**2*k**2*x**2 + 2*cos(k*x)*sin(k*x)**2 - 18*cos(k*x
)*k**2*x**2 + 40*cos(k*x) + 6*sin(k*x)**3*k*x + 36*sin(k*x)*k*x + 16)/(27*
k**3)
```

3.144. $\int x^2 \sin^3(kx)\,dx$

3.145 $\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$

3.145.1 Optimal result 932
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3.145.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx = \text{Int}(x \cos(k \csc(x)) \cot(x) \csc(x), x)$$

output
```
CannotIntegrate(x*cos(k*csc(x))*cot(x)*csc(x),x)
```

3.145.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx = \int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

input
```
Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

output
```
Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

3.145.3 Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot(x)\csc(x)\cos(k\csc(x))\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int x\cot(x)\csc(x)\cos(k\csc(x))dx$$

input
```
Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

output
```
$Aborted
```

3.145.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.145.4 Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2}dx$$

input
```
int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

output
```
int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

3.145.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx = \int \frac{x\cos\left(x\right)\cos\left(\frac{k}{\sin(x)}\right)}{\sin\left(x\right)^{2}}\,dx$$

input
```
integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")
```

output
```
integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)
```

3.145.6 Sympy [N/A]

Not integrable

Time = 53.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx = \int \frac{x\cos\left(x\right)\cos\left(\frac{k}{\sin\left(x\right)}\right)}{\sin^{2}\left(x\right)}\,dx$$

input
```
integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)
```

output
```
Integral(x*cos(x)*cos(k/sin(x))/sin(x)**2, x)
```

3.145.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 21.82

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx =$$

$$-\frac{\left(xe^{\left(\frac{4\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^{2}+\sin(2\,x)^{2}-2\,\cos(2\,x)+1}+\frac{4\,k\sin(2\,x)\sin(x)}{\cos(2\,x)^{2}+\sin(2\,x)^{2}-2\,\cos(2\,x)+1}\right)}+xe^{\left(\frac{4\,k\cos(x)}{\cos(2\,x)^{2}+\sin(2\,x)^{2}-2\,\cos(2\,x)+1}\right)}\right)e^{\left(-\frac{2\,k\cos(2\,x)\,\mathrm{c}}{\cos(2\,x)^{2}+\sin(2\,x)^{2}-}\right.}}{2\,k}$$

input
```
integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")
```

output
```
-1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
+ 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4
*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)))*e^(-2*k*cos(2*x)*co
s(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos
(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)
^2 - 2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin
(x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k
```

3.145.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx = \int \frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2}\,dx$$

input
```
integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="giac")
```

output
```
integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2, x)
```

3.145.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx = \int \frac{x\cos\left(\frac{k}{\sin(x)}\right)\cos(x)}{\sin(x)^2}\,dx$$

input
```
int((x*cos(k/sin(x))*cos(x))/sin(x)^2,x)
```

output
```
int((x*cos(k/sin(x))*cos(x))/sin(x)^2, x)
```

3.145.10 Reduce [F]

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx=\int\frac{\cos\left(\frac{k}{\sin(x)}\right)\cos(x)\,x}{\sin(x)^2}dx$$

input
```
int((cos(k/sin(x))*cos(x)*x)/sin(x)**2,x)
```

output
```
int((cos(k/sin(x))*cos(x)*x)/sin(x)**2,x)
```

3.146 $\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx$

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3.146.9 Mupad [B] (verification not implemented) . . . 941
3.146.10Reduce [B] (verification not implemented) . . . 942

3.146.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = -x-\cot\left(\frac{x}{2}\right)$$

output
```
-x-cot(1/2*x)
```

3.146.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = -x-\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[Cot[x/2]*Cot[x],x]
```

output
```
-x - Cot[x/2]
```

3.146.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4889, 27, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cot\left(\frac{x}{2}\right)\cot(x)dx$$

$$\Big\downarrow \text{ 4889}$$

$$2\int \frac{\cot^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}{2\left(\tan^2\left(\frac{x}{2}\right)+1\right)}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\left(1-\tan^2\left(\frac{x}{2}\right)\right)\cot^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 359}$$

$$-2\int \frac{1}{\tan^2\left(\frac{x}{2}\right)+1}d\tan\left(\frac{x}{2}\right)-\cot\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)$$

input
```
Int[Cot[x/2]*Cot[x],x]
```

output
```
-2*ArcTan[Tan[x/2]] - Cot[x/2]
```

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.146.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$-x-\cot\left(\frac{x}{2}\right)$	11
norman	$\frac{-1-x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$	17
risch	$-x-\frac{2i}{{\mathrm e}^{i x}-1}$	17
parallelrisch	$\frac{-1-x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$	17

input
```
int(cos(x)/sin(x)/tan(1/2*x),x,method=_RETURNVERBOSE)
```

output
```
-x-cot(1/2*x)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = -\frac{x\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="fricas")
```

output
```
-(x*tan(1/2*x) + 1)/tan(1/2*x)
```

3.146.6 Sympy [F]

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = \int \frac{\cos(x)}{\sin(x)\tan\left(\frac{x}{2}\right)}\,dx$$

input
```
integrate(cos(x)/sin(x)/tan(1/2*x),x)
```

output
```
Integral(cos(x)/(sin(x)*tan(x/2)), x)
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(10) = 20.

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = -\frac{x\cos(x)^2+x\sin(x)^2-2\,x\cos(x)+x+2\,\sin(x)}{\cos(x)^2+\sin(x)^2-2\,\cos(x)+1}$$

input
```
integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="maxima")
```

output
```
-(x*cos(x)^2 + x*sin(x)^2 - 2*x*cos(x) + x + 2*sin(x))/(cos(x)^2 + sin(x)^
2 - 2*cos(x) + 1)
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = -x - \frac{1}{2\tan\left(\frac{1}{4}x\right)} + \frac{1}{2}\tan\left(\frac{1}{4}x\right)$$

input
```
integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="giac")
```

output
```
-x - 1/2/tan(1/4*x) + 1/2*tan(1/4*x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = -x - \cot\left(\frac{x}{2}\right)$$

input
```
int(cos(x)/(tan(x/2)*sin(x)),x)
```

output
```
- x - cot(x/2)
```

3.146.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx = \frac{-\tan\left(\frac{x}{2}\right)x-1}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(cos(x)/(sin(x)*tan(x/2)),x)
```

output
```
( - (tan(x/2)*x + 1))/tan(x/2)
```

3.147 $\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$

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3.147.2 Mathematica [A] (verified) 943
3.147.3 Rubi [A] (verified) 944
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3.147.8 Giac [A] (verification not implemented) 948
3.147.9 Mupad [B] (verification not implemented) 948
3.147.10Reduce [B] (verification not implemented) 949

3.147.1 Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx=-\frac{2c\arctan\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(b+c\sin(ax)\right)}$$

output
```
-2*c*arctan((c+b*tan(1/2*a*x))/(b^2-c^2)^(1/2))/a/(b^2-c^2)^(3/2)-b*cos(a*
x)/a/(b^2-c^2)/(b+c*sin(a*x))
```

3.147.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx=-\frac{\frac{2c\arctan\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{b\cos(ax)}{(b-c)(b+c)(b+c\sin(ax))}}{a}$$

input
```
Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

output
```
-(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) +
(b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)
```

3.147.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(ax)}{(c\sin(ax)+b)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(ax)}{(c\sin(ax)+b)^2}dx$$
$$\Big\downarrow \; 3233$$
$$-\frac{\int \frac{c}{b+c\sin(ax)}dx}{b^2-c^2}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$
$$\Big\downarrow \; 27$$
$$-\frac{c\int \frac{1}{b+c\sin(ax)}dx}{b^2-c^2}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$
$$\Big\downarrow \; 3042$$
$$-\frac{c\int \frac{1}{b+c\sin(ax)}dx}{b^2-c^2}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$
$$\Big\downarrow \; 3139$$
$$-\frac{2c\int \frac{1}{b\tan^2\left(\frac{ax}{2}\right)+2c\tan\left(\frac{ax}{2}\right)+b}d\tan\left(\frac{ax}{2}\right)}{a\left(b^2-c^2\right)}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$
$$\Big\downarrow \; 1083$$
$$\frac{4c\int \frac{1}{-\left(2c+2b\tan\left(\frac{ax}{2}\right)\right)^2-4\left(b^2-c^2\right)}d\left(2c+2b\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$
$$\Big\downarrow \; 217$$
$$-\frac{2c\arctan\left(\frac{2b\tan\left(\frac{ax}{2}\right)+2c}{2\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$

input
```
Int[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

output
```
(-2*c*ArcTan[(2*c + 2*b*Tan[(a*x)/2])/(2*Sqrt[b^2 - c^2])])/(a*(b^2 - c^2)
^(3/2)) - (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.147.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{\frac{-8c\tan\left(\frac{ax}{2}\right)-8b}{\left(4b^{2}-4c^{2}\right)\left(b\left(\tan^{2}\left(\frac{ax}{2}\right)\right)+2c\tan\left(\frac{ax}{2}\right)+b\right)}-\frac{8c\arctan\left(\frac{2b\tan\left(\frac{ax}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(4b^{2}-4c^{2}\right)\sqrt{b^{2}-c^{2}}}}{a}$	107
default	$\frac{\frac{-8c\tan\left(\frac{ax}{2}\right)-8b}{\left(4b^{2}-4c^{2}\right)\left(b\left(\tan^{2}\left(\frac{ax}{2}\right)\right)+2c\tan\left(\frac{ax}{2}\right)+b\right)}-\frac{8c\arctan\left(\frac{2b\tan\left(\frac{ax}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(4b^{2}-4c^{2}\right)\sqrt{b^{2}-c^{2}}}}{a}$	107
risch	$\frac{2ib\left(c-ib\,e^{iax}\right)}{c\left(-b^{2}+c^{2}\right)a\left(c\,e^{2iax}-c+2ib\,e^{iax}\right)}-\frac{ic\ln\left(e^{iax}+\frac{i\left(b\sqrt{b^{2}-c^{2}}+b^{2}-c^{2}\right)}{\sqrt{b^{2}-c^{2}}\,c}\right)}{\sqrt{b^{2}-c^{2}}\,(b+c)(b-c)a}+\frac{ic\ln\left(e^{iax}+\frac{i\left(b\sqrt{b^{2}-c^{2}}-b^{2}+c^{2}\right)}{\sqrt{b^{2}-c^{2}}\,c}\right)}{\sqrt{b^{2}-c^{2}}\,(b+c)(b-c)a}$	213

input
```
int(sin(a*x)/(b+c*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(4*(-2*c*tan(1/2*a*x)-2*b)/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2
*a*x)+b)-8*c/(4*b^2-4*c^2)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+2*
c)/(b^2-c^2)^(1/2)))
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.05

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$$
$$= \left[\frac{(c^2\sin(ax)+bc)\sqrt{-b^2+c^2}\log\left(\frac{(2b^2-c^2)\cos(ax)^2-2bc\sin(ax)-b^2-c^2+2(b\cos(ax)\sin(ax)+c\cos(ax))\sqrt{-b^2+c^2}}{c^2\cos(ax)^2-2bc\sin(ax)-b^2-c^2}\right)-2(b^3}{2(ab^5-2ab^3c^2+abc^4+(ab^4c-2ab^2c^3+ac^5)\sin(ax))}\right.$$

input
```
integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="fricas")
```

output
```
[1/2*((c^2*sin(a*x) + b*c)*sqrt(-b^2 + c^2)*log(((2*b^2 - c^2)*cos(a*x)^2
- 2*b*c*sin(a*x) - b^2 - c^2 + 2*(b*cos(a*x)*sin(a*x) + c*cos(a*x))*sqrt(-
b^2 + c^2))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)) - 2*(b^3 - b*c^
2)*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c
^5)*sin(a*x)), ((c^2*sin(a*x) + b*c)*sqrt(b^2 - c^2)*arctan(-(b*sin(a*x) +
 c)/(sqrt(b^2 - c^2)*cos(a*x))) - (b^3 - b*c^2)*cos(a*x))/(a*b^5 - 2*a*b^3
*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x))]
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(ax)}{(b + c\sin(ax))^2} \, dx = \text{Timed out}$$

input
```
integrate(sin(a*x)/(b+c*sin(a*x))**2,x)
```

output
```
Timed out
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(ax)}{(b + c\sin(ax))^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ax}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ax\right)+c}{\sqrt{b^2-c^2}}\right)\right)c}{(b^2-c^2)^{\frac{3}{2}}}+\frac{c\tan\left(\frac{1}{2}ax\right)+b}{\left(b\tan\left(\frac{1}{2}ax\right)^2+2c\tan\left(\frac{1}{2}ax\right)+b\right)(b^2-c^2)}\right)}{a}$$

input
```
integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*a*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*a*x) + c)/sqrt(
b^2 - c^2)))*c/(b^2 - c^2)^(3/2) + (c*tan(1/2*a*x) + b)/((b*tan(1/2*a*x)^2
 + 2*c*tan(1/2*a*x) + b)*(b^2 - c^2)))/a
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.73

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx = -\frac{\frac{2b}{b^2-c^2}+\frac{2c\tan\left(\frac{ax}{2}\right)}{b^2-c^2}}{a\left(b\tan\left(\frac{ax}{2}\right)^2+2c\tan\left(\frac{ax}{2}\right)+b\right)} - \frac{2c\,\mathrm{atan}\left(\frac{\left(\frac{2c^2}{(b+c)^{3/2}(b-c)^{3/2}}+\frac{2bc\tan\left(\frac{ax}{2}\right)}{(b+c)^{3/2}(b-c)^{3/2}}\right)(b^2-c^2)}{2c}\right)}{a\,(b+c)^{3/2}\,(b-c)^{3/2}}$$

input
```
int(sin(a*x)/(b + c*sin(a*x))^2,x)
```

output
```
- ((2*b)/(b^2 - c^2) + (2*c*tan((a*x)/2))/(b^2 - c^2))/(a*(b + 2*c*tan((a*
x)/2) + b*tan((a*x)/2)^2)) - (2*c*atan((((2*c^2)/((b + c)^(3/2)*(b - c)^(3
/2)) + (2*b*c*tan((a*x)/2))/((b + c)^(3/2)*(b - c)^(3/2)))*(b^2 - c^2))/(2
*c)))/(a*(b + c)^(3/2)*(b - c)^(3/2))
```

3.147.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$$
$$= \frac{-2\sqrt{b^2-c^2}\,atan\left(\frac{\tan\left(\frac{ax}{2}\right)b+c}{\sqrt{b^2-c^2}}\right)\sin(ax)\,c^2-2\sqrt{b^2-c^2}\,atan\left(\frac{\tan\left(\frac{ax}{2}\right)b+c}{\sqrt{b^2-c^2}}\right)bc-\cos(ax)\,b^3+\cos(ax)\,b\,c^2}{a\left(\sin(ax)\,b^4c-2\sin(ax)\,b^2c^3+\sin(ax)\,c^5+b^5-2b^3c^2+b\,c^4\right)}$$

input
```
int(sin(a*x)/(sin(a*x)**2*c**2 + 2*sin(a*x)*b*c + b**2),x)
```

output
```
( - 2*sqrt(b**2 - c**2)*atan((tan((a*x)/2)*b + c)/sqrt(b**2 - c**2))*sin(a
*x)*c**2 - 2*sqrt(b**2 - c**2)*atan((tan((a*x)/2)*b + c)/sqrt(b**2 - c**2)
)*b*c - cos(a*x)*b**3 + cos(a*x)*b*c**2)/(a*(sin(a*x)*b**4*c - 2*sin(a*x)*
b**2*c**3 + sin(a*x)*c**5 + b**5 - 2*b**3*c**2 + b*c**4))
```

3.148 $\int \sin(\log(x))\,dx$

3.148.1 Optimal result 950
3.148.2 Mathematica [A] (verified) 950
3.148.3 Rubi [A] (verified) 951
3.148.4 Maple [A] (verified) 952
3.148.5 Fricas [A] (verification not implemented) 952
3.148.6 Sympy [A] (verification not implemented) 953
3.148.7 Maxima [A] (verification not implemented) 953
3.148.8 Giac [A] (verification not implemented) 953
3.148.9 Mupad [B] (verification not implemented) 954
3.148.10Reduce [B] (verification not implemented) 954

3.148.1 Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

output
```
-1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

3.148.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

input
```
Integrate[Sin[Log[x]],x]
```

output
```
-1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

3.148.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4978}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\log(x))\,dx$$

$$\Big\downarrow \; 4978$$

$$\frac{1}{2}x\sin(\log(x))-\frac{1}{2}x\cos(\log(x))$$

input
```
Int[Sin[Log[x]],x]
```

output
```
-1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

3.148.3.1 Defintions of rubi rules used

rule 4978
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

3.148.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$-\frac{x(\cos(\ln(x))-\sin(\ln(x)))}{2}$	13
lookup	$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(-\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22
norman	$\frac{x\tan\left(\frac{\ln(x)}{2}\right)-\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{\ln(x)}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{\ln(x)}{2}\right)}$	34

input
```
int(sin(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(cos(ln(x))-sin(ln(x)))
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x))\,dx = -\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(sin(log(x)),x, algorithm="fricas")
```

output
```
-1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sin(\log(x))\,dx = \frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

input
```
integrate(sin(ln(x)),x)
```

output
```
x*sin(log(x))/2 - x*cos(log(x))/2
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \sin(\log(x))\,dx = -\frac{1}{2}\,x(\cos(\log(x)) - \sin(\log(x)))$$

input
```
integrate(sin(log(x)),x, algorithm="maxima")
```

output
```
-1/2*x*(cos(log(x)) - sin(log(x)))
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x))\,dx = -\frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

input
```
integrate(sin(log(x)),x, algorithm="giac")
```

output
```
-1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x))\,dx = -\frac{\sqrt{2}\,x\,\cos\left(\frac{\pi}{4}+\ln(x)\right)}{2}$$

input
```
int(sin(log(x)),x)
```

output
```
-(2^(1/2)*x*cos(pi/4 + log(x)))/2
```

3.148.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \sin(\log(x))\,dx = \frac{x(-\cos(\log(x))+\sin(\log(x)))}{2}$$

input
```
int(sin(log(x)),x)
```

output
```
(x*( - cos(log(x)) + sin(log(x))))/2
```

3.149 $\int \cos(\log(x))\,dx$

3.149.1 Optimal result 955
3.149.2 Mathematica [A] (verified) 955
3.149.3 Rubi [A] (verified) 956
3.149.4 Maple [A] (verified) 956
3.149.5 Fricas [A] (verification not implemented) 957
3.149.6 Sympy [A] (verification not implemented) 957
3.149.7 Maxima [A] (verification not implemented) 957
3.149.8 Giac [A] (verification not implemented) 958
3.149.9 Mupad [B] (verification not implemented) 958
3.149.10Reduce [B] (verification not implemented) 958

3.149.1 Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

output
```
1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

3.149.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

input
```
Integrate[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

3.149.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\log(x))\,dx$$

$$\Big\downarrow \; 4979$$

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

input
```
Int[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

3.149.3.1 Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

3.149.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$	11
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

input
```
int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(cos(ln(x))+sin(ln(x)))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(cos(log(x)),x, algorithm="fricas")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x))\,dx = \frac{x\sin\left(\log\left(x\right)\right)}{2} + \frac{x\cos\left(\log\left(x\right)\right)}{2}$$

input
```
integrate(cos(ln(x)),x)
```

output
```
x*sin(log(x))/2 + x*cos(log(x))/2
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x(\cos\left(\log\left(x\right)\right) + \sin\left(\log\left(x\right)\right))$$

input
```
integrate(cos(log(x)),x, algorithm="maxima")
```

output
```
1/2*x*(cos(log(x)) + sin(log(x)))
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(cos(log(x)),x, algorithm="giac")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4} + \ln\left(x\right)\right)}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(2^(1/2)*x*sin(pi/4 + log(x)))/2
```

3.149.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{x(\cos\left(\log(x)\right) + \sin\left(\log(x)\right))}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(x*(cos(log(x)) + sin(log(x))))/2
```

3.150 $\int e^x \, dx$

3.150.1 Optimal result 959
3.150.2 Mathematica [A] (verified) 959
3.150.3 Rubi [A] (verified) 960
3.150.4 Maple [A] (verified) 961
3.150.5 Fricas [A] (verification not implemented) 961
3.150.6 Sympy [A] (verification not implemented) 962
3.150.7 Maxima [A] (verification not implemented) 962
3.150.8 Giac [A] (verification not implemented) 962
3.150.9 Mupad [B] (verification not implemented) 963
3.150.10 Reduce [B] (verification not implemented) 963

3.150.1 Optimal result

Integrand size = 3, antiderivative size = 3

$$\int e^x \, dx = e^x$$

output
```
exp(x)
```

3.150.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x \, dx = e^x$$

input
```
Integrate[E^x,x]
```

output
```
E^x
```

3.150.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \, dx$$

$$\Big\downarrow \text{2624}$$

$$e^x$$

input
```
Int[E^x,x]
```

output
```
E^x
```

3.150.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.150.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

method	result	size
gosper	e^x	3
lookup	e^x	3
derivativedivides	e^x	3
default	e^x	3
norman	e^x	3
risch	e^x	3
parallelrisch	e^x	3
meijerg	$-1+e^x$	5

input
```
int(exp(x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx=e^x$$

input
```
integrate(exp(x),x, algorithm="fricas")
```

output
```
e^x
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

input
```
integrate(exp(x),x)
```

output
```
exp(x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

input
```
integrate(exp(x),x, algorithm="maxima")
```

output
```
e^x
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

input
```
integrate(exp(x),x, algorithm="giac")
```

output
```
e^x
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = \mathrm{e}^x$$

input
```
int(exp(x),x)
```

output
```
exp(x)
```

3.150.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x\,dx = e^x$$

input
```
int(e**x,x)
```

output
```
e**x
```

3.151 $\int a^x\,dx$

3.151.1 Optimal result 964
3.151.2 Mathematica [A] (verified) 964
3.151.3 Rubi [A] (verified) 965
3.151.4 Maple [A] (verified) 966
3.151.5 Fricas [A] (verification not implemented) 966
3.151.6 Sympy [A] (verification not implemented) 967
3.151.7 Maxima [A] (verification not implemented) 967
3.151.8 Giac [A] (verification not implemented) 967
3.151.9 Mupad [B] (verification not implemented) 968
3.151.10 Reduce [B] (verification not implemented) 968

3.151.1 Optimal result

Integrand size = 3, antiderivative size = 8

$$\int a^x\,dx=\frac{a^x}{\log(a)}$$

output
```
a^x/ln(a)
```

3.151.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx=\frac{a^x}{\log(a)}$$

input
```
Integrate[a^x,x]
```

output
```
a^x/Log[a]
```

3.151.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x\,dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{a^x}{\log(a)}$$

input
```
Int[a^x,x]
```

output
```
a^x/Log[a]
```

3.151.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.151.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
gosper	$\frac{a^{x}}{\ln(a)}$	9
derivativedivides	$\frac{a^{x}}{\ln(a)}$	9
default	$\frac{a^{x}}{\ln(a)}$	9
risch	$\frac{a^{x}}{\ln(a)}$	9
parallelrisch	$\frac{a^{x}}{\ln(a)}$	9
norman	$\frac{e^{x\ln(a)}}{\ln(a)}$	11
meijerg	$-\frac{1-e^{x\ln(a)}}{\ln(a)}$	16

input
```
int(a^x,x,method=_RETURNVERBOSE)
```

output
```
a^x/ln(a)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^{x}\,dx = \frac{a^{x}}{\log(a)}$$

input
```
integrate(a^x,x, algorithm="fricas")
```

output
```
a^x/log(a)
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(a**x,x)
```

output
```
Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

input
```
integrate(a^x,x, algorithm="maxima")
```

output
```
a^x/log(a)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

input
```
integrate(a^x,x, algorithm="giac")
```

output
```
a^x/log(a)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\ln(a)}$$

input
```
int(a^x,x)
```

output
```
a^x/log(a)
```

3.151.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\log(a)}$$

input
```
int(a**x,x)
```

output
```
a**x/log(a)
```

3.152 $\int e^{ax} dx$

3.152.1 Optimal result 969
3.152.2 Mathematica [A] (verified) 969
3.152.3 Rubi [A] (verified) 970
3.152.4 Maple [A] (verified) 971
3.152.5 Fricas [A] (verification not implemented) 971
3.152.6 Sympy [A] (verification not implemented) 972
3.152.7 Maxima [A] (verification not implemented) 972
3.152.8 Giac [A] (verification not implemented) 972
3.152.9 Mupad [B] (verification not implemented) 973
3.152.10 Reduce [B] (verification not implemented) 973

3.152.1 Optimal result

Integrand size = 5, antiderivative size = 9

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

output
```
exp(a*x)/a
```

3.152.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

input
```
Integrate[E^(a*x),x]
```

output
```
E^(a*x)/a
```

3.152.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{2624\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{ax}\,dx$$

$$\Big\downarrow \; 2624$$

$$\frac{e^{ax}}{a}$$

input
```
Int[E^(a*x),x]
```

output
```
E^(a*x)/a
```

3.152.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.152.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{ax}}{a}$	9
derivativedivides	$\frac{e^{ax}}{a}$	9
default	$\frac{e^{ax}}{a}$	9
norman	$\frac{e^{ax}}{a}$	9
risch	$\frac{e^{ax}}{a}$	9
parallelrisch	$\frac{e^{ax}}{a}$	9
meijerg	$-\frac{1-e^{ax}}{a}$	14

input
```
int(exp(a*x),x,method=_RETURNVERBOSE)
```

output
```
exp(a*x)/a
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax}\,dx=\frac{e^{(ax)}}{a}$$

input
```
integrate(exp(a*x),x, algorithm="fricas")
```

output
```
e^(a*x)/a
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{ax}\,dx = \begin{cases} \frac{e^{ax}}{a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a*x),x)
```

output
```
Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax}\,dx = \frac{e^{(ax)}}{a}$$

input
```
integrate(exp(a*x),x, algorithm="maxima")
```

output
```
e^(a*x)/a
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax}\,dx = \frac{e^{(ax)}}{a}$$

input
```
integrate(exp(a*x),x, algorithm="giac")
```

output
```
e^(a*x)/a
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax}\,dx = \frac{\mathrm{e}^{a\,x}}{a}$$

input
```
int(exp(a*x),x)
```

output
```
exp(a*x)/a
```

3.152.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

input
```
int(e**(a*x),x)
```

output
```
e**(a*x)/a
```

3.153 $\int \frac{e^{ax}}{x} dx$

3.153.1 Optimal result 974
3.153.2 Mathematica [A] (verified) 974
3.153.3 Rubi [A] (verified) 975
3.153.4 Maple [A] (verified) 975
3.153.5 Fricas [A] (verification not implemented) 976
3.153.6 Sympy [A] (verification not implemented) 976
3.153.7 Maxima [A] (verification not implemented) 976
3.153.8 Giac [A] (verification not implemented) 977
3.153.9 Mupad [B] (verification not implemented) 977
3.153.10Reduce [B] (verification not implemented) 977

3.153.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int \frac{e^{ax}}{x} dx = \text{ExpIntegralEi}(ax)$$

output
```
Ei(a*x)
```

3.153.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} dx = \text{ExpIntegralEi}(ax)$$

input
```
Integrate[E^(a*x)/x,x]
```

output
```
ExpIntegralEi[a*x]
```

3.153.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{ax}}{x}\,dx$$

$$\Big\downarrow \; 2609$$

$$\text{ExpIntegralEi}(ax)$$

input
```
Int[E^(a*x)/x,x]
```

output
```
ExpIntegralEi[a*x]
```

3.153.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.153.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 2.25

method	result	size
derivativedivides	$-\operatorname{Ei}_1(-ax)$	9
default	$-\operatorname{Ei}_1(-ax)$	9
risch	$-\operatorname{Ei}_1(-ax)$	9
meijerg	$\ln(x)+\ln(-a)-\ln(-ax)-\operatorname{Ei}_1(-ax)$	23

input
```
int(exp(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-a*x)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} \, dx = \text{Ei}(ax)$$

input
```
integrate(exp(a*x)/x,x, algorithm="fricas")
```

output
```
Ei(a*x)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{ax}}{x} \, dx = \text{Ei}\left(ax\right)$$

input
```
integrate(exp(a*x)/x,x)
```

output
```
Ei(a*x)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} \, dx = \text{Ei}(ax)$$

input
```
integrate(exp(a*x)/x,x, algorithm="maxima")
```

output
```
Ei(a*x)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x}\,dx = \mathrm{Ei}(ax)$$

input
```
integrate(exp(a*x)/x,x, algorithm="giac")
```

output
```
Ei(a*x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x}\,dx = \mathrm{ei}(a\,x)$$

input
```
int(exp(a*x)/x,x)
```

output
```
ei(a*x)
```

3.153.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x}\,dx = ei(ax)$$

input
```
int(e**(a*x)/x,x)
```

output
```
ei(a*x)
```

3.154 $\int \frac{1}{a+be^{mx}}\,dx$

3.154.1 Optimal result . . . 978
3.154.2 Mathematica [A] (verified) . . . 978
3.154.3 Rubi [A] (verified) . . . 979
3.154.4 Maple [A] (verified) . . . 980
3.154.5 Fricas [A] (verification not implemented) . . . 981
3.154.6 Sympy [A] (verification not implemented) . . . 981
3.154.7 Maxima [A] (verification not implemented) . . . 981
3.154.8 Giac [A] (verification not implemented) . . . 982
3.154.9 Mupad [B] (verification not implemented) . . . 982
3.154.10Reduce [B] (verification not implemented) . . . 982

3.154.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{a+be^{mx}}\,dx=\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

output
```
x/a-ln(a+b*exp(m*x))/a/m
```

3.154.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{a+be^{mx}}\,dx=\frac{\log\left(e^{mx}\right)}{am}-\frac{\log\left(a^2m+abe^{mx}m\right)}{am}$$

input
```
Integrate[(a + b*E^(m*x))^(-1),x]
```

output
```
Log[E^(m*x)]/(a*m) - Log[a^2*m + a*b*E^(m*x)*m]/(a*m)
```

3.154.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+be^{mx}}\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{e^{-mx}}{a+be^{mx}}de^{mx}}{m}$$

$$\Big\downarrow \ 47$$

$$\frac{\frac{\int e^{-mx}de^{mx}}{a}-\frac{b\int \frac{1}{a+be^{mx}}de^{mx}}{a}}{m}$$

$$\Big\downarrow \ 14$$

$$\frac{\frac{\log(e^{mx})}{a}-\frac{b\int \frac{1}{a+be^{mx}}de^{mx}}{a}}{m}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{\log(e^{mx})}{a}-\frac{\log(a+be^{mx})}{a}}{m}$$

input
```
Int[(a + b*E^(m*x))^(-1),x]
```

output
```
(Log[E^(m*x)]/a - Log[a + b*E^(m*x)]/a)/m
```

3.154.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.154.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{-mx+\ln(a+b\,\mathrm{e}^{mx})}{am}$	23
norman	$\frac{x}{a}-\frac{\ln(a+b\,\mathrm{e}^{mx})}{am}$	24
risch	$\frac{x}{a}-\frac{\ln\left(\mathrm{e}^{mx}+\frac{a}{b}\right)}{am}$	26
derivativedivides	$\frac{-\frac{\ln(a+b\,\mathrm{e}^{mx})}{a}+\frac{\ln(\mathrm{e}^{mx})}{a}}{m}$	29
default	$\frac{-\frac{\ln(a+b\,\mathrm{e}^{mx})}{a}+\frac{\ln(\mathrm{e}^{mx})}{a}}{m}$	29

input
```
int(1/(a+b*exp(m*x)),x,method=_RETURNVERBOSE)
```

output
```
-(-m*x+ln(a+b*exp(m*x)))/a/m
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+be^{mx}}\,dx = \frac{mx-\log\left(be^{(mx)}+a\right)}{am}$$

input
```
integrate(1/(a+b*exp(m*x)),x, algorithm="fricas")
```

output
```
(m*x - log(b*e^(m*x) + a))/(a*m)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+be^{mx}}\,dx = \frac{x}{a}-\frac{\log\left(\frac{a}{b}+e^{mx}\right)}{am}$$

input
```
integrate(1/(a+b*exp(m*x)),x)
```

output
```
x/a - log(a/b + exp(m*x))/(a*m)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+be^{mx}}\,dx = \frac{x}{a}-\frac{\log\left(be^{(mx)}+a\right)}{am}$$

input
```
integrate(1/(a+b*exp(m*x)),x, algorithm="maxima")
```

output
```
x/a - log(b*e^(m*x) + a)/(a*m)
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+be^{mx}}\,dx = \frac{\frac{mx}{a} - \frac{\log\left(\left|be^{(mx)}+a\right|\right)}{a}}{m}$$

input
```
integrate(1/(a+b*exp(m*x)),x, algorithm="giac")
```

output
```
(m*x/a - log(abs(b*e^(m*x) + a))/a)/m
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+be^{mx}}\,dx = -\frac{\ln\left(a+b\,\mathrm{e}^{m\,x}\right)-m\,x}{a\,m}$$

input
```
int(1/(a + b*exp(m*x)),x)
```

output
```
-(log(a + b*exp(m*x)) - m*x)/(a*m)
```

3.154.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+be^{mx}}\,dx = \frac{-\log(e^{mx}b+a)+mx}{am}$$

input
```
int(1/(e**(m*x)*b + a),x)
```

output
```
( - log(e**(m*x)*b + a) + m*x)/(a*m)
```

3.155 $\int \frac{e^{2x}}{1+e^x} dx$

3.155.1 Optimal result 983
3.155.2 Mathematica [A] (verified) 983
3.155.3 Rubi [A] (verified) 984
3.155.4 Maple [A] (verified) 985
3.155.5 Fricas [A] (verification not implemented) 985
3.155.6 Sympy [A] (verification not implemented) 986
3.155.7 Maxima [A] (verification not implemented) 986
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3.155.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

output
```
exp(x)-ln(1+exp(x))
```

3.155.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

input
```
Integrate[E^(2*x)/(1 + E^x),x]
```

output
```
E^x - Log[1 + E^x]
```

3.155.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{e^x+1}dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^x}{e^x+1}de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{-e^x-1}+1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$e^x-\log\left(e^x+1\right)$$

input
```
Int[E^(2*x)/(1 + E^x),x]
```

output
```
E^x - Log[1 + E^x]
```

3.155.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

3.155.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11
norman	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11
risch	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11

input
```
int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)-ln(1+exp(x))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

output
```
e^x - log(e^x + 1)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x)
```

output
```
exp(x) - log(exp(x) + 1)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

output
```
e^x - log(e^x + 1)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

output
```
e^x - log(e^x + 1)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = \mathrm{e}^x - \ln\left(\mathrm{e}^x + 1\right)$$

input
```
int(exp(2*x)/(exp(x) + 1),x)
```

output
```
exp(x) - log(exp(x) + 1)
```

3.155.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log(e^x + 1)$$

input
```
int(e**(2*x)/(e**x + 1),x)
```

output
```
e**x - log(e**x + 1)
```

3.156 $\int e^{2x+ax}\,dx$

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3.156.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int e^{2x+ax}\,dx = \frac{e^{(2+a)x}}{2+a}$$

output
```
exp((2+a)*x)/(2+a)
```

3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{2x+ax}\,dx = \frac{e^{(2+a)x}}{2+a}$$

input
```
Integrate[E^(2*x + a*x),x]
```

output
```
E^((2 + a)*x)/(2 + a)
```

3.156.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{ax+2x}\,dx$$

$$\Big\downarrow \; 2624$$

$$\frac{e^{(a+2)x}}{a+2}$$

input
```
Int[E^(2*x + a*x),x]
```

output
```
E^((2 + a)*x)/(2 + a)
```

3.156.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.156.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{(2+a)x}}{2+a}$	13
parallelrisch	$\frac{e^{(2+a)x}}{2+a}$	13
gosper	$\frac{e^{ax+2x}}{2+a}$	15
derivativedivides	$\frac{e^{ax+2x}}{2+a}$	15
default	$\frac{e^{ax+2x}}{2+a}$	15
norman	$\frac{e^{ax+2x}}{2+a}$	15
meijerg	$\frac{1-e^{-x(-a-2)}}{-a-2}$	22

input
```
int(exp(a*x+2*x),x,method=_RETURNVERBOSE)
```

output
```
exp((2+a)*x)/(2+a)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int e^{2x+ax}\,dx = \frac{e^{((a+2)x)}}{a+2}$$

input
```
integrate(exp(a*x+2*x),x, algorithm="fricas")
```

output
```
e^((a + 2)*x)/(a + 2)
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax}\,dx = \begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a \neq -2 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a*x+2*x),x)
```

output
```
Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a, -2)), (x, True))
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax}\,dx = \frac{e^{(ax+2\,x)}}{a+2}$$

input
```
integrate(exp(a*x+2*x),x, algorithm="maxima")
```

output
```
e^(a*x + 2*x)/(a + 2)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax}\,dx = \frac{e^{(ax+2\,x)}}{a+2}$$

input
```
integrate(exp(a*x+2*x),x, algorithm="giac")
```

output
```
e^(a*x + 2*x)/(a + 2)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax}\,dx=\frac{\mathrm{e}^{2\,x+a\,x}}{a+2}$$

input
```
int(exp(2*x + a*x),x)
```

output
```
exp(2*x + a*x)/(a + 2)
```

3.156.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int e^{2x+ax}\,dx=\frac{e^{ax+2x}}{a+2}$$

input
```
int(e**(a*x + 2*x),x)
```

output
```
e**(a*x + 2*x)/(a + 2)
```

3.157 $\int \frac{1}{be^{-mx}+ae^{mx}}\,dx$

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3.157.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{be^{-mx}+ae^{mx}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

output
```
arctan(exp(m*x)*a^(1/2)/b^(1/2))/m/a^(1/2)/b^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{be^{-mx}+ae^{mx}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

input
```
Integrate[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

3.157.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2720, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ae^{mx}+be^{-mx}}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{e^{2mx}a+b}de^{mx}}{m}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

input
```
Int[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

3.157.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.157.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{a\,e^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$	22
default	$\frac{\arctan\left(\frac{a\,e^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$	22
risch	$-\frac{\ln\left(e^{mx}-\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,m}+\frac{\ln\left(e^{mx}+\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,m}$	53

input
```
int(1/(b/exp(m*x)+a*exp(m*x)),x,method=_RETURNVERBOSE)
```

output
```
1/m/(a*b)^(1/2)*arctan(a*exp(m*x)/(a*b)^(1/2))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{1}{be^{-mx}+ae^{mx}}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ae^{(2\,mx)}-2\,\sqrt{-ab}e^{(mx)}-b}{ae^{(2\,mx)}+b}\right)}{2\,abm}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}e^{(-mx)}}{a}\right)}{abm}\right]$$

input
```
integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((a*e^(2*m*x) - 2*sqrt(-a*b)*e^(m*x) - b)/(a*e^(2*m*x)
 + b))/(a*b*m), -sqrt(a*b)*arctan(sqrt(a*b)*e^(-m*x)/a)/(a*b*m)]
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{be^{-mx}+ae^{mx}}\,dx = \frac{\operatorname{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(2ib+e^{mx}\right)\right)\right)}{m}$$

input
```
integrate(1/(b/exp(m*x)+a*exp(m*x)),x)
```

output
```
RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2*_i*b + exp(m*x))))/m
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{be^{-mx} + ae^{mx}}\,dx = -\frac{\arctan\left(\frac{be^{(-mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}m}$$

input
```
integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="maxima")
```

output
```
-arctan(b*e^(-m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{be^{-mx} + ae^{mx}}\,dx = \frac{\arctan\left(\frac{ae^{(mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}m}$$

input
```
integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="giac")
```

output
```
arctan(a*e^(m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{be^{-mx} + ae^{mx}} \, dx = \frac{\operatorname{atan}\left(\frac{a\,\mathrm{e}^{m\,x}}{\sqrt{a\,b}}\right)}{m\,\sqrt{a\,b}}$$

input
```
int(1/(a*exp(m*x) + b*exp(-m*x)),x)
```

output
```
atan((a*exp(m*x))/(a*b)^(1/2))/(m*(a*b)^(1/2))
```

3.157.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{be^{-mx} + ae^{mx}} \, dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{mx}a}{\sqrt{b}\,\sqrt{a}}\right)}{abm}$$

input
```
int(e**(m*x)/(e**(2*m*x)*a + b),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((e**(m*x)*a)/(sqrt(b)*sqrt(a))))/(a*b*m)
```

3.158 $\int e^{ax}x\,dx$

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3.158.3 Rubi [A] (verified) 999
3.158.4 Maple [A] (verified) 1000
3.158.5 Fricas [A] (verification not implemented) 1000
3.158.6 Sympy [A] (verification not implemented) 1001
3.158.7 Maxima [A] (verification not implemented) 1001
3.158.8 Giac [A] (verification not implemented) 1001
3.158.9 Mupad [B] (verification not implemented) 1002
3.158.10 Reduce [B] (verification not implemented) 1002

3.158.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int e^{ax}x\,dx=-\frac{e^{ax}}{a^2}+\frac{e^{ax}x}{a}$$

output
```
-exp(a*x)/a^2+exp(a*x)*x/a
```

3.158.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int e^{ax}x\,dx=\frac{e^{ax}(-1+ax)}{a^2}$$

input
```
Integrate[E^(a*x)*x,x]
```

output
```
(E^(a*x)*(-1 + a*x))/a^2
```

3.158.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{ax}\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{xe^{ax}}{a}-\frac{\int e^{ax}dx}{a}$$

$$\Big\downarrow \text{2624}$$

$$\frac{xe^{ax}}{a}-\frac{e^{ax}}{a^2}$$

input
```
Int[E^(a*x)*x,x]
```

output
```
-(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

3.158.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.158.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(ax-1)e^{ax}}{a^2}$	14
risch	$\frac{(ax-1)e^{ax}}{a^2}$	14
derivativedivides	$\frac{a\,e^{ax}x-e^{ax}}{a^2}$	19
default	$\frac{a\,e^{ax}x-e^{ax}}{a^2}$	19
meijerg	$\frac{1-\frac{(-2ax+2)e^{ax}}{2}}{a^2}$	19
parallelrisch	$\frac{a\,e^{ax}x-e^{ax}}{a^2}$	19
norman	$-\frac{e^{ax}}{a^2}+\frac{e^{ax}x}{a}$	20
parts	$-\frac{e^{ax}}{a^2}+\frac{e^{ax}x}{a}$	20

input
```
int(exp(a*x)*x,x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*exp(a*x)/a^2
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax}x\,dx=\frac{(ax-1)e^{(ax)}}{a^2}$$

input
```
integrate(exp(a*x)*x,x, algorithm="fricas")
```

output
```
(a*x - 1)*e^(a*x)/a^2
```

3.158.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{ax}x\,dx = \begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a*x)*x,x)
```

output
```
Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax}x\,dx = \frac{(ax-1)e^{(ax)}}{a^2}$$

input
```
integrate(exp(a*x)*x,x, algorithm="maxima")
```

output
```
(a*x - 1)*e^(a*x)/a^2
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax}x\,dx = \frac{(ax-1)e^{(ax)}}{a^2}$$

input
```
integrate(exp(a*x)*x,x, algorithm="giac")
```

output
```
(a*x - 1)*e^(a*x)/a^2
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax}x\,dx = \frac{\mathrm{e}^{a\,x}\,(a\,x-1)}{a^2}$$

input
```
int(x*exp(a*x),x)
```

output
```
(exp(a*x)*(a*x - 1))/a^2
```

3.158.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int e^{ax}x\,dx = \frac{e^{ax}(ax-1)}{a^2}$$

input
```
int(e**(a*x)*x,x)
```

output
```
(e**(a*x)*(a*x - 1))/a**2
```

3.159 $\int e^x x^{20} \, dx$

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3.159.1 Optimal result

Integrand size = 7, antiderivative size = 163

$$\begin{aligned}\int e^x x^{20} \, dx = {} & 2432902008176640000e^x - 2432902008176640000e^x x \\ & + 1216451004088320000e^x x^2 - 405483668029440000e^x x^3 \\ & + 101370917007360000e^x x^4 - 20274183401472000e^x x^5 \\ & + 3379030566912000e^x x^6 - 482718652416000e^x x^7 + 60339831552000e^x x^8 \\ & - 6704425728000e^x x^9 + 670442572800e^x x^{10} - 60949324800e^x x^{11} \\ & + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} \\ & + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20}\end{aligned}$$

output
```
2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+121645100408832000
0*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-2
0274183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*ex
p(x)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*e
xp(x)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)
*x^13+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp
(x)*x^17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20
```

3.159.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^x x^{20}\,dx = e^x\left(2432902008176640000 - 2432902008176640000x + 1216451004088320000x^2 - 405483668029440000x^3 + 101370917007360000x^4 - 20274183401472000x^5 + 3379030566912000x^6 - 482718652416000x^7 + 60339831552000x^8 - 6704425728000x^9 + 670442572800x^{10} - 60949324800x^{11} + 5079110400x^{12} - 390700800x^{13} + 27907200x^{14} - 1860480x^{15} + 116280x^{16} - 6840x^{17} + 380x^{18} - 20x^{19} + x^{20}\right)$$

input
```
Integrate[E^x*x^20,x]
```

output
```
E^x*(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2
 - 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5
 + 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 67044
25728000*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 39
0700800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 38
0*x^18 - 20*x^19 + x^20)
```

3.159.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.23, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 3.000$, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^{20}\,dx$$

$$\Big\downarrow \text{2607}$$

$$e^x x^{20} - 20\int e^x x^{19}dx$$

$$\Big\downarrow \text{2607}$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\int e^x x^{18} dx\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\int e^x x^{17} dx\right)\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\int e^x x^{16} dx\right)\right)\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\int e^x x^{15} dx\right)\right)\right)\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\int e^x x^{14} dx\right)\right)\right)\right)\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\int e^x x^{13} dx\right)\right)\right)\right)\right)\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\int e^x x^{12} dx\right)\right)\right)\right)\right)\right)\right)$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\int e^x x^{11} d\right.\right.\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} - \right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2607

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} - \right.\right.\right.\right.\right.\right.\right.\right.\right.$$

3.159. $\int e^x x^{20}\, dx$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} -\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 2607$$

3.159. $\int e^x x^{20}\, dx$

$$e^x x^{20} - 20\left(e^x x^{19} - 19\left(e^x x^{18} - 18\left(e^x x^{17} - 17\left(e^x x^{16} - 16\left(e^x x^{15} - 15\left(e^x x^{14} - 14\left(e^x x^{13} - 13\left(e^x x^{12} - 12\left(e^x x^{11} - \right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$\big\downarrow$ 2624

$$e^x x^{20} - 20(e^x x^{19} - 19(e^x x^{18} - 18(e^x x^{17} - 17(e^x x^{16} - 16(e^x x^{15} - 15(e^x x^{14} - 14(e^x x^{13} - 13(e^x x^{12} - 12(e^x x^{11} - 11(e^x$$

input
```
Int[E^x*x^20,x]
```

output
```
E^x*x^20 - 20*(E^x*x^19 - 19*(E^x*x^18 - 18*(E^x*x^17 - 17*(E^x*x^16 - 16*
(E^x*x^15 - 15*(E^x*x^14 - 14*(E^x*x^13 - 13*(E^x*x^12 - 12*(E^x*x^11 - 11
*(E^x*x^10 - 10*(E^x*x^9 - 9*(E^x*x^8 - 8*(E^x*x^7 - 7*(E^x*x^6 - 6*(E^x*x
^5 - 5*(E^x*x^4 - 4*(E^x*x^3 - 3*(E^x*x^2 - 2*(-E^x + E^x*x))))))))))))))
))))
```

3.159.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.159.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

method	result
gosper	$(x^{20}-20x^{19}+380x^{18}-6840x^{17}+116280x^{16}-1860480x^{15}+27907200x^{14}-390700800$
risch	$(x^{20}-20x^{19}+380x^{18}-6840x^{17}+116280x^{16}-1860480x^{15}+27907200x^{14}-390700800$
meijerg	$-2432902008176640000+\frac{(21x^{20}-420x^{19}+7980x^{18}-143640x^{17}+2441880x^{16}-39070080x^{15}+586051200x^{14}-82}{}$
default	$2432902008176640000\,\mathrm{e}^{x}-2432902008176640000\,\mathrm{e}^{x}x+1216451004088320000\,\mathrm{e}^{x}x^{2}-40$
parallelrisch	$2432902008176640000\,\mathrm{e}^{x}-2432902008176640000\,\mathrm{e}^{x}x+1216451004088320000\,\mathrm{e}^{x}x^{2}-40$
parts	$2432902008176640000\,\mathrm{e}^{x}-2432902008176640000\,\mathrm{e}^{x}x+1216451004088320000\,\mathrm{e}^{x}x^{2}-40$

input
```
int(exp(x)*x^20,x,method=_RETURNVERBOSE)
```

output
```
(x^20-20*x^19+380*x^18-6840*x^17+116280*x^16-1860480*x^15+27907200*x^14-39
0700800*x^13+5079110400*x^12-60949324800*x^11+670442572800*x^10-6704425728
000*x^9+60339831552000*x^8-482718652416000*x^7+3379030566912000*x^6-202741
83401472000*x^5+101370917007360000*x^4-405483668029440000*x^3+121645100408
8320000*x^2-2432902008176640000*x+2432902008176640000)*exp(x)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^{x}x^{20}\,dx$$
$$=(x^{20}-20\,x^{19}+380\,x^{18}-6840\,x^{17}+116280\,x^{16}-1860480\,x^{15}+27907200\,x^{14}-390700800\,x^{13}+5079$$

input
```
integrate(exp(x)*x^20,x, algorithm="fricas")
```

output
```
(x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 2790
7200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 67044257
2800*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 +
 3379030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 4
05483668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x +
2432902008176640000)*e^x
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^x x^{20}\,dx = \left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^{9} + 60339831552000x^{8} - 482718652416000x^{7} + 3379030566912000x^{6} - 20274183401472000x^{5} + 101370917007360000x^{4} - 405483668029440000x^{3} + 1216451004088320000x^{2} - 2432902008176640000x + 2432902008176640000\right)e^x$$

input
```
integrate(exp(x)*x**20,x)
```

output
```
(x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x**15
+ 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x**11
+ 670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 48271865
2416000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 1013709170
07360000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 - 24329
02008176640000*x + 2432902008176640000)*exp(x)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20}\,dx$$
$$= \left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079\right.$$

input
```
integrate(exp(x)*x^20,x, algorithm="maxima")
```

output
```
(x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 2790
7200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 67044257
2800*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 +
 3379030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 4
05483668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x +
2432902008176640000)*e^x
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20}\,dx$$
$$= \left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079\right.$$

input
```
integrate(exp(x)*x^20,x, algorithm="giac")
```

output
```
(x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 2790
7200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 67044257
2800*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 +
 3379030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 4
05483668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x +
2432902008176640000)*e^x
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20}\,dx = \mathrm{e}^x\,\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60949324800\,x^{11} + 670442572800\,x^{10} - 6704425728000\,x^9 + 60339831552000\,x^8 - 482718652416000\,x^7 + 3379030566912000\,x^6 - 20274183401472000\,x^5 + 101370917007360000\,x^4 - 405483668029440000\,x^3 + 1216451004088320000\,x^2 - 2432902008176640000\,x + 2432902008176640000\right)$$

input
```
int(x^20*exp(x),x)
```

output
```
exp(x)*(1216451004088320000*x^2 - 2432902008176640000*x - 4054836680294400
00*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000
*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 6704425728000*x^9 + 6704
42572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 2790
7200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 +
x^20 + 2432902008176640000)
```

3.159.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^x x^{20}\,dx = e^x\left(x^{20}-20x^{19}+380x^{18}-6840x^{17}+116280x^{16}-1860480x^{15}+27907200x^{14} - 390700800x^{13}+5079110400x^{12}-60949324800x^{11}+670442572800x^{10} - 6704425728000x^9+60339831552000x^8-482718652416000x^7 + 3379030566912000x^6-20274183401472000x^5+101370917007360000x^4 - 405483668029440000x^3+1216451004088320000x^2 - 2432902008176640000x+2432902008176640000\right)$$

input
```
int(e**x*x**20,x)
```

output
```
e**x*(x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x
**15 + 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x
**11 + 670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 482
718652416000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 10137
0917007360000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 -
2432902008176640000*x + 2432902008176640000)
```

3.160 $\int a^x b^{-x} \, dx$

3.160.1 Optimal result . . . 1012
3.160.2 Mathematica [A] (verified) . . . 1012
3.160.3 Rubi [A] (verified) . . . 1013
3.160.4 Maple [A] (verified) . . . 1014
3.160.5 Fricas [A] (verification not implemented) . . . 1014
3.160.6 Sympy [F(-2)] . . . 1015
3.160.7 Maxima [F(-2)] . . . 1015
3.160.8 Giac [C] (verification not implemented) . . . 1015
3.160.9 Mupad [B] (verification not implemented) . . . 1016
3.160.10Reduce [B] (verification not implemented) . . . 1016

3.160.1 Optimal result

Integrand size = 9, antiderivative size = 18

$$\int a^x b^{-x} \, dx = \frac{a^x b^{-x}}{\log(a) - \log(b)}$$

output
```
a^x/(b^x)/(ln(a)-ln(b))
```

3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} \, dx = \frac{a^x b^{-x}}{\log(a) - \log(b)}$$

input
```
Integrate[a^x/b^x,x]
```

output
```
a^x/(b^x*(Log[a] - Log[b]))
```

3.160.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x b^{-x} \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int e^{x(\log(a)-\log(b))} dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

input
```
Int[a^x/b^x,x]
```

output
```
a^x/(b^x*(Log[a] - Log[b]))
```

3.160.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

3.160.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$	19
risch	$\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$	19
parallelrisch	$\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$	19
norman	$\frac{{\mathrm e}^{x \ln(a)} {\mathrm e}^{-x \ln(b)}}{\ln(a)-\ln(b)}$	23
meijerg	$-\frac{1-{\mathrm e}^{x \ln(a) \left(1-\frac{\ln(b)}{\ln(a)}\right)}}{\ln(a) \left(1-\frac{\ln(b)}{\ln(a)}\right)}$	38

input
```
int(a^x/(b^x),x,method=_RETURNVERBOSE)
```

output
```
a^x/(b^x)/(ln(a)-ln(b))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^{x} b^{-x}\,dx = \frac{a^{x}}{b^{x} (\log(a) - \log(b))}$$

input
```
integrate(a^x/(b^x),x, algorithm="fricas")
```

output
```
a^x/(b^x*(log(a) - log(b)))
```

3.160.6 Sympy [F(-2)]

Exception generated.

$$\int a^x b^{-x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(a**x/(b**x),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

3.160.7 Maxima [F(-2)]

Exception generated.

$$\int a^x b^{-x} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^x/(b^x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?
` for more
```

3.160.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.00

$$\int a^x b^{-x} \, dx$$
$$= 2\left(\frac{2\left(\log\left(|a|\right) - \log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\,\pi x\text{sgn}(a) + \frac{1}{2}\,\pi x\text{sgn}(b)\right)}{\left(\pi\text{sgn}\left(a\right) - \pi\text{sgn}\left(b\right)\right)^2 + 4\left(\log\left(|a|\right) - \log\left(|b|\right)\right)^2} - \frac{\left(\pi\text{sgn}(a) - \pi\text{sgn}(b)\right)\sin\left(-\frac{1}{2}\,\pi x\text{sgn}(a) + \frac{1}{2}\right.}{\left(\pi\text{sgn}\left(a\right) - \pi\text{sgn}\left(b\right)\right)^2 + 4\left(\log\left(|a|\right) - \text{lo}\right.}\right.$$
$$+ i\left(\frac{i\,e^{\left(\frac{1}{2}\,i\,\pi x\text{sgn}(a) - \frac{1}{2}\,i\,\pi x\text{sgn}(b)\right)}}{i\,\pi\text{sgn}\left(a\right) - i\,\pi\text{sgn}\left(b\right) + 2\,\log\left(|a|\right) - 2\,\log\left(|b|\right)} - \frac{i\,e^{\left(-\frac{1}{2}\,i\,\pi x\text{sgn}(a) + \frac{1}{2}\,i\,\pi x\text{sgn}(b)\right)}}{-i\,\pi\text{sgn}\left(a\right) + i\,\pi\text{sgn}\left(b\right) + 2\,\log\left(|a|\right) - 2\,\log\left(|b\right.}\right.$$

input
```
integrate(a^x/(b^x),x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/(
(pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a)
- pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(
b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))
) + I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn
(b) + 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*
x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^
(x*(log(abs(a)) - log(abs(b))))
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} \, dx = \frac{a^x}{b^x \, (\ln (a) - \ln (b))}$$

input
```
int(a^x/b^x,x)
```

output
```
a^x/(b^x*(log(a) - log(b)))
```

3.160.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} \, dx = \frac{a^x}{b^x (\log (a) - \log (b))}$$

input
```
int(a**x/b**x,x)
```

output
```
a**x/(b**x*(log(a) - log(b)))
```

3.161 $\int a^x b^x \, dx$

3.161.1 Optimal result . . . 1017
3.161.2 Mathematica [A] (verified) . . . 1017
3.161.3 Rubi [A] (verified) . . . 1018
3.161.4 Maple [A] (verified) . . . 1019
3.161.5 Fricas [A] (verification not implemented) . . . 1019
3.161.6 Sympy [B] (verification not implemented) . . . 1020
3.161.7 Maxima [F(-2)] . . . 1020
3.161.8 Giac [C] (verification not implemented) . . . 1020
3.161.9 Mupad [B] (verification not implemented) . . . 1021
3.161.10Reduce [B] (verification not implemented) . . . 1022

3.161.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int a^x b^x \, dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

output
```
a^x*b^x/(ln(a)+ln(b))
```

3.161.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

input
```
Integrate[a^x*b^x,x]
```

output
```
(a^x*b^x)/(Log[a] + Log[b])
```

3.161.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x b^x \, dx$$

$$\Big\downarrow \; 2725$$

$$\int e^{x(\log(a)+\log(b))} dx$$

$$\Big\downarrow \; 2624$$

$$\frac{a^x b^x}{\log(a)+\log(b)}$$

input
```
Int[a^x*b^x,x]
```

output
```
(a^x*b^x)/(Log[a] + Log[b])
```

3.161.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

3.161.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{a^{x} b^{x}}{\ln(a)+\ln(b)}$	15
risch	$\frac{a^{x} b^{x}}{\ln(a)+\ln(b)}$	15
parallelrisch	$\frac{a^{x} b^{x}}{\ln(a)+\ln(b)}$	15
norman	$\frac{{\mathrm e}^{x \ln(a)} {\mathrm e}^{x \ln(b)}}{\ln(a)+\ln(b)}$	19
meijerg	$-\frac{1-{\mathrm e}^{x \ln(b) \left(1+\frac{\ln(a)}{\ln(b)}\right)}}{\ln(b) \left(1+\frac{\ln(a)}{\ln(b)}\right)}$	36

input
```
int(a^x*b^x,x,method=_RETURNVERBOSE)
```

output
```
a^x*b^x/(ln(a)+ln(b))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

input
```
integrate(a^x*b^x,x, algorithm="fricas")
```

output
```
a^x*b^x/(log(a) + log(b))
```

3.161.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int a^x b^x\,dx = \begin{cases} \frac{a^x b^x}{\log(a)+\log(b)} & \text{for } a \neq \frac{1}{b} \\ \frac{b^x \left(\frac{1}{b}\right)^x}{\log\left(\frac{1}{b}\right)+\log(b)} & \text{otherwise} \end{cases}$$

input
```
integrate(a**x*b**x,x)
```

output
```
Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (b**x*(1/b)**x/(log(1
/b) + log(b)), True))
```

3.161.7 Maxima [F(-2)]

Exception generated.

$$\int a^x b^x\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^x*b^x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(log(b)/log(a)>0)', see `assume?`
 for more
```

3.161.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 16.93

$$\int a^x b^x \, dx$$
$$= 2\left(\frac{2\left(\log\left(|a|\right) + \log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\mathrm{sgn}(a) - \frac{1}{2}\pi x\mathrm{sgn}(b) + \pi x\right)}{\left(2\pi - \pi\mathrm{sgn}\left(a\right) - \pi\mathrm{sgn}\left(b\right)\right)^2 + 4\left(\log\left(|a|\right) + \log\left(|b|\right)\right)^2} + \frac{\left(2\pi - \pi\mathrm{sgn}(a) - \pi\mathrm{sgn}(b)\right)\sin\left(-\frac{1}{2}\pi \cdots\right.}{\left(2\pi - \pi\mathrm{sgn}\left(a\right) - \pi\mathrm{sgn}\left(b\right)\right)^2 + \cdots}\right.$$
$$\left. + i\left(\frac{i\,e^{\left(\frac{1}{2}i\pi x\mathrm{sgn}(a) + \frac{1}{2}i\pi x\mathrm{sgn}(b) - i\pi x\right)}}{-2i\pi + i\pi\mathrm{sgn}\left(a\right) + i\pi\mathrm{sgn}\left(b\right) + 2\log\left(|a|\right) + 2\log\left(|b|\right)} - \frac{i\,e^{\left(-\frac{1}{2}i\pi x\mathrm{sgn}(a) - \frac{1}{2}i\pi x\mathrm{sgn}(b) + i\pi \cdots\right.}}{2i\pi - i\pi\mathrm{sgn}\left(a\right) - i\pi\mathrm{sgn}\left(b\right) + 2\log\left(|a \cdots\right.}\right.\right.$$

input
```
integrate(a^x*b^x,x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) +
pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2)
 + (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) +
 pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2
))*e^(x*(log(abs(a)) + log(abs(b)))) + I*(I*e^(1/2*I*pi*x*sgn(a) + 1/2*I*p
i*x*sgn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a))
+ 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)/(
2*I*pi - I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))*e^(x*
(log(abs(a)) + log(abs(b))))
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{a^x\, b^x}{\ln\left(a\right) + \ln\left(b\right)}$$

input
```
int(a^x*b^x,x)
```

output
```
(a^x*b^x)/(log(a) + log(b))
```

3.161.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{b^x a^x}{\log(a) + \log(b)}$$

input
```
int(b**x*a**x,x)
```

output
```
(b**x*a**x)/(log(a) + log(b))
```

3.162 $\int \frac{a^x}{x^2} \, dx$

3.162.1 Optimal result . . . 1023
3.162.2 Mathematica [A] (verified) . . . 1023
3.162.3 Rubi [A] (verified) . . . 1024
3.162.4 Maple [A] (verified) . . . 1025
3.162.5 Fricas [A] (verification not implemented) . . . 1025
3.162.6 Sympy [F] . . . 1025
3.162.7 Maxima [A] (verification not implemented) . . . 1026
3.162.8 Giac [F] . . . 1026
3.162.9 Mupad [B] (verification not implemented) . . . 1026
3.162.10 Reduce [B] (verification not implemented) . . . 1027

3.162.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \frac{a^x}{x^2} \, dx = -\frac{a^x}{x} + \text{ExpIntegralEi}(x \log(a)) \log(a)$$

output
```
-a^x/x+Ei(x*ln(a))*ln(a)
```

3.162.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^x}{x^2} \, dx = -\frac{a^x}{x} + \text{ExpIntegralEi}(x \log(a)) \log(a)$$

input
```
Integrate[a^x/x^2,x]
```

output
```
-(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

3.162.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^x}{x^2} \, dx$$

$$\Big\downarrow \text{2608}$$

$$\log(a) \int \frac{a^x}{x} dx - \frac{a^x}{x}$$

$$\Big\downarrow \text{2609}$$

$$\log(a) \operatorname{ExpIntegralEi}(x \log(a)) - \frac{a^x}{x}$$

input
```
Int[a^x/x^2,x]
```

output
```
-(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

3.162.3.1 Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.162.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{a^{x}}{x}-\ln\left(a\right)\operatorname{Ei}_{1}\left(-x\ln\left(a\right)\right)$
meijerg	$-\ln\left(a\right)\left(\frac{1}{x\ln(a)}+1-\ln\left(x\right)-i\pi-\ln\left(\ln\left(a\right)\right)-\frac{2+2x\ln(a)}{2x\ln(a)}+\frac{\mathrm{e}^{x\ln(a)}}{x\ln(a)}+\ln\left(-x\ln\left(a\right)\right)+\operatorname{Ei}_{1}\left(-x\ln\right.\right.$

input
```
int(a^x/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^x/x-ln(a)*Ei(1,-x*ln(a))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{a^x}{x^2}\,dx=\frac{x\mathrm{Ei}(x\log{(a)})\log{(a)}-a^x}{x}$$

input
```
integrate(a^x/x^2,x, algorithm="fricas")
```

output
```
(x*Ei(x*log(a))*log(a) - a^x)/x
```

3.162.6 Sympy [F]

$$\int \frac{a^x}{x^2}\,dx=\int \frac{a^x}{x^2}\,dx$$

input
```
integrate(a**x/x**2,x)
```

output
```
Integral(a**x/x**2, x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{a^x}{x^2}\,dx = \Gamma(-1, -x\log(a))\log(a)$$

input
```
integrate(a^x/x^2,x, algorithm="maxima")
```

output
```
gamma(-1, -x*log(a))*log(a)
```

3.162.8 Giac [F]

$$\int \frac{a^x}{x^2}\,dx = \int \frac{a^x}{x^2}\,dx$$

input
```
integrate(a^x/x^2,x, algorithm="giac")
```

output
```
integrate(a^x/x^2, x)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{a^x}{x^2}\,dx = -\ln(a)\ \mathrm{expint}(-x\ \ln(a)) - \frac{a^x}{x}$$

input
```
int(a^x/x^2,x)
```

output
```
- log(a)*expint(-x*log(a)) - a^x/x
```

3.162.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{a^x}{x^2}\,dx = \frac{ei(\log(a)\,x)\log(a)\,x - a^x}{x}$$

input
```
int(a**x/x**2,x)
```

output
```
(ei(log(a)*x)*log(a)*x - a**x)/x
```

3.163 $\int \frac{a^x x}{(1+bx)^2} \, dx$

3.163.1 Optimal result . . . 1028
3.163.2 Mathematica [A] (verified) . . . 1028
3.163.3 Rubi [A] (verified) . . . 1029
3.163.4 Maple [A] (verified) . . . 1030
3.163.5 Fricas [A] (verification not implemented) . . . 1030
3.163.6 Sympy [F] . . . 1030
3.163.7 Maxima [F] . . . 1031
3.163.8 Giac [F] . . . 1031
3.163.9 Mupad [F(-1)] . . . 1031
3.163.10Reduce [F] . . . 1032

3.163.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a^x x}{(1+bx)^2} \, dx = \frac{a^x}{b^2(1+bx)} + \frac{a^{-1/b} \operatorname{ExpIntegralEi}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2} - \frac{a^{-1/b} \operatorname{ExpIntegralEi}\left(\frac{(1+bx)\log(a)}{b}\right)\log(a)}{b^3}$$

output
```
a^x/b^2/(b*x+1)+Ei((b*x+1)*ln(a)/b)/(a^(1/b))/b^2-Ei((b*x+1)*ln(a)/b)*ln(a
)/(a^(1/b))/b^3
```

3.163.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int \frac{a^x x}{(1+bx)^2} \, dx = \frac{\frac{a^x b}{1+bx} + a^{-1/b} \operatorname{ExpIntegralEi}\left(\frac{(1+bx)\log(a)}{b}\right)(b-\log(a))}{b^3}$$

input
```
Integrate[(a^x*x)/(1 + b*x)^2,x]
```

output
```
((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Log[a]))/a^
b^(-1))/b^3
```

3.163.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xa^x}{(bx+1)^2} \, dx$$

$$\Big\downarrow \text{2629}$$

$$\int \left(\frac{a^x}{b(bx+1)} - \frac{a^x}{b(bx+1)^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{-1/b} \log(a) \text{ExpIntegralEi}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^3} + \frac{a^{-1/b} \text{ExpIntegralEi}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^2} + \frac{a^x}{b^2(bx+1)}$$

input
```
Int[(a^x*x)/(1 + b*x)^2,x]
```

output
```
a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-1)*b^2) -
 (ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*b^3)
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

3.163.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{a^{-\frac{1}{b}} \mathrm{Ei}_1\left(-x\ln(a)-\frac{\ln(a)}{b}\right)}{b^2}+\frac{\ln(a)a^x}{b^3\left(x\ln(a)+\frac{\ln(a)}{b}\right)}+\frac{\ln(a)a^{-\frac{1}{b}} \mathrm{Ei}_1\left(-x\ln(a)-\frac{\ln(a)}{b}\right)}{b^3}$	79

input
```
int(a^x*x/(b*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)+ln(a)/b^3*a^x/(x*ln(a)+ln(a)/b)+ln(
a)/b^3*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \frac{a^x b + \frac{(b^2x-(bx+1)\log(a)+b)\mathrm{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{a^{\left(\frac{1}{b}\right)}}}{b^4x+b^3}$$

input
```
integrate(a^x*x/(b*x+1)^2,x, algorithm="fricas")
```

output
```
(a^x*b + (b^2*x - (b*x + 1)*log(a) + b)*Ei((b*x + 1)*log(a)/b)/a^(1/b))/(b
^4*x + b^3)
```

3.163.6 Sympy [F]

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \int \frac{a^x x}{(bx+1)^2}\,dx$$

input
```
integrate(a**x*x/(b*x+1)**2,x)
```

output
```
Integral(a**x*x/(b*x + 1)**2, x)
```

3.163.7 Maxima [F]

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \int \frac{a^x x}{(bx+1)^2}\,dx$$

input
```
integrate(a^x*x/(b*x+1)^2,x, algorithm="maxima")
```

output
```
a^x*x/(b^2*x^2*log(a) + 2*b*x*log(a) + log(a)) + integrate((b*x - 1)*a^x/(
b^3*x^3*log(a) + 3*b^2*x^2*log(a) + 3*b*x*log(a) + log(a)), x)
```

3.163.8 Giac [F]

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \int \frac{a^x x}{(bx+1)^2}\,dx$$

input
```
integrate(a^x*x/(b*x+1)^2,x, algorithm="giac")
```

output
```
integrate(a^x*x/(b*x + 1)^2, x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \int \frac{a^x\,x}{(b\,x+1)^2}\,dx$$

input
```
int((a^x*x)/(b*x + 1)^2,x)
```

output
```
int((a^x*x)/(b*x + 1)^2, x)
```

3.163.10 Reduce [F]

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \left(\int \frac{a^x x}{\log(a)\,b^2x^2 + 2\log(a)\,bx + \log(a) - b^3x^2 - 2b^2x - b}dx\right)(\log(a) - b)$$

input
```
int((a**x*x)/(b**2*x**2 + 2*b*x + 1),x)
```

output
```
int((a**x*x)/(log(a)*b**2*x**2 + 2*log(a)*b*x + log(a) - b**3*x**2 - 2*b**
2*x - b),x)*(log(a) - b)
```

3.164 $\int \frac{e^{ax}x}{(1+ax)^2}\,dx$

3.164.1 Optimal result 1033
3.164.2 Mathematica [A] (verified) 1033
3.164.3 Rubi [A] (verified) 1034
3.164.4 Maple [A] (verified) 1035
3.164.5 Fricas [A] (verification not implemented) 1035
3.164.6 Sympy [A] (verification not implemented) 1036
3.164.7 Maxima [A] (verification not implemented) 1036
3.164.8 Giac [B] (verification not implemented) 1036
3.164.9 Mupad [B] (verification not implemented) 1037
3.164.10 Reduce [B] (verification not implemented) 1037

3.164.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{e^{ax}x}{(1+ax)^2}\,dx = \frac{e^{ax}}{a^2(1+ax)}$$

output
```
exp(a*x)/a^2/(a*x+1)
```

3.164.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2}\,dx = \frac{e^{ax}}{a^2(1+ax)}$$

input
```
Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]
```

output
```
E^(a*x)/(a^2*(1 + a*x))
```

3.164.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2627}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{ax}}{(ax+1)^2} \, dx$$
$$\Big\downarrow \text{ 2627}$$
$$\frac{e^{ax}}{a^2(ax+1)}$$

input
```
Int[(E^(a*x)*x)/(1 + a*x)^2,x]
```

output
```
E^(a*x)/(a^2*(1 + a*x))
```

3.164.3.1 Defintions of rubi rules used

rule 2627
```
Int[(F_)^(v_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)), x_Symbol] :>
 Simp[g*(d + e*x)^(m + 1)*(F^v/(D[v, x]*e*Log[F])), x] /; FreeQ[{F, d, e, f
, g, m}, x] && LinearQ[v, x] && EqQ[e*g*(m + 1) - D[v, x]*(e*f - d*g)*Log[F
], 0]
```

3.164.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{ax}}{a^2(ax+1)}$	16
derivativedivides	$\frac{e^{ax}}{a^2(ax+1)}$	16
default	$\frac{e^{ax}}{a^2(ax+1)}$	16
norman	$\frac{e^{ax}}{a^2(ax+1)}$	16
risch	$\frac{e^{ax}}{a^2(ax+1)}$	16
parallelrisch	$\frac{e^{ax}}{a^2(ax+1)}$	16

input
```
int(exp(a*x)*x/(a*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(a*x)/a^2/(a*x+1)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2}\,dx = \frac{e^{(ax)}}{a^3x+a^2}$$

input
```
integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="fricas")
```

output
```
e^(a*x)/(a^3*x + a^2)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{ax}x}{(1+ax)^2} \, dx = \frac{e^{ax}}{a^3x+a^2}$$

input
```
integrate(exp(a*x)*x/(a*x+1)**2,x)
```

output
```
exp(a*x)/(a**3*x + a**2)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2} \, dx = \frac{e^{(ax)}}{a^3x+a^2}$$

input
```
integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="maxima")
```

output
```
e^(a*x)/(a^3*x + a^2)
```

3.164.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(15) = 30.

