Computer algebra independent integration tests
`0_Independent_test_suites/Hearn_Problems`

Nasser M. Abbasi

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

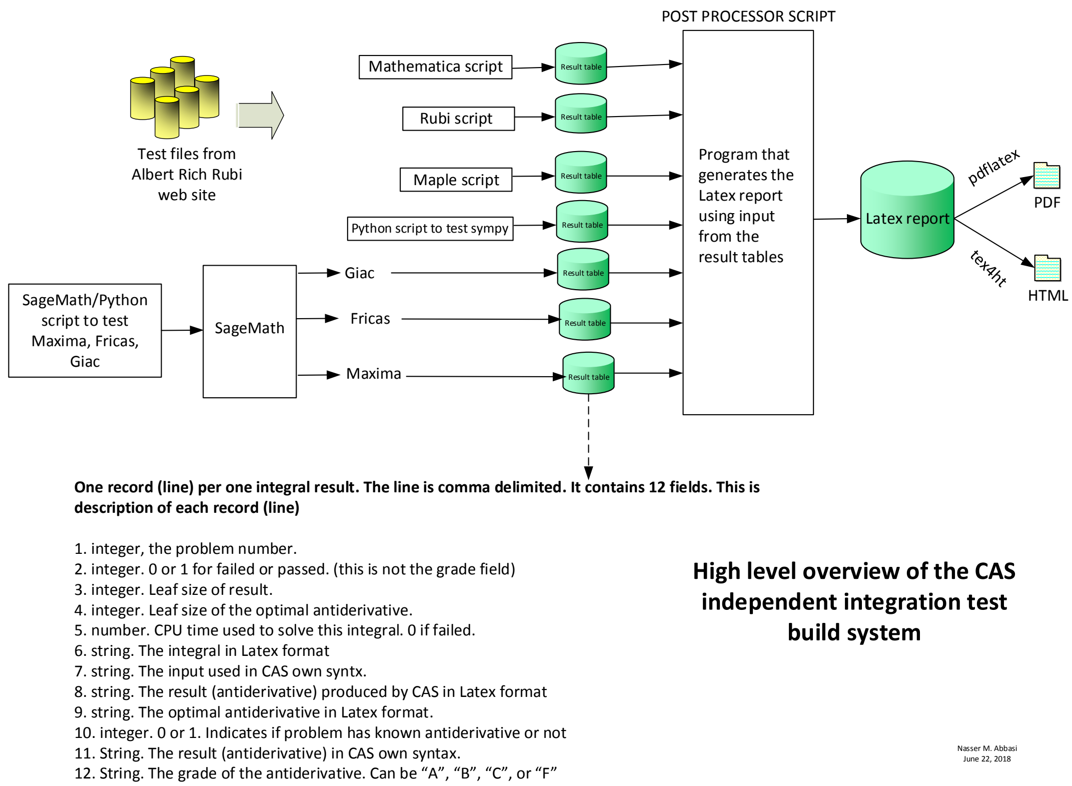
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.24 (279)	% 1.76 (5)
Rubi in Sympy	% 83.45 (237)	% 16.55 (47)
Mathematica	% 100. (284)	% 0. (0)
Maple	% 99.3 (282)	% 0.7 (2)
Maxima	% 84.15 (239)	% 15.85 (45)
Fricas	% 96.48 (274)	% 3.52 (10)
Sympy	% 85.92 (244)	% 14.08 (40)
Giac	% 89.44 (254)	% 10.56 (30)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

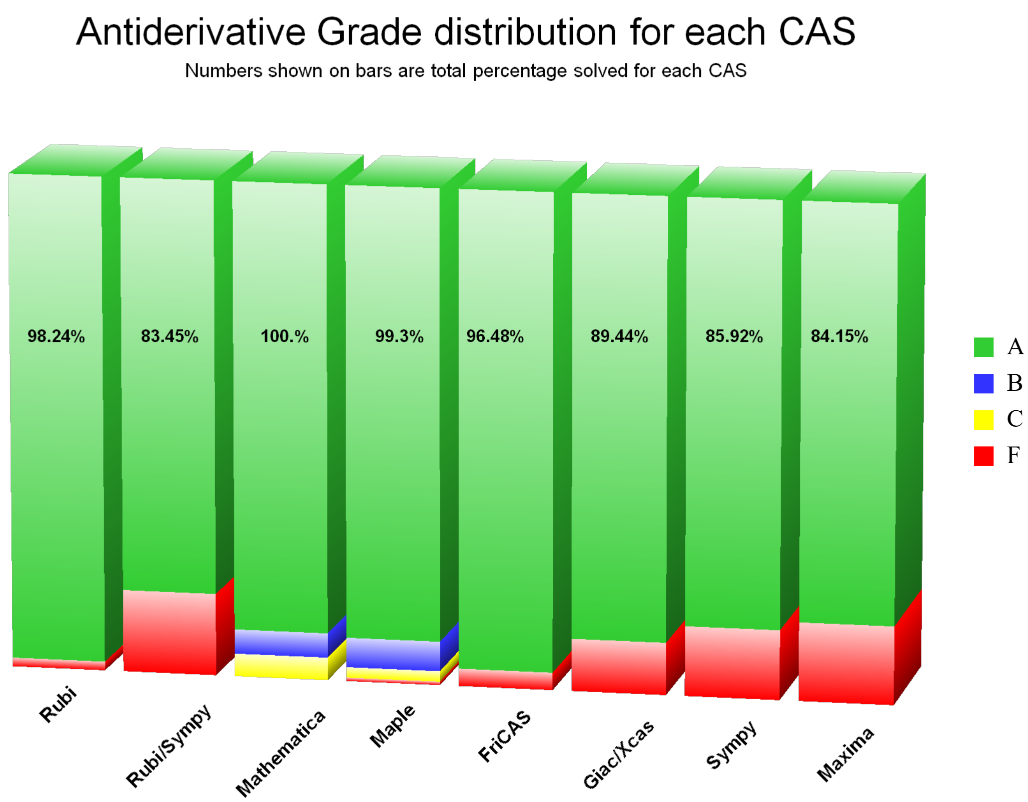
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

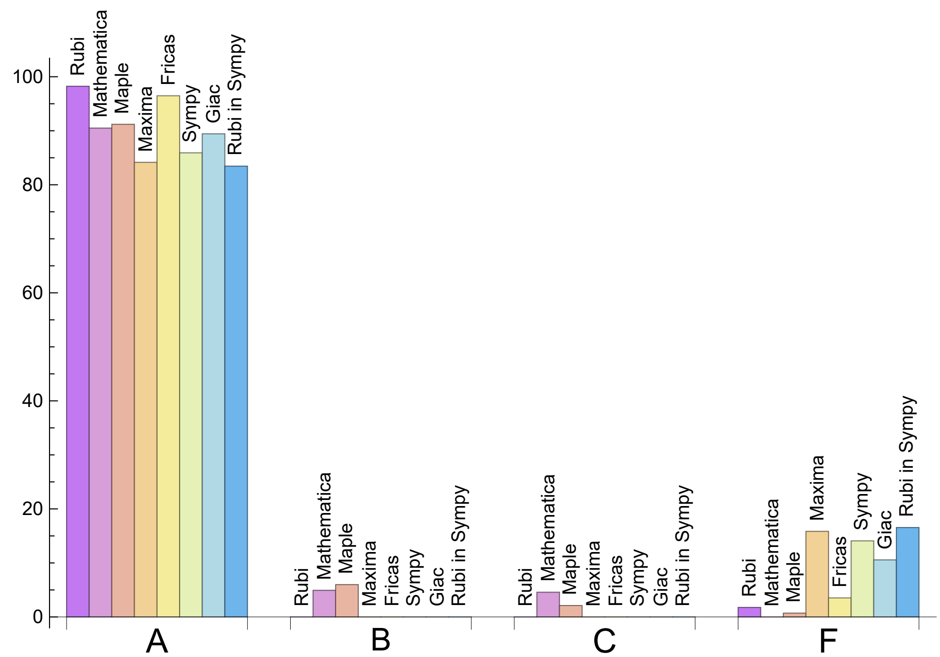
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.24	0.	0.	1.76
Rubi in Sympy	83.45	0.	0.	16.55
Mathematica	90.49	4.93	4.58	0.
Maple	91.2	5.99	2.11	0.7
Maxima	84.15	0.	0.	15.85
Fricas	96.48	0.	0.	3.52
Sympy	85.92	0.	0.	14.08
Giac	89.44	0.	0.	10.56

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	39.01	0.98	22.	1.
Rubi in Sympy	3.11	33.02	0.88	17.	0.86
Mathematica	0.14	66.57	1.52	23.	1.
Maple	0.02	4314.34	46.78	21.	0.94
Maxima	1.41	47.83	1.64	24.	1.17
Fricas	0.22	57.06	1.63	26.	1.25
Sympy	2.55	86.44	1.9	20.	1.
Giac	0.21	47.35	1.78	24.	1.22

1.8 list of integrals that has no closed form antiderivative

{75, 145, 170, 273, 281}

1.9 list of integrals not solved by each system

Not solved by Rubi {169, 274, 278, 279, 284}

Not solved by Rubi in Sympy {1, 3, 26, 27, 54, 58, 65, 66, 68, 69, 72, 73, 94, 99, 128, 129, 130, 138, 139, 140, 141, 142, 146, 156, 159, 168, 169, 174, 200, 201, 219, 225, 226, 229, 233, 234, 242, 250, 252, 265, 271, 272, 274, 278, 279, 282, 284}

Not solved by Mathematica {}

Not solved by Maple {86, 251}

Not solved by Maxima {8, 21, 35, 39, 40, 42, 43, 44, 45, 49, 51, 56, 64, 86, 122, 123, 147, 157, 160, 161, 163, 175, 176, 180, 181, 185, 186, 187, 196, 203, 205, 206, 207, 208, 211, 212, 213, 214, 224, 239, 251, 257, 278, 279, 284}

Not solved by Fricas {60, 61, 63, 86, 102, 103, 104, 105, 174, 257}

Not solved by Sympy {50, 56, 60, 61, 63, 86, 123, 128, 129, 145, 146, 147, 162, 163, 172, 176, 196, 197, 198, 204, 207, 208, 211, 212, 213, 214, 235, 236, 237, 238, 242, 249, 251, 257, 265, 275, 278, 279, 280, 282}

Not solved by Giac {33, 34, 44, 45, 47, 48, 56, 63, 86, 128, 129, 162, 163, 164, 168, 169, 198, 200, 201, 203, 235, 236, 237, 238, 257, 259, 274, 278, 279, 284}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {281}

Maple {281}

Maxima {145}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {20, 279}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	1	10	16	0
normalized size	1	1.	1.	0.81	1.	0.06	0.62	1.	0.
time (sec)	N/A	0.008	0.	0.01	1.398	0.2	0.025	0.211	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	1	17	22	17
normalized size	1	1.	1.	0.77	1.	0.05	0.77	1.	0.77
time (sec)	N/A	0.023	0.006	0.001	1.368	0.169	0.033	0.214	2.091

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	1	15	22	0
normalized size	1	1.	1.	0.77	1.	0.05	0.68	1.	0.
time (sec)	N/A	0.011	0.024	0.001	1.409	0.187	0.038	0.212	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	4	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	2.	1.
time (sec)	N/A	0.002	0.	0.004	1.387	0.199	0.023	0.211	0.024

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	43	62	29	31	24
normalized size	1	1.	0.67	0.75	1.19	1.72	0.81	0.86	0.67
time (sec)	N/A	0.027	0.04	0.026	1.412	0.208	0.122	0.211	2.229

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	32	45	24	46	24
normalized size	1	1.	0.88	0.78	1.	1.41	0.75	1.44	0.75
time (sec)	N/A	0.033	0.025	0.008	1.376	0.21	0.154	0.214	2.063

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	66	54	46	144	57	26
normalized size	1	1.	0.85	1.65	1.35	1.15	3.6	1.42	0.65
time (sec)	N/A	0.064	0.023	0.005	1.374	0.202	1.033	0.21	4.942

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	1	124	46	34
normalized size	1	1.	1.12	1.03	0.	0.03	3.65	1.35	1.
time (sec)	N/A	0.038	0.053	0.005	0.	0.198	0.258	0.213	2.058

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	19	26	19	14
normalized size	1	1.	1.	0.94	1.19	1.19	1.62	1.19	0.88
time (sec)	N/A	0.016	0.034	0.002	1.513	0.198	0.201	0.211	1.672

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	17	19	19	22	19	19
normalized size	1	1.	0.84	0.89	1.	1.	1.16	1.	1.
time (sec)	N/A	0.025	0.056	0.002	1.518	0.194	0.093	0.21	0.67

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	53	96	39	108	34
normalized size	1	1.	0.71	0.73	1.08	1.96	0.8	2.2	0.69
time (sec)	N/A	0.07	0.031	0.012	1.513	0.198	0.199	0.212	4.58

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	69	105	109	4216	109	39
normalized size	1	1.	0.91	1.01	1.54	1.6	62.	1.6	0.57
time (sec)	N/A	0.117	0.043	0.007	1.372	0.281	145.64	0.21	9.139

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	44	58	43	121	88	36
normalized size	1	1.	0.72	0.94	1.23	0.91	2.57	1.87	0.77
time (sec)	N/A	0.051	0.013	0.005	1.39	0.207	0.842	0.225	4.791

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	41	54	41	393	54	26
normalized size	1	1.	0.75	1.02	1.35	1.02	9.82	1.35	0.65
time (sec)	N/A	0.055	0.02	0.009	1.49	0.217	1.522	0.212	6.194

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	26	19	27	19
normalized size	1	1.	1.	0.74	0.96	0.96	0.7	1.	0.7
time (sec)	N/A	0.045	0.006	0.003	1.501	0.221	0.13	0.222	2.531

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	46	55	41	47	37
normalized size	1	1.	0.98	0.85	1.12	1.34	1.	1.15	0.9
time (sec)	N/A	0.048	0.015	0.006	1.532	0.213	0.172	0.216	3.22

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	42	54	31	49	31
normalized size	1	1.	0.79	0.74	0.98	1.26	0.72	1.14	0.72
time (sec)	N/A	0.2	0.027	0.006	1.562	0.223	0.162	0.216	11.235

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	131	73	97	73
normalized size	1	1.	0.75	0.68	1.14	1.54	0.86	1.14	0.86
time (sec)	N/A	0.077	0.057	0.004	1.552	0.223	0.193	0.21	5.899

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	131	73	97	73
normalized size	1	1.	0.75	0.68	1.14	1.54	0.86	1.14	0.86
time (sec)	N/A	0.079	0.014	0.002	1.555	0.22	0.192	0.212	6.412

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	72	77	70	72	63
normalized size	1	1.	1.09	0.81	1.07	1.15	1.04	1.07	0.94
time (sec)	N/A	0.072	0.154	0.004	1.544	0.21	0.237	0.209	4.831

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	0	27	20	24	12
normalized size	1	1.	0.94	1.06	0.	1.5	1.11	1.33	0.67
time (sec)	N/A	0.011	0.012	0.004	0.	0.227	0.035	0.209	0.861

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	57	72	201	113	31
normalized size	1	1.	0.85	0.92	1.46	1.85	5.15	2.9	0.79
time (sec)	N/A	0.031	0.018	0.002	1.383	0.223	0.836	0.206	3.559

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	92	130	597	208	51
normalized size	1	1.	0.95	1.22	1.53	2.17	9.95	3.47	0.85
time (sec)	N/A	0.049	0.03	0.006	1.406	0.23	1.574	0.208	6.073

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	14	7	15	7
normalized size	1	1.	1.	1.1	1.4	1.4	0.7	1.5	0.7
time (sec)	N/A	0.006	0.001	0.001	1.376	0.197	0.037	0.203	0.654

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	18	10	16	8
normalized size	1	1.	1.	1.08	1.33	1.5	0.83	1.33	0.67
time (sec)	N/A	0.005	0.004	0.	1.396	0.214	0.518	0.202	0.638

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	23	14	26	0
normalized size	1	1.	1.	1.06	1.33	1.28	0.78	1.44	0.
time (sec)	N/A	0.022	0.003	0.003	1.422	0.202	0.477	0.203	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	39	39	26	41	0
normalized size	1	1.	1.	0.97	1.26	1.26	0.84	1.32	0.
time (sec)	N/A	0.034	0.005	0.002	1.43	0.192	0.502	0.2	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	22	10	27	12
normalized size	1	1.	1.	1.06	1.33	1.22	0.56	1.5	0.67
time (sec)	N/A	0.012	0.005	0.003	1.377	0.213	0.165	0.199	1.706

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	35	19	41	24
normalized size	1	1.	1.	1.04	1.36	1.25	0.68	1.46	0.86
time (sec)	N/A	0.03	0.006	0.008	1.412	0.214	0.598	0.199	2.834

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	61	85	36	70	39
normalized size	1	1.	0.83	1.02	1.45	2.02	0.86	1.67	0.93
time (sec)	N/A	0.048	0.054	0.008	1.376	0.21	0.746	0.202	4.268

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	14	20	14	5
normalized size	1	1.	1.	1.1	1.4	1.4	2.	1.4	0.5
time (sec)	N/A	0.007	0.003	0.003	1.52	0.231	0.103	0.2	0.726

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	22	28	24	15	31	5
normalized size	1	1.	1.	2.2	2.8	2.4	1.5	3.1	0.5
time (sec)	N/A	0.008	0.004	0.003	1.408	0.225	0.115	0.198	0.898

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	58	89	77	78	0	73
normalized size	1	1.	0.85	0.74	1.14	0.99	1.	0.	0.94
time (sec)	N/A	0.095	0.075	0.004	1.575	0.214	0.652	0.	4.751

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	54	76	81	71	0	68
normalized size	1	1.	0.88	0.73	1.03	1.09	0.96	0.	0.92
time (sec)	N/A	0.075	0.025	0.003	1.562	0.239	0.603	0.	3.973

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	92	0	120	20	140	109
normalized size	1	1.	0.77	0.8	0.	1.04	0.17	1.22	0.95
time (sec)	N/A	0.129	0.038	0.002	0.	0.212	0.168	0.211	9.59

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	35	50	80	46	53	34
normalized size	1	1.	1.23	1.	1.43	2.29	1.31	1.51	0.97
time (sec)	N/A	0.027	0.025	0.002	1.517	0.22	0.623	0.213	0.933

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	36	55	77	48	53	31
normalized size	1	1.	1.23	1.03	1.57	2.2	1.37	1.51	0.89
time (sec)	N/A	0.031	0.022	0.002	1.573	0.228	0.625	0.213	0.959

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	171	120	111	204	219	151	128	168
normalized size	1	1.01	0.71	0.66	1.21	1.3	0.89	0.76	0.99
time (sec)	N/A	0.21	0.075	0.004	1.548	0.223	0.769	0.21	11.799

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	238	24	100	71
normalized size	1	1.	0.93	0.77	0.	3.26	0.33	1.37	0.97
time (sec)	N/A	0.129	0.08	0.031	0.	0.226	0.629	0.23	1.77

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	240	24	100	71
normalized size	1	1.	0.93	0.77	0.	3.29	0.33	1.37	0.97
time (sec)	N/A	0.052	0.04	0.019	0.	0.226	0.63	0.255	1.704

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	54	101	128	158	109	70
normalized size	1	1.	1.15	0.75	1.4	1.78	2.19	1.51	0.97
time (sec)	N/A	0.127	0.056	0.004	1.501	0.245	0.781	0.225	2.011

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	225	24	136	60
normalized size	1	1.	1.	0.9	0.	3.36	0.36	2.03	0.9
time (sec)	N/A	0.102	0.049	0.022	0.	0.212	0.788	0.25	1.478

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	182	92	69	60
normalized size	1	1.	1.	0.9	0.	2.72	1.37	1.03	0.9
time (sec)	N/A	0.03	0.027	0.019	0.	0.223	0.291	0.204	1.106

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	386	0	501	24	0	178
normalized size	1	1.	0.46	1.97	0.	2.56	0.12	0.	0.91
time (sec)	N/A	0.305	0.091	0.065	0.	0.215	0.818	0.	8.221

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	386	0	501	24	0	178
normalized size	1	1.	0.46	1.97	0.	2.56	0.12	0.	0.91
time (sec)	N/A	0.286	0.12	0.039	0.	0.212	0.844	0.	8.097

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	66	88	101	83	90	68
normalized size	1	1.	1.03	0.9	1.21	1.38	1.14	1.23	0.93
time (sec)	N/A	0.187	0.021	0.005	1.539	0.216	0.349	0.199	16.738

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	122	111	151	207	14	0	131
normalized size	1	1.	0.88	0.8	1.09	1.5	0.1	0.	0.95
time (sec)	N/A	0.405	0.054	0.056	1.575	0.225	0.989	0.	23.235

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	115	95	144	244	14	0	138
normalized size	1	1.	0.83	0.69	1.04	1.77	0.1	0.	1.
time (sec)	N/A	0.564	0.037	0.083	1.501	0.218	0.388	0.	38.371

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	1345	14	323	529
normalized size	1	1.	0.62	0.06	0.	3.97	0.04	0.95	1.56
time (sec)	N/A	0.681	0.008	0.01	0.	0.233	1.937	0.204	25.612

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	66	119	151	0	122	83
normalized size	1	1.	1.01	0.68	1.23	1.56	0.	1.26	0.86
time (sec)	N/A	0.096	0.036	0.001	1.569	0.219	0.	0.199	6.336

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	257	165	277	529
normalized size	1	1.	0.15	0.11	0.	0.93	0.6	1.01	1.92
time (sec)	N/A	0.59	0.017	0.009	0.	0.23	0.289	0.208	26.032

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	54	63	46	54	42
normalized size	1	1.	5.31	0.84	1.1	1.29	0.94	1.1	0.86
time (sec)	N/A	0.075	0.164	0.003	1.574	0.221	0.24	0.206	4.007

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	5	11	5
normalized size	1	1.	1.	1.12	1.38	1.38	0.62	1.38	0.62
time (sec)	N/A	0.004	0.013	0.001	1.375	0.22	0.061	0.2	0.461

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	0
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.
time (sec)	N/A	0.007	0.001	0.006	1.367	0.219	0.068	0.198	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	12
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.71
time (sec)	N/A	0.011	0.002	0.001	1.366	0.234	0.07	0.2	1.05

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	34	0	34	0	0	20
normalized size	1	1.	0.73	1.31	0.	1.31	0.	0.	0.77
time (sec)	N/A	0.019	0.012	0.01	0.	0.22	0.	0.	1.622

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	20	15	20	15
normalized size	1	1.	1.	1.07	1.07	1.33	1.	1.33	1.
time (sec)	N/A	0.008	0.002	0.001	1.404	0.22	0.08	0.199	0.543

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	104	96	139	133	139	0
normalized size	1	1.	1.	0.82	0.76	1.09	1.05	1.09	0.
time (sec)	N/A	0.205	0.003	0.001	1.411	0.224	0.315	0.208	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.62
time (sec)	N/A	0.018	0.001	0.001	1.4	0.223	0.068	0.203	1.427

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	4	0	0	4	2
normalized size	1	1.	1.	4.5	2.	0.	0.	2.	1.
time (sec)	N/A	0.004	0.088	0.004	1.445	0.	0.	0.214	0.024

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	11	7	0	0	7	3
normalized size	1	1.	1.	2.75	1.75	0.	0.	1.75	0.75
time (sec)	N/A	0.006	0.003	0.001	1.514	0.	0.	0.215	0.477

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	3	5	3
normalized size	1	1.	1.	1.33	1.33	1.33	1.	1.67	1.
time (sec)	N/A	0.018	0.001	0.	1.45	0.22	0.076	0.215	1.004

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	8	0	0	0	14
normalized size	1	1.	1.	0.88	0.47	0.	0.	0.	0.82
time (sec)	N/A	0.049	0.01	0.006	1.483	0.	0.	0.	2.33

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	16	15	16	8
normalized size	1	1.	1.	1.08	0.	1.33	1.25	1.33	0.67
time (sec)	N/A	0.025	0.004	0.001	0.	0.219	1.174	0.199	1.635

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	34	34	22	32	0
normalized size	1	1.	0.97	0.86	1.17	1.17	0.76	1.1	0.
time (sec)	N/A	0.021	0.002	0.001	1.401	0.238	0.103	0.204	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	48	63	63	44	63	0
normalized size	1	1.	0.85	0.89	1.17	1.17	0.81	1.17	0.
time (sec)	N/A	0.052	0.014	0.001	1.427	0.204	0.132	0.199	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	51	46	24	49	26
normalized size	1	1.	0.93	1.03	1.76	1.59	0.83	1.69	0.9
time (sec)	N/A	0.025	0.019	0.008	1.415	0.217	0.654	0.199	3.358

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	47	59	53	42	78	0
normalized size	1	1.	1.	1.02	1.28	1.15	0.91	1.7	0.
time (sec)	N/A	0.047	0.005	0.002	1.38	0.219	0.619	0.201	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	58	77	66	54	127	0
normalized size	1	1.	1.	0.98	1.31	1.12	0.92	2.15	0.
time (sec)	N/A	0.063	0.005	0.002	1.379	0.22	0.598	0.201	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	31	36	31	20
normalized size	1	1.	1.	1.04	1.35	1.35	1.57	1.35	0.87
time (sec)	N/A	0.02	0.004	0.003	1.55	0.222	0.552	0.2	2.325

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	29	38	31	31	38	31
normalized size	1	1.	1.41	1.07	1.41	1.15	1.15	1.41	1.15
time (sec)	N/A	0.071	0.003	0.001	1.385	0.216	0.554	0.199	1.227

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	49	49	53	49	0
normalized size	1	1.	1.	0.84	1.11	1.11	1.2	1.11	0.
time (sec)	N/A	0.047	0.004	0.003	1.571	0.228	0.583	0.202	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	59	63	59	0
normalized size	1	1.	1.	0.83	1.09	1.09	1.17	1.09	0.
time (sec)	N/A	0.053	0.005	0.002	1.533	0.218	0.595	0.204	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	42	42	29	45	20
normalized size	1	1.	1.	1.28	1.68	1.68	1.16	1.8	0.8
time (sec)	N/A	0.024	0.005	0.005	1.404	0.223	0.554	0.2	2.811

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.251	0.018	0.	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	3	5	3
normalized size	1	1.	1.	1.25	1.25	1.25	0.75	1.25	0.75
time (sec)	N/A	0.005	0.004	0.001	1.401	0.227	0.03	0.198	0.027

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	3	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.004	0.004	0.	1.386	0.224	0.031	0.199	0.023

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	4	15	5	8	5
normalized size	1	1.	1.	1.2	0.8	3.	1.	1.6	1.
time (sec)	N/A	0.004	0.007	0.001	1.382	0.232	0.039	0.2	0.034

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	3	23	3
normalized size	1	1.	1.	1.33	1.33	7.33	1.	7.67	1.
time (sec)	N/A	0.005	0.007	0.	1.388	0.219	0.048	0.203	0.031

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	26	34	50	75	35	17
normalized size	1	1.	1.29	1.24	1.62	2.38	3.57	1.67	0.81
time (sec)	N/A	0.041	0.043	0.019	1.546	0.228	0.653	0.203	1.998

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	20	23	15	23	3
normalized size	1	1.	11.	2.33	6.67	7.67	5.	7.67	1.
time (sec)	N/A	0.006	0.007	0.006	1.409	0.214	0.09	0.201	0.027

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	20	26	15	23	5
normalized size	1	1.	3.4	1.8	4.	5.2	3.	4.6	1.
time (sec)	N/A	0.005	0.007	0.003	1.405	0.228	0.092	0.201	0.03

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.011	0.003	0.006	1.397	0.228	0.035	0.199	0.495

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	15	22	15
normalized size	1	1.	1.	0.85	1.1	1.1	0.75	1.1	0.75
time (sec)	N/A	0.028	0.005	0.005	1.391	0.24	0.768	0.21	1.366

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	15	8	15	8
normalized size	1	1.	1.15	0.85	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.011	0.003	0.036	1.398	0.232	0.04	0.209	0.643

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	39
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.018	0.236	0.299	0.	0.	0.	0.	0.724

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	24	17	20	17
normalized size	1	1.	1.	0.84	1.05	1.26	0.89	1.05	0.89
time (sec)	N/A	0.034	0.006	0.009	1.388	0.229	1.59	0.206	2.29

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.011	0.003	0.005	1.363	0.264	0.034	0.211	0.502

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	12	14	8	12	8
normalized size	1	1.	1.36	1.	1.09	1.27	0.73	1.09	0.73
time (sec)	N/A	0.011	0.003	0.019	1.398	0.248	0.039	0.217	0.648

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	9	5	3	5
normalized size	1	1.	1.	1.5	1.5	4.5	2.5	1.5	2.5
time (sec)	N/A	0.01	0.003	0.02	1.423	0.232	0.033	0.231	0.481

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	15	14	20	15	10
normalized size	1	1.	1.	0.47	1.	0.93	1.33	1.	0.67
time (sec)	N/A	0.013	0.008	0.008	1.388	0.237	0.76	0.221	0.966

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	7	11	7
normalized size	1	1.	1.	1.12	1.38	1.38	0.88	1.38	0.88
time (sec)	N/A	0.014	0.003	0.	1.369	0.22	0.175	0.21	0.785

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	20	20	17	20	17
normalized size	1	1.	0.88	1.06	1.18	1.18	1.	1.18	1.
time (sec)	N/A	0.033	0.01	0.003	1.372	0.221	0.384	0.203	1.446

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	26	36	26	0
normalized size	1	1.	1.	1.	1.04	1.04	1.44	1.04	0.
time (sec)	N/A	0.023	0.005	0.003	1.399	0.216	0.405	0.199	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	35	39	56	35	36
normalized size	1	1.	0.71	0.9	0.85	0.95	1.37	0.85	0.88
time (sec)	N/A	0.048	0.037	0.019	1.345	0.251	0.84	0.202	1.651

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	31	31	39	31	32
normalized size	1	1.	0.94	0.7	0.94	0.94	1.18	0.94	0.97
time (sec)	N/A	0.034	0.006	0.02	1.427	0.228	0.818	0.201	1.363

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	9	7	9	7
normalized size	1	1.	1.	1.14	1.29	1.29	1.	1.29	1.
time (sec)	N/A	0.014	0.004	0.005	1.419	0.217	0.17	0.199	0.763

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	19	19	17	19	17
normalized size	1	1.	0.88	1.06	1.19	1.19	1.06	1.19	1.06
time (sec)	N/A	0.035	0.009	0.004	1.396	0.213	0.387	0.199	1.34

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	26	36	26	0
normalized size	1	1.	1.	1.	1.04	1.04	1.44	1.04	0.
time (sec)	N/A	0.025	0.005	0.005	1.411	0.219	0.405	0.2	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	35	39	56	35	36
normalized size	1	1.	0.71	0.9	0.85	0.95	1.37	0.85	0.88
time (sec)	N/A	0.051	0.035	0.016	1.411	0.245	0.843	0.2	1.606

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	31	34	39	31	32
normalized size	1	1.	0.94	0.7	0.94	1.03	1.18	0.94	0.97
time (sec)	N/A	0.036	0.007	0.009	1.419	0.221	0.817	0.199	1.34

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	0	2	3	2
normalized size	1	1.	1.	1.5	9.	0.	1.	1.5	1.
time (sec)	N/A	0.017	0.006	0.002	1.502	0.	0.833	0.198	0.636

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	0	12	3	2
normalized size	1	1.	1.	1.5	9.	0.	6.	1.5	1.
time (sec)	N/A	0.019	0.005	0.004	1.501	0.	1.836	0.202	0.641

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	20	0	17	18	7
normalized size	1	1.	1.	1.1	2.	0.	1.7	1.8	0.7
time (sec)	N/A	0.036	0.007	0.003	1.528	0.	2.469	0.198	1.612

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	23	0	10	15	10
normalized size	1	1.	1.	0.8	1.53	0.	0.67	1.	0.67
time (sec)	N/A	0.055	0.007	0.008	1.479	0.	2.231	0.199	2.14

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	27	24	12	22	10
normalized size	1	1.	1.	1.42	2.25	2.	1.	1.83	0.83
time (sec)	N/A	0.01	0.005	0.002	1.414	0.222	0.083	0.199	0.493

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	15	15	14	15	8
normalized size	1	1.	2.	1.09	1.36	1.36	1.27	1.36	0.73
time (sec)	N/A	0.009	0.009	0.002	1.391	0.211	0.135	0.199	0.614

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	14	14	12	14	7
normalized size	1	1.	2.1	1.1	1.4	1.4	1.2	1.4	0.7
time (sec)	N/A	0.008	0.009	0.004	1.39	0.209	0.139	0.205	0.612

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	15	24	19	18	10
normalized size	1	1.	1.	1.42	1.25	2.	1.58	1.5	0.83
time (sec)	N/A	0.009	0.043	0.001	1.39	0.216	0.136	0.204	0.635

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	29	15	36	29	76	8
normalized size	1	1.	1.	2.64	1.36	3.27	2.64	6.91	0.73
time (sec)	N/A	0.009	0.047	0.003	1.406	0.223	0.614	0.219	0.644

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	21	35	41	17	69	10
normalized size	1	1.	3.17	1.75	2.92	3.42	1.42	5.75	0.83
time (sec)	N/A	0.009	0.016	0.004	1.39	0.233	0.908	0.212	0.613

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	68	19	35	38	34	38	8
normalized size	1	1.	6.18	1.73	3.18	3.45	3.09	3.45	0.73
time (sec)	N/A	0.009	0.018	0.003	1.378	0.232	1.031	0.216	0.611

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	31	46	24	19
normalized size	1	1.	0.92	1.08	1.28	1.24	1.84	0.96	0.76
time (sec)	N/A	0.018	0.033	0.003	1.4	0.215	0.272	0.201	0.71

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	30	30	37	34	19
normalized size	1	1.	1.07	0.81	1.11	1.11	1.37	1.26	0.7
time (sec)	N/A	0.021	0.01	0.002	1.413	0.227	0.611	0.204	1.179

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	30	30	46	24	19
normalized size	1	1.	0.92	1.08	1.2	1.2	1.84	0.96	0.76
time (sec)	N/A	0.018	0.022	0.004	1.382	0.244	0.263	0.2	0.712

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	22	30	28	36	30	19
normalized size	1	1.	1.12	0.85	1.15	1.08	1.38	1.15	0.73
time (sec)	N/A	0.021	0.009	0.004	1.388	0.224	0.609	0.202	1.14

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	24	58	14	14
normalized size	1	1.	1.	1.1	1.4	2.4	5.8	1.4	1.4
time (sec)	N/A	0.016	0.006	0.004	1.385	0.208	2.197	0.199	0.7

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	12	3	41	7
normalized size	1	1.	0.67	0.56	1.33	1.33	0.33	4.56	0.78
time (sec)	N/A	0.016	0.006	0.004	1.386	0.22	0.216	0.197	0.488

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	14	7	11	8
normalized size	1	1.	0.67	0.75	1.17	1.17	0.58	0.92	0.67
time (sec)	N/A	0.015	0.008	0.007	1.396	0.217	0.645	0.201	0.504

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	20	24	8	14	8
normalized size	1	1.	2.3	1.1	2.	2.4	0.8	1.4	0.8
time (sec)	N/A	0.013	0.008	0.007	1.352	0.222	0.664	0.201	0.506

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	20	23	8	14	7
normalized size	1	1.	2.27	1.	1.82	2.09	0.73	1.27	0.64
time (sec)	N/A	0.015	0.009	0.014	1.351	0.215	0.683	0.203	0.502

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	1	114	65	31
normalized size	1	1.	1.	0.98	0.	0.02	2.85	1.62	0.78
time (sec)	N/A	0.071	0.047	0.014	0.	0.235	27.066	0.202	3.28

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	43	0	1	0	81	37
normalized size	1	1.	0.94	0.91	0.	0.02	0.	1.72	0.79
time (sec)	N/A	0.1	0.079	0.033	0.	0.242	0.	0.204	3.613

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	158	73	105	61	65
normalized size	1	1.	0.64	2.16	2.16	1.	1.44	0.84	0.89
time (sec)	N/A	0.078	0.133	0.004	1.401	0.241	1.528	0.2	2.628

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	16	20	15	10
normalized size	1	1.	1.	0.8	1.	1.07	1.33	1.	0.67
time (sec)	N/A	0.015	0.007	0.015	1.39	0.247	0.748	0.198	0.948

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	153	73	105	61	65
normalized size	1	1.	0.64	2.16	2.1	1.	1.44	0.84	0.89
time (sec)	N/A	0.075	0.134	0.007	1.421	0.229	1.601	0.202	2.713

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	19	42	14	39	14
normalized size	1	1.	1.	1.57	1.36	3.	1.	2.79	1.
time (sec)	N/A	0.013	0.005	0.004	1.364	0.215	0.086	0.203	0.496

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	138	655	181	0	0	0
normalized size	1	1.	0.97	1.33	6.3	1.74	0.	0.	0.
time (sec)	N/A	0.336	0.185	0.036	1.688	0.247	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	133	237	1034	221	0	0	0
normalized size	1	1.	0.87	1.55	6.76	1.44	0.	0.	0.
time (sec)	N/A	0.62	0.435	0.043	2.2	0.262	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	144	28	19	31	0
normalized size	1	1.	1.	1.33	9.6	1.87	1.27	2.07	0.
time (sec)	N/A	0.023	0.008	0.005	1.561	0.255	0.188	0.208	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	18	26	15	10
normalized size	1	1.	1.	0.8	1.	1.2	1.73	1.	0.67
time (sec)	N/A	0.014	0.009	0.025	1.378	0.225	0.696	0.2	1.052

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	26	14	14	20
normalized size	1	1.	0.58	0.79	0.58	1.08	0.58	0.58	0.83
time (sec)	N/A	0.039	0.007	0.004	1.416	0.221	0.053	0.199	1.621

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	12	24	12	12	15
normalized size	1	1.	0.86	2.14	1.71	3.43	1.71	1.71	2.14
time (sec)	N/A	0.034	0.005	0.02	1.397	0.23	0.055	0.201	1.577

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	69	34	30	104	454	29
normalized size	1	1.	0.69	2.16	1.06	0.94	3.25	14.19	0.91
time (sec)	N/A	0.021	0.018	0.014	1.426	0.222	1.885	0.219	1.553

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	71	32	27	107	455	29
normalized size	1	1.	0.65	2.29	1.03	0.87	3.45	14.68	0.94
time (sec)	N/A	0.018	0.016	0.012	1.43	0.242	1.89	0.215	1.562

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	50	137	81	81	308	1	88
normalized size	1	1.	0.6	1.63	0.96	0.96	3.67	0.01	1.05
time (sec)	N/A	0.083	0.06	0.016	1.45	0.25	5.818	0.22	6.05

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	142	78	78	304	1	88
normalized size	1	1.	0.59	1.71	0.94	0.94	3.66	0.01	1.06
time (sec)	N/A	0.077	0.054	0.015	1.408	0.253	5.653	0.216	5.921

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	225	144	155	665	1	0
normalized size	1	1.	0.58	1.39	0.89	0.96	4.1	0.01	0.
time (sec)	N/A	0.282	0.11	0.024	1.409	0.243	16.629	0.229	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	93	231	142	150	668	1	0
normalized size	1	1.	0.58	1.43	0.88	0.93	4.15	0.01	0.
time (sec)	N/A	0.271	0.093	0.023	1.39	0.224	16.633	0.226	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	169	437	251	274	1355	1	0
normalized size	1	1.	0.65	1.67	0.96	1.05	5.19	0.	0.
time (sec)	N/A	0.679	0.211	0.031	1.489	0.25	45.431	0.255	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	168	441	248	273	1352	1	0
normalized size	1	1.	0.65	1.7	0.95	1.05	5.2	0.	0.
time (sec)	N/A	0.663	0.175	0.03	1.45	0.244	45.556	0.256	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	23	114	18	0
normalized size	1	1.	1.	0.8	1.04	0.92	4.56	0.72	0.
time (sec)	N/A	0.047	0.014	0.037	1.393	0.246	21.373	0.2	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	34	114	30	27
normalized size	1	1.	1.	0.77	1.	1.13	3.8	1.	0.9
time (sec)	N/A	0.047	0.014	0.032	1.404	0.252	21.234	0.204	2.363

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	55	64	74	80	100	81	85
normalized size	1	1.	0.65	0.75	0.87	0.94	1.18	0.95	1.
time (sec)	N/A	0.104	0.085	0.019	1.427	0.237	3.497	0.199	4.894

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	324	0	0	0	0
normalized size	1	0.	0.	0.	23.14	0.	0.	0.	0.
time (sec)	N/A	0.686	1.077	0.344	0.132	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	55	22	0	24	0
normalized size	1	1.	1.	0.92	4.58	1.83	0.	2.	0.
time (sec)	N/A	0.057	0.011	0.034	1.382	0.241	0.	0.212	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	143	0	1	0	132	61
normalized size	1	1.	0.99	1.86	0.	0.01	0.	1.71	0.79
time (sec)	N/A	0.162	0.325	0.032	0.	0.238	0.	0.236	7.738

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	18	15	18	15
normalized size	1	1.	1.	0.82	0.94	1.06	0.88	1.06	0.88
time (sec)	N/A	0.008	0.004	0.	1.43	0.228	0.538	0.219	0.503

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	18	15	18	15
normalized size	1	1.	1.	0.82	0.82	1.06	0.88	1.06	0.88
time (sec)	N/A	0.007	0.004	0.	1.401	0.24	0.522	0.216	0.474

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	3	3	2	3	2
normalized size	1	1.	1.	1.	1.	1.	0.67	1.	0.67
time (sec)	N/A	0.002	0.	0.001	1.393	0.206	0.045	0.215	0.458

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	8	11	5
normalized size	1	1.	1.	1.12	1.38	1.38	1.	1.38	0.62
time (sec)	N/A	0.005	0.001	0.001	1.404	0.214	0.066	0.198	0.565

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	11	7	11	5
normalized size	1	1.	1.	1.	1.22	1.22	0.78	1.22	0.56
time (sec)	N/A	0.005	0.001	0.001	1.388	0.214	0.061	0.198	0.606

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	5	5	3	5	3
normalized size	1	1.	1.	2.25	1.25	1.25	0.75	1.25	0.75
time (sec)	N/A	0.017	0.002	0.002	1.486	0.202	1.296	0.198	1.34

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	31	30	15	32	22
normalized size	1	1.	1.	1.29	1.29	1.25	0.62	1.33	0.92
time (sec)	N/A	0.031	0.007	0.003	1.432	0.209	0.125	0.199	3.019

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.67
time (sec)	N/A	0.035	0.006	0.002	1.418	0.22	0.082	0.198	3.309

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	19	16	14	19	0
normalized size	1	1.	1.	1.15	1.46	1.23	1.08	1.46	0.
time (sec)	N/A	0.015	0.003	0.001	1.408	0.212	0.071	0.2	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	0	1	26	28	29
normalized size	1	1.	1.	0.71	0.	0.03	0.84	0.9	0.94
time (sec)	N/A	0.051	0.014	0.006	0.	0.222	0.19	0.2	5.51

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	14	18	18	19	18	15
normalized size	1	1.	0.67	0.67	0.86	0.86	0.9	0.86	0.71
time (sec)	N/A	0.017	0.004	0.001	1.386	0.213	0.091	0.199	1.58

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	102	102	136	136	102	136	0
normalized size	1	1.	0.63	0.63	0.83	0.83	0.63	0.83	0.
time (sec)	N/A	0.378	0.011	0.002	1.425	0.214	0.12	0.2	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	24	17	292	15
normalized size	1	1.	1.	1.06	0.	1.33	0.94	16.22	0.83
time (sec)	N/A	0.033	0.006	0.002	0.	0.212	0.963	0.222	1.864

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	19	24	320	15
normalized size	1	1.	1.	1.07	0.	1.36	1.71	22.86	1.07
time (sec)	N/A	0.024	0.004	0.002	0.	0.221	0.9	0.215	1.706

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	14	26	0	0	14
normalized size	1	1.	1.	1.24	0.82	1.53	0.	0.	0.82
time (sec)	N/A	0.03	0.007	0.011	1.48	0.226	0.	0.	2.243

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	43	79	0	73	0	0	54
normalized size	1	1.	0.67	1.23	0.	1.14	0.	0.	0.84
time (sec)	N/A	0.153	0.051	0.02	0.	0.251	0.	0.	7.419

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	22	22	12	0	12
normalized size	1	1.	1.	1.	1.38	1.38	0.75	0.	0.75
time (sec)	N/A	0.051	0.01	0.003	1.367	0.199	0.124	0.	2.816

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	17	15	8
normalized size	1	1.	1.	0.92	1.15	1.15	1.31	1.15	0.62
time (sec)	N/A	0.013	0.003	0.002	1.414	0.211	0.082	0.198	1.11

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	12	9	8	12	8
normalized size	1	1.	1.	0.73	1.09	0.82	0.73	1.09	0.73
time (sec)	N/A	0.007	0.002	0.002	1.459	0.238	0.288	0.201	0.487

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	8	5	8	5
normalized size	1	1.	1.	0.78	0.89	0.89	0.56	0.89	0.56
time (sec)	N/A	0.011	0.002	0.001	1.394	0.197	0.067	0.207	1.013

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	16	18	23	23	14	0	0
normalized size	1	1.	0.59	0.67	0.85	0.85	0.52	0.	0.
time (sec)	N/A	0.158	0.008	0.001	1.423	0.207	0.081	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	A	A	A	A	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	25	23	30	49	31	0	0
normalized size	1	0.	1.	0.92	1.2	1.96	1.24	0.	0.
time (sec)	N/A	1.534	0.041	0.041	1.94	0.227	0.64	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.012	0.015	0.	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	14	14	8	14	8
normalized size	1	1.	1.	1.09	1.27	1.27	0.73	1.27	0.73
time (sec)	N/A	0.026	0.004	0.008	1.599	0.218	2.878	0.198	2.111

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	21	20	20	0	20	20
normalized size	1	1.	0.77	0.95	0.91	0.91	0.	0.91	0.91
time (sec)	N/A	0.076	0.011	0.009	1.413	0.221	0.	0.198	4.459

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	17	28	15	15	10	15	17
normalized size	1	1.	0.74	1.22	0.65	0.65	0.43	0.65	0.74
time (sec)	N/A	0.019	0.005	0.003	1.476	0.21	0.078	0.198	2.281

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	0	12	16	0
normalized size	1	1.	1.	0.81	1.	0.	0.75	1.	0.
time (sec)	N/A	0.008	0.	0.001	1.548	0.	0.028	0.199	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	48	0	1	920	65	49
normalized size	1	1.	0.75	0.84	0.	0.02	16.14	1.14	0.86
time (sec)	N/A	0.062	0.042	0.008	0.	0.229	6.461	0.2	3.845

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	110	305	0	1	0	189	97
normalized size	1	1.	0.95	2.63	0.	0.01	0.	1.63	0.84
time (sec)	N/A	0.138	0.082	0.011	0.	0.25	0.	0.218	7.757

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	16	12	16	12
normalized size	1	1.	1.	0.81	1.	1.	0.75	1.	0.75
time (sec)	N/A	0.006	0.005	0.002	1.347	0.202	0.032	0.199	0.636

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	21	35	41	202	34	31
normalized size	1	1.	1.	0.62	1.03	1.21	5.94	1.	0.91
time (sec)	N/A	0.024	0.012	0.002	1.342	0.202	1.695	0.198	2.432

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	46	32	55	57	666	62	49
normalized size	1	1.	0.87	0.6	1.04	1.08	12.57	1.17	0.92
time (sec)	N/A	0.038	0.015	0.004	1.36	0.2	2.709	0.199	4.031

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	0	1	68	43	31
normalized size	1	1.	1.	0.8	0.	0.03	1.94	1.23	0.89
time (sec)	N/A	0.034	0.016	0.005	0.	0.23	2.242	0.2	2.256

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	0	1	44	55	32
normalized size	1	1.	1.	0.95	0.	0.03	1.13	1.41	0.82
time (sec)	N/A	0.035	0.033	0.007	0.	0.23	2.921	0.205	2.395

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	16	10	16	10
normalized size	1	1.	1.	0.93	1.14	1.14	0.71	1.14	0.71
time (sec)	N/A	0.008	0.003	0.001	1.334	0.212	0.033	0.199	0.656

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	35	26	162	31	29
normalized size	1	1.	0.72	0.66	1.09	0.81	5.06	0.97	0.91
time (sec)	N/A	0.031	0.012	0.002	1.319	0.202	1.729	0.198	2.398

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	55	42	600	62	48
normalized size	1	1.	0.69	0.63	1.08	0.82	11.76	1.22	0.94
time (sec)	N/A	0.042	0.016	0.003	1.351	0.198	2.565	0.198	3.915

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	0	1	24	28	22
normalized size	1	1.	1.	0.78	0.	0.04	1.04	1.22	0.96
time (sec)	N/A	0.02	0.01	0.004	0.	0.207	1.676	0.2	1.545

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	0	1	44	63	32
normalized size	1	1.	1.	0.98	0.	0.02	1.07	1.54	0.78
time (sec)	N/A	0.037	0.023	0.005	0.	0.219	3.456	0.201	2.274

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	25	0	34	26	28	15
normalized size	1	1.	1.04	1.09	0.	1.48	1.13	1.22	0.65
time (sec)	N/A	0.013	0.015	0.001	0.	0.225	0.041	0.199	0.895

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	61	78	216	122	37
normalized size	1	1.	0.79	0.9	1.27	1.62	4.5	2.54	0.77
time (sec)	N/A	0.036	0.021	0.003	1.369	0.235	0.919	0.204	3.754

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	42	70	50	230	70	56
normalized size	1	1.	0.87	0.76	1.27	0.91	4.18	1.27	1.02
time (sec)	N/A	0.067	0.064	0.006	1.552	0.237	2.109	0.202	3.586

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	19	19	2	20	10
normalized size	1	1.	3.17	0.92	1.58	1.58	0.17	1.67	0.83
time (sec)	N/A	0.006	0.004	0.002	1.419	0.204	0.148	0.202	0.149

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	50	96	158	119	42	34
normalized size	1	1.	0.72	1.16	2.23	3.67	2.77	0.98	0.79
time (sec)	N/A	0.02	0.025	0.005	1.364	0.211	3.969	0.227	1.733

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	20	22	46
normalized size	1	1.	1.	0.77	1.	1.	0.91	1.	2.09
time (sec)	N/A	0.017	0.009	0.002	1.43	0.234	0.39	0.199	2.866

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	15	15	8	15	8
normalized size	1	1.	1.	1.38	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.006	0.004	0.002	1.386	0.211	1.78	0.201	0.911

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	22	24	19	8	5
normalized size	1	1.	1.	0.88	2.75	3.	2.38	1.	0.62
time (sec)	N/A	0.01	0.008	0.011	1.502	0.22	1.499	0.211	1.213

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	34	34	8	34	12
normalized size	1	1.	1.	0.79	2.43	2.43	0.57	2.43	0.86
time (sec)	N/A	0.019	0.03	0.008	1.37	0.324	1.503	0.203	1.564

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	0	30	0	30	22
normalized size	1	1.	1.	0.78	0.	1.67	0.	1.67	1.22
time (sec)	N/A	0.036	0.009	0.008	0.	0.212	0.	0.206	2.04

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	37	23	23	74	0	20	20
normalized size	1	1.	1.23	0.77	0.77	2.47	0.	0.67	0.67
time (sec)	N/A	0.049	0.044	0.01	1.536	0.223	0.	0.219	3.84

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	19	46	0	0	10
normalized size	1	1.	1.	0.88	1.12	2.71	0.	0.	0.59
time (sec)	N/A	0.037	0.024	0.004	1.508	0.215	0.	0.	2.079

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	19	2	19	2
normalized size	1	1.	1.	1.5	1.5	9.5	1.	9.5	1.
time (sec)	N/A	0.003	0.004	0.002	1.552	0.238	0.135	0.235	0.077

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	15	19	19	17	0	0
normalized size	1	1.	1.5	0.75	0.95	0.95	0.85	0.	0.
time (sec)	N/A	1.704	0.025	0.002	1.533	0.202	1.608	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	41	27	19	0	0
normalized size	1	1.	1.	0.88	1.71	1.12	0.79	0.	0.
time (sec)	N/A	0.512	0.075	0.004	1.725	0.202	2.02	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	71	24	42	39	8	42	24
normalized size	1	1.	2.63	0.89	1.56	1.44	0.3	1.56	0.89
time (sec)	N/A	0.016	0.012	0.002	1.366	0.21	0.195	0.206	0.906

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	161	25	0	1	51	0	73
normalized size	1	1.	1.96	0.3	0.	0.01	0.62	0.	0.89
time (sec)	N/A	0.138	0.274	0.049	0.	0.24	2.274	0.	8.188

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	39	54	39	0	14	10
normalized size	1	1.	1.	3.25	4.5	3.25	0.	1.17	0.83
time (sec)	N/A	0.082	0.009	0.058	1.486	0.219	0.	0.199	3.217

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	0	1	66	46	37
normalized size	1	1.	1.	0.82	0.	0.02	1.65	1.15	0.92
time (sec)	N/A	0.026	0.015	0.002	0.	0.221	1.8	0.206	1.813

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	58	66	0	55	42	51	37
normalized size	1	1.	1.26	1.43	0.	1.2	0.91	1.11	0.8
time (sec)	N/A	0.073	0.05	0.006	0.	0.212	1.953	0.203	5.012

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	48	52	0	47	0	54	39
normalized size	1	1.	1.3	1.41	0.	1.27	0.	1.46	1.05
time (sec)	N/A	0.045	0.086	0.005	0.	0.217	0.	0.217	3.873

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	72	74	0	74	0	69	65
normalized size	1	1.	1.18	1.21	0.	1.21	0.	1.13	1.07
time (sec)	N/A	0.056	0.144	0.004	0.	0.228	0.	0.234	5.634

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	26	29	26	15
normalized size	1	1.	1.	0.87	1.13	1.13	1.26	1.13	0.65
time (sec)	N/A	0.01	0.006	0.002	1.316	0.208	0.765	0.2	1.397

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	32	36	22	19
normalized size	1	1.	1.	0.89	1.14	1.14	1.29	0.79	0.68
time (sec)	N/A	0.01	0.008	0.003	1.359	0.214	0.99	0.202	1.792

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	66	47	0	1	0	81	56
normalized size	1	1.	1.18	0.84	0.	0.02	0.	1.45	1.
time (sec)	N/A	0.078	0.089	0.01	0.	0.231	0.	0.264	4.303

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	70	0	1	0	97	75
normalized size	1	1.	1.04	0.86	0.	0.01	0.	1.2	0.93
time (sec)	N/A	0.067	0.113	0.004	0.	0.233	0.	0.258	4.027

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	59	56	0	55	0	42	39
normalized size	1	1.	1.34	1.27	0.	1.25	0.	0.95	0.89
time (sec)	N/A	0.094	0.13	0.011	0.	0.221	0.	0.263	7.004

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	80	78	0	85	0	42	65
normalized size	1	1.	1.18	1.15	0.	1.25	0.	0.62	0.96
time (sec)	N/A	0.114	0.304	0.011	0.	0.228	0.	0.26	9.442

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	30	10	15	10
normalized size	1	1.	1.	0.92	1.15	2.31	0.77	1.15	0.77
time (sec)	N/A	0.027	0.009	0.003	1.417	0.218	0.37	0.201	1.418

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	15	17	20	17
normalized size	1	1.	1.	0.84	1.05	0.79	0.89	1.05	0.89
time (sec)	N/A	0.012	0.003	0.003	1.469	0.208	0.083	0.203	1.079

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	21	27	16	10	16	10
normalized size	1	1.	1.	1.75	2.25	1.33	0.83	1.33	0.83
time (sec)	N/A	0.016	0.018	0.005	1.381	0.228	0.187	0.198	0.875

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	14	14	7	14	7
normalized size	1	1.	1.	1.	1.27	1.27	0.64	1.27	0.64
time (sec)	N/A	0.039	0.011	0.008	1.478	0.217	0.52	0.203	2.556

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	30	20	32	0
normalized size	1	1.	1.	0.89	1.15	1.11	0.74	1.19	0.
time (sec)	N/A	0.041	0.006	0.002	1.353	0.203	0.528	0.202	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	16	27	127	165	89	29
normalized size	1	1.	0.64	0.48	0.82	3.85	5.	2.7	0.88
time (sec)	N/A	0.014	0.015	0.002	1.329	0.203	72.813	0.204	1.221

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	12	7	15	7
normalized size	1	1.	1.	1.11	1.33	1.33	0.78	1.67	0.78
time (sec)	N/A	0.006	0.003	0.003	1.357	0.196	0.075	0.202	0.89

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	38	38	39	43	17
normalized size	1	1.	1.	0.74	2.	2.	2.05	2.26	0.89
time (sec)	N/A	0.014	0.007	0.003	1.505	0.208	0.759	0.201	1.116

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	19	15	15	15
normalized size	1	1.	0.84	0.68	0.79	1.	0.79	0.79	0.79
time (sec)	N/A	0.007	0.004	0.002	1.369	0.221	0.663	0.201	0.976

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	112	85	0	138	155	139	78
normalized size	1	1.	1.27	0.97	0.	1.57	1.76	1.58	0.89
time (sec)	N/A	0.087	0.058	0.005	0.	0.219	2.428	0.495	3.195

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	46	12	7	23	0
normalized size	1	1.	1.	1.11	5.11	1.33	0.78	2.56	0.
time (sec)	N/A	0.019	0.005	0.006	1.388	0.217	0.176	0.199	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	46	12	7	23	0
normalized size	1	1.	1.	1.11	5.11	1.33	0.78	2.56	0.
time (sec)	N/A	0.018	0.005	0.004	1.365	0.217	0.169	0.229	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	35	7	18	7
normalized size	1	1.	1.	0.89	1.	3.89	0.78	2.	0.78
time (sec)	N/A	0.008	0.006	0.004	1.349	0.215	0.127	0.225	1.412

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	18	35	22	31	12
normalized size	1	1.	1.	1.33	1.5	2.92	1.83	2.58	1.
time (sec)	N/A	0.052	0.018	0.009	1.334	0.23	2.108	0.226	6.597

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	30	43	76	30	0
normalized size	1	1.	1.	0.82	1.07	1.54	2.71	1.07	0.
time (sec)	N/A	0.038	0.014	0.008	1.334	0.227	3.052	0.212	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	18	15	20	15
normalized size	1	1.	1.	0.76	0.95	0.86	0.71	0.95	0.71
time (sec)	N/A	0.018	0.003	0.002	1.608	0.212	0.316	0.198	1.971

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	18	15	23	15
normalized size	1	1.	1.	0.76	0.95	0.86	0.71	1.1	0.71
time (sec)	N/A	0.018	0.003	0.002	1.625	0.209	0.326	0.199	1.944

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	23	30	26	26	30	26
normalized size	1	1.	1.39	1.	1.3	1.13	1.13	1.3	1.13
time (sec)	N/A	0.033	0.003	0.002	1.431	0.214	0.543	0.217	1.031

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	38	32	19	0
normalized size	1	1.	1.	0.83	1.06	2.11	1.78	1.06	0.
time (sec)	N/A	0.028	0.024	0.006	1.37	0.239	0.741	0.199	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	38	32	19	0
normalized size	1	1.	1.	0.83	1.06	2.11	1.78	1.06	0.
time (sec)	N/A	0.023	0.015	0.006	1.396	0.232	0.741	0.199	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	58	32	0	0	14
normalized size	1	1.	3.33	1.42	4.83	2.67	0.	0.	1.17
time (sec)	N/A	0.011	0.02	0.034	1.544	0.214	0.	0.	0.491

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	23	58	35	0	0	12
normalized size	1	1.	3.	1.64	4.14	2.5	0.	0.	0.86
time (sec)	N/A	0.015	0.019	0.028	1.593	0.219	0.	0.	0.524

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	16	22	38	14	0	0	12
normalized size	1	1.	1.33	1.83	3.17	1.17	0.	0.	1.
time (sec)	N/A	0.012	0.006	0.021	1.596	0.211	0.	0.	0.496

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	27	24	0	0	14
normalized size	1	1.	1.29	1.57	1.93	1.71	0.	0.	1.
time (sec)	N/A	0.015	0.008	0.026	1.557	0.218	0.	0.	0.517

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	0	18	63	18	17
normalized size	1	1.	0.81	0.67	0.	0.86	3.	0.86	0.81
time (sec)	N/A	0.015	0.024	0.002	0.	0.209	0.697	0.2	0.687

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	31	24	24	20	26	20
normalized size	1	1.	0.92	1.29	1.	1.	0.83	1.08	0.83
time (sec)	N/A	0.024	0.008	0.003	1.383	0.21	0.131	0.23	1.496

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	45	22	7	22	7
normalized size	1	1.	1.	0.75	3.75	1.83	0.58	1.83	0.58
time (sec)	N/A	0.012	0.007	0.005	1.363	0.23	1.502	0.212	1.102

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	32	0	32	0
normalized size	1	1.	1.	2.88	1.	1.	0.	1.	0.
time (sec)	N/A	0.027	0.013	0.018	1.372	0.204	0.	0.2	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	54	31	35	31
normalized size	1	1.	1.	0.82	1.06	1.64	0.94	1.06	0.94
time (sec)	N/A	0.025	0.02	0.003	1.501	0.206	0.134	0.2	1.977

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	15	3	15	3
normalized size	1	1.	1.	1.33	1.33	5.	1.	5.	1.
time (sec)	N/A	0.005	0.003	0.001	1.404	0.219	0.041	0.2	0.035

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	65	19	46	14
normalized size	1	1.	1.5	1.17	1.83	5.42	1.58	3.83	1.17
time (sec)	N/A	0.015	0.005	0.004	1.553	0.247	0.052	0.208	0.52

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	24	7	15	10
normalized size	1	1.	1.	1.33	1.33	8.	2.33	5.	3.33
time (sec)	N/A	0.006	0.003	0.	1.4	0.224	0.115	0.198	21.24

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	24	12	16	17
normalized size	1	1.	1.	1.33	1.33	8.	4.	5.33	5.67
time (sec)	N/A	0.006	0.005	0.001	1.462	0.222	0.401	0.228	35.117

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	8	11	5
normalized size	1	1.	1.	1.12	1.38	1.38	1.	1.38	0.62
time (sec)	N/A	0.005	0.001	0.002	1.365	0.214	0.067	0.207	0.56

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	32	14	14	0	15	39
normalized size	1	1.	0.59	0.65	0.29	0.29	0.	0.31	0.8
time (sec)	N/A	0.038	0.014	0.003	1.48	0.207	0.	0.197	2.403

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	16	16	12	18	0
normalized size	1	1.	1.06	0.81	1.	1.	0.75	1.12	0.
time (sec)	N/A	0.017	0.004	0.002	1.386	0.204	0.056	0.198	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	41	0	54	32
normalized size	1	1.	0.9	0.	0.	1.02	0.	1.35	0.8
time (sec)	N/A	0.043	0.028	0.003	0.	0.216	0.	0.202	2.506

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	67	63	78	74	109	77	0
normalized size	1	1.	0.52	0.49	0.61	0.58	0.85	0.6	0.
time (sec)	N/A	0.292	0.189	0.009	1.371	0.234	8.013	0.202	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	23	23	27	20	27
normalized size	1	1.	0.6	0.67	0.77	0.77	0.9	0.67	0.9
time (sec)	N/A	0.055	0.027	0.004	1.355	0.219	1.023	0.2	3.061

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	9	23	17	9	7
normalized size	1	1.	0.82	0.73	0.82	2.09	1.55	0.82	0.64
time (sec)	N/A	0.003	0.002	0.001	1.33	0.188	0.094	0.199	0.531

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	43	53	41	45	37
normalized size	1	1.	1.	0.82	1.08	1.32	1.02	1.12	0.92
time (sec)	N/A	0.042	0.013	0.001	1.525	0.233	0.163	0.199	2.984

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	23	23	15	24	7
normalized size	1	1.	2.88	2.75	2.88	2.88	1.88	3.	0.88
time (sec)	N/A	0.009	0.004	0.001	1.319	0.197	0.086	0.202	1.109

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	277	1210	0	0	0	0	321
normalized size	1	1.	1.22	5.33	0.	0.	0.	0.	1.41
time (sec)	N/A	0.321	0.294	0.014	0.	0.	0.	0.	24.514

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	26	34	32	32	34	32
normalized size	1	1.	1.25	1.08	1.42	1.33	1.33	1.42	1.33
time (sec)	N/A	0.018	0.005	0.002	1.321	0.222	2.813	0.201	0.667

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	71	54	77	81	73	0	70
normalized size	1	1.	0.91	0.69	0.99	1.04	0.94	0.	0.9
time (sec)	N/A	0.09	0.035	0.003	1.509	0.213	0.606	0.	4.805

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	19	2	19	2
normalized size	1	1.	1.	1.5	1.5	9.5	1.	9.5	1.
time (sec)	N/A	0.003	0.005	0.	1.544	0.205	0.137	0.2	0.076

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	27	109	24	34	24
normalized size	1	1.	1.	0.78	1.	4.04	0.89	1.26	0.89
time (sec)	N/A	0.009	0.01	0.003	1.48	0.216	0.235	0.207	0.593

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	8	15	8
normalized size	1	1.	1.	1.1	1.4	2.2	0.8	1.5	0.8
time (sec)	N/A	0.011	0.004	0.004	1.354	0.19	0.063	0.211	1.063

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	19	12	19	12
normalized size	1	1.	1.	0.94	1.19	1.19	0.75	1.19	0.75
time (sec)	N/A	0.009	0.005	0.003	1.524	0.221	0.126	0.201	1.015

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	45	32	32	51	27
normalized size	1	1.	0.72	0.85	1.12	0.8	0.8	1.27	0.68
time (sec)	N/A	0.049	0.02	0.003	1.496	0.232	0.447	0.202	3.041

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	18	39	0	20	0
normalized size	1	1.	1.38	0.67	0.86	1.86	0.	0.95	0.
time (sec)	N/A	0.178	0.022	0.041	1.329	0.233	0.	0.211	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	22	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.57	0.71
time (sec)	N/A	0.011	0.003	0.006	1.403	0.228	0.056	0.204	0.494

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	24	14	24	14
normalized size	1	1.	1.	0.94	1.22	1.33	0.78	1.33	0.78
time (sec)	N/A	0.037	0.006	0.006	1.41	0.205	0.114	0.205	2.816

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	22	7	22	7
normalized size	1	1.	1.	0.7	0.8	2.2	0.7	2.2	0.7
time (sec)	N/A	0.005	0.006	0.001	1.498	0.207	0.15	0.201	0.522

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	19	3	19	3
normalized size	1	1.	1.	0.83	0.83	3.17	0.5	3.17	0.5
time (sec)	N/A	0.004	0.005	0.002	1.5	0.201	0.141	0.204	0.512

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	22	22	22	19
normalized size	1	1.	1.	0.81	1.05	1.05	1.05	1.05	0.9
time (sec)	N/A	0.03	0.014	0.002	1.643	0.194	0.11	0.199	0.684

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	63	99	46	62	0
normalized size	1	1.	0.96	0.79	1.19	1.87	0.87	1.17	0.
time (sec)	N/A	0.053	0.031	0.007	1.503	0.197	0.222	0.2	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	66	55	55	762	59	0
normalized size	1	1.	1.	1.35	1.12	1.12	15.55	1.2	0.
time (sec)	N/A	0.129	0.04	0.006	1.372	0.23	35.828	0.203	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	4.25	0.023	0.	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	A	A	A	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	28	28	36	59	26	0	0
normalized size	1	0.	1.	1.	1.29	2.11	0.93	0.	0.
time (sec)	N/A	0.442	9.876	0.03	1.542	0.225	0.788	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	136	133	216	196	0	154	180
normalized size	1	1.	0.68	0.67	1.09	0.98	0.	0.77	0.9
time (sec)	N/A	0.182	0.254	0.016	1.445	0.258	0.	0.21	13.122

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	20	24	15	20	15
normalized size	1	1.	1.	0.89	1.11	1.33	0.83	1.11	0.83
time (sec)	N/A	0.09	0.011	0.004	1.393	0.204	0.541	0.211	0.49

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	28	43	36	50	20
normalized size	1	1.	1.	0.92	1.17	1.79	1.5	2.08	0.83
time (sec)	N/A	0.025	0.045	0.002	1.399	0.206	0.804	0.22	0.615

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	B	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	5137	1197351	0	242	0	0	0
normalized size	1	0.	54.65	12737.8	0.	2.57	0.	0.	0.
time (sec)	N/A	2.88	6.138	0.703	0.	0.295	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	C	F	A	F	F	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	630	352	0	301	0	0	0
normalized size	1	0.	4.44	2.48	0.	2.12	0.	0.	0.
time (sec)	N/A	5.567	2.385	0.177	0.	0.282	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	97	1088	231	219	0	105	19
normalized size	1	1.	4.62	51.81	11.	10.43	0.	5.	0.9
time (sec)	N/A	0.437	0.082	0.095	1.655	0.214	0.	0.35	18.515

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	0	3168	4640	0	0	0	0	0
normalized size	1	0.	66.	96.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	6.073	0.504	0.	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	181	1356	435	363	0	637	0
normalized size	1	1.	0.55	4.11	1.32	1.1	0.	1.93	0.
time (sec)	N/A	1.419	0.219	0.086	1.594	0.226	0.	0.216	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	5	5	5
normalized size	1	1.	1.	1.25	1.25	1.25	1.25	1.25	1.25
time (sec)	N/A	0.017	0.002	0.007	1.371	0.226	0.623	0.202	1.363

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	A	F	A	A	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	0	71	102	0	185	76	0	0
normalized size	1	0.	1.	1.44	0.	2.61	1.07	0.	0.
time (sec)	N/A	1.213	0.061	0.017	0.	0.203	0.31	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [1.429]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	0	1.	6	0.
2	A	3	2	1.	13	0.154
3	A	2	1	1.	10	0.1
4	A	1	1	1.	3	0.333
5	A	2	1	1.	11	0.091
6	A	2	1	1.	14	0.071
7	A	2	1	1.	20	0.05
8	A	2	2	1.	12	0.167
9	A	3	3	1.	13	0.231
10	A	2	2	1.	10	0.2
11	A	6	5	1.	13	0.385
12	A	2	1	1.	23	0.043
13	A	4	3	1.	20	0.15
14	A	3	2	1.	22	0.091
15	A	5	4	1.	14	0.286
16	A	6	6	1.	9	0.667
17	A	3	2	1.	16	0.125
18	A	9	6	1.	7	0.857
19	A	9	6	1.	11	0.546
20	A	9	5	1.	10	0.5
21	A	1	1	1.	7	0.143
22	A	2	1	1.	9	0.111
23	A	2	1	1.	11	0.091
24	A	1	1	1.	7	0.143
25	A	1	1	1.	7	0.143
26	A	2	1	1.	9	0.111
27	A	2	1	1.	11	0.091
28	A	3	3	1.	11	0.273
29	A	2	1	1.	11	0.091
30	A	2	1	1.	11	0.091
31	A	1	1	1.	9	0.111
32	A	1	1	1.	11	0.091
33	A	6	6	1.	9	0.667
34	A	6	6	1.	7	0.857
35	A	6	6	1.	11	0.546

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	3	3	1.	7	0.429
37	A	3	3	1.	9	0.333
38	A	9	6	1.01	9	0.667
39	A	3	3	1.	12	0.25
40	A	3	3	1.	12	0.25
41	A	3	2	1.	12	0.167
42	A	3	2	1.	12	0.167
43	A	3	2	1.	12	0.167
44	A	9	5	1.	10	0.5
45	A	9	5	1.	12	0.417
46	A	10	6	1.	7	0.857
47	A	10	6	1.	7	0.857
48	A	10	6	1.	7	0.857
49	A	19	6	1.	7	0.857
50	A	13	10	1.	7	1.429
51	A	19	6	1.	12	0.5
52	A	7	7	1.	11	0.636
53	A	1	1	1.	2	0.5
54	A	1	1	1.	4	0.25
55	A	1	1	1.	6	0.167
56	A	1	1	1.	6	0.167
57	A	2	2	1.	4	0.5
58	A	11	2	1.	8	0.25
59	A	2	2	1.	8	0.25
60	A	1	1	1.	4	0.25
61	A	2	2	1.	6	0.333
62	A	2	2	1.	8	0.25
63	A	3	3	1.	8	0.375
64	A	2	2	1.	8	0.25
65	A	2	1	1.	8	0.125
66	A	4	4	1.	10	0.4
67	A	2	2	1.	10	0.2
68	A	3	2	1.	8	0.25
69	A	3	2	1.	10	0.2
70	A	3	3	1.	8	0.375
71	A	3	3	1.	10	0.3

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	4	3	1.	12	0.25
73	A	4	3	1.	12	0.25
74	A	3	3	1.	10	0.3
75	A	0	0	0.	0	0.
76	A	1	1	1.	2	0.5
77	A	1	1	1.	2	0.5
78	A	1	1	1.	2	0.5
79	A	1	1	1.	2	0.5
80	A	2	2	1.	6	0.333
81	A	1	1	1.	2	0.5
82	A	1	1	1.	2	0.5
83	A	2	2	1.	4	0.5
84	A	3	3	1.	8	0.375
85	A	2	1	1.	4	0.25
86	A	1	1	1.	4	0.25
87	A	3	2	1.	11	0.182
88	A	2	2	1.	4	0.5
89	A	2	1	1.	4	0.25
90	A	2	2	1.	4	0.5
91	A	1	1	1.	7	0.143
92	A	2	2	1.	4	0.5
93	A	3	2	1.	6	0.333
94	A	2	2	1.	6	0.333
95	A	4	4	1.	8	0.5
96	A	3	3	1.	6	0.5
97	A	2	2	1.	4	0.5
98	A	3	2	1.	6	0.333
99	A	2	2	1.	6	0.333
100	A	4	4	1.	8	0.5
101	A	3	3	1.	6	0.5
102	A	1	1	1.	6	0.167
103	A	1	1	1.	6	0.167
104	A	2	2	1.	6	0.333
105	A	3	2	1.	8	0.25
106	A	2	2	1.	4	0.5
107	A	1	1	1.	6	0.167

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	1	1	1.	6	0.167
109	A	1	1	1.	6	0.167
110	A	1	1	1.	6	0.167
111	A	1	1	1.	6	0.167
112	A	1	1	1.	6	0.167
113	A	2	2	1.	8	0.25
114	A	2	1	1.	8	0.125
115	A	2	2	1.	8	0.25
116	A	2	1	1.	8	0.125
117	A	2	2	1.	8	0.25
118	A	1	1	1.	6	0.167
119	A	1	1	1.	8	0.125
120	A	1	1	1.	6	0.167
121	A	1	1	1.	8	0.125
122	A	3	3	1.	8	0.375
123	A	3	3	1.	10	0.3
124	A	4	4	1.	12	0.333
125	A	1	1	1.	7	0.143
126	A	4	4	1.	12	0.333
127	A	2	2	1.	4	0.5
128	A	17	8	1.	8	1.
129	A	34	9	1.	8	1.125
130	A	3	3	1.	6	0.5
131	A	1	1	1.	9	0.111
132	A	3	3	1.	9	0.333
133	A	3	2	1.	9	0.222
134	A	1	1	1.	6	0.167
135	A	1	1	1.	6	0.167
136	A	4	3	1.	7	0.429
137	A	4	3	1.	7	0.429
138	A	11	5	1.	9	0.556
139	A	11	5	1.	9	0.556
140	A	25	5	1.	9	0.556
141	A	25	5	1.	9	0.556
142	A	5	2	1.	11	0.182
143	A	5	2	1.	11	0.182

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	6	4	1.	10	0.4
145	A	0	0	0.	0	0.
146	A	4	3	1.	9	0.333
147	A	5	5	1.	15	0.333
148	A	1	1	1.	3	0.333
149	A	1	1	1.	3	0.333
150	A	1	1	1.	3	0.333
151	A	1	1	1.	3	0.333
152	A	1	1	1.	5	0.2
153	A	1	1	1.	9	0.111
154	A	4	4	1.	11	0.364
155	A	3	2	1.	13	0.154
156	A	2	2	1.	9	0.222
157	A	2	2	1.	18	0.111
158	A	2	2	1.	7	0.286
159	A	21	2	1.	7	0.286
160	A	2	2	1.	9	0.222
161	A	2	2	1.	7	0.286
162	A	2	2	1.	7	0.286
163	A	5	3	1.	12	0.25
164	A	1	1	1.	14	0.071
165	A	1	1	1.	7	0.143
166	A	1	1	1.	5	0.2
167	A	1	1	1.	7	0.143
168	A	7	3	1.	12	0.25
169	F	0	0	N/A	0	N/A
170	A	0	0	0.	0	0.
171	A	2	3	1.	6	0.5
172	A	5	5	1.	7	0.714
173	A	2	2	1.	10	0.2
174	A	1	0	1.	13	0.
175	A	4	4	1.	12	0.333
176	A	4	3	1.	19	0.158
177	A	1	1	1.	9	0.111
178	A	2	1	1.	11	0.091
179	A	2	1	1.	13	0.077

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
180	A	3	3	1.	13	0.231
181	A	3	3	1.	13	0.231
182	A	1	1	1.	9	0.111
183	A	2	1	1.	11	0.091
184	A	2	1	1.	13	0.077
185	A	2	2	1.	13	0.154
186	A	3	3	1.	13	0.231
187	A	1	1	1.	11	0.091
188	A	2	1	1.	13	0.077
189	A	6	6	1.	18	0.333
190	A	2	2	1.	9	0.222
191	A	4	3	1.	13	0.231
192	A	3	3	1.	6	0.5
193	A	1	1	1.	13	0.077
194	A	2	2	1.	13	0.154
195	A	3	3	1.	13	0.231
196	A	3	3	1.	14	0.214
197	A	3	3	1.	18	0.167
198	A	1	1	1.	20	0.05
199	A	1	1	1.	9	0.111
200	A	3	2	1.	65	0.031
201	A	5	4	1.	68	0.059
202	A	5	2	1.	21	0.095
203	A	6	6	1.	21	0.286
204	A	4	3	1.	23	0.13
205	A	2	2	1.	16	0.125
206	A	3	3	1.	25	0.12
207	A	2	2	1.	24	0.083
208	A	2	2	1.	29	0.069
209	A	1	1	1.	18	0.056
210	A	1	1	1.	23	0.043
211	A	3	3	1.	24	0.125
212	A	3	3	1.	22	0.136
213	A	3	3	1.	26	0.115
214	A	3	3	1.	31	0.097
215	A	3	3	1.	16	0.188

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	1	1	1.	8	0.125
217	A	2	2	1.	6	0.333
218	A	1	1	1.	20	0.05
219	A	3	2	1.	13	0.154
220	A	2	2	1.	13	0.154
221	A	3	3	1.	9	0.333
222	A	2	2	1.	13	0.154
223	A	2	1	1.	11	0.091
224	A	5	5	1.	13	0.385
225	A	2	2	1.	4	0.5
226	A	2	2	1.	4	0.5
227	A	1	1	1.	4	0.25
228	A	1	1	1.	28	0.036
229	A	4	2	1.	11	0.182
230	A	3	3	1.	4	0.75
231	A	3	3	1.	4	0.75
232	A	3	3	1.	8	0.375
233	A	3	2	1.	7	0.286
234	A	3	2	1.	7	0.286
235	A	1	1	1.	8	0.125
236	A	1	1	1.	10	0.1
237	A	1	1	1.	8	0.125
238	A	1	1	1.	10	0.1
239	A	3	3	1.	17	0.176
240	A	4	3	1.	13	0.231
241	A	2	2	1.	11	0.182
242	A	4	3	1.	13	0.231
243	A	3	3	1.	8	0.375
244	A	1	1	1.	2	0.5
245	A	3	2	1.	4	0.5
246	A	1	1	1.	2	0.5
247	A	1	1	1.	2	0.5
248	A	1	1	1.	3	0.333
249	A	4	3	1.	12	0.25
250	A	2	1	1.	13	0.077
251	A	3	3	1.	14	0.214

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	14	8	1.	12	0.667
253	A	4	3	1.	7	0.429
254	A	1	1	1.	5	0.2
255	A	6	6	1.	9	0.667
256	A	2	2	1.	9	0.222
257	A	10	3	1.	15	0.2
258	A	6	2	1.	11	0.182
259	A	6	6	1.	7	0.857
260	A	1	1	1.	9	0.111
261	A	2	2	1.	9	0.222
262	A	2	1	1.	7	0.143
263	A	2	2	1.	2	1.
264	A	4	3	1.	6	0.5
265	A	4	2	1.	17	0.118
266	A	2	2	1.	4	0.5
267	A	2	1	1.	19	0.053
268	A	1	1	1.	11	0.091
269	A	1	1	1.	9	0.111
270	A	2	2	1.	12	0.167
271	A	3	2	1.	24	0.083
272	A	2	1	1.	29	0.034
273	A	0	0	0.	0	0.
274	F	0	0	N/A	0	N/A
275	A	4	3	1.	21	0.143
276	A	1	1	1.	2	0.5
277	A	1	1	1.	4	0.25
278	F	0	0	N/A	0	N/A
279	F	0	0	N/A	0	N/A
280	A	1	1	1.	85	0.012
281	A	0	0	0.	0	0.
282	A	20	8	1.	107	0.075
283	A	2	2	1.	7	0.286
284	F	0	0	N/A	0	N/A

3 Listing of integrals

3.1 $\int (1 + x + x^2)\ dx$

Optimal. Leaf size=16

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

```
[Out] x + x^2/2 + x^3/3
```

Rubi [A] time = 0.00804981, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

```
[In]  Int[1 + x + x^2,x]

[Out] x + x^2/2 + x^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3} + x + \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2+x+1,x)

[Out] x**3/3 + x + Integral(x, x)
```

Mathematica [A] time = 0.000303984, size = 16, normalized size = 1.

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

```
[In]  Integrate[1 + x + x^2,x]
```

```
[Out] x + x^2/2 + x^3/3
```

Maple [A] time = 0.01, size = 13, normalized size = 0.8

$$x + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2+x+1,x)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

Maxima [A] time = 1.39807, size = 16, normalized size = 1.

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2 + x + 1,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Fricas [A] time = 0.200323, size = 1, normalized size = 0.06

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2 + x + 1,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] time = 0.02528, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2+x+1,x)
```

```
[Out] x**3/3 + x**2/2 + x
```

GIAC/XCAS [A] time = 0.210829, size = 16, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2 + x + 1,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

3.2 $\int x^2 \left(x+2x^2\right)^2 \, dx$

Optimal. Leaf size=22

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Rubi [A] time = 0.0233712, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(x + 2*x^2)^2,x]

[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Rubi in Sympy [A] time = 2.09129, size = 17, normalized size = 0.77

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*x**2+x)**2,x)

[Out] 4*x**7/7 + 2*x**6/3 + x**5/5
```

Mathematica [A] time = 0.0057053, size = 22, normalized size = 1.

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$\frac{x^5}{5} + \frac{2\,x^6}{3} + \frac{4\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^2+x)^2,x)
```

```
[Out] 1/5*x^5+2/3*x^6+4/7*x^7
```

Maxima [A] time = 1.36762, size = 22, normalized size = 1.

$$\frac{4}{7}\,x^7 + \frac{2}{3}\,x^6 + \frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + x)^2*x^2,x, algorithm="maxima")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Fricas [A] time = 0.1694, size = 1, normalized size = 0.05

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + x)^2*x^2,x, algorithm="fricas")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Sympy [A] time = 0.032833, size = 17, normalized size = 0.77

$$\frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*x**2+x)**2,x)
```

```
[Out] 4*x**7/7 + 2*x**6/3 + x**5/5
```

GIAC/XCAS [A] time = 0.214127, size = 22, normalized size = 1.