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \frac{e^{ax}x}{(1+ax)^2} \, dx = -\frac{e^{\left(-(ax+1)\left(\frac{1}{ax+1}-1\right)\right)}}{(ax+1)a^2\left(\frac{1}{ax+1}-1\right)-a^2}$$

input
```
integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="giac")
```

output
```
-e^(-(a*x + 1)*(1/(a*x + 1) - 1))/((a*x + 1)*a^2*(1/(a*x + 1) - 1) - a^2)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{ax}x}{(1+ax)^2} \, dx = \frac{\mathrm{e}^{a\,x}}{a^2\,\left(a\,x+1\right)}$$

input
```
int((x*exp(a*x))/(a*x + 1)^2,x)
```

output
```
exp(a*x)/(a^2*(a*x + 1))
```

3.164.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2} \, dx = \frac{e^{ax}}{a^2 (ax+1)}$$

input
```
int((e**(a*x)*x)/(a**2*x**2 + 2*a*x + 1),x)
```

output
```
e**(a*x)/(a**2*(a*x + 1))
```

3.165 $\int k^{x^2} x \, dx$

3.165.1 Optimal result 1038
3.165.2 Mathematica [A] (verified) 1038
3.165.3 Rubi [A] (verified) 1039
3.165.4 Maple [A] (verified) 1040
3.165.5 Fricas [A] (verification not implemented) 1040
3.165.6 Sympy [A] (verification not implemented) 1041
3.165.7 Maxima [A] (verification not implemented) 1041
3.165.8 Giac [A] (verification not implemented) 1041
3.165.9 Mupad [B] (verification not implemented) 1042
3.165.10 Reduce [B] (verification not implemented) 1042

3.165.1 Optimal result

Integrand size = 7, antiderivative size = 13

$$\int k^{x^2} x \, dx = \frac{k^{x^2}}{2 \log(k)}$$

output
```
1/2*k^(x^2)/ln(k)
```

3.165.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int k^{x^2} x \, dx = \frac{k^{x^2}}{2 \log(k)}$$

input
```
Integrate[k^x^2*x,x]
```

output
```
k^x^2/(2*Log[k])
```

3.165.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xk^{x^2}\,dx$$

$$\Big\downarrow \; 2638$$

$$\frac{k^{x^2}}{2\log(k)}$$

input
```
Int[k^x^2*x,x]
```

output
```
k^x^2/(2*Log[k])
```

3.165.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

3.165.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{k^{x^2}}{2\ln(k)}$	12
derivativedivides	$\frac{k^{x^2}}{2\ln(k)}$	12
default	$\frac{k^{x^2}}{2\ln(k)}$	12
risch	$\frac{k^{x^2}}{2\ln(k)}$	12
parallelrisch	$\frac{k^{x^2}}{2\ln(k)}$	12
norman	$\frac{e^{\ln(k)x^2}}{2\ln(k)}$	14
meijerg	$-\frac{1-e^{\ln(k)x^2}}{2\ln(k)}$	18

input
```
int(k^(x^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*k^(x^2)/ln(k)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2}x\,dx = \frac{k^{(x^2)}}{2\,\log{(k)}}$$

input
```
integrate(k^(x^2)*x,x, algorithm="fricas")
```

output
```
1/2*k^(x^2)/log(k)
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int k^{x^2} x\,dx = \begin{cases} \frac{k^{x^2}}{2\log(k)} & \text{for } \log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(k**(x**2)*x,x)
```

output
```
Piecewise((k**(x**2)/(2*log(k)), Ne(log(k), 0)), (x**2/2, True))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x\,dx = \frac{k^{(x^2)}}{2\log(k)}$$

input
```
integrate(k^(x^2)*x,x, algorithm="maxima")
```

output
```
1/2*k^(x^2)/log(k)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x\,dx = \frac{k^{(x^2)}}{2\log(k)}$$

input
```
integrate(k^(x^2)*x,x, algorithm="giac")
```

output
```
1/2*k^(x^2)/log(k)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x\,dx = \frac{k^{x^2}}{2\ln(k)}$$

input
```
int(k^(x^2)*x,x)
```

output
```
k^(x^2)/(2*log(k))
```

3.165.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x\,dx = \frac{k^{x^2}}{2\log(k)}$$

input
```
int(k**(x**2)*x,x)
```

output
```
k**(x**2)/(2*log(k))
```

3.166 $\int e^{x^2} dx$

3.166.1 Optimal result . . . 1043
3.166.2 Mathematica [A] (verified) . . . 1043
3.166.3 Rubi [A] (verified) . . . 1044
3.166.4 Maple [A] (verified) . . . 1044
3.166.5 Fricas [A] (verification not implemented) . . . 1045
3.166.6 Sympy [A] (verification not implemented) . . . 1045
3.166.7 Maxima [C] (verification not implemented) . . . 1045
3.166.8 Giac [C] (verification not implemented) . . . 1046
3.166.9 Mupad [B] (verification not implemented) . . . 1046
3.166.10Reduce [B] (verification not implemented) . . . 1047

3.166.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

output
```
1/2*erfi(x)*Pi^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

input
```
Integrate[E^x^2,x]
```

output
```
(Sqrt[Pi]*Erfi[x])/2
```

3.166.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\,dx$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

input
```
Int[E^x^2,x]
```

output
```
(Sqrt[Pi]*Erfi[x])/2
```

3.166.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

3.166.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8
meijerg	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8
risch	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8

input
```
int(exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*erfi(x)*Pi^(1/2)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

input
```
integrate(exp(x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*erfi(x)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{x^2} dx = \frac{\sqrt{\pi}\operatorname{erfi}(x)}{2}$$

input
```
integrate(exp(x**2),x)
```

output
```
sqrt(pi)*erfi(x)/2
```

3.166.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2} dx = -\frac{1}{2}i\sqrt{\pi}\operatorname{erf}(i x)$$

input
```
integrate(exp(x^2),x, algorithm="maxima")
```

output
```
-1/2*I*sqrt(pi)*erf(I*x)
```

3.166.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2}\,dx = \frac{1}{2}i\sqrt{\pi}\,\mathrm{erf}\left(-i\,x\right)$$

input
```
integrate(exp(x^2),x, algorithm="giac")
```

output
```
1/2*I*sqrt(pi)*erf(-I*x)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfi}(x)}{2}$$

input
```
int(exp(x^2),x)
```

output
```
(pi^(1/2)*erfi(x))/2
```

3.166.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}(ix)\,i}{2}$$

input
```
int(e**(x**2),x)
```

output
```
( - sqrt(pi)*erf(i*x)*i)/2
```

3.167 $\int e^{x^2} x\,dx$

3.167.1 Optimal result 1048
3.167.2 Mathematica [A] (verified) 1048
3.167.3 Rubi [A] (verified) 1049
3.167.4 Maple [A] (verified) 1050
3.167.5 Fricas [A] (verification not implemented) 1050
3.167.6 Sympy [A] (verification not implemented) 1051
3.167.7 Maxima [A] (verification not implemented) 1051
3.167.8 Giac [A] (verification not implemented) 1051
3.167.9 Mupad [B] (verification not implemented) 1052
3.167.10 Reduce [B] (verification not implemented) 1052

3.167.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

output
```
1/2*exp(x^2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

input
```
Integrate[E^x^2*x,x]
```

output
```
E^x^2/2
```

3.167.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x \, dx$$

$$\Big\downarrow \; 2638$$

$$\frac{e^{x^2}}{2}$$

input
```
Int[E^x^2*x,x]
```

output
```
E^x^2/2
```

3.167.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

3.167.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
parallelrisch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(x\operatorname{erfi}(x)-\frac{e^{x^2}}{\sqrt{\pi}}\right)}{2}$	29

input
```
int(exp(x^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x^2)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2}x\,dx=\frac{1}{2}\,e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="fricas")
```

output
```
1/2*e^(x^2)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x,x)
```

output
```
exp(x**2)/2
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x \, dx = \frac{1}{2} e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="maxima")
```

output
```
1/2*e^(x^2)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x \, dx = \frac{1}{2} e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="giac")
```

output
```
1/2*e^(x^2)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{\mathrm{e}^{x^2}}{2}$$

input
```
int(x*exp(x^2),x)
```

output
```
exp(x^2)/2
```

3.167.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

input
```
int(e**(x**2)*x,x)
```

output
```
e**(x**2)/2
```

3.168 $\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx$

3.168.1 Optimal result . . . 1053
3.168.2 Mathematica [A] (verified) . . . 1053
3.168.3 Rubi [A] (verified) . . . 1054
3.168.4 Maple [A] (verified) . . . 1055
3.168.5 Fricas [A] (verification not implemented) . . . 1055
3.168.6 Sympy [A] (verification not implemented) . . . 1056
3.168.7 Maxima [C] (verification not implemented) . . . 1056
3.168.8 Giac [A] (verification not implemented) . . . 1056
3.168.9 Mupad [B] (verification not implemented) . . . 1057
3.168.10 Reduce [B] (verification not implemented) . . . 1057

3.168.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = -e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$$

output
```
-exp(1/x)-exp(1/x)/x^2+exp(1/x)/x
```

3.168.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = \frac{e^{\frac{1}{x}}(-1+x-x^2)}{x^2}$$

input
```
Integrate[(E^x^(-1)*(1 + x))/x^4,x]
```

output
```
(E^x^(-1)*(-1 + x - x^2))/x^2
```

3.168.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{x}}(x+1)}{x^4} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^{\frac{1}{x}}}{x^4}+\frac{e^{\frac{1}{x}}}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

input
```
Int[(E^x^(-1)*(1 + x))/x^4,x]
```

output
```
-E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.168.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{(x^2-x+1)\mathrm{e}^{\frac{1}{x}}}{x^2}$	18
risch	$-\frac{(x^2-x+1)\mathrm{e}^{\frac{1}{x}}}{x^2}$	18
derivativedivides	$-\mathrm{e}^{\frac{1}{x}}-\frac{\mathrm{e}^{\frac{1}{x}}}{x^2}+\frac{\mathrm{e}^{\frac{1}{x}}}{x}$	25
default	$-\mathrm{e}^{\frac{1}{x}}-\frac{\mathrm{e}^{\frac{1}{x}}}{x^2}+\frac{\mathrm{e}^{\frac{1}{x}}}{x}$	25
parallelrisch	$-\frac{x^2\mathrm{e}^{\frac{1}{x}}-x\,\mathrm{e}^{\frac{1}{x}}+\mathrm{e}^{\frac{1}{x}}}{x^2}$	26
norman	$\frac{x^2\mathrm{e}^{\frac{1}{x}}-x\,\mathrm{e}^{\frac{1}{x}}-x^3\mathrm{e}^{\frac{1}{x}}}{x^3}$	30
meijerg	$1-\frac{\left(\frac{3}{x^2}-\frac{6}{x}+6\right)\mathrm{e}^{\frac{1}{x}}}{3}+\frac{\left(2-\frac{2}{x}\right)\mathrm{e}^{\frac{1}{x}}}{2}$	34

input
```
int(exp(1/x)*(1+x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-(x^2-x+1)*exp(1/x)/x^2
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = -\frac{(x^2-x+1)e^{\frac{1}{x}}}{x^2}$$

input
```
integrate(exp(1/x)*(1+x)/x^4,x, algorithm="fricas")
```

output
```
-(x^2 - x + 1)*e^(1/x)/x^2
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = \frac{(-x^2+x-1)\,e^{\frac{1}{x}}}{x^2}$$

input
```
integrate(exp(1/x)*(1+x)/x**4,x)
```

output
```
(-x**2 + x - 1)*exp(1/x)/x**2
```

3.168.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = -\Gamma\left(3,-\frac{1}{x}\right)+\Gamma\left(2,-\frac{1}{x}\right)$$

input
```
integrate(exp(1/x)*(1+x)/x^4,x, algorithm="maxima")
```

output
```
-gamma(3, -1/x) + gamma(2, -1/x)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = \frac{e^{\frac{1}{x}}}{x}-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}$$

input
```
integrate(exp(1/x)*(1+x)/x^4,x, algorithm="giac")
```

output
```
e^(1/x)/x - e^(1/x)/x^2 - e^(1/x)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = -\frac{e^{1/x}\,(x^2-x+1)}{x^2}$$

input
```
int((exp(1/x)*(x + 1))/x^4,x)
```

output
```
-(exp(1/x)*(x^2 - x + 1))/x^2
```

3.168.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx = \frac{e^{\frac{1}{x}}(-x^2+x-1)}{x^2}$$

input
```
int((e**(1/x)*(x + 1))/x**4,x)
```

output
```
(e**(1/x)*( - x**2 + x - 1))/x**2
```

3.169 $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx$

3.169.1 Optimal result . . . 1058
3.169.2 Mathematica [A] (verified) . . . 1058
3.169.3 Rubi [F] . . . 1059
3.169.4 Maple [A] (verified) . . . 1060
3.169.5 Fricas [A] (verification not implemented) . . . 1060
3.169.6 Sympy [A] (verification not implemented) . . . 1060
3.169.7 Maxima [A] (verification not implemented) . . . 1061
3.169.8 Giac [A] (verification not implemented) . . . 1061
3.169.9 Mupad [F(-1)] . . . 1061
3.169.10Reduce [B] (verification not implemented) . . . 1062

3.169.1 Optimal result

Integrand size = 37, antiderivative size = 25

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx = -\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

output
```
-exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

3.169.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx = -\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

input
```
Integrate[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

output
```
-(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

3.169.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x^2-e^{x^2}x+1}(2x^3+x)}{(1-e^{x^2}x)^2}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{e^{2x^2-e^{x^2}x+1}x(2x^2+1)}{(1-e^{x^2}x)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^{2x^2-e^{x^2}x+1}x}{(e^{x^2}x-1)^2}+\frac{2e^{2x^2-e^{x^2}x+1}x^3}{(e^{x^2}x-1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\int \frac{e^{2x^2-e^{x^2}x+1}x}{(e^{x^2}x-1)^2}dx+2\int \frac{e^{2x^2-e^{x^2}x+1}x^3}{(e^{x^2}x-1)^2}dx$$

input
```
Int[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

output
```
$Aborted
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.169.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$	23

input
```
int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x,method=_RETURNVER
BOSE)
```

output
```
-exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx = -\frac{e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3\,x^2\right)}-e^{\left(2\,x^2\right)}}$$

input
```
integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm=
"fricas")
```

output
```
-e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx = -\frac{e^{2x^2-xe^{x^2}+1}}{xe^{3x^2}-e^{2x^2}}$$

input
```
integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)
```

output
```
-exp(2*x**2 - x*exp(x**2) + 1)/(x*exp(3*x**2) - exp(2*x**2))
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx = -\frac{e^{\left(-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(x^2\right)}-1}$$

input
```
integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm=
"maxima")
```

output
```
-e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx = -\frac{e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3\,x^2\right)}-e^{\left(2\,x^2\right)}}$$

input
```
integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm=
"giac")
```

output
```
-e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx = \int \frac{\mathrm{e}^{2\,x^2-x\,\mathrm{e}^{x^2}+1}\,\left(2\,x^3+x\right)}{\left(x\,\mathrm{e}^{x^2}-1\right)^2}\,dx$$

input
```
int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2,x)
```

output
```
int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2, x)
```

3.169.10 Reduce [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx = -\frac{e}{e^{e^{x^2}x}\left(e^{x^2}x-1\right)}$$

input
```
int((e**(2*x**2)*e*x*(2*x**2 + 1))/(e**(e**(x**2)*x)*(e**(2*x**2)*x**2 - 2
*e**(x**2)*x + 1)),x)
```

output
```
( - e)/(e**(e**(x**2)*x)*(e**(x**2)*x - 1))
```

3.170 $\int e^{e^{e^{e^x}}}\,dx$

3.170.1 Optimal result 1063
3.170.2 Mathematica [N/A] 1063
3.170.3 Rubi [N/A] 1064
3.170.4 Maple [N/A] 1065
3.170.5 Fricas [N/A] 1065
3.170.6 Sympy [N/A] 1065
3.170.7 Maxima [N/A] 1066
3.170.8 Giac [N/A] 1066
3.170.9 Mupad [N/A] 1066
3.170.10 Reduce [F] 1067

3.170.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int e^{e^{e^{e^x}}}\,dx=\text{Int}\left(e^{e^{e^{e^x}}},x\right)$$

output
```
CannotIntegrate(exp(exp(exp(exp(x)))),x)
```

3.170.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int e^{e^{e^{e^x}}}\,dx=\int e^{e^{e^{e^x}}}\,dx$$

input
```
Integrate[E^E^E^E^x,x]
```

output
```
Integrate[E^E^E^E^x, x]
```

3.170.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2720, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{e^{e^{e^x}}} \, dx$$

$$\Big\downarrow \; 2720$$

$$\int e^{e^{e^{e^x}}-x} \, de^x$$

$$\Big\downarrow \; 7299$$

$$\int e^{e^{e^{e^x}}-x} \, de^x$$

input
```
Int[E^E^E^E^x,x]
```

output
```
$Aborted
```

3.170.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.170.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \mathrm{e}^{\mathrm{e}^{\mathrm{e}^{\mathrm{e}^{x}}}}dx$$

input
```
int(exp(exp(exp(exp(x)))),x)
```

output
```
int(exp(exp(exp(exp(x)))),x)
```

3.170.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}}\,dx = \int e^{\left(e^{\left(e^{\left(e^{x}\right)}\right)}\right)}\,dx$$

input
```
integrate(exp(exp(exp(exp(x)))),x, algorithm="fricas")
```

output
```
integral(e^(e^(e^(e^x))), x)
```

3.170.6 Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{e^{e^{e^x}}}\,dx = \int e^{e^{e^{e^{x}}}}\,dx$$

input
```
integrate(exp(exp(exp(exp(x)))),x)
```

output
```
Integral(exp(exp(exp(exp(x)))), x)
```

3.170.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)} dx$$

input
```
integrate(exp(exp(exp(exp(x)))),x, algorithm="maxima")
```

output
```
integrate(e^(e^(e^(e^x))), x)
```

3.170.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)} dx$$

input
```
integrate(exp(exp(exp(exp(x)))),x, algorithm="giac")
```

output
```
integrate(e^(e^(e^(e^x))), x)
```

3.170.9 Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int \mathrm{e}^{\mathrm{e}^{\mathrm{e}^{\mathrm{e}^x}}} dx$$

input
```
int(exp(exp(exp(exp(x)))),x)
```

output
```
int(exp(exp(exp(exp(x)))), x)
```

3.170.10 Reduce [F]

$$\int e^{e^{e^{e^x}}}\,dx=\int e^{e^{e^{e^x}}}\,dx$$

input
```
int(e**(e**(e**(e**x))),x)
```

output
```
int(e**(e**(e**(e**x))),x)
```

3.171 $\int e^x \log(x)\,dx$

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3.171.5 Fricas [A] (verification not implemented) 1070
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3.171.8 Giac [A] (verification not implemented) 1071
3.171.9 Mupad [B] (verification not implemented) 1071
3.171.10 Reduce [B] (verification not implemented) 1072

3.171.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int e^x \log(x)\,dx = -\text{ExpIntegralEi}(x) + e^x \log(x)$$

output
```
-Ei(x)+exp(x)*ln(x)
```

3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^x \log(x)\,dx = -\text{ExpIntegralEi}(x) + e^x \log(x)$$

input
```
Integrate[E^x*Log[x],x]
```

output
```
-ExpIntegralEi[x] + E^x*Log[x]
```

3.171.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3034, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \log(x) \, dx$$

$$\Big\downarrow \text{3034}$$

$$e^x \log(x) - \int \frac{e^x}{x} dx$$

$$\Big\downarrow \text{2609}$$

$$e^x \log(x) - \text{ExpIntegralEi}(x)$$

input
```
Int[E^x*Log[x],x]
```

output
```
-ExpIntegralEi[x] + E^x*Log[x]
```

3.171.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

3.171.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
risch	$e^x \ln(x) + \mathrm{Ei}_1(-x)$	12

input
```
int(exp(x)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*ln(x)+Ei(1,-x)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x)\,dx = e^x \log(x) - \mathrm{Ei}(x)$$

input
```
integrate(exp(x)*log(x),x, algorithm="fricas")
```

output
```
e^x*log(x) - Ei(x)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^x \log(x)\,dx = e^x \log(x) - \mathrm{Ei}(x)$$

input
```
integrate(exp(x)*ln(x),x)
```

output
```
exp(x)*log(x) - Ei(x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x)\,dx = e^x \log(x) - \operatorname{Ei}(x)$$

input
```
integrate(exp(x)*log(x),x, algorithm="maxima")
```

output
```
e^x*log(x) - Ei(x)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x)\,dx = e^x \log(x) - \operatorname{Ei}(x)$$

input
```
integrate(exp(x)*log(x),x, algorithm="giac")
```

output
```
e^x*log(x) - Ei(x)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x)\,dx = \mathrm{e}^x\,\ln(x) - \mathrm{ei}(x)$$

input
```
int(exp(x)*log(x),x)
```

output
```
exp(x)*log(x) - ei(x)
```

3.171.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^x \log(x)\,dx = -ei(x) + e^x\log(x)$$

input
```
int(e**x*log(x),x)
```

output
```
- ei(x) + e**x*log(x)
```

3.172 $\int e^x x \log(x)\,dx$

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3.172.9 Mupad [B] (verification not implemented) 1076
3.172.10Reduce [B] (verification not implemented) 1077

3.172.1 Optimal result

Integrand size = 7, antiderivative size = 22

$$\int e^x x \log(x)\,dx = -e^x + \text{ExpIntegralEi}(x) - e^x \log(x) + e^x x \log(x)$$

output
```
-exp(x)+Ei(x)-exp(x)*ln(x)+exp(x)*x*ln(x)
```

3.172.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int e^x x \log(x)\,dx = -e^x + \text{ExpIntegralEi}(x) + e^x(-1+x)\log(x)$$

input
```
Integrate[E^x*x*Log[x],x]
```

output
```
-E^x + ExpIntegralEi[x] + E^x*(-1 + x)*Log[x]
```

3.172.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3034, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \log(x) \, dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{e^x(x-1)}{x}dx - e^x \log(x) + e^x x \log(x)$$

$$\Big\downarrow \text{2629}$$

$$-\int \left(e^x - \frac{e^x}{x}\right) dx - e^x \log(x) + e^x x \log(x)$$

$$\Big\downarrow \text{2009}$$

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

input
```
Int[E^x*x*Log[x],x]
```

output
```
-E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

3.172.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
risch	$(-1+x)\,\mathrm{e}^{x}\ln(x)-\mathrm{Ei}_1(-x)-\mathrm{e}^{x}$	21

input
```
int(exp(x)*x*ln(x),x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*exp(x)*ln(x)-Ei(1,-x)-exp(x)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x x \log(x)\,dx = (x-1)e^x\log(x) + \mathrm{Ei}(x) - e^x$$

input
```
integrate(exp(x)*x*log(x),x, algorithm="fricas")
```

output
```
(x - 1)*e^x*log(x) + Ei(x) - e^x
```

3.172.6 Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int e^x x \log(x)\,dx = (xe^x - e^x)\log(x) - e^x + \mathrm{Ei}(x)$$

input
```
integrate(exp(x)*x*ln(x),x)
```

output
```
(x*exp(x) - exp(x))*log(x) - exp(x) + Ei(x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x x \log(x) \, dx = (x-1)e^x \log(x) + \text{Ei}(x) - e^x$$

input
```
integrate(exp(x)*x*log(x),x, algorithm="maxima")
```

output
```
(x - 1)*e^x*log(x) + Ei(x) - e^x
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x x \log(x) \, dx = (x-1)e^x \log(x) + \text{Ei}(x) - e^x$$

input
```
integrate(exp(x)*x*log(x),x, algorithm="giac")
```

output
```
(x - 1)*e^x*log(x) + Ei(x) - e^x
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int e^x x \log(x) \, dx = \text{ei}(x) - \frac{x\,e^x + x\,e^x\,\ln(x) - x^2\,e^x\,\ln(x)}{x}$$

input
```
int(x*exp(x)*log(x),x)
```

output
```
ei(x) - (x*exp(x) + x*exp(x)*log(x) - x^2*exp(x)*log(x))/x
```

3.172.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^x x \log(x)\,dx = ei(x) + e^x \log(x)\,x - e^x \log(x) - e^x$$

input
```
int(e**x*log(x)*x,x)
```

output
```
ei(x) + e**x*log(x)*x - e**x*log(x) - e**x
```

3.173 $\int e^{2x} \log(e^x)\,dx$

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3.173.2 Mathematica [A] (verified) . . . 1078
3.173.3 Rubi [A] (verified) . . . 1079
3.173.4 Maple [A] (verified) . . . 1080
3.173.5 Fricas [A] (verification not implemented) . . . 1080
3.173.6 Sympy [A] (verification not implemented) . . . 1081
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3.173.9 Mupad [B] (verification not implemented) . . . 1082
3.173.10Reduce [B] (verification not implemented) . . . 1082

3.173.1 Optimal result

Integrand size = 10, antiderivative size = 23

$$\int e^{2x} \log(e^x)\,dx = -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x}\log(e^x)$$

output
```
-1/4*exp(2*x)+1/2*exp(2*x)*ln(exp(x))
```

3.173.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{2x} \log(e^x)\,dx = \frac{1}{4}e^{2x}(-1 + 2\log(e^x))$$

input
```
Integrate[E^(2*x)*Log[E^x],x]
```

output
```
(E^(2*x)*(-1 + 2*Log[E^x]))/4
```

3.173.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3034, 27, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x} \log(e^x)\, dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{1}{2} e^{2x} \log(e^x) - \int \frac{e^{2x}}{2} dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} e^{2x} \log(e^x) - \frac{\int e^{2x} dx}{2}$$

$$\Big\downarrow \text{2624}$$

$$\frac{1}{2} e^{2x} \log(e^x) - \frac{e^{2x}}{4}$$

input
```
Int[E^(2*x)*Log[E^x],x]
```

output
```
-1/4*E^(2*x) + (E^(2*x)*Log[E^x])/2
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

3.173.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

method	result	size
norman	$-\frac{e^{2x}}{4}+\frac{e^{2x}\ln\left(e^{x}\right)}{2}$	17
risch	$-\frac{e^{2x}}{4}+\frac{e^{2x}\ln\left(e^{x}\right)}{2}$	17
parallelrisch	$-\frac{e^{2x}}{4}+\frac{e^{2x}\ln\left(e^{x}\right)}{2}$	17
default	$\frac{e^{2x}x}{2}-\frac{e^{2x}}{4}+\frac{e^{2x}\left(\ln\left(e^{x}\right)-x\right)}{2}$	28

input
```
int(exp(2*x)*ln(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)^2+1/2*exp(x)^2*ln(exp(x))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x}\log\left(e^{x}\right)\,dx=\frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

input
```
integrate(exp(2*x)*log(exp(x)),x, algorithm="fricas")
```

output
```
1/4*(2*x - 1)*e^(2*x)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.43

$$\int e^{2x}\log\left(e^{x}\right)\,dx = \frac{(2x-1)\,e^{2x}}{4}$$

input
```
integrate(exp(2*x)*ln(exp(x)),x)
```

output
```
(2*x - 1)*exp(2*x)/4
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x}\log\left(e^{x}\right)\,dx = \frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

input
```
integrate(exp(2*x)*log(exp(x)),x, algorithm="maxima")
```

output
```
1/4*(2*x - 1)*e^(2*x)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x}\log\left(e^{x}\right)\,dx = \frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

input
```
integrate(exp(2*x)*log(exp(x)),x, algorithm="giac")
```

output
```
1/4*(2*x - 1)*e^(2*x)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x} \log(e^x) \, dx = \frac{e^{2x}(2x-1)}{4}$$

input
```
int(log(exp(x))*exp(2*x),x)
```

output
```
(exp(2*x)*(2*x - 1))/4
```

3.173.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int e^{2x} \log(e^x) \, dx = \frac{e^{2x}(2x-1)}{4}$$

input
```
int(e**(2*x)*x,x)
```

output
```
(e**(2*x)*(2*x - 1))/4
```

3.174 $\int \left(2x+\sqrt{2}x^2\right)\,dx$

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3.174.10 Reduce [B] (verification not implemented) . . . 1087

3.174.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=x^2+\frac{\sqrt{2}x^3}{3}$$

output
```
x^2+1/3*x^3*2^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=x^2+\frac{\sqrt{2}x^3}{3}$$

input
```
Integrate[2*x + Sqrt[2]*x^2,x]
```

output
```
x^2 + (Sqrt[2]*x^3)/3
```

3.174.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{2}x^2+2x\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{2}x^3}{3}+x^2$$

input
```
Int[2*x + Sqrt[2]*x^2,x]
```

output
```
x^2 + (Sqrt[2]*x^3)/3
```

3.174.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.174.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
norman	$x^2+\frac{x^3\sqrt{2}}{3}$	13
risch	$x^2+\frac{x^3\sqrt{2}}{3}$	13
parallelrisch	$x^2+\frac{x^3\sqrt{2}}{3}$	13
parts	$x^2+\frac{x^3\sqrt{2}}{3}$	13
default	$\sqrt{2}\left(\frac{x^3}{3}+\frac{x^2\sqrt{2}}{2}\right)$	19
gosper	$\frac{x^2\left(2x+3\sqrt{2}\right)\left(x\sqrt{2}+2\right)}{6x+6\sqrt{2}}$	29

input
```
int(2*x+x^2*2^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^2+1/3*x^3*2^(1/2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=\frac{1}{3}\,\sqrt{2}x^3+x^2$$

input
```
integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*x^3 + x^2
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=\frac{\sqrt{2}x^3}{3}+x^2$$

input
```
integrate(2*x+x**2*2**(1/2),x)
```

output
```
sqrt(2)*x**3/3 + x**2
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=\frac{1}{3}\,\sqrt{2}x^3+x^2$$

input
```
integrate(2*x+x^2*2^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(2)*x^3 + x^2
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=\frac{1}{3}\,\sqrt{2}x^3+x^2$$

input
```
integrate(2*x+x^2*2^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*x^3 + x^2
```

3.174.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=\frac{x^2\,\left(\sqrt{2}\,x+3\right)}{3}$$

input
```
int(2*x + 2^(1/2)*x^2,x)
```

output
```
(x^2*(2^(1/2)*x + 3))/3
```

3.174.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \left(2x+\sqrt{2}x^2\right)\,dx=\frac{x^2(\sqrt{2}\,x+3)}{3}$$

input
```
int(x*(sqrt(2)*x + 2),x)
```

output
```
(x**2*(sqrt(2)*x + 3))/3
```

3.175 $\int \frac{\log(x)}{\sqrt{b+ax}} dx$

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3.175.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = -\frac{4\sqrt{b+ax}}{a} + \frac{4\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{b+ax}\log(x)}{a}$$

output
```
4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+
b)^(1/2)/a
```

3.175.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = \frac{4\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right) + 2\sqrt{b+ax}(-2+\log(x))}{a}$$

input
```
Integrate[Log[x]/Sqrt[b + a*x],x]
```

output
```
(4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 + Log[x]))
/a
```

3.175.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2756, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{\sqrt{ax+b}}dx$$

$$\Big\downarrow \text{ 2756}$$

$$\frac{2\log(x)\sqrt{ax+b}}{a}-\frac{2\int \frac{\sqrt{b+ax}}{x}dx}{a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2\log(x)\sqrt{ax+b}}{a}-\frac{2\left(b\int \frac{1}{x\sqrt{b+ax}}dx+2\sqrt{ax+b}\right)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\log(x)\sqrt{ax+b}}{a}-\frac{2\left(\frac{2b\int \frac{1}{\frac{b+ax}{a}-\frac{b}{a}}d\sqrt{b+ax}}{a}+2\sqrt{ax+b}\right)}{a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\log(x)\sqrt{ax+b}}{a}-\frac{2\left(2\sqrt{ax+b}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\right)}{a}$$

input
```
Int[Log[x]/Sqrt[b + a*x],x]
```

output
```
(-2*(2*Sqrt[b + a*x] - 2*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]]))/a + (2*S
qrt[b + a*x]*Log[x])/a
```

3.175.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.175.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2\sqrt{ax+b}\,\ln(x)-4\sqrt{ax+b}+4\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$	43
default	$\frac{2\sqrt{ax+b}\,\ln(x)-4\sqrt{ax+b}+4\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$	43

input
```
int(ln(x)/(a*x+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a*((a*x+b)^(1/2)*ln(x)-2*(a*x+b)^(1/2)+2*b^(1/2)*arctanh((a*x+b)^(1/2)/b
^(1/2)))
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

$$\int \frac{\log(x)}{\sqrt{b+ax}}\,dx$$
$$= \left[\frac{2\left(\sqrt{ax+b}(\log(x)-2)+\sqrt{b}\log\left(\frac{ax+2\sqrt{ax+b}\sqrt{b}+2b}{x}\right)\right)}{a}, \frac{2\left(\sqrt{ax+b}(\log(x)-2)-2\sqrt{-b}\arctan\left(\frac{\sqrt{ax+}}{}\right.\right.}{a}\right.$$

input
```
integrate(log(x)/(a*x+b)^(1/2),x, algorithm="fricas")
```

output
```
[2*(sqrt(a*x + b)*(log(x) - 2) + sqrt(b)*log((a*x + 2*sqrt(a*x + b)*sqrt(b
) + 2*b)/x))/a, 2*(sqrt(a*x + b)*(log(x) - 2) - 2*sqrt(-b)*arctan(sqrt(a*x
 + b)*sqrt(-b)/b))/a]
```

3.175.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 1166, normalized size of antiderivative = 20.46

$$\int \frac{\log(x)}{\sqrt{b+ax}}\,dx = \text{Too large to display}$$

input
```
integrate(ln(x)/(a*x+b)**(1/2),x)
```

output
```
Piecewise((4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x +
 b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/
a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/
a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*s
qrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x
 + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*p
i*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a) < 1)), (4*sq
rt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/
sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*l
og(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x
 + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt
(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sq
rt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log
(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), Abs(x + b/a) < 1)
, (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*lo
g(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x +
 b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi
*sqrt(x + b/a)/sqrt(a), (1/Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1))
, (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*lo
g(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(...
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\log(x)}{\sqrt{b+ax}}\,dx = \frac{2\left(\sqrt{ax+b}\log\left(x\right) - \sqrt{b}\log\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) - 2\sqrt{ax+b}\right)}{a}$$

input
```
integrate(log(x)/(a*x+b)^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(a*x + b)*log(x) - sqrt(b)*log((sqrt(a*x + b) - sqrt(b))/(sqrt(a*x
+ b) + sqrt(b))) - 2*sqrt(a*x + b))/a
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{\log(x)}{\sqrt{b+ax}}\,dx = -\frac{2\left(\frac{2\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{ax+b}\log\left(x\right) + 2\,\sqrt{ax+b}\right)}{a}$$

input
```
integrate(log(x)/(a*x+b)^(1/2),x, algorithm="giac")
```

output
```
-2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*log(x) + 2
*sqrt(a*x + b))/a
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\log(x)}{\sqrt{b+ax}}\,dx = \frac{2\,\sqrt{b}\,\ln\left(\frac{2\,b+a\,x+2\,\sqrt{b}\,\sqrt{b+a\,x}}{x}\right)}{a} + \frac{2\,\left(\ln\left(x\right)-2\right)\,\sqrt{b+a\,x}}{a}$$

input
```
int(log(x)/(b + a*x)^(1/2),x)
```

output
```
(2*b^(1/2)*log((2*b + a*x + 2*b^(1/2)*(b + a*x)^(1/2))/x))/a + (2*(log(x)
- 2)*(b + a*x)^(1/2))/a
```

3.175.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\log(x)}{\sqrt{b+ax}}\,dx = \frac{2\sqrt{ax+b}\log(x) - 4\sqrt{ax+b} - 4\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right) + 2\sqrt{b}\log(x)}{a}$$

input
```
int(log(x)/sqrt(a*x + b),x)
```

output
```
(2*(sqrt(a*x + b)*log(x) - 2*sqrt(a*x + b) - 2*sqrt(b)*log(sqrt(a*x + b) -
 sqrt(b)) + sqrt(b)*log(x)))/a
```

3.176 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

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3.176.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx=\frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}-\frac{(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

output
```
-1/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(
3/2)/d^(3/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b+1/4*(-a*d+b*c)*(b*x+a)^(1/2
)*(d*x+c)^(1/2)/b/d
```

3.176.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx =\frac{\sqrt{a+bx}\sqrt{c+dx}(bc+ad+2bdx)}{4bd}-\frac{(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(b*c + a*d + 2*b*d*x))/(4*b*d) - ((b*c - a*d)
^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^
(3/2))
```

3.176.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{4b}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

$$\Big\downarrow \; 60$$

$$\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}\right)}{4b}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

$$\Big\downarrow \; 66$$

$$\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}\right)}{4b}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

$$\Big\downarrow \; 221$$

$$\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4b}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

output
```
((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*b) + ((b*c - a*d)*((Sqrt[a + b*x]*Sqrt[
c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/(Sqrt[b]*d^(3/2))))/(4*b)
```

3.176.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.176.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{dx+c}\,\sqrt{bx+a}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}+(ad+bc)x+ac}\right)}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}\right)}{4d}$	140

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*(1/b*(d*x+c)^(1/2)*(b*x
+a)^(1/2)-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.59

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$
$$= \left[\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx}\right.}{16\,b^2d^2}\right.$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b
*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x +
 a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*
b^2*d^2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

3.176.6 Sympy [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = \int \sqrt{a+bx}\sqrt{c+dx}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

3.176.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(90) = 180.

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.00

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = -\frac{\frac{4\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)a|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,a}{d^2}\right)\right)}{}}{4\,b}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x +
2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^
2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a
*b*d)))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

3.176.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$
$$= \left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}$$
$$-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

input
```
int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

output
```
(x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b
*c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b
*c)^2)/(8*b^(3/2)*d^(3/2))
```

3.176.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.69

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$
$$= \frac{\sqrt{dx+c}\,\sqrt{bx+a}\,ab\,d^2+\sqrt{dx+c}\,\sqrt{bx+a}\,b^2cd+2\sqrt{dx+c}\,\sqrt{bx+a}\,b^2d^2x-\sqrt{d}\,\sqrt{b}\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{d}}{\sqrt{ad-bc}}\right.}{4b^2d^2}$$

input
```
int(sqrt(c + d*x)*sqrt(a + b*x),x)
```

output
```
(sqrt(c + d*x)*sqrt(a + b*x)*a*b*d**2 + sqrt(c + d*x)*sqrt(a + b*x)*b**2*c
*d + 2*sqrt(c + d*x)*sqrt(a + b*x)*b**2*d**2*x - sqrt(d)*sqrt(b)*log((sqrt
(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*d**2 + 2*
sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a
*d - b*c))*a*b*c*d - sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*
sqrt(c + d*x))/sqrt(a*d - b*c))*b**2*c**2)/(4*b**2*d**2)
```

3.177 $\int \sqrt{a+bx}\,dx$

3.177.1 Optimal result . . . 1100
3.177.2 Mathematica [A] (verified) . . . 1100
3.177.3 Rubi [A] (verified) . . . 1101
3.177.4 Maple [A] (verified) . . . 1102
3.177.5 Fricas [A] (verification not implemented) . . . 1102
3.177.6 Sympy [A] (verification not implemented) . . . 1103
3.177.7 Maxima [A] (verification not implemented) . . . 1103
3.177.8 Giac [A] (verification not implemented) . . . 1103
3.177.9 Mupad [B] (verification not implemented) . . . 1104
3.177.10 Reduce [B] (verification not implemented) . . . 1104

3.177.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{3/2}}{3b}$$

output
```
2/3*(b*x+a)^(3/2)/b
```

3.177.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{3/2}}{3b}$$

input
```
Integrate[Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2))/(3*b)
```

3.177.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2(a+bx)^{3/2}}{3b}$$

input
```
Int[Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2))/(3*b)
```

3.177.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.177.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
trager	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
risch	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13

input
```
int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{\frac{3}{2}}}{3b}$$

input
```
integrate((b*x+a)**(1/2),x)
```

output
```
2*(a + b*x)**(3/2)/(3*b)
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

3.177.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx=\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3\,b}$$

input
```
int((a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(3/2))/(3*b)
```

3.177.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a+bx}\,dx=\frac{2\sqrt{bx+a}\,(bx+a)}{3b}$$

input
```
int(sqrt(a + b*x),x)
```

output
```
(2*sqrt(a + b*x)*(a + b*x))/(3*b)
```

3.178 $\int x\sqrt{a+bx}\,dx$

3.178.1 Optimal result . . . 1105
3.178.2 Mathematica [A] (verified) . . . 1105
3.178.3 Rubi [A] (verified) . . . 1106
3.178.4 Maple [A] (verified) . . . 1107
3.178.5 Fricas [A] (verification not implemented) . . . 1107
3.178.6 Sympy [B] (verification not implemented) . . . 1108
3.178.7 Maxima [A] (verification not implemented) . . . 1108
3.178.8 Giac [B] (verification not implemented) . . . 1109
3.178.9 Mupad [B] (verification not implemented) . . . 1109
3.178.10Reduce [B] (verification not implemented) . . . 1109

3.178.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\sqrt{a+bx}\,dx=-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}$$

output
```
-2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2
```

3.178.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\sqrt{a+bx}\,dx=\frac{2\sqrt{a+bx}(-2a^2+abx+3b^2x^2)}{15b^2}$$

input
```
Integrate[x*Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)
```

3.178.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{(a+bx)^{3/2}}{b}-\frac{a\sqrt{a+bx}}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

input
```
Int[x*Sqrt[a + b*x],x]
```

output
```
(-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

3.178.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.178.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
trager	$-\frac{2(-3x^2b^2-axb+2a^2)\sqrt{bx+a}}{15b^2}$	32
risch	$-\frac{2(-3x^2b^2-axb+2a^2)\sqrt{bx+a}}{15b^2}$	32

input
```
int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\sqrt{a+bx}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

input
```
integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

3.178.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.

Time = 0.67 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int x\sqrt{a+bx}\,dx = -\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x} - \frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x} + \frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

input
```
integrate(x*(b*x+a)**(1/2),x)
```

output
```
-4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*
a**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 +
15*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b*
*2*x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**
3*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2} - \frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

input
```
integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int x\sqrt{a+bx}\,dx = \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a}{b}+\frac{3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2}{b}\right)}{15b}$$

input
```
integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 1
0*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

3.178.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x\sqrt{a+bx}\,dx = -\frac{10\,a\,(a+b\,x)^{3/2}-6\,(a+b\,x)^{5/2}}{15\,b^2}$$

input
```
int(x*(a + b*x)^(1/2),x)
```

output
```
-(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

3.178.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x\sqrt{a+bx}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2+abx-2a^2)}{15b^2}$$

input
```
int(sqrt(a + b*x)*x,x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.179 $\int x^2\sqrt{a+bx}\,dx$

3.179.1 Optimal result . . . 1110
3.179.2 Mathematica [A] (verified) . . . 1110
3.179.3 Rubi [A] (verified) . . . 1111
3.179.4 Maple [A] (verified) . . . 1112
3.179.5 Fricas [A] (verification not implemented) . . . 1112
3.179.6 Sympy [B] (verification not implemented) . . . 1113
3.179.7 Maxima [A] (verification not implemented) . . . 1114
3.179.8 Giac [B] (verification not implemented) . . . 1115
3.179.9 Mupad [B] (verification not implemented) . . . 1115
3.179.10 Reduce [B] (verification not implemented) . . . 1116

3.179.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2\sqrt{a+bx}\,dx=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}$$

output
```
2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3
```

3.179.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

input
```
Integrate[x^2*Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

3.179.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

input
```
Int[x^2*Sqrt[a + b*x],x]
```

output
```
(2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a +
b*x)^(7/2))/(7*b^3)
```

3.179.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.179.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}(15x^2b^2-12axb+8a^2)}{105b^3}$	32
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}(15x^2b^2-12axb+8a^2)}{105b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{b^3}$	38
trager	$\frac{2(15x^3b^3+3a\,b^2x^2-4a^2bx+8a^3)\sqrt{bx+a}}{105b^3}$	43
risch	$\frac{2(15x^3b^3+3a\,b^2x^2-4a^2bx+8a^3)\sqrt{bx+a}}{105b^3}$	43

input
```
int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{a+bx}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.

Time = 1.00 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int x^2\sqrt{a+bx}\,dx = \frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

input
```
integrate(x**2*(b*x+a)**(1/2),x)
```

output
```
16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b
**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b*
*4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1
+ b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*
b**6*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*
b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(
105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
 - 48*a**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5
*x**2 + 105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*
a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 1
6*a**(17/2)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**
2 + 105*a**5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**
8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a
**(13/2)*b**5*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*
a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x
/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*
x**3)
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3} - \frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3} + \frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*
a^2/b^3
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int x^2\sqrt{a+bx}\,dx$$
$$= \frac{2\left(\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a}{b^2}+\frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}a^3\right)}{b^2}\right)}{105\,b}$$

input
```
integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{a+bx}\,dx = \frac{30\,(a+b\,x)^{7/2}-84\,a\,(a+b\,x)^{5/2}+70\,a^2\,(a+b\,x)^{3/2}}{105\,b^3}$$

input
```
int(x^2*(a + b*x)^(1/2),x)
```

output
```
(30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*
b^3)
```

3.179.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2\sqrt{a+bx}\,dx = \frac{2\sqrt{bx+a}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105b^3}$$

input
```
int(sqrt(a + b*x)*x**2,x)
```

output
```
(2*sqrt(a + b*x)*(8*a**3 - 4*a**2*b*x + 3*a*b**2*x**2 + 15*b**3*x**3))/(10
5*b**3)
```

3.180 $\int \frac{\sqrt{a+bx}}{x} dx$

3.180.1 Optimal result 1117
3.180.2 Mathematica [A] (verified) 1117
3.180.3 Rubi [A] (verified) 1118
3.180.4 Maple [A] (verified) 1119
3.180.5 Fricas [A] (verification not implemented) 1119
3.180.6 Sympy [B] (verification not implemented) 1120
3.180.7 Maxima [A] (verification not implemented) 1120
3.180.8 Giac [A] (verification not implemented) 1121
3.180.9 Mupad [B] (verification not implemented) 1121
3.180.10 Reduce [B] (verification not implemented) 1121

3.180.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*x]/x,x]
```

output
```
2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

3.180.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x} \, dx$$

$$\Big\downarrow \ 60$$

$$a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}$$

$$\Big\downarrow \ 73$$

$$\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}$$

$$\Big\downarrow \ 221$$

$$2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[a + b*x]/x,x]
```

output
```
2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

3.180.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.180.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28
default	$-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28
pseudoelliptic	$-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28

input
```
int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\sqrt{bx+a},\, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

input
```
integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2
*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.

Time = 0.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{a+bx}}{x}\,dx = -2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(1/2)/x,x)
```

output
```
-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/
(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + 2\sqrt{bx+a}$$

input
```
integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(
b*x + a)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{bx+a}$$

input
```
integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+b\,x} - 2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

input
```
int((a + b*x)^(1/2)/x,x)
```

output
```
2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

3.180.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{bx+a} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right) - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)$$

input
```
int(sqrt(a + b*x)/x,x)
```

output
```
2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))
```

3.181 $\int \frac{\sqrt{a+bx}}{x^2} dx$

3.181.1 Optimal result 1122
3.181.2 Mathematica [A] (verified) 1122
3.181.3 Rubi [A] (verified) 1123
3.181.4 Maple [A] (verified) 1124
3.181.5 Fricas [A] (verification not implemented) 1124
3.181.6 Sympy [A] (verification not implemented) 1125
3.181.7 Maxima [A] (verification not implemented) 1125
3.181.8 Giac [A] (verification not implemented) 1126
3.181.9 Mupad [B] (verification not implemented) 1126
3.181.10Reduce [B] (verification not implemented) 1126

3.181.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

3.181.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b*x]/x^2,x]
```

output
```
-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

3.181.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^2} \, dx$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2} b \int \frac{1}{x\sqrt{a+bx}} dx - \frac{\sqrt{a+bx}}{x}$$

$$\Big\downarrow \ 73$$

$$\int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx} - \frac{\sqrt{a+bx}}{x}$$

$$\Big\downarrow \ 221$$

$$-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{a+bx}}{x}$$

input
```
Int[Sqrt[a + b*x]/x^2,x]
```

output
```
-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

3.181.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.181.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$	32
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{bx+a}\,\sqrt{a}}{x\sqrt{a}}$	36
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37
default	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37

input
```
int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx$$
$$=\left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2ax},\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}a}{ax}\right]$$

input
```
integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*
x + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x
 + a)*a)/(a*x)]
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate((b*x+a)**(1/2)/x**2,x)
```

output
```
-sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sq
rt(a)
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$$

input
```
integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - s
qrt(b*x + a)/x
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}b}{x}}{b}$$

input
```
integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

3.181.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+b\,x}}{x} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int((a + b*x)^(1/2)/x^2,x)
```

output
```
- (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.181.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{-2\sqrt{bx+a}\,a+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2ax}$$

input
```
int(sqrt(a + b*x)/x**2,x)
```

output
```
( - 2*sqrt(a + b*x)*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x - sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a*x)
```

3.182 $\int \frac{1}{\sqrt{a+bx}}\,dx$

3.182.1 Optimal result 1127
3.182.2 Mathematica [A] (verified) 1127
3.182.3 Rubi [A] (verified) 1128
3.182.4 Maple [A] (verified) 1129
3.182.5 Fricas [A] (verification not implemented) 1129
3.182.6 Sympy [A] (verification not implemented) 1130
3.182.7 Maxima [A] (verification not implemented) 1130
3.182.8 Giac [A] (verification not implemented) 1130
3.182.9 Mupad [B] (verification not implemented) 1131
3.182.10 Reduce [B] (verification not implemented) 1131

3.182.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

output
```
2*(b*x+a)^(1/2)/b
```

3.182.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

input
```
Integrate[1/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x])/b
```

3.182.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2\sqrt{a+bx}}{b}$$

input
```
Int[1/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x])/b
```

3.182.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.182.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2\sqrt{bx+a}}{b}$	13
derivativedivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13

input
```
int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)/b
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x + a)/b
```

3.182.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

input
```
integrate(1/(b*x+a)**(1/2),x)
```

output
```
2*sqrt(a + b*x)/b
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{bx+a}}{b}$$

input
```
integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{bx+a}}{b}$$

input
```
integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x + a)/b
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{a+b\,x}}{b}$$

input
```
int(1/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(1/2))/b
```

3.182.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
int(1/sqrt(a + b*x),x)
```

output
```
(2*sqrt(a + b*x))/b
```

3.183 $\int \frac{x}{\sqrt{a+bx}} \, dx$

3.183.1 Optimal result 1132
3.183.2 Mathematica [A] (verified) 1132
3.183.3 Rubi [A] (verified) 1133
3.183.4 Maple [A] (verified) 1134
3.183.5 Fricas [A] (verification not implemented) 1134
3.183.6 Sympy [B] (verification not implemented) 1135
3.183.7 Maxima [A] (verification not implemented) 1135
3.183.8 Giac [A] (verification not implemented) 1136
3.183.9 Mupad [B] (verification not implemented) 1136
3.183.10Reduce [B] (verification not implemented) 1136

3.183.1 Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x}{\sqrt{a+bx}} \, dx = -\frac{2a\sqrt{a+bx}}{b^2} + \frac{2(a+bx)^{3/2}}{3b^2}$$

output
```
2/3*(b*x+a)^(3/2)/b^2-2*a*(b*x+a)^(1/2)/b^2
```

3.183.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{a+bx}} \, dx = \frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

input
```
Integrate[x/Sqrt[a + b*x],x]
```

output
```
(2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

3.183.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{\sqrt{a+bx}}{b}-\frac{a}{b\sqrt{a+bx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

input
```
Int[x/Sqrt[a + b*x],x]
```

output
```
(-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

3.183.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.183.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
trager	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
risch	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^{2}}$	26
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^{2}}$	26

input
```
int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

input
```
integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

3.183.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.

Time = 0.61 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{x}{\sqrt{a+bx}}\,dx = -\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

input
```
integrate(x/(b*x+a)**(1/2),x)
```

output
```
-4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**
2*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b
**3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*
sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2} - \frac{2\,\sqrt{bx+a}a}{b^2}$$

input
```
integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)}{3\,b^2}$$

input
```
integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{a+bx}}\,dx = -\frac{6\,a\,\sqrt{a+b\,x} - 2\,(a+b\,x)^{3/2}}{3\,b^2}$$

input
```
int(x/(a + b*x)^(1/2),x)
```

output
```
-(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

3.183.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.56

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(bx-2a)}{3b^2}$$

input
```
int(x/sqrt(a + b*x),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a + b*x))/(3*b**2)
```

3.184 $\int \frac{x^2}{\sqrt{a+bx}} dx$

3.184.1 Optimal result 1137
3.184.2 Mathematica [A] (verified) 1137
3.184.3 Rubi [A] (verified) 1138
3.184.4 Maple [A] (verified) 1139
3.184.5 Fricas [A] (verification not implemented) 1139
3.184.6 Sympy [B] (verification not implemented) 1140
3.184.7 Maxima [A] (verification not implemented) 1141
3.184.8 Giac [A] (verification not implemented) 1141
3.184.9 Mupad [B] (verification not implemented) 1142
3.184.10Reduce [B] (verification not implemented) 1142

3.184.1 Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}$$

output
```
-4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3+2*a^2*(b*x+a)^(1/2)/b^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}(8a^2-4abx+3b^2x^2)}{15b^3}$$

input
```
Integrate[x^2/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

3.184.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \text{ 53}$$
$$\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

input
```
Int[x^2/Sqrt[a + b*x],x]
```

output
```
(2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(
5/2))/(5*b^3)
```

3.184.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.184.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4axb+8a^2)}{15b^3}$	32
trager	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4axb+8a^2)}{15b^3}$	32
risch	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4axb+8a^2)}{15b^3}$	32
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4axb+8a^2)}{15b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37

input
```
int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(48) = 96$.

Time = 0.99 (sec) , antiderivative size = 600, normalized size of antiderivative = 11.76

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

input

```
integrate(x**2/(b*x+a)**(1/2),x)
```

output
```
16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5
*x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x +
 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)
/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) -
 48*a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*
a**5*b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 4
5*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2
*x**2/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x*
*3) + 10*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*
x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a*
*8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**
(13/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*
b**5*x**2 + 15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15
*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3} - \frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3} + \frac{2\,\sqrt{bx+a}a^2}{b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/
b^3
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

3.184.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{6\,{\left(a+b\,x\right)}^{5/2}-20\,a\,{\left(a+b\,x\right)}^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

input
```
int(x^2/(a + b*x)^(1/2),x)
```

output
```
(6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^
3)
```

3.184.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$$

input
```
int(x**2/sqrt(a + b*x),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2 - 4*a*b*x + 3*b**2*x**2))/(15*b**3)
```

3.185 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

3.185.1 Optimal result 1143
3.185.2 Mathematica [A] (verified) 1143
3.185.3 Rubi [A] (verified) 1144
3.185.4 Maple [A] (verified) 1145
3.185.5 Fricas [A] (verification not implemented) 1145
3.185.6 Sympy [A] (verification not implemented) 1145
3.185.7 Maxima [A] (verification not implemented) 1146
3.185.8 Giac [A] (verification not implemented) 1146
3.185.9 Mupad [B] (verification not implemented) 1147
3.185.10 Reduce [B] (verification not implemented) 1147

3.185.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

3.185.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \; 73$$

$$\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}$$

$$\Big\downarrow \; 221$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

3.185.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.185.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

input
```
int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(s
qrt(b*x + a)*sqrt(-a)/a)/a]
```

3.185.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x)^(1/2)),x)
```

output
```
-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.185.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/(sqrt(a + b*x)*x),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.186 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

3.186.1 Optimal result 1148
3.186.2 Mathematica [A] (verified) 1148
3.186.3 Rubi [A] (verified) 1149
3.186.4 Maple [A] (verified) 1150
3.186.5 Fricas [A] (verification not implemented) 1150
3.186.6 Sympy [A] (verification not implemented) 1151
3.186.7 Maxima [A] (verification not implemented) 1151
3.186.8 Giac [A] (verification not implemented) 1152
3.186.9 Mupad [B] (verification not implemented) 1152
3.186.10 Reduce [B] (verification not implemented) 1152

3.186.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x
```

3.186.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

output
```
-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

3.186.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx}}dx$$

$$\Big\downarrow \; 52$$

$$-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}$$

$$\Big\downarrow \; 73$$

$$-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}$$

$$\Big\downarrow \; 221$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x]),x]
```

output
```
-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

3.186.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.186.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{ax}$	34
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}}x}$	36
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

input
```
int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx=\left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2a^2x},\ -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

input
```
integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*
x + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(
b*x + a)*a)/(a^2*x)]
```

3.186.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}} + \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/x**2/(b*x+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))
)/a**(3/2)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{bx+a}b}{(bx+a)a-a^2} - \frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))
/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx+a}b}{ax}}{b}$$

input
```
integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))
/b
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+b\,x}}{a\,x}$$

input
```
int(1/(x^2*(a + b*x)^(1/2)),x)
```

output
```
(b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

3.186.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \frac{-2\sqrt{bx+a}\,a - \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx + \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2a^2x}$$

input
```
int(1/(sqrt(a + b*x)*x**2),x)
```

output
```
( - 2*sqrt(a + b*x)*a - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x + sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a**2*x)
```

3.187 $\int (a + bx)^{p/2}\, dx$

3.187.1 Optimal result . . . 1153
3.187.2 Mathematica [A] (verified) . . . 1153
3.187.3 Rubi [A] (verified) . . . 1154
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3.187.9 Mupad [B] (verification not implemented) . . . 1156
3.187.10Reduce [B] (verification not implemented) . . . 1156

3.187.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int (a+bx)^{p/2}\, dx = \frac{2(a+bx)^{\frac{2+p}{2}}}{b(2+p)}$$

output
```
2*(b*x+a)^(1+1/2*p)/b/(2+p)
```

3.187.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int (a+bx)^{p/2}\, dx = \frac{(a+bx)^{1+\frac{p}{2}}}{b\left(1+\frac{p}{2}\right)}$$

input
```
Integrate[(a + b*x)^(p/2),x]
```

output
```
(a + b*x)^(1 + p/2)/(b*(1 + p/2))
```

3.187.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{p/2}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

input
```
Int[(a + b*x)^(p/2),x]
```

output
```
(2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

3.187.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.187.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$	25
risch	$\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$	25

input
```
int(((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+bx)^{p/2}\,dx = \frac{2\,(bx+a)\sqrt{bx+a}^{p}}{bp+2\,b}$$

input
```
integrate(((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)
```

3.187.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (a+bx)^{p/2}\,dx = \frac{\begin{cases} \frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for } p \neq -2 \\ \log{(a+bx)} & \text{otherwise} \end{cases}}{b}$$

input
```
integrate(((b*x+a)**(1/2))**p,x)
```

output
```
Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p, -2)), (log(a + b*x), True
))/b
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a+bx)^{p/2}\,dx = \frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b(p+2)}$$

input
```
integrate(((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a+bx)^{p/2}\,dx = \frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b(p+2)}$$

input
```
integrate(((b*x+a)^(1/2))^p,x, algorithm="giac")
```

output
```
2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

3.187.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a+bx)^{p/2}\,dx = \frac{2\,(a+b\,x)^{\frac{p}{2}+1}}{b\,(p+2)}$$

input
```
int((a + b*x)^(p/2),x)
```

output
```
(2*(a + b*x)^(p/2 + 1))/(b*(p + 2))
```

3.187.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (a+bx)^{p/2}\,dx = \frac{2(bx+a)^{\frac{p}{2}}\,(bx+a)}{b\,(p+2)}$$

input
```
int((a + b*x)**(p/2),x)
```

output
```
(2*(a + b*x)**(p/2)*(a + b*x))/(b*(p + 2))
```

3.187. $\int (a+bx)^{p/2}\,dx$

3.188 $\int x(a+bx)^{p/2}\,dx$

3.188.1 Optimal result 1158
3.188.2 Mathematica [A] (verified) 1158
3.188.3 Rubi [A] (verified) 1159
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3.188.6 Sympy [B] (verification not implemented) 1160
3.188.7 Maxima [A] (verification not implemented) 1161
3.188.8 Giac [A] (verification not implemented) 1161
3.188.9 Mupad [B] (verification not implemented) 1162
3.188.10Reduce [B] (verification not implemented) 1162

3.188.1 Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x(a+bx)^{p/2}\,dx=-\frac{2a(a+bx)^{\frac{2+p}{2}}}{b^2(2+p)}+\frac{2(a+bx)^{\frac{4+p}{2}}}{b^2(4+p)}$$

output
```
-2*a*(b*x+a)^(1+1/2*p)/b^2/(2+p)+2*(b*x+a)^(2+1/2*p)/b^2/(4+p)
```

3.188.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int x(a+bx)^{p/2}\,dx=\frac{2(a+bx)^{1+\frac{p}{2}}(-2a+b(2+p)x)}{b^2(2+p)(4+p)}$$

input
```
Integrate[x*(a + b*x)^(p/2),x]
```

output
```
(2*(a + b*x)^(1 + p/2)*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p))
```

3.188.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{p/2} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{(a+bx)^{\frac{p}{2}+1}}{b} - \frac{a(a+bx)^{p/2}}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)} - \frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

input
```
Int[x*(a + b*x)^(p/2),x]
```

output
```
(-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p)/2))/(b^
2*(4 + p))
```

3.188.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.188.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{p}{2}}(-xpb-2bx+2a)(bx+a)}{b^2(p^2+6p+8)}$	43
risch	$-\frac{2(-x^2b^2p-xapb-2x^2b^2+2a^2)(bx+a)^{\frac{p}{2}}}{b^2(4+p)(2+p)}$	54

input
```
int(x*((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)
```

output
```
-2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x(a+bx)^{p/2}\,dx = \frac{2\,(abpx+(b^2p+2\,b^2)x^2-2\,a^2)\sqrt{bx+a}^p}{b^2p^2+6\,b^2p+8\,b^2}$$

input
```
integrate(x*((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^2 + 6*b^2
*p + 8*b^2)
```

3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 4.50

$$\int x(a+bx)^{p/2}\,dx = \begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for } b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p=-4 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b} & \text{for } p=-2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2}+\frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2}+\frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2}+\frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*((b*x+a)**(1/2))**p,x)
```

output
```
Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) +
 a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -4)), (-a
*log(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*(a + b*x)**(p/2)/(b**2*p**2
 + 6*b**2*p + 8*b**2) + 2*a*b*p*x*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p +
 8*b**2) + 2*b**2*p*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2)
+ 4*b**2*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int x(a+bx)^{p/2}\,dx=\frac{2\,(b^2(p+2)x^2+abpx-2\,a^2)(bx+a)^{\frac{1}{2}\,p}}{(p^2+6\,p+8)b^2}$$

input
```
integrate(x*((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6*p + 8)*b
^2)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int x(a+bx)^{p/2}\,dx=\frac{2\left((bx+a)^{\frac{1}{2}\,p}b^2px^2+(bx+a)^{\frac{1}{2}\,p}abpx+2\,(bx+a)^{\frac{1}{2}\,p}b^2x^2-2\,(bx+a)^{\frac{1}{2}\,p}a^2\right)}{b^2p^2+6\,b^2p+8\,b^2}$$

input
```
integrate(x*((b*x+a)^(1/2))^p,x, algorithm="giac")
```

output
```
2*((b*x + a)^(1/2*p)*b^2*p*x^2 + (b*x + a)^(1/2*p)*a*b*p*x + 2*(b*x + a)^(
1/2*p)*b^2*x^2 - 2*(b*x + a)^(1/2*p)*a^2)/(b^2*p^2 + 6*b^2*p + 8*b^2)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int x(a+bx)^{p/2}\,dx = \begin{cases} -\frac{a\ln(a+b\,x)-b\,x}{b^2} & \text{if } p=-2 \\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } p=-4 \\ \frac{2\left(\frac{(a+b\,x)^{\frac{p}{2}+2}}{p+4}-\frac{a\,(a+b\,x)^{\frac{p}{2}+1}}{p+2}\right)}{b^2} & \text{if } p\neq -2\land p\neq -4 \end{cases}$$

input
```
int(x*(a + b*x)^(p/2),x)
```

output
```
piecewise(p == -2, -(a*log(a + b*x) - b*x)/b^2, p == -4, (log(a + b*x) + a
/(a + b*x))/b^2, p ~= -2 & p ~= -4, (2*((a + b*x)^(p/2 + 2)/(p + 4) - (a*(
a + b*x)^(p/2 + 1))/(p + 2)))/b^2)
```

3.188.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x(a+bx)^{p/2}\,dx = \frac{2(bx+a)^{\frac{p}{2}}\left(b^2p\,x^2+abpx+2b^2x^2-2a^2\right)}{b^2\left(p^2+6p+8\right)}$$

input
```
int((a + b*x)**(p/2)*x,x)
```

output
```
(2*(a + b*x)**(p/2)*( - 2*a**2 + a*b*p*x + b**2*p*x**2 + 2*b**2*x**2))/(b*
*2*(p**2 + 6*p + 8))
```

3.189 $\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

3.189.1 Optimal result . . . 1163
3.189.2 Mathematica [A] (verified) . . . 1163
3.189.3 Rubi [A] (verified) . . . 1164
3.189.4 Maple [A] (verified) . . . 1166
3.189.5 Fricas [A] (verification not implemented) . . . 1167
3.189.6 Sympy [B] (verification not implemented) . . . 1167
3.189.7 Maxima [A] (verification not implemented) . . . 1168
3.189.8 Giac [A] (verification not implemented) . . . 1168
3.189.9 Mupad [B] (verification not implemented) . . . 1168
3.189.10Reduce [B] (verification not implemented) . . . 1169

3.189.1 Optimal result

Integrand size = 18, antiderivative size = 55

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx = \frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}} - x\arctan\left(1-\sqrt{2}x\right) - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

output
```
x*arctan(-1+x*2^(1/2))-1/2*arctan(-1+x*2^(1/2))*2^(1/2)-1/4*ln(1+x^2-x*2^(
1/2))*2^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx = \frac{1}{4}\left(2\left(\sqrt{2}-2x\right)\arctan\left(1-\sqrt{2}x\right)-\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

output
```
(2*(Sqrt[2] - 2*x)*ArcTan[1 - Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2
])/4
```

3.189.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5726, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\frac{2x-\sqrt{2}}{\sqrt{2}}\right)dx$$

$$\Big\downarrow \text{5726}$$

$$-\int \frac{x}{\sqrt{2}\left(x^2-\sqrt{2}x+1\right)}dx-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{x}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{\int \frac{1}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(1-\sqrt{2}x\right)}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{1}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\left(1-\sqrt{2}x\right)}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{1}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}-\frac{\int \frac{1-\sqrt{2}x}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)-\frac{\int \frac{1-\sqrt{2}x}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{-\frac{\int \frac{1-\sqrt{2}x}{x^2-\sqrt{2}x+1}dx}{\sqrt{2}}-\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{\frac{1}{2}\log\left(x^2-\sqrt{2}x+1\right)-\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-x\arctan\left(1-\sqrt{2}x\right)$$

input
```
Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

output
```
-(x*ArcTan[1 - Sqrt[2]*x]) - (-ArcTan[1 - Sqrt[2]*x] + Log[1 - Sqrt[2]*x +
 x^2]/2)/Sqrt[2]
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5726
```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

3.189.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\sqrt{2}\left(\left(-1+x\sqrt{2}\right)\arctan\left(-1+x\sqrt{2}\right)-\frac{\ln\left(\left(-1+x\sqrt{2}\right)^{2}+1\right)}{2}\right)}{2}$
default	$\frac{\sqrt{2}\left(\left(-1+x\sqrt{2}\right)\arctan\left(-1+x\sqrt{2}\right)-\frac{\ln\left(\left(-1+x\sqrt{2}\right)^{2}+1\right)}{2}\right)}{2}$
parts	$x\arctan\left(\frac{\left(2x-\sqrt{2}\right)\sqrt{2}}{2}\right)-2\sqrt{2}\left(\frac{\ln\left(1+x^{2}-x\sqrt{2}\right)}{8}+\frac{\arctan\left(\frac{\left(2x-\sqrt{2}\right)\sqrt{2}}{2}\right)}{4}\right)$
risch	$\frac{ix\ln\left(1+\frac{i\left(-2x+\sqrt{2}\right)\sqrt{2}}{2}\right)}{2}-\frac{ix\ln\left(1-\frac{i\left(-2x+\sqrt{2}\right)\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(4-4x\sqrt{2}+4x^{2}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{\left(2x-\sqrt{2}\right)\sqrt{2}}{2}\right)}{2}$
parallelrisch	$\frac{\sqrt{2}\,\ln\left(1+x^{2}-x\sqrt{2}\right)x^{2}+6\sqrt{2}\,\arctan\left(\frac{\left(2x-\sqrt{2}\right)\sqrt{2}}{2}\right)x^{2}-4x^{3}\arctan\left(\frac{\left(2x-\sqrt{2}\right)\sqrt{2}}{2}\right)+\ln\left(1+x^{2}-x\sqrt{2}\right)\sqrt{2}+2\sqrt{2}\,\text{arcta}}{4x\sqrt{2}-4x^{2}-4}$

input
```
int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*((-1+x*2^(1/2))*arctan(-1+x*2^(1/2))-1/2*ln((-1+x*2^(1/2))^2+1
))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx = \frac{1}{2}\left(2\,x-\sqrt{2}\right)\arctan\left(\sqrt{2}x-1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*x - sqrt(2))*arctan(sqrt(2)*x - 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*
x + 1)
```

3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(48) = 96.