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + x)^2*x^2,x, algorithm="giac")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

3.3 $\int x\left(1+2x+x^2\right)\,dx$

Optimal. Leaf size=22

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Rubi [A] time = 0.0108596, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + 2*x + x^2),x]

[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4}+\frac{2x^3}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+2*x+1),x)

[Out] x**4/4 + 2*x**3/3 + Integral(x, x)
```

Mathematica [A] time = 0.0241846, size = 22, normalized size = 1.

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$\frac{x^2}{2} + \frac{2\,x^3}{3} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+1),x)
```

```
[Out] 1/2*x^2+2/3*x^3+1/4*x^4
```

Maxima [A] time = 1.40873, size = 22, normalized size = 1.

$$\frac{1}{4}\,x^4 + \frac{2}{3}\,x^3 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x + 1)*x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Fricas [A] time = 0.186816, size = 1, normalized size = 0.05

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x + 1)*x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Sympy [A] time = 0.038237, size = 15, normalized size = 0.68

$$\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+2*x+1),x)
```

```
[Out] x**4/4 + 2*x**3/3 + x**2/2
```

GIAC/XCAS [A] time = 0.211591, size = 22, normalized size = 1.

$$\frac{1}{4}\,x^4 + \frac{2}{3}\,x^3 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 1)*x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

3.4 $\int \frac{1}{x}\,dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] Log[x]
```

Rubi [A] time = 0.00231508, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1),x]

[Out] Log[x]
```

Rubi in Sympy [A] time = 0.023605, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x,x)

[Out] log(x)
```

Mathematica [A] time = 0.000232308, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1),x]

[Out] Log[x]
```

Maple [A] time = 0.004, size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x,x)

[Out] ln(x)
```

Maxima [A] time = 1.38653, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="maxima")

[Out] log(x)
```

Fricas [A] time = 0.199388, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="fricas")

[Out] log(x)
```

Sympy [A] time = 0.022619, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x)

[Out] log(x)
```

GIAC/XCAS [A] time = 0.211278, size = 4, normalized size = 2.

$$\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="giac")
```

```
[Out] ln(abs(x))
```

3.5 $\int \frac{(1+x)^3}{(-1+x)^4} dx$

Optimal. Leaf size=36

$$\frac{6}{1-x}-\frac{6}{(1-x)^2}+\frac{8}{3(1-x)^3}+\log(1-x)$$

```
[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

Rubi [A] time = 0.0275, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{6}{1-x}-\frac{6}{(1-x)^2}+\frac{8}{3(1-x)^3}+\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)^3/(-1 + x)^4,x]

[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

Rubi in Sympy [A] time = 2.22928, size = 24, normalized size = 0.67

$$\log\left(-x+1\right)+\frac{6}{-x+1}-\frac{6}{\left(-x+1\right)^2}+\frac{8}{3\left(-x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)**3/(-1+x)**4,x)

[Out] log(-x + 1) + 6/(-x + 1) - 6/(-x + 1)**2 + 8/(3*(-x + 1)**3)
```

Mathematica [A] time = 0.0399364, size = 24, normalized size = 0.67

$$\log(x-1)-\frac{2\left(9x^2-9x+4\right)}{3(x-1)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)^3/(-1 + x)^4,x]
```

```
[Out] (-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]
```

Maple [A] time = 0.026, size = 27, normalized size = 0.8

$$-6\ (-1+x)^{-2}-6\ (-1+x)^{-1}-\frac{8}{3\ (-1+x)^{3}}+\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)^3/(-1+x)^4,x)
```

```
[Out] -6/(-1+x)^2-6/(-1+x)-8/3/(-1+x)^3+ln(-1+x)
```

Maxima [A] time = 1.41242, size = 43, normalized size = 1.19

$$-\frac{2\left(9\,x^{2}-9\,x+4\right)}{3\,(x^{3}-3\,x^{2}+3\,x-1)}+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^3/(x - 1)^4,x, algorithm="maxima")
```

```
[Out] -2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)
```

Fricas [A] time = 0.207759, size = 62, normalized size = 1.72

$$-\frac{18\,x^{2}-3\left(x^{3}-3\,x^{2}+3\,x-1\right)\log\left(x-1\right)-18\,x+8}{3\,(x^{3}-3\,x^{2}+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^3/(x - 1)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*log(x - 1) - 18*x + 8)/(
x^3 - 3*x^2 + 3*x - 1)
```

Sympy [A] time = 0.12178, size = 29, normalized size = 0.81

$$-\frac{18x^{2}-18x+8}{3x^{3}-9x^{2}+9x-3}+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)**3/(-1+x)**4,x)
```

```
[Out] -(18*x**2 - 18*x + 8)/(3*x**3 - 9*x**2 + 9*x - 3) + log(x - 1)
```

GIAC/XCAS [A] time = 0.211012, size = 31, normalized size = 0.86

$$-\frac{2\left(9\,x^2-9\,x+4\right)}{3\,(x-1)^3}+\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^3/(x - 1)^4,x, algorithm="giac")
```

```
[Out] -2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + ln(abs(x - 1))
```

3.6 $\int \frac{1}{(-1+x)x(1+x)^2} \, dx$

Optimal. Leaf size=32

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

```
[Out] -1/(2*(1 + x)) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

Rubi [A] time = 0.0326283, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)*x*(1 + x)^2),x]

[Out] -1/(2*(1 + x)) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

Rubi in Sympy [A] time = 2.0632, size = 24, normalized size = 0.75

$$-\log(x)+\frac{\log(-x+1)}{4}+\frac{3\log(x+1)}{4}-\frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)/x/(1+x)**2,x)

[Out] -log(x) + log(-x + 1)/4 + 3*log(x + 1)/4 - 1/(2*(x + 1))
```

Mathematica [A] time = 0.0249273, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-\frac{2}{x+1}+\log(1-x)-4\log(x)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] (-2/(1 + x) + Log[1 - x] - 4*Log[x] + 3*Log[1 + x])/4
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$-\ln(x) - \frac{1}{2+2x} + \frac{3\ln(1+x)}{4} + \frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)/x/(1+x)^2,x)
```

```
[Out] -ln(x)-1/2/(1+x)+3/4*ln(1+x)+1/4*ln(-1+x)
```

Maxima [A] time = 1.37561, size = 32, normalized size = 1.

$$-\frac{1}{2(x+1)} + \frac{3}{4}\log(x+1) + \frac{1}{4}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^2*(x - 1)*x),x, algorithm="maxima")
```

```
[Out] -1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1) - log(x)
```

Fricas [A] time = 0.209791, size = 45, normalized size = 1.41

$$\frac{3(x+1)\log(x+1) + (x+1)\log(x-1) - 4(x+1)\log(x) - 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^2*(x - 1)*x),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) - 4*(x + 1)*log(x)
 - 2)/(x + 1)
```

Sympy [A] time = 0.154346, size = 24, normalized size = 0.75

$$-\log(x) + \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} - \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)/x/(1+x)**2,x)
```

```
[Out] -log(x) + log(x - 1)/4 + 3*log(x + 1)/4 - 1/(2*x + 2)
```

GIAC/XCAS [A] time = 0.213504, size = 46, normalized size = 1.44

$$-\frac{1}{2\,(x+1)} - \ln\left(\left|-\frac{1}{x+1}+1\right|\right) + \frac{1}{4}\ln\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^2*(x - 1)*x),x, algorithm="giac")
```

```
[Out] -1/2/(x + 1) - ln(abs(-1/(x + 1) + 1)) + 1/4*ln(abs(-2/(x + 1) +
1))
```

3.7 $\int \frac{b+ax}{(-p+x)(-q+x)}\,dx$

Optimal. Leaf size=40

$$\frac{(ap+b)\log(p-x)}{p-q}-\frac{(aq+b)\log(q-x)}{p-q}$$

```
[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

Rubi [A] time = 0.0637384, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{(ap+b)\log(p-x)}{p-q}-\frac{(aq+b)\log(q-x)}{p-q}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + a*x)/((-p + x)*(-q + x)),x]

[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

Rubi in Sympy [A] time = 4.94215, size = 26, normalized size = 0.65

$$\frac{(ap+b)\log\left(p-x\right)}{p-q}-\frac{(aq+b)\log\left(q-x\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x+b)/(-p+x)/(-q+x),x)

[Out] (a*p + b)*log(p - x)/(p - q) - (a*q + b)*log(q - x)/(p - q)
```

Mathematica [A] time = 0.0225985, size = 34, normalized size = 0.85

$$\frac{(ap+b)\log(x-p)-(aq+b)\log(x-q)}{p-q}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + a*x)/((-p + x)*(-q + x)),x]
```

```
[Out] ((b + a*p)*Log[-p + x] - (b + a*q)*Log[-q + x])/(p - q)
```

Maple [A] time = 0.005, size = 66, normalized size = 1.7

$$\frac{\ln(-p+x)\,ap}{p-q}+\frac{\ln(-p+x)\,b}{p-q}-\frac{\ln(-q+x)\,aq}{p-q}-\frac{\ln(-q+x)\,b}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] 1/(p-q)*ln(-p+x)*a*p+1/(p-q)*ln(-p+x)*b-1/(p-q)*ln(-q+x)*a*q-1/(p
-q)*ln(-q+x)*b
```

Maxima [A] time = 1.37392, size = 54, normalized size = 1.35

$$\frac{(ap+b)\log(-p+x)}{p-q}-\frac{(aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)/((p - x)*(q - x)),x, algorithm="maxima")
```

```
[Out] (a*p + b)*log(-p + x)/(p - q) - (a*q + b)*log(-q + x)/(p - q)
```

Fricas [A] time = 0.201657, size = 46, normalized size = 1.15

$$\frac{(ap+b)\log(-p+x)-(aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)/((p - x)*(q - x)),x, algorithm="fricas")
```

```
[Out] ((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)
```

Sympy [A] time = 1.03251, size = 144, normalized size = 3.6

$$\frac{(ap+b)\log\left(x+\frac{-2apq-bp-bq-\frac{p^2(ap+b)}{p-q}+\frac{2pq(ap+b)}{p-q}-\frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p-q}-\frac{(aq+b)\log\left(x+\frac{-2apq-bp-bq+\frac{p^2(aq+b)}{p-q}-\frac{2pq(aq+b)}{p-q}+\frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] (a*p + b)*log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q)
+ 2*p*q*(a*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q +
2*b))/(p - q) - (a*q + b)*log(x + (-2*a*p*q - b*p - b*q + p**2*(a
*q + b)/(p - q) - 2*p*q*(a*q + b)/(p - q) + q**2*(a*q + b)/(p - q
))/(a*p + a*q + 2*b))/(p - q)
```

GIAC/XCAS [A] time = 0.209679, size = 57, normalized size = 1.42

$$\frac{(ap+b)\ln\left(|-p+x|\right)}{p-q}-\frac{(aq+b)\ln\left(|-q+x|\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)/((p - x)*(q - x)),x, algorithm="giac")
```

```
[Out] (a*p + b)*ln(abs(-p + x))/(p - q) - (a*q + b)*ln(abs(-q + x))/(p
- q)
```

3.8 $\int \frac{1}{c+bx+ax^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0378399, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + b*x + a*x^2)^(-1),x]

[Out] (-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 2.05828, size = 34, normalized size = 1.

$$-\frac{2\,\mathrm{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*x**2+b*x+c),x)

[Out] -2*atanh((2*a*x + b)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.052531, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + b*x + a*x^2)^(-1),x]

[Out] (2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.005, size = 35, normalized size = 1.

$$2\frac{1}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*x^2+b*x+c),x)

[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^2 + b*x + c),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.198196, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{b^3-4\,abc+2\,(ab^2-4\,a^2c)\,x-(2\,a^2x^2+2\,abx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right)}{\sqrt{b^2-4\,ac}},\ \frac{2\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)}{\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^2 + b*x + c),x, algorithm="fricas")

[Out] [log(-(b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x - (2*a^2*x^2 + 2*a*b
*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 + b*x + c))/sqrt(b^2
- 4*a*c), 2*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))
/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 0.258405, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x**2+b*x+c),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2))
+ b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**
2))*log(x + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c
- b**2)) + b)/(2*a))
```

GIAC/XCAS [A] time = 0.212984, size = 46, normalized size = 1.35

$$\frac{2\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^2 + b*x + c),x, algorithm="giac")
```

```
[Out] 2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.9 $\int \frac{b+ax}{1+x^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}a\log\left(x^2+1\right)+b\tan^{-1}(x)$$

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Rubi [A] time = 0.0155256, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{1}{2}a\log\left(x^2+1\right)+b\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(b + a*x)/(1 + x^2),x]

[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Rubi in Sympy [A] time = 1.67246, size = 14, normalized size = 0.88

$$\frac{a\log\left(x^2+1\right)}{2}+b\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x+b)/(x**2+1),x)

[Out] a*log(x**2 + 1)/2 + b*atan(x)
```

Mathematica [A] time = 0.0337243, size = 16, normalized size = 1.

$$\frac{1}{2}a\log\left(x^2+1\right)+b\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + a*x)/(1 + x^2),x]
```

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$b\arctan(x) + \frac{a\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+b)/(x^2+1),x)
```

```
[Out] b*arctan(x)+1/2*a*ln(x^2+1)
```

Maxima [A] time = 1.51319, size = 19, normalized size = 1.19

$$b\arctan(x) + \frac{1}{2}a\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

Fricas [A] time = 0.197975, size = 19, normalized size = 1.19

$$b\arctan(x) + \frac{1}{2}a\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

Sympy [A] time = 0.200584, size = 26, normalized size = 1.62

$$\left(\frac{a}{2} - \frac{ib}{2}\right)\log(x-i) + \left(\frac{a}{2} + \frac{ib}{2}\right)\log(x+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+b)/(x**2+1),x)
```

```
[Out] (a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)
```

GIAC/XCAS [A] time = 0.210547, size = 19, normalized size = 1.19

$$b\arctan(x) + \frac{1}{2}a\ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] b*arctan(x) + 1/2*a*ln(x^2 + 1)
```

3.10 $\int \frac{1}{3-2x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0249369, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 2*x + x^2)^(-1),x]

[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])
```

Rubi in Sympy [A] time = 0.670154, size = 19, normalized size = 1.

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x}{2}-\frac{1}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x+3),x)

[Out] sqrt(2)*atan(sqrt(2)*(x/2 - 1/2))/2
```

Mathematica [A] time = 0.0561077, size = 16, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 2*x + x^2)^(-1),x]

[Out] ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(2x-2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x+3),x)

[Out] 1/2*2^(1/2)*arctan(1/4*(2*x-2)*2^(1/2))
```

Maxima [A] time = 1.51756, size = 19, normalized size = 1.

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*x + 3),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Fricas [A] time = 0.194157, size = 19, normalized size = 1.

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*x + 3),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Sympy [A] time = 0.092613, size = 22, normalized size = 1.16

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}-\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x+3),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2
```

GIAC/XCAS [A] time = 0.209883, size = 19, normalized size = 1.

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*x + 3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

3.11 $\int \frac{1}{(-1+x)^2(1+x^2)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{4(x^2+1)}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\tan^{-1}(x)$$

```
[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Lo
g[1 + x^2]/4
```

Rubi [A] time = 0.070354, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$-\frac{1}{4(x^2+1)}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Lo
g[1 + x^2]/4
```

Rubi in Sympy [A] time = 4.57951, size = 34, normalized size = 0.69

$$-\frac{\log(-x+1)}{2}+\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{4}-\frac{1}{4(x^2+1)}+\frac{1}{4(-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**2/(x**2+1)**2,x)
```

```
[Out] -log(-x + 1)/2 + log(x**2 + 1)/4 + atan(x)/4 - 1/(4*(x**2 + 1)) +
 1/(4*(-x + 1))
```

Mathematica [A] time = 0.0307283, size = 35, normalized size = 0.71

$$\frac{1}{4}\left(-\frac{1}{x^2+1}+\log\left(x^2+1\right)+\frac{1}{1-x}-2\log(x-1)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] ((1 - x)^(-1) - (1 + x^2)^(-1) + ArcTan[x] - 2*Log[-1 + x] + Log[
1 + x^2])/4
```

Maple [A] time = 0.012, size = 36, normalized size = 0.7

$$-\frac{1}{4x^2+4}+\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{4}-\frac{1}{-4+4x}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^2/(x^2+1)^2,x)
```

```
[Out] -1/4/(x^2+1)+1/4*ln(x^2+1)+1/4*arctan(x)-1/4/(-1+x)-1/2*ln(-1+x)
```

Maxima [A] time = 1.51304, size = 53, normalized size = 1.08

$$-\frac{x^2+x}{4(x^3-x^2+x-1)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*arctan(x) + 1/4*log(x^2
+ 1) - 1/2*log(x - 1)
```

Fricas [A] time = 0.197811, size = 96, normalized size = 1.96

$$-\frac{x^2-(x^3-x^2+x-1)\arctan(x)-(x^3-x^2+x-1)\log(x^2+1)+2(x^3-x^2+x-1)\log(x-1)+x}{4(x^3-x^2+x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^3 - x^2 + x - 1)*arctan(x) - (x^3 - x^2 + x - 1)*l
og(x^2 + 1) + 2*(x^3 - x^2 + x - 1)*log(x - 1) + x)/(x^3 - x^2 +
x - 1)
```

Sympy [A] time = 0.199228, size = 39, normalized size = 0.8

$$-\frac{x^2+x}{4x^3-4x^2+4x-4}-\frac{\log(x-1)}{2}+\frac{\log(x^2+1)}{4}+\frac{\operatorname{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**2/(x**2+1)**2,x)
```

```
[Out] -(x**2 + x)/(4*x**3 - 4*x**2 + 4*x - 4) - log(x - 1)/2 + log(x**2
 + 1)/4 + atan(x)/4
```

GIAC/XCAS [A] time = 0.211832, size = 108, normalized size = 2.2

$$\frac{1}{16}\pi-\frac{1}{4}\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{\frac{2}{x-1}+1}{8\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)}-\frac{1}{4(x-1)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\ln\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*(x - 1)^2),x, algorithm="giac")
```

```
[Out] 1/16*pi - 1/4*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/8*(2/
(x - 1) + 1)/(2/(x - 1) + 2/(x - 1)^2 + 1) - 1/4/(x - 1) + 1/4*ar
ctan(x) + 1/4*ln(2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.12 $\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx$

Optimal. Leaf size=68

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

```
[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c
)) + (c*Log[c - x])/((a - c)*(b - c))
```

Rubi [A] time = 0.116886, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c
)) + (c*Log[c - x])/((a - c)*(b - c))
```

Rubi in Sympy [A] time = 9.13852, size = 39, normalized size = 0.57

$$\frac{a\log\left(a-x\right)}{\left(a-b\right)\left(a-c\right)}-\frac{b\log\left(b-x\right)}{\left(a-b\right)\left(b-c\right)}+\frac{c\log\left(c-x\right)}{\left(a-c\right)\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] a*log(a - x)/((a - b)*(a - c)) - b*log(b - x)/((a - b)*(b - c)) +
 c*log(c - x)/((a - c)*(b - c))
```

Mathematica [A] time = 0.0426886, size = 62, normalized size = 0.91

$$\frac{a(b-c)\log(x-a)+b(c-a)\log(x-b)+c(a-b)\log(x-c)}{(a-b)(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-
c + x])/((a - b)*(a - c)*(b - c))
```

Maple [A] time = 0.007, size = 69, normalized size = 1.

$$\frac{c \ln(-c+x)}{(a-c)(b-c)} + \frac{a \ln(-a+x)}{(a-b)(a-c)} - \frac{b \ln(-b+x)}{(b-c)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] c/(a-c)/(b-c)*ln(-c+x)+a/(a-b)/(a-c)*ln(-a+x)-b/(b-c)/(a-b)*ln(-b
+x)
```

Maxima [A] time = 1.37171, size = 105, normalized size = 1.54

$$\frac{a \log(-a+x)}{a^2 - ab - (a-b)c} - \frac{b \log(-b+x)}{ab - b^2 - (a-b)c} + \frac{c \log(-c+x)}{ab - (a+b)c + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((a - x)*(b - x)*(c - x)),x, algorithm="maxima")
```

```
[Out] a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2
- (a - b)*c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)
```

Fricas [A] time = 0.281224, size = 109, normalized size = 1.6

$$\frac{(a-b)c \log(-c+x) + (ab - ac) \log(-a+x) - (ab - bc) \log(-b+x)}{a^2 b - ab^2 + (a-b)c^2 - (a^2 - b^2)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((a - x)*(b - x)*(c - x)),x, algorithm="fricas")
```

```
[Out] ((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*lo
g(-b + x))/(a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)
```

Sympy [A] time = 145.64, size = 4216, normalized size = 62.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] a*log(x + (-a**8*b**3/((a - b)**2*(a - c)**2) + a**8*b**2*c/((a -
 b)**2*(a - c)**2) + a**8*b*c**2/((a - b)**2*(a - c)**2) - a**8*c
**3/((a - b)**2*(a - c)**2) + a**7*b**4/((a - b)**2*(a - c)**2) +
 8*a**7*b**3*c/((a - b)**2*(a - c)**2) - 18*a**7*b**2*c**2/((a -
b)**2*(a - c)**2) + 8*a**7*b*c**3/((a - b)**2*(a - c)**2) + a**7*
c**4/((a - b)**2*(a - c)**2) + a**6*b**5/((a - b)**2*(a - c)**2)
- 18*a**6*b**4*c/((a - b)**2*(a - c)**2) + 17*a**6*b**3*c**2/((a
- b)**2*(a - c)**2) - a**6*b**3/((a - b)*(a - c)) + 17*a**6*b**2*
c**3/((a - b)**2*(a - c)**2) + a**6*b**2*c/((a - b)*(a - c)) - 18
*a**6*b*c**4/((a - b)**2*(a - c)**2) + a**6*b*c**2/((a - b)*(a -
c)) + a**6*c**5/((a - b)**2*(a - c)**2) - a**6*c**3/((a - b)*(a -
 c)) - a**5*b**6/((a - b)**2*(a - c)**2) + 8*a**5*b**5*c/((a - b)
**2*(a - c)**2) + 17*a**5*b**4*c**2/((a - b)**2*(a - c)**2) + 2*a
**5*b**4/((a - b)*(a - c)) - 48*a**5*b**3*c**3/((a - b)**2*(a - c
)**2) - a**5*b**3*c/((a - b)*(a - c)) + 17*a**5*b**2*c**4/((a - b
)**2*(a - c)**2) - 2*a**5*b**2*c**2/((a - b)*(a - c)) + 8*a**5*b*
c**5/((a - b)**2*(a - c)**2) - a**5*b*c**3/((a - b)*(a - c)) - a*
*5*c**6/((a - b)**2*(a - c)**2) + 2*a**5*c**4/((a - b)*(a - c)) +
 a**4*b**6*c/((a - b)**2*(a - c)**2) - 18*a**4*b**5*c**2/((a - b)
**2*(a - c)**2) - a**4*b**5/((a - b)*(a - c)) + 17*a**4*b**4*c**3
/((a - b)**2*(a - c)**2) - a**4*b**4*c/((a - b)*(a - c)) + 17*a**
4*b**3*c**4/((a - b)**2*(a - c)**2) + 2*a**4*b**3*c**2/((a - b)*(
a - c)) - 18*a**4*b**2*c**5/((a - b)**2*(a - c)**2) + 2*a**4*b**2
*c**3/((a - b)*(a - c)) + a**4*b**2*c + a**4*b*c**6/((a - b)**2*(
a - c)**2) - a**4*b*c**4/((a - b)*(a - c)) + a**4*b*c**2 - a**4*c
**5/((a - b)*(a - c)) + a**3*b**6*c**2/((a - b)**2*(a - c)**2) +
8*a**3*b**5*c**3/((a - b)**2*(a - c)**2) + a**3*b**5*c/((a - b)*(
a - c)) - 18*a**3*b**4*c**4/((a - b)**2*(a - c)**2) - 2*a**3*b**4
*c**2/((a - b)*(a - c)) + 8*a**3*b**3*c**5/((a - b)**2*(a - c)**2
) + 2*a**3*b**3*c**3/((a - b)*(a - c)) - 4*a**3*b**3*c + a**3*b**
2*c**6/((a - b)**2*(a - c)**2) - 2*a**3*b**2*c**4/((a - b)*(a - c
)) + 2*a**3*b**2*c**2 + a**3*b*c**5/((a - b)*(a - c)) - 4*a**3*b*
c**3 - a**2*b**6*c**3/((a - b)**2*(a - c)**2) + a**2*b**5*c**4/((
a - b)**2*(a - c)**2) + a**2*b**5*c**2/((a - b)*(a - c)) + a**2*b
**4*c**5/((a - b)**2*(a - c)**2) - a**2*b**4*c**3/((a - b)*(a - c
)) + a**2*b**4*c - a**2*b**3*c**6/((a - b)**2*(a - c)**2) - a**2*
b**3*c**4/((a - b)*(a - c)) + 2*a**2*b**3*c**2 + a**2*b**2*c**5/(
(a - b)*(a - c)) + 2*a**2*b**2*c**3 + a**2*b*c**4 - a*b**5*c**3/(
(a - b)*(a - c)) + 2*a*b**4*c**4/((a - b)*(a - c)) + a*b**4*c**2
- a*b**3*c**5/((a - b)*(a - c)) - 4*a*b**3*c**3 + a*b**2*c**4)/(2
*a**3*b**3 - 3*a**3*b**2*c - 3*a**3*b*c**2 + 2*a**3*c**3 - 3*a**2
*b**3*c + 12*a**2*b**2*c**2 - 3*a**2*b*c**3 - 3*a*b**3*c**2 - 3*a
*b**2*c**3 + 2*b**3*c**3))/((a - b)*(a - c)) - b*log(x + (-a**6*b
**5/((a - b)**2*(b - c)**2) + a**6*b**4*c/((a - b)**2*(b - c)**2)
 + a**6*b**3*c**2/((a - b)**2*(b - c)**2) - a**6*b**2*c**3/((a -
```

```
b)**2*(b - c)**2) + a**5*b**6/((a - b)**2*(b - c)**2) + 8*a**5*b*
*5*c/((a - b)**2*(b - c)**2) - 18*a**5*b**4*c**2/((a - b)**2*(b -
 c)**2) + a**5*b**4/((a - b)*(b - c)) + 8*a**5*b**3*c**3/((a - b)
**2*(b - c)**2) - a**5*b**3*c/((a - b)*(b - c)) + a**5*b**2*c**4/
((a - b)**2*(b - c)**2) - a**5*b**2*c**2/((a - b)*(b - c)) + a**5
*b*c**3/((a - b)*(b - c)) + a**4*b**7/((a - b)**2*(b - c)**2) - 1
8*a**4*b**6*c/((a - b)**2*(b - c)**2) + 17*a**4*b**5*c**2/((a - b
)**2*(b - c)**2) - 2*a**4*b**5/((a - b)*(b - c)) + 17*a**4*b**4*c
**3/((a - b)**2*(b - c)**2) + a**4*b**4*c/((a - b)*(b - c)) - 18*
a**4*b**3*c**4/((a - b)**2*(b - c)**2) + 2*a**4*b**3*c**2/((a - b
)*(b - c)) + a**4*b**2*c**5/((a - b)**2*(b - c)**2) + a**4*b**2*c
**3/((a - b)*(b - c)) + a**4*b**2*c - 2*a**4*b*c**4/((a - b)*(b -
 c)) + a**4*b*c**2 - a**3*b**8/((a - b)**2*(b - c)**2) + 8*a**3*b
**7*c/((a - b)**2*(b - c)**2) + 17*a**3*b**6*c**2/((a - b)**2*(b
- c)**2) + a**3*b**6/((a - b)*(b - c)) - 48*a**3*b**5*c**3/((a -
b)**2*(b - c)**2) + a**3*b**5*c/((a - b)*(b - c)) + 17*a**3*b**4*
c**4/((a - b)**2*(b - c)**2) - 2*a**3*b**4*c**2/((a - b)*(b - c))
 + 8*a**3*b**3*c**5/((a - b)**2*(b - c)**2) - 2*a**3*b**3*c**3/((
a - b)*(b - c)) - 4*a**3*b**3*c - a**3*b**2*c**6/((a - b)**2*(b -
 c)**2) + a**3*b**2*c**4/((a - b)*(b - c)) + 2*a**3*b**2*c**2 + a
**3*b*c**5/((a - b)*(b - c)) - 4*a**3*b*c**3 + a**2*b**8*c/((a -
b)**2*(b - c)**2) - 18*a**2*b**7*c**2/((a - b)**2*(b - c)**2) + 1
7*a**2*b**6*c**3/((a - b)**2*(b - c)**2) - a**2*b**6*c/((a - b)*(
b - c)) + 17*a**2*b**5*c**4/((a - b)**2*(b - c)**2) + 2*a**2*b**5
*c**2/((a - b)*(b - c)) - 18*a**2*b**4*c**5/((a - b)**2*(b - c)**
2) - 2*a**2*b**4*c**3/((a - b)*(b - c)) + a**2*b**4*c + a**2*b**3
*c**6/((a - b)**2*(b - c)**2) + 2*a**2*b**3*c**4/((a - b)*(b - c)
) + 2*a**2*b**3*c**2 - a**2*b**2*c**5/((a - b)*(b - c)) + 2*a**2*
b**2*c**3 + a**2*b*c**4 + a*b**8*c**2/((a - b)**2*(b - c)**2) + 8
*a*b**7*c**3/((a - b)**2*(b - c)**2) - 18*a*b**6*c**4/((a - b)**2
*(b - c)**2) - a*b**6*c**2/((a - b)*(b - c)) + 8*a*b**5*c**5/((a
- b)**2*(b - c)**2) + a*b**5*c**3/((a - b)*(b - c)) + a*b**4*c**6
/((a - b)**2*(b - c)**2) + a*b**4*c**4/((a - b)*(b - c)) + a*b**4
*c**2 - a*b**3*c**5/((a - b)*(b - c)) - 4*a*b**3*c**3 + a*b**2*c*
*4 - b**8*c**3/((a - b)**2*(b - c)**2) + b**7*c**4/((a - b)**2*(b
 - c)**2) + b**6*c**5/((a - b)**2*(b - c)**2) + b**6*c**3/((a - b
)*(b - c)) - b**5*c**6/((a - b)**2*(b - c)**2) - 2*b**5*c**4/((a
- b)*(b - c)) + b**4*c**5/((a - b)*(b - c)))/(2*a**3*b**3 - 3*a**
3*b**2*c - 3*a**3*b*c**2 + 2*a**3*c**3 - 3*a**2*b**3*c + 12*a**2*
b**2*c**2 - 3*a**2*b*c**3 - 3*a*b**3*c**2 - 3*a*b**2*c**3 + 2*b**
3*c**3))/((a - b)*(b - c)) + c*log(x + (-a**6*b**3*c**2/((a - c)*
*2*(b - c)**2) + a**6*b**2*c**3/((a - c)**2*(b - c)**2) + a**6*b*
c**4/((a - c)**2*(b - c)**2) - a**6*c**5/((a - c)**2*(b - c)**2)
+ a**5*b**4*c**2/((a - c)**2*(b - c)**2) + 8*a**5*b**3*c**3/((a -
 c)**2*(b - c)**2) - a**5*b**3*c/((a - c)*(b - c)) - 18*a**5*b**2
*c**4/((a - c)**2*(b - c)**2) + a**5*b**2*c**2/((a - c)*(b - c))
+ 8*a**5*b*c**5/((a - c)**2*(b - c)**2) + a**5*b*c**3/((a - c)*(b
 - c)) + a**5*c**6/((a - c)**2*(b - c)**2) - a**5*c**4/((a - c)*(
b - c)) + a**4*b**5*c**2/((a - c)**2*(b - c)**2) - 18*a**4*b**4*c
**3/((a - c)**2*(b - c)**2) + 2*a**4*b**4*c/((a - c)*(b - c)) + 1
7*a**4*b**3*c**4/((a - c)**2*(b - c)**2) - a**4*b**3*c**2/((a - c
)*(b - c)) + 17*a**4*b**2*c**5/((a - c)**2*(b - c)**2) - 2*a**4*b
**2*c**3/((a - c)*(b - c)) + a**4*b**2*c - 18*a**4*b*c**6/((a - c
)**2*(b - c)**2) - a**4*b*c**4/((a - c)*(b - c)) + a**4*b*c**2 +
a**4*c**7/((a - c)**2*(b - c)**2) + 2*a**4*c**5/((a - c)*(b - c))
 - a**3*b**6*c**2/((a - c)**2*(b - c)**2) + 8*a**3*b**5*c**3/((a
```

```
- c)**2*(b - c)**2) - a**3*b**5*c/((a - c)*(b - c)) + 17*a**3*b**
4*c**4/((a - c)**2*(b - c)**2) - a**3*b**4*c**2/((a - c)*(b - c))
 - 48*a**3*b**3*c**5/((a - c)**2*(b - c)**2) + 2*a**3*b**3*c**3/(
(a - c)*(b - c)) - 4*a**3*b**3*c + 17*a**3*b**2*c**6/((a - c)**2*
(b - c)**2) + 2*a**3*b**2*c**4/((a - c)*(b - c)) + 2*a**3*b**2*c*
*2 + 8*a**3*b*c**7/((a - c)**2*(b - c)**2) - a**3*b*c**5/((a - c)
*(b - c)) - 4*a**3*b*c**3 - a**3*c**8/((a - c)**2*(b - c)**2) - a
**3*c**6/((a - c)*(b - c)) + a**2*b**6*c**3/((a - c)**2*(b - c)**
2) - 18*a**2*b**5*c**4/((a - c)**2*(b - c)**2) + a**2*b**5*c**2/(
(a - c)*(b - c)) + 17*a**2*b**4*c**5/((a - c)**2*(b - c)**2) - 2*
a**2*b**4*c**3/((a - c)*(b - c)) + a**2*b**4*c + 17*a**2*b**3*c**
6/((a - c)**2*(b - c)**2) + 2*a**2*b**3*c**4/((a - c)*(b - c)) +
2*a**2*b**3*c**2 - 18*a**2*b**2*c**7/((a - c)**2*(b - c)**2) - 2*
a**2*b**2*c**5/((a - c)*(b - c)) + 2*a**2*b**2*c**3 + a**2*b*c**8
/((a - c)**2*(b - c)**2) + a**2*b*c**6/((a - c)*(b - c)) + a**2*b
*c**4 + a*b**6*c**4/((a - c)**2*(b - c)**2) + 8*a*b**5*c**5/((a -
 c)**2*(b - c)**2) + a*b**5*c**3/((a - c)*(b - c)) - 18*a*b**4*c*
*6/((a - c)**2*(b - c)**2) - a*b**4*c**4/((a - c)*(b - c)) + a*b*
*4*c**2 + 8*a*b**3*c**7/((a - c)**2*(b - c)**2) - a*b**3*c**5/((a
 - c)*(b - c)) - 4*a*b**3*c**3 + a*b**2*c**8/((a - c)**2*(b - c)*
*2) + a*b**2*c**6/((a - c)*(b - c)) + a*b**2*c**4 - b**6*c**5/((a
 - c)**2*(b - c)**2) + b**5*c**6/((a - c)**2*(b - c)**2) - b**5*c
**4/((a - c)*(b - c)) + b**4*c**7/((a - c)**2*(b - c)**2) + 2*b**
4*c**5/((a - c)*(b - c)) - b**3*c**8/((a - c)**2*(b - c)**2) - b*
*3*c**6/((a - c)*(b - c)))/(2*a**3*b**3 - 3*a**3*b**2*c - 3*a**3*
b*c**2 + 2*a**3*c**3 - 3*a**2*b**3*c + 12*a**2*b**2*c**2 - 3*a**2
*b*c**3 - 3*a*b**3*c**2 - 3*a*b**2*c**3 + 2*b**3*c**3))/((a - c)*
(b - c))
```

GIAC/XCAS [A] time = 0.209653, size = 109, normalized size = 1.6

$$\frac{a\ln\left(|-a+x|\right)}{a^2-ab-ac+bc}-\frac{b\ln\left(|-b+x|\right)}{ab-b^2-ac+bc}+\frac{c\ln\left(|-c+x|\right)}{ab-ac-bc+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((a - x)*(b - x)*(c - x)),x, algorithm="giac")
```

```
[Out] a*ln(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*ln(abs(-b + x))/(a*
b - b^2 - a*c + b*c) + c*ln(abs(-c + x))/(a*b - a*c - b*c + c^2)
```

3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx$

Optimal. Leaf size=47

$$\frac{\log(b^2+x^2)}{2(a^2-b^2)}-\frac{\log(a^2+x^2)}{2(a^2-b^2)}$$

```
[Out] -Log[a^2 + x^2]/(2*(a^2 - b^2)) + Log[b^2 + x^2]/(2*(a^2 - b^2))
```

Rubi [A] time = 0.0508699, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\log(b^2+x^2)}{2(a^2-b^2)}-\frac{\log(a^2+x^2)}{2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] -Log[a^2 + x^2]/(2*(a^2 - b^2)) + Log[b^2 + x^2]/(2*(a^2 - b^2))
```

Rubi in Sympy [A] time = 4.79107, size = 36, normalized size = 0.77

$$-\frac{\log(a^2+x^2)}{2(a^2-b^2)}+\frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] -log(a**2 + x**2)/(2*(a**2 - b**2)) + log(b**2 + x**2)/(2*(a**2 -
 b**2))
```

Mathematica [A] time = 0.0130995, size = 34, normalized size = 0.72

$$\frac{\log(b^2+x^2)-\log(a^2+x^2)}{2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))
```

Maple [A] time = 0.005, size = 44, normalized size = 0.9

$$-\frac{\ln\left(a^2+x^2\right)}{2\,a^2-2\,b^2}+\frac{\ln\left(b^2+x^2\right)}{2\,a^2-2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^2+x^2)/(b^2+x^2),x)
```

```
[Out] -1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

Maxima [A] time = 1.38987, size = 58, normalized size = 1.23

$$-\frac{\log\left(a^2+x^2\right)}{2\,(a^2-b^2)}+\frac{\log\left(b^2+x^2\right)}{2\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((a^2 + x^2)*(b^2 + x^2)),x, algorithm="maxima")
```

```
[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

Fricas [A] time = 0.20656, size = 43, normalized size = 0.91

$$-\frac{\log\left(a^2+x^2\right)-\log\left(b^2+x^2\right)}{2\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((a^2 + x^2)*(b^2 + x^2)),x, algorithm="fricas")
```

```
[Out] -1/2*(log(a^2 + x^2) - log(b^2 + x^2))/(a^2 - b^2)
```

Sympy [A] time = 0.842479, size = 121, normalized size = 2.57

$$\frac{\log\left(-\frac{a^4}{2(a-b)(a+b)}+\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}-\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2(a-b)(a+b)}-\frac{\log\left(\frac{a^4}{2(a-b)(a+b)}-\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}+\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**
2/2 - b**4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b
)) - log(a**4/(2*(a - b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) +
 a**2/2 + b**4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a
 + b))
```

GIAC/XCAS [A] time = 0.22537, size = 88, normalized size = 1.87

$$\frac{\ln\left(\frac{\left|a^2+b^2+2\,x^2-\left|a^2-b^2\right|\right|}{a^2+b^2+2\,x^2+\left|a^2-b^2\right|}\right)}{2\left|a^2-b^2\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((a^2 + x^2)*(b^2 + x^2)),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(a^2 + b^2 + 2*x^2 - abs(a^2 - b^2))/(a^2 + b^2 + 2*x^2
 + abs(a^2 - b^2)))/abs(a^2 - b^2)
```

3.14 $\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx$

Optimal. Leaf size=40

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

```
[Out] (a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

Rubi [A] time = 0.054887, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]

[Out] (a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

Rubi in Sympy [A] time = 6.19423, size = 26, normalized size = 0.65

$$\frac{a\,\mathrm{atan}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\,\mathrm{atan}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**2+x**2)/(b**2+x**2),x)

[Out] a*atan(x/a)/(a**2 - b**2) - b*atan(x/b)/(a**2 - b**2)
```

Mathematica [A] time = 0.0195599, size = 30, normalized size = 0.75

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)-b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)
```

Maple [A] time = 0.009, size = 41, normalized size = 1.

$$\frac{a}{a^2-b^2}\arctan\left(\frac{x}{a}\right)-\frac{b}{a^2-b^2}\arctan\left(\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^2+x^2)/(b^2+x^2),x)

[Out] a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

Maxima [A] time = 1.48984, size = 54, normalized size = 1.35

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((a^2 + x^2)*(b^2 + x^2)),x, algorithm="maxima")

[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

Fricas [A] time = 0.216918, size = 41, normalized size = 1.02

$$\frac{a\arctan\left(\frac{x}{a}\right)-b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((a^2 + x^2)*(b^2 + x^2)),x, algorithm="fricas")

[Out] (a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)
```

Sympy [A] time = 1.52227, size = 393, normalized size = 9.82

$$-\frac{ia\log\left(-\frac{2ia^7}{(a-b)^3(a+b)^3}+\frac{4ia^5b^2}{(a-b)^3(a+b)^3}-\frac{2ia^3b^4}{(a-b)^3(a+b)^3}+\frac{ia^3}{(a-b)(a+b)}+\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}+\frac{ia\log\left(\frac{2ia^7}{(a-b)^3(a+b)^3}-\frac{4ia^5b^2}{(a-b)^3(a+b)^3}+\frac{2ia^3b^4}{(a-b)^3(a+b)^3}-\frac{ia^3}{(a-b)(a+b)}-\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}-\frac{ib\log\left(-\frac{2ia^4b^3}{(a-b)^3(a+b)^3}+\frac{4ia^2b^5}{(a-b)^3(a+b)^3}+\frac{ia^2b}{(a-b)(a+b)}-\frac{2ib^7}{(a-b)^3(a+b)^3}+\frac{ib^3}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}+\frac{ib\log\left(\frac{2ia^4b^3}{(a-b)^3(a+b)^3}-\frac{4ia^2b^5}{(a-b)^3(a+b)^3}-\frac{ia^2b}{(a-b)(a+b)}+\frac{2ib^7}{(a-b)^3(a+b)^3}-\frac{ib^3}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] -I*a*log(-2*I*a**7/((a - b)**3*(a + b)**3) + 4*I*a**5*b**2/((a -
b)**3*(a + b)**3) - 2*I*a**3*b**4/((a - b)**3*(a + b)**3) + I*a**
3/((a - b)*(a + b)) + I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*
(a + b)) + I*a*log(2*I*a**7/((a - b)**3*(a + b)**3) - 4*I*a**5*b*
*2/((a - b)**3*(a + b)**3) + 2*I*a**3*b**4/((a - b)**3*(a + b)**3
) - I*a**3/((a - b)*(a + b)) - I*a*b**2/((a - b)*(a + b)) + x)/(2
*(a - b)*(a + b)) - I*b*log(-2*I*a**4*b**3/((a - b)**3*(a + b)**3
) + 4*I*a**2*b**5/((a - b)**3*(a + b)**3) + I*a**2*b/((a - b)*(a
+ b)) - 2*I*b**7/((a - b)**3*(a + b)**3) + I*b**3/((a - b)*(a + b
)) + x)/(2*(a - b)*(a + b)) + I*b*log(2*I*a**4*b**3/((a - b)**3*(
a + b)**3) - 4*I*a**2*b**5/((a - b)**3*(a + b)**3) - I*a**2*b/((a
 - b)*(a + b)) + 2*I*b**7/((a - b)**3*(a + b)**3) - I*b**3/((a -
b)*(a + b)) + x)/(2*(a - b)*(a + b))
```

GIAC/XCAS [A] time = 0.212004, size = 54, normalized size = 1.35

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((a^2 + x^2)*(b^2 + x^2)),x, algorithm="giac")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

3.15 $\int \frac{x}{(-1+x)(1+x^2)}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0448895, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((-1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Rubi in Sympy [A] time = 2.53133, size = 19, normalized size = 0.7

$$\frac{\log(-x+1)}{2}-\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-1+x)/(x**2+1),x)

[Out] log(-x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Mathematica [A] time = 0.00578465, size = 27, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((-1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.003, size = 20, normalized size = 0.7

$$-\frac{\ln\left(x^2+1\right)}{4}+\frac{\arctan\left(x\right)}{2}+\frac{\ln\left(-1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-1+x)/(x^2+1),x)
```

```
[Out] -1/4*ln(x^2+1)+1/2*arctan(x)+1/2*ln(-1+x)
```

Maxima [A] time = 1.50145, size = 26, normalized size = 0.96

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x - 1)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 0.220811, size = 26, normalized size = 0.96

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Sympy [A] time = 0.129761, size = 19, normalized size = 0.7

$$\frac{\log\left(x-1\right)}{2}-\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-1+x)/(x**2+1),x)
```

```
[Out] log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

GIAC/XCAS [A] time = 0.221868, size = 27, normalized size = 1.

$$\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\ln\left(x^2 + 1\right) + \frac{1}{2}\ln\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x - 1)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*ln(x^2 + 1) + 1/2*ln(abs(x - 1))
```

3.16 $\int \frac{x}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x +
 x^2]/6
```

Rubi [A] time = 0.0484013, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x +
 x^2]/6
```

Rubi in Sympy [A] time = 3.2198, size = 37, normalized size = 0.9

$$-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x/3
 - 1/3))/3
```

Mathematica [A] time = 0.0148475, size = 40, normalized size = 0.98

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x
^2]/6
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^3+1),x)
```

```
[Out] 1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))-1/3*ln(1
+x)
```

Maxima [A] time = 1.53201, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1)
- 1/3*log(x + 1)
```

Fricas [A] time = 0.213383, size = 55, normalized size = 1.34

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^2-x+1\right)-2\sqrt{3}\log(x+1)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*log(x^2 - x + 1) - 2*sqrt(3)*log(x + 1) + 6
*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.172225, size = 41, normalized size = 1.

$$-\frac{\log(x+1)}{3}+\frac{\log(x^2-x+1)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.216293, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{1}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*ln(x^2 - x + 1) -
 1/3*ln(abs(x + 1))
```

3.17 $\int \frac{x^3}{(-1+x)^2(1+x^3)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(x+1)$$

```
[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^
2]/3
```

Rubi [A] time = 0.199665, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^
2]/3
```

Rubi in Sympy [A] time = 11.2349, size = 31, normalized size = 0.72

$$\frac{3\log\left(-x+1\right)}{4}-\frac{\log\left(x+1\right)}{12}-\frac{\log\left(x^2-x+1\right)}{3}+\frac{1}{2\left(-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-1+x)**2/(x**3+1),x)
```

```
[Out] 3*log(-x + 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 + 1/(2*(-x
+ 1))
```

Mathematica [A] time = 0.0265317, size = 34, normalized size = 0.79

$$\frac{1}{12}\left(-\frac{6}{x-1}+9\log(x-1)-\log(x+1)-4\log\left((x-1)^2+x\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] (-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x]
)/12
```

Maple [A] time = 0.006, size = 32, normalized size = 0.7

$$-\frac{\ln\left(x^2-x+1\right)}{3}-\frac{\ln\left(1+x\right)}{12}-\frac{1}{2\,x-2}+\frac{3\,\ln\left(-1+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-1+x)^2/(x^3+1),x)
```

```
[Out] -1/3*ln(x^2-x+1)-1/12*ln(1+x)-1/2/(-1+x)+3/4*ln(-1+x)
```

Maxima [A] time = 1.5625, size = 42, normalized size = 0.98

$$-\frac{1}{2\left(x-1\right)}-\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{1}{12}\,\log\left(x+1\right)+\frac{3}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^3 + 1)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] -1/2/(x - 1) - 1/3*log(x^2 - x + 1) - 1/12*log(x + 1) + 3/4*log(x
 - 1)
```

Fricas [A] time = 0.2232, size = 54, normalized size = 1.26

$$-\frac{4\left(x-1\right)\log\left(x^2-x+1\right)+\left(x-1\right)\log\left(x+1\right)-9\left(x-1\right)\log\left(x-1\right)+6}{12\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^3 + 1)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(x - 1)*log(x^2 - x + 1) + (x - 1)*log(x + 1) - 9*(x - 1
)*log(x - 1) + 6)/(x - 1)
```

Sympy [A] time = 0.161605, size = 31, normalized size = 0.72

$$\frac{3\log\left(x-1\right)}{4}-\frac{\log\left(x+1\right)}{12}-\frac{\log\left(x^{2}-x+1\right)}{3}-\frac{1}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-1+x)**2/(x**3+1),x)
```

```
[Out] 3*log(x - 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 - 1/(2*x - 2
)
```

GIAC/XCAS [A] time = 0.216075, size = 49, normalized size = 1.14

$$-\frac{1}{2\left(x-1\right)}-\frac{1}{3}\ln\left(\frac{1}{x-1}+\frac{1}{\left(x-1\right)^{2}}+1\right)-\frac{1}{12}\ln\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^3 + 1)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] -1/2/(x - 1) - 1/3*ln(1/(x - 1) + 1/(x - 1)^2 + 1) - 1/12*ln(abs(
-2/(x - 1) - 1))
```

3.18 $\int \frac{1}{1+x^4} \, dx$

Optimal. Leaf size=85

$$-\frac{\log \left(x^2-\sqrt{2} x+1\right)}{4 \sqrt{2}}+\frac{\log \left(x^2+\sqrt{2} x+1\right)}{4 \sqrt{2}}-\frac{\tan ^{-1}\left(1-\sqrt{2} x\right)}{2 \sqrt{2}}+\frac{\tan ^{-1}\left(\sqrt{2} x+1\right)}{2 \sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0773511, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$-\frac{\log \left(x^2-\sqrt{2} x+1\right)}{4 \sqrt{2}}+\frac{\log \left(x^2+\sqrt{2} x+1\right)}{4 \sqrt{2}}-\frac{\tan ^{-1}\left(1-\sqrt{2} x\right)}{2 \sqrt{2}}+\frac{\tan ^{-1}\left(\sqrt{2} x+1\right)}{2 \sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 5.89908, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2} \log \left(x^2-\sqrt{2} x+1\right)}{8}+\frac{\sqrt{2} \log \left(x^2+\sqrt{2} x+1\right)}{8}+\frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2} x-1\right)}{4}+\frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2} x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*
x + 1)/4
```

Mathematica [A] time = 0.0573826, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqr
t[2]*x + x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0.004, size = 58, normalized size = 0.7

$$\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+1),x)
```

```
[Out] 1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/4*arctan(x*2^(1/2)-1)*2^(1/2)+1
/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))
```

Maxima [A] time = 1.55178, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$
$$+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*
x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 0.222759, size = 131, normalized size = 1.54

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x +
 1) + 1)) - 1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 -
sqrt(2)*x + 1) - 1)) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8
*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.192601, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*
x + 1)/4
```

GIAC/XCAS [A] time = 0.21022, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*ln(x^2 + sqrt(2)*x
 + 1) - 1/8*sqrt(2)*ln(x^2 - sqrt(2)*x + 1)
```

3.19 $\int \frac{x^2}{1+x^4}\,dx$

Optimal. Leaf size=85

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0791276, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + x^4),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 6.41178, size = 73, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*
x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x
 + 1)/4
```

Mathematica [A] time = 0.0138946, size = 64, normalized size = 0.75

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)-\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] + Log[1 - Sqr
t[2]*x + x^2] - Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0.002, size = 58, normalized size = 0.7

$$\frac{\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4+1),x)
```

```
[Out] 1/4*arctan(x*2^(1/2)-1)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/
(1+x^2+x*2^(1/2)))+1/4*arctan(1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.55491, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)\\-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*
x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 0.220334, size = 131, normalized size = 1.54

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x +
 1) + 1)) - 1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 -
sqrt(2)*x + 1) - 1)) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8
*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.192362, size = 73, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*
x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x
 + 1)/4
```

GIAC/XCAS [A] time = 0.211907, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*ln(x^2 + sqrt(2)*x
 + 1) + 1/8*sqrt(2)*ln(x^2 - sqrt(2)*x + 1)
```

3.20 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rubi [A] time = 0.0717443, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rubi in Sympy [A] time = 4.83082, size = 63, normalized size = 0.94

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(sqrt(3)
*(2*x/3 - 1/3))/6 + sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/6
```

Mathematica [C] time = 0.153813, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)-\sqrt{1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I
*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] time = 0.004, size = 54, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^
2-x+1)+1/6*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Maxima [A] time = 1.54357, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1
)
```

Fricas [A] time = 0.210363, size = 77, normalized size = 1.15

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-\sqrt{3}\log\left(x^2-x+1\right)+2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - sqrt(3)*log(x^2 - x + 1)
 + 2*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*arctan(1/3*sqrt(3)*(2*x -
1)))
```

Sympy [A] time = 0.236678, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(
3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.209445, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\ln\left(x^2+x+1\right)-\frac{1}{4}\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/4*ln(x^2 + x + 1) - 1/4*ln(x^2 - x + 1)
```

3.21 $\int (a+bx)^p \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

```
[Out] (a + b*x)^(1 + p)/(b*(1 + p))
```

Rubi [A] time = 0.0112612, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^p,x]

[Out] (a + b*x)^(1 + p)/(b*(1 + p))
```

Rubi in Sympy [A] time = 0.860915, size = 12, normalized size = 0.67

$$\frac{(a+bx)^{p+1}}{b\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**p,x)

[Out] (a + b*x)**(p + 1)/(b*(p + 1))
```

Mathematica [A] time = 0.0120771, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{p+1}}{bp+b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b + b*p)
```

Maple [A] time = 0.004, size = 19, normalized size = 1.1

$$\frac{(bx+a)^{1+p}}{b\,(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^p,x)
```

```
[Out] (b*x+a)^(1+p)/b/(1+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227131, size = 27, normalized size = 1.5

$$\frac{(bx+a)(bx+a)^p}{bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^p/(b*p + b)
```

Sympy [A] time = 0.035379, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log{(a+bx)} & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x),
 True))/b
```

GIAC/XCAS [A] time = 0.208772, size = 24, normalized size = 1.33

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p,x, algorithm="giac")
```

```
[Out] (b*x + a)^(p + 1)/(b*(p + 1))
```

3.22 $\int x(a+bx)^p\,dx$

Optimal. Leaf size=39

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

```
[Out] -((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(
2 + p))
```

Rubi [A] time = 0.0311446, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x)^p,x]
```

```
[Out] -((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(
2 + p))
```

Rubi in Sympy [A] time = 3.55946, size = 31, normalized size = 0.79

$$-\frac{a\left(a+bx\right)^{p+1}}{b^2\left(p+1\right)}+\frac{\left(a+bx\right)^{p+2}}{b^2\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x+a)**p,x)
```

```
[Out] -a*(a + b*x)**(p + 1)/(b**2*(p + 1)) + (a + b*x)**(p + 2)/(b**2*(
p + 2))
```

Mathematica [A] time = 0.0184611, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{p+1}(b(p+1)x-a)}{b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x)^p,x]

[Out] ((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))
```

Maple [A] time = 0.002, size = 36, normalized size = 0.9

$$-\frac{(bx+a)^{1+p}(-xpb-bx+a)}{b^2(p^2+3p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x+a)^p,x)

[Out] -(b*x+a)^(1+p)*(-b*p*x-b*x+a)/b^2/(p^2+3*p+2)
```

Maxima [A] time = 1.38277, size = 57, normalized size = 1.46

$$\frac{\left(b^2(p+1)x^2+abpx-a^2\right)(bx+a)^p}{(p^2+3p+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p*x,x, algorithm="maxima")

[Out] (b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^
2)
```

Fricas [A] time = 0.222538, size = 72, normalized size = 1.85

$$\frac{\left(abpx+\left(b^2p+b^2\right)x^2-a^2\right)(bx+a)^p}{b^2p^2+3b^2p+2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p*x,x, algorithm="fricas")

[Out] (a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*
p + 2*b^2)
```

Sympy [A] time = 0.83637, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p = -2 \\ -\frac{a \log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p = -1 \\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3
*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), E
q(p, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b
*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + a*b*p*x*(a + b*x)**p/(b*
*2*p**2 + 3*b**2*p + 2*b**2) + b**2*p*x**2*(a + b*x)**p/(b**2*p**
2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)**p/(b**2*p**2 + 3*b*
*2*p + 2*b**2), True))
```

GIAC/XCAS [A] time = 0.206119, size = 113, normalized size = 2.9

$$\frac{b^2px^2e^{(p\ln(bx+a))} + abpxe^{(p\ln(bx+a))} + b^2x^2e^{(p\ln(bx+a))} - a^2e^{(p\ln(bx+a))}}{b^2p^2 + 3\,b^2p + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p*x,x, algorithm="giac")
```

```
[Out] (b^2*p*x^2*e^(p*ln(b*x + a)) + a*b*p*x*e^(p*ln(b*x + a)) + b^2*x^
2*e^(p*ln(b*x + a)) - a^2*e^(p*ln(b*x + a)))/(b^2*p^2 + 3*b^2*p +
 2*b^2)
```

3.23 $\int x^2(a+bx)^p\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

```
[Out] (a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(
b^3*(2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Rubi [A] time = 0.0488915, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x)^p,x]
```

```
[Out] (a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(
b^3*(2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Rubi in Sympy [A] time = 6.07311, size = 51, normalized size = 0.85

$$\frac{a^2\,(a+bx)^{p+1}}{b^3\,(p+1)}-\frac{2a\,(a+bx)^{p+2}}{b^3\,(p+2)}+\frac{(a+bx)^{p+3}}{b^3\,(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x+a)**p,x)
```

```
[Out] a**2*(a + b*x)**(p + 1)/(b**3*(p + 1)) - 2*a*(a + b*x)**(p + 2)/(
b**3*(p + 2)) + (a + b*x)**(p + 3)/(b**3*(p + 3))
```

Mathematica [A] time = 0.0297459, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{p+1}\left(2a^2-2ab(p+1)x+b^2\left(p^2+3p+2\right)x^2\right)}{b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)
*x^2))/(b^3*(1 + p)*(2 + p)*(3 + p))
```

Maple [A]   time = 0.006, size = 73, normalized size = 1.2

$$\frac{(bx+a)^{1+p}\left(b^2p^2x^2+3\,b^2px^2-2\,abpx+2\,x^2b^2-2\,axb+2\,a^2\right)}{b^3\left(p^3+6\,p^2+11\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x+a)^p,x)
```

```
[Out] (b*x+a)^(1+p)*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*
x+2*a^2)/b^3/(p^3+6*p^2+11*p+6)
```

Maxima [A]   time = 1.40643, size = 92, normalized size = 1.53

$$\frac{\left(\left(p^2+3\,p+2\right)b^3x^3+\left(p^2+p\right)ab^2x^2-2\,a^2bpx+2\,a^3\right)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p*x^2,x, algorithm="maxima")
```

```
[Out] ((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*
a^3)*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Fricas [A]   time = 0.229888, size = 130, normalized size = 2.17

$$-\frac{\left(2\,a^2bpx-\left(b^3p^2+3\,b^3p+2\,b^3\right)x^3-2\,a^3-\left(ab^2p^2+ab^2p\right)x^2\right)(bx+a)^p}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p*x^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*
p^2 + a*b^2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p +
```

```
6*b^3)
```

Sympy [A] time = 1.57396, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^p x^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} - \frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{3b^3px^3(a+}{b^3p^3+6b^3p^2+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b
**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*
x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x
 + 2*b**5*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x
**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**
2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x) + b*
*2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3 -
a*x/b**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**
3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p
/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**
2*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a
*b**2*p*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p +
6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 +
11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b**3*p
**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3), True))
```

GIAC/XCAS [A] time = 0.207982, size = 208, normalized size = 3.47

$$\frac{b^3p^2x^3e^{(p\ln(bx+a))} + ab^2p^2x^2e^{(p\ln(bx+a))} + 3\,b^3px^3e^{(p\ln(bx+a))} + ab^2px^2e^{(p\ln(bx+a))} + 2\,b^3x^3e^{(p\ln(bx+a))} - 2\,a^2bpxe^{(p\ln(bx+a))} + 2}{b^3p^3 + 6\,b^3p^2 + 11\,b^3p + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^p*x^2,x, algorithm="giac")
```

```
[Out] (b^3*p^2*x^3*e^(p*ln(b*x + a)) + a*b^2*p^2*x^2*e^(p*ln(b*x + a))
+ 3*b^3*p*x^3*e^(p*ln(b*x + a)) + a*b^2*p*x^2*e^(p*ln(b*x + a)) +
```

```
 2*b^3*x^3*e^(p*ln(b*x + a)) - 2*a^2*b*p*x*e^(p*ln(b*x + a)) + 2*
a^3*e^(p*ln(b*x + a)))/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

3.24 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] Log[a + b*x]/b
```

Rubi [A] time = 0.00583681, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b
```

Rubi in Sympy [A] time = 0.654205, size = 7, normalized size = 0.7

$$\frac{\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x+a),x)

[Out] log(a + b*x)/b
```

Mathematica [A] time = 0.00114778, size = 10, normalized size = 1.

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A] time = 1.37599, size = 14, normalized size = 1.4

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x + a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A] time = 0.197009, size = 14, normalized size = 1.4

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x + a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 0.03691, size = 7, normalized size = 0.7

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

GIAC/XCAS [A] time = 0.20298, size = 15, normalized size = 1.5

$$\frac{\ln (|bx + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x + a),x, algorithm="giac")
```

```
[Out] ln(abs(b*x + a))/b
```

3.25 $\int \frac{1}{(a+bx)^2} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -(1/(b*(a + b*x)))
```

Rubi [A] time = 0.00547363, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^(-2),x]

[Out] -(1/(b*(a + b*x)))
```

Rubi in Sympy [A] time = 0.638036, size = 8, normalized size = 0.67

$$-\frac{1}{b\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x+a)**2,x)

[Out] -1/(b*(a + b*x))
```

Mathematica [A] time = 0.00382892, size = 12, normalized size = 1.

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A] time = 0., size = 13, normalized size = 1.1

$$-\frac{1}{b\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A] time = 1.39585, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^(-2),x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A] time = 0.213944, size = 18, normalized size = 1.5

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^(-2),x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A] time = 0.51837, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

GIAC/XCAS [A] time = 0.202497, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^(-2),x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

3.26 $\int \frac{x}{a+bx}\,dx$

Optimal. Leaf size=18

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0215182, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\log\left(a+bx\right)}{b^2}+\int\frac{1}{b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + Integral(1/b, x)
```

Mathematica [A] time = 0.00325391, size = 18, normalized size = 1.

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A] time = 0.003, size = 19, normalized size = 1.1

$$\frac{x}{b} - \frac{a\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x+a),x)
```

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A] time = 1.42164, size = 24, normalized size = 1.33

$$\frac{x}{b} - \frac{a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x + a),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A] time = 0.201912, size = 23, normalized size = 1.28

$$\frac{bx - a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x + a),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A] time = 0.477018, size = 14, normalized size = 0.78

$$-\frac{a\log(a+bx)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

GIAC/XCAS [A] time = 0.203463, size = 26, normalized size = 1.44

$$\frac{x}{b} - \frac{a\ln\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x + a),x, algorithm="giac")
```

```
[Out] x/b - a*ln(abs(b*x + a))/b^2
```

3.27 $\int \frac{x^2}{a+bx} \, dx$

Optimal. Leaf size=31

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rubi [A] time = 0.0336065, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x),x]

[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log{(a+bx)}}{b^3} + \frac{\int x \, dx}{b} - \frac{\int a \, dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x+a),x)

[Out] a**2*log(a + b*x)/b**3 + Integral(x, x)/b - Integral(a, x)/b**2
```

Mathematica [A] time = 0.00463847, size = 31, normalized size = 1.

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A] time = 0.002, size = 30, normalized size = 1.

$$-\frac{ax}{b^2}+\frac{x^2}{2\,b}+\frac{a^2\ln\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x+a),x)
```

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Maxima [A] time = 1.43013, size = 39, normalized size = 1.26

$$\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b*x + a),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A] time = 0.191815, size = 39, normalized size = 1.26

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b*x + a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] time = 0.501758, size = 26, normalized size = 0.84

$$\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

GIAC/XCAS [A] time = 0.200361, size = 41, normalized size = 1.32

$$\frac{a^2 \ln\left(|bx + a|\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b*x + a),x, algorithm="giac")
```

```
[Out] a^2*ln(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.28 $\int \frac{1}{x(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rubi [A] time = 0.0124125, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x)),x]

[Out] Log[x]/a - Log[a + b*x]/a
```

Rubi in Sympy [A] time = 1.70585, size = 12, normalized size = 0.67

$$\frac{\log{(x)}}{a}-\frac{\log{(a+bx)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x+a),x)

[Out] log(x)/a - log(a + b*x)/a
```

Mathematica [A] time = 0.00496038, size = 18, normalized size = 1.

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A] time = 0.003, size = 19, normalized size = 1.1

$$\frac{\ln(x)}{a} - \frac{\ln(bx + a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A] time = 1.37705, size = 24, normalized size = 1.33

$$-\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x + a)*x),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A] time = 0.212876, size = 22, normalized size = 1.22

$$-\frac{\log(bx + a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x + a)*x),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] time = 0.165344, size = 10, normalized size = 0.56

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

GIAC/XCAS [A] time = 0.198912, size = 27, normalized size = 1.5

$$-\frac{\ln\left(|bx+a|\right)}{a}+\frac{\ln\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)*x),x, algorithm="giac")
```

```
[Out] -ln(abs(b*x + a))/a + ln(abs(x))/a
```

3.29 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] time = 0.0304201, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]

[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi in Sympy [A] time = 2.83375, size = 24, normalized size = 0.86

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x**2/(b*x+a),x)

[Out] -1/(a*x) - b*log(x)/a**2 + b*log(a + b*x)/a**2
```

Mathematica [A] time = 0.00592833, size = 28, normalized size = 1.

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A] time = 0.008, size = 29, normalized size = 1.

$$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x+a),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A] time = 1.41238, size = 38, normalized size = 1.36

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)*x^2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A] time = 0.213899, size = 35, normalized size = 1.25

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)*x^2),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] time = 0.598399, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x+a),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

GIAC/XCAS [A] time = 0.199222, size = 41, normalized size = 1.46

$$\frac{b\ln\left(|bx + a|\right)}{a^2} - \frac{b\ln\left(|x|\right)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)*x^2),x, algorithm="giac")
```

```
[Out] b*ln(abs(b*x + a))/a^2 - b*ln(abs(x))/a^2 - 1/(a*x)
```

3.30 $\int \frac{1}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a
+ b*x])/a^3
```

Rubi [A] time = 0.0476493, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a
+ b*x])/a^3
```

Rubi in Sympy [A] time = 4.2685, size = 39, normalized size = 0.93

$$-\frac{b}{a^2\left(a+bx\right)}-\frac{1}{a^2x}-\frac{2b\log\left(x\right)}{a^3}+\frac{2b\log\left(a+bx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] -b/(a**2*(a + b*x)) - 1/(a**2*x) - 2*b*log(x)/a**3 + 2*b*log(a +
b*x)/a**3
```

Mathematica [A] time = 0.0535642, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x)^2),x]

[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A] time = 0.008, size = 43, normalized size = 1.

$$-\frac{1}{a^2x} - \frac{b}{a^2(bx+a)} - 2\,\frac{b\ln(x)}{a^3} + 2\,\frac{b\ln(bx+a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x+a)^2,x)

[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A] time = 1.37597, size = 61, normalized size = 1.45

$$-\frac{2\,bx+a}{a^2bx^2+a^3x} + \frac{2\,b\log(bx+a)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)^2*x^2),x, algorithm="maxima")

[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log
(x)/a^3
```

Fricas [A] time = 0.209714, size = 85, normalized size = 2.02

$$-\frac{2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log(bx+a) + 2\left(b^2x^2 + abx\right)\log(x)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)^2*x^2),x, algorithm="fricas")

[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 +
 a*b*x)*log(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] time = 0.746439, size = 36, normalized size = 0.86

$$-\frac{a+2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] -(a + 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x)
)/a**3
```

GIAC/XCAS [A] time = 0.202174, size = 70, normalized size = 1.67

$$-\frac{2\,b\ln\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -2*b*ln(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(
a/(b*x + a) - 1))
```

3.31 $\int \frac{1}{c^2+x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] ArcTan[x/c]/c
```

Rubi [A] time = 0.00715066, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c^2 + x^2)^(-1),x]

[Out] ArcTan[x/c]/c
```

Rubi in Sympy [A] time = 0.726118, size = 5, normalized size = 0.5

$$\frac{\operatorname{atan}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c**2+x**2),x)

[Out] atan(x/c)/c
```

Mathematica [A] time = 0.00314959, size = 10, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/c]/c
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$\frac{1}{c}\arctan\left(\frac{x}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c^2+x^2),x)
```

```
[Out] arctan(x/c)/c
```

Maxima [A] time = 1.51966, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2 + x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/c)/c
```

Fricas [A] time = 0.231272, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2 + x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/c)/c
```

Sympy [A] time = 0.102591, size = 20, normalized size = 2.

$$\frac{-\frac{i\log\left(-ic+x\right)}{2}+\frac{i\log\left(ic+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c**2+x**2),x)
```

```
[Out] (-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c
```

GIAC/XCAS [A] time = 0.199612, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2 + x^2),x, algorithm="giac")
```

```
[Out] arctan(x/c)/c
```

3.32 $\int \frac{1}{c^2-x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] ArcTanh[x/c]/c
```

Rubi [A] time = 0.00771223, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c^2 - x^2)^(-1),x]

[Out] ArcTanh[x/c]/c
```

Rubi in Sympy [A] time = 0.898045, size = 5, normalized size = 0.5

$$\frac{\operatorname{atanh}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c**2-x**2),x)

[Out] atanh(x/c)/c
```

Mathematica [A] time = 0.00355437, size = 10, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

Maple [B] time = 0.003, size = 22, normalized size = 2.2

$$-\frac{\ln\left(-c+x\right)}{2\,c}+\frac{\ln\left(c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c^2-x^2),x)
```

```
[Out] -1/2/c*ln(-c+x)+1/2/c*ln(c+x)
```

Maxima [A] time = 1.40752, size = 28, normalized size = 2.8

$$\frac{\log\left(c+x\right)}{2\,c}-\frac{\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2 - x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c + x)/c - 1/2*log(-c + x)/c
```

Fricas [A] time = 0.224707, size = 24, normalized size = 2.4

$$\frac{\log\left(c+x\right)-\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2 - x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(c + x) - log(-c + x))/c
```

Sympy [A] time = 0.114832, size = 15, normalized size = 1.5

$$-\frac{\frac{\log\left(-c+x\right)}{2}-\frac{\log\left(c+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c**2-x**2),x)
```

```
[Out] -(log(-c + x)/2 - log(c + x)/2)/c
```

GIAC/XCAS [A] time = 0.197995, size = 31, normalized size = 3.1

$$\frac{\ln\left(|c+x|\right)}{2\,c}-\frac{\ln\left(|-c+x|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2 - x^2),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(c + x))/c - 1/2*ln(abs(-c + x))/c
```

3.33 $\int \frac{1}{-1+2x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) + Log[1 -
2^(1/3)*x]/(3*2^(1/3)) - Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(6*2^(1
/3))
```

Rubi [A] time = 0.0952237, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) + Log[1 -
2^(1/3)*x]/(3*2^(1/3)) - Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(6*2^(1
/3))
```

Rubi in Sympy [A] time = 4.7508, size = 73, normalized size = 0.94

$$\frac{2^{\frac{2}{3}}\log\left(-\sqrt[3]{2}x+1\right)}{6}-\frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^2+\sqrt[3]{2}x+1\right)}{12}-\frac{2^{\frac{2}{3}}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**3-1),x)
```

```
[Out] 2**(2/3)*log(-2**(1/3)*x + 1)/6 - 2**(2/3)*log(2**(2/3)*x**2 + 2*
*(1/3)*x + 1)/12 - 2**(2/3)*sqrt(3)*atan(sqrt(3)*(2*2**(1/3)*x/3
+ 1/3))/6
```

Mathematica [A] time = 0.0745608, size = 66, normalized size = 0.85

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)-2\log\left(1-\sqrt[3]{2}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]] - 2*Log[1 - 2^(1/3)
*x] + Log[1 + 2^(1/3)*x + 2^(2/3)*x^2])/(6*2^(1/3))
```

Maple [A] time = 0.004, size = 58, normalized size = 0.7

$$\frac{2^{\frac{2}{3}}}{6}\ln\left(x-\frac{2^{\frac{2}{3}}}{2}\right)-\frac{2^{\frac{2}{3}}}{12}\ln\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)-\frac{2^{\frac{2}{3}}\sqrt{3}}{6}\arctan\left(\frac{\left(1+2\sqrt[3]{2}x\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^3-1),x)
```

```
[Out] 1/6*2^(2/3)*ln(x-1/2*2^(2/3))-1/12*2^(2/3)*ln(x^2+1/2*2^(2/3)*x+1
/2*2^(1/3))-1/6*arctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/
2)
```

Maxima [A] time = 1.57509, size = 89, normalized size = 1.14

$$-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x+1\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{1}{2}\cdot 2^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x^3 - 1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*2^(2/3)*x + 2^
(1/3))) - 1/12*2^(2/3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) + 1/6*2^(
2/3)*log(1/2*2^(2/3)*(2^(1/3)*x - 1))
```

Fricas [A] time = 0.213589, size = 77, normalized size = 0.99

$$-\frac{1}{36}\sqrt{3}2^{\frac{2}{3}}\left(\sqrt{3}\log\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x+1\right)-2\sqrt{3}\log\left(2^{\frac{1}{3}}x-1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}\left(2\cdot 2^{\frac{1}{3}}x+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*2^(2/3)*(sqrt(3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) -
 2*sqrt(3)*log(2^(1/3)*x - 1) + 6*arctan(1/3*sqrt(3)*(2*2^(1/3)*x
 + 1)))
```

Sympy [A] time = 0.652428, size = 78, normalized size = 1.

$$\frac{2^{\frac{2}{3}}\log\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt[3]{2}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**3-1),x)
```

```
[Out] 2**(2/3)*log(x - 2**(2/3)/2)/6 - 2**(2/3)*log(x**2 + 2**(2/3)*x/2
 + 2**(1/3)/2)/6 - 2**(2/3)*sqrt(3)*atan(2*2**(1/3)*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x^3 - 1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.34 $\int \frac{1}{-2+x^3} \, dx$

Optimal. Leaf size=74

$$-\frac{\log \left(x^2+\sqrt[3]{2}x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log \left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3
) - x]/(3*2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))
```

Rubi [A] time = 0.074852, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$

$$-\frac{\log \left(x^2+\sqrt[3]{2}x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log \left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3
) - x]/(3*2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))
```

Rubi in Sympy [A] time = 3.97276, size = 68, normalized size = 0.92

$$\frac{\sqrt[3]{2}\log \left(-x+\sqrt[3]{2}\right)}{6}-\frac{\sqrt[3]{2}\log \left(x^2+\sqrt[3]{2}x+2^{\frac{2}{3}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2^{\frac{2}{3}}x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-2),x)
```

```
[Out] 2**(1/3)*log(-x + 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x +
2**(2/3))/12 - 2**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(2/3)*x/3 + 1/3)
)/6
```

Mathematica [A] time = 0.0253695, size = 65, normalized size = 0.88

$$-\frac{\log \left(\sqrt[3]{2}x^2+2^{2/3}x+2\right)-2\log \left(2-2^{2/3}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] - 2*Log[2 - 2^(2/3)*x
] + Log[2 + 2^(2/3)*x + 2^(1/3)*x^2])/(6*2^(2/3))
```

Maple [A] time = 0.003, size = 54, normalized size = 0.7

$$\frac{\sqrt[3]{2}\ln\left(x-\sqrt[3]{2}\right)}{6}-\frac{\ln\left(2^{\frac{2}{3}}+\sqrt[3]{2}x+x^{2}\right)\sqrt[3]{2}}{12}-\frac{\sqrt[3]{2}\sqrt{3}}{6}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}x\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-2),x)
```

```
[Out] 1/6*2^(1/3)*ln(x-2^(1/3))-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-
1/6*arctan(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

Maxima [A] time = 1.56196, size = 76, normalized size = 1.03

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^{2}+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(x-2^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3)))
- 1/12*2^(1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(x
 - 2^(1/3))
```

Fricas [A] time = 0.239128, size = 81, normalized size = 1.09

$$-\frac{1}{72}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}\log\left(4^{\frac{2}{3}}x^{2}+2\cdot 4^{\frac{1}{3}}x+4\right)-2\sqrt{3}\log\left(4^{\frac{1}{3}}x-2\right)+6\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 2),x, algorithm="fricas")
```

```
[Out] -1/72*4^(2/3)*sqrt(3)*(sqrt(3)*log(4^(2/3)*x^2 + 2*4^(1/3)*x + 4)
 - 2*sqrt(3)*log(4^(1/3)*x - 2) + 6*arctan(1/3*4^(1/3)*sqrt(3)*x
+ 1/3*sqrt(3)))
```

Sympy [A] time = 0.602572, size = 71, normalized size = 0.96

$$\frac{\sqrt[3]{2}\log\left(x-\sqrt[3]{2}\right)}{6}-\frac{\sqrt[3]{2}\log\left(x^2+\sqrt[3]{2}x+2^{\frac{2}{3}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-2),x)
```

```
[Out] 2**(1/3)*log(x - 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x + 2
**(2/3))/12 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x/3 + sqrt(3
)/3)/6
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.35 $\int \frac{1}{-b+ax^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

```
[Out] -(ArcTan[(b^(1/3) + 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1
/3)*b^(2/3))) + Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Lo
g[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.129419, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-b + a*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1
/3)*b^(2/3))) + Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Lo
g[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rubi in Sympy [A] time = 9.58983, size = 109, normalized size = 0.95

$$\frac{\log\left(-\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}}-\frac{\log\left(a^{\frac{2}{3}}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}\right)}{6\sqrt[3]{a}b^{\frac{2}{3}}}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{a}x}{3}+\frac{\sqrt[3]{b}}{3}\right)}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*x**3-b),x)
```

```
[Out] log(-a**(1/3)*x + b**(1/3))/(3*a**(1/3)*b**(2/3)) - log(a**(2/3)*
x**2 + a**(1/3)*b**(1/3)*x + b**(2/3))/(6*a**(1/3)*b**(2/3)) - sq
rt(3)*atan(sqrt(3)*(2*a**(1/3)*x/3 + b**(1/3)/3)/b**(1/3))/(3*a**
```

```
(1/3)*b**(2/3))
```

Mathematica [A] time = 0.0378316, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)-2\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-b + a*x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b
^(1/3) - a^(1/3)*x] + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x
^2])/(6*a^(1/3)*b^(2/3))
```

Maple [A] time = 0.002, size = 92, normalized size = 0.8

$$\frac{1}{3a}\ln\left(x-\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{1}{6a}\ln\left(x^2+x\sqrt[3]{\frac{b}{a}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{b}{a}}}+1\right)\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*x^3-b),x)
```

```
[Out] 1/3/a/(1/a*b)^(2/3)*ln(x-(1/a*b)^(1/3))-1/6/a/(1/a*b)^(2/3)*ln(x^
2+x*(1/a*b)^(1/3)+(1/a*b)^(2/3))-1/3/a/(1/a*b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^3 - b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212289, size = 120, normalized size = 1.04

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(\left(ab^2\right)^{\frac{2}{3}}x^2 + \left(ab^2\right)^{\frac{1}{3}}bx + b^2\right) - 2\sqrt{3}\log\left(\left(ab^2\right)^{\frac{1}{3}}x - b\right) + 6\arctan\left(\frac{2\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}x + \sqrt{3}b}{3b}\right)\right)}{18\left(ab^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^3 - b),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log((a*b^2)^(2/3)*x^2 + (a*b^2)^(1/3)*b*x
+ b^2) - 2*sqrt(3)*log((a*b^2)^(1/3)*x - b) + 6*arctan(1/3*(2*sqr
t(3)*(a*b^2)^(1/3)*x + sqrt(3)*b)/b))/(a*b^2)^(1/3)
```

Sympy [A] time = 0.167712, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3ab^2 - 1, \left(t \mapsto t\log\left(-3tb + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x**3-b),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))
```

GIAC/XCAS [A] time = 0.211178, size = 140, normalized size = 1.22

$$\frac{\left(\frac{b}{a}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3b} - \frac{\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3ab} - \frac{\left(a^2b\right)^{\frac{1}{3}}\ln\left(x^2 + x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^3 - b),x, algorithm="giac")
```

```
[Out] 1/3*(b/a)^(1/3)*ln(abs(x - (b/a)^(1/3)))/b - 1/3*sqrt(3)*(a^2*b)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*b) -
 1/6*(a^2*b)^(1/3)*ln(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*b)
```

3.36 $\int \frac{1}{-2+x^4}\,dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}$$

```
[Out] -ArcTan[x/2^(1/4)]/(2*2^(3/4)) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

Rubi [A] time = 0.0265055, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^4)^(-1),x]

[Out] -ArcTan[x/2^(1/4)]/(2*2^(3/4)) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

Rubi in Sympy [A] time = 0.933167, size = 34, normalized size = 0.97

$$-\frac{\sqrt[4]{2}\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}-\frac{\sqrt[4]{2}\,\mathrm{atanh}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-2),x)

[Out] -2**(1/4)*atan(2**(3/4)*x/2)/4 - 2**(1/4)*atanh(2**(3/4)*x/2)/4
```

Mathematica [A] time = 0.025382, size = 43, normalized size = 1.23

$$-\frac{-\log\left(2-2^{3/4}x\right)+\log\left(2^{3/4}x+2\right)+2\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^4)^(-1),x]
```

```
[Out] -(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/
(4*2^(3/4))
```

Maple [A] time = 0.002, size = 35, normalized size = 1.

$$-\frac{\sqrt[4]{2}}{4}\arctan\left(\frac{x2^{\frac{3}{4}}}{2}\right)-\frac{\sqrt[4]{2}}{8}\ln\left(\frac{x+\sqrt[4]{2}}{x-\sqrt[4]{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-2),x)
```

```
[Out] -1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/8*2^(1/4)*ln((x+2^(1/4))/(x-
2^(1/4)))
```

Maxima [A] time = 1.51745, size = 50, normalized size = 1.43

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x-2^{\frac{1}{4}}\right)}{2\,x+2\cdot 2^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 2),x, algorithm="maxima")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) + 1/8*2^(1/4)*log(2*(x - 2^(1/
4))/((2*2^(1/4)) + 2*x))
```

Fricas [A] time = 0.219564, size = 80, normalized size = 2.29

$$\frac{1}{32}\cdot 8^{\frac{3}{4}}\left(4\arctan\left(\frac{2}{8^{\frac{1}{4}}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}x^{2}+2\right)}+8^{\frac{1}{4}}x}\right)-\log\left(8^{\frac{1}{4}}x+2\right)+\log\left(8^{\frac{1}{4}}x-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 2),x, algorithm="fricas")
```

```
[Out] 1/32*8^(3/4)*(4*arctan(2/(8^(1/4)*sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2)
*x^2 + 2)) + 8^(1/4)*x)) - log(8^(1/4)*x + 2) + log(8^(1/4)*x - 2
))
```

Sympy [A] time = 0.623137, size = 46, normalized size = 1.31

$$\frac{\sqrt[4]{2}\log\left(x-\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\log\left(x+\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-2),x)
```

```
[Out] 2**(1/4)*log(x - 2**(1/4))/8 - 2**(1/4)*log(x + 2**(1/4))/8 - 2**
(1/4)*atan(2**(3/4)*x/2)/4
```

GIAC/XCAS [A] time = 0.213099, size = 53, normalized size = 1.51

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\ln\left(\left|x+2^{\frac{1}{4}}\right|\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\ln\left(\left|x-2^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 2),x, algorithm="giac")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) - 1/8*2^(1/4)*ln(abs(x + 2^(1/
4))) + 1/8*2^(1/4)*ln(abs(x - 2^(1/4)))
```

3.37 $\int \frac{1}{-1+5x^4} \, dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

```
[Out] -ArcTan[5^(1/4)*x]/(2*5^(1/4)) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

Rubi [A] time = 0.0306963, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -ArcTan[5^(1/4)*x]/(2*5^(1/4)) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

Rubi in Sympy [A] time = 0.958668, size = 31, normalized size = 0.89

$$-\frac{5^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{5}x\right)}{10}-\frac{5^{\frac{3}{4}}\operatorname{atanh}\left(\sqrt[4]{5}x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**4-1),x)
```

```
[Out] -5**(3/4)*atan(5**(1/4)*x)/10 - 5**(3/4)*atanh(5**(1/4)*x)/10
```

Mathematica [A] time = 0.0219758, size = 43, normalized size = 1.23

$$-\frac{-\log\left(1-\sqrt[4]{5}x\right)+\log\left(\sqrt[4]{5}x+1\right)+2\tan^{-1}\left(\sqrt[4]{5}x\right)}{4\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/
(4*5^(1/4))
```

Maple [A] time = 0.002, size = 36, normalized size = 1.

$$-\frac{\arctan\left(\sqrt[4]{5}x\right)5^{\frac{3}{4}}}{10}-\frac{5^{\frac{3}{4}}}{20}\ln\left(1\left(x+\frac{5^{\frac{3}{4}}}{5}\right)\left(x-\frac{5^{\frac{3}{4}}}{5}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^4-1),x)
```

```
[Out] -1/10*arctan(5^(1/4)*x)*5^(3/4)-1/20*5^(3/4)*ln((x+1/5*5^(3/4))/(
x-1/5*5^(3/4)))
```

Maxima [A] time = 1.57251, size = 55, normalized size = 1.57

$$-\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5^{\frac{1}{4}}x\right)+\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\frac{\sqrt{5}x-5^{\frac{1}{4}}}{\sqrt{5}x+5^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x^4 - 1),x, algorithm="maxima")
```

```
[Out] -1/10*5^(3/4)*arctan(5^(1/4)*x) + 1/20*5^(3/4)*log((sqrt(5)*x - 5
^(1/4))/(sqrt(5)*x + 5^(1/4)))
```

Fricas [A] time = 0.228475, size = 77, normalized size = 2.2

$$\frac{1}{20}\cdot 5^{\frac{3}{4}}\left(4\arctan\left(\frac{1}{5^{\frac{1}{4}}\sqrt{\frac{1}{5}}\sqrt{\sqrt{5}\left(\sqrt{5}x^{2}+1\right)}+5^{\frac{1}{4}}x}\right)-\log\left(5^{\frac{1}{4}}x+1\right)+\log\left(5^{\frac{1}{4}}x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/20*5^(3/4)*(4*arctan(1/(5^(1/4)*sqrt(1/5)*sqrt(sqrt(5)*(sqrt(5)
*x^2 + 1)) + 5^(1/4)*x)) - log(5^(1/4)*x + 1) + log(5^(1/4)*x - 1
))
```

Sympy [A] time = 0.624876, size = 48, normalized size = 1.37

$$\frac{5^{\frac{3}{4}}\log\left(x-\frac{5^{\frac{3}{4}}}{5}\right)}{20}-\frac{5^{\frac{3}{4}}\log\left(x+\frac{5^{\frac{3}{4}}}{5}\right)}{20}-\frac{5^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{5}x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x**4-1),x)
```

```
[Out] 5**(3/4)*log(x - 5**(3/4)/5)/20 - 5**(3/4)*log(x + 5**(3/4)/5)/20
 - 5**(3/4)*atan(5**(1/4)*x)/10
```

GIAC/XCAS [A] time = 0.212633, size = 53, normalized size = 1.51

$$-\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5\left(\frac{1}{5}\right)^{\frac{3}{4}}x\right)-\frac{1}{20}\cdot 5^{\frac{3}{4}}\ln\left(\left|x+\left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)+\frac{1}{20}\cdot 5^{\frac{3}{4}}\ln\left(\left|x-\left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5*x^4 - 1),x, algorithm="giac")
```

```
[Out] -1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*ln(abs(x + (
1/5)^(1/4))) + 1/20*5^(3/4)*ln(abs(x - (1/5)^(1/4)))
```

3.38 $\int \frac{1}{7+3x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{\log\left(\sqrt{3}x^2-\sqrt{2}\sqrt[4]{21}x+\sqrt{7}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\log\left(\sqrt{3}x^2+\sqrt{2}\sqrt[4]{21}x+\sqrt{7}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

```
[Out] -ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) +
ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - L
og[Sqrt[7] - Sqrt[2]*21^(1/4)*x + Sqrt[3]*x^2]/(4*Sqrt[2]*3^(1/4)
*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*21^(1/4)*x + Sqrt[3]*x^2]/(4*Sq
rt[2]*3^(1/4)*7^(3/4))
```

Rubi [A] time = 0.20955, antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$

$$-\frac{\log\left(3x^2-\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\log\left(3x^2+\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 3*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) +
ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - L
og[Sqrt[21] - Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/
4)*7^(3/4)) + Log[Sqrt[21] + Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(
4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Rubi in Sympy [A] time = 11.7986, size = 168, normalized size = 0.99

$$-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\log\left(3x^2-\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}x+\sqrt{21}\right)}{168}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\log\left(3x^2+\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}x+\sqrt{21}\right)}{168}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{84}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+7),x)
```

```
[Out] -sqrt(2)*3**(3/4)*7**(1/4)*log(3*x**2 - sqrt(2)*3**(3/4)*7**(1/4)
*x + sqrt(21))/168 + sqrt(2)*3**(3/4)*7**(1/4)*log(3*x**2 + sqrt(
2)*3**(3/4)*7**(1/4)*x + sqrt(21))/168 + sqrt(2)*3**(3/4)*7**(1/4
)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt(2)*3**(3/4)*7
**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84
```

Mathematica [A] time = 0.0751019, size = 120, normalized size = 0.71

$$\frac{-\log\left(\sqrt{21}x^2-\sqrt{2}\sqrt[4]{3}7^{3/4}x+7\right)+\log\left(\sqrt{21}x^2+\sqrt{2}\sqrt[4]{3}7^{3/4}x+7\right)-2\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 3*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*
Sqrt[2]*x] - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] +
Log[7 + Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(
1/4)*7^(3/4))
```

Maple [A] time = 0.004, size = 111, normalized size = 0.7

$$\frac{\sqrt{3}\sqrt[4]{21}\sqrt{2}}{84}\arctan\left(\frac{\sqrt{2}\sqrt{3}21^{\frac{3}{4}}x}{21}-1\right)+\frac{\sqrt{3}\sqrt[4]{21}\sqrt{2}}{168}\ln\left(1\left(x^2+\frac{\sqrt{3}\sqrt[4]{21}x\sqrt{2}}{3}+\frac{\sqrt{21}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{21}x\sqrt{2}}{3}+\frac{\sqrt{21}}{3}\right)^{-1}\right)+\frac{\sqrt{3}\sqrt[4]{21}\sqrt{2}}{84}\arctan\left(\frac{\sqrt{2}\sqrt{3}21^{\frac{3}{4}}x}{21}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+7),x)
```

```
[Out] 1/84*3^(1/2)*21^(1/4)*2^(1/2)*arctan(1/21*2^(1/2)*3^(1/2)*21^(3/4
)*x-1)+1/168*3^(1/2)*21^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*21^(1/4
)*x*2^(1/2)+1/3*21^(1/2))/(x^2-1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3
*21^(1/2)))+1/84*3^(1/2)*21^(1/4)*2^(1/2)*arctan(1/21*2^(1/2)*3^(
1/2)*21^(3/4)*x+1)
```

Maxima [A] time = 1.54757, size = 204, normalized size = 1.21

$$\frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}x+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}x-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{168}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\log\left(\sqrt{3}x^2+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{168}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\log\left(\sqrt{3}x^2-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^4 + 7),x, algorithm="maxima")
```

```
[Out] 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7^(3/4)*3^(3/4)*sqrt(2)*
(2*sqrt(3)*x + 7^(1/4)*3^(1/4)*sqrt(2))) + 1/84*7^(1/4)*3^(3/4)*s
qrt(2)*arctan(1/42*7^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)*x - 7^(1/4)
*3^(1/4)*sqrt(2))) + 1/168*7^(1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^
2 + 7^(1/4)*3^(1/4)*sqrt(2)*x + sqrt(7)) - 1/168*7^(1/4)*3^(3/4)*
sqrt(2)*log(sqrt(3)*x^2 - 7^(1/4)*3^(1/4)*sqrt(2)*x + sqrt(7))
```

Fricas [A] time = 0.222757, size = 219, normalized size = 1.3

$$-\frac{1}{8232}\cdot 1029^{\frac{3}{4}}\left(4\sqrt{2}\arctan\left(\frac{7}{1029^{\frac{1}{4}}\sqrt{2}\sqrt{\frac{1}{21}}\sqrt{\sqrt{21}\left(\sqrt{21}x^2+1029^{\frac{1}{4}}\sqrt{2}x+7\right)}+1029^{\frac{1}{4}}\sqrt{2}x+7}\right)+4\sqrt{2}\arctan\left(\frac{}{1029^{\frac{1}{4}}\sqrt{2}\sqrt{\frac{1}{21}}\sqrt{\sqrt{21}\left(\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^4 + 7),x, algorithm="fricas")
```

```
[Out] -1/8232*1029^(3/4)*(4*sqrt(2)*arctan(7/(1029^(1/4)*sqrt(2)*sqrt(1
/21)*sqrt(sqrt(21)*(sqrt(21)*x^2 + 1029^(1/4)*sqrt(2)*x + 7)) + 1
029^(1/4)*sqrt(2)*x + 7)) + 4*sqrt(2)*arctan(7/(1029^(1/4)*sqrt(2
)*sqrt(1/21)*sqrt(sqrt(21)*(sqrt(21)*x^2 - 1029^(1/4)*sqrt(2)*x +
 7)) + 1029^(1/4)*sqrt(2)*x - 7)) - sqrt(2)*log(7*sqrt(21)*x^2 +
7*1029^(1/4)*sqrt(2)*x + 49) + sqrt(2)*log(7*sqrt(21)*x^2 - 7*102
9^(1/4)*sqrt(2)*x + 49))
```

Sympy [A] time = 0.768793, size = 151, normalized size = 0.89

$$-\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2-\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{84}+\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2+\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{84}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{42}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+7),x)
```

```
[Out] -189**(1/4)*sqrt(2)*log(x**2 - 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/
3)/84 + 189**(1/4)*sqrt(2)*log(x**2 + 189**(1/4)*sqrt(2)*x/3 + sq
rt(21)/3)/84 + sqrt(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7*
*(3/4)*x/7 - 1)/42 + sqrt(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1
/4)*7**(3/4)*x/7 + 1)/42
```

GIAC/XCAS [A] time = 0.209669, size = 128, normalized size = 0.76

$$\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{168}\cdot 756^{\frac{1}{4}}\ln\left(x^2+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{7}{3}}\right)-\frac{1}{168}\cdot 756^{\frac{1}{4}}\ln\left(x^2-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{7}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^4 + 7),x, algorithm="giac")
```

```
[Out] 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x + (7/3)^(1/4)
*sqrt(2))) + 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x
- (7/3)^(1/4)*sqrt(2))) + 1/168*756^(1/4)*ln(x^2 + (7/3)^(1/4)*sq
rt(2)*x + sqrt(7/3)) - 1/168*756^(1/4)*ln(x^2 - (7/3)^(1/4)*sqrt(
2)*x + sqrt(7/3))
```

3.39 $\int \frac{1}{-1+3x^2+x^4}\,dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)$$

```
[Out] -(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) -
 Sqrt[(3 + Sqrt[13])/26]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

Rubi [A] time = 0.129424, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) -
 Sqrt[(3 + Sqrt[13])/26]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

Rubi in Sympy [A] time = 1.76998, size = 71, normalized size = 0.97

$$-\frac{\sqrt{26}\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{3+\sqrt{13}}}\right)}{13\sqrt{3+\sqrt{13}}}-\frac{\sqrt{26}\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-3+\sqrt{13}}}\right)}{13\sqrt{-3+\sqrt{13}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+3*x**2-1),x)
```

```
[Out] -sqrt(26)*atan(sqrt(2)*x/sqrt(3 + sqrt(13)))/(13*sqrt(3 + sqrt(13
))) - sqrt(26)*atanh(sqrt(2)*x/sqrt(-3 + sqrt(13)))/(13*sqrt(-3 +
 sqrt(13)))
```

Mathematica [A] time = 0.0795209, size = 68, normalized size = 0.93

$$-\frac{\sqrt{\sqrt{13}-3}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)+\sqrt{3+\sqrt{13}}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x] + Sqrt[3
+ Sqrt[13]]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x])/Sqrt[26])
```

Maple [A] time = 0.031, size = 56, normalized size = 0.8

$$-\frac{2\sqrt{13}}{13\sqrt{-6+2\sqrt{13}}}Artanh\left(2\frac{x}{\sqrt{-6+2\sqrt{13}}}\right)-\frac{2\sqrt{13}}{13\sqrt{6+2\sqrt{13}}}\arctan\left(2\frac{x}{\sqrt{6+2\sqrt{13}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+3*x^2-1),x)
```

```
[Out] -2/13*13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctanh(2*x/(-6+2*13^(1/2))^
(1/2))-2/13*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctan(2*x/(6+2*13^(1/2
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4+3x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 3*x^2 - 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 3*x^2 - 1), x)
```

Fricas [A] time = 0.22627, size = 238, normalized size = 3.26

$$\frac{2}{13}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{13}\left(3\sqrt{13}-13\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{-\sqrt{13}\left(3\sqrt{13}-13\right)}\left(\sqrt{13}+3\right)}{2\left(\sqrt{13}\sqrt{\frac{1}{26}}\sqrt{\sqrt{13}\left(\sqrt{13}(2x^2+3)+13\right)}+\sqrt{13}x\right)}\right)$$
$$-\frac{1}{26}\sqrt{\frac{1}{2}}\sqrt{\sqrt{13}\left(3\sqrt{13}+13\right)}\log\left(\sqrt{\frac{1}{2}}\sqrt{\sqrt{13}\left(3\sqrt{13}+13\right)}\left(\sqrt{13}-3\right)+2\sqrt{13}x\right)$$
$$+\frac{1}{26}\sqrt{\frac{1}{2}}\sqrt{\sqrt{13}\left(3\sqrt{13}+13\right)}\log\left(-\sqrt{\frac{1}{2}}\sqrt{\sqrt{13}\left(3\sqrt{13}+13\right)}\left(\sqrt{13}-3\right)+2\sqrt{13}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 3*x^2 - 1),x, algorithm="fricas")
```

```
[Out] 2/13*sqrt(1/2)*sqrt(-sqrt(13)*(3*sqrt(13) - 13))*arctan(1/2*sqrt(
1/2)*sqrt(-sqrt(13)*(3*sqrt(13) - 13))*(sqrt(13) + 3)/(sqrt(13)*s
qrt(1/26)*sqrt(sqrt(13)*(sqrt(13)*(2*x^2 + 3) + 13)) + sqrt(13)*x
)) - 1/26*sqrt(1/2)*sqrt(sqrt(13)*(3*sqrt(13) + 13))*log(sqrt(1/2
)*sqrt(sqrt(13)*(3*sqrt(13) + 13))*(sqrt(13) - 3) + 2*sqrt(13)*x)
 + 1/26*sqrt(1/2)*sqrt(sqrt(13)*(3*sqrt(13) + 13))*log(-sqrt(1/2)
*sqrt(sqrt(13)*(3*sqrt(13) + 13))*(sqrt(13) - 3) + 2*sqrt(13)*x)
```

Sympy [A] time = 0.628545, size = 24, normalized size = 0.33

$$\operatorname{RootSum}\left(2704t^4-156t^2-1,\left(t\mapsto t\log\left(312t^3-22t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+3*x**2-1),x)
```

```
[Out] RootSum(2704*_t**4 - 156*_t**2 - 1, Lambda(_t, _t*log(312*_t**3 -
 22*_t + x)))
```

GIAC/XCAS [A] time = 0.230336, size = 100, normalized size = 1.37

$$-\frac{1}{26}\sqrt{26\sqrt{13}-78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}+78}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)$$
$$+\frac{1}{52}\sqrt{26\sqrt{13}+78}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 3*x^2 - 1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) - 78)*arctan(x/sqrt(1/2*sqrt(13) + 3/2)) -
 1/52*sqrt(26*sqrt(13) + 78)*ln(abs(x + sqrt(1/2*sqrt(13) - 3/2))
) + 1/52*sqrt(26*sqrt(13) + 78)*ln(abs(x - sqrt(1/2*sqrt(13) - 3/
2)))
```

3.40 $\int \frac{1}{-1-3x^2+x^4}\,dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

```
[Out] -(Sqrt[(3 + Sqrt[13])/26]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sq
rt[2/(13*(3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

Rubi [A] time = 0.0519112, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[13])/26]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sq
rt[2/(13*(3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

Rubi in Sympy [A] time = 1.70357, size = 71, normalized size = 0.97

$$-\frac{\sqrt{26}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{-3+\sqrt{13}}}\right)}{13\sqrt{-3+\sqrt{13}}}-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt{3+\sqrt{13}}}\right)}{13\sqrt{3+\sqrt{13}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-3*x**2-1),x)
```

```
[Out] -sqrt(26)*atan(sqrt(2)*x/sqrt(-3 + sqrt(13)))/(13*sqrt(-3 + sqrt(
13))) - sqrt(26)*atanh(sqrt(2)*x/sqrt(3 + sqrt(13)))/(13*sqrt(3 +
 sqrt(13)))
```

Mathematica [A] time = 0.0404103, size = 68, normalized size = 0.93

$$-\frac{\sqrt{3+\sqrt{13}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)+\sqrt{\sqrt{13}-3}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x] + Sqrt[-3
 + Sqrt[13]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x])/Sqrt[26])
```

Maple [A] time = 0.019, size = 56, normalized size = 0.8

$$-\frac{2\sqrt{13}}{13\sqrt{-6+2\sqrt{13}}}\arctan\left(2\frac{x}{\sqrt{-6+2\sqrt{13}}}\right)-\frac{2\sqrt{13}}{13\sqrt{6+2\sqrt{13}}}Artanh\left(2\frac{x}{\sqrt{6+2\sqrt{13}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-3*x^2-1),x)
```

```
[Out] -2/13*13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctan(2*x/(-6+2*13^(1/2))^(
1/2))-2/13*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctanh(2*x/(6+2*13^(1/2
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4-3x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^2 - 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 3*x^2 - 1), x)
```

Fricas [A] time = 0.225579, size = 240, normalized size = 3.29

$$\frac{2}{13}\sqrt{\frac{1}{2}}\sqrt{\sqrt{13}\left(3\sqrt{13}+13\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{13}\left(3\sqrt{13}+13\right)}\left(\sqrt{13}-3\right)}{2\left(\sqrt{13}\sqrt{\frac{1}{26}}\sqrt{\sqrt{13}\left(\sqrt{13}(2x^2-3)+13\right)}+\sqrt{13}x\right)}\right) - \frac{1}{26}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{13}\left(3\sqrt{13}-13\right)}\log\left(\sqrt{\frac{1}{2}}\sqrt{-\sqrt{13}\left(3\sqrt{13}-13\right)}\left(\sqrt{13}+3\right)+2\sqrt{13}x\right) + \frac{1}{26}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{13}\left(3\sqrt{13}-13\right)}\log\left(-\sqrt{\frac{1}{2}}\sqrt{-\sqrt{13}\left(3\sqrt{13}-13\right)}\left(\sqrt{13}+3\right)+2\sqrt{13}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^2 - 1),x, algorithm="fricas")
```

```
[Out] 2/13*sqrt(1/2)*sqrt(sqrt(13)*(3*sqrt(13) + 13))*arctan(1/2*sqrt(1
/2)*sqrt(sqrt(13)*(3*sqrt(13) + 13))*(sqrt(13) - 3)/(sqrt(13)*sqr
t(1/26)*sqrt(sqrt(13)*(sqrt(13)*(2*x^2 - 3) + 13)) + sqrt(13)*x))
 - 1/26*sqrt(1/2)*sqrt(-sqrt(13)*(3*sqrt(13) - 13))*log(sqrt(1/2)
*sqrt(-sqrt(13)*(3*sqrt(13) - 13))*(sqrt(13) + 3) + 2*sqrt(13)*x)
 + 1/26*sqrt(1/2)*sqrt(-sqrt(13)*(3*sqrt(13) - 13))*log(-sqrt(1/2
)*sqrt(-sqrt(13)*(3*sqrt(13) - 13))*(sqrt(13) + 3) + 2*sqrt(13)*x
)
```

Sympy [A] time = 0.629646, size = 24, normalized size = 0.33

$$\operatorname{RootSum}\left(2704t^4+156t^2-1,\left(t\mapsto t\log\left(-312t^3-22t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-3*x**2-1),x)
```

```
[Out] RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3
- 22*_t + x)))
```

GIAC/XCAS [A] time = 0.255243, size = 100, normalized size = 1.37

$$-\frac{1}{26}\sqrt{26\sqrt{13}+78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}-78}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}-78}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^2 - 1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) -
 1/52*sqrt(26*sqrt(13) - 78)*ln(abs(x + sqrt(1/2*sqrt(13) + 3/2))
) + 1/52*sqrt(26*sqrt(13) - 78)*ln(abs(x - sqrt(1/2*sqrt(13) + 3/
2)))
```

3.41 $\int \frac{1}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=72

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + S
qrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rubi [A] time = 0.126972, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + S
qrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rubi in Sympy [A] time = 2.01087, size = 70, normalized size = 0.97

$$\frac{\sqrt{10}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{5}+3}}\right)}{5\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{10}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{5}+3}}\right)}{5\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-3*x**2+1),x)
```

```
[Out] sqrt(10)*atanh(sqrt(2)*x/sqrt(-sqrt(5) + 3))/(5*sqrt(-sqrt(5) + 3
)) - sqrt(10)*atanh(sqrt(2)*x/sqrt(sqrt(5) + 3))/(5*sqrt(sqrt(5)
+ 3))
```

Mathematica [A] time = 0.0555974, size = 83, normalized size = 1.15

$$\frac{1}{20}\left(-\left(5+\sqrt{5}\right)\log\left(-2x+\sqrt{5}-1\right)-\left(\sqrt{5}-5\right)\log\left(-2x+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}-1\right)+\left(\sqrt{5}-5\right)\log\left(2x+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) - (-5 + Sqrt[5])*Log[1
+ Sqrt[5] - 2*x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x] + (-5 +
Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/20
```

Maple [A] time = 0.004, size = 54, normalized size = 0.8

$$\frac{\ln\left(x^2-x-1\right)}{4}+\frac{\sqrt{5}}{10}Artanh\left(\frac{(-1+2\,x)\sqrt{5}}{5}\right)-\frac{\ln\left(x^2+x-1\right)}{4}+\frac{\sqrt{5}}{10}Artanh\left(\frac{(1+2\,x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-3*x^2+1),x)
```

```
[Out] 1/4*ln(x^2-x-1)+1/10*5^(1/2)*arctanh(1/5*(-1+2*x)*5^(1/2))-1/4*ln
(x^2+x-1)+1/10*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

Maxima [A] time = 1.50115, size = 101, normalized size = 1.4

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/20
*sqrt(5)*log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) - 1/4*log(x
^2 + x - 1) + 1/4*log(x^2 - x - 1)
```

Fricas [A] time = 0.244633, size = 128, normalized size = 1.78

$$-\frac{1}{20}\sqrt{5}\left(\sqrt{5}\log\left(x^2+x-1\right)-\sqrt{5}\log\left(x^2-x-1\right)-\log\left(\frac{\sqrt{5}\left(2\,x^2+2\,x+3\right)+10\,x+5}{x^2+x-1}\right)-\log\left(\frac{\sqrt{5}\left(2\,x^2-2\,x+3\right)+10\,x}{x^2-x-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(5)*(sqrt(5)*log(x^2 + x - 1) - sqrt(5)*log(x^2 - x - 1
) - log((sqrt(5)*(2*x^2 + 2*x + 3) + 10*x + 5)/(x^2 + x - 1)) - l
og((sqrt(5)*(2*x^2 - 2*x + 3) + 10*x - 5)/(x^2 - x - 1)))
```

Sympy [A] time = 0.781282, size = 158, normalized size = 2.19

$$\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x-\frac{7}{2}-\frac{7\sqrt{5}}{10}+120\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)^{3}\right)+\left(-\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x-\frac{7}{2}+120\left(-\frac{\sqrt{5}}{20}+\frac{1}{4}\right)^{3}+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7\sqrt{5}}{10}+120\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)^{3}+\frac{7}{2}\right)+\left(-\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x+120\left(-\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^{3}+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-3*x**2+1),x)
```

```
[Out] (sqrt(5)/20 + 1/4)*log(x - 7/2 - 7*sqrt(5)/10 + 120*(sqrt(5)/20 +
 1/4)**3) + (-sqrt(5)/20 + 1/4)*log(x - 7/2 + 120*(-sqrt(5)/20 +
1/4)**3 + 7*sqrt(5)/10) + (-1/4 + sqrt(5)/20)*log(x - 7*sqrt(5)/1
0 + 120*(-1/4 + sqrt(5)/20)**3 + 7/2) + (-1/4 - sqrt(5)/20)*log(x
 + 120*(-1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/10 + 7/2)
```

GIAC/XCAS [A] time = 0.224573, size = 109, normalized size = 1.51

$$-\frac{1}{20}\sqrt{5}\ln\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)-\frac{1}{20}\sqrt{5}\ln\left(\frac{\left|2x-\sqrt{5}-1\right|}{\left|2x+\sqrt{5}-1\right|}\right)-\frac{1}{4}\ln\left(\left|x^{2}+x-1\right|\right)+\frac{1}{4}\ln\left(\left|x^{2}-x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(5)*ln(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) -
 1/20*sqrt(5)*ln(abs(2*x - sqrt(5) - 1)/abs(2*x + sqrt(5) - 1)) -
 1/4*ln(abs(x^2 + x - 1)) + 1/4*ln(abs(x^2 - x - 1))
```

3.42 $\int \frac{1}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[
x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.102278, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[
x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rubi in Sympy [A] time = 1.47789, size = 60, normalized size = 0.9

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{x}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{x}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-4*x**2+1),x)
```

```
[Out] sqrt(3)*atanh(x/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)) - sqrt
(3)*atanh(x/sqrt(sqrt(3) + 2))/(6*sqrt(sqrt(3) + 2))
```

Mathematica [A] time = 0.049046, size = 67, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[
x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A] time = 0.022, size = 60, normalized size = 0.9

$$\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\mathit{Artanh}\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)-\frac{\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\mathit{Artanh}\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-4*x^2+1),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))-1/3*
3^(1/2)/(6^(1/2)+2^(1/2))*arctanh(2*x/(6^(1/2)+2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4-4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 4*x^2 + 1), x)
```

Fricas [A] time = 0.212098, size = 225, normalized size = 3.36

$$-\frac{1}{12}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\log\left(\sqrt{3}x+\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\left(\sqrt{3}+2\right)\right)+\frac{1}{12}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\log\left(\sqrt{3}x-\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\left(\sqrt{3}+2\right)\right)-\frac{1}{12}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\log\left(\sqrt{3}x+\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\left(\sqrt{3}-2\right)\right)+\frac{1}{12}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\log\left(\sqrt{3}x-\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\left(\sqrt{3}-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3)*(2*sqrt(3) - 3))*log(sqrt(3)*x + sqrt(sqrt(3)*
(2*sqrt(3) - 3))*(sqrt(3) + 2)) + 1/12*sqrt(sqrt(3)*(2*sqrt(3) -
3))*log(sqrt(3)*x - sqrt(sqrt(3)*(2*sqrt(3) - 3))*(sqrt(3) + 2))
- 1/12*sqrt(sqrt(3)*(2*sqrt(3) + 3))*log(sqrt(3)*x + sqrt(sqrt(3)
*(2*sqrt(3) + 3))*(sqrt(3) - 2)) + 1/12*sqrt(sqrt(3)*(2*sqrt(3) +
 3))*log(sqrt(3)*x - sqrt(sqrt(3)*(2*sqrt(3) + 3))*(sqrt(3) - 2))
```

Sympy [A] time = 0.788284, size = 24, normalized size = 0.36

$$\operatorname{RootSum}\left(2304t^4-192t^2+1,\left(t\mapsto t\log\left(384t^3-28t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-4*x**2+1),x)
```

```
[Out] RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 -
 28*_t + x)))
```

GIAC/XCAS [A] time = 0.249576, size = 136, normalized size = 2.03

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(\left|x+\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(\left|x+\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(\left|x-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(\left|x-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*ln(abs(x + 1/2*sqrt(6) + 1/2*sqrt(2)))
 + 1/24*(sqrt(6) + 3*sqrt(2))*ln(abs(x + 1/2*sqrt(6) - 1/2*sqrt(2
))) - 1/24*(sqrt(6) + 3*sqrt(2))*ln(abs(x - 1/2*sqrt(6) + 1/2*sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*ln(abs(x - 1/2*sqrt(6) - 1/2*
sqrt(2)))
```

3.43 $\int \frac{1}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/
Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.0296746, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/
Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rubi in Sympy [A] time = 1.10556, size = 60, normalized size = 0.9

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{x}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{x}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+4*x**2+1),x)
```

```
[Out] sqrt(3)*atan(x/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)) - sqrt(
3)*atan(x/sqrt(sqrt(3) + 2))/(6*sqrt(sqrt(3) + 2))
```

Mathematica [A] time = 0.0271374, size = 67, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/
Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A] time = 0.019, size = 60, normalized size = 0.9

$$-\frac{\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)+\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+4*x^2+1),x)
```

```
[Out] -1/3*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))+1/3*
3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4+4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 4*x^2 + 1), x)
```

Fricas [A] time = 0.222937, size = 182, normalized size = 2.72

$$\frac{1}{3}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\arctan\left(\frac{\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\left(\sqrt{3}+2\right)}{\sqrt{3}\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(\sqrt{3}(x^2+2)+3\right)}+\sqrt{3}x}\right)+\frac{1}{3}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\arctan\left(\frac{\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\left(\sqrt{3}-2\right)}{\sqrt{3}\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(\sqrt{3}(x^2+2)-3\right)}+\sqrt{3}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(sqrt(3)*(2*sqrt(3) - 3))*arctan(sqrt(sqrt(3)*(2*sqrt(3)
- 3))*(sqrt(3) + 2)/(sqrt(3)*sqrt(1/3)*sqrt(sqrt(3)*(sqrt(3)*(x^2
 + 2) + 3)) + sqrt(3)*x)) + 1/3*sqrt(sqrt(3)*(2*sqrt(3) + 3))*arc
tan(sqrt(sqrt(3)*(2*sqrt(3) + 3))*(sqrt(3) - 2)/(sqrt(3)*sqrt(1/3
)*sqrt(sqrt(3)*(sqrt(3)*(x^2 + 2) - 3)) + sqrt(3)*x))
```

Sympy [A] time = 0.291233, size = 92, normalized size = 1.37

$$-2\sqrt{-\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{-\sqrt{3}+2}+2\sqrt{-\sqrt{3}+2}}\right)-2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+4*x**2+1),x)
```

```
[Out] -2*sqrt(-sqrt(3)/48 + 1/24)*atan(x/(sqrt(3)*sqrt(-sqrt(3) + 2) +
2*sqrt(-sqrt(3) + 2))) - 2*sqrt(sqrt(3)/48 + 1/24)*atan(x/(-2*sqr
t(sqrt(3) + 2) + sqrt(3)*sqrt(sqrt(3) + 2)))
```

GIAC/XCAS [A] time = 0.203876, size = 69, normalized size = 1.03

$$\frac{1}{12}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - 3*sqrt(2))*arctan(2*x/(sqrt(6) + sqrt(2))) + 1/12
*(sqrt(6) + 3*sqrt(2))*arctan(2*x/(sqrt(6) - sqrt(2)))
```

3.44 $\int \frac{1}{2+x^2+x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/S
qrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[
-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sq
rt[-1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + Log[Sq
rt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(-1 + 2*Sqrt[2])]
)
```

Rubi [A] time = 0.305195, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/S
qrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[
-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sq
rt[-1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + Log[Sq
rt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(-1 + 2*Sqrt[2])]
)
```

Rubi in Sympy [A] time = 8.22135, size = 178, normalized size = 0.91

$$-\frac{\sqrt{2}\log\left(x^2 - x\sqrt{-1+2\sqrt{2}} + \sqrt{2}\right)}{8\sqrt{-1+2\sqrt{2}}} + \frac{\sqrt{2}\log\left(x^2 + x\sqrt{-1+2\sqrt{2}} + \sqrt{2}\right)}{8\sqrt{-1+2\sqrt{2}}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{4\sqrt{1+2\sqrt{2}}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{4\sqrt{1+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+x**2+2),x)
```

```
[Out] -sqrt(2)*log(x**2 - x*sqrt(-1 + 2*sqrt(2)) + sqrt(2))/(8*sqrt(-1
+ 2*sqrt(2))) + sqrt(2)*log(x**2 + x*sqrt(-1 + 2*sqrt(2)) + sqrt(
2))/(8*sqrt(-1 + 2*sqrt(2))) + sqrt(2)*atan((2*x - sqrt(-1 + 2*sq
rt(2)))/sqrt(1 + 2*sqrt(2)))/(4*sqrt(1 + 2*sqrt(2))) + sqrt(2)*at
an((2*x + sqrt(-1 + 2*sqrt(2)))/sqrt(1 + 2*sqrt(2)))/(4*sqrt(1 +
2*sqrt(2)))
```

Mathematica [C] time = 0.0906976, size = 91, normalized size = 0.46

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1+i\sqrt{7}\right)}} - \frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))
/2] + (I*ArcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7
]))/2]
```

Maple [B] time = 0.065, size = 386, normalized size = 2.

$$\frac{\ln\left(x^{2}+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\sqrt{2}}{56}+\frac{\ln\left(x^{2}+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}}{14}-\frac{\left(-1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{-1+2\sqrt{2}}{7\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{\ln\left(x^{2}+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\sqrt{2}}{56}-\frac{\ln\left(x^{2}+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}}{14}-\frac{\left(-1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{-1+2\sqrt{2}}{7\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+x^2+2),x)
```

```
[Out] 1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*
2^(1/2)+1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2)
)^(1/2)-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2)
)/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)-1/7/(1+2*2^(1/2))^(
1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2
*2^(1/2))+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2
))/(1+2*2^(1/2))^(1/2))*2^(1/2)-1/56*ln(x^2+2^(1/2)-x*(-1+2*2^(1/
2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)-1/14*ln(x^2+2^(1/2)-x*(-1
+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)-1/28/(1+2*2^(1/2))^(1/2)*
arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1
/2))*2^(1/2)-1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(
1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))+1/2/(1+2*2^(1/2))^(1/2)
*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{4}+x^{2}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + x^2 + 2), x)
```

Fricas [A] time = 0.215352, size = 501, normalized size = 2.56

$$\frac{98^{\frac{3}{4}}\sqrt{7}\left(\sqrt{7}\left(\sqrt{2}+4\right)\log\left(14\sqrt{2}x^2+14\cdot 98^{\frac{1}{4}}x\sqrt{\frac{\sqrt{2}+4}{4\sqrt{2}+9}}+28\right)-\sqrt{7}\left(\sqrt{2}+4\right)\log\left(14\sqrt{2}x^2-14\cdot 98^{\frac{1}{4}}x\sqrt{\frac{\sqrt{2}+4}{4\sqrt{2}+9}}+28\right)-28\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/2744*98^(3/4)*sqrt(7)*(sqrt(7)*(sqrt(2) + 4)*log(14*sqrt(2)*x^2
 + 14*98^(1/4)*x*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9)) + 28) - sqrt
(7)*(sqrt(2) + 4)*log(14*sqrt(2)*x^2 - 14*98^(1/4)*x*sqrt((sqrt(2
) + 4)/(4*sqrt(2) + 9)) + 28) - 28*sqrt(2)*arctan(7*(2*sqrt(2) +
1)/(98^(1/4)*sqrt(7)*sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2)*x^2 + 98^(1/
4)*x*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9)) + 2))*(sqrt(2) + 4)*sqrt
((sqrt(2) + 4)/(4*sqrt(2) + 9)) + 98^(1/4)*sqrt(7)*(sqrt(2)*x + 4
*x)*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9)) + 7*sqrt(7))) - 28*sqrt(2
)*arctan(7*(2*sqrt(2) + 1)/(98^(1/4)*sqrt(7)*sqrt(1/2)*sqrt(sqrt(
2)*(sqrt(2)*x^2 - 98^(1/4)*x*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9))
+ 2))*(sqrt(2) + 4)*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9)) + 98^(1/4
)*sqrt(7)*(sqrt(2)*x + 4*x)*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9)) -
 7*sqrt(7))))/((sqrt(2) + 4)*sqrt((sqrt(2) + 4)/(4*sqrt(2) + 9)))
```

Sympy [A] time = 0.817976, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(1568t^4-28t^2+1,\left(t\mapsto t\log\left(112t^3+6t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 - 28*_t**2 + 1, Lambda(_t, _t*log(112*_t**3 +
6*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 2),x, algorithm="giac")

[Out] integrate(1/(x^4 + x^2 + 2), x)
```

3.45 $\int \frac{1}{2-x^2+x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)$$

```
[Out] -(Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] - 2*x)/Sqr
t[-1 + 2*Sqrt[2]]])/2 + (Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1
+ 2*Sqrt[2]] + 2*x)/Sqrt[-1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt
[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(1 + 2*Sqrt[2])]) + Log[Sqrt[2
] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.286099, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] - 2*x)/Sqr
t[-1 + 2*Sqrt[2]]])/2 + (Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1
+ 2*Sqrt[2]] + 2*x)/Sqrt[-1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt
[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(1 + 2*Sqrt[2])]) + Log[Sqrt[2
] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt[2*(1 + 2*Sqrt[2])])
```

Rubi in Sympy [A] time = 8.09686, size = 178, normalized size = 0.91

$$-\frac{\sqrt{2}\log\left(x^2-x\sqrt{1+2\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{1+2\sqrt{2}}}+\frac{\sqrt{2}\log\left(x^2+x\sqrt{1+2\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{1+2\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{4\sqrt{-1+2\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{4\sqrt{-1+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-x**2+2),x)
```

```
[Out] -sqrt(2)*log(x**2 - x*sqrt(1 + 2*sqrt(2)) + sqrt(2))/(8*sqrt(1 +
2*sqrt(2))) + sqrt(2)*log(x**2 + x*sqrt(1 + 2*sqrt(2)) + sqrt(2))
/(8*sqrt(1 + 2*sqrt(2))) + sqrt(2)*atan((2*x - sqrt(1 + 2*sqrt(2)
))/sqrt(-1 + 2*sqrt(2)))/(4*sqrt(-1 + 2*sqrt(2))) + sqrt(2)*atan(
(2*x + sqrt(1 + 2*sqrt(2)))/sqrt(-1 + 2*sqrt(2)))/(4*sqrt(-1 + 2*
sqrt(2)))
```

Mathematica [C] time = 0.119972, size = 91, normalized size = 0.46

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1+i\sqrt{7}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]
))/2] + (I*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sq
rt[7]))/2]
```

Maple [B] time = 0.039, size = 386, normalized size = 2.

$$\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}\sqrt{2}}{56}-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}}{14}$$
$$+\frac{\left(1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)-\frac{1+2\sqrt{2}}{7\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)$$
$$+\frac{\sqrt{2}}{2\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}\sqrt{2}}{56}$$
$$+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}}{14}+\frac{\left(1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)$$
$$-\frac{1+2\sqrt{2}}{7\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-x^2+2),x)
```

```
[Out] 1/56*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2)*2^
(1/2)-1/14*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1
/2)+1/28/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-
1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))*2^(1/2)-1/7/(-1+2*2^(1/2))^(1/2
)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(
1/2))+1/2/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(
-1+2*2^(1/2))^(1/2))*2^(1/2)-1/56*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^
(1/2))*(1+2*2^(1/2))^(1/2)*2^(1/2)+1/14*ln(x^2+2^(1/2)+x*(1+2*2^(
1/2))^(1/2))*(1+2*2^(1/2))^(1/2)+1/28/(-1+2*2^(1/2))^(1/2)*arctan
((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))*2^
(1/2)-1/7/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1+2*2^(1/2))^(1/2))/(
-1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))+1/2/(-1+2*2^(1/2))^(1/2)*arcta
n((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 - x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 2), x)
```

Fricas [A] time = 0.21233, size = 501, normalized size = 2.56

$$\frac{98^{\frac{3}{4}}\sqrt{7}\left(\sqrt{7}\left(\sqrt{2}-4\right)\log\left(14\sqrt{2}x^2+14\cdot 98^{\frac{1}{4}}x\sqrt{\frac{\sqrt{2}-4}{4\sqrt{2}-9}}+28\right)-\sqrt{7}\left(\sqrt{2}-4\right)\log\left(14\sqrt{2}x^2-14\cdot 98^{\frac{1}{4}}x\sqrt{\frac{\sqrt{2}-4}{4\sqrt{2}-9}}+28\right)-28\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/2744*98^(3/4)*sqrt(7)*(sqrt(7)*(sqrt(2) - 4)*log(14*sqrt(2)*x^2
 + 14*98^(1/4)*x*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9)) + 28) - sqrt
(7)*(sqrt(2) - 4)*log(14*sqrt(2)*x^2 - 14*98^(1/4)*x*sqrt((sqrt(2
) - 4)/(4*sqrt(2) - 9)) + 28) - 28*sqrt(2)*arctan(7*(2*sqrt(2) -
1)/(98^(1/4)*sqrt(7)*sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2)*x^2 + 98^(1/
4)*x*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9)) + 2))*(sqrt(2) - 4)*sqrt
```

```
((sqrt(2) - 4)/(4*sqrt(2) - 9)) + 98^(1/4)*sqrt(7)*(sqrt(2)*x - 4
*x)*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9)) - 7*sqrt(7))) - 28*sqrt(2
)*arctan(7*(2*sqrt(2) - 1)/(98^(1/4)*sqrt(7)*sqrt(1/2)*sqrt(sqrt(
2)*(sqrt(2)*x^2 - 98^(1/4)*x*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9))
+ 2))*(sqrt(2) - 4)*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9)) + 98^(1/4
)*sqrt(7)*(sqrt(2)*x - 4*x)*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9)) +
 7*sqrt(7))))/((sqrt(2) - 4)*sqrt((sqrt(2) - 4)/(4*sqrt(2) - 9)))
```

Sympy [A] time = 0.843832, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(1568t^4 + 28t^2 + 1, \left(t \mapsto t \log\left(-112t^3 + 6t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + 28*_t**2 + 1, Lambda(_t, _t*log(-112*_t**3 +
 6*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 - x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 - x^2 + 2), x)
```

3.46 $\int \frac{1}{-1+x^6}\,dx$

Optimal. Leaf size=73

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]
/(2*Sqrt[3]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x
^2]/12
```

Rubi [A] time = 0.187223, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]
/(2*Sqrt[3]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x
^2]/12
```

Rubi in Sympy [A] time = 16.7378, size = 68, normalized size = 0.93

$$\frac{\log\left(x^2-x+1\right)}{12}-\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}-\frac{\mathrm{atanh}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-1),x)
```

```
[Out] log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 - sqrt(3)*atan(sqrt(3
)*(2*x/3 - 1/3))/6 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/6 - atan
h(x)/3
```

Mathematica [A] time = 0.0213089, size = 75, normalized size = 1.03

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*
x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Lo
g[1 + x + x^2])/12
```

Maple [A] time = 0.005, size = 66, normalized size = 0.9

$$-\frac{\ln\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(-1+x)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-1),x)
```

```
[Out] -1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*ln(
-1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))-1
/6*ln(1+x)
```

Maxima [A] time = 1.53919, size = 88, normalized size = 1.21

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x
```

```
+ 1) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

Fricas [A] time = 0.215803, size = 101, normalized size = 1.38

$$-\frac{1}{36}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-\sqrt{3}\log\left(x^2-x+1\right)+2\sqrt{3}\log\left(x+1\right)-2\sqrt{3}\log\left(x-1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+6\ \mathrm{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - sqrt(3)*log(x^2 - x + 1
) + 2*sqrt(3)*log(x + 1) - 2*sqrt(3)*log(x - 1) + 6*arctan(1/3*sq
rt(3)*(2*x + 1)) + 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.348727, size = 83, normalized size = 1.14

$$\frac{\log\left(x-1\right)}{6}-\frac{\log\left(x+1\right)}{6}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x
 + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*at
an(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.199146, size = 90, normalized size = 1.23

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\ln\left(x^2+x+1\right)+\frac{1}{12}\ln\left(x^2-x+1\right)-\frac{1}{6}\ln\left(|x+1|\right)+\frac{1}{6}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - 1/12*ln(x^2 + x + 1) + 1/12*ln(x^2 - x +
1) - 1/6*ln(abs(x + 1)) + 1/6*ln(abs(x - 1))
```

3.47 $\int \frac{1}{-2+x^6} \, dx$

Optimal. Leaf size=138

$$\frac{\log\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

```
[Out] ArcTan[1/Sqrt[3] - (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - Arc
Tan[1/Sqrt[3] + (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTan
h[x/2^(1/6)]/(3*2^(5/6)) + Log[2^(1/3) - 2^(1/6)*x + x^2]/(12*2^(
5/6)) - Log[2^(1/3) + 2^(1/6)*x + x^2]/(12*2^(5/6))
```

Rubi [A] time = 0.405045, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$

$$\frac{\log\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^6)^(-1),x]
```

```
[Out] ArcTan[1/Sqrt[3] - (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - Arc
Tan[1/Sqrt[3] + (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTan
h[x/2^(1/6)]/(3*2^(5/6)) + Log[2^(1/3) - 2^(1/6)*x + x^2]/(12*2^(
5/6)) - Log[2^(1/3) + 2^(1/6)*x + x^2]/(12*2^(5/6))
```

Rubi in Sympy [A] time = 23.2348, size = 131, normalized size = 0.95

$$\frac{\sqrt[6]{2}\log\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}{24}-\frac{\sqrt[6]{2}\log\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}{24}-\frac{\sqrt[6]{2}\sqrt{3}\,\operatorname{atan}\left(2^{\frac{5}{6}}\sqrt{3}\left(\frac{x}{3}-\frac{\sqrt[6]{2}}{6}\right)\right)}{12}-\frac{\sqrt[6]{2}\sqrt{3}\,\operatorname{atan}\left(2^{\frac{5}{6}}\sqrt{3}\left(\frac{x}{3}+\frac{\sqrt[6]{2}}{6}\right)\right)}{12}-\frac{\sqrt[6]{2}\,\operatorname{atanh}\left(\frac{2^{\frac{5}{6}}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-2),x)
```

```
[Out] 2**(1/6)*log(x**2 - 2**(1/6)*x + 2**(1/3))/24 - 2**(1/6)*log(x**2
 + 2**(1/6)*x + 2**(1/3))/24 - 2**(1/6)*sqrt(3)*atan(2**(5/6)*sqr
```

```
t(3)*(x/3 - 2**(1/6)/6))/12 - 2**(1/6)*sqrt(3)*atan(2**(5/6)*sqrt
(3)*(x/3 + 2**(1/6)/6))/12 - 2**(1/6)*atanh(2**(5/6)*x/2)/6
```

Mathematica [A] time = 0.0541312, size = 122, normalized size = 0.88

$$-\frac{-\log\left(2^{2/3}x^2-2^{5/6}x+2\right)+\log\left(2^{2/3}x^2+2^{5/6}x+2\right)-2\log\left(2-2^{5/6}x\right)+2\log\left(2^{5/6}x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{5/6}x-1}{\sqrt{3}}\right)+2\sqrt{3}}{12\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^6)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(-1 + 2^(5/6)*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(
1 + 2^(5/6)*x)/Sqrt[3]] - 2*Log[2 - 2^(5/6)*x] + 2*Log[2 + 2^(5/6
)*x] - Log[2 - 2^(5/6)*x + 2^(2/3)*x^2] + Log[2 + 2^(5/6)*x + 2^(
2/3)*x^2])/(12*2^(5/6))
```

Maple [A] time = 0.056, size = 111, normalized size = 0.8

$$-\frac{\sqrt[6]{2}\ln\left(x+\sqrt[6]{2}\right)}{12}+\frac{\ln\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)\sqrt[6]{2}}{24}-\frac{\sqrt{3}\sqrt[6]{2}}{12}\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)-\frac{\ln\left(\sqrt[3]{2}+\sqrt[6]{2}x+x^2\right)\sqrt[6]{2}}{24}-\frac{\sqrt{3}\sqrt[6]{2}}{12}\arctan\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)+\frac{\sqrt[6]{2}\ln\left(x-\sqrt[6]{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-2),x)
```

```
[Out] -1/12*2^(1/6)*ln(x+2^(1/6))+1/24*ln(2^(1/3)-2^(1/6)*x+x^2)*2^(1/6
)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
-1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arctan(1/3*3^(1/2)+1
/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3^(1/2)+1/12*2^(1/6)*ln(x-2^(1/6))
```

Maxima [A] time = 1.5752, size = 151, normalized size = 1.09

$$-\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right)-\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2-2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(x+2^{\frac{1}{6}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(x-2^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6)))
 - 1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6)
)) - 1/24*2^(1/6)*log(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*l
og(x^2 - 2^(1/6)*x + 2^(1/3)) - 1/12*2^(1/6)*log(x + 2^(1/6)) + 1
/12*2^(1/6)*log(x - 2^(1/6))
```

Fricas [A] time = 0.224712, size = 207, normalized size = 1.5

$$\frac{1}{384} \cdot 32^{\frac{5}{6}} \left(4\sqrt{3} \arctan\left(\frac{2\sqrt{3}}{2 \cdot 32^{\frac{1}{6}} x + 32^{\frac{1}{6}} \sqrt{4^{\frac{2}{3}} \left(4^{\frac{1}{3}} x^2 + 32^{\frac{1}{6}} x + 2\right)} + 2} \right) + 4\sqrt{3} \arctan\left(\frac{2\sqrt{3}}{2 \cdot 32^{\frac{1}{6}} x + 32^{\frac{1}{6}} \sqrt{4^{\frac{2}{3}} \left(4^{\frac{1}{3}} x^2 - 32^{\frac{1}{6}} x + 2\right)} - 2} \right) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 2),x, algorithm="fricas")
```

```
[Out] 1/384*32^(5/6)*(4*sqrt(3)*arctan(2*sqrt(3)/(2*32^(1/6)*x + 32^(1/
6)*sqrt(4^(2/3)*(4^(1/3)*x^2 + 32^(1/6)*x + 2)) + 2)) + 4*sqrt(3)
*arctan(2*sqrt(3)/(2*32^(1/6)*x + 32^(1/6)*sqrt(4^(2/3)*(4^(1/3)*
x^2 - 32^(1/6)*x + 2)) - 2)) - log(2*4^(1/3)*x^2 + 2*32^(1/6)*x +
 4) + log(2*4^(1/3)*x^2 - 2*32^(1/6)*x + 4) - 2*log(32^(1/6)*x +
2) + 2*log(32^(1/6)*x - 2))
```

Sympy [A] time = 0.989179, size = 14, normalized size = 0.1

$$\operatorname{RootSum}\left(1492992 t^{6} - 1, \left(t \mapsto t \log\left(-12 t + x\right) \right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-2),x)
```

```
[Out] RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.48 $\int \frac{1}{2+x^6} \, dx$

Optimal. Leaf size=138

$$-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(2^{5/6}x+\sqrt{3}\right)}{6\ 2^{5/6}}$$

```
[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^
(5/6)) + ArcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) -
2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1
/6)*Sqrt[3]*x + x^2]/(4*2^(5/6)*Sqrt[3])
```

Rubi [A] time = 0.564451, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(2^{5/6}x+\sqrt{3}\right)}{6\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^6)^(-1),x]
```

```
[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^
(5/6)) + ArcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) -
2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1
/6)*Sqrt[3]*x + x^2]/(4*2^(5/6)*Sqrt[3])
```

Rubi in Sympy [A] time = 38.3713, size = 138, normalized size = 1.

$$-\frac{\sqrt[6]{2}\sqrt{3}\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{24}+\frac{\sqrt[6]{2}\sqrt{3}\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{24}+\frac{\sqrt[6]{2}\,\mathrm{atan}\left(\frac{2^{\frac{5}{6}}x}{2}\right)}{6}+\frac{\sqrt[6]{2}\,\mathrm{atan}\left(2^{\frac{5}{6}}\left(x-\frac{\sqrt[6]{2}\sqrt{3}}{2}\right)\right)}{12}+\frac{\sqrt[6]{2}\,\mathrm{atan}\left(2^{\frac{5}{6}}\left(x+\frac{\sqrt[6]{2}\sqrt{3}}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6+2),x)
```

```
[Out] -2**(1/6)*sqrt(3)*log(x**2 - 2**(1/6)*sqrt(3)*x + 2**(1/3))/24 +
2**(1/6)*sqrt(3)*log(x**2 + 2**(1/6)*sqrt(3)*x + 2**(1/3))/24 + 2
**(1/6)*atan(2**(5/6)*x/2)/6 + 2**(1/6)*atan(2**(5/6)*(x - 2**(1/
6)*sqrt(3)/2))/12 + 2**(1/6)*atan(2**(5/6)*(x + 2**(1/6)*sqrt(3)/
2))/12
```

Mathematica [A] time = 0.0372502, size = 115, normalized size = 0.83

$$\frac{-\sqrt{3}\log\left(2^{2/3}x^2-2^{5/6}\sqrt{3}x+2\right)+\sqrt{3}\log\left(2^{2/3}x^2+2^{5/6}\sqrt{3}x+2\right)+4\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)-2\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)+2\tan^{-1}\left(2^{5/6}x\right.}{12\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^6)^(-1),x]
```

```
[Out] (4*ArcTan[x/2^(1/6)] - 2*ArcTan[Sqrt[3] - 2^(5/6)*x] + 2*ArcTan[S
qrt[3] + 2^(5/6)*x] - Sqrt[3]*Log[2 - 2^(5/6)*Sqrt[3]*x + 2^(2/3)
*x^2] + Sqrt[3]*Log[2 + 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2])/(12*2^(
5/6))
```

Maple [A] time = 0.083, size = 95, normalized size = 0.7

$$\frac{\sqrt[6]{2}}{6}\arctan\left(\frac{x2^{\frac{5}{6}}}{2}\right)+\frac{\arctan\left(x2^{\frac{5}{6}}-\sqrt{3}\right)\sqrt[6]{2}}{12}+\frac{\arctan\left(x2^{\frac{5}{6}}+\sqrt{3}\right)\sqrt[6]{2}}{12}-\frac{\ln\left(\sqrt[3]{2}+x^2-\sqrt[6]{2}x\sqrt{3}\right)\sqrt[6]{2}\sqrt{3}}{24}+\frac{\ln\left(\sqrt[3]{2}+x^2+\sqrt[6]{2}x\sqrt{3}\right)\sqrt[6]{2}\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6+2),x)
```

```
[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*
2^(1/6)+1/12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^
2-2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*
x*3^(1/2))*2^(1/6)*3^(1/2)
```

Maxima [A] time = 1.50116, size = 144, normalized size = 1.04

$$\frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2+\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2-\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2x+\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2x-\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{6}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/2
4*sqrt(3)*2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2
^(1/6)*arctan(1/2*2^(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)
*arctan(1/2*2^(5/6)*(2*x - sqrt(3)*2^(1/6))) + 1/6*2^(1/6)*arctan
(1/2*2^(5/6)*x)
```

Fricas [A] time = 0.217881, size = 244, normalized size = 1.77

$$\frac{1}{384}\cdot 32^{\frac{5}{6}}\left(\sqrt{3}\log\left(2\cdot 4^{\frac{1}{3}}x^2+2\cdot 32^{\frac{1}{6}}\sqrt{3}x+4\right)-\sqrt{3}\log\left(2\cdot 4^{\frac{1}{3}}x^2-2\cdot 32^{\frac{1}{6}}\sqrt{3}x+4\right)-4\arctan\left(\frac{2}{2\cdot 32^{\frac{1}{6}}x+32^{\frac{1}{6}}\sqrt{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}x^2+32^{\frac{1}{6}}\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 2),x, algorithm="fricas")
```

```
[Out] 1/384*32^(5/6)*(sqrt(3)*log(2*4^(1/3)*x^2 + 2*32^(1/6)*sqrt(3)*x
+ 4) - sqrt(3)*log(2*4^(1/3)*x^2 - 2*32^(1/6)*sqrt(3)*x + 4) - 4*
arctan(2/(2*32^(1/6)*x + 32^(1/6)*sqrt(4^(2/3)*(4^(1/3)*x^2 + 32^
(1/6)*sqrt(3)*x + 2)) + 2*sqrt(3))) - 4*arctan(2/(2*32^(1/6)*x +
32^(1/6)*sqrt(4^(2/3)*(4^(1/3)*x^2 - 32^(1/6)*sqrt(3)*x + 2)) - 2
*sqrt(3))) - 8*arctan(4/(2*32^(1/6)*x + 32^(1/6)*sqrt(4^(2/3)*(4^
(1/3)*x^2 + 2)))))
```

Sympy [A] time = 0.387704, size = 14, normalized size = 0.1

$$\operatorname{RootSum}\left(1492992t^6+1,\left(t\mapsto t\log\left(12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6+2),x)
```

```
[Out] RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.49 $\int \frac{1}{1+x^8}\,dx$

Optimal. Leaf size=339

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]
]]/(4*Sqrt[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/S
qrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sq
rt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sqrt[2])]) - (Sqr
t[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/16 + (Sqrt[2 -
 Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 + Sqrt
[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*
Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rubi [A] time = 0.680687, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]
]]/(4*Sqrt[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/S
qrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sq
rt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sqrt[2])]) - (Sqr
t[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/16 + (Sqrt[2 -
 Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 + Sqrt
```

```
[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*
Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rubi in Sympy [A] time = 25.6116, size = 529, normalized size = 1.56

$$\frac{\sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)}{8\sqrt{-\sqrt{2}+2}}-\frac{\sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)}{8\sqrt{-\sqrt{2}+2}}-\frac{\sqrt{2}\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)}{8\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)}{8\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\left(-\frac{(1+\sqrt{2})\sqrt{\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{4\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\left(-\frac{(1+\sqrt{2})\sqrt{\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{4\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\left(\frac{(-\sqrt{2}+1)\sqrt{-\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{-\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{4\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\left(\frac{(-\sqrt{2}+1)\sqrt{-\sqrt{2}+2}}{2}+\sqrt{2}\sqrt{-\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{4\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8+1),x)
```

```
[Out] sqrt(2)*(-sqrt(2)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(2) + 2) + 1)/(
8*sqrt(-sqrt(2) + 2)) - sqrt(2)*(-sqrt(2)/2 + 1/2)*log(x**2 + x*s
qrt(-sqrt(2) + 2) + 1)/(8*sqrt(-sqrt(2) + 2)) - sqrt(2)*(1/2 + sq
rt(2)/2)*log(x**2 - x*sqrt(sqrt(2) + 2) + 1)/(8*sqrt(sqrt(2) + 2)
) + sqrt(2)*(1/2 + sqrt(2)/2)*log(x**2 + x*sqrt(sqrt(2) + 2) + 1)
/(8*sqrt(sqrt(2) + 2)) + sqrt(2)*(-(1 + sqrt(2))*sqrt(sqrt(2) + 2
)/2 + sqrt(2)*sqrt(sqrt(2) + 2))*atan((2*x - sqrt(sqrt(2) + 2))/s
qrt(-sqrt(2) + 2))/(4*sqrt(-sqrt(2) + 2)*sqrt(sqrt(2) + 2)) + sqr
t(2)*(-(1 + sqrt(2))*sqrt(sqrt(2) + 2)/2 + sqrt(2)*sqrt(sqrt(2) +
 2))*atan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2))/(4*sqrt(-
sqrt(2) + 2)*sqrt(sqrt(2) + 2)) + sqrt(2)*((-sqrt(2) + 1)*sqrt(-s
qrt(2) + 2)/2 + sqrt(2)*sqrt(-sqrt(2) + 2))*atan((2*x - sqrt(-sqr
t(2) + 2))/sqrt(sqrt(2) + 2))/(4*sqrt(-sqrt(2) + 2)*sqrt(sqrt(2)
+ 2)) + sqrt(2)*((-sqrt(2) + 1)*sqrt(-sqrt(2) + 2)/2 + sqrt(2)*sq
rt(-sqrt(2) + 2))*atan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
2))/(4*sqrt(-sqrt(2) + 2)*sqrt(sqrt(2) + 2))
```

Mathematica [A] time = 0.00770615, size = 209, normalized size = 0.62

$$-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)$$
$$-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)$$
$$+\frac{1}{4}\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x-\cos\left(\frac{\pi}{8}\right)\right)\right)+\frac{1}{4}\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x+\cos\left(\frac{\pi}{8}\right)\right)\right)$$
$$+\frac{1}{4}\cos\left(\frac{\pi}{8}\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x-\sin\left(\frac{\pi}{8}\right)\right)\right)+\frac{1}{4}\cos\left(\frac{\pi}{8}\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^8)^(-1),x]
```

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/
8]*(x + Sin[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*C
os[Pi/8]])/8 + (Cos[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcT
an[(x - Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/
8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1 + x^2 - 2*x*Sin[Pi/8]]*Sin[P
i/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])/8
```

Maple [C] time = 0.01, size = 22, normalized size = 0.1

$$\frac{1}{8}\sum_{\_R=RootOf(\_Z^8+1)}\frac{\ln(x-\_R)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8+1),x)
```

```
[Out] 1/8*sum(1/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 1), x)
```

Fricas [A] time = 0.232609, size = 1345, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 1),x, algorithm="fricas")
```

```
[Out] -1/64*sqrt(2)*(4*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(
(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)
*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(
-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 4
*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan((sqrt(sqrt(2) +
2) + sqrt(-sqrt(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)*sqrt(x^2 - 1/2*
sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) +
1) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(sqrt(2) +
 2) - sqrt(-sqrt(2) + 2))*arctan(-(sqrt(sqrt(2) + 2) - sqrt(-sqrt
(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(
sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(
2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(sqrt(2) + 2) - sqrt(-sqr
t(2) + 2))*arctan(-(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(2*sq
rt(2)*x + 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) - sqrt(
-sqrt(2) + 2))) + 4*sqrt(2)*sqrt(sqrt(2) + 2)*arctan(sqrt(sqrt(2)
 + 2)/(2*x + 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(
2) + 2))) + 4*sqrt(2)*sqrt(sqrt(2) + 2)*arctan(sqrt(sqrt(2) + 2)/
(2*x + 2*sqrt(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2
))) + 4*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan(sqrt(-sqrt(2) + 2)/(2*x
 + 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))) +
4*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan(sqrt(-sqrt(2) + 2)/(2*x + 2*s
qrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))) - (sqrt(
sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 + 1/2*sqrt(2)*x*sqrt(s
qrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - (sqrt(sqrt(
2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2
) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + (sqrt(sqrt(2) +
2) - sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2
) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + (sqrt(sqrt(2) + 2) +
sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1
/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(2)*sqrt(sqrt(2) + 2)*
log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(2)*sqrt(sqrt(2) + 2)*lo
g(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(2)*sqrt(-sqrt(2) + 2)*log
(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(2)*sqrt(-sqrt(2) + 2)*log
(x^2 - x*sqrt(-sqrt(2) + 2) + 1))
```

Sympy [A] time = 1.93723, size = 14, normalized size = 0.04

$$\operatorname{RootSum}\left(16777216t^{8} + 1, \left(t \mapsto t\log\left(8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))
```

GIAC/XCAS [A] time = 0.20441, size = 323, normalized size = 0.95

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{\sqrt{2}+2}\ln\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{\sqrt{2}+2}\ln\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{-\sqrt{2}+2}\ln\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-\sqrt{2}+2}\ln\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt
(2) + 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2
))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt
(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arcta
n((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/16*sqrt(sqrt(
2) + 2)*ln(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(sqrt(2) + 2
)*ln(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) + 2)*ln(
x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*ln(x^2
- x*sqrt(-sqrt(2) + 2) + 1)
```

3.50 $\int \frac{1}{-1+x^8} \, dx$

Optimal. Leaf size=97

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqr
t[2]*x]/(4*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*
Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rubi [A] time = 0.0964736, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^8)^(-1),x]
```

```
[Out] -ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqr
t[2]*x]/(4*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*
Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rubi in Sympy [A] time = 6.33561, size = 83, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{16}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{16}-\frac{\operatorname{atan}(x)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{8}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{8}-\frac{\operatorname{atanh}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8-1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 - sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/16 - atan(x)/4 - sqrt(2)*atan(sqrt(2)*x - 1)/8 - sqrt(2)*
atan(sqrt(2)*x + 1)/8 - atanh(x)/4
```

Mathematica [A] time = 0.0362864, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+2\log(1-x)-2\log(x+1)-4\tan^{-1}(x)+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^8)^(-1),x]
```

```
[Out] (-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTa
n[1 + Sqrt[2]*x] + 2*Log[1 - x] - 2*Log[1 + x] + Sqrt[2]*Log[1 -
Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

Maple [A] time = 0.001, size = 66, normalized size = 0.7

$$-\frac{\mathit{Artanh}(x)}{4}-\frac{\arctan(x)}{4}-\frac{\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{8}-\frac{\sqrt{2}}{16}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)-\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8-1),x)
```

```
[Out] -1/4*arctanh(x)-1/4*arctan(x)-1/8*arctan(x*2^(1/2)-1)*2^(1/2)-1/1
6*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))-1/8*arctan(1+x*
2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.56852, size = 119, normalized size = 1.23

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{1}{4}\arctan(x)-\frac{1}{8}\log(x+1)+\frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2
)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x)
- 1/8*log(x + 1) + 1/8*log(x - 1)
```

Fricas [A] time = 0.219247, size = 151, normalized size = 1.56

$$-\frac{1}{16}\sqrt{2}\left(2\sqrt{2}\arctan(x)+\sqrt{2}\log(x+1)-\sqrt{2}\log(x-1)-4\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-4\arctan\left(\frac{}{\sqrt{2}x+\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 1),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(2*sqrt(2)*arctan(x) + sqrt(2)*log(x + 1) - sqrt(2)
*log(x - 1) - 4*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*
x + 1) + 1)) - 4*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)
*x + 1) - 1)) + log(x^2 + sqrt(2)*x + 1) - log(x^2 - sqrt(2)*x +
1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8-1),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.199278, size = 122, normalized size = 1.26

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)-\frac{1}{4}\arctan(x)-\frac{1}{8}\ln(|x+1|)+\frac{1}{8}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*ln(x^2 + sqrt(2)
*x + 1) + 1/16*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) -
1/8*ln(abs(x + 1)) + 1/8*ln(abs(x - 1))
```

3.51 $\int \frac{1}{1-x^4+x^8}\, dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6])
- ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6])
 + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]
) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6
]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[
6])
```

Rubi [A] time = 0.590313, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4 + x^8)^(-1),x]
```


```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6])
- ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6])
 + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]
) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6
]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[
```

```
6])
```

Rubi in Sympy [A] time = 26.0321, size = 529, normalized size = 1.92

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}$$
$$-\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(-\frac{(1+\sqrt{3})\sqrt{\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(-\frac{(1+\sqrt{3})\sqrt{\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(\frac{(-\sqrt{3}+1)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(\frac{(-\sqrt{3}+1)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(
12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 + x*
sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(1/2 +
sqrt(3)/2)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) +
 2)) + sqrt(3)*(1/2 + sqrt(3)/2)*log(x**2 + x*sqrt(sqrt(3) + 2) +
 1)/(12*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(1 + sqrt(3))*sqrt(sqrt(3)
 + 2)/2 + sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2
))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) +
 sqrt(3)*(-(1 + sqrt(3))*sqrt(sqrt(3) + 2)/2 + sqrt(3)*sqrt(sqrt(
3) + 2))*atan((2*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sq
rt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-sqrt(3) + 1)*sqr
t(-sqrt(3) + 2)/2 + sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x - sqrt(
-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt
(3) + 2)) + sqrt(3)*((-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)/2 + sqrt(3
)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3
) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0170151, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.009, size = 30, normalized size = 0.1

$$\frac{\sum_{\_R=RootOf\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.229715, size = 257, normalized size = 0.93

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}x+2}{\sqrt{3}\sqrt{2}x+2x^2+2\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}x-2}{\sqrt{3}\sqrt{2}x-2x^2-2\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan((sqrt(3)*sqrt(2)*x + 2)/(sqrt(3)*sqrt
(2)*x + 2*x^2 + 2*sqrt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 +
1))) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*x - 2)/(sqrt(
3)*sqrt(2)*x - 2*x^2 - 2*sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3
*x^2 + 1))) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3
 + x) + 3*x^2 + 1) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(
2)*(x^3 + x) + 3*x^2 + 1)
```

Sympy [A] time = 0.289391, size = 165, normalized size = 0.6

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2
 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*
atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x
**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**
4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24
```

GIAC/XCAS [A] time = 0.208233, size = 277, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))
) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/12*sqrt(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) -
sqrt(2))) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6
) - sqrt(2))) + 1/24*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) +
 1) - 1/24*sqrt(6)*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24
*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*sqrt(6)*l
n(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.52 $\int \frac{x^7}{1+x^{12}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{12}\log\left(x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[
1 - x^4 + x^8]/24
```

Rubi [A] time = 0.0749672, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636

$$-\frac{1}{12}\log\left(x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 + x^12),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[
1 - x^4 + x^8]/24
```

Rubi in Sympy [A] time = 4.0068, size = 42, normalized size = 0.86

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(sqrt(3
)*(2*x**4/3 - 1/3))/12
```

Mathematica [B] time = 0.163987, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(-2\log\left(x^2-\sqrt{2}x+1\right)-2\log\left(x^2+\sqrt{2}x+1\right)+\log\left(2x^2-\sqrt{6}x+\sqrt{2}x+2\right)\right.$$
$$+\log\left(2x^2+\sqrt{2}\left(\sqrt{3}-1\right)x+2\right)+\log\left(2x^2-\left(\sqrt{2}+\sqrt{6}\right)x+2\right)$$
$$+\log\left(2x^2+\left(\sqrt{2}+\sqrt{6}\right)x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{-2\sqrt{2}x+\sqrt{3}+1}{1-\sqrt{3}}\right)$$
$$\left.-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt{2}x-\sqrt{3}+1}{1+\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt{2}x+\sqrt{3}-1}{1+\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt{2}x+\sqrt{3}+1}{\sqrt{3}-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*
Sqrt[3]*ArcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqr
t[3]*ArcTan[(-1 + Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[
3]*ArcTan[(1 + Sqrt[3] + 2*Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 -
 Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x + x^2] + Log[2 + Sqrt[2]*
x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqrt[3])*x + 2*x^2
] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2] + S
qrt[6])*x + 2*x^2])/24
```

Maple [A] time = 0.003, size = 41, normalized size = 0.8

$$\frac{\ln\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2\,x^4-1\right)\sqrt{3}}{3}\right)-\frac{\ln\left(x^4+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^12+1),x)
```

```
[Out] 1/24*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))-1/1
2*ln(x^4+1)
```

Maxima [A] time = 1.57401, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^12 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4
 + 1) - 1/12*log(x^4 + 1)
```

Fricas [A] time = 0.220663, size = 63, normalized size = 1.29

$$\frac{1}{72}\sqrt{3}\left(\sqrt{3}\log\left(x^8 - x^4 + 1\right) - 2\sqrt{3}\log\left(x^4 + 1\right) + 6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^12 + 1),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(sqrt(3)*log(x^8 - x^4 + 1) - 2*sqrt(3)*log(x^4 + 1)
 + 6*arctan(1/3*sqrt(3)*(2*x^4 - 1)))
```

Sympy [A] time = 0.240174, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4 + 1\right)}{12} + \frac{\log\left(x^8 - x^4 + 1\right)}{24} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt
(3)*x**4/3 - sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.206088, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) + \frac{1}{24}\ln\left(x^8 - x^4 + 1\right) - \frac{1}{12}\ln\left(x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^12 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*ln(x^8 - x^4
+ 1) - 1/12*ln(x^4 + 1)
```

3.53 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$x\log(x)-x$$

```
[Out] -x + x*Log[x]
```

Rubi [A] time = 0.00405194, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x\log(x)-x$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Rubi in Sympy [A] time = 0.46123, size = 5, normalized size = 0.62

$$x\log\left(x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Mathematica [A] time = 0.013382, size = 8, normalized size = 1.

$$x\log(x)-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$-x + x\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x),x)
```

```
[Out] -x+x*ln(x)
```

Maxima [A] time = 1.37487, size = 11, normalized size = 1.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A] time = 0.220285, size = 11, normalized size = 1.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A] time = 0.060553, size = 5, normalized size = 0.62

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

GIAC/XCAS [A] time = 0.199781, size = 11, normalized size = 1.38

$$x\ln(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x),x, algorithm="giac")
```

```
[Out] x*ln(x) - x
```

3.54 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.00724857, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2 \log{(x)}}{2} - \frac{\int x\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - Integral(x, x)/2
```

Mathematica [A] time = 0.0010809, size = 17, normalized size = 1.

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0.006, size = 14, normalized size = 0.8

$$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 1.36746, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2\log(x)-\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 0.218629, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2\log(x)-\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.067969, size = 12, normalized size = 0.71

$$\frac{x^2\log(x)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

GIAC/XCAS [A] time = 0.197566, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\ln\left(x\right) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*ln(x) - 1/4*x^2
```

3.55 $\int x^2 \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

```
[Out] -x^3/9 + (x^3*Log[x])/3
```

Rubi [A] time = 0.0114205, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Log[x],x]

[Out] -x^3/9 + (x^3*Log[x])/3
```

Rubi in Sympy [A] time = 1.05041, size = 12, normalized size = 0.71

$$\frac{x^3 \log(x)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*ln(x),x)

[Out] x**3*log(x)/3 - x**3/9
```

Mathematica [A] time = 0.00158424, size = 17, normalized size = 1.

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Log[x],x]
```

```
[Out] -x^3/9 + (x^3*Log[x])/3
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*ln(x),x)
```

```
[Out] -1/9*x^3+1/3*x^3*ln(x)
```

Maxima [A] time = 1.36633, size = 18, normalized size = 1.06

$$\frac{1}{3}\,x^3\log(x)-\frac{1}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Fricas [A] time = 0.234303, size = 18, normalized size = 1.06

$$\frac{1}{3}\,x^3\log(x)-\frac{1}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Sympy [A] time = 0.07007, size = 12, normalized size = 0.71

$$\frac{x^3\log(x)}{3}-\frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*ln(x),x)
```

```
[Out] x**3*log(x)/3 - x**3/9
```

GIAC/XCAS [A] time = 0.200327, size = 18, normalized size = 1.06

$$\frac{1}{3}\,x^{3}\ln\left(x\right) - \frac{1}{9}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*ln(x) - 1/9*x^3
```

3.56 $\int x^p \log(x) \, dx$

Optimal. Leaf size=26

$$\frac{x^{p+1}\log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

```
[Out] -(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

Rubi [A] time = 0.0187475, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x^{p+1}\log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^p*Log[x],x]

[Out] -(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

Rubi in Sympy [A] time = 1.62196, size = 20, normalized size = 0.77

$$\frac{x^{p+1}\log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**p*ln(x),x)

[Out] x**(p + 1)*log(x)/(p + 1) - x**(p + 1)/(p + 1)**2
```

Mathematica [A] time = 0.0122582, size = 19, normalized size = 0.73

$$\frac{x^{p+1}((p+1)\log(x)-1)}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^p*Log[x],x]
```

```
[Out] (x^(1 + p)*(-1 + (1 + p)*Log[x]))/(1 + p)^2
```

Maple [A] time = 0.01, size = 34, normalized size = 1.3

$$\frac{x \ln (x)\, \mathrm{e}^{\ln(x) p}}{1+p}-\frac{x \mathrm{e}^{\ln(x) p}}{p^{2}+2 p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^p*ln(x),x)
```

```
[Out] 1/(1+p)*x*ln(x)*exp(ln(x)*p)-1/(p^2+2*p+1)*x*exp(ln(x)*p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^p*log(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.21999, size = 34, normalized size = 1.31

$$\frac{((p+1) x \log (x)-x) x^{p}}{p^{2}+2 p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^p*log(x),x, algorithm="fricas")
```

```
[Out] ((p + 1)*x*log(x) - x)*x^p/(p^2 + 2*p + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**p*ln(x),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^p \log(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^p*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^p*log(x), x)
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi [A] time = 0.00811349, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi in Sympy [A] time = 0.54262, size = 15, normalized size = 1.

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Mathematica [A] time = 0.00155384, size = 15, normalized size = 1.

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] time = 0.001, size = 16, normalized size = 1.1

$$2x - 2x\ln(x) + x(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^2,x)
```

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A] time = 1.40363, size = 16, normalized size = 1.07

$$\left(\log(x)^2 - 2\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A] time = 0.219668, size = 20, normalized size = 1.33

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="fricas")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] time = 0.079892, size = 15, normalized size = 1.

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

<u>GIAC/XCAS</u> [A] time = 0.199126, size = 20, normalized size = 1.33

$$x\ln(x)^2 - 2x\ln(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*ln(x)^2 - 2*x*ln(x) + 2*x
```

3.58 $\int x^9 \log^{11}(x)\, dx$

Optimal. Leaf size=127

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x)}{12500}+\frac{2079x^{10}\log^{5}(x)}{62500}-\frac{2079x^{10}\log^{4}(x)}{125000}+\frac{2079x^{10}\log^{3}(x)}{312500}-\frac{6237x^{10}\log^{2}(x)}{3125000}+\frac{6237x^{10}\log(x)}{15625000}$$

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10
*Log[x]^2)/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log
[x]^4)/125000 + (2079*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/
12500 + (99*x^10*Log[x]^7)/1250 - (99*x^10*Log[x]^8)/1000 + (11*x
^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/100 + (x^10*Log[x]^11)/10
```

Rubi [A] time = 0.204647, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x)}{12500}+\frac{2079x^{10}\log^{5}(x)}{62500}-\frac{2079x^{10}\log^{4}(x)}{125000}+\frac{2079x^{10}\log^{3}(x)}{312500}-\frac{6237x^{10}\log^{2}(x)}{3125000}+\frac{6237x^{10}\log(x)}{15625000}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10
*Log[x]^2)/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log
[x]^4)/125000 + (2079*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/
12500 + (99*x^10*Log[x]^7)/1250 - (99*x^10*Log[x]^8)/1000 + (11*x
^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/100 + (x^10*Log[x]^11)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^{10}\log(x)^{11}}{10}-\frac{11x^{10}\log(x)^{10}}{100}+\frac{11x^{10}\log(x)^{9}}{100}-\frac{99x^{10}\log(x)^{8}}{1000}+\frac{99x^{10}\log(x)^{7}}{1250}-\frac{693x^{10}\log(x)^{6}}{12500}+\frac{2079x^{10}\log(x)^{5}}{62500}-\frac{2079x^{10}\log(x)^{4}}{125000}+\frac{2079x^{10}\log(x)^{3}}{312500}-\frac{6237x^{10}\log(x)^{2}}{3125000}+\frac{6237x^{10}\log(x)}{15625000}-\frac{6237\int x^{9}\,dx}{15625000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*ln(x)**11,x)
```

```
[Out] x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**
9/100 - 99*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x
**10*log(x)**6/12500 + 2079*x**10*log(x)**5/62500 - 2079*x**10*lo
g(x)**4/125000 + 2079*x**10*log(x)**3/312500 - 6237*x**10*log(x)*
*2/3125000 + 6237*x**10*log(x)/15625000 - 6237*Integral(x**9, x)/
15625000
```

Mathematica [A] time = 0.00337934, size = 127, normalized size = 1.

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x)}{12500}+\frac{2079x^{10}\log^{5}(x)}{62500}-\frac{2079x^{10}\log^{4}(x)}{125000}+\frac{2079x^{10}\log^{3}(x)}{312500}-\frac{6237x^{10}\log^{2}(x)}{3125000}+\frac{6237x^{10}\log(x)}{15625000}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10
*Log[x]^2)/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log
[x]^4)/125000 + (2079*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/
12500 + (99*x^10*Log[x]^7)/1250 - (99*x^10*Log[x]^8)/1000 + (11*x
^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/100 + (x^10*Log[x]^11)/10
```

Maple [A] time = 0.001, size = 104, normalized size = 0.8

$$-\frac{6237\,x^{10}}{156250000}+\frac{6237\,x^{10}\ln(x)}{15625000}-\frac{6237\,x^{10}(\ln(x))^{2}}{3125000}+\frac{2079\,x^{10}(\ln(x))^{3}}{312500}-\frac{2079\,x^{10}(\ln(x))^{4}}{125000}+\frac{2079\,x^{10}(\ln(x))^{5}}{62500}-\frac{693\,x^{10}(\ln(x))^{6}}{12500}+\frac{99\,x^{10}(\ln(x))^{7}}{1250}-\frac{99\,x^{10}(\ln(x))^{8}}{1000}+\frac{11\,x^{10}(\ln(x))^{9}}{100}-\frac{11\,x^{10}(\ln(x))^{10}}{100}+\frac{x^{10}(\ln(x))^{11}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*ln(x)^11,x)
```

```
[Out] -6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*l
n(x)^2+2079/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/625
```

```
00*x^10*ln(x)^5-693/12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/10
00*x^10*ln(x)^8+11/100*x^10*ln(x)^9-11/100*x^10*ln(x)^10+1/10*x^1
0*ln(x)^11
```

Maxima [A] time = 1.41108, size = 96, normalized size = 0.76

$$\frac{1}{156250000}\left(15625000\log(x)^{11} - 17187500\log(x)^{10} + 17187500\log(x)^{9} - 15468750\log(x)^{8} + 12375000\log(x)^{7} - 86625\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9*log(x)^11,x, algorithm="maxima")
```

```
[Out] 1/156250000*(15625000*log(x)^11 - 17187500*log(x)^10 + 17187500*l
og(x)^9 - 15468750*log(x)^8 + 12375000*log(x)^7 - 8662500*log(x)^
6 + 5197500*log(x)^5 - 2598750*log(x)^4 + 1039500*log(x)^3 - 3118
50*log(x)^2 + 62370*log(x) - 6237)*x^10
```

Fricas [A] time = 0.223714, size = 139, normalized size = 1.09

$$\frac{1}{10}x^{10}\log(x)^{11} - \frac{11}{100}x^{10}\log(x)^{10} + \frac{11}{100}x^{10}\log(x)^{9} - \frac{99}{1000}x^{10}\log(x)^{8} + \frac{99}{1250}x^{10}\log(x)^{7} - \frac{693}{12500}x^{10}\log(x)^{6} + \frac{2079}{62500}x^{10}\log(x)^{5} - \frac{2079}{125000}x^{10}\log(x)^{4} + \frac{2079}{312500}x^{10}\log(x)^{3} - \frac{6237}{3125000}x^{10}\log(x)^{2} + \frac{6237}{15625000}x^{10}\log(x) - \frac{6237}{156250000}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9*log(x)^11,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^
9 - 99/1000*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^1
0*log(x)^6 + 2079/62500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4
 + 2079/312500*x^10*log(x)^3 - 6237/3125000*x^10*log(x)^2 + 6237/
15625000*x^10*log(x) - 6237/156250000*x^10
```

Sympy [A] time = 0.314924, size = 133, normalized size = 1.05

$$\frac{x^{10}\log(x)^{11}}{10} - \frac{11x^{10}\log(x)^{10}}{100} + \frac{11x^{10}\log(x)^{9}}{100} - \frac{99x^{10}\log(x)^{8}}{1000} + \frac{99x^{10}\log(x)^{7}}{1250} - \frac{693x^{10}\log(x)^{6}}{12500} + \frac{2079x^{10}\log(x)^{5}}{62500} - \frac{2079x^{10}\log(x)^{4}}{125000} + \frac{2079x^{10}\log(x)^{3}}{312500} - \frac{6237x^{10}\log(x)^{2}}{3125000} + \frac{6237x^{10}\log(x)}{15625000} - \frac{6237x^{10}}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*ln(x)**11,x)
```

```
[Out] x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**
9/100 - 99*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x
**10*log(x)**6/12500 + 2079*x**10*log(x)**5/62500 - 2079*x**10*lo
g(x)**4/125000 + 2079*x**10*log(x)**3/312500 - 6237*x**10*log(x)*
*2/3125000 + 6237*x**10*log(x)/15625000 - 6237*x**10/156250000
```

<u>GIAC/XCAS</u> [A] time = 0.207787, size = 139, normalized size = 1.09

$$\frac{1}{10}\,x^{10}\ln(x)^{11} - \frac{11}{100}\,x^{10}\ln(x)^{10} + \frac{11}{100}\,x^{10}\ln(x)^{9} - \frac{99}{1000}\,x^{10}\ln(x)^{8} + \frac{99}{1250}\,x^{10}\ln(x)^{7} - \frac{693}{12500}\,x^{10}\ln(x)^{6} + \frac{2079}{62500}\,x^{10}\ln(x)^{5} - \frac{2079}{125000}\,x^{10}\ln(x)^{4} + \frac{2079}{312500}\,x^{10}\ln(x)^{3} - \frac{6237}{3125000}\,x^{10}\ln(x)^{2} + \frac{6237}{15625000}\,x^{10}\ln(x) - \frac{6237}{156250000}\,x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9*log(x)^11,x, algorithm="giac")
```

```
[Out] 1/10*x^10*ln(x)^11 - 11/100*x^10*ln(x)^10 + 11/100*x^10*ln(x)^9 -
 99/1000*x^10*ln(x)^8 + 99/1250*x^10*ln(x)^7 - 693/12500*x^10*ln(
x)^6 + 2079/62500*x^10*ln(x)^5 - 2079/125000*x^10*ln(x)^4 + 2079/
312500*x^10*ln(x)^3 - 6237/3125000*x^10*ln(x)^2 + 6237/15625000*x
^10*ln(x) - 6237/156250000*x^10
```

3.59 $\int \frac{\log^2(x)}{x} \, dx$

Optimal. Leaf size=8

$$\frac{\log^3(x)}{3}$$

```
[Out] Log[x]^3/3
```

Rubi [A] time = 0.0176202, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^2/x,x]

[Out] Log[x]^3/3
```

Rubi in Sympy [A] time = 1.42685, size = 5, normalized size = 0.62

$$\frac{\log(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**2/x,x)

[Out] log(x)**3/3
```

Mathematica [A] time = 0.000927631, size = 8, normalized size = 1.

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^2/x,x]
```

```
[Out] Log[x]^3/3
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$\frac{(\ln(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^2/x,x)
```

```
[Out] 1/3*ln(x)^3
```

Maxima [A] time = 1.40021, size = 8, normalized size = 1.

$$\frac{1}{3}\log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*log(x)^3
```

Fricas [A] time = 0.22338, size = 8, normalized size = 1.

$$\frac{1}{3}\log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*log(x)^3
```

Sympy [A] time = 0.067876, size = 5, normalized size = 0.62

$$\frac{\log(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**2/x,x)
```

```
[Out] log(x)**3/3
```

GIAC/XCAS [A] time = 0.203481, size = 8, normalized size = 1.

$$\frac{1}{3}\ln\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2/x,x, algorithm="giac")
```

```
[Out] 1/3*ln(x)^3
```

3.60 $\int \frac{1}{\log(x)}\, dx$

Optimal. Leaf size=2

$$\text{LogIntegral}(x)$$

```
[Out] LogIntegral[x]
```

Rubi [A] time = 0.00381644, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rubi in Sympy [A] time = 0.023573, size = 2, normalized size = 1.

$$\text{li}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/ln(x),x)
```

```
[Out] li(x)
```

Mathematica [A] time = 0.0875745, size = 2, normalized size = 1.

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Maple [B] time = 0.004, size = 9, normalized size = 4.5

$$-Ei(1, -\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/ln(x),x)
```

```
[Out] -Ei(1,-ln(x))
```

Maxima [A] time = 1.44452, size = 4, normalized size = 2.

$$\mathrm{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x),x, algorithm="maxima")
```

```
[Out] Ei(log(x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$log_i ntegral(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/ln(x),x)
```

```
[Out] Integral(1/log(x), x)
```

GIAC/XCAS [A] time = 0.213899, size = 4, normalized size = 2.

$$\mathrm{Ei}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(ln(x))
```

3.61 $\int \frac{1}{\log(1+x)} dx$

Optimal. Leaf size=4

$$\text{LogIntegral}(x+1)$$

```
[Out] LogIntegral[1 + x]
```

Rubi [A] time = 0.00561218, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\text{LogIntegral}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[1 + x]^(-1),x]

[Out] LogIntegral[1 + x]
```

Rubi in Sympy [A] time = 0.477194, size = 3, normalized size = 0.75

$$\text{li}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/ln(1+x),x)

[Out] li(x + 1)
```

Mathematica [A] time = 0.00283505, size = 4, normalized size = 1.

$$\text{LogIntegral}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[1 + x]^(-1),x]

[Out] LogIntegral[1 + x]
```

Maple [B] time = 0.001, size = 11, normalized size = 2.8

$$-Ei\left(1,-\ln\left(1+x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/ln(1+x),x)
```

```
[Out] -Ei(1,-ln(1+x))
```

Maxima [A] time = 1.51406, size = 7, normalized size = 1.75

$$\mathrm{Ei}\left(\log\left(x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x + 1),x, algorithm="maxima")
```

```
[Out] Ei(log(x + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$log_{i}ntegral\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x + 1),x, algorithm="fricas")
```

```
[Out] log_integral(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/ln(1+x),x)
```

```
[Out] Integral(1/log(x + 1), x)
```

GIAC/XCAS [A] time = 0.215299, size = 7, normalized size = 1.75

$$\mathrm{Ei}\left(\ln\left(x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x + 1),x, algorithm="giac")
```

```
[Out] Ei(ln(x + 1))
```

3.62 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] Log[Log[x]]
```

Rubi [A] time = 0.0177911, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rubi in Sympy [A] time = 1.00407, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

Mathematica [A] time = 0.000800917, size = 3, normalized size = 1.

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x),x)
```

```
[Out] ln(ln(x))
```

Maxima [A] time = 1.44975, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A] time = 0.219574, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A] time = 0.075804, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

GIAC/XCAS [A] time = 0.21496, size = 5, normalized size = 1.67

$$\ln(|\ln(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="giac")
```

```
[Out] ln(abs(ln(x)))
```

3.63 $\int \frac{1}{x^2 \log^2(x)} \, dx$

Optimal. Leaf size=17

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Rubi [A] time = 0.0486889, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Log[x]^2),x]

[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Rubi in Sympy [A] time = 2.3296, size = 14, normalized size = 0.82

$$-\operatorname{Ei}(-\log(x)) - \frac{1}{x \log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/ln(x)**2,x)

[Out] -Ei(-log(x)) - 1/(x*log(x))
```

Mathematica [A] time = 0.00988236, size = 17, normalized size = 1.

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Log[x]^2),x]
```

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Maple [A] time = 0.006, size = 15, normalized size = 0.9

$$-\frac{1}{x\ln(x)}+Ei(1,\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(x)^2,x)
```

```
[Out] -1/x/ln(x)+Ei(1,ln(x))
```

Maxima [A] time = 1.48322, size = 8, normalized size = 0.47

$$-(-1,\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*log(x)^2),x, algorithm="maxima")
```

```
[Out] -gamma(-1, log(x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x\log(x)\,log_integral\left(\frac{1}{x}\right)+1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*log(x)^2),x, algorithm="fricas")
```

```
[Out] -(x*log(x)*log_integral(1/x) + 1)/(x*log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x^2\log(x)}\,dx-\frac{1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/ln(x)**2,x)
```

```
[Out] -Integral(1/(x**2*log(x)), x) - 1/(x*log(x))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*log(x)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log(x)^2), x)
```

3.64 $\int \frac{\log^p(x)}{x} \, dx$

Optimal. Leaf size=12

$$\frac{\log^{p+1}(x)}{p+1}$$

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0249641, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^p/x,x]
```

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Rubi in Sympy [A] time = 1.63509, size = 8, normalized size = 0.67

$$\frac{\log(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**p/x,x)
```

```
[Out] log(x)**(p + 1)/(p + 1)
```

Mathematica [A] time = 0.00353357, size = 12, normalized size = 1.

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^p/x,x]
```

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$\frac{(\ln(x))^{1+p}}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^p/x,x)

[Out] ln(x)^(1+p)/(1+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^p/x,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218832, size = 16, normalized size = 1.33

$$\frac{\log(x)^{p}\log(x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^p/x,x, algorithm="fricas")

[Out] log(x)^p*log(x)/(p + 1)
```

Sympy [A] time = 1.17423, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**p/x,x)
```

```
[Out] Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), Tru
e))
```

GIAC/XCAS [A] time = 0.198559, size = 16, normalized size = 1.33

$$\frac{\ln(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^p/x,x, algorithm="giac")
```

```
[Out] ln(x)^(p + 1)/(p + 1)
```

3.65 $\int (b + ax) \log(x) \, dx$

Optimal. Leaf size=29

$$\frac{1}{2} \log(x) \left(ax^2 + 2bx\right) - \frac{ax^2}{4} - bx$$

```
[Out] -(b*x) - (a*x^2)/4 + ((2*b*x + a*x^2)*Log[x])/2
```

Rubi [A] time = 0.0211842, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{1}{2} \log(x) \left(ax^2 + 2bx\right) - \frac{ax^2}{4} - bx$$

Antiderivative was successfully verified.

```
[In]  Int[(b + a*x)*Log[x],x]
```

```
[Out] -(b*x) - (a*x^2)/4 + ((2*b*x + a*x^2)*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a \int x \, dx}{2} - bx - \frac{b^2 \log(x)}{2a} + \frac{(ax + b)^2 \log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x+b)*ln(x),x)
```

```
[Out] -a*Integral(x, x)/2 - b*x - b**2*log(x)/(2*a) + (a*x + b)**2*log(
x)/(2*a)
```

Mathematica [A] time = 0.00214773, size = 28, normalized size = 0.97

$$-\frac{ax^2}{4} + \frac{1}{2} ax^2 \log(x) - bx + bx \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + a*x)*Log[x],x]
```

```
[Out] -(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$\frac{ax^2\ln(x)}{2} - \frac{ax^2}{4} + \ln(x)\,xb - bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+b)*ln(x),x)
```

```
[Out] 1/2*a*x^2*ln(x)-1/4*a*x^2+ln(x)*x*b-b*x
```

Maxima [A] time = 1.40092, size = 34, normalized size = 1.17

$$-\frac{1}{4}\,ax^2 - bx + \frac{1}{2}\left(ax^2 + 2\,bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

Fricas [A] time = 0.238247, size = 34, normalized size = 1.17

$$-\frac{1}{4}\,ax^2 - bx + \frac{1}{2}\left(ax^2 + 2\,bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)*log(x),x, algorithm="fricas")
```

```
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

Sympy [A] time = 0.102519, size = 22, normalized size = 0.76

$$-\frac{ax^2}{4} - bx + \left(\frac{ax^2}{2} + bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+b)*ln(x),x)
```

```
[Out] -a*x**2/4 - b*x + (a*x**2/2 + b*x)*log(x)
```

GIAC/XCAS [A] time = 0.204208, size = 32, normalized size = 1.1

$$\frac{1}{2}\,ax^2\ln\left(x\right) - \frac{1}{4}\,ax^2 + bx\ln\left(x\right) - bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*ln(x) - 1/4*a*x^2 + b*x*ln(x) - b*x
```

3.66 $\int (b+ax)^2 \log(x)\,dx$

Optimal. Leaf size=54

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*Log[x])/(3*a) + ((b +
 a*x)^3*Log[x])/(3*a)
```

Rubi [A] time = 0.0522001, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

Antiderivative was successfully verified.

```
[In]  Int[(b + a*x)^2*Log[x],x]
```

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*Log[x])/(3*a) + ((b +
 a*x)^3*Log[x])/(3*a)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2x^3}{9}-ab\int x\,dx-b^2x-\frac{b^3\log(x)}{3a}+\frac{(ax+b)^3\log(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x+b)**2*ln(x),x)
```

```
[Out] -a**2*x**3/9 - a*b*Integral(x, x) - b**2*x - b**3*log(x)/(3*a) +
(a*x + b)**3*log(x)/(3*a)
```

Mathematica [A] time = 0.0143132, size = 46, normalized size = 0.85

$$\frac{1}{18}x\left(6\log(x)\left(a^2x^2+3abx+3b^2\right)-2a^2x^2-9abx-18b^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + a*x)^2*Log[x],x]
```

```
[Out] (x*(-18*b^2 - 9*a*b*x - 2*a^2*x^2 + 6*(3*b^2 + 3*a*b*x + a^2*x^2)
*Log[x]))/18
```

Maple [A] time = 0.001, size = 48, normalized size = 0.9

$$\frac{a^2x^3\ln(x)}{3} - \frac{a^2x^3}{9} + abx^2\ln(x) - \frac{abx^2}{2} + \ln(x)\,xb^2 - b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+b)^2*ln(x),x)
```

```
[Out] 1/3*a^2*x^3*ln(x)-1/9*a^2*x^3+a*b*x^2*ln(x)-1/2*a*b*x^2+ln(x)*x*b
^2-b^2*x
```

Maxima [A] time = 1.42692, size = 63, normalized size = 1.17

$$-\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)^2*log(x),x, algorithm="maxima")
```

```
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3
*b^2*x)*log(x)
```

Fricas [A] time = 0.204077, size = 63, normalized size = 1.17

$$-\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)^2*log(x),x, algorithm="fricas")
```

```
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3
*b^2*x)*log(x)
```

Sympy [A] time = 0.132458, size = 44, normalized size = 0.81

$$-\frac{a^2x^3}{9} - \frac{abx^2}{2} - b^2x + \left(\frac{a^2x^3}{3} + abx^2 + b^2x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+b)**2*ln(x),x)
```

```
[Out] -a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b*
*2*x)*log(x)
```

GIAC/XCAS [A] time = 0.199139, size = 63, normalized size = 1.17

$$\frac{1}{3}a^2x^3\ln(x) - \frac{1}{9}a^2x^3 + abx^2\ln(x) - \frac{1}{2}abx^2 + b^2x\ln(x) - b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + b)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*a^2*x^3*ln(x) - 1/9*a^2*x^3 + a*b*x^2*ln(x) - 1/2*a*b*x^2 + b
^2*x*ln(x) - b^2*x
```

3.67 $\int \frac{\log(x)}{(b+ax)^2} \, dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{b(ax+b)} - \frac{\log(ax+b)}{ab}$$

```
[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

Rubi [A] time = 0.0253494, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x\log(x)}{b(ax+b)} - \frac{\log(ax+b)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]/(b + a*x)^2,x]

[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

Rubi in Sympy [A] time = 3.35815, size = 26, normalized size = 0.9

$$-\frac{\log(x)}{a(ax+b)} + \frac{\log(x)}{ab} - \frac{\log(ax+b)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)/(a*x+b)**2,x)

[Out] -log(x)/(a*(a*x + b)) + log(x)/(a*b) - log(a*x + b)/(a*b)
```

Mathematica [A] time = 0.0190655, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{ax+b} - \frac{\log(ax+b)}{a}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]/(b + a*x)^2,x]
```

```
[Out] ((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b
```

Maple [A] time = 0.008, size = 30, normalized size = 1.

$$\frac{x\ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)/(a*x+b)^2,x)
```

```
[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

Maxima [A] time = 1.41451, size = 51, normalized size = 1.76

$$-\frac{\frac{\log(ax+b)}{b} - \frac{\log(x)}{b}}{a} - \frac{\log(x)}{(ax+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(a*x + b)^2,x, algorithm="maxima")
```

```
[Out] -(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)
```

Fricas [A] time = 0.21726, size = 46, normalized size = 1.59

$$\frac{ax\log(x) - (ax+b)\log(ax+b)}{a^2bx + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(a*x + b)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)
```

Sympy [A] time = 0.654088, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{a^2x + ab} + \frac{\log(x) - \log\left(x + \frac{b}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/(a*x+b)**2,x)
```

```
[Out] -log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)
```

GIAC/XCAS [A] time = 0.199195, size = 49, normalized size = 1.69

$$-\frac{\ln(x)}{(ax+b)a} + \frac{\ln\left(\left|-\frac{b}{ax+b} + 1\right|\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(a*x + b)^2,x, algorithm="giac")
```

```
[Out] -ln(x)/((a*x + b)*a) + ln(abs(-b/(a*x + b) + 1))/(a*b)
```

3.68 $\int x \log(b + ax)\, dx$

Optimal. Leaf size=46

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a
*x])/2
```

Rubi [A] time = 0.046683, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a
*x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2 \log(ax+b)}{2} - \frac{\int x\, dx}{2} + \frac{\int b\, dx}{2a} - \frac{b^2 \log(ax+b)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(a*x+b),x)
```

```
[Out] x**2*log(a*x + b)/2 - Integral(x, x)/2 + Integral(b, x)/(2*a) - b
**2*log(a*x + b)/(2*a**2)
```

Mathematica [A] time = 0.00491238, size = 46, normalized size = 1.

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a
*x])/2
```

Maple [A] time = 0.002, size = 47, normalized size = 1.

$$-\frac{b^{2}\ln\left(ax+b\right)}{2\,a^{2}}+\frac{bx}{2\,a}+\frac{3\,b^{2}}{4\,a^{2}}+\frac{x^{2}\ln\left(ax+b\right)}{2}-\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(a*x+b),x)
```

```
[Out] -1/2*b^2*ln(a*x+b)/a^2+1/2*b*x/a+3/4/a^2*b^2+1/2*x^2*ln(a*x+b)-1/
4*x^2
```

Maxima [A] time = 1.37992, size = 59, normalized size = 1.28

$$\frac{1}{2}\,x^{2}\log\left(ax+b\right)-\frac{1}{4}\,a\left(\frac{2\,b^{2}\log\left(ax+b\right)}{a^{3}}+\frac{ax^{2}-2\,bx}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(a*x + b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2
*b*x)/a^2)
```

Fricas [A] time = 0.219275, size = 53, normalized size = 1.15

$$-\frac{a^{2}x^{2}-2\,abx-2\,\left(a^{2}x^{2}-b^{2}\right)\log\left(ax+b\right)}{4\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(a*x + b),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2
```

Sympy [A] time = 0.619149, size = 42, normalized size = 0.91

$$-a\left(\frac{x^2}{4a}-\frac{bx}{2a^2}+\frac{b^2\log(ax+b)}{2a^3}\right)+\frac{x^2\log(ax+b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(a*x+b),x)
```

```
[Out] -a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**
2*log(a*x + b)/2
```

GIAC/XCAS [A] time = 0.20108, size = 78, normalized size = 1.7

$$\frac{(ax+b)^2\ln(ax+b)}{2\,a^2}-\frac{(ax+b)b\ln(ax+b)}{a^2}-\frac{(ax+b)^2}{4\,a^2}+\frac{(ax+b)b}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(a*x + b),x, algorithm="giac")
```

```
[Out] 1/2*(a*x + b)^2*ln(a*x + b)/a^2 - (a*x + b)*b*ln(a*x + b)/a^2 - 1
/4*(a*x + b)^2/a^2 + (a*x + b)*b/a^2
```

3.69 $\int x^2 \log(b + ax)\, dx$

Optimal. Leaf size=59

$$\frac{b^3 \log(ax+b)}{3a^3} - \frac{b^2 x}{3a^2} + \frac{1}{3}x^3 \log(ax+b) + \frac{bx^2}{6a} - \frac{x^3}{9}$$

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*
a^3) + (x^3*Log[b + a*x])/3
```

Rubi [A] time = 0.0633704, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{b^3 \log(ax+b)}{3a^3} - \frac{b^2 x}{3a^2} + \frac{1}{3}x^3 \log(ax+b) + \frac{bx^2}{6a} - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Log[b + a*x],x]
```

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*
a^3) + (x^3*Log[b + a*x])/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ab^2 \int \frac{1}{a^3}\, dx}{3} + \frac{x^3 \log(ax+b)}{3} - \frac{x^3}{9} + \frac{b \int x\, dx}{3a} + \frac{b^3 \log(ax+b)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*ln(a*x+b),x)
```

```
[Out] -a*b**2*Integral(a**(-3), x)/3 + x**3*log(a*x + b)/3 - x**3/9 + b
*Integral(x, x)/(3*a) + b**3*log(a*x + b)/(3*a**3)
```

Mathematica [A] time = 0.00542915, size = 59, normalized size = 1.

$$\frac{b^3 \log(ax+b)}{3a^3} - \frac{b^2 x}{3a^2} + \frac{1}{3}x^3 \log(ax+b) + \frac{bx^2}{6a} - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Log[b + a*x],x]
```

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*
a^3) + (x^3*Log[b + a*x])/3
```

Maple [A] time = 0.002, size = 58, normalized size = 1.

$$\frac{x^3 \ln(ax+b)}{3} + \frac{b^3 \ln(ax+b)}{3\,a^3} - \frac{x^3}{9} + \frac{bx^2}{6\,a} - \frac{b^2x}{3\,a^2} - \frac{11\,b^3}{18\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*ln(a*x+b),x)
```

```
[Out] 1/3*x^3*ln(a*x+b)+1/3*b^3*ln(a*x+b)/a^3-1/9*x^3+1/6*b*x^2/a-1/3*b
^2*x/a^2-11/18/a^3*b^3
```

Maxima [A] time = 1.37941, size = 77, normalized size = 1.31

$$\frac{1}{3}\,x^3 \log(ax+b) + \frac{1}{18}\,a\left(\frac{6\,b^3 \log(ax+b)}{a^4} - \frac{2\,a^2x^3 - 3\,abx^2 + 6\,b^2x}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(a*x + b),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^
3 - 3*a*b*x^2 + 6*b^2*x)/a^3)
```

Fricas [A] time = 0.219923, size = 66, normalized size = 1.12

$$-\frac{2\,a^3x^3 - 3\,a^2bx^2 + 6\,ab^2x - 6\,\left(a^3x^3 + b^3\right)\log(ax+b)}{18\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(a*x + b),x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*lo
g(a*x + b))/a^3
```

Sympy [A] time = 0.597722, size = 54, normalized size = 0.92

$$-a\left(\frac{x^3}{9a}-\frac{bx^2}{6a^2}+\frac{b^2x}{3a^3}-\frac{b^3\log(ax+b)}{3a^4}\right)+\frac{x^3\log(ax+b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*ln(a*x+b),x)
```

```
[Out] -a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x
 + b)/(3*a**4)) + x**3*log(a*x + b)/3
```

GIAC/XCAS [A] time = 0.200539, size = 127, normalized size = 2.15

$$\frac{(ax+b)^3\ln(ax+b)}{3\,a^3}-\frac{(ax+b)^2b\ln(ax+b)}{a^3}+\frac{(ax+b)b^2\ln(ax+b)}{a^3}-\frac{(ax+b)^3}{9\,a^3}+\frac{(ax+b)^2b}{2\,a^3}-\frac{(ax+b)b^2}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(a*x + b),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + b)^3*ln(a*x + b)/a^3 - (a*x + b)^2*b*ln(a*x + b)/a^3 +
 (a*x + b)*b^2*ln(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x +
 b)^2*b/a^3 - (a*x + b)*b^2/a^3
```

3.70 $\int \log\left(a^2+x^2\right)\,dx$

Optimal. Leaf size=23

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Rubi [A] time = 0.0195372, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In]  Int[Log[a^2 + x^2],x]

[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Rubi in Sympy [A] time = 2.32502, size = 20, normalized size = 0.87

$$2a\,\text{atan}\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(a**2+x**2),x)

[Out] 2*a*atan(x/a) + x*log(a**2 + x**2) - 2*x
```

Mathematica [A] time = 0.0040673, size = 23, normalized size = 1.