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.18

$$\begin{aligned}\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx = {} & \frac{4x^3\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}-\frac{\sqrt{2}x^2\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4} \\ & -\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4} \\ & +\frac{2x\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}+\frac{8x\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4} \\ & -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}-\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}\end{aligned}$$

input
```
integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)
```

output
```
4*x**3*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*x**2*log(x
**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 6*sqrt(2)*x**2*atan(sqrt
(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 2*x*log(x**2 - sqrt(2)*x + 1)/(4*x
**2 - 4*sqrt(2)*x + 4) + 8*x*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4
) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 2*sqrt(
2)*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx = \frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^2+1\right)\right)$$

input
```
integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
 log(1/2*(2*x - sqrt(2))^2 + 1))
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx = \frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^2+1\right)\right)$$

input
```
integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
 log(1/2*(2*x - sqrt(2))^2 + 1))
```

3.189.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx = \operatorname{atan}\left(\frac{\sqrt{2}\,\left(2x-\sqrt{2}\right)}{2}\right)\,\left(x-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}\,\ln\left(\left(2x-\sqrt{2}\right)^2+2\right)}{4}$$

input
```
int(atan((2^(1/2)*(2*x - 2^(1/2)))/2),x)
```

output
```
atan((2^(1/2)*(2*x - 2^(1/2)))/2)*(x - 2^(1/2)/2) - (2^(1/2)*log((2*x - 2^
(1/2))^2 + 2))/4
```

3.189.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx = -\frac{\sqrt{2}\,atan\left(\sqrt{2}\,x-1\right)}{2} + atan\left(\sqrt{2}\,x-1\right)x - \frac{\sqrt{2}\log\left(-\sqrt{2}\,x+x^2+1\right)}{4}$$

input
```
int(atan(sqrt(2)*x - 1),x)
```

output
```
( - 2*sqrt(2)*atan(sqrt(2)*x - 1) + 4*atan(sqrt(2)*x - 1)*x - sqrt(2)*log(
 - sqrt(2)*x + x**2 + 1))/4
```

3.190 $\int \frac{1}{\sqrt{-1+x^2}}\,dx$

3.190.1 Optimal result 1170
3.190.2 Mathematica [B] (verified) 1170
3.190.3 Rubi [A] (verified) 1171
3.190.4 Maple [A] (verified) 1172
3.190.5 Fricas [A] (verification not implemented) 1172
3.190.6 Sympy [A] (verification not implemented) 1172
3.190.7 Maxima [A] (verification not implemented) 1173
3.190.8 Giac [B] (verification not implemented) 1173
3.190.9 Mupad [B] (verification not implemented) 1173
3.190.10 Reduce [B] (verification not implemented) 1174

3.190.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
arctanh(x/(x^2-1)^(1/2))
```

3.190.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)$$

input
```
Integrate[1/Sqrt[-1 + x^2],x]
```

output
```
-1/2*Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]/2
```

3.190.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}}dx$$
$$\Big\downarrow \text{224}$$
$$\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}$$
$$\Big\downarrow \text{219}$$
$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

input
```
Int[1/Sqrt[-1 + x^2],x]
```

output
```
ArcTanh[x/Sqrt[-1 + x^2]]
```

3.190.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.190.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\ln\left(x+\sqrt{x^{2}-1}\right)$	11
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{x^{2}-1}}{x}\right)$	13
trager	$-\ln\left(-\sqrt{x^{2}-1}+x\right)$	15
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{2}-1\right)}\, \arcsin\left(x\right)}{\sqrt{\operatorname{signum}\left(x^{2}-1\right)}}$	22

input
```
int(1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x+(x^2-1)^(1/2))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x^2}}\, dx = -\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate(1/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 - 1))
```

3.190.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+x^2}}\, dx = \log\left(x+\sqrt{x^2-1}\right)$$

input
```
integrate(1/(x**2-1)**(1/2),x)
```

output
```
log(x + sqrt(x**2 - 1))
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \log\left(2\,x + 2\,\sqrt{x^2-1}\right)$$

input
```
integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - 1))
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2-1}x + \frac{1}{2}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate(1/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

3.190.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \ln\left(x+\sqrt{x^2-1}\right)$$

input
```
int(1/(x^2 - 1)^(1/2),x)
```

output
```
log(x + (x^2 - 1)^(1/2))
```

3.190.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \log\left(\sqrt{x^2-1}+x\right)$$

input
```
int(1/sqrt(x**2 - 1),x)
```

output
```
log(sqrt(x**2 - 1) + x)
```

3.191 $\int \sqrt{x}\sqrt{1+x}\,dx$

3.191.1 Optimal result 1175
3.191.2 Mathematica [A] (verified) 1175
3.191.3 Rubi [A] (verified) 1176
3.191.4 Maple [A] (verified) 1177
3.191.5 Fricas [A] (verification not implemented) 1178
3.191.6 Sympy [C] (verification not implemented) 1178
3.191.7 Maxima [B] (verification not implemented) 1178
3.191.8 Giac [A] (verification not implemented) 1179
3.191.9 Mupad [B] (verification not implemented) 1179
3.191.10 Reduce [B] (verification not implemented) 1180

3.191.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \sqrt{x}\sqrt{1+x}\,dx = \frac{1}{4}\sqrt{x}\sqrt{1+x} + \frac{1}{2}x^{3/2}\sqrt{1+x} - \frac{\operatorname{arcsinh}(\sqrt{x})}{4}$$

output
```
-1/4*arcsinh(x^(1/2))+1/2*x^(3/2)*(1+x)^(1/2)+1/4*x^(1/2)*(1+x)^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\sqrt{1+x}\,dx = \frac{1}{4}\left(\sqrt{x}\sqrt{1+x}(1+2x) + \log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)$$

input
```
Integrate[Sqrt[x]*Sqrt[1 + x],x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(1 + 2*x) + Log[-Sqrt[x] + Sqrt[1 + x]])/4
```

3.191.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{x+1}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}x^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx\right)+\frac{1}{2}\sqrt{x+1}x^{3/2}$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}\right)+\frac{1}{2}\sqrt{x+1}x^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{x+1}x^{3/2}$$

input
```
Int[Sqrt[x]*Sqrt[1 + x],x]
```

output
```
(x^(3/2)*Sqrt[1 + x])/2 + (Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]])/4
```

3.191.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.191.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
meijerg	$-\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,(3+6x)\sqrt{1+x}}{6}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}}{2\sqrt{\pi}}$	34
risch	$\frac{(1+2x)\sqrt{x}\,\sqrt{1+x}}{4}-\frac{\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{8\sqrt{1+x}\,\sqrt{x}}$	45
default	$\frac{\sqrt{x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{x}\,\sqrt{1+x}}{4}-\frac{\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{8\sqrt{1+x}\,\sqrt{x}}$	50

input
```
int(x^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/Pi^(1/2)*(-1/6*Pi^(1/2)*x^(1/2)*(3+6*x)*(1+x)^(1/2)+1/2*Pi^(1/2)*arcs
inh(x^(1/2)))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{x}\sqrt{1+x}\,dx = \frac{1}{4}\,(2\,x+1)\sqrt{x+1}\sqrt{x} + \frac{1}{8}\,\log\left(2\,\sqrt{x+1}\sqrt{x} - 2\,x - 1\right)$$

input
```
integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/8*log(2*sqrt(x + 1)*sqrt(x) - 2*x -
1)
```

3.191.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.77

$$\int \sqrt{x}\sqrt{1+x}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\sqrt{x+1}\right)}{4} + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}} - \frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}} + \frac{\sqrt{x+1}}{4\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{i\operatorname{asin}\left(\sqrt{x+1}\right)}{4} - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}} + \frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}} - \frac{i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(1+x)**(1/2),x)
```

output
```
Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3*(x + 1)*
*(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1) > 1), (I*asin(sqr
t(x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3*I*(x + 1)**(3/2)/(4*sqrt(-
x)) - I*sqrt(x + 1)/(4*sqrt(-x)), True))
```

3.191.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \sqrt{x}\sqrt{1+x}\,dx = \frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} + \frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^2}{x^2} - \frac{2\,(x+1)}{x} + 1\right)} - \frac{1}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{1}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

input
```
integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x +
1)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x)
 - 1)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\sqrt{1+x}\,dx = \frac{1}{4}\,(2\,x-3)\sqrt{x+1}\sqrt{x}+\sqrt{x+1}\sqrt{x}+\frac{1}{4}\,\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

input
```
integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*x - 3)*sqrt(x + 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 1/4*log(sqrt(x +
 1) - sqrt(x))
```

3.191.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\sqrt{1+x}\,dx = \sqrt{x}\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x+1}-\frac{\ln\left(x+\sqrt{x}\,\sqrt{x+1}+\frac{1}{2}\right)}{8}$$

input
```
int(x^(1/2)*(x + 1)^(1/2),x)
```

output
```
x^(1/2)*(x/2 + 1/4)*(x + 1)^(1/2) - log(x + x^(1/2)*(x + 1)^(1/2) + 1/2)/8
```

3.191.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \sqrt{x}\sqrt{1+x}\,dx = \frac{\sqrt{x}\,\sqrt{x+1}\,x}{2} + \frac{\sqrt{x}\,\sqrt{x+1}}{4} - \frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4}$$

input
```
int(sqrt(x)*sqrt(x + 1),x)
```

output
```
(2*sqrt(x)*sqrt(x + 1)*x + sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x)
))/4
```

3.192 $\int \sin\left(\sqrt{x}\right)\,dx$

3.192.1 Optimal result 1181
3.192.2 Mathematica [A] (verified) 1181
3.192.3 Rubi [A] (verified) 1182
3.192.4 Maple [A] (verified) 1183
3.192.5 Fricas [A] (verification not implemented) 1184
3.192.6 Sympy [A] (verification not implemented) 1184
3.192.7 Maxima [A] (verification not implemented) 1184
3.192.8 Giac [A] (verification not implemented) 1185
3.192.9 Mupad [B] (verification not implemented) 1185
3.192.10Reduce [B] (verification not implemented) 1185

3.192.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

output
```
2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
Integrate[Sin[Sqrt[x]],x]
```

output
```
-2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

3.192.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3842, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\sqrt{x}\right)\, dx$$

$$\Big\downarrow \text{3842}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3777}$$

$$2\left(\int \cos\left(\sqrt{x}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\int \sin\left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$2\left(\sin\left(\sqrt{x}\right)-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

input
```
Int[Sin[Sqrt[x]],x]
```

output
```
2*(-(Sqrt[x]*Cos[Sqrt[x]]) + Sin[Sqrt[x]])
```

3.192.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

3.192.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

input
```
int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="fricas")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.192.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x**(1/2)),x)
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="maxima")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = 2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

3.192.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
int(sin(sqrt(x)),x)
```

output
```
2*( - sqrt(x)*cos(sqrt(x)) + sin(sqrt(x)))
```

3.193 $\int \frac{x}{(1-x^2)^{9/8}}\,dx$

3.193.1 Optimal result 1186
3.193.2 Mathematica [A] (verified) 1186
3.193.3 Rubi [A] (verified) 1187
3.193.4 Maple [A] (verified) 1188
3.193.5 Fricas [A] (verification not implemented) 1188
3.193.6 Sympy [A] (verification not implemented) 1189
3.193.7 Maxima [A] (verification not implemented) 1189
3.193.8 Giac [A] (verification not implemented) 1189
3.193.9 Mupad [B] (verification not implemented) 1190
3.193.10Reduce [F] 1190

3.193.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = \frac{4}{\sqrt[8]{1-x^2}}$$

output
```
4/(-x^2+1)^(1/8)
```

3.193.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = \frac{4}{\sqrt[8]{1-x^2}}$$

input
```
Integrate[x/(1 - x^2)^(9/8),x]
```

output
```
4/(1 - x^2)^(1/8)
```

3.193.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{4}{\sqrt[8]{1-x^2}}$$

input
```
Int[x/(1 - x^2)^(9/8),x]
```

output
```
4/(1 - x^2)^(1/8)
```

3.193.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.193.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$	12
default	$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$	12
risch	$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$	12
pseudoelliptic	$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$	12
meijerg	$\frac{x^{2} {}_{2}F_{1}\left(1,\frac{9}{8};2;x^{2}\right)}{2}$	15
gosper	$-\frac{4(-1+x)(1+x)}{\left(-x^{2}+1\right)^{\frac{9}{8}}}$	18
trager	$-\frac{4\left(-x^{2}+1\right)^{\frac{7}{8}}}{x^{2}-1}$	19

input
```
int(x/(-x^2+1)^(9/8),x,method=_RETURNVERBOSE)
```

output
```
4/(-x^2+1)^(1/8)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x}{\left(1-x^{2}\right)^{9/8}}\,dx=-\frac{4\left(-x^{2}+1\right)^{\frac{7}{8}}}{x^{2}-1}$$

input
```
integrate(x/(-x^2+1)^(9/8),x, algorithm="fricas")
```

output
```
-4*(-x^2 + 1)^(7/8)/(x^2 - 1)
```

3.193.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = \frac{4}{\sqrt[8]{1-x^2}}$$

input
```
integrate(x/(-x**2+1)**(9/8),x)
```

output
```
4/(1 - x**2)**(1/8)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = \frac{4}{(-x^2+1)^{\frac{1}{8}}}$$

input
```
integrate(x/(-x^2+1)^(9/8),x, algorithm="maxima")
```

output
```
4/(-x^2 + 1)^(1/8)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = \frac{4}{(-x^2+1)^{\frac{1}{8}}}$$

input
```
integrate(x/(-x^2+1)^(9/8),x, algorithm="giac")
```

output
```
4/(-x^2 + 1)^(1/8)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = \frac{4}{(1-x^2)^{1/8}}$$

input
```
int(x/(1 - x^2)^(9/8),x)
```

output
```
4/(1 - x^2)^(1/8)
```

3.193.10 Reduce [F]

$$\int \frac{x}{(1-x^2)^{9/8}}\,dx = -\left(\int \frac{x}{(-x^2+1)^{\frac{1}{8}}\,x^2 - (-x^2+1)^{\frac{1}{8}}}dx\right)$$

input
```
int(( - x)/(( - x**2 + 1)**(1/8)*(x**2 - 1)),x)
```

output
```
- int(x/(( - x**2 + 1)**(1/8)*x**2 - ( - x**2 + 1)**(1/8)),x)
```

3.194 $\int \frac{x}{\sqrt{1-x^4}} \, dx$

3.194.1 Optimal result 1191
3.194.2 Mathematica [A] (verified) 1191
3.194.3 Rubi [A] (verified) 1192
3.194.4 Maple [A] (verified) 1193
3.194.5 Fricas [B] (verification not implemented) 1193
3.194.6 Sympy [C] (verification not implemented) 1194
3.194.7 Maxima [B] (verification not implemented) 1194
3.194.8 Giac [A] (verification not implemented) 1194
3.194.9 Mupad [B] (verification not implemented) 1195
3.194.10 Reduce [B] (verification not implemented) 1195

3.194.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{x}{\sqrt{1-x^4}} \, dx = \frac{\arcsin(x^2)}{2}$$

output
```
1/2*arcsin(x^2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^4}} \, dx = \frac{\arcsin(x^2)}{2}$$

input
```
Integrate[x/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x^2]/2
```

3.194.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^4}} \, dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2} \int \frac{1}{\sqrt{1-x^4}} dx^2$$

$$\Big\downarrow \ 223$$

$$\frac{\arcsin \left(x^2\right)}{2}$$

input
```
Int[x/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x^2]/2
```

3.194.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.194.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arcsin(x^2)}{2}$	7
meijerg	$\frac{\arcsin(x^2)}{2}$	7
elliptic	$\frac{\arcsin(x^2)}{2}$	7
pseudoelliptic	$\frac{\arcsin(x^2)}{2}$	7
trager	$\frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(\mathrm{RootOf}(\_Z^2+1)\sqrt{-x^4+1}+x^2\right)}{2}$	30

input
```
int(x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x^2)
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

input
```
integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

3.194.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{i\operatorname{acosh}(x^2)}{2} & \text{for } |x^4| > 1 \\ \frac{\operatorname{asin}(x^2)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\arcsin\left(x^2\right)$$

input
```
integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*arcsin(x^2)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{2}$$

input
```
int(x/(1 - x^4)^(1/2),x)
```

output
```
atan(x^2/(1 - x^4)^(1/2))/2
```

3.194.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{asin(x^2)}{2}$$

input
```
int(x/sqrt( - x**4 + 1),x)
```

output
```
asin(x**2)/2
```

3.195 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

3.195.1 Optimal result 1196
3.195.2 Mathematica [A] (verified) 1196
3.195.3 Rubi [A] (verified) 1197
3.195.4 Maple [A] (verified) 1198
3.195.5 Fricas [B] (verification not implemented) 1199
3.195.6 Sympy [A] (verification not implemented) 1199
3.195.7 Maxima [B] (verification not implemented) 1199
3.195.8 Giac [B] (verification not implemented) 1200
3.195.9 Mupad [B] (verification not implemented) 1200
3.195.10 Reduce [B] (verification not implemented) 1200

3.195.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\operatorname{arctanh}\left(\sqrt{1+x^4}\right)$$

output
```
-1/2*arctanh((x^4+1)^(1/2))
```

3.195.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\operatorname{arctanh}\left(\sqrt{1+x^4}\right)$$

input
```
Integrate[1/(x*Sqrt[1 + x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[1 + x^4]]
```

3.195.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\sqrt{x^4+1}}dx^4$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\int \frac{1}{x^8-1}d\sqrt{x^4+1}$$

$$\Big\downarrow \text{220}$$

$$-\frac{1}{2}\text{arctanh}\left(\sqrt{x^4+1}\right)$$

input
```
Int[1/(x*Sqrt[1 + x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[1 + x^4]]
```

3.195.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.195.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
trager	$\frac{\ln\left(\frac{-1+\sqrt{x^4+1}}{x^2}\right)}{2}$	17
meijerg	$\frac{(-2\ln(2)+4\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	37

input
```
int(1/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/(x^4+1)^(1/2))
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

3.195.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

input
```
integrate(1/x/(x**4+1)**(1/2),x)
```

output
```
-asinh(x**(-2))/2
```

3.195.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int(1/(x*(x^4 + 1)^(1/2)),x)
```

output
```
-atanh((x^4 + 1)^(1/2))/2
```

3.195.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = \frac{\log\left(\sqrt{x^4+1}+x^2-1\right)}{2}-\frac{\log\left(\sqrt{x^4+1}+x^2+1\right)}{2}$$

input
```
int(1/(sqrt(x**4 + 1)*x),x)
```

output
```
(log(sqrt(x**4 + 1) + x**2 - 1) - log(sqrt(x**4 + 1) + x**2 + 1))/2
```

3.196 $\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx$

3.196.1 Optimal result . . . 1201
3.196.2 Mathematica [A] (verified) . . . 1201
3.196.3 Rubi [A] (verified) . . . 1202
3.196.4 Maple [A] (verified) . . . 1203
3.196.5 Fricas [A] (verification not implemented) . . . 1203
3.196.6 Sympy [F] . . . 1204
3.196.7 Maxima [F] . . . 1204
3.196.8 Giac [A] (verification not implemented) . . . 1204
3.196.9 Mupad [B] (verification not implemented) . . . 1205
3.196.10 Reduce [B] (verification not implemented) . . . 1205

3.196.1 Optimal result

Integrand size = 14, antiderivative size = 18

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\text{arcsinh}\left(\frac{1+2x^2}{\sqrt{3}}\right)$$

output
```
1/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))
```

3.196.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{2}\log\left(-1-2x^2+2\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[x/Sqrt[1 + x^2 + x^4],x]
```

output
```
-1/2*Log[-1 - 2*x^2 + 2*Sqrt[1 + x^2 + x^4]]
```

3.196.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1432, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d(2x^2+1)}{2\sqrt{3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

input
```
Int[x/Sqrt[1 + x^2 + x^4],x]
```

output
```
ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

3.196.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.196.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$	14
elliptic	$\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$	14
pseudoelliptic	$\frac{\operatorname{arcsinh}\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{2}$	16
trager	$-\frac{\ln\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)}{2}$	23

input
```
int(x/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{2}\log\left(-2\,x^2+2\,\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

3.196.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x}{\sqrt{(x^2-x+1)\,(x^2+x+1)}}\,dx$$

input
```
integrate(x/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.196.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(x^4 + x^2 + 1), x)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{2}\log\left(-2\,x^2+2\,\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \frac{\ln\left(\sqrt{x^4+x^2+1}+x^2+\frac{1}{2}\right)}{2}$$

input
```
int(x/(x^2 + x^4 + 1)^(1/2),x)
```

output
```
log((x^2 + x^4 + 1)^(1/2) + x^2 + 1/2)/2
```

3.196.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \frac{\log\left(\frac{2\sqrt{x^4+x^2+1}+2x^2+1}{\sqrt{3}}\right)}{2}$$

input
```
int(x/sqrt(x**4 + x**2 + 1),x)
```

output
```
log((2*sqrt(x**4 + x**2 + 1) + 2*x**2 + 1)/sqrt(3))/2
```

3.197 $\int \frac{1}{x\sqrt{-1+x^2-x^4}} \, dx$

3.197.1 Optimal result 1206
3.197.2 Mathematica [A] (verified) 1206
3.197.3 Rubi [A] (verified) 1207
3.197.4 Maple [A] (verified) 1208
3.197.5 Fricas [C] (verification not implemented) 1208
3.197.6 Sympy [F] 1209
3.197.7 Maxima [C] (verification not implemented) 1209
3.197.8 Giac [F] 1209
3.197.9 Mupad [B] (verification not implemented) 1210
3.197.10 Reduce [F] 1210

3.197.1 Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}} \, dx = -\frac{1}{2}\arctan\left(\frac{2-x^2}{2\sqrt{-1+x^2-x^4}}\right)$$

output
```
-1/2*arctan(1/2*(-x^2+2)/(-x^4+x^2-1)^(1/2))
```

3.197.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}} \, dx = \frac{1}{2}\arctan\left(\frac{-1+\frac{x^2}{2}}{\sqrt{-1+x^2-x^4}}\right)$$

input
```
Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

output
```
ArcTan[(-1 + x^2/2)/Sqrt[-1 + x^2 - x^4]]/2
```

3.197.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-x^4+x^2-1}}dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{-x^4+x^2-1}}dx^2$$
$$\Big\downarrow \text{1154}$$
$$-\int \frac{1}{-x^4-4}d\left(-\frac{2-x^2}{\sqrt{-x^4+x^2-1}}\right)$$
$$\Big\downarrow \text{217}$$
$$-\frac{1}{2}\arctan\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

input
```
Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

output
```
-1/2*ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]
```

3.197.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.197.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
elliptic	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
pseudoelliptic	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{-x^4+x^2-1}+2\operatorname{RootOf}\left(\_Z^2+1\right)}{x^2}\right)}{2}$	48

input
```
int(1/x/(-x^4+x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))
```

3.197.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \frac{1}{4}i\log\left(\frac{x^2+2i\sqrt{-x^4+x^2-1}-2}{2\,x^2}\right) - \frac{1}{4}i\log\left(\frac{x^2-2i\sqrt{-x^4+x^2-1}-2}{2\,x^2}\right)$$

input
```
integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*I*log(1/2*(x^2 + 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2) - 1/4*I*log(1/2*(x
^2 - 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2)
```

3.197.6 Sympy [F]

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \int \frac{1}{x\sqrt{-x^4+x^2-1}}\,dx$$

input
```
integrate(1/x/(-x**4+x**2-1)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)
```

3.197.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = -\frac{1}{2}i\,\operatorname{arsinh}\left(-\frac{1}{3}\sqrt{3}+\frac{2\sqrt{3}}{3x^2}\right)$$

input
```
integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)
```

3.197.8 Giac [F]

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2-1}x}\,dx$$

input
```
integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 - 1)*x), x)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \frac{\ln\left(\frac{1}{x^2}\right)\,1\mathrm{i}}{2} + \frac{\ln\left(x^2-2+\sqrt{-x^4+x^2-1}\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(1/(x*(x^2 - x^4 - 1)^(1/2)),x)
```

output
```
(log(1/x^2)*1i)/2 + (log((x^2 - x^4 - 1)^(1/2)*2i + x^2 - 2)*1i)/2
```

3.197.10 Reduce [F]

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2-1}\,x}dx$$

input
```
int(1/(sqrt( - x**4 + x**2 - 1)*x),x)
```

output
```
int(1/(sqrt( - x**4 + x**2 - 1)*x),x)
```

3.198 $\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx$

3.198.1 Optimal result 1211
3.198.2 Mathematica [A] (verified) 1211
3.198.3 Rubi [A] (verified) 1212
3.198.4 Maple [A] (verified) 1213
3.198.5 Fricas [A] (verification not implemented) 1213
3.198.6 Sympy [F] 1213
3.198.7 Maxima [A] (verification not implemented) 1214
3.198.8 Giac [B] (verification not implemented) 1214
3.198.9 Mupad [B] (verification not implemented) 1214
3.198.10 Reduce [B] (verification not implemented) 1215

3.198.1 Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = \frac{\sqrt{1+x^2}}{1-x}$$

output
```
(x^2+1)^(1/2)/(1-x)
```

3.198.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = \frac{\sqrt{1+x^2}}{1-x}$$

input
```
Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]
```

output
```
Sqrt[1 + x^2]/(1 - x)
```

3.198.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{677\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(1-x)^2\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 677}$$

$$\frac{\sqrt{x^2+1}}{1-x}$$

input
```
Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]
```

output
```
Sqrt[1 + x^2]/(1 - x)
```

3.198.3.1 Defintions of rubi rules used

rule 677
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]
```

3.198.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{\sqrt{x^2+1}}{-1+x}$	15
trager	$-\frac{\sqrt{x^2+1}}{-1+x}$	15
risch	$-\frac{\sqrt{x^2+1}}{-1+x}$	15
default	$-\frac{\sqrt{(-1+x)^2+2x}}{-1+x}$	19

input
```
int((1+x)/(1-x)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x^2+1)^(1/2)/(-1+x)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = -\frac{x+\sqrt{x^2+1}-1}{x-1}$$

input
```
integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(x + sqrt(x^2 + 1) - 1)/(x - 1)
```

3.198.6 Sympy [F]

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = \int \frac{x+1}{(x-1)^2\sqrt{x^2+1}}\,dx$$

input
```
integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)
```

output
```
Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = -\frac{\sqrt{x^2+1}}{x-1}$$

input
```
integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x^2 + 1)/(x - 1)
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = -\frac{\sqrt{\frac{2}{x-1}+\frac{2}{(x-1)^2}+1}}{\operatorname{sgn}\left(\frac{1}{x-1}\right)} + \operatorname{sgn}\left(\frac{1}{x-1}\right)$$

input
```
integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2/(x - 1) + 2/(x - 1)^2 + 1)/sgn(1/(x - 1)) + sgn(1/(x - 1))
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = -\frac{\sqrt{x^2+1}}{x-1}$$

input
```
int((x + 1)/((x^2 + 1)^(1/2)*(x - 1)^2),x)
```

output
```
-(x^2 + 1)^(1/2)/(x - 1)
```

3.198.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = -\frac{\sqrt{x^2+1}}{x-1}$$

input
```
int((x + 1)/(sqrt(x**2 + 1)*(x**2 - 2*x + 1)),x)
```

output
```
( - sqrt(x**2 + 1))/(x - 1)
```

3.199 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

3.199.1 Optimal result . . . 1216
3.199.2 Mathematica [B] (verified) . . . 1216
3.199.3 Rubi [A] (verified) . . . 1217
3.199.4 Maple [A] (verified) . . . 1217
3.199.5 Fricas [B] (verification not implemented) . . . 1218
3.199.6 Sympy [A] (verification not implemented) . . . 1218
3.199.7 Maxima [A] (verification not implemented) . . . 1219
3.199.8 Giac [B] (verification not implemented) . . . 1219
3.199.9 Mupad [B] (verification not implemented) . . . 1219
3.199.10Reduce [B] (verification not implemented) . . . 1220

3.199.1 Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)
```

3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. $2(2) = 4$.

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[1/Sqrt[1 + x^2],x]
```

output
```
-Log[-x + Sqrt[1 + x^2]]
```

3.199.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{222}$$
$$\text{arcsinh}(x)$$

input
```
Int[1/Sqrt[1 + x^2],x]
```

output
```
ArcSinh[x]
```

3.199.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.199.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\text{arcsinh}\left(x\right)$	3
meijerg	$\text{arcsinh}\left(x\right)$	3
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^{2}+1}}{x}\right)$	13
trager	$-\ln\left(x-\sqrt{x^{2}+1}\right)$	15

input
```
int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 1))
```

3.199.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
integrate(1/(x**2+1)**(1/2),x)
```

output
```
asinh(x)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(x)
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.199.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
int(1/(x^2 + 1)^(1/2),x)
```

output
```
asinh(x)
```

3.199.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \log\left(\sqrt{x^2+1}+x\right)$$

input
```
int(1/sqrt(x**2 + 1),x)
```

output
```
log(sqrt(x**2 + 1) + x)
```

3.200 $\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx$

3.200.1 Optimal result 1221
3.200.2 Mathematica [A] (verified) 1221
3.200.3 Rubi [A] (verified) 1222
3.200.4 Maple [A] (verified) 1223
3.200.5 Fricas [A] (verification not implemented) 1223
3.200.6 Sympy [A] (verification not implemented) 1224
3.200.7 Maxima [A] (verification not implemented) 1224
3.200.8 Giac [F] 1224
3.200.9 Mupad [B] (verification not implemented) 1225
3.200.10Reduce [B] (verification not implemented) 1225

3.200.1 Optimal result

Integrand size = 65, antiderivative size = 20

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx=\sqrt{x}+\sqrt{1+x}+\sqrt{2+x}$$

output
```
x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx=\frac{1}{2}\left(2\sqrt{x}+2\sqrt{1+x}+2\sqrt{2+x}\right)$$

input
```
Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2
+ x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

output
```
(2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2
```

3.200.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {27, 2035, 7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\sqrt{x+1}+\sqrt{x+2}\sqrt{x+1}+\sqrt{x}\sqrt{x+2}}{2\sqrt{x}\sqrt{x+1}\sqrt{x+2}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{\sqrt{x}\sqrt{x+1}+\sqrt{x+2}\sqrt{x+1}+\sqrt{x}\sqrt{x+2}}{\sqrt{x}\sqrt{x+1}\sqrt{x+2}}dx$$

$$\Big\downarrow \ 2035$$

$$\int \frac{\sqrt{x}\sqrt{x+1}+\sqrt{x+2}\sqrt{x+1}+\sqrt{x}\sqrt{x+2}}{\sqrt{x+1}\sqrt{x+2}}d\sqrt{x}$$

$$\Big\downarrow \ 7239$$

$$\int \left(\sqrt{x}\left(\frac{1}{\sqrt{x+2}}+\frac{1}{\sqrt{x+1}}\right)+1\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

input
```
Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/
(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

output
```
Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.200.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$\sqrt{x}+\sqrt{1+x}+\sqrt{2+x}$	15

input
```
int(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/
x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx=\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

input
```
integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(
1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

3.200.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx = \sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

input
```
integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2
+x)**(1/2))/x**(1/2)/(1+x)**(1/2)/(2+x)**(1/2),x)
```

output
```
sqrt(x) + sqrt(x + 1) + sqrt(x + 2)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx = \sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

input
```
integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(
1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

3.200.8 Giac [F]

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx = \int \frac{\sqrt{x+2}\sqrt{x+1}+\sqrt{x+2}\sqrt{x}+\sqrt{x+1}\sqrt{x}}{2\,\sqrt{x+2}\sqrt{x+1}\sqrt{x}}\,dx$$

input
```
integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(
1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)
*sqrt(x))/(sqrt(x + 2)*sqrt(x + 1)*sqrt(x)), x)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx=\sqrt{x+1}+\sqrt{x+2}+\sqrt{x}$$

input
```
int(((x^(1/2)*(x + 1)^(1/2))/2 + (x^(1/2)*(x + 2)^(1/2))/2 + ((x + 1)^(1/2
)*(x + 2)^(1/2))/2)/(x^(1/2)*(x + 1)^(1/2)*(x + 2)^(1/2)),x)
```

output
```
(x + 1)^(1/2) + (x + 2)^(1/2) + x^(1/2)
```

3.200.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx=\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

input
```
int((sqrt(x + 1)*sqrt(x + 2) + sqrt(x)*sqrt(x + 2) + sqrt(x)*sqrt(x + 1))/
(2*sqrt(x)*sqrt(x + 1)*sqrt(x + 2)),x)
```

output
```
sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

3.201 $\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$

3.201.1 Optimal result 1226
3.201.2 Mathematica [A] (verified) 1226
3.201.3 Rubi [A] (verified) 1227
3.201.4 Maple [A] (verified) 1228
3.201.5 Fricas [A] (verification not implemented) 1228
3.201.6 Sympy [A] (verification not implemented) 1229
3.201.7 Maxima [A] (verification not implemented) 1229
3.201.8 Giac [F] 1229
3.201.9 Mupad [B] (verification not implemented) 1230
3.201.10 Reduce [B] (verification not implemented) 1230

3.201.1 Optimal result

Integrand size = 68, antiderivative size = 24

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx = -\sqrt{1+x^3}+\sqrt{1-2x+x^5}$$

output
```
-(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 14.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx = -\sqrt{1+x^3}+\sqrt{1-2x+x^5}$$

input
```
Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x
^5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

output
```
-Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

3.201.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {27, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2\sqrt{x^3+1}-3\sqrt{x^5-2x+1}x^2+5\sqrt{x^3+1}x^4}{2\sqrt{x^3+1}\sqrt{x^5-2x+1}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int -\frac{-5\sqrt{x^3+1}x^4+3\sqrt{x^5-2x+1}x^2+2\sqrt{x^3+1}}{\sqrt{x^3+1}\sqrt{x^5-2x+1}}dx$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{2}\int \frac{-5\sqrt{x^3+1}x^4+3\sqrt{x^5-2x+1}x^2+2\sqrt{x^3+1}}{\sqrt{x^3+1}\sqrt{x^5-2x+1}}dx$$

$$\Big\downarrow \; 7293$$

$$-\frac{1}{2}\int \left(-\frac{5x^4}{\sqrt{x^5-2x+1}}+\frac{3x^2}{\sqrt{x^3+1}}+\frac{2}{\sqrt{x^5-2x+1}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(2\sqrt{x^5-2x+1}-2\sqrt{x^3+1}\right)$$

input
```
Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(
2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

output
```
(-2*Sqrt[1 + x^3] + 2*Sqrt[1 - 2*x + x^5])/2
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.201.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$	21
elliptic	$-\sqrt{x^3+1}+\frac{(-1+x)(x^4+x^3+x^2+x-1)}{\sqrt{x^5-2x+1}}$	37

input
```
int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^
3+1)^(1/2)/(x^5-2*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx = \sqrt{x^5-2\,x+1}-\sqrt{x^3+1}$$

input
```
integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2
))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^5 - 2*x + 1) - sqrt(x^3 + 1)
```

3.201.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx=-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$$

input
```
integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-2*x+
1)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)
```

output
```
-sqrt(x**3 + 1) + sqrt(x**5 - 2*x + 1)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$$
$$=\sqrt{x^4+x^3+x^2+x-1}\sqrt{x-1}-\sqrt{x^3+1}$$

input
```
integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2
))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^4 + x^3 + x^2 + x - 1)*sqrt(x - 1) - sqrt(x^3 + 1)
```

3.201.8 Giac [F]

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$$
$$=\int \frac{5\,\sqrt{x^3+1}x^4-3\,\sqrt{x^5-2\,x+1}x^2-2\,\sqrt{x^3+1}}{2\,\sqrt{x^5-2\,x+1}\sqrt{x^3+1}}\,dx$$

input
```
integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2
))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^
3 + 1))/(sqrt(x^5 - 2*x + 1)*sqrt(x^3 + 1)), x)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx = \sqrt{x^5-2\,x+1}-\sqrt{x^3+1}$$

input
```
int(-((x^3 + 1)^(1/2) + (3*x^2*(x^5 - 2*x + 1)^(1/2))/2 - (5*x^4*(x^3 + 1)
^(1/2))/2)/((x^3 + 1)^(1/2)*(x^5 - 2*x + 1)^(1/2)),x)
```

output
```
(x^5 - 2*x + 1)^(1/2) - (x^3 + 1)^(1/2)
```

3.201.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx = \sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

input
```
int(( - 3*sqrt(x**5 - 2*x + 1)*x**2 + 5*sqrt(x**3 + 1)*x**4 - 2*sqrt(x**3
+ 1))/(2*sqrt(x**3 + 1)*sqrt(x**5 - 2*x + 1)),x)
```

output
```
sqrt(x**5 - 2*x + 1) - sqrt(x**3 + 1)
```

3.202 $\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx$

3.202.1 Optimal result 1231
3.202.2 Mathematica [B] (verified) 1231
3.202.3 Rubi [A] (verified) 1232
3.202.4 Maple [A] (verified) 1232
3.202.5 Fricas [A] (verification not implemented) 1233
3.202.6 Sympy [A] (verification not implemented) 1233
3.202.7 Maxima [A] (verification not implemented) 1233
3.202.8 Giac [B] (verification not implemented) 1234
3.202.9 Mupad [B] (verification not implemented) 1234
3.202.10 Reduce [B] (verification not implemented) 1234

3.202.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = 10\text{arctanh}\left(\frac{x}{\sqrt{-4+x^2}}\right)+\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
10*arctanh(x/(x^2-4)^(1/2))+arctanh(x/(x^2-1)^(1/2))
```

3.202.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. $2(27) = 54$.

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = -5\log\left(1-\frac{x}{\sqrt{-4+x^2}}\right)+5\log\left(1+\frac{x}{\sqrt{-4+x^2}}\right) -\frac{1}{2}\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)$$

input
```
Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

output
```
-5*Log[1 - x/Sqrt[-4 + x^2]] + 5*Log[1 + x/Sqrt[-4 + x^2]] - Log[1 - x/Sqr
t[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2
```

3.202.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\sqrt{x^2-1}}+\frac{10}{\sqrt{x^2-4}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$10\text{arctanh}\left(\frac{x}{\sqrt{x^2-4}}\right)+\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

input
```
Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

output
```
10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

3.202.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.202.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$\ln\left(x+\sqrt{x^2-1}\right)+10\ln\left(x+\sqrt{x^2-4}\right)$	24
meijerg	$\frac{10\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{4}\right)}\,\arcsin\left(\frac{x}{2}\right)}{\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{4}\right)}}+\frac{\sqrt{-\operatorname{signum}\left(x^2-1\right)}\,\arcsin\left(x\right)}{\sqrt{\operatorname{signum}\left(x^2-1\right)}}$	51

input
```
int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = -\log\left(-x+\sqrt{x^2-1}\right)-10\log\left(-x+\sqrt{x^2-4}\right)$$

input
```
integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 - 1)) - 10*log(-x + sqrt(x^2 - 4))
```

3.202.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = 10\log\left(x+\sqrt{x^2-4}\right)+\log\left(x+\sqrt{x^2-1}\right)$$

input
```
integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)
```

output
```
10*log(x + sqrt(x**2 - 4)) + log(x + sqrt(x**2 - 1))
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = \log\left(2x+2\sqrt{x^2-1}\right)+10\log\left(2x+2\sqrt{x^2-4}\right)$$

input
```
integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - 1)) + 10*log(2*x + 2*sqrt(x^2 - 4))
```

3.202.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = \frac{1}{2}\sqrt{x^2-1}x+5\sqrt{x^2-4}x+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-1}\right|\right) + 20\log\left(\left|-x+\sqrt{x^2-4}\right|\right)$$

input
```
integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 1)*x + 5*sqrt(x^2 - 4)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
 + 20*log(abs(-x + sqrt(x^2 - 4)))
```

3.202.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = \ln\left(x+\sqrt{x^2-1}\right)+10\ln\left(x+\sqrt{x^2-4}\right)$$

input
```
int(1/(x^2 - 1)^(1/2) + 10/(x^2 - 4)^(1/2),x)
```

output
```
log(x + (x^2 - 1)^(1/2)) + 10*log(x + (x^2 - 4)^(1/2))
```

3.202.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx = \log\left(\sqrt{x^2-1}+x\right)+10\log\left(\frac{\sqrt{x^2-4}}{2}+\frac{x}{2}\right)$$

input
```
int((sqrt(x**2 - 4) + 10*sqrt(x**2 - 1))/(sqrt(x**2 - 1)*sqrt(x**2 - 4)),x
)
```

output
```
log(sqrt(x**2 - 1) + x) + 10*log((sqrt(x**2 - 4) + x)/2)
```

3.203 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

3.203.1 Optimal result 1235
3.203.2 Mathematica [A] (verified) 1235
3.203.3 Rubi [A] (verified) 1236
3.203.4 Maple [C] (verified) 1238
3.203.5 Fricas [A] (verification not implemented) 1239
3.203.6 Sympy [C] (verification not implemented) 1239
3.203.7 Maxima [F] 1240
3.203.8 Giac [F] 1240
3.203.9 Mupad [F(-1)] 1240
3.203.10 Reduce [B] (verification not implemented) 1241

3.203.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

output
```
-2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2
)^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

output
```
2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/S
qrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

3.203.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2544, 25, 363, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{a^2+x^2}+x}}{x} \, dx$$

$$\Big\downarrow \text{2544}$$

$$\int -\frac{\left(\sqrt{a^2+x^2}+x\right)^2+a^2}{\sqrt{\sqrt{a^2+x^2}+x}\left(a^2-\left(\sqrt{a^2+x^2}+x\right)^2\right)}d\left(\sqrt{a^2+x^2}+x\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{a^2+\left(x+\sqrt{a^2+x^2}\right)^2}{\sqrt{x+\sqrt{a^2+x^2}}\left(a^2-\left(x+\sqrt{a^2+x^2}\right)^2\right)}d\left(x+\sqrt{a^2+x^2}\right)$$

$$\Big\downarrow \text{363}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-2a^2\int \frac{1}{\sqrt{x+\sqrt{a^2+x^2}}\left(a^2-\left(x+\sqrt{a^2+x^2}\right)^2\right)}d\left(x+\sqrt{a^2+x^2}\right)$$

$$\Big\downarrow \text{266}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\int \frac{1}{a^2-\left(x+\sqrt{a^2+x^2}\right)^2}d\sqrt{x+\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{756}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\left(\frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}+\frac{\int \frac{1}{a+x+\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}\right)$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\left(\frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}+\frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\left(\frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}\right)$$

input
```
Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

output
```
2*Sqrt[x + Sqrt[a^2 + x^2]] - 4*a^2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt
[a]]/(2*a^(3/2)) + ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]/(2*a^(3/2)))
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

3.203.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.30

method	result	size
meijerg	$2\sqrt{2}\,\sqrt{x}\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{4};\frac{1}{2},\frac{3}{4};-\frac{a^2}{x^2}\right)$	25

input
```
int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) + \sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right) + \sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}a+\left(\sqrt{-a}(a+x)-\sqrt{a^2+x^2}\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x +
sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt
(-a)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a
 + (sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2))
)/x) + 2*sqrt(x + sqrt(a^2 + x^2))]
```

3.203.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \frac{\sqrt{x}\,\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

output
```
sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**
2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

3.203.7 Maxima [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.203.8 Giac [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

input
```
int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

output
```
int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

3.203.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$
$$=\sqrt{a}\,atan\left(\frac{\sqrt{a}\,\sqrt{\sqrt{a^2+x^2}+x}\,\sqrt{a^2+x^2}-\sqrt{a}\,\sqrt{\sqrt{a^2+x^2}+x}\,a-\sqrt{a}\,\sqrt{\sqrt{a^2+x^2}+x}\,x}{2a^2}\right)$$
$$+2\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\log\left(\sqrt{\sqrt{a^2+x^2}+x}-\sqrt{a}\right)$$
$$-\sqrt{a}\log\left(\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\right)$$

input
```
int(sqrt(sqrt(a**2 + x**2) + x)/x,x)
```

output
```
sqrt(a)*atan((sqrt(a)*sqrt(sqrt(a**2 + x**2) + x)*sqrt(a**2 + x**2) - sqrt
(a)*sqrt(sqrt(a**2 + x**2) + x)*a - sqrt(a)*sqrt(sqrt(a**2 + x**2) + x)*x)
/(2*a**2)) + 2*sqrt(sqrt(a**2 + x**2) + x) + sqrt(a)*log(sqrt(sqrt(a**2 +
x**2) + x) - sqrt(a)) - sqrt(a)*log(sqrt(sqrt(a**2 + x**2) + x) + sqrt(a))
```

3.204 $\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx$

3.204.1 Optimal result 1242
3.204.2 Mathematica [A] (verified) 1242
3.204.3 Rubi [A] (verified) 1243
3.204.4 Maple [A] (verified) 1244
3.204.5 Fricas [B] (verification not implemented) 1245
3.204.6 Sympy [B] (verification not implemented) 1245
3.204.7 Maxima [B] (verification not implemented) 1245
3.204.8 Giac [A] (verification not implemented) 1246
3.204.9 Mupad [B] (verification not implemented) 1246
3.204.10Reduce [B] (verification not implemented) 1247

3.204.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \log\left(1+\sqrt{1+x^3}\right)$$

output
```
ln(1+(x^3+1)^(1/2))
```

3.204.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \log\left(1+\sqrt{1+x^3}\right)$$

input
```
Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

output
```
Log[1 + Sqrt[1 + x^3]]
```

3.204.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {27, 2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2}{2\left(x^3+\sqrt{x^3+1}+1\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}\int \frac{x^2}{x^3+\sqrt{x^3+1}+1}dx$$

$$\Big\downarrow \text{ 2586}$$

$$\frac{1}{2}\int \frac{1}{x^3+\sqrt{x^3+1}+1}dx^3$$

$$\Big\downarrow \text{ 7267}$$

$$\int \frac{1}{\sqrt{x^3+1}+1}d\sqrt{x^3+1}$$

$$\Big\downarrow \text{ 16}$$

$$\log\left(\sqrt{x^3+1}+1\right)$$

input
```
Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

output
```
Log[1 + Sqrt[1 + x^3]]
```

3.204.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.204.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result	size
trager	$\frac{\ln\left(-x^{3}-2\sqrt{x^{3}+1}-2\right)}{2}$	20
default	$-\frac{\ln(1+x)}{2}+\frac{3\ln(x)}{2}-\frac{\ln(x^{2}-x+1)}{2}+\frac{\ln(x^{3}+1)}{2}+\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)$	39
elliptic	$\frac{\left(1+\sqrt{x^{3}+1}\right)\sqrt{x^{3}+1}\left(\frac{3\ln(x)}{2}+\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)\right)}{1+x^{3}+\sqrt{x^{3}+1}}$	45

input
```
int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-x^3-2*(x^3+1)^(1/2)-2)
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \frac{3}{2}\,\log\left(x\right) + \frac{1}{2}\,\log\left(\sqrt{x^3+1}+1\right) - \frac{1}{2}\,\log\left(\sqrt{x^3+1}-1\right)$$

input
```
integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="fricas")
```

output
```
3/2*log(x) + 1/2*log(sqrt(x^3 + 1) + 1) - 1/2*log(sqrt(x^3 + 1) - 1)
```

3.204.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

Time = 49.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = -\frac{\log\left(2\sqrt{x^3+1}\right)}{2} + \frac{\log\left(2\sqrt{x^3+1}+2\right)}{2} + \frac{\log\left(3x^3+3\sqrt{x^3+1}+3\right)}{2}$$

input
```
integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)
```

output
```
-log(2*sqrt(x**3 + 1))/2 + log(2*sqrt(x**3 + 1) + 2)/2 + log(3*x**3 + 3*sq
rt(x**3 + 1) + 3)/2
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = -\frac{1}{2}\,\log\left(x^2-x+1\right) + \log\left(\frac{x^3+\sqrt{x^2-x+1}\sqrt{x+1}+1}{\sqrt{x+1}}\right)$$

input
```
integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="maxima")
```

output
```
-1/2*log(x^2 - x + 1) + log((x^3 + sqrt(x^2 - x + 1)*sqrt(x + 1) + 1)/sqrt
(x + 1))
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \log\left(\sqrt{x^3+1}+1\right)$$

input
```
integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="giac")
```

output
```
log(sqrt(x^3 + 1) + 1)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 14.08

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \frac{3\ln(x)}{2} + \frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((3*x^2)/(2*((x^3 + 1)^(1/2) + x^3 + 1)),x)
```

output
```
(3*log(x))/2 + (3*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)
*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2
+ 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.204.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \frac{\log\left(2\sqrt{x^3+1}+x^3+2\right)}{2}$$

input
```
int((3*x**2)/(2*(sqrt(x**3 + 1) + x**3 + 1)),x)
```

output
```
log(2*sqrt(x**3 + 1) + x**3 + 2)/2
```

3.205 $\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr$

3.205.1 Optimal result 1248
3.205.2 Mathematica [A] (verified) 1248
3.205.3 Rubi [A] (verified) 1249
3.205.4 Maple [A] (verified) 1250
3.205.5 Fricas [A] (verification not implemented) 1250
3.205.6 Sympy [C] (verification not implemented) 1251
3.205.7 Maxima [A] (verification not implemented) 1251
3.205.8 Giac [A] (verification not implemented) 1251
3.205.9 Mupad [B] (verification not implemented) 1252
3.205.10Reduce [B] (verification not implemented) 1252

3.205.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

output
```
1/2*arctanh(r*2^(1/2)*h^(1/2)/(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

input
```
Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```

output
```
ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

3.205.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2hr^2-\alpha^2}}dr$$
$$\Big\downarrow \text{224}$$
$$\int \frac{1}{1-\frac{2hr^2}{2hr^2-\alpha^2}}d\frac{r}{\sqrt{2hr^2-\alpha^2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

input
```
Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```

output
```
ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

3.205.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.205.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln\left(\sqrt{h}\, r\sqrt{2}+\sqrt{2h\, r^{2}-\alpha^{2}}\right)\sqrt{2}}{2\sqrt{h}}$	33
pseudoelliptic	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{2h\, r^{2}-\alpha^{2}}\, \sqrt{2}}{2r\sqrt{h}}\right)}{2\sqrt{h}}$	35

input
```
int(1/(2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr = \left[\frac{\sqrt{2}\log\left(4\,hr^{2}+2\,\sqrt{2}\sqrt{2\,hr^{2}-\alpha^{2}}\sqrt{h}r-\alpha^{2}\right)}{4\,\sqrt{h}},\ -\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{h}}\arctan\left(\frac{\sqrt{2}hr\sqrt{-\frac{1}{h}}}{\sqrt{2\,hr^{2}-\alpha^{2}}}\right)\right]$$

input
```
integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

output
```
[1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h)*r - a
lpha^2)/sqrt(h), -1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*h*r*sqrt(-1/h)/sqr
t(2*h*r^2 - alpha^2))]
```

3.205.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}}\,dr = \begin{cases} \frac{\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{for } \left|\frac{hr^2}{\alpha^2}\right| > \frac{1}{2} \\ -\frac{\sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*h*r**2-alpha**2)**(1/2),r)
```

output
```
Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), Abs(h*r**2/
alpha**2) > 1/2), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), T
rue))
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}}\,dr = \frac{\sqrt{2}\log\left(4\,hr + 2\,\sqrt{2}\sqrt{2\,hr^2 - \alpha^2}\sqrt{h}\right)}{2\,\sqrt{h}}$$

input
```
integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(4*h*r + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h))/sqrt(h)
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}}\,dr = \frac{\sqrt{2}\alpha^2\log\left(\left|-\sqrt{2}\sqrt{h}r + \sqrt{2\,hr^2 - \alpha^2}\right|\right)}{4\,\sqrt{h}} + \frac{1}{2}\,\sqrt{2\,hr^2 - \alpha^2}r$$

input
```
integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

output
```
1/4*sqrt(2)*alpha^2*log(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2)))
/sqrt(h) + 1/2*sqrt(2*h*r^2 - alpha^2)*r
```

3.205.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr = \frac{\sqrt{2}\,\ln\left(\sqrt{2\,h\,r^2-\alpha^2}+\sqrt{2}\,\sqrt{h}\,r\right)}{2\,\sqrt{h}}$$

input
```
int(1/(2*h*r^2 - alpha^2)^(1/2),r)
```

output
```
(2^(1/2)*log((2*h*r^2 - alpha^2)^(1/2) + 2^(1/2)*h^(1/2)*r))/(2*h^(1/2))
```

3.205.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr = \frac{\sqrt{h}\,\sqrt{2}\log\left(\frac{\sqrt{2h\,r^2-\alpha^2}+\sqrt{h}\,\sqrt{2}\,r}{\alpha}\right)}{2h}$$

input
```
int(1/sqrt( - alpha**2 + 2*h*r**2),r)
```

output
```
(sqrt(h)*sqrt(2)*log((sqrt( - alpha**2 + 2*h*r**2) + sqrt(h)*sqrt(2)*r)/al
pha))/(2*h)
```

3.206 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr$

3.206.1 Optimal result 1253
3.206.2 Mathematica [A] (verified) 1253
3.206.3 Rubi [A] (verified) 1254
3.206.4 Maple [A] (verified) 1255
3.206.5 Fricas [A] (verification not implemented) 1255
3.206.6 Sympy [A] (verification not implemented) 1256
3.206.7 Maxima [A] (verification not implemented) 1256
3.206.8 Giac [A] (verification not implemented) 1256
3.206.9 Mupad [B] (verification not implemented) 1257
3.206.10 Reduce [B] (verification not implemented) 1257

3.206.1 Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr=\frac{\arctan\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

output
```
arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha
^2+epsilon^2)^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr=\frac{\arctan\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

output
```
ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqr
t[alpha^2 + epsilon^2]
```

3.206.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{r^2\sqrt{-\alpha^2-\epsilon^2+2hr^2}}dr^2$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{r^4}{2h}+\frac{\alpha^2+\epsilon^2}{2h}}d\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{2h}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

output
```
ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqr
t[alpha^2 + epsilon^2]
```

3.206.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.206.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{2h\,r^2-\alpha^2-\epsilon^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$	41
default	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{2h\,r^2-\alpha^2-\epsilon^2}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$	66

input
```
int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha
^2+epsilon^2)^(1/2)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr = -\frac{\arctan\left(\frac{\sqrt{\alpha^2+\epsilon^2}}{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

output
```
-arctan(sqrt(alpha^2 + epsilon^2)/sqrt(2*h*r^2 - alpha^2 - epsilon^2))/sqr
t(alpha^2 + epsilon^2)
```

3.206.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{\text{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}{2\sqrt{h}r}\right)}{\sqrt{\text{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}$$

input
```
integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

output
```
-asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h)*r))/sqr
t(polar_lift(-alpha**2 - epsilon**2))
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr = -\frac{\arcsin\left(\frac{\sqrt{2}\alpha^2}{2\sqrt{(\alpha^2+\epsilon^2)h}r}+\frac{\sqrt{2}\epsilon^2}{2\sqrt{(\alpha^2+\epsilon^2)h}r}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

output
```
-arcsin(1/2*sqrt(2)*alpha^2/(sqrt((alpha^2 + epsilon^2)*h)*r) + 1/2*sqrt(2
)*epsilon^2/(sqrt((alpha^2 + epsilon^2)*h)*r))/sqrt(alpha^2 + epsilon^2)
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr = \frac{\arctan\left(\frac{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

output
```
arctan(sqrt(2*h*r^2 - alpha^2 - epsilon^2)/sqrt(alpha^2 + epsilon^2))/sqrt
(alpha^2 + epsilon^2)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr=\frac{\operatorname{atan}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2\,h\,r^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
int(1/(r*(2*h*r^2 - alpha^2 - epsilon^2)^(1/2)),r)
```

output
```
atan((2*h*r^2 - alpha^2 - epsilon^2)^(1/2)/(alpha^2 + epsilon^2)^(1/2))/(a
lpha^2 + epsilon^2)^(1/2)
```

3.206.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr=\frac{2\sqrt{\alpha^2+\epsilon^2}\,atan\left(\frac{\sqrt{2h\,r^2-\alpha^2-\epsilon^2}+\sqrt{h}\,\sqrt{2}\,r}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\alpha^2+\epsilon^2}$$

input
```
int(1/(sqrt( - alpha**2 - epsilon**2 + 2*h*r**2)*r),r)
```

output
```
(2*sqrt(alpha**2 + epsilon**2)*atan((sqrt( - alpha**2 - epsilon**2 + 2*h*r
**2) + sqrt(h)*sqrt(2)*r)/sqrt(alpha**2 + epsilon**2)))/(alpha**2 + epsilo
n**2)
```

3.207 $\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr$

3.207.1 Optimal result 1258
3.207.2 Mathematica [A] (verified) 1258
3.207.3 Rubi [A] (verified) 1259
3.207.4 Maple [A] (verified) 1260
3.207.5 Fricas [A] (verification not implemented) 1260
3.207.6 Sympy [F] 1260
3.207.7 Maxima [A] (verification not implemented) 1261
3.207.8 Giac [A] (verification not implemented) 1261
3.207.9 Mupad [B] (verification not implemented) 1261
3.207.10 Reduce [B] (verification not implemented) 1262

3.207.1 Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr=-\frac{\arctan\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha}$$

output
```
-arctan((alpha^2+k*r)/alpha/(2*h*r^2-alpha^2-2*k*r)^(1/2))/alpha
```

3.207.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr=-\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{h}r-\sqrt{-\alpha^2-2kr+2hr^2}}{\alpha}\right)}{\alpha}$$

input
```
Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

output
```
(-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])/alpha])/
alpha
```

3.207.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr}}dr$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{-4\alpha^2-\frac{4(\alpha^2+kr)^2}{-\alpha^2+2hr^2-2kr}}d\left(-\frac{2(\alpha^2+kr)}{\sqrt{-\alpha^2+2hr^2-2kr}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

input
```
Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

output
```
-(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])]/alpha)
```

3.207.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.207.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2-2kr+2\sqrt{-\alpha^2}\,\sqrt{2h\,r^2-\alpha^2-2kr}}{r}\right)}{\sqrt{-\alpha^2}}$	52

input
```
int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
-1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*r^2-alpha
^2-2*k*r)^(1/2))/r)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = -\frac{\arctan\left(\frac{\sqrt{2\,hr^2-\alpha^2-2\,kr}(\alpha^2+kr)}{2\,\alpha hr^2-\alpha^3-2\,\alpha kr}\right)}{\alpha}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

output
```
-arctan(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*(alpha^2 + k*r)/(2*alpha*h*r^2 - a
lpha^3 - 2*alpha*k*r))/alpha
```

3.207.6 Sympy [F]

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = \int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr}}\,dr$$

input
```
integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)
```

output
```
Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = -\frac{\arcsin\left(\frac{k}{\sqrt{2\,\alpha^2h+k^2}}+\frac{\alpha^2}{\sqrt{2\,\alpha^2h+k^2}r}\right)}{\alpha}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

output
```
-arcsin(k/sqrt(2*alpha^2*h + k^2) + alpha^2/(sqrt(2*alpha^2*h + k^2)*r))/a
lpha
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = \frac{2\,\arctan\left(-\frac{\sqrt{2}\sqrt{h}r-\sqrt{2\,hr^2-\alpha^2-2\,kr}}{\alpha}\right)}{\alpha}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

output
```
2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/alpha)/alp
ha
```

3.207.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = -\frac{\ln\left(\frac{\sqrt{-\alpha^2}\,\sqrt{-\alpha^2+2\,h\,r^2-2\,k\,r}}{r}-\frac{\alpha^2}{r}-k\right)}{\sqrt{-\alpha^2}}$$

input
```
int(1/(r*(2*h*r^2 - 2*k*r - alpha^2)^(1/2)),r)
```

output
```
-log(((-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2)^(1/2))/r - alpha^2/r -
k)/(-alpha^2)^(1/2)
```

3.207.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = \frac{2atan\left(\frac{\sqrt{h}\,\sqrt{2h\,r^2-\alpha^2-2kr}\,\sqrt{2}+2hr}{\sqrt{h}\,\sqrt{2}\,\alpha}\right)}{\alpha}$$

input
```
int(1/(sqrt( - alpha**2 + 2*h*r**2 - 2*k*r)*r),r)
```

output
```
(2*atan((sqrt(h)*sqrt( - alpha**2 + 2*h*r**2 - 2*k*r)*sqrt(2) + 2*h*r)/(sq
rt(h)*sqrt(2)*alpha)))/alpha
```

3.208 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr$

3.208.1 Optimal result 1263
3.208.2 Mathematica [A] (verified) 1263
3.208.3 Rubi [A] (verified) 1264
3.208.4 Maple [A] (verified) 1265
3.208.5 Fricas [A] (verification not implemented) 1265
3.208.6 Sympy [F] 1265
3.208.7 Maxima [A] (verification not implemented) 1266
3.208.8 Giac [A] (verification not implemented) 1266
3.208.9 Mupad [B] (verification not implemented) 1266
3.208.10Reduce [B] (verification not implemented) 1267