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[a^2 + x^2],x]

[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$-2\,x+2\,a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^{2}+x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(a^2+x^2),x)
```

```
[Out] -2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

Maxima [A] time = 1.5502, size = 31, normalized size = 1.35

$$2\,a\arctan\left(\frac{x}{a}\right)+x\log\left(a^{2}+x^{2}\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Fricas [A] time = 0.221842, size = 31, normalized size = 1.35

$$2\,a\arctan\left(\frac{x}{a}\right)+x\log\left(a^{2}+x^{2}\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Sympy [A] time = 0.552446, size = 36, normalized size = 1.57

$$-2a\left(\frac{i\log\left(-ia+x\right)}{2}-\frac{i\log\left(ia+x\right)}{2}\right)+x\log\left(a^{2}+x^{2}\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(a**2+x**2),x)
```

```
[Out] -2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2)
- 2*x
```

GIAC/XCAS [A] time = 0.200245, size = 31, normalized size = 1.35

$$2\,a\arctan\left(\frac{x}{a}\right) + x\ln\left(a^2 + x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(a^2 + x^2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(x/a) + x*ln(a^2 + x^2) - 2*x
```

3.71 $\int x \log \left(a^2+x^2\right) \, dx$

Optimal. Leaf size=27

$$\frac{1}{2} \left(a^2+x^2\right) \log \left(a^2+x^2\right)-\frac{x^2}{2}$$

```
[Out] -x^2/2 + ((a^2 + x^2)*Log[a^2 + x^2])/2
```

Rubi [A] time = 0.0706951, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{2} \left(a^2+x^2\right) \log \left(a^2+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[a^2 + x^2],x]

[Out] -x^2/2 + ((a^2 + x^2)*Log[a^2 + x^2])/2
```

Rubi in Sympy [A] time = 1.22678, size = 31, normalized size = 1.15

$$\frac{a^2 \log \left(a^2+x^2\right)}{2}+\frac{x^2 \log \left(a^2+x^2\right)}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(a**2+x**2),x)

[Out] a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2
```

Mathematica [A] time = 0.00324879, size = 38, normalized size = 1.41

$$\frac{1}{2}a^2 \log \left(a^2+x^2\right)+\frac{1}{2}x^2 \log \left(a^2+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[a^2 + x^2],x]
```

```
[Out] -x^2/2 + (a^2*Log[a^2 + x^2])/2 + (x^2*Log[a^2 + x^2])/2
```

Maple [A] time = 0.001, size = 29, normalized size = 1.1

$$\frac{(a^2+x^2)\ln(a^2+x^2)}{2}-\frac{x^2}{2}-\frac{a^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(a^2+x^2),x)
```

```
[Out] 1/2*(a^2+x^2)*ln(a^2+x^2)-1/2*x^2-1/2*a^2
```

Maxima [A] time = 1.38547, size = 38, normalized size = 1.41

$$-\frac{1}{2}a^2-\frac{1}{2}x^2+\frac{1}{2}(a^2+x^2)\log(a^2+x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Fricas [A] time = 0.215803, size = 31, normalized size = 1.15

$$-\frac{1}{2}x^2+\frac{1}{2}(a^2+x^2)\log(a^2+x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Sympy [A] time = 0.553694, size = 31, normalized size = 1.15

$$\frac{a^2\log(a^2+x^2)}{2}+\frac{x^2\log(a^2+x^2)}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(a**2+x**2),x)
```

```
[Out] a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2
```

GIAC/XCAS [A] time = 0.198737, size = 38, normalized size = 1.41

$$-\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}\left(a^2 + x^2\right)\ln\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(a^2 + x^2),x, algorithm="giac")
```

```
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*ln(a^2 + x^2)
```

3.72 $\int x^2 \log \left(a^2+x^2\right) \, dx$

Optimal. Leaf size=44

$$-\frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3 \log \left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 +
x^2])/3
```

Rubi [A] time = 0.046763, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3 \log \left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Log[a^2 + x^2],x]
```

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 +
x^2])/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a^3 \operatorname{atan}\left(\frac{x}{a}\right)}{3}+\frac{x^3 \log \left(a^2+x^2\right)}{3}-\frac{2x^3}{9}+\frac{2\int a^2\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*ln(a**2+x**2),x)
```

```
[Out] -2*a**3*atan(x/a)/3 + x**3*log(a**2 + x**2)/3 - 2*x**3/9 + 2*Inte
gral(a**2, x)/3
```

Mathematica [A] time = 0.00437289, size = 44, normalized size = 1.

$$-\frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3 \log \left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^2*Log[a^2 + x^2],x]
```

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 +
x^2])/3
```

Maple [A] time = 0.003, size = 37, normalized size = 0.8

$$\frac{2\,a^{2}x}{3}-\frac{2\,x^{3}}{9}-\frac{2\,a^{3}}{3}\arctan\left(\frac{x}{a}\right)+\frac{x^{3}\ln\left(a^{2}+x^{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*ln(a^2+x^2),x)
```

```
[Out] 2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

Maxima [A] time = 1.57122, size = 49, normalized size = 1.11

$$-\frac{2}{3}\,a^{3}\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^{3}\log\left(a^{2}+x^{2}\right)+\frac{2}{3}\,a^{2}x-\frac{2}{9}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^2*log(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x
^3
```

Fricas [A] time = 0.228245, size = 49, normalized size = 1.11

$$-\frac{2}{3}\,a^{3}\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^{3}\log\left(a^{2}+x^{2}\right)+\frac{2}{3}\,a^{2}x-\frac{2}{9}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^2*log(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x
^3
```

Sympy [A] time = 0.583052, size = 53, normalized size = 1.2

$$-2a^3\left(-\frac{i\log(-ia+x)}{6}+\frac{i\log(ia+x)}{6}\right)+\frac{2a^2x}{3}+\frac{x^3\log(a^2+x^2)}{3}-\frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*ln(a**2+x**2),x)
```

```
[Out] -2*a**3*(-I*log(-I*a + x)/6 + I*log(I*a + x)/6) + 2*a**2*x/3 + x*
*3*log(a**2 + x**2)/3 - 2*x**3/9
```

GIAC/XCAS [A] time = 0.201867, size = 49, normalized size = 1.11

$$-\frac{2}{3}a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}x^3\ln\left(a^2+x^2\right)+\frac{2}{3}a^2x-\frac{2}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(a^2 + x^2),x, algorithm="giac")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*ln(a^2 + x^2) + 2/3*a^2*x - 2/9*x^
3
```

3.73 $\int x^4 \log\left(a^2+x^2\right)\, dx$

Optimal. Leaf size=54

$$\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2x^5}{25}$$

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/
5 + (x^5*Log[a^2 + x^2])/5
```

Rubi [A] time = 0.0527166, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/
5 + (x^5*Log[a^2 + x^2])/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a^5\operatorname{atan}\left(\frac{x}{a}\right)}{5}+\frac{2a^2x^3}{15}+\frac{x^5\log\left(a^2+x^2\right)}{5}-\frac{2x^5}{25}-\frac{2\int a^4\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*ln(a**2+x**2),x)
```

```
[Out] 2*a**5*atan(x/a)/5 + 2*a**2*x**3/15 + x**5*log(a**2 + x**2)/5 - 2
*x**5/25 - 2*Integral(a**4, x)/5
```

Mathematica [A] time = 0.0052746, size = 54, normalized size = 1.

$$\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/
5 + (x^5*Log[a^2 + x^2])/5
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$-\frac{2\,a^4x}{5}+\frac{2\,a^2x^3}{15}-\frac{2\,x^5}{25}+\frac{2\,a^5}{5}\arctan\left(\frac{x}{a}\right)+\frac{x^5\ln\left(a^2+x^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*ln(a^2+x^2),x)
```

```
[Out] -2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a
^2+x^2)
```

Maxima [A] time = 1.53296, size = 59, normalized size = 1.09

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*log(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a
^2*x^3 - 2/25*x^5
```

Fricas [A] time = 0.218425, size = 59, normalized size = 1.09

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*log(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a
^2*x^3 - 2/25*x^5
```

Sympy [A] time = 0.595494, size = 63, normalized size = 1.17

$$-2a^5\left(\frac{i\log\left(-ia+x\right)}{10}-\frac{i\log\left(ia+x\right)}{10}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{x^5\log\left(a^2+x^2\right)}{5}-\frac{2x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*ln(a**2+x**2),x)
```

```
[Out] -2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2
*a**2*x**3/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25
```

GIAC/XCAS [A] time = 0.204426, size = 59, normalized size = 1.09

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\ln\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*log(a^2 + x^2),x, algorithm="giac")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*ln(a^2 + x^2) - 2/5*a^4*x + 2/15*a^
2*x^3 - 2/25*x^5
```

3.74 $\int \log(-a^2+x^2)\,dx$

Optimal. Leaf size=25

$$x\log(x^2-a^2)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Rubi [A] time = 0.0244179, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$x\log(x^2-a^2)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In]  Int[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Rubi in Sympy [A] time = 2.81063, size = 20, normalized size = 0.8

$$2a\,\mathrm{atanh}\left(\frac{x}{a}\right)+x\log(-a^2+x^2)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(-a**2+x**2),x)
```

```
[Out] 2*a*atanh(x/a) + x*log(-a**2 + x**2) - 2*x
```

Mathematica [A] time = 0.00501925, size = 25, normalized size = 1.

$$x\log(x^2-a^2)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Maple [A] time = 0.005, size = 32, normalized size = 1.3

$$x\ln\left(-a^2+x^2\right)-2x-a\ln\left(-a+x\right)+a\ln\left(a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(-a^2+x^2),x)

[Out] x*ln(-a^2+x^2)-2*x-a*ln(-a+x)+a*ln(a+x)
```

Maxima [A] time = 1.40368, size = 42, normalized size = 1.68

$$x\log\left(-a^2+x^2\right)+a\log\left(a+x\right)-a\log\left(-a+x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(-a^2 + x^2),x, algorithm="maxima")

[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

Fricas [A] time = 0.223262, size = 42, normalized size = 1.68

$$x\log\left(-a^2+x^2\right)+a\log\left(a+x\right)-a\log\left(-a+x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(-a^2 + x^2),x, algorithm="fricas")

[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

Sympy [A] time = 0.554331, size = 29, normalized size = 1.16

$$-2a\left(\frac{\log\left(-a+x\right)}{2}-\frac{\log\left(a+x\right)}{2}\right)+x\log\left(-a^2+x^2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(-a**2+x**2),x)
```

```
[Out] -2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x
```

GIAC/XCAS [A] time = 0.200116, size = 45, normalized size = 1.8

$$x\ln\left(-a^2+x^2\right)+a\ln\left(|a+x|\right)-a\ln\left(|-a+x|\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(-a^2 + x^2),x, algorithm="giac")
```

```
[Out] x*ln(-a^2 + x^2) + a*ln(abs(a + x)) - a*ln(abs(-a + x)) - 2*x
```

3.75 $\int \log(\log(\log(\log(x)))) \, dx$

Optimal. Leaf size=8

$$\text{Int}(\log(\log(\log(\log(x)))), x)$$

```
[Out] CannotIntegrate[Log[Log[Log[Log[x]]]], x]
```

Rubi [A] time = 0.0098494, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}(\log(\log(\log(\log(x)))), x)$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[Log[Log[x]]]],x]
```

```
[Out] Defer[Int][Log[Log[Log[Log[x]]]], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(ln(ln(ln(ln(x)))),x)
```

```
[Out] Integral(log(log(log(log(x)))), x)
```

Mathematica [A] time = 0.250813, size = 0, normalized size = 0.

$$\int \log(\log(\log(\log(x)))) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[Log[Log[x]]]],x]
```

```
[Out] Integrate[Log[Log[Log[Log[x]]]], x]
```

Maple [A] time = 0.018, size = 0, normalized size = 0.

$$\int \ln\left(\ln\left(\ln\left(\ln\left(x\right)\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(ln(ln(ln(x)))),x)
```

```
[Out] int(ln(ln(ln(ln(x)))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$x\log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right) - \int \frac{1}{\log\left(x\right)\log\left(\log\left(x\right)\right)\log\left(\log\left(\log\left(x\right)\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(log(log(x)))),x, algorithm="maxima")
```

```
[Out] x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log
(log(x)))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(log(log(x)))),x, algorithm="fricas")
```

```
[Out] integral(log(log(log(log(x)))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$x\log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right) - \int \frac{1}{\log\left(x\right)\log\left(\log\left(x\right)\right)\log\left(\log\left(\log\left(x\right)\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(ln(ln(ln(x)))),x)
```

```
[Out] x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(
log(x)))), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\log \left(\log \left(\log \left(x\right)\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(log(log(x)))),x, algorithm="giac")
```

```
[Out] integrate(log(log(log(log(x)))), x)
```

3.76 $\int \sin(x)\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -Cos[x]
```

Rubi [A] time = 0.0049239, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x],x]

[Out] -Cos[x]
```

Rubi in Sympy [A] time = 0.027205, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x),x)

[Out] -cos(x)
```

Mathematica [A] time = 0.00416458, size = 4, normalized size = 1.

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x],x]

[Out] -Cos[x]
```

Maple [A] time = 0.001, size = 5, normalized size = 1.3

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x),x)

[Out] -cos(x)
```

Maxima [A] time = 1.40122, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x, algorithm="maxima")

[Out] -cos(x)
```

Fricas [A] time = 0.227302, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x, algorithm="fricas")

[Out] -cos(x)
```

Sympy [A] time = 0.03024, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x)

[Out] -cos(x)
```

GIAC/XCAS [A] time = 0.198074, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

3.77 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] Sin[x]
```

Rubi [A] time = 0.00430505, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x],x]

[Out] Sin[x]
```

Rubi in Sympy [A] time = 0.023192, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x),x)

[Out] sin(x)
```

Mathematica [A] time = 0.00407082, size = 2, normalized size = 1.

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x],x]

[Out] Sin[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x),x)

[Out] sin(x)
```

Maxima [A] time = 1.38628, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="maxima")

[Out] sin(x)
```

Fricas [A] time = 0.224468, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="fricas")

[Out] sin(x)
```

Sympy [A] time = 0.030524, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x)

[Out] sin(x)
```

GIAC/XCAS [A] time = 0.198681, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.78 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -Log[Cos[x]]
```

Rubi [A] time = 0.00440457, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x],x]

[Out] -Log[Cos[x]]
```

Rubi in Sympy [A] time = 0.034337, size = 5, normalized size = 1.

$$-\log\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x),x)

[Out] -log(cos(x))
```

Mathematica [A] time = 0.00700219, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x],x]

[Out] -Log[Cos[x]]
```

Maple [A] time = 0.001, size = 6, normalized size = 1.2

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x),x)
```

```
[Out] -ln(cos(x))
```

Maxima [A] time = 1.38163, size = 4, normalized size = 0.8

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x, algorithm="maxima")
```

```
[Out] log(sec(x))
```

Fricas [A] time = 0.231983, size = 15, normalized size = 3.

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.039367, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

GIAC/XCAS [A] time = 0.200068, size = 8, normalized size = 1.6

$$-\ln(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x, algorithm="giac")

[Out] -ln(abs(cos(x)))
```

3.79 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.00479751, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x],x]

[Out] Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.031322, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/tan(x),x)

[Out] log(sin(x))
```

Mathematica [A] time = 0.00716954, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x],x]

[Out] Log[Sin[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/tan(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 1.38813, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [A] time = 0.219127, size = 22, normalized size = 7.33

$$\frac{1}{2}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(x)^2/(tan(x)^2 + 1))
```

Sympy [A] time = 0.047514, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(x),x)
```

```
[Out] log(sin(x))
```

GIAC/XCAS [A] time = 0.202874, size = 23, normalized size = 7.67

$$-\frac{1}{2}\ln\left(\tan\left(x\right)^{2}+1\right)+\frac{1}{2}\ln\left(\tan\left(x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(x),x, algorithm="giac")
```

```
[Out] -1/2*ln(tan(x)^2 + 1) + 1/2*ln(tan(x)^2)
```

3.80 $\int \frac{1}{(1+\tan(x))^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

```
[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

Rubi [A] time = 0.0414381, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Tan[x])^(-2),x]

[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

Rubi in Sympy [A] time = 1.99797, size = 17, normalized size = 0.81

$$\frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{2}-\frac{1}{2\left(\tan\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+tan(x))**2,x)

[Out] log(sin(x) + cos(x))/2 - 1/(2*(tan(x) + 1))
```

Mathematica [A] time = 0.0430105, size = 27, normalized size = 1.29

$$\frac{\tan(x)+\log(\sin(x)+\cos(x))+\tan(x)\log(\sin(x)+\cos(x))}{2\tan(x)+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Tan[x])^(-2),x]
```

```
[Out] (Log[Cos[x] + Sin[x]] + Tan[x] + Log[Cos[x] + Sin[x]]*Tan[x])/(2
+ 2*Tan[x])
```

Maple [A] time = 0.019, size = 26, normalized size = 1.2

$$-\frac{\ln\left(1+(\tan(x))^2\right)}{4}-\frac{1}{2+2\,\tan(x)}+\frac{\ln(1+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+tan(x))^2,x)
```

```
[Out] -1/4*ln(1+tan(x)^2)-1/2/(1+tan(x))+1/2*ln(1+tan(x))
```

Maxima [A] time = 1.54612, size = 34, normalized size = 1.62

$$-\frac{1}{2(\tan(x)+1)}-\frac{1}{4}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(x) + 1)^(-2),x, algorithm="maxima")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)
```

Fricas [A] time = 0.228009, size = 50, normalized size = 2.38

$$\frac{(\tan(x)+1)\log\left(\frac{\tan(x)^2+2\,\tan(x)+1}{\tan(x)^2+1}\right)+\tan(x)-1}{4(\tan(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(x) + 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) +
 tan(x) - 1)/(tan(x) + 1)
```

Sympy [A] time = 0.652921, size = 75, normalized size = 3.57

$$\frac{2\log(\tan(x)+1)\tan(x)}{4\tan(x)+4}+\frac{2\log(\tan(x)+1)}{4\tan(x)+4}-\frac{\log\left(\tan^2(x)+1\right)\tan(x)}{4\tan(x)+4}-\frac{\log\left(\tan^2(x)+1\right)}{4\tan(x)+4}-\frac{2}{4\tan(x)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+tan(x))**2,x)
```

```
[Out] 2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*ta
n(x) + 4) - log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)
**2 + 1)/(4*tan(x) + 4) - 2/(4*tan(x) + 4)
```

GIAC/XCAS [A] time = 0.202713, size = 35, normalized size = 1.67

$$-\frac{1}{2\left(\tan\left(x\right)+1\right)}-\frac{1}{4}\ln\left(\tan\left(x\right)^{2}+1\right)+\frac{1}{2}\ln\left(\left|\tan\left(x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(x) + 1)^(-2),x, algorithm="giac")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*ln(tan(x)^2 + 1) + 1/2*ln(abs(tan(x) + 1)
)
```

3.81 $\int \sec(x)\,dx$

Optimal. Leaf size=3

$$\tanh^{-1}(\sin(x))$$

```
[Out] ArcTanh[Sin[x]]
```

Rubi [A] time = 0.00614975, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x],x]

[Out] ArcTanh[Sin[x]]
```

Rubi in Sympy [A] time = 0.027307, size = 3, normalized size = 1.

$$\operatorname{atanh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(x),x)

[Out] atanh(sin(x))
```

Mathematica [B] time = 0.00722906, size = 33, normalized size = 11.

$$\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x],x]

[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.006, size = 7, normalized size = 2.3

$$\ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(x),x)
```

```
[Out] ln(sec(x)+tan(x))
```

Maxima [A] time = 1.40877, size = 20, normalized size = 6.67

$$\frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Fricas [A] time = 0.214421, size = 23, normalized size = 7.67

$$\frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [A] time = 0.089604, size = 15, normalized size = 5.

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

GIAC/XCAS [A] time = 0.200707, size = 23, normalized size = 7.67

$$\frac{1}{2}\ln\left(\sin\left(x\right)+1\right)-\frac{1}{2}\ln\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x),x, algorithm="giac")
```

```
[Out] 1/2*ln(sin(x) + 1) - 1/2*ln(-sin(x) + 1)
```

3.82 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]
```

Rubi [A] time = 0.0045684, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x],x]

[Out] -ArcTanh[Cos[x]]
```

Rubi in Sympy [A] time = 0.029763, size = 5, normalized size = 1.

$$-\operatorname{atanh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sin(x),x)

[Out] -atanh(cos(x))
```

Mathematica [B] time = 0.00671292, size = 17, normalized size = 3.4

$$\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x],x]

[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.003, size = 9, normalized size = 1.8

$$\ln(\csc(x) - \cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sin(x),x)
```

```
[Out] ln(csc(x)-cot(x))
```

Maxima [A] time = 1.40525, size = 20, normalized size = 4.

$$-\frac{1}{2}\log(\cos(x)+1) + \frac{1}{2}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [A] time = 0.227691, size = 26, normalized size = 5.2

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [A] time = 0.091734, size = 15, normalized size = 3.

$$\frac{\log(\cos(x)-1)}{2} - \frac{\log(\cos(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

GIAC/XCAS [A] time = 0.201473, size = 23, normalized size = 4.6

$$-\frac{1}{2}\ln\left(\cos\left(x\right)+1\right)+\frac{1}{2}\ln\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x),x, algorithm="giac")
```

```
[Out] -1/2*ln(cos(x) + 1) + 1/2*ln(-cos(x) + 1)
```

3.83 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.010595, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.494602, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin{(x)}\cos{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00276625, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0.006, size = 11, normalized size = 0.8

$$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2,x)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.3966, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 0.228131, size = 14, normalized size = 1.

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.034846, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.198973, size = 14, normalized size = 1.

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.84 $\int x^3 \sin(x^2)\, dx$

Optimal. Leaf size=20

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi [A] time = 0.0282478, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sin[x^2],x]

[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi in Sympy [A] time = 1.36637, size = 15, normalized size = 0.75

$$-\frac{x^2\cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*sin(x**2),x)

[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Mathematica [A] time = 0.00461543, size = 20, normalized size = 1.

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$-\frac{x^2 \cos \left(x^2\right)}{2}+\frac{\sin \left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A] time = 1.39064, size = 22, normalized size = 1.1

$$-\frac{1}{2} x^2 \cos \left(x^2\right)+\frac{1}{2} \sin \left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A] time = 0.240209, size = 22, normalized size = 1.1

$$-\frac{1}{2} x^2 \cos \left(x^2\right)+\frac{1}{2} \sin \left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] time = 0.768155, size = 15, normalized size = 0.75

$$-\frac{x^2 \cos \left(x^2\right)}{2}+\frac{\sin \left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

GIAC/XCAS [A] time = 0.209948, size = 22, normalized size = 1.1

$$-\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.85 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0111025, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^3,x]

[Out] -Cos[x] + Cos[x]^3/3
```

Rubi in Sympy [A] time = 0.642657, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)
```

Mathematica [A] time = 0.00280369, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0.036, size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.39823, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.232486, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.040446, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)
```

GIAC/XCAS [A]   time = 0.208975, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos\left(x\right)^{3} - \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="giac")

[Out] 1/3*cos(x)^3 - cos(x)
```

3.86 $\int \sin^p(x)\,dx$

Optimal. Leaf size=44

$$\frac{\cos(x)\sin^{p+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

```
[Out] (Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Si
n[x]^(1 + p))/((1 + p)*Sqrt[Cos[x]^2])
```

Rubi [A] time = 0.0176983, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\cos(x)\sin^{p+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^p,x]
```

```
[Out] (Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Si
n[x]^(1 + p))/((1 + p)*Sqrt[Cos[x]^2])
```

Rubi in Sympy [A] time = 0.724414, size = 39, normalized size = 0.89

$$\frac{\sin^{p+1}(x)\cos(x){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{p}{2}+\frac{1}{2}\\ \frac{p}{2}+\frac{3}{2}\end{matrix}\middle|\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**p,x)
```

```
[Out] sin(x)**(p + 1)*cos(x)*hyper((1/2, p/2 + 1/2), (p/2 + 3/2,), sin(
x)**2)/((p + 1)*sqrt(cos(x)**2))
```

Mathematica [A] time = 0.236121, size = 44, normalized size = 1.

$$-\cos(x)\sin^{p+1}(x)\sin^2(x)^{\frac{1}{2}(-p-1)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3}{2},\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^p,x]
```

```
[Out] -(Cos[x]*Hypergeometric2F1[1/2, (1 - p)/2, 3/2, Cos[x]^2]*Sin[x]^
(1 + p)*(Sin[x]^2)^((-1 - p)/2))
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int (\sin (x))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^p,x)
```

```
[Out] int(sin(x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (x)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^p,x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin (x)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^p,x, algorithm="fricas")
```

```
[Out] integral(sin(x)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^p(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**p,x)
```

```
[Out] Integral(sin(x)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^p,x, algorithm="giac")
```

```
[Out] integrate(sin(x)^p, x)
```

3.87 $\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5}+\frac{2 \sin^3(x)}{3}+\sin(x)$$

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi [A] time = 0.033788, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\sin^5(x)}{5}+\frac{2 \sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi in Sympy [A] time = 2.29008, size = 17, normalized size = 0.89

$$\frac{\sin^5\left(x\right)}{5}+\frac{2 \sin^3\left(x\right)}{3}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*(1+sin(x)**2)**2,x)
```

```
[Out] sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)
```

Mathematica [A] time = 0.0055981, size = 19, normalized size = 1.

$$\frac{\sin^5(x)}{5}+\frac{2 \sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Maple [A] time = 0.009, size = 16, normalized size = 0.8

$$\sin(x) + \frac{2(\sin(x))^3}{3} + \frac{(\sin(x))^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^2)^2,x)
```

```
[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

Maxima [A] time = 1.38825, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2 + 1)^2*cos(x),x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.22883, size = 24, normalized size = 1.26

$$\frac{1}{15}\left(3\cos(x)^4 - 16\cos(x)^2 + 28\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2 + 1)^2*cos(x),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)
```

Sympy [A] time = 1.58961, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*(1+sin(x)**2)**2,x)
```

```
[Out] sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)
```

GIAC/XCAS [A] time = 0.205921, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^2 + 1)^2*cos(x),x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

3.88 $\int \cos^2(x) \, dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.011256, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.501664, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin{(x)}\cos{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00275889, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.005, size = 11, normalized size = 0.8

$$\frac{x}{2} + \frac{\cos\left(x\right)\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.36333, size = 14, normalized size = 1.

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.263716, size = 14, normalized size = 1.

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right) + \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.033964, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

<u>GIAC/XCAS</u> [A] time = 0.210773, size = 14, normalized size = 1.

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.89 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi [A] time = 0.0112266, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3,x]

[Out] Sin[x] - Sin[x]^3/3
```

Rubi in Sympy [A] time = 0.647618, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3,x)

[Out] -sin(x)**3/3 + sin(x)
```

Mathematica [A] time = 0.002924, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4} + \frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A] time = 0.019, size = 11, normalized size = 1.

$$\frac{\left(2+(\cos(x))^{2}\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 1.39839, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.248205, size = 14, normalized size = 1.27

$$\frac{1}{3}\left(\cos(x)^{2}+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.038639, size = 8, normalized size = 0.73

$$-\frac{\sin^{3}(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

GIAC/XCAS [A] time = 0.217396, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

3.90 $\int \sec^2(x)\,dx$

Optimal. Leaf size=2

$$\tan(x)$$

[Out] Tan[x]

Rubi [A] time = 0.0097854, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\tan(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2,x]

[Out] Tan[x]

Rubi in Sympy [A] time = 0.480767, size = 5, normalized size = 2.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/cos(x)**2,x)

[Out] sin(x)/cos(x)

Mathematica [A] time = 0.00260562, size = 2, normalized size = 1.

$$\tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2,x]

[Out] Tan[x]

Maple [A] time = 0.02, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(x)^2,x)
```

```
[Out] tan(x)
```

Maxima [A] time = 1.42325, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(-2),x, algorithm="maxima")
```

```
[Out] tan(x)
```

Fricas [A] time = 0.231937, size = 9, normalized size = 4.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(-2),x, algorithm="fricas")
```

```
[Out] sin(x)/cos(x)
```

Sympy [A] time = 0.032949, size = 5, normalized size = 2.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

GIAC/XCAS [A] time = 0.231017, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(-2),x, algorithm="giac")
```

```
[Out] tan(x)
```

3.91 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rubi [A] time = 0.0130863, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6
```

Rubi in Sympy [A] time = 0.965702, size = 10, normalized size = 0.67

$$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)*sin(2*x),x)

[Out] sin(x)/2 - sin(3*x)/6
```

Mathematica [A] time = 0.00750168, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Maple [A] time = 0.008, size = 7, normalized size = 0.5

$$\frac{2\ (\sin (x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)*sin(2*x),x)
```

```
[Out] 2/3*sin(x)^3
```

Maxima [A] time = 1.38784, size = 15, normalized size = 1.

$$-\frac{1}{6}\ \sin(3\,x) + \frac{1}{2}\ \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A] time = 0.237343, size = 14, normalized size = 0.93

$$-\frac{2}{3}\left(\cos(x)^2 - 1\right)\ \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.759978, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3} + \frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

GIAC/XCAS [A] time = 0.221298, size = 15, normalized size = 1.

$$-\frac{1}{6}\sin\left(3x\right)+\frac{1}{2}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

3.92 $\int x \sin(x) \, dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi [A] time = 0.0138175, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[x],x]

[Out] -(x*Cos[x]) + Sin[x]
```

Rubi in Sympy [A] time = 0.784814, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x),x)

[Out] -x*cos(x) + sin(x)
```

Mathematica [A] time = 0.0033435, size = 8, normalized size = 1.

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x],x]

[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$-x\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

Maxima [A] time = 1.36869, size = 11, normalized size = 1.38

$$-x\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

Fricas [A] time = 0.219934, size = 11, normalized size = 1.38

$$-x\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

Sympy [A] time = 0.175468, size = 7, normalized size = 0.88

$$-x\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

GIAC/XCAS [A] time = 0.210033, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

3.93 $\int x^2 \sin(x)\,dx$

Optimal. Leaf size=17

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rubi [A] time = 0.0325199, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[x],x]

[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rubi in Sympy [A] time = 1.44643, size = 17, normalized size = 1.

$$-x^2\cos(x) + 2x\sin(x) + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(x),x)

[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Mathematica [A] time = 0.0101591, size = 15, normalized size = 0.88

$$2x\sin(x) - \left(x^2 - 2\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[x],x]

[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A] time = 0.003, size = 18, normalized size = 1.1

$$2\ \cos\left(x\right)-x^{2}\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(x),x)
```

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Maxima [A] time = 1.3719, size = 20, normalized size = 1.18

$$-\left(x^{2}-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Fricas [A] time = 0.221423, size = 20, normalized size = 1.18

$$-\left(x^{2}-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A] time = 0.38429, size = 17, normalized size = 1.

$$-x^{2}\cos\left(x\right)+2x\sin\left(x\right)+2\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

GIAC/XCAS [A] time = 0.202646, size = 20, normalized size = 1.18

$$-\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.94 $\int x \sin^2(x)\, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rubi [A] time = 0.0227348, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x\sin(x)\cos(x)}{2}+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x)**2,x)
```

```
[Out] -x*sin(x)*cos(x)/2 + sin(x)**2/4 + Integral(x, x)/2
```

Mathematica [A] time = 0.00524772, size = 25, normalized size = 1.

$$\frac{x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A] time = 0.003, size = 25, normalized size = 1.

$$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)^2,x)
```

```
[Out] x*(1/2*x-1/2*cos(x)*sin(x))-1/4*x^2+1/4*sin(x)^2
```

Maxima [A] time = 1.39919, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Fricas [A] time = 0.216381, size = 26, normalized size = 1.04

$$-\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2-\frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A] time = 0.405213, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4}+\frac{x^2\cos^2(x)}{4}-\frac{x\sin(x)\cos(x)}{2}+\frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)*
*2/4
```

GIAC/XCAS [A] time = 0.199457, size = 26, normalized size = 1.04

$$\frac{1}{4}\,x^2 - \frac{1}{4}\,x\sin\left(2\,x\right) - \frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.95 $\int x^2 \sin^2(x)\, dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin
[x]^2)/2
```

Rubi [A] time = 0.0477853, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin
[x]^2)/2
```

Rubi in Sympy [A] time = 1.65059, size = 36, normalized size = 0.88

$$\frac{x^3}{6}-\frac{x^2\sin(x)\cos(x)}{2}+\frac{x\sin^2(x)}{2}-\frac{x}{4}+\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/2 - x/4 + sin(x)*cos(
x)/4
```

Mathematica [A] time = 0.03695, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(3-6x^2\right)\sin(2x)-6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[x]^2,x]

[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [A] time = 0.019, size = 37, normalized size = 0.9

$$x^{2}\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x(\cos(x))^{2}}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}-\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(x)^2,x)

[Out] x^2*(1/2*x-1/2*cos(x)*sin(x))-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/
4*x-1/3*x^3
```

Maxima [A] time = 1.34507, size = 35, normalized size = 0.85

$$\frac{1}{6}x^{3}-\frac{1}{4}x\cos(2x)-\frac{1}{8}\left(2x^{2}-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^2,x, algorithm="maxima")

[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 0.251267, size = 39, normalized size = 0.95

$$\frac{1}{6}x^{3}-\frac{1}{2}x\cos(x)^{2}-\frac{1}{4}\left(2x^{2}-1\right)\cos(x)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^2,x, algorithm="fricas")

[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A] time = 0.840045, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6}+\frac{x^3\cos^2(x)}{6}-\frac{x^2\sin(x)\cos(x)}{2}+\frac{x\sin^2(x)}{4}-\frac{x\cos^2(x)}{4}+\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*si
n(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

GIAC/XCAS [A] time = 0.201769, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3-\frac{1}{4}x\cos(2x)-\frac{1}{8}\left(2x^2-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.96 $\int x \sin^3(x) \, dx$

Optimal. Leaf size=33

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3
/9
```

Rubi [A] time = 0.0338782, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3
/9
```

Rubi in Sympy [A] time = 1.36319, size = 32, normalized size = 0.97

$$-\frac{x\sin^2(x)\cos(x)}{3}-\frac{2x\cos(x)}{3}+\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x)/3 - 2*x*cos(x)/3 + sin(x)**3/9 + 2*sin(x)/3
```

Mathematica [A] time = 0.0063939, size = 31, normalized size = 0.94

$$\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x]^3,x]

[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A] time = 0.02, size = 23, normalized size = 0.7

$$-\frac{x\left(2+(\sin(x))^2\right)\cos(x)}{3}+\frac{(\sin(x))^3}{9}+\frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)^3,x)

[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Maxima [A] time = 1.42682, size = 31, normalized size = 0.94

$$\frac{1}{12}x\cos(3x)-\frac{3}{4}x\cos(x)-\frac{1}{36}\sin(3x)+\frac{3}{4}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3,x, algorithm="maxima")

[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Fricas [A] time = 0.228395, size = 31, normalized size = 0.94

$$\frac{1}{3}x\cos(x)^3-x\cos(x)-\frac{1}{9}\left(\cos(x)^2-7\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3,x, algorithm="fricas")

[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A] time = 0.817574, size = 39, normalized size = 1.18

$$-x\sin^2(x)\cos(x)-\frac{2x\cos^3(x)}{3}+\frac{7\sin^3(x)}{9}+\frac{2\sin(x)\cos^2(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*
cos(x)**2/3
```

GIAC/XCAS [A] time = 0.201063, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.97 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$x\sin(x)+\cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0140927, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]
```

Rubi in Sympy [A] time = 0.763251, size = 7, normalized size = 1.

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(x),x)

[Out] x*sin(x) + cos(x)
```

Mathematica [A] time = 0.00354317, size = 7, normalized size = 1.

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0.005, size = 8, normalized size = 1.1

$$\cos(x) + x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 1.41925, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 0.21711, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.170127, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

GIAC/XCAS [A] time = 0.198825, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.98 $\int x^2 \cos(x)\,dx$

Optimal. Leaf size=16

$$x^2 \sin(x) - 2\sin(x) + 2x\cos(x)$$

```
[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]
```

Rubi [A] time = 0.0347383, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^2 \sin(x) - 2\sin(x) + 2x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[x],x]

[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]
```

Rubi in Sympy [A] time = 1.34024, size = 17, normalized size = 1.06

$$x^2 \sin(x) + 2x\cos(x) - 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(x),x)

[Out] x**2*sin(x) + 2*x*cos(x) - 2*sin(x)
```

Mathematica [A] time = 0.00926927, size = 14, normalized size = 0.88

$$\left(x^2-2\right)\sin(x) + 2x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[x],x]

[Out] 2*x*Cos[x] + (-2 + x^2)*Sin[x]
```

Maple [A] time = 0.004, size = 17, normalized size = 1.1

$$2\,x\cos\left(x\right)-2\,\sin\left(x\right)+x^{2}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(x),x)
```

```
[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

Maxima [A] time = 1.39582, size = 19, normalized size = 1.19

$$2\,x\cos\left(x\right)+\left(x^{2}-2\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x),x, algorithm="maxima")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Fricas [A] time = 0.213297, size = 19, normalized size = 1.19

$$2\,x\cos\left(x\right)+\left(x^{2}-2\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Sympy [A] time = 0.387365, size = 17, normalized size = 1.06

$$x^{2}\sin\left(x\right)+2x\cos\left(x\right)-2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(x),x)
```

```
[Out] x**2*sin(x) + 2*x*cos(x) - 2*sin(x)
```

GIAC/XCAS [A] time = 0.19912, size = 19, normalized size = 1.19

$$2\,x\cos\left(x\right) + \left(x^{2} - 2\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x),x, algorithm="giac")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

3.99 $\int x\cos^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0245117, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[x]^2,x]

[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x\sin(x)\cos(x)}{2}+\frac{\cos^2(x)}{4}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(x)**2,x)

[Out] x*sin(x)*cos(x)/2 + cos(x)**2/4 + Integral(x, x)/2
```

Mathematica [A] time = 0.00512229, size = 25, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[x]^2,x]
```

```
[Out] x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4
```

Maple [A] time = 0.005, size = 25, normalized size = 1.

$$x\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}-\frac{(\sin(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x)^2,x)
```

```
[Out] x*(1/2*x+1/2*cos(x)*sin(x))-1/4*x^2-1/4*sin(x)^2
```

Maxima [A] time = 1.41086, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2+\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

Fricas [A] time = 0.218863, size = 26, normalized size = 1.04

$$\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2+\frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2
```

Sympy [A] time = 0.405291, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4}+\frac{x^2\cos^2(x)}{4}+\frac{x\sin(x)\cos(x)}{2}-\frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 - sin(x)*
*2/4
```

GIAC/XCAS [A] time = 0.200048, size = 26, normalized size = 1.04

$$\frac{1}{4}\,x^{2} + \frac{1}{4}\,x \sin\left(2\,x\right) + \frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

3.100 $\int x^2 \cos^2(x)\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + x^3/6 + (x*Cos[x]^2)/2 - (Cos[x]*Sin[x])/4 + (x^2*Cos[x]*S
in[x])/2
```

Rubi [A] time = 0.0511026, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (x*Cos[x]^2)/2 - (Cos[x]*Sin[x])/4 + (x^2*Cos[x]*S
in[x])/2
```

Rubi in Sympy [A] time = 1.60591, size = 36, normalized size = 0.88

$$\frac{x^3}{6}+\frac{x^2\sin(x)\cos(x)}{2}+\frac{x\cos^2(x)}{2}-\frac{x}{4}-\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(x)**2,x)
```

```
[Out] x**3/6 + x**2*sin(x)*cos(x)/2 + x*cos(x)**2/2 - x/4 - sin(x)*cos(
x)/4
```

Mathematica [A] time = 0.0349165, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(6x^2-3\right)\sin(2x)+6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[x]^2,x]
```

```
[Out] (4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24
```

Maple [A] time = 0.016, size = 37, normalized size = 0.9

$$x^2\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)+\frac{x(\cos(x))^2}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(x)^2,x)
```

```
[Out] x^2*(1/2*x+1/2*cos(x)*sin(x))+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/
4*x-1/3*x^3
```

Maxima [A] time = 1.41139, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3+\frac{1}{4}x\cos(2x)+\frac{1}{8}\left(2x^2-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 0.245236, size = 39, normalized size = 0.95

$$\frac{1}{6}x^3+\frac{1}{2}x\cos(x)^2+\frac{1}{4}\left(2x^2-1\right)\cos(x)\sin(x)-\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x*cos(x)^2 + 1/4*(2*x^2 - 1)*cos(x)*sin(x) - 1/4*x
```

Sympy [A] time = 0.843016, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6}+\frac{x^3\cos^2(x)}{6}+\frac{x^2\sin(x)\cos(x)}{2}-\frac{x\sin^2(x)}{4}+\frac{x\cos^2(x)}{4}-\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 + x**2*sin(x)*cos(x)/2 - x*si
n(x)**2/4 + x*cos(x)**2/4 - sin(x)*cos(x)/4
```

GIAC/XCAS [A] time = 0.199854, size = 35, normalized size = 0.85

$$\frac{1}{6}\,x^3+\frac{1}{4}\,x\cos(2\,x)+\frac{1}{8}\left(2\,x^2-1\right)\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

3.101 $\int x \cos^3(x)\, dx$

Optimal. Leaf size=33

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

```
[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/
3
```

Rubi [A] time = 0.0358407, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[x]^3,x]
```

```
[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/
3
```

Rubi in Sympy [A] time = 1.33991, size = 32, normalized size = 0.97

$$\frac{x\sin(x)\cos^2(x)}{3}+\frac{2x\sin(x)}{3}+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(x)**3,x)
```

```
[Out] x*sin(x)*cos(x)**2/3 + 2*x*sin(x)/3 + cos(x)**3/9 + 2*cos(x)/3
```

Mathematica [A] time = 0.00656925, size = 31, normalized size = 0.94

$$\frac{3}{4}x\sin(x)+\frac{1}{12}x\sin(3x)+\frac{3\cos(x)}{4}+\frac{1}{36}\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[x]^3,x]
```

```
[Out] (3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12
```

Maple [A] time = 0.009, size = 23, normalized size = 0.7

$$\frac{x\left(2+(\cos(x))^2\right)\sin(x)}{3}+\frac{(\cos(x))^3}{9}+\frac{2\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x)^3,x)
```

```
[Out] 1/3*x*(2+cos(x)^2)*sin(x)+1/9*cos(x)^3+2/3*cos(x)
```

Maxima [A] time = 1.41896, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\sin(3\,x)+\frac{3}{4}\,x\sin(x)+\frac{1}{36}\,\cos(3\,x)+\frac{3}{4}\,\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

Fricas [A] time = 0.221458, size = 34, normalized size = 1.03

$$\frac{1}{9}\,\cos(x)^3+\frac{1}{3}\,\left(x\cos(x)^2+2\,x\right)\sin(x)+\frac{2}{3}\,\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*cos(x)^3 + 1/3*(x*cos(x)^2 + 2*x)*sin(x) + 2/3*cos(x)
```

Sympy [A] time = 0.81658, size = 39, normalized size = 1.18

$$\frac{2x\sin^3(x)}{3}+x\sin(x)\cos^2(x)+\frac{2\sin^2(x)\cos(x)}{3}+\frac{7\cos^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)**3,x)
```

```
[Out] 2*x*sin(x)**3/3 + x*sin(x)*cos(x)**2 + 2*sin(x)**2*cos(x)/3 + 7*c
os(x)**3/9
```

GIAC/XCAS [A] time = 0.198689, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\sin\left(3\,x\right)+\frac{3}{4}\,x\sin\left(x\right)+\frac{1}{36}\,\cos\left(3\,x\right)+\frac{3}{4}\,\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

3.102 $\int \frac{\sin(x)}{x} \, dx$

Optimal. Leaf size=2

$$\mathrm{Si}(x)$$

```
[Out] SinIntegral[x]
```

Rubi [A] time = 0.0168023, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\mathrm{Si}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/x,x]
```

```
[Out] SinIntegral[x]
```

Rubi in Sympy [A] time = 0.635571, size = 2, normalized size = 1.

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/x,x)
```

```
[Out] Si(x)
```

Mathematica [A] time = 0.00586657, size = 2, normalized size = 1.

$$\mathrm{Si}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/x,x]
```

```
[Out] SinIntegral[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$Si(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/x,x)
```

```
[Out] Si(x)
```

Maxima [A] time = 1.50223, size = 18, normalized size = 9.

$$-\frac{1}{2}i\,\mathrm{Ei}\,(i\,x)+\frac{1}{2}i\,\mathrm{Ei}\,(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x,x, algorithm="fricas")
```

```
[Out] sin_integral(x)
```

Sympy [A] time = 0.832742, size = 2, normalized size = 1.

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x,x)
```

```
[Out] Si(x)
```

GIAC/XCAS [A] time = 0.198433, size = 3, normalized size = 1.5

$$\mathrm{Si}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x,x, algorithm="giac")
```

```
[Out] Si(x)
```

3.103 $\int \frac{\cos(x)}{x}\,dx$

Optimal. Leaf size=2

$$\text{CosIntegral}(x)$$

```
[Out] CosIntegral[x]
```

Rubi [A] time = 0.0194114, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\text{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/x,x]

[Out] CosIntegral[x]
```

Rubi in Sympy [A] time = 0.641312, size = 2, normalized size = 1.

$$\text{Ci}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/x,x)

[Out] Ci(x)
```

Mathematica [A] time = 0.00520292, size = 2, normalized size = 1.

$$\text{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/x,x]

[Out] CosIntegral[x]
```

Maple [A] time = 0.004, size = 3, normalized size = 1.5

$$Ci(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/x,x)

[Out] Ci(x)
```

Maxima [A] time = 1.50075, size = 18, normalized size = 9.

$$\frac{1}{2}\operatorname{Ei}(ix)+\frac{1}{2}\operatorname{Ei}(-ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/x,x, algorithm="maxima")

[Out] 1/2*Ei(I*x) + 1/2*Ei(-I*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\operatorname{Ci}(-x)+\frac{1}{2}\operatorname{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/x,x, algorithm="fricas")

[Out] 1/2*cos_integral(-x) + 1/2*cos_integral(x)
```

Sympy [A] time = 1.83591, size = 12, normalized size = 6.

$$-\log(x)+\frac{\log(x^2)}{2}+\operatorname{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/x,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x)
```

GIAC/XCAS [A] time = 0.202163, size = 3, normalized size = 1.5

$$\mathrm{Ci}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/x,x, algorithm="giac")
```

```
[Out] Ci(x)
```

3.104 $\int \frac{\sin(x)}{x^2}\,dx$

Optimal. Leaf size=10

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

```
[Out] CosIntegral[x] - Sin[x]/x
```

Rubi [A] time = 0.03616, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/x^2,x]

[Out] CosIntegral[x] - Sin[x]/x
```

Rubi in Sympy [A] time = 1.61192, size = 7, normalized size = 0.7

$$\text{Ci}\,(x) - \frac{\sin\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/x**2,x)

[Out] Ci(x) - sin(x)/x
```

Mathematica [A] time = 0.00655581, size = 10, normalized size = 1.

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/x^2,x]
```

```
[Out] CosIntegral[x] - Sin[x]/x
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$Ci(x) - \frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/x^2,x)
```

```
[Out] Ci(x)-sin(x)/x
```

Maxima [A] time = 1.52798, size = 20, normalized size = 2.

$$\frac{1}{2}\,(-1, i\,x) + \frac{1}{2}\,(-1, -i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{x\,\mathrm{Ci}(-x) + x\,\mathrm{Ci}(x) - 2\,\sin(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos_integral(-x) + x*cos_integral(x) - 2*sin(x))/x
```

Sympy [A] time = 2.46907, size = 17, normalized size = 1.7

$$-\log(x) + \frac{\log(x^2)}{2} + \mathrm{Ci}(x) - \frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x) - sin(x)/x
```

GIAC/XCAS [A]   time = 0.198277, size = 18, normalized size = 1.8

$$\frac{x\mathrm{Ci}\left(x\right) - \sin\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/x^2,x, algorithm="giac")
```

```
[Out] (x*Ci(x) - sin(x))/x
```

3.105 $\int \frac{\sin^2(x)}{x} \, dx$

Optimal. Leaf size=15

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

```
[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Rubi [A] time = 0.0554678, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/x,x]

[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Rubi in Sympy [A] time = 2.13974, size = 10, normalized size = 0.67

$$\frac{\log(x)}{2} - \frac{\text{Ci}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x)**2/x,x)

[Out] log(x)/2 - Ci(2*x)/2
```

Mathematica [A] time = 0.00742617, size = 15, normalized size = 1.

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/x,x]
```

```
[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Maple [A] time = 0.008, size = 12, normalized size = 0.8

$$-\frac{Ci(2x)}{2}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2/x,x)

[Out] -1/2*Ci(2*x)+1/2*ln(x)
```

Maxima [A] time = 1.47929, size = 23, normalized size = 1.53

$$-\frac{1}{4}\operatorname{Ei}(2ix)-\frac{1}{4}\operatorname{Ei}(-2ix)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/x,x, algorithm="maxima")

[Out] -1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\operatorname{Ci}(2x)-\frac{1}{4}\operatorname{Ci}(-2x)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/x,x, algorithm="fricas")

[Out] -1/4*cos_integral(2*x) - 1/4*cos_integral(-2*x) + 1/2*log(x)
```

Sympy [A] time = 2.23092, size = 10, normalized size = 0.67

$$\frac{\log(x)}{2}-\frac{\operatorname{Ci}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2/x,x)
```

```
[Out] log(x)/2 - Ci(2*x)/2
```

GIAC/XCAS [A] time = 0.199283, size = 15, normalized size = 1.

$$-\frac{1}{2}\,\mathrm{Ci}\left(2\,x\right) + \frac{1}{2}\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/x,x, algorithm="giac")
```

```
[Out] -1/2*Ci(2*x) + 1/2*ln(x)
```

3.106 $\int \tan^3(x)\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rubi [A] time = 0.0104932, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^3,x]

[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rubi in Sympy [A] time = 0.493171, size = 10, normalized size = 0.83

$$\log(\cos(x))+\frac{\tan^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**3,x)

[Out] log(cos(x)) + tan(x)**2/2
```

Mathematica [A] time = 0.00474791, size = 12, normalized size = 1.

$$\frac{\sec^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

Maple [A] time = 0.002, size = 17, normalized size = 1.4

$$\frac{(\tan(x))^2}{2} - \frac{\ln\left(1 + (\tan(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^3,x)
```

```
[Out] 1/2*tan(x)^2-1/2*ln(1+tan(x)^2)
```

Maxima [A] time = 1.41401, size = 27, normalized size = 2.25

$$-\frac{1}{2\left(\sin(x)^2 - 1\right)} + \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 0.221943, size = 24, normalized size = 2.

$$\frac{1}{2}\tan(x)^2 + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.083302, size = 12, normalized size = 1.

$$\log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**3,x)
```

```
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

GIAC/XCAS [A] time = 0.19938, size = 22, normalized size = 1.83

$$\frac{1}{2}\tan(x)^2 - \frac{1}{2}\ln\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*tan(x)^2 - 1/2*ln(tan(x)^2 + 1)
```

3.107 $\int \sin(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b)
```

Rubi [A] time = 0.00890929, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x],x]

[Out] -(Cos[a + b*x]/b)
```

Rubi in Sympy [A] time = 0.613678, size = 8, normalized size = 0.73

$$-\frac{\cos\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(b*x+a),x)

[Out] -cos(a + b*x)/b
```

Mathematica [A] time = 0.00858322, size = 22, normalized size = 2.

$$\frac{\sin(a)\sin(bx)}{b}-\frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x],x]
```

```
[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

Maple [A] time = 0.002, size = 12, normalized size = 1.1

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(b*x+a),x)

[Out] -cos(b*x+a)/b
```

Maxima [A] time = 1.39067, size = 15, normalized size = 1.36

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a),x, algorithm="maxima")

[Out] -cos(b*x + a)/b
```

Fricas [A] time = 0.211339, size = 15, normalized size = 1.36

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a),x, algorithm="fricas")

[Out] -cos(b*x + a)/b
```

Sympy [A] time = 0.134514, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

GIAC/XCAS [A] time = 0.198686, size = 15, normalized size = 1.36

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)/b
```

3.108 $\int \cos(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sin(a+bx)}{b}$$

```
[Out] Sin[a + b*x]/b
```

Rubi [A] time = 0.00802069, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[a + b*x],x]

[Out] Sin[a + b*x]/b
```

Rubi in Sympy [A] time = 0.612479, size = 7, normalized size = 0.7

$$\frac{\sin(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(b*x+a),x)

[Out] sin(a + b*x)/b
```

Mathematica [B] time = 0.0086357, size = 21, normalized size = 2.1

$$\frac{\sin(a)\cos(bx)}{b} + \frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

Maple [A] time = 0.004, size = 11, normalized size = 1.1

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(b*x+a),x)
```

```
[Out] sin(b*x+a)/b
```

Maxima [A] time = 1.3898, size = 14, normalized size = 1.4

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a),x, algorithm="maxima")
```

```
[Out] sin(b*x + a)/b
```

Fricas [A] time = 0.208757, size = 14, normalized size = 1.4

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/b
```

Sympy [A] time = 0.139349, size = 12, normalized size = 1.2

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for}\ b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

GIAC/XCAS [A] time = 0.20459, size = 14, normalized size = 1.4

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a),x, algorithm="giac")
```

```
[Out] sin(b*x + a)/b
```

3.109 $\int \tan(a + bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a + bx))}{b}$$

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rubi [A] time = 0.00853043, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[a + b*x],x]

[Out] -(Log[Cos[a + b*x]]/b)
```

Rubi in Sympy [A] time = 0.634915, size = 10, normalized size = 0.83

$$-\frac{\log\left(\cos\left(a + bx\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(b*x+a),x)

[Out] -log(cos(a + b*x))/b
```

Mathematica [A] time = 0.0427468, size = 12, normalized size = 1.

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Maple [A] time = 0.001, size = 17, normalized size = 1.4

$$\frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a),x)
```

```
[Out] 1/2/b*ln(1+tan(b*x+a)^2)
```

Maxima [A] time = 1.39011, size = 15, normalized size = 1.25

$$\frac{\log(\sec(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x + a),x, algorithm="maxima")
```

```
[Out] log(sec(b*x + a))/b
```

Fricas [A] time = 0.216467, size = 24, normalized size = 2.

$$-\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x + a),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(b*x + a)^2 + 1))/b
```

Sympy [A] time = 0.136158, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\log\left(\tan^2(a+bx)+1\right)}{2b} & \text{for } b \neq 0 \\ x\tan(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a),
True))
```

GIAC/XCAS [A] time = 0.204447, size = 18, normalized size = 1.5

$$-\frac{\ln\left(\left|\cos\left(bx+a\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(b*x + a),x, algorithm="giac")
```

```
[Out] -ln(abs(cos(b*x + a)))/b
```

3.110 $\int \cot(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a+bx))}{b}$$

```
[Out] Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.00877841, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rubi in Sympy [A] time = 0.644061, size = 8, normalized size = 0.73

$$\frac{\log(\sin(a+bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/tan(b*x+a),x)
```

```
[Out] log(sin(a + b*x))/b
```

Mathematica [A] time = 0.0467217, size = 11, normalized size = 1.

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Maple [B] time = 0.003, size = 29, normalized size = 2.6

$$\frac{\ln(\tan(bx+a))}{b} - \frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/tan(b*x+a),x)
```

```
[Out] 1/b*ln(tan(b*x+a))-1/2/b*ln(1+tan(b*x+a)^2)
```

Maxima [A] time = 1.40633, size = 15, normalized size = 1.36

$$\frac{\log(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(b*x + a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

Fricas [A] time = 0.223112, size = 36, normalized size = 3.27

$$\frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(b*x + a),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b
```

Sympy [A] time = 0.613739, size = 29, normalized size = 2.64

$$\begin{cases} -\frac{\log(\tan^2(a+bx)+1)}{2b} + \frac{\log(\tan(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\tan(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b,
 Ne(b, 0)), (x/tan(a), True))
```

GIAC/XCAS [A] time = 0.218593, size = 76, normalized size = 6.91

$$\frac{\ln\left(\frac{\left|-\cos\left(bx+a\right)+1\right|}{\left|\cos\left(bx+a\right)+1\right|}\right)-2\,\ln\left(\left|-\frac{\cos\left(bx+a\right)-1}{\cos\left(bx+a\right)+1}+1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(b*x + a),x, algorithm="giac")
```

```
[Out] 1/2*(ln(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*ln(abs(
-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.111 $\int \csc(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b)
```

Rubi [A] time = 0.00855027, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[a + b*x],x]

[Out] -(ArcTanh[Cos[a + b*x]]/b)
```

Rubi in Sympy [A] time = 0.613439, size = 10, normalized size = 0.83

$$-\frac{\operatorname{atanh}(\cos(a+bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sin(b*x+a),x)

[Out] -atanh(cos(a + b*x))/b
```

Mathematica [B] time = 0.0162692, size = 38, normalized size = 3.17

$$\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[a + b*x],x]
```

```
[Out] -(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b
```

Maple [A] time = 0.004, size = 21, normalized size = 1.8

$$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sin(b*x+a),x)
```

```
[Out] 1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 1.39045, size = 35, normalized size = 2.92

$$-\frac{\log(\cos(bx+a)+1)-\log(\cos(bx+a)-1)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(b*x + a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 0.232736, size = 41, normalized size = 3.42

$$-\frac{\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(b*x + a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))
/b
```

Sympy [A] time = 0.908169, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\sin(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))
```

GIAC/XCAS [A] time = 0.212015, size = 69, normalized size = 5.75

$$\frac{\ln\left(\left|-\frac{\cos(bx+a)}{b}+\frac{1}{|b|}\right|\right)}{2\,|b|}-\frac{\ln\left(\left|-\frac{\cos(bx+a)}{b}-\frac{1}{|b|}\right|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(b*x + a),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(-cos(b*x + a)/b + 1/abs(b)))/abs(b) - 1/2*ln(abs(-cos(
b*x + a)/b - 1/abs(b)))/abs(b)
```

3.112 $\int \sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Rubi [A] time = 0.00900368, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b
```

Rubi in Sympy [A] time = 0.611393, size = 8, normalized size = 0.73

$$\frac{\operatorname{atanh}(\sin(a+bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(b*x+a),x)

[Out] atanh(sin(a + b*x))/b
```

Mathematica [B] time = 0.017952, size = 68, normalized size = 6.18

$$\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)+\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[a + b*x],x]
```

```
[Out] -(Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]]/b) + Log[Cos[a/2 +
 (b*x)/2] + Sin[a/2 + (b*x)/2]]/b
```

Maple [A] time = 0.003, size = 19, normalized size = 1.7

$$\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(b*x+a),x)
```

```
[Out] 1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.37825, size = 35, normalized size = 3.18

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(b*x + a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 0.232269, size = 38, normalized size = 3.45

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(b*x + a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [A] time = 1.03065, size = 34, normalized size = 3.09

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b} & \text{for}\ b\neq 0\\ \frac{x}{\cos\left(a\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) +
1)/b, Ne(b, 0)), (x/cos(a), True))
```

GIAC/XCAS [A] time = 0.216057, size = 38, normalized size = 3.45

$$\frac{\ln\left(|\sin\left(bx+a\right)+1|\right)-\ln\left(|\sin\left(bx+a\right)-1|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(b*x + a),x, algorithm="giac")
```

```
[Out] 1/2*(ln(abs(sin(b*x + a) + 1)) - ln(abs(sin(b*x + a) - 1)))/b
```

3.113 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0181875, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi in Sympy [A] time = 0.709956, size = 19, normalized size = 0.76

$$\frac{x}{2}-\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(b*x+a)**2,x)
```

```
[Out] x/2 - sin(a + b*x)*cos(a + b*x)/(2*b)
```

Mathematica [A] time = 0.0329464, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[a + b*x]^2,x]
```

```
[Out] -(-2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.003, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.40042, size = 32, normalized size = 1.28

$$\frac{2\,bx+2\,a-\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.215333, size = 31, normalized size = 1.24

$$\frac{bx-\cos(bx+a)\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.271641, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b\neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*
x)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

GIAC/XCAS [A] time = 0.200558, size = 24, normalized size = 0.96

$$\frac{1}{2}\,x - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

3.114 $\int \sin^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0212904, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[a + b*x]^3,x]

[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rubi in Sympy [A] time = 1.17945, size = 19, normalized size = 0.7

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(b*x+a)**3,x)

[Out] cos(a + b*x)**3/(3*b) - cos(a + b*x)/b
```

Mathematica [A] time = 0.00971692, size = 29, normalized size = 1.07

$$\frac{\cos(3(a+bx))}{12b}-\frac{3\cos(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

Maple [A] time = 0.002, size = 22, normalized size = 0.8

$$-\frac{\left(2+\left(\sin\left(bx+a\right)\right)^{2}\right)\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3,x)
```

```
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

Maxima [A] time = 1.41262, size = 30, normalized size = 1.11

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x + a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Fricas [A] time = 0.227319, size = 30, normalized size = 1.11

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x + a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Sympy [A] time = 0.611178, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2\cos^{3}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ x\sin^{3}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3
*b), Ne(b, 0)), (x*sin(a)**3, True))
```

GIAC/XCAS [A] time = 0.204047, size = 34, normalized size = 1.26

$$\frac{\cos\left(bx+a\right)^{3}}{3\,b}-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(b*x + a)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

3.115 $\int \cos^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0182586, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[a + b*x]^2,x]

[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi in Sympy [A] time = 0.712147, size = 19, normalized size = 0.76

$$\frac{x}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(b*x+a)**2,x)

[Out] x/2 + sin(a + b*x)*cos(a + b*x)/(2*b)
```

Mathematica [A] time = 0.0222123, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.004, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\cos\left(bx+a\right)\sin\left(bx+a\right)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.38177, size = 30, normalized size = 1.2

$$\frac{2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.244026, size = 30, normalized size = 1.2

$$\frac{bx+\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.263376, size = 46, normalized size = 1.84

$$\begin{cases}\frac{x\sin^{2}\left(a+bx\right)}{2}+\frac{x\cos^{2}\left(a+bx\right)}{2}+\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b} & \text{for}\ b\neq 0\\ x\cos^{2}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*
x)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

GIAC/XCAS [A] time = 0.20044, size = 24, normalized size = 0.96

$$\frac{1}{2}x + \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

3.116 $\int \cos^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0208488, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3,x]

[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rubi in Sympy [A] time = 1.13957, size = 19, normalized size = 0.73

$$-\frac{\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(b*x+a)**3,x)

[Out] -sin(a + b*x)**3/(3*b) + sin(a + b*x)/b
```

Mathematica [A] time = 0.00946222, size = 29, normalized size = 1.12

$$\frac{3\sin(a+bx)}{4b}+\frac{\sin(3(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3,x]
```

```
[Out] (3*Sin[a + b*x])/(4*b) + Sin[3*(a + b*x)]/(12*b)
```

Maple [A] time = 0.004, size = 22, normalized size = 0.9

$$\frac{\left(2+\left(\cos\left(bx+a\right)\right)^{2}\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(b*x+a)^3,x)
```

```
[Out] 1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A] time = 1.38757, size = 30, normalized size = 1.15

$$-\frac{\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Fricas [A] time = 0.223909, size = 28, normalized size = 1.08

$$\frac{\left(\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [A] time = 0.608875, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2\sin^{3}\left(a+bx\right)}{3b}+\frac{\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)}{b} & \text{for}\ b\neq 0\\ x\cos^{3}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2
/b, Ne(b, 0)), (x*cos(a)**3, True))
```

GIAC/XCAS [A] time = 0.201955, size = 30, normalized size = 1.15

$$-\frac{\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.117 $\int \sec^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tan(a+bx)}{b}$$

```
[Out] Tan[a + b*x]/b
```

Rubi [A] time = 0.0161463, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[a + b*x]^2,x]

[Out] Tan[a + b*x]/b
```

Rubi in Sympy [A] time = 0.699536, size = 14, normalized size = 1.4

$$\frac{\sin(a+bx)}{b\cos(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(b*x+a)**2,x)

[Out] sin(a + b*x)/(b*cos(a + b*x))
```

Mathematica [A] time = 0.006096, size = 10, normalized size = 1.

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Maple [A] time = 0.004, size = 11, normalized size = 1.1

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(b*x+a)^2,x)
```

```
[Out] tan(b*x+a)/b
```

Maxima [A] time = 1.38495, size = 14, normalized size = 1.4

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^(-2),x, algorithm="maxima")
```

```
[Out] tan(b*x + a)/b
```

Fricas [A] time = 0.208153, size = 24, normalized size = 2.4

$$\frac{\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^(-2),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [A] time = 2.19692, size = 58, normalized size = 5.8

$$\begin{cases} \tilde{\infty}x & \text{for } \left(a=-\frac{\pi}{2}\vee a=-bx-\frac{\pi}{2}\right)\wedge\left(a=-bx-\frac{\pi}{2}\vee b=0\right) \\ \frac{x}{\cos^2(a)} & \text{for } b=0 \\ -\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)-b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2)
 | Eq(a, -b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b
*x/2)/(b*tan(a/2 + b*x/2)**2 - b), True))
```

GIAC/XCAS [A] time = 0.199033, size = 14, normalized size = 1.4

$$\frac{\tan\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x + a)^(-2),x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

3.118 $\int \frac{1}{1+\cos(x)} \, dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x) + 1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0156868, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sin(x)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cos[x])^(-1),x]

[Out] Sin[x]/(1 + Cos[x])
```

Rubi in Sympy [A] time = 0.487695, size = 7, normalized size = 0.78

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+cos(x)),x)

[Out] sin(x)/(cos(x) + 1)
```

Mathematica [A] time = 0.00577345, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

Maple [A] time = 0.004, size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+cos(x)),x)
```

```
[Out] tan(1/2*x)
```

Maxima [A] time = 1.38645, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 0.219667, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.216179, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

GIAC/XCAS [A] time = 0.197388, size = 41, normalized size = 4.56

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.119 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.0153937, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Cos[x])^(-1),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi in Sympy [A] time = 0.504328, size = 8, normalized size = 0.67

$$-\frac{\sin(x)}{-\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(x)),x)

[Out] -sin(x)/(-cos(x) + 1)
```

Mathematica [A] time = 0.00773047, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0.007, size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 1.39632, size = 14, normalized size = 1.17

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 0.216823, size = 14, normalized size = 1.17

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.64492, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

GIAC/XCAS [A] time = 0.200905, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.120 $\int \frac{1}{1+\sin(x)} \, dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rubi [A] time = 0.0132518, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rubi in Sympy [A] time = 0.505654, size = 8, normalized size = 0.8

$$-\frac{\cos(x)}{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+sin(x)),x)
```

```
[Out] -cos(x)/(sin(x) + 1)
```

Mathematica [B] time = 0.00779831, size = 23, normalized size = 2.3

$$\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.007, size = 11, normalized size = 1.1

$$-2\,\left(1+\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+sin(x)),x)
```

```
[Out] -2/(1+tan(1/2*x))
```

Maxima [A] time = 1.35161, size = 20, normalized size = 2.

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A] time = 0.222419, size = 24, normalized size = 2.4

$$-\frac{\cos\left(x\right)-\sin\left(x\right)+1}{\cos\left(x\right)+\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A] time = 0.66411, size = 8, normalized size = 0.8

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+sin(x)),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

GIAC/XCAS [A] time = 0.201414, size = 14, normalized size = 1.4

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x) + 1),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) + 1)
```

3.121 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] Cos[x]/(1 - Sin[x])
```

Rubi [A] time = 0.015044, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sin[x])^(-1),x]

[Out] Cos[x]/(1 - Sin[x])
```

Rubi in Sympy [A] time = 0.502373, size = 7, normalized size = 0.64

$$\frac{\cos(x)}{-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-sin(x)),x)

[Out] cos(x)/(-sin(x) + 1)
```

Mathematica [B] time = 0.00889265, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.014, size = 11, normalized size = 1.

$$-2\left(-1+\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-sin(x)),x)
```

```
[Out] -2/(-1+tan(1/2*x))
```

Maxima [A] time = 1.35077, size = 20, normalized size = 1.82

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sin(x) - 1),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 0.214962, size = 23, normalized size = 2.09

$$\frac{\cos\left(x\right)+\sin\left(x\right)+1}{\cos\left(x\right)-\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sin(x) - 1),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] time = 0.682731, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-sin(x)),x)

[Out] -2/(tan(x/2) - 1)
```

GIAC/XCAS [A] time = 0.202533, size = 14, normalized size = 1.27

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sin(x) - 1),x, algorithm="giac")

[Out] -2/(tan(1/2*x) - 1)
```

3.122 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0705924, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi in Sympy [A] time = 3.27996, size = 31, normalized size = 0.78

$$\frac{2\,\text{atan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*sin(x)),x)
```

```
[Out] 2*atan((a*tan(x/2) + b)/sqrt(a**2 - b**2))/sqrt(a**2 - b**2)
```

Mathematica [A] time = 0.0465735, size = 40, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sin[x])^(-1),x]

[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A] time = 0.014, size = 39, normalized size = 1.

$$2\,{\frac {1}{\sqrt {{a}^{2}-{b}^{2}}}\arctan \left(1/2\,{\frac {2\,a\tan \left(x/2 \right) +2\,b}{\sqrt {{a}^{2}-{b}^{2}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*sin(x)),x)

[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*sin(x) + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234622, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\left(a^{3}-ab^{2}\right)\cos\left(x\right)\sin\left(x\right)+2\left(a^{2}b-b^{3}\right)\cos\left(x\right)-\left(\left(2\,a^{2}-b^{2}\right)\cos\left(x\right)^{2}-2\,ab\sin\left(x\right)-a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(x\right)^{2}-2\,ab\sin\left(x\right)-a^{2}-b^{2}}\right)}{2\sqrt{-a^{2}+b^{2}}},\ -\frac{\arctan\left(-\frac{a\sin\left(x\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(x\right)}\right)}{\sqrt{a^{2}-b^{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*sin(x) + a),x, algorithm="fricas")

[Out] [1/2*log(-(2*(a^3 - a*b^2)*cos(x)*sin(x) + 2*(a^2*b - b^3)*cos(x)
 - ((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)*sqrt(-a^2
+ b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2))/sqrt(-a^2 + b^
```

```
2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x)))/sqrt(a^2 -
b^2)]
```

Sympy [A] time = 27.0663, size = 114, normalized size = 2.85

$$\begin{cases} \tilde{\infty} \log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{2}{b\tan\left(\frac{x}{2}\right) - b} & \text{for } a = -b \\ -\frac{2}{b\tan\left(\frac{x}{2}\right) + b} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a = 0 \\ -\frac{\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2-b^2} + \frac{\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (2/(b*tan(x/2
) - b), Eq(a, -b)), (-2/(b*tan(x/2) + b), Eq(a, b)), (log(tan(x/2
))/b, Eq(a, 0)), (-sqrt(-a**2 + b**2)*log(tan(x/2) + b/a - sqrt(-
a**2 + b**2)/a)/(a**2 - b**2) + sqrt(-a**2 + b**2)*log(tan(x/2) +
 b/a + sqrt(-a**2 + b**2)/a)/(a**2 - b**2), True))
```

GIAC/XCAS [A] time = 0.201879, size = 65, normalized size = 1.62

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sign}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right) + b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*sin(x) + a),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sign(a) + arctan((a*tan(1/2*x) + b)/s
qrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

3.123 $\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

```
[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]])/Sqrt[1 -
 a^2 + b^2]
```

Rubi [A] time = 0.0997582, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]])/Sqrt[1 -
 a^2 + b^2]
```

Rubi in Sympy [A] time = 3.61321, size = 37, normalized size = 0.79

$$-\frac{2\,\mathrm{atanh}\left(\frac{b+(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] -2*atanh((b + (a - 1)*tan(x/2))/sqrt(-a**2 + b**2 + 1))/sqrt(-a**
2 + b**2 + 1)
```

Mathematica [A] time = 0.0788115, size = 44, normalized size = 0.94

$$\frac{2\tan^{-1}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1
+ a^2 - b^2]
```

Maple [A] time = 0.033, size = 43, normalized size = 0.9

$$2\,\frac{1}{\sqrt{a^2-b^2-1}}\arctan\left(1/2\,\frac{2\,(a-1)\tan(x/2)+2\,b}{\sqrt{a^2-b^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2-1)^(1/2)*arctan(1/2*(2*(a-1)*tan(1/2*x)+2*b)/(a^2-b^2-
1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*sin(x) + a + cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242274, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\,ab^3-4\,\left(ab^3-\left(a^3-a\right)b\right)\cos(x)^2-2\,\left(a^3-a\right)b-2\,\left(b^5-\left(a^2-2\right)b^3-\left(a^2-1\right)b\right)\cos(x)+2\,\left(b^4-\left(a^2-2\right)b^2-a^2+\left(a^3b^2-ab^4-a^3+a\right)\cos(x)+1\right)\sin(x)+\left(b^4+(a^2+\right.}{(b^2-1)\cos(x)^2-a^2-b^2-2\,a\cos(x)-2\,(ab+b\,\mathrm{co}}\right.}{2\,\sqrt{-a^2+b^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*sin(x) + a + cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a*b^3 - 4*(a*b^3 - (a^3 - a)*b)*cos(x)^2 - 2*(a^3 -
a)*b - 2*(b^5 - (a^2 - 2)*b^3 - (a^2 - 1)*b)*cos(x) + 2*(b^4 - (a
^2 - 2)*b^2 - a^2 + (a^3*b^2 - a*b^4 - a^3 + a)*cos(x) + 1)*sin(x
) + (b^4 + (a^2 + 3)*b^2 - (2*a^2*b^2 - b^4 - 2*a^2 + 1)*cos(x)^2
 - a^2 + 2*(a*b^2 + a)*cos(x) + 2*(a*b^3 + a*b - (b^3 - (2*a^2 -
1)*b)*cos(x))*sin(x) + 2)*sqrt(-a^2 + b^2 + 1))/((b^2 - 1)*cos(x)
^2 - a^2 - b^2 - 2*a*cos(x) - 2*(a*b + b*cos(x))*sin(x)))/sqrt(-a
^2 + b^2 + 1), arctan(-(a*b*sin(x) + b^2 + a*cos(x) + 1)*sqrt(a^2
 - b^2 - 1)/((b^3 - (a^2 - 1)*b)*cos(x) + (a^2 - b^2 - 1)*sin(x))
)/sqrt(a^2 - b^2 - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.203599, size = 81, normalized size = 1.72

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sign}\left(2\,a-2\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b-\tan\left(\frac{1}{2}\,x\right)}{\sqrt{a^2-b^2-1}}\right)\right)}{\sqrt{a^2-b^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*sin(x) + a + cos(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sign(2*a - 2) + arctan((a*tan(1/2*x)
+ b - tan(1/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)
```

3.124 $\int x^2 \sin^2(a + bx)\, dx$

Optimal. Leaf size=73

$$\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

```
[Out] -x/(4*b^2) + x^3/6 + (Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - (x^2*C
os[a + b*x]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
```

Rubi [A] time = 0.077704, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[a + b*x]^2,x]
```

```
[Out] -x/(4*b^2) + x^3/6 + (Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - (x^2*C
os[a + b*x]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
```

Rubi in Sympy [A] time = 2.62825, size = 65, normalized size = 0.89

$$\frac{x^3}{6}-\frac{x^2\sin{(a+bx)}\cos{(a+bx)}}{2b}+\frac{x\sin^2{(a+bx)}}{2b^2}-\frac{x}{4b^2}+\frac{\sin{(a+bx)}\cos{(a+bx)}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(b*x+a)**2,x)
```

```
[Out] x**3/6 - x**2*sin(a + b*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2
/(2*b**2) - x/(4*b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3)
```

Mathematica [A] time = 0.133366, size = 47, normalized size = 0.64

$$\frac{\left(3-6b^2x^2\right)\sin(2(a+bx))-6bx\cos(2(a+bx))+4b^3x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a +
b*x)])/(24*b^3)
```

Maple [B] time = 0.004, size = 158, normalized size = 2.2

$$\frac{1}{b^3}\left((bx+a)^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)(\cos(bx+a))^2}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}-\frac{(bx+}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(
b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(
b*x+a)^3-2*a*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-
1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+a^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1
/2*b*x+1/2*a))
```

Maxima [A] time = 1.40111, size = 158, normalized size = 2.16

$$\frac{4(bx+a)^3+6(2bx+2a-\sin(2bx+2a))a^2-6\left(2(bx+a)^2-2(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\right)a-6(bx+a)\mathrm{c}}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(b*x + a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2 - 6*
(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))
*a - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))/b^3
```

Fricas [A] time = 0.240634, size = 73, normalized size = 1.

$$\frac{2b^3x^3-6bx\cos(bx+a)^2-3\left(2b^2x^2-1\right)\cos(bx+a)\sin(bx+a)+3bx}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(b*x + a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*
x + a)*sin(b*x + a) + 3*b*x)/b^3
```

Sympy [A] time = 1.52803, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2{(a+bx)}}{6} + \frac{x^3\cos^2{(a+bx)}}{6} - \frac{x^2\sin{(a+bx)}\cos{(a+bx)}}{2b} + \frac{x\sin^2{(a+bx)}}{4b^2} - \frac{x\cos^2{(a+bx)}}{4b^2} + \frac{\sin{(a+bx)}\cos{(a+bx)}}{4b^3} & \text{for } b \neq 0 \\ \frac{x^3\sin^2{(a)}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2
*sin(a + b*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x
*cos(a + b*x)**2/(4*b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), N
e(b, 0)), (x**3*sin(a)**2/3, True))
```

GIAC/XCAS [A] time = 0.199986, size = 61, normalized size = 0.84

$$\frac{1}{6}\,x^3 - \frac{x\cos\left(2\,bx + 2\,a\right)}{4\,b^2} - \frac{\left(2\,b^2x^2 - 1\right)\,\sin\left(2\,bx + 2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(b*x + a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*
b*x + 2*a)/b^3
```

3.125 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Rubi [A] time = 0.0152933, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6
```

Rubi in Sympy [A] time = 0.947779, size = 10, normalized size = 0.67

$$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(2*x),x)

[Out] sin(x)/2 + sin(3*x)/6
```

Mathematica [A] time = 0.00680412, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Maple [A] time = 0.015, size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(2*x),x)
```

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

Maxima [A] time = 1.39047, size = 15, normalized size = 1.

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A] time = 0.247074, size = 16, normalized size = 1.07

$$\frac{1}{3}\left(2\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A] time = 0.748108, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3} + \frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*cos(2*x),x)
```

```
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

GIAC/XCAS [A] time = 0.198428, size = 15, normalized size = 1.

$$\frac{1}{6}\,\sin\left(3\,x\right) + \frac{1}{2}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

3.126 $\int x^2 \cos^2(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

```
[Out] -x/(4*b^2) + x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) - (Cos[a + b*x]*S
in[a + b*x])/(4*b^3) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0753563, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[a + b*x]^2,x]
```

```
[Out] -x/(4*b^2) + x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) - (Cos[a + b*x]*S
in[a + b*x])/(4*b^3) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi in Sympy [A] time = 2.71277, size = 65, normalized size = 0.89

$$\frac{x^3}{6}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{x}{4b^2}-\frac{\sin(a+bx)\cos(a+bx)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(b*x+a)**2,x)
```

```
[Out] x**3/6 + x**2*sin(a + b*x)*cos(a + b*x)/(2*b) + x*cos(a + b*x)**2
/(2*b**2) - x/(4*b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3)
```

Mathematica [A] time = 0.134142, size = 47, normalized size = 0.64

$$\frac{\left(6b^2x^2-3\right)\sin(2(a+bx))+6bx\cos(2(a+bx))+4b^3x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a +
 b*x)])/(24*b^3)
```

Maple [B] time = 0.007, size = 158, normalized size = 2.2

$$\frac{1}{b^3}\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)(\cos(bx+a))^2}{2}-\frac{\cos(bx+a)\sin(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{(bx+a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(b*x+a)^2,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b
*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b
*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/
4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a))
```

Maxima [A] time = 1.42097, size = 153, normalized size = 2.1

$$\frac{4(bx+a)^3+6(2bx+2a+\sin(2bx+2a))a^2-6\left(2(bx+a)^2+2(bx+a)\sin(2bx+2a)+\cos(2bx+2a)\right)a+6(bx+a)\text{co}}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(b*x + a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2 - 6*
(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))
*a + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))/b^3
```

Fricas [A] time = 0.229324, size = 73, normalized size = 1.

$$\frac{2b^3x^3+6bx\cos(bx+a)^2+3\left(2b^2x^2-1\right)\cos(bx+a)\sin(bx+a)-3bx}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(b*x + a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 6*b*x*cos(b*x + a)^2 + 3*(2*b^2*x^2 - 1)*cos(b*
x + a)*sin(b*x + a) - 3*b*x)/b^3
```

Sympy [A] time = 1.6014, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x\sin^2(a+bx)}{4b^2}+\frac{x\cos^2(a+bx)}{4b^2}-\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for } b\neq 0 \\ \frac{x^3\cos^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2
*sin(a + b*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x
*cos(a + b*x)**2/(4*b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), N
e(b, 0)), (x**3*cos(a)**2/3, True))
```

GIAC/XCAS [A] time = 0.20154, size = 61, normalized size = 0.84

$$\frac{1}{6}x^3+\frac{x\cos(2\,bx+2\,a)}{4\,b^2}+\frac{\left(2\,b^2x^2-1\right)\sin(2\,bx+2\,a)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(b*x + a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*
b*x + 2*a)/b^3
```

3.127 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rubi [A] time = 0.0131011, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^3,x]

[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.496474, size = 14, normalized size = 1.

$$-\log(\sin(x))-\frac{1}{2\tan^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/tan(x)**3,x)

[Out] -log(sin(x)) - 1/(2*tan(x)**2)
```

Mathematica [A] time = 0.00503621, size = 14, normalized size = 1.

$$-\frac{1}{2}\csc^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^3,x]

[Out] -Csc[x]^2/2 - Log[Sin[x]]
```

Maple [A] time = 0.004, size = 22, normalized size = 1.6

$$-\frac{1}{2\,(\tan(x))^2}-\ln(\tan(x))+\frac{\ln\left(1+(\tan(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/tan(x)^3,x)

[Out] -1/2/tan(x)^2-ln(tan(x))+1/2*ln(1+tan(x)^2)
```

Maxima [A] time = 1.36448, size = 19, normalized size = 1.36

$$-\frac{1}{2\,\sin(x)^2}-\frac{1}{2}\,\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^(-3),x, algorithm="maxima")

[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

Fricas [A] time = 0.214642, size = 42, normalized size = 3.

$$-\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^2+\tan(x)^2+1}{2\,\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^(-3),x, algorithm="fricas")

[Out] -1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x
)^2
```

Sympy [A] time = 0.085874, size = 14, normalized size = 1.

$$-\log(\sin(x))-\frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(x)**3,x)
```

```
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

GIAC/XCAS [A] time = 0.202866, size = 39, normalized size = 2.79

$$\frac{\tan(x)^2 - 1}{2 \tan(x)^2} + \frac{1}{2} \ln\left(\tan(x)^2 + 1\right) - \frac{1}{2} \ln\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^(-3),x, algorithm="giac")
```

```
[Out] 1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*ln(tan(x)^2 + 1) - 1/2*ln(tan(x
)^2)
```

3.128 $\int x^3 \tan^4(x)\,dx$

Optimal. Leaf size=104

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}+\frac{1}{3}x^3\tan^3(x)$$
$$-x^3\tan(x)-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)-\frac{1}{2}x^2\tan^2(x)+x\tan(x)+\log(\cos(x))$$

```
[Out] -x^2/2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log
[Cos[x]] + (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2
*I)*x)] + x*Tan[x] - x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^
3)/3
```