3.208.1 Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = -\frac{\arctan\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

output
```
-arctan((alpha^2+epsilon^2+k*r)/(alpha^2+epsilon^2)^(1/2)/(2*h*r^2-alpha^2
-epsilon^2-2*k*r)^(1/2))/(alpha^2+epsilon^2)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = -\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{h}r-\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

output
```
(-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^
2])/Sqrt[alpha^2 + epsilon^2]])/Sqrt[alpha^2 + epsilon^2]
```

3.208.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\,dr$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{-\frac{4(\alpha^2+\epsilon^2+kr)^2}{-\alpha^2-\epsilon^2+2hr^2-2kr}-4\left(\alpha^2+\epsilon^2\right)}d\left(-\frac{2(\alpha^2+\epsilon^2+kr)}{\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

output
```
-(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alph
a^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsilon^2])
```

3.208.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.208.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^{2}-2\epsilon^{2}-2kr+2\sqrt{-\alpha^{2}-\epsilon^{2}}\,\sqrt{2h\,r^{2}-\alpha^{2}-\epsilon^{2}-2kr}}{r}\right)}{\sqrt{-\alpha^{2}-\epsilon^{2}}}$	74

input
```
int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
-1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*(-alpha^2
-epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/r)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = -\frac{\arctan\left(-\frac{\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\,(\alpha^2+\epsilon^2+kr)\sqrt{\alpha^2+\epsilon^2}}{\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\,(\alpha^2+\epsilon^2)hr^2+2\,(\alpha^2+\epsilon^2)kr}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="fricas
")
```

output
```
-arctan(-sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*(alpha^2 + epsilon^2
+ k*r)*sqrt(alpha^2 + epsilon^2)/(alpha^4 + 2*alpha^2*epsilon^2 + epsilon^
4 - 2*(alpha^2 + epsilon^2)*h*r^2 + 2*(alpha^2 + epsilon^2)*k*r))/sqrt(alp
ha^2 + epsilon^2)
```

3.208.6 Sympy [F]

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = \int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\,dr$$

input
```
integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)
```

output
```
Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = -\frac{\arcsin\left(\frac{k}{\sqrt{2\,(\alpha^2+\epsilon^2)h+k^2}}+\frac{\alpha^2}{\sqrt{2\,(\alpha^2+\epsilon^2)h+k^2}r}+\frac{\epsilon^2}{\sqrt{2\,(\alpha^2+\epsilon^2)h+k^2}r}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="maxima
")
```

output
```
-arcsin(k/sqrt(2*(alpha^2 + epsilon^2)*h + k^2) + alpha^2/(sqrt(2*(alpha^2
 + epsilon^2)*h + k^2)*r) + epsilon^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^
2)*r))/sqrt(alpha^2 + epsilon^2)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{h}r-\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="giac")
```

output
```
2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)
)/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = -\frac{\ln\left(\frac{\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-\alpha^2-\epsilon^2+2\,h\,r^2-2\,k\,r}}{r}-\frac{\alpha^2+\epsilon^2}{r}-k\right)}{\sqrt{-\alpha^2-\epsilon^2}}$$

input
```
int(1/(r*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)^(1/2)),r)
```

output
```
-log(((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2
)^(1/2))/r - (alpha^2 + epsilon^2)/r - k)/(- alpha^2 - epsilon^2)^(1/2)
```

3.208.10 Reduce [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = \frac{2\sqrt{\alpha^2+\epsilon^2}\,atan\left(\frac{\sqrt{h}\,\sqrt{2h\,r^2-\alpha^2-\epsilon^2-2kr}\,\sqrt{2}+2hr}{\sqrt{h}\,\sqrt{\alpha^2+\epsilon^2}\,\sqrt{2}}\right)}{\alpha^2+\epsilon^2}$$

input
```
int(1/(sqrt( - alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)*r),r)
```

output
```
(2*sqrt(alpha**2 + epsilon**2)*atan((sqrt(h)*sqrt( - alpha**2 - epsilon**2
 + 2*h*r**2 - 2*k*r)*sqrt(2) + 2*h*r)/(sqrt(h)*sqrt(alpha**2 + epsilon**2)
*sqrt(2))))/(alpha**2 + epsilon**2)
```

3.209 $\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr$

3.209.1 Optimal result 1268
3.209.2 Mathematica [A] (verified) 1268
3.209.3 Rubi [A] (verified) 1269
3.209.4 Maple [A] (verified) 1270
3.209.5 Fricas [A] (verification not implemented) 1270
3.209.6 Sympy [A] (verification not implemented) 1271
3.209.7 Maxima [A] (verification not implemented) 1271
3.209.8 Giac [A] (verification not implemented) 1271
3.209.9 Mupad [B] (verification not implemented) 1272
3.209.10 Reduce [B] (verification not implemented) 1272

3.209.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2+2er^2}}{2e}$$

output
```
1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

3.209.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2+2er^2}}{2e}$$

input
```
Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

output
```
Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

3.209.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{2er^2-\alpha^2}}\,dr$$
$$\Big\downarrow \; 241$$
$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

input
```
Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

output
```
Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

3.209.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.209.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gosper	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
derivativedivides	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
default	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
trager	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
pseudoelliptic	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
risch	$-\frac{-2e\,r^{2}+\alpha^{2}}{2e\sqrt{2e\,r^{2}-\alpha^{2}}}$	30

input
```
int(r/(2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr = \frac{\sqrt{2\,er^{2}-\alpha^{2}}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

output
```
1/2*sqrt(2*e*r^2 - alpha^2)/e
```

3.209.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\begin{cases}\frac{\sqrt{-\alpha^2+2er^2}}{2e} & \text{for } e\neq 0\\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise}\end{cases}$$

input
```
integrate(r/(2*e*r**2-alpha**2)**(1/2),r)
```

output
```
Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alp
ha**2)), True))
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

output
```
1/2*sqrt(2*e*r^2 - alpha^2)/e
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

output
```
1/2*sqrt(2*e*r^2 - alpha^2)/e
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}}\, dr = \frac{\sqrt{2\, e\, r^2 - \alpha^2}}{2\, e}$$

input
```
int(r/(2*e*r^2 - alpha^2)^(1/2),r)
```

output
```
(2*e*r^2 - alpha^2)^(1/2)/(2*e)
```

3.209.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}}\, dr = \frac{\sqrt{2e\, r^2 - \alpha^2}}{2e}$$

input
```
int(r/sqrt( - alpha**2 + 2*e*r**2),r)
```

output
```
sqrt( - alpha**2 + 2*e*r**2)/(2*e)
```

3.210 $\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}} \, dr$

3.210.1 Optimal result 1273
3.210.2 Mathematica [A] (verified) 1273
3.210.3 Rubi [A] (verified) 1274
3.210.4 Maple [A] (verified) 1275
3.210.5 Fricas [A] (verification not implemented) 1275
3.210.6 Sympy [A] (verification not implemented) 1276
3.210.7 Maxima [A] (verification not implemented) 1276
3.210.8 Giac [A] (verification not implemented) 1276
3.210.9 Mupad [B] (verification not implemented) 1277
3.210.10Reduce [B] (verification not implemented) 1277

3.210.1 Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}} \, dr = \frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

output
```
1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

3.210.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}} \, dr = \frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

input
```
Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

output
```
Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

3.210.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-\epsilon^2}}\,dr$$
$$\Big\downarrow \text{241}$$
$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

input
```
Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

output
```
Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

3.210.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.210.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
derivativedivides	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
default	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
trager	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
pseudoelliptic	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
risch	$-\frac{-2e\,r^{2}+\alpha^{2}+\epsilon^{2}}{2e\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}$	38

input
```
int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr=\frac{\sqrt{2\,er^{2}-\alpha^{2}-\epsilon^{2}}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

output
```
1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

3.210.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr = \begin{cases} \frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2-\epsilon^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)
```

output
```
Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0)), (r**2
/(2*sqrt(-alpha**2 - epsilon**2)), True))
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr = \frac{\sqrt{2\,er^2-\alpha^2-\epsilon^2}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

output
```
1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr = \frac{\sqrt{2\,er^2-\alpha^2-\epsilon^2}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

output
```
1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2-\epsilon^2+2\,e\,r^2}}{2\,e}$$

input
```
int(r/(2*e*r^2 - alpha^2 - epsilon^2)^(1/2),r)
```

output
```
(2*e*r^2 - alpha^2 - epsilon^2)^(1/2)/(2*e)
```

3.210.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr=\frac{\sqrt{2e\,r^2-\alpha^2-\epsilon^2}}{2e}$$

input
```
int(r/sqrt( - alpha**2 + 2*e*r**2 - epsilon**2),r)
```

output
```
sqrt( - alpha**2 + 2*e*r**2 - epsilon**2)/(2*e)
```

3.211 $\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$

3.211.1 Optimal result 1278
3.211.2 Mathematica [A] (verified) 1278
3.211.3 Rubi [A] (verified) 1279
3.211.4 Maple [A] (verified) 1280
3.211.5 Fricas [A] (verification not implemented) 1280
3.211.6 Sympy [F] 1281
3.211.7 Maxima [F(-2)] 1281
3.211.8 Giac [A] (verification not implemented) 1282
3.211.9 Mupad [B] (verification not implemented) 1282
3.211.10Reduce [F] 1282

3.211.1 Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr=-\frac{\arctan\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

output
```
-1/4*arctan(1/2*(-2*k*r^2+e)*2^(1/2)/k^(1/2)/(-2*k*r^4+2*e*r^2-alpha^2)^(1
/2))*2^(1/2)/k^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr=-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{k}r^2}{\sqrt{-\alpha^2}-\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{\sqrt{2}\sqrt{k}}$$

input
```
Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

output
```
-(ArcTan[(Sqrt[2]*Sqrt[k]*r^2)/(Sqrt[-alpha^2] - Sqrt[-alpha^2 + 2*e*r^2 -
 2*k*r^4])]/(Sqrt[2]*Sqrt[k]))
```

3.211.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1432, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}dr$$
$$\Big\downarrow \; 1432$$
$$\frac{1}{2}\int \frac{1}{\sqrt{-2kr^4+2er^2-\alpha^2}}dr^2$$
$$\Big\downarrow \; 1092$$
$$\int \frac{1}{-8k-r^4}d\frac{2(e-2kr^2)}{\sqrt{-\alpha^2+2er^2-2kr^4}}$$
$$\Big\downarrow \; 217$$
$$-\frac{\arctan\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

input
```
Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

output
```
-1/2*ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r
^4])]/(Sqrt[2]*Sqrt[k])
```

3.211.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.211.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\left(-2k\,r^{2}+e\right)\sqrt{2}}{2\sqrt{k}\,\sqrt{-2k\,r^{4}+2e\,r^{2}-\alpha^{2}}}\right)\sqrt{2}}{4\sqrt{k}}$	46
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{k}\,\left(r^{2}-\frac{e}{2k}\right)}{\sqrt{-2k\,r^{4}+2e\,r^{2}-\alpha^{2}}}\right)}{4\sqrt{k}}$	47
elliptic	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{k}\,\left(r^{2}-\frac{e}{2k}\right)}{\sqrt{-2k\,r^{4}+2e\,r^{2}-\alpha^{2}}}\right)}{4\sqrt{k}}$	47

input
```
int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
-1/4*arctan(1/2*(-2*k*r^2+e)*2^(1/2)/k^(1/2)/(-2*k*r^4+2*e*r^2-alpha^2)^(1
/2))*2^(1/2)/k^(1/2)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.71

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$$
$$=\left[-\frac{\sqrt{2}\sqrt{-k}\log\left(-8\,k^2r^4+8\,ekr^2-2\,\alpha^2k+2\,\sqrt{2}\sqrt{-2\,kr^4+2\,er^2-\alpha^2}(2\,kr^2-e)\sqrt{-k}-e^2\right)}{8\,k},\right.$$
$$\left.-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,kr^4+2\,er^2-\alpha^2}(2\,kr^2-e)\sqrt{k}}{2\,(2\,k^2r^4-2\,ekr^2+\alpha^2k)}\right)}{4\,\sqrt{k}}\right]$$

input
```
integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

output
```
[-1/8*sqrt(2)*sqrt(-k)*log(-8*k^2*r^4 + 8*e*k*r^2 - 2*alpha^2*k + 2*sqrt(2
)*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*sqrt(-k) - e^2)/k, -1/4
*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 -
e)*sqrt(k)/(2*k^2*r^4 - 2*e*k*r^2 + alpha^2*k))/sqrt(k)]
```

3.211.6 Sympy [F]

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\, dr = \int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\, dr$$

input
```
integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```

output
```
Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)
```

3.211.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\, dr = \text{Exception raised: ValueError}$$

input
```
integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*alpha^2*k-e^2>0)', see `assume
?` for mor
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr = -\frac{\sqrt{2}\log\left(\left|\sqrt{2}\left(\sqrt{2}\sqrt{-k}r^2-\sqrt{-2\,kr^4+2\,er^2-\alpha^2}\right)\sqrt{-k}+e\right|\right)}{4\sqrt{-k}}$$

input
```
integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*e*r
^2 - alpha^2))*sqrt(-k) + e))/sqrt(-k)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr = \frac{\sqrt{2}\ln\left(\sqrt{-\alpha^2-2\,k\,r^4+2\,e\,r^2}+\frac{\sqrt{2}\,(e-2\,k\,r^2)}{2\,\sqrt{-k}}\right)}{4\,\sqrt{-k}}$$

input
```
int(r/(2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2),r)
```

output
```
(2^(1/2)*log((2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2) + (2^(1/2)*(e - 2*k*r^2))
/(2*(-k)^(1/2))))/(4*(-k)^(1/2))
```

3.211.10 Reduce [F]

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr = \int \frac{r}{\sqrt{-2k\,r^4+2e\,r^2-\alpha^2}}dr$$

input
```
int(r/sqrt( - alpha**2 + 2*e*r**2 - 2*k*r**4),r)
```

output
```
int(r/sqrt( - alpha**2 + 2*e*r**2 - 2*k*r**4),r)
```

3.212 $\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$

3.212.1 Optimal result . . . 1283
3.212.2 Mathematica [A] (verified) . . . 1283
3.212.3 Rubi [A] (verified) . . . 1284
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3.212.5 Fricas [A] (verification not implemented) . . . 1286
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3.212.8 Giac [A] (verification not implemented) . . . 1287
3.212.9 Mupad [B] (verification not implemented) . . . 1288
3.212.10Reduce [B] (verification not implemented) . . . 1288

3.212.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr=\frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}-\frac{k\text{arctanh}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2-2kr+2er^2}}\right)}{2\sqrt{2}e^{3/2}}$$

output
```
-1/4*k*arctanh(1/2*(-2*e*r+k)*2^(1/2)/e^(1/2)/(2*e*r^2-alpha^2-2*k*r)^(1/2
))/e^(3/2)*2^(1/2)+1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e
```

3.212.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr=\frac{1}{2}\left(\frac{\sqrt{-\alpha^2-2kr+2er^2}}{e}-\frac{\sqrt{2}k\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}r}{\sqrt{-\alpha^2}-\sqrt{-\alpha^2-2kr+2er^2}}\right)}{e^{3/2}}\right)$$

input
```
Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

output
```
(Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/e - (Sqrt[2]*k*ArcTanh[(Sqrt[2]*Sqrt[e]*
r)/(Sqrt[-alpha^2] - Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/e^(3/2))/2
```

3.212.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr}}dr$$

$$\Big\downarrow \text{1160}$$

$$\frac{k\int \frac{1}{\sqrt{-\alpha^2+2er^2-2kr}}dr}{2e}+\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{k\int \frac{1}{8e-\frac{4(k-2er)^2}{-\alpha^2+2er^2-2kr}}d\left(-\frac{2(k-2er)}{\sqrt{-\alpha^2+2er^2-2kr}}\right)}{e}+\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\text{arctanh}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}e^{3/2}}$$

input
```
Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

output
```
Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*S
qrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))
```

3.212.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.212.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-2kr}}{2e}+\frac{k\ln\left(\frac{(2er-k)\sqrt{2}}{2\sqrt{e}}+\sqrt{2e\,r^{2}-\alpha^{2}-2kr}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$	70
risch	$-\frac{-2e\,r^{2}+\alpha^{2}+2kr}{2e\sqrt{2e\,r^{2}-\alpha^{2}-2kr}}+\frac{k\ln\left(\frac{(2er-k)\sqrt{2}}{2\sqrt{e}}+\sqrt{2e\,r^{2}-\alpha^{2}-2kr}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$	84

input
```
int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*ln(1/2*(2*e*r-k)*2^(1/2)
/e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$$
$$= \left[\frac{\sqrt{2}\sqrt{e}k\log\left(8\,e^2r^2-2\,\alpha^2e-8\,ekr+2\,\sqrt{2}\sqrt{2\,er^2-\alpha^2-2\,kr}(2\,er-k)\sqrt{e}+k^2\right)+4\,\sqrt{2\,er^2-\alpha^2-2\,\ldots}}{8\,e^2}\right.$$
$$\left.-\frac{\sqrt{2}\sqrt{-e}k\arctan\left(\frac{\sqrt{2}\sqrt{2\,er^2-\alpha^2-2\,kr}(2\,er-k)\sqrt{-e}}{2\,(2\,e^2r^2-\alpha^2e-2\,ekr)}\right)-2\,\sqrt{2\,er^2-\alpha^2-2\,kr}e}{4\,e^2}\right]$$

input
```
integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*sqrt(e)*k*log(8*e^2*r^2 - 2*alpha^2*e - 8*e*k*r + 2*sqrt(2)*
sqrt(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(e) + k^2) + 4*sqrt(2*e*r^
2 - alpha^2 - 2*k*r)*e)/e^2, -1/4*(sqrt(2)*sqrt(-e)*k*arctan(1/2*sqrt(2)*s
qrt(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(-e)/(2*e^2*r^2 - alpha^2*e
 - 2*e*k*r)) - 2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)*e)/e^2]
```

3.212.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.12

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$$
$$= \begin{cases} \frac{k\left(\begin{cases} \frac{\sqrt{2}\log\left(2\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}+4er-2k\right)}{2\sqrt{e}} & \text{for } \alpha^2+\frac{k^2}{2e}\neq 0 \\ \frac{\sqrt{2}\left(r-\frac{k}{2e}\right)\log\left(r-\frac{k}{2e}\right)}{2\sqrt{e\left(r-\frac{k}{2e}\right)^2}} & \text{otherwise} \end{cases}\right)}{2e}+\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e} & \text{for } e\neq 0 \\ \frac{\alpha^2\sqrt{-\alpha^2-2kr}+\frac{\left(-\alpha^2-2kr\right)^{\frac{3}{2}}}{3}}{2k^2} & \text{for } k\neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

output
```
Piecewise((k*Piecewise((sqrt(2)*log(2*sqrt(2)*sqrt(e)*sqrt(-alpha**2 + 2*e
*r**2 - 2*k*r) + 4*e*r - 2*k)/(2*sqrt(e)), Ne(alpha**2 + k**2/(2*e), 0)),
(sqrt(2)*(r - k/(2*e))*log(r - k/(2*e))/(2*sqrt(e*(r - k/(2*e))**2)), True
))/(2*e) + sqrt(-alpha**2 + 2*e*r**2 - 2*k*r)/(2*e), Ne(e, 0)), ((alpha**2
*sqrt(-alpha**2 - 2*k*r) + (-alpha**2 - 2*k*r)**(3/2)/3)/(2*k**2), Ne(k, 0
)), (r**2/(2*sqrt(-alpha**2)), True))
```

3.212.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr = \text{Exception raised: ValueError}$$

input
```
integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(k^2+2*alpha^2*e>0)', see `assume
?` for mor
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr = -\frac{\sqrt{2}k\log\left(\left|\sqrt{2}\left(\sqrt{2}\sqrt{e}r-\sqrt{2\,er^2-\alpha^2-2\,kr}\right)\sqrt{e}-k\right|\right)}{4\,e^{\frac{3}{2}}} + \frac{\sqrt{2\,er^2-\alpha^2-2\,kr}}{2\,e}$$

input
```
integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

output
```
-1/4*sqrt(2)*k*log(abs(sqrt(2)*(sqrt(2)*sqrt(e)*r - sqrt(2*e*r^2 - alpha^2
 - 2*k*r))*sqrt(e) - k))/e^(3/2) + 1/2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)/e
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}}\,dr = \frac{\sqrt{-\alpha^2 + 2\,e\,r^2 - 2\,k\,r}}{2\,e} + \frac{\sqrt{2}\,k\,\ln\left(\sqrt{-\alpha^2 + 2\,e\,r^2 - 2\,k\,r} - \frac{\sqrt{2}\,(k-2\,e\,r)}{2\,\sqrt{e}}\right)}{4\,e^{3/2}}$$

input
```
int(r/(2*e*r^2 - 2*k*r - alpha^2)^(1/2),r)
```

output
```
(2*e*r^2 - 2*k*r - alpha^2)^(1/2)/(2*e) + (2^(1/2)*k*log((2*e*r^2 - 2*k*r
- alpha^2)^(1/2) - (2^(1/2)*(k - 2*e*r))/(2*e^(1/2))))/(4*e^(3/2))
```

3.212.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}}\,dr = \frac{2\sqrt{2e\,r^2 - \alpha^2 - 2kr}\,e + \sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e}\,\sqrt{2e\,r^2-\alpha^2-2kr}\,\sqrt{2}+2er-k}{\sqrt{2\alpha^2e+k^2}}\right)k}{4e^2}$$

input
```
int(r/sqrt( - alpha**2 + 2*e*r**2 - 2*k*r),r)
```

output
```
(2*sqrt( - alpha**2 + 2*e*r**2 - 2*k*r)*e + sqrt(e)*sqrt(2)*log((sqrt(e)*s
qrt( - alpha**2 + 2*e*r**2 - 2*k*r)*sqrt(2) + 2*e*r - k)/sqrt(2*alpha**2*e
 + k**2))*k)/(4*e**2)
```

3.213 $\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$

3.213.1 Optimal result 1289
3.213.2 Mathematica [A] (verified) 1289
3.213.3 Rubi [A] (verified) 1290
3.213.4 Maple [A] (verified) 1291
3.213.5 Fricas [A] (verification not implemented) 1291
3.213.6 Sympy [F] 1292
3.213.7 Maxima [F(-2)] 1292
3.213.8 Giac [A] (verification not implemented) 1292
3.213.9 Mupad [B] (verification not implemented) 1293
3.213.10Reduce [F] 1293

3.213.1 Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = -\frac{\arctan\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

output
```
-1/2*arctan((-h*r^2+alpha^2)/alpha/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/alpha
```

3.213.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-k}r^2}{\alpha}-\frac{\sqrt{-\alpha^2+2hr^2-2kr^4}}{\alpha}\right)}{\alpha}$$

input
```
Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

output
```
-(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2)/alpha - Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]
/alpha]/alpha)
```

3.213.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{r^2\sqrt{-2kr^4+2hr^2-\alpha^2}}dr^2$$

$$\Big\downarrow \text{ 1154}$$

$$-\int \frac{1}{-r^4-4\alpha^2}d\left(-\frac{2(\alpha^2-hr^2)}{\sqrt{-2kr^4+2hr^2-\alpha^2}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\arctan\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

input
```
Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

output
```
-1/2*ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]/
alpha
```

3.213.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.213.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2+2h\,r^2+2\sqrt{-\alpha^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$	56
elliptic	$-\frac{\ln\left(\frac{-2\alpha^2+2h\,r^2+2\sqrt{-\alpha^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$	56
pseudoelliptic	$-\frac{\ln(2)+\ln\left(\frac{h\,r^2+\sqrt{-\alpha^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}-\alpha^2}{r^2}\right)}{2\sqrt{-\alpha^2}}$	57

input
```
int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

output
```
-1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(-2*k*r^4+
2*h*r^2-alpha^2)^(1/2))/r^2)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = -\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\,(hr^2-\alpha^2)}{2\,\alpha kr^4-2\,\alpha hr^2+\alpha^3}\right)}{2\,\alpha}$$

input
```
integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

output
```
-1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*(h*r^2 - alpha^2)/(2*alpha*
k*r^4 - 2*alpha*h*r^2 + alpha^3))/alpha
```

3.213.6 Sympy [F]

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = \int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$$

input
```
integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

output
```
Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r**4)), r)
```

3.213.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = \text{Exception raised: ValueError}$$

input
```
integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*alpha^2*k-h^2>0)', see `assume
?` for mor
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = \frac{\arctan\left(-\frac{\sqrt{2}\sqrt{-k}r^2-\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}}{\alpha}\right)}{\alpha}$$

input
```
integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

output
```
arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)
/alpha
```

3.213.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = -\frac{\ln\left(\frac{1}{r^2}\right)+\ln\left(h\,r^2-\alpha^2+\sqrt{-\alpha^2}\,\sqrt{-\alpha^2-2\,k\,r^4+2\,h\,r^2}\right)}{2\,\sqrt{-\alpha^2}}$$

input
```
int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)),r)
```

output
```
-(log(1/r^2) + log(h*r^2 - alpha^2 + (-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r^4 -
alpha^2)^(1/2)))/(2*(-alpha^2)^(1/2))
```

3.213.10 Reduce [F]

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = \int \frac{1}{\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}\,r}dr$$

input
```
int(1/(sqrt( - alpha**2 + 2*h*r**2 - 2*k*r**4)*r),r)
```

output
```
int(1/(sqrt( - alpha**2 + 2*h*r**2 - 2*k*r**4)*r),r)
```

3.214 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$

3.214.1 Optimal result 1294
3.214.2 Mathematica [A] (verified) 1294
3.214.3 Rubi [A] (verified) 1295
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3.214.5 Fricas [A] (verification not implemented) 1296
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3.214.8 Giac [A] (verification not implemented) 1298
3.214.9 Mupad [B] (verification not implemented) 1298
3.214.10Reduce [F] 1298

3.214.1 Optimal result

Integrand size = 31, antiderivative size = 68

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr=-\frac{\arctan\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

output
```
-1/2*arctan((-h*r^2+alpha^2+epsilon^2)/(alpha^2+epsilon^2)^(1/2)/(-2*k*r^4
+2*h*r^2-alpha^2-epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr=-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-k}r^2-\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

output
```
-(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2 - Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k
*r^4])/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2])
```

3.214.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}dr$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int \frac{1}{r^2\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}}dr^2$$
$$\Big\downarrow \text{1154}$$
$$-\int \frac{1}{-r^4-4(\alpha^2+\epsilon^2)}d\left(-\frac{2(\alpha^2+\epsilon^2-hr^2)}{\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}}\right)$$
$$\Big\downarrow \text{217}$$
$$-\frac{\arctan\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

input
```
Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

output
```
-1/2*ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[
-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/Sqrt[alpha^2 + epsilon^2]
```

3.214.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.214.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2h\,r^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$	78
elliptic	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2h\,r^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$	78
pseudoelliptic	$-\frac{\ln(2)+\ln\left(\frac{h\,r^2+\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}-\alpha^2-\epsilon^2}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$	79

input
```
int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE
)
```

output
```
-1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^2+2*(-alp
ha^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/r^2)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = -\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}\,(hr^2-\alpha^2-\epsilon^2)\sqrt{\alpha^2+\epsilon^2}}{2\,(\alpha^2+\epsilon^2)kr^4+\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\,(\alpha^2+\epsilon^2)hr^2}\right)}{2\,\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fri
cas")
```

output
```
-1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*(h*r^2 - alpha^
2 - epsilon^2)*sqrt(alpha^2 + epsilon^2)/(2*(alpha^2 + epsilon^2)*k*r^4 +
alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2)
)/sqrt(alpha^2 + epsilon^2)
```

3.214.6 Sympy [F]

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = \int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$$

input
```
integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

output
```
Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)), r)
```

3.214.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = \text{Exception raised: ValueError}$$

input
```
integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*epsilon^2*k+2*alpha^2*k>0)', s
ee `assume
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = \frac{\arctan\left(-\frac{\sqrt{2}\sqrt{-k}r^2-\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input
```
integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="gia
c")
```

output
```
arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilo
n^2))/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = -\frac{\ln\left(h-\frac{\alpha^2+\epsilon^2}{r^2}+\frac{\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-\alpha^2-\epsilon^2-2\,k\,r^4+2\,h\,r^2}}{r^2}\right)}{2\,\sqrt{-\alpha^2-\epsilon^2}}$$

input
```
int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2)),r)
```

output
```
-log(h - (alpha^2 + epsilon^2)/r^2 + ((- alpha^2 - epsilon^2)^(1/2)*(2*h*r
^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2))/r^2)/(2*(- alpha^2 - epsilon^2)
^(1/2))
```

3.214.10 Reduce [F]

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = \int \frac{1}{\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}\,r}dr$$

input
```
int(1/(sqrt( - alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)*r),r)
```

output
```
int(1/(sqrt( - alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)*r),r)
```

3.215 $\int a\cos(5+3x)\sin^2(5+3x)\,dx$

3.215.1 Optimal result 1299
3.215.2 Mathematica [A] (verified) 1299
3.215.3 Rubi [A] (verified) 1300
3.215.4 Maple [A] (verified) 1301
3.215.5 Fricas [A] (verification not implemented) 1302
3.215.6 Sympy [A] (verification not implemented) 1302
3.215.7 Maxima [A] (verification not implemented) 1302
3.215.8 Giac [A] (verification not implemented) 1303
3.215.9 Mupad [B] (verification not implemented) 1303
3.215.10 Reduce [B] (verification not implemented) 1303

3.215.1 Optimal result

Integrand size = 16, antiderivative size = 13

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx=\frac{1}{9}a\sin^3(5+3x)$$

output
```
1/9*a*sin(5+3*x)^3
```

3.215.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx=\frac{1}{9}a\sin^3(5+3x)$$

input
```
Integrate[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

output
```
(a*Sin[5 + 3*x]^3)/9
```

3.215.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {27, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a\sin^2(3x+5)\cos(3x+5)\,dx$$
$$\Big\downarrow \ 27$$
$$a\int \cos(3x+5)\sin^2(3x+5)dx$$
$$\Big\downarrow \ 3042$$
$$a\int \cos(3x+5)\sin(3x+5)^2dx$$
$$\Big\downarrow \ 3044$$
$$\frac{1}{3}a\int \sin^2(3x+5)d\sin(3x+5)$$
$$\Big\downarrow \ 15$$
$$\frac{1}{9}a\sin^3(3x+5)$$

input
```
Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

output
```
(a*Sin[5 + 3*x]^3)/9
```

3.215.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.215.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{a\left(\sin^3(5+3x)\right)}{9}$	12
default	$\frac{a\left(\sin^3(5+3x)\right)}{9}$	12
risch	$\frac{a\sin(5+3x)}{12}-\frac{a\sin(15+9x)}{36}$	20
parallelrisch	$a\left(-\frac{\sin(15+9x)}{36}+\frac{\sin(5+3x)}{12}\right)$	20
norman	$\frac{8a\left(\tan^3\left(\frac{5}{2}+\frac{3x}{2}\right)\right)}{9\left(1+\tan^2\left(\frac{5}{2}+\frac{3x}{2}\right)\right)^3}$	24

input
```
int(a*cos(5+3*x)*sin(5+3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*a*sin(5+3*x)^3
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx = -\frac{1}{9}\left(a\cos\left(3\,x+5\right)^2-a\right)\sin\left(3\,x+5\right)$$

input
```
integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="fricas")
```

output
```
-1/9*(a*cos(3*x + 5)^2 - a)*sin(3*x + 5)
```

3.215.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx = \frac{a\sin^3\left(3x+5\right)}{9}$$

input
```
integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)
```

output
```
a*sin(3*x + 5)**3/9
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx = \frac{1}{9}\,a\sin\left(3\,x+5\right)^3$$

input
```
integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="maxima")
```

output
```
1/9*a*sin(3*x + 5)^3
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx = \frac{1}{9}\,a\sin\left(3\,x+5\right)^3$$

input
```
integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="giac")
```

output
```
1/9*a*sin(3*x + 5)^3
```

3.215.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx = \frac{a\sin\left(3\,x+5\right)^3}{9}$$

input
```
int(a*cos(3*x + 5)*sin(3*x + 5)^2,x)
```

output
```
(a*sin(3*x + 5)^3)/9
```

3.215.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a\cos(5+3x)\sin^2(5+3x)\,dx = \frac{\sin\left(3x+5\right)^3 a}{9}$$

input
```
int(cos(3*x + 5)*sin(3*x + 5)**2*a,x)
```

output
```
(sin(3*x + 5)**3*a)/9
```

3.216 $\int \frac{\log(x^2)}{x^3} dx$

3.216.1 Optimal result 1304
3.216.2 Mathematica [A] (verified) 1304
3.216.3 Rubi [A] (verified) 1305
3.216.4 Maple [A] (verified) 1306
3.216.5 Fricas [A] (verification not implemented) 1306
3.216.6 Sympy [A] (verification not implemented) 1306
3.216.7 Maxima [A] (verification not implemented) 1307
3.216.8 Giac [A] (verification not implemented) 1307
3.216.9 Mupad [B] (verification not implemented) 1307
3.216.10 Reduce [B] (verification not implemented) 1308

3.216.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

output
```
-1/2/x^2-1/2*ln(x^2)/x^2
```

3.216.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

input
```
Integrate[Log[x^2]/x^3,x]
```

output
```
-1/2*1/x^2 - Log[x^2]/(2*x^2)
```

3.216.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x^2)}{x^3} \, dx$$

$$\Big\downarrow \text{2741}$$

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

input
```
Int[Log[x^2]/x^3,x]
```

output
```
-1/2*1/x^2 - Log[x^2]/(2*x^2)
```

3.216.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.216.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$-\frac{1+\ln(x^2)}{2x^2}$	12
norman	$\frac{-\frac{1}{2}-\frac{\ln(x^2)}{2}}{x^2}$	13
default	$-\frac{1}{2x^2}-\frac{\ln(x^2)}{2x^2}$	16
risch	$-\frac{1}{2x^2}-\frac{\ln(x^2)}{2x^2}$	16
parts	$-\frac{1}{2x^2}-\frac{\ln(x^2)}{2x^2}$	16

input
```
int(ln(x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*(1+ln(x^2))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\log(x^2)}{x^3}\,dx = -\frac{\log(x^2)+1}{2\,x^2}$$

input
```
integrate(log(x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(log(x^2) + 1)/x^2
```

3.216.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\log(x^2)}{x^3}\,dx = -\frac{\log(x^2)}{2x^2}-\frac{1}{2x^2}$$

input
```
integrate(ln(x**2)/x**3,x)
```

output
```
-log(x**2)/(2*x**2) - 1/(2*x**2)
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\log(x^2)}{x^3}\,dx = -\frac{\log(x^2)}{2\,x^2} - \frac{1}{2\,x^2}$$

input
```
integrate(log(x^2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(x^2)/x^2 - 1/2/x^2
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\log(x^2)}{x^3}\,dx = -\frac{\log(x^2)}{2\,x^2} - \frac{1}{2\,x^2}$$

input
```
integrate(log(x^2)/x^3,x, algorithm="giac")
```

output
```
-1/2*log(x^2)/x^2 - 1/2/x^2
```

3.216.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\log(x^2)}{x^3}\,dx = -\frac{\ln(x^2)+1}{2\,x^2}$$

input
```
int(log(x^2)/x^3,x)
```

output
```
-(log(x^2) + 1)/(2*x^2)
```

3.216.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\log(x^2)}{x^3}\,dx = \frac{-\log(x^2)-1}{2x^2}$$

input
```
int(log(x**2)/x**3,x)
```

output
```
( - (log(x**2) + 1))/(2*x**2)
```

3.217 $\int x \sin(a+x)\,dx$

3.217.1 Optimal result 1309
3.217.2 Mathematica [A] (verified) 1309
3.217.3 Rubi [A] (verified) 1310
3.217.4 Maple [A] (verified) 1311
3.217.5 Fricas [A] (verification not implemented) 1311
3.217.6 Sympy [A] (verification not implemented) 1312
3.217.7 Maxima [A] (verification not implemented) 1312
3.217.8 Giac [A] (verification not implemented) 1312
3.217.9 Mupad [B] (verification not implemented) 1313
3.217.10Reduce [B] (verification not implemented) 1313

3.217.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int x \sin(a+x)\,dx = -x\cos(a+x) + \sin(a+x)$$

output
```
-x*cos(a+x)+sin(a+x)
```

3.217.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \sin(a+x)\,dx = -x\cos(a+x) + \sin(a+x)$$

input
```
Integrate[x*Sin[a + x],x]
```

output
```
-(x*Cos[a + x]) + Sin[a + x]
```

3.217.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x \sin(a+x)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\int \cos(a+x)dx - x\cos(a+x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(a+x+\frac{\pi}{2}\right)dx - x\cos(a+x)$$

$$\Big\downarrow \text{ 3117}$$

$$\sin(a+x) - x\cos(a+x)$$

input
```
Int[x*Sin[a + x],x]
```

output
```
-(x*Cos[a + x]) + Sin[a + x]
```

3.217.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.217.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
risch	$-x\cos(a+x)+\sin(a+x)$	13
parallelrisch	$-x\cos(a+x)+\sin(a+x)$	13
parts	$-x\cos(a+x)+\sin(a+x)$	13
derivativedivides	$a\cos(a+x)+\sin(a+x)-(a+x)\cos(a+x)$	21
default	$a\cos(a+x)+\sin(a+x)-(a+x)\cos(a+x)$	21
norman	$\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)-x+2\tan\left(\frac{a}{2}+\frac{x}{2}\right)}{1+\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)}$	42
meijerg	$2\sin(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)+2\cos(a)\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	53

input
```
int(x*sin(a+x),x,method=_RETURNVERBOSE)
```

output
```
-x*cos(a+x)+sin(a+x)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sin(a+x)\,dx = -x\cos(a+x)+\sin(a+x)$$

input
```
integrate(x*sin(a+x),x, algorithm="fricas")
```

output
```
-x*cos(a + x) + sin(a + x)
```

3.217.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x\sin(a+x)\,dx=-x\cos(a+x)+\sin(a+x)$$

input
```
integrate(x*sin(a+x),x)
```

output
```
-x*cos(a + x) + sin(a + x)
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x\sin(a+x)\,dx=-(a+x)\cos(a+x)+a\cos(a+x)+\sin(a+x)$$

input
```
integrate(x*sin(a+x),x, algorithm="maxima")
```

output
```
-(a + x)*cos(a + x) + a*cos(a + x) + sin(a + x)
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sin(a+x)\,dx=-x\cos(a+x)+\sin(a+x)$$

input
```
integrate(x*sin(a+x),x, algorithm="giac")
```

output
```
-x*cos(a + x) + sin(a + x)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sin(a+x)\,dx = \sin(a+x) - x\,\cos(a+x)$$

input
```
int(x*sin(a + x),x)
```

output
```
sin(a + x) - x*cos(a + x)
```

3.217.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sin(a+x)\,dx = -\cos(a+x)\,x + \sin(a+x)$$

input
```
int(sin(a + x)*x,x)
```

output
```
- cos(a + x)*x + sin(a + x)
```

3.218 $\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx$

3.218.1 Optimal result . . . 1314
3.218.2 Mathematica [A] (verified) . . . 1314
3.218.3 Rubi [A] (verified) . . . 1315
3.218.4 Maple [A] (verified) . . . 1315
3.218.5 Fricas [A] (verification not implemented) . . . 1316
3.218.6 Sympy [A] (verification not implemented) . . . 1316
3.218.7 Maxima [A] (verification not implemented) . . . 1316
3.218.8 Giac [A] (verification not implemented) . . . 1317
3.218.9 Mupad [B] (verification not implemented) . . . 1317
3.218.10 Reduce [B] (verification not implemented) . . . 1317

3.218.1 Optimal result

Integrand size = 20, antiderivative size = 11

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{e^{-x}x}{\log(x)}$$

output
```
x/exp(x)/ln(x)
```

3.218.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{e^{-x}x}{\log(x)}$$

input
```
Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

output
```
x/(E^x*Log[x])
```

3.218.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2630}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}((1-x)\log(x)-1)}{\log^2(x)}\,dx$$

$$\Big\downarrow \text{ 2630}$$

$$\frac{e^{-x}x}{\log(x)}$$

input
```
Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

output
```
x/(E^x*Log[x])
```

3.218.3.1 Defintions of rubi rules used

rule 2630
```
Int[Log[(d_.)*(x_)]^(n_.)*(F_)^(v_)*((e_) + Log[(d_.)*(x_)]*(h_.)*((f_.) +
(g_.)*(x_))), x_Symbol] :> Simp[e*x*F^v*(Log[d*x]^(n + 1)/(n + 1)), x] /; F
reeQ[{F, d, e, f, g, h, n}, x] && LinearQ[v, x] && EqQ[e, f*h*(n + 1)] && E
qQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]
```

3.218.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{x\,{\rm e}^{-x}}{\ln(x)}$	11
risch	$\frac{x\,{\rm e}^{-x}}{\ln(x)}$	11
parallelrisch	$\frac{x\,{\rm e}^{-x}}{\ln(x)}$	11

input
```
int((-1+(1-x)*ln(x))/exp(x)/ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/exp(x)/ln(x)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx=\frac{xe^{(-x)}}{\log(x)}$$

input
```
integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="fricas")
```

output
```
x*e^(-x)/log(x)
```

3.218.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx=\frac{xe^{-x}}{\log(x)}$$

input
```
integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)
```

output
```
x*exp(-x)/log(x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx=\frac{xe^{(-x)}}{\log(x)}$$

input
```
integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="maxima")
```

output
```
x*e^(-x)/log(x)
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{xe^{(-x)}}{\log(x)}$$

input
```
integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="giac")
```

output
```
x*e^(-x)/log(x)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{x\,\mathrm{e}^{-x}}{\ln(x)}$$

input
```
int(-(exp(-x)*(log(x)*(x - 1) + 1))/log(x)^2,x)
```

output
```
(x*exp(-x))/log(x)
```

3.218.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{x}{e^x\log(x)}$$

input
```
int(( - log(x)*x + log(x) - 1)/(e**x*log(x)**2),x)
```

output
```
x/(e**x*log(x))
```

3.219 $\int \frac{x^3}{b+ax^2}\,dx$

3.219.1 Optimal result 1318
3.219.2 Mathematica [A] (verified) 1318
3.219.3 Rubi [A] (verified) 1319
3.219.4 Maple [A] (verified) 1320
3.219.5 Fricas [A] (verification not implemented) 1320
3.219.6 Sympy [A] (verification not implemented) 1321
3.219.7 Maxima [A] (verification not implemented) 1321
3.219.8 Giac [A] (verification not implemented) 1321
3.219.9 Mupad [B] (verification not implemented) 1322
3.219.10Reduce [B] (verification not implemented) 1322

3.219.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{x^2}{2a} - \frac{b\log(b+ax^2)}{2a^2}$$

output
```
1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

3.219.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{x^2}{2a} - \frac{b\log(b+ax^2)}{2a^2}$$

input
```
Integrate[x^3/(b + a*x^2),x]
```

output
```
x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

3.219.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^2+b}\,dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{x^2}{ax^2+b}dx^2$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{2}\int \left(\frac{1}{a}-\frac{b}{a\,(ax^2+b)}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{x^2}{a}-\frac{b\log\left(ax^2+b\right)}{a^2}\right)$$

input
```
Int[x^3/(b + a*x^2),x]
```

output
```
(x^2/a - (b*Log[b + a*x^2])/a^2)/2
```

3.219.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.219.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-a\,x^2+b\ln\left(a\,x^2+b\right)}{2a^2}$	23
default	$\frac{x^2}{2a}-\frac{b\ln\left(a\,x^2+b\right)}{2a^2}$	24
norman	$\frac{x^2}{2a}-\frac{b\ln\left(a\,x^2+b\right)}{2a^2}$	24
risch	$\frac{x^2}{2a}-\frac{b\ln\left(a\,x^2+b\right)}{2a^2}$	24

input
```
int(x^3/(a*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*x^2+b*ln(a*x^2+b))/a^2
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{ax^2-b\log\left(ax^2+b\right)}{2\,a^2}$$

input
```
integrate(x^3/(a*x^2+b),x, algorithm="fricas")
```

output
```
1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

3.219.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

input
```
integrate(x**3/(a*x**2+b),x)
```

output
```
x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{x^2}{2\,a} - \frac{b\log\left(ax^2+b\right)}{2\,a^2}$$

input
```
integrate(x^3/(a*x^2+b),x, algorithm="maxima")
```

output
```
1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{x^2}{2\,a} - \frac{b\log\left(|ax^2+b|\right)}{2\,a^2}$$

input
```
integrate(x^3/(a*x^2+b),x, algorithm="giac")
```

output
```
1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2
```

3.219.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{b+ax^2}\,dx = -\frac{b\,\ln\left(a\,x^2+b\right)-a\,x^2}{2\,a^2}$$

input
```
int(x^3/(b + a*x^2),x)
```

output
```
-(b*log(b + a*x^2) - a*x^2)/(2*a^2)
```

3.219.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{-\log(a\,x^2+b)\,b+a\,x^2}{2a^2}$$

input
```
int(x**3/(a*x**2 + b),x)
```

output
```
( - log(a*x**2 + b)*b + a*x**2)/(2*a**2)
```

3.220 $\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx$

3.220.1 Optimal result 1323
3.220.2 Mathematica [A] (verified) 1323
3.220.3 Rubi [A] (verified) 1324
3.220.4 Maple [A] (verified) 1325
3.220.5 Fricas [B] (verification not implemented) 1325
3.220.6 Sympy [C] (verification not implemented) 1326
3.220.7 Maxima [A] (verification not implemented) 1326
3.220.8 Giac [B] (verification not implemented) 1327
3.220.9 Mupad [B] (verification not implemented) 1327
3.220.10 Reduce [B] (verification not implemented) 1327

3.220.1 Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx = \frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{4x^{3/2}}{15(1+x)^{3/2}}$$

output
```
2/5*x^(3/2)/(1+x)^(5/2)+4/15*x^(3/2)/(1+x)^(3/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx = \frac{2x^{3/2}(5+2x)}{15(1+x)^{5/2}}$$

input
```
Integrate[Sqrt[x]/(1 + x)^(7/2),x]
```

output
```
(2*x^(3/2)*(5 + 2*x))/(15*(1 + x)^(5/2))
```

3.220.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(x+1)^{7/2}} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{2}{5} \int \frac{\sqrt{x}}{(x+1)^{5/2}} dx + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

$$\Big\downarrow \; 48$$

$$\frac{4x^{3/2}}{15(x+1)^{3/2}} + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

input
```
Int[Sqrt[x]/(1 + x)^(7/2),x]
```

output
```
(2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

3.220.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

3.220.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
meijerg	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
risch	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
default	$-\frac{2\sqrt{x}}{5(1+x)^{\frac{5}{2}}}+\frac{2\sqrt{x}}{15(1+x)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{15\sqrt{1+x}}$	32

input
```
int(x^(1/2)/(1+x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*x^(3/2)*(5+2*x)/(1+x)^(5/2)
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \frac{2\left(2\,x^3+\left(2\,x^2+5\,x\right)\sqrt{x+1}\sqrt{x}+6\,x^2+6\,x+2\right)}{15\left(x^3+3\,x^2+3\,x+1\right)}$$

input
```
integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(2*x^3 + (2*x^2 + 5*x)*sqrt(x + 1)*sqrt(x) + 6*x^2 + 6*x + 2)/(x^3 +
3*x^2 + 3*x + 1)
```

3.220.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.06

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \begin{cases} \frac{4i\sqrt{-1+\frac{1}{x+1}}(x+1)^2}{-15x+15(x+1)^2-15} - \frac{2i\sqrt{-1+\frac{1}{x+1}}(x+1)}{-15x+15(x+1)^2-15} - \frac{8i\sqrt{-1+\frac{1}{x+1}}}{-15x+15(x+1)^2-15} + \frac{6i\sqrt{-1+\frac{1}{x+1}}}{(x+1)\left(-15x+15(x+1)^2-15\right)} & \text{for } | \\ \frac{4\sqrt{1-\frac{1}{x+1}}}{15} + \frac{2\sqrt{1-\frac{1}{x+1}}}{15(x+1)} - \frac{2\sqrt{1-\frac{1}{x+1}}}{5(x+1)^2} & \text{othe} \end{cases}$$

input
```
integrate(x**(1/2)/(1+x)**(7/2),x)
```

output
```
Piecewise((4*I*sqrt(-1 + 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1)**2 - 15
) - 2*I*sqrt(-1 + 1/(x + 1))*(x + 1)/(-15*x + 15*(x + 1)**2 - 15) - 8*I*sq
rt(-1 + 1/(x + 1))/(-15*x + 15*(x + 1)**2 - 15) + 6*I*sqrt(-1 + 1/(x + 1))
/((x + 1)*(-15*x + 15*(x + 1)**2 - 15)), 1/Abs(x + 1) > 1), (4*sqrt(1 - 1/
(x + 1))/15 + 2*sqrt(1 - 1/(x + 1))/(15*(x + 1)) - 2*sqrt(1 - 1/(x + 1))/(
5*(x + 1)**2), True))
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \frac{2\,x^{\frac{5}{2}}\left(\frac{5\,(x+1)}{x} - 3\right)}{15\,(x+1)^{\frac{5}{2}}}$$

input
```
integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="maxima")
```

output
```
2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(21) = 42$.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \frac{8\left(15\left(\sqrt{x+1}-\sqrt{x}\right)^6-5\left(\sqrt{x+1}-\sqrt{x}\right)^4+5\left(\sqrt{x+1}-\sqrt{x}\right)^2+1\right)}{15\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2+1\right)^5}$$

input
```
integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="giac")
```

output
```
8/15*(15*(sqrt(x + 1) - sqrt(x))^6 - 5*(sqrt(x + 1) - sqrt(x))^4 + 5*(sqrt
(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 + 1)^5
```

3.220.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \frac{2\,x^{3/2}\,(2\,x+5)}{15\,(x+1)^{5/2}}$$

input
```
int(x^(1/2)/(x + 1)^(7/2),x)
```

output
```
(2*x^(3/2)*(2*x + 5))/(15*(x + 1)^(5/2))
```

3.220.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \frac{-\frac{4\sqrt{x+1}\,x^2}{15}-\frac{8\sqrt{x+1}\,x}{15}-\frac{4\sqrt{x+1}}{15}+\frac{4\sqrt{x}\,x^2}{15}+\frac{2\sqrt{x}\,x}{3}}{\sqrt{x+1}\,(x^2+2x+1)}$$

input
```
int(sqrt(x)/(sqrt(x + 1)*(x**3 + 3*x**2 + 3*x + 1)),x)
```

output
```
(2*( - 2*sqrt(x + 1)*x**2 - 4*sqrt(x + 1)*x - 2*sqrt(x + 1) + 2*sqrt(x)*x*
*2 + 5*sqrt(x)*x))/(15*sqrt(x + 1)*(x**2 + 2*x + 1))
```

3.221 $\int \frac{1}{x(1+x)}\,dx$

3.221.1 Optimal result 1328
3.221.2 Mathematica [A] (verified) 1328
3.221.3 Rubi [A] (verified) 1329
3.221.4 Maple [A] (verified) 1330
3.221.5 Fricas [A] (verification not implemented) 1330
3.221.6 Sympy [A] (verification not implemented) 1330
3.221.7 Maxima [A] (verification not implemented) 1331
3.221.8 Giac [A] (verification not implemented) 1331
3.221.9 Mupad [B] (verification not implemented) 1331
3.221.10 Reduce [B] (verification not implemented) 1332

3.221.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{1}{x(1+x)}\,dx = \log(x) - \log(1+x)$$

output
```
ln(x)-ln(1+x)
```

3.221.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)}\,dx = \log(x) - \log(1+x)$$

input
```
Integrate[1/(x*(1 + x)),x]
```

output
```
Log[x] - Log[1 + x]
```

3.221.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)}dx$$

$$\Big\downarrow \text{ 47}$$

$$\int \frac{1}{x}dx-\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{ 14}$$

$$\log(x)-\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{ 16}$$

$$\log(x)-\log(x+1)$$

input
```
Int[1/(x*(1 + x)),x]
```

output
```
Log[x] - Log[1 + x]
```

3.221.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

3.221.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\ln(x) - \ln(1+x)$	10
norman	$\ln(x) - \ln(1+x)$	10
meijerg	$\ln(x) - \ln(1+x)$	10
risch	$\ln(x) - \ln(1+x)$	10
parallelrisch	$\ln(x) - \ln(1+x)$	10

input
```
int(1/x/(1+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-ln(1+x)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)}\,dx = -\log(x+1) + \log(x)$$

input
```
integrate(1/x/(1+x),x, algorithm="fricas")
```

output
```
-log(x + 1) + log(x)
```

3.221.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)}\,dx = \log(x) - \log(x+1)$$

input
```
integrate(1/x/(1+x),x)
```

output
```
log(x) - log(x + 1)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)}\,dx = -\log(x+1) + \log(x)$$

input
```
integrate(1/x/(1+x),x, algorithm="maxima")
```

output
```
-log(x + 1) + log(x)
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(1+x)}\,dx = -\log(|x+1|) + \log(|x|)$$

input
```
integrate(1/x/(1+x),x, algorithm="giac")
```

output
```
-log(abs(x + 1)) + log(abs(x))
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+x)}\,dx = -\ln\left(\frac{1}{x}+1\right)$$

input
```
int(1/(x*(x + 1)),x)
```

output
```
-log(1/x + 1)
```

3.221.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)}\,dx = -\log(x+1)+\log(x)$$

input
```
int(1/(x*(x + 1)),x)
```

output
```
 - log(x + 1) + log(x)
```

3.222 $\int \frac{1}{\sqrt{x}(-1+2x)}\,dx$

3.222.1 Optimal result 1333
3.222.2 Mathematica [A] (verified) 1333
3.222.3 Rubi [A] (verified) 1334
3.222.4 Maple [A] (verified) 1335
3.222.5 Fricas [B] (verification not implemented) 1335
3.222.6 Sympy [B] (verification not implemented) 1336
3.222.7 Maxima [B] (verification not implemented) 1336
3.222.8 Giac [B] (verification not implemented) 1336
3.222.9 Mupad [B] (verification not implemented) 1337
3.222.10Reduce [B] (verification not implemented) 1337

3.222.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = -\sqrt{2}\text{arctanh}\left(\sqrt{2}\sqrt{x}\right)$$

output
```
-arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = -\sqrt{2}\text{arctanh}\left(\sqrt{2}\sqrt{x}\right)$$

input
```
Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

3.222.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(2x-1)}dx$$
$$\Big\downarrow \text{ 73}$$
$$2\int \frac{1}{2x-1}d\sqrt{x}$$
$$\Big\downarrow \text{ 220}$$
$$-\sqrt{2}\text{arctanh}\left(\sqrt{2}\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[x]*(-1 + 2*x)),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

3.222.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.222.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{x}\right)\sqrt{2}$	14
default	$-\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{x}\right)\sqrt{2}$	14
meijerg	$-\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{x}\right)\sqrt{2}$	14
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)-4\sqrt{x}}{2x-1}\right)}{2}$	40

input
```
int(1/x^(1/2)/(2*x-1),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{x}-2\,x-1}{2\,x-1}\right)$$

input
```
integrate(1/x^(1/2)/(-1+2*x),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))
```

3.222.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = \frac{\sqrt{2}\log\left(\sqrt{x}-\frac{\sqrt{2}}{2}\right)}{2} - \frac{\sqrt{2}\log\left(\sqrt{x}+\frac{\sqrt{2}}{2}\right)}{2}$$

input
```
integrate(1/x**(1/2)/(-1+2*x),x)
```

output
```
sqrt(2)*log(sqrt(x) - sqrt(2)/2)/2 - sqrt(2)*log(sqrt(x) + sqrt(2)/2)/2
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{x}}{\sqrt{2}+2\sqrt{x}}\right)$$

input
```
integrate(1/x^(1/2)/(-1+2*x),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - 2*sqrt(x))/(sqrt(2) + 2*sqrt(x)))
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{1}{2}\sqrt{2}+\sqrt{x}\right) + \frac{1}{2}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sqrt{x}\right|\right)$$

input
```
integrate(1/x^(1/2)/(-1+2*x),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*log(abs(-1/2*sqrt(2)
 + sqrt(x)))
```

3.222.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = -\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2\,x}\right)$$

input
```
int(1/(x^(1/2)*(2*x - 1)),x)
```

output
```
-2^(1/2)*atanh((2*x)^(1/2))
```

3.222.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = \frac{\sqrt{2}\,\left(\log\left(2\sqrt{x}-\sqrt{2}\right)-\log\left(2\sqrt{x}+\sqrt{2}\right)\right)}{2}$$

input
```
int(1/(sqrt(x)*(2*x - 1)),x)
```

output
```
(sqrt(2)*(log(2*sqrt(x) - sqrt(2)) - log(2*sqrt(x) + sqrt(2))))/2
```

3.223 $\int \sqrt{x}(1+x^2)\,dx$

3.223.1 Optimal result 1338
3.223.2 Mathematica [A] (verified) 1338
3.223.3 Rubi [A] (verified) 1339
3.223.4 Maple [A] (verified) 1340
3.223.5 Fricas [A] (verification not implemented) 1340
3.223.6 Sympy [A] (verification not implemented) 1341
3.223.7 Maxima [A] (verification not implemented) 1341
3.223.8 Giac [A] (verification not implemented) 1341
3.223.9 Mupad [B] (verification not implemented) 1342
3.223.10 Reduce [B] (verification not implemented) 1342

3.223.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \sqrt{x}(1+x^2)\,dx=\frac{2x^{3/2}}{3}+\frac{2x^{7/2}}{7}$$

output
```
2/3*x^(3/2)+2/7*x^(7/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(1+x^2)\,dx=\frac{2}{21}x^{3/2}(7+3x^2)$$

input
```
Integrate[Sqrt[x]*(1 + x^2),x]
```

output
```
(2*x^(3/2)*(7 + 3*x^2))/21
```

3.223.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(x^2+1)\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(x^{5/2}+\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{7/2}}{7}+\frac{2x^{3/2}}{3}$$

input
```
Int[Sqrt[x]*(1 + x^2),x]
```

output
```
(2*x^(3/2))/3 + (2*x^(7/2))/7
```

3.223.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.223.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{7}{2}}}{7}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{7}{2}}}{7}$	12
gosper	$\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$	13
trager	$\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$	13
risch	$\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$	13

input
```
int(x^(1/2)*(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)+2/7*x^(7/2)
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(1+x^2)\,dx=\frac{2}{21}\left(3\,x^3+7\,x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(x^2+1),x, algorithm="fricas")
```

output
```
2/21*(3*x^3 + 7*x)*sqrt(x)
```

3.223.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(1+x^2)\ dx = \frac{2x^{\frac{7}{2}}}{7} + \frac{2x^{\frac{3}{2}}}{3}$$

input
```
integrate(x**(1/2)*(x**2+1),x)
```

output
```
2*x**(7/2)/7 + 2*x**(3/2)/3
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \sqrt{x}(1+x^2)\ dx = \frac{2}{7}\,x^{\frac{7}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(x^2+1),x, algorithm="maxima")
```

output
```
2/7*x^(7/2) + 2/3*x^(3/2)
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \sqrt{x}(1+x^2)\ dx = \frac{2}{7}\,x^{\frac{7}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(x^2+1),x, algorithm="giac")
```

output
```
2/7*x^(7/2) + 2/3*x^(3/2)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \sqrt{x}(1+x^2)\,dx = \frac{2\,x^{3/2}\,(3\,x^2+7)}{21}$$

input
```
int(x^(1/2)*(x^2 + 1),x)
```

output
```
(2*x^(3/2)*(3*x^2 + 7))/21
```

3.223.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \sqrt{x}(1+x^2)\,dx = \frac{2\sqrt{x}\,x(3x^2+7)}{21}$$

input
```
int(sqrt(x)*(x**2 + 1),x)
```

output
```
(2*sqrt(x)*x*(3*x**2 + 7))/21
```

3.224 $\int \frac{\sqrt[3]{-a+x}}{x}\,dx$

3.224.1 Optimal result . . . 1343
3.224.2 Mathematica [A] (verified) . . . 1343
3.224.3 Rubi [A] (verified) . . . 1344
3.224.4 Maple [A] (verified) . . . 1346
3.224.5 Fricas [A] (verification not implemented) . . . 1347
3.224.6 Sympy [C] (verification not implemented) . . . 1347
3.224.7 Maxima [A] (verification not implemented) . . . 1348
3.224.8 Giac [A] (verification not implemented) . . . 1348
3.224.9 Mupad [B] (verification not implemented) . . . 1349
3.224.10 Reduce [B] (verification not implemented) . . . 1349

3.224.1 Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = 3\sqrt[3]{-a+x}+\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+x}}{\sqrt{3}\sqrt[3]{a}}\right) + \frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)$$

output
```
3*(-a+x)^(1/3)+1/2*a^(1/3)*ln(x)-3/2*a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+a^(1
/3)*arctan(1/3*(a^(1/3)-2*(-a+x)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = 3\sqrt[3]{-a+x}+\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right) + \frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+x}+(-a+x)^{2/3}\right)$$

input
```
Integrate[(-a + x)^(1/3)/x,x]
```

output
```
3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3))
/Sqrt[3]] - a^(1/3)*Log[a^(1/3) + (-a + x)^(1/3)] + (a^(1/3)*Log[a^(2/3) -
 a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)])/2
```

3.224.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x-a}}{x}dx$$

$$\Big\downarrow \ 60$$

$$3\sqrt[3]{x-a}-a\int \frac{1}{x(x-a)^{2/3}}dx$$

$$\Big\downarrow \ 70$$

$$3\sqrt[3]{x-a}-a\left(\frac{3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{x-a}}d\sqrt[3]{x-a}}{2a^{2/3}}+\frac{3\int \frac{1}{a^{2/3}-\sqrt[3]{x-a}\sqrt[3]{a}+(x-a)^{2/3}}d\sqrt[3]{x-a}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)$$

$$\Big\downarrow \ 16$$

$$3\sqrt[3]{x-a}-a\left(\frac{3\int \frac{1}{a^{2/3}-\sqrt[3]{x-a}\sqrt[3]{a}+(x-a)^{2/3}}d\sqrt[3]{x-a}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)}{2a^{2/3}}\right)$$

$$\Big\downarrow \ 1082$$

$$3\sqrt[3]{x-a}-a\left(\frac{3\int \frac{1}{-(x-a)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{x-a}}{\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)}{2a^{2/3}}\right)$$

$$\Big\downarrow \ 217$$

$$3\sqrt[3]{x-a}-a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{x-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)}{2a^{2/3}}\right)$$

input
```
Int[(-a + x)^(1/3)/x,x]
```

output
```
3*(-a + x)^(1/3) - a*(-((Sqrt[3]*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3))/S
qrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) + (-a + x)^(1/3)])
/(2*a^(2/3)))
```

3.224.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.224. $\int \frac{\sqrt[3]{-a+x}}{x}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.224.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
derivativedivides	$3(-a+x)^{\frac{1}{3}}-3\left(\frac{\ln\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((-a+x)^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(-a+x)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$
default	$3(-a+x)^{\frac{1}{3}}-3\left(\frac{\ln\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((-a+x)^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(-a+x)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$

input
```
int((-a+x)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3*(-a+x)^(1/3)-3*(1/3/a^(2/3)*ln(a^(1/3)+(-a+x)^(1/3))-1/6/a^(2/3)*ln((-a+
x)^(2/3)-a^(1/3)*(-a+x)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/a^(1/3)*(-a+x)^(1/3)-1))*a
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = \sqrt{3}(-a)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}(-a)^{\frac{2}{3}}(-a+x)^{\frac{1}{3}}}{3\,a}\right) - \frac{1}{2}(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right) + (-a)^{\frac{1}{3}}\log\left(-(-a)^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)+3(-a+x)^{\frac{1}{3}}$$

input
```
integrate((-a+x)^(1/3)/x,x, algorithm="fricas")
```

output
```
sqrt(3)*(-a)^(1/3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-a)^(2/3)*(-a + x)^
(1/3))/a) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-
a + x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^
(1/3)
```

3.224.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = \frac{4\sqrt[3]{a}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{-a+x}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{-a+x}\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-a+x)**(1/3)/x,x)
```

output
```
4*a**(1/3)*exp(-I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3)
)*gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a + x)**(1/3)*exp_polar
(I*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(I*pi/3)*log(1
- (-a + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3))
+ 4*(-a + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\left(a^{\frac{1}{3}}-2\,(-a+x)^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + \frac{1}{2}\,a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right) - a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)+3\,(-a+x)^{\frac{1}{3}}$$

input
```
integrate((-a+x)^(1/3)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*a^(1/3)*arctan(-1/3*sqrt(3)*(a^(1/3) - 2*(-a + x)^(1/3))/a^(1/3))
 + 1/2*a^(1/3)*log(a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) - a^
(1/3)*log(a^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = -\sqrt{3}(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left((-a)^{\frac{1}{3}}+2\,(-a+x)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right) - \frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right) + (-a)^{\frac{1}{3}}\log\left(\left|-(-a)^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right|\right)+3\,(-a+x)^{\frac{1}{3}}$$

input
```
integrate((-a+x)^(1/3)/x,x, algorithm="giac")
```

output
```
-sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(1/3))/(-a
)^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a
 + x)^(2/3)) + (-a)^(1/3)*log(abs(-(-a)^(1/3) + (-a + x)^(1/3))) + 3*(-a +
 x)^(1/3)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = (-a)^{1/3}\ln\left(-9\,(-a)^{4/3} -9\,a\,(x-a)^{1/3}\right)+3\,(x-a)^{1/3}+\frac{(-a)^{1/3}\ln\left(\frac{9\,(-a)^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+9\,a\,(x-a)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{(-a)^{1}}{}$$

input
```
int((x - a)^(1/3)/x,x)
```

output
```
(-a)^(1/3)*log(- 9*(-a)^(4/3) - 9*a*(x - a)^(1/3)) + 3*(x - a)^(1/3) + ((-
a)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i - 1))/2 + 9*a*(x - a)^(1/3))*(3^(1/
2)*1i - 1))/2 - ((-a)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i + 1))/2 - 9*a*(x
 - a)^(1/3))*(3^(1/2)*1i + 1))/2
```

3.224.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(-a+x)^{\frac{1}{6}}-a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)a+2\sqrt{3}\,atan\left(\frac{2(-a+x)^{\frac{1}{6}}+a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)a+6a^{\frac{2}{3}}(-a+x)^{\frac{1}{3}}-2\log\left((-a+x)^{\frac{1}{3}}+a\right.}{2a^{\frac{2}{3}}}$$

input
```
int(( - a + x)**(1/3)/x,x)
```

output
```
( - 2*sqrt(3)*atan((2*( - a + x)**(1/6) - a**(1/6)*sqrt(3))/a**(1/6))*a +
2*sqrt(3)*atan((2*( - a + x)**(1/6) + a**(1/6)*sqrt(3))/a**(1/6))*a + 6*a*
*(2/3)*( - a + x)**(1/3) - 2*log(( - a + x)**(1/3) + a**(1/3))*a + log( -
a**(1/6)*( - a + x)**(1/6)*sqrt(3) + ( - a + x)**(1/3) + a**(1/3))*a + log
(a**(1/6)*( - a + x)**(1/6)*sqrt(3) + ( - a + x)**(1/3) + a**(1/3))*a)/(2*
a**(2/3))
```

3.224. $\int \frac{\sqrt[3]{-a+x}}{x}\,dx$

3.225 $\int x \sinh(x)\,dx$

3.225.1 Optimal result 1351
3.225.2 Mathematica [A] (verified) 1351
3.225.3 Rubi [C] (verified) 1352
3.225.4 Maple [A] (verified) 1353
3.225.5 Fricas [A] (verification not implemented) 1354
3.225.6 Sympy [A] (verification not implemented) 1354
3.225.7 Maxima [B] (verification not implemented) 1354
3.225.8 Giac [A] (verification not implemented) 1355
3.225.9 Mupad [B] (verification not implemented) 1355
3.225.10Reduce [B] (verification not implemented) 1355

3.225.1 Optimal result

Integrand size = 4, antiderivative size = 9

$$\int x \sinh(x)\,dx = x\cosh(x) - \sinh(x)$$

output
```
x*cosh(x)-sinh(x)
```

3.225.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x)\,dx = x\cosh(x) - \sinh(x)$$

input
```
Integrate[x*Sinh[x],x]
```

output
```
x*Cosh[x] - Sinh[x]
```

3.225.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix\sin(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x\sin(ix)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(ix\cosh(x) - i\int \cosh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(ix\cosh(x) - i\int \sin\left(ix+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{3117}$$

$$-i(ix\cosh(x) - i\sinh(x))$$

input
```
Int[x*Sinh[x],x]
```

output
```
(-I)*(I*x*Cosh[x] - I*Sinh[x])
```

3.225.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.225.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$x\cosh\left(x\right)-\sinh\left(x\right)$	10
meijerg	$x\cosh\left(x\right)-\sinh\left(x\right)$	10
parallelrisch	$x\cosh\left(x\right)-\sinh\left(x\right)$	10
parts	$x\cosh\left(x\right)-\sinh\left(x\right)$	10
risch	$\left(-\frac{1}{2}+\frac{x}{2}\right)\mathrm{e}^{x}+\left(\frac{1}{2}+\frac{x}{2}\right)\mathrm{e}^{-x}$	20

input
```
int(x*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
x*cosh(x)-sinh(x)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x)\, dx = x \cosh{(x)} - \sinh{(x)}$$

input
```
integrate(x*sinh(x),x, algorithm="fricas")
```

output
```
x*cosh(x) - sinh(x)
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x \sinh(x)\, dx = x \cosh{(x)} - \sinh{(x)}$$

input
```
integrate(x*sinh(x),x)
```

output
```
x*cosh(x) - sinh(x)
```

3.225.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.78

$$\int x \sinh(x)\, dx = \frac{1}{2}\, x^2 \sinh{(x)} + \frac{1}{4}\, \left(x^2 + 2\,x + 2\right)e^{(-x)} - \frac{1}{4}\, \left(x^2 - 2\,x + 2\right)e^{x}$$

input
```
integrate(x*sinh(x),x, algorithm="maxima")
```

output
```
1/2*x^2*sinh(x) + 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int x \sinh(x)\, dx = \frac{1}{2}\,(x+1)e^{(-x)} + \frac{1}{2}\,(x-1)e^{x}$$

input
```
integrate(x*sinh(x),x, algorithm="giac")
```

output
```
1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x)\, dx = x \cosh(x) - \sinh(x)$$

input
```
int(x*sinh(x),x)
```

output
```
x*cosh(x) - sinh(x)
```

3.225.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x)\, dx = \cosh(x)\, x - \sinh(x)$$

input
```
int(sinh(x)*x,x)
```

output
```
cosh(x)*x - sinh(x)
```

3.226 $\int x \cosh(x)\,dx$

3.226.1 Optimal result 1356
3.226.2 Mathematica [A] (verified) 1356
3.226.3 Rubi [A] (verified) 1357
3.226.4 Maple [A] (verified) 1358
3.226.5 Fricas [A] (verification not implemented) 1359
3.226.6 Sympy [A] (verification not implemented) 1359
3.226.7 Maxima [B] (verification not implemented) 1359
3.226.8 Giac [A] (verification not implemented) 1360
3.226.9 Mupad [B] (verification not implemented) 1360
3.226.10Reduce [B] (verification not implemented) 1360

3.226.1 Optimal result

Integrand size = 4, antiderivative size = 9

$$\int x \cosh(x)\,dx = -\cosh(x) + x\sinh(x)$$

output
```
-cosh(x)+x*sinh(x)
```

3.226.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \cosh(x)\,dx = -\cosh(x) + x\sinh(x)$$

input
```
Integrate[x*Cosh[x],x]
```

output
```
-Cosh[x] + x*Sinh[x]
```

3.226.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{3777}$$

$$x\sinh(x)-i\int -i\sinh(x)dx$$

$$\Big\downarrow \text{26}$$

$$x\sinh(x)-\int \sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\sinh(x)-\int -i\sin(ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\sinh(x)+i\int \sin(ix)dx$$

$$\Big\downarrow \text{3118}$$

$$x\sinh(x)-\cosh(x)$$

input
```
Int[x*Cosh[x],x]
```

output
```
-Cosh[x] + x*Sinh[x]
```

3.226.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.226.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-\cosh\left(x\right)+x\sinh\left(x\right)$	10
parts	$-\cosh\left(x\right)+x\sinh\left(x\right)$	10
risch	$\left(-\frac{1}{2}+\frac{x}{2}\right){\mathrm e}^{x}+\left(-\frac{1}{2}-\frac{x}{2}\right){\mathrm e}^{-x}$	20
parallelrisch	$\frac{2-2x\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)-1}$	21
meijerg	$-2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(x\right)}{2\sqrt{\pi}}-\frac{x\sinh\left(x\right)}{2\sqrt{\pi}}\right)$	27

input
```
int(x*cosh(x),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)+x*sinh(x)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x\cosh(x)\,dx = x\sinh\left(x\right) - \cosh\left(x\right)$$

input
```
integrate(x*cosh(x),x, algorithm="fricas")
```

output
```
x*sinh(x) - cosh(x)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x\cosh(x)\,dx = x\sinh\left(x\right) - \cosh\left(x\right)$$

input
```
integrate(x*cosh(x),x)
```

output
```
x*sinh(x) - cosh(x)
```

3.226.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.78

$$\int x\cosh(x)\,dx = \frac{1}{2}\,x^2\cosh\left(x\right) - \frac{1}{4}\left(x^2 + 2\,x + 2\right)e^{(-x)} - \frac{1}{4}\left(x^2 - 2\,x + 2\right)e^x$$

input
```
integrate(x*cosh(x),x, algorithm="maxima")
```

output
```
1/2*x^2*cosh(x) - 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int x\cosh(x)\,dx = -\frac{1}{2}\,(x+1)e^{(-x)} + \frac{1}{2}\,(x-1)e^{x}$$

input
```
integrate(x*cosh(x),x, algorithm="giac")
```

output
```
-1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

3.226.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x\cosh(x)\,dx = x\sinh(x) - \cosh(x)$$

input
```
int(x*cosh(x),x)
```

output
```
x*sinh(x) - cosh(x)
```

3.226.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x\cosh(x)\,dx = -\cosh(x) + \sinh(x)\,x$$

input
```
int(cosh(x)*x,x)
```

output
```
 - cosh(x) + sinh(x)*x
```

3.227 $\int \tanh(2x)\,dx$

3.227.1 Optimal result 1361
3.227.2 Mathematica [A] (verified) 1361
3.227.3 Rubi [A] (verified) 1362
3.227.4 Maple [A] (verified) 1363
3.227.5 Fricas [B] (verification not implemented) 1363
3.227.6 Sympy [A] (verification not implemented) 1363
3.227.7 Maxima [A] (verification not implemented) 1364
3.227.8 Giac [A] (verification not implemented) 1364
3.227.9 Mupad [B] (verification not implemented) 1364
3.227.10 Reduce [B] (verification not implemented) 1365

3.227.1 Optimal result

Integrand size = 4, antiderivative size = 9

$$\int \tanh(2x)\,dx = \frac{1}{2}\log(\cosh(2x))$$

output
```
1/2*ln(cosh(2*x))
```

3.227.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \tanh(2x)\,dx = \frac{1}{2}\log(\cosh(2x))$$

input
```
Integrate[Tanh[2*x],x]
```

output
```
Log[Cosh[2*x]]/2
```

3.227.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(2ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(2ix)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\log(\cosh(2x))$$

input
```
Int[Tanh[2*x],x]
```

output
```
Log[Cosh[2*x]]/2
```

3.227.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.227.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln(\cosh(2x))}{2}$	8
default	$\frac{\ln(\cosh(2x))}{2}$	8
risch	$-x+\frac{\ln(e^{4x}+1)}{2}$	14
parallelrisch	$-x+\ln\left(\frac{1}{\sqrt{1-\tanh(2x)}}\right)$	16

input
```
int(sinh(2*x)/cosh(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(cosh(2*x))
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(7) = 14.