Rubi [A] time = 0.336239, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}+\frac{1}{3}x^3\tan^3(x)$$
$$-x^3\tan(x)-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)-\frac{1}{2}x^2\tan^2(x)+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Tan[x]^4,x]
```

```
[Out] -x^2/2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log
[Cos[x]] + (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2
*I)*x)] + x*Tan[x] - x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^
3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4}+\frac{x^3\tan^3(x)}{3}-x^3\tan(x)+\frac{4ix^3}{3}-4x^2\log\left(e^{2ix}+1\right)-\frac{x^2\tan^2(x)}{2}$$
$$+x\tan(x)+4ix\,\text{Li}_2\left(-e^{2ix}\right)+\log(\cos(x))-2\,\text{Li}_3\left(-e^{2ix}\right)-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*tan(x)**4,x)
```

```
[Out] x**4/4 + x**3*tan(x)**3/3 - x**3*tan(x) + 4*I*x**3/3 - 4*x**2*log
(exp(2*I*x) + 1) - x**2*tan(x)**2/2 + x*tan(x) + 4*I*x*polylog(2,
```

```
 -exp(2*I*x)) + log(cos(x)) - 2*polylog(3, -exp(2*I*x)) - Integra
l(x, x)
```

Mathematica [A] time = 0.184673, size = 101, normalized size = 0.97

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}-\frac{4}{3}x^3\tan(x)+\frac{1}{3}x^3\tan(x)\sec^2(x)-4x^2\log\left(1+e^{2ix}\right)-\frac{1}{2}x^2\sec^2(x)+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Tan[x]^4,x]
```

```
[Out] ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]]
+ (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] -
 (x^2*Sec[x]^2)/2 + x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*T
an[x])/3
```

Maple [A] time = 0.036, size = 138, normalized size = 1.3

$$\frac{x^4}{4}-\frac{\frac{2i}{3}x\left(6x^2e^{4ix}+6x^2e^{2ix}-3e^{4ix}-3ixe^{4ix}+4x^2-6e^{2ix}-3ixe^{2ix}-3\right)}{\left(1+e^{2ix}\right)^3}-2\ln\left(e^{ix}\right)+\ln\left(1+e^{2ix}\right)+\frac{8i}{3}x^3-4x^2\ln\left(1+e^{2ix}\right)+4ix\,polylog\left(2,-e^{2ix}\right)-2\,polylog\left(3,-e^{2ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*tan(x)^4,x)
```

```
[Out] 1/4*x^4-2/3*I*x*(6*x^2*exp(4*I*x)+6*x^2*exp(2*I*x)-3*exp(4*I*x)-3
*I*x*exp(4*I*x)+4*x^2-6*exp(2*I*x)-3*I*x*exp(2*I*x)-3)/(1+exp(2*I
*x))^3-2*ln(exp(I*x))+ln(1+exp(2*I*x))+8/3*I*x^3-4*x^2*ln(1+exp(2
*I*x))+4*I*x*polylog(2,-exp(2*I*x))-2*polylog(3,-exp(2*I*x))
```

Maxima [A] time = 1.68842, size = 655, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -(3*I*x^4 + (48*x^2 + 12*(4*x^2 - 1)*cos(6*x) + 36*(4*x^2 - 1)*co
s(4*x) + 36*(4*x^2 - 1)*cos(2*x) + (48*I*x^2 - 12*I)*sin(6*x) + (
144*I*x^2 - 36*I)*sin(4*x) + (144*I*x^2 - 36*I)*sin(2*x) - 12)*ar
ctan2(sin(2*x), cos(2*x) + 1) + (3*I*x^4 - 32*x^3 + 24*x)*cos(6*x
) + (9*I*x^4 - 48*x^3 - 24*I*x^2 + 48*x)*cos(4*x) + (9*I*x^4 - 48
*x^3 - 24*I*x^2 + 24*x)*cos(2*x) - (48*x*cos(6*x) + 144*x*cos(4*x
) + 144*x*cos(2*x) + 48*I*x*sin(6*x) + 144*I*x*sin(4*x) + 144*I*x
*sin(2*x) + 48*x)*dilog(-e^(2*I*x)) + (-24*I*x^2 + (-24*I*x^2 + 6
*I)*cos(6*x) + (-72*I*x^2 + 18*I)*cos(4*x) + (-72*I*x^2 + 18*I)*c
os(2*x) + 6*(4*x^2 - 1)*sin(6*x) + 18*(4*x^2 - 1)*sin(4*x) + 18*(
4*x^2 - 1)*sin(2*x) + 6*I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*
x) + 1) + (-24*I*cos(6*x) - 72*I*cos(4*x) - 72*I*cos(2*x) + 24*si
n(6*x) + 72*sin(4*x) + 72*sin(2*x) - 24*I)*polylog(3, -e^(2*I*x))
 - (3*x^4 + 32*I*x^3 - 24*I*x)*sin(6*x) - (9*x^4 + 48*I*x^3 - 24*
x^2 - 48*I*x)*sin(4*x) - (9*x^4 + 48*I*x^3 - 24*x^2 - 24*I*x)*sin
(2*x))/(-12*I*cos(6*x) - 36*I*cos(4*x) - 36*I*cos(2*x) + 12*sin(6
*x) + 36*sin(4*x) + 36*sin(2*x) - 12*I)
```

Fricas [A] time = 0.247047, size = 181, normalized size = 1.74

$$\frac{1}{3}x^3\tan(x)^3+\frac{1}{4}x^4-\frac{1}{2}x^2\tan(x)^2-\frac{1}{2}x^2+2ix\mathrm{Li}_2\left(-\frac{2i}{\tan(x)+i}+1\right)-2ix\mathrm{Li}_2\left(\frac{2i}{\tan(x)-i}+1\right)-\frac{1}{2}\left(4x^2-1\right)\log\left(\frac{2i}{\tan(x)+i}\right)-\frac{1}{2}\left(4x^2-1\right)\log\left(-\frac{2i}{\tan(x)-i}\right)-\left(x^3-x\right)\tan(x)-\mathrm{Li}_3\left(\frac{\tan(x)+i}{\tan(x)-i}\right)-\mathrm{Li}_3\left(\frac{\tan(x)-i}{\tan(x)+i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*tan(x)^3 + 1/4*x^4 - 1/2*x^2*tan(x)^2 - 1/2*x^2 + 2*I*x*d
ilog(-2*I/(tan(x) + I) + 1) - 2*I*x*dilog(2*I/(tan(x) - I) + 1) -
 1/2*(4*x^2 - 1)*log(2*I/(tan(x) + I)) - 1/2*(4*x^2 - 1)*log(-2*I
/(tan(x) - I)) - (x^3 - x)*tan(x) - polylog(3, (tan(x) + I)/(tan(
x) - I)) - polylog(3, (tan(x) - I)/(tan(x) + I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tan^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*tan(x)**4,x)
```

```
[Out] Integral(x**3*tan(x)**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \tan(x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*tan(x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^4, x)
```

3.129 $\int x^3 \tan^6(x)\, dx$

Optimal. Leaf size=153

$$-\frac{23}{5}ix\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10}\text{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{19x^2}{20}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-\frac{3}{20}x^2\tan^4(x)+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{\tan^2(x)}{20}-\frac{19}{10}x\tan(x)-2\log(\cos(x))$$

```
[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*
x)])/5 - 2*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] +
(23*PolyLog[3, -E^((2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x]
- Tan[x]^2/20 + (4*x^2*Tan[x]^2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x
]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[x]^5)/5
```

Rubi [A] time = 0.620417, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125

$$-\frac{23}{5}ix\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10}\text{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{19x^2}{20}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-\frac{3}{20}x^2\tan^4(x)+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{\tan^2(x)}{20}-\frac{19}{10}x\tan(x)-2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Tan[x]^6,x]
```

```
[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*
x)])/5 - 2*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] +
(23*PolyLog[3, -E^((2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x]
- Tan[x]^2/20 + (4*x^2*Tan[x]^2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x
]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[x]^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^4}{4}+\frac{x^3\tan^5(x)}{5}-\frac{x^3\tan^3(x)}{3}+x^3\tan(x)-\frac{23ix^3}{15}+\frac{23x^2\log\left(e^{2ix}+1\right)}{5}-\frac{3x^2\tan^4(x)}{20}+\frac{4x^2\tan^2(x)}{5}+\frac{x\tan^3(x)}{10}-\frac{19x\tan(x)}{10}-\frac{23ix\,\text{Li}_2\left(-e^{2ix}\right)}{5}-2\log(\cos(x))-\frac{\tan^2(x)}{20}+\frac{23\,\text{Li}_3\left(-e^{2ix}\right)}{10}+\frac{19\int x\,dx}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*tan(x)**6,x)
```

```
[Out] -x**4/4 + x**3*tan(x)**5/5 - x**3*tan(x)**3/3 + x**3*tan(x) - 23*
I*x**3/15 + 23*x**2*log(exp(2*I*x) + 1)/5 - 3*x**2*tan(x)**4/20 +
 4*x**2*tan(x)**2/5 + x*tan(x)**3/10 - 19*x*tan(x)/10 - 23*I*x*po
lylog(2, -exp(2*I*x))/5 - 2*log(cos(x)) - tan(x)**2/20 + 23*polyl
og(3, -exp(2*I*x))/10 + 19*Integral(x, x)/10
```

Mathematica [A] time = 0.434922, size = 133, normalized size = 0.87

$$\frac{1}{60}\left(-276ix\text{PolyLog}\left(2,-e^{2ix}\right)+138\text{PolyLog}\left(3,-e^{2ix}\right)-15x^4-92ix^3+92x^3\tan(x)+12x^3\tan(x)\sec^4(x)-44x^3\tan(x)\sec^2(x)+276x^2\log\left(1+e^{2ix}\right)-9x^2\sec^4(x)+66x^2\sec^2(x)-120x\tan(x)-3\sec^2(x)-120\log(\cos(x))+6x\tan(x)\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Tan[x]^6,x]
```

```
[Out] ((-92*I)*x^3 - 15*x^4 + 276*x^2*Log[1 + E^((2*I)*x)] - 120*Log[Co
s[x]] - (276*I)*x*PolyLog[2, -E^((2*I)*x)] + 138*PolyLog[3, -E^((
2*I)*x)] - 3*Sec[x]^2 + 66*x^2*Sec[x]^2 - 9*x^2*Sec[x]^4 - 120*x*
Tan[x] + 92*x^3*Tan[x] + 6*x*Sec[x]^2*Tan[x] - 44*x^3*Sec[x]^2*Ta
n[x] + 12*x^3*Sec[x]^4*Tan[x])/60
```

Maple [A] time = 0.043, size = 237, normalized size = 1.6

$$-\frac{x^4}{4}+\frac{\frac{i}{15}\left(-162\,ix^2e^{4\,ix}+90\,x^3e^{8\,ix}+9\,ie^{6\,ix}+3\,ie^{2\,ix}+180\,x^3e^{6\,ix}-66\,xe^{8\,ix}-66\,ix^2e^{2\,ix}+3\,ie^{8\,ix}+280\,x^3e^{4\,ix}-246\,xe^{6\,ix}-\right.}{\left(1+e^{2\,ix}\right)^5}+4\,\ln\left(e^{ix}\right)-2\,\ln\left(1+e^{2\,ix}\right)-\frac{46\,i}{15}x^3+\frac{23\,x^2\ln\left(1+e^{2\,ix}\right)}{5}-\frac{23\,i}{5}x\,polylog\left(2,-e^{2\,ix}\right)+\frac{23\,polylog\left(3,-e^{2\,ix}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*tan(x)^6,x)
```

```
[Out] -1/4*x^4+1/15*I*(-162*I*x^2*exp(4*I*x)+90*x^3*exp(8*I*x)+9*I*exp(
6*I*x)+3*I*exp(2*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)-66*I*x^2
*exp(2*I*x)+3*I*exp(8*I*x)+280*x^3*exp(4*I*x)-246*x*exp(6*I*x)-66
*I*x^2*exp(8*I*x)-162*I*x^2*exp(6*I*x)+140*x^3*exp(2*I*x)-354*x*e
xp(4*I*x)+9*I*exp(4*I*x)+46*x^3-234*x*exp(2*I*x)-60*x)/(1+exp(2*I
*x))^5+4*ln(exp(I*x))-2*ln(1+exp(2*I*x))-46/15*I*x^3+23/5*x^2*ln(
1+exp(2*I*x))-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-ex
```

```
p(2*I*x))
```

Maxima [A] time = 2.19969, size = 1034, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*tan(x)^6,x, algorithm="maxima")
```

```
[Out] (15*I*x^4 + (276*x^2 + 12*(23*x^2 - 10)*cos(10*x) + 60*(23*x^2 -
10)*cos(8*x) + 120*(23*x^2 - 10)*cos(6*x) + 120*(23*x^2 - 10)*cos
(4*x) + 60*(23*x^2 - 10)*cos(2*x) + (276*I*x^2 - 120*I)*sin(10*x)
 + (1380*I*x^2 - 600*I)*sin(8*x) + (2760*I*x^2 - 1200*I)*sin(6*x)
 + (2760*I*x^2 - 1200*I)*sin(4*x) + (1380*I*x^2 - 600*I)*sin(2*x)
 - 120)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^4 - 184*x^3 + 2
40*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x + 12*I)
*cos(8*x) + (150*I*x^4 - 1120*x^3 - 648*I*x^2 + 1416*x + 36*I)*co
s(6*x) + (150*I*x^4 - 720*x^3 - 648*I*x^2 + 984*x + 36*I)*cos(4*x
) + (75*I*x^4 - 360*x^3 - 264*I*x^2 + 264*x + 12*I)*cos(2*x) - (2
76*x*cos(10*x) + 1380*x*cos(8*x) + 2760*x*cos(6*x) + 2760*x*cos(4
*x) + 1380*x*cos(2*x) + 276*I*x*sin(10*x) + 1380*I*x*sin(8*x) + 2
760*I*x*sin(6*x) + 2760*I*x*sin(4*x) + 1380*I*x*sin(2*x) + 276*x)
*dilog(-e^(2*I*x)) + (-138*I*x^2 + (-138*I*x^2 + 60*I)*cos(10*x)
+ (-690*I*x^2 + 300*I)*cos(8*x) + (-1380*I*x^2 + 600*I)*cos(6*x)
+ (-1380*I*x^2 + 600*I)*cos(4*x) + (-690*I*x^2 + 300*I)*cos(2*x)
+ 6*(23*x^2 - 10)*sin(10*x) + 30*(23*x^2 - 10)*sin(8*x) + 60*(23*
x^2 - 10)*sin(6*x) + 60*(23*x^2 - 10)*sin(4*x) + 30*(23*x^2 - 10)
*sin(2*x) + 60*I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) +
 (-138*I*cos(10*x) - 690*I*cos(8*x) - 1380*I*cos(6*x) - 1380*I*co
s(4*x) - 690*I*cos(2*x) + 138*sin(10*x) + 690*sin(8*x) + 1380*sin
(6*x) + 1380*sin(4*x) + 690*sin(2*x) - 138*I)*polylog(3, -e^(2*I*
x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x) - (75*x^4 + 560*I*
x^3 - 264*x^2 - 936*I*x + 12)*sin(8*x) - (150*x^4 + 1120*I*x^3 -
648*x^2 - 1416*I*x + 36)*sin(6*x) - (150*x^4 + 720*I*x^3 - 648*x^
2 - 984*I*x + 36)*sin(4*x) - (75*x^4 + 360*I*x^3 - 264*x^2 - 264*
I*x + 12)*sin(2*x))/(-60*I*cos(10*x) - 300*I*cos(8*x) - 600*I*cos
(6*x) - 600*I*cos(4*x) - 300*I*cos(2*x) + 60*sin(10*x) + 300*sin(
8*x) + 600*sin(6*x) + 600*sin(4*x) + 300*sin(2*x) - 60*I)
```

Fricas [A] time = 0.262202, size = 221, normalized size = 1.44

$$\frac{1}{5}\,x^3\tan\left(x\right)^5-\frac{3}{20}\,x^2\tan\left(x\right)^4-\frac{1}{4}\,x^4-\frac{1}{30}\,\left(10\,x^3-3\,x\right)\,\tan\left(x\right)^3+\frac{1}{20}\,\left(16\,x^2-1\right)\,\tan\left(x\right)^2+\frac{19}{20}\,x^2-\frac{23}{10}i\,x\mathrm{Li}_2\left(-\frac{2i}{\tan\left(x\right)+i}+1\right)+\frac{23}{10}i\,x\mathrm{Li}_2\left(\frac{2i}{\tan\left(x\right)-i}+1\right)+\frac{1}{10}\,\left(23\,x^2-10\right)\,\log\left(\frac{2i}{\tan\left(x\right)+i}\right)+\frac{1}{10}\,\left(23\,x^2-10\right)\,\log\left(-\frac{2i}{\tan\left(x\right)-i}\right)+\frac{1}{10}\,\left(10\,x^3-19\,x\right)\,\tan\left(x\right)+\frac{23}{20}\,\mathrm{Li}_3\left(\frac{\tan\left(x\right)+i}{\tan\left(x\right)-i}\right)+\frac{23}{20}\,\mathrm{Li}_3\left(\frac{\tan\left(x\right)-i}{\tan\left(x\right)+i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3
*x)*tan(x)^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 - 23/10*I*x
*dilog(-2*I/(tan(x) + I) + 1) + 23/10*I*x*dilog(2*I/(tan(x) - I)
+ 1) + 1/10*(23*x^2 - 10)*log(2*I/(tan(x) + I)) + 1/10*(23*x^2 -
10)*log(-2*I/(tan(x) - I)) + 1/10*(10*x^3 - 19*x)*tan(x) + 23/20*
polylog(3, (tan(x) + I)/(tan(x) - I)) + 23/20*polylog(3, (tan(x)
- I)/(tan(x) + I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tan^6\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*tan(x)**6,x)
```

```
[Out] Integral(x**3*tan(x)**6, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tan\left(x\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*tan(x)^6,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^6, x)
```

3.130 $\int x \tan^2(x) \, dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rubi [A] time = 0.0228647, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[x*Tan[x]^2,x]

[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x \tan(x) + \log(\cos(x)) - \int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*tan(x)**2,x)

[Out] x*tan(x) + log(cos(x)) - Integral(x, x)
```

Mathematica [A] time = 0.00776663, size = 15, normalized size = 1.

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Maple [A] time = 0.005, size = 20, normalized size = 1.3

$$x\tan(x)-\frac{x^2}{2}-\frac{\ln\left(1+(\tan(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*tan(x)^2,x)
```

```
[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

Maxima [A] time = 1.56143, size = 144, normalized size = 9.6

$$-\frac{x^2\cos(2x)^2+x^2\sin(2x)^2+2x^2\cos(2x)+x^2-\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\log\left(\cos(2x)^2+\sin(2x)^2+2\ \mathrm{co}\right.}{2\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (c
os(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x
)^2 + 2*cos(2*x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 +
2*cos(2*x) + 1)
```

Fricas [A] time = 0.254882, size = 28, normalized size = 1.87

$$-\frac{1}{2}x^2+x\tan(x)+\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.187614, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x\tan(x) - \frac{\log\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)**2,x)
```

```
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

GIAC/XCAS [A] time = 0.208255, size = 31, normalized size = 2.07

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\ln\left(\frac{4}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*ln(4/(tan(x)^2 + 1))
```

3.131 $\int \cos(3x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Rubi [A] time = 0.0139151, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[3*x]*Sin[2*x],x]

[Out] Cos[x]/2 - Cos[5*x]/10
```

Rubi in Sympy [A] time = 1.05248, size = 10, normalized size = 0.67

$$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3*x)*sin(2*x),x)

[Out] cos(x)/2 - cos(5*x)/10
```

Mathematica [A] time = 0.00869426, size = 15, normalized size = 1.

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Maple [A] time = 0.025, size = 12, normalized size = 0.8

$$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3*x)*sin(2*x),x)
```

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

Maxima [A] time = 1.37778, size = 15, normalized size = 1.

$$-\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

Fricas [A] time = 0.224957, size = 18, normalized size = 1.2

$$-\frac{8}{5}\cos(x)^5 + 2\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 2*cos(x)^3
```

Sympy [A] time = 0.69644, size = 26, normalized size = 1.73

$$\frac{3\sin(2x)\sin(3x)}{5} + \frac{2\cos(2x)\cos(3x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*sin(2*x),x)
```

```
[Out] 3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5
```

GIAC/XCAS [A] time = 0.200464, size = 15, normalized size = 1.

$$-\frac{1}{10}\cos\left(5x\right)+\frac{1}{2}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

3.132 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0388725, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[x]^2,x]

[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi in Sympy [A] time = 1.62067, size = 20, normalized size = 0.83

$$\frac{x}{8}-\frac{\sin(x)\cos^3(x)}{4}+\frac{\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)**2,x)

[Out] x/8 - sin(x)*cos(x)**3/4 + sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00687931, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0.004, size = 19, normalized size = 0.8

$$\frac{x}{8}+\frac{\cos\left(x\right)\sin\left(x\right)}{8}-\frac{\left(\cos\left(x\right)\right)^{3}\sin\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 1.41576, size = 14, normalized size = 0.58

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 0.220733, size = 26, normalized size = 1.08

$$-\frac{1}{8}\left(2\,\cos\left(x\right)^{3}-\cos\left(x\right)\right)\,\sin\left(x\right)+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.053056, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

GIAC/XCAS [A] time = 0.198791, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.133 $\int \csc^2(x)\sec^2(x)\,dx$

Optimal. Leaf size=7

$$\tan(x) - \cot(x)$$

```
[Out] -Cot[x] + Tan[x]
```

Rubi [A] time = 0.0336683, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^2*Sec[x]^2,x]

[Out] -Cot[x] + Tan[x]
```

Rubi in Sympy [A] time = 1.5766, size = 15, normalized size = 2.14

$$\frac{2\sin(x)}{\cos(x)} - \frac{1}{\sin(x)\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(x)**2/sin(x)**2,x)

[Out] 2*sin(x)/cos(x) - 1/(sin(x)*cos(x))
```

Mathematica [A] time = 0.00503269, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^2*Sec[x]^2,x]

[Out] -2*Cot[2*x]
```

Maple [A] time = 0.02, size = 15, normalized size = 2.1

$$\frac{1}{\cos(x)\sin(x)} - 2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(x)^2/sin(x)^2,x)
```

```
[Out] 1/sin(x)/cos(x)-2*cot(x)
```

Maxima [A] time = 1.39672, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/tan(x) + tan(x)
```

Fricas [A] time = 0.230417, size = 24, normalized size = 3.43

$$-\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

Sympy [A] time = 0.054691, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x)**2/sin(x)**2,x)
```

```
[Out] -2*cos(2*x)/sin(2*x)
```

GIAC/XCAS [A] time = 0.2013, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/tan(x) + tan(x)
```

3.134 $\int d^x \sin(x)\,dx$

Optimal. Leaf size=32

$$\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\cos(x)}{\log^2(d)+1}$$

```
[Out] -((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^
2)
```

Rubi [A] time = 0.0206888, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*Sin[x],x]
```

```
[Out] -((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^
2)
```

Rubi in Sympy [A] time = 1.55313, size = 29, normalized size = 0.91

$$\frac{d^x\log(d)\sin(x)}{\log(d)^2+1}-\frac{d^x\cos(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*sin(x),x)
```

```
[Out] d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*cos(x)/(log(d)**2 + 1)
```

Mathematica [A] time = 0.0184598, size = 22, normalized size = 0.69

$$\frac{d^x(\log(d)\sin(x)-\cos(x))}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*Sin[x],x]
```

```
[Out] (d^x*(-Cos[x] + Log[d]*Sin[x]))/(1 + Log[d]^2)
```

Maple [B] time = 0.014, size = 69, normalized size = 2.2

$$1\left(\frac{e^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{e^{x\ln(d)}}{1+(\ln(d))^2}+2\,\frac{\ln(d)\,e^{x\ln(d)}\tan(x/2)}{1+(\ln(d))^2}\right)\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*exp(x*ln(d
))+2*ln(d)/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x))/(1+tan(1/2*x)^2)
```

Maxima [A] time = 1.42554, size = 34, normalized size = 1.06

$$\frac{d^x\log(d)\sin(x)-d^x\cos(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*sin(x),x, algorithm="maxima")
```

```
[Out] (d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)
```

Fricas [A] time = 0.221829, size = 30, normalized size = 0.94

$$\frac{(\log(d)\sin(x)-\cos(x))d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*sin(x),x, algorithm="fricas")
```

```
[Out] (log(d)*sin(x) - cos(x))*d^x/(log(d)^2 + 1)
```

Sympy [A] time = 1.88459, size = 104, normalized size = 3.25

$$\begin{cases} \frac{xe^{-ix}\sin(x)}{2} - \frac{ixe^{-ix}\cos(x)}{2} - \frac{e^{-ix}\cos(x)}{2} & \text{for } d = e^{-i} \\ \frac{xe^{ix}\sin(x)}{2} + \frac{ixe^{ix}\cos(x)}{2} - \frac{e^{ix}\cos(x)}{2} & \text{for } d = e^{i} \\ \frac{d^x\log(d)\sin(x)}{\log(d)^2+1} - \frac{d^x\cos(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*sin(x),x)
```

```
[Out] Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I
*x)*cos(x)/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x
)*cos(x)/2 - exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(
x)/(log(d)**2 + 1) - d**x*cos(x)/(log(d)**2 + 1), True))
```

GIAC/XCAS [A] time = 0.219074, size = 454, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*sin(x),x, algorithm="giac")
```

```
[Out] ((pi - pi*sign(d) - 2)*cos(1/2*pi*x*sign(d) - 1/2*pi*x + x)/((pi
- pi*sign(d) - 2)^2 + 4*ln(abs(d))^2) + 2*ln(abs(d))*sin(1/2*pi*x
*sign(d) - 1/2*pi*x + x)/((pi - pi*sign(d) - 2)^2 + 4*ln(abs(d))^
2))*e^(x*ln(abs(d))) - ((pi - pi*sign(d) + 2)*cos(1/2*pi*x*sign(d
) - 1/2*pi*x - x)/((pi - pi*sign(d) + 2)^2 + 4*ln(abs(d))^2) + 2*
ln(abs(d))*sin(1/2*pi*x*sign(d) - 1/2*pi*x - x)/((pi - pi*sign(d)
 + 2)^2 + 4*ln(abs(d))^2))*e^(x*ln(abs(d))) + 1/2*(2*I*e^(1/2*I*p
i*x*sign(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sign(d) + 4*ln(
abs(d)) + 4*I) + 2*I*e^(-1/2*I*pi*x*sign(d) + 1/2*I*pi*x - I*x)/(
2*I*pi - 2*I*pi*sign(d) + 4*ln(abs(d)) - 4*I))*e^(x*ln(abs(d))) +
 1/2*(-2*I*e^(1/2*I*pi*x*sign(d) - 1/2*I*pi*x - I*x)/(-2*I*pi + 2
*I*pi*sign(d) + 4*ln(abs(d)) - 4*I) - 2*I*e^(-1/2*I*pi*x*sign(d)
+ 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sign(d) + 4*ln(abs(d)) + 4*I
))*e^(x*ln(abs(d)))
```

3.135 $\int d^x \cos(x)\,dx$

Optimal. Leaf size=31

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d)\cos(x)}{\log^2(d)+1}$$

```
[Out] (d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.0179357, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d)\cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*Cos[x],x]
```

```
[Out] (d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [A] time = 1.56179, size = 29, normalized size = 0.94

$$\frac{d^x \log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x \sin(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*cos(x),x)
```

```
[Out] d**x*log(d)*cos(x)/(log(d)**2 + 1) + d**x*sin(x)/(log(d)**2 + 1)
```

Mathematica [A] time = 0.0164772, size = 20, normalized size = 0.65

$$\frac{d^x(\log(d)\cos(x)+\sin(x))}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*Log[d] + Sin[x]))/(1 + Log[d]^2)
```

Maple [B] time = 0.012, size = 71, normalized size = 2.3

$$1\left(\frac{\ln(d)\,\mathrm{e}^{x\ln(d)}}{1+(\ln(d))^2}+2\,\frac{\mathrm{e}^{x\ln(d)}\tan(x/2)}{1+(\ln(d))^2}-\frac{\ln(d)\,\mathrm{e}^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*cos(x),x)
```

```
[Out] (ln(d)/(1+ln(d)^2)*exp(x*ln(d))+2/(1+ln(d)^2)*exp(x*ln(d))*tan(1/
2*x)-ln(d)/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2)/(1+tan(1/2*x)^2
)
```

Maxima [A] time = 1.42967, size = 32, normalized size = 1.03

$$\frac{d^x\cos(x)\log(d)+d^x\sin(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*cos(x),x, algorithm="maxima")
```

```
[Out] (d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)
```

Fricas [A] time = 0.241784, size = 27, normalized size = 0.87

$$\frac{(\cos(x)\log(d)+\sin(x))d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(d) + sin(x))*d^x/(log(d)^2 + 1)
```

Sympy [A] time = 1.89035, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for } d = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for } d = e^{i} \\ \frac{d^x\log(d)\cos(x)}{\log(d)^2+1} + \frac{d^x\sin(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(
-I*x)*cos(x)/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(
I*x)*cos(x)/2 - I*exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)
*cos(x)/(log(d)**2 + 1) + d**x*sin(x)/(log(d)**2 + 1), True))
```

GIAC/XCAS [A] time = 0.214781, size = 455, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*cos(x),x, algorithm="giac")
```

```
[Out] (2*cos(1/2*pi*x*sign(d) - 1/2*pi*x + x)*ln(abs(d))/((pi - pi*sign
(d) - 2)^2 + 4*ln(abs(d))^2) - (pi - pi*sign(d) - 2)*sin(1/2*pi*x
*sign(d) - 1/2*pi*x + x)/((pi - pi*sign(d) - 2)^2 + 4*ln(abs(d))^
2))*e^(x*ln(abs(d))) + (2*cos(1/2*pi*x*sign(d) - 1/2*pi*x - x)*ln
(abs(d))/((pi - pi*sign(d) + 2)^2 + 4*ln(abs(d))^2) - (pi - pi*si
gn(d) + 2)*sin(1/2*pi*x*sign(d) - 1/2*pi*x - x)/((pi - pi*sign(d)
 + 2)^2 + 4*ln(abs(d))^2))*e^(x*ln(abs(d))) - 1/2*I*(-2*I*e^(1/2*
I*pi*x*sign(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sign(d) + 4*
ln(abs(d)) + 4*I) + 2*I*e^(-1/2*I*pi*x*sign(d) + 1/2*I*pi*x - I*x
)/(2*I*pi - 2*I*pi*sign(d) + 4*ln(abs(d)) - 4*I))*e^(x*ln(abs(d))
) - 1/2*I*(-2*I*e^(1/2*I*pi*x*sign(d) - 1/2*I*pi*x - I*x)/(-2*I*p
i + 2*I*pi*sign(d) + 4*ln(abs(d)) - 4*I) + 2*I*e^(-1/2*I*pi*x*sig
n(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sign(d) + 4*ln(abs(d))
+ 4*I))*e^(x*ln(abs(d)))
```

3.136 $\int d^x x \sin(x)\,dx$

Optimal. Leaf size=84

$$\frac{xd^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{xd^x\cos(x)}{\log^2(d)+1}+\frac{2d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}$$

```
[Out] (2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[
d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1
+ Log[d]^2)^2 + (d^x*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.0826123, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{xd^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{xd^x\cos(x)}{\log^2(d)+1}+\frac{2d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*x*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[
d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1
+ Log[d]^2)^2 + (d^x*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [A] time = 6.05047, size = 88, normalized size = 1.05

$$\frac{d^x x\log(d)\sin(x)}{\log(d)^2+1}-\frac{d^x x\cos(x)}{\log(d)^2+1}-\frac{d^x\log(d)^2\sin(x)}{\left(\log(d)^2+1\right)^2}+\frac{2d^x\log(d)\cos(x)}{\left(\log(d)^2+1\right)^2}+\frac{d^x\sin(x)}{\left(\log(d)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*x*sin(x),x)
```

```
[Out] d**x*x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*x*cos(x)/(log(d)**2 +
 1) - d**x*log(d)**2*sin(x)/(log(d)**2 + 1)**2 + 2*d**x*log(d)*co
s(x)/(log(d)**2 + 1)**2 + d**x*sin(x)/(log(d)**2 + 1)**2
```

Mathematica [A] time = 0.06012, size = 50, normalized size = 0.6

$$\frac{d^x\left(\sin(x)\left(x\log^3(d)+x\log(d)-\log^2(d)+1\right)-\cos(x)\left(x\log^2(d)-2\log(d)+x\right)\right)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*x*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log
[d]^2 + x*Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2
```

Maple [A] time = 0.016, size = 137, normalized size = 1.6

$$1\left(\frac{xe^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2+2\frac{\ln(d)\,e^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}-\frac{xe^{x\ln(d)}}{1+(\ln(d))^2}-2\frac{\ln(d)\,e^{x\ln(d)}\,(\tan(x/2))^2}{\left(1+(\ln(d))^2\right)^2}-2\frac{\left((\ln(d))^2-1\right)e^{x\ln(d)}\tan(x/\right.}{\left(1+(\ln(d))^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*x*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*x*exp(x*ln(d))*tan(1/2*x)^2+2/(1+ln(d)^2)^2*ln(d)*
exp(x*ln(d))-1/(1+ln(d)^2)*x*exp(x*ln(d))-2/(1+ln(d)^2)^2*ln(d)*e
xp(x*ln(d))*tan(1/2*x)^2-2*(ln(d)^2-1)/(1+ln(d)^2)^2*exp(x*ln(d))
*tan(1/2*x)+2*ln(d)/(1+ln(d)^2)*x*exp(x*ln(d))*tan(1/2*x))/(1+tan
(1/2*x)^2)
```

Maxima [A] time = 1.44978, size = 81, normalized size = 0.96

$$-\frac{\left(\left(\log(d)^2+1\right)x-2\,\log(d)\right)d^x\cos(x)-\left(\left(\log(d)^3+\log(d)\right)x-\log(d)^2+1\right)d^x\sin(x)}{\log(d)^4+2\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^2 + 1)*x - 2*log(d))*d^x*cos(x) - ((log(d)^3 + log(d))
*x - log(d)^2 + 1)*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

Fricas [A] time = 0.250232, size = 81, normalized size = 0.96

$$-\frac{\left(x\cos(x)\log(d)^2+x\cos(x)-2\,\cos(x)\log(d)-\left(x\log(d)^3+x\log(d)-\log(d)^2+1\right)\,\sin(x)\right)d^x}{\log(d)^4+2\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x*sin(x),x, algorithm="fricas")
```

```
[Out] -(x*cos(x)*log(d)^2 + x*cos(x) - 2*cos(x)*log(d) - (x*log(d)^3 +
x*log(d) - log(d)^2 + 1)*sin(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

Sympy [A] time = 5.81825, size = 308, normalized size = 3.67

$$\begin{cases} \frac{x^2 e^{-ix}\sin(x)}{4} - \frac{ix^2 e^{-ix}\cos(x)}{4} + \frac{ixe^{-ix}\sin(x)}{4} - \frac{xe^{-ix}\cos(x)}{4} + \frac{ie^{-ix}\cos(x)}{4} \\ \frac{x^2 e^{ix}\sin(x)}{4} + \frac{ix^2 e^{ix}\cos(x)}{4} - \frac{ixe^{ix}\sin(x)}{4} - \frac{xe^{ix}\cos(x)}{4} - \frac{ie^{ix}\cos(x)}{4} \\ \frac{d^x x\log(d)^3\sin(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x x\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\log(d)\sin(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x x\cos(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1} + \frac{2d^x\log(d)\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x s}{\log(d)^4+2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*x*sin(x),x)
```

```
[Out] Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 +
I*x*exp(-I*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x
)/4, Eq(d, exp(-I))), (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*c
os(x)/4 - I*x*exp(I*x)*sin(x)/4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x
)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)**3*sin(x)/(log(d)**4 +
 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(log(d)**4 + 2*log(d)
**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d
**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin(x
)/(log(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4
 + 2*log(d)**2 + 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1),
True))
```

GIAC/XCAS [A] time = 0.220464, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x*sin(x),x, algorithm="giac")
```

```
[Out] Done
```

3.137 $\int d^x x \cos(x)\, dx$

Optimal. Leaf size=83

$$\frac{xd^x\sin(x)}{\log^2(d)+1}-\frac{2d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\cos(x)}{\left(\log^2(d)+1\right)^2}$$

```
[Out] (d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]
^2)^2 + (d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[
x])/(1 + Log[d]^2)^2 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.076787, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{xd^x\sin(x)}{\log^2(d)+1}-\frac{2d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*x*Cos[x],x]
```

```
[Out] (d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]
^2)^2 + (d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[
x])/(1 + Log[d]^2)^2 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [A] time = 5.92063, size = 88, normalized size = 1.06

$$\frac{d^x x\log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x x\sin(x)}{\log(d)^2+1}-\frac{d^x\log(d)^2\cos(x)}{\left(\log(d)^2+1\right)^2}-\frac{2d^x\log(d)\sin(x)}{\left(\log(d)^2+1\right)^2}+\frac{d^x\cos(x)}{\left(\log(d)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*x*cos(x),x)
```

```
[Out] d**x*x*log(d)*cos(x)/(log(d)**2 + 1) + d**x*x*sin(x)/(log(d)**2 +
 1) - d**x*log(d)**2*cos(x)/(log(d)**2 + 1)**2 - 2*d**x*log(d)*si
n(x)/(log(d)**2 + 1)**2 + d**x*cos(x)/(log(d)**2 + 1)**2
```

Mathematica [A] time = 0.0541693, size = 49, normalized size = 0.59

$$\frac{d^x\left(\sin(x)\left(x\log^2(d)-2\log(d)+x\right)+\cos(x)\left(x\log^3(d)+x\log(d)-\log^2(d)+1\right)\right)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(1 + x*Log[d] - Log[d]^2 + x*Log[d]^3) + (x - 2*Log[
d] + x*Log[d]^2)*Sin[x]))/(1 + Log[d]^2)^2
```

Maple [A] time = 0.015, size = 142, normalized size = 1.7

$$1\left(\frac{\left((\ln(d))^2-1\right)\mathrm{e}^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}\left(\tan\left(\frac{x}{2}\right)\right)^2+\frac{x\ln(d)\,\mathrm{e}^{x\ln(d)}}{1+(\ln(d))^2}-\frac{\left((\ln(d))^2-1\right)\mathrm{e}^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}-4\frac{\ln(d)\,\mathrm{e}^{x\ln(d)}\tan(x/2)}{\left(1+(\ln(d))^2\right)^2}+2\frac{x\mathrm{e}^{x\ln(d)}\tan(x}{1+(\ln(d))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*x*cos(x),x)
```

```
[Out] ((ln(d)^2-1)/(1+ln(d)^2)^2*exp(x*ln(d))*tan(1/2*x)^2+ln(d)/(1+ln(
d)^2)*x*exp(x*ln(d))-(ln(d)^2-1)/(1+ln(d)^2)^2*exp(x*ln(d))-4/(1+
ln(d)^2)^2*ln(d)*exp(x*ln(d))*tan(1/2*x)+2/(1+ln(d)^2)*x*exp(x*ln
(d))*tan(1/2*x)-ln(d)/(1+ln(d)^2)*x*exp(x*ln(d))*tan(1/2*x)^2)/(1
+tan(1/2*x)^2)
```

Maxima [A] time = 1.40774, size = 78, normalized size = 0.94

$$\frac{\left(\left(\log(d)^3+\log(d)\right)x-\log(d)^2+1\right)d^x\cos(x)+\left(\left(\log(d)^2+1\right)x-2\,\log(d)\right)d^x\sin(x)}{\log(d)^4+2\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*cos(x) + ((log(d)^2 +
 1)*x - 2*log(d))*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

Fricas [A] time = 0.252959, size = 78, normalized size = 0.94

$$\frac{\left(x\cos(x)\log(d)^3+x\cos(x)\log(d)-\cos(x)\log(d)^2+\left(x\log(d)^2+x-2\,\log(d)\right)\,\sin(x)+\cos(x)\right)d^x}{\log(d)^4+2\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x*cos(x),x, algorithm="fricas")
```

```
[Out] (x*cos(x)*log(d)^3 + x*cos(x)*log(d) - cos(x)*log(d)^2 + (x*log(d
)^2 + x - 2*log(d))*sin(x) + cos(x))*d^x/(log(d)^4 + 2*log(d)^2 +
 1)
```

Sympy [A] time = 5.65339, size = 304, normalized size = 3.66

$$\begin{cases} \frac{ix^2e^{-ix}\sin(x)}{4} + \frac{x^2e^{-ix}\cos(x)}{4} + \frac{xe^{-ix}\sin(x)}{4} + \frac{ixe^{-ix}\cos(x)}{4} + \frac{e^{-ix}\cos(x)}{4} \\ -\frac{ix^2e^{ix}\sin(x)}{4} + \frac{x^2e^{ix}\cos(x)}{4} + \frac{xe^{ix}\sin(x)}{4} - \frac{ixe^{ix}\cos(x)}{4} + \frac{e^{ix}\cos(x)}{4} \\ \frac{d^x x\log(d)^3\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\log(d)\cos(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x x\sin(x)}{\log(d)^4+2\log(d)^2+1} - \frac{d^x\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1} - \frac{2d^x\log(d)\sin(x)}{\log(d)^4+2\log(d)^2+1} + \frac{d^x c}{\log(d)^4+2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*x*cos(x),x)
```

```
[Out] Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 +
x*exp(-I*x)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/
4, Eq(d, exp(-I))), (-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*co
s(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*c
os(x)/4, Eq(d, exp(I))), (d**x*x*log(d)**3*cos(x)/(log(d)**4 + 2*
log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log(d)**4 + 2*log(d)**2
 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x
*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)/(
log(d)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 +
2*log(d)**2 + 1) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), Tru
e))
```

GIAC/XCAS [A] time = 0.216181, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x*cos(x),x, algorithm="giac")
```

```
[Out] Done
```

3.138 $\int d^x x^2 \sin(x)\,dx$

Optimal. Leaf size=162

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2 d^x \cos(x)}{\log^2(d)+1}-\frac{2xd^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{4xd^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\cos(x)}{\left(\log^2(d)+1\right)^3}$$

```
[Out] (2*d^x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*Cos[x]*Log[d]^2)/(1 + Lo
g[d]^2)^3 + (4*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^2*C
os[x])/(1 + Log[d]^2) - (6*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 +
(2*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*x*Sin[x])/(1 +
Log[d]^2)^2 - (2*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d^x*x
^2*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.282436, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2 d^x \cos(x)}{\log^2(d)+1}-\frac{2xd^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{4xd^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\cos(x)}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*x^2*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*Cos[x]*Log[d]^2)/(1 + Lo
g[d]^2)^3 + (4*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^2*C
os[x])/(1 + Log[d]^2) - (6*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 +
(2*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*x*Sin[x])/(1 +
Log[d]^2)^2 - (2*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d^x*x
^2*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^x x^2 \log(d)\sin(x)}{\log(d)^2+1}-\frac{d^x x^2\cos(x)}{\log(d)^2+1}-2\int x\left(\frac{d^x\log(d)\sin(x)}{\log(d)^2+1}-\frac{d^x\cos(x)}{\log(d)^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*x**2*sin(x),x)
```

```
[Out] d**x*x**2*log(d)*sin(x)/(log(d)**2 + 1) - d**x*x**2*cos(x)/(log(d
)**2 + 1) - 2*Integral(x*(d**x*log(d)*sin(x)/(log(d)**2 + 1) - d*
*x*cos(x)/(log(d)**2 + 1)), x)
```

Mathematica [A] time = 0.110228, size = 94, normalized size = 0.58

$$\frac{d^x\left(\sin(x)\left(x^2\log^5(d)+2\left(x^2+1\right)\log^3(d)+\left(x^2-6\right)\log(d)-2x\log^4(d)+2x\right)-\cos(x)\left(x^2\log^4(d)+2\left(x^2+3\right)\log^2(d)-4x\right.\right.}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*x^2*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*
x*Log[d]^3 + x^2*Log[d]^4)) + (2*x + (-6 + x^2)*Log[d] + 2*(1 + x
^2)*Log[d]^3 - 2*x*Log[d]^4 + x^2*Log[d]^5)*Sin[x]))/(1 + Log[d]^
2)^3
```

Maple [A] time = 0.024, size = 225, normalized size = 1.4

$$1\left(\frac{x^2e^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{x^2e^{x\ln(d)}}{1+(\ln(d))^2}-2\frac{\left(3(\ln(d))^2-1\right)e^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^3}+4\frac{x\ln(d)e^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}+2\frac{\left(3(\ln(d))^2-1\right)e^{x\ln(d)}(\tan(}{\left(1+(\ln(d))^2\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*x^2*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*x^2*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*x^2*ex
p(x*ln(d))-2*(3*ln(d)^2-1)/(1+ln(d)^2)^3*exp(x*ln(d))+4/(1+ln(d)^
2)^2*ln(d)*x*exp(x*ln(d))+2*(3*ln(d)^2-1)/(1+ln(d)^2)^3*exp(x*ln(
d))*tan(1/2*x)^2-4/(1+ln(d)^2)^2*ln(d)*x*exp(x*ln(d))*tan(1/2*x)^
2-4*(ln(d)^2-1)/(1+ln(d)^2)^2*x*exp(x*ln(d))*tan(1/2*x)+2*ln(d)/(
1+ln(d)^2)*x^2*exp(x*ln(d))*tan(1/2*x)+4*ln(d)*(ln(d)^2-3)/(1+ln(
d)^2)^3*exp(x*ln(d))*tan(1/2*x))/(1+tan(1/2*x)^2)
```

Maxima [A] time = 1.40864, size = 144, normalized size = 0.89

$$-\frac{\left(\left(\log(d)^4+2\log(d)^2+1\right)x^2-4\left(\log(d)^3+\log(d)\right)x+6\log(d)^2-2\right)d^x\cos(x)-\left(\left(\log(d)^5+2\log(d)^3+\log(d)\right)x^2+2\right.}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*
log(d)^2 - 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2
+ 2*log(d)^3 - 2*(log(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)
^6 + 3*log(d)^4 + 3*log(d)^2 + 1)
```

Fricas [A] time = 0.242909, size = 155, normalized size = 0.96

$$-\frac{\left(x^2\cos\left(x\right)\log\left(d\right)^4-4\,x\cos\left(x\right)\log\left(d\right)^3+2\,\left(x^2+3\right)\,\cos\left(x\right)\log\left(d\right)^2-4\,x\cos\left(x\right)\log\left(d\right)+\left(x^2-2\right)\,\cos\left(x\right)-\left(x^2\log\left(d\right)^5-\right.}{\log\left(d\right)^6+3\,\log\left(d\right)^4+3\,\log\left(d\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2*cos(x)*log(d)^4 - 4*x*cos(x)*log(d)^3 + 2*(x^2 + 3)*cos(x)*
log(d)^2 - 4*x*cos(x)*log(d) + (x^2 - 2)*cos(x) - (x^2*log(d)^5 -
 2*x*log(d)^4 + 2*(x^2 + 1)*log(d)^3 + (x^2 - 6)*log(d) + 2*x)*si
n(x))*d^x/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)
```

Sympy [A] time = 16.6294, size = 665, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*x**2*sin(x),x)
```

```
[Out] Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 +
I*x**2*exp(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)
*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, ex
p(-I))), (x**3*exp(I*x)*sin(x)/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x
**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x
)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))),
(d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**
2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(
d)**4 + 3*log(d)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**
6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log
(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - d**x*x**2*cos(x)/(log(d
)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x*log(d)**4*sin(x)
/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*log(d)**3
*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*lo
g(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*
x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log
```

```
(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**
x*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) -
6*d**x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1)
+ 2*d**x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1), True
))
```

GIAC/XCAS [A] time = 0.229414, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^2*sin(x),x, algorithm="giac")
```

```
[Out] Done
```

3.139 $\int d^x x^2 \cos(x)\,dx$

Optimal. Leaf size=161

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1}+\frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x \sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x \cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x \log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^3}$$

```
[Out] (-6*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (2*d^x*Cos[x]*Log[d]^3)
/(1 + Log[d]^2)^3 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^2 - (2*d^x*x*
Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d])/(1 +
Log[d]^2) - (2*d^x*Sin[x])/(1 + Log[d]^2)^3 + (6*d^x*Log[d]^2*Sin
[x])/(1 + Log[d]^2)^3 - (4*d^x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^2
+ (d^x*x^2*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.271059, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1}+\frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x \sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x \cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x \log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*x^2*Cos[x],x]
```

```
[Out] (-6*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (2*d^x*Cos[x]*Log[d]^3)
/(1 + Log[d]^2)^3 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^2 - (2*d^x*x*
Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d])/(1 +
Log[d]^2) - (2*d^x*Sin[x])/(1 + Log[d]^2)^3 + (6*d^x*Log[d]^2*Sin
[x])/(1 + Log[d]^2)^3 - (4*d^x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^2
+ (d^x*x^2*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^x x^2 \log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x x^2 \sin(x)}{\log(d)^2+1}-2\int x\left(\frac{d^x \log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x \sin(x)}{\log(d)^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*x**2*cos(x),x)
```

```
[Out] d**x*x**2*log(d)*cos(x)/(log(d)**2 + 1) + d**x*x**2*sin(x)/(log(d
)**2 + 1) - 2*Integral(x*(d**x*log(d)*cos(x)/(log(d)**2 + 1) + d*
*x*sin(x)/(log(d)**2 + 1)), x)
```

Mathematica [A] time = 0.0932392, size = 93, normalized size = 0.58

$$\frac{d^x\left(\sin(x)\left(x^2\log^4(d)+2\left(x^2+3\right)\log^2(d)-4x\log^3(d)-4x\log(d)+x^2-2\right)+\cos(x)\left(x^2\log^5(d)+2\left(x^2+1\right)\log^3(d)+\left(x^2\right.\right.\right.}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*x^2*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*
x*Log[d]^4 + x^2*Log[d]^5) + (-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)
*Log[d]^2 - 4*x*Log[d]^3 + x^2*Log[d]^4)*Sin[x]))/(1 + Log[d]^2)^
3
```

Maple [A] time = 0.023, size = 231, normalized size = 1.4

$$1\left(\frac{\ln(d)\,x^2e^{x\ln(d)}}{1+(\ln(d))^2}+2\,\frac{x^2e^{x\ln(d)}\tan(x/2)}{1+(\ln(d))^2}-2\,\frac{\left((\ln(d))^2-1\right)xe^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}+4\,\frac{\left(3\,(\ln(d))^2-1\right)e^{x\ln(d)}\tan(x/2)}{\left(1+(\ln(d))^2\right)^3}+2\,\frac{\ln(d)\left((\ln(d)\right.}{\left(1+(\ln\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*x^2*cos(x),x)
```

```
[Out] (ln(d)/(1+ln(d)^2)*x^2*exp(x*ln(d))+2/(1+ln(d)^2)*x^2*exp(x*ln(d)
)*tan(1/2*x)-2*(ln(d)^2-1)/(1+ln(d)^2)^2*x*exp(x*ln(d))+4*(3*ln(d
)^2-1)/(1+ln(d)^2)^3*exp(x*ln(d))*tan(1/2*x)+2*ln(d)*(ln(d)^2-3)/
(1+ln(d)^2)^3*exp(x*ln(d))-8/(1+ln(d)^2)^2*ln(d)*x*exp(x*ln(d))*t
an(1/2*x)+2*(ln(d)^2-1)/(1+ln(d)^2)^2*x*exp(x*ln(d))*tan(1/2*x)^2
-ln(d)/(1+ln(d)^2)*x^2*exp(x*ln(d))*tan(1/2*x)^2-2*ln(d)*(ln(d)^2
-3)/(1+ln(d)^2)^3*exp(x*ln(d))*tan(1/2*x)^2)/(1+tan(1/2*x)^2)
```

Maxima [A] time = 1.3895, size = 142, normalized size = 0.88

$$\frac{\left(\left(\log(d)^5+2\,\log(d)^3+\log(d)\right)x^2+2\,\log(d)^3-2\left(\log(d)^4-1\right)x-6\,\log(d)\right)d^x\cos(x)+\left(\left(\log(d)^4+2\,\log(d)^2+1\right)x^2-\right.}{\log(d)^6+3\,\log(d)^4+3\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^2*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4
 - 1)*x - 6*log(d))*d^x*cos(x) + ((log(d)^4 + 2*log(d)^2 + 1)*x^2
 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2 - 2)*d^x*sin(x))/(log(d)^
6 + 3*log(d)^4 + 3*log(d)^2 + 1)
```

Fricas [A] time = 0.223876, size = 150, normalized size = 0.93

$$\frac{\left(x^2 \cos(x) \log(d)^5 - 2\,x \cos(x) \log(d)^4 + 2\,\left(x^2+1\right)\,\cos(x) \log(d)^3 + \left(x^2-6\right)\,\cos(x) \log(d) + 2\,x \cos(x) + \left(x^2 \log(d)^4 - 4\right.}{\log(d)^6 + 3\,\log(d)^4 + 3\,\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^2*cos(x),x, algorithm="fricas")
```

```
[Out] (x^2*cos(x)*log(d)^5 - 2*x*cos(x)*log(d)^4 + 2*(x^2 + 1)*cos(x)*l
og(d)^3 + (x^2 - 6)*cos(x)*log(d) + 2*x*cos(x) + (x^2*log(d)^4 -
4*x*log(d)^3 + 2*(x^2 + 3)*log(d)^2 + x^2 - 4*x*log(d) - 2)*sin(x
))*d^x/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)
```

Sympy [A] time = 16.6334, size = 668, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*x**2*cos(x),x)
```

```
[Out] Piecewise((I*x**3*exp(-I*x)*sin(x)/6 + x**3*exp(-I*x)*cos(x)/6 +
x**2*exp(-I*x)*sin(x)/4 + I*x**2*exp(-I*x)*cos(x)/4 - I*x*exp(-I*
x)*sin(x)/4 + x*exp(-I*x)*cos(x)/4 - I*exp(-I*x)*cos(x)/4, Eq(d,
exp(-I))), (-I*x**3*exp(I*x)*sin(x)/6 + x**3*exp(I*x)*cos(x)/6 +
x**2*exp(I*x)*sin(x)/4 - I*x**2*exp(I*x)*cos(x)/4 + I*x*exp(I*x)*
sin(x)/4 + x*exp(I*x)*cos(x)/4 + I*exp(I*x)*cos(x)/4, Eq(d, exp(I
))), (d**x*x**2*log(d)**5*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log
(d)**2 + 1) + d**x*x**2*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4
 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*cos(x)/(log(d)**6 + 3
*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**2*sin(x)/(log
(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)*cos(x)
/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*sin(x)/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x*log(d)**4*c
os(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*x*log(
d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x
*x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*
d**x*x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**
```

```
x*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) +
6*d**x*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 +
1) - 6*d**x*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
+ 1) - 2*d**x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1),
 True))
```

GIAC/XCAS [A] time = 0.225789, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^2*cos(x),x, algorithm="giac")
```

```
[Out] Done
```

3.140 $\int d^x x^3 \sin(x)\,dx$

Optimal. Leaf size=261

$$\frac{x^3 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3 d^x \cos(x)}{\log^2(d)+1}+\frac{3x^2 d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6x^2 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{18x d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{36 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x \sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x \log^4(d)\sin(x)}{\left(\log^2(d)+1\right)^4}+\frac{6x d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{18x d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{6x d^x \cos(x)}{\left(\log^2(d)+1\right)^3}-\frac{24 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^4}+\frac{24 d^x \log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^4}$$

```
[Out] (-24*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (24*d^x*Cos[x]*Log[d]^
3)/(1 + Log[d]^2)^4 + (6*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (18*d^x
*x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3 + (6*d^x*x^2*Cos[x]*Log[d])/
(1 + Log[d]^2)^2 - (d^x*x^3*Cos[x])/(1 + Log[d]^2) - (6*d^x*Sin[x
])/(1 + Log[d]^2)^4 + (36*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^4 -
 (6*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (18*d^x*x*Log[d]*Sin[
x])/(1 + Log[d]^2)^3 + (6*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3
 + (3*d^x*x^2*Sin[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Log[d]^2*Sin[
x])/(1 + Log[d]^2)^2 + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.679133, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$

$$\frac{x^3 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3 d^x \cos(x)}{\log^2(d)+1}+\frac{3x^2 d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6x^2 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{18x d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{36 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x \sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x \log^4(d)\sin(x)}{\left(\log^2(d)+1\right)^4}+\frac{6x d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{18x d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{6x d^x \cos(x)}{\left(\log^2(d)+1\right)^3}-\frac{24 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^4}+\frac{24 d^x \log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*x^3*Sin[x],x]
```

```
[Out] (-24*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (24*d^x*Cos[x]*Log[d]^
3)/(1 + Log[d]^2)^4 + (6*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (18*d^x
*x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3 + (6*d^x*x^2*Cos[x]*Log[d])/
(1 + Log[d]^2)^2 - (d^x*x^3*Cos[x])/(1 + Log[d]^2) - (6*d^x*Sin[x
```

```
])/(1 + Log[d]^2)^4 + (36*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^4 -
 (6*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (18*d^x*x*Log[d]*Sin[
x])/(1 + Log[d]^2)^3 + (6*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3
 + (3*d^x*x^2*Sin[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Log[d]^2*Sin[
x])/(1 + Log[d]^2)^2 + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^x x^3 \log(d)\sin(x)}{\log(d)^2+1} - \frac{d^x x^3 \cos(x)}{\log(d)^2+1} - 3\int x^2 \left(\frac{d^x \log(d)\sin(x)}{\log(d)^2+1} - \frac{d^x \cos(x)}{\log(d)^2+1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*x**3*sin(x),x)
```

```
[Out] d**x*x**3*log(d)*sin(x)/(log(d)**2 + 1) - d**x*x**3*cos(x)/(log(d
)**2 + 1) - 3*Integral(x**2*(d**x*log(d)*sin(x)/(log(d)**2 + 1) -
 d**x*cos(x)/(log(d)**2 + 1)), x)
```

Mathematica [A] time = 0.210981, size = 169, normalized size = 0.65

$$\frac{d^x \left(\sin(x)\left(x^3\log^7(d)-3x^2\log^6(d)+3x\left(x^2+2\right)\log^5(d)-3\left(x^2+2\right)\log^4(d)+3x\left(x^2-4\right)\log^3(d)+3\left(x^2+12\right)\log^2(d)+x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*x^3*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2
)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x
^2*Log[d]^5 + x^3*Log[d]^6)) + (3*(-2 + x^2) + x*(-18 + x^2)*Log[
d] + 3*(12 + x^2)*Log[d]^2 + 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2
)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*x^2*Log[d]^6 + x^3*Log[d]
^7)*Sin[x]))/(1 + Log[d]^2)^4
```

Maple [A] time = 0.031, size = 437, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*x^3*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*x^3*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*x^3*ex
p(x*ln(d))+6*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))-6*(ln(d
)^2-1)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)-6*(3*ln(
d)^2-1)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))-12*(ln(d
)^4-6*ln(d)^2+1)/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)/(1+ln(d)^2)*exp(
x*ln(d))*tan(1/2*x)+24/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*ln(d)*(ln(
d)^2-1)/(1+ln(d)^2)*exp(x*ln(d))+2*ln(d)/(1+ln(d)^2)*x^3*exp(x*ln
(d))*tan(1/2*x)-6*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))*ta
n(1/2*x)^2+6*(3*ln(d)^2-1)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*ex
p(x*ln(d))*tan(1/2*x)^2-24/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*ln(d)*
(ln(d)^2-1)/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2+12*ln(d)*(ln(d)
^2-3)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)
)/(1+tan(1/2*x)^2)
```

Maxima [A] time = 1.48924, size = 251, normalized size = 0.96

$$-\frac{\left(\left(\log\left(d\right)^{6} + 3\,\log\left(d\right)^{4} + 3\,\log\left(d\right)^{2} + 1\right)x^{3} - 6\,\left(\log\left(d\right)^{5} + 2\,\log\left(d\right)^{3} + \log\left(d\right)\right)x^{2} - 24\,\log\left(d\right)^{3} + 6\,\left(3\,\log\left(d\right)^{4} + 2\,\log\left(d\right)^{2} - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2
*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^
2 - 1)*x + 24*log(d))*d^x*cos(x) - ((log(d)^7 + 3*log(d)^5 + 3*lo
g(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(d)^6 + log(d)^4 - log(
d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))*x + 36*log(d
)^2 - 6)*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(
d)^2 + 1)
```

Fricas [A] time = 0.249742, size = 274, normalized size = 1.05

$$-\frac{\left(x^{3}\cos\left(x\right)\log\left(d\right)^{6} - 6\,x^{2}\cos\left(x\right)\log\left(d\right)^{5} + 3\,\left(x^{3} + 6\,x\right)\,\cos\left(x\right)\log\left(d\right)^{4} - 12\,\left(x^{2} + 2\right)\,\cos\left(x\right)\log\left(d\right)^{3} + 3\,\left(x^{3} + 4\,x\right)\,\cos\left(x\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3*cos(x)*log(d)^6 - 6*x^2*cos(x)*log(d)^5 + 3*(x^3 + 6*x)*cos
(x)*log(d)^4 - 12*(x^2 + 2)*cos(x)*log(d)^3 + 3*(x^3 + 4*x)*cos(x
)*log(d)^2 - 6*(x^2 - 4)*cos(x)*log(d) + (x^3 - 6*x)*cos(x) - (x^
3*log(d)^7 - 3*x^2*log(d)^6 + 3*(x^3 + 2*x)*log(d)^5 - 3*(x^2 + 2
)*log(d)^4 + 3*(x^3 - 4*x)*log(d)^3 + 3*(x^2 + 12)*log(d)^2 + 3*x
^2 + (x^3 - 18*x)*log(d) - 6)*sin(x))*d^x/(log(d)^8 + 4*log(d)^6
```

```
+ 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [A] time = 45.4307, size = 1355, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*x**3*sin(x),x)
```

```
[Out] Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 +
I*x**3*exp(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(
-I*x)*sin(x)/8 + 3*I*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*si
n(x)/8 + 3*x*exp(-I*x)*cos(x)/8 - 3*I*exp(-I*x)*cos(x)/8, Eq(d, e
xp(-I))), (x**4*exp(I*x)*sin(x)/8 + I*x**4*exp(I*x)*cos(x)/8 - I*
x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)/4 + 3*x**2*exp(I*x)
*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x)*sin(x)/8
+ 3*x*exp(I*x)*cos(x)/8 + 3*I*exp(I*x)*cos(x)/8, Eq(d, exp(I))),
(d**x*x**3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**
4 + 4*log(d)**2 + 1) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*
log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**
5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1
) - 3*d**x*x**3*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log
(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*sin(x)/(log(d)*
*8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*l
og(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)
**2 + 1) + d**x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*l
og(d)**4 + 4*log(d)**2 + 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log
(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*s
in(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
 6*d**x*x**2*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*sin(x)/(log(d)**8
+ 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x**2*log
(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) + 3*d**x*x**2*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 +
6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)*cos(x)/(log(d
)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**2
*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
 + 6*d**x*x*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)*
*4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)**4*cos(x)/(log(d)**8 + 4
*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3
*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
 - 12*d**x*x*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)*sin(x)/(log(d)**8 + 4*l
og(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x*cos(x)/(log(
d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*log
(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) + 24*d**x*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*lo
g(d)**4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*sin(x)/(log(d)**8
+ 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 24*d**x*log(d)*c
os(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) -
 6*d**x*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)*
```

```
*2 + 1), True))
```

GIAC/XCAS [A] time = 0.254913, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^3*sin(x),x, algorithm="giac")
```

```
[Out] Done
```

3.141 $\int d^x x^3 \cos(x) \, dx$

Optimal. Leaf size=260

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1}+\frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{6x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18x d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{6x d^x \sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{24 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{24 d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{18x d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{36 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^4}-\frac{6 d^x \cos(x)}{\left(\log^2(d)+1\right)^4}-\frac{6 d^x \log^4(d)\cos(x)}{\left(\log^2(d)+1\right)^4}+\frac{6x d^x \log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^3}$$

```
[Out] (-6*d^x*Cos[x])/(1 + Log[d]^2)^4 + (36*d^x*Cos[x]*Log[d]^2)/(1 +
Log[d]^2)^4 - (6*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 - (18*d^x*
x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (6*d^x*x*Cos[x]*Log[d]^3)/(1
+ Log[d]^2)^3 + (3*d^x*x^2*Cos[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*
Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x*x^3*Cos[x]*Log[d])/(1 +
Log[d]^2) + (24*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^4 - (24*d^x*Log
[d]^3*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Sin[x])/(1 + Log[d]^2)^
3 + (18*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x^2*Log[
d]*Sin[x])/(1 + Log[d]^2)^2 + (d^x*x^3*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.662917, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1}+\frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{6x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18x d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{6x d^x \sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{24 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{24 d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{18x d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{36 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^4}-\frac{6 d^x \cos(x)}{\left(\log^2(d)+1\right)^4}-\frac{6 d^x \log^4(d)\cos(x)}{\left(\log^2(d)+1\right)^4}+\frac{6x d^x \log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[d^x*x^3*Cos[x],x]
```

```
[Out] (-6*d^x*Cos[x])/(1 + Log[d]^2)^4 + (36*d^x*Cos[x]*Log[d]^2)/(1 +
Log[d]^2)^4 - (6*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 - (18*d^x*
x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (6*d^x*x*Cos[x]*Log[d]^3)/(1
+ Log[d]^2)^3 + (3*d^x*x^2*Cos[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*
```

```
Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x*x^3*Cos[x]*Log[d])/(1 +
Log[d]^2) + (24*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^4 - (24*d^x*Log
[d]^3*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Sin[x])/(1 + Log[d]^2)^
3 + (18*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x^2*Log[
d]*Sin[x])/(1 + Log[d]^2)^2 + (d^x*x^3*Sin[x])/(1 + Log[d]^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^x x^3 \log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x x^3 \sin(x)}{\log(d)^2+1}-3\int x^2\left(\frac{d^x \log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x \sin(x)}{\log(d)^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**x*x**3*cos(x),x)
```

```
[Out] d**x*x**3*log(d)*cos(x)/(log(d)**2 + 1) + d**x*x**3*sin(x)/(log(d
)**2 + 1) - 3*Integral(x**2*(d**x*log(d)*cos(x)/(log(d)**2 + 1) +
 d**x*sin(x)/(log(d)**2 + 1)), x)
```

Mathematica [A] time = 0.175438, size = 168, normalized size = 0.65

$$d^x\left(\sin(x)\left(x^3\log^6(d)-6x^2\log^5(d)+3x\left(x^2+6\right)\log^4(d)-12\left(x^2+2\right)\log^3(d)+3x\left(x^2+4\right)\log^2(d)-6\left(x^2-4\right)\log(d)+x\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[d^x*x^3*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*
Log[d]^2 + 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(
2 + x^2)*Log[d]^5 - 3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2
) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*L
og[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Log[d]^6)
*Sin[x]))/(1 + Log[d]^2)^4
```

Maple [A] time = 0.03, size = 441, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^x*x^3*cos(x),x)
```

```
[Out] (ln(d)/(1+ln(d)^2)*x^3*exp(x*ln(d))+2/(1+ln(d)^2)*x^3*exp(x*ln(d)
)*tan(1/2*x)-3*(ln(d)^2-1)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))
-6*(ln(d)^4-6*ln(d)^2+1)/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)/(1+ln(d)
^2)*exp(x*ln(d))+3*(ln(d)^2-1)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln
(d))*tan(1/2*x)^2+6*(ln(d)^4-6*ln(d)^2+1)/(ln(d)^6+3*ln(d)^4+3*ln
(d)^2+1)/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2-ln(d)/(1+ln(d)^2)*
x^3*exp(x*ln(d))*tan(1/2*x)^2-12*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*
exp(x*ln(d))*tan(1/2*x)+12*(3*ln(d)^2-1)/(1+ln(d)^2)/(ln(d)^4+2*l
n(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)+6*ln(d)*(ln(d)^2-3)/(1+ln(d)^
2)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))-48*ln(d)*(ln(d)^2-1)/(ln(
d)^4+2*ln(d)^2+1)/(1+ln(d)^2)^2*exp(x*ln(d))*tan(1/2*x)-6*ln(d)*(
ln(d)^2-3)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))*tan(1
/2*x)^2)/(1+tan(1/2*x)^2)
```

Maxima [A] time = 1.44989, size = 248, normalized size = 0.95

$$\frac{\left(\left(\log\left(d\right)^{7} + 3\,\log\left(d\right)^{5} + 3\,\log\left(d\right)^{3} + \log\left(d\right)\right)x^{3} - 6\,\log\left(d\right)^{4} - 3\,\left(\log\left(d\right)^{6} + \log\left(d\right)^{4} - \log\left(d\right)^{2} - 1\right)x^{2} + 6\,\left(\log\left(d\right)^{5} - 2\,\log\left(d\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4
- 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*lo
g(d)^3 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 +
 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3 + lo
g(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^2 - 1)*x + 24*
log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d
)^2 + 1)
```

Fricas [A] time = 0.243986, size = 273, normalized size = 1.05

$$\frac{\left(x^{3} \cos\left(x\right) \log\left(d\right)^{7} - 3\,x^{2} \cos\left(x\right) \log\left(d\right)^{6} + 3\,\left(x^{3} + 2\,x\right) \cos\left(x\right) \log\left(d\right)^{5} - 3\,\left(x^{2} + 2\right) \cos\left(x\right) \log\left(d\right)^{4} + 3\,\left(x^{3} - 4\,x\right) \cos\left(x\right) \log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^3*cos(x),x, algorithm="fricas")
```

```
[Out] (x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(
x)*log(d)^5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*
log(d)^3 + 3*(x^2 + 12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log
(d) + 3*(x^2 - 2)*cos(x) + (x^3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^
3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log(d)^3 + x^3 + 3*(x^3 + 4*x)*l
og(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d^x/(log(d)^8 + 4*log
```

```
(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [A] time = 45.5564, size = 1352, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**x*x**3*cos(x),x)
```

```
[Out] Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 +
x**3*exp(-I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*ex
p(-I*x)*sin(x)/8 + 3*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(
x)/8 - 3*I*x*exp(-I*x)*cos(x)/8 - 3*exp(-I*x)*cos(x)/8, Eq(d, exp
(-I))), (-I*x**4*exp(I*x)*sin(x)/8 + x**4*exp(I*x)*cos(x)/8 + x**
3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/4 + 3*I*x**2*exp(I*x
)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*sin(x)/8 + 3
*I*x*exp(I*x)*cos(x)/8 - 3*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d*
*x*x**3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log
(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*c
os(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
 3*d**x*x**3*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*cos(x)/(log(d)**8
+ 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(
d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
 + 1) + d**x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(
d)**4 + 4*log(d)**2 + 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)
**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*cos(
x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*
d**x*x**2*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*cos(x)/(log(d)**8 + 4
*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x**2*log(d)
**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
 1) + 3*d**x*x**2*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*l
og(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)*sin(x)/(log(d)**
8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**2*co
s(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
6*d**x*x*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
+ 4*log(d)**2 + 1) + 18*d**x*x*log(d)**4*sin(x)/(log(d)**8 + 4*lo
g(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*co
s(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
12*d**x*x*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)*cos(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x*sin(x)/(log(d)*
*8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*log(d)
**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
 1) - 24*d**x*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d
)**4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*cos(x)/(log(d)**8 + 4
*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 24*d**x*log(d)*sin(
x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*
d**x*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
```

```
+ 1), True))
```

GIAC/XCAS [A] time = 0.255998, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d^x*x^3*cos(x),x, algorithm="giac")
```

```
[Out] Done
```

3.142 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi [A] time = 0.0472429, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]*Sin[2*x]*Sin[3*x],x]

[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}-\frac{\int^{\sin(3x)} x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)*sin(2*x)*sin(3*x),x)

[Out] -cos(2*x)/8 - cos(4*x)/16 - Integral(x, (x, sin(3*x)))/3
```

Mathematica [A] time = 0.0144485, size = 25, normalized size = 1.

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A] time = 0.037, size = 20, normalized size = 0.8

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)*sin(2*x)*sin(3*x),x)

[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A] time = 1.39326, size = 26, normalized size = 1.04

$$\frac{1}{24}\cos(6x)-\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(2*x)*sin(x),x, algorithm="maxima")

[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A] time = 0.24567, size = 23, normalized size = 0.92

$$\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(2*x)*sin(x),x, algorithm="fricas")

[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [A] time = 21.3735, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{3\sin(x)\sin(2x)\cos(3x)}{8}+\frac{\sin(x)\sin(3x)\cos(2x)}{6}+\frac{\sin(2x)\sin(3x)\cos(x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*s
in(2*x)*cos(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 3*sin(
x)*sin(2*x)*cos(3*x)/8 + sin(x)*sin(3*x)*cos(2*x)/6 + sin(2*x)*si
n(3*x)*cos(x)/24
```

GIAC/XCAS [A] time = 0.199975, size = 18, normalized size = 0.72

$$-\frac{4}{3}\sin(x)^{6}+\frac{3}{2}\sin(x)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.143 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi [A] time = 0.0474909, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Cos[2*x]*Cos[3*x],x]

[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi in Sympy [A] time = 2.36328, size = 27, normalized size = 0.9

$$\frac{x}{4}+\frac{\sin(2x)}{4}+\frac{\sin(3x)\cos(3x)}{12}+\frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(2*x)*cos(3*x),x)

[Out] x/4 + sin(2*x)/4 + sin(3*x)*cos(3*x)/12 + sin(4*x)/8
```

Mathematica [A] time = 0.0142066, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]

[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A] time = 0.032, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sin\left(2\,x\right)}{8}+\frac{\sin\left(4\,x\right)}{16}+\frac{\sin\left(6\,x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A] time = 1.40386, size = 30, normalized size = 1.

$$\frac{1}{4}\,x+\frac{1}{24}\,\sin\left(6\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(2*x)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A] time = 0.252403, size = 34, normalized size = 1.13

$$\frac{1}{12}\left(16\,\cos\left(x\right)^{5}-10\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(2*x)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [A] time = 21.234, size = 114, normalized size = 3.8

$$-\frac{x\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{4}+\frac{x\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{4}+\frac{x\sin\left(2x\right)\sin\left(3x\right)\cos\left(x\right)}{4}+\frac{x\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4}-\frac{\sin\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{24}-\frac{\sin\left(2x\right)\cos\left(x\right)\cos\left(3x\right)}{6}+\frac{3\sin\left(3x\right)\cos\left(x\right)\cos\left(2x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*
sin(2*x)*sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 - sin(x
)*cos(2*x)*cos(3*x)/24 - sin(2*x)*cos(x)*cos(3*x)/6 + 3*sin(3*x)*
cos(x)*cos(2*x)/8
```

GIAC/XCAS [A] time = 0.204228, size = 30, normalized size = 1.

$$\frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(2*x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.144 $\int x^2 \sin^3(kx)\,dx$

Optimal. Leaf size=85

$$-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

```
[Out] (14*Cos[k*x])/(9*k^3) - (2*x^2*Cos[k*x])/(3*k) - (2*Cos[k*x]^3)/(
27*k^3) + (4*x*Sin[k*x])/(3*k^2) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k
) + (2*x*Sin[k*x]^3)/(9*k^2)
```

Rubi [A] time = 0.103559, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[k*x]^3,x]
```

```
[Out] (14*Cos[k*x])/(9*k^3) - (2*x^2*Cos[k*x])/(3*k) - (2*Cos[k*x]^3)/(
27*k^3) + (4*x*Sin[k*x])/(3*k^2) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k
) + (2*x*Sin[k*x]^3)/(9*k^2)
```

Rubi in Sympy [A] time = 4.89368, size = 85, normalized size = 1.

$$-\frac{x^2\sin^2(kx)\cos(kx)}{3k}-\frac{2x^2\cos(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(k*x)**3,x)
```

```
[Out] -x**2*sin(k*x)**2*cos(k*x)/(3*k) - 2*x**2*cos(k*x)/(3*k) + 2*x*si
n(k*x)**3/(9*k**2) + 4*x*sin(k*x)/(3*k**2) - 2*cos(k*x)**3/(27*k*
*3) + 14*cos(k*x)/(9*k**3)
```

Mathematica [A] time = 0.0852879, size = 55, normalized size = 0.65

$$\frac{-81\left(k^2x^2-2\right)\cos(kx)+\left(9k^2x^2-2\right)\cos(3kx)-6kx(\sin(3kx)-27\sin(kx))}{108k^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[k*x]^3,x]
```

```
[Out] (-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*
x*(-27*Sin[k*x] + Sin[3*k*x]))/(108*k^3)
```

Maple [A] time = 0.019, size = 64, normalized size = 0.8

$$\frac{1}{k^3}\left(-\frac{k^2x^2\left(2+(\sin(kx))^2\right)\cos(kx)}{3}+\frac{4\cos(kx)}{3}+\frac{4kx\sin(kx)}{3}+\frac{2kx(\sin(kx))^3}{9}+\frac{\left(4+2(\sin(kx))^2\right)\cos(kx)}{27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(k*x)^3,x)
```

```
[Out] 1/k^3*(-1/3*k^2*x^2*(2+sin(k*x)^2)*cos(k*x)+4/3*cos(k*x)+4/3*k*x*
sin(k*x)+2/9*k*x*sin(k*x)^3+2/27*(2+sin(k*x)^2)*cos(k*x))
```

Maxima [A] time = 1.42707, size = 74, normalized size = 0.87

$$-\frac{6kx\sin(3kx)-162kx\sin(kx)-\left(9k^2x^2-2\right)\cos(3kx)+81\left(k^2x^2-2\right)\cos(kx)}{108k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(k*x)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(6*k*x*sin(3*k*x) - 162*k*x*sin(k*x) - (9*k^2*x^2 - 2)*cos
(3*k*x) + 81*(k^2*x^2 - 2)*cos(k*x))/k^3
```

Fricas [A] time = 0.236567, size = 80, normalized size = 0.94

$$\frac{\left(9k^2x^2-2\right)\cos(kx)^3-3\left(9k^2x^2-14\right)\cos(kx)-6\left(kx\cos(kx)^2-7kx\right)\sin(kx)}{27k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(k*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*k^2*x^2 - 2)*cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*cos(k*x) -
6*(k*x*cos(k*x)^2 - 7*k*x)*sin(k*x))/k^3
```

Sympy [A] time = 3.49676, size = 100, normalized size = 1.18

$$\begin{cases} -\frac{x^2\sin^2(kx)\cos(kx)}{k} - \frac{2x^2\cos^3(kx)}{3k} + \frac{14x\sin^3(kx)}{9k^2} + \frac{4x\sin(kx)\cos^2(kx)}{3k^2} + \frac{14\sin^2(kx)\cos(kx)}{9k^3} + \frac{40\cos^3(kx)}{27k^3} & \text{for } k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(k*x)**3,x)
```

```
[Out] Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k
) + 14*x*sin(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2)
 + 14*sin(k*x)**2*cos(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), N
e(k, 0)), (0, True))
```

GIAC/XCAS [A] time = 0.198624, size = 81, normalized size = 0.95

$$-\frac{x\sin(3kx)}{18k^2} + \frac{3x\sin(kx)}{2k^2} + \frac{(9k^2x^2-2)\cos(3kx)}{108k^3} - \frac{3(k^2x^2-2)\cos(kx)}{4k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(k*x)^3,x, algorithm="giac")
```

```
[Out] -1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 -
2)*cos(3*k*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3
```

3.145 $\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$

Optimal. Leaf size=14

$$\mathrm{Int}(x\cot(x)\csc(x)\cos(k\csc(x)),x)$$

```
[Out] CannotIntegrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Rubi [A] time = 0.686283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\mathrm{Int}(x\cos(k\csc(x))\cot(x)\csc(x),x)$$

Verification is Not applicable to the result.

```
[In] Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

```
[Out] Defer[Int][x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)
```

```
[Out] Integral(x*cos(x)*cos(k/sin(x))/sin(x)**2, x)
```

Mathematica [A] time = 1.07669, size = 0, normalized size = 0.

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

```
[Out] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Maple [A] time = 0.344, size = 0, normalized size = 0.

$$\int \frac{x\cos(x)}{(\sin(x))^2}\cos\left(\frac{k}{\sin(x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

```
[Out] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

Maxima [A] time = 0.132088, size = 324, normalized size = 23.14

$$-\frac{\left(xe^{\left(\frac{4\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}+\frac{4\,k\sin(2\,x)\sin(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}+xe^{\left(\frac{4\,k\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}\right)e^{\left(-\frac{2\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}-\frac{2\,k\sin(2\,x)\sin(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}-\right.}}{2\,k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2
*x) + 1) + 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2
*x) + 1)) + x*e^(4*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x)
 + 1)))*e^(-2*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(
2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(
2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
)*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x))/(cos(2
*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\cos(x)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)
```

<u>Sympy</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)
```

```
[Out] Timed out
```

<u>GIAC/XCAS</u> [A] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.146 $\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx$

Optimal. Leaf size=12

$$-x-\cot\left(\frac{x}{2}\right)$$

```
[Out] -x - Cot[x/2]
```

Rubi [A] time = 0.0573134, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x/2]*Cot[x],x]

[Out] -x - Cot[x/2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\tan\left(\frac{x}{2}\right)\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)/sin(x)/tan(1/2*x),x)

[Out] Integral(1/(tan(x/2)*tan(x)), x)
```

Mathematica [A] time = 0.0106212, size = 12, normalized size = 1.

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x/2]*Cot[x],x]

[Out] -x - Cot[x/2]
```

Maple [A] time = 0.034, size = 11, normalized size = 0.9

$$-x-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/sin(x)/tan(1/2*x),x)
```

```
[Out] -x-cot(1/2*x)
```

Maxima [A] time = 1.38156, size = 55, normalized size = 4.58

$$-\frac{x\cos\left(x\right)^{2}+x\sin\left(x\right)^{2}-2\,x\cos\left(x\right)+x+2\,\sin\left(x\right)}{\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)*tan(1/2*x)),x, algorithm="maxima")
```

```
[Out] -(x*cos(x)^2 + x*sin(x)^2 - 2*x*cos(x) + x + 2*sin(x))/(cos(x)^2
+ sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 0.241092, size = 22, normalized size = 1.83

$$-\frac{x\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)*tan(1/2*x)),x, algorithm="fricas")
```

```
[Out] -(x*tan(1/2*x) + 1)/tan(1/2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(x\right)}{\sin\left(x\right)\tan\left(\frac{x}{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(x)/tan(1/2*x),x)

[Out] Integral(cos(x)/(sin(x)*tan(x/2)), x)
```

GIAC/XCAS [A] time = 0.211679, size = 24, normalized size = 2.

$$-x - \frac{1}{2\tan\left(\frac{1}{4}x\right)} + \frac{1}{2}\tan\left(\frac{1}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)*tan(1/2*x)),x, algorithm="giac")

[Out] -x - 1/2/tan(1/4*x) + 1/2*tan(1/4*x)
```

3.147 $\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$

```
[Out] (-2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(a*(b^2 - c^2
)^(3/2)) - (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

Rubi [A] time = 0.162121, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] (-2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(a*(b^2 - c^2
)^(3/2)) - (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

Rubi in Sympy [A] time = 7.73782, size = 61, normalized size = 0.79

$$-\frac{b\cos(ax)}{a\left(b+c\sin(ax)\right)\left(b^2-c^2\right)}-\frac{2c\,\mathrm{atan}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(a*x)/(b+c*sin(a*x))**2,x)
```

```
[Out] -b*cos(a*x)/(a*(b + c*sin(a*x))*(b**2 - c**2)) - 2*c*atan((b*tan(
a*x/2) + c)/sqrt(b**2 - c**2))/(a*(b**2 - c**2)**(3/2))
```

Mathematica [A] time = 0.325073, size = 76, normalized size = 0.99

$$-\frac{\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{b\cos(ax)}{(b-c)(b+c)(c\sin(ax)+b)}}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] -(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)
^(3/2) + (b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)
```

Maple [A] time = 0.032, size = 143, normalized size = 1.9

$$-8\,\frac{c\tan(1/2\,ax)}{a\left(4\,b^2-4\,c^2\right)\left(b\left(\tan(1/2\,ax)\right)^2+2\,c\tan(1/2\,ax)+b\right)}-8\,\frac{b}{a\left(4\,b^2-4\,c^2\right)\left(b\left(\tan(1/2\,ax)\right)^2+2\,c\tan(1/2\,ax)+b\right)}-8\,\frac{c}{a\left(4\,b^2-4\,c^2\right)\sqrt{b^2-c^2}}\arctan\left(1/2\,\frac{2\,b\tan(1/2\,ax)+2\,c}{\sqrt{b^2-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(a*x)/(b+c*sin(a*x))^2,x)
```

```
[Out] -8/a/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)*c*tan(1/
2*a*x)-8/a/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)*b-
8/a*c/(4*b^2-4*c^2)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+
2*c)/(b^2-c^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(a*x)/(c*sin(a*x) + b)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2379, size = 1, normalized size = 0.01

$$\left[-\frac{2\sqrt{-b^2+c^2}\,b\cos(ax)+\left(c^2\sin(ax)+bc\right)\log\left(-\frac{2\left(b^3-bc^2\right)\cos(ax)\sin(ax)+2\left(b^2c-c^3\right)\cos(ax)-\left(\left(2b^2-c^2\right)\cos(ax)^2-2bc\sin(ax)-b^2-c^2\right)\sqrt{\cdots}}{c^2\cos(ax)^2-2bc\sin(ax)-b^2-c^2}\cdots\right.}{2\left(ab^3-abc^2+\left(ab^2c-ac^3\right)\sin(ax)\right)\sqrt{-b^2+c^2}}\right.$$

$$\left.-\frac{\sqrt{b^2-c^2}\,b\cos(ax)-\left(c^2\sin(ax)+bc\right)\arctan\left(-\frac{b\sin(ax)+c}{\sqrt{b^2-c^2}\cos(ax)}\right)}{\left(ab^3-abc^2+\left(ab^2c-ac^3\right)\sin(ax)\right)\sqrt{b^2-c^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(a*x)/(c*sin(a*x) + b)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(-b^2 + c^2)*b*cos(a*x) + (c^2*sin(a*x) + b*c)*log(-
(2*(b^3 - b*c^2)*cos(a*x)*sin(a*x) + 2*(b^2*c - c^3)*cos(a*x) - (
(2*b^2 - c^2)*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)*sqrt(-b^2
+ c^2))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)))/((a*b^3 -
 a*b*c^2 + (a*b^2*c - a*c^3)*sin(a*x))*sqrt(-b^2 + c^2)), -(sqrt(
b^2 - c^2)*b*cos(a*x) - (c^2*sin(a*x) + b*c)*arctan(-(b*sin(a*x)
+ c)/(sqrt(b^2 - c^2)*cos(a*x))))/((a*b^3 - a*b*c^2 + (a*b^2*c -
a*c^3)*sin(a*x))*sqrt(b^2 - c^2))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(a*x)/(b+c*sin(a*x))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.235833, size = 132, normalized size = 1.71

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ax}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sign}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ax\right)+c}{\sqrt{b^2-c^2}}\right)\right)c}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{c\tan\left(\frac{1}{2}ax\right)+b}{\left(b\tan\left(\frac{1}{2}ax\right)^2+2c\tan\left(\frac{1}{2}ax\right)+b\right)\left(b^2-c^2\right)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(a*x)/(c*sin(a*x) + b)^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*a*x/pi + 1/2)*sign(b) + arctan((b*tan(1/2*a*x)
+ c)/sqrt(b^2 - c^2)))*c/(b^2 - c^2)^(3/2) + (c*tan(1/2*a*x) + b)
/((b*tan(1/2*a*x)^2 + 2*c*tan(1/2*a*x) + b)*(b^2 - c^2)))/a
```

3.148 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.00795542, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi in Sympy [A] time = 0.502601, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

Mathematica [A] time = 0.00443272, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(ln(x)),x)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 1.43017, size = 16, normalized size = 0.94

$$-\frac{1}{2}\,x(\cos(\log(x))-\sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Fricas [A] time = 0.228173, size = 18, normalized size = 1.06

$$-\frac{1}{2}\,x\cos(\log(x))+\frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.537742, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2}-\frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

GIAC/XCAS [A] time = 0.218705, size = 18, normalized size = 1.06

$$-\frac{1}{2}\,x\cos\left(\ln\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(ln(x)) + 1/2*x*sin(ln(x))
```

3.149 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.00728153, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi in Sympy [A] time = 0.473577, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2}+\frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(ln(x)),x)

[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Mathematica [A] time = 0.00439241, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{x\cos\left(\ln\left(x\right)\right)}{2}+\frac{x\sin\left(\ln\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(ln(x)),x)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 1.40124, size = 14, normalized size = 0.82

$$\frac{1}{2}\,x(\cos\left(\log\left(x\right)\right)+\sin\left(\log\left(x\right)\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A] time = 0.239583, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.521858, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}+\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(ln(x)),x)

[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

GIAC/XCAS [A] time = 0.215739, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos\left(\ln\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="giac")

[Out] 1/2*x*cos(ln(x)) + 1/2*x*sin(ln(x))
```

3.150 $\int e^x \, dx$

Optimal. Leaf size=3

$$e^x$$

```
[Out] E^x
```

Rubi [A] time = 0.00238067, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x,x]
```

```
[Out] E^x
```

Rubi in Sympy [A] time = 0.458163, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x),x)
```

```
[Out] exp(x)
```

Mathematica [A] time = 0.00018303, size = 3, normalized size = 1.

$$e^x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x,x]
```

```
[Out] E^x
```

Maple [A] time = 0.001, size = 3, normalized size = 1.

$$\mathrm{e}^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x),x)

[Out] exp(x)
```

Maxima [A] time = 1.39299, size = 3, normalized size = 1.

$$e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x,x, algorithm="maxima")

[Out] e^x
```

Fricas [A] time = 0.205797, size = 3, normalized size = 1.

$$e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x,x, algorithm="fricas")

[Out] e^x
```

Sympy [A] time = 0.044535, size = 2, normalized size = 0.67

$$e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x),x)

[Out] exp(x)
```

GIAC/XCAS [A] time = 0.215366, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x,x, algorithm="giac")
```

```
[Out] e^x
```

3.151 $\int a^x \, dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

```
[Out] a^x/Log[a]
```

Rubi [A] time = 0.00475623, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[a^x,x]

[Out] a^x/Log[a]
```

Rubi in Sympy [A] time = 0.564855, size = 5, normalized size = 0.62

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**x,x)

[Out] a**x/log(a)
```

Mathematica [A] time = 0.000856275, size = 8, normalized size = 1.

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^x,x]
```

```
[Out] a^x/Log[a]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
```

```
[Out] a^x/ln(a)
```

Maxima [A] time = 1.40368, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="maxima")
```

```
[Out] a^x/log(a)
```

Fricas [A] time = 0.213548, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="fricas")
```

```
[Out] a^x/log(a)
```

Sympy [A] time = 0.065949, size = 8, normalized size = 1.

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for}\ \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x,x)
```

```
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

GIAC/XCAS [A] time = 0.198347, size = 11, normalized size = 1.38

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x,x, algorithm="giac")
```

```
[Out] a^x/ln(a)
```

3.152 $\int e^{ax}\,dx$

Optimal. Leaf size=9

$$\frac{e^{ax}}{a}$$

```
[Out] E^(a*x)/a
```

Rubi [A] time = 0.00468807, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(a*x),x]

[Out] E^(a*x)/a
```

Rubi in Sympy [A] time = 0.606267, size = 5, normalized size = 0.56

$$\frac{e^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x),x)

[Out] exp(a*x)/a
```

Mathematica [A] time = 0.00139801, size = 9, normalized size = 1.

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(a*x),x]
```

```
[Out] E^(a*x)/a
```

Maple [A] time = 0.001, size = 9, normalized size = 1.

$$\frac{e^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(a*x),x)
```

```
[Out] exp(a*x)/a
```

Maxima [A] time = 1.38826, size = 11, normalized size = 1.22

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x),x, algorithm="maxima")
```

```
[Out] e^(a*x)/a
```

Fricas [A] time = 0.213676, size = 11, normalized size = 1.22

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x),x, algorithm="fricas")
```

```
[Out] e^(a*x)/a
```

Sympy [A] time = 0.060735, size = 7, normalized size = 0.78

$$\begin{cases} \frac{e^{ax}}{a} & \text{for}\: a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x),x)

[Out] Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))
```

GIAC/XCAS [A] time = 0.198151, size = 11, normalized size = 1.22

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x),x, algorithm="giac")

[Out] e^(a*x)/a
```

3.153 $\int \frac{e^{ax}}{x} \, dx$

Optimal. Leaf size=4

$$\text{ExpIntegralEi}(ax)$$

```
[Out] ExpIntegralEi[a*x]
```

Rubi [A] time = 0.0167793, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\text{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(a*x)/x,x]

[Out] ExpIntegralEi[a*x]
```

Rubi in Sympy [A] time = 1.34027, size = 3, normalized size = 0.75

$$\text{Ei}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x)/x,x)

[Out] Ei(a*x)
```

Mathematica [A] time = 0.00170743, size = 4, normalized size = 1.

$$\text{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(a*x)/x,x]

[Out] ExpIntegralEi[a*x]
```

Maple [A] time = 0.002, size = 9, normalized size = 2.3

$$-Ei(1, -ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(a*x)/x,x)
```

```
[Out] -Ei(1,-a*x)
```

Maxima [A] time = 1.48587, size = 5, normalized size = 1.25

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x)/x,x, algorithm="maxima")
```

```
[Out] Ei(a*x)
```

Fricas [A] time = 0.201593, size = 5, normalized size = 1.25

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x)/x,x, algorithm="fricas")
```

```
[Out] Ei(a*x)
```

Sympy [A] time = 1.29583, size = 3, normalized size = 0.75

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x)/x,x)
```

```
[Out] Ei(a*x)
```

GIAC/XCAS [A] time = 0.19808, size = 5, normalized size = 1.25

$$\mathrm{Ei}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x)/x,x, algorithm="giac")
```

```
[Out] Ei(a*x)
```

3.154 $\int \frac{1}{a+be^{mx}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Rubi [A] time = 0.0308992, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*E^(m*x))^(-1),x]
```

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Rubi in Sympy [A] time = 3.01905, size = 22, normalized size = 0.92

$$-\frac{\log\left(a+be^{mx}\right)}{am}+\frac{\log\left(e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*exp(m*x)),x)
```

```
[Out] -log(a + b*exp(m*x))/(a*m) + log(exp(m*x))/(a*m)
```

Mathematica [A] time = 0.00689755, size = 24, normalized size = 1.

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*E^(m*x))^(-1),x]
```

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Maple [A] time = 0.003, size = 31, normalized size = 1.3

$$\frac{\ln(e^{mx})}{ma} - \frac{\ln(a + be^{mx})}{ma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*exp(m*x)),x)
```

```
[Out] 1/m/a*ln(exp(m*x))-ln(a+b*exp(m*x))/a/m
```

Maxima [A] time = 1.43155, size = 31, normalized size = 1.29

$$\frac{x}{a} - \frac{\log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*e^(m*x) + a),x, algorithm="maxima")
```

```
[Out] x/a - log(b*e^(m*x) + a)/(a*m)
```

Fricas [A] time = 0.20871, size = 30, normalized size = 1.25

$$\frac{mx - \log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*e^(m*x) + a),x, algorithm="fricas")
```

```
[Out] (m*x - log(b*e^(m*x) + a))/(a*m)
```

Sympy [A] time = 0.125135, size = 15, normalized size = 0.62

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*exp(m*x)),x)
```

```
[Out] x/a - log(a/b + exp(m*x))/(a*m)
```

GIAC/XCAS [A] time = 0.199144, size = 32, normalized size = 1.33

$$\frac{x}{a} - \frac{\ln\left(\left|be^{(mx)} + a\right|\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*e^(m*x) + a),x, algorithm="giac")
```

```
[Out] x/a - ln(abs(b*e^(m*x) + a))/(a*m)
```

3.155 $\int \frac{e^{2x}}{1+e^x}\,dx$

Optimal. Leaf size=12

$$e^x - \log\left(e^x + 1\right)$$

```
[Out] E^x - Log[1 + E^x]
```

Rubi [A] time = 0.0347527, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$e^x - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Rubi in Sympy [A] time = 3.30917, size = 8, normalized size = 0.67

$$e^x - \log\left(e^x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/(1+exp(x)),x)

[Out] exp(x) - log(exp(x) + 1)
```

Mathematica [A] time = 0.00552995, size = 12, normalized size = 1.

$$e^x - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$e^{x} - \ln(1 + e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A] time = 1.41787, size = 14, normalized size = 1.17

$$e^{x} - \log(e^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A] time = 0.220167, size = 14, normalized size = 1.17

$$e^{x} - \log(e^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A] time = 0.0825, size = 8, normalized size = 0.67

$$e^{x} - \log(e^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.197874, size = 14, normalized size = 1.17

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="giac")

[Out] e^x - ln(e^x + 1)
```

3.156 $\int e^{2x+ax}\,dx$

Optimal. Leaf size=13

$$\frac{e^{(a+2)x}}{a+2}$$

```
[Out] E^((2 + a)*x)/(2 + a)
```

Rubi [A] time = 0.0147195, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x + a*x),x]
```

```
[Out] E^((2 + a)*x)/(2 + a)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{ax+2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x+2*x),x)
```

```
[Out] Integral(exp(a*x + 2*x), x)
```

Mathematica [A] time = 0.00333198, size = 13, normalized size = 1.

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x + a*x),x]
```

```
[Out] E^((2 + a)*x)/(2 + a)
```

Maple [A] time = 0.001, size = 15, normalized size = 1.2

$$\frac{e^{ax+2x}}{2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x+2*x),x)
```

```
[Out] 1/(2+a)*exp(a*x+2*x)
```

Maxima [A] time = 1.40839, size = 19, normalized size = 1.46

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(a*x + 2*x),x, algorithm="maxima")
```

```
[Out] e^(a*x + 2*x)/(a + 2)
```

Fricas [A] time = 0.211677, size = 16, normalized size = 1.23

$$\frac{e^{((a+2)x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(a*x + 2*x),x, algorithm="fricas")
```

```
[Out] e^((a + 2)*x)/(a + 2)
```

Sympy [A] time = 0.071039, size = 14, normalized size = 1.08

$$\begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a+2\neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x+2*x),x)

[Out] Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a + 2, 0)), (x, True))
```

GIAC/XCAS [A]   time = 0.199825, size = 19, normalized size = 1.46

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x + 2*x),x, algorithm="giac")

[Out] e^(a*x + 2*x)/(a + 2)
```

3.157 $\int \frac{1}{be^{-mx}+ae^{mx}} dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Rubi [A] time = 0.0514737, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

Antiderivative was successfully verified.

```
[In]  Int[(b/E^(m*x) + a*E^(m*x))^(-1),x]

[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Rubi in Sympy [A] time = 5.50959, size = 29, normalized size = 0.94

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{b}e^{-mx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b/exp(m*x)+a*exp(m*x)),x)

[Out] -atan(sqrt(b)*exp(-m*x)/sqrt(a))/(sqrt(a)*sqrt(b)*m)
```

Mathematica [A] time = 0.0136105, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Maple [A] time = 0.006, size = 22, normalized size = 0.7

$$\frac{1}{m}\arctan\left(ae^{mx}\frac{1}{\sqrt{ba}}\right)\frac{1}{\sqrt{ba}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] 1/m/(b*a)^(1/2)*arctan(exp(m*x)*a/(b*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*e^(m*x) + b*e^(-m*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222215, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\,abe^{(mx)}+\sqrt{-ab}\left(ae^{(2\,mx)}-b\right)}{ae^{(2\,mx)}+b}\right)}{2\,\sqrt{-ab}m}, -\frac{\arctan\left(\frac{be^{(-mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}m}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*e^(m*x) + b*e^(-m*x)),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*e^(m*x) + sqrt(-a*b)*(a*e^(2*m*x) - b))/(a*e^(2*m
*x) + b))/(sqrt(-a*b)*m), -arctan(b*e^(-m*x)/sqrt(a*b))/(sqrt(a*b
)*m)]
```

Sympy [A] time = 0.190262, size = 26, normalized size = 0.84

$$\frac{\text{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(-2ia+e^{-mx}\right)\right)\right)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(-2*_i*a + exp(-m*x))))
/m
```

GIAC/XCAS [A] time = 0.20011, size = 28, normalized size = 0.9

$$\frac{\arctan\left(\frac{ae^{(mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*e^(m*x) + b*e^(-m*x)),x, algorithm="giac")
```

```
[Out] arctan(a*e^(m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

3.158 $\int e^{ax} x \, dx$

Optimal. Leaf size=21

$$\frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

```
[Out] -(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

Rubi [A] time = 0.0173146, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(a*x)*x,x]

[Out] -(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

Rubi in Sympy [A] time = 1.58013, size = 15, normalized size = 0.71

$$\frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x)*x,x)

[Out] x*exp(a*x)/a - exp(a*x)/a**2
```

Mathematica [A] time = 0.00369356, size = 14, normalized size = 0.67

$$\frac{e^{ax}(ax - 1)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(a*x)*x,x]
```

```
[Out] (E^(a*x)*(-1 + a*x))/a^2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.7

$$\frac{(ax-1)\,\mathrm{e}^{ax}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(a*x)*x,x)
```

```
[Out] (a*x-1)*exp(a*x)/a^2
```

Maxima [A] time = 1.38634, size = 18, normalized size = 0.86

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(a*x),x, algorithm="maxima")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

Fricas [A] time = 0.212855, size = 18, normalized size = 0.86

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(a*x),x, algorithm="fricas")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

Sympy [A] time = 0.091366, size = 19, normalized size = 0.9

$$\begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for}\ a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x)*x,x)
```

```
[Out] Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))
```

GIAC/XCAS [A] time = 0.19931, size = 18, normalized size = 0.86

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(a*x),x, algorithm="giac")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

3.159 $\int e^x x^{20}\,dx$

Optimal. Leaf size=163

$$e^x x^{20} - 20e^x x^{19} + 380e^x x^{18} - 6840e^x x^{17} + 116280e^x x^{16} - 1860480e^x x^{15} + 27907200e^x x^{14} - 390700800e^x x^{13} + 5079110400e^x x^{12} - 60949324800e^x x^{11} + 670442572800e^x x^{10} - 6704425728000e^x x^9 + 60339831552000e^x x^8 - 482718652416000e^x x^7 + 3379030566912000e^x x^6 - 20274183401472000e^x x^5 + 101370917007360000e^x x^4 - 405483668029440000e^x x^3 + 1216451004088320000e^x x^2 - 2432902008176640000e^x x + 2432902008176640000e^x$$

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 12164510040
88320000*E^x*x^2 - 405483668029440000*E^x*x^3 + 10137091700736000
0*E^x*x^4 - 20274183401472000*E^x*x^5 + 3379030566912000*E^x*x^6
- 482718652416000*E^x*x^7 + 60339831552000*E^x*x^8 - 670442572800
0*E^x*x^9 + 670442572800*E^x*x^10 - 60949324800*E^x*x^11 + 507911
0400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*x^14 - 1860480*
E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 - 20*E^
x*x^19 + E^x*x^20
```

Rubi [A] time = 0.378312, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$e^x x^{20} - 20e^x x^{19} + 380e^x x^{18} - 6840e^x x^{17} + 116280e^x x^{16} - 1860480e^x x^{15} + 27907200e^x x^{14} - 390700800e^x x^{13} + 5079110400e^x x^{12} - 60949324800e^x x^{11} + 670442572800e^x x^{10} - 6704425728000e^x x^9 + 60339831552000e^x x^8 - 482718652416000e^x x^7 + 3379030566912000e^x x^6 - 20274183401472000e^x x^5 + 101370917007360000e^x x^4 - 405483668029440000e^x x^3 + 1216451004088320000e^x x^2 - 2432902008176640000e^x x + 2432902008176640000e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x^20,x]
```

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 12164510040
88320000*E^x*x^2 - 405483668029440000*E^x*x^3 + 10137091700736000
0*E^x*x^4 - 20274183401472000*E^x*x^5 + 3379030566912000*E^x*x^6
- 482718652416000*E^x*x^7 + 60339831552000*E^x*x^8 - 670442572800
0*E^x*x^9 + 670442572800*E^x*x^10 - 60949324800*E^x*x^11 + 507911
0400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*x^14 - 1860480*
E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 - 20*E^
x*x^19 + E^x*x^20
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x^{20}e^x - 20x^{19}e^x + 380x^{18}e^x - 6840x^{17}e^x + 116280x^{16}e^x - 1860480x^{15}e^x + 27907200x^{14}e^x - 390700800x^{13}e^x + 5079110400x^{12}e^x - 60949324800x^{11}e^x + 670442572800x^{10}e^x - 6704425728000\int x^9 e^x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x**20,x)
```

```
[Out] x**20*exp(x) - 20*x**19*exp(x) + 380*x**18*exp(x) - 6840*x**17*ex
p(x) + 116280*x**16*exp(x) - 1860480*x**15*exp(x) + 27907200*x**1
4*exp(x) - 390700800*x**13*exp(x) + 5079110400*x**12*exp(x) - 609
49324800*x**11*exp(x) + 670442572800*x**10*exp(x) - 6704425728000
*Integral(x**9*exp(x), x)
```

Mathematica [A] time = 0.0112535, size = 102, normalized size = 0.63

$$e^x \left(x^{20}-20x^{19}+380x^{18}-6840x^{17}+116280x^{16}-1860480x^{15}+27907200x^{14}\right.$$
$$-390700800x^{13}+5079110400x^{12}-60949324800x^{11}+670442572800x^{10}$$
$$-6704425728000x^9+60339831552000x^8-482718652416000x^7+3379030566912000x^6$$
$$-20274183401472000x^5+101370917007360000x^4-405483668029440000x^3$$
$$\left.+1216451004088320000x^2-2432902008176640000x+2432902008176640000\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x^20,x]
```

```
[Out] E^x*(2432902008176640000 - 2432902008176640000*x + 12164510040883
20000*x^2 - 405483668029440000*x^3 + 101370917007360000*x^4 - 202
74183401472000*x^5 + 3379030566912000*x^6 - 482718652416000*x^7 +
 60339831552000*x^8 - 6704425728000*x^9 + 670442572800*x^10 - 609
49324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200*x^14
- 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x
^20)
```

Maple [A] time = 0.002, size = 102, normalized size = 0.6

$$\left(x^{20}-20\,x^{19}+380\,x^{18}-6840\,x^{17}+116280\,x^{16}-1860480\,x^{15}+27907200\,x^{14}\right.$$
$$-390700800\,x^{13}+5079110400\,x^{12}-60949324800\,x^{11}+670442572800\,x^{10}$$
$$-6704425728000\,x^9+60339831552000\,x^8-482718652416000\,x^7+3379030566912000\,x^6$$
$$-20274183401472000\,x^5+101370917007360000\,x^4-405483668029440000\,x^3$$
$$\left.+1216451004088320000\,x^2-2432902008176640000\,x+2432902008176640000\right)\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x^20,x)
```

```
[Out] (x^20-20*x^19+380*x^18-6840*x^17+116280*x^16-1860480*x^15+2790720
0*x^14-390700800*x^13+5079110400*x^12-60949324800*x^11+6704425728
```

```
00*x^10-6704425728000*x^9+60339831552000*x^8-482718652416000*x^7+
3379030566912000*x^6-20274183401472000*x^5+101370917007360000*x^4
-405483668029440000*x^3+1216451004088320000*x^2-24329020081766400
00*x+2432902008176640000)*exp(x)
```

Maxima [A] time = 1.42532, size = 136, normalized size = 0.83

$(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 6094932$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^20*e^x,x, algorithm="maxima")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^
15 + 27907200*x^14 - 390700800*x^13 + 5079110400*x^12 - 609493248
00*x^11 + 670442572800*x^10 - 6704425728000*x^9 + 60339831552000*
x^8 - 482718652416000*x^7 + 3379030566912000*x^6 - 20274183401472
000*x^5 + 101370917007360000*x^4 - 405483668029440000*x^3 + 12164
51004088320000*x^2 - 2432902008176640000*x + 2432902008176640000)
*e^x
```

Fricas [A] time = 0.214026, size = 136, normalized size = 0.83

$(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 6094932$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^20*e^x,x, algorithm="fricas")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^
15 + 27907200*x^14 - 390700800*x^13 + 5079110400*x^12 - 609493248
00*x^11 + 670442572800*x^10 - 6704425728000*x^9 + 60339831552000*
x^8 - 482718652416000*x^7 + 3379030566912000*x^6 - 20274183401472
000*x^5 + 101370917007360000*x^4 - 405483668029440000*x^3 + 12164
51004088320000*x^2 - 2432902008176640000*x + 2432902008176640000)
*e^x
```

Sympy [A] time = 0.119834, size = 102, normalized size = 0.63

$$\begin{aligned}&\left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14}\right.\\&- 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10}\\&- 6704425728000x^{9} + 60339831552000x^{8} - 482718652416000x^{7} + 3379030566912000x^{6}\\&- 20274183401472000x^{5} + 101370917007360000x^{4} - 405483668029440000x^{3}\\&\left.+ 1216451004088320000x^{2} - 2432902008176640000x + 2432902008176640000\right) e^{x}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x**20,x)
```

```
[Out] (x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 18604
80*x**15 + 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 -
60949324800*x**11 + 670442572800*x**10 - 6704425728000*x**9 + 603
39831552000*x**8 - 482718652416000*x**7 + 3379030566912000*x**6 -
 20274183401472000*x**5 + 101370917007360000*x**4 - 4054836680294
40000*x**3 + 121645100408832000*x**2 - 243290200817664000*x + 2
43290200817664000)*exp(x)
```

GIAC/XCAS [A]   time = 0.200304, size = 136, normalized size = 0.83

$(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 6094932$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^20*e^x,x, algorithm="giac")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^
15 + 27907200*x^14 - 390700800*x^13 + 5079110400*x^12 - 609493248
00*x^11 + 670442572800*x^10 - 6704425728000*x^9 + 60339831552000*
x^8 - 482718652416000*x^7 + 3379030566912000*x^6 - 20274183401472
000*x^5 + 101370917007360000*x^4 - 405483668029440000*x^3 + 12164
51004088320000*x^2 - 2432902008176640000*x + 2432902008176640000)
*e^x
```

3.160 $\int a^x b^{-x} \, dx$

Optimal. Leaf size=18

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Rubi [A] time = 0.0332808, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In]  Int[a^x/b^x,x]

[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Rubi in Sympy [A] time = 1.86403, size = 15, normalized size = 0.83

$$\frac{e^{x(\log{(a)} - \log{(b)})}}{\log{(a)} - \log{(b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**x/(b**x),x)

[Out] exp(x*(log(a) - log(b)))/(log(a) - log(b))
```

Mathematica [A] time = 0.0056509, size = 18, normalized size = 1.

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^x/b^x,x]
```

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Maple [A] time = 0.002, size = 19, normalized size = 1.1

$$\frac{a^x}{b^x\left(\ln\left(a\right)-\ln\left(b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/(b^x),x)
```

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/b^x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.211712, size = 24, normalized size = 1.33

$$\frac{a^x}{b^x(\log\left(a\right)-\log\left(b\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/b^x,x, algorithm="fricas")
```

```
[Out] a^x/(b^x*(log(a) - log(b)))
```

Sympy [A] time = 0.963115, size = 17, normalized size = 0.94

$$\begin{cases}\frac{a^x}{b^x\log\left(a\right)-b^x\log\left(b\right)} & \text{for } a\neq b\\ x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x/(b**x),x)
```

```
[Out] Piecewise((a**x/(b**x*log(a) - b**x*log(b)), Ne(a, b)), (x, True)
)
```

GIAC/XCAS [A] time = 0.222027, size = 292, normalized size = 16.22

$$2\left(\frac{2\left(\ln\left(|a|\right)-\ln\left(|b|\right)\right)\cos\left(-\frac{1}{2}\,\pi x\mathrm{sign}\left(a\right)+\frac{1}{2}\,\pi x\mathrm{sign}\left(b\right)\right)}{\left(\pi\mathrm{sign}\left(a\right)-\pi\mathrm{sign}\left(b\right)\right)^{2}+4\left(\ln\left(|a|\right)-\ln\left(|b|\right)\right)^{2}}-\frac{\left(\pi\mathrm{sign}\left(a\right)-\pi\mathrm{sign}\left(b\right)\right)\sin\left(-\frac{1}{2}\,\pi x\mathrm{sign}\left(a\right)+\frac{1}{2}\,\pi x\mathrm{sign}\left(b\right)\right)}{\left(\pi\mathrm{sign}\left(a\right)-\pi\mathrm{sign}\left(b\right)\right)^{2}+4\left(\ln\left(|a|\right)-\ln\left(|b|\right)\right)^{2}}\right)e^{\left(x\left(\ln\left(|a|\right)-\ln\left(|b|\right)\right)\right)}-\frac{1}{2}\,i\left(-\frac{2i\,e^{\left(\frac{1}{2}\,i\,\pi x\mathrm{sign}(a)-\frac{1}{2}\,i\,\pi x\mathrm{sign}(b)\right)}}{i\,\pi\mathrm{sign}\left(a\right)-i\,\pi\mathrm{sign}\left(b\right)+2\ln\left(|a|\right)-2\ln\left(|b|\right)}+\frac{2i\,e^{\left(-\frac{1}{2}\,i\,\pi x\mathrm{sign}(a)+\frac{1}{2}\,i\,\pi x\mathrm{sign}(b)\right)}}{-i\,\pi\mathrm{sign}\left(a\right)+i\,\pi\mathrm{sign}\left(b\right)+2\ln\left(|a|\right)-2\ln\left(|b|\right)}\right)e^{\left(x\left(\ln\left(|a|\right)-\ln\left(|b|\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x/b^x,x, algorithm="giac")
```

```
[Out] 2*(2*(ln(abs(a)) - ln(abs(b)))*cos(-1/2*pi*x*sign(a) + 1/2*pi*x*s
ign(b))/((pi*sign(a) - pi*sign(b))^2 + 4*(ln(abs(a)) - ln(abs(b))
)^2) - (pi*sign(a) - pi*sign(b))*sin(-1/2*pi*x*sign(a) + 1/2*pi*x
*sign(b))/((pi*sign(a) - pi*sign(b))^2 + 4*(ln(abs(a)) - ln(abs(b
)))^2))*e^(x*(ln(abs(a)) - ln(abs(b)))) - 1/2*I*(-2*I*e^(1/2*I*pi
*x*sign(a) - 1/2*I*pi*x*sign(b))/(I*pi*sign(a) - I*pi*sign(b) + 2
*ln(abs(a)) - 2*ln(abs(b))) + 2*I*e^(-1/2*I*pi*x*sign(a) + 1/2*I*
pi*x*sign(b))/(-I*pi*sign(a) + I*pi*sign(b) + 2*ln(abs(a)) - 2*ln
(abs(b))))*e^(x*(ln(abs(a)) - ln(abs(b))))
```

3.161 $\int a^x b^x \, dx$

Optimal. Leaf size=14

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Rubi [A] time = 0.0240266, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In]  Int[a^x*b^x,x]

[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Rubi in Sympy [A] time = 1.70607, size = 15, normalized size = 1.07

$$\frac{e^{x(\log(a)+\log(b))}}{\log(a) + \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**x*b**x,x)

[Out] exp(x*(log(a) + log(b)))/(log(a) + log(b))
```

Mathematica [A] time = 0.00426025, size = 14, normalized size = 1.

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Maple [A] time = 0.002, size = 15, normalized size = 1.1

$$\frac{a^x b^x}{\ln(a) + \ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^x*b^x,x)

[Out] a^x*b^x/(ln(a)+ln(b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*b^x,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221151, size = 19, normalized size = 1.36

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*b^x,x, algorithm="fricas")

[Out] a^x*b^x/(log(a) + log(b))
```

Sympy [A] time = 0.900027, size = 24, normalized size = 1.71

$$\begin{cases} \frac{a^x b^x}{\log(a)+\log(b)} & \text{for } a \neq \frac{1}{b} \\ \tilde{\infty} b^x \left(\frac{1}{b}\right)^x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x*b**x,x)
```

```
[Out] Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (zoo*b**x*(1
/b)**x, True))
```

GIAC/XCAS [A] time = 0.214541, size = 320, normalized size = 22.86

$$2\left(\frac{2(\ln(|a|)+\ln(|b|))\cos\left(-\frac{1}{2}\pi x\text{sign}(a)-\frac{1}{2}\pi x\text{sign}(b)+\pi x\right)}{(2\pi-\pi\text{sign}(a)-\pi\text{sign}(b))^2+4(\ln(|a|)+\ln(|b|))^2}+\frac{(2\pi-\pi\text{sign}(a)-\pi\text{sign}(b))\sin\left(-\frac{1}{2}\pi x\text{sign}(a)-\frac{1}{2}\pi x\text{s}\right.}{(2\pi-\pi\text{sign}(a)-\pi\text{sign}(b))^2+4(\ln(|a|)+\ln(|}\right.$$
$$-\frac{1}{2}i\left(-\frac{2ie^{\left(\frac{1}{2}i\pi x\text{sign}(a)+\frac{1}{2}i\pi x\text{sign}(b)-i\pi x\right)}}{-2i\pi+i\pi\text{sign}(a)+i\pi\text{sign}(b)+2\ln(|a|)+2\ln(|b|)}+\frac{2ie^{\left(-\frac{1}{2}i\pi x\text{sign}(a)-\frac{1}{2}i\pi x\text{sign}(b)+i\pi x\right)}}{2i\pi-i\pi\text{sign}(a)-i\pi\text{sign}(b)+2\ln(|a|)+2\ln(|b|)}\right)e^{(x(\ln}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*b^x,x, algorithm="giac")
```

```
[Out] 2*(2*(ln(abs(a)) + ln(abs(b)))*cos(-1/2*pi*x*sign(a) - 1/2*pi*x*s
ign(b) + pi*x)/((2*pi - pi*sign(a) - pi*sign(b))^2 + 4*(ln(abs(a)
) + ln(abs(b)))^2) + (2*pi - pi*sign(a) - pi*sign(b))*sin(-1/2*pi
*x*sign(a) - 1/2*pi*x*sign(b) + pi*x)/((2*pi - pi*sign(a) - pi*si
gn(b))^2 + 4*(ln(abs(a)) + ln(abs(b)))^2))*e^(x*(ln(abs(a)) + ln(
abs(b)))) - 1/2*I*(-2*I*e^(1/2*I*pi*x*sign(a) + 1/2*I*pi*x*sign(b
) - I*pi*x)/(-2*I*pi + I*pi*sign(a) + I*pi*sign(b) + 2*ln(abs(a))
 + 2*ln(abs(b))) + 2*I*e^(-1/2*I*pi*x*sign(a) - 1/2*I*pi*x*sign(b
) + I*pi*x)/(2*I*pi - I*pi*sign(a) - I*pi*sign(b) + 2*ln(abs(a))
+ 2*ln(abs(b))))*e^(x*(ln(abs(a)) + ln(abs(b))))
```

3.162 $\int \frac{a^x}{x^2} \, dx$

Optimal. Leaf size=17

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Rubi [A] time = 0.0301664, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

```
[In] Int[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Rubi in Sympy [A] time = 2.24261, size = 14, normalized size = 0.82

$$-\frac{a^x}{x} + \log(a)\,\text{Ei}(x\log(a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(a**x/x**2,x)
```

```
[Out] -a**x/x + log(a)*Ei(x*log(a))
```

Mathematica [A] time = 0.00659357, size = 17, normalized size = 1.

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Maple [A] time = 0.011, size = 21, normalized size = 1.2

$$-\frac{a^x}{x} - \ln(a)\, Ei(1, -x\ln(a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^x/x^2,x)
```

```
[Out] -a^x/x-ln(a)*Ei(1,-x*ln(a))
```

Maxima [A] time = 1.48018, size = 14, normalized size = 0.82

$$(-1, -x\log(a))\log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x/x^2,x, algorithm="maxima")
```

```
[Out] gamma(-1, -x*log(a))*log(a)
```

Fricas [A] time = 0.226367, size = 26, normalized size = 1.53

$$\frac{x\mathrm{Ei}\,(x\log(a))\log(a) - a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x/x^2,x, algorithm="fricas")
```

```
[Out] (x*Ei(x*log(a))*log(a) - a^x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x/x**2,x)
```

```
[Out] Integral(a**x/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x/x^2,x, algorithm="giac")
```

```
[Out] integrate(a^x/x^2, x)
```

3.163 $\int \frac{a^x x}{(1+bx)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{a^{-1/b}\log(a)\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^3}+\frac{a^{-1/b}\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^2}+\frac{a^x}{b^2(bx+1)}$$

```
[Out] a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-
1)*b^2) - (ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*
b^3)
```

Rubi [A] time = 0.153365, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^{-1/b}\log(a)\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^3}+\frac{a^{-1/b}\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^2}+\frac{a^x}{b^2(bx+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-
1)*b^2) - (ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*
b^3)
```

Rubi in Sympy [A] time = 7.41885, size = 54, normalized size = 0.84

$$\frac{a^x}{b^2\,(bx+1)}+\frac{a^{-\frac{1}{b}}\operatorname{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^2}-\frac{a^{-\frac{1}{b}}\log(a)\operatorname{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**x*x/(b*x+1)**2,x)
```

```
[Out] a**x/(b**2*(b*x + 1)) + a**(-1/b)*Ei((b*x + 1)*log(a)/b)/b**2 - a
**(-1/b)*log(a)*Ei((b*x + 1)*log(a)/b)/b**3
```

Mathematica [A] time = 0.0507419, size = 43, normalized size = 0.67

$$\frac{a^{-1/b}(b-\log(a))\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)+\frac{ba^x}{bx+1}}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] ((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Lo
g[a]))/a^b^(-1))/b^3
```

Maple [A] time = 0.02, size = 79, normalized size = 1.2

$$-\frac{1}{b^2}a^{-b^{-1}}Ei\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)+\frac{a^x\ln(a)}{b^3}\left(x\ln(a)+\frac{\ln(a)}{b}\right)^{-1}+\frac{\ln(a)}{b^3}a^{-b^{-1}}Ei\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^x*x/(b*x+1)^2,x)
```

```
[Out] -1/b^2*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)+ln(a)/b^3*a^x/(x*ln(a)+ln(
a)/b)+ln(a)/b^3*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^x x}{b^2x^2\log(a)+2\,bx\log(a)+\log(a)}+\int\frac{(bx-1)a^x}{b^3x^3\log(a)+3\,b^2x^2\log(a)+3\,bx\log(a)+\log(a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*x/(b*x + 1)^2,x, algorithm="maxima")
```

```
[Out] a^x*x/(b^2*x^2*log(a) + 2*b*x*log(a) + log(a)) + integrate((b*x -
 1)*a^x/(b^3*x^3*log(a) + 3*b^2*x^2*log(a) + 3*b*x*log(a) + log(a
)), x)
```

Fricas [A] time = 0.25096, size = 73, normalized size = 1.14

$$\frac{a^x b+\frac{\left(b^2x-(bx+1)\log(a)+b\right)\mathrm{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{a^{\left(\frac{1}{b}\right)}}}{b^4x+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*x/(b*x + 1)^2,x, algorithm="fricas")
```

```
[Out] (a^x*b + (b^2*x - (b*x + 1)*log(a) + b)*Ei((b*x + 1)*log(a)/b)/a^
(1/b))/(b^4*x + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x x}{(bx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x*x/(b*x+1)**2,x)
```

```
[Out] Integral(a**x*x/(b*x + 1)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x x}{(bx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*x/(b*x + 1)^2,x, algorithm="giac")
```

```
[Out] integrate(a^x*x/(b*x + 1)^2, x)
```

3.164 $\int \frac{e^{ax}x}{(1+ax)^2}\,dx$

Optimal. Leaf size=16

$$\frac{e^{ax}}{a^2(ax+1)}$$

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Rubi [A] time = 0.0508024, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^(a*x)*x)/(1 + a*x)^2,x]

[Out] E^(a*x)/(a^2*(1 + a*x))
```

Rubi in Sympy [A] time = 2.81589, size = 12, normalized size = 0.75

$$\frac{e^{ax}}{a^2\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x)*x/(a*x+1)**2,x)

[Out] exp(a*x)/(a**2*(a*x + 1))
```

Mathematica [A] time = 0.0102228, size = 16, normalized size = 1.

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]
```

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Maple [A] time = 0.003, size = 16, normalized size = 1.

$$\frac{e^{ax}}{a^2 (ax + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(a*x)*x/(a*x+1)^2,x)
```

```
[Out] exp(a*x)/a^2/(a*x+1)
```

Maxima [A] time = 1.3671, size = 22, normalized size = 1.38

$$\frac{e^{(ax)}}{a^3 x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(a*x)/(a*x + 1)^2,x, algorithm="maxima")
```

```
[Out] e^(a*x)/(a^3*x + a^2)
```

Fricas [A] time = 0.198744, size = 22, normalized size = 1.38

$$\frac{e^{(ax)}}{a^3 x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(a*x)/(a*x + 1)^2,x, algorithm="fricas")
```

```
[Out] e^(a*x)/(a^3*x + a^2)
```

Sympy [A] time = 0.124263, size = 12, normalized size = 0.75

$$\frac{e^{ax}}{a^3 x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x)*x/(a*x+1)**2,x)
```

```
[Out] exp(a*x)/(a**3*x + a**2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(a*x)/(a*x + 1)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.165 $\int k^{x^2} x \, dx$

Optimal. Leaf size=13

$$\frac{k^{x^2}}{2\log(k)}$$

```
[Out] k^x^2/(2*Log[k])
```

Rubi [A] time = 0.0131667, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

```
[In]  Int[k^x^2*x,x]
```

```
[Out] k^x^2/(2*Log[k])
```

Rubi in Sympy [A] time = 1.10979, size = 8, normalized size = 0.62

$$\frac{k^{x^2}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(k**(x**2)*x,x)
```

```
[Out] k**(x**2)/(2*log(k))
```

Mathematica [A] time = 0.00272018, size = 13, normalized size = 1.

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[k^x^2*x,x]
```

```
[Out] k^x^2/(2*Log[k])
```

Maple [A] time = 0.002, size = 12, normalized size = 0.9

$$\frac{k^{x^2}}{2\ln(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(k^(x^2)*x,x)

[Out] 1/2*k^(x^2)/ln(k)
```

Maxima [A] time = 1.41374, size = 15, normalized size = 1.15

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(k^(x^2)*x,x, algorithm="maxima")

[Out] 1/2*k^(x^2)/log(k)
```

Fricas [A] time = 0.210551, size = 15, normalized size = 1.15

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(k^(x^2)*x,x, algorithm="fricas")

[Out] 1/2*k^(x^2)/log(k)
```

Sympy [A] time = 0.082227, size = 17, normalized size = 1.31

$$\begin{cases} \frac{k^{x^2}}{2\log(k)} & \text{for}\ 2\log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(k**(x**2)*x,x)
```

```
[Out] Piecewise((k**(x**2)/(2*log(k)), Ne(2*log(k), 0)), (x**2/2, True)
)
```

GIAC/XCAS [A] time = 0.19762, size = 15, normalized size = 1.15

$$\frac{k^{(x^2)}}{2\ln(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(k^(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*k^(x^2)/ln(k)
```

3.166 $\int e^{x^2} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rubi [A] time = 0.00665885, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rubi in Sympy [A] time = 0.486834, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\text{erfi}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x**2),x)

[Out] sqrt(pi)*erfi(x)/2
```

Mathematica [A] time = 0.00213845, size = 11, normalized size = 1.

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$\frac{\mathit{erfi}(x)\,\sqrt{\pi}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2),x)
```

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Maxima [A] time = 1.45931, size = 12, normalized size = 1.09

$$-\frac{1}{2}i\,\sqrt{\pi}\,\mathrm{erf}\,(i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Fricas [A] time = 0.237532, size = 9, normalized size = 0.82

$$\frac{1}{2}\,\sqrt{\pi}\,\mathrm{erfi}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A] time = 0.288247, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\mathrm{erfi}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

GIAC/XCAS [A] time = 0.201048, size = 12, normalized size = 1.09

$$\frac{1}{2}i\sqrt{\pi}\,\mathrm{erf}\left(-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

3.167 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

```
[Out] E^x^2/2
```

Rubi [A] time = 0.0111309, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Rubi in Sympy [A] time = 1.01296, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Mathematica [A] time = 0.00192534, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Maple [A] time = 0.001, size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*x,x)
```

```
[Out] 1/2*exp(x^2)
```

Maxima [A] time = 1.39441, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

Fricas [A] time = 0.197369, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.06744, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

GIAC/XCAS [A] time = 0.206934, size = 8, normalized size = 0.89

$$\frac{1}{2}\, e^{\left(x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

3.168 $\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx$

Optimal. Leaf size=27

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

```
[Out] -E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

Rubi [A] time = 0.157809, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x^(-1)*(1 + x))/x^4,x]

[Out] -E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)\,e^{\frac{1}{x}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1/x)*(1+x)/x**4,x)

[Out] Integral((x + 1)*exp(1/x)/x**4, x)
```

Mathematica [A] time = 0.00798998, size = 16, normalized size = 0.59

$$e^{\frac{1}{x}}\left(-\frac{1}{x^2}+\frac{1}{x}-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x^(-1)*(1 + x))/x^4,x]
```

```
[Out] E^x^(-1)*(-1 - x^(-2) + x^(-1))
```

Maple [A] time = 0.001, size = 18, normalized size = 0.7

$$-\frac{(x^2 - x + 1)\,e^{x^{-1}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1/x)*(1+x)/x^4,x)
```

```
[Out] -(x^2-x+1)*exp(1/x)/x^2
```

Maxima [A] time = 1.42257, size = 23, normalized size = 0.85

$$-\left(3, -\frac{1}{x}\right) + \left(2, -\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)*e^(1/x)/x^4,x, algorithm="maxima")
```

```
[Out] -gamma(3, -1/x) + gamma(2, -1/x)
```

Fricas [A] time = 0.20705, size = 23, normalized size = 0.85

$$-\frac{(x^2 - x + 1)\,e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)*e^(1/x)/x^4,x, algorithm="fricas")
```

```
[Out] -(x^2 - x + 1)*e^(1/x)/x^2
```

Sympy [A] time = 0.080872, size = 14, normalized size = 0.52

$$\frac{(-x^2 + x - 1)\,e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(1/x)*(1+x)/x**4,x)
```

```
[Out] (-x**2 + x - 1)*exp(1/x)/x**2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)*e^(1/x)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.169 $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{e^{1-e^{x^2}x}}{e^{x^2}x-1}$$

```
[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

Rubi [F] time = 1.53434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]

[Out] Defer[Int][(E^(1 - E^x^2*x + 2*x^2)*x)/(-1 + E^x^2*x)^2, x] + 2*D
efer[Int][(E^(1 - E^x^2*x + 2*x^2)*x^3)/(-1 + E^x^2*x)^2, x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)

[Out] Timed out
```

Mathematica [A] time = 0.0405127, size = 25, normalized size = 1.

$$-\frac{e^{1-e^{x^2}x}}{e^{x^2}x-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]

[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

Maple [A] time = 0.041, size = 23, normalized size = 0.9

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x)

[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

Maxima [A] time = 1.94034, size = 30, normalized size = 1.2

$$-\frac{e^{\left(-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(x^2\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + x)*e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(x^2) - 1)^2,x, algorithm="m

[Out] -e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)
```

Fricas [A] time = 0.226716, size = 49, normalized size = 1.96

$$-\frac{e^{\left(2x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3x^2\right)}-e^{\left(2x^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + x)*e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(x^2) - 1)^2,x, algorithm="f

[Out] -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

Sympy [A] time = 0.640331, size = 31, normalized size = 1.24

$$-\frac{e^{2x^2-xe^{x^2}+1}}{xe^{3x^2}-e^{2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)
```

```
[Out] -exp(2*x**2 - x*exp(x**2) + 1)/(x*exp(3*x**2) - exp(2*x**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^3+x\right)e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{\left(xe^{\left(x^2\right)}-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + x)*e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(x^2) - 1)^2,x, algorithm="g
```

```
[Out] integrate((2*x^3 + x)*e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(x^2) - 1)^2
, x)
```

3.170 $\int e^{e^{e^{e^x}}}\,dx$

Optimal. Leaf size=12

$$\text{Int}\left(e^{e^{e^{e^x}}},x\right)$$

```
[Out] CannotIntegrate[E^E^E^E^x, x]
```

Rubi [A] time = 0.0711793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(e^{e^{e^{e^x}}},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[E^E^E^E^x,x]
```

```
[Out] Defer[Subst][Defer[Int][E^E^E^x/x, x], x, E^x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int^{e^x} \frac{e^{e^{e^x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(exp(exp(exp(x)))),x)
```

```
[Out] Integral(exp(exp(exp(x)))/x, (x, exp(x)))
```

Mathematica [A] time = 0.0115539, size = 0, normalized size = 0.

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^E^E^E^x,x]
```

```
[Out] Integrate[E^E^E^E^x, x]
```

Maple [A] time = 0.015, size = 0, normalized size = 0.

$$\int e^{e^{e^{e^{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(exp(exp(exp(x)))),x)
```

```
[Out] int(exp(exp(exp(exp(x)))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(e^{\left(e^{\left(e^{x}\right)}\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(e^(e^(e^x))),x, algorithm="maxima")
```

```
[Out] integrate(e^(e^(e^(e^x))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{\left(e^{\left(e^{\left(e^{x}\right)}\right)}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(e^(e^(e^x))),x, algorithm="fricas")
```

```
[Out] integral(e^(e^(e^(e^x))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int e^{e^{e^{e^{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(exp(exp(exp(x)))),x)

[Out] Integral(exp(exp(exp(exp(x)))), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(e^{\left(e^{\left(e^{x}\right)}\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(e^(e^(e^x))),x, algorithm="giac")

[Out] integrate(e^(e^(e^(e^x))), x)
```

3.171 $\int e^x \log(x)\,dx$

Optimal. Leaf size=11

$$e^x \log(x) - \text{ExpIntegralEi}(x)$$

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Rubi [A] time = 0.0257913, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$e^x \log(x) - \text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Log[x],x]

[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Rubi in Sympy [A] time = 2.11083, size = 8, normalized size = 0.73

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*ln(x),x)

[Out] exp(x)*log(x) - Ei(x)
```

Mathematica [A] time = 0.0040497, size = 11, normalized size = 1.

$$e^x \log(x) - \text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Log[x],x]

[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Maple [A] time = 0.008, size = 12, normalized size = 1.1

$$e^x \ln(x) + Ei(1, -x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*ln(x),x)
```

```
[Out] exp(x)*ln(x)+Ei(1,-x)
```

Maxima [A] time = 1.59863, size = 14, normalized size = 1.27

$$e^x \log(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*log(x),x, algorithm="maxima")
```

```
[Out] e^x*log(x) - Ei(x)
```

Fricas [A] time = 0.21789, size = 14, normalized size = 1.27

$$e^x \log(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*log(x),x, algorithm="fricas")
```

```
[Out] e^x*log(x) - Ei(x)
```

Sympy [A] time = 2.87832, size = 8, normalized size = 0.73

$$e^x \log(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*ln(x),x)
```

```
[Out] exp(x)*log(x) - Ei(x)
```

<u>GIAC/XCAS</u> [A] time = 0.197816, size = 14, normalized size = 1.27

$$e^x \ln(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*log(x),x, algorithm="giac")

[Out] e^x*ln(x) - Ei(x)
```

3.172 $\int e^x x \log(x)\, dx$

Optimal. Leaf size=22

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

```
[Out] -E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

Rubi [A] time = 0.0760769, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x*Log[x],x]

[Out] -E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

Rubi in Sympy [A] time = 4.45879, size = 20, normalized size = 0.91

$$xe^x \log(x) - e^x \log(x) - e^x + \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x*ln(x),x)

[Out] x*exp(x)*log(x) - exp(x)*log(x) - exp(x) + Ei(x)
```

Mathematica [A] time = 0.0109994, size = 17, normalized size = 0.77

$$\text{ExpIntegralEi}(x) - e^x + e^x(x-1)\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x*Log[x],x]

[Out] -E^x + ExpIntegralEi[x] + E^x*(-1 + x)*Log[x]
```

Maple [A] time = 0.009, size = 21, normalized size = 1.

$$(-1+x)\,\mathrm{e}^{x}\ln(x)-Ei(1,-x)-\mathrm{e}^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x*ln(x),x)
```

```
[Out] (-1+x)*exp(x)*ln(x)-Ei(1,-x)-exp(x)
```

Maxima [A] time = 1.41295, size = 20, normalized size = 0.91

$$(x-1)e^{x}\log(x)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x*log(x),x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Fricas [A] time = 0.220526, size = 20, normalized size = 0.91

$$(x-1)e^{x}\log(x)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x*log(x),x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{x}\log(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x*ln(x),x)
```

```
[Out] Integral(x*exp(x)*log(x), x)
```

GIAC/XCAS [A] time = 0.198027, size = 20, normalized size = 0.91

$$(x-1)e^{x}\ln(x)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x*log(x),x, algorithm="giac")
```

```
[Out] (x - 1)*e^x*ln(x) + Ei(x) - e^x
```

3.173 $\int e^{2x}\log\left(e^x\right)\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}e^{2x}\log\left(e^x\right)-\frac{e^{2x}}{4}$$

```
[Out] -E^(2*x)/4 + (E^(2*x)*Log[E^x])/2
```

Rubi [A] time = 0.0192684, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}e^{2x}\log\left(e^x\right)-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)*Log[E^x],x]
```

```
[Out] -E^(2*x)/4 + (E^(2*x)*Log[E^x])/2
```

Rubi in Sympy [A] time = 2.28131, size = 17, normalized size = 0.74

$$\frac{e^{2x}\log\left(e^x\right)}{2}-\frac{e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)*ln(exp(x)),x)
```

```
[Out] exp(2*x)*log(exp(x))/2 - exp(2*x)/4
```

Mathematica [A] time = 0.00468039, size = 17, normalized size = 0.74

$$\frac{1}{4}e^{2x}\left(2\log\left(e^x\right)-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)*Log[E^x],x]
```

```
[Out] (E^(2*x)*(-1 + 2*Log[E^x]))/4
```

Maple [A] time = 0.003, size = 28, normalized size = 1.2

$$\frac{e^{2x}x}{2}-\frac{e^{2x}}{4}+\frac{e^{2x}\left(\ln\left(e^{x}\right)-x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)*ln(exp(x)),x)
```

```
[Out] 1/2*exp(2*x)*x-1/4*exp(2*x)+1/2*exp(2*x)*(ln(exp(x))-x)
```

Maxima [A] time = 1.4755, size = 15, normalized size = 0.65

$$\frac{1}{4}\left(2\,x-1\right)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)*log(e^x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Fricas [A] time = 0.209962, size = 15, normalized size = 0.65

$$\frac{1}{4}\left(2\,x-1\right)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)*log(e^x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Sympy [A] time = 0.077773, size = 10, normalized size = 0.43

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)*ln(exp(x)),x)
```

```
[Out] (2*x - 1)*exp(2*x)/4
```

<u>GIAC/XCAS</u> [A] time = 0.19792, size = 15, normalized size = 0.65

$$\frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)*log(e^x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

3.174 $\int \left(2x+\sqrt{2}x^2\right)\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{2}x^3}{3}+x^2$$

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Rubi [A] time = 0.00750072, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{\sqrt{2}x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In]  Int[2*x + Sqrt[2]*x^2,x]

[Out] x^2 + (Sqrt[2]*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}x^3}{3}+2\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2*x+x**2*2**(1/2),x)

[Out] sqrt(2)*x**3/3 + 2*Integral(x, x)
```

Mathematica [A] time = 0.000056637, size = 16, normalized size = 1.

$$\frac{\sqrt{2}x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$x^2 + \frac{x^3\sqrt{2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2*x+x^2*2^(1/2),x)
```

```
[Out] x^2+1/3*x^3*2^(1/2)
```

Maxima [A] time = 1.54767, size = 16, normalized size = 1.

$$\frac{1}{3}\sqrt{2}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2)*x^2 + 2*x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2)*x^2 + 2*x,x, algorithm="fricas")
```

```
[Out] Exception raised: SyntaxError
```

Sympy [A] time = 0.02803, size = 12, normalized size = 0.75

$$\frac{\sqrt{2}x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x+x**2*2**(1/2),x)

[Out] sqrt(2)*x**3/3 + x**2
```

GIAC/XCAS [A] time = 0.198786, size = 16, normalized size = 1.

$$\frac{1}{3}\sqrt{2}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2)*x^2 + 2*x,x, algorithm="giac")

[Out] 1/3*sqrt(2)*x^3 + x^2
```

3.175 $\int \frac{\log(x)}{\sqrt{b+ax}}\,dx$

Optimal. Leaf size=57

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

```
[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])
/a + (2*Sqrt[b + a*x]*Log[x])/a
```

Rubi [A] time = 0.0615337, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])
/a + (2*Sqrt[b + a*x]*Log[x])/a
```

Rubi in Sympy [A] time = 3.8454, size = 49, normalized size = 0.86

$$\frac{4\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}+\frac{2\sqrt{ax+b}\log(x)}{a}-\frac{4\sqrt{ax+b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)/(a*x+b)**(1/2),x)
```

```
[Out] 4*sqrt(b)*atanh(sqrt(a*x + b)/sqrt(b))/a + 2*sqrt(a*x + b)*log(x)
/a - 4*sqrt(a*x + b)/a
```

Mathematica [A] time = 0.0417066, size = 43, normalized size = 0.75

$$\frac{2(\log(x)-2)\sqrt{ax+b}+4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 +
 Log[x]))/a
```

Maple [A] time = 0.008, size = 48, normalized size = 0.8

$$4\,\frac{\sqrt{b}}{a}\mathit{Artanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-4\,\frac{\sqrt{ax+b}}{a}+2\,\frac{\ln(x)\sqrt{ax+b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)/(a*x+b)^(1/2),x)
```

```
[Out] 4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln
(x)*(a*x+b)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/sqrt(a*x + b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228977, size = 1, normalized size = 0.02

$$\left[\frac{2\left(\sqrt{ax+b}(\log(x)-2)+\sqrt{b}\log\left(\frac{ax+2\sqrt{ax+b}\sqrt{b}+2b}{x}\right)\right)}{a}, \frac{2\left(\sqrt{ax+b}(\log(x)-2)+2\sqrt{-b}\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/sqrt(a*x + b),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(a*x + b)*(log(x) - 2) + sqrt(b)*log((a*x + 2*sqrt(a*x +
b)*sqrt(b) + 2*b)/x))/a, 2*(sqrt(a*x + b)*(log(x) - 2) + 2*sqrt(-
```

```
b)*arctan(sqrt(a*x + b)/sqrt(-b)))/a]
```

Sympy [A] time = 6.46125, size = 920, normalized size = 16.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/(a*x+b)**(1/2),x)
```

```
[Out] Piecewise((4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2
*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b
/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a)
 - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sqrt(a), (Abs(x
 + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt
(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a)
 - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)
*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), Abs(
x + b/a) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/
a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(
x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sq
rt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sqrt(a), (
Abs(1/(x + b/a)) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*a
tanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a
)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt
(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt
(a), Abs(1/(x + b/a)) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqr
t(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2
*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a
)/sqrt(a) + meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(
b/a)/sqrt(a) + I*pi*meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b
/a)/sqrt(a) + meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*lo
g(b/a)/sqrt(a) + I*pi*meijerg(((3/2, 1), ()), ((), (1/2, 0)), x +
 b/a)/sqrt(a), Abs(b/(a*(x + b/a))) > 1), (4*sqrt(b)*atanh(sqrt(b
)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)
))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4
*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt(x + b/a)/sqrt(a) + meijerg((
(1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(b/a)/sqrt(a) + I*pi*m
eijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)/sqrt(a) + meijerg
(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi
*meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.200254, size = 65, normalized size = 1.14

$$-\frac{2\left(\frac{2\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{ax+b}\ln\left(x\right) + 2\,\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/sqrt(a*x + b),x, algorithm="giac")
```

```
[Out] -2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*l
n(x) + 2*sqrt(a*x + b))/a
```

3.176 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3
/2)*Sqrt[c + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a
 + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rubi [A] time = 0.138142, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3
/2)*Sqrt[c + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a
 + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rubi in Sympy [A] time = 7.75652, size = 97, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}{2d}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(ad-bc)}{4bd}-\frac{(ad-bc)^2\,\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{\frac{3}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] sqrt(a + b*x)*(c + d*x)**(3/2)/(2*d) + sqrt(a + b*x)*sqrt(c + d*x
)*(a*d - b*c)/(4*b*d) - (a*d - b*c)**2*atanh(sqrt(d)*sqrt(a + b*x
)/(sqrt(b)*sqrt(c + d*x)))/(4*b**(3/2)*d**(3/2))
```

Mathematica [A] time = 0.0821976, size = 110, normalized size = 0.95

$$\sqrt{a+bx}\sqrt{c+dx}\left(\frac{ad+bc}{4bd}+\frac{x}{2}\right)-\frac{(bc-ad)^2\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}+ad+bc+2bdx\right)}{8b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c + a*d)/(4*b*d) + x/2)*Sqrt[a + b*x]*Sqrt[c + d*x] - ((b*c -
 a*d)^2*Log[b*c + a*d + 2*b*d*x + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]
*Sqrt[c + d*x]])/(8*b^(3/2)*d^(3/2))
```

Maple [B] time = 0.011, size = 305, normalized size = 2.6

$$\frac{1}{2d}\sqrt{bx+a}(dx+c)^{\frac{3}{2}}+\frac{a}{4b}\sqrt{bx+a}\sqrt{dx+c}-\frac{c}{4d}\sqrt{bx+a}\sqrt{dx+c}$$
$$-\frac{da^2}{8b}\sqrt{(bx+a)(dx+c)}\ln\left(1\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{bdx^2+(ad+bc)x+ac}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{bd}}$$
$$+\frac{ac}{4}\sqrt{(bx+a)(dx+c)}\ln\left(1\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{bdx^2+(ad+bc)x+ac}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{bd}}$$
$$-\frac{c^2b}{8d}\sqrt{(bx+a)(dx+c)}\ln\left(1\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{bdx^2+(ad+bc)x+ac}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)+1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/
2)*a-1/4/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c-1/8*d/b*((b*x+a)*(d*x+c)
)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b
*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2+1/4*((
b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2
*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1
/2)*a*c-1/8/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)
*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)
^(1/2))/(b*d)^(1/2)*c^2*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*sqrt(d*x + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249978, size = 1, normalized size = 0.01

$$\left[\frac{4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(-4\,\left(2\,b^2d^2x+b^2cd+abd^2\right)\sqrt{bx+a}\sqrt{dx+c}+\left(8\,b^2c\right.\right.}{16\,\sqrt{bd}bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*sqrt(d*x + c),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x +
 c) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-4*(2*b^2*d^2*x + b^2*c
*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x)*sqrt(b*d)))/
(sqrt(b*d)*b*d), 1/8*(2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*
(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x + a)*sqrt(d*x + c)*b*d
)))/(sqrt(-b*d)*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

GIAC/XCAS [A] time = 0.21768, size = 189, normalized size = 1.63

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^4d^2}+\frac{bcd-ad^2}{b^4d^4}\right)+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\ln\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}b^3d^3}\right)|b|}{96\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*sqrt(d*x + c),x, algorithm="giac")
```

```
[Out] 1/96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x +
 a)/(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*ln(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^3))*abs(b)/b^3
```

3.177 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rubi [A] time = 0.00632222, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rubi in Sympy [A] time = 0.635795, size = 12, normalized size = 0.75

$$\frac{2\left(a+bx\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

Mathematica [A] time = 0.00548195, size = 16, normalized size = 1.

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2}{3\,b}\,(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^(1/2),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Maxima [A] time = 1.3472, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A] time = 0.201732, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A] time = 0.032253, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

GIAC/XCAS [A] time = 0.199239, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

3.178 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rubi [A] time = 0.0238743, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rubi in Sympy [A] time = 2.43236, size = 31, normalized size = 0.91

$$-\frac{2a\left(a+bx\right)^{\frac{3}{2}}}{3b^2}+\frac{2\left(a+bx\right)^{\frac{5}{2}}}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -2*a*(a + b*x)**(3/2)/(3*b**2) + 2*(a + b*x)**(5/2)/(5*b**2)
```

Mathematica [A] time = 0.0123065, size = 34, normalized size = 1.

$$\frac{2\sqrt{a+bx}\left(-2a^2+abx+3b^2x^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)
```

Maple [A] time = 0.002, size = 21, normalized size = 0.6

$$-\frac{-6\,bx+4\,a}{15\,b^{2}}\left(bx+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x+a)^(1/2),x)

[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Maxima [A] time = 1.34248, size = 35, normalized size = 1.03

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^{2}}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*x,x, algorithm="maxima")

[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Fricas [A] time = 0.202072, size = 41, normalized size = 1.21

$$\frac{2\left(3\,b^{2}x^{2}+abx-2\,a^{2}\right)\sqrt{bx+a}}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*x,x, algorithm="fricas")

[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [A] time = 1.69484, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(
9/2)/(15*a**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a
)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 1
5*a*b**3*x) + 8*a**(5/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**2*b**2
+ 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**2*b*
*2 + 15*a*b**3*x)
```

GIAC/XCAS [A] time = 0.198204, size = 34, normalized size = 1.

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*x,x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/b^2
```

3.179 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) +
 (2*(a + b*x)^(7/2))/(7*b^3)
```

Rubi [A] time = 0.0381906, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) +
 (2*(a + b*x)^(7/2))/(7*b^3)
```

Rubi in Sympy [A] time = 4.03145, size = 49, normalized size = 0.92

$$\frac{2a^2\left(a+bx\right)^{\frac{3}{2}}}{3b^3}-\frac{4a\left(a+bx\right)^{\frac{5}{2}}}{5b^3}+\frac{2\left(a+bx\right)^{\frac{7}{2}}}{7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 2*a**2*(a + b*x)**(3/2)/(3*b**3) - 4*a*(a + b*x)**(5/2)/(5*b**3)
+ 2*(a + b*x)**(7/2)/(7*b**3)
```

Mathematica [A] time = 0.0152769, size = 46, normalized size = 0.87

$$\frac{2\sqrt{a+bx}\left(8a^3-4a^2bx+3ab^2x^2+15b^3x^3\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^3 - 4*a^2*b*x + 3*a*b^2*x^2 + 15*b^3*x^3))/
(105*b^3)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.6

$$\frac{30\,b^{2}x^{2}-24\,axb+16\,a^{2}}{105\,b^{3}}\left(bx+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.36013, size = 55, normalized size = 1.04

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{3}}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^{3}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*x^2,x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x +
a)^(3/2)*a^2/b^3
```

Fricas [A] time = 0.20006, size = 57, normalized size = 1.08

$$\frac{2\left(15\,b^{3}x^{3}+3\,ab^{2}x^{2}-4\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*x^2,x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a
)/b^3
```

Sympy [A] time = 2.70939, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 3
15*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*
b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
 + 40*a**(21/2)*b*x*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**
4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a**(21/2)*b*x
/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5
*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a**8*b*
*3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) -
 48*a**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a*
*6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(
1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
+ 105*a**5*b**6*x**3) - 16*a**(17/2)*b**3*x**3/(105*a**8*b**3 + 3
15*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 100*a
**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6
*b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1
```

```
+ b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3)
```

GIAC/XCAS [A] time = 0.198964, size = 62, normalized size = 1.17

$$\frac{2\left(15\,(bx+a)^{\frac{7}{2}}b^{12}-42\,(bx+a)^{\frac{5}{2}}ab^{12}+35\,(bx+a)^{\frac{3}{2}}a^{2}b^{12}\right)}{105\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)*x^2,x, algorithm="giac")
```

```
[Out] 2/105*(15*(b*x + a)^(7/2)*b^12 - 42*(b*x + a)^(5/2)*a*b^12 + 35*(
b*x + a)^(3/2)*a^2*b^12)/b^15
```

3.180 $\int \frac{\sqrt{a+bx}}{x}\,dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0337963, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x]/x,x]

[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi in Sympy [A] time = 2.25595, size = 31, normalized size = 0.89

$$-2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2\sqrt{a+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**(1/2)/x,x)

[Out] -2*sqrt(a)*atanh(sqrt(a + b*x)/sqrt(a)) + 2*sqrt(a + b*x)
```

Mathematica [A] time = 0.0158728, size = 35, normalized size = 1.

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.005, size = 28, normalized size = 0.8

$$-2\,\mathit{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^(1/2)/x,x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.22965, size = 1, normalized size = 0.03

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\sqrt{bx+a},\,-2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*
x + a), -2*sqrt(-a)*arctan(sqrt(b*x + a)/sqrt(-a)) + 2*sqrt(b*x +
 a)]
```

Sympy [A] time = 2.24177, size = 68, normalized size = 1.94

$$-2\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x
)*sqrt(a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

GIAC/XCAS [A] time = 0.200071, size = 43, normalized size = 1.23

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

3.181 $\int \frac{\sqrt{a+bx}}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0350426, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x]/x^2,x]

[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 2.39471, size = 32, normalized size = 0.82

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**(1/2)/x**2,x)

[Out] -sqrt(a + b*x)/x - b*atanh(sqrt(a + b*x)/sqrt(a))/sqrt(a)
```

Mathematica [A] time = 0.0326559, size = 39, normalized size = 1.

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.007, size = 37, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229515, size = 1, normalized size = 0.03

$$\left[\frac{bx\log\left(\frac{(bx+2\,a)\sqrt{a}-2\,\sqrt{bx+a}a}{x}\right)-2\,\sqrt{bx+a}\sqrt{a}}{2\,\sqrt{a}x},\frac{bx\arctan\left(\frac{a}{\sqrt{bx+a}\sqrt{-a}}\right)-\sqrt{bx+a}\sqrt{-a}}{\sqrt{-a}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(b*x*log(((b*x + 2*a)*sqrt(a) - 2*sqrt(b*x + a)*a)/x) - 2*sq
rt(b*x + a)*sqrt(a))/(sqrt(a)*x), (b*x*arctan(a/(sqrt(b*x + a)*sq
rt(-a))) - sqrt(b*x + a)*sqrt(-a))/(sqrt(-a)*x)]
```

Sympy [A] time = 2.92092, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqr
t(x)))/sqrt(a)
```

GIAC/XCAS [A] time = 0.205324, size = 55, normalized size = 1.41

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx+a}b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)
/b
```

3.182 $\int \frac{1}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

```
[Out] (2*Sqrt[a + b*x])/b
```

Rubi [A] time = 0.00767447, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b
```

Rubi in Sympy [A] time = 0.656061, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x+a)**(1/2),x)

[Out] 2*sqrt(a + b*x)/b
```

Mathematica [A] time = 0.00346158, size = 14, normalized size = 1.

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$2\,\frac{\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

Maxima [A] time = 1.3342, size = 16, normalized size = 1.14

$$\frac{2\,\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A] time = 0.212023, size = 16, normalized size = 1.14

$$\frac{2\,\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A] time = 0.033094, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

GIAC/XCAS [A] time = 0.199442, size = 16, normalized size = 1.14

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

3.183 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rubi [A] time = 0.0309536, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rubi in Sympy [A] time = 2.39822, size = 29, normalized size = 0.91

$$-\frac{2a\sqrt{a+bx}}{b^2}+\frac{2\left(a+bx\right)^{\frac{3}{2}}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -2*a*sqrt(a + b*x)/b**2 + 2*(a + b*x)**(3/2)/(3*b**2)
```

Mathematica [A] time = 0.0115927, size = 23, normalized size = 0.72

$$\frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Maple [A] time = 0.002, size = 21, normalized size = 0.7

$$-\frac{-2\,bx+4\,a}{3\,b^{2}}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x+a)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Maxima [A] time = 1.31949, size = 35, normalized size = 1.09

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^{2}}-\frac{2\,\sqrt{bx+a}a}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Fricas [A] time = 0.202241, size = 26, normalized size = 0.81

$$\frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x - 2*a)/b^2
```

Sympy [A] time = 1.7286, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^{2}b^{2}+3ab^{3}x}+\frac{4a^{\frac{7}{2}}}{3a^{2}b^{2}+3ab^{3}x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^{2}b^{2}+3ab^{3}x}+\frac{4a^{\frac{5}{2}}bx}{3a^{2}b^{2}+3ab^{3}x}+\frac{2a^{\frac{3}{2}}b^{2}x^{2}\sqrt{1+\frac{bx}{a}}}{3a^{2}b^{2}+3ab^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/
2)/(3*a**2*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3
*a**2*b**2 + 3*a*b**3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3
*x) + 2*a**(3/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x)
```

GIAC/XCAS [A] time = 0.197865, size = 31, normalized size = 0.97

$$\frac{2\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

3.184 $\int \frac{x^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a
 + b*x)^(5/2))/(5*b^3)
```

Rubi [A] time = 0.0418243, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a
 + b*x)^(5/2))/(5*b^3)
```

Rubi in Sympy [A] time = 3.91541, size = 48, normalized size = 0.94

$$\frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a\,(a+bx)^{\frac{3}{2}}}{3b^3}+\frac{2\,(a+bx)^{\frac{5}{2}}}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(a + b*x)/b**3 - 4*a*(a + b*x)**(3/2)/(3*b**3) + 2*(a
+ b*x)**(5/2)/(5*b**3)
```

Mathematica [A] time = 0.015539, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{6\,b^2x^2-8\,axb+16\,a^2}{15\,b^3}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.35141, size = 55, normalized size = 1.08

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x
+ a)*a^2/b^3
```

Fricas [A] time = 0.197783, size = 42, normalized size = 0.82

$$\frac{2\left(3\,b^2x^2-4\,abx+8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [A] time = 2.56468, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*
a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3
+ 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a*
*(19/2)*b*x*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a
**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(19/2)*b*x/(15*a**8*b*
*3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 30
*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4
*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x
**2/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 10*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3
 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a
**(15/2)*b**3*x**3/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4*x**4*sqrt(1 + b*x/a
)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b*
*6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

GIAC/XCAS [A] time = 0.198209, size = 62, normalized size = 1.22

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^8-10\,(bx+a)^{\frac{3}{2}}ab^8+15\,\sqrt{bx+a}a^2b^8\right)}{15\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2)*b^8 - 10*(b*x + a)^(3/2)*a*b^8 + 15*sqrt(
b*x + a)*a^2*b^8)/b^11
```

3.185 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0203436, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 1.54545, size = 22, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x+a)**(1/2),x)

[Out] -2*atanh(sqrt(a + b*x)/sqrt(a))/sqrt(a)
```

Mathematica [A] time = 0.0097966, size = 23, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-2\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.207207, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{(bx+2a)\sqrt{a}-2\sqrt{bx+a}a}{x}\right)}{\sqrt{a}}, \frac{2\arctan\left(\frac{a}{\sqrt{bx+a}\sqrt{-a}}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x),x, algorithm="fricas")
```

```
[Out] [log(((b*x + 2*a)*sqrt(a) - 2*sqrt(b*x + a)*a)/x)/sqrt(a), 2*arct
an(a/(sqrt(b*x + a)*sqrt(-a)))/sqrt(-a)]
```

Sympy [A] time = 1.67585, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

GIAC/XCAS [A] time = 0.199591, size = 28, normalized size = 1.22

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.186 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/
2)
```

Rubi [A] time = 0.0367683, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/
2)
```

Rubi in Sympy [A] time = 2.2741, size = 32, normalized size = 0.78

$$-\frac{\sqrt{a+bx}}{ax}+\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x)/(a*x) + b*atanh(sqrt(a + b*x)/sqrt(a))/a**(3/2)
```

Mathematica [A] time = 0.0230001, size = 41, normalized size = 1.

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/
2)
```

Maple [A] time = 0.005, size = 40, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{bx+a}}{axb}+1/2\,\frac{1}{a^{3/2}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/x/b+1/2/a^(3/2)*arctanh((b*x+a)^(1/2)/a
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218636, size = 1, normalized size = 0.02

$$\left[\frac{bx\log\left(\frac{(bx+2\,a)\sqrt{a}+2\,\sqrt{bx+a}a}{x}\right)-2\,\sqrt{bx+a}\sqrt{a}}{2\,a^{\frac{3}{2}}x},-\frac{bx\arctan\left(\frac{a}{\sqrt{bx+a}\sqrt{-a}}\right)+\sqrt{bx+a}\sqrt{-a}}{\sqrt{-a}ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*x*log(((b*x + 2*a)*sqrt(a) + 2*sqrt(b*x + a)*a)/x) - 2*sq
rt(b*x + a)*sqrt(a))/(a^(3/2)*x), -(b*x*arctan(a/(sqrt(b*x + a)*s
qrt(-a))) + sqrt(b*x + a)*sqrt(-a))/(sqrt(-a)*a*x)]
```

Sympy [A] time = 3.45636, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)
*sqrt(x)))/a**(3/2)
```

GIAC/XCAS [A] time = 0.200883, size = 63, normalized size = 1.54

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x^2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)
*b/(a*x))/b
```

3.187 $\int (a+bx)^{p/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

```
[Out] (2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

Rubi [A] time = 0.0129017, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^(p/2),x]

[Out] (2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

Rubi in Sympy [A] time = 0.89481, size = 15, normalized size = 0.65

$$\frac{2\left(a+bx\right)^{\frac{p}{2}+1}}{b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**(1/2))**p,x)

[Out] 2*(a + b*x)**(p/2 + 1)/(b*(p + 2))
```

Mathematica [A] time = 0.0145989, size = 24, normalized size = 1.04

$$\frac{2(a+bx)^{\frac{p}{2}+1}}{bp+2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2))/(2*b + b*p)
```

Maple [A] time = 0.001, size = 25, normalized size = 1.1

$$2\,\frac{(bx+a)\left(\sqrt{bx+a}\right)^{p}}{b\,(2+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)^(1/2))^p,x)
```

```
[Out] 2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.225041, size = 34, normalized size = 1.48

$$\frac{2\,(bx+a)\sqrt{bx+a}^{p}}{bp+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)^p,x, algorithm="fricas")
```

```
[Out] 2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)
```

Sympy [A] time = 0.04057, size = 26, normalized size = 1.13

$$\frac{\begin{cases}\frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for}\ \frac{p}{2}\neq -1\\ \log\left(a+bx\right) & \text{otherwise}\end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p/2, -1)), (log(a +
 b*x), True))/b
```

GIAC/XCAS [A] time = 0.199137, size = 28, normalized size = 1.22

$$\frac{2\,(bx + a)^{\frac{1}{2}\,p+1}}{b(p + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)^p,x, algorithm="giac")
```

```
[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

3.188 $\int x(a+bx)^{p/2}\,dx$

Optimal. Leaf size=48

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

```
[Out] (-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p
)/2))/(b^2*(4 + p))
```

Rubi [A] time = 0.0360198, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x)^(p/2),x]
```

```
[Out] (-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p
)/2))/(b^2*(4 + p))
```

Rubi in Sympy [A] time = 3.7542, size = 37, normalized size = 0.77

$$-\frac{2a\,(a+bx)^{\frac{p}{2}+1}}{b^2\,(p+2)}+\frac{2\,(a+bx)^{\frac{p}{2}+2}}{b^2\,(p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((b*x+a)**(1/2))**p,x)
```

```
[Out] -2*a*(a + b*x)**(p/2 + 1)/(b**2*(p + 2)) + 2*(a + b*x)**(p/2 + 2)
/(b**2*(p + 4))
```

Mathematica [A] time = 0.0207298, size = 38, normalized size = 0.79

$$\frac{2(a+bx)^{\frac{p}{2}+1}(b(p+2)x-2a)}{b^2(p+2)(p+4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2)*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p)
)
```

Maple [A] time = 0.003, size = 43, normalized size = 0.9

$$-2\,\frac{\left(\sqrt{bx+a}\right)^{p}(-xpb-2\,bx+2\,a)\,(bx+a)}{b^{2}\,(p^{2}+6\,p+8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((b*x+a)^(1/2))^p,x)
```

```
[Out] -2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)
```

Maxima [A] time = 1.36924, size = 61, normalized size = 1.27

$$\frac{2\,\left(b^{2}(p+2)x^{2}+abpx-2\,a^{2}\right)(bx+a)^{\frac{1}{2}\,p}}{(p^{2}+6\,p+8)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)^p*x,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6
*p + 8)*b^2)
```

Fricas [A] time = 0.234875, size = 78, normalized size = 1.62

$$\frac{2\,\left(abpx+\left(b^{2}p+2\,b^{2}\right)x^{2}-2\,a^{2}\right)\sqrt{bx+a}^{p}}{b^{2}p^{2}+6\,b^{2}p+8\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)^p*x,x, algorithm="fricas")
```

```
[Out] 2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^
2 + 6*b^2*p + 8*b^2)
```

Sympy [A] time = 0.919282, size = 216, normalized size = 4.5

$$\begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for } b = 0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p = -4 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p = -2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 +
b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x
), Eq(p, -4)), (-a*log(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*
(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 2*a*b*p*x*(a +
 b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 2*b**2*p*x**2*(a +
 b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 4*b**2*x**2*(a + b
*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))
```

GIAC/XCAS [A] time = 0.203847, size = 122, normalized size = 2.54

$$\frac{2\left(b^2px^2e^{\left(\frac{1}{2}p\ln(bx+a)\right)} + abpxe^{\left(\frac{1}{2}p\ln(bx+a)\right)} + 2b^2x^2e^{\left(\frac{1}{2}p\ln(bx+a)\right)} - 2a^2e^{\left(\frac{1}{2}p\ln(bx+a)\right)}\right)}{b^2p^2 + 6b^2p + 8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x + a)^p*x,x, algorithm="giac")
```

```
[Out] 2*(b^2*p*x^2*e^(1/2*p*ln(b*x + a)) + a*b*p*x*e^(1/2*p*ln(b*x + a)
) + 2*b^2*x^2*e^(1/2*p*ln(b*x + a)) - 2*a^2*e^(1/2*p*ln(b*x + a))
)/(b^2*p^2 + 6*b^2*p + 8*b^2)
```

3.189 $\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)\,dx$

Optimal. Leaf size=55

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 -
 Sqrt[2]*x + x^2]/(2*Sqrt[2])
```

Rubi [A] time = 0.0666246, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 -
 Sqrt[2]*x + x^2]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 3.58589, size = 56, normalized size = 1.02

$$x\,\mathrm{atan}\left(\sqrt{2}\left(x-\frac{\sqrt{2}}{2}\right)\right)-\frac{\sqrt{2}\log\left(4x^2-4\sqrt{2}x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)
```

```
[Out] x*atan(sqrt(2)*(x - sqrt(2)/2)) - sqrt(2)*log(4*x**2 - 4*sqrt(2)*
x + 4)/4 - sqrt(2)*atan(sqrt(2)*x - 1)/2
```

Mathematica [A] time = 0.063924, size = 48, normalized size = 0.87

$$\frac{1}{4}\left(2\left(\sqrt{2}-2x\right)\tan^{-1}\left(1-\sqrt{2}x\right)-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] (2*(Sqrt[2] - 2*x)*ArcTan[1 - Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2
]*x + x^2])/4
```

Maple [A] time = 0.006, size = 42, normalized size = 0.8

$$x\arctan\left(x\sqrt{2}-1\right)-\frac{\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{2}-\frac{\sqrt{2}\ln\left(\left(x\sqrt{2}-1\right)^{2}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x)
```

```
[Out] x*arctan(x*2^(1/2)-1)-1/2*arctan(x*2^(1/2)-1)*2^(1/2)-1/4*2^(1/2)
*ln((x*2^(1/2)-1)^2+1)
```

Maxima [A] time = 1.55182, size = 70, normalized size = 1.27

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^{2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(1/2*sqrt(2)*(2*x - sqrt(2))),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sq
rt(2))) - log(1/2*(2*x - sqrt(2))^2 + 1))
```

Fricas [A] time = 0.237161, size = 50, normalized size = 0.91

$$\frac{1}{4}\sqrt{2}\left(2\left(\sqrt{2}x-1\right)\arctan\left(\sqrt{2}x-1\right)-\log\left(x^{2}-\sqrt{2}x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(1/2*sqrt(2)*(2*x - sqrt(2))),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(2*(sqrt(2)*x - 1)*arctan(sqrt(2)*x - 1) - log(x^2 -
sqrt(2)*x + 1))
```

Sympy [A] time = 2.10909, size = 230, normalized size = 4.18

$$\frac{4x^3 \operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}-\frac{\sqrt{2}x^2 \log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}-\frac{6\sqrt{2}x^2 \operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}+\frac{2x\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}+\frac{8x\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}-\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)
```

```
[Out] 4*x**3*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*x
**2*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 6*sqrt
(2)*x**2*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 2*x*log
(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 8*x*atan(sqrt
(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*log(x**2 - sqrt(2
)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 2*sqrt(2)*atan(sqrt(2)*x -
1)/(4*x**2 - 4*sqrt(2)*x + 4)
```

GIAC/XCAS [A] time = 0.20187, size = 70, normalized size = 1.27

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\ln\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(1/2*sqrt(2)*(2*x - sqrt(2))),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sq
rt(2))) - ln(1/2*(2*x - sqrt(2))^2 + 1))
```

3.190 $\int \frac{1}{\sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=12

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.00633214, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-1 + x^2],x]

[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi in Sympy [A] time = 0.149264, size = 10, normalized size = 0.83

$$\operatorname{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2),x)

[Out] atanh(x/sqrt(x**2 - 1))
```

Mathematica [B] time = 0.00413546, size = 38, normalized size = 3.17

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-1 + x^2],x]
```

```
[Out] -Log[1 - x/Sqrt[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2),x)

[Out] ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 1.41897, size = 19, normalized size = 1.58

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1),x, algorithm="maxima")

[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.203619, size = 19, normalized size = 1.58

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A] time = 0.148382, size = 2, normalized size = 0.17

$$\operatorname{acosh}{\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2),x)
```

```
[Out] acosh(x)
```

GIAC/XCAS [A] time = 0.20183, size = 20, normalized size = 1.67

$$-\ln\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(x^2 - 1)))
```

3.191 $\int \sqrt{x}\sqrt{1+x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\sqrt{x+1}x^{3/2}+\frac{1}{4}\sqrt{x+1}\sqrt{x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (Sqrt[x]*Sqrt[1 + x])/4 + (x^(3/2)*Sqrt[1 + x])/2 - ArcSinh[Sqrt[
x]]/4
```

Rubi [A] time = 0.0203311, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{1}{2}\sqrt{x+1}x^{3/2}+\frac{1}{4}\sqrt{x+1}\sqrt{x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x])/4 + (x^(3/2)*Sqrt[1 + x])/2 - ArcSinh[Sqrt[
x]]/4
```

Rubi in Sympy [A] time = 1.73303, size = 34, normalized size = 0.79

$$\frac{\sqrt{x}\,(x+1)^{\frac{3}{2}}}{2}-\frac{\sqrt{x}\sqrt{x+1}}{4}-\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(1+x)**(1/2),x)
```

```
[Out] sqrt(x)*(x + 1)**(3/2)/2 - sqrt(x)*sqrt(x + 1)/4 - asinh(sqrt(x))
/4
```

Mathematica [A] time = 0.0250975, size = 31, normalized size = 0.72

$$\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}(2x+1)-\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*Sqrt[1 + x],x]

[Out] (Sqrt[x]*Sqrt[1 + x]*(1 + 2*x) - ArcSinh[Sqrt[x]])/4
```

Maple [A] time = 0.005, size = 50, normalized size = 1.2

$$\frac{1}{2}\sqrt{x}\,(1+x)^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}\sqrt{1+x}-\frac{1}{8}\sqrt{x\,(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(1+x)^(1/2),x)

[Out] 1/2*x^(1/2)*(1+x)^(3/2)-1/4*x^(1/2)*(1+x)^(1/2)-1/8*(x*(1+x))^(1/
2)/(1+x)^(1/2)/x^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 1.36398, size = 96, normalized size = 2.23

$$\frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^{2}}{x^{2}}-\frac{2\,(x+1)}{x}+1\right)}-\frac{1}{8}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{8}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*sqrt(x),x, algorithm="maxima")

[Out] 1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2
- 2*(x + 1)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(s
qrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 0.211217, size = 158, normalized size = 3.67

$$\frac{128\,x^{4}+256\,x^{3}-4\left(32\,x^{3}+48\,x^{2}+18\,x+1\right)\sqrt{x+1}\sqrt{x}+152\,x^{2}+4\left(4\,(2\,x+1)\sqrt{x+1}\sqrt{x}-8\,x^{2}-8\,x-1\right)\log\left(2\sqrt{x+1}\sqrt{\right.}{32\left(4\,(2\,x+1)\sqrt{x+1}\sqrt{x}-8\,x^{2}-8\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*sqrt(x),x, algorithm="fricas")
```

```
[Out] 1/32*(128*x^4 + 256*x^3 - 4*(32*x^3 + 48*x^2 + 18*x + 1)*sqrt(x +
 1)*sqrt(x) + 152*x^2 + 4*(4*(2*x + 1)*sqrt(x + 1)*sqrt(x) - 8*x^
2 - 8*x - 1)*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1) + 24*x - 1)/(4*
(2*x + 1)*sqrt(x + 1)*sqrt(x) - 8*x^2 - 8*x - 1)
```

Sympy [A] time = 3.96857, size = 119, normalized size = 2.77

$$\begin{cases} -\frac{\operatorname{acosh}\left(\sqrt{x+1}\right)}{4} + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}} - \frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}} + \frac{\sqrt{x+1}}{4\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{i\operatorname{asin}\left(\sqrt{x+1}\right)}{4} - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}} + \frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}} - \frac{i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3
*(x + 1)**(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1)
 > 1), (I*asin(sqrt(x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3
*I*(x + 1)**(3/2)/(4*sqrt(-x)) - I*sqrt(x + 1)/(4*sqrt(-x)), True
))
```

GIAC/XCAS [A] time = 0.227168, size = 42, normalized size = 0.98

$$\frac{1}{4}\,(2\,x+1)\sqrt{x+1}\sqrt{x} + \frac{1}{4}\,\ln\left(\left|-\sqrt{x+1} + \sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*sqrt(x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/4*ln(abs(-sqrt(x + 1) + sqr
t(x)))
```

3.192 $\int \sin\left(\sqrt{x}\right)\, dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0173069, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[Sqrt[x]],x]

[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.86635, size = 46, normalized size = 2.09

$$-\sqrt{x}e^{i\sqrt{x}} - \sqrt{x}e^{-i\sqrt{x}} - ie^{i\sqrt{x}} + ie^{-i\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x**(1/2)),x)

[Out] -sqrt(x)*exp(I*sqrt(x)) - sqrt(x)*exp(-I*sqrt(x)) - I*exp(I*sqrt(
x)) + I*exp(-I*sqrt(x))
```

Mathematica [A] time = 0.00893873, size = 22, normalized size = 1.