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.89

$$\int \tanh(2x)\,dx = -x+\frac{1}{2}\log\left(\frac{2\cosh(2x)}{\cosh(2x)-\sinh(2x)}\right)$$

input
```
integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")
```

output
```
-x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))
```

3.227.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x)\,dx = \frac{\log(\cosh(2x))}{2}$$

input
```
integrate(sinh(2*x)/cosh(2*x),x)
```

output
```
log(cosh(2*x))/2
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x)\,dx = \frac{1}{2}\,\log\left(\cosh\left(2\,x\right)\right)$$

input
```
integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")
```

output
```
1/2*log(cosh(2*x))
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \tanh(2x)\,dx = -x + \frac{1}{2}\,\log\left(e^{(4\,x)} + 1\right)$$

input
```
integrate(sinh(2*x)/cosh(2*x),x, algorithm="giac")
```

output
```
-x + 1/2*log(e^(4*x) + 1)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x)\,dx = \frac{\ln\left(\cosh(2\,x)\right)}{2}$$

input
```
int(sinh(2*x)/cosh(2*x),x)
```

output
```
log(cosh(2*x))/2
```

3.227.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x)\,dx = \frac{\log(\cosh(2x))}{2}$$

input
```
int(sinh(2*x)/cosh(2*x),x)
```

output
```
log(cosh(2*x))/2
```

3.228 $\int \frac{-1+i\mathbf{eps}\sinh(x)}{ia-x+i\mathbf{eps}\cosh(x)}\,dx$

3.228.1 Optimal result 1366
3.228.2 Mathematica [A] (verified) 1366
3.228.3 Rubi [A] (verified) 1367
3.228.4 Maple [A] (verified) 1367
3.228.5 Fricas [B] (verification not implemented) 1368
3.228.6 Sympy [B] (verification not implemented) 1368
3.228.7 Maxima [A] (verification not implemented) 1369
3.228.8 Giac [B] (verification not implemented) 1369
3.228.9 Mupad [B] (verification not implemented) 1369
3.228.10Reduce [B] (verification not implemented) 1370

3.228.1 Optimal result

Integrand size = 28, antiderivative size = 12

$$\int \frac{-1+i\text{eps}\sinh(x)}{ia-x+i\text{eps}\cosh(x)}\,dx = \log(a+ix+\text{eps}\cosh(x))$$

output
```
ln(a+I*x+eps*cosh(x))
```

3.228.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+i\text{eps}\sinh(x)}{ia-x+i\text{eps}\cosh(x)}\,dx = \log(a+ix+\text{eps}\cosh(x))$$

input
```
Integrate[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
```

output
```
Log[a + I*x + eps*Cosh[x]]
```

3.228.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-1+i\text{eps}\sinh(x)}{ia+i\text{eps}\cosh(x)-x}\,dx$$

$$\Big\downarrow \ 7235$$

$$\log(a+\text{eps}\cosh(x)+ix)$$

input
```
Int[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
```

output
```
Log[a + I*x + eps*Cosh[x]]
```

3.228.3.1 Defintions of rubi rules used

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

3.228.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\ln\left(ia-x+i\,\text{eps}\cosh\left(x\right)\right)$	16
default	$\ln\left(ia-x+i\,\text{eps}\cosh\left(x\right)\right)$	16
risch	$-x+\ln\left(1+\frac{2(ix+a)\mathrm{e}^{x}}{\text{eps}}+\mathrm{e}^{2x}\right)$	25

input
```
int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(I*a-x+I*eps*cosh(x))
```

3.228.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} \, dx = -x + \log\left(\frac{epse^{(2x)} + 2(a + ix)e^x + eps}{eps}\right)$$

input
```
integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="fricas")
```

output
```
-x + log((eps*e^(2*x) + 2*(a + I*x)*e^x + eps)/eps)
```

3.228.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} \, dx = -x + \log\left(e^{2x} + 1 + \frac{(2a + 2ix)\, e^x}{eps}\right)$$

input
```
integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
```

output
```
-x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{-1 + i\text{eps}\sinh(x)}{ia - x + i\text{eps}\cosh(x)}\,dx = \log\left(i\,eps\cosh\left(x\right) + i\,a - x\right)$$

input
```
integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="maxima")
```

output
```
log(I*eps*cosh(x) + I*a - x)
```

3.228.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{-1 + i\text{eps}\sinh(x)}{ia - x + i\text{eps}\cosh(x)}\,dx = -x + \log\left(epse^{(2\,x)} + 2\,ae^{x} + 2i\,xe^{x} + eps\right)$$

input
```
integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="giac")
```

output
```
-x + log(eps*e^(2*x) + 2*a*e^x + 2*I*x*e^x + eps)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{-1 + i\text{eps}\sinh(x)}{ia - x + i\text{eps}\cosh(x)}\,dx = \ln\left(x - a\,1\mathrm{i} - \text{eps}\cosh(x)\ 1\mathrm{i}\right)$$

input
```
int((eps*sinh(x)*1i - 1)/(a*1i - x + eps*cosh(x)*1i),x)
```

output
```
log(x - a*1i - eps*cosh(x)*1i)
```

3.228.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{-1+i\text{eps}\sinh(x)}{ia-x+i\text{eps}\cosh(x)}\,dx = \log(\cosh(x)\,epsi+ai-x)$$

input
```
int((sinh(x)*eps*i - 1)/(cosh(x)*eps*i + a*i - x),x)
```

output
```
log(cosh(x)*eps*i + a*i - x)
```

3.229 $\int \cos^2(x)\sin(3+2x)\,dx$

3.229.1 Optimal result . . . 1371
3.229.2 Mathematica [A] (verified) . . . 1371
3.229.3 Rubi [A] (verified) . . . 1372
3.229.4 Maple [A] (verified) . . . 1373
3.229.5 Fricas [A] (verification not implemented) . . . 1373
3.229.6 Sympy [B] (verification not implemented) . . . 1374
3.229.7 Maxima [A] (verification not implemented) . . . 1374
3.229.8 Giac [A] (verification not implemented) . . . 1374
3.229.9 Mupad [B] (verification not implemented) . . . 1375
3.229.10Reduce [B] (verification not implemented) . . . 1375

3.229.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \cos^2(x)\sin(3+2x)\,dx = -\frac{1}{4}\cos(3+2x) - \frac{1}{16}\cos(3+4x) + \frac{1}{4}x\sin(3)$$

output
```
-1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

3.229.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\sin(3+2x)\,dx = -\frac{1}{4}\cos(3+2x) - \frac{1}{16}\cos(3+4x) + \frac{1}{4}x\sin(3)$$

input
```
Integrate[Cos[x]^2*Sin[3 + 2*x],x]
```

output
```
-1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

3.229.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x+3)\cos^2(x)\,dx$$

$$\Big\downarrow \text{ 5085}$$

$$\int \left(\frac{1}{2}\sin(2x+3)+\frac{1}{4}\sin(4x+3)+\frac{\sin(3)}{4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

input
```
Int[Cos[x]^2*Sin[3 + 2*x],x]
```

output
```
-1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

3.229.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.229.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result
default	$-\frac{\cos(3+2x)}{4}-\frac{\cos(4x+3)}{16}+\frac{x\sin(3)}{4}$
risch	$-\frac{\cos(3+2x)}{4}-\frac{\cos(4x+3)}{16}+\frac{x\sin(3)}{4}$
parallelrisch	$-\frac{\cos(3+2x)}{4}-\frac{\cos(4x+3)}{16}-\frac{\cos(3)}{16}+\frac{1}{8}+\frac{x\sin(3)}{4}$
norman	$\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right)+\left(\tan^3\left(\frac{x}{2}\right)\right)x+x\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{3}{2}+x\right)\right)-3\left(\tan^3\left(\frac{x}{2}\right)\right)\tan\left(\frac{3}{2}+x\right)+2\left(\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{3}{2}+x\right)\right)+3\tan\left(\frac{x}{2}\right)\tan\left(\frac{3}{2}+\right.}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(1+\tan^2\left(\frac{3}{2}\right.\right.}$

input
```
int(cos(x)^2*sin(3+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(3+2*x)-1/16*cos(4*x+3)+1/4*x*sin(3)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \cos^2(x)\sin(3+2x)\,dx = -\frac{1}{2}\cos(3)\cos(x)^4+\frac{1}{4}\,x\sin(3) + \frac{1}{4}\left(2\cos(x)^3\sin(3)+\cos(x)\sin(3)\right)\sin(x)$$

input
```
integrate(cos(x)^2*sin(3+2*x),x, algorithm="fricas")
```

output
```
-1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + cos(x)*sin(
3))*sin(x)
```

3.229.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \cos^2(x)\sin(3+2x)\,dx = -\frac{x\sin^2(x)\sin(2x+3)}{4} - \frac{x\sin(x)\cos(x)\cos(2x+3)}{2} + \frac{x\sin(2x+3)\cos^2(x)}{4} - \frac{\sin(x)\sin(2x+3)\cos(x)}{4} - \frac{\cos^2(x)\cos(2x+3)}{2}$$

input
```
integrate(cos(x)**2*sin(3+2*x),x)
```

output
```
-x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*sin(2*x +
 3)*cos(x)**2/4 - sin(x)*sin(2*x + 3)*cos(x)/4 - cos(x)**2*cos(2*x + 3)/2
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin(3+2x)\,dx = \frac{1}{4}\,x\sin(3) - \frac{1}{16}\,\cos(4\,x+3) - \frac{1}{4}\,\cos(2\,x+3)$$

input
```
integrate(cos(x)^2*sin(3+2*x),x, algorithm="maxima")
```

output
```
1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin(3+2x)\,dx = \frac{1}{4}\,x\sin(3) - \frac{1}{16}\,\cos(4\,x+3) - \frac{1}{4}\,\cos(2\,x+3)$$

input
```
integrate(cos(x)^2*sin(3+2*x),x, algorithm="giac")
```

output
```
1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin(3+2x)\,dx = \frac{x\,\sin(3)}{4} - \frac{\cos(4\,x+3)}{16} - \frac{\cos(2\,x+3)}{4}$$

input
```
int(sin(2*x + 3)*cos(x)^2,x)
```

output
```
(x*sin(3))/4 - cos(4*x + 3)/16 - cos(2*x + 3)/4
```

3.229.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

$$\int \cos^2(x)\sin(3+2x)\,dx = -\frac{\cos(2x+3)\cos(x)\sin(x)\,x}{2} + \frac{\cos(2x+3)\sin(x)^2}{4} - \frac{3\cos(2x+3)}{8} - \frac{\sin(2x+3)\sin(x)^2\,x}{2} + \frac{\sin(2x+3)\,x}{4} + \frac{3}{8}$$

input
```
int(cos(x)**2*sin(2*x + 3),x)
```

output
```
( - 4*cos(2*x + 3)*cos(x)*sin(x)*x + 2*cos(2*x + 3)*sin(x)**2 - 3*cos(2*x
+ 3) - 4*sin(2*x + 3)*sin(x)**2*x + 2*sin(2*x + 3)*x + 3)/8
```

3.230 $\int x \arctan(x)\,dx$

3.230.1 Optimal result 1376
3.230.2 Mathematica [A] (verified) 1376
3.230.3 Rubi [A] (verified) 1377
3.230.4 Maple [A] (verified) 1378
3.230.5 Fricas [A] (verification not implemented) 1379
3.230.6 Sympy [A] (verification not implemented) 1379
3.230.7 Maxima [A] (verification not implemented) 1379
3.230.8 Giac [A] (verification not implemented) 1380
3.230.9 Mupad [B] (verification not implemented) 1380
3.230.10Reduce [B] (verification not implemented) 1380

3.230.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \arctan(x)\,dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x)$$

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

3.230.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x \arctan(x)\,dx = \frac{1}{2}\left(-x + \left(1 + x^2\right)\arctan(x)\right)$$

input
```
Integrate[x*ArcTan[x],x]
```

output
```
(-x + (1 + x^2)*ArcTan[x])/2
```

3.230.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(x)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx-x\right)+\frac{1}{2}x^2\arctan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}(\arctan(x)-x)$$

input
```
Int[x*ArcTan[x],x]
```

output
```
(x^2*ArcTan[x])/2 + (-x + ArcTan[x])/2
```

3.230.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.230.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
parallelrisch	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
parts	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

input
```
int(x*arctan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x\arctan(x)\,dx = \frac{1}{2}\left(x^2+1\right)\arctan\left(x\right) - \frac{1}{2}\,x$$

input
```
integrate(x*arctan(x),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

3.230.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{x^2\,\mathrm{atan}\left(x\right)}{2} - \frac{x}{2} + \frac{\mathrm{atan}\left(x\right)}{2}$$

input
```
integrate(x*atan(x),x)
```

output
```
x**2*atan(x)/2 - x/2 + atan(x)/2
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{1}{2}\,x^2\arctan\left(x\right) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(x\right)$$

input
```
integrate(x*arctan(x),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{1}{2}\,x^2\arctan\left(x\right) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(x\right)$$

input
```
integrate(x*arctan(x),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x\arctan(x)\,dx = \mathrm{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

input
```
int(x*atan(x),x)
```

output
```
atan(x)*(x^2/2 + 1/2) - x/2
```

3.230.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{atan(x)\,x^2}{2}+\frac{atan(x)}{2}-\frac{x}{2}$$

input
```
int(atan(x)*x,x)
```

output
```
(atan(x)*x**2 + atan(x) - x)/2
```

3.231 $\int x \cot^{-1}(x)\,dx$

3.231.1 Optimal result 1381
3.231.2 Mathematica [A] (verified) 1381
3.231.3 Rubi [A] (verified) 1382
3.231.4 Maple [A] (verified) 1383
3.231.5 Fricas [A] (verification not implemented) 1384
3.231.6 Sympy [A] (verification not implemented) 1384
3.231.7 Maxima [A] (verification not implemented) 1384
3.231.8 Giac [A] (verification not implemented) 1385
3.231.9 Mupad [B] (verification not implemented) 1385
3.231.10Reduce [B] (verification not implemented) 1385

3.231.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \cot^{-1}(x)\,dx = \frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{\arctan(x)}{2}$$

output
```
1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

3.231.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x \cot^{-1}(x)\,dx = \frac{1}{2}\left(x + x^2 \cot^{-1}(x) - \arctan(x)\right)$$

input
```
Integrate[x*ArcCot[x],x]
```

output
```
(x + x^2*ArcCot[x] - ArcTan[x])/2
```

3.231.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5362, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(x) \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2} \int \frac{x^2}{x^2+1} dx + \frac{1}{2} x^2 \cot^{-1}(x)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(x - \int \frac{1}{x^2+1} dx \right) + \frac{1}{2} x^2 \cot^{-1}(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} (x - \arctan(x)) + \frac{1}{2} x^2 \cot^{-1}(x)$$

input
```
Int[x*ArcCot[x],x]
```

output
```
(x^2*ArcCot[x])/2 + (x - ArcTan[x])/2
```

3.231.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.231.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x}{2}+\frac{x^{2} \operatorname{arccot}\left(x\right)}{2}-\frac{\arctan\left(x\right)}{2}$	16
parallelrisch	$\frac{x^{2} \operatorname{arccot}\left(x\right)}{2}+\frac{x}{2}+\frac{\operatorname{arccot}\left(x\right)}{2}$	16
parts	$\frac{x}{2}+\frac{x^{2} \operatorname{arccot}\left(x\right)}{2}-\frac{\arctan\left(x\right)}{2}$	16
risch	$\frac{i x^{2} \ln\left(i x+1\right)}{4}-\frac{i x^{2} \ln\left(-i x+1\right)}{4}+\frac{\pi \,x^{2}}{4}+\frac{x}{2}-\frac{\arctan\left(x\right)}{2}$	41

input
```
int(x*arccot(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x \cot^{-1}(x)\,dx = \frac{1}{2}\left(x^2+1\right)\operatorname{arccot}(x) + \frac{1}{2}\,x$$

input
```
integrate(x*arccot(x),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*arccot(x) + 1/2*x
```

3.231.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x)\,dx = \frac{x^2 \operatorname{acot}(x)}{2} + \frac{x}{2} + \frac{\operatorname{acot}(x)}{2}$$

input
```
integrate(x*acot(x),x)
```

output
```
x**2*acot(x)/2 + x/2 + acot(x)/2
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x)\,dx = \frac{1}{2}\,x^2 \operatorname{arccot}(x) + \frac{1}{2}\,x - \frac{1}{2}\,\arctan(x)$$

input
```
integrate(x*arccot(x),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x \cot^{-1}(x)\,dx = \frac{1}{2}\,x^2 \arctan\left(\frac{1}{x}\right) + \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(\frac{1}{x}\right)$$

input
```
integrate(x*arccot(x),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(1/x) + 1/2*x + 1/2*arctan(1/x)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x)\,dx = \frac{x}{2} - \frac{\mathrm{atan}(x)}{2} + \frac{x^2\,\mathrm{acot}(x)}{2}$$

input
```
int(x*acot(x),x)
```

output
```
x/2 - atan(x)/2 + (x^2*acot(x))/2
```

3.231.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x)\,dx = \frac{acot(x)\,x^2}{2} + \frac{acot(x)}{2} + \frac{x}{2}$$

input
```
int(acot(x)*x,x)
```

output
```
(acot(x)*x**2 + acot(x) + x)/2
```

3.232 $\int x \log(a+x^2)\, dx$

3.232.1 Optimal result 1386
3.232.2 Mathematica [A] (verified) 1386
3.232.3 Rubi [A] (verified) 1387
3.232.4 Maple [A] (verified) 1388
3.232.5 Fricas [A] (verification not implemented) 1388
3.232.6 Sympy [A] (verification not implemented) 1389
3.232.7 Maxima [A] (verification not implemented) 1389
3.232.8 Giac [A] (verification not implemented) 1389
3.232.9 Mupad [B] (verification not implemented) 1390
3.232.10Reduce [B] (verification not implemented) 1390

3.232.1 Optimal result

Integrand size = 8, antiderivative size = 23

$$\int x \log(a+x^2)\, dx = -\frac{x^2}{2} + \frac{1}{2}(a+x^2)\log(a+x^2)$$

output
```
-1/2*x^2+1/2*(x^2+a)*ln(x^2+a)
```

3.232.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x \log(a+x^2)\, dx = \frac{1}{2}\left(-x^2 + (a+x^2)\log(a+x^2)\right)$$

input
```
Integrate[x*Log[a + x^2],x]
```

output
```
(-x^2 + (a + x^2)*Log[a + x^2])/2
```

3.232.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(a+x^2\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2} \int \log \left(x^2+a\right)dx^2$$

$$\Big\downarrow \text{2836}$$

$$\frac{1}{2} \int \log \left(x^2+a\right)d\left(x^2+a\right)$$

$$\Big\downarrow \text{2732}$$

$$\frac{1}{2} \left(\left(a+x^2\right) \log \left(a+x^2\right)-a-x^2\right)$$

input
```
Int[x*Log[a + x^2],x]
```

output
```
(-a - x^2 + (a + x^2)*Log[a + x^2])/2
```

3.232.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

3.232.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{(x^2+a)\ln(x^2+a)}{2}-\frac{x^2}{2}-\frac{a}{2}$	23
default	$\frac{(x^2+a)\ln(x^2+a)}{2}-\frac{x^2}{2}-\frac{a}{2}$	23
norman	$-\frac{x^2}{2}+\frac{\ln(x^2+a)a}{2}+\frac{\ln(x^2+a)x^2}{2}$	27
risch	$-\frac{x^2}{2}+\frac{\ln(x^2+a)a}{2}+\frac{\ln(x^2+a)x^2}{2}$	27
parts	$-\frac{x^2}{2}+\frac{\ln(x^2+a)a}{2}+\frac{\ln(x^2+a)x^2}{2}$	27
parallelrisch	$\frac{\ln(x^2+a)x^2}{2}-\frac{x^2}{2}+\frac{\ln(x^2+a)a}{2}+\frac{a}{2}$	30

input
```
int(x*ln(x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2+a)*ln(x^2+a)-1/2*x^2-1/2*a
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\log\left(a+x^2\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{2}\left(x^2+a\right)\log\left(x^2+a\right)$$

input
```
integrate(x*log(x^2+a),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a)
```

3.232.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x\log\left(a+x^2\right)\,dx=\frac{a\log\left(a+x^2\right)}{2}+\frac{x^2\log\left(a+x^2\right)}{2}-\frac{x^2}{2}$$

input
```
integrate(x*ln(x**2+a),x)
```

output
```
a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x\log\left(a+x^2\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(x^2+a\right)\log\left(x^2+a\right)-\frac{1}{2}\,a$$

input
```
integrate(x*log(x^2+a),x, algorithm="maxima")
```

output
```
-1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x\log\left(a+x^2\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(x^2+a\right)\log\left(x^2+a\right)-\frac{1}{2}\,a$$

input
```
integrate(x*log(x^2+a),x, algorithm="giac")
```

output
```
-1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

3.232.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int x \log \left(a+x^2\right) \, dx=\frac{a \ln \left(x+\sqrt{-a}\right)}{2}+\frac{x^2 \ln \left(x^2+a\right)}{2}+\frac{a \ln \left(x-\sqrt{-a}\right)}{2}-\frac{x^2}{2}$$

input
```
int(x*log(a + x^2),x)
```

output
```
(a*log(x + (-a)^(1/2)))/2 + (x^2*log(a + x^2))/2 + (a*log(x - (-a)^(1/2)))
/2 - x^2/2
```

3.232.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x \log \left(a+x^2\right) \, dx=\frac{\log(x^2+a)\, a}{2}+\frac{\log(x^2+a)\, x^2}{2}-\frac{x^2}{2}$$

input
```
int(log(a + x**2)*x,x)
```

output
```
(log(a + x**2)*a + log(a + x**2)*x**2 - x**2)/2
```

3.233 $\int \cos(x)\sin(a+x)\,dx$

3.233.1 Optimal result . . . 1391
3.233.2 Mathematica [A] (verified) . . . 1391
3.233.3 Rubi [A] (verified) . . . 1392
3.233.4 Maple [A] (verified) . . . 1393
3.233.5 Fricas [A] (verification not implemented) . . . 1393
3.233.6 Sympy [B] (verification not implemented) . . . 1393
3.233.7 Maxima [A] (verification not implemented) . . . 1394
3.233.8 Giac [A] (verification not implemented) . . . 1394
3.233.9 Mupad [B] (verification not implemented) . . . 1394
3.233.10Reduce [B] (verification not implemented) . . . 1395

3.233.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \cos(x)\sin(a+x)\,dx = -\frac{1}{4}\cos(a+2x)+\frac{1}{2}x\sin(a)$$

output
```
-1/4*cos(a+2*x)+1/2*x*sin(a)
```

3.233.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(a+x)\,dx = \frac{1}{4}(-\cos(a+2x)+2x\sin(a))$$

input
```
Integrate[Cos[x]*Sin[a + x],x]
```

output
```
(-Cos[a + 2*x] + 2*x*Sin[a])/4
```

3.233.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sin(a+x)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin(a+2x)+\frac{\sin(a)}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

input
```
Int[Cos[x]*Sin[a + x],x]
```

output
```
-1/4*Cos[a + 2*x] + (x*Sin[a])/2
```

3.233.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.233.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\cos(a+2x)}{4}+\frac{x\sin(a)}{2}$	15
risch	$-\frac{\cos(a+2x)}{4}+\frac{x\sin(a)}{2}$	15
parallelrisch	$-\frac{\cos(a+2x)}{4}+\frac{\cos(a)}{4}+\frac{x\sin(a)}{2}$	19
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)\tan\left(\frac{a}{2}+\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)-x\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{a}{2}+\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)}$	91

input
```
int(cos(x)*sin(a+x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(a+2*x)+1/2*x*sin(a)
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \cos(x)\sin(a+x)\,dx = -\frac{1}{2}\cos\left(a+x\right)^2\cos\left(a\right) - \frac{1}{2}\cos\left(a+x\right)\sin\left(a+x\right)\sin\left(a\right)+\frac{1}{2}x\sin\left(a\right)$$

input
```
integrate(cos(x)*sin(a+x),x, algorithm="fricas")
```

output
```
-1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) + 1/2*x*sin(a)
```

3.233.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \cos(x)\sin(a+x)\,dx = -\frac{x\sin\left(x\right)\cos\left(a+x\right)}{2}+\frac{x\sin\left(a+x\right)\cos\left(x\right)}{2}-\frac{\cos\left(x\right)\cos\left(a+x\right)}{2}$$

input
```
integrate(cos(x)*sin(a+x),x)
```

output
```
-x*sin(x)*cos(a + x)/2 + x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)/2
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(x)\sin(a+x)\,dx = \frac{1}{2}\,x\sin\left(a\right) - \frac{1}{4}\,\cos\left(a+2\,x\right)$$

input
```
integrate(cos(x)*sin(a+x),x, algorithm="maxima")
```

output
```
1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(x)\sin(a+x)\,dx = \frac{1}{2}\,x\sin\left(a\right) - \frac{1}{4}\,\cos\left(a+2\,x\right)$$

input
```
integrate(cos(x)*sin(a+x),x, algorithm="giac")
```

output
```
1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.233.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(x)\sin(a+x)\,dx = \frac{x\,\sin\left(a\right)}{2} - \frac{\cos\left(a+2\,x\right)}{4}$$

input
```
int(sin(a + x)*cos(x),x)
```

output
```
(x*sin(a))/2 - cos(a + 2*x)/4
```

3.233.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \cos(x)\sin(a+x)\,dx = -\frac{\cos(a+x)\cos(x)}{2} - \frac{\cos(a+x)\sin(x)\,x}{2} + \frac{\cos(x)\sin(a+x)\,x}{2}$$

input
```
int(cos(x)*sin(a + x),x)
```

output
```
( - cos(a + x)*cos(x) - cos(a + x)*sin(x)*x + cos(x)*sin(a + x)*x)/2
```

3.234 $\int \cos(a+x)\sin(x)\,dx$

3.234.1 Optimal result 1396
3.234.2 Mathematica [A] (verified) 1396
3.234.3 Rubi [A] (verified) 1397
3.234.4 Maple [A] (verified) 1398
3.234.5 Fricas [A] (verification not implemented) 1398
3.234.6 Sympy [B] (verification not implemented) 1399
3.234.7 Maxima [A] (verification not implemented) 1399
3.234.8 Giac [A] (verification not implemented) 1399
3.234.9 Mupad [B] (verification not implemented) 1400
3.234.10Reduce [B] (verification not implemented) 1400

3.234.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \cos(a+x)\sin(x)\,dx=-\frac{1}{4}\cos(a+2x)-\frac{1}{2}x\sin(a)$$

output
```
-1/4*cos(a+2*x)-1/2*x*sin(a)
```

3.234.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cos(a+x)\sin(x)\,dx=\frac{1}{4}(-\cos(a+2x)-2x\sin(a))$$

input
```
Integrate[Cos[a + x]*Sin[x],x]
```

output
```
(-Cos[a + 2*x] - 2*x*Sin[a])/4
```

3.234.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(a+x)\,dx$$
$$\Big\downarrow \text{5085}$$
$$\int \left(\frac{1}{2}\sin(a+2x)-\frac{\sin(a)}{2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

input
```
Int[Cos[a + x]*Sin[x],x]
```

output
```
-1/4*Cos[a + 2*x] - (x*Sin[a])/2
```

3.234.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.234.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\cos(a+2x)}{4}-\frac{x\sin(a)}{2}$	15
risch	$-\frac{\cos(a+2x)}{4}-\frac{x\sin(a)}{2}$	15
parallelrisch	$-\frac{\cos(a+2x)}{4}+\frac{\cos(a)}{4}-\frac{x\sin(a)}{2}$	19
meijerg	$\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}-\frac{\sin(a)\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	45
norman	$\frac{x\tan\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{a}{2}+\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)\tan\left(\frac{a}{2}+\frac{x}{2}\right)-x\tan\left(\frac{a}{2}+\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)}$	91

input
```
int(cos(a+x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(a+2*x)-1/2*x*sin(a)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \cos(a+x)\sin(x)\,dx = -\frac{1}{2}\cos(a+x)^2\cos(a) - \frac{1}{2}\cos(a+x)\sin(a+x)\sin(a) - \frac{1}{2}x\sin(a)$$

input
```
integrate(cos(a+x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) - 1/2*x*sin(a)
```

3.234.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \cos(a+x)\sin(x)\,dx = \frac{x\sin(x)\cos(a+x)}{2} - \frac{x\sin(a+x)\cos(x)}{2} - \frac{\cos(x)\cos(a+x)}{2}$$

input
```
integrate(cos(a+x)*sin(x),x)
```

output
```
x*sin(x)*cos(a + x)/2 - x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)/2
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(a+x)\sin(x)\,dx = -\frac{1}{2}\,x\sin(a) - \frac{1}{4}\cos(a+2\,x)$$

input
```
integrate(cos(a+x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(a+x)\sin(x)\,dx = -\frac{1}{2}\,x\sin(a) - \frac{1}{4}\cos(a+2\,x)$$

input
```
integrate(cos(a+x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(a+x)\sin(x)\,dx = -\frac{\cos(a+2x)}{4} - \frac{x\sin(a)}{2}$$

input
```
int(cos(a + x)*sin(x),x)
```

output
```
- cos(a + 2*x)/4 - (x*sin(a))/2
```

3.234.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \cos(a+x)\sin(x)\,dx = -\frac{\cos(a+x)\cos(x)}{2} + \frac{\cos(a+x)\sin(x)x}{2} - \frac{\cos(x)\sin(a+x)x}{2}$$

input
```
int(cos(a + x)*sin(x),x)
```

output
```
( - cos(a + x)*cos(x) + cos(a + x)*sin(x)*x - cos(x)*sin(a + x)*x)/2
```

3.235 $\int \sqrt{1+\sin(x)}\,dx$

3.235.1 Optimal result 1401
3.235.2 Mathematica [B] (verified) 1401
3.235.3 Rubi [A] (verified) 1402
3.235.4 Maple [A] (verified) 1403
3.235.5 Fricas [B] (verification not implemented) 1403
3.235.6 Sympy [F] 1403
3.235.7 Maxima [F] 1404
3.235.8 Giac [B] (verification not implemented) 1404
3.235.9 Mupad [B] (verification not implemented) 1404
3.235.10 Reduce [F] 1405

3.235.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \sqrt{1+\sin(x)}\,dx = -\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

output
```
-2*cos(x)/(1+sin(x))^(1/2)
```

3.235.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(12) = 24.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \sqrt{1+\sin(x)}\,dx = \frac{2\left(-\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{1+\sin(x)}}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sqrt[1 + Sin[x]],x]
```

output
```
(2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

3.235.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)+1}dx$$

$$\Big\downarrow \text{3125}$$

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

input
```
Int[Sqrt[1 + Sin[x]],x]
```

output
```
(-2*Cos[x])/Sqrt[1 + Sin[x]]
```

3.235.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.235.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{2(\sin(x)-1)\sqrt{\sin(x)+1}}{\cos(x)}$	17
risch	$-\frac{i\sqrt{2}\,\sqrt{2\sin(x)+2}\,(\mathrm{e}^{ix}-i)\,(i+\mathrm{e}^{ix})}{\mathrm{e}^{2ix}-1+2i\mathrm{e}^{ix}}$	48

input
```
int((sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(sin(x)-1)*(sin(x)+1)^(1/2)/cos(x)
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \sqrt{1+\sin(x)}\,dx = -\frac{2\left(\cos\left(x\right)-\sin\left(x\right)+1\right)\sqrt{\sin\left(x\right)+1}}{\cos\left(x\right)+\sin\left(x\right)+1}$$

input
```
integrate((1+sin(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

3.235.6 Sympy [F]

$$\int \sqrt{1+\sin(x)}\,dx = \int \sqrt{\sin\left(x\right)+1}\,dx$$

input
```
integrate((1+sin(x))**(1/2),x)
```

output
```
Integral(sqrt(sin(x) + 1), x)
```

3.235.7 Maxima [F]

$$\int \sqrt{1+\sin(x)}\,dx = \int \sqrt{\sin(x)+1}\,dx$$

input
```
integrate((1+sin(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(x) + 1), x)
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \sqrt{1+\sin(x)}\,dx = 2\sqrt{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)$$

input
```
integrate((1+sin(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)
```

3.235.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \sqrt{1+\sin(x)}\,dx = \frac{2(\sin(x)-1)\sqrt{\sin(x)+1}}{\cos(x)}$$

input
```
int((sin(x) + 1)^(1/2),x)
```

output
```
(2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)
```

3.235.10 Reduce [F]

$$\int \sqrt{1+\sin(x)}\,dx = \int \sqrt{\sin(x)+1}dx$$

input
```
int(sqrt(sin(x) + 1),x)
```

output
```
int(sqrt(sin(x) + 1),x)
```

3.236 $\int \sqrt{1-\sin(x)}\,dx$

3.236.1 Optimal result . . . 1406
3.236.2 Mathematica [B] (verified) . . . 1406
3.236.3 Rubi [A] (verified) . . . 1407
3.236.4 Maple [A] (verified) . . . 1408
3.236.5 Fricas [B] (verification not implemented) . . . 1408
3.236.6 Sympy [F] . . . 1408
3.236.7 Maxima [F] . . . 1409
3.236.8 Giac [B] (verification not implemented) . . . 1409
3.236.9 Mupad [B] (verification not implemented) . . . 1409
3.236.10Reduce [F] . . . 1410

3.236.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

output
```
2*cos(x)/(1-sin(x))^(1/2)
```

3.236.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{1-\sin(x)}}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sqrt[1 - Sin[x]],x]
```

output
```
(2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])
```

3.236.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\sin(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{1-\sin(x)}dx$$
$$\Big\downarrow \text{ 3125}$$
$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

input
```
Int[Sqrt[1 - Sin[x]],x]
```

output
```
(2*Cos[x])/Sqrt[1 - Sin[x]]
```

3.236.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.236.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

method	result	size
default	$-\frac{2(\sin(x)-1)(\sin(x)+1)}{\cos(x)\sqrt{-\sin(x)+1}}$	23
risch	$\frac{i\sqrt{2-2\sin(x)}\,\sqrt{-i(2i\mathrm{e}^{2ix}-\mathrm{e}^{3ix}+\mathrm{e}^{ix})}\,\sqrt{2}\,(\mathrm{e}^{ix}-i)\,(i+\mathrm{e}^{ix})}{(2i\mathrm{e}^{ix}-\mathrm{e}^{2ix}+1)\sqrt{i(\mathrm{e}^{3ix}-2i\mathrm{e}^{2ix}-\mathrm{e}^{ix})}}$	102

input
```
int((-sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(sin(x)-1)*(sin(x)+1)/cos(x)/(-sin(x)+1)^(1/2)
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\sqrt{-\sin\left(x\right)+1}}{\cos\left(x\right)-\sin\left(x\right)+1}$$

input
```
integrate((1-sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)
```

3.236.6 Sympy [F]

$$\int \sqrt{1-\sin(x)}\,dx = \int \sqrt{1-\sin\left(x\right)}\,dx$$

input
```
integrate((1-sin(x))**(1/2),x)
```

output
```
Integral(sqrt(1 - sin(x)), x)
```

3.236.7 Maxima [F]

$$\int \sqrt{1-\sin(x)}\,dx = \int \sqrt{-\sin\left(x\right) + 1}\,dx$$

input
```
integrate((1-sin(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-sin(x) + 1), x)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \sqrt{1-\sin(x)}\,dx$$
$$= -2\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\right) - \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\right)\right)$$

input
```
integrate((1-sin(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*(cos(-1/4*pi + 1/2*x)*sgn(sin(-1/4*pi + 1/2*x)) - sgn(sin(-1/4*
pi + 1/2*x)))
```

3.236.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\sqrt{1-\sin\left(x\right)}\,\left(\sin\left(x\right)+1\right)}{\cos\left(x\right)}$$

input
```
int((1 - sin(x))^(1/2),x)
```

output
```
(2*(1 - sin(x))^(1/2)*(sin(x) + 1))/cos(x)
```

3.236.10 Reduce [F]

$$\int \sqrt{1-\sin(x)}\,dx = \int \sqrt{-\sin\left(x\right)+1}dx$$

input
```
int(sqrt( - sin(x) + 1),x)
```

output
```
int(sqrt( - sin(x) + 1),x)
```

3.237 $\int \sqrt{1+\cos(x)}\,dx$

3.237.1 Optimal result 1411
3.237.2 Mathematica [A] (verified) 1411
3.237.3 Rubi [A] (verified) 1412
3.237.4 Maple [B] (verified) 1413
3.237.5 Fricas [A] (verification not implemented) 1413
3.237.6 Sympy [B] (verification not implemented) 1413
3.237.7 Maxima [A] (verification not implemented) 1414
3.237.8 Giac [A] (verification not implemented) 1414
3.237.9 Mupad [B] (verification not implemented) 1414
3.237.10 Reduce [F] 1415

3.237.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \sqrt{1+\cos(x)}\,dx = \frac{2\sin(x)}{\sqrt{1+\cos(x)}}$$

output
```
2*sin(x)/(1+cos(x))^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \sqrt{1+\cos(x)}\,dx = 2\sqrt{1+\cos(x)}\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[1 + Cos[x]],x]
```

output
```
2*Sqrt[1 + Cos[x]]*Tan[x/2]
```

3.237.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(x)+1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{3125}$$
$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

input
```
Int[Sqrt[1 + Cos[x]],x]
```

output
```
(2*Sin[x])/Sqrt[1 + Cos[x]]
```

3.237.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

method	result	size
default	$\frac{4\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{2\cos(x)+2}}$	22
risch	$-\frac{i\sqrt{2}\,\sqrt{\left(e^{ix}+1\right)^{2}e^{-ix}}\,\left(e^{ix}-1\right)}{e^{ix}+1}$	40

input
```
int((cos(x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1+\cos(x)}\,dx = \frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

input
```
integrate((1+cos(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sin(x)/sqrt(cos(x) + 1)
```

3.237.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int \sqrt{1+\cos(x)}\,dx = 2\sqrt{1-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan\left(\frac{x}{2}\right)$$

input
```
integrate((1+cos(x))**(1/2),x)
```

output
```
2*sqrt(1 - tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \sqrt{1+\cos(x)}\,dx = 2\sqrt{2}\sin\left(\frac{1}{2}x\right)$$

input
```
integrate((1+cos(x))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sin(1/2*x)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \sqrt{1+\cos(x)}\,dx = 2\sqrt{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)$$

input
```
integrate((1+cos(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sgn(cos(1/2*x))*sin(1/2*x)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1+\cos(x)}\,dx = \frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

input
```
int((cos(x) + 1)^(1/2),x)
```

output
```
(2*sin(x))/(cos(x) + 1)^(1/2)
```

3.237.10 Reduce [F]

$$\int \sqrt{1+\cos(x)}\,dx = \int \sqrt{\cos{(x)}+1}dx$$

input
```
int(sqrt(cos(x) + 1),x)
```

output
```
int(sqrt(cos(x) + 1),x)
```

3.238 $\int \sqrt{1-\cos(x)}\,dx$

3.238.1 Optimal result 1416
3.238.2 Mathematica [A] (verified) 1416
3.238.3 Rubi [A] (verified) 1417
3.238.4 Maple [A] (verified) 1418
3.238.5 Fricas [A] (verification not implemented) 1418
3.238.6 Sympy [F] 1418
3.238.7 Maxima [A] (verification not implemented) 1419
3.238.8 Giac [A] (verification not implemented) 1419
3.238.9 Mupad [B] (verification not implemented) 1419
3.238.10Reduce [F] 1420

3.238.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{1-\cos(x)}\,dx = -\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

output
```
-2*sin(x)/(1-cos(x))^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1-\cos(x)}\,dx = -2\sqrt{1-\cos(x)}\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[1 - Cos[x]],x]
```

output
```
-2*Sqrt[1 - Cos[x]]*Cot[x/2]
```

3.238.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3125}$$

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

input
```
Int[Sqrt[1 - Cos[x]],x]
```

output
```
(-2*Sin[x])/Sqrt[1 - Cos[x]]
```

3.238.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.238.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

method	result	size
default	$-\frac{4\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{2-2\cos\left(x\right)}}$	22
risch	$-\frac{i\sqrt{2}\,\sqrt{-\left({\mathrm e}^{i x}-1\right)^{2} {\mathrm e}^{-i x}}\,\left({\mathrm e}^{i x}+1\right)}{{\mathrm e}^{i x}-1}$	41

input
```
int((1-cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1-\cos(x)}\,dx=-\frac{2\left(\cos\left(x\right)+1\right)\sqrt{-\cos\left(x\right)+1}}{\sin\left(x\right)}$$

input
```
integrate((1-cos(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)
```

3.238.6 Sympy [F]

$$\int \sqrt{1-\cos(x)}\,dx=\int \sqrt{1-\cos\left(x\right)}\,dx$$

input
```
integrate((1-cos(x))**(1/2),x)
```

output
```
Integral(sqrt(1 - cos(x)), x)
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \sqrt{1-\cos(x)}\,dx = -\frac{2\sqrt{2}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

input
```
integrate((1-cos(x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \sqrt{1-\cos(x)}\,dx = -2\sqrt{2}\left(\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right) - \mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\right)$$

input
```
integrate((1-cos(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))
```

3.238.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{1-\cos(x)}\,dx = -\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

input
```
int((1 - cos(x))^(1/2),x)
```

output
```
-(2*sin(x))/(1 - cos(x))^(1/2)
```

3.238.10 Reduce [F]

$$\int \sqrt{1-\cos(x)}\,dx = \int \sqrt{-\cos\left(x\right)+1}dx$$

input
```
int(sqrt( - cos(x) + 1),x)
```

output
```
int(sqrt( - cos(x) + 1),x)
```

3.239 $\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx$

3.239.1 Optimal result 1421
3.239.2 Mathematica [A] (verified) 1421
3.239.3 Rubi [A] (verified) 1422
3.239.4 Maple [A] (verified) 1423
3.239.5 Fricas [A] (verification not implemented) 1423
3.239.6 Sympy [B] (verification not implemented) 1424
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3.239.8 Giac [A] (verification not implemented) 1424
3.239.9 Mupad [B] (verification not implemented) 1425
3.239.10 Reduce [B] (verification not implemented) 1425

3.239.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3}$$

output
```
2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2}{3}\left((-1+x)^{3/2}+x^{3/2}\right)$$

input
```
Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

output
```
(2*((-1 + x)^(3/2) + x^(3/2)))/3
```

3.239.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2531, 15, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}-\sqrt{x-1}}dx$$

$$\Big\downarrow \text{2531}$$

$$\int \sqrt{x-1}dx+\int \sqrt{x}dx$$

$$\Big\downarrow \text{15}$$

$$\int \sqrt{x-1}dx+\frac{2x^{3/2}}{3}$$

$$\Big\downarrow \text{17}$$

$$\frac{2x^{3/2}}{3}+\frac{2}{3}(x-1)^{3/2}$$

input
```
Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

output
```
(2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

3.239.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

3.239.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{2(-1+x)^{\frac{3}{2}}}{3}+\frac{2x^{\frac{3}{2}}}{3}$	14

input
```
int(1/(-(-1+x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx=\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.239.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x-1}}{-3\sqrt{x}+3\sqrt{x-1}} - \frac{4x}{-3\sqrt{x}+3\sqrt{x-1}} + \frac{2}{-3\sqrt{x}+3\sqrt{x-1}}$$

input
```
integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)
```

output
```
2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt(x) + 3*s
qrt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))
```

3.239.7 Maxima [F]

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \int -\frac{1}{\sqrt{x-1}-\sqrt{x}}\,dx$$

input
```
integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x - 1) - sqrt(x)), x)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2}{3}\,(x-1)^{\frac{3}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

output
```
2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\,x\,\sqrt{x-1}}{3} - \frac{2\,\sqrt{x-1}}{3} + \frac{2\,x^{3/2}}{3}$$

input
```
int(-1/((x - 1)^(1/2) - x^(1/2)),x)
```

output
```
(2*x*(x - 1)^(1/2))/3 - (2*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

3.239.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\sqrt{x-1}\,x}{3} - \frac{2\sqrt{x-1}}{3} + \frac{2\sqrt{x}\,x}{3}$$

input
```
int(( - 1)/(sqrt(x - 1) - sqrt(x)),x)
```

output
```
(2*(sqrt(x - 1)*x - sqrt(x - 1) + sqrt(x)*x))/3
```

3.240 $\int \frac{1}{1-\sqrt{1+x}}\,dx$

3.240.1 Optimal result 1426
3.240.2 Mathematica [A] (verified) 1426
3.240.3 Rubi [A] (warning: unable to verify) 1427
3.240.4 Maple [A] (verified) 1428
3.240.5 Fricas [A] (verification not implemented) 1429
3.240.6 Sympy [A] (verification not implemented) 1429
3.240.7 Maxima [A] (verification not implemented) 1429
3.240.8 Giac [A] (verification not implemented) 1430
3.240.9 Mupad [B] (verification not implemented) 1430
3.240.10 Reduce [B] (verification not implemented) 1430

3.240.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{1}{1-\sqrt{1+x}}\,dx=-2\sqrt{1+x}-2\log\left(1-\sqrt{1+x}\right)$$

output
```
-2*ln(1-(1+x)^(1/2))-2*(1+x)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{1-\sqrt{1+x}}\,dx=-2\sqrt{1+x}-2\log\left(-1+\sqrt{1+x}\right)$$

input
```
Integrate[(1 - Sqrt[1 + x])^(-1),x]
```

output
```
-2*Sqrt[1 + x] - 2*Log[-1 + Sqrt[1 + x]]
```

3.240.3 Rubi [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 239}$$
$$\int \frac{1}{1-\sqrt{x+1}}d(x+1)$$
$$\Big\downarrow \text{ 774}$$
$$2\int -\frac{\sqrt{x+1}}{x}d\sqrt{x+1}$$
$$\Big\downarrow \text{ 49}$$
$$2\int \left(-1-\frac{1}{x}\right)d\sqrt{x+1}$$
$$\Big\downarrow \text{ 2009}$$
$$2(-x-\log(-x)-1)$$

input
```
Int[(1 - Sqrt[1 + x])^(-1),x]
```

output
```
2*(-1 - x - Log[-x])
```

3.240.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.240.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-2\sqrt{1+x}-2\ln\left(-1+\sqrt{1+x}\right)$	19
trager	$-2\sqrt{1+x}-\ln\left(2\sqrt{1+x}-2-x\right)$	24
default	$-\ln\left(x\right)-2\sqrt{1+x}-\ln\left(-1+\sqrt{1+x}\right)+\ln\left(1+\sqrt{1+x}\right)$	31

input
```
int(1/(1-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*(1+x)^(1/2)-2*ln(-1+(1+x)^(1/2))
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-\sqrt{1+x}}\,dx = -2\sqrt{x+1} - 2\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate(1/(1-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
-2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

3.240.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\sqrt{1+x}}\,dx = -2\sqrt{x+1} - 2\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate(1/(1-(1+x)**(1/2)),x)
```

output
```
-2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-\sqrt{1+x}}\,dx = -2\sqrt{x+1} - 2\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate(1/(1-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
-2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{1-\sqrt{1+x}}\,dx = -2\sqrt{x+1} - 2\log\left(\left|\sqrt{x+1}-1\right|\right)$$

input
```
integrate(1/(1-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(x + 1) - 2*log(abs(sqrt(x + 1) - 1))
```

3.240.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-\sqrt{1+x}}\,dx = -2\,\ln\left(\sqrt{x+1}-1\right) - 2\sqrt{x+1}$$

input
```
int(-1/((x + 1)^(1/2) - 1),x)
```

output
```
- 2*log((x + 1)^(1/2) - 1) - 2*(x + 1)^(1/2)
```

3.240.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\sqrt{1+x}}\,dx = -2\sqrt{x+1} - 2\log\left(\sqrt{x+1}-1\right)$$

input
```
int(( - 1)/(sqrt(x + 1) - 1),x)
```

output
```
 - 2*(sqrt(x + 1) + log(sqrt(x + 1) - 1))
```

3.241 $\int \frac{x}{\sqrt{36+x^4}} \, dx$

3.241.1 Optimal result . . . 1431
3.241.2 Mathematica [A] (verified) . . . 1431
3.241.3 Rubi [A] (verified) . . . 1432
3.241.4 Maple [A] (verified) . . . 1433
3.241.5 Fricas [A] (verification not implemented) . . . 1433
3.241.6 Sympy [A] (verification not implemented) . . . 1434
3.241.7 Maxima [B] (verification not implemented) . . . 1434
3.241.8 Giac [A] (verification not implemented) . . . 1434
3.241.9 Mupad [B] (verification not implemented) . . . 1435
3.241.10 Reduce [B] (verification not implemented) . . . 1435

3.241.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{x}{\sqrt{36+x^4}} \, dx = \frac{1}{2}\text{arcsinh}\left(\frac{x^2}{6}\right)$$

output
```
1/2*arcsinh(1/6*x^2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{36+x^4}} \, dx = \frac{1}{2}\log\left(x^2+\sqrt{36+x^4}\right)$$

input
```
Integrate[x/Sqrt[36 + x^4],x]
```

output
```
Log[x^2 + Sqrt[36 + x^4]]/2
```

3.241.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4+36}}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+36}}dx^2$$
$$\Big\downarrow \text{222}$$
$$\frac{1}{2}\text{arcsinh}\left(\frac{x^2}{6}\right)$$

input
```
Int[x/Sqrt[36 + x^4],x]
```

output
```
ArcSinh[x^2/6]/2
```

3.241.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.241.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
elliptic	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
pseudoelliptic	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
trager	$\frac{\ln\left(x^2+\sqrt{x^4+36}\right)}{2}$	15

input
```
int(x/(x^4+36)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(1/6*x^2)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{36+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

input
```
integrate(x/(x^4+36)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 + 36))
```

3.241.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt{36+x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

input
```
integrate(x/(x**4+36)**(1/2),x)
```

output
```
asinh(x**2/6)/2
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(8) = 16.

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{x}{\sqrt{36+x^4}}\,dx = \frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}-1\right)$$

input
```
integrate(x/(x^4+36)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{36+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

input
```
integrate(x/(x^4+36)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 + 36))
```

3.241.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{36+x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

input
```
int(x/(x^4 + 36)^(1/2),x)
```

output
```
asinh(x^2/6)/2
```

3.241.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{x}{\sqrt{36+x^4}}\,dx = \frac{\log\left(\frac{\sqrt{x^4+36}}{6}+\frac{x^2}{6}\right)}{2}$$

input
```
int(x/sqrt(x**4 + 36),x)
```

output
```
log((sqrt(x**4 + 36) + x**2)/6)/2
```

3.242 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

3.242.1 Optimal result 1436
3.242.2 Mathematica [A] (verified) 1436
3.242.3 Rubi [A] (verified) 1437
3.242.4 Maple [A] (verified) 1438
3.242.5 Fricas [A] (verification not implemented) 1439
3.242.6 Sympy [F] 1439
3.242.7 Maxima [A] (verification not implemented) 1439
3.242.8 Giac [A] (verification not implemented) 1440
3.242.9 Mupad [B] (verification not implemented) 1440
3.242.10Reduce [B] (verification not implemented) 1440

3.242.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

output
```
6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

input
```
Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

output
```
(6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

3.242.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\left(\sqrt[6]{x}+1\right)\sqrt[3]{x}}dx$$
$$\Big\downarrow \text{798}$$
$$6\int \frac{\sqrt{x}}{\sqrt[6]{x}+1}d\sqrt[6]{x}$$
$$\Big\downarrow \text{49}$$
$$6\int \left(\sqrt[3]{x}-\sqrt[6]{x}+\frac{1}{-\sqrt[6]{x}-1}+1\right)d\sqrt[6]{x}$$
$$\Big\downarrow \text{2009}$$
$$6\left(\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}+\sqrt[6]{x}-\log\left(\sqrt[6]{x}+1\right)\right)$$

input
```
Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

output
```
6*(x^(1/6) - x^(1/3)/2 + Sqrt[x]/3 - Log[1 + x^(1/6)])
```

3.242.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.242.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+2\ln\left(x^{\frac{1}{6}}-1\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{x}+\ln$

input
```
int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.242.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

input
```
integrate(1/(x**(1/3)+x**(1/2)),x)
```

output
```
Integral(1/(x**(1/3) + sqrt(x)), x)
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 6\,\ln\left(x^{1/6}+1\right) - 3\,x^{1/3} + 6\,x^{1/6}$$

input
```
int(1/(x^(1/2) + x^(1/3)),x)
```

output
```
2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.242.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 6x^{\frac{1}{6}} - 3x^{\frac{1}{3}} + 2\sqrt{x} - 6\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
int(1/(x**(1/3) + sqrt(x)),x)
```

output
```
6*x**(1/6) - 3*x**(1/3) + 2*sqrt(x) - 6*log(x**(1/6) + 1)
```

3.243 $\int \log(2+3x^2)\, dx$

3.243.1 Optimal result 1441
3.243.2 Mathematica [A] (verified) 1441
3.243.3 Rubi [A] (verified) 1442
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3.243.5 Fricas [A] (verification not implemented) 1443
3.243.6 Sympy [A] (verification not implemented) 1444
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3.243.8 Giac [A] (verification not implemented) 1444
3.243.9 Mupad [B] (verification not implemented) 1445
3.243.10 Reduce [B] (verification not implemented) 1445

3.243.1 Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \log(2+3x^2)\, dx = -2x + 2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)$$

output
```
-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \log(2+3x^2)\, dx = -2x + 2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)$$

input
```
Integrate[Log[2 + 3*x^2],x]
```

output
```
-2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

3.243.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2898, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(3x^2+2\right)\,dx$$

$$\Big\downarrow \text{2898}$$

$$x\log\left(3x^2+2\right)-6\int \frac{x^2}{3x^2+2}dx$$

$$\Big\downarrow \text{262}$$

$$x\log\left(3x^2+2\right)-6\left(\frac{x}{3}-\frac{2}{3}\int \frac{1}{3x^2+2}dx\right)$$

$$\Big\downarrow \text{216}$$

$$x\log\left(3x^2+2\right)-6\left(\frac{x}{3}-\frac{1}{3}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)\right)$$

input
```
Int[Log[2 + 3*x^2],x]
```

output
```
-6*(x/3 - (Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/3) + x*Log[2 + 3*x^2]
```

3.243.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

3.243.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
risch	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
parts	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27

input
```
int(ln(3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \log\left(2+3x^2\right)\,dx=\frac{2}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)+x\log\left(3\,x^2+2\right)-2\,x$$

input
```
integrate(log(3*x^2+2),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

3.243.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \log \left(2+3x^2\right)\,dx = x\log \left(3x^2+2\right) - 2x + \frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

input
```
integrate(ln(3*x**2+2),x)
```

output
```
x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\,dx = x\log \left(3\,x^2+2\right) + \frac{2}{3}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) - 2\,x$$

input
```
integrate(log(3*x^2+2),x, algorithm="maxima")
```

output
```
x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\,dx = x\log \left(3\,x^2+2\right) + \frac{2}{3}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) - 2\,x$$

input
```
integrate(log(3*x^2+2),x, algorithm="giac")
```

output
```
x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.243.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\, dx = \frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{3} - 2\,x + x\,\ln\left(3\,x^2+2\right)$$

input
```
int(log(3*x^2 + 2),x)
```

output
```
(2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

3.243.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\, dx = \frac{2\sqrt{6}\,atan\left(\frac{3x}{\sqrt{6}}\right)}{3} + \log\left(3x^2+2\right)x - 2x$$

input
```
int(log(3*x**2 + 2),x)
```

output
```
(2*sqrt(6)*atan((3*x)/sqrt(6)) + 3*log(3*x**2 + 2)*x - 6*x)/3
```

3.244 $\int \cot(x)\,dx$

3.244.1 Optimal result . . . 1446
3.244.2 Mathematica [B] (verified) . . . 1446
3.244.3 Rubi [A] (verified) . . . 1447
3.244.4 Maple [A] (verified) . . . 1448
3.244.5 Fricas [B] (verification not implemented) . . . 1448
3.244.6 Sympy [A] (verification not implemented) . . . 1449
3.244.7 Maxima [A] (verification not implemented) . . . 1449
3.244.8 Giac [A] (verification not implemented) . . . 1449
3.244.9 Mupad [B] (verification not implemented) . . . 1450
3.244.10 Reduce [B] (verification not implemented) . . . 1450

3.244.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x)\,dx = \log(\sin(x))$$

output
```
ln(sin(x))
```

3.244.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x)\,dx = \log(\cos(x)) + \log(\tan(x))$$

input
```
Integrate[Cot[x],x]
```

output
```
Log[Cos[x]] + Log[Tan[x]]
```

3.244.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\log(\sin(x))$$

input
```
Int[Cot[x],x]
```

output
```
Log[Sin[x]]
```

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.244.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right)+\ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(e^{2ix}-1)$	14

input
```
int(cot(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x))
```

3.244.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \cot(x)\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

input
```
integrate(cot(x),x, algorithm="fricas")
```

output
```
1/2*log(-1/2*cos(2*x) + 1/2)
```

3.244.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log\left(\sin\left(x\right)\right)$$

input
```
integrate(cot(x),x)
```

output
```
log(sin(x))
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log\left(\sin\left(x\right)\right)$$

input
```
integrate(cot(x),x, algorithm="maxima")
```

output
```
log(sin(x))
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \cot(x)\,dx = \log\left(\left|\sin\left(x\right)\right|\right)$$

input
```
integrate(cot(x),x, algorithm="giac")
```

output
```
log(abs(sin(x)))
```

3.244.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \ln(\sin(x))$$

input
```
int(cot(x),x)
```

output
```
log(sin(x))
```

3.244.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \cot(x)\,dx = -\log\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) + \log\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(cot(x),x)
```

output
```
- log(tan(x/2)**2 + 1) + log(tan(x/2))
```

3.245 $\int \cot^4(x)\,dx$

3.245.1 Optimal result 1451
3.245.2 Mathematica [C] (verified) 1451
3.245.3 Rubi [A] (verified) 1452
3.245.4 Maple [A] (verified) 1453
3.245.5 Fricas [B] (verification not implemented) 1454
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3.245.8 Giac [B] (verification not implemented) 1455
3.245.9 Mupad [B] (verification not implemented) 1455
3.245.10 Reduce [B] (verification not implemented) 1455

3.245.1 Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \cot^4(x)\,dx = x+\cot(x)-\frac{\cot^3(x)}{3}$$

output
```
x+cot(x)-1/3*cot(x)^3
```

3.245.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \cot^4(x)\,dx = -\frac{1}{3}\cot^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(x)\right)$$

input
```
Integrate[Cot[x]^4,x]
```

output
```
-1/3*(Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x]^2])
```

3.245.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3954}$$

$$-\int \cot^2(x)dx-\frac{1}{3}\cot^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{3}\cot^3(x)$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{1}{3}\cot^3(x)+\cot(x)$$

$$\Big\downarrow \text{24}$$

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

input
```
Int[Cot[x]^4,x]
```

output
```
x + Cot[x] - Cot[x]^3/3
```

3.245.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.245.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x+\cot\left(x\right)-\frac{\left(\cot^{3}\left(x\right)\right)}{3}$	11
derivativedivides	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
default	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
norman	$\frac{-\frac{1}{3}+\tan^{2}\left(x\right)+x\left(\tan^{3}\left(x\right)\right)}{\tan\left(x\right)^{3}}$	18
risch	$x+\frac{4i\left(3\,{\mathrm e}^{4ix}-3\,{\mathrm e}^{2ix}+2\right)}{3\left({\mathrm e}^{2ix}-1\right)^{3}}$	31

input
```
int(cot(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x+cot(x)-1/3*cot(x)^3
```

3.245.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \cot^4(x)\,dx = \frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

input
```
integrate(cot(x)^4,x, algorithm="fricas")
```

output
```
1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*
x) - 1)*sin(2*x))
```

3.245.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cot^4(x)\,dx = x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

input
```
integrate(cot(x)**4,x)
```

output
```
x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot^4(x)\,dx = x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

input
```
integrate(cot(x)^4,x, algorithm="maxima")
```

output
```
x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \cot^4(x)\,dx = \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(cot(x)^4,x, algorithm="giac")
```

output
```
1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(
1/2*x)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x)\,dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

input
```
int(cot(x)^4,x)
```

output
```
x + cot(x) - cot(x)^3/3
```

3.245.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x)\,dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

input
```
int(cot(x)**4,x)
```

output
```
( - cot(x)**3 + 3*cot(x) + 3*x)/3
```

3.246 $\int \tanh(x)\,dx$

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3.246.2 Mathematica [A] (verified) . . . 1456
3.246.3 Rubi [A] (verified) . . . 1457
3.246.4 Maple [A] (verified) . . . 1458
3.246.5 Fricas [B] (verification not implemented) . . . 1458
3.246.6 Sympy [B] (verification not implemented) . . . 1458
3.246.7 Maxima [A] (verification not implemented) . . . 1459
3.246.8 Giac [B] (verification not implemented) . . . 1459
3.246.9 Mupad [B] (verification not implemented) . . . 1460
3.246.10Reduce [B] (verification not implemented) . . . 1460

3.246.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

output
```
ln(cosh(x))
```

3.246.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

input
```
Integrate[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

3.246.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -i\tan(ix)dx$$
$$\Big\downarrow \; 26$$
$$-i\int \tan(ix)dx$$
$$\Big\downarrow \; 3956$$
$$\log(\cosh(x))$$

input
```
Int[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

3.246.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.246.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
derivativedivides	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln(1+e^{2x})$	12
parallelrisch	$-\ln(1-\tanh(x))-x$	14

input
```
int(tanh(x),x,method=_RETURNVERBOSE)
```

output
```
ln(cosh(x))
```

3.246.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x)\,dx = -x + \log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(tanh(x),x, algorithm="fricas")
```

output
```
-x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.246.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x)\,dx = x - \log(\tanh(x)+1)$$

input
```
integrate(tanh(x),x)
```

output
```
x - log(tanh(x) + 1)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\, dx = \log\left(\cosh\left(x\right)\right)$$

input
```
integrate(tanh(x),x, algorithm="maxima")
```

output
```
log(cosh(x))
```

3.246.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x)\, dx = -x + \log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(tanh(x),x, algorithm="giac")
```

output
```
-x + log(e^(2*x) + 1)
```

3.246.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \ln\left(\cosh(x)\right)$$

input
```
int(tanh(x),x)
```

output
```
log(cosh(x))
```

3.246.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \tanh(x)\,dx = \log\left(e^{2x}+1\right) - x$$

input
```
int(tanh(x),x)
```

output
```
log(e**(2*x) + 1) - x
```

3.247 $\int \coth(x)\,dx$

3.247.1 Optimal result 1461
3.247.2 Mathematica [B] (verified) 1461
3.247.3 Rubi [A] (verified) 1462
3.247.4 Maple [A] (verified) 1463
3.247.5 Fricas [B] (verification not implemented) 1463
3.247.6 Sympy [B] (verification not implemented) 1463
3.247.7 Maxima [A] (verification not implemented) 1464
3.247.8 Giac [B] (verification not implemented) 1464
3.247.9 Mupad [B] (verification not implemented) 1465
3.247.10Reduce [B] (verification not implemented) 1465

3.247.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \coth(x)\,dx = \log(\sinh(x))$$

output
```
ln(sinh(x))
```

3.247.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \coth(x)\,dx = \log(\cosh(x)) + \log(\tanh(x))$$

input
```
Integrate[Coth[x],x]
```

output
```
Log[Cosh[x]] + Log[Tanh[x]]
```

3.247.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\tan\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3956$$

$$\log(\sinh(x))$$

input
```
Int[Coth[x],x]
```

output
```
Log[Sinh[x]]
```

3.247.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.247.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
derivativedivides	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x+\ln(\mathrm{e}^{2x}-1)$	12
parallelrisch	$\ln(\tanh(x))-\ln(1-\tanh(x))-x$	17

input
```
int(coth(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sinh(x))
```

3.247.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \coth(x)\,dx = -x + \log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(coth(x),x, algorithm="fricas")
```

output
```
-x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

3.247.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = x - \log(\tanh(x)+1) + \log(\tanh(x))$$

input
```
integrate(coth(x),x)
```

output
```
x - log(tanh(x) + 1) + log(tanh(x))
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \log\left(\sinh\left(x\right)\right)$$

input
```
integrate(coth(x),x, algorithm="maxima")
```

output
```
log(sinh(x))
```

3.247.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = -x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate(coth(x),x, algorithm="giac")
```

output
```
-x + log(abs(e^(2*x) - 1))
```

3.247.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \ln(\sinh(x))$$

input
```
int(coth(x),x)
```

output
```
log(sinh(x))
```

3.247.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \coth(x)\,dx = \log(e^x - 1) + \log(e^x + 1) - x$$

input
```
int(coth(x),x)
```

output
```
log(e**x - 1) + log(e**x + 1) - x
```

3.248 $\int b^x\,dx$

3.248.1 Optimal result 1466
3.248.2 Mathematica [A] (verified) 1466
3.248.3 Rubi [A] (verified) 1467
3.248.4 Maple [A] (verified) 1468
3.248.5 Fricas [A] (verification not implemented) 1468
3.248.6 Sympy [A] (verification not implemented) 1469
3.248.7 Maxima [A] (verification not implemented) 1469
3.248.8 Giac [A] (verification not implemented) 1469
3.248.9 Mupad [B] (verification not implemented) 1470
3.248.10 Reduce [B] (verification not implemented) 1470

3.248.1 Optimal result

Integrand size = 3, antiderivative size = 8

$$\int b^x\,dx=\frac{b^x}{\log(b)}$$

output
```
b^x/ln(b)
```

3.248.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx=\frac{b^x}{\log(b)}$$

input
```
Integrate[b^x,x]
```

output
```
b^x/Log[b]
```

3.248.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int b^x\,dx$$
$$\Big\downarrow \text{2624}$$
$$\frac{b^x}{\log(b)}$$

input
```
Int[b^x,x]
```

output
```
b^x/Log[b]
```

3.248.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.248.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
gosper	$\frac{b^x}{\ln(b)}$	9
derivativedivides	$\frac{b^x}{\ln(b)}$	9
default	$\frac{b^x}{\ln(b)}$	9
risch	$\frac{b^x}{\ln(b)}$	9
parallelrisch	$\frac{b^x}{\ln(b)}$	9
norman	$\frac{e^{x\ln(b)}}{\ln(b)}$	11
meijerg	$-\frac{1-e^{x\ln(b)}}{\ln(b)}$	16

input
```
int(b^x,x,method=_RETURNVERBOSE)
```

output
```
b^x/ln(b)
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx = \frac{b^x}{\log(b)}$$

input
```
integrate(b^x,x, algorithm="fricas")
```

output
```
b^x/log(b)
```

3.248.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx = \begin{cases} \frac{b^x}{\log(b)} & \text{for } \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(b**x,x)
```

output
```
Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx = \frac{b^x}{\log(b)}$$

input
```
integrate(b^x,x, algorithm="maxima")
```

output
```
b^x/log(b)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx = \frac{b^x}{\log(b)}$$

input
```
integrate(b^x,x, algorithm="giac")
```

output
```
b^x/log(b)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx = \frac{b^x}{\ln(b)}$$

input
```
int(b^x,x)
```

output
```
b^x/log(b)
```

3.248.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x\,dx = \frac{b^x}{\log(b)}$$

input
```
int(b**x,x)
```

output
```
b**x/log(b)
```

3.249 $\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx$

3.249.1 Optimal result . . . 1471
3.249.2 Mathematica [A] (verified) . . . 1471
3.249.3 Rubi [A] (verified) . . . 1472
3.249.4 Maple [A] (verified) . . . 1473
3.249.5 Fricas [A] (verification not implemented) . . . 1474
3.249.6 Sympy [F] . . . 1474
3.249.7 Maxima [A] (verification not implemented) . . . 1474
3.249.8 Giac [A] (verification not implemented) . . . 1475
3.249.9 Mupad [F(-1)] . . . 1475
3.249.10Reduce [B] (verification not implemented) . . . 1475

3.249.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx=-\frac{x\sqrt{2+\frac{1}{x^4}+x^4}}{1+x^4}+\frac{x^5\sqrt{2+\frac{1}{x^4}+x^4}}{3(1+x^4)}$$

output
```
-x*(2+1/x^4+x^4)^(1/2)/(x^4+1)+1/3*x^5*(2+1/x^4+x^4)^(1/2)/(x^4+1)
```

3.249.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx=\frac{x(-3+x^4)\sqrt{2+\frac{1}{x^4}+x^4}}{3(1+x^4)}$$

input
```
Integrate[Sqrt[2 + x^(-4) + x^4],x]
```

output
```
(x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))
```

3.249.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1689, 1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^4+\frac{1}{x^4}+2}\,dx$$

$$\Big\downarrow \text{1689}$$

$$\frac{x^2\sqrt{x^4+\frac{1}{x^4}+2}\int \frac{\sqrt{x^8+2x^4+1}}{x^2}dx}{\sqrt{x^8+2x^4+1}}$$

$$\Big\downarrow \text{1384}$$

$$\frac{x^2\sqrt{x^4+\frac{1}{x^4}+2}\int \frac{x^4+1}{x^2}dx}{x^4+1}$$

$$\Big\downarrow \text{802}$$

$$\frac{x^2\sqrt{x^4+\frac{1}{x^4}+2}\int \left(x^2+\frac{1}{x^2}\right)dx}{x^4+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2\left(\frac{x^3}{3}-\frac{1}{x}\right)\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

input
```
Int[Sqrt[2 + x^(-4) + x^4],x]
```

output
```
(x^2*(-x^(-1) + x^3/3)*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)
```

3.249.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1689
```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] :> Simp[x^
(n*FracPart[p])*((a + b/x^n + c*x^n)^FracPart[p]/(b + a*x^n + c*x^(2*n))^Fr
acPart[p])   Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x], x] /; FreeQ[{a, b,
c, n, p}, x] && EqQ[mn, -n] &&  !IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.249.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x\left(x^{4}-3\right)\sqrt{\frac{x^{8}+2x^{4}+1}{x^{4}}}}{3x^{4}+3}$	32
default	$\frac{x\left(x^{4}-3\right)\sqrt{\frac{x^{8}+2x^{4}+1}{x^{4}}}}{3x^{4}+3}$	32
risch	$\frac{\sqrt{\frac{\left(x^{4}+1\right)^{2}}{x^{4}}}\, x^{5}}{3x^{4}+3}-\frac{\sqrt{\frac{\left(x^{4}+1\right)^{2}}{x^{4}}}\, x}{x^{4}+1}$	50

input
```
int((2+1/x^4+x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx=\frac{x^4-3}{3\,x}$$

input
```
integrate((2+1/x^4+x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^4 - 3)/x
```

3.249.6 Sympy [F]

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx=\int \sqrt{x^4+2+\frac{1}{x^4}}\,dx$$

input
```
integrate((2+1/x**4+x**4)**(1/2),x)
```

output
```
Integral(sqrt(x**4 + 2 + x**(-4)), x)
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx=\frac{x^4-3}{3\,x}$$

input
```
integrate((2+1/x^4+x^4)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^4 - 3)/x
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx = \frac{1}{3}\,x^3-\frac{1}{x}$$

input
```
integrate((2+1/x^4+x^4)^(1/2),x, algorithm="giac")
```

output
```
1/3*x^3 - 1/x
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx = \int \sqrt{\frac{1}{x^4}+x^4+2}\,dx$$

input
```
int((1/x^4 + x^4 + 2)^(1/2),x)
```

output
```
int((1/x^4 + x^4 + 2)^(1/2), x)
```

3.249.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx = \frac{x^4-3}{3x}$$

input
```
int((x**4 + 1)/x**2,x)
```

output
```
(x**4 - 3)/(3*x)
```

3.250 $\int \frac{1+2x}{2+3x} dx$

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3.250.2 Mathematica [A] (verified) 1476
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3.250.9 Mupad [B] (verification not implemented) 1479
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3.250.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1+2x}{2+3x} \, dx = \frac{2x}{3} - \frac{1}{9} \log(2+3x)$$

output
```
2/3*x-1/9*ln(2+3*x)
```

3.250.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1+2x}{2+3x} \, dx = \frac{1}{9}(4+6x-\log(2+3x))$$

input
```
Integrate[(1 + 2*x)/(2 + 3*x),x]
```

output
```
(4 + 6*x - Log[2 + 3*x])/9
```

3.250.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{3x+2} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2}{3}-\frac{1}{3(3x+2)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x}{3}-\frac{1}{9}\log(3x+2)$$

input
```
Int[(1 + 2*x)/(2 + 3*x),x]
```

output
```
(2*x)/3 - Log[2 + 3*x]/9
```

3.250.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.250.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{2x}{3}-\frac{\ln\left(\frac{2}{3}+x\right)}{9}$	11
default	$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$	13
norman	$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$	13
meijerg	$-\frac{\ln\left(1+\frac{3x}{2}\right)}{9}+\frac{2x}{3}$	13
risch	$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$	13

input
```
int((1+2*x)/(2+3*x),x,method=_RETURNVERBOSE)
```

output
```
2/3*x-1/9*ln(2/3+x)
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x}\,dx = \frac{2}{3}\,x - \frac{1}{9}\,\log\left(3\,x+2\right)$$

input
```
integrate((1+2*x)/(2+3*x),x, algorithm="fricas")
```

output
```
2/3*x - 1/9*log(3*x + 2)
```

3.250.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x}\,dx = \frac{2x}{3} - \frac{\log\left(3x+2\right)}{9}$$

input
```
integrate((1+2*x)/(2+3*x),x)
```

output
```
2*x/3 - log(3*x + 2)/9
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x} \, dx = \frac{2}{3} \, x - \frac{1}{9} \, \log \left(3 \, x + 2\right)$$

input
```
integrate((1+2*x)/(2+3*x),x, algorithm="maxima")
```

output
```
2/3*x - 1/9*log(3*x + 2)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{2+3x} \, dx = \frac{2}{3} \, x - \frac{1}{9} \, \log \left(|3 \, x + 2|\right)$$

input
```
integrate((1+2*x)/(2+3*x),x, algorithm="giac")
```

output
```
2/3*x - 1/9*log(abs(3*x + 2))
```

3.250.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1+2x}{2+3x} \, dx = \frac{2\,x}{3} - \frac{\ln\left(x+\frac{2}{3}\right)}{9}$$

input
```
int((2*x + 1)/(3*x + 2),x)
```

output
```
(2*x)/3 - log(x + 2/3)/9
```

3.250.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x}\,dx = -\frac{\log(3x+2)}{9} + \frac{2x}{3}$$

input
```
int((2*x + 1)/(3*x + 2),x)
```

output
```
( - log(3*x + 2) + 6*x)/9
```

3.251 $\int x \log \left(x+\sqrt{1+x^2}\right) dx$

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3.251.9 Mupad [B] (verification not implemented) . . . 1485
3.251.10Reduce [B] (verification not implemented) . . . 1485

3.251.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int x \log \left(x+\sqrt{1+x^2}\right) dx = -\frac{1}{4}x\sqrt{1+x^2}+\frac{\operatorname{arcsinh}(x)}{4}+\frac{1}{2}x^2\log \left(x+\sqrt{1+x^2}\right)$$

output
```
1/4*arcsinh(x)+1/2*x^2*ln(x+(x^2+1)^(1/2))-1/4*x*(x^2+1)^(1/2)
```

3.251.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int x \log \left(x+\sqrt{1+x^2}\right) dx = \frac{1}{4}\left(-x\sqrt{1+x^2}+\operatorname{arcsinh}(x)+2x^2\log \left(x+\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[x*Log[x + Sqrt[1 + x^2]],x]
```

output
```
(-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/4
```

3.251.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3016, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(\sqrt{x^2+1}+x\right) \, dx$$

$$\Big\downarrow \text{3016}$$

$$\frac{1}{2} x^2 \log \left(\sqrt{x^2+1}+x\right)-\frac{1}{2} \int \frac{x^2}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{\sqrt{x^2+1}}dx-\frac{1}{2} x \sqrt{x^2+1}\right)+\frac{1}{2} x^2 \log \left(\sqrt{x^2+1}+x\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2} \left(\frac{\text{arcsinh}(x)}{2}-\frac{1}{2} x \sqrt{x^2+1}\right)+\frac{1}{2} x^2 \log \left(\sqrt{x^2+1}+x\right)$$

input
```
Int[x*Log[x + Sqrt[1 + x^2]],x]
```

output
```
(-1/2*(x*Sqrt[1 + x^2]) + ArcSinh[x]/2)/2 + (x^2*Log[x + Sqrt[1 + x^2]])/2
```

3.251.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3016
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]]*((g_.)*(x_))
^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[a + c*x^2]]/(
g*(m + 1))), x] - Simp[a*c*(f^2/(g*(m + 1)))   Int[(g*x)^(m + 1)/(d*e*(a +
c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

3.251.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

method	result	size
parts	$\frac{x^2 \ln\left(x+\sqrt{x^2+1}\right)}{2}+\frac{x^3\sqrt{x^2+1}}{8}-\frac{x\sqrt{x^2+1}}{8}+\frac{\operatorname{arcsinh}(x)}{4}-\frac{x\left(x^2+1\right)^{\frac{3}{2}}}{8}$	53

input
```
int(x*ln(x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(x+(x^2+1)^(1/2))+1/8*x^3*(x^2+1)^(1/2)-1/8*x*(x^2+1)^(1/2)+1/4*
arcsinh(x)-1/8*x*(x^2+1)^(3/2)
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int x \log\left(x+\sqrt{1+x^2}\right)\,dx = \frac{1}{4}\left(2\,x^2+1\right)\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,\sqrt{x^2+1}x$$

input
```
integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x
```

3.251.6 Sympy [F]

$$\int x \log \left(x+\sqrt{1+x^2}\right) \, dx = \int x \log \left(x+\sqrt{x^2+1}\right) dx$$

input
```
integrate(x*ln(x+(x**2+1)**(1/2)),x)
```

output
```
Integral(x*log(x + sqrt(x**2 + 1)), x)
```

3.251.7 Maxima [F]

$$\int x \log \left(x+\sqrt{1+x^2}\right) \, dx = \int x \log \left(x+\sqrt{x^2+1}\right) dx$$

input
```
integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3 + (x^2 +
 1)^(3/2) + x), x) + 1/4*log(x^2 + 1)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x \log \left(x+\sqrt{1+x^2}\right) \, dx = \frac{1}{2}\,x^2 \log \left(x+\sqrt{x^2+1}\right) - \frac{1}{4}\,\sqrt{x^2+1}x - \frac{1}{4}\,\log \left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*log(-x + sqrt(x
^2 + 1))
```

3.251.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x \log \left(x+\sqrt{1+x^2}\right) \, dx = x \ln \left(x+\sqrt{x^2+1}\right) \left(\frac{x}{2}+\frac{1}{4\,x}\right)-\frac{x\,\sqrt{x^2+1}}{4}$$

input
```
int(x*log(x + (x^2 + 1)^(1/2)),x)
```

output
```
x*log(x + (x^2 + 1)^(1/2))*(x/2 + 1/(4*x)) - (x*(x^2 + 1)^(1/2))/4
```

3.251.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x \log \left(x+\sqrt{1+x^2}\right) \, dx = -\frac{\sqrt{x^2+1}\,x}{4}+\frac{\log\left(\sqrt{x^2+1}+x\right)x^2}{2}+\frac{\log\left(\sqrt{x^2+1}+x\right)}{4}$$

input
```
int(log(sqrt(x**2 + 1) + x)*x,x)
```

output
```
( - sqrt(x**2 + 1)*x + 2*log(sqrt(x**2 + 1) + x)*x**2 + log(sqrt(x**2 + 1)
 + x))/4
```

3.252 $\int x(1+e^x \sin(x))^2 \, dx$

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3.252.7 Maxima [A] (verification not implemented) 1489
3.252.8 Giac [A] (verification not implemented) 1490
3.252.9 Mupad [B] (verification not implemented) 1490
3.252.10 Reduce [B] (verification not implemented) 1491

3.252.1 Optimal result

Integrand size = 12, antiderivative size = 128

$$\int x(1+e^x \sin(x))^2 \, dx = -\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)-\frac{1}{32}e^{2x}\cos(2x) +e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x) -\frac{1}{16}e^{2x}\sin^2(x)+\frac{1}{4}e^{2x}x\sin^2(x)+\frac{1}{32}e^{2x}\sin(2x)$$

output
```
-3/32*exp(2*x)+1/8*exp(2*x)*x+1/2*x^2+exp(x)*cos(x)-exp(x)*x*cos(x)-1/32*e
xp(2*x)*cos(2*x)+exp(x)*x*sin(x)+1/16*exp(2*x)*cos(x)*sin(x)-1/4*exp(2*x)*
x*cos(x)*sin(x)-1/16*exp(2*x)*sin(x)^2+1/4*exp(2*x)*x*sin(x)^2+1/32*exp(2*
x)*sin(2*x)
```

3.252.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int x(1+e^x \sin(x))^2 \, dx = \frac{1}{8}\left(4x^2+e^{2x}(-1+2x)-8e^x(-1+x)\cos(x)-e^{2x}x\cos(2x) +8e^x x\sin(x)-e^{2x}(-1+2x)\cos(x)\sin(x)\right)$$

input
```
Integrate[x*(1 + E^x*Sin[x])^2,x]
```

output
```
(4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*Cos[2*x] +
 8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8
```

3.252.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(e^x \sin(x)+1)^2\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x+e^{2x}x\sin^2(x)+2e^x x\sin(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^x x\sin(x)+\frac{1}{32}e^{2x}\sin(2x)-e^x x\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)-\frac{1}{4}e^{2x}x\sin(x)\cos(x)+\frac{1}{16}e^{2x}\sin(x)\cos(x)$$

input
```
Int[x*(1 + E^x*Sin[x])^2,x]
```

output
```
(-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(
2*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x
*Cos[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2
*x)*Sin[2*x])/32
```

3.252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.252.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.49

method	result
default	$\frac{e^{2x}x}{4}-\frac{e^{2x}}{8}-\frac{x\,e^{2x}\cos(2x)}{8}+\frac{\left(-\frac{x}{4}+\frac{1}{8}\right)e^{2x}\sin(2x)}{2}+2\left(-\frac{x}{2}+\frac{1}{2}\right)e^{x}\cos(x)+e^{x}x\sin(x)+\frac{x^{2}}{2}$
parts	$\frac{e^{2x}x}{4}-\frac{e^{2x}}{8}-\frac{x\,e^{2x}\cos(2x)}{8}+\frac{\left(-\frac{x}{4}+\frac{1}{8}\right)e^{2x}\sin(2x)}{2}+2\left(-\frac{x}{2}+\frac{1}{2}\right)e^{x}\cos(x)+e^{x}x\sin(x)+\frac{x^{2}}{2}$
risch	$\frac{x^{2}}{2}+\left(-\frac{1}{8}+\frac{x}{4}\right)e^{2x}+\left(-\frac{1}{64}+\frac{i}{64}\right)(-1+i+4x)\,e^{(2+2i)x}+\left(-\frac{1}{4}-\frac{i}{4}\right)(-1+i+2x)\,e^{(1+i)x}+(-$

input
```
int(x*(1+exp(x)*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(x)^2*x-1/8*exp(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2
*x)*sin(2*x)+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/2*x^2
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.43

$$\int x(1+e^x\sin(x))^2\,dx = -(x-1)\cos(x)\,e^x+\frac{1}{2}\,x^2-\frac{1}{8}\left(2\,x\cos(x)^2-3\,x+1\right)e^{(2\,x)} - \frac{1}{8}\left((2\,x-1)\cos(x)\,e^{(2\,x)}-8\,xe^x\right)\sin(x)$$

input
```
integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="fricas")
```

output
```
-(x - 1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x)^2 - 3*x + 1)*e^(2*x) - 1/8
*((2*x - 1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)
```

3.252.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

$$\int x(1+e^x\sin(x))^2\,dx = \frac{x^2}{2} + \frac{3xe^{2x}\sin^2(x)}{8} - \frac{xe^{2x}\sin(x)\cos(x)}{4} + \frac{xe^{2x}\cos^2(x)}{8} + xe^x\sin(x) - xe^x\cos(x) - \frac{e^{2x}\sin^2(x)}{8} + \frac{e^{2x}\sin(x)\cos(x)}{8} - \frac{e^{2x}\cos^2(x)}{8} + e^x\cos(x)$$

input
```
integrate(x*(1+exp(x)*sin(x))**2,x)
```

output
```
x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 + x*exp(2*x
)*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(2*x)*sin(x)**2/8 +
 exp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/8 + exp(x)*cos(x)
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

$$\int x(1+e^x\sin(x))^2\,dx = -\frac{1}{8}\,x\cos(2\,x)\,e^{(2\,x)} - (x-1)\cos(x)\,e^x - \frac{1}{16}\,(2\,x-1)e^{(2\,x)}\sin(2\,x) + xe^x\sin(x) + \frac{1}{2}\,x^2 + \frac{1}{8}\,(2\,x-1)e^{(2\,x)}$$

input
```
integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="maxima")
```

output
```
-1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(2*x)*sin(
2*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int x(1+e^x\sin(x))^2\,dx = \frac{1}{2}\,x^2 - \frac{1}{16}\,(2\,x\cos(2\,x) + (2\,x-1)\sin(2\,x))e^{(2\,x)} + \frac{1}{8}\,(2\,x-1)e^{(2\,x)} - ((x-1)\cos(x) - x\sin(x))e^x$$

input
```
integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) + 1/8*(2*x - 1)
*e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x
```

3.252.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.54

$$\int x(1+e^x\sin(x))^2\,dx = \frac{3\,x\,e^{2\,x}}{8} - \frac{e^{2\,x}}{8} + e^x\,\cos(x) + \frac{x^2}{2} - \frac{x\,e^{2\,x}\cos(x)^2}{4} + \frac{e^{2\,x}\,\cos(x)\,\sin(x)}{8} - x\,e^x\,\cos(x) + x\,e^x\,\sin(x) - \frac{x\,e^{2\,x}\,\cos(x)\,\sin(x)}{4}$$

input
```
int(x*(exp(x)*sin(x) + 1)^2,x)
```

output
```
(3*x*exp(2*x))/8 - exp(2*x)/8 + exp(x)*cos(x) + x^2/2 - (x*exp(2*x)*cos(x)
^2)/4 + (exp(2*x)*cos(x)*sin(x))/8 - x*exp(x)*cos(x) + x*exp(x)*sin(x) - (
x*exp(2*x)*cos(x)*sin(x))/4
```

3.252.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

$$\int x(1+e^x\sin(x))^2\,dx = -\frac{e^{2x}\cos(x)\sin(x)\,x}{4} + \frac{e^{2x}\cos(x)\sin(x)}{8} - e^x\cos(x)\,x + e^x\cos(x) + \frac{e^{2x}\sin(x)^2\,x}{4} + \frac{e^{2x}x}{8} - \frac{e^{2x}}{8} + e^x\sin(x)\,x + \frac{x^2}{2}$$

input
```
int(x*(e**(2*x)*sin(x)**2 + 2*e**x*sin(x) + 1),x)
```

output
```
( - 2*e**(2*x)*cos(x)*sin(x)*x + e**(2*x)*cos(x)*sin(x) - 8*e**x*cos(x)*x
+ 8*e**x*cos(x) + 2*e**(2*x)*sin(x)**2*x + e**(2*x)*x - e**(2*x) + 8*e**x*
sin(x)*x + 4*x**2)/8
```

3.253 $\int e^x x \cos(x) \, dx$

3.253.1 Optimal result 1492
3.253.2 Mathematica [A] (verified) 1492
3.253.3 Rubi [A] (verified) 1493
3.253.4 Maple [A] (verified) 1494
3.253.5 Fricas [A] (verification not implemented) 1494
3.253.6 Sympy [A] (verification not implemented) 1494
3.253.7 Maxima [A] (verification not implemented) 1495
3.253.8 Giac [A] (verification not implemented) 1495
3.253.9 Mupad [B] (verification not implemented) 1495
3.253.10 Reduce [B] (verification not implemented) 1496

3.253.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)$$

output
```
1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

3.253.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x (x \cos(x) + (-1 + x) \sin(x))$$

input
```
Integrate[E^x*x*Cos[x],x]
```

output
```
(E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

3.253.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \cos(x)\,dx$$

$$\Big\downarrow \text{4969}$$

$$-\int \left(\frac{1}{2}e^x \cos(x)+\frac{1}{2}e^x \sin(x)\right)dx+\frac{1}{2}e^x x \sin(x)+\frac{1}{2}e^x x \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}e^x \sin(x)+\frac{1}{2}e^x x \sin(x)+\frac{1}{2}e^x x \cos(x)$$

input
```
Int[E^x*x*Cos[x],x]
```

output
```
(E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

3.253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.253.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{e^{x}\left(\left(-1+x\right)\sin\left(x\right)+x\cos\left(x\right)\right)}{2}$	16
default	$\frac{e^{x}x\cos\left(x\right)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^{x}\sin\left(x\right)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)\left(-1+i+2x\right)e^{\left(1+i\right)x}+\left(\frac{1}{8}+\frac{i}{8}\right)\left(-1-i+2x\right)e^{\left(1-i\right)x}$	36
norman	$\frac{e^{x}x\tan\left(\frac{x}{2}\right)+\frac{e^{x}x}{2}-e^{x}\tan\left(\frac{x}{2}\right)-\frac{e^{x}x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	45

input
```
int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*((-1+x)*sin(x)+x*cos(x))
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^{x}x\cos(x)\,dx=\frac{1}{2}\,x\cos\left(x\right)e^{x}+\frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

3.253.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{x}x\cos(x)\,dx=\frac{xe^{x}\sin\left(x\right)}{2}+\frac{xe^{x}\cos\left(x\right)}{2}-\frac{e^{x}\sin\left(x\right)}{2}$$

input
```
integrate(exp(x)*x*cos(x),x)
```

output
```
x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{1}{2} x \cos{(x)} e^x + \frac{1}{2} (x-1)e^x \sin{(x)}$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^x x \cos(x) \, dx = \frac{1}{2} (x \cos{(x)} + (x-1) \sin{(x)})e^x$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

output
```
1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.253.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{\mathrm{e}^x \left(x \cos{(x)} - \sin{(x)} + x \sin{(x)}\right)}{2}$$

input
```
int(x*exp(x)*cos(x),x)
```

output
```
(exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.253.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x)\, dx = \frac{e^x(\cos(x)\, x + \sin(x)\, x - \sin(x))}{2}$$

input
```
int(e**x*cos(x)*x,x)
```

output
```
(e**x*(cos(x)*x + sin(x)*x - sin(x)))/2
```

3.254 $\int \frac{1}{(-3+x)^4} dx$

3.254.1 Optimal result 1497
3.254.2 Mathematica [A] (verified) 1497
3.254.3 Rubi [A] (verified) 1498
3.254.4 Maple [A] (verified) 1499
3.254.5 Fricas [B] (verification not implemented) 1499
3.254.6 Sympy [B] (verification not implemented) 1500
3.254.7 Maxima [A] (verification not implemented) 1500
3.254.8 Giac [A] (verification not implemented) 1500
3.254.9 Mupad [B] (verification not implemented) 1501
3.254.10 Reduce [B] (verification not implemented) 1501

3.254.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int \frac{1}{(-3+x)^4} dx = \frac{1}{3(3-x)^3}$$

output
```
1/3/(3-x)^3
```

3.254.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3(-3+x)^3}$$

input
```
Integrate[(-3 + x)^(-4),x]
```

output
```
-1/3*1/(-3 + x)^3
```

3.254.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-3)^4}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{1}{3(3-x)^3}$$

input
```
Int[(-3 + x)^(-4),x]
```

output
```
1/(3*(3 - x)^3)
```

3.254.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.254.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{1}{3(-3+x)^{3}}$	8
default	$-\frac{1}{3(-3+x)^{3}}$	8
norman	$-\frac{1}{3(-3+x)^{3}}$	8
risch	$-\frac{1}{3(-3+x)^{3}}$	8
parallelrisch	$-\frac{1}{3(-3+x)^{3}}$	8
meijerg	$\frac{x\left(\frac{1}{9}x^{2}-x+3\right)}{243\left(1-\frac{x}{3}\right)^{3}}$	21

input
```
int(1/(-3+x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(-3+x)^3
```

3.254.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-3+x)^4}\,dx = -\frac{1}{3\,(x^3-9\,x^2+27\,x-27)}$$

input
```
integrate(1/(-3+x)^4,x, algorithm="fricas")
```

output
```
-1/3/(x^3 - 9*x^2 + 27*x - 27)
```

3.254.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-3+x)^4}\,dx = -\frac{1}{3x^3 - 27x^2 + 81x - 81}$$

input
```
integrate(1/(-3+x)**4,x)
```

output
```
-1/(3*x**3 - 27*x**2 + 81*x - 81)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3+x)^4}\,dx = -\frac{1}{3\,(x-3)^3}$$

input
```
integrate(1/(-3+x)^4,x, algorithm="maxima")
```

output
```
-1/3/(x - 3)^3
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3+x)^4}\,dx = -\frac{1}{3\,(x-3)^3}$$

input
```
integrate(1/(-3+x)^4,x, algorithm="giac")
```

output
```
-1/3/(x - 3)^3
```

3.254.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3+x)^4}\,dx = -\frac{1}{3\,(x-3)^3}$$

input
```
int(1/(x - 3)^4,x)
```

output
```
-1/(3*(x - 3)^3)
```

3.254.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{1}{(-3+x)^4}\,dx = -\frac{1}{3x^3-27x^2+81x-81}$$

input
```
int(1/(x**4 - 12*x**3 + 54*x**2 - 108*x + 81),x)
```

output
```
( - 1)/(3*(x**3 - 9*x**2 + 27*x - 27))
```

3.255 $\int \frac{x}{-1+x^3} dx$

3.255.1 Optimal result . . . 1502
3.255.2 Mathematica [A] (verified) . . . 1502
3.255.3 Rubi [A] (verified) . . . 1503
3.255.4 Maple [A] (verified) . . . 1505
3.255.5 Fricas [A] (verification not implemented) . . . 1505
3.255.6 Sympy [A] (verification not implemented) . . . 1506
3.255.7 Maxima [A] (verification not implemented) . . . 1506
3.255.8 Giac [A] (verification not implemented) . . . 1506
3.255.9 Mupad [B] (verification not implemented) . . . 1507
3.255.10Reduce [B] (verification not implemented) . . . 1507

3.255.1 Optimal result

Integrand size = 9, antiderivative size = 40

$$\int \frac{x}{-1+x^3} dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

output
```
1/3*ln(1-x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x}{-1+x^3} dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

input
```
Integrate[x/(-1 + x^3),x]
```

output
```
ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

3.255.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {821, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^3-1} \, dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\int \frac{1}{x-1}dx-\frac{1}{3}\int -\frac{1-x}{x^2+x+1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\int -\frac{1-x}{x^2+x+1}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{1-x}{x^2+x+1}dx+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-3\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{3}\log(1-x)$$

input
```
Int[x/(-1 + x^3),x]
```

output
```
Log[1 - x]/3 + (Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 + x + x^2]/2)/3
```

3.255.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.255.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{3}$	31
default	$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	33
meijerg	$\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}$	63

input
```
int(x/(x^3-1),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(-1+x)-1/6*ln(x^2+x+1)+1/3*3^(1/2)*arctan(2/3*3^(1/2)*(x+1/2))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x}{-1+x^3}\,dx=\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate(x/(x^3-1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

3.255.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x}{-1+x^3}\,dx = \frac{\log{(x-1)}}{3} - \frac{\log{(x^2+x+1)}}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**3-1),x)
```

output
```
log(x - 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/
3)/3
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x}{-1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\,\log\left(x^2+x+1\right) + \frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(x/(x^3-1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x}{-1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\,\log\left(x^2+x+1\right) + \frac{1}{3}\,\log\left(|x-1|\right)$$

input
```
integrate(x/(x^3-1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(abs(x - 1))
```

3.255.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{x}{-1+x^3}\,dx = \frac{\ln(x-1)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(x/(x^3 - 1),x)
```

output
```
log(x - 1)/3 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + log(
x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6)
```

3.255.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x}{-1+x^3}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} - \frac{\log(x^2+x+1)}{6} + \frac{\log(x-1)}{3}$$

input
```
int(x/(x**3 - 1),x)
```

output
```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - log(x**2 + x + 1) + 2*log(x - 1))/6
```

3.256 $\int \frac{x}{-1+x^4} dx$

3.256.1 Optimal result 1508
3.256.2 Mathematica [B] (verified) 1508
3.256.3 Rubi [A] (verified) 1509
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3.256.9 Mupad [B] (verification not implemented) 1512
3.256.10 Reduce [B] (verification not implemented) 1512

3.256.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{x}{-1+x^4} dx = -\frac{1}{2}\operatorname{arctanh}(x^2)$$

output
```
-1/2*arctanh(x^2)
```

3.256.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{x}{-1+x^4} dx = \frac{1}{4}\log(1-x^2) - \frac{1}{4}\log(1+x^2)$$

input
```
Integrate[x/(-1 + x^4),x]
```

output
```
Log[1 - x^2]/4 - Log[1 + x^2]/4
```

3.256.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {807, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4-1}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^4-1}dx^2$$

$$\Big\downarrow \text{220}$$

$$-\frac{1}{2}\text{arctanh}\left(x^2\right)$$

input
```
Int[x/(-1 + x^4),x]
```

output
```
-1/2*ArcTanh[x^2]
```

3.256.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.256.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$-\frac{\operatorname{arctanh}(x^2)}{2}$	7
risch	$-\frac{\ln(x^2+1)}{4}+\frac{\ln(x^2-1)}{4}$	18
default	$\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{4}$	22
norman	$\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{4}$	22
parallelrisch	$\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{4}$	22

input
```
int(x/(x^4-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(x^2)
```

3.256.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{-1+x^4}\,dx = -\frac{1}{4}\,\log\left(x^2+1\right)+\frac{1}{4}\,\log\left(x^2-1\right)$$

input
```
integrate(x/(x^4-1),x, algorithm="fricas")
```

output
```
-1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

3.256.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{x}{-1+x^4}\,dx = \frac{\log\left(x^2-1\right)}{4} - \frac{\log\left(x^2+1\right)}{4}$$

input
```
integrate(x/(x**4-1),x)
```

output
```
log(x**2 - 1)/4 - log(x**2 + 1)/4
```

3.256.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{-1+x^4}\,dx = -\frac{1}{4}\log\left(x^2+1\right) + \frac{1}{4}\log\left(x^2-1\right)$$

input
```
integrate(x/(x^4-1),x, algorithm="maxima")
```

output
```
-1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{-1+x^4}\,dx = -\frac{1}{4}\log\left(x^2+1\right) + \frac{1}{4}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x/(x^4-1),x, algorithm="giac")
```

output
```
-1/4*log(x^2 + 1) + 1/4*log(abs(x^2 - 1))
```

3.256.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{-1+x^4}\,dx = -\frac{\operatorname{atanh}(x^2)}{2}$$

input
```
int(x/(x^4 - 1),x)
```

output
```
-atanh(x^2)/2
```

3.256.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{x}{-1+x^4}\,dx = -\frac{\log(x^2+1)}{4}+\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

input
```
int(x/(x**4 - 1),x)
```

output
```
( - log(x**2 + 1) + log(x - 1) + log(x + 1))/4
```

3.257 $\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx$

3.257.1 Optimal result . . . 1513
3.257.2 Mathematica [A] (verified) . . . 1514
3.257.3 Rubi [A] (verified) . . . 1514
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3.257.8 Giac [F] . . . 1517
3.257.9 Mupad [F(-1)] . . . 1518
3.257.10Reduce [F] . . . 1518

3.257.1 Optimal result

Integrand size = 15, antiderivative size = 227

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \frac{1}{8}(2+i\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{16}(4+(1-i)2^{3/4})\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}(2+\sqrt[4]{-2})\log(x)\log\left(1-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}(2-\sqrt[4]{-2})\log(x)\text{lo}$$

output
```
1/8*(2+I*(-2)^(1/4))*ln(x)*ln(1-(1/2+1/2*I)*x*2^(1/4))+1/16*(4+(1-I)*2^(3/
4))*ln(x)*ln(1+(1/2+1/2*I)*x*2^(1/4))+1/8*(2+(-2)^(1/4))*ln(x)*ln(1-1/2*(-
1)^(3/4)*x*2^(3/4))+1/8*(2-(-2)^(1/4))*ln(x)*ln(1+1/2*(-1)^(3/4)*x*2^(3/4)
)+1/16*(4+(1-I)*2^(3/4))*polylog(2,(-1/2-1/2*I)*x*2^(1/4))+1/8*(2+I*(-2)^(
1/4))*polylog(2,(1/2+1/2*I)*x*2^(1/4))+1/8*(2-(-2)^(1/4))*polylog(2,-1/2*(
-1)^(3/4)*x*2^(3/4))+1/8*(2+(-2)^(1/4))*polylog(2,1/2*(-1)^(3/4)*x*2^(3/4)
)
```

3.257.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \frac{1}{8}\left((2+i\sqrt[4]{-2})\log(x)\log\left(1-\sqrt[4]{-\frac{1}{2}}x\right) + \left(2+\frac{1-i}{\sqrt[4]{2}}\right)\log(x)\log\left(1+\sqrt[4]{-\frac{1}{2}}x\right) - (-2+\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1-i)x}{2^{3/4}}\right) + (2+\sqrt[4]{-2})\log(x)\log\left(1+\frac{(1-i)x}{2^{3/4}}\right) + \left(2+\frac{1-i}{\sqrt[4]{2}}\right)\operatorname{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right) + (2+\sqrt[4]{-2})\operatorname{PolyLog}\left(2\right.\right.$$

input
```
Integrate[((1 + x^3)*Log[x])/(2 + x^4),x]
```

output
```
((2 + I*(-2)^(1/4))*Log[x]*Log[1 - (-1/2)^(1/4)*x] + (2 + (1 - I)/2^(1/4))
*Log[x]*Log[1 + (-1/2)^(1/4)*x] - (-2 + (-2)^(1/4))*Log[x]*Log[1 - ((1 - I
)*x)/2^(3/4)] + (2 + (-2)^(1/4))*Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + (2
+ (1 - I)/2^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*Pol
yLog[2, ((-1 + I)*x)/2^(3/4)] - (-2 + (-2)^(1/4))*PolyLog[2, ((1 - I)*x)/2
^(3/4)] + (2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8
```

3.257.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+1)\log(x)}{x^4+2}\,dx$$

$$\Big\downarrow \text{2804}$$

$$\int \left(\frac{(\sqrt[4]{-2}-2)\log(x)}{8(\sqrt[4]{-2}-x)} + \frac{(\sqrt[4]{-2}-2i)\log(x)}{8(\sqrt[4]{-2}-ix)} + \frac{(\sqrt[4]{-2}+2i)\log(x)}{8(\sqrt[4]{-2}+ix)} + \frac{(2+\sqrt[4]{-2})\log(x)}{8(x+\sqrt[4]{-2})}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}(2-i\sqrt[4]{-2})\operatorname{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}(2+i\sqrt[4]{-2})\operatorname{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}(2-\sqrt[4]{-2})\operatorname{PolyLog}\left(2,-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}(2+\sqrt[4]{-2})\operatorname{PolyLog}\left(2,\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}(2+i\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}(2-i\sqrt[4]{-2})\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}(2+\sqrt[4]{-2})\log(x)\log\left(1-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}(2-\sqrt[4]{-2})\log(x)\log\left(\frac{(-1)^{3/4}x}{\sqrt[4]{2}}+1\right)$$

input
```
Int[((1 + x^3)*Log[x])/(2 + x^4),x]
```

output
```
((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + ((2 - I*(-2)^
(1/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)])/8 + ((2 + (-2)^(1/4))*Log[x]*L
og[1 - ((-1)^(3/4)*x)/2^(1/4)])/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1)
^(3/4)*x)/2^(1/4)])/8 + ((2 - I*(-2)^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4
)])/8 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2 - (-2
)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (-2)^(1/4))*Poly
Log[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```

3.257.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.257.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(171) = 342.

Time = 0.37 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.74

method	result
default	$\frac{\left(\left(\frac{2^{\frac{3}{4}}}{2}+\frac{i 2^{\frac{3}{4}}}{2}\right)^{3}+1\right)\left(\ln(x)\ln\left(\frac{\frac{2^{\frac{3}{4}}}{2}+\frac{i 2^{\frac{3}{4}}}{2}-x}{\frac{2^{\frac{3}{4}}}{2}+\frac{i 2^{\frac{3}{4}}}{2}}\right)+\operatorname{dilog}\left(\frac{\frac{2^{\frac{3}{4}}}{2}+\frac{i 2^{\frac{3}{4}}}{2}-x}{\frac{2^{\frac{3}{4}}}{2}+\frac{i 2^{\frac{3}{4}}}{2}}\right)\right)}{4\left(\frac{2^{\frac{3}{4}}}{2}+\frac{i 2^{\frac{3}{4}}}{2}\right)^{3}}+\frac{\left(\left(\frac{i 2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}\right)^{3}+1\right)\left(\ln(x)\ln\left(\frac{\frac{i 2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}-x}{\frac{i 2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}}\right)+\mathrm{di}\right.}{4\left(\frac{i 2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}\right)^{3}}$
parts	Expression too large to display
risch	Expression too large to display

input
```
int((x^3+1)*ln(x)/(x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((1/2*2^(3/4)+1/2*I*2^(3/4))^3+1)/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*(ln(x)
*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))+dilog((1/2*
2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4))))+1/4*((1/2*I*2^(3/4)
-1/2*2^(3/4))^3+1)/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*(ln(x)*ln((1/2*I*2^(3/4)-
1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+dilog((1/2*I*2^(3/4)-1/2*2^(3/
4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4))))+1/4*((-1/2*2^(3/4)-1/2*I*2^(3/4))^3+1)
/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*(ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(
-1/2*2^(3/4)-1/2*I*2^(3/4)))+dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^
(3/4)-1/2*I*2^(3/4))))+1/4*((-1/2*I*2^(3/4)+1/2*2^(3/4))^3+1)/(-1/2*I*2^(3
/4)+1/2*2^(3/4))^3*(ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4
)+1/2*2^(3/4)))+dilog((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2
^(3/4))))
```

3.257.5 Fricas [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \frac{(x^3+1)\log(x)}{x^4+2}\,dx$$

input
```
integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="fricas")
```

output
```
integral((x^3 + 1)*log(x)/(x^4 + 2), x)
```

3.257.6 Sympy [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \frac{(x+1)\,(x^2-x+1)\log{(x)}}{x^4+2}\,dx$$

input
```
integrate((x**3+1)*ln(x)/(x**4+2),x)
```

output
```
Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)
```

3.257.7 Maxima [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \frac{(x^3+1)\log{(x)}}{x^4+2}\,dx$$

input
```
integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="maxima")
```

output
```
integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

3.257.8 Giac [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \frac{(x^3+1)\log{(x)}}{x^4+2}\,dx$$

input
```
integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="giac")
```

output
```
integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \frac{\ln(x)\ (x^3+1)}{x^4+2}\,dx$$

input
```
int((log(x)*(x^3 + 1))/(x^4 + 2),x)
```

output
```
int((log(x)*(x^3 + 1))/(x^4 + 2), x)
```

3.257.10 Reduce [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = -2\left(\int \frac{\log(x)}{x^5+2x}dx\right) + \int \frac{\log(x)}{x^4+2}dx + \frac{\log(x)^2}{2}$$

input
```
int((log(x)*(x**3 + 1))/(x**4 + 2),x)
```

output
```
( - 4*int(log(x)/(x**5 + 2*x),x) + 2*int(log(x)/(x**4 + 2),x) + log(x)**2)
/2
```

3.258 $\int (\log(x) + \log(1 + x) + \log(2 + x))\, dx$

3.258.1 Optimal result 1519
3.258.2 Mathematica [A] (verified) 1519
3.258.3 Rubi [A] (verified) 1520
3.258.4 Maple [A] (verified) 1520
3.258.5 Fricas [A] (verification not implemented) 1521
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3.258.8 Giac [A] (verification not implemented) 1522
3.258.9 Mupad [B] (verification not implemented) 1522
3.258.10 Reduce [B] (verification not implemented) 1523

3.258.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int (\log(x) + \log(1 + x) + \log(2 + x))\, dx = -3x + x\log(x) + (1 + x)\log(1 + x) + (2 + x)\log(2 + x)$$

output
```
-3*x+x*ln(x)+(1+x)*ln(1+x)+(2+x)*ln(2+x)
```

3.258.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (\log(x) + \log(1 + x) + \log(2 + x))\, dx = -3x + x\log(x) + (1 + x)\log(1 + x) + (2 + x)\log(2 + x)$$

input
```
Integrate[Log[x] + Log[1 + x] + Log[2 + x],x]
```

output
```
-3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

3.258.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\log(x)+\log(x+1)+\log(x+2))\,dx$$

$$\Big\downarrow \text{2009}$$

$$-3x+x\log(x)+(x+1)\log(x+1)+(x+2)\log(x+2)$$

input
```
Int[Log[x] + Log[1 + x] + Log[2 + x],x]
```

output
```
-3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

3.258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.258.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
default	$-3x+x\ln(x)+(1+x)\ln(1+x)-3+(2+x)\ln(2+x)$	26
parts	$-3x+x\ln(x)+(1+x)\ln(1+x)-3+(2+x)\ln(2+x)$	26
risch	$-3x+x\ln(x)+\ln(1+x)x+\ln(1+x)+\ln(2+x)x+2\ln(2+x)$	31
parallelrisch	$x\ln(x)+\ln(1+x)x+\ln(2+x)x-3x+\ln(1+x)+2\ln(2+x)+9$	32

input
```
int(ln(x)+ln(1+x)+ln(2+x),x,method=_RETURNVERBOSE)
```

output
```
-3*x+x*ln(x)+(1+x)*ln(1+x)-3+(2+x)*ln(2+x)
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (\log(x) + \log(1+x) + \log(2+x))\,dx = (x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3\,x$$

input
```
integrate(log(x)+log(1+x)+log(2+x),x, algorithm="fricas")
```

output
```
(x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x
```

3.258.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int (\log(x) + \log(1+x) + \log(2+x))\,dx = x\log(x) - 3x + \left(x + \frac{1}{2}\right)\log(x+1) + (x+1)\log(x+2) + \frac{\log(x+1)}{2} + \log(x+2)$$

input
```
integrate(ln(x)+ln(1+x)+ln(2+x),x)
```

output
```
x*log(x) - 3*x + (x + 1/2)*log(x + 1) + (x + 1)*log(x + 2) + log(x + 1)/2
+ log(x + 2)
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (\log(x) + \log(1+x) + \log(2+x))\,dx = (x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x - 3$$

input
```
integrate(log(x)+log(1+x)+log(2+x),x, algorithm="maxima")
```

output
```
(x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (\log(x) + \log(1+x) + \log(2+x))\,dx = (x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x - 3$$

input
```
integrate(log(x)+log(1+x)+log(2+x),x, algorithm="giac")
```

output
```
(x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

3.258.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int (\log(x) + \log(1+x) + \log(2+x))\,dx = \ln(x+1) - 3x + 2\ln(x+2) + x\ln(x+1) + x\ln(x) + \frac{\ln(x+2)\,(x^3+3x^2+2x)}{(x+1)\,(x+2)}$$

input
```
int(log(x + 1) + log(x + 2) + log(x),x)
```

output
```
log(x + 1) - 3*x + 2*log(x + 2) + x*log(x + 1) + x*log(x) + (log(x + 2)*(2
*x + 3*x^2 + x^3))/((x + 1)*(x + 2))
```

3.258.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int (\log(x)+\log(1+x)+\log(2+x))\,dx = \log(x+2)\,x+2\log(x+2)+\log(x+1)\,x+\log(x+1)+\log(x)\,x-3x$$

input
```
int(log(x + 2) + log(x + 1) + log(x),x)
```

output
```
log(x + 2)*x + 2*log(x + 2) + log(x + 1)*x + log(x + 1) + log(x)*x - 3*x
```

3.259 $\int \frac{1}{5+x^3} dx$

3.259.1 Optimal result 1524
3.259.2 Mathematica [A] (verified) 1524
3.259.3 Rubi [A] (verified) 1525
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3.259.9 Mupad [B] (verification not implemented) 1529
3.259.10Reduce [B] (verification not implemented) 1530

3.259.1 Optimal result

Integrand size = 7, antiderivative size = 78

$$\int \frac{1}{5+x^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}} + \frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}} - \frac{\log\left(5^{2/3}-\sqrt[3]{5}x+x^2\right)}{6\ 5^{2/3}}$$

output
```
1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)-1/15*arc
tan(1/15*(5^(1/3)-2*x)*5^(2/3)*3^(1/2))*5^(1/3)*3^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{1}{5+x^3} dx = \frac{2\sqrt{3}\arctan\left(\frac{-5+2\ 5^{2/3}x}{5\sqrt{3}}\right) + 2\log\left(5+5^{2/3}x\right) - \log\left(5-5^{2/3}x+\sqrt[3]{5}x^2\right)}{6\ 5^{2/3}}$$

input
```
Integrate[(5 + x^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-5 + 2*5^(2/3)*x)/(5*Sqrt[3])] + 2*Log[5 + 5^(2/3)*x] -
 Log[5 - 5^(2/3)*x + 5^(1/3)*x^2])/(6*5^(2/3))
```

3.259.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3+5} \, dx$$
$$\Big\downarrow \text{ 750}$$
$$\frac{\int \frac{2\sqrt[3]{5}-x}{x^2-\sqrt[3]{5}x+5^{2/3}}dx}{3\ 5^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{5}}dx}{3\ 5^{2/3}}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{\int \frac{2\sqrt[3]{5}-x}{x^2-\sqrt[3]{5}x+5^{2/3}}dx}{3\ 5^{2/3}}+\frac{\log \left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}$$
$$\Big\downarrow \text{ 1142}$$
$$\frac{\frac{3}{2}\sqrt[3]{5}\int \frac{1}{x^2-\sqrt[3]{5}x+5^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}}dx}{3\ 5^{2/3}}+\frac{\log \left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\frac{3}{2}\sqrt[3]{5}\int \frac{1}{x^2-\sqrt[3]{5}x+5^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}}dx}{3\ 5^{2/3}}+\frac{\log \left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}$$
$$\Big\downarrow \text{ 1082}$$
$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{5}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{5}}\right)}{3\ 5^{2/3}}+\frac{\log \left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}$$
$$\Big\downarrow \text{ 217}$$
$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{5}}}{\sqrt{3}}\right)}{3\ 5^{2/3}}+\frac{\log \left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{5}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{3\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}$$

input
```
Int[(5 + x^3)^(-1),x]
```

output
```
Log[5^(1/3) + x]/(3*5^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/5^(1/3))/Sqrt[
3]]) - Log[5^(2/3) - 5^(1/3)*x + x^2]/2)/(3*5^(2/3))
```

3.259.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.259.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.28

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}+5\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3}$	22
default	$\frac{\ln\left(5^{\frac{1}{3}}+x\right)5^{\frac{1}{3}}}{15}-\frac{\ln\left(5^{\frac{2}{3}}-5^{\frac{1}{3}}x+x^{2}\right)5^{\frac{1}{3}}}{30}+\frac{5^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,5^{\frac{2}{3}}x}{5}-1\right)}{3}\right)}{15}$	54
meijerg	$\frac{5^{\frac{1}{3}}\left(\frac{x\ln\left(1+\frac{5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{5}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{x\ln\left(1-\frac{5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{5}+\frac{5^{\frac{1}{3}}\left(x^{3}\right)^{\frac{2}{3}}}{5}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{10-5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{15}$	96

input
```
int(1/(x^3+5),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3+5))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{1}{5+x^3}\,dx = \frac{1}{15}\cdot 25^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{75}\cdot 25^{\frac{1}{6}}\left(2\cdot 25^{\frac{2}{3}}\sqrt{3}x - 5\cdot 25^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{150}\cdot 25^{\frac{2}{3}}\log\left(5\,x^2 - 25^{\frac{2}{3}}x + 5\cdot 25^{\frac{1}{3}}\right) + \frac{1}{75}\cdot 25^{\frac{2}{3}}\log\left(5\,x + 25^{\frac{2}{3}}\right)$$

input
```
integrate(1/(x^3+5),x, algorithm="fricas")
```

output
```
1/15*25^(1/6)*sqrt(3)*arctan(1/75*25^(1/6)*(2*25^(2/3)*sqrt(3)*x - 5*25^(1
/3)*sqrt(3))) - 1/150*25^(2/3)*log(5*x^2 - 25^(2/3)*x + 5*25^(1/3)) + 1/75
*25^(2/3)*log(5*x + 25^(2/3))
```

3.259.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{1}{5+x^3}\,dx = \frac{\sqrt[3]{5}\log\left(x+\sqrt[3]{5}\right)}{15} - \frac{\sqrt[3]{5}\log\left(x^2-\sqrt[3]{5}x+5^{\frac{2}{3}}\right)}{30} + \frac{\sqrt{3}\cdot\sqrt[3]{5}\,\mathrm{atan}\left(\frac{2\sqrt{3}\cdot 5^{\frac{2}{3}}x}{15}-\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
integrate(1/(x**3+5),x)
```

output
```
5**(1/3)*log(x + 5**(1/3))/15 - 5**(1/3)*log(x**2 - 5**(1/3)*x + 5**(2/3))
/30 + sqrt(3)*5**(1/3)*atan(2*sqrt(3)*5**(2/3)*x/15 - sqrt(3)/3)/15
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{1}{5+x^3}\,dx = \frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^2-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(x+5^{\frac{1}{3}}\right)$$

input
```
integrate(1/(x^3+5),x, algorithm="maxima")
```

output
```
1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5
^(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(x + 5^(1/3))
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{1}{5+x^3}\,dx = \frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^2-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(\left|x+5^{\frac{1}{3}}\right|\right)$$

input
```
integrate(1/(x^3+5),x, algorithm="giac")
```

output
```
1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5
^(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(abs(x + 5^(1/3)))
```

3.259.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{1}{5+x^3}\,dx = \frac{5^{1/3}\ln\left(x+5^{1/3}\right)}{15}+\frac{5^{1/3}\ln\left(x+\frac{5^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{30}-\frac{5^{1/3}\ln\left(x-\frac{5^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{30}$$

input
```
int(1/(x^3 + 5),x)
```

output
```
(5^(1/3)*log(x + 5^(1/3)))/15 + (5^(1/3)*log(x + (5^(1/3)*(3^(1/2)*1i - 1)
)/2)*(3^(1/2)*1i - 1))/30 - (5^(1/3)*log(x - (5^(1/3)*(3^(1/2)*1i + 1))/2)
*(3^(1/2)*1i + 1))/30
```

3.259.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{1}{5+x^3}\,dx = \frac{5^{\frac{1}{3}}\left(-2\sqrt{3}\,atan\left(\frac{\left(5^{\frac{1}{3}}-2x\right)5^{\frac{2}{3}}}{5\sqrt{3}}\right)-\log\left(5^{\frac{2}{3}}-5^{\frac{1}{3}}x+x^2\right)+2\log\left(5^{\frac{1}{3}}+x\right)\right)}{30}$$

input
```
int(1/(x**3 + 5),x)
```

output
```
(5**(1/3)*( - 2*sqrt(3)*atan((5**(1/3) - 2*x)/(sqrt(3)*5**(1/3))) - log(5*
*(2/3) - 5**(1/3)*x + x**2) + 2*log(5**(1/3) + x)))/30
```

3.260 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

3.260.1 Optimal result 1531
3.260.2 Mathematica [B] (verified) 1531
3.260.3 Rubi [A] (verified) 1532
3.260.4 Maple [A] (verified) 1532
3.260.5 Fricas [B] (verification not implemented) 1533
3.260.6 Sympy [A] (verification not implemented) 1533
3.260.7 Maxima [A] (verification not implemented) 1534
3.260.8 Giac [B] (verification not implemented) 1534
3.260.9 Mupad [B] (verification not implemented) 1534
3.260.10Reduce [B] (verification not implemented) 1535

3.260.1 Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)
```

3.260.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = -\log \left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[1/Sqrt[1 + x^2],x]
```

output
```
-Log[-x + Sqrt[1 + x^2]]
```

3.260.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}} dx$$
$$\Big\downarrow \text{222}$$
$$\text{arcsinh}(x)$$

input
```
Int[1/Sqrt[1 + x^2],x]
```

output
```
ArcSinh[x]
```

3.260.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.260.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\text{arcsinh}\left(x\right)$	3
meijerg	$\text{arcsinh}\left(x\right)$	3
trager	$\ln\left(x+\sqrt{x^{2}+1}\right)$	11
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^{2}+1}}{x}\right)$	13

input
```
int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)
```

3.260.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 1))
```

3.260.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
integrate(1/(x**2+1)**(1/2),x)
```

output
```
asinh(x)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(x)
```

3.260.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(2) = 4$.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.260.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
int(1/(x^2 + 1)^(1/2),x)
```

output
```
asinh(x)
```

3.260.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \log\left(\sqrt{x^2+1}+x\right)$$

input
```
int(1/sqrt(x**2 + 1),x)
```

output
```
log(sqrt(x**2 + 1) + x)
```

3.261 $\int \sqrt{3+x^2}\,dx$

3.261.1 Optimal result 1536
3.261.2 Mathematica [A] (verified) 1536
3.261.3 Rubi [A] (verified) 1537
3.261.4 Maple [A] (verified) 1538
3.261.5 Fricas [A] (verification not implemented) 1538
3.261.6 Sympy [A] (verification not implemented) 1539
3.261.7 Maxima [A] (verification not implemented) 1539
3.261.8 Giac [A] (verification not implemented) 1539
3.261.9 Mupad [B] (verification not implemented) 1540
3.261.10Reduce [B] (verification not implemented) 1540

3.261.1 Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \sqrt{3+x^2}\,dx = \frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)$$

output
```
3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \sqrt{3+x^2}\,dx = \frac{1}{2}x\sqrt{3+x^2}-\frac{3}{2}\log\left(-x+\sqrt{3+x^2}\right)$$

input
```
Integrate[Sqrt[3 + x^2],x]
```

output
```
(x*Sqrt[3 + x^2])/2 - (3*Log[-x + Sqrt[3 + x^2]])/2
```

3.261.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+3}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{2}\int \frac{1}{\sqrt{x^2+3}}dx+\frac{1}{2}\sqrt{x^2+3}x$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{2}\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{x^2+3}x$$

input
```
Int[Sqrt[3 + x^2],x]
```

output
```
(x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

3.261.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.261.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{3 \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{x^2+3}}{2}$	21
risch	$\frac{3 \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{x^2+3}}{2}$	21
trager	$\frac{x\sqrt{x^2+3}}{2}+\frac{3\ln\left(x+\sqrt{x^2+3}\right)}{2}$	24
meijerg	$-\frac{3\left(-\frac{2\sqrt{\pi}\, x\sqrt{3}\,\sqrt{\frac{x^2}{3}+1}}{3}-2\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)\right)}{4\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{x^2+3}}{2}+\frac{3\ln\left(\frac{x+\sqrt{x^2+3}}{x}\right)}{4}-\frac{3\ln\left(\frac{\sqrt{x^2+3}-x}{x}\right)}{4}$	46

input
```
int((x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{3+x^2}\,dx = \frac{1}{2}\sqrt{x^2+3}x - \frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

input
```
integrate((x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

3.261.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sqrt{3+x^2}\,dx = \frac{x\sqrt{x^2+3}}{2} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

input
```
integrate((x**2+3)**(1/2),x)
```

output
```
x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{3+x^2}\,dx = \frac{1}{2}\sqrt{x^2+3}x + \frac{3}{2}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}x\right)$$

input
```
integrate((x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{3+x^2}\,dx = \frac{1}{2}\sqrt{x^2+3}x - \frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

input
```
integrate((x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{3+x^2}\, dx = \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{3}\,x}{3}\right)}{2} + \frac{x\,\sqrt{x^2+3}}{2}$$

input
```
int((x^2 + 3)^(1/2),x)
```

output
```
(3*asinh((3^(1/2)*x)/3))/2 + (x*(x^2 + 3)^(1/2))/2
```

3.261.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sqrt{3+x^2}\, dx = \frac{\sqrt{x^2+3}\,x}{2} + \frac{3\log\left(\frac{\sqrt{x^2+3}+x}{\sqrt{3}}\right)}{2}$$

input
```
int(sqrt(x**2 + 3),x)
```

output
```
(sqrt(x**2 + 3)*x + 3*log((sqrt(x**2 + 3) + x)/sqrt(3)))/2
```

3.262 $\int \frac{x}{(1+x)^2} \, dx$

3.262.1 Optimal result 1541
3.262.2 Mathematica [A] (verified) 1541
3.262.3 Rubi [A] (verified) 1542
3.262.4 Maple [A] (verified) 1543
3.262.5 Fricas [A] (verification not implemented) 1543
3.262.6 Sympy [A] (verification not implemented) 1543
3.262.7 Maxima [A] (verification not implemented) 1544
3.262.8 Giac [A] (verification not implemented) 1544
3.262.9 Mupad [B] (verification not implemented) 1544
3.262.10 Reduce [B] (verification not implemented) 1545

3.262.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{1+x} + \log(1+x)$$

output
```
1/(1+x)+ln(1+x)
```

3.262.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{1+x} + \log(1+x)$$

input
```
Integrate[x/(1 + x)^2,x]
```

output
```
(1 + x)^(-1) + Log[1 + x]
```

3.262.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^2}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{x+1}+\log(x+1)$$

input
```
Int[x/(1 + x)^2,x]
```

output
```
(1 + x)^(-1) + Log[1 + x]
```

3.262.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.262.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
norman	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
risch	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
meijerg	$-\frac{x}{1+x}+\ln\left(1+x\right)$	14
parallelrisch	$\frac{\ln(1+x)x+1+\ln(1+x)}{1+x}$	19

input
```
int(x/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(1+x)+ln(1+x)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x}{(1+x)^2}\,dx = \frac{(x+1)\log\left(x+1\right)+1}{x+1}$$

input
```
integrate(x/(1+x)^2,x, algorithm="fricas")
```

output
```
((x + 1)*log(x + 1) + 1)/(x + 1)
```

3.262.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)^2}\,dx = \log\left(x+1\right)+\frac{1}{x+1}$$

input
```
integrate(x/(1+x)**2,x)
```

output
```
log(x + 1) + 1/(x + 1)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{x+1} + \log(x+1)$$

input
```
integrate(x/(1+x)^2,x, algorithm="maxima")
```

output
```
1/(x + 1) + log(x + 1)
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{x+1} + \log(|x+1|)$$

input
```
integrate(x/(1+x)^2,x, algorithm="giac")
```

output
```
1/(x + 1) + log(abs(x + 1))
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \ln(x+1) + \frac{1}{x+1}$$

input
```
int(x/(x + 1)^2,x)
```

output
```
log(x + 1) + 1/(x + 1)
```

3.262.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{x}{(1+x)^2} \, dx = \frac{\log(x+1)\, x + \log(x+1) - x}{x+1}$$

input
```
int(x/(x**2 + 2*x + 1),x)
```

output
```
(log(x + 1)*x + log(x + 1) - x)/(x + 1)
```

3.263 $\int \arcsin(x)\,dx$

3.263.1 Optimal result 1546
3.263.2 Mathematica [A] (verified) 1546
3.263.3 Rubi [A] (verified) 1547
3.263.4 Maple [A] (verified) 1548
3.263.5 Fricas [A] (verification not implemented) 1548
3.263.6 Sympy [A] (verification not implemented) 1548
3.263.7 Maxima [A] (verification not implemented) 1549
3.263.8 Giac [A] (verification not implemented) 1549
3.263.9 Mupad [B] (verification not implemented) 1549
3.263.10 Reduce [B] (verification not implemented) 1550