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[Sqrt[x]],x]

[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] time = 0.002, size = 17, normalized size = 0.8

$$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x^(1/2)),x)
```

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.43035, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A] time = 0.233666, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] time = 0.389953, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x**(1/2)),x)
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

GIAC/XCAS [A] time = 0.198767, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.193 $\int \frac{x}{(1-x^2)^{9/8}} \, dx$

Optimal. Leaf size=13

$$\frac{4}{\sqrt[8]{1-x^2}}$$

```
[Out] 4/(1 - x^2)^(1/8)
```

Rubi [A] time = 0.00582945, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - x^2)^(9/8),x]

[Out] 4/(1 - x^2)^(1/8)
```

Rubi in Sympy [A] time = 0.911313, size = 8, normalized size = 0.62

$$\frac{4}{\sqrt[8]{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+1)**(9/8),x)

[Out] 4/(-x**2 + 1)**(1/8)
```

Mathematica [A] time = 0.00365069, size = 13, normalized size = 1.

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 - x^2)^(9/8),x]
```

```
[Out] 4/(1 - x^2)^(1/8)
```

Maple [A] time = 0.002, size = 18, normalized size = 1.4

$$-4(-1+x)(1+x)\left(-x^2+1\right)^{-\frac{9}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+1)^(9/8),x)

[Out] -4*(-1+x)*(1+x)/(-x^2+1)^(9/8)
```

Maxima [A] time = 1.3864, size = 15, normalized size = 1.15

$$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^2 + 1)^(9/8),x, algorithm="maxima")

[Out] 4/(-x^2 + 1)^(1/8)
```

Fricas [A] time = 0.211273, size = 15, normalized size = 1.15

$$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^2 + 1)^(9/8),x, algorithm="fricas")

[Out] 4/(-x^2 + 1)^(1/8)
```

Sympy [A] time = 1.78034, size = 8, normalized size = 0.62

$$\frac{4}{\sqrt[8]{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+1)**(9/8),x)
```

```
[Out] 4/(-x**2 + 1)**(1/8)
```

GIAC/XCAS [A] time = 0.201328, size = 15, normalized size = 1.15

$$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^2 + 1)^(9/8),x, algorithm="giac")
```

```
[Out] 4/(-x^2 + 1)^(1/8)
```

3.194 $\int \frac{x}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

```
[Out] ArcSin[x^2]/2
```

Rubi [A] time = 0.00989643, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[1 - x^4],x]

[Out] ArcSin[x^2]/2
```

Rubi in Sympy [A] time = 1.21311, size = 5, normalized size = 0.62

$$\frac{\operatorname{asin}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**4+1)**(1/2),x)

[Out] asin(x**2)/2
```

Mathematica [A] time = 0.00803221, size = 8, normalized size = 1.

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

Maple [A] time = 0.011, size = 7, normalized size = 0.9

$$\frac{\arcsin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x^2)
```

Maxima [A] time = 1.50193, size = 22, normalized size = 2.75

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(-x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

Fricas [A] time = 0.219887, size = 24, normalized size = 3.

$$-\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(-x^4 + 1),x, algorithm="fricas")
```

```
[Out] -arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [A] time = 1.49906, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i\operatorname{acosh}\left(x^2\right)}{2} & \text{for}\ \left|x^4\right| > 1 \\ \frac{\operatorname{asin}\left(x^2\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True)
)
```

GIAC/XCAS [A] time = 0.211213, size = 8, normalized size = 1.

$$\frac{1}{2} \arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x^2)
```

3.195 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Rubi [A] time = 0.019239, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[1 + x^4]),x]

[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Rubi in Sympy [A] time = 1.56426, size = 12, normalized size = 0.86

$$-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**4+1)**(1/2),x)

[Out] -atanh(sqrt(x**4 + 1))/2
```

Mathematica [A] time = 0.0297667, size = 14, normalized size = 1.

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Maple [A] time = 0.008, size = 11, normalized size = 0.8

$$-\frac{1}{2}Artanh\left(\frac{1}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))
```

Maxima [A] time = 1.36977, size = 34, normalized size = 2.43

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Fricas [A] time = 0.323611, size = 34, normalized size = 2.43

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Sympy [A] time = 1.50319, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**4+1)**(1/2),x)
```

```
[Out] -asinh(x**(-2))/2
```

GIAC/XCAS [A] time = 0.203154, size = 34, normalized size = 2.43

$$-\frac{1}{4}\ln\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\ln\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/4*ln(sqrt(x^4 + 1) + 1) + 1/4*ln(sqrt(x^4 + 1) - 1)
```

3.196 $\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Rubi [A] time = 0.036424, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[1 + x^2 + x^4],x]

[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Rubi in Sympy [A] time = 2.03971, size = 22, normalized size = 1.22

$$\frac{\operatorname{atanh}\left(\frac{2x^2+1}{2\sqrt{x^4+x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+x**2+1)**(1/2),x)

[Out] atanh((2*x**2 + 1)/(2*sqrt(x**4 + x**2 + 1)))/2
```

Mathematica [A] time = 0.00912336, size = 18, normalized size = 1.

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Maple [A] time = 0.008, size = 14, normalized size = 0.8

$$\frac{1}{2}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x^2+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^4 + x^2 + 1), x)
```

Fricas [A] time = 0.211738, size = 30, normalized size = 1.67

$$-\frac{1}{2}\log\left(-2\,x^2+2\sqrt{x^4+x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

GIAC/XCAS [A] time = 0.2064, size = 30, normalized size = 1.67

$$-\frac{1}{2} \ln\left(-2 x^2 + 2 \sqrt{x^4 + x^2 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*ln(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

3.197 $\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

```
[Out] -ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

Rubi [A] time = 0.0486982, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]

[Out] -ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

Rubi in Sympy [A] time = 3.84005, size = 20, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-x**4+x**2-1)**(1/2),x)

[Out] atan((x**2 - 2)/(2*sqrt(-x**4 + x**2 - 1)))/2
```

Mathematica [C] time = 0.0435708, size = 37, normalized size = 1.23

$$\frac{1}{2}i\log\left(x^2+2i\sqrt{-x^4+x^2-1}-2\right)-i\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] (-I)*Log[x] + (I/2)*Log[-2 + x^2 + (2*I)*Sqrt[-1 + x^2 - x^4]]
```

Maple [A] time = 0.01, size = 23, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\frac{x^2-2}{2}\frac{1}{\sqrt{-x^4+x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-x^4+x^2-1)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))
```

Maxima [A] time = 1.53616, size = 23, normalized size = 0.77

$$-\frac{1}{2}i\,\operatorname{arsinh}\left(-\frac{1}{3}\sqrt{3}+\frac{2\sqrt{3}}{3\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 - 1)*x),x, algorithm="maxima")
```

```
[Out] -1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)
```

Fricas [A] time = 0.222753, size = 74, normalized size = 2.47

$$\frac{1}{4}i\,\log\left(\frac{x^2+2i\sqrt{-x^4+x^2-1}-2}{2\,x^2}\right)-\frac{1}{4}i\,\log\left(\frac{x^2-2i\sqrt{-x^4+x^2-1}-2}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 - 1)*x),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(1/2*(x^2 + 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2) - 1/4*I*l
og(1/2*(x^2 - 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{-x^4+x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-x**4+x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)
```

GIAC/XCAS [A] time = 0.219106, size = 20, normalized size = 0.67

$$\frac{1}{2}i\ \arcsin\left(\frac{1}{3}\sqrt{3}\left(\frac{2i}{x^2}-i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^4 + x^2 - 1)*x),x, algorithm="giac")
```

```
[Out] 1/2*I*arcsin(1/3*sqrt(3)*(2*I/x^2 - I))
```

3.198 $\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{x^2+1}}{1-x}$$

```
[Out] Sqrt[1 + x^2]/(1 - x)
```

Rubi [A] time = 0.0373708, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\sqrt{x^2+1}}{1-x}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]

[Out] Sqrt[1 + x^2]/(1 - x)
```

Rubi in Sympy [A] time = 2.07873, size = 10, normalized size = 0.59

$$\frac{\sqrt{x^2+1}}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)

[Out] sqrt(x**2 + 1)/(-x + 1)
```

Mathematica [A] time = 0.0239648, size = 17, normalized size = 1.

$$\frac{\sqrt{x^2+1}}{1-x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]
```

```
[Out] Sqrt[1 + x^2]/(1 - x)
```

Maple [A] time = 0.004, size = 15, normalized size = 0.9

$$-\frac{1}{-1+x}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(1-x)^2/(x^2+1)^(1/2),x)
```

```
[Out] -(x^2+1)^(1/2)/(-1+x)
```

Maxima [A] time = 1.50834, size = 19, normalized size = 1.12

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 1)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 + 1)/(x - 1)
```

Fricas [A] time = 0.2151, size = 46, normalized size = 2.71

$$\frac{x-\sqrt{x^2+1}+1}{x^2-\sqrt{x^2+1}(x-1)-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 1)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] (x - sqrt(x^2 + 1) + 1)/(x^2 - sqrt(x^2 + 1)*(x - 1) - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(x-1)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+1}(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 1)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 1)*(x - 1)^2), x)
```

3.199 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.00301008, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rubi in Sympy [A] time = 0.077355, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Mathematica [A] time = 0.00438121, size = 2, normalized size = 1.

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$Arcsinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2),x)

[Out] arcsinh(x)
```

Maxima [A] time = 1.55203, size = 3, normalized size = 1.5

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="maxima")

[Out] arcsinh(x)
```

Fricas [A] time = 0.237867, size = 19, normalized size = 9.5

$$-\log\left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.135368, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)
```

<u>GIAC/XCAS</u> [A] time = 0.234949, size = 19, normalized size = 9.5

$$-\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(x^2 + 1))
```

3.200 $\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Rubi [A] time = 1.70373, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(2*Sqrt[
```

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{x+1}+\sqrt{x+2}+2\int^{\sqrt{x}}\frac{1}{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(
```

```
[Out] sqrt(x + 1) + sqrt(x + 2) + 2*Integral(1/2, (x, sqrt(x)))
```

Mathematica [A] time = 0.0252207, size = 30, normalized size = 1.5

$$\frac{1}{2}\left(2\sqrt{x}+2\sqrt{x+1}+2\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(2
```

```
[Out] (2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\sqrt{x}+\sqrt{1+x}+\sqrt{2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1
```

```
[Out] x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

Maxima [A] time = 1.53255, size = 19, normalized size = 0.95

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*sqrt(x)
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Fricas [A] time = 0.201664, size = 19, normalized size = 0.95

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*sqrt(x)
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Sympy [A] time = 1.60816, size = 17, normalized size = 0.85

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2+x)*
```

```
[Out] sqrt(x) + sqrt(x + 1) + sqrt(x + 2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+2}\sqrt{x+1} + \sqrt{x+2}\sqrt{x} + \sqrt{x+1}\sqrt{x}}{2\sqrt{x+2}\sqrt{x+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*sqrt(x)
```

```
[Out] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sq
rt(x + 1)*sqrt(x))/(sqrt(x + 2)*sqrt(x + 1)*sqrt(x)), x)
```

3.201 $\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$

Optimal. Leaf size=24

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Rubi [A] time = 0.51216, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 68, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*Sqrt[1
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\sqrt{x^3+1}+\frac{5\int \frac{x^4}{\sqrt{x^5-2x+1}}\,dx}{2}-\int \frac{1}{\sqrt{x^5-2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-
2*x+1)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)
```

```
[Out] -sqrt(x**3 + 1) + 5*Integral(x**4/sqrt(x**5 - 2*x + 1), x)/2 - In
tegral(1/sqrt(x**5 - 2*x + 1), x)
```

Mathematica [A] time = 0.0749122, size = 24, normalized size = 1.

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Maple [A] time = 0.004, size = 21, normalized size = 0.9

$$-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)
2*x+1)^(1/2),x)
```

```
[Out] -(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

Maxima [A] time = 1.72452, size = 41, normalized size = 1.71

$$\sqrt{x^4+x^3+x^2+x-1}\sqrt{x-1}-\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3 + 1))/
```

```
[Out] sqrt(x^4 + x^3 + x^2 + x - 1)*sqrt(x - 1) - sqrt(x^3 + 1)
```

Fricas [A] time = 0.201973, size = 27, normalized size = 1.12

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3 + 1))/
```

```
[Out] sqrt(x^5 - 2*x + 1) - sqrt(x^3 + 1)
```

Sympy [A] time = 2.02019, size = 19, normalized size = 0.79

$$-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-
2*x+1)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)
```

```
[Out] -sqrt(x**3 + 1) + sqrt(x**5 - 2*x + 1)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\sqrt{x^3+1}x^4 - 3\sqrt{x^5-2x+1}x^2 - 2\sqrt{x^3+1}}{2\sqrt{x^5-2x+1}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3 + 1))/
```

```
[Out] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 -
2*sqrt(x^3 + 1))/(sqrt(x^5 - 2*x + 1)*sqrt(x^3 + 1)), x)
```

3.202 $\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) \, dx$

Optimal. Leaf size=27

$$10 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0163863, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$10 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]

[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi in Sympy [A] time = 0.905945, size = 24, normalized size = 0.89

$$10 \operatorname{atanh}\left(\frac{x}{\sqrt{x^2-4}}\right)+\operatorname{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)

[Out] 10*atanh(x/sqrt(x**2 - 4)) + atanh(x/sqrt(x**2 - 1))
```

Mathematica [B] time = 0.0119472, size = 71, normalized size = 2.63

$$-5 \log\left(1-\frac{x}{\sqrt{x^2-4}}\right)+5 \log\left(\frac{x}{\sqrt{x^2-4}}+1\right)-\frac{1}{2} \log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\frac{1}{2} \log\left(\frac{x}{\sqrt{x^2-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] -5*Log[1 - x/Sqrt[-4 + x^2]] + 5*Log[1 + x/Sqrt[-4 + x^2]] - Log[
1 - x/Sqrt[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2
```

Maple [A] time = 0.002, size = 24, normalized size = 0.9

$$\ln\left(x+\sqrt{x^2-1}\right)+10\ln\left(x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x)
```

```
[Out] ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))
```

Maxima [A] time = 1.36558, size = 42, normalized size = 1.56

$$\log\left(2x+2\sqrt{x^2-1}\right)+10\log\left(2x+2\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1) + 10/sqrt(x^2 - 4),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1)) + 10*log(2*x + 2*sqrt(x^2 - 4))
```

Fricas [A] time = 0.210409, size = 39, normalized size = 1.44

$$-\log\left(-x+\sqrt{x^2-1}\right)-10\log\left(-x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1) + 10/sqrt(x^2 - 4),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1)) - 10*log(-x + sqrt(x^2 - 4))
```

Sympy [A] time = 0.194793, size = 8, normalized size = 0.3

$$10\operatorname{acosh}\left(\frac{x}{2}\right)+\operatorname{acosh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)
```

```
[Out] 10*acosh(x/2) + acosh(x)
```

GIAC/XCAS [A] time = 0.206359, size = 42, normalized size = 1.56

$$-\ln\left(\left|-x+\sqrt{x^2-1}\right|\right)-10\ln\left(\left|-x+\sqrt{x^2-4}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1) + 10/sqrt(x^2 - 4),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(x^2 - 1))) - 10*ln(abs(-x + sqrt(x^2 - 4)))
```

3.203 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]]
```

Rubi [A] time = 0.137614, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]]
```

Rubi in Sympy [A] time = 8.18798, size = 73, normalized size = 0.89

$$-2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*atan(sqrt(x + sqrt(a**2 + x**2))/sqrt(a)) - 2*sqrt(a)*
atanh(sqrt(x + sqrt(a**2 + x**2))/sqrt(a)) + 2*sqrt(x + sqrt(a**2
 + x**2))
```

Mathematica [A] time = 0.274323, size = 161, normalized size = 1.96

$$-\frac{\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-2\sqrt{\sqrt{a^2+x^2}+x}-\sqrt{a}\log\left(\sqrt{a}-\sqrt{\sqrt{a^2+x^2}+x}\right)+\sqrt{a}\log\left(\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\right)+2\sqrt{a}\tan^{-}\right.}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] -((Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-2*Sqrt[x + Sqrt[a^2 +
x^2]] + 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - Sqr
t[a]*Log[Sqrt[a] - Sqrt[x + Sqrt[a^2 + x^2]]] + Sqrt[a]*Log[Sqrt[
a] + Sqrt[x + Sqrt[a^2 + x^2]]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
)
```

Maple [C] time = 0.049, size = 25, normalized size = 0.3

$$2\sqrt{2}\sqrt{x}{}_3F_2(-1/4,-1/4,1/4;\ 1/2,3/4;\ -\frac{a^2}{x^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^2*a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Fricas [A] time = 0.240301, size = 1, normalized size = 0.01

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(-\frac{a-2\sqrt{a}\sqrt{x+\sqrt{a^2+x^2}}+x+\sqrt{a^2+x^2}}{a-x-\sqrt{a^2+x^2}}\right)+2\sqrt{x+\sqrt{a^2+x^2}},-2\sqrt{-a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{-a}}\right)+\sqrt{-a}\log\left(-\frac{a+2\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}-x-\sqrt{a^2+x^2}}{a+x+\sqrt{a^2+x^2}}\right)+2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*l
og(-(a - 2*sqrt(a)*sqrt(x + sqrt(a^2 + x^2)) + x + sqrt(a^2 + x^2
))/(a - x - sqrt(a^2 + x^2))) + 2*sqrt(x + sqrt(a^2 + x^2)), -2*s
qrt(-a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(-a)) + sqrt(-a)*log
(-(a + 2*sqrt(-a)*sqrt(x + sqrt(a^2 + x^2)) - x - sqrt(a^2 + x^2)
)/(a + x + sqrt(a^2 + x^2))) + 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [A] time = 2.27418, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}^2\left(-\frac{1}{4}\right)\ \left(\frac{1}{4}\right)\ {}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2,
3/4), a**2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.204 $\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)} \, dx$

Optimal. Leaf size=12

$$\log \left(\sqrt{x^3 + 1} + 1\right)$$

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Rubi [A] time = 0.0817713, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\log \left(\sqrt{x^3 + 1} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]

[Out] Log[1 + Sqrt[1 + x^3]]
```

Rubi in Sympy [A] time = 3.21734, size = 10, normalized size = 0.83

$$\log \left(\sqrt{x^3 + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)

[Out] log(sqrt(x**3 + 1) + 1)
```

Mathematica [A] time = 0.00919247, size = 12, normalized size = 1.

$$\log \left(\sqrt{x^3 + 1} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Maple [B] time = 0.058, size = 39, normalized size = 3.3

$$\frac{3\ln(x)}{2}-\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\ln(1+x)}{2}+\frac{\ln\left(x^3+1\right)}{2}+\mathit{Artanh}\left(\sqrt{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x)
```

```
[Out] 3/2*ln(x)-1/2*ln(x^2-x+1)-1/2*ln(1+x)+1/2*ln(x^3+1)+arctanh((x^3+
1)^(1/2))
```

Maxima [A] time = 1.48647, size = 54, normalized size = 4.5

$$-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(\frac{x^3+\sqrt{x^2-x+1}\sqrt{x+1}+1}{\sqrt{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3/2*x^2/(x^3 + sqrt(x^3 + 1) + 1),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 - x + 1) + log((x^3 + sqrt(x^2 - x + 1)*sqrt(x + 1)
+ 1)/sqrt(x + 1))
```

Fricas [A] time = 0.218551, size = 39, normalized size = 3.25

$$\frac{3}{2}\log(x)+\frac{1}{2}\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{2}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3/2*x^2/(x^3 + sqrt(x^3 + 1) + 1),x, algorithm="fricas")
```

```
[Out] 3/2*log(x) + 1/2*log(sqrt(x^3 + 1) + 1) - 1/2*log(sqrt(x^3 + 1) -
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\int \frac{x^2}{x^3+\sqrt{x^3+1}+1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)
```

```
[Out] 3*Integral(x**2/(x**3 + sqrt(x**3 + 1) + 1), x)/2
```

GIAC/XCAS [A] time = 0.198971, size = 14, normalized size = 1.17

$$\ln\left(\sqrt{x^3+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3/2*x^2/(x^3 + sqrt(x^3 + 1) + 1),x, algorithm="giac")
```

```
[Out] ln(sqrt(x^3 + 1) + 1)
```

3.205 $\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sq
rt[h])
```

Rubi [A] time = 0.0255692, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sq
rt[h])
```

Rubi in Sympy [A] time = 1.81254, size = 37, normalized size = 0.92

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2+2hr^2}}\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] sqrt(2)*atanh(sqrt(2)*sqrt(h)*r/sqrt(-alpha**2 + 2*h*r**2))/(2*sq
rt(h))
```

Mathematica [A] time = 0.0147973, size = 40, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sq
rt[h])
```

Maple [A] time = 0.002, size = 33, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\ln\left(\sqrt{h}r\sqrt{2}+\sqrt{2\,hr^2-\alpha^2}\right)\frac{1}{\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*h*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*h*r^2 - alpha^2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221142, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{2}\log\left(4\,hr^2+2\,\sqrt{2}\sqrt{2\,hr^2-\alpha^2}\sqrt{h}r-\alpha^2\right)}{4\,\sqrt{h}},\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{h}}\arctan\left(\frac{\sqrt{2}r}{\sqrt{2\,hr^2-\alpha^2}\sqrt{-\frac{1}{h}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*h*r^2 - alpha^2),r, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt
(h)*r - alpha^2)/sqrt(h), 1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*r
```

```
/(sqrt(2*h*r^2 - alpha^2)*sqrt(-1/h)))]
```

Sympy [A] time = 1.80004, size = 66, normalized size = 1.65

$$\begin{cases} \frac{\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{for } 2\left|\frac{hr^2}{\alpha^2}\right| > 1 \\ -\frac{\sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), 2*
Abs(h*r**2/alpha**2) > 1), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alp
ha)/(2*sqrt(h)), True))
```

GIAC/XCAS [A] time = 0.205761, size = 46, normalized size = 1.15

$$-\frac{\sqrt{2}\ln\left(\left|-\sqrt{2}\sqrt{h}r + \sqrt{2\,hr^2 - \alpha^2}\right|\right)}{2\,\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*h*r^2 - alpha^2),r, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*ln(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2))
)/sqrt(h)
```

3.206 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilo
n^2]]/Sqrt[alpha^2 + epsilon^2]
```

Rubi [A] time = 0.0734454, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilo
n^2]]/Sqrt[alpha^2 + epsilon^2]
```

Rubi in Sympy [A] time = 5.01208, size = 37, normalized size = 0.8

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] atan(sqrt(-alpha**2 - epsilon**2 + 2*h*r**2)/sqrt(alpha**2 + epsi
lon**2))/sqrt(alpha**2 + epsilon**2)
```

Mathematica [C] time = 0.0500831, size = 58, normalized size = 1.26

$$-\frac{i\log\left(\frac{2\left(\sqrt{-\alpha^2-\epsilon^2+2hr^2}-i\sqrt{\alpha^2+\epsilon^2}\right)}{r}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ((-I)*Log[(2*((-I)*Sqrt[alpha^2 + epsilon^2] + Sqrt[-alpha^2 - ep
silon^2 + 2*h*r^2]))/r])/Sqrt[alpha^2 + epsilon^2]
```

Maple [A] time = 0.006, size = 66, normalized size = 1.4

$$-1\ln\left(\frac{1}{r}\left(-2\,\alpha^2-2\,\epsilon^2+2\,\sqrt{-\alpha^2-\epsilon^2}\sqrt{2\,hr^2-\alpha^2-\epsilon^2}\right)\right)\frac{1}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*(-alph
a^2-epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2)^(1/2))/r)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - epsilon^2)*r),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212135, size = 55, normalized size = 1.2

$$-\frac{\arctan\left(\frac{\sqrt{\alpha^2+\epsilon^2}}{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - epsilon^2)*r),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(alpha^2 + epsilon^2)/sqrt(2*h*r^2 - alpha^2 - epsilo
n^2))/sqrt(alpha^2 + epsilon^2)
```

Sympy [A] time = 1.9533, size = 42, normalized size = 0.91

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{\text{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}{2\sqrt{h}r}\right)}{\sqrt{\text{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] -asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h
)*r))/sqrt(polar_lift(-alpha**2 - epsilon**2))
```

GIAC/XCAS [A] time = 0.203348, size = 51, normalized size = 1.11

$$\frac{1000000000000.0\,\arctan\left(\frac{1000000000000.0\,\sqrt{1.9999999999999998\,hr^2-0.9999999999999999\,\alpha^2-1\times10^{-24}}}{\sqrt{1\times10^{24}\,\alpha^2+1.0}}\right)}{\sqrt{1\times10^{24}\,\alpha^2+1.0}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*h*r^2 - alpha^2 - epsilon^2)*r),r, algorithm="giac")
```

```
[Out] 1000000000000.0*arctan(1000000000000.0*sqrt(1.9999999999999998*h*
r^2 - 0.9999999999999999*alpha^2 - 1e-24)/sqrt((1e+24)*alpha^2 +
1.0))/sqrt((1e+24)*alpha^2 + 1.0)
```

3.207 $\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

```
[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])
]/alpha)
```

Rubi [A] time = 0.0448111, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])
]/alpha)
```

Rubi in Sympy [A] time = 3.87299, size = 39, normalized size = 1.05

$$-\frac{\operatorname{atan}\left(-\frac{-2\alpha^2-2kr}{2\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] -atan(-(-2*alpha**2 - 2*k*r)/(2*alpha*sqrt(-alpha**2 + 2*h*r**2 -
 2*k*r)))/alpha
```

Mathematica [C] time = 0.0859734, size = 48, normalized size = 1.3

$$-\frac{i\log\left(\frac{2\left(\sqrt{2r(hr-k)-\alpha^2}-\frac{i\left(\alpha^2+kr\right)}{\alpha}\right)}{r}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] ((-I)*Log[(2*(((-I)*(alpha^2 + k*r))/alpha + Sqrt[-alpha^2 + 2*r*
(-k + h*r)]))/r])/alpha
```

Maple [A] time = 0.005, size = 52, normalized size = 1.4

$$-1\ln\left(\frac{1}{r}\left(-2\,\alpha^2-2\,kr+2\,\sqrt{-\alpha^2}\sqrt{2\,hr^2-\alpha^2-2\,kr}\right)\right)\frac{1}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r)
```

```
[Out] -1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*
r^2-alpha^2-2*k*r)^(1/2))/r)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*r),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217262, size = 47, normalized size = 1.27

$$-\frac{\arctan\left(\frac{\alpha^2+kr}{\sqrt{2\,hr^2-\alpha^2-2\,kr}\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*r),r, algorithm="fricas")
```

```
[Out] -arctan((alpha^2 + k*r)/(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*alpha))/
alpha
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)
```

GIAC/XCAS [A] time = 0.217428, size = 54, normalized size = 1.46

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{h}r - \sqrt{2\,hr^2 - \alpha^2 - 2\,kr}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*r),r, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/a
lpha)/alpha
```

3.208 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*S
qrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsi
lon^2])
```

Rubi [A] time = 0.0564588, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*S
qrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsi
lon^2])
```

Rubi in Sympy [A] time = 5.63441, size = 65, normalized size = 1.07

$$-\frac{\operatorname{atan}\left(-\frac{-2\alpha^2-2\epsilon^2-2kr}{2\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)
```

```
[Out] -atan(-(-2*alpha**2 - 2*epsilon**2 - 2*k*r)/(2*sqrt(alpha**2 + ep
silon**2)*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)))/sqrt(
alpha**2 + epsilon**2)
```

Mathematica [C] time = 0.144264, size = 72, normalized size = 1.18

$$-\frac{i\log\left(\frac{2\left(\sqrt{-\alpha^2-\epsilon^2+2r(hr-k)}-\frac{i\left(\alpha^2+\epsilon^2+kr\right)}{\sqrt{\alpha^2+\epsilon^2}}\right)}{r}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] ((-I)*Log[(2*(((-I)*(alpha^2 + epsilon^2 + k*r))/Sqrt[alpha^2 + e
psilon^2] + Sqrt[-alpha^2 - epsilon^2 + 2*r*(-k + h*r)]))/r])/Sqr
t[alpha^2 + epsilon^2]
```

Maple [A] time = 0.004, size = 74, normalized size = 1.2

$$-1\ln\left(\frac{1}{r}\left(-2\,\alpha^2-2\,\epsilon^2-2\,kr+2\,\sqrt{-\alpha^2-\epsilon^2}\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\right)\right)\frac{1}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*
(-alpha^2-epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2
))/r)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*r),r, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227985, size = 74, normalized size = 1.21

$$-\frac{\arctan\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*r),r, algorithm="fricas
```

```
[Out] -arctan((alpha^2 + epsilon^2 + k*r)/(sqrt(2*h*r^2 - alpha^2 - eps
ilon^2 - 2*k*r)*sqrt(alpha^2 + epsilon^2)))/sqrt(alpha^2 + epsilo
n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r
)
```

GIAC/XCAS [A] time = 0.23366, size = 69, normalized size = 1.13

$$\frac{2000000000000.0\,\arctan\left(\frac{(6.5536\times10^{-08})\left(-2.157918643757774\times10^{19}\sqrt{h}r+1.52587890625\times10^{19}\sqrt{2.0\,hr^2-\alpha^2-2.0\,kr-1\times10^{-24}}\right)}{\sqrt{1\times10^{24}\,\alpha^2+1.0}}\right)}{\sqrt{1\times10^{24}\,\alpha^2+1.0}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*r),r, algorithm="giac")
```

```
[Out] 2000000000000.0*arctan((6.5536e-08)*(-(2.157918643757774e+19)*sqr
t(h)*r + (1.52587890625e+19)*sqrt(2.0*h*r^2 - alpha^2 - 2.0*k*r -
 1e-24))/sqrt((1e+24)*alpha^2 + 1.0))/sqrt((1e+24)*alpha^2 + 1.0)
```

3.209 $\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr$

Optimal. Leaf size=23

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Rubi [A] time = 0.00983596, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]

[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Rubi in Sympy [A] time = 1.39683, size = 15, normalized size = 0.65

$$\frac{\sqrt{-\alpha^2+2er^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(2*e*r**2-alpha**2)**(1/2),r)

[Out] sqrt(-alpha**2 + 2*e*r**2)/(2*e)
```

Mathematica [A] time = 0.00633438, size = 23, normalized size = 1.

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Maple [A] time = 0.002, size = 20, normalized size = 0.9

$$\frac{1}{2\,e}\sqrt{2\,er^2-\alpha^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(2*e*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

Maxima [A] time = 1.316, size = 26, normalized size = 1.13

$$\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e
```

Fricas [A] time = 0.2078, size = 26, normalized size = 1.13

$$\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e
```

Sympy [A] time = 0.764982, size = 29, normalized size = 1.26

$$\begin{cases} \frac{\sqrt{-\alpha^2+2er^2}}{2e} & \text{for}\ e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*
sqrt(-alpha**2)), True))
```

GIAC/XCAS [A] time = 0.200474, size = 26, normalized size = 1.13

$$\frac{1}{2}\sqrt{2\,r^2e-\alpha^2}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2),r, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)
```

3.210 $\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}} dr$

Optimal. Leaf size=28

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Rubi [A] time = 0.010266, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]

[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Rubi in Sympy [A] time = 1.79172, size = 19, normalized size = 0.68

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)

[Out] sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e)
```

Mathematica [A] time = 0.00782838, size = 28, normalized size = 1.

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.9

$$\frac{1}{2\,e}\sqrt{2\,er^{2}-\alpha^{2}-\epsilon^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

Maxima [A] time = 1.35853, size = 32, normalized size = 1.14

$$\frac{\sqrt{2\,er^{2}-\alpha^{2}-\epsilon^{2}}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2 - epsilon^2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

Fricas [A] time = 0.213591, size = 32, normalized size = 1.14

$$\frac{\sqrt{2\,er^{2}-\alpha^{2}-\epsilon^{2}}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2 - epsilon^2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

Sympy [A] time = 0.990328, size = 36, normalized size = 1.29

$$\begin{cases} \frac{\sqrt{-\alpha^{2}+2er^{2}-\epsilon^{2}}}{2e} & \text{for}\: e \neq 0 \\ \frac{r^{2}}{2\sqrt{-\alpha^{2}-\epsilon^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0
)), (r**2/(2*sqrt(-alpha**2 - epsilon**2)), True))
```

GIAC/XCAS [A] time = 0.201905, size = 22, normalized size = 0.79

$$0.183939720586 \sqrt{-\alpha^2 + 5.43656365692 r^2 - 1 \times 10^{-24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2 - epsilon^2),r, algorithm="giac")
```

```
[Out] 0.183939720586*sqrt(-alpha^2 + 5.43656365692*r^2 - 1e-24)
```

3.211 $\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

```
[Out] -ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 -
2*k*r^4])]/(2*Sqrt[2]*Sqrt[k])
```

Rubi [A] time = 0.0779085, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] -ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 -
2*k*r^4])]/(2*Sqrt[2]*Sqrt[k])
```

Rubi in Sympy [A] time = 4.3033, size = 56, normalized size = 1.

$$-\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}(2e-4kr^2)}{4\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{4\sqrt{k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*(2*e - 4*k*r**2)/(4*sqrt(k)*sqrt(-alpha**2
+ 2*e*r**2 - 2*k*r**4)))/(4*sqrt(k))
```

Mathematica [C] time = 0.0891028, size = 66, normalized size = 1.18

$$\frac{i\log\left(2\sqrt{-\alpha^2+2er^2-2kr^4}-\frac{i\sqrt{2}(2kr^2-e)}{\sqrt{k}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] ((I/2)*Log[((-I)*Sqrt[2]*(-e + 2*k*r^2))/Sqrt[k] + 2*Sqrt[-alpha^
2 + 2*e*r^2 - 2*k*r^4]])/(Sqrt[2]*Sqrt[k])
```

Maple [A] time = 0.01, size = 47, normalized size = 0.8

$$\frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}\sqrt{k}\left(r^2-\frac{e}{2\,k}\right)\frac{1}{\sqrt{-2\,kr^4+2\,er^2-\alpha^2}}\right)\frac{1}{\sqrt{k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/4*2^(1/2)/k^(1/2)*arctan(2^(1/2)*k^(1/2)*(r^2-1/2*e/k)/(-2*k*r^
4+2*e*r^2-alpha^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23133, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{2}\log\left(-\sqrt{2}\left(8\,k^2r^4-8\,ekr^2+2\,\alpha^2k+e^2\right)\sqrt{-k}-4\,\sqrt{-2\,kr^4+2\,er^2-\alpha^2}\left(2\,k^2r^2-ek\right)\right)}{8\,\sqrt{-k}},\ \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,kr^2-e\right)}{2\,\sqrt{-2\,kr^4+2\,er^2-\alpha^2}\sqrt{k}}\right)}{4\,\sqrt{k}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2),r, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*log(-sqrt(2)*(8*k^2*r^4 - 8*e*k*r^2 + 2*alpha^2*k +
e^2)*sqrt(-k) - 4*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k^2*r^2 -
```

```
 e*k))/sqrt(-k), 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*k*r^2 - e)/(sq
rt(-2*k*r^4 + 2*e*r^2 - alpha^2)*sqrt(k)))/sqrt(k)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\, dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)
```

GIAC/XCAS [A] time = 0.264474, size = 81, normalized size = 1.45

$$-\frac{\sqrt{2}\ln\left(\left|\sqrt{2}\left(\sqrt{2}\sqrt{-k}r^2 - \sqrt{-2\,kr^4 + 2\,r^2e - \alpha^2}\right)\sqrt{-k} + e\right|\right)}{4\,\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*ln(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4
 + 2*r^2*e - alpha^2))*sqrt(-k) + e))/sqrt(-k)
```

3.212 $\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$

Optimal. Leaf size=81

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}e^{3/2}}$$

```
[Out] Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(
Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^
(3/2))
```

Rubi [A] time = 0.0665779, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(
Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^
(3/2))
```

Rubi in Sympy [A] time = 4.02737, size = 75, normalized size = 0.93

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}+\frac{\sqrt{2}k\,\mathrm{atanh}\left(\frac{\sqrt{2}(4er-2k)}{4\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{4e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] sqrt(-alpha**2 + 2*e*r**2 - 2*k*r)/(2*e) + sqrt(2)*k*atanh(sqrt(2
)*(4*e*r - 2*k)/(4*sqrt(e)*sqrt(-alpha**2 + 2*e*r**2 - 2*k*r)))/(
4*e**(3/2))
```

Mathematica [A] time = 0.113445, size = 84, normalized size = 1.04

$$\frac{\sqrt{2}k\log\left(\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}+2er-k\right)+2\sqrt{e}\sqrt{2r(er-k)-\alpha^2}}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] (2*Sqrt[e]*Sqrt[-alpha^2 + 2*r*(-k + e*r)] + Sqrt[2]*k*Log[-k + 2
*e*r + Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]])/(4*e^(3
/2))
```

Maple [A] time = 0.004, size = 70, normalized size = 0.9

$$\frac{1}{2e}\sqrt{2\,er^2-\alpha^2-2\,kr}+\frac{k\sqrt{2}}{4}\ln\left(\frac{(2\,er-k)\,\sqrt{2}}{2}\frac{1}{\sqrt{e}}+\sqrt{2\,er^2-\alpha^2-2\,kr}\right)e^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*ln(1/2*(2*e*r-k
)*2^(1/2)/e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2 - 2*k*r),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233027, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(k\log\left(\sqrt{2}\left(8\,e^2r^2-2\,\alpha^2e-8\,ekr+k^2\right)\sqrt{e}+4\left(2\,e^2r-ek\right)\sqrt{2\,er^2-\alpha^2-2\,kr}\right)+2\,\sqrt{2}\sqrt{2\,er^2-\alpha^2-2\,kr}\sqrt{e}\right)}{8\,e^{\frac{3}{2}}},\ \frac{\sqrt{2}\left(k\,\mathrm{ar}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2 - 2*k*r),r, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*(k*log(sqrt(2)*(8*e^2*r^2 - 2*alpha^2*e - 8*e*k*r +
k^2)*sqrt(e) + 4*(2*e^2*r - e*k)*sqrt(2*e*r^2 - alpha^2 - 2*k*r))
 + 2*sqrt(2)*sqrt(2*e*r^2 - alpha^2 - 2*k*r)*sqrt(e))/e^(3/2), 1/
4*sqrt(2)*(k*arctan(1/2*sqrt(2)*(2*e*r - k)*sqrt(-e)/(sqrt(2*e*r^
2 - alpha^2 - 2*k*r)*e)) + sqrt(2)*sqrt(2*e*r^2 - alpha^2 - 2*k*r
)*sqrt(-e))/(sqrt(-e)*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r), r)
```

GIAC/XCAS [A] time = 0.258274, size = 97, normalized size = 1.2

$$-\frac{1}{4}\sqrt{2}ke^{\left(-\frac{3}{2}\right)}\ln\left(\left|-\sqrt{2}\left(\sqrt{2}re^{\frac{1}{2}} - \sqrt{2\,r^2e - \alpha^2 - 2\,kr}\right)e^{\frac{1}{2}} + k\right|\right) + \frac{1}{2}\sqrt{2\,r^2e - \alpha^2 - 2\,kr}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*e*r^2 - alpha^2 - 2*k*r),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*k*e^(-3/2)*ln(abs(-sqrt(2)*(sqrt(2)*r*e^(1/2) - sqrt
(2*r^2*e - alpha^2 - 2*k*r))*e^(1/2) + k)) + 1/2*sqrt(2*r^2*e - a
lpha^2 - 2*k*r)*e^(-1)
```

3.213 $\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

```
[Out] -ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^
4])]/(2*alpha)
```

Rubi [A] time = 0.0942891, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^
4])]/(2*alpha)
```

Rubi in Sympy [A] time = 7.00375, size = 39, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{-2\alpha^2+2hr^2}{2\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] atan((-2*alpha**2 + 2*h*r**2)/(2*alpha*sqrt(-alpha**2 + 2*h*r**2
- 2*k*r**4)))/(2*alpha)
```

Mathematica [C] time = 0.129645, size = 59, normalized size = 1.34

$$-\frac{i\log\left(\frac{2\alpha\sqrt{2r^2(h-kr^2)-\alpha^2}-2i\alpha^2+2ihr^2}{\alpha r^2}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] ((-I/2)*Log[((-2*I)*alpha^2 + (2*I)*h*r^2 + 2*alpha*Sqrt[-alpha^2
 + 2*r^2*(h - k*r^2)])/(alpha*r^2)])/alpha
```

Maple [A] time = 0.011, size = 56, normalized size = 1.3

$$-\frac{1}{2}\ln\left(\frac{1}{r^2}\left(-2\,\alpha^2+2\,hr^2+2\,\sqrt{-\alpha^2}\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\right)\right)\frac{1}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r)
```

```
[Out] -1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(
-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/r^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*r),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221156, size = 55, normalized size = 1.25

$$\frac{\arctan\left(\frac{hr^2-\alpha^2}{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\alpha}\right)}{2\,\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*r),r, algorithm="fricas")
```

```
[Out] 1/2*arctan((h*r^2 - alpha^2)/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*
alpha))/alpha
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}}\, dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r**4)), r)
```

GIAC/XCAS [A] time = 0.263174, size = 42, normalized size = 0.95

$$-\frac{\arcsin\left(-\frac{h-\frac{\alpha^2}{r^2}}{\sqrt{-2\,\alpha^2 k+h^2}}\right)}{2\,|\alpha|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*r),r, algorithm="giac")
```

```
[Out] -1/2*arcsin(-(h - alpha^2/r^2)/sqrt(-2*alpha^2*k + h^2))/abs(alph
a)
```

3.214 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=68

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*
Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2
+ epsilon^2])
```

Rubi [A] time = 0.11392, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*
Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2
+ epsilon^2])
```

Rubi in Sympy [A] time = 9.44171, size = 65, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{-2\alpha^2-2\epsilon^2+2hr^2}{2\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] atan((-2*alpha**2 - 2*epsilon**2 + 2*h*r**2)/(2*sqrt(alpha**2 + e
psilon**2)*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)))/(
2*sqrt(alpha**2 + epsilon**2))
```

Mathematica [C] time = 0.30411, size = 80, normalized size = 1.18

$$-\frac{i\log\left(\frac{2\left(\sqrt{-\alpha^2-\epsilon^2+2r^2(h-kr^2)}-\frac{i(\alpha^2+\epsilon^2-hr^2)}{\sqrt{\alpha^2+\epsilon^2}}\right)}{r^2}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] ((-I/2)*Log[(2*(((-I)*(alpha^2 + epsilon^2 - h*r^2))/Sqrt[alpha^2
 + epsilon^2] + Sqrt[-alpha^2 - epsilon^2 + 2*r^2*(h - k*r^2)]))/
r^2])/Sqrt[alpha^2 + epsilon^2]
```

Maple [A] time = 0.011, size = 78, normalized size = 1.2

$$-\frac{1}{2}\ln\left(\frac{1}{r^2}\left(-2\alpha^2-2\epsilon^2+2hr^2+2\sqrt{-\alpha^2-\epsilon^2}\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}\right)\right)\frac{1}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] -1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^
2+2*(-alpha^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^
2)^(1/2))/r^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*r),r, algorithm="max
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227722, size = 85, normalized size = 1.25

$$\frac{\arctan\left(\frac{hr^2-\alpha^2-\epsilon^2}{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}\sqrt{\alpha^2+\epsilon^2}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*r),r, algorithm="fri
```

```
[Out] 1/2*arctan((h*r^2 - alpha^2 - epsilon^2)/(sqrt(-2*k*r^4 + 2*h*r^2
 - alpha^2 - epsilon^2)*sqrt(alpha^2 + epsilon^2)))/sqrt(alpha^2
+ epsilon^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4))
, r)
```

GIAC/XCAS [A] time = 0.260408, size = 42, normalized size = 0.62

$$-\frac{\arcsin\left(-\frac{h-\frac{\alpha^2}{r^2}}{\sqrt{-2\,\alpha^2 k + h^2}}\right)}{2\,|\alpha|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*r),r, algorithm="gia
```

```
[Out] -1/2*arcsin(-(h - alpha^2/r^2)/sqrt(-2*alpha^2*k + h^2))/abs(alph
a)
```

3.215 $\int a\cos(5+3x)\sin^2(5+3x)\,dx$

Optimal. Leaf size=13

$$\frac{1}{9}a\sin^3(3x+5)$$

```
[Out] (a*Sin[5 + 3*x]^3)/9
```

Rubi [A] time = 0.0274779, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

```
[In]  Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]

[Out] (a*Sin[5 + 3*x]^3)/9
```

Rubi in Sympy [A] time = 1.41787, size = 10, normalized size = 0.77

$$\frac{a\sin^3(3x+5)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)

[Out] a*sin(3*x + 5)**3/9
```

Mathematica [A] time = 0.00948142, size = 13, normalized size = 1.

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

```
[In]  Integrate[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

```
[Out] (a*Sin[5 + 3*x]^3)/9
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{a\left(\sin\left(5+3\,x\right)\right)^{3}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a*cos(5+3*x)*sin(5+3*x)^2,x)
```

```
[Out] 1/9*a*sin(5+3*x)^3
```

Maxima [A] time = 1.417, size = 15, normalized size = 1.15

$$\frac{1}{9}\,a\sin\left(3\,x+5\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*cos(3*x + 5)*sin(3*x + 5)^2,x, algorithm="maxima")
```

```
[Out] 1/9*a*sin(3*x + 5)^3
```

Fricas [A] time = 0.217906, size = 30, normalized size = 2.31

$$-\frac{1}{9}\,\left(a\cos\left(3\,x+5\right)^{2}-a\right)\,\sin\left(3\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*cos(3*x + 5)*sin(3*x + 5)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(a*cos(3*x + 5)^2 - a)*sin(3*x + 5)
```

Sympy [A] time = 0.370279, size = 10, normalized size = 0.77

$$\frac{a\sin^{3}\left(3x+5\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)
```

```
[Out] a*sin(3*x + 5)**3/9
```

GIAC/XCAS [A] time = 0.200689, size = 15, normalized size = 1.15

$$\frac{1}{9}\,a\sin\left(3\,x+5\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*cos(3*x + 5)*sin(3*x + 5)^2,x, algorithm="giac")
```

```
[Out] 1/9*a*sin(3*x + 5)^3
```

3.216 $\int \frac{\log(x^2)}{x^3} dx$

Optimal. Leaf size=19

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

```
[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Rubi [A] time = 0.0115642, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x^2]/x^3,x]

[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Rubi in Sympy [A] time = 1.07926, size = 17, normalized size = 0.89

$$-\frac{\log(x^2)}{2x^2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x**2)/x**3,x)

[Out] -log(x**2)/(2*x**2) - 1/(2*x**2)
```

Mathematica [A] time = 0.00320815, size = 19, normalized size = 1.

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x^2]/x^3,x]
```

```
[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$-\frac{1}{2\,x^{2}}-\frac{\ln\left(x^{2}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x^2)/x^3,x)
```

```
[Out] -1/2/x^2-1/2*ln(x^2)/x^2
```

Maxima [A] time = 1.46935, size = 20, normalized size = 1.05

$$-\frac{\log\left(x^{2}\right)}{2\,x^{2}}-\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

Fricas [A] time = 0.207688, size = 15, normalized size = 0.79

$$-\frac{\log\left(x^{2}\right)+1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(log(x^2) + 1)/x^2
```

Sympy [A] time = 0.082511, size = 17, normalized size = 0.89

$$-\frac{\log\left(x^{2}\right)}{2x^{2}}-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x**2)/x**3,x)

[Out] -log(x**2)/(2*x**2) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.202928, size = 20, normalized size = 1.05

$$-\frac{\ln\left(x^{2}\right)}{2\,x^{2}}-\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x^2)/x^3,x, algorithm="giac")

[Out] -1/2*ln(x^2)/x^2 - 1/2/x^2
```

3.217 $\int x \sin(a+x)\,dx$

Optimal. Leaf size=12

$$\sin(a+x) - x\cos(a+x)$$

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Rubi [A] time = 0.0155563, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\sin(a+x) - x\cos(a+x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[a + x],x]

[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Rubi in Sympy [A] time = 0.875497, size = 10, normalized size = 0.83

$$-x\cos(a+x) + \sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(a+x),x)

[Out] -x*cos(a + x) + sin(a + x)
```

Mathematica [A] time = 0.0182112, size = 12, normalized size = 1.

$$\sin(a+x) - x\cos(a+x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[a + x],x]

[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Maple [A] time = 0.005, size = 21, normalized size = 1.8

$$a\cos(a+x)+\sin(a+x)-(a+x)\cos(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(a+x),x)

[Out] a*cos(a+x)+sin(a+x)-(a+x)*cos(a+x)
```

Maxima [A] time = 1.38148, size = 27, normalized size = 2.25

$$-(a+x)\cos(a+x)+a\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(a + x),x, algorithm="maxima")

[Out] -(a + x)*cos(a + x) + a*cos(a + x) + sin(a + x)
```

Fricas [A] time = 0.228348, size = 16, normalized size = 1.33

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(a + x),x, algorithm="fricas")

[Out] -x*cos(a + x) + sin(a + x)
```

Sympy [A] time = 0.186598, size = 10, normalized size = 0.83

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(a+x),x)

[Out] -x*cos(a + x) + sin(a + x)
```

GIAC/XCAS [A] time = 0.198172, size = 16, normalized size = 1.33

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(a + x),x, algorithm="giac")
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

3.218 $\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{e^{-x}x}{\log(x)}$$

```
[Out] x/(E^x*Log[x])
```

Rubi [A] time = 0.0392392, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

```
[Out] x/(E^x*Log[x])
```

Rubi in Sympy [A] time = 2.55625, size = 7, normalized size = 0.64

$$\frac{xe^{-x}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)
```

```
[Out] x*exp(-x)/log(x)
```

Mathematica [A] time = 0.0110276, size = 11, normalized size = 1.

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

```
[Out] x/(E^x*Log[x])
```

Maple [A] time = 0.008, size = 11, normalized size = 1.

$$\frac{x}{e^{x} \ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+(1-x)*ln(x))/exp(x)/ln(x)^2,x)
```

```
[Out] x/exp(x)/ln(x)
```

Maxima [A] time = 1.47767, size = 14, normalized size = 1.27

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((x - 1)*log(x) + 1)*e^(-x)/log(x)^2,x, algorithm="maxima")
```

```
[Out] x*e^(-x)/log(x)
```

Fricas [A] time = 0.217211, size = 14, normalized size = 1.27

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((x - 1)*log(x) + 1)*e^(-x)/log(x)^2,x, algorithm="fricas")
```

```
[Out] x*e^(-x)/log(x)
```

Sympy [A] time = 0.519782, size = 7, normalized size = 0.64

$$\frac{xe^{-x}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)
```

```
[Out] x*exp(-x)/log(x)
```

GIAC/XCAS [A] time = 0.202974, size = 14, normalized size = 1.27

$$\frac{xe^{(-x)}}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-((x - 1)*log(x) + 1)*e^(-x)/log(x)^2,x, algorithm="giac")
```

```
[Out] x*e^(-x)/ln(x)
```

3.219 $\int \frac{x^3}{b+ax^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rubi [A] time = 0.0409316, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b + a*x^2),x]

[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^2} \frac{1}{a} \, dx}{2} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(a*x**2+b),x)

[Out] Integral(1/a, (x, x**2))/2 - b*log(a*x**2 + b)/(2*a**2)
```

Mathematica [A] time = 0.00617023, size = 27, normalized size = 1.

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b + a*x^2),x]
```

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Maple [A] time = 0.002, size = 24, normalized size = 0.9

$$\frac{x^2}{2a} - \frac{b\ln\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2+b),x)
```

```
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Maxima [A] time = 1.35339, size = 31, normalized size = 1.15

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2 + b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

Fricas [A] time = 0.202517, size = 30, normalized size = 1.11

$$\frac{ax^2 - b\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2 + b),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

Sympy [A] time = 0.527808, size = 20, normalized size = 0.74

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a*x**2+b),x)
```

```
[Out] x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

GIAC/XCAS [A] time = 0.202026, size = 32, normalized size = 1.19

$$\frac{x^2}{2\,a} - \frac{b\ln\left(\left|ax^2+b\right|\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^2 + b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a - 1/2*b*ln(abs(a*x^2 + b))/a^2
```

3.220 $\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx$

Optimal. Leaf size=33

$$\frac{4x^{3/2}}{15(x+1)^{3/2}} + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

```
[Out] (2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

Rubi [A] time = 0.0138937, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{4x^{3/2}}{15(x+1)^{3/2}} + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(1 + x)^(7/2),x]

[Out] (2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

Rubi in Sympy [A] time = 1.22108, size = 29, normalized size = 0.88

$$\frac{4x^{\frac{3}{2}}}{15\,(x+1)^{\frac{3}{2}}} + \frac{2x^{\frac{3}{2}}}{5\,(x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(1+x)**(7/2),x)

[Out] 4*x**(3/2)/(15*(x + 1)**(3/2)) + 2*x**(3/2)/(5*(x + 1)**(5/2))
```

Mathematica [A] time = 0.014717, size = 21, normalized size = 0.64

$$\frac{2x^{3/2}(2x+5)}{15(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(1 + x)^(7/2),x]
```

```
[Out] (2*x^(3/2)*(5 + 2*x))/(15*(1 + x)^(5/2))
```

Maple [A] time = 0.002, size = 16, normalized size = 0.5

$$\frac{4\,x+10}{15}x^{\frac{3}{2}}\left(1+x\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(1+x)^(7/2),x)
```

```
[Out] 2/15*x^(3/2)*(2*x+5)/(1+x)^(5/2)
```

Maxima [A] time = 1.32939, size = 27, normalized size = 0.82

$$\frac{2\,x^{\frac{5}{2}}\left(\frac{5\,(x+1)}{x}-3\right)}{15\,(x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + 1)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)
```

Fricas [A] time = 0.203264, size = 127, normalized size = 3.85

$$\frac{2\left(60\,x^{3}-5\left(12\,x^{2}+11\,x+2\right)\sqrt{x+1}\sqrt{x}+85\,x^{2}+30\,x+2\right)}{15\left(16\,x^{5}+60\,x^{4}+85\,x^{3}-\left(16\,x^{4}+52\,x^{3}+61\,x^{2}+30\,x+5\right)\sqrt{x+1}\sqrt{x}+55\,x^{2}+15\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + 1)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(60*x^3 - 5*(12*x^2 + 11*x + 2)*sqrt(x + 1)*sqrt(x) + 85*x^2
 + 30*x + 2)/(16*x^5 + 60*x^4 + 85*x^3 - (16*x^4 + 52*x^3 + 61*x^
2 + 30*x + 5)*sqrt(x + 1)*sqrt(x) + 55*x^2 + 15*x + 1)
```

Sympy [A] time = 72.8128, size = 165, normalized size = 5.

$$\begin{cases} \frac{4i\sqrt{-1+\frac{1}{x+1}}}{15} + \frac{2i\sqrt{-1+\frac{1}{x+1}}}{15(x+1)} - \frac{2i\sqrt{-1+\frac{1}{x+1}}}{5(x+1)^2} & \text{for } \left|\frac{1}{x+1}\right| > 1 \\ \frac{4\sqrt{1-\frac{1}{x+1}}(x+1)^2}{-15x+15(x+1)^2-15} - \frac{2\sqrt{1-\frac{1}{x+1}}(x+1)}{-15x+15(x+1)^2-15} - \frac{8\sqrt{1-\frac{1}{x+1}}}{-15x+15(x+1)^2-15} + \frac{6\sqrt{1-\frac{1}{x+1}}}{(x+1)\left(-15x+15(x+1)^2-15\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(1+x)**(7/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 1/(x + 1))/15 + 2*I*sqrt(-1 + 1/(x + 1))
/(15*(x + 1)) - 2*I*sqrt(-1 + 1/(x + 1))/(5*(x + 1)**2), Abs(1/(x
 + 1)) > 1), (4*sqrt(1 - 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1
)**2 - 15) - 2*sqrt(1 - 1/(x + 1))*(x + 1)/(-15*x + 15*(x + 1)**2
 - 15) - 8*sqrt(1 - 1/(x + 1))/(-15*x + 15*(x + 1)**2 - 15) + 6*s
qrt(1 - 1/(x + 1))/((x + 1)*(-15*x + 15*(x + 1)**2 - 15)), True))
```

GIAC/XCAS [A] time = 0.204204, size = 89, normalized size = 2.7

$$\frac{8\left(15\left(\sqrt{x+1}-\sqrt{x}\right)^{6}-5\left(\sqrt{x+1}-\sqrt{x}\right)^{4}+5\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)}{15\left(\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + 1)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*(sqrt(x + 1) - sqrt(x))^6 - 5*(sqrt(x + 1) - sqrt(x))^4
+ 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 + 1
)^5
```

3.221 $\int \frac{1}{x(1+x)}\,dx$

Optimal. Leaf size=9

$$\log(x) - \log(x+1)$$

```
[Out] Log[x] - Log[1 + x]
```

Rubi [A] time = 0.00628095, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x)),x]

[Out] Log[x] - Log[1 + x]
```

Rubi in Sympy [A] time = 0.890321, size = 7, normalized size = 0.78

$$\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+x),x)

[Out] log(x) - log(x + 1)
```

Mathematica [A] time = 0.00279249, size = 9, normalized size = 1.

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x)),x]

[Out] Log[x] - Log[1 + x]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$\ln(x) - \ln(1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+x),x)
```

```
[Out] ln(x)-ln(1+x)
```

Maxima [A] time = 1.35689, size = 12, normalized size = 1.33

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)*x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + log(x)
```

Fricas [A] time = 0.19573, size = 12, normalized size = 1.33

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)*x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + log(x)
```

Sympy [A] time = 0.075456, size = 7, normalized size = 0.78

$$\log(x) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+x),x)
```

```
[Out] log(x) - log(x + 1)
```

GIAC/XCAS [A] time = 0.201843, size = 15, normalized size = 1.67

$$-\ln\left(|x+1|\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)*x),x, algorithm="giac")
```

```
[Out] -ln(abs(x + 1)) + ln(abs(x))
```

3.222 $\int \frac{1}{\sqrt{x}(-1+2x)} \, dx$

Optimal. Leaf size=19

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Rubi [A] time = 0.014124, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(-1 + 2*x)),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Rubi in Sympy [A] time = 1.11609, size = 17, normalized size = 0.89

$$-\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(2*x-1),x)

[Out] -sqrt(2)*atanh(sqrt(2)*sqrt(x))
```

Mathematica [A] time = 0.00697179, size = 19, normalized size = 1.

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-Artanh\left(\sqrt{2}\sqrt{x}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(-1+2*x),x)
```

```
[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

Maxima [A] time = 1.50511, size = 38, normalized size = 2.

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-2\sqrt{x}\right)}{2\sqrt{2}+4\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x - 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-2*(sqrt(2) - 2*sqrt(x))/((2*sqrt(2)) + 4*sqrt(x)
))
```

Fricas [A] time = 0.208419, size = 38, normalized size = 2.

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{x}-2x-1}{2x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x - 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))
```

Sympy [A] time = 0.75927, size = 39, normalized size = 2.05

$$\begin{cases} -\sqrt{2}\operatorname{acoth}\left(\sqrt{2}\sqrt{x}\right) & \text{for } 2|x| > 1 \\ -\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(2*x-1),x)
```

```
[Out] Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(x)), 2*Abs(x) > 1), (-sqrt
(2)*atanh(sqrt(2)*sqrt(x)), True))
```

GIAC/XCAS [A] time = 0.201376, size = 43, normalized size = 2.26

$$-\frac{1}{2}\sqrt{2}\ln\left(\frac{1}{2}\sqrt{2}+\sqrt{x}\right)+\frac{1}{2}\sqrt{2}\ln\left(\left|-\frac{1}{2}\sqrt{2}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x - 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*ln(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*ln(abs(-1/2*
sqrt(2) + sqrt(x)))
```

3.223 $\int \sqrt{x}\left(1+x^2\right)\,dx$

Optimal. Leaf size=19

$$\frac{2x^{7/2}}{7}+\frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3 + (2*x^(7/2))/7
```

Rubi [A] time = 0.00736761, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2x^{7/2}}{7}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(1 + x^2),x]

[Out] (2*x^(3/2))/3 + (2*x^(7/2))/7
```

Rubi in Sympy [A] time = 0.976133, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}}{7}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(x**2+1),x)

[Out] 2*x**(7/2)/7 + 2*x**(3/2)/3
```

Mathematica [A] time = 0.00417738, size = 16, normalized size = 0.84

$$\frac{2}{21}x^{3/2}\left(3x^2+7\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(1 + x^2),x]
```

```
[Out] (2*x^(3/2)*(7 + 3*x^2))/21
```

Maple [A] time = 0.002, size = 13, normalized size = 0.7

$$\frac{6\,x^2+14}{21}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^2+1),x)
```

```
[Out] 2/21*x^(3/2)*(3*x^2+7)
```

Maxima [A] time = 1.36906, size = 15, normalized size = 0.79

$$\frac{2}{7}\,x^{\frac{7}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

Fricas [A] time = 0.221029, size = 19, normalized size = 1.

$$\frac{2}{21}\left(3\,x^3+7\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^3 + 7*x)*sqrt(x)
```

Sympy [A] time = 0.66284, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}}{7}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(x**2+1),x)
```

```
[Out] 2*x**(7/2)/7 + 2*x**(3/2)/3
```

GIAC/XCAS [A] time = 0.201137, size = 15, normalized size = 0.79

$$\frac{2}{7}\,x^{\frac{7}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

3.224 $\int \frac{\sqrt[3]{-a+x}}{x}\,dx$

Optimal. Leaf size=88

$$3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)$$

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*(-a + x)^(
1/3))/(Sqrt[3]*a^(1/3))] + (a^(1/3)*Log[x])/2 - (3*a^(1/3)*Log[a^
(1/3) + (-a + x)^(1/3)])/2
```

Rubi [A] time = 0.0867618, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*(-a + x)^(
1/3))/(Sqrt[3]*a^(1/3))] + (a^(1/3)*Log[x])/2 - (3*a^(1/3)*Log[a^
(1/3) + (-a + x)^(1/3)])/2
```

Rubi in Sympy [A] time = 3.19455, size = 78, normalized size = 0.89

$$\frac{\sqrt[3]{a}\log(x)}{2}-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)}{2}+\sqrt{3}\sqrt[3]{a}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{-a+x}}{3}\right)}{\sqrt[3]{a}}\right)+3\sqrt[3]{-a+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-a+x)**(1/3)/x,x)
```

```
[Out] a**(1/3)*log(x)/2 - 3*a**(1/3)*log(a**(1/3) + (-a + x)**(1/3))/2
+ sqrt(3)*a**(1/3)*atan(sqrt(3)*(a**(1/3)/3 - 2*(-a + x)**(1/3)/3
)/a**(1/3)) + 3*(-a + x)**(1/3)
```

Mathematica [A] time = 0.0575432, size = 112, normalized size = 1.27

$$\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{x-a}+(x-a)^{2/3}\right)+3\sqrt[3]{x-a}-\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{x-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*(-a + x)^(1/3))
/a^(1/3))/Sqrt[3]] - a^(1/3)*Log[a^(1/3) + (-a + x)^(1/3)] + (a^(
1/3)*Log[a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)])/2
```

Maple [A] time = 0.005, size = 85, normalized size = 1.

$$3\sqrt[3]{-a+x}-\sqrt[3]{a}\ln\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)+\frac{1}{2}\sqrt[3]{a}\ln\left((-a+x)^{\frac{2}{3}}-\sqrt[3]{-a+x}\sqrt[3]{a}+a^{\frac{2}{3}}\right)$$
$$-\sqrt[3]{a}\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{-a+x}}{\sqrt[3]{a}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-a+x)^(1/3)/x,x)
```

```
[Out] 3*(-a+x)^(1/3)-a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+1/2*a^(1/3)*ln((-
a+x)^(2/3)-(-a+x)^(1/3)*a^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/a^(1/3)*(-a+x)^(1/3)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a + x)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219499, size = 138, normalized size = 1.57

$$-\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left((-a)^{\frac{1}{3}}+2\,(-a+x)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\log\left(-(-a)^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)+3\,(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a + x)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(
1/3))/(-a)^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-
a + x)^(1/3) + (-a + x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a
 + x)^(1/3)) + 3*(-a + x)^(1/3)
```

Sympy [A] time = 2.4282, size = 155, normalized size = 1.76

$$\frac{4\sqrt[3]{a}e^{\frac{5i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{-a+x}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-a+x}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a+x)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*exp(5*I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3
)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a +
x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4
*a**(1/3)*exp(I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(5*I*pi/3)
/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*(-a + x)**(1/3)*gamma(4/
3)/gamma(7/3)
```

GIAC/XCAS [A] time = 0.495071, size = 139, normalized size = 1.58

$$-\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left((-a)^{\frac{1}{3}}+2\,(-a+x)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\ln\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\ln\left(\left|-(-a)^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right|\right)+3\,(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a + x)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(
1/3))/(-a)^(1/3)) - 1/2*(-a)^(1/3)*ln((-a)^(2/3) + (-a)^(1/3)*(-a
 + x)^(1/3) + (-a + x)^(2/3)) + (-a)^(1/3)*ln(abs(-(-a)^(1/3) + (
-a + x)^(1/3))) + 3*(-a + x)^(1/3)
```

3.225 $\int x \sinh(x)\, dx$

Optimal. Leaf size=9

$$x\cosh(x) - \sinh(x)$$

```
[Out] x*Cosh[x] - Sinh[x]
```

Rubi [A] time = 0.0188656, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x\cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[x],x]

[Out] x*Cosh[x] - Sinh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x\cosh(x) - \int \cosh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*sinh(x),x)

[Out] x*cosh(x) - Integral(cosh(x), x)
```

Mathematica [A] time = 0.00505029, size = 9, normalized size = 1.

$$x\cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[x],x]

[Out] x*Cosh[x] - Sinh[x]
```

Maple [A] time = 0.006, size = 10, normalized size = 1.1

$$x\cosh(x)-\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sinh(x),x)
```

```
[Out] x*cosh(x)-sinh(x)
```

Maxima [A] time = 1.38799, size = 46, normalized size = 5.11

$$\frac{1}{2}x^2\sinh(x)+\frac{1}{4}\left(x^2+2x+2\right)e^{(-x)}-\frac{1}{4}\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh(x) + 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2
)*e^x
```

Fricas [A] time = 0.216624, size = 12, normalized size = 1.33

$$x\cosh(x)-\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sinh(x),x, algorithm="fricas")
```

```
[Out] x*cosh(x) - sinh(x)
```

Sympy [A] time = 0.176317, size = 7, normalized size = 0.78

$$x\cosh(x)-\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sinh(x),x)
```

```
[Out] x*cosh(x) - sinh(x)
```

GIAC/XCAS [A] time = 0.199279, size = 23, normalized size = 2.56

$$\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

3.226 $\int x\cosh(x)\,dx$

Optimal. Leaf size=9

$$x\sinh(x) - \cosh(x)$$

```
[Out] -Cosh[x] + x*Sinh[x]
```

Rubi [A] time = 0.0177194, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x\sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x],x]

[Out] -Cosh[x] + x*Sinh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x\sinh(x) - \int \sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*cosh(x),x)

[Out] x*sinh(x) - Integral(sinh(x), x)
```

Mathematica [A] time = 0.00497574, size = 9, normalized size = 1.

$$x\sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x],x]

[Out] -Cosh[x] + x*Sinh[x]
```

Maple [A] time = 0.004, size = 10, normalized size = 1.1

$$-\cosh(x) + x\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cosh(x),x)
```

```
[Out] -cosh(x)+x*sinh(x)
```

Maxima [A] time = 1.36487, size = 46, normalized size = 5.11

$$\frac{1}{2}x^2\cosh(x) - \frac{1}{4}\left(x^2 + 2x + 2\right)e^{(-x)} - \frac{1}{4}\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*cosh(x) - 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2
)*e^x
```

Fricas [A] time = 0.217322, size = 12, normalized size = 1.33

$$x\sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cosh(x),x, algorithm="fricas")
```

```
[Out] x*sinh(x) - cosh(x)
```

Sympy [A] time = 0.169484, size = 7, normalized size = 0.78

$$x\sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cosh(x),x)
```

```
[Out] x*sinh(x) - cosh(x)
```

GIAC/XCAS [A] time = 0.228665, size = 23, normalized size = 2.56

$$-\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

3.227 $\int \tanh(2x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log(\cosh(2x))$$

```
[Out] Log[Cosh[2*x]]/2
```

Rubi [A] time = 0.00752568, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tanh[2*x],x]

[Out] Log[Cosh[2*x]]/2
```

Rubi in Sympy [A] time = 1.41185, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(2x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sinh(2*x)/cosh(2*x),x)

[Out] log(cosh(2*x))/2
```

Mathematica [A] time = 0.0064003, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tanh[2*x],x]

[Out] Log[Cosh[2*x]]/2
```

Maple [A] time = 0.004, size = 8, normalized size = 0.9

$$\frac{\ln\left(\cosh\left(2x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sinh(2*x)/cosh(2*x),x)
```

```
[Out] 1/2*ln(cosh(2*x))
```

Maxima [A] time = 1.34916, size = 9, normalized size = 1.

$$\frac{1}{2}\,\log\left(\cosh\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(2*x))
```

Fricas [A] time = 0.215119, size = 35, normalized size = 3.89

$$-x+\frac{1}{2}\,\log\left(\frac{2\,\cosh\left(2x\right)}{\cosh\left(2x\right)-\sinh\left(2x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))
```

Sympy [A] time = 0.126503, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(2x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)/cosh(2*x),x)
```

```
[Out] log(cosh(2*x))/2
```

GIAC/XCAS [A] time = 0.225222, size = 18, normalized size = 2.

$$-x + \frac{1}{2}\ln\left(e^{(4x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)/cosh(2*x),x, algorithm="giac")
```

```
[Out] -x + 1/2*ln(e^(4*x) + 1)
```

3.228 $\int \frac{-1+i\mathbf{eps}\sinh(x)}{ia-x+i\mathbf{eps}\cosh(x)}\,dx$

Optimal. Leaf size=12

$$\log(a+\mathrm{eps}\cosh(x)+ix)$$

```
[Out] Log[a + I*x + eps*Cosh[x]]
```

Rubi [A] time = 0.0516475, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\log(a+\mathrm{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]

[Out] Log[a + I*x + eps*Cosh[x]]
```

Rubi in Sympy [A] time = 6.59654, size = 12, normalized size = 1.

$$\log\left(-ia-ieps\cosh\left(x\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)

[Out] log(-I*a - I*eps*cosh(x) + x)
```

Mathematica [A] time = 0.0177242, size = 12, normalized size = 1.

$$\log(a+\mathrm{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]

[Out] Log[a + I*x + eps*Cosh[x]]
```

Maple [A] time = 0.009, size = 16, normalized size = 1.3

$$\ln \left(ia - x + ieps \cosh \left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)

[Out] ln(I*a-x+I*eps*cosh(x))
```

Maxima [A] time = 1.33436, size = 18, normalized size = 1.5

$$\log \left(i\, eps \cosh \left(x\right) + i\, a - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((I*eps*sinh(x) - 1)/(I*eps*cosh(x) + I*a - x),x, algorithm="maxima")

[Out] log(I*eps*cosh(x) + I*a - x)
```

Fricas [A] time = 0.229574, size = 35, normalized size = 2.92

$$-x + \log \left(\frac{eps e^{\left(2\, x\right)} + 2\, {\left(a + i\, x\right)} e^{x} + eps}{eps}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((I*eps*sinh(x) - 1)/(I*eps*cosh(x) + I*a - x),x, algorithm="fricas")

[Out] -x + log((eps*e^(2*x) + 2*(a + I*x)*e^x + eps)/eps)
```

Sympy [A] time = 2.10794, size = 22, normalized size = 1.83

$$-x + \log \left(e^{2x} + 1 + \frac{\left(2a + 2ix\right) e^{x}}{eps}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
```

```
[Out] -x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)
```

GIAC/XCAS [A] time = 0.226171, size = 31, normalized size = 2.58

$$-x + \ln\left(epse^{(2x)} + 2\,ae^{x} + 2i\,xe^{x} + eps\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*eps*sinh(x) - 1)/(I*eps*cosh(x) + I*a - x),x, algorithm="giac")
```

```
[Out] -x + ln(eps*e^(2*x) + 2*a*e^x + 2*I*x*e^x + eps)
```

3.229 $\int \cos^2(x)\sin(3+2x)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

```
[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Rubi [A] time = 0.0381084, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[3 + 2*x],x]

[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\cos(2x+3)}{4}-\frac{\cos(4x+3)}{16}+\sin(3)\int\frac{1}{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(3+2*x),x)

[Out] -cos(2*x + 3)/4 - cos(4*x + 3)/16 + sin(3)*Integral(1/4, x)
```

Mathematica [A] time = 0.0142453, size = 28, normalized size = 1.

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[3 + 2*x],x]
```

```
[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Maple [A] time = 0.008, size = 23, normalized size = 0.8

$$-\frac{\cos(3+2x)}{4}-\frac{\cos(3+4x)}{16}+\frac{x\sin(3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(3+2*x),x)
```

```
[Out] -1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

Maxima [A] time = 1.33387, size = 30, normalized size = 1.07

$$\frac{1}{4}x\sin(3)-\frac{1}{16}\cos(4x+3)-\frac{1}{4}\cos(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(2*x + 3),x, algorithm="maxima")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

Fricas [A] time = 0.227144, size = 43, normalized size = 1.54

$$-\frac{1}{2}\cos(3)\cos(x)^4+\frac{1}{4}x\sin(3)+\frac{1}{4}\left(2\cos(x)^3\sin(3)+\cos(x)\sin(3)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(2*x + 3),x, algorithm="fricas")
```

```
[Out] -1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + co
s(x)*sin(3))*sin(x)
```

Sympy [A] time = 3.05223, size = 76, normalized size = 2.71

$$-\frac{x\sin^2(x)\sin(2x+3)}{4}-\frac{x\sin(x)\cos(x)\cos(2x+3)}{2}+\frac{x\sin(2x+3)\cos^2(x)}{4}-\frac{\sin^2(x)\cos(2x+3)}{2}+\frac{3\sin(x)\sin(2x+3)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(3+2*x),x)
```

```
[Out] -x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*
sin(2*x + 3)*cos(x)**2/4 - sin(x)**2*cos(2*x + 3)/2 + 3*sin(x)*si
n(2*x + 3)*cos(x)/4
```

GIAC/XCAS [A] time = 0.211553, size = 30, normalized size = 1.07

$$\frac{1}{4}\,x\sin\left(3\right) - \frac{1}{16}\,\cos\left(4\,x + 3\right) - \frac{1}{4}\,\cos\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(2*x + 3),x, algorithm="giac")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

3.230 $\int x \tan^{-1}(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi [A] time = 0.018239, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]

[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi in Sympy [A] time = 1.97074, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}(x)}{2}-\frac{x}{2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*atan(x),x)

[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Mathematica [A] time = 0.00341262, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]

[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$-\frac{x}{2}+\frac{\arctan\left(x\right)}{2}+\frac{x^{2}\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arctan(x),x)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A] time = 1.60849, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^{2}\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A] time = 0.212079, size = 18, normalized size = 0.86

$$\frac{1}{2}\,\left(x^{2}+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] time = 0.31586, size = 15, normalized size = 0.71

$$\frac{x^{2}\operatorname{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

GIAC/XCAS [A] time = 0.198055, size = 20, normalized size = 0.95

$$\frac{1}{2}x^2\arctan\left(x\right) - \frac{1}{2}x + \frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.231 $\int x \cot^{-1}(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Rubi [A] time = 0.0183174, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*ArcCot[x],x]

[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Rubi in Sympy [A] time = 1.94362, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{acot}(x)}{2}+\frac{x}{2}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*acot(x),x)

[Out] x**2*acot(x)/2 + x/2 - atan(x)/2
```

Mathematica [A] time = 0.00345294, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*ArcCot[x],x]

[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{x}{2}+\frac{x^{2}\operatorname{arccot}(x)}{2}-\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arccot(x),x)
```

```
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

Maxima [A] time = 1.62479, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^{2}\operatorname{arccot}(x)+\frac{1}{2}\,x-\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arccot(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)
```

Fricas [A] time = 0.208875, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(x^{2}+1\right)\operatorname{arccot}(x)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arccot(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arccot(x) + 1/2*x
```

Sympy [A] time = 0.326281, size = 15, normalized size = 0.71

$$\frac{x^{2}\operatorname{acot}(x)}{2}+\frac{x}{2}+\frac{\operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*acot(x),x)

[Out] x**2*acot(x)/2 + x/2 + acot(x)/2
```

GIAC/XCAS [A] time = 0.199066, size = 23, normalized size = 1.1

$$\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)+\frac{1}{2}\,x-\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arccot(x),x, algorithm="giac")

[Out] 1/2*x^2*arctan(1/x) + 1/2*x - 1/2*arctan(x)
```

3.232 $\int x \log(a+x^2)\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\left(a+x^2\right)\log\left(a+x^2\right)-\frac{x^2}{2}$$

```
[Out] -x^2/2 + ((a + x^2)*Log[a + x^2])/2
```

Rubi [A] time = 0.033045, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{1}{2}\left(a+x^2\right)\log\left(a+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[a + x^2],x]

[Out] -x^2/2 + ((a + x^2)*Log[a + x^2])/2
```

Rubi in Sympy [A] time = 1.03066, size = 26, normalized size = 1.13

$$\frac{a\log\left(a+x^2\right)}{2}+\frac{x^2\log\left(a+x^2\right)}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x**2+a),x)

[Out] a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2
```

Mathematica [A] time = 0.00337806, size = 32, normalized size = 1.39

$$\frac{1}{2}x^2\log\left(a+x^2\right)+\frac{1}{2}a\log\left(a+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[a + x^2],x]
```

```
[Out] -x^2/2 + (a*Log[a + x^2])/2 + (x^2*Log[a + x^2])/2
```

Maple [A] time = 0.002, size = 23, normalized size = 1.

$$\frac{(x^2+a)\ln(x^2+a)}{2} - \frac{x^2}{2} - \frac{a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x^2+a),x)
```

```
[Out] 1/2*(x^2+a)*ln(x^2+a)-1/2*x^2-1/2*a
```

Maxima [A] time = 1.43127, size = 30, normalized size = 1.3

$$-\frac{1}{2}x^2 + \frac{1}{2}(x^2+a)\log(x^2+a) - \frac{1}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x^2 + a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

Fricas [A] time = 0.213565, size = 26, normalized size = 1.13

$$-\frac{1}{2}x^2 + \frac{1}{2}(x^2+a)\log(x^2+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a)
```

Sympy [A] time = 0.542983, size = 26, normalized size = 1.13

$$\frac{a\log(a+x^2)}{2} + \frac{x^2\log(a+x^2)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x**2+a),x)
```

```
[Out] a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2
```

GIAC/XCAS [A] time = 0.217452, size = 30, normalized size = 1.3

$$-\frac{1}{2}\,x^2+\frac{1}{2}\left(x^2+a\right)\ln\left(x^2+a\right)-\frac{1}{2}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*ln(x^2 + a) - 1/2*a
```

3.233 $\int \cos(x)\sin(a+x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

```
[Out] -Cos[a + 2*x]/4 + (x*Sin[a])/2
```

Rubi [A] time = 0.0281243, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Sin[a + x],x]

[Out] -Cos[a + 2*x]/4 + (x*Sin[a])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sin(a)\int\frac{1}{2}\,dx-\frac{\cos(a+2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*sin(a+x),x)

[Out] sin(a)*Integral(1/2, x) - cos(a + 2*x)/4
```

Mathematica [A] time = 0.0239456, size = 18, normalized size = 1.

$$\frac{1}{4}(2x\sin(a)-\cos(a+2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Sin[a + x],x]
```

```
[Out] (-Cos[a + 2*x] + 2*x*Sin[a])/4
```

Maple [A] time = 0.006, size = 15, normalized size = 0.8

$$-\frac{\cos(a+2x)}{4}+\frac{x\sin(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*sin(a+x),x)
```

```
[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)
```

Maxima [A] time = 1.37006, size = 19, normalized size = 1.06

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(a + x),x, algorithm="maxima")
```

```
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Fricas [A] time = 0.238898, size = 38, normalized size = 2.11

$$-\frac{1}{2}\cos(a+x)^2\cos(a)-\frac{1}{2}\cos(a+x)\sin(a+x)\sin(a)+\frac{1}{2}x\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(a + x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) + 1/2
*x*sin(a)
```

Sympy [A] time = 0.74076, size = 32, normalized size = 1.78

$$-\frac{x\sin(x)\cos(a+x)}{2}+\frac{x\sin(a+x)\cos(x)}{2}-\frac{\cos(x)\cos(a+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(a+x),x)
```

```
[Out] -x*sin(x)*cos(a + x)/2 + x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x
)/2
```

GIAC/XCAS [A] time = 0.198968, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x\sin\left(a\right) - \frac{1}{4}\,\cos\left(a + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(a + x),x, algorithm="giac")
```

```
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.234 $\int \cos(a+x)\sin(x)\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

```
[Out] -Cos[a + 2*x]/4 - (x*Sin[a])/2
```

Rubi [A] time = 0.0228637, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[a + x]*Sin[x],x]

[Out] -Cos[a + 2*x]/4 - (x*Sin[a])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sin(a)\int\left(-\frac{1}{2}\right)dx-\frac{\cos(a+2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(a+x)*sin(x),x)

[Out] sin(a)*Integral(-1/2, x) - cos(a + 2*x)/4
```

Mathematica [A] time = 0.0145576, size = 18, normalized size = 1.

$$\frac{1}{4}(-2x\sin(a)-\cos(a+2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[a + x]*Sin[x],x]
```

```
[Out] (-Cos[a + 2*x] - 2*x*Sin[a])/4
```

Maple [A] time = 0.006, size = 15, normalized size = 0.8

$$-\frac{\cos (a+2 x)}{4}-\frac{x \sin (a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+x)*sin(x),x)
```

```
[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)
```

Maxima [A] time = 1.39566, size = 19, normalized size = 1.06

$$-\frac{1}{2} x \sin (a)-\frac{1}{4} \cos (a+2 x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a + x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Fricas [A] time = 0.231597, size = 38, normalized size = 2.11

$$-\frac{1}{2} \cos (a+x)^2 \cos (a)-\frac{1}{2} \cos (a+x) \sin (a+x) \sin (a)-\frac{1}{2} x \sin (a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a + x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) - 1/2
*x*sin(a)
```

Sympy [A] time = 0.740763, size = 32, normalized size = 1.78

$$\frac{x \sin (x) \cos (a+x)}{2}-\frac{x \sin (a+x) \cos (x)}{2}-\frac{\cos (x) \cos (a+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(a+x)*sin(x),x)
```

```
[Out] x*sin(x)*cos(a + x)/2 - x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)
/2
```

GIAC/XCAS [A] time = 0.199018, size = 19, normalized size = 1.06

$$-\frac{1}{2}\,x\sin\left(a\right)-\frac{1}{4}\,\cos\left(a+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(a + x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.235 $\int \sqrt{1+\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

```
[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rubi [A] time = 0.0110596, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Sin[x]],x]

[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rubi in Sympy [A] time = 0.490841, size = 14, normalized size = 1.17

$$-\frac{2\cos{(x)}}{\sqrt{\sin{(x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+sin(x))**(1/2),x)

[Out] -2*cos(x)/sqrt(sin(x) + 1)
```

Mathematica [B] time = 0.0197196, size = 40, normalized size = 3.33

$$\frac{2\sqrt{\sin(x)+1}\left(\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.034, size = 17, normalized size = 1.4

$$2\,\frac{(-1+\sin(x))\sqrt{1+\sin(x)}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x))^(1/2),x)
```

```
[Out] 2*(-1+sin(x))*(1+sin(x))^(1/2)/cos(x)
```

Maxima [A] time = 1.54354, size = 58, normalized size = 4.83

$$-\frac{2}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}+\frac{2\,\sin(x)}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -2/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*sin(x)/(sqrt(sin(x)^2/(c
os(x) + 1)^2 + 1)*(cos(x) + 1))
```

Fricas [A] time = 0.214015, size = 32, normalized size = 2.67

$$-\frac{2\,(\cos(x)-\sin(x)+1)\sqrt{\sin(x)+1}}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] -2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x) + 1), x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x) + 1), x)
```

3.236 $\int \sqrt{1-\sin(x)}\,dx$

Optimal. Leaf size=14

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

```
[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]
```

Rubi [A] time = 0.0146549, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - Sin[x]],x]

[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]
```

Rubi in Sympy [A] time = 0.523708, size = 12, normalized size = 0.86

$$\frac{2\cos(x)}{\sqrt{-\sin(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-sin(x))**(1/2),x)

[Out] 2*cos(x)/sqrt(-sin(x) + 1)
```

Mathematica [B] time = 0.0192303, size = 42, normalized size = 3.

$$\frac{2\sqrt{1-\sin(x)}\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.028, size = 23, normalized size = 1.6

$$-2\frac{(-1+\sin(x))(1+\sin(x))}{\cos(x)\sqrt{1-\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x))^(1/2),x)
```

```
[Out] -2*(-1+sin(x))*(1+sin(x))/cos(x)/(1-sin(x))^(1/2)
```

Maxima [A] time = 1.59325, size = 58, normalized size = 4.14

$$-\frac{2}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}-\frac{2\sin(x)}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -2/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1) - 2*sin(x)/(sqrt(sin(x)^2/(c
os(x) + 1)^2 + 1)*(cos(x) + 1))
```

Fricas [A] time = 0.219188, size = 35, normalized size = 2.5

$$\frac{2(\cos(x)+\sin(x)+1)\sqrt{-\sin(x)+1}}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-sin(x) + 1),x, algorithm="fricas")
```

```
[Out] 2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-sin(x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sin(x) + 1), x)
```

3.237 $\int \sqrt{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

```
[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]
```

Rubi [A] time = 0.0124195, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Cos[x]],x]
```

```
[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]
```

Rubi in Sympy [A] time = 0.496345, size = 12, normalized size = 1.

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+cos(x))**(1/2),x)
```

```
[Out] 2*sin(x)/sqrt(cos(x) + 1)
```

Mathematica [A] time = 0.0061168, size = 16, normalized size = 1.33

$$2\sqrt{\cos(x)+1}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Cos[x]],x]
```

```
[Out] 2*Sqrt[1 + Cos[x]]*Tan[x/2]
```

Maple [B] time = 0.021, size = 22, normalized size = 1.8

$$2\,\frac{\cos(x/2)\sin(x/2)\sqrt{2}}{\sqrt{(\cos(x/2))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+cos(x))^(1/2),x)
```

```
[Out] 2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)
```

Maxima [A] time = 1.59605, size = 38, normalized size = 3.17

$$\frac{2\sqrt{2}\sin(x)}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sin(x)/(sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))
```

Fricas [A] time = 0.210835, size = 14, normalized size = 1.17

$$\frac{2\,\sin(x)}{\sqrt{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] 2*sin(x)/sqrt(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(x) + 1), x)
```

3.238 $\int \sqrt{1-\cos(x)}\,dx$

Optimal. Leaf size=14

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

```
[Out] (-2*Sin[x])/Sqrt[1 - Cos[x]]
```

Rubi [A] time = 0.0151198, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - Cos[x]],x]

[Out] (-2*Sin[x])/Sqrt[1 - Cos[x]]
```

Rubi in Sympy [A] time = 0.517319, size = 14, normalized size = 1.

$$-\frac{2\sin(x)}{\sqrt{-\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-cos(x))**(1/2),x)

[Out] -2*sin(x)/sqrt(-cos(x) + 1)
```

Mathematica [A] time = 0.00816821, size = 18, normalized size = 1.29

$$-2\sqrt{1-\cos(x)}\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - Cos[x]],x]
```

```
[Out] -2*Sqrt[1 - Cos[x]]*Cot[x/2]
```

Maple [A] time = 0.026, size = 22, normalized size = 1.6

$$-2\,\frac{\sin\left(x/2\right)\cos\left(x/2\right)\sqrt{2}}{\sqrt{\left(\sin\left(x/2\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-cos(x))^(1/2),x)
```

```
[Out] -2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)
```

Maxima [A] time = 1.55677, size = 27, normalized size = 1.93

$$-\frac{2\,\sqrt{2}}{\sqrt{\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-cos(x) + 1),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.218304, size = 24, normalized size = 1.71

$$-\frac{2\left(\cos\left(x\right)+1\right)\sqrt{-\cos\left(x\right)+1}}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-cos(x) + 1),x, algorithm="fricas")
```

```
[Out] -2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\cos\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\cos(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-cos(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(x) + 1), x)
```

3.239 $\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}} \, dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3} + \frac{2}{3}(x-1)^{3/2}$$

```
[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi [A] time = 0.0154869, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{2x^{3/2}}{3} + \frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi in Sympy [A] time = 0.686521, size = 17, normalized size = 0.81

$$\frac{2x^{\frac{3}{2}}}{3} + \frac{2(x-1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + 2*(x - 1)**(3/2)/3
```

Mathematica [A] time = 0.0240752, size = 17, normalized size = 0.81

$$\frac{2}{3}\left(x^{3/2} + (x-1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*((-1 + x)^(3/2) + x^(3/2)))/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{2}{3}(-1+x)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(-1+x)^(1/2)+x^(1/2)),x)
```

```
[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{x-1}-\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(x - 1) - sqrt(x)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x - 1) - sqrt(x)), x)
```

Fricas [A] time = 0.208959, size = 18, normalized size = 0.86

$$\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(x - 1) - sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [A] time = 0.697352, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x-1}}{-3\sqrt{x}+3\sqrt{x-1}}-\frac{4x}{-3\sqrt{x}+3\sqrt{x-1}}+\frac{2}{-3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt
(x) + 3*sqrt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))
```

GIAC/XCAS [A] time = 0.199536, size = 18, normalized size = 0.86

$$\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x - 1) - sqrt(x)),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.240 $\int \frac{1}{1-\sqrt{1+x}}\,dx$

Optimal. Leaf size=24

$$-2\sqrt{x+1}-2\log\left(1-\sqrt{x+1}\right)$$

```
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Rubi [A] time = 0.0236266, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-2\sqrt{x+1}-2\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[1 + x])^(-1),x]

[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Rubi in Sympy [A] time = 1.49626, size = 20, normalized size = 0.83

$$-2\sqrt{x+1}-2\log\left(-\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-(1+x)**(1/2)),x)

[Out] -2*sqrt(x + 1) - 2*log(-sqrt(x + 1) + 1)
```

Mathematica [A] time = 0.00826196, size = 22, normalized size = 0.92

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[1 + x])^(-1),x]

[Out] -2*Sqrt[1 + x] - 2*Log[-1 + Sqrt[1 + x]]
```

Maple [A] time = 0.003, size = 31, normalized size = 1.3

$$-\ln(x) - 2\sqrt{1+x} - \ln\left(-1+\sqrt{1+x}\right) + \ln\left(1+\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(1+x)^(1/2)),x)
```

```
[Out] -ln(x)-2*(1+x)^(1/2)-ln(-1+(1+x)^(1/2))+ln(1+(1+x)^(1/2))
```

Maxima [A] time = 1.38275, size = 24, normalized size = 1.

$$-2\sqrt{x+1} - 2\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x + 1) - 1),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Fricas [A] time = 0.209711, size = 24, normalized size = 1.

$$-2\sqrt{x+1} - 2\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x + 1) - 1),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Sympy [A] time = 0.13108, size = 20, normalized size = 0.83

$$-2\sqrt{x+1} - 2\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(1+x)**(1/2)),x)
```

```
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

GIAC/XCAS [A] time = 0.230195, size = 26, normalized size = 1.08

$$-2\sqrt{x+1} - 2\ln\left(\left|\sqrt{x+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x + 1) - 1),x, algorithm="giac")
```

```
[Out] -2*sqrt(x + 1) - 2*ln(abs(sqrt(x + 1) - 1))
```

3.241 $\int \frac{x}{\sqrt{36+x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

```
[Out] ArcSinh[x^2/6]/2
```

Rubi [A] time = 0.0117975, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[36 + x^4],x]

[Out] ArcSinh[x^2/6]/2
```

Rubi in Sympy [A] time = 1.10235, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+36)**(1/2),x)

[Out] asinh(x**2/6)/2
```

Mathematica [A] time = 0.00703099, size = 12, normalized size = 1.

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[36 + x^4],x]
```

```
[Out] ArcSinh[x^2/6]/2
```

Maple [A] time = 0.005, size = 9, normalized size = 0.8

$$\frac{1}{2}Arcsinh\left(\frac{x^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4+36)^(1/2),x)
```

```
[Out] 1/2*arcsinh(1/6*x^2)
```

Maxima [A] time = 1.36289, size = 45, normalized size = 3.75

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^4 + 36),x, algorithm="maxima")
```

```
[Out] 1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)
```

Fricas [A] time = 0.229911, size = 22, normalized size = 1.83

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^4 + 36),x, algorithm="fricas")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

Sympy [A] time = 1.50191, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+36)**(1/2),x)
```

```
[Out] asinh(x**2/6)/2
```

GIAC/XCAS [A] time = 0.212432, size = 22, normalized size = 1.83

$$-\frac{1}{2}\ln\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^4 + 36),x, algorithm="giac")
```

```
[Out] -1/2*ln(-x^2 + sqrt(x^4 + 36))
```

3.242 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi [A] time = 0.026716, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$6\sqrt[6]{x}+2\sqrt{x}-6\log\left(\sqrt[6]{x}+1\right)-6\int^{\sqrt[6]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + 2*sqrt(x) - 6*log(x**(1/6) + 1) - 6*Integral(x, (x,
x**(1/6)))
```

Mathematica [A] time = 0.0128294, size = 32, normalized size = 1.