3.263.1 Optimal result

Integrand size = 2, antiderivative size = 16

$$\int \arcsin(x)\,dx = \sqrt{1-x^2} + x\arcsin(x)$$

output
```
x*arcsin(x)+(-x^2+1)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arcsin(x)\,dx = \sqrt{1-x^2} + x\arcsin(x)$$

input
```
Integrate[ArcSin[x],x]
```

output
```
Sqrt[1 - x^2] + x*ArcSin[x]
```

3.263.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5130, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(x)\,dx$$

$$\Big\downarrow \text{ 5130}$$

$$x\arcsin(x)-\int \frac{x}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 241}$$

$$x\arcsin(x)+\sqrt{1-x^2}$$

input
```
Int[ArcSin[x],x]
```

output
```
Sqrt[1 - x^2] + x*ArcSin[x]
```

3.263.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

3.263.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
lookup	$\arcsin\left(x\right) x+\sqrt{-x^{2}+1}$	15
default	$\arcsin\left(x\right) x+\sqrt{-x^{2}+1}$	15
parts	$\arcsin\left(x\right) x+\sqrt{-x^{2}+1}$	15

input
```
int(arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x)*x+(-x^2+1)^(1/2)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\arcsin\left(x\right)+\sqrt{-x^{2}+1}$$

input
```
integrate(arcsin(x),x, algorithm="fricas")
```

output
```
x*arcsin(x) + sqrt(-x^2 + 1)
```

3.263.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arcsin(x)\,dx = x\operatorname{asin}\left(x\right)+\sqrt{1-x^{2}}$$

input
```
integrate(asin(x),x)
```

output
```
x*asin(x) + sqrt(1 - x**2)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{-x^2+1}$$

input
```
integrate(arcsin(x),x, algorithm="maxima")
```

output
```
x*arcsin(x) + sqrt(-x^2 + 1)
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{-x^2+1}$$

input
```
integrate(arcsin(x),x, algorithm="giac")
```

output
```
x*arcsin(x) + sqrt(-x^2 + 1)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\,\mathrm{asin}(x) + \sqrt{1-x^2}$$

input
```
int(asin(x),x)
```

output
```
x*asin(x) + (1 - x^2)^(1/2)
```

3.263.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \arcsin(x)\,dx = asin(x)\,x + \sqrt{-x^2 + 1}$$

input
```
int(asin(x),x)
```

output
```
asin(x)*x + sqrt( - x**2 + 1)
```

3.264 $\int x^2 \arcsin(x)\,dx$

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3.264.1 Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^2 \arcsin(x)\,dx = \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3 \arcsin(x)$$

output
```
-1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int x^2 \arcsin(x)\,dx = \frac{1}{9}\left(\sqrt{1-x^2}\left(2+x^2\right) + 3x^3 \arcsin(x)\right)$$

input
```
Integrate[x^2*ArcSin[x],x]
```

output
```
(Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

3.264.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5138, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arcsin(x)\,dx$$

$$\Big\downarrow \text{5138}$$

$$\frac{1}{3}x^3\arcsin(x)-\frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3\arcsin(x)-\frac{1}{6}\int \frac{x^2}{\sqrt{1-x^2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}x^3\arcsin(x)-\frac{1}{6}\int \left(\frac{1}{\sqrt{1-x^2}}-\sqrt{1-x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\arcsin(x)+\frac{1}{6}\left(2\sqrt{1-x^2}-\frac{2}{3}\left(1-x^2\right)^{3/2}\right)$$

input
```
Int[x^2*ArcSin[x],x]
```

output
```
(2*Sqrt[1 - x^2] - (2*(1 - x^2)^(3/2))/3)/6 + (x^3*ArcSin[x])/3
```

3.264.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.264.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^3 \arcsin(x)}{3}+\frac{x^2\sqrt{-x^2+1}}{9}+\frac{2\sqrt{-x^2+1}}{9}$	34
parts	$\frac{x^3 \arcsin(x)}{3}+\frac{x^2\sqrt{-x^2+1}}{9}+\frac{2\sqrt{-x^2+1}}{9}$	34

input
```
int(x^2*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,x^3 \arcsin{(x)} + \frac{1}{9}\,(x^2+2)\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x),x, algorithm="fricas")
```

output
```
1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

3.264.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^2 \arcsin(x)\,dx = \frac{x^3 \operatorname{asin}{(x)}}{3} + \frac{x^2\sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

input
```
integrate(x**2*asin(x),x)
```

output
```
x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,x^3 \arcsin{(x)} + \frac{1}{9}\,\sqrt{-x^2+1}x^2 + \frac{2}{9}\,\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x),x, algorithm="maxima")
```

output
```
1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\left(x^2-1\right)x\arcsin\left(x\right) + \frac{1}{3}\,x\arcsin\left(x\right) - \frac{1}{9}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x),x, algorithm="giac")
```

output
```
1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*s
qrt(-x^2 + 1)
```

3.264.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x)\,dx = \frac{x^3\,\mathrm{asin}(x)}{3} + \frac{\sqrt{1-x^2}\,\left(x^2+2\right)}{9}$$

input
```
int(x^2*asin(x),x)
```

output
```
(x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.264.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int x^2 \arcsin(x)\,dx = \frac{asin(x)\,x^3}{3} + \frac{\sqrt{-x^2+1}\,x^2}{9} + \frac{2\sqrt{-x^2+1}}{9}$$

input
```
int(asin(x)*x**2,x)
```

output
```
(3*asin(x)*x**3 + sqrt( - x**2 + 1)*x**2 + 2*sqrt( - x**2 + 1))/9
```

3.265 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

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3.265.9 Mupad [B] (verification not implemented) 1560
3.265.10Reduce [B] (verification not implemented) 1560

3.265.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -\log(\cos(x)-\sin(x))+\log(2\cos(x)-\sin(x))$$

output
```
-ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))
```

3.265.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = 2\left(-\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{1}{2}\log(2\cos(x)-\sin(x))\right)$$

input
```
Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

output
```
2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

3.265.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4889, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{-3\tan(x)+\sec^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{-3\tan(x)+\sec(x)^2+1}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{\tan^2(x)-3\tan(x)+2}d\tan(x)$$

$$\Big\downarrow \text{1081}$$

$$\int \left(\frac{1}{\tan(x)-2}+\frac{1}{1-\tan(x)}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\log(2-\tan(x))-\log(1-\tan(x))$$

input
```
Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

output
```
-Log[1 - Tan[x]] + Log[2 - Tan[x]]
```

3.265.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.265.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$\ln(\tan(x)-2)-\ln(\tan(x)-1)$	14
risch	$\ln\left(\mathrm{e}^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)-\ln\left(\mathrm{e}^{2ix}-i\right)$	23

input
```
int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(x)-2)-ln(tan(x)-1)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = \frac{1}{2}\log\left(\frac{3}{4}\cos(x)^2-\cos(x)\sin(x)+\frac{1}{4}\right) - \frac{1}{2}\log(-2\cos(x)\sin(x)+1)$$

input
```
integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

output
```
1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1
)
```

3.265.6 Sympy [F]

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = \int \frac{\sec^2(x)}{-3\tan(x)+\sec^2(x)+1}\,dx$$

input
```
integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

output
```
Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -\log(\tan(x)-1)+\log(\tan(x)-2)$$

input
```
integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

output
```
-log(tan(x) - 1) + log(tan(x) - 2)
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -\log\left(|\tan\left(x\right)-1|\right)+\log\left(|\tan\left(x\right)-2|\right)$$

input
```
integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
```

output
```
-log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

3.265.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -2\,\mathrm{atanh}(2\tan(x)-3)$$

input
```
int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)
```

output
```
-2*atanh(2*tan(x) - 3)
```

3.265.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = \log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)-\log\left(-\sqrt{2}+\tan\left(\frac{x}{2}\right)+1\right) + \log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)+1\right)-\log\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
int(sec(x)**2/(sec(x)**2 - 3*tan(x) + 1),x)
```

output
```
log( - sqrt(5) + 2*tan(x/2) + 1) - log( - sqrt(2) + tan(x/2) + 1) + log(sq
rt(5) + 2*tan(x/2) + 1) - log(sqrt(2) + tan(x/2) + 1)
```

3.266 $\int \cos^2(x)\,dx$

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3.266.10Reduce [B] (verification not implemented) 1565

3.266.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.266.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

3.266.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x) \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \ 3115$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \ 24$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.266.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.266.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin\left(2x\right)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin\left(2x\right)}{4}$	11
norman	$\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	45

input
```
int(1/sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,\cos\left(x\right)\sin\left(x\right) + \frac{1}{2}\,x$$

input
```
integrate(1/sec(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

3.266.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(1/sec(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(1/sec(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{\tan(x)}{2\,(\tan(x)^2 + 1)}$$

input
```
integrate(1/sec(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

3.266.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

input
```
int(1/sec(x)**2,x)
```

output
```
(cos(x)*sin(x) + x)/2
```

3.267 $\int \frac{-2-3x+5x^2}{(-2+x)x^2} \, dx$

3.267.1 Optimal result 1566
3.267.2 Mathematica [A] (verified) 1566
3.267.3 Rubi [A] (verified) 1567
3.267.4 Maple [A] (verified) 1568
3.267.5 Fricas [A] (verification not implemented) 1568
3.267.6 Sympy [A] (verification not implemented) 1568
3.267.7 Maxima [A] (verification not implemented) 1569
3.267.8 Giac [A] (verification not implemented) 1569
3.267.9 Mupad [B] (verification not implemented) 1569
3.267.10 Reduce [B] (verification not implemented) 1570

3.267.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2} \, dx = -\frac{1}{x} + 3\log(2-x) + 2\log(x)$$

output
```
-1/x+3*ln(2-x)+2*ln(x)
```

3.267.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2} \, dx = -\frac{1}{x} + 3\log(2-x) + 2\log(x)$$

input
```
Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

output
```
-x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

3.267.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2-3x-2}{(x-2)x^2}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{1}{x^2}+\frac{2}{x}+\frac{3}{x-2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

input
```
Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

output
```
-x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

3.267.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.267.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{x}+2\ln\left(x\right)+3\ln\left(-2+x\right)$	17
norman	$-\frac{1}{x}+2\ln\left(x\right)+3\ln\left(-2+x\right)$	17
risch	$-\frac{1}{x}+2\ln\left(x\right)+3\ln\left(-2+x\right)$	17
parallelrisch	$\frac{2x\ln(x)+3\ln(-2+x)x-1}{x}$	19
meijerg	$-\frac{1}{x}+2\ln\left(x\right)-2\ln\left(2\right)+2i\pi+3\ln\left(1-\frac{x}{2}\right)$	27

input
```
int((5*x^2-3*x-2)/(-2+x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+2*ln(x)+3*ln(-2+x)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = \frac{3\,x\log\left(x-2\right)+2\,x\log\left(x\right)-1}{x}$$

input
```
integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")
```

output
```
(3*x*log(x - 2) + 2*x*log(x) - 1)/x
```

3.267.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = 2\log\left(x\right)+3\log\left(x-2\right)-\frac{1}{x}$$

input
```
integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)
```

output
```
2*log(x) + 3*log(x - 2) - 1/x
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = -\frac{1}{x}+3\,\log\left(x-2\right)+2\,\log\left(x\right)$$

input
```
integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")
```

output
```
-1/x + 3*log(x - 2) + 2*log(x)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = -\frac{1}{x}+3\,\log\left(|x-2|\right)+2\,\log\left(|x|\right)$$

input
```
integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="giac")
```

output
```
-1/x + 3*log(abs(x - 2)) + 2*log(abs(x))
```

3.267.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = 3\,\ln\left(x-2\right)+2\,\ln\left(x\right)-\frac{1}{x}$$

input
```
int(-(3*x - 5*x^2 + 2)/(x^2*(x - 2)),x)
```

output
```
3*log(x - 2) + 2*log(x) - 1/x
```

3.267.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = \frac{3\log(x-2)\,x+2\log(x)\,x-1}{x}$$

input
```
int((5*x**2 - 3*x - 2)/(x**2*(x - 2)),x)
```

output
```
(3*log(x - 2)*x + 2*log(x)*x - 1)/x
```

3.268 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

3.268.1 Optimal result 1571
3.268.2 Mathematica [A] (verified) 1571
3.268.3 Rubi [A] (verified) 1572
3.268.4 Maple [A] (verified) 1572
3.268.5 Fricas [B] (verification not implemented) 1573
3.268.6 Sympy [A] (verification not implemented) 1573
3.268.7 Maxima [A] (verification not implemented) 1574
3.268.8 Giac [B] (verification not implemented) 1574
3.268.9 Mupad [B] (verification not implemented) 1574
3.268.10 Reduce [B] (verification not implemented) 1575

3.268.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\text{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
1/2*arcsinh(2/3*x)
```

3.268.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = -\frac{1}{2}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[1/Sqrt[9 + 4*x^2],x]
```

output
```
-1/2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

3.268.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{222\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2+9}} \, dx$$
$$\Big\downarrow \; 222$$
$$\frac{1}{2}\text{arcsinh}\left(\frac{2x}{3}\right)$$

input
```
Int[1/Sqrt[9 + 4*x^2],x]
```

output
```
ArcSinh[(2*x)/3]/2
```

3.268.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.268.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\text{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\text{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
trager	$-\frac{\ln\left(2x-\sqrt{4x^2+9}\right)}{2}$	19
pseudoelliptic	$-\frac{\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)}{4}+\frac{\ln\left(\frac{\sqrt{4x^2+9}+2x}{x}\right)}{4}$	42

input
```
int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(2/3*x)
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = -\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 + 9))
```

3.268.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

input
```
integrate(1/(4*x**2+9)**(1/2),x)
```

output
```
asinh(2*x/3)/2
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsinh(2/3*x)
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\,\sqrt{4\,x^2+9}x - \frac{9}{4}\,\log\left(-2\,x + \sqrt{4\,x^2+9}\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

3.268.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{2}$$

input
```
int(1/(4*x^2 + 9)^(1/2),x)
```

output
```
asinh((2*x)/3)/2
```

3.268.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{2}$$

input
```
int(1/sqrt(4*x**2 + 9),x)
```

output
```
log((sqrt(4*x**2 + 9) + 2*x)/3)/2
```

3.269 $\int \frac{1}{\sqrt{4+x^2}} dx$

3.269.1 Optimal result 1576
3.269.2 Mathematica [B] (verified) 1576
3.269.3 Rubi [A] (verified) 1577
3.269.4 Maple [A] (verified) 1577
3.269.5 Fricas [B] (verification not implemented) 1578
3.269.6 Sympy [A] (verification not implemented) 1578
3.269.7 Maxima [A] (verification not implemented) 1579
3.269.8 Giac [B] (verification not implemented) 1579
3.269.9 Mupad [B] (verification not implemented) 1579
3.269.10 Reduce [B] (verification not implemented) 1580

3.269.1 Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{1}{\sqrt{4+x^2}} dx = \operatorname{arcsinh}\left(\frac{x}{2}\right)$$

output
```
arcsinh(1/2*x)
```

3.269.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{4+x^2}} dx = -\log\left(-x+\sqrt{4+x^2}\right)$$

input
```
Integrate[1/Sqrt[4 + x^2],x]
```

output
```
-Log[-x + Sqrt[4 + x^2]]
```

3.269.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+4}}dx$$
$$\Big\downarrow \text{222}$$
$$\text{arcsinh}\left(\frac{x}{2}\right)$$

input
```
Int[1/Sqrt[4 + x^2],x]
```

output
```
ArcSinh[x/2]
```

3.269.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.269.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\text{arcsinh}\left(\frac{x}{2}\right)$	5
meijerg	$\text{arcsinh}\left(\frac{x}{2}\right)$	5
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^2+4}}{x}\right)$	13
trager	$-\ln\left(x-\sqrt{x^2+4}\right)$	15

input
```
int(1/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(1/2*x)
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+4}\right)$$

input
```
integrate(1/(x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 4))
```

3.269.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{2}\right)$$

input
```
integrate(1/(x**2+4)**(1/2),x)
```

output
```
asinh(x/2)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{2}\,x\right)$$

input
```
integrate(1/(x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/2*x)
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+4}x - 2\,\log\left(-x+\sqrt{x^2+4}\right)$$

input
```
integrate(1/(x^2+4)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 4)*x - 2*log(-x + sqrt(x^2 + 4))
```

3.269.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{2}\right)$$

input
```
int(1/(x^2 + 4)^(1/2),x)
```

output
```
asinh(x/2)
```

3.269.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \log\left(\frac{\sqrt{x^2+4}}{2}+\frac{x}{2}\right)$$

input
```
int(1/sqrt(x**2 + 4),x)
```

output
```
log((sqrt(x**2 + 4) + x)/2)
```

3.270 $\int \frac{1}{10-12x+9x^2}\,dx$

3.270.1 Optimal result 1581
3.270.2 Mathematica [A] (verified) 1581
3.270.3 Rubi [A] (verified) 1582
3.270.4 Maple [A] (verified) 1583
3.270.5 Fricas [A] (verification not implemented) 1583
3.270.6 Sympy [A] (verification not implemented) 1583
3.270.7 Maxima [A] (verification not implemented) 1584
3.270.8 Giac [A] (verification not implemented) 1584
3.270.9 Mupad [B] (verification not implemented) 1584
3.270.10Reduce [B] (verification not implemented) 1585

3.270.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{10-12x+9x^2}\,dx = -\frac{\arctan\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

output
```
-1/18*arctan(1/6*(2-3*x)*6^(1/2))*6^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{\arctan\left(\frac{-2+3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

input
```
Integrate[(10 - 12*x + 9*x^2)^(-1),x]
```

output
```
ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

3.270.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{9x^2-12x+10}\,dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(18x-12)^2-216}d(18x-12)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{18x-12}{6\sqrt{6}}\right)}{3\sqrt{6}}$$

input
```
Int[(10 - 12*x + 9*x^2)^(-1),x]
```

output
```
ArcTan[(-12 + 18*x)/(6*Sqrt[6])]/(3*Sqrt[6])
```

3.270.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.270.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{6}\,\arctan\left(\frac{(18x-12)\sqrt{6}}{36}\right)}{18}$	17
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{(-2+3x)\sqrt{6}}{6}\right)}{18}$	17

input
```
int(1/(9*x^2-12*x+10),x,method=_RETURNVERBOSE)
```

output
```
1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{1}{18}\,\sqrt{6}\arctan\left(\frac{1}{6}\,\sqrt{6}(3\,x-2)\right)$$

input
```
integrate(1/(9*x^2-12*x+10),x, algorithm="fricas")
```

output
```
1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

3.270.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}-\frac{\sqrt{6}}{3}\right)}{18}$$

input
```
integrate(1/(9*x**2-12*x+10),x)
```

output
```
sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

input
```
integrate(1/(9*x^2-12*x+10),x, algorithm="maxima")
```

output
```
1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

input
```
integrate(1/(9*x^2-12*x+10),x, algorithm="giac")
```

output
```
1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

3.270.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,(3x-2)}{6}\right)}{18}$$

input
```
int(1/(9*x^2 - 12*x + 10),x)
```

output
```
(6^(1/2)*atan((6^(1/2)*(3*x - 2))/6))/18
```

3.270.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{10-12x+9x^2}\,dx = \frac{\sqrt{6}\,atan\left(\frac{3x-2}{\sqrt{6}}\right)}{18}$$

input
```
int(1/(9*x**2 - 12*x + 10),x)
```

output
```
(sqrt(6)*atan((3*x - 2)/sqrt(6)))/18
```

3.271 $\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx$

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3.271.1 Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = \frac{1}{2(1-x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{4}\log\left(1+x^2\right)$$

output
```
1/2/(1-x)-1/3/x^3-1/x^2-2/x-5/2*ln(1-x)+2*ln(x)+1/4*ln(x^2+1)
```

3.271.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = -\frac{1}{2(-1+x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

output
```
-1/2*1/(-1 + x) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] +
 Log[1 + x^2]/4
```

3.271.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2026, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-2x^7+2x^6-2x^5+x^4}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^4(x^4-2x^3+2x^2-2x+1)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1}{x^4}+\frac{2}{x^3}+\frac{x}{2(x^2+1)}+\frac{2}{x^2}-\frac{5}{2(x-1)}+\frac{1}{2(x-1)^2}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log(x^2+1)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

input
```
Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

output
```
1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + L
og[1 + x^2]/4
```

3.271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.271.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2(-1+x)}-\frac{5\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}+2\ln(x)$	42
norman	$\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)}+2\ln(x)-\frac{5\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}$	42
risch	$\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)}+2\ln(x)-\frac{5\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}$	42
parallelrisch	$\frac{24x^4\ln(x)-30\ln(-1+x)x^4+3\ln(x^2+1)x^4+4-24x^3\ln(x)+30\ln(-1+x)x^3-3\ln(x^2+1)x^3-30x^3+12x^2+8x}{12x^3(-1+x)}$	80

input
```
int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+x)-5/2*ln(-1+x)+1/4*ln(x^2+1)-1/3/x^3-1/x^2-2/x+2*ln(x)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int\frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = -\frac{30\,x^3-12\,x^2-3\,(x^4-x^3)\log(x^2+1)+30\,(x^4-x^3)\log(x-1)-24\,(x^4-x^3)\log(x)-8\,x-4}{12\,(x^4-x^3)}$$

input
```
integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="fricas")
```

output
```
-1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x^3)*log(x
 - 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

3.271.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = 2\log(x) - \frac{5\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{-15x^3+6x^2+4x+2}{6x^4-6x^3}$$

input
```
integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)
```

output
```
2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 + (-15*x**3 + 6*x**2 + 4*x + 2
)/(6*x**4 - 6*x**3)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = -\frac{15\,x^3-6\,x^2-4\,x-2}{6\,(x^4-x^3)} + \frac{1}{4}\,\log\left(x^2+1\right) - \frac{5}{2}\,\log\left(x-1\right) + 2\,\log\left(x\right)$$

input
```
integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="maxima")
```

output
```
-1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) - 5/2*log(x
 - 1) + 2*log(x)
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = -\frac{15\,x^3-6\,x^2-4\,x-2}{6\,(x-1)x^3} + \frac{1}{4}\,\log\left(x^2+1\right) - \frac{5}{2}\,\log\left(|x-1|\right) + 2\,\log\left(|x|\right)$$

input
```
integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="giac")
```

output
```
-1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*log(x^2 + 1) - 5/2*log
(abs(x - 1)) + 2*log(abs(x))
```

3.271.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = \frac{\ln\left(x^2+1\right)}{4} - \frac{5\,\ln\left(x-1\right)}{2} + 2\,\ln\left(x\right) - \frac{-\frac{5\,x^3}{2}+x^2+\frac{2\,x}{3}+\frac{1}{3}}{x^3-x^4}$$

input
```
int(1/(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8),x)
```

output
```
log(x^2 + 1)/4 - (5*log(x - 1))/2 + 2*log(x) - ((2*x)/3 + x^2 - (5*x^3)/2
+ 1/3)/(x^3 - x^4)
```

3.271.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = \frac{3\log(x^2+1)\,x^4-3\log(x^2+1)\,x^3-30\log(x-1)\,x^4+30\log(x-1)\,x^3+24\log(x)\,x^4-24\log(x)\,x^3-}{12x^3\,(x-1)}$$

input
```
int(1/(x**4*(x**4 - 2*x**3 + 2*x**2 - 2*x + 1)),x)
```

output
```
(3*log(x**2 + 1)*x**4 - 3*log(x**2 + 1)*x**3 - 30*log(x - 1)*x**4 + 30*log
(x - 1)*x**3 + 24*log(x)*x**4 - 24*log(x)*x**3 - 30*x**4 + 12*x**2 + 8*x +
 4)/(12*x**3*(x - 1))
```

3.272 $\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx$

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3.272.9 Mupad [B] (verification not implemented) 1595
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3.272.1 Optimal result

Integrand size = 29, antiderivative size = 49

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx = ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x) - \frac{1}{3}d\log(x)-\frac{1}{4}(a-b+c-d)\log(1+x)$$

output
```
a*x+1/12*(27*a+9*b+3*c+d)*ln(3-x)-1/3*d*ln(x)-1/4*(a-b+c-d)*ln(1+x)
```

3.272.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx = ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x) - \frac{1}{3}d\log(x)+\frac{1}{4}(-a+b-c+d)\log(1+x)$$

input
```
Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

output
```
a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-a + b - c
 + d)*Log[1 + x])/4
```

3.272.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^3+bx^2+cx+d}{(x-3)x(x+1)} dx$$
$$\Big\downarrow \text{2115}$$
$$\int \left(\frac{27a+9b+3c+d}{12(x-3)}+\frac{-a+b-c+d}{4(x+1)}+a-\frac{d}{3x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

input
```
Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

output
```
a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c
- d)*Log[1 + x])/4
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.272.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result
default	$ax-\frac{d\ln(x)}{3}+\left(-\frac{a}{4}+\frac{b}{4}-\frac{c}{4}+\frac{d}{4}\right)\ln(1+x)+\left(\frac{9a}{4}+\frac{3b}{4}+\frac{c}{4}+\frac{d}{12}\right)\ln(-3+x)$
norman	$ax-\frac{d\ln(x)}{3}+\left(-\frac{a}{4}+\frac{b}{4}-\frac{c}{4}+\frac{d}{4}\right)\ln(1+x)+\left(\frac{9a}{4}+\frac{3b}{4}+\frac{c}{4}+\frac{d}{12}\right)\ln(-3+x)$
parallelrisch	$ax-\frac{d\ln(x)}{3}-\frac{\ln(1+x)a}{4}+\frac{\ln(1+x)b}{4}-\frac{\ln(1+x)c}{4}+\frac{\ln(1+x)d}{4}+\frac{9\ln(-3+x)a}{4}+\frac{3\ln(-3+x)b}{4}+\frac{\ln(-3+x)c}{4}+$
risch	$ax+\frac{9\ln(3-x)a}{4}+\frac{3\ln(3-x)b}{4}+\frac{\ln(3-x)c}{4}+\frac{\ln(3-x)d}{12}-\frac{d\ln(x)}{3}-\frac{\ln(-1-x)a}{4}+\frac{\ln(-1-x)b}{4}-\frac{\ln(-1-x)c}{4}+$

input
```
int((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x,method=_RETURNVERBOSE)
```

output
```
a*x-1/3*d*ln(x)+(-1/4*a+1/4*b-1/4*c+1/4*d)*ln(1+x)+(9/4*a+3/4*b+1/4*c+1/12
*d)*ln(-3+x)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx = ax-\frac{1}{4}(a-b+c-d)\log(x+1) + \frac{1}{12}(27\,a+9\,b+3\,c+d)\log(x-3)-\frac{1}{3}\,d\log(x)$$

input
```
integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="fricas")
```

output
```
a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
 3) - 1/3*d*log(x)
```

3.272.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(49) = 98$.

Time = 55.06 (sec) , antiderivative size = 762, normalized size of antiderivative = 15.55

$$\int \frac{d + cx + bx^2 + ax^3}{(-3 + x)x(1 + x)}\,dx = ax - \frac{d\log(x)}{3} - \frac{(a-b+c-d)\log\left(x + \frac{-1512a^2d+1134a^2(a-b+c-d)-864abd+648ab(a-b+c-d)-432acd+324ac(a-b+c-d)-144ad^2+81a(a-}{1215a^3-567a^2b+1593a^2c-2691a^2}\right.}{} + \frac{(27a+9b+3c+d)\log\left(x + \frac{-1512a^2d-378a^2\cdot(27a+9b+3c+d)-864abd-216ab(27a+9b+3c+d)-432acd-108ac(27a+9b+3c+}{12}\right.}{}$$

input
```
integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

output
```
a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a**2*(a -
b + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a*c*d + 324*a*c*(a
- b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d)**2 - 216*b**2*d + 162*b**
2*(a - b + c - d) - 288*b*d**2 + 108*b*d*(a - b + c - d) + 81*b*(a - b + c
 - d)**2 - 72*c**2*d + 54*c**2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a -
b + c - d) - 27*c*(a - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d)
+ 117*d*(a - b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a
**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351
*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*
d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/4 + (27*a + 9*b + 3*c +
 d)*log(x + (-1512*a**2*d - 378*a**2*(27*a + 9*b + 3*c + d) - 864*a*b*d -
216*a*b*(27*a + 9*b + 3*c + d) - 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d
) - 144*a*d**2 + 9*a*(27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*
a + 9*b + 3*c + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*
a + 9*b + 3*c + d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) + 144*c
*d**2 + 24*c*d*(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*c + d)**2 - 13
6*d**3 + 18*d**2*(27*a + 9*b + 3*c + d) + 13*d*(27*a + 9*b + 3*c + d)**2)/
(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b
*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*
c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*...
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx = ax - \frac{1}{4}\,(a-b+c-d)\log\left(x+1\right) + \frac{1}{12}\,(27\,a+9\,b+3\,c+d)\log\left(x-3\right) - \frac{1}{3}\,d\log\left(x\right)$$

input
```
integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="maxima")
```

output
```
a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
 3) - 1/3*d*log(x)
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx = ax - \frac{1}{4}\,(a-b+c-d)\log\left(|x+1|\right) + \frac{1}{12}\,(27\,a+9\,b+3\,c+d)\log\left(|x-3|\right) - \frac{1}{3}\,d\log\left(|x|\right)$$

input
```
integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="giac")
```

output
```
a*x - 1/4*(a - b + c - d)*log(abs(x + 1)) + 1/12*(27*a + 9*b + 3*c + d)*lo
g(abs(x - 3)) - 1/3*d*log(abs(x))
```

3.272.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx = \ln\left(x-3\right)\left(\frac{9\,a}{4}+\frac{3\,b}{4}+\frac{c}{4}+\frac{d}{12}\right) - \ln\left(x+1\right)\left(\frac{a}{4}-\frac{b}{4}+\frac{c}{4}-\frac{d}{4}\right) + a\,x - \frac{d\,\ln\left(x\right)}{3}$$

input
```
int((d + c*x + a*x^3 + b*x^2)/(x*(x + 1)*(x - 3)),x)
```

output
```
log(x - 3)*((9*a)/4 + (3*b)/4 + c/4 + d/12) - log(x + 1)*(a/4 - b/4 + c/4
- d/4) + a*x - (d*log(x))/3
```

3.272.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx = \frac{9\log(x-3)\,a}{4}+\frac{3\log(x-3)\,b}{4}+\frac{\log(x-3)\,c}{4}+\frac{\log(x-3)\,d}{12}-\frac{\log(x+1)\,a}{4}+\frac{\log(x+1)\,b}{4}-\frac{\log(x+1)\,c}{4}+\frac{\log(x+1)\,d}{4}-\frac{\log(x)\,d}{3}+ax$$

input
```
int((a*x**3 + b*x**2 + c*x + d)/(x*(x**2 - 2*x - 3)),x)
```

output
```
(27*log(x - 3)*a + 9*log(x - 3)*b + 3*log(x - 3)*c + log(x - 3)*d - 3*log(
x + 1)*a + 3*log(x + 1)*b - 3*log(x + 1)*c + 3*log(x + 1)*d - 4*log(x)*d +
 12*a*x)/12
```

3.273 $\int \frac{1}{(2-\log(1+x^2))^5} dx$

3.273.1 Optimal result . . . 1597
3.273.2 Mathematica [N/A] . . . 1597
3.273.3 Rubi [N/A] . . . 1598
3.273.4 Maple [N/A] . . . 1598
3.273.5 Fricas [N/A] . . . 1599
3.273.6 Sympy [N/A] . . . 1599
3.273.7 Maxima [N/A] . . . 1600
3.273.8 Giac [N/A] . . . 1600
3.273.9 Mupad [N/A] . . . 1601
3.273.10Reduce [F] . . . 1601

3.273.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{(2-\log(1+x^2))^5} dx = \text{Int}\left(\frac{1}{(2-\log(1+x^2))^5}, x\right)$$

output
```
Unintegrable(1/(2-ln(x^2+1))^5,x)
```

3.273.2 Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{(2-\log(1+x^2))^5} dx = \int \frac{1}{(2-\log(1+x^2))^5} dx$$

input
```
Integrate[(2 - Log[1 + x^2])^(-5),x]
```

output
```
Integrate[(2 - Log[1 + x^2])^(-5), x]
```

3.273.3 Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-\log(x^2+1))^5} \, dx$$
$$\Big\downarrow \text{ 2902}$$
$$\int \frac{1}{(2-\log(x^2+1))^5}dx$$

input
```
Int[(2 - Log[1 + x^2])^(-5),x]
```

output
```
$Aborted
```

3.273.3.1 Defintions of rubi rules used

rule 2902
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Unintegrable[(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d,
 e, n, p, q}, x]
```

3.273.4 Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-\ln(x^2+1))^5}dx$$

input
```
int(1/(2-ln(x^2+1))^5,x)
```

output
```
int(1/(2-ln(x^2+1))^5,x)
```

3.273.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.50

$$\int \frac{1}{(2-\log(1+x^2))^5}\,dx = \int -\frac{1}{(\log(x^2+1)-2)^5}\,dx$$

input
```
integrate(1/(2-log(x^2+1))^5,x, algorithm="fricas")
```

output
```
integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)^3 - 80*l
og(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)
```

3.273.6 Sympy [N/A]

Not integrable

Time = 7.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 22.50

$$\int \frac{1}{(2-\log(1+x^2))^5}\,dx = -\frac{\int \frac{120x^2}{x^8\log(x^2+1)-2x^8}\,dx + \int \frac{30x^4}{x^8\log(x^2+1)-2x^8}\,dx + \int \frac{x^8}{x^8\log(x^2+1)-2x^8}\,dx + \int \frac{105}{x^8\log(x^2+1)-2x^8}\,dx}{384} + \frac{\frac{2x^8}{3}+\frac{7x^6}{6}+\frac{5x^4}{2}+\frac{9x^2}{2}+\left(\frac{x^8}{48}-\frac{5x^4}{24}-\frac{x^2}{2}-\frac{5}{16}\right)\log(x^2+1)^3+\left(-\frac{x^8}{12}+\frac{x^6}{24}+\frac{11x^4}{8}+\frac{25x^2}{8}+\frac{15}{8}\right)\log(x^2}{8x^7\log(x^2+1)^4-64x^7\log(x^2+1)^3+192x^7\log(x^2+1)^2-256x^7}$$

input
```
integrate(1/(2-ln(x**2+1))**5,x)
```

output
```
-(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(30*x**4/(
x**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*log(x**2 + 1) - 2*x
**8), x) + Integral(105/(x**8*log(x**2 + 1) - 2*x**8), x))/384 + (2*x**8/3
 + 7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x**8/48 - 5*x**4/24 - x**2/2 - 5/16)*
log(x**2 + 1)**3 + (-x**8/12 + x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log
(x**2 + 1)**2 + (x**8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2
+ 1) + 5/2)/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7
*log(x**2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)
```

3.273.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 17.25

$$\int \frac{1}{(2-\log(1+x^2))^5}\,dx = \int -\frac{1}{(\log(x^2+1)-2)^5}\,dx$$

input
```
integrate(1/(2-log(x^2+1))^5,x, algorithm="maxima")
```

output
```
1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*log(x^2 +
1)^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 + 1)^2 + 216*x^2 + 4
*(3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2 + 1) + 120)/(x^7*log(x^2 +
 1)^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1)
 + 16*x^7) - integrate(1/384*(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 +
 1) - 2*x^8), x)
```

3.273.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{(2-\log(1+x^2))^5}\,dx = \int -\frac{1}{(\log(x^2+1)-2)^5}\,dx$$

input
```
integrate(1/(2-log(x^2+1))^5,x, algorithm="giac")
```

output
```
integrate(-1/(log(x^2 + 1) - 2)^5, x)
```

3.273.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx = \int -\frac{1}{\left(\ln\left(x^2+1\right)-2\right)^5}\,dx$$

input
```
int(-1/(log(x^2 + 1) - 2)^5,x)
```

output
```
int(-1/(log(x^2 + 1) - 2)^5, x)
```

3.273.10 Reduce [F]

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx = -\left(\int \frac{1}{\log\left(x^2+1\right)^5-10\log\left(x^2+1\right)^4+40\log\left(x^2+1\right)^3-80\log\left(x^2+1\right)^2+80\log\left(x^2+1\right)-32}dx\right)$$

input
```
int(( - 1)/(log(x**2 + 1)**5 - 10*log(x**2 + 1)**4 + 40*log(x**2 + 1)**3 -
 80*log(x**2 + 1)**2 + 80*log(x**2 + 1) - 32),x)
```

output
```
 - int(1/(log(x**2 + 1)**5 - 10*log(x**2 + 1)**4 + 40*log(x**2 + 1)**3 - 8
0*log(x**2 + 1)**2 + 80*log(x**2 + 1) - 32),x)
```

3.274 $\int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx$

3.274.1 Optimal result . . . 1602
3.274.2 Mathematica [A] (verified) . . . 1602
3.274.3 Rubi [A] (verified) . . . 1603
3.274.4 Maple [A] (verified) . . . 1604
3.274.5 Fricas [A] (verification not implemented) . . . 1604
3.274.6 Sympy [A] (verification not implemented) . . . 1605
3.274.7 Maxima [A] (verification not implemented) . . . 1605
3.274.8 Giac [A] (verification not implemented) . . . 1606
3.274.9 Mupad [B] (verification not implemented) . . . 1606
3.274.10 Reduce [B] (verification not implemented) . . . 1607

3.274.1 Optimal result

Integrand size = 54, antiderivative size = 28

$$\int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx = e^{x^2}\log(x) - \frac{\log(x)}{x+\log^2(x)} + \log\left(x+\log^2(x)\right)$$

output
```
exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

3.274.2 Mathematica [A] (verified)

Time = 37.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx = e^{x^2}\log(x) - \frac{\log(x)}{x+\log^2(x)} + \log\left(x+\log^2(x)\right)$$

input
```
Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1
 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

output
```
E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

3.274.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{\log(x)-2}{\left(x+\log^2(x)\right)^2}+\frac{\frac{1}{x}+\frac{2\log(x)}{x}+1}{x+\log^2(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

input
```
Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(
-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

output
```
E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.274.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result	size
default	$\mathrm{e}^{x^2}\ln(x)-\frac{\ln(x)}{x+\ln(x)^2}+\ln\left(x+\ln(x)^2\right)$	28
risch	$\mathrm{e}^{x^2}\ln(x)-\frac{\ln(x)}{x+\ln(x)^2}+\ln\left(x+\ln(x)^2\right)$	28
parts	$\mathrm{e}^{x^2}\ln(x)-\frac{\ln(x)}{x+\ln(x)^2}+\ln\left(x+\ln(x)^2\right)$	28
parallelrisch	$\frac{\mathrm{e}^{x^2}\ln(x)^3+\mathrm{e}^{x^2}x\ln(x)+\ln(x)^2\ln\left(x+\ln(x)^2\right)+\ln\left(x+\ln(x)^2\right)x-\ln(x)}{x+\ln(x)^2}$	53

input
```
int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)^2+(1+1/x+2*ln(x)/
x)/(x+ln(x)^2),x,method=_RETURNVERBOSE)
```

output
```
exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int\left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx =\frac{e^{(x^2)}\log(x)^3+\left(\log(x)^2+x\right)\log\left(\log(x)^2+x\right)+\left(xe^{(x^2)}-1\right)\log(x)}{\log(x)^2+x}$$

input
```
integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x
+2*log(x)/x)/(x+log(x)^2),x, algorithm="fricas")
```

output
```
(e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2) - 1)*log
(x))/(log(x)^2 + x)
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right) dx$$
$$= e^{x^2}\log(x) + \log\left(x+\log(x)^2\right) - \frac{\log(x)}{x+\log(x)^2}$$

input
```
integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/
x+2*ln(x)/x)/(x+ln(x)**2),x)
```

output
```
exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right) dx$$
$$= e^{(x^2)}\log(x) - \frac{\log(x)}{\log(x)^2+x} + \log\left(\log(x)^2+x\right)$$

input
```
integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x
+2*log(x)/x)/(x+log(x)^2),x, algorithm="maxima")
```

output
```
e^(x^2)*log(x) - log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx$$
$$=e^{(x^2)}\log\left(x\right)-\frac{3\log\left(x\right)}{\log\left(x\right)^2+x}+\log\left(\log\left(x\right)^2+x\right)$$

input
```
integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x
+2*log(x)/x)/(x+log(x)^2),x, algorithm="giac")
```

output
```
e^(x^2)*log(x) - 3*log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx$$
$$=\ln\left(\ln\left(x\right)^2+x\right)+\mathrm{e}^{x^2}\ln\left(x\right)-\frac{\ln\left(x\right)}{\ln\left(x\right)^2+x}$$

input
```
int((log(x) - 2)/(x + log(x)^2)^2 + ((2*log(x))/x + 1/x + 1)/(x + log(x)^2
) + exp(x^2)/x + 2*x*exp(x^2)*log(x),x)
```

output
```
log(x + log(x)^2) + exp(x^2)*log(x) - log(x)/(x + log(x)^2)
```

3.274.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx$$
$$=\frac{e^{x^2}\log(x)^3+e^{x^2}\log(x)\,x+\log\left(\log(x)^2+x\right)\log(x)^2+\log\left(\log(x)^2+x\right)x-\log(x)}{\log(x)^2+x}$$

input
```
int((2*e**(x**2)*log(x)**5*x**2 + e**(x**2)*log(x)**4 + 4*e**(x**2)*log(x)
**3*x**3 + 2*e**(x**2)*log(x)**2*x + 2*e**(x**2)*log(x)*x**4 + e**(x**2)*x
**2 + 2*log(x)**3 + log(x)**2*x + log(x)**2 + 3*log(x)*x + x**2 - x)/(x*(l
og(x)**4 + 2*log(x)**2*x + x**2)),x)
```

output
```
(e**(x**2)*log(x)**3 + e**(x**2)*log(x)*x + log(log(x)**2 + x)*log(x)**2 +
 log(log(x)**2 + x)*x - log(x))/(log(x)**2 + x)
```

3.275 $\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$

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3.275.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\begin{aligned}\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz = {} & \frac{24e^{\frac{x}{2}+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4}-\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4}\\ & +\frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4}\\ & -\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2}+\frac{e^{\frac{x}{2}+xz}x^5\sin^4(\pi z)}{16\pi^2+x^2}\end{aligned}$$

output
```
24*exp(1/2*x+x*z)*Pi^4*x^3/(64*Pi^4+20*Pi^2*x^2+x^4)-24*exp(1/2*x+x*z)*Pi^
3*x^4*cos(Pi*z)*sin(Pi*z)/(64*Pi^4+20*Pi^2*x^2+x^4)+12*exp(1/2*x+x*z)*Pi^2
*x^5*sin(Pi*z)^2/(64*Pi^4+20*Pi^2*x^2+x^4)-4*exp(1/2*x+x*z)*Pi*x^4*cos(Pi*
z)*sin(Pi*z)^3/(16*Pi^2+x^2)+exp(1/2*x+x*z)*x^5*sin(Pi*z)^4/(16*Pi^2+x^2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\begin{aligned}&\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz\\ &=\frac{e^{x\left(\frac{1}{2}+z\right)}x^4(192\pi^4+60\pi^2x^2+3x^4-4x^2(16\pi^2+x^2)\cos(2\pi z)+x^2(4\pi^2+x^2)\cos(4\pi z)-128\pi^3x\sin(2\pi z)}{8\left(64\pi^4x+20\pi^2x^3+x^5\right)}\end{aligned}$$

input
```
Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

output
```
(E^(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*Pi^2 + x^
2)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi^3*x*Sin[2*Pi*z] -
 8*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] + 4*Pi*x^3*Sin[4*Pi*z]))/(8*
(64*Pi^4*x + 20*Pi^2*x^3 + x^5))
```

3.275.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {27, 4934, 4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{xz+\frac{x}{2}} \sin^4(\pi z)\,dz$$

$$\Big\downarrow \; 27$$

$$x^4 \int e^{zx+\frac{x}{2}} \sin^4(\pi z)dz$$

$$\Big\downarrow \; 4934$$

$$x^4 \left(\frac{12\pi^2 \int e^{zx+\frac{x}{2}} \sin^2(\pi z)dz}{x^2+16\pi^2} + \frac{xe^{xz+\frac{x}{2}} \sin^4(\pi z)}{x^2+16\pi^2} - \frac{4\pi e^{xz+\frac{x}{2}} \sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}\right)$$

$$\Big\downarrow \; 4934$$

$$x^4 \left(\frac{12\pi^2 \left(\frac{2\pi^2 \int e^{zx+\frac{x}{2}}dz}{x^2+4\pi^2} + \frac{xe^{xz+\frac{x}{2}} \sin^2(\pi z)}{x^2+4\pi^2} - \frac{2\pi e^{xz+\frac{x}{2}} \sin(\pi z)\cos(\pi z)}{x^2+4\pi^2}\right)}{x^2+16\pi^2} + \frac{xe^{xz+\frac{x}{2}} \sin^4(\pi z)}{x^2+16\pi^2} - \frac{4\pi e^{xz+\frac{x}{2}} \sin^3(\pi z)\cos(\pi z}{x^2+16\pi^2}\right.$$

$$\Big\downarrow \; 2624$$

$$x^4 \left(\frac{xe^{xz+\frac{x}{2}} \sin^4(\pi z)}{x^2+16\pi^2} - \frac{4\pi e^{xz+\frac{x}{2}} \sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2} + \frac{12\pi^2 \left(\frac{2\pi^2 e^{xz+\frac{x}{2}}}{x(x^2+4\pi^2)} + \frac{xe^{xz+\frac{x}{2}} \sin^2(\pi z)}{x^2+4\pi^2} - \frac{2\pi e^{xz+\frac{x}{2}} \sin(\pi z)\cos(\pi z)}{x^2+4\pi^2}\right)}{x^2+16\pi^2}\right)$$

input
```
Int[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

output
```
x^4*((-4*E^(x/2 + x*z)*Pi*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2
 + x*z)*x*Sin[Pi*z]^4)/(16*Pi^2 + x^2) + (12*Pi^2*((2*E^(x/2 + x*z)*Pi^2)/
(x*(4*Pi^2 + x^2)) - (2*E^(x/2 + x*z)*Pi*Cos[Pi*z]*Sin[Pi*z])/(4*Pi^2 + x^
2) + (E^(x/2 + x*z)*x*Sin[Pi*z]^2)/(4*Pi^2 + x^2)))/(16*Pi^2 + x^2))
```

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

3.275.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.63

method	result
default	$x^{4} \left(\frac{3 \,{\mathrm e}^{x \left(\frac{1}{2}+z \right)}}{8 x}+\frac{x \,{\mathrm e}^{\frac{1}{2} x +x z} \cos \left(4 \pi z \right)}{128 \pi^{2}+8 x^{2}}+\frac{\pi \,{\mathrm e}^{\frac{1}{2} x +x z} \sin \left(4 \pi z \right)}{32 \pi^{2}+2 x^{2}}-\frac{x \,{\mathrm e}^{\frac{1}{2} x +x z} \cos \left(2 \pi z \right)}{2 \left(4 \pi^{2}+x^{2}\right)}-\frac{\pi \,{\mathrm e}^{\frac{1}{2} x +x z} \sin \left(2 \pi z \right)}{4 \pi^{2}+x^{2}}\right)$
risch	$\frac{3 x^{3} {\mathrm e}^{\frac{x \left(1+2 z \right)}{2}}}{8}+\frac{x^{5} {\mathrm e}^{\frac{1}{2} x +x z} \cos \left(4 \pi z \right)}{128 \pi^{2}+8 x^{2}}+\frac{x^{4} {\mathrm e}^{\frac{1}{2} x +x z} \pi \sin \left(4 \pi z \right)}{32 \pi^{2}+2 x^{2}}-\frac{x^{5} {\mathrm e}^{\frac{1}{2} x +x z} \cos \left(2 \pi z \right)}{2 \left(4 \pi^{2}+x^{2}\right)}-\frac{x^{4} {\mathrm e}^{\frac{1}{2} x +x z} \pi \sin \left(2 \pi z \right)}{4 \pi^{2}+x^{2}}$
parallelrisch	$\frac{x^{3} {\mathrm e}^{\frac{x \left(1+2 z \right)}{2}} \left(-128 \pi^{3} x \sin \left(2 \pi z \right)+16 \pi^{3} x \sin \left(4 \pi z \right)+4 \pi^{2} x^{2} \cos \left(4 \pi z \right)-64 \pi^{2} x^{2} \cos \left(2 \pi z \right)-8 x^{3} \pi \sin \left(2 \pi z \right)+4 x^{3} \pi \sin \left(4 \pi z \right)+x^{4} \cos\right.}{512 \pi^{4}+160 \pi^{2} x^{2}+8 x^{4}}$
norman	$\frac{24 \,{\mathrm e}^{\frac{1}{2} x +x z} \pi^{4} x^{3}}{64 \pi^{4}+20 \pi^{2} x^{2}+x^{4}}-\frac{48 \pi^{3} x^{4} {\mathrm e}^{\frac{1}{2} x +x z} \tan \left(\frac{\pi z}{2}\right)}{64 \pi^{4}+20 \pi^{2} x^{2}+x^{4}}+\frac{48 \pi^{3} x^{4} {\mathrm e}^{\frac{1}{2} x +x z} \left(\tan^{7}\left(\frac{\pi z}{2}\right)\right)}{64 \pi^{4}+20 \pi^{2} x^{2}+x^{4}}+\frac{16 \left(9 \pi^{4}+10 \pi^{2} x^{2}+x^{4}\right) x^{3} {\mathrm e}^{\frac{1}{2} x +x z} \left(\tan^{4}\left(\frac{\pi z}{2}\right)\right)}{64 \pi^{4}+20 \pi^{2} x^{2}+x^{4}}+$

input
```
int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z,method=_RETURNVERBOSE)
```

output
```
x^4*(3/8*exp(x*(1/2+z))/x+1/8*x/(16*Pi^2+x^2)*exp(1/2*x+x*z)*cos(4*Pi*z)+1
/2*Pi/(16*Pi^2+x^2)*exp(1/2*x+x*z)*sin(4*Pi*z)-1/2*x/(4*Pi^2+x^2)*exp(1/2*
x+x*z)*cos(2*Pi*z)-Pi/(4*Pi^2+x^2)*exp(1/2*x+x*z)*sin(2*Pi*z))
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$$
$$= \frac{4\left(\left(4\,\pi^3x^4+\pi x^6\right)\cos\left(\pi z\right)^3-\left(10\,\pi^3x^4+\pi x^6\right)\cos\left(\pi z\right)\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(\pi z\right)+\left(24\,\pi^4x^3+16\,\pi^2x^5+x^7+(4\right.}{64\,\pi^4+20\,\pi^2x^2+x^4}$$

input
```
integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="fricas")
```

output
```
(4*((4*pi^3*x^4 + pi*x^6)*cos(pi*z)^3 - (10*pi^3*x^4 + pi*x^6)*cos(pi*z))*
e^(x*z + 1/2*x)*sin(pi*z) + (24*pi^4*x^3 + 16*pi^2*x^5 + x^7 + (4*pi^2*x^5
 + x^7)*cos(pi*z)^4 - 2*(10*pi^2*x^5 + x^7)*cos(pi*z)^2)*e^(x*z + 1/2*x))/
(64*pi^4 + 20*pi^2*x^2 + x^4)
```

3.275.6 Sympy [A] (verification not implemented)

Time = 151.38 (sec) , antiderivative size = 1277, normalized size of antiderivative = 6.42

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz = \text{Too large to display}$$

input
```
integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

output
```
x**4*Piecewise((3*z*sin(pi*z)**4/8 + 3*z*sin(pi*z)**2*cos(pi*z)**2/4 + 3*z
*cos(pi*z)**4/8 - 5*sin(pi*z)**3*cos(pi*z)/(8*pi) - 3*sin(pi*z)*cos(pi*z)*
*3/(8*pi), Eq(x, 0)), (z*exp(-4*I*pi*z)*sin(pi*z)**4/16 - I*z*exp(-4*I*pi*
z)*sin(pi*z)**3*cos(pi*z)/4 - 3*z*exp(-4*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2
/8 + I*z*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(-4*I*pi*z)*cos(pi
*z)**4/16 + 7*I*exp(-4*I*pi*z)*sin(pi*z)**4/(24*pi) + 11*exp(-4*I*pi*z)*si
n(pi*z)**3*cos(pi*z)/(48*pi) + 5*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(48
*pi) - I*exp(-4*I*pi*z)*cos(pi*z)**4/(24*pi), Eq(x, -4*I*pi)), (-z*exp(-2*
I*pi*z)*sin(pi*z)**4/4 + I*z*exp(-2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 + I*z
*exp(-2*I*pi*z)*sin(pi*z)*cos(pi*z)**3/2 + z*exp(-2*I*pi*z)*cos(pi*z)**4/4
 - 5*I*exp(-2*I*pi*z)*sin(pi*z)**4/(24*pi) + exp(-2*I*pi*z)*sin(pi*z)**3*c
os(pi*z)/(3*pi) - I*exp(-2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/(2*pi) - I*ex
p(-2*I*pi*z)*cos(pi*z)**4/(8*pi), Eq(x, -2*I*pi)), (-z*exp(2*I*pi*z)*sin(p
i*z)**4/4 - I*z*exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 - I*z*exp(2*I*pi*z)
*sin(pi*z)*cos(pi*z)**3/2 + z*exp(2*I*pi*z)*cos(pi*z)**4/4 + 5*I*exp(2*I*p
i*z)*sin(pi*z)**4/(24*pi) + exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/(3*pi) +
I*exp(2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/(2*pi) + I*exp(2*I*pi*z)*cos(pi*
z)**4/(8*pi), Eq(x, 2*I*pi)), (z*exp(4*I*pi*z)*sin(pi*z)**4/16 + I*z*exp(4
*I*pi*z)*sin(pi*z)**3*cos(pi*z)/4 - 3*z*exp(4*I*pi*z)*sin(pi*z)**2*cos(pi*
z)**2/8 - I*z*exp(4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(4*I*pi*z)*...
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$$
$$= \frac{\left((4\,\pi^2x^2+x^4)\cos\left(4\,\pi z\right)e^{\left(xz+\frac{1}{2}\,x\right)}-4\,(16\,\pi^2x^2+x^4)\cos\left(2\,\pi z\right)e^{\left(xz+\frac{1}{2}\,x\right)}+4\,(4\,\pi^3x+\pi x^3)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(4\right.\right.}{8\,(64\,\pi^4x+20\,\pi^2x^3+x^5}$$

input
```
integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="maxima")
```

output
```
1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*x^2 + x^4
)*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(
4*pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(2*pi*z) + 3*(64*pi^4
+ 20*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.57

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$$
$$=\frac{1}{8}\left(\left(\frac{x\cos(4\pi z)}{16\pi^2+x^2}+\frac{4\pi\sin(4\pi z)}{16\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}x\right)}-4\left(\frac{x\cos(2\pi z)}{4\pi^2+x^2}+\frac{2\pi\sin(2\pi z)}{4\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}x\right)}+\frac{3e^{\left(xz+\frac{1}{2}x\right)}}{x}\right)$$

input
```
integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="giac")
```

output
```
1/8*((x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 + x^2))*e^
(x*z + 1/2*x) - 4*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*sin(2*pi*z)/(4*pi^2
 + x^2))*e^(x*z + 1/2*x) + 3*e^(x*z + 1/2*x)/x)*x^4
```

3.275.9 Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$$
$$=\frac{x^3\,\mathrm{e}^{\frac{x}{2}+x\,z}\left(24\,\Pi^4-\frac{x^4\,\cos(2\,\Pi\,z)}{2}+\frac{x^4\,\cos(4\,\Pi\,z)}{8}+\frac{3\,x^4}{8}+\frac{15\,\Pi^2\,x^2}{2}-\Pi\,x^3\,\sin(2\,\Pi\,z)-16\,\Pi^3\,x\,\sin(2\,\Pi\,z)+\frac{\Pi\,x^3}{}\right.}{64\,\Pi^4+20\,\Pi^2\,x^2+x^4}$$

input
```
int(x^4*exp(x/2 + x*z)*sin(Pi*z)^4,z)
```

output
```
(x^3*exp(x/2 + x*z)*(24*Pi^4 - (x^4*cos(2*Pi*z))/2 + (x^4*cos(4*Pi*z))/8 +
 (3*x^4)/8 + (15*Pi^2*x^2)/2 - Pi*x^3*sin(2*Pi*z) - 16*Pi^3*x*sin(2*Pi*z)
+ (Pi*x^3*sin(4*Pi*z))/2 + 2*Pi^3*x*sin(4*Pi*z) - 8*Pi^2*x^2*cos(2*Pi*z) +
 (Pi^2*x^2*cos(4*Pi*z))/2))/(64*Pi^4 + x^4 + 20*Pi^2*x^2)
```

3.275.10 Reduce [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.61

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$$
$$= \frac{e^{xz+\frac{1}{2}x}x^3\left(-16\cos(\pi z)\sin(\pi z)^3\pi^3x-4\cos(\pi z)\sin(\pi z)^3\pi\,x^3-24\cos(\pi z)\sin(\pi z)\pi^3x+4\sin(\pi z)^4\pi^2\right.}{64\pi^4+20\pi^2x^2+x^4}$$

input
```
int(e**((2*x*z + x)/2)*sin(pi*z)**4*x***4,z)
```

output
```
(e**((2*x*z + x)/2)*x***3*( - 16*cos(pi*z)*sin(pi*z)**3*pi**3*x - 4*cos(pi*
z)*sin(pi*z)**3*pi*x***3 - 24*cos(pi*z)*sin(pi*z)*pi**3*x + 4*sin(pi*z)**4*
pi**2*x***2 + sin(pi*z)**4*x***4 + 12*sin(pi*z)**2*pi**2*x***2 + 24*pi**4))/(
64*pi**4 + 20*pi**2*x***2 + x***4)
```

3.276 $\int \mathbf{erf}(x)\,dx$

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3.276.1 Optimal result

Integrand size = 2, antiderivative size = 18

$$\int \operatorname{erf}(x)\,dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

output
```
x*erf(x)+1/exp(x^2)/Pi^(1/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \operatorname{erf}(x)\,dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

input
```
Integrate[Erf[x],x]
```

output
```
1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

3.276.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(x)\,dx$$
$$\Big\downarrow \text{6903}$$
$$x\text{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

input
```
Int[Erf[x],x]
```

output
```
1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

3.276.3.1 Defintions of rubi rules used

rule 6903
```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b),
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

3.276.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
default	$x\ \text{erf}\left(x\right)+\frac{{\mathrm e}^{-x^{2}}}{\sqrt{\pi}}$	16
parts	$x\ \text{erf}\left(x\right)+\frac{{\mathrm e}^{-x^{2}}}{\sqrt{\pi}}$	16
parallelrisch	$\frac{x\sqrt{\pi}\ \text{erf}\left(x\right)+{\mathrm e}^{-x^{2}}}{\sqrt{\pi}}$	19
meijerg	$\frac{-2+2\,{\mathrm e}^{-x^{2}}+2x\sqrt{\pi}\ \text{erf}\left(x\right)}{2\sqrt{\pi}}$	24

input
```
int(erf(x),x,method=_RETURNVERBOSE)
```

output
```
x*erf(x)+1/Pi^(1/2)*exp(-x^2)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \operatorname{erf}(x)\, dx = \frac{\pi x \operatorname{erf}(x) + \sqrt{\pi} e^{(-x^2)}}{\pi}$$

input
```
integrate(erf(x),x, algorithm="fricas")
```

output
```
(pi*x*erf(x) + sqrt(pi)*e^(-x^2))/pi
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x)\, dx = x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

input
```
integrate(erf(x),x)
```

output
```
x*erf(x) + exp(-x**2)/sqrt(pi)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x)\, dx = x \operatorname{erf}(x) + \frac{e^{(-x^2)}}{\sqrt{\pi}}$$

input
```
integrate(erf(x),x, algorithm="maxima")
```

output
```
x*erf(x) + e^(-x^2)/sqrt(pi)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x)\,dx = x \operatorname{erf}(x) + \frac{e^{(-x^2)}}{\sqrt{\pi}}$$

input
```
integrate(erf(x),x, algorithm="giac")
```

output
```
x*erf(x) + e^(-x^2)/sqrt(pi)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x)\,dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x \operatorname{erf}(x)$$

input
```
int(erf(x),x)
```

output
```
exp(-x^2)/pi^(1/2) + x*erf(x)
```

3.276.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \operatorname{erf}(x)\,dx = \frac{\sqrt{\pi}\, e^{x^2} \operatorname{erf}(x)\, x + 1}{\sqrt{\pi}\, e^{x^2}}$$

input
```
int(erf(x),x)
```

output
```
(sqrt(pi)*e**(x**2)*erf(x)*x + 1)/(sqrt(pi)*e**(x**2))
```

3.277 $\int \operatorname{erf}(a+x)\,dx$

3.277.1 Optimal result 1619
3.277.2 Mathematica [A] (verified) 1619
3.277.3 Rubi [A] (verified) 1620
3.277.4 Maple [A] (verified) 1621
3.277.5 Fricas [A] (verification not implemented) 1621
3.277.6 Sympy [A] (verification not implemented) 1621
3.277.7 Maxima [A] (verification not implemented) 1622
3.277.8 Giac [A] (verification not implemented) 1622
3.277.9 Mupad [B] (verification not implemented) 1622
3.277.10Reduce [B] (verification not implemented) 1623

3.277.1 Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \operatorname{erf}(a+x)\,dx = \frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a+x)\operatorname{erf}(a+x)$$

output
```
(a+x)*erf(a+x)+1/exp((a+x)^2)/Pi^(1/2)
```

3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \operatorname{erf}(a+x)\,dx = \frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a+x)\operatorname{erf}(a+x)$$

input
```
Integrate[Erf[a + x],x]
```

output
```
1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

3.277.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(a+x)\,dx$$

$$\Big\downarrow \text{6903}$$

$$(a+x)\text{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

input
```
Int[Erf[a + x],x]
```

output
```
1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

3.277.3.1 Defintions of rubi rules used

rule 6903
```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b),
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

3.277.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$(a+x)\,\mathrm{erf}\,(a+x)+\frac{\mathrm{e}^{-(a+x)^{2}}}{\sqrt{\pi}}$	22
default	$(a+x)\,\mathrm{erf}\,(a+x)+\frac{\mathrm{e}^{-(a+x)^{2}}}{\sqrt{\pi}}$	22
parallelrisch	$\frac{x\ \mathrm{erf}(a+x)\sqrt{\pi}+a\ \mathrm{erf}(a+x)\sqrt{\pi}+\mathrm{e}^{-(a+x)^{2}}}{\sqrt{\pi}}$	32
parts	$x\ \mathrm{erf}\,(a+x)-\frac{2\left(-\frac{\mathrm{e}^{-a^{2}-2ax-x^{2}}}{2}-\frac{a\ \mathrm{erf}(a+x)\sqrt{\pi}}{2}\right)}{\sqrt{\pi}}$	42

input
```
int(erf(a+x),x,method=_RETURNVERBOSE)
```

output
```
(a+x)*erf(a+x)+1/Pi^(1/2)*exp(-(a+x)^2)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \mathrm{erf}(a+x)\,dx=\frac{(\pi a+\pi x)\,\mathrm{erf}\,(a+x)+\sqrt{\pi}e^{\left(-a^{2}-2\,ax-x^{2}\right)}}{\pi}$$

input
```
integrate(erf(a+x),x, algorithm="fricas")
```

output
```
((pi*a + pi*x)*erf(a + x) + sqrt(pi)*e^(-a^2 - 2*a*x - x^2))/pi
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \mathrm{erf}(a+x)\,dx=a\,\mathrm{erf}\,(a+x)+x\,\mathrm{erf}\,(a+x)+\frac{e^{-a^{2}}e^{-x^{2}}e^{-2ax}}{\sqrt{\pi}}$$

input
```
integrate(erf(a+x),x)
```

output
```
a*erf(a + x) + x*erf(a + x) + exp(-a**2)*exp(-x**2)*exp(-2*a*x)/sqrt(pi)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \operatorname{erf}(a+x)\,dx = (a+x)\operatorname{erf}(a+x) + \frac{e^{\left(-(a+x)^2\right)}}{\sqrt{\pi}}$$

input
```
integrate(erf(a+x),x, algorithm="maxima")
```

output
```
(a + x)*erf(a + x) + e^(-(a + x)^2)/sqrt(pi)
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \operatorname{erf}(a+x)\,dx = x\operatorname{erf}(a+x) + \frac{\sqrt{\pi}a\operatorname{erf}(a+x) + e^{\left(-a^2-2\,ax-x^2\right)}}{\sqrt{\pi}}$$

input
```
integrate(erf(a+x),x, algorithm="giac")
```

output
```
x*erf(a + x) + (sqrt(pi)*a*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/sqrt(pi)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \operatorname{erf}(a+x)\,dx = \operatorname{erf}(a+x)\;(a+x) + \frac{\mathrm{e}^{-(a+x)^2}}{\sqrt{\pi}}$$

input
```
int(erf(a + x),x)
```

output
```
erf(a + x)*(a + x) + exp(-(a + x)^2)/pi^(1/2)
```

3.277.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \operatorname{erf}(a+x)\,dx = \frac{\sqrt{\pi}\,e^{a^2+2ax+x^2}\operatorname{erf}(a+x)\,a+\sqrt{\pi}\,e^{a^2+2ax+x^2}\operatorname{erf}(a+x)\,x+1}{\sqrt{\pi}\,e^{a^2+2ax+x^2}}$$

input
```
int(erf(a + x),x)
```

output
```
(sqrt(pi)*e**(a**2 + 2*a*x + x**2)*erf(a + x)*a + sqrt(pi)*e**(a**2 + 2*a*
x + x**2)*erf(a + x)*x + 1)/(sqrt(pi)*e**(a**2 + 2*a*x + x**2))
```

3.278 $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$

3.278.1 Optimal result . . . 1624
3.278.2 Mathematica [C] (warning: unable to verify) . . . 1624
3.278.3 Rubi [F] . . . 1625
3.278.4 Maple [F(-1)] . . . 1626
3.278.5 Fricas [B] (verification not implemented) . . . 1626
3.278.6 Sympy [F] . . . 1627
3.278.7 Maxima [F] . . . 1627
3.278.8 Giac [F] . . . 1627
3.278.9 Mupad [F(-1)] . . . 1628
3.278.10Reduce [B] (verification not implemented) . . . 1628

3.278.1 Optimal result

Integrand size = 59, antiderivative size = 94

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2(-1+2x^2)} - \operatorname{arctanh}\left(\frac{x(2+x)(7-x+27x^2+33x^3)}{(2+37x^2+31x^3)\sqrt{1+2x^2+4x^3+x^4}}\right)$$

output
```
-arctanh(x*(2+x)*(33*x^3+27*x^2-x+7)/(31*x^3+37*x^2+2)/(x^4+4*x^3+2*x^2+1)
^(1/2))+1/2*(1+2*x)*(x^4+4*x^3+2*x^2+1)^(1/2)/(2*x^2-1)
```

3.278.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.14 (sec) , antiderivative size = 5141, normalized size of antiderivative = 54.69

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2
*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]
```

output
```
Result too large to show
```

3.278.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{(2x^2-1)^2\sqrt{x^4+4x^3+2x^2+1}}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{x^2}{2\sqrt{x^4+4x^3+2x^2+1}}+\frac{x}{\sqrt{x^4+4x^3+2x^2+1}}+\frac{2x+15}{4(2x^2-1)\sqrt{x^4+4x^3+2x^2+1}}+\frac{-17x-1}{2(2x^2-1)^2\sqrt{x^4+4}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{9}{4}\int \frac{1}{\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{13}{4}\int \frac{1}{\left(\sqrt{2}-2x\right)^2\sqrt{x^4+4x^3+2x^2+1}}dx+\int \frac{x}{\sqrt{x^4+4x^3+2x^2+1}}dx+\frac{1}{2}\int \frac{x^2}{\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{13}{4}\int \frac{1}{\left(2x+\sqrt{2}\right)^2\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{1}{8}\left(15+\sqrt{2}\right)\int \frac{1}{\left(1-\sqrt{2}x\right)\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{13}{8}\int \frac{1}{\left(1-\sqrt{2}x\right)\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{1}{8}\left(15-\sqrt{2}\right)\int \frac{1}{\left(\sqrt{2}x+1\right)\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{13}{8}\int \frac{1}{\left(\sqrt{2}x+1\right)\sqrt{x^4+4x^3+2x^2+1}}dx-\frac{17}{2}\int \frac{x}{(2x^2-1)^2\sqrt{x^4+4x^3+2x^2+1}}dx$$

input
```
Int[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[
1 + 2*x^2 + 4*x^3 + x^4]),x]
```

output
```
$Aborted
```

3.278.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.278.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1
/2),x)
```

output
```
int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1
/2),x)
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.90

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \frac{(2x^2-1)\log\left(\frac{1025x^{10}+6138x^9+12307x^8+10188x^7+4503x^6+3134x^5+1589x^4+140x^3+176x^2-(1023x^8+4104x^7+5084x^6+2182x}{32x^{10}-80x^8+80x^6-40x^4+10x^2-1}\right.}{2(2x^2-1)}$$

input
```
integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2
+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((2*x^2 - 1)*log((1025*x^10 + 6138*x^9 + 12307*x^8 + 10188*x^7 + 4503*
x^6 + 3134*x^5 + 1589*x^4 + 140*x^3 + 176*x^2 - (1023*x^8 + 4104*x^7 + 508
4*x^6 + 2182*x^5 + 805*x^4 + 624*x^3 + 10*x^2 + 28*x)*sqrt(x^4 + 4*x^3 + 2
*x^2 + 1) + 2)/(32*x^10 - 80*x^8 + 80*x^6 - 40*x^4 + 10*x^2 - 1)) + sqrt(x
^4 + 4*x^3 + 2*x^2 + 1)*(2*x + 1))/(2*x^2 - 1)
```

3.278.6 Sympy [F]

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{\sqrt{(x+1)(x^3+3x^2-x+1)}(2x^2-1)^2}\,dx$$

input
```
integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x
**3+2*x**2+1)**(1/2),x)
```

output
```
Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt((x + 1
)*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)
```

3.278.7 Maxima [F]

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int \frac{2\,x^6+4\,x^5+7\,x^4-3\,x^3-x^2-8\,x-8}{\sqrt{x^4+4\,x^3+2\,x^2+1}(2\,x^2-1)^2}\,dx$$

input
```
integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2
+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^
3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

3.278.8 Giac [F]

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int \frac{2\,x^6+4\,x^5+7\,x^4-3\,x^3-x^2-8\,x-8}{\sqrt{x^4+4\,x^3+2\,x^2+1}(2\,x^2-1)^2}\,dx$$

input
```
integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2
+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^
3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int -\frac{-2\,x^6-4\,x^5-7\,x^4+3\,x^3+x^2+8\,x+8}{\left(2\,x^2-1\right)^2\sqrt{x^4+4\,x^3+2\,x^2+1}}\,dx$$

input
```
int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2
 + 4*x^3 + x^4 + 1)^(1/2)),x)
```

output
```
int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2
 + 4*x^3 + x^4 + 1)^(1/2)), x)
```

3.278.10 Reduce [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 786, normalized size of antiderivative = 8.36

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \text{Too large to display}$$

input
```
int((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt(x**4 + 4*x*
*3 + 2*x**2 + 1)*(4*x**4 - 4*x**2 + 1)),x)
```

output
```
(2*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*x + sqrt(x**4 + 4*x**3 + 2*x**2 + 1) -
 4*log(x**2 + 4*x + 2)*x**2 + 2*log(x**2 + 4*x + 2) + 10*log( - 16*sqrt(x*
*4 + 4*x**3 + 2*x**2 + 1)*sqrt(2) - 13*sqrt(x**4 + 4*x**3 + 2*x**2 + 1) +
28*sqrt(2)*x**2 + 32*sqrt(2)*x - 6*sqrt(2) + 35*x**2 + 26*x - 11)*x**2 - 5
*log( - 16*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqrt(2) - 13*sqrt(x**4 + 4*x**
3 + 2*x**2 + 1) + 28*sqrt(2)*x**2 + 32*sqrt(2)*x - 6*sqrt(2) + 35*x**2 + 2
6*x - 11) + 4*log( - 2*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqrt(2) - sqrt(x**
4 + 4*x**3 + 2*x**2 + 1) + 4*sqrt(2)*x**2 + 16*sqrt(2)*x + 10*sqrt(2) - 5*
x**2 - 20*x - 9)*x**2 - 2*log( - 2*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqrt(2
) - sqrt(x**4 + 4*x**3 + 2*x**2 + 1) + 4*sqrt(2)*x**2 + 16*sqrt(2)*x + 10*
sqrt(2) - 5*x**2 - 20*x - 9) - 2*log(sqrt(2) + 2*x)*x**2 + log(sqrt(2) + 2
*x) + 2*log(sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqrt(2) + 4*sqrt(x**4 + 4*x**
3 + 2*x**2 + 1) - 2*sqrt(2)*x - 4*sqrt(2) - x - 2)*x**2 - log(sqrt(x**4 +
4*x**3 + 2*x**2 + 1)*sqrt(2) + 4*sqrt(x**4 + 4*x**3 + 2*x**2 + 1) - 2*sqrt
(2)*x - 4*sqrt(2) - x - 2) + 2*log(25*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqr
t(2)*x + 6*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqrt(2) + 44*sqrt(x**4 + 4*x**
3 + 2*x**2 + 1)*x + 38*sqrt(x**4 + 4*x**3 + 2*x**2 + 1) - 7*sqrt(2)*x**3 -
 36*sqrt(2)*x**2 - 21*sqrt(2)*x - 10*sqrt(2) - 28*x**3 - 130*x**2 - 84*x +
 2)*x**2 - log(25*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*sqrt(2)*x + 6*sqrt(x**4
 + 4*x**3 + 2*x**2 + 1)*sqrt(2) + 44*sqrt(x**4 + 4*x**3 + 2*x**2 + 1)*x...
```

3.278. $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$

3.279 $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$

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3.279.1 Optimal result

Integrand size = 47, antiderivative size = 142

$$\begin{aligned}\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy = & -\frac{1}{4}\text{arctanh}\left(\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right)\\ & -\frac{1}{2}\text{arctanh}\left(\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right)\\ & +\frac{9}{4}\text{arctanh}\left(\frac{(11+7y)\sqrt{1-5y-5y^2}}{3(7+5y)\sqrt{1-y-y^2}}\right)\end{aligned}$$

output

```
-1/4*arctanh((1-3*y)*(-5*y^2-5*y+1)^(1/2)/(1-5*y)/(-y^2-y+1)^(1/2))-1/2*ar
ctanh((4+3*y)*(-5*y^2-5*y+1)^(1/2)/(6+5*y)/(-y^2-y+1)^(1/2))+9/4*arctanh(1
/3*(11+7*y)*(-5*y^2-5*y+1)^(1/2)/(7+5*y)/(-y^2-y+1)^(1/2))
```

3.279.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.38 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.44

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$$

$$= \frac{\left(-1-\frac{2}{\sqrt{5}}\right)\left(1+\sqrt{5}+2y\right)^2\sqrt{\frac{5+3\sqrt{5}+10y}{5+5\sqrt{5}+10y}}\left(20\left(-4\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}+\sqrt{5}\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-}{1+\sqrt{5}+}}\right.\right.}{}$$

input
```
Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y
- y^2]),y]
```

output
```
((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(5 + 5
*Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)
]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + Sqrt[5]*Sqrt[(-5 + 3*Sq
rt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5]
+ 2*y)] + 5*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt
[-((-3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((
-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2
*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*Sqrt[5]
 + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16] + Sqrt[(-5 + 3*Sqrt[5] -
10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]*
(9*Sqrt[5]*EllipticPi[5/8 - Sqrt[5]/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*
y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[
(-3*(-5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5]
+ 2*y)])/Sqrt[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, Arc
Sin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16]
)))/(16*Sqrt[1 - 5*y - 5*y^2]*Sqrt[1 - y - y^2])
```

3.279.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y(y+1)(y+2)\sqrt{-y^2-y+1}}\,dy$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{-5y^2-5y+1}}{2y\sqrt{-y^2-y+1}}+\frac{\sqrt{-5y^2-5y+1}}{(y+1)\sqrt{-y^2-y+1}}-\frac{3\sqrt{-5y^2-5y+1}}{2(y+2)\sqrt{-y^2-y+1}}\right)dy$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\int \frac{\sqrt{-5y^2-5y+1}}{y\sqrt{-y^2-y+1}}dy+\int \frac{\sqrt{-5y^2-5y+1}}{(y+1)\sqrt{-y^2-y+1}}dy-\frac{3}{2}\int \frac{\sqrt{-5y^2-5y+1}}{(y+2)\sqrt{-y^2-y+1}}dy$$

input
```
Int[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2]
),y]
```

output
```
$Aborted
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.279.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.48

method	result
default	$-\frac{300\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}\,\sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}}\,\left(-10y-5+3\sqrt{5}\right)^2\sqrt{\frac{-2y+\sqrt{5}-1}{-10y-5+3\sqrt{5}}}\,\sqrt{5}\,\sqrt{\frac{2y+1+\sqrt{5}}{-10y-5+3\sqrt{5}}}\left(\Pi\left(2\sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}}\right.\right.}{\sqrt{5y^4+10y^3-y^2-6y+1}\,\sqrt{\left(10y+5+3\sqrt{5}\right)\left(-10y-5+3\sqrt{5}\right)\left(-2y+\sqrt{5}-1\right)\left(\right.}}$
elliptic	Expression too large to display

input
```
int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y,method=_
RETURNVERBOSE)
```

output
```
-300*(-5*y^2-5*y+1)^(1/2)*(-y^2-y+1)^(1/2)*(-(10*y+5+3*5^(1/2))/(-10*y-5+3
*5^(1/2)))^(1/2)*(-10*y-5+3*5^(1/2))^2*((-2*y+5^(1/2)-1)/(-10*y-5+3*5^(1/2
)))^(1/2)*5^(1/2)*((2*y+1+5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2)*(EllipticPi(
2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(3*5^(1/2)-5)/(5+3*
5^(1/2)),1/4)+2*EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/
2),-1/4*(5+3*5^(1/2))/(3*5^(1/2)-5),1/4)-3*EllipticPi(2*(-(10*y+5+3*5^(1/2
))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(5+5^(1/2))/(5^(1/2)-5),1/4))/(5*y^4+10
*y^3-y^2-6*y+1)^(1/2)/((10*y+5+3*5^(1/2))*(-10*y-5+3*5^(1/2))*(-2*y+5^(1/2
)-1)*(2*y+1+5^(1/2)))^(1/2)/(3*5^(1/2)-5)/(5+3*5^(1/2))/(5+5^(1/2))/(5^(1/
2)-5)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.57

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$$
$$= \frac{9}{8}\log\left(-\frac{235\,y^4+935\,y^3-3\,(35\,y^2+104\,y+77)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}+1086\,y^2+131\,y-2}{y^4+8\,y^3+24\,y^2+32\,y+16}\right.$$
$$+\frac{1}{4}\log\left(\frac{35\,y^4+125\,y^3+(15\,y^2+38\,y+24)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}+131\,y^2+16\,y-26}{y^4+4\,y^3+6\,y^2+4\,y+1}\right)$$
$$+\frac{1}{8}\log\left(\frac{35\,y^4+15\,y^3+(15\,y^2-8\,y+1)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}-34\,y^2+11\,y-1}{y^4}\right)$$

input
```
integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, a
lgorithm="fricas")
```

output
```
9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 - y + 1)*s
qrt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8*y^3 + 24*y^2 + 32
*y + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2 + 38*y + 24)*sqrt(-y^2 - y
 + 1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 +
 4*y + 1)) + 1/8*log((35*y^4 + 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y +
 1)*sqrt(-5*y^2 - 5*y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

3.279.6 Sympy [F]

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy = \int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y\,(y+1)\,(y+2)\,\sqrt{-y^2-y+1}}\,dy$$

input
```
integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2),
y)
```

output
```
Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqrt(-y**2 -
 y + 1)), y)
```

3.279.7 Maxima [F]

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy = \int \frac{\sqrt{-5\,y^2-5\,y+1}(2\,y+1)}{\sqrt{-y^2-y+1}(y+2)(y+1)y}\,dy$$

input
```
integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, a
lgorithm="maxima")
```

output
```
integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y
+ 1)*y), y)
```

3.279.8 Giac [F]

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy = \int \frac{\sqrt{-5\,y^2-5\,y+1}(2\,y+1)}{\sqrt{-y^2-y+1}(y+2)(y+1)y}\,dy$$

input
```
integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, a
lgorithm="giac")
```

output
```
integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y
+ 1)*y), y)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy = \int \frac{(2\,y+1)\,\sqrt{-5\,y^2-5\,y+1}}{y\,(y+1)\,(y+2)\,\sqrt{-y^2-y+1}}\,dy$$

input
```
int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(
1/2)),y)
```

output
```
int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(
1/2)), y)
```

3.279.10 Reduce [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.97

$$
\begin{aligned}
&\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy \\
&= -2\log\left(y^2-2y+1\right)-\frac{\log(y^2+4y+4)}{2}+\log\Big(-3\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}\,i \\
&\quad +4\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}-15\sqrt{-y^2-y+1}\,iy+3\sqrt{-y^2-y+1}\,i \\
&\quad -5\sqrt{-y^2-y+1}\,y-4\sqrt{-y^2-y+1}-\sqrt{-5y^2-5y+1}\,iy-3\sqrt{-5y^2-5y+1}\,i \\
&\quad -7\sqrt{-5y^2-5y+1}\,y+4\sqrt{-5y^2-5y+1}-5i\,y^2+11iy+3i-10y^2+2y-4\Big) \\
&\quad +\frac{\log\left(-2\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}\,i-5\sqrt{-y^2-y+1}\,y-4\sqrt{-y^2-y+1}+3\sqrt{-5y^2-5y+1}\,y\right.}{2} \\
&\quad +\log\Big(\sqrt{-y^2-y+1}-\sqrt{-5y^2-5y+1}-4iy-4i\Big) \\
&\quad +\frac{\log\left(\sqrt{-y^2-y+1}\,i-\sqrt{-5y^2-5y+1}\,i+2y\right)}{2} \\
&\quad +\log\Big(6\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}\,i+8\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1} \\
&\quad -20\sqrt{-y^2-y+1}\,iy-4\sqrt{-y^2-y+1}\,i+15\sqrt{-y^2-y+1}\,y+3\sqrt{-y^2-y+1} \\
&\quad -12\sqrt{-5y^2-5y+1}\,iy+4\sqrt{-5y^2-5y+1}\,i+9\sqrt{-5y^2-5y+1}\,y \\
&\quad -3\sqrt{-5y^2-5y+1}-10i\,y^2-22iy+14i-30y^2+4y+2\Big) \\
&\quad +\log\Big(6\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}-15\sqrt{-y^2-y+1}\,iy-3\sqrt{-y^2-y+1}\,i \\
&\quad -9\sqrt{-5y^2-5y+1}\,iy+3\sqrt{-5y^2-5y+1}\,i-10y^2-22y+14\Big) \\
&\quad +\frac{9\log\left(3\sqrt{-y^2-y+1}\,i+\sqrt{-5y^2-5y+1}\,i-2y-4\right)}{2} \\
&\quad -\log(y-1)-\frac{9\log(y+2)}{2}-\log(y+1)-\frac{3\log(y)}{2}
\end{aligned}
$$

input
```
int((sqrt( - 5*y**2 - 5*y + 1)*(2*y + 1))/(sqrt( - y**2 - y + 1)*y*(y**2 +
 3*y + 2)),y)
```

output
```
( - 4*log(y**2 - 2*y + 1) - log(y**2 + 4*y + 4) + 2*log( - 3*sqrt( - 5*y**
2 - 5*y + 1)*sqrt( - y**2 - y + 1)*i + 4*sqrt( - 5*y**2 - 5*y + 1)*sqrt( -
 y**2 - y + 1) - 15*sqrt( - y**2 - y + 1)*i*y + 3*sqrt( - y**2 - y + 1)*i
- 5*sqrt( - y**2 - y + 1)*y - 4*sqrt( - y**2 - y + 1) - sqrt( - 5*y**2 - 5
*y + 1)*i*y - 3*sqrt( - 5*y**2 - 5*y + 1)*i - 7*sqrt( - 5*y**2 - 5*y + 1)*
y + 4*sqrt( - 5*y**2 - 5*y + 1) - 5*i*y**2 + 11*i*y + 3*i - 10*y**2 + 2*y
- 4) + log( - 2*sqrt( - 5*y**2 - 5*y + 1)*sqrt( - y**2 - y + 1)*i - 5*sqrt
( - y**2 - y + 1)*y - 4*sqrt( - y**2 - y + 1) + 3*sqrt( - 5*y**2 - 5*y + 1
)*y + 4*sqrt( - 5*y**2 - 5*y + 1) - 5*i*y**2 - 6*i*y + 2*i) + 2*log(sqrt(
- y**2 - y + 1) - sqrt( - 5*y**2 - 5*y + 1) - 4*i*y - 4*i) + log(sqrt( - y
**2 - y + 1)*i - sqrt( - 5*y**2 - 5*y + 1)*i + 2*y) + 2*log(6*sqrt( - 5*y*
*2 - 5*y + 1)*sqrt( - y**2 - y + 1)*i + 8*sqrt( - 5*y**2 - 5*y + 1)*sqrt(
- y**2 - y + 1) - 20*sqrt( - y**2 - y + 1)*i*y - 4*sqrt( - y**2 - y + 1)*i
 + 15*sqrt( - y**2 - y + 1)*y + 3*sqrt( - y**2 - y + 1) - 12*sqrt( - 5*y**
2 - 5*y + 1)*i*y + 4*sqrt( - 5*y**2 - 5*y + 1)*i + 9*sqrt( - 5*y**2 - 5*y
+ 1)*y - 3*sqrt( - 5*y**2 - 5*y + 1) - 10*i*y**2 - 22*i*y + 14*i - 30*y**2
 + 4*y + 2) + 2*log(6*sqrt( - 5*y**2 - 5*y + 1)*sqrt( - y**2 - y + 1) - 15
*sqrt( - y**2 - y + 1)*i*y - 3*sqrt( - y**2 - y + 1)*i - 9*sqrt( - 5*y**2
- 5*y + 1)*i*y + 3*sqrt( - 5*y**2 - 5*y + 1)*i - 10*y**2 - 22*y + 14) + 9*
log(3*sqrt( - y**2 - y + 1)*i + sqrt( - 5*y**2 - 5*y + 1)*i - 2*y - 4) ...
```

3.279. $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$

3.280 $\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$

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3.280.8 Giac [B] (verification not implemented) 1642
3.280.9 Mupad [B] (verification not implemented) 1643
3.280.10Reduce [B] (verification not implemented) 1644

3.280.1 Optimal result

Integrand size = 85, antiderivative size = 21

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx = \log\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)$$

output
```
ln(1+(x^2-4)^(1/2)+(x^2-1)^(1/2))
```

3.280.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx = 2\text{arctanh}\left(1-\frac{2}{3}\sqrt{-4+x^2}+\frac{2}{3}\sqrt{-1+x^2}\right)$$

input
```
Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^
2*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]
)),x]
```

output
```
2*ArcTanh[1 - (2*Sqrt[-4 + x^2])/3 + (2*Sqrt[-1 + x^2])/3]
```

3.280.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = $\{7235\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(\sqrt{x^2-4}x^2+\sqrt{x^2-1}x^2-\sqrt{x^2-4}-4\sqrt{x^2-1}\right)}{(x^4-5x^2+4)\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)}dx$$

$$\Big\downarrow \text{7235}$$

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

input
```
Int[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt
[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]
```

output
```
Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

3.280.3.1 Defintions of rubi rules used

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

3.280.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(17) = 34.

Time = 0.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 11.90

method	result
elliptic	$\frac{\sqrt{(x^2-4)(x^2-1)}\left(\frac{\ln(x^2-5)}{4}+\frac{\ln\left(-\frac{5}{2}+x^2+\sqrt{x^4-5x^2+4}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{5x^2-17}{4\sqrt{(x^2-5)^2+5x^2-21}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{8+2\sqrt{5}\,(x-\sqrt{5})}{4\sqrt{(x-\sqrt{5})^2+2\sqrt{5}\,(x-\sqrt{5})+4}}\right)}{4}\right.}{\sqrt{x^2-4}\,\sqrt{}}$
default	$\frac{\ln(x^2-5)}{4}-\frac{\sqrt{(-2+x)^2+4x-8}+2\ln\left(x+\sqrt{(-2+x)^2+4x-8}\right)}{4\left(2+\sqrt{5}\right)\left(-2+\sqrt{5}\right)}-\frac{\sqrt{(2+x)^2-4x-8}-2\ln\left(x+\sqrt{(2+x)^2-4x-8}\right)}{4\left(2+\sqrt{5}\right)\left(-2+\sqrt{5}\right)}+\frac{\sqrt{(x-\sqrt{5}}}{}$

input
```
int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))
/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
((x^2-4)*(x^2-1))^(1/2)/(x^2-4)^(1/2)/(x^2-1)^(1/2)*(1/4*ln(x^2-5)+1/4*ln(
-5/2+x^2+(x^4-5*x^2+4)^(1/2))+1/4*arctanh(1/4*(5*x^2-17)/((x^2-5)^2+5*x^2-
21)^(1/2))+1/4*arctanh(1/4*(8+2*5^(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(1
/2)*(x-5^(1/2))+4)^(1/2))+1/4*arctanh(1/4*(8-2*5^(1/2)*(x+5^(1/2)))/((x+5^
(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2))-1/4*arctanh(1/2*(2+2*5^(1/2)*(x-5
^(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))-1/4*arctanh(1/2*(2
-2*5^(1/2)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2)))
```

3.280.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 7.71

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{\left(4-5x^2+x^4\right)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$$
$$= -\frac{1}{4}\log\left(4x^4-\left(4x^2-11\right)\sqrt{x^2-1}\sqrt{x^2-4}-21x^2+23\right) - \frac{1}{4}\log\left(x^2-\sqrt{x^2-1}(x+2)+2x-1\right) + \frac{1}{4}\log\left(x^2-\sqrt{x^2-4}(x+1)+x-4\right) - \frac{1}{4}\log\left(x^2-\sqrt{x^2-4}(x-1)-x-4\right) + \frac{1}{4}\log\left(x^2-\sqrt{x^2-1}(x-2)-2x-1\right) + \frac{1}{4}\log\left(x^2-5\right) + \frac{1}{4}\log\left(-x^2+\sqrt{x^2-1}\sqrt{x^2-4}+7\right)$$

input
```
integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^
(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="fricas"
)
```

output
```
-1/4*log(4*x^4 - (4*x^2 - 11)*sqrt(x^2 - 1)*sqrt(x^2 - 4) - 21*x^2 + 23) -
 1/4*log(x^2 - sqrt(x^2 - 1)*(x + 2) + 2*x - 1) + 1/4*log(x^2 - sqrt(x^2 -
 4)*(x + 1) + x - 4) - 1/4*log(x^2 - sqrt(x^2 - 4)*(x - 1) - x - 4) + 1/4*
log(x^2 - sqrt(x^2 - 1)*(x - 2) - 2*x - 1) + 1/4*log(x^2 - 5) + 1/4*log(-x
^2 + sqrt(x^2 - 1)*sqrt(x^2 - 4) + 7)
```

3.280.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{\left(4-5x^2+x^4\right)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx = \text{Timed out}$$

input
```
integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*
(x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)
```

output
```
Timed out
```

3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.14

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$$
$$= \frac{1}{4}\log(x+1)+\frac{3}{8}\log(x-1)+\frac{1}{8}\log(x-2)$$
$$+\frac{1}{4}\log\left(\frac{2x^4+4(x^2-3)\sqrt{x+1}\sqrt{x-1}-7x^2+2\left((x^2-1)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+(2x^2-3)\sqrt{x-\right.}{2\left((x^2-1)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+(2x^2-3)\sqrt{x-2}\right)}\right.$$
$$+\frac{1}{4}\log\left(\frac{(x^2-1)\sqrt{x+1}\sqrt{x-1}+2x^2-3}{(x^2-1)\sqrt{x-1}}\right)$$

input
```
integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^
(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="maxima"
)
```

output
```
1/4*log(x + 1) + 3/8*log(x - 1) + 1/8*log(x - 2) + 1/4*log(1/2*(2*x^4 + 4*
(x^2 - 3)*sqrt(x + 1)*sqrt(x - 1) - 7*x^2 + 2*((x^2 - 1)*sqrt(x + 1)*sqrt(
x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))*sqrt(x + 2) + 3)/((x^2 - 1)*
sqrt(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))) + 1/4*log(
((x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 2*x^2 - 3)/((x^2 - 1)*sqrt(x - 1)))
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.62

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$$
$$= -\frac{1}{2}\log\left(\sqrt{x^2-1}-\sqrt{x^2-4}+1\right)-\frac{1}{2}\log\left(\sqrt{x^2-1}-\sqrt{x^2-4}\right)$$
$$+\frac{1}{2}\log\left(\sqrt{x^2-1}+2\right)+\frac{1}{2}\log\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}-3\right|\right)$$

input
```
integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^
(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*log(sqrt(x^2 - 1) - sqrt(x^2 - 4) + 1) - 1/2*log(sqrt(x^2 - 1) - sqrt
(x^2 - 4)) + 1/2*log(sqrt(x^2 - 1) + 2) + 1/2*log(abs(-sqrt(x^2 - 1) + sqr
t(x^2 - 4) - 3))
```

3.280.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 8.19

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx = \frac{\ln\left(x-\sqrt{5}\right)}{4}-\operatorname{atanh}\left(\frac{\sqrt{3}-\sqrt{x^2-1}}{\sqrt{x^2-4}}\right)+\frac{\operatorname{atanh}\left(\frac{\sqrt{x^2-1}}{2}\right)}{2}+\frac{\ln\left(x+\sqrt{5}\right)}{4}-\frac{7\operatorname{atanh}\left(\frac{4\left(\sqrt{3}-\sqrt{x^2-1}\right)}{\sqrt{x^2-4}\left(\frac{\left(\sqrt{3}-\sqrt{x^2-1}\right)^2}{x^2-4}+1\right)}\right)}{4}+\frac{5\operatorname{atanh}\left(\frac{12150\left(\sqrt{3}-\sqrt{x^2-1}\right)}{\sqrt{x^2-4}\left(\frac{6075\left(\sqrt{3}-\sqrt{x^2-1}\right)^2}{2\left(x^2-4\right)}+\frac{6075}{2}\right)}\right)}{4}-\frac{\operatorname{atanh}\left(\sqrt{x^2-4}\right)}{2}$$

input
```
int(-(x*(4*(x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) - x^2*(x^2 - 1)^(1/2) - x^2*(
x^2 - 4)^(1/2)))/((x^4 - 5*x^2 + 4)*((x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) + 1
)),x)
```

output
```
log(x - 5^(1/2))/4 - atanh((3^(1/2) - (x^2 - 1)^(1/2))/(x^2 - 4)^(1/2)) +
atanh((x^2 - 1)^(1/2)/2)/2 + log(x + 5^(1/2))/4 - (7*atanh((4*(3^(1/2) - (
x^2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((3^(1/2) - (x^2 - 1)^(1/2))^2/(x^2 - 4)
 + 1))))/4 + (5*atanh((12150*(3^(1/2) - (x^2 - 1)^(1/2)))/((x^2 - 4)^(1/2)
*((6075*(3^(1/2) - (x^2 - 1)^(1/2))^2)/(2*(x^2 - 4)) + 6075/2))))/4 - atan
h((x^2 - 4)^(1/2))/2
```

3.280.10 Reduce [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$$
$$=\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

input
```
int((x*(sqrt(x**2 - 4)*x**2 - sqrt(x**2 - 4) + sqrt(x**2 - 1)*x**2 - 4*sqr
t(x**2 - 1)))/(sqrt(x**2 - 4)*x**4 - 5*sqrt(x**2 - 4)*x**2 + 4*sqrt(x**2 -
 4) + sqrt(x**2 - 1)*x**4 - 5*sqrt(x**2 - 1)*x**2 + 4*sqrt(x**2 - 1) + x**
4 - 5*x**2 + 4),x)
```

output
```
log(sqrt(x**2 - 4) + sqrt(x**2 - 1) + 1)
```

3.281 $\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx$

3.281.1 Optimal result 1645
3.281.2 Mathematica [C] (warning: unable to verify) 1646
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3.281.10Reduce [F] 1650

3.281.1 Optimal result

Integrand size = 40, antiderivative size = 4030

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx = \text{Too large to display}$$

output
```
1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(-1/3*(x^4+2*x^2+4*x+1)^(1/2)+1/3*(1+x)*(x^
4+2*x^2+4*x+1)^(1/2)+4*I*(-13+3*33^(1/2))^(1/3)*(x^4+2*x^2+4*x+1)^(1/2)/(4
*2^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(1/2))^(1/3)+6*I*x*(-13+3*33^(1/2))^(1
/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))-8*2^(2/3)*EllipticE((26-6*
33^(1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2))*(
-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/2)/((39+1
3*I*3^(1/2)-9*I*11^(1/2)-9*33^(1/2)+4*(3-I*3^(1/2))*(-26+6*33^(1/2))^(1/3)
)/(39-13*I*3^(1/2)+9*I*11^(1/2)-9*33^(1/2)+4*(3+I*3^(1/2))*(-26+6*33^(1/2)
)^(1/3)))^(1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*
11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/
2))^(2/3))^(1/2),((84+28*I*3^(1/2)-12*I*11^(1/2)-12*33^(1/2)+(3-I*3^(1/2)-
3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3))/(84-28*I*3^(1/2)+12*I*11^
(1/2)-12*33^(1/2)+(3+I*3^(1/2)+3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(
1/3)))^(1/2))*(x^4+2*x^2+4*x+1)^(1/2)*3^(1/2)/(-13+3*33^(1/2)+4*(-26+6*33^
(1/2))^(1/3))^(1/2)*(I*(-19899+x*(59697-10335*33^(1/2))+3445*33^(1/2)+(-26
+6*33^(1/2))^(2/3)*(-2574+466*33^(1/2))+(-26+6*33^(1/2))^(1/3)*(-19899+344
5*33^(1/2)))/(-39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3*I+3^(1/2))*(
-26+6*33^(1/2))^(1/3))/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/
2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6
*33^(1/2))^(2/3)))^(1/2)/(4*2^(2/3)-(-13+3*33^(1/2))^(1/3)+3*x*(-13+3*3...
```

3.281.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 13.55 (sec) , antiderivative size = 3168, normalized size of antiderivative = 0.79

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

output
```
(Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 -
(2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])^2*(-(EllipticF[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])
*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 +
 #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#
1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
 0]))]*Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) + EllipticPi[(1 + Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0])/(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1
 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - R
oot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & ,
 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^
2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]
 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*(1 + Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0]))*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root
[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2,
 0]))]*(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + ...
```

3.281.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}x^2-\sqrt{2}\int \sqrt{x^4+2x^2+4x+1}dx$$

input
```
Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

output
```
$Aborted
```

3.281. $\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx$

3.281.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.281.4 Maple [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 4640, normalized size of antiderivative = 1.15

method	result	size
default	Expression too large to display	4640
parts	Expression too large to display	4640
elliptic	Expression too large to display	4646

input
```
int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x,method=_RETURNVERB
OSE)
```

output
```
1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(1/3*x*(x^4+2*x^2+4*x+1)^(1/2)+4/3*(-1/6*(2
6+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-4/3+1/2*I*3^(1/2)*(-1/3*(26+
6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-
4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+
6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(
1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/
3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x
+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)
+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8
/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^
(1/3)+4/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1
/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(
(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33
^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(
1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26
+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6
*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-
1/3))^(1/2)/(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*...
```

3.281.5 Fricas [F]

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)\,dx$$
$$=\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\sqrt{x^4+2\,x^2+4\,x+1}\,dx$$

input
```
integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="
fricas")
```

output
```
integral(2*sqrt(2)*x - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1) - x, x)
```

3.281.6 Sympy [F]

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)\,dx$$
$$=\int \left(x\left(-1+2\sqrt{2}\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\right)\,dx$$

input
```
integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)
```

output
```
Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x + 1), x)
```

3.281.7 Maxima [F]

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)\,dx$$
$$=\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\sqrt{x^4+2\,x^2+4\,x+1}\,dx$$

input
```
integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="
maxima")
```

output
```
1/2*x^2*(2*sqrt(2) - 1) - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*x + 1)*sqrt
(x + 1), x)
```

3.281.8 Giac [F]

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx$$
$$=\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\,dx$$

input
```
integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="
giac")
```

output
```
integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx$$
$$=\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\,dx$$

input
```
int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2),x)
```

output
```
int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2), x)
```

3.281.10 Reduce [F]

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx = \text{Too large to display}$$

input
```
int( - sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2) + 2*sqrt(2)*x - x,x)
```

output
```
( - 30*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2)*x - 5*sqrt(x**4 + 2*x**2 + 4*
x + 1)*sqrt(2) - 14*sqrt(2)*int(sqrt(x**4 + 2*x**2 + 4*x + 1)/(x**4 + 2*x*
*2 + 4*x + 1),x) + 10*sqrt(2)*int((sqrt(x**4 + 2*x**2 + 4*x + 1)*x**3)/(x*
*4 + 2*x**2 + 4*x + 1),x) - 60*sqrt(2)*int((sqrt(x**4 + 2*x**2 + 4*x + 1)*
x**2)/(x**4 + 2*x**2 + 4*x + 1),x) + 10*sqrt(2)*int((sqrt(x**4 + 2*x**2 +
4*x + 1)*x)/(x**4 + 2*x**2 + 4*x + 1),x) - 48*sqrt(2)*log(x**2 - 2) + 105*
sqrt(2)*log( - 46*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2) - 65*sqrt(x**4 + 2
*x**2 + 4*x + 1) - 62*sqrt(2)*x**2 - 32*sqrt(2)*x - 8*sqrt(2) - 87*x**2 -
44*x - 11) + 21*sqrt(2)*log( - 41*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2)*x
- 58*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2) + 58*sqrt(x**4 + 2*x**2 + 4*x +
 1)*x + 82*sqrt(x**4 + 2*x**2 + 4*x + 1) - 55*sqrt(2)*x**3 - 106*sqrt(2)*x
**2 - 47*sqrt(2)*x - 10*sqrt(2) + 78*x**3 + 150*x**2 + 66*x + 14) + 6*sqrt
(2)*log( - 4*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2) + 5*sqrt(x**4 + 2*x**2
+ 4*x + 1) + 2*sqrt(2)*x**2 - 8*sqrt(2)*x - 8*sqrt(2) - 3*x**2 + 10*x + 11
) + 3*sqrt(2)*log( - 3*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2) + 4*sqrt(x**4
 + 2*x**2 + 4*x + 1) + 4*sqrt(2)*x**2 + 2*sqrt(2)*x - 6*x**2 - 3*x) - 90*s
qrt(2)*log(sqrt(x**4 + 2*x**2 + 4*x + 1) + x**2) + 90*sqrt(2)*log(sqrt(x**
4 + 2*x**2 + 4*x + 1) - x**2) - 24*sqrt(2)*log(sqrt(2) + 2*x + 2) + 15*sqr
t(2)*log(46*sqrt(x**4 + 2*x**2 + 4*x + 1)*sqrt(2) + 65*sqrt(x**4 + 2*x**2
+ 4*x + 1) - 62*sqrt(2)*x**2 - 32*sqrt(2)*x - 8*sqrt(2) - 87*x**2 - 44*...
```

3.281. $\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)\,dx$

3.282 $\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\mathbf{mc}^8+4\mathbf{mc}^9+24\mathbf{mc}^6x-48\mathbf{mc}^7x-144\mathbf{mc}^5x^2-24\right.\right.}{}$

3.282.1 Optimal result 1652
3.282.2 Mathematica [A] (verified) 1653
3.282.3 Rubi [A] (verified) 1654
3.282.4 Maple [C] (warning: unable to verify) 1655
3.282.5 Fricas [A] (verification not implemented) 1656
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3.282.1 Optimal result

Integrand size = 107, antiderivative size = 330

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+1\right.}{384x^2}$$

$$= \frac{e^{-\frac{x}{y}}(3-4\mathrm{mc})\mathrm{mc}^8\pi^2}{384x}+\frac{3}{8}e^{-\frac{x}{y}}\mathrm{mc}^5\pi^2y+\frac{1}{48}e^{-\frac{x}{y}}(3-22\mathrm{mc})\mathrm{mc}^2\pi^2xy$$
$$-\frac{1}{128}e^{-\frac{x}{y}}(1+4\mathrm{mc})\pi^2x^2y+\frac{1}{48}e^{-\frac{x}{y}}(3-22\mathrm{mc})\mathrm{mc}^2\pi^2y^2$$
$$+\frac{1}{4}e^{-\frac{x}{y}}\mathrm{mc}^3\pi^2y^2-\frac{1}{64}e^{-\frac{x}{y}}(1+4\mathrm{mc})\pi^2xy^2-\frac{1}{64}e^{-\frac{x}{y}}(1+4\mathrm{mc})\pi^2y^3$$
$$+\frac{1}{16}(1-2\mathrm{mc})\mathrm{mc}^6\pi^2\,\mathrm{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{(3-4\mathrm{mc})\mathrm{mc}^8\pi^2\,\mathrm{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}$$
$$+\frac{1}{32}\mathrm{mc}^3\pi^2\left(3\mathrm{mc}-12\mathrm{mc}^2-8y\right)y\,\mathrm{ExpIntegralEi}\left(-\frac{x}{y}\right)$$
$$-\frac{1}{32}e^{-\frac{x}{y}}\mathrm{mc}^3\pi^2(3(1-4\mathrm{mc})\mathrm{mc}-8x)y\log\left(\frac{x}{\mathrm{mc}^2}\right)+\frac{1}{4}e^{-\frac{x}{y}}\mathrm{mc}^3\pi^2y^2\log\left(\frac{x}{\mathrm{mc}^2}\right)$$

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4\right)+12\mathrm{mc}^3\pi^2\left(3\mathrm{mc}-12\mathrm{mc}^2-8x\right)x^2\log\right.}{384x^2}$$

output
```
1/384*(3-4*mc)*mc^8*Pi^2/exp(x/y)/x+3/8*mc^5*Pi^2*y/exp(x/y)+1/48*(3-22*mc
)*mc^2*Pi^2*x*y/exp(x/y)-1/128*(1+4*mc)*Pi^2*x^2*y/exp(x/y)+1/48*(3-22*mc)
*mc^2*Pi^2*y^2/exp(x/y)+1/4*mc^3*Pi^2*y^2/exp(x/y)-1/64*(1+4*mc)*Pi^2*x*y^
2/exp(x/y)-1/64*(1+4*mc)*Pi^2*y^3/exp(x/y)+1/16*(1-2*mc)*mc^6*Pi^2*Ei(-x/y
)+1/384*(3-4*mc)*mc^8*Pi^2*Ei(-x/y)/y+1/32*mc^3*Pi^2*(-12*mc^2+3*mc-8*y)*y
*Ei(-x/y)-1/32*mc^3*Pi^2*(3*(1-4*mc)*mc-8*x)*y*ln(x/mc^2)/exp(x/y)+1/4*mc^
3*Pi^2*y^2*ln(x/mc^2)/exp(x/y)
```

3.282.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{x}{y}}(\pi^2(-3\text{mc}^8+4\text{mc}^9+24\text{mc}^6x-48\text{mc}^7x-144\text{mc}^5x^2-24\text{mc}^2x^3+176\text{mc}^3x^3+3x^4+12\text{mc}x^4)+1}{384x^2}$$

$$=\frac{1}{384}\pi^2\left(-\frac{\text{mc}^3(-3\text{mc}^5+4\text{mc}^6-24\text{mc}^3y+48\text{mc}^4y-36\text{mc}y^2+144\text{mc}^2y^2+96y^3)\,\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{y}\right.$$

$$+\frac{e^{-\frac{x}{y}}(3\text{mc}^8-4\text{mc}^9+144\text{mc}^5xy+24\text{mc}^2xy(x+y)-16\text{mc}^3xy(11x+5y)-3xy(x^2+2xy+2y^2)-1}{x}$$

input
```
Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 -
 24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12
*mc^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

output
```
(Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*y^2 + 144
*mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)])/y) + (3*mc^8 - 4*mc^9 + 144*mc^
5*x*y + 24*mc^2*x*y*(x + y) - 16*mc^3*x*y*(11*x + 5*y) - 3*x*y*(x^2 + 2*x*
y + 2*y^2) - 12*mc*x*y*(x^2 + 2*x*y + 2*y^2) + 12*mc^3*x*y*(-3*mc + 12*mc^
2 + 8*(x + y))*Log[x/mc^2])/(E^(x/y)*x)))/384
```

3.282.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {27, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{x}{y}}\left(12\pi^2\text{mc}^3x^2\left(-12\text{mc}^2+3\text{mc}-8x\right)\log\left(\frac{x}{\text{mc}^2}\right)+\pi^2\left(4\text{mc}^9-3\text{mc}^8-48\text{mc}^7x+24\text{mc}^6x-144\text{mc}^5x^2+176\right.}{384x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{384}\int -\frac{e^{-\frac{x}{y}}\left(\pi^2\left((3-4\text{mc})\text{mc}^8+48x\text{mc}^7-24x\text{mc}^6+144x^2\text{mc}^5-176x^3\text{mc}^3+24x^3\text{mc}^2-12x^4\text{mc}-3x^4\right)-1\right.}{x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{384}\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left((3-4\text{mc})\text{mc}^8+48x\text{mc}^7-24x\text{mc}^6+144x^2\text{mc}^5-176x^3\text{mc}^3+24x^3\text{mc}^2-12x^4\text{mc}-3x^4\right)-1\right.}{x^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{384}\int \left(12e^{-\frac{x}{y}}\pi^2\left(12\text{mc}^2-3\text{mc}+8x\right)\log\left(\frac{x}{\text{mc}^2}\right)\text{mc}^3+\frac{e^{-\frac{x}{y}}\pi^2\left(\text{mc}^2-x\right)\left((3-4\text{mc})\text{mc}^6-(21-44\text{mc})x\text{mc}^4\right.}{x^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{384}\left(\frac{\pi^2(3-4\text{mc})\text{mc}^8\,\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{y}+24\pi^2(1-2\text{mc})\text{mc}^6\,\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+12\pi^2\text{mc}^3y(-12\text{mc}^2\right.$$

input
```
Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc
^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2
- 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

output
```
(((3 - 4*mc)*mc^8*Pi^2)/(E^(x/y)*x) + (144*mc^5*Pi^2*y)/E^(x/y) + (8*(3 -
22*mc)*mc^2*Pi^2*x*y)/E^(x/y) - (3*(1 + 4*mc)*Pi^2*x^2*y)/E^(x/y) + (8*(3
- 22*mc)*mc^2*Pi^2*y^2)/E^(x/y) + (96*mc^3*Pi^2*y^2)/E^(x/y) - (6*(1 + 4*m
c)*Pi^2*x*y^2)/E^(x/y) - (6*(1 + 4*mc)*Pi^2*y^3)/E^(x/y) + 24*(1 - 2*mc)*m
c^6*Pi^2*ExpIntegralEi[-(x/y)] + ((3 - 4*mc)*mc^8*Pi^2*ExpIntegralEi[-(x/y
)])/y + 12*mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpIntegralEi[-(x/y)] - (12*
mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2])/E^(x/y) + (96*mc^3*Pi^2*y
^2*Log[x/mc^2])/E^(x/y))/384
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.282.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.70 (sec) , antiderivative size = 1356, normalized size of antiderivative = 4.11

method	result	size
risch	Expression too large to display	1356

input
```
int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x
^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/
mc^2))/exp(x/y)/x^2,x,method=_RETURNVERBOSE)
```

3.282.

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\text{mc}^8+4\text{mc}^9+24\text{mc}^6x-48\text{mc}^7x-144\text{mc}^5x^2-24\text{mc}^2x^3+176\text{mc}^3x^3+3x^4+12\text{mc}x^4\right)+12\text{mc}^3\pi^2\left(3\text{mc}-12\text{mc}^2-8x\right)x^2\log\right.}{384x^2}$$

output
```
1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^2*csgn(I*x)*exp(-x/y)*x+1/8*I*y*Pi^3*mc^3
*csgn(I/mc^2)*csgn(I/mc^2*x)^2*exp(-x/y)*x-1/4*I*y*Pi^3*mc^3*csgn(I*mc)*cs
gn(I*mc^2)^2*exp(-x/y)*x+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*exp(-
x/y)*x+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)-
3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)-1/8*I*y
^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)*exp(-x/y)-3/64*I*y*Pi^3
*exp(-x/y)*mc^4*csgn(I*mc)^2*csgn(I*mc^2)+3/32*I*y*Pi^3*exp(-x/y)*mc^4*csg
n(I*mc)*csgn(I*mc^2)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2*x)^2*csgn(
I*x)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)^2+1/8*I*y*Pi
^3*mc^3*csgn(I*mc^2)^3*exp(-x/y)*x-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^3*exp(
-x/y)*x-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*exp(-x/y)+1/8*I*y^2*
Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/m
c^2*x)^2*csgn(I*x)*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*
x)^2*exp(-x/y)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*mc^2)-3/8*
I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)*csgn(I*mc^2)^2+3/16*I*y*Pi^3*exp(-x/y)*
mc^5*csgn(I/mc^2*x)^2*csgn(I*x)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*
csgn(I/mc^2*x)^2-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)*e
xp(-x/y)*x-1/128/y*Pi^2*mc^8*Ei(1,x/y)+1/96/y*Pi^2*mc^9*Ei(1,x/y)-1/128*y*
Pi^2*exp(-x/y)*x^2-1/64*y^2*Pi^2*x*exp(-x/y)-1/16*y^3*Pi^2*mc*exp(-x/y)+1/
16*y^2*Pi^2*mc^2*exp(-x/y)+3/8*y*Pi^2*exp(-x/y)*mc^5-1/96*Pi^2*mc^9/x*e...
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+1\right.}{384x^2}$$

$$= \frac{12\,(8\,\pi^2mc^3xy^3+(8\,\pi^2mc^3x^2+3\,\pi^2(4\,mc^5-mc^4)x)y^2)e^{\left(-\frac{x}{y}\right)}\log\left(\frac{x}{mc^2}\right)-(96\,\pi^2mc^3xy^3+36\,\pi^2(4\,mc^5}{}$$

input
```
integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*
mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2
*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="fricas")
```

3.282. $\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+12\mathrm{mc}^3\pi^2(3\mathrm{mc}-12\mathrm{mc}^2-8x)x^2\log\right.}{384x^2}$

output
```
1/384*(12*(8*pi^2*mc^3*x*y^3 + (8*pi^2*mc^3*x^2 + 3*pi^2*(4*mc^5 - mc^4)*x
)*y^2)*e^(-x/y)*log(x/mc^2) - (96*pi^2*mc^3*x*y^3 + 36*pi^2*(4*mc^5 - mc^4
)*x*y^2 + 24*pi^2*(2*mc^7 - mc^6)*x*y + pi^2*(4*mc^9 - 3*mc^8)*x)*Ei(-x/y)
 - (6*pi^2*(4*mc + 1)*x*y^4 + pi^2*(4*mc^9 - 3*mc^8)*y + 2*(3*pi^2*(4*mc +
 1)*x^2 + 4*pi^2*(10*mc^3 - 3*mc^2)*x)*y^3 - (144*pi^2*mc^5*x - 3*pi^2*(4*
mc + 1)*x^3 - 8*pi^2*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^(-x/y))/(x*y)
```

3.282.6 Sympy [A] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+1\right.}{384x^2}$$

$$= -\frac{\pi^2 mc^9 \operatorname{E}_2\left(\frac{x}{y}\right)}{96x} + \frac{\pi^2 mc^8 \operatorname{E}_2\left(\frac{x}{y}\right)}{128x} - \frac{\pi^2 mc^7 \operatorname{Ei}\left(-\frac{x}{y}\right)}{8} + \frac{\pi^2 mc^6 \operatorname{Ei}\left(-\frac{x}{y}\right)}{16}$$
$$+ \frac{3\pi^2 mc^5 y e^{-\frac{x}{y}}}{8} - \frac{3\pi^2 mc^5\left(y \operatorname{Ei}\left(-\frac{x}{y}\right) - y e^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right)\right)}{8}$$
$$+ \frac{3\pi^2 mc^4\left(y \operatorname{Ei}\left(-\frac{x}{y}\right) - y e^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right)\right)}{32} + \frac{11\pi^2 mc^3\left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)}{24}$$
$$- \frac{\pi^2 mc^3\left(y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) - y^2 e^{-\frac{x}{y}} + \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)\log\left(\frac{x}{mc^2}\right)\right)}{4}$$
$$- \frac{\pi^2 mc^2\left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)}{16} + \frac{\pi^2 mc\left(-x^2 y e^{-\frac{x}{y}} - 2xy^2 e^{-\frac{x}{y}} - 2y^3 e^{-\frac{x}{y}}\right)}{32}$$
$$+ \frac{\pi^2\left(-x^2 y e^{-\frac{x}{y}} - 2xy^2 e^{-\frac{x}{y}} - 2y^3 e^{-\frac{x}{y}}\right)}{128}$$

input
```
integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x*
*2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-12*mc*
*2+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)
```

output

```
-pi**2*mc**9*expint(2, x/y)/(96*x) + pi**2*mc**8*expint(2, x/y)/(128*x) -
pi**2*mc**7*Ei(-x/y)/8 + pi**2*mc**6*Ei(-x/y)/16 + 3*pi**2*mc**5*y*exp(-x/
y)/8 - 3*pi**2*mc**5*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/8 + 3*pi**2*m
c**4*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/32 + 11*pi**2*mc**3*(-x*y*exp
(-x/y) - y**2*exp(-x/y))/24 - pi**2*mc**3*(y**2*Ei(-x/y) - y**2*exp(-x/y)
+ (-x*y*exp(-x/y) - y**2*exp(-x/y))*log(x/mc**2))/4 - pi**2*mc**2*(-x*y*ex
p(-x/y) - y**2*exp(-x/y))/16 + pi**2*mc*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(
-x/y) - 2*y**3*exp(-x/y))/32 + pi**2*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/
y) - 2*y**3*exp(-x/y))/128
```

3.282.7 Maxima [F]

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\text{mc}^8 + 4\text{mc}^9 + 24\text{mc}^6x - 48\text{mc}^7x - 144\text{mc}^5x^2 - 24\text{mc}^2x^3 + 176\text{mc}^3x^3 + 3x^4 + 12\text{mc}x^4) + 1\right.}{384x^2}$$

$$= \int -\frac{\left(12\,\pi^2(12\,mc^2 - 3\,mc + 8\,x)mc^3x^2\log\left(\frac{x}{mc^2}\right) - \pi^2(4\,mc^9 - 3\,mc^8 - 48\,mc^7x + 24\,mc^6x - 144\,mc^5x\right.}{384\,x^2}$$

input

```
integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*
mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2
*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="maxima")
```

output

```
-1/96*pi^2*mc^9*gamma(-1, x/y)/y - 1/8*pi^2*mc^7*Ei(-x/y) + 1/128*pi^2*mc^
8*gamma(-1, x/y)/y + 3/8*pi^2*mc^5*y*e^(-x/y)*log(x/mc^2) + 1/16*pi^2*mc^6
*Ei(-x/y) - 3/8*pi^2*mc^5*y*Ei(-x/y) + 3/8*pi^2*mc^5*y*e^(-x/y) - 3/32*pi^
2*mc^4*y*e^(-x/y)*log(x/mc^2) + 3/32*pi^2*mc^4*y*Ei(-x/y) - 11/24*pi^2*(x*
y + y^2)*mc^3*e^(-x/y) + 1/4*pi^2*((x*y + y^2)*e^(-x/y)*log(x) + integrate
((2*x^2*log(mc) - x*y - y^2)*e^(-x/y)/x, x))*mc^3 + 1/16*pi^2*(x*y + y^2)*
mc^2*e^(-x/y) - 1/32*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^(-x/y) - 1/128*pi
^2*(x^2*y + 2*x*y^2 + 2*y^3)*e^(-x/y)
```

3.282.

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\text{mc}^8+4\text{mc}^9+24\text{mc}^6x-48\text{mc}^7x-144\text{mc}^5x^2-24\text{mc}^2x^3+176\text{mc}^3x^3+3x^4+12\text{mc}x^4)+12\text{mc}^3\pi^2(3\text{mc}-12\text{mc}^2-8x)x^2\log\right.}{384x^2}$$

3.282.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.43

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+1\right.}{384x^2}$$

$$= -\frac{4\,\pi^2 mc^9 x\mathrm{Ei}\left(-\frac{x}{y}\right)+4\,\pi^2 mc^9 y e^{\left(-\frac{x}{y}\right)}-3\,\pi^2 mc^8 x\mathrm{Ei}\left(-\frac{x}{y}\right)+48\,\pi^2 mc^7 xy\mathrm{Ei}\left(-\frac{x}{y}\right)-3\,\pi^2 mc^8 y e^{\left(-\frac{x}{y}\right)}-}{}$$

input
```
integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*
mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2
*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="giac")
```

output
```
-1/384*(4*pi^2*mc^9*x*Ei(-x/y) + 4*pi^2*mc^9*y*e^(-x/y) - 3*pi^2*mc^8*x*Ei
(-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) - 3*pi^2*mc^8*y*e^(-x/y) - 144*pi^2*mc^
5*x*y^2*e^(-x/y)*log(x/mc^2) - 24*pi^2*mc^6*x*y*Ei(-x/y) + 144*pi^2*mc^5*x
*y^2*Ei(-x/y) - 144*pi^2*mc^5*x*y^2*e^(-x/y) + 36*pi^2*mc^4*x*y^2*e^(-x/y)
*log(x/mc^2) - 96*pi^2*mc^3*x^2*y^2*e^(-x/y)*log(x/mc^2) - 96*pi^2*mc^3*x*
y^3*e^(-x/y)*log(x/mc^2) - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x*y^
3*Ei(-x/y) + 176*pi^2*mc^3*x^2*y^2*e^(-x/y) + 80*pi^2*mc^3*x*y^3*e^(-x/y)
- 24*pi^2*mc^2*x^2*y^2*e^(-x/y) + 12*pi^2*mc*x^3*y^2*e^(-x/y) - 24*pi^2*mc
^2*x*y^3*e^(-x/y) + 24*pi^2*mc*x^2*y^3*e^(-x/y) + 24*pi^2*mc*x*y^4*e^(-x/y
) + 3*pi^2*x^3*y^2*e^(-x/y) + 6*pi^2*x^2*y^3*e^(-x/y) + 6*pi^2*x*y^4*e^(-x
/y))/(x*y)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+1\right.}{384x^2}$$

$$= \mathrm{ei}\left(-\frac{x}{y}\right)\left(\frac{\frac{\Pi^2\,\mathrm{mc}^8}{128}-\frac{\Pi^2\,\mathrm{mc}^9}{96}}{y}+\frac{\Pi^2\,\mathrm{mc}^6}{16}-\frac{\Pi^2\,\mathrm{mc}^7}{8}+y\left(\frac{3\,\Pi^2\,\mathrm{mc}^4}{32}-\frac{3\,\Pi^2\,\mathrm{mc}^5}{8}\right)\right.$$
$$\left.-\frac{\Pi^2\,\mathrm{mc}^3\,y^2}{4}\right)$$
$$-\frac{2\,\Pi^2\,x^2\,y\,\mathrm{e}^{-\frac{x}{y}}\,(-72\,\mathrm{mc}^5+40\,\mathrm{mc}^3\,y-12\,\mathrm{mc}^2\,y+12\,\mathrm{mc}\,y^2+3\,y^2)+2\,\Pi^2\,x^3\,y\,\mathrm{e}^{-\frac{x}{y}}\,(88\,\mathrm{mc}^3-12\,\mathrm{mc}^2+1}{}$$

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$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+12\mathrm{mc}^3\pi^2(3\mathrm{mc}-12\mathrm{mc}^2-8x)x^2\log\right.}{384x^2}$

input
```
int((exp(-x/y)*((Pi^2*(176*mc^3*x^3 - 24*mc^2*x^3 - 144*mc^5*x^2 + 12*mc*x
^4 + 24*mc^6*x - 48*mc^7*x - 3*mc^8 + 4*mc^9 + 3*x^4))/384 - (Pi^2*mc^3*x^
2*log(x/mc^2)*(8*x - 3*mc + 12*mc^2))/32))/x^2,x)
```

output
```
ei(-x/y)*(((Pi^2*mc^8)/128 - (Pi^2*mc^9)/96)/y + (Pi^2*mc^6)/16 - (Pi^2*mc
^7)/8 + y*((3*Pi^2*mc^4)/32 - (3*Pi^2*mc^5)/8) - (Pi^2*mc^3*y^2)/4) - (2*P
i^2*x^2*y*exp(-x/y)*(12*mc*y^2 - 12*mc^2*y + 40*mc^3*y - 72*mc^5 + 3*y^2)
+ 2*Pi^2*x^3*y*exp(-x/y)*(3*y + 12*mc*y - 12*mc^2 + 88*mc^3) + Pi^2*mc^8*x
*exp(-x/y)*(4*mc - 3) + 3*Pi^2*x^4*y*exp(-x/y)*(4*mc + 1) - 96*Pi^2*mc^3*x
^3*y*log(x/mc^2)*exp(-x/y) - 12*Pi^2*mc^3*x^2*y*log(x/mc^2)*exp(-x/y)*(8*y
 - 3*mc + 12*mc^2))/(384*x^2)
```

3.282.10 Reduce [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+1\right.}{384x^2}$$

$$=\frac{\pi^2\left(-4e^{\frac{x}{y}}ei\left(-\frac{x}{y}\right)mc^9x+3e^{\frac{x}{y}}ei\left(-\frac{x}{y}\right)mc^8x-48e^{\frac{x}{y}}ei\left(-\frac{x}{y}\right)mc^7xy+24e^{\frac{x}{y}}ei\left(-\frac{x}{y}\right)mc^6xy-144e^{\frac{x}{y}}ei\left(-\right.\right.}{}$$

input
```
int((pi**2*( - 144*log(x/mc**2)*mc**5*x**2 + 36*log(x/mc**2)*mc**4*x**2 -
96*log(x/mc**2)*mc**3*x**3 + 4*mc**9 - 3*mc**8 - 48*mc**7*x + 24*mc**6*x -
 144*mc**5*x**2 + 176*mc**3*x**3 - 24*mc**2*x**3 + 12*mc*x**4 + 3*x**4))/(
384*e**(x/y)*x**2),x)
```

output
```
(pi**2*( - 4*e**(x/y)*ei(( - x)/y)*mc**9*x + 3*e**(x/y)*ei(( - x)/y)*mc**8
*x - 48*e**(x/y)*ei(( - x)/y)*mc**7*x*y + 24*e**(x/y)*ei(( - x)/y)*mc**6*x
*y - 144*e**(x/y)*ei(( - x)/y)*mc**5*x*y**2 + 36*e**(x/y)*ei(( - x)/y)*mc*
*4*x*y**2 - 96*e**(x/y)*ei(( - x)/y)*mc**3*x*y**3 + 144*log(x/mc**2)*mc**5
*x*y**2 - 36*log(x/mc**2)*mc**4*x*y**2 + 96*log(x/mc**2)*mc**3*x**2*y**2 +
 96*log(x/mc**2)*mc**3*x*y**3 - 4*mc**9*y + 3*mc**8*y + 144*mc**5*x*y**2 -
 176*mc**3*x**2*y**2 - 80*mc**3*x*y**3 + 24*mc**2*x**2*y**2 + 24*mc**2*x*y
**3 - 12*mc*x**3*y**2 - 24*mc*x**2*y**3 - 24*mc*x*y**4 - 3*x**3*y**2 - 6*x
**2*y**3 - 6*x*y**4))/(384*e**(x/y)*x*y)
```

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$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2(-3\mathrm{mc}^8+4\mathrm{mc}^9+24\mathrm{mc}^6x-48\mathrm{mc}^7x-144\mathrm{mc}^5x^2-24\mathrm{mc}^2x^3+176\mathrm{mc}^3x^3+3x^4+12\mathrm{mc}x^4)+12\mathrm{mc}^3\pi^2(3\mathrm{mc}-12\mathrm{mc}^2-8x)x^2\log\right.}{384x^2}$$

3.283 $\int \sec(x)\sin(2x)\,dx$

3.283.1 Optimal result 1661
3.283.2 Mathematica [A] (verified) 1661
3.283.3 Rubi [A] (verified) 1662
3.283.4 Maple [A] (verified) 1663
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3.283.8 Giac [A] (verification not implemented) 1664
3.283.9 Mupad [B] (verification not implemented) 1665
3.283.10 Reduce [F] 1665

3.283.1 Optimal result

Integrand size = 7, antiderivative size = 4

$$\int \sec(x)\sin(2x)\,dx = -2\cos(x)$$

output
```
-2*cos(x)
```

3.283.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x)\sin(2x)\,dx = -2\cos(x)$$

input
```
Integrate[Sec[x]*Sin[2*x],x]
```

output
```
-2*Cos[x]
```

3.283.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4775, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2x)}{\cos(x)}dx$$

$$\Big\downarrow \text{ 4775}$$

$$2\int \sin(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \sin(x)dx$$

$$\Big\downarrow \text{ 3118}$$

$$-2\cos(x)$$

input
```
Int[Sec[x]*Sin[2*x],x]
```

output
```
-2*Cos[x]
```

3.283.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.283.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-2\cos(x)$	5
default	$-2\cos(x)$	5
risch	$-2\cos(x)$	5

input
```
int(sin(2*x)/cos(x),x,method=_RETURNVERBOSE)
```

output
```
-2*cos(x)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x)\sin(2x)\,dx = -2\,\cos(x)$$

input
```
integrate(sin(2*x)/cos(x),x, algorithm="fricas")
```

output
```
-2*cos(x)
```

3.283.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \sec(x)\sin(2x)\,dx = -2\cos(x)$$

input
```
integrate(sin(2*x)/cos(x),x)
```

output
```
-2*cos(x)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x)\sin(2x)\,dx = -2\cos(x)$$

input
```
integrate(sin(2*x)/cos(x),x, algorithm="maxima")
```

output
```
-2*cos(x)
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x)\sin(2x)\,dx = -2\cos(x)$$

input
```
integrate(sin(2*x)/cos(x),x, algorithm="giac")
```

output
```
-2*cos(x)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x)\sin(2x)\,dx = -2\cos(x)$$

input
```
int(sin(2*x)/cos(x),x)
```

output
```
-2*cos(x)
```

3.283.10 Reduce [F]

$$\int \sec(x)\sin(2x)\,dx = \int \frac{\sin(2x)}{\cos(x)}dx$$

input
```
int(sin(2*x)/cos(x),x)
```

output
```
int(sin(2*x)/cos(x),x)
```

3.284 $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$

3.284.1 Optimal result 1666
3.284.2 Mathematica [A] (verified) 1666
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3.284.1 Optimal result

Integrand size = 71, antiderivative size = 71

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx = \frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(1+x+\sqrt{2}x+\sqrt{2}x^2-x^7\right) - \left(-1+\sqrt{2}\right)\log\left(-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}x^2+x^7\right)\right)$$

output
```
-1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))*(2^(1/2)-1)+1/2*ln(1+x-x^7+x*2^(
1/2)+x^2*2^(1/2))*(1+2^(1/2))
```

3.284.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx = \frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(1+x+\sqrt{2}x+\sqrt{2}x^2-x^7\right) - \left(-1+\sqrt{2}\right)\log\left(-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}x^2+x^7\right)\right)$$

input
```
Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 +
 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

output
```
((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 + Sqrt[2])
*Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

3.284.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{7x^{13}+10x^8+4x^7-7x^6-4x^3-4x^2+3x+3}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{1}{14}\int \frac{28(5x^8+6x^7+x^2+2x+1)}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx+\frac{1}{2}\log\left(x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{5x^8+6x^7+x^2+2x+1}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx+\frac{1}{2}\log\left(x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1\right)$$

$$\Big\downarrow \text{ 7293}$$

$$2\int \left(\frac{5x^8}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}+\frac{6x^7}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}+\frac{}{x^{14}-2x^8-}\right.$$
$$\frac{1}{2}\log\left(x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\int \frac{1}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx+2\int \frac{x}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx+\int\right.$$
$$\frac{1}{2}\log\left(x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1\right)$$

input
```
Int[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x -
 x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

output
```
$Aborted
```

3.284.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.284.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result
default	$\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right) \ln \left(x^{7}-x^{2} \sqrt{2}+\left(-1-\sqrt{2}\right) x-1\right)+\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right) \ln \left(-1+x^{7}+x \left(\sqrt{2}-1\right)+x^{2} \sqrt{2}\right.$
risch	$\frac{\ln \left(x^{7}-x^{2} \sqrt{2}+\left(-1-\sqrt{2}\right) x-1\right)}{2}+\frac{\ln \left(x^{7}-x^{2} \sqrt{2}+\left(-1-\sqrt{2}\right) x-1\right) \sqrt{2}}{2}-\frac{\ln \left(-1+x^{7}+x \left(\sqrt{2}-1\right)+x^{2} \sqrt{2}\right) \sqrt{2}}{2}+\frac{\ln \left(-1+x^{7}+x\right.}{}$

input
```
int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-
4*x^3-x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2+1/2*2^(1/2))*ln(x^7-x^2*2^(1/2)+(-1-2^(1/2))*x-1)+(-1/2*2^(1/2)+1/2)*
ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.93

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$
$$= \frac{1}{2}\sqrt{2}\log\left(\frac{x^{14}-2\,x^8-2\,x^7+2\,x^4+4\,x^3+3\,x^2-2\,\sqrt{2}(x^9+x^8-x^3-2\,x^2-x)+2\,x+1}{x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1}\right)$$
$$+\frac{1}{2}\log\left(x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1\right)$$

input
```
integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-
2*x^4-4*x^3-x^2+2*x+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2*sqrt(2)*
(x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4
*x^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^
2 + 2*x + 1)
```

3.284.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$
$$= \left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^7-\sqrt{2}x^2-2x\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)-1\right)$$
$$+\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)\log\left(x^7+\sqrt{2}x^2-2x\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)-1\right)$$

input
```
integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-2*x**
8-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)
```

output
```
(1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2) - 1) + (
1/2 - sqrt(2)/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(1/2 - sqrt(2)/2) - 1)
```

3.284.7 Maxima [F]

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx = \int \frac{7\,x^{13}+10\,x^8+4\,x^7-7\,x^6-4\,x^3-4\,x^2+3\,x+3}{x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1}\,dx$$

input
```
integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-
2*x^4-4*x^3-x^2+2*x+1),x, algorithm="maxima")
```

output
```
integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^1
4 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\left|x^7+\sqrt{2}x^2+\sqrt{2}x-x-1\right|\right) + \frac{1}{2}\sqrt{2}\log\left(\left|x^7-\sqrt{2}x^2-\sqrt{2}x-x-1\right|\right) + \frac{1}{2}\log\left(\left|x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1\right|\right)$$

input
```
integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-
2*x^4-4*x^3-x^2+2*x+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(x^7 + sqrt(2)*x^2 + sqrt(2)*x - x - 1)) + 1/2*sqrt(2)
*log(abs(x^7 - sqrt(2)*x^2 - sqrt(2)*x - x - 1)) + 1/2*log(abs(x^14 - 2*x^
8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1))
```

3.284.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.45

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx = \frac{\ln\left(\sqrt{2}\,x-x+\sqrt{2}\,x^2+x^7-1\right)}{2}+\frac{\ln\left(x^7-\sqrt{2}\,x-\sqrt{2}\,x^2-x-1\right)}{2} - \frac{\sqrt{2}\,\ln\left(\sqrt{2}\,x-x+\sqrt{2}\,x^2+x^7-1\right)}{2}+\frac{\sqrt{2}\,\ln\left(x^7-\sqrt{2}\,x-\sqrt{2}\,x^2-x-1\right)}{2}$$

input
```
int(-(3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13 + 3)/(x^2 - 2*
x + 4*x^3 + 2*x^4 + 2*x^7 + 2*x^8 - x^14 - 1),x)
```

output
```
log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1)/2 + log(x^7 - 2^(1/2)*x - 2^(1/
2)*x^2 - x - 1)/2 - (2^(1/2)*log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1))/2
 + (2^(1/2)*log(x^7 - 2^(1/2)*x - 2^(1/2)*x^2 - x - 1))/2
```

3.284.10 Reduce [F]

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$
$$= 7\left(\int \frac{x^{13}}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$+10\left(\int \frac{x^8}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$+4\left(\int \frac{x^7}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$-7\left(\int \frac{x^6}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$-4\left(\int \frac{x^3}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$-4\left(\int \frac{x^2}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$+3\left(\int \frac{x}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$
$$+3\left(\int \frac{1}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}dx\right)$$

input
```
int((7*x**13 + 10*x**8 + 4*x**7 - 7*x**6 - 4*x**3 - 4*x**2 + 3*x + 3)/(x**
14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x**2 + 2*x + 1),x)
```

output
```
7*int(x**13/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x**2 + 2*x + 1),x
) + 10*int(x**8/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x**2 + 2*x +
1),x) + 4*int(x**7/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x**2 + 2*x
 + 1),x) - 7*int(x**6/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x**2 +
2*x + 1),x) - 4*int(x**3/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x**2
 + 2*x + 1),x) - 4*int(x**2/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x
**2 + 2*x + 1),x) + 3*int(x/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x
**2 + 2*x + 1),x) + 3*int(1/(x**14 - 2*x**8 - 2*x**7 - 2*x**4 - 4*x**3 - x
**2 + 2*x + 1),x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1673

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
```

```
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
```

```
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
   if expnType_result <= expnType_optimal:
      if result.has(I):
         if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
               grade = "A"
               grade_annotation =""
            else:
               grade = "B"
               grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
         else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
      else: # result do not contain complex, this assumes optimal do not as well
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =""
         else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
   else:
      grade = "C"
      grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)
```

```
    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
```

```
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
```

```
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```