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Maple [B] time = 0.018, size = 92, normalized size = 2.9

$$-\ln\left(\sqrt[3]{x}+\sqrt[6]{x}+1\right)+2\ln\left(-1+\sqrt[6]{x}\right)+\ln\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\ln\left(1+\sqrt[6]{x}\right)+2\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(1+\sqrt{x}\right)+6\sqrt[6]{x}-\ln\left(-1+x\right)+\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)-2\ln\left(-1+\sqrt[3]{x}\right)-3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] -ln(x^(1/3)+x^(1/6)+1)+2*ln(-1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1)-2*l
n(1+x^(1/6))+2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))+6*x^(1/6)-ln(-
1+x)+ln(x^(2/3)+x^(1/3)+1)-2*ln(-1+x^(1/3))-3*x^(1/3)
```

Maxima [A] time = 1.37249, size = 32, normalized size = 1.

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A] time = 0.204472, size = 32, normalized size = 1.

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

GIAC/XCAS [A] time = 0.199955, size = 32, normalized size = 1.

$$2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\ln\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*ln(x^(1/6) + 1)
```

3.243 $\int \log \left(2+3 x^2\right) \, dx$

Optimal. Leaf size=33

$$x \log \left(3 x^2+2\right)-2 x+2 \sqrt{\frac{2}{3}} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)$$

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rubi [A] time = 0.0251145, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$x \log \left(3 x^2+2\right)-2 x+2 \sqrt{\frac{2}{3}} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rubi in Sympy [A] time = 1.97707, size = 31, normalized size = 0.94

$$x \log \left(3 x^2+2\right)-2 x+\frac{2 \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Mathematica [A] time = 0.0204792, size = 33, normalized size = 1.

$$x \log \left(3 x^2+2\right)-2 x+2 \sqrt{\frac{2}{3}} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A] time = 0.003, size = 27, normalized size = 0.8

$$-2\,x+x\ln\left(3\,x^{2}+2\right)+\frac{2\sqrt{6}}{3}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(3*x^2+2),x)
```

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.50082, size = 35, normalized size = 1.06

$$x\log\left(3\,x^{2}+2\right)+\frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Fricas [A] time = 0.205749, size = 54, normalized size = 1.64

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3}x\log\left(3\,x^{2}+2\right)-2\sqrt{3}x+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*x*log(3*x^2 + 2) - 2*sqrt(3)*x + 2*sqrt(2)*a
rctan(1/2*sqrt(3)*sqrt(2)*x))
```

Sympy [A] time = 0.134252, size = 31, normalized size = 0.94

$$x\log\left(3x^2+2\right)-2x+\frac{2\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

GIAC/XCAS [A] time = 0.199679, size = 35, normalized size = 1.06

$$x\ln\left(3\,x^2+2\right)+\frac{2}{3}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] x*ln(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.244 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.00476487, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x],x]

[Out] Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.035339, size = 3, normalized size = 1.

$$\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x),x)

[Out] log(sin(x))
```

Mathematica [A] time = 0.00335982, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x],x]

[Out] Log[Sin[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 1.40366, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [A] time = 0.219285, size = 15, normalized size = 5.

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] time = 0.041301, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

GIAC/XCAS [A] time = 0.199943, size = 15, normalized size = 5.

$$\frac{1}{2}\ln\left(-\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="giac")
```

```
[Out] 1/2*ln(-cos(x)^2 + 1)
```

3.245 $\int \cot^4(x)\, dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0149954, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^4,x]

[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi in Sympy [A] time = 0.520162, size = 14, normalized size = 1.17

$$x+\frac{1}{\tan(x)}-\frac{1}{3\tan^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**4,x)

[Out] x + 1/tan(x) - 1/(3*tan(x)**3)
```

Mathematica [A] time = 0.00525028, size = 18, normalized size = 1.5

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0.004, size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3} + \cot(x) - \frac{\pi}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

Maxima [A] time = 1.55278, size = 22, normalized size = 1.83

$$x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [A] time = 0.246982, size = 65, normalized size = 5.42

$$\frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)
/((cos(2*x) - 1)*sin(2*x))
```

Sympy [A] time = 0.052207, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.207786, size = 46, normalized size = 3.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^{3}+x+\frac{15\tan\left(\frac{1}{2}x\right)^{2}-1}{24\tan\left(\frac{1}{2}x\right)^{3}}-\frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 -
 5/8*tan(1/2*x)
```

3.246 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

```
[Out] Log[Cosh[x]]
```

Rubi [A] time = 0.00632222, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tanh[x],x]

[Out] Log[Cosh[x]]
```

Rubi in Sympy [A] time = 21.2397, size = 10, normalized size = 3.33

$$-\frac{\log\left(-\tanh^{2}(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tanh(x),x)

[Out] -log(-tanh(x)**2 + 1)/2
```

Mathematica [A] time = 0.00318447, size = 3, normalized size = 1.

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tanh[x],x]

[Out] Log[Cosh[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tanh(x),x)

[Out] ln(cosh(x))
```

Maxima [A] time = 1.3996, size = 4, normalized size = 1.33

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x, algorithm="maxima")

[Out] log(cosh(x))
```

Fricas [A] time = 0.223663, size = 24, normalized size = 8.

$$-x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x, algorithm="fricas")

[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [A] time = 0.114938, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x)
```

```
[Out] x - log(tanh(x) + 1)
```

GIAC/XCAS [A] time = 0.19794, size = 15, normalized size = 5.

$$-x + \ln\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + ln(e^(2*x) + 1)
```

3.247 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

```
[Out] Log[Sinh[x]]
```

Rubi [A] time = 0.00608192, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Coth[x],x]

[Out] Log[Sinh[x]]
```

Rubi in Sympy [A] time = 35.1172, size = 17, normalized size = 5.67

$$-\frac{\log\left(-\tanh^2(x)+1\right)}{2}+\frac{\log\left(\tanh^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(coth(x),x)

[Out] -log(-tanh(x)**2 + 1)/2 + log(tanh(x)**2)/2
```

Mathematica [A] time = 0.00534756, size = 3, normalized size = 1.

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Coth[x],x]

[Out] Log[Sinh[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(coth(x),x)

[Out] ln(sinh(x))
```

Maxima [A] time = 1.46243, size = 4, normalized size = 1.33

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x, algorithm="maxima")

[Out] log(sinh(x))
```

Fricas [A] time = 0.221777, size = 24, normalized size = 8.

$$-x+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x, algorithm="fricas")

[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [A] time = 0.401351, size = 12, normalized size = 4.

$$x-\log(\tanh(x)+1)+\log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x)
```

```
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

GIAC/XCAS [A] time = 0.227696, size = 16, normalized size = 5.33

$$-x + \ln\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x, algorithm="giac")
```

```
[Out] -x + ln(abs(e^(2*x) - 1))
```

3.248 $\int b^x \, dx$

Optimal. Leaf size=8

$$\frac{b^x}{\log(b)}$$

```
[Out] b^x/Log[b]
```

Rubi [A] time = 0.00494214, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[b^x,x]

[Out] b^x/Log[b]
```

Rubi in Sympy [A] time = 0.560434, size = 5, normalized size = 0.62

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(b**x,x)

[Out] b**x/log(b)
```

Mathematica [A] time = 0.000841235, size = 8, normalized size = 1.

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b^x,x]
```

```
[Out] b^x/Log[b]
```

Maple [A] time = 0.002, size = 9, normalized size = 1.1

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^x,x)
```

```
[Out] b^x/ln(b)
```

Maxima [A] time = 1.36529, size = 11, normalized size = 1.38

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="maxima")
```

```
[Out] b^x/log(b)
```

Fricas [A] time = 0.214388, size = 11, normalized size = 1.38

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="fricas")
```

```
[Out] b^x/log(b)
```

Sympy [A] time = 0.066678, size = 8, normalized size = 1.

$$\begin{cases} \frac{b^x}{\log(b)} & \text{for}\ \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b**x,x)
```

```
[Out] Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))
```

GIAC/XCAS [A] time = 0.207368, size = 11, normalized size = 1.38

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b^x,x, algorithm="giac")
```

```
[Out] b^x/ln(b)
```

3.249 $\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

```
[Out] -((x*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)) + (x^5*Sqrt[2 + x^(-4) +
x^4])/(3*(1 + x^4))
```

Rubi [A] time = 0.0376486, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^(-4) + x^4],x]
```

```
[Out] -((x*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)) + (x^5*Sqrt[2 + x^(-4) +
x^4])/(3*(1 + x^4))
```

Rubi in Sympy [A] time = 2.40323, size = 39, normalized size = 0.8

$$\frac{x\sqrt{x^4+2+\frac{1}{x^4}}}{3}-\frac{4x\sqrt{x^4+2+\frac{1}{x^4}}}{3\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+1/x**4+x**4)**(1/2),x)
```

```
[Out] x*sqrt(x**4 + 2 + x**(-4))/3 - 4*x*sqrt(x**4 + 2 + x**(-4))/(3*(x
**4 + 1))
```

Mathematica [A] time = 0.0135004, size = 29, normalized size = 0.59

$$\frac{x\left(x^4-3\right)\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^(-4) + x^4],x]

[Out] (x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.7

$$\frac{\left(x^4-3\right)x}{3x^4+3}\sqrt{\frac{x^8+2x^4+1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+1/x^4+x^4)^(1/2),x)

[Out] 1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)
```

Maxima [A] time = 1.48046, size = 14, normalized size = 0.29

$$\frac{x^4-3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 1/x^4 + 2),x, algorithm="maxima")

[Out] 1/3*(x^4 - 3)/x
```

Fricas [A] time = 0.207197, size = 14, normalized size = 0.29

$$\frac{x^4-3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 1/x^4 + 2),x, algorithm="fricas")

[Out] 1/3*(x^4 - 3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 2 + \frac{1}{x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+1/x**4+x**4)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 2 + x**(-4)), x)
```

GIAC/XCAS [A] time = 0.197295, size = 15, normalized size = 0.31

$$\frac{1}{3}\,x^3 - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 1/x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 1/x
```

3.250 $\int \frac{1+2x}{2+3x} \, dx$

Optimal. Leaf size=16

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

```
[Out] (2*x)/3 - Log[2 + 3*x]/9
```

Rubi [A] time = 0.0167143, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/(2 + 3*x),x]

[Out] (2*x)/3 - Log[2 + 3*x]/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(3x+2)}{9} + \int \frac{2}{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(2+3*x),x)

[Out] -log(3*x + 2)/9 + Integral(2/3, x)
```

Mathematica [A] time = 0.00415242, size = 17, normalized size = 1.06

$$\frac{1}{9}(6x - \log(3x+2) + 4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/(2 + 3*x),x]
```

```
[Out] (4 + 6*x - Log[2 + 3*x])/9
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(2+3*x),x)
```

```
[Out] 2/3*x-1/9*ln(2+3*x)
```

Maxima [A] time = 1.38567, size = 16, normalized size = 1.

$$\frac{2}{3}x-\frac{1}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(3*x + 2),x, algorithm="maxima")
```

```
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Fricas [A] time = 0.204146, size = 16, normalized size = 1.

$$\frac{2}{3}x-\frac{1}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(3*x + 2),x, algorithm="fricas")
```

```
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Sympy [A] time = 0.056242, size = 12, normalized size = 0.75

$$\frac{2x}{3}-\frac{\log(3x+2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(2+3*x),x)
```

```
[Out] 2*x/3 - log(3*x + 2)/9
```

GIAC/XCAS [A] time = 0.197999, size = 18, normalized size = 1.12

$$\frac{2}{3}\,x - \frac{1}{9}\,\ln\left(\left|3\,x + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(3*x + 2),x, algorithm="giac")
```

```
[Out] 2/3*x - 1/9*ln(abs(3*x + 2))
```

3.251 $\int x \log \left(x+\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=40

$$-\frac{1}{4}\sqrt{x^2+1}x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

```
[Out] -(x*Sqrt[1 + x^2])/4 + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]]
)/2
```

Rubi [A] time = 0.0427209, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{1}{4}\sqrt{x^2+1}x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -(x*Sqrt[1 + x^2])/4 + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]]
)/2
```

Rubi in Sympy [A] time = 2.50616, size = 32, normalized size = 0.8

$$\frac{x^2\log\left(x+\sqrt{x^2+1}\right)}{2}-\frac{x\sqrt{x^2+1}}{4}+\frac{\operatorname{asinh}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] x**2*log(x + sqrt(x**2 + 1))/2 - x*sqrt(x**2 + 1)/4 + asinh(x)/4
```

Mathematica [A] time = 0.0280494, size = 36, normalized size = 0.9

$$\frac{1}{4}\left(-\sqrt{x^2+1}x+2x^2\log\left(\sqrt{x^2+1}+x\right)+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] (-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/
4
```

Maple [F] time = 0.003, size = 0, normalized size = 0.

$$\int x\ln\left(x+\sqrt{x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x+(x^2+1)^(1/2)),x)
```

```
[Out] int(x*ln(x+(x^2+1)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,x^2\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,x^2-\int\frac{x^2}{2\left(x^3+(x^2+1)^{\frac{3}{2}}+x\right)}\,dx+\frac{1}{4}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x + sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3
 + (x^2 + 1)^(3/2) + x), x) + 1/4*log(x^2 + 1)
```

Fricas [A] time = 0.215754, size = 41, normalized size = 1.02

$$\frac{1}{4}\,\left(2\,x^2+1\right)\,\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,\sqrt{x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x + sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\log\left(x+\sqrt{x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral(x*log(x + sqrt(x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.201787, size = 54, normalized size = 1.35

$$\frac{1}{2}\,x^2\ln\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,\sqrt{x^2+1}x-\frac{1}{4}\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x + sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*ln(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*ln(-x +
 sqrt(x^2 + 1))
```

3.252 $\int x(1+e^x\sin(x))^2\,dx$

Optimal. Leaf size=128

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^x x\sin(x)+\frac{1}{32}e^{2x}\sin(2x)$$
$$-e^x x\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)-\frac{1}{4}e^{2x}x\sin(x)\cos(x)+\frac{1}{16}e^{2x}\sin(x)\cos(x)$$

```
[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[
x] - (E^(2*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x
])/16 - (E^(2*x)*x*Cos[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^
(2*x)*x*Sin[x]^2)/4 + (E^(2*x)*Sin[2*x])/32
```

Rubi [A] time = 0.292078, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^x x\sin(x)+\frac{1}{32}e^{2x}\sin(2x)$$
$$-e^x x\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)-\frac{1}{4}e^{2x}x\sin(x)\cos(x)+\frac{1}{16}e^{2x}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[
x] - (E^(2*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x
])/16 - (E^(2*x)*x*Cos[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^
(2*x)*x*Sin[x]^2)/4 + (E^(2*x)*Sin[2*x])/32
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(e^x\sin(x)+1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+exp(x)*sin(x))**2,x)
```

```
[Out] Integral(x*(exp(x)*sin(x) + 1)**2, x)
```

Mathematica [A] time = 0.188626, size = 67, normalized size = 0.52

$$\frac{1}{8}\left(4x^2+e^{2x}(2x-1)+8e^x x\sin(x)-e^{2x}x\cos(2x)-8e^x(x-1)\cos(x)-e^{2x}(2x-1)\sin(x)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*C
os[2*x] + 8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8
```

Maple [A] time = 0.009, size = 63, normalized size = 0.5

$$\frac{x^2}{2}+2\,(-x/2+1/2)\,\mathrm{e}^x\cos(x)+\mathrm{e}^x x\sin(x)+\frac{(\mathrm{e}^x)^2 x}{4}-\frac{(\mathrm{e}^x)^2}{8}-\frac{x\mathrm{e}^{2x}\cos(2x)}{8}+\frac{\mathrm{e}^{2x}\sin(2x)}{2}\left(-\frac{x}{4}+\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+exp(x)*sin(x))^2,x)
```

```
[Out] 1/2*x^2+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/4*exp(x)^2
*x-1/8*exp(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2*x)
*sin(2*x)
```

Maxima [A] time = 1.3714, size = 78, normalized size = 0.61

$$-\frac{1}{8}x\cos(2x)e^{(2x)}-(x-1)\cos(x)e^x-\frac{1}{16}(2x-1)e^{(2x)}\sin(2x)+xe^x\sin(x)+\frac{1}{2}x^2+\frac{1}{8}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^x*sin(x) + 1)^2*x,x, algorithm="maxima")
```

```
[Out] -1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(
2*x)*sin(2*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)
```

Fricas [A] time = 0.233844, size = 74, normalized size = 0.58

$$-(x-1)\cos(x)e^x+\frac{1}{2}x^2-\frac{1}{8}\left(2x\cos(x)^2-3x+1\right)e^{(2x)}-\frac{1}{8}\left((2x-1)\cos(x)e^{(2x)}-8xe^x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^x*sin(x) + 1)^2*x,x, algorithm="fricas")
```

```
[Out] -(x - 1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x)^2 - 3*x + 1)*e^(2
*x) - 1/8*((2*x - 1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)
```

Sympy [A] time = 8.01265, size = 109, normalized size = 0.85

$$\frac{x^2}{2} + \frac{3xe^{2x}\sin^2(x)}{8} - \frac{xe^{2x}\sin(x)\cos(x)}{4} + \frac{xe^{2x}\cos^2(x)}{8} + xe^x\sin(x) - xe^x\cos(x) - \frac{e^{2x}\sin^2(x)}{8} + \frac{e^{2x}\sin(x)\cos(x)}{8} - \frac{e^{2x}\cos^2(x)}{8} + e^x\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+exp(x)*sin(x))**2,x)
```

```
[Out] x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 +
x*exp(2*x)*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(
2*x)*sin(x)**2/8 + exp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/
8 + exp(x)*cos(x)
```

GIAC/XCAS [A] time = 0.202307, size = 77, normalized size = 0.6

$$\frac{1}{2}x^2 - \frac{1}{16}(2x\cos(2x) + (2x-1)\sin(2x))e^{(2x)} + \frac{1}{8}(2x-1)e^{(2x)} - ((x-1)\cos(x) - x\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^x*sin(x) + 1)^2*x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) + 1/8*
(2*x - 1)*e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x
```

3.253 $\int e^x x \cos(x)\, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi [A] time = 0.0545267, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x*Cos[x],x]
```



```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi in Sympy [A] time = 3.06087, size = 27, normalized size = 0.9

$$\frac{xe^x \sin(x)}{2} + \frac{xe^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Mathematica [A] time = 0.0272331, size = 18, normalized size = 0.6

$$\frac{1}{2}e^x((x-1)\sin(x) + x\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x*Cos[x],x]
```

```
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$\frac{e^{x} x \cos(x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right) e^{x} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x*cos(x),x)

[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A] time = 1.35525, size = 23, normalized size = 0.77

$$\frac{1}{2} x \cos(x) e^{x}+\frac{1}{2} (x-1) e^{x} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*e^x,x, algorithm="maxima")

[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A] time = 0.219123, size = 23, normalized size = 0.77

$$\frac{1}{2} x \cos(x) e^{x}+\frac{1}{2} (x-1) e^{x} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*e^x,x, algorithm="fricas")

[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] time = 1.02271, size = 27, normalized size = 0.9

$$\frac{x e^{x} \sin(x)}{2}+\frac{x e^{x} \cos(x)}{2}-\frac{e^{x} \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

GIAC/XCAS [A] time = 0.200187, size = 20, normalized size = 0.67

$$\frac{1}{2}\left(x\cos\left(x\right) + \left(x - 1\right)\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*e^x,x, algorithm="giac")
```

```
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.254 $\int \frac{1}{(-3+x)^4}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3(3-x)^3}$$

```
[Out] 1/(3*(3 - x)^3)
```

Rubi [A] time = 0.00346382, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{3(3-x)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + x)^(-4),x]

[Out] 1/(3*(3 - x)^3)
```

Rubi in Sympy [A] time = 0.53133, size = 7, normalized size = 0.64

$$\frac{1}{3\,(-x+3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3+x)**4,x)

[Out] 1/(3*(-x + 3)**3)
```

Mathematica [A] time = 0.0019471, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-3)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + x)^(-4),x]
```

```
[Out] -1/(3*(-3 + x)^3)
```

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$-\frac{1}{3\left(-3+x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3+x)^4,x)
```

```
[Out] -1/3/(-3+x)^3
```

Maxima [A] time = 1.32975, size = 9, normalized size = 0.82

$$-\frac{1}{3\,(x-3)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)^(-4),x, algorithm="maxima")
```

```
[Out] -1/3/(x - 3)^3
```

Fricas [A] time = 0.18834, size = 23, normalized size = 2.09

$$-\frac{1}{3\,(x^{3}-9\,x^{2}+27\,x-27)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)^(-4),x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 9*x^2 + 27*x - 27)
```

Sympy [A] time = 0.094065, size = 17, normalized size = 1.55

$$-\frac{1}{3x^{3}-27x^{2}+81x-81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3+x)**4,x)
```

```
[Out] -1/(3*x**3 - 27*x**2 + 81*x - 81)
```

GIAC/XCAS [A] time = 0.198529, size = 9, normalized size = 0.82

$$-\frac{1}{3\,(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)^(-4),x, algorithm="giac")
```

```
[Out] -1/3/(x - 3)^3
```

3.255 $\int \frac{x}{-1+x^3}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^
2]/6
```

Rubi [A] time = 0.0415191, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-1 + x^3),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^
2]/6
```

Rubi in Sympy [A] time = 2.98416, size = 37, normalized size = 0.92

$$\frac{\log\left(-x+1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**3-1),x)
```

```
[Out] log(-x + 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x/3
 + 1/3))/3
```

Mathematica [A] time = 0.012976, size = 40, normalized size = 1.

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-1 + x^3),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^
2]/6
```

Maple [A] time = 0.001, size = 33, normalized size = 0.8

$$-\frac{\ln\left(x^{2}+x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(-1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^3-1),x)
```

```
[Out] -1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(-
1+x)
```

Maxima [A] time = 1.52479, size = 43, normalized size = 1.08

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^{2}+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1)
+ 1/3*log(x - 1)
```

Fricas [A] time = 0.232537, size = 53, normalized size = 1.32

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^{2}+x+1\right)-2\sqrt{3}\log(x-1)-6\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - 2*sqrt(3)*log(x - 1) -
6*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.163032, size = 41, normalized size = 1.02

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^{2}+x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.199399, size = 45, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{1}{6}\ln\left(x^{2}+x+1\right)+\frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*ln(x^2 + x + 1) +
 1/3*ln(abs(x - 1))
```

3.256 $\int \frac{x}{-1+x^4}\,dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

```
[Out] -ArcTanh[x^2]/2
```

Rubi [A] time = 0.00911792, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-1 + x^4),x]

[Out] -ArcTanh[x^2]/2
```

Rubi in Sympy [A] time = 1.10944, size = 7, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4-1),x)

[Out] -atanh(x**2)/2
```

Mathematica [B] time = 0.00402059, size = 23, normalized size = 2.88

$$\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{4}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-1 + x^4),x]
```

```
[Out] Log[1 - x^2]/4 - Log[1 + x^2]/4
```

Maple [B] time = 0.001, size = 22, normalized size = 2.8

$$\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-1),x)
```

```
[Out] 1/4*ln(-1+x)+1/4*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.31853, size = 23, normalized size = 2.88

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Fricas [A] time = 0.197185, size = 23, normalized size = 2.88

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Sympy [A] time = 0.08554, size = 15, normalized size = 1.88

$$\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4-1),x)
```

```
[Out] log(x**2 - 1)/4 - log(x**2 + 1)/4
```

GIAC/XCAS [A] time = 0.20203, size = 24, normalized size = 3.

$$-\frac{1}{4}\ln\left(x^2+1\right)+\frac{1}{4}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 1),x, algorithm="giac")
```

```
[Out] -1/4*ln(x^2 + 1) + 1/4*ln(abs(x^2 - 1))
```

3.257 $\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx$

Optimal. Leaf size=227

$$\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)$$
$$+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{}\right.$$

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + ((4
+ (1 - I)*2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)])/16 + ((2
+ (-2)^(1/4))*Log[x]*Log[1 - ((-1)^(3/4)*x)/2^(1/4)])/8 + ((2 - (
-2)^(1/4))*Log[x]*Log[1 + ((-1)^(3/4)*x)/2^(1/4)])/8 + ((4 + (1 -
 I)*2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)])/16 + ((2 + I*(-2)^
(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2 - (-2)^(1/4))*Pol
yLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (-2)^(1/4))*PolyLog
[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```

Rubi [A] time = 0.320781, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\left(\frac{1}{16}-\frac{i}{16}\right)\left(2^{3/4}+(2+2i)\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)$$
$$+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{}\right.$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^3)*Log[x])/(2 + x^4),x]
```

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + (1/1
6 - I/16)*((2 + 2*I) + 2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4
)] + ((2 + (-2)^(1/4))*Log[x]*Log[1 - ((-1)^(3/4)*x)/2^(1/4)])/8
+ ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1)^(3/4)*x)/2^(1/4)])/8 + (
1/16 - I/16)*((2 + 2*I) + 2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4
)] + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2
 - (-2)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (
-2)^(1/4))*PolyLog[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```

Rubi in Sympy [A] time = 24.5135, size = 321, normalized size = 1.41

$$\left(\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}+\frac{2^{\frac{3}{4}}i}{16}\right)\log(x)\log\left(\sqrt[4]{2}(-1+i)\left(-\frac{x}{2}-\frac{2^{\frac{3}{4}}(1+i)}{4}\right)\right)$$
$$+\left(-\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}-\frac{2^{\frac{3}{4}}i}{16}\right)\log(x)\log\left(\sqrt[4]{2}(-1+i)\left(\frac{x}{2}-\frac{2^{\frac{3}{4}}(1+i)}{4}\right)\right)$$
$$+\left(\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}-\frac{2^{\frac{3}{4}}i}{16}\right)\log(x)\log\left(\sqrt[4]{2}(-1+i)\left(-\frac{ix}{2}-\frac{2^{\frac{3}{4}}(1+i)}{4}\right)\right)$$
$$+\left(-\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}+\frac{2^{\frac{3}{4}}i}{16}\right)\log(x)\log\left(\sqrt[4]{2}(-1+i)\left(\frac{ix}{2}-\frac{2^{\frac{3}{4}}(1+i)}{4}\right)\right)$$
$$+\left(-\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}-\frac{2^{\frac{3}{4}}i}{16}\right)\mathrm{Li}_2\left(-\frac{\sqrt[4]{2}x(-1+i)}{2}\right)+\left(\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}+\frac{2^{\frac{3}{4}}i}{16}\right)\mathrm{Li}_2\left(\frac{\sqrt[4]{2}x(-1+i)}{2}\right)$$
$$+\left(-\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}+\frac{2^{\frac{3}{4}}i}{16}\right)\mathrm{Li}_2\left(\frac{\sqrt[4]{2}x(1+i)}{2}\right)+\left(\frac{2^{\frac{3}{4}}}{16}+\frac{1}{4}-\frac{2^{\frac{3}{4}}i}{16}\right)\mathrm{Li}_2\left(\frac{\sqrt[4]{2}ix(-1+i)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)*ln(x)/(x**4+2),x)
```

```
[Out] (2**(3/4)/16 + 1/4 + 2**(3/4)*I/16)*log(x)*log(2**(1/4)*(-1 + I)*
(-x/2 - 2**(3/4)*(1 + I)/4)) + (-2**(3/4)/16 + 1/4 - 2**(3/4)*I/1
6)*log(x)*log(2**(1/4)*(-1 + I)*(x/2 - 2**(3/4)*(1 + I)/4)) + (2*
*(3/4)/16 + 1/4 - 2**(3/4)*I/16)*log(x)*log(2**(1/4)*(-1 + I)*(-I
*x/2 - 2**(3/4)*(1 + I)/4)) + (-2**(3/4)/16 + 1/4 + 2**(3/4)*I/16
)*log(x)*log(2**(1/4)*(-1 + I)*(I*x/2 - 2**(3/4)*(1 + I)/4)) + (-
2**(3/4)/16 + 1/4 - 2**(3/4)*I/16)*polylog(2, -2**(1/4)*x*(-1 + I
)/2) + (2**(3/4)/16 + 1/4 + 2**(3/4)*I/16)*polylog(2, 2**(1/4)*x*
(-1 + I)/2) + (-2**(3/4)/16 + 1/4 + 2**(3/4)*I/16)*polylog(2, 2**
(1/4)*x*(1 + I)/2) + (2**(3/4)/16 + 1/4 - 2**(3/4)*I/16)*polylog(
2, 2**(1/4)*I*x*(-1 + I)/2)
```

Mathematica [A] time = 0.293678, size = 277, normalized size = 1.22

$$\frac{1}{8}\left(-(-1)^{3/4}\sqrt[4]{2}\left(\text{PolyLog}\left(2,-\sqrt[4]{-\frac{1}{2}}x\right)+\log(x)\log\left(\sqrt[4]{-\frac{1}{2}}x+1\right)\right)\right.$$
$$+2\left(\text{PolyLog}\left(2,-\sqrt[4]{-\frac{1}{2}}x\right)+\log(x)\log\left(\sqrt[4]{-\frac{1}{2}}x+1\right)\right)$$
$$+(-1)^{3/4}\sqrt[4]{2}\left(\text{PolyLog}\left(2,\sqrt[4]{-\frac{1}{2}}x\right)+\log(x)\log\left(1-\sqrt[4]{-\frac{1}{2}}x\right)\right)+2\left(\text{PolyLog}\left(2,\sqrt[4]{-\frac{1}{2}}x\right)+\log(x)\log\left(1-\sqrt[4]{-\frac{1}{2}}x\right)\right)+\sqrt[4]{-2}\left(\text{Pol}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^3)*Log[x])/(2 + x^4),x]
```

```
[Out] (2*(Log[x]*Log[1 + (-1/2)^(1/4)*x] + PolyLog[2, -((-1/2)^(1/4)*x)
]) - (-1)^(3/4)*2^(1/4)*(Log[x]*Log[1 + (-1/2)^(1/4)*x] + PolyLog
[2, -((-1/2)^(1/4)*x)]) + 2*(Log[x]*Log[1 - (-1/2)^(1/4)*x] + Pol
yLog[2, (-1/2)^(1/4)*x]) + (-1)^(3/4)*2^(1/4)*(Log[x]*Log[1 - (-1
/2)^(1/4)*x] + PolyLog[2, (-1/2)^(1/4)*x]) + 2*(Log[x]*Log[1 + ((
1 - I)*x)/2^(3/4)] + PolyLog[2, ((-1 + I)*x)/2^(3/4)]) + (-2)^(1/
4)*(Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + PolyLog[2, ((-1 + I)*x)
/2^(3/4)]) + 2*(Log[x]*Log[1 - ((1 - I)*x)/2^(3/4)] + PolyLog[2,
((1 - I)*x)/2^(3/4)]) - (-2)^(1/4)*(Log[x]*Log[1 - ((1 - I)*x)/2^
(3/4)] + PolyLog[2, ((1 - I)*x)/2^(3/4)]))/8
```

Maple [C] time = 0.014, size = 1210, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)*ln(x)/(x^4+2),x)
```

```
[Out] -1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-
x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)*ln(x)-1/4/(1/2*2^(3/4)+1/
2*I*2^(3/4))^3*dilog((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1
/2*I*2^(3/4)))*2^(1/4)-1/4*I/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*ln((-
1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4
)*ln(x)-1/4*I/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*dilog((-1/2*I*2^(3/4
)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4)+1/4/(1/2*2
^(3/4)+1/2*I*2^(3/4))^3*ln(x)*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1
/2*2^(3/4)+1/2*I*2^(3/4)))+1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*dilo
g((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))+1/4*
I/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/
(1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)*ln(x)-1/4*I/(-1/2*2^(3/4)-1/
2*I*2^(3/4))^3*dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)
-1/2*I*2^(3/4)))*2^(1/4)+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*ln((1/
2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))*2^(1/4)*l
n(x)+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*dilog((1/2*I*2^(3/4)-1/2*2
^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))*2^(1/4)+1/4/(1/2*I*2^(3/4)
-1/2*2^(3/4))^3*ln(x)*ln((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(
3/4)-1/2*2^(3/4)))+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*dilog((1/2*I
*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1/4/(-1/2*2^
(3/4)-1/2*I*2^(3/4))^3*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^
(3/4)-1/2*I*2^(3/4)))*2^(1/4)*ln(x)+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/
4))^3*dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^
(3/4)))*2^(1/4)+1/4*I/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*dilog((1/2*I*
2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))*2^(1/4)-1/4*I
/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)
/(-1/2*2^(3/4)-1/2*I*2^(3/4)))*2^(1/4)*ln(x)+1/4/(-1/2*2^(3/4)-1/
2*I*2^(3/4))^3*ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3
/4)-1/2*I*2^(3/4)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*dilog((-1/
2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))+1/4*I/(1
```

```
/2*2^(3/4)+1/2*I*2^(3/4))^3*dilog((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(
1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)+1/4*I/(1/2*I*2^(3/4)-1/2*2^(3
/4))^3*ln((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4
)))*2^(1/4)*ln(x)-1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*ln((-1/2*I*2
^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4)*ln(x)
-1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*dilog((-1/2*I*2^(3/4)+1/2*2^(
3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4)+1/4/(-1/2*I*2^(3/4)
+1/2*2^(3/4))^3*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2
^(3/4)+1/2*2^(3/4)))+1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*dilog((-1
/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3+1)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)*log(x)/(x^4 + 2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^3+1)\log(x)}{x^4+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)*log(x)/(x^4 + 2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)(x^2-x+1)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)*ln(x)/(x**4+2),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3 + 1)\log(x)}{x^4 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)*log(x)/(x^4 + 2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

3.258 $\int (\log(x)+\log(1+x)+\log(2+x))\,dx$

Optimal. Leaf size=24

$$-3x+x\log(x)+(x+1)\log(x+1)+(x+2)\log(x+2)$$

```
[Out] -3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

Rubi [A] time = 0.0179568, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$

$$-3x+x\log(x)+(x+1)\log(x+1)+(x+2)\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x] + Log[1 + x] + Log[2 + x],x]

[Out] -3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

Rubi in Sympy [A] time = 0.66689, size = 32, normalized size = 1.33

$$x\log(x)+x\log(x+1)+x\log(x+2)-3x+\log(x+1)+2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)+ln(1+x)+ln(2+x),x)

[Out] x*log(x) + x*log(x + 1) + x*log(x + 2) - 3*x + log(x + 1) + 2*log
(x + 2)
```

Mathematica [A] time = 0.00470535, size = 30, normalized size = 1.25

$$-3x+x\log(x)+x\log(x+1)+x\log(x+2)+\log(x+1)+2\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x] + Log[1 + x] + Log[2 + x],x]

[Out] -3*x + x*Log[x] + Log[1 + x] + x*Log[1 + x] + 2*Log[2 + x] + x*Lo
g[2 + x]
```

Maple [A] time = 0.002, size = 26, normalized size = 1.1

$$x\ln(x) - 3x + (1 + x)\ln(1 + x) - 3 + (2 + x)\ln(2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)+ln(1+x)+ln(2+x),x)
```

```
[Out] x*ln(x)-3*x+(1+x)*ln(1+x)-3+(2+x)*ln(2+x)
```

Maxima [A] time = 1.32051, size = 34, normalized size = 1.42

$$(x + 2)\log(x + 2) + (x + 1)\log(x + 1) + x\log(x) - 3x - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x + 2) + log(x + 1) + log(x),x, algorithm="maxima")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

Fricas [A] time = 0.22204, size = 32, normalized size = 1.33

$$(x + 2)\log(x + 2) + (x + 1)\log(x + 1) + x\log(x) - 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x + 2) + log(x + 1) + log(x),x, algorithm="fricas")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x
```

Sympy [A] time = 2.81347, size = 32, normalized size = 1.33

$$x\log(x) + x\log(x + 1) + x\log(x + 2) - 3x + \log(x + 1) + 2\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)+ln(1+x)+ln(2+x),x)
```

```
[Out] x*log(x) + x*log(x + 1) + x*log(x + 2) - 3*x + log(x + 1) + 2*log
(x + 2)
```

GIAC/XCAS [A] time = 0.201223, size = 34, normalized size = 1.42

$$(x + 2)\ln(x + 2) + (x + 1)\ln(x + 1) + x\ln(x) - 3x - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x + 2) + log(x + 1) + log(x),x, algorithm="giac")
```

```
[Out] (x + 2)*ln(x + 2) + (x + 1)*ln(x + 1) + x*ln(x) - 3*x - 3
```

3.259 $\int \frac{1}{5+x^3} dx$

Optimal. Leaf size=78

$$-\frac{\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{6\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}}$$

```
[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) +
Log[5^(1/3) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*
5^(2/3))
```

Rubi [A] time = 0.0895831, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$-\frac{\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{6\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) +
Log[5^(1/3) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*
5^(2/3))
```

Rubi in Sympy [A] time = 4.80512, size = 70, normalized size = 0.9

$$\frac{\sqrt[3]{5}\log\left(x+\sqrt[3]{5}\right)}{15}-\frac{\sqrt[3]{5}\log\left(x^2-\sqrt[3]{5}x+5^{\frac{2}{3}}\right)}{30}-\frac{\sqrt{3}\sqrt[3]{5}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\cdot 5^{\frac{2}{3}}x}{15}+\frac{1}{3}\right)\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3+5),x)
```

```
[Out] 5**(1/3)*log(x + 5**(1/3))/15 - 5**(1/3)*log(x**2 - 5**(1/3)*x +
5**(2/3))/30 - sqrt(3)*5**(1/3)*atan(sqrt(3)*(-2*5**(2/3)*x/15 +
1/3))/15
```

Mathematica [A] time = 0.0348343, size = 71, normalized size = 0.91

$$\frac{-\log\left(\sqrt[3]{5}x^2-5^{2/3}x+5\right)+2\log\left(5^{2/3}x+5\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\ 5^{2/3}x-5}{5\sqrt{3}}\right)}{6\ 5^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-5 + 2*5^(2/3)*x)/(5*Sqrt[3])] + 2*Log[5 + 5^(
2/3)*x] - Log[5 - 5^(2/3)*x + 5^(1/3)*x^2])/(6*5^(2/3))
```

Maple [A] time = 0.003, size = 54, normalized size = 0.7

$$\frac{\ln\left(\sqrt[3]{5}+x\right)\sqrt[3]{5}}{15}-\frac{\ln\left(5^{\frac{2}{3}}-\sqrt[3]{5}x+x^2\right)\sqrt[3]{5}}{30}+\frac{\sqrt[3]{5}\sqrt{3}}{15}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,5^{2/3}x}{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3+5),x)
```

```
[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)
+1/15*5^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/5*5^(2/3)*x-1))
```

Maxima [A] time = 1.50871, size = 77, normalized size = 0.99

$$\frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^2-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(x+5^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + 5),x, algorithm="maxima")
```

```
[Out] 1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3)))
 - 1/30*5^(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log
(x + 5^(1/3))
```

Fricas [A] time = 0.2128, size = 81, normalized size = 1.04

$$-\frac{1}{450}\cdot 25^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}\log\left(25^{\frac{2}{3}}x^2-5\cdot 25^{\frac{1}{3}}x+25\right)-2\sqrt{3}\log\left(25^{\frac{1}{3}}x+5\right)-6\arctan\left(\frac{2}{15}\cdot 25^{\frac{1}{3}}\sqrt{3}x-\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + 5),x, algorithm="fricas")
```

```
[Out] -1/450*25^(2/3)*sqrt(3)*(sqrt(3)*log(25^(2/3)*x^2 - 5*25^(1/3)*x
+ 25) - 2*sqrt(3)*log(25^(1/3)*x + 5) - 6*arctan(2/15*25^(1/3)*sq
rt(3)*x - 1/3*sqrt(3)))
```

Sympy [A] time = 0.605939, size = 73, normalized size = 0.94

$$\frac{\sqrt[3]{5}\log\left(x+\sqrt[3]{5}\right)}{15}-\frac{\sqrt[3]{5}\log\left(x^2-\sqrt[3]{5}x+5^{\frac{2}{3}}\right)}{30}+\frac{\sqrt{3}\sqrt[3]{5}\operatorname{atan}\left(\frac{2\sqrt{3}\cdot5^{\frac{2}{3}}x}{15}-\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3+5),x)
```

```
[Out] 5**(1/3)*log(x + 5**(1/3))/15 - 5**(1/3)*log(x**2 - 5**(1/3)*x +
5**(2/3))/30 + sqrt(3)*5**(1/3)*atan(2*sqrt(3)*5**(2/3)*x/15 - sq
rt(3)/3)/15
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + 5),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.260 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.00320783, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]
```

Rubi in Sympy [A] time = 0.076226, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)
```

Mathematica [A] time = 0.00498694, size = 2, normalized size = 1.

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\mathit{Arcsinh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2),x)

[Out] arcsinh(x)
```

Maxima [A] time = 1.54356, size = 3, normalized size = 1.5

$$\operatorname{arsinh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="maxima")

[Out] arcsinh(x)
```

Fricas [A] time = 0.204743, size = 19, normalized size = 9.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.136914, size = 2, normalized size = 1.

$$\operatorname{asinh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)
```

GIAC/XCAS [A] time = 0.200105, size = 19, normalized size = 9.5

$$-\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(x^2 + 1))
```

3.261 $\int \sqrt{3+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{x^2+3}x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Rubi [A] time = 0.00890737, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{2}\sqrt{x^2+3}x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3 + x^2],x]

[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Rubi in Sympy [A] time = 0.592704, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+3}}{2}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3)**(1/2),x)

[Out] x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2
```

Mathematica [A] time = 0.00983404, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+3}x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3 + x^2],x]
```

```
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\frac{3}{2}Arcsinh\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{2}\sqrt{x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3)^(1/2),x)
```

```
[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

Maxima [A] time = 1.47996, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+3}x+\frac{3}{2}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)
```

Fricas [A] time = 0.215627, size = 109, normalized size = 4.04

$$-\frac{2x^4+6x^2+3\left(2x^2-2\sqrt{x^2+3}x+3\right)\log\left(-x+\sqrt{x^2+3}\right)-\left(2x^3+3x\right)\sqrt{x^2+3}}{2\left(2x^2-2\sqrt{x^2+3}x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 6*x^2 + 3*(2*x^2 - 2*sqrt(x^2 + 3)*x + 3)*log(-x +
sqrt(x^2 + 3)) - (2*x^3 + 3*x)*sqrt(x^2 + 3))/(2*x^2 - 2*sqrt(x^2
 + 3)*x + 3)
```

Sympy [A] time = 0.235009, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+3}}{2}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2
```

GIAC/XCAS [A] time = 0.207324, size = 34, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^2+3}x-\frac{3}{2}\ln\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*ln(-x + sqrt(x^2 + 3))
```

3.262 $\int \frac{x}{(1+x)^2}\,dx$

Optimal. Leaf size=10

$$\frac{1}{x+1}+\log(x+1)$$

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rubi [A] time = 0.0111296, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{x+1}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rubi in Sympy [A] time = 1.06328, size = 8, normalized size = 0.8

$$\log(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

Mathematica [A] time = 0.00397579, size = 10, normalized size = 1.

$$\frac{1}{x+1}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A] time = 0.004, size = 11, normalized size = 1.1

$$(1+x)^{-1}+\ln\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)^2,x)

[Out] 1/(1+x)+ln(1+x)
```

Maxima [A] time = 1.35402, size = 14, normalized size = 1.4

$$\frac{1}{x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 1)^2,x, algorithm="maxima")

[Out] 1/(x + 1) + log(x + 1)
```

Fricas [A] time = 0.190112, size = 22, normalized size = 2.2

$$\frac{(x+1)\log\left(x+1\right)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 1)^2,x, algorithm="fricas")

[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A] time = 0.063238, size = 8, normalized size = 0.8

$$\log\left(x+1\right)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)**2,x)

[Out] log(x + 1) + 1/(x + 1)
```

GIAC/XCAS [A] time = 0.210926, size = 15, normalized size = 1.5

$$\frac{1}{x+1} + \ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 1)^2,x, algorithm="giac")

[Out] 1/(x + 1) + ln(abs(x + 1))
```

3.263 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rubi [A] time = 0.00889489, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x],x]

[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rubi in Sympy [A] time = 1.01464, size = 12, normalized size = 0.75

$$x\operatorname{asin}(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x),x)

[Out] x*asin(x) + sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.0052122, size = 16, normalized size = 1.

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x],x]

[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x),x)
```

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Maxima [A] time = 1.52434, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x),x, algorithm="maxima")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Fricas [A] time = 0.221187, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x),x, algorithm="fricas")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Sympy [A] time = 0.126117, size = 12, normalized size = 0.75

$$x \operatorname{asin}(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x),x)
```

```
[Out] x*asin(x) + sqrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.201211, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

3.264 $\int x^2 \sin^{-1}(x)\, dx$

Optimal. Leaf size=40

$$\frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{\sqrt{1-x^2}}{3}$$

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rubi [A] time = 0.0492255, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rubi in Sympy [A] time = 3.04072, size = 27, normalized size = 0.68

$$\frac{x^3 \operatorname{asin}(x)}{3} - \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{9} + \frac{\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 - (-x**2 + 1)**(3/2)/9 + sqrt(-x**2 + 1)/3
```

Mathematica [A] time = 0.0198844, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(3x^3 \sin^{-1}(x) + \sqrt{1-x^2}\left(x^2+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*ArcSin[x],x]
```

```
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A] time = 0.003, size = 34, normalized size = 0.9

$$\frac{x^3 \arcsin(x)}{3} + \frac{x^2}{9}\sqrt{-x^2+1} + \frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arcsin(x),x)
```

```
[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49594, size = 45, normalized size = 1.12

$$\frac{1}{3}\,x^3 \arcsin(x) + \frac{1}{9}\,\sqrt{-x^2+1}x^2 + \frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

Fricas [A] time = 0.231502, size = 32, normalized size = 0.8

$$\frac{1}{3}\,x^3 \arcsin(x) + \frac{1}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] time = 0.447424, size = 32, normalized size = 0.8

$$\frac{x^3 \operatorname{asin}(x)}{3} + \frac{x^2\sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(-x**2 + 1)/9 + 2*sqrt(-x**2 + 1)/9
```

GIAC/XCAS [A] time = 0.20164, size = 51, normalized size = 1.27

$$\frac{1}{3}\left(x^{2}-1\right)x\arcsin\left(x\right)+\frac{1}{3}\,x\arcsin\left(x\right)-\frac{1}{9}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{1}{3}\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2
) + 1/3*sqrt(-x^2 + 1)
```

3.265 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rubi [A] time = 0.177549, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]

[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)

[Out] Timed out
```

Mathematica [A] time = 0.0215125, size = 29, normalized size = 1.38

$$2\left(\frac{1}{2}\log(2\cos(x)-\sin(x))-\frac{1}{2}\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]

[Out] 2*(-Log[Cos[x] - Sin[x]]/2 + Log[2*Cos[x] - Sin[x]]/2)
```

Maple [A] time = 0.041, size = 14, normalized size = 0.7

$$\ln\left(-2+\tan\left(x\right)\right)-\ln\left(-1+\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)
```

```
[Out] ln(-2+tan(x))-ln(-1+tan(x))
```

Maxima [A] time = 1.32892, size = 18, normalized size = 0.86

$$-\log\left(\tan\left(x\right)-1\right)+\log\left(\tan\left(x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2/(sec(x)^2 - 3*tan(x) + 1),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

Fricas [A] time = 0.233114, size = 39, normalized size = 1.86

$$\frac{1}{2}\,\log\left(\frac{3}{4}\,\cos\left(x\right)^{2}-\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\right)-\frac{1}{2}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2/(sec(x)^2 - 3*tan(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*s
in(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(x\right)}{-3\tan\left(x\right)+\sec^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

GIAC/XCAS [A] time = 0.210711, size = 20, normalized size = 0.95

$$-\ln\left(|\tan\left(x\right) - 1|\right) + \ln\left(|\tan\left(x\right) - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2/(sec(x)^2 - 3*tan(x) + 1),x, algorithm="giac")
```

```
[Out] -ln(abs(tan(x) - 1)) + ln(abs(tan(x) - 2))
```

3.266 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0107597, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.493882, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sec(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00290481, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.006, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sec(x)^2,x)

[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.40299, size = 14, normalized size = 1.

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(-2),x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.228423, size = 14, normalized size = 1.

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(-2),x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.05551, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.203797, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.267 $\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

```
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Rubi [A] time = 0.0365555, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]

[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Rubi in Sympy [A] time = 2.81582, size = 14, normalized size = 0.78

$$2\log(x)+3\log(-x+2)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)

[Out] 2*log(x) + 3*log(-x + 2) - 1/x
```

Mathematica [A] time = 0.00601856, size = 18, normalized size = 1.

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

```
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Maple [A] time = 0.006, size = 17, normalized size = 0.9

$$-x^{-1} + 2\ln(x) + 3\ln(-2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2-3*x-2)/(-2+x)/x^2,x)
```

```
[Out] -1/x+2*ln(x)+3*ln(-2+x)
```

Maxima [A] time = 1.41041, size = 22, normalized size = 1.22

$$-\frac{1}{x} + 3\log(x-2) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 - 3*x - 2)/((x - 2)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x + 3*log(x - 2) + 2*log(x)
```

Fricas [A] time = 0.204945, size = 24, normalized size = 1.33

$$\frac{3x\log(x-2) + 2x\log(x) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 - 3*x - 2)/((x - 2)*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x - 2) + 2*x*log(x) - 1)/x
```

Sympy [A] time = 0.113655, size = 14, normalized size = 0.78

$$2\log(x) + 3\log(x-2) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)
```

```
[Out] 2*log(x) + 3*log(x - 2) - 1/x
```

GIAC/XCAS [A] time = 0.205338, size = 24, normalized size = 1.33

$$-\frac{1}{x} + 3\ln\left(|x-2|\right) + 2\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 - 3*x - 2)/((x - 2)*x^2),x, algorithm="giac")
```

```
[Out] -1/x + 3*ln(abs(x - 2)) + 2*ln(abs(x))
```

3.268 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] ArcSinh[(2*x)/3]/2
```

Rubi [A] time = 0.00502021, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[9 + 4*x^2],x]

[Out] ArcSinh[(2*x)/3]/2
```

Rubi in Sympy [A] time = 0.521502, size = 7, normalized size = 0.7

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*x**2+9)**(1/2),x)

[Out] asinh(2*x/3)/2
```

Mathematica [A] time = 0.00607744, size = 10, normalized size = 1.

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Maple [A] time = 0.001, size = 7, normalized size = 0.7

$$\frac{1}{2}\mathit{Arcsinh}\left(\frac{2\,x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Maxima [A] time = 1.49827, size = 8, normalized size = 0.8

$$\frac{1}{2}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^2 + 9),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Fricas [A] time = 0.206729, size = 22, normalized size = 2.2

$$-\frac{1}{2}\,\log\left(-2\,x + \sqrt{4\,x^{2} + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^2 + 9),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 0.149618, size = 7, normalized size = 0.7

$$\frac{\operatorname{asinh}{\left(\frac{2x}{3}\right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**2+9)**(1/2),x)
```

```
[Out] asinh(2*x/3)/2
```

GIAC/XCAS [A] time = 0.201141, size = 22, normalized size = 2.2

$$-\frac{1}{2}\ln\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^2 + 9),x, algorithm="giac")
```

```
[Out] -1/2*ln(-2*x + sqrt(4*x^2 + 9))
```

3.269 $\int \frac{1}{\sqrt{4+x^2}} \, dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] ArcSinh[x/2]
```

Rubi [A] time = 0.00359629, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4 + x^2],x]

[Out] ArcSinh[x/2]
```

Rubi in Sympy [A] time = 0.5122, size = 3, normalized size = 0.5

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4)**(1/2),x)

[Out] asinh(x/2)
```

Mathematica [A] time = 0.00512773, size = 6, normalized size = 1.

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4 + x^2],x]

[Out] ArcSinh[x/2]
```

Maple [A] time = 0.002, size = 5, normalized size = 0.8

$$Arcsinh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4)^(1/2),x)

[Out] arcsinh(1/2*x)
```

Maxima [A] time = 1.49965, size = 5, normalized size = 0.83

$$\operatorname{arsinh}\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4),x, algorithm="maxima")

[Out] arcsinh(1/2*x)
```

Fricas [A] time = 0.201026, size = 19, normalized size = 3.17

$$-\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 + 4))
```

Sympy [A] time = 0.14089, size = 3, normalized size = 0.5

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4)**(1/2),x)
```

```
[Out] asinh(x/2)
```

GIAC/XCAS [A] time = 0.203885, size = 19, normalized size = 3.17

$$-\ln\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(x^2 + 4))
```

3.270 $\int \frac{1}{10-12x+9x^2}\,dx$

Optimal. Leaf size=21

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

```
[Out] -ArcTan[(2 - 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Rubi [A] time = 0.0304269, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(10 - 12*x + 9*x^2)^(-1),x]

[Out] -ArcTan[(2 - 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Rubi in Sympy [A] time = 0.683545, size = 19, normalized size = 0.9

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\sqrt{6}\left(\frac{x}{2}-\frac{1}{3}\right)\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2-12*x+10),x)

[Out] sqrt(6)*atan(sqrt(6)*(x/2 - 1/3))/18
```

Mathematica [A] time = 0.013934, size = 21, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(10 - 12*x + 9*x^2)^(-1),x]

[Out] ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Maple [A] time = 0.002, size = 17, normalized size = 0.8

$$\frac{\sqrt{6}}{18}\arctan\left(\frac{(18\,x-12)\,\sqrt{6}}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2-12*x+10),x)

[Out] 1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))
```

Maxima [A] time = 1.6426, size = 22, normalized size = 1.05

$$\frac{1}{18}\,\sqrt{6}\arctan\left(\frac{1}{6}\,\sqrt{6}(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x^2 - 12*x + 10),x, algorithm="maxima")

[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Fricas [A] time = 0.193802, size = 22, normalized size = 1.05

$$\frac{1}{18}\,\sqrt{6}\arctan\left(\frac{1}{6}\,\sqrt{6}(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x^2 - 12*x + 10),x, algorithm="fricas")

[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Sympy [A] time = 0.109521, size = 22, normalized size = 1.05

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}-\frac{\sqrt{6}}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2-12*x+10),x)

[Out] sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18
```

GIAC/XCAS [A] time = 0.19889, size = 22, normalized size = 1.05

$$\frac{1}{18}\,\sqrt{6}\arctan\left(\frac{1}{6}\,\sqrt{6}(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x^2 - 12*x + 10),x, algorithm="giac")

[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

3.271 $\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

```
[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*L
og[x] + Log[1 + x^2]/4
```

Rubi [A] time = 0.0530346, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*L
og[x] + Log[1 + x^2]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8-2x^7+2x^6-2x^5+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)
```

```
[Out] Integral(1/(x**8 - 2*x**7 + 2*x**6 - 2*x**5 + x**4), x)
```

Mathematica [A] time = 0.0305456, size = 51, normalized size = 0.96

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2(x-1)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] -1/(2*(-1 + x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2
*Log[x] + Log[1 + x^2]/4
```

Maple [A] time = 0.007, size = 42, normalized size = 0.8

$$-\frac{1}{3\,x^{3}}-x^{-2}-2\,x^{-1}+2\,\ln\left(x\right)+\frac{\ln\left(x^{2}+1\right)}{4}-\frac{1}{2\,x-2}-\frac{5\,\ln\left(-1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x)
```

```
[Out] -1/3/x^3-1/x^2-2/x+2*ln(x)+1/4*ln(x^2+1)-1/2/(-1+x)-5/2*ln(-1+x)
```

Maxima [A] time = 1.5032, size = 63, normalized size = 1.19

$$-\frac{15\,x^{3}-6\,x^{2}-4\,x-2}{6\,(x^{4}-x^{3})}+\frac{1}{4}\,\log\left(x^{2}+1\right)-\frac{5}{2}\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8 - 2*x^7 + 2*x^6 - 2*x^5 + x^4),x, algorithm="maxima")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) -
5/2*log(x - 1) + 2*log(x)
```

Fricas [A] time = 0.197411, size = 99, normalized size = 1.87

$$-\frac{30\,x^{3}-12\,x^{2}-3\,\left(x^{4}-x^{3}\right)\,\log\left(x^{2}+1\right)+30\,\left(x^{4}-x^{3}\right)\,\log\left(x-1\right)-24\,\left(x^{4}-x^{3}\right)\,\log\left(x\right)-8\,x-4}{12\,(x^{4}-x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8 - 2*x^7 + 2*x^6 - 2*x^5 + x^4),x, algorithm="fricas")
```

```
[Out] -1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x
^3)*log(x - 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

Sympy [A] time = 0.22168, size = 46, normalized size = 0.87

$$2\log(x) - \frac{5\log(x-1)}{2} + \frac{\log(x^2+1)}{4} - \frac{15x^3 - 6x^2 - 4x - 2}{6x^4 - 6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 - (15*x**3 - 6*x**2 -
 4*x - 2)/(6*x**4 - 6*x**3)
```

GIAC/XCAS [A] time = 0.200122, size = 62, normalized size = 1.17

$$-\frac{15\,x^3 - 6\,x^2 - 4\,x - 2}{6\,(x-1)x^3} + \frac{1}{4}\ln\left(x^2+1\right) - \frac{5}{2}\ln(|x-1|) + 2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 2*x^7 + 2*x^6 - 2*x^5 + x^4),x, algorithm="giac")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*ln(x^2 + 1) -
 5/2*ln(abs(x - 1)) + 2*ln(abs(x))
```

3.272 $\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx$

Optimal. Leaf size=49

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a
 - b + c - d)*Log[1 + x])/4
```

Rubi [A] time = 0.129281, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a
 - b + c - d)*Log[1 + x])/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0399784, size = 49, normalized size = 1.

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)+\frac{1}{4}\log(x+1)(-a+b-c+d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-
a + b - c + d)*Log[1 + x])/4
```

Maple [A] time = 0.006, size = 66, normalized size = 1.4

$$ax - \frac{d\ln(x)}{3} + \frac{9\ln(-3+x)a}{4} + \frac{3\ln(-3+x)b}{4} + \frac{\ln(-3+x)c}{4} + \frac{\ln(-3+x)d}{12} - \frac{\ln(1+x)a}{4} + \frac{\ln(1+x)b}{4} - \frac{\ln(1+x)c}{4} + \frac{\ln(1+x)d}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x-1/3*d*ln(x)+9/4*ln(-3+x)*a+3/4*ln(-3+x)*b+1/4*ln(-3+x)*c+1/12
*ln(-3+x)*d-1/4*ln(1+x)*a+1/4*ln(1+x)*b-1/4*ln(1+x)*c+1/4*ln(1+x)
*d
```

Maxima [A] time = 1.37212, size = 55, normalized size = 1.12

$$ax - \frac{1}{4}(a - b + c - d)\log(x+1) + \frac{1}{12}(27a + 9b + 3c + d)\log(x-3) - \frac{1}{3}d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^3 + b*x^2 + c*x + d)/((x + 1)*(x - 3)*x),x, algorithm="maxima")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d
)*log(x - 3) - 1/3*d*log(x)
```

Fricas [A] time = 0.230329, size = 55, normalized size = 1.12

$$ax - \frac{1}{4}(a - b + c - d)\log(x+1) + \frac{1}{12}(27a + 9b + 3c + d)\log(x-3) - \frac{1}{3}d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^3 + b*x^2 + c*x + d)/((x + 1)*(x - 3)*x),x, algorithm="fricas")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d
)*log(x - 3) - 1/3*d*log(x)
```

Sympy [A] time = 35.8284, size = 762, normalized size = 15.55

$$ax - \frac{d\log(x)}{3}$$

$$-\frac{(a-b+c-d)\log\left(x + \frac{-1512a^2d+1134a^2(a-b+c-d)-864abd+648ab(a-b+c-d)-432acd+324ac(a-b+c-d)-144ad^2+81a(a-b+c-d)^2-216b^2d+162b}{1215a^3-567a^2b+1593a^2c-2691a^2d-567ab^2+378abc-1638}\right.}{}$$

$$+\frac{(27a+9b+3c+d)\log\left(x + \frac{-1512a^2d-378a^2(27a+9b+3c+d)-864abd-216ab(27a+9b+3c+d)-432acd-108ac(27a+9b+3c+d)-144ad^2+9a(27a+9b+3}{1215a^3-567a^2b+1593a^2c-2691a^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a
**2*(a - b + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a
*c*d + 324*a*c*(a - b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d
)**2 - 216*b**2*d + 162*b**2*(a - b + c - d) - 288*b*d**2 + 108*b
*d*(a - b + c - d) + 81*b*(a - b + c - d)**2 - 72*c**2*d + 54*c**
2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a - b + c - d) - 27*c*(a
 - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d) + 117*d*(a
- b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**
2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*
d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 2
70*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3)
)/4 + (27*a + 9*b + 3*c + d)*log(x + (-1512*a**2*d - 378*a**2*(27
*a + 9*b + 3*c + d) - 864*a*b*d - 216*a*b*(27*a + 9*b + 3*c + d)
- 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d) - 144*a*d**2 + 9*a*(
27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*a + 9*b + 3*c
 + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*a +
9*b + 3*c + d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) +
144*c*d**2 + 24*c*d*(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*
c + d)**2 - 136*d**3 + 18*d**2*(27*a + 9*b + 3*c + d) + 13*d*(27*
a + 9*b + 3*c + d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 26
91*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 70
2*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c*
*2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35
*d**3))/12
```

GIAC/XCAS [A] time = 0.202564, size = 59, normalized size = 1.2

$$ax - \frac{1}{4}(a-b+c-d)\ln(|x+1|) + \frac{1}{12}(27a+9b+3c+d)\ln(|x-3|) - \frac{1}{3}d\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3 + b*x^2 + c*x + d)/((x + 1)*(x - 3)*x),x, algorithm="giac")
```

```
[Out] a*x - 1/4*(a - b + c - d)*ln(abs(x + 1)) + 1/12*(27*a + 9*b + 3*c
 + d)*ln(abs(x - 3)) - 1/3*d*ln(abs(x))
```

3.273 $\int \frac{1}{(2-\log(1+x^2))^5} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{(2-\log(x^2+1))^5}, x\right)$$

```
[Out] Unintegrable[(2 - Log[1 + x^2])^(-5), x]
```

Rubi [A] time = 0.00711418, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{1}{(2-\log(1+x^2))^5}, x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(2 - Log[1 + x^2])^(-5),x]

[Out] Defer[Int][(2 - Log[1 + x^2])^(-5), x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\log(x^2+1)+2)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2-ln(x**2+1))**5,x)

[Out] Integral((-log(x**2 + 1) + 2)**(-5), x)
```

Mathematica [A] time = 4.24961, size = 0, normalized size = 0.

$$\int \frac{1}{(2-\log(1+x^2))^5} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(2 - Log[1 + x^2])^(-5),x]
```

```
[Out] Integrate[(2 - Log[1 + x^2])^(-5), x]
```

Maple [A] time = 0.023, size = 0, normalized size = 0.

$$\int \left(2-\ln \left(x^{2}+1\right)\right)^{-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2-ln(x^2+1))^5,x)
```

```
[Out] int(1/(2-ln(x^2+1))^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{32\,x^{8}+56\,x^{6}+120\,x^{4}+\left(x^{8}-10\,x^{4}-24\,x^{2}-15\right)\log\left(x^{2}+1\right)^{3}-2\left(2\,x^{8}-x^{6}-33\,x^{4}-75\,x^{2}-45\right)\log\left(x^{2}+1\right)^{2}+216\,x^{2}}{384\left(x^{7}\log\left(x^{2}+1\right)^{4}-8\,x^{7}\log\left(x^{2}+1\right)^{3}+24\,x^{7}\log\left(x^{2}+1\right)^{2}-32\,x^{7}\log\left(x^{2}+\right.\right.}$$

$$-\int \frac{x^{8}+30\,x^{4}+120\,x^{2}+105}{384\left(x^{8}\log\left(x^{2}+1\right)-2\,x^{8}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(log(x^2 + 1) - 2)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*l
og(x^2 + 1)^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 +
1)^2 + 216*x^2 + 4*(3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2
 + 1) + 120)/(x^7*log(x^2 + 1)^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*
log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1) + 16*x^7) - integrate(1/384*
(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 + 1) - 2*x^8), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\log\left(x^{2}+1\right)^{5}-10\,\log\left(x^{2}+1\right)^{4}+40\,\log\left(x^{2}+1\right)^{3}-80\,\log\left(x^{2}+1\right)^{2}+80\,\log\left(x^{2}+1\right)-32},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(log(x^2 + 1) - 2)^5,x, algorithm="fricas")
```

```
[Out] integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)
^3 - 80*log(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{120x^2}{x^8\log(x^2+1)-2x^8}\,dx + \int \frac{30x^4}{x^8\log(x^2+1)-2x^8}\,dx + \int \frac{x^8}{x^8\log(x^2+1)-2x^8}\,dx + \int \frac{105}{x^8\log(x^2+1)-2x^8}\,dx}{384} + \frac{\frac{2x^8}{3} + \frac{7x^6}{6} + \frac{5x^4}{2} + \frac{9x^2}{2} + \left(\frac{x^8}{48} - \frac{5x^4}{24} - \frac{x^2}{2} - \frac{5}{16}\right)\log\left(x^2+1\right)^3 + \left(-\frac{x^8}{12} + \frac{x^6}{24} + \frac{11x^4}{8} + \frac{25x^2}{8} + \frac{15}{8}\right)\log\left(x^2+1\right)^2 + \left(\frac{x^8}{4} - \frac{x^6}{6} - \frac{19x}{6}\right.}{8x^7\log(x^2+1)^4 - 64x^7\log(x^2+1)^3 + 192x^7\log(x^2+1)^2 - 256x^7\log(x^2+1) + 128x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2-ln(x**2+1))**5,x)
```

```
[Out] -(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(
30*x**4/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*l
og(x**2 + 1) - 2*x**8), x) + Integral(105/(x**8*log(x**2 + 1) - 2
*x**8), x))/384 + (2*x**8/3 + 7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x
**8/48 - 5*x**4/24 - x**2/2 - 5/16)*log(x**2 + 1)**3 + (-x**8/12
+ x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log(x**2 + 1)**2 + (x**
8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2 + 1) + 5/2)
/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7*l
og(x**2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(\log(x^2+1)-2)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(log(x^2 + 1) - 2)^5,x, algorithm="giac")
```

```
[Out] integrate(-1/(log(x^2 + 1) - 2)^5, x)
```

3.274 $\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)\,dx$

Optimal. Leaf size=28

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Rubi [F] time = 0.44171, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-
1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - 2*Defer[Int][(x + Log[x]^2)^(-2), x] + Defer[Int][
Log[x]/(x + Log[x]^2)^2, x] + Defer[Int][(x + Log[x]^2)^(-1), x]
+ Defer[Int][1/(x*(x + Log[x]^2)), x] + 2*Defer[Int][Log[x]/(x*(x
 + Log[x]^2)), x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{x^2}\log(x)+\int\frac{\log(x)-2}{\left(x+\log(x)^2\right)^2}\,dx+\int\frac{1+\frac{2\log(x)}{x}+\frac{1}{x}}{x+\log(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1
```

```
[Out] exp(x**2)*log(x) + Integral((log(x) - 2)/(x + log(x)**2)**2, x) +
 Integral((1 + 2*log(x)/x + 1/x)/(x + log(x)**2), x)
```

Mathematica [A] time = 9.8761, size = 28, normalized size = 1.

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-
1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Maple [A] time = 0.03, size = 28, normalized size = 1.

$$e^{x^2}\ln(x)-\frac{\ln(x)}{x+(\ln(x))^2}+\ln\left(x+(\ln(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)^2+(1+1/x+2*ln(x)/x)/(
```

```
[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

Maxima [A] time = 1.54193, size = 36, normalized size = 1.29

$$e^{\left(x^2\right)}\log(x)-\frac{\log(x)}{\log(x)^2+x}+\log\left(\log(x)^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*e^(x^2)*log(x) + (2*log(x)/x + 1/x + 1)/(log(x)^2 + x) + e^(x^2)/x +
```

```
[Out] e^(x^2)*log(x) - log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

Fricas [A] time = 0.225221, size = 59, normalized size = 2.11

$$\frac{e^{\left(x^2\right)}\log(x)^3+\left(\log(x)^2+x\right)\log\left(\log(x)^2+x\right)+\left(xe^{\left(x^2\right)}-1\right)\log(x)}{\log(x)^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*e^(x^2)*log(x) + (2*log(x)/x + 1/x + 1)/(log(x)^2 + x) + e^(x^2)/x +
```

```
[Out] (e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2)
 - 1)*log(x))/(log(x)^2 + x)
```

Sympy [A] time = 0.788409, size = 26, normalized size = 0.93

$$e^{x^2}\log(x) + \log\left(x + \log(x)^2\right) - \frac{\log(x)}{x + \log(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/x+2*
```

```
[Out] exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int 2\,xe^{\left(x^2\right)}\log(x) + \frac{\frac{2\log(x)}{x} + \frac{1}{x} + 1}{\log(x)^2 + x} + \frac{e^{\left(x^2\right)}}{x} + \frac{\log(x) - 2}{\left(\log(x)^2 + x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*e^(x^2)*log(x) + (2*log(x)/x + 1/x + 1)/(log(x)^2 + x) + e^(x^2)/x +
```

```
[Out] integrate(2*x*e^(x^2)*log(x) + (2*log(x)/x + 1/x + 1)/(log(x)^2 +
 x) + e^(x^2)/x + (log(x) - 2)/(log(x)^2 + x)^2, x)
```

3.275 $\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$

Optimal. Leaf size=199

$$\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}$$

```
[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E
^(x/2 + x*z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2
 + x^4) + (12*E^(x/2 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*P
i^2*x^2 + x^4) - (4*E^(x/2 + x*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(
16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[Pi*z]^4)/(16*Pi^2 + x^2)
```

Rubi [A] time = 0.181946, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E
^(x/2 + x*z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2
 + x^4) + (12*E^(x/2 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*P
i^2*x^2 + x^4) - (4*E^(x/2 + x*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(
16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[Pi*z]^4)/(16*Pi^2 + x^2)
```

Rubi in Sympy [A] time = 13.1217, size = 180, normalized size = 0.9

$$\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{(x^2+4\pi^2)(x^2+16\pi^2)}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{(x^2+4\pi^2)(x^2+16\pi^2)}+\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{(x^2+4\pi^2)(x^2+16\pi^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

```
[Out] x**5*exp(x*z + x/2)*sin(pi*z)**4/(x**2 + 16*pi**2) + 12*pi**2*x**
5*exp(x*z + x/2)*sin(pi*z)**2/((x**2 + 4*pi**2)*(x**2 + 16*pi**2)
) - 4*pi*x**4*exp(x*z + x/2)*sin(pi*z)**3*cos(pi*z)/(x**2 + 16*pi
**2) - 24*pi**3*x**4*exp(x*z + x/2)*sin(pi*z)*cos(pi*z)/((x**2 +
4*pi**2)*(x**2 + 16*pi**2)) + 24*pi**4*x**3*exp(x*z + x/2)/((x**2
 + 4*pi**2)*(x**2 + 16*pi**2))
```

Mathematica [A] time = 0.254416, size = 136, normalized size = 0.68

$$\frac{x^4e^{x\left(z+\frac{1}{2}\right)}\left(3x^4-8\pi x^3\sin(2\pi z)+4\pi x^3\sin(4\pi z)-4\left(x^2+16\pi^2\right)x^2\cos(2\pi z)+\left(x^2+4\pi^2\right)x^2\cos(4\pi z)+60\pi^2x^2-128\pi^3x}{8\left(x^5+20\pi^2x^3+64\pi^4x\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (E^(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*
Pi^2 + x^2)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi
^3*x*Sin[2*Pi*z] - 8*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] +
 4*Pi*x^3*Sin[4*Pi*z]))/(8*(64*Pi^4*x + 20*Pi^2*x^3 + x^5))
```

Maple [A] time = 0.016, size = 133, normalized size = 0.7

$$\frac{3\,x^3}{8}e^{\frac{x}{2}+xz}-\frac{x^4}{2}\left(\frac{x\cos(2\,\pi\,z)}{4\,\pi^2+x^2}e^{\frac{x}{2}+xz}+2\,\frac{\pi\,e^{x/2+xz}\sin(2\,\pi\,z)}{4\,\pi^2+x^2}\right)+\frac{x^4}{8}\left(\frac{x\cos(4\,\pi\,z)}{16\,\pi^2+x^2}e^{\frac{x}{2}+xz}+4\,\frac{\pi\,e^{x/2+xz}\sin(4\,\pi\,z)}{16\,\pi^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z)
```

```
[Out] 3/8*x^3*exp(1/2*x+x*z)-1/2*x^4*(x/(4*Pi^2+x^2)*exp(1/2*x+x*z)*cos
(2*Pi*z)+2*Pi/(4*Pi^2+x^2)*exp(1/2*x+x*z)*sin(2*Pi*z))+1/8*x^4*(x
/(16*Pi^2+x^2)*exp(1/2*x+x*z)*cos(4*Pi*z)+4*Pi/(16*Pi^2+x^2)*exp(
1/2*x+x*z)*sin(4*Pi*z))
```

Maxima [A] time = 1.44461, size = 216, normalized size = 1.09

$$\frac{\left(\left(4\,\pi^2x^2+x^4\right)\cos(4\,\pi z)\,e^{\left(xz+\frac{1}{2}\,x\right)}-4\left(16\,\pi^2x^2+x^4\right)\cos(2\,\pi z)\,e^{\left(xz+\frac{1}{2}\,x\right)}+4\left(4\,\pi^3x+\pi x^3\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin(4\,\pi z)-8\left(16\,\pi^3x\right.}{8\left(64\,\pi^4x+20\,\pi^2x^3+x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*e^(x*z + 1/2*x)*sin(pi*z)^4,z, algorithm="maxima")
```

```
[Out] 1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*
x^2 + x^4)*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^
(x*z + 1/2*x)*sin(4*pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x
)*sin(2*pi*z) + 3*(64*pi^4 + 20*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*
x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)
```

Fricas [A] time = 0.25848, size = 196, normalized size = 0.98

$$\frac{4\left(\left(\left(4\,\pi^{3}x^{4}+\pi x^{6}\right)\cos\left(\pi z\right)^{3}-\left(10\,\pi^{3}x^{4}+\pi x^{6}\right)\cos\left(\pi z\right)\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(\pi z\right)+\left(24\,\pi^{4}x^{3}+16\,\pi^{2}x^{5}+x^{7}+\left(4\,\pi^{2}x^{5}+x^{7}\right)\cos\left(\pi\right.\right.\right.}{64\,\pi^{4}+20\,\pi^{2}x^{2}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*e^(x*z + 1/2*x)*sin(pi*z)^4,z, algorithm="fricas")
```

```
[Out] (4*((4*pi^3*x^4 + pi*x^6)*cos(pi*z)^3 - (10*pi^3*x^4 + pi*x^6)*co
s(pi*z))*e^(x*z + 1/2*x)*sin(pi*z) + (24*pi^4*x^3 + 16*pi^2*x^5 +
 x^7 + (4*pi^2*x^5 + x^7)*cos(pi*z)^4 - 2*(10*pi^2*x^5 + x^7)*cos
(pi*z)^2)*e^(x*z + 1/2*x))/(64*pi^4 + 20*pi^2*x^2 + x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.209834, size = 154, normalized size = 0.77

$$\frac{1}{8}\left(\left(\frac{x\cos\left(4\,\pi z\right)}{16\,\pi^{2}+x^{2}}+\frac{4\,\pi\sin\left(4\,\pi z\right)}{16\,\pi^{2}+x^{2}}\right)e^{\left(xz+\frac{1}{2}\,x\right)}-4\left(\frac{x\cos\left(2\,\pi z\right)}{4\,\pi^{2}+x^{2}}+\frac{2\,\pi\sin\left(2\,\pi z\right)}{4\,\pi^{2}+x^{2}}\right)e^{\left(xz+\frac{1}{2}\,x\right)}+\frac{3\,e^{\left(xz+\frac{1}{2}\,x\right)}}{x}\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*e^(x*z + 1/2*x)*sin(pi*z)^4,z, algorithm="giac")
```

```
[Out] 1/8*((x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 +
 x^2))*e^(x*z + 1/2*x) - 4*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*s
in(2*pi*z)/(4*pi^2 + x^2))*e^(x*z + 1/2*x) + 3*e^(x*z + 1/2*x)/x)
*x^4
```

3.276 $\int \mathrm{Erf}(x)\,dx$

Optimal. Leaf size=18

$$x\mathrm{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Rubi [A] time = 0.0899616, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$x\mathrm{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In]  Int[Erf[x],x]

[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Rubi in Sympy [A] time = 0.49045, size = 15, normalized size = 0.83

$$x\operatorname{erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(erf(x),x)

[Out] x*erf(x) + exp(-x**2)/sqrt(pi)
```

Mathematica [A] time = 0.0109546, size = 18, normalized size = 1.

$$x\mathrm{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Erf[x],x]
```

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$xErf(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(x),x)
```

```
[Out] x*erf(x)+1/Pi^(1/2)*exp(-x^2)
```

Maxima [A] time = 1.3928, size = 20, normalized size = 1.11

$$x \operatorname{erf}(x) + \frac{e^{(-x^2)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="maxima")
```

```
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

Fricas [A] time = 0.203514, size = 24, normalized size = 1.33

$$\frac{\sqrt{\pi}x \operatorname{erf}(x) + e^{(-x^2)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*x*erf(x) + e^(-x^2))/sqrt(pi)
```

Sympy [A] time = 0.540621, size = 15, normalized size = 0.83

$$x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(erf(x),x)
```

```
[Out] x*erf(x) + exp(-x**2)/sqrt(pi)
```

GIAC/XCAS [A] time = 0.210945, size = 20, normalized size = 1.11

$$x\,\mathrm{erf}\left(x\right) + \frac{e^{\left(-x^{2}\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(erf(x),x, algorithm="giac")
```

```
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

3.277 $\int \text{Erf}(a + x)\, dx$

Optimal. Leaf size=24

$$(a+x)\text{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

```
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Rubi [A] time = 0.024982, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$(a+x)\text{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In]  Int[Erf[a + x],x]
```

```
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Rubi in Sympy [A] time = 0.61507, size = 20, normalized size = 0.83

$$(a+x)\operatorname{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(erf(a+x),x)
```

```
[Out] (a + x)*erf(a + x) + exp(-(a + x)**2)/sqrt(pi)
```

Mathematica [A] time = 0.0447285, size = 24, normalized size = 1.

$$(a+x)\text{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Erf[a + x],x]
```

```
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Maple [A] time = 0.002, size = 22, normalized size = 0.9

$$(a+x)\,Erf\,(a+x)+\frac{\mathrm{e}^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(erf(a+x),x)
```

```
[Out] (a+x)*erf(a+x)+1/Pi^(1/2)*exp(-(a+x)^2)
```

Maxima [A] time = 1.39889, size = 28, normalized size = 1.17

$$(a+x)\,\mathrm{erf}\,(a+x)+\frac{e^{\left(-(a+x)^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(erf(a + x),x, algorithm="maxima")
```

```
[Out] (a + x)*erf(a + x) + e^(-(a + x)^2)/sqrt(pi)
```

Fricas [A] time = 0.206208, size = 43, normalized size = 1.79

$$\frac{\sqrt{\pi}(a+x)\,\mathrm{erf}\,(a+x)+e^{\left(-a^2-2\,ax-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(erf(a + x),x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*(a + x)*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/sqrt(pi)
```

Sympy [A] time = 0.804102, size = 36, normalized size = 1.5

$$a\,\mathrm{erf}\,(a+x)+x\,\mathrm{erf}\,(a+x)+\frac{e^{-a^2}e^{-x^2}e^{-2ax}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(erf(a+x),x)
```

```
[Out] a*erf(a + x) + x*erf(a + x) + exp(-a**2)*exp(-x**2)*exp(-2*a*x)/s
qrt(pi)
```

GIAC/XCAS [A] time = 0.219522, size = 50, normalized size = 2.08

$$x \operatorname{erf}(a+x) + \frac{\sqrt{\pi} a \operatorname{erf}(a+x) + e^{\left(-a^2 - 2\,ax - x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(erf(a + x),x, algorithm="giac")
```

```
[Out] x*erf(a + x) + (sqrt(pi)*a*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/s
qrt(pi)
```

3.278 $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$

Optimal. Leaf size=94

$$\frac{(2x+1)\sqrt{x^4+4x^3+2x^2+1}}{2(2x^2-1)}-\tanh^{-1}\left(\frac{x(x+2)\left(33x^3+27x^2-x+7\right)}{(31x^3+37x^2+2)\sqrt{x^4+4x^3+2x^2+1}}\right)$$

```
[Out] ((1 + 2*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4])/(2*(-1 + 2*x^2)) - ArcT
anh[(x*(2 + x)*(7 - x + 27*x^2 + 33*x^3))/((2 + 37*x^2 + 31*x^3)*
Sqrt[1 + 2*x^2 + 4*x^3 + x^4])]
```

Rubi [F] time = 2.88034, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[1 + 2*x^2
```

```
[Out] (9*Defer[Int][1/Sqrt[1 + 2*x^2 + 4*x^3 + x^4], x])/4 - (13*Defer[
Int][1/((Sqrt[2] - 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/4 +
 Defer[Int][x/Sqrt[1 + 2*x^2 + 4*x^3 + x^4], x] + Defer[Int][x^2/
Sqrt[1 + 2*x^2 + 4*x^3 + x^4], x]/2 - (13*Defer[Int][1/((Sqrt[2]
+ 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/4 - (13*Defer[Int][1
/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/8 - ((15 +
Sqrt[2])*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x
^4]), x])/8 - (13*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*x^2 +
4*x^3 + x^4]), x])/8 - ((15 - Sqrt[2])*Defer[Int][1/((1 + Sqrt[2]
*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/8 - (17*Defer[Int][x/((-1
 + 2*x^2)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-
1)**2/(x**4+4*x**3+2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 6.13796, size = 5137, normalized size = 54.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-
1 + 2*x^2)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.703, size = 1197351, normalized size = 12737.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^6 + 4\,x^5 + 7\,x^4 - 3\,x^3 - x^2 - 8\,x - 8}{\sqrt{x^4 + 4\,x^3 + 2\,x^2 + 1}(2\,x^2 - 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3 + 2*x^2
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x
^4 + 4*x^3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

Fricas [A] time = 0.294648, size = 242, normalized size = 2.57

$$\frac{(2\,x^2 - 1)\log\left(\frac{1025\,x^{10}+6138\,x^9+12307\,x^8+10188\,x^7+4503\,x^6+3134\,x^5+1589\,x^4+140\,x^3+176\,x^2-(1023\,x^8+4104\,x^7+5084\,x^6+2182\,x^5+805\,x^4+624\,x^3+10\,x^2+2}{32\,x^{10}-80\,x^8+80\,x^6-40\,x^4+10\,x^2-1}\right.}{2\,(2\,x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3 + 2*x^2
```

```
[Out] 1/2*((2*x^2 - 1)*log((1025*x^10 + 6138*x^9 + 12307*x^8 + 10188*x^
7 + 4503*x^6 + 3134*x^5 + 1589*x^4 + 140*x^3 + 176*x^2 - (1023*x^
8 + 4104*x^7 + 5084*x^6 + 2182*x^5 + 805*x^4 + 624*x^3 + 10*x^2 +
 28*x)*sqrt(x^4 + 4*x^3 + 2*x^2 + 1) + 2)/(32*x^10 - 80*x^8 + 80*
x^6 - 40*x^4 + 10*x^2 - 1)) + sqrt(x^4 + 4*x^3 + 2*x^2 + 1)*(2*x
+ 1))/(2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^6 + 4x^5 + 7x^4 - 3x^3 - x^2 - 8x - 8}{\sqrt{(x+1)(x^3+3x^2-x+1)}\,(2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x**3+
```

```
[Out] Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sq
rt((x + 1)*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^6 + 4\,x^5 + 7\,x^4 - 3\,x^3 - x^2 - 8\,x - 8}{\sqrt{x^4 + 4\,x^3 + 2\,x^2 + 1}(2\,x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3 + 2*x^2
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x
^4 + 4*x^3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

3.279 $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$

Optimal. Leaf size=142

$$-\frac{1}{4}\tanh^{-1}\left(\frac{(1-3y)\sqrt{-5y^2-5y+1}}{(1-5y)\sqrt{-y^2-y+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{(3y+4)\sqrt{-5y^2-5y+1}}{(5y+6)\sqrt{-y^2-y+1}}\right)+\frac{9}{4}\tanh^{-1}\left(\frac{(7y+11)\sqrt{-5y^2-5y+1}}{3(5y+7)\sqrt{-y^2-y+1}}\right)$$

```
[Out] -ArcTanh[((1 - 3*y)*Sqrt[1 - 5*y - 5*y^2])/((1 - 5*y)*Sqrt[1 - y
- y^2])]/4 - ArcTanh[((4 + 3*y)*Sqrt[1 - 5*y - 5*y^2])/((6 + 5*y)
*Sqrt[1 - y - y^2])]/2 + (9*ArcTanh[((11 + 7*y)*Sqrt[1 - 5*y - 5*
y^2])/(3*(7 + 5*y)*Sqrt[1 - y - y^2])])/4
```

Rubi [F] time = 5.56654, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}},y\right)$$

Verification is Not applicable to the result.

```
[In] Int[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2]),y]
```

```
[Out] Defer[Int][Sqrt[1 - 5*y - 5*y^2]/(y*Sqrt[1 - y - y^2]), y]/2 + De
fer[Int][Sqrt[1 - 5*y - 5*y^2]/((1 + y)*Sqrt[1 - y - y^2]), y] -
(3*Defer[Int][Sqrt[1 - 5*y - 5*y^2]/((2 + y)*Sqrt[1 - y - y^2]),
y])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-
y+1)**(1/2),y)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.3852, size = 630, normalized size = 4.44

$$\frac{\left(-1-\frac{2}{\sqrt{5}}\right)\left(2y+\sqrt{5}+1\right)^2\sqrt{\frac{10y+3\sqrt{5}+5}{10y+5\sqrt{5}+5}}\left(20\left(\sqrt{5}\sqrt{\frac{-10y+3\sqrt{5}-5}{2y+\sqrt{5}+1}}\sqrt{\frac{-2y+\sqrt{5}-1}{2y+\sqrt{5}+1}}-4\sqrt{\frac{-10y+3\sqrt{5}-5}{2y+\sqrt{5}+1}}\sqrt{\frac{-2y+\sqrt{5}-1}{2y+\sqrt{5}+1}}-2\sqrt{5}\sqrt{-\frac{2\sqrt{5}y+\sqrt{5}-5}{2y+\sqrt{5}+1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2]),y
```

```
[Out] ((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*
y)/(5 + 5*Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(
1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)
] + Sqrt[5]*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqr
t[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + 5*Sqrt[-((-5 + Sqrt
[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2
*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((-5 + Sqrt[5
] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*S
qrt[5]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*
Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16] + Sqrt[(-
5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2
*y)/(1 + Sqrt[5] + 2*y)]*(9*Sqrt[5]*EllipticPi[5/8 - Sqrt[5]/8, A
rcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[1
5]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[(-3*(-5 + Sqrt[5]))/8,
 ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt
[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, ArcSin[
(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 1
5/16])))/(16*Sqrt[1 - 5*y - 5*y^2]*Sqrt[1 - y - y^2])
```

Maple [C] time = 0.177, size = 352, normalized size = 2.5

$$-1200\,\frac{\sqrt{-5\,y^2-5\,y+1}\sqrt{-y^2-y+1}\left(10\,y+5+3\,\sqrt{5}\right)^2\sqrt{5}}{\sqrt{5\,y^4+10\,y^3-y^2-6\,y+1}\sqrt{\left(-10\,y+3\,\sqrt{5}-5\right)\left(10\,y+5+3\,\sqrt{5}\right)\left(-2\,y+\sqrt{5}-1\right)\left(2\,y+1+\sqrt{5}\right)}\left(5+3\,\sqrt{5}\right)\left(3\,\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y)
```

```
[Out] -1200*(-5*y^2-5*y+1)^(1/2)*(-y^2-y+1)^(1/2)*(-(-10*y+3*5^(1/2)-5)
/(10*y+5+3*5^(1/2)))^(1/2)*(10*y+5+3*5^(1/2))^2*5^(1/2)*((-2*y+5^
(1/2)-1)/(10*y+5+3*5^(1/2)))^(1/2)*((2*y+1+5^(1/2))/(10*y+5+3*5^(
1/2)))^(1/2)*(EllipticPi(1/2*(-(-10*y+3*5^(1/2)-5)/(10*y+5+3*5^(1
/2)))^(1/2),-4*(5+3*5^(1/2))/(3*5^(1/2)-5),4)+2*EllipticPi(1/2*(-
(-10*y+3*5^(1/2)-5)/(10*y+5+3*5^(1/2)))^(1/2),-4*(3*5^(1/2)-5)/(5
+3*5^(1/2)),4)-3*EllipticPi(1/2*(-(-10*y+3*5^(1/2)-5)/(10*y+5+3*5
```

```
^(1/2)))^(1/2),-4*(-5+5^(1/2))/(5+5^(1/2)),4))/(5*y^4+10*y^3-y^2-
6*y+1)^(1/2)/((-10*y+3*5^(1/2)-5)*(10*y+5+3*5^(1/2))*(-2*y+5^(1/2
)-1)*(2*y+1+5^(1/2)))^(1/2)/(5+3*5^(1/2))/(3*5^(1/2)-5)/(-5+5^(1/
2))/(5+5^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-5\,y^2-5\,y+1}(2\,y+1)}{\sqrt{-y^2-y+1}(y+2)(y+1)y}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y + 1)*y),y
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y
 + 2)*(y + 1)*y), y)
```

Fricas [A] time = 0.281836, size = 301, normalized size = 2.12

$$\frac{9}{8}\,\log\left(-\frac{235\,y^4+935\,y^3-3\,\left(35\,y^2+104\,y+77\right)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}+1086\,y^2+131\,y-281}{y^4+8\,y^3+24\,y^2+32\,y+16}\right)$$
$$+\frac{1}{4}\,\log\left(\frac{35\,y^4+125\,y^3+\left(15\,y^2+38\,y+24\right)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}+131\,y^2+16\,y-26}{y^4+4\,y^3+6\,y^2+4\,y+1}\right)$$
$$+\frac{1}{8}\,\log\left(\frac{35\,y^4+15\,y^3+\left(15\,y^2-8\,y+1\right)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}-34\,y^2+11\,y-1}{y^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y + 1)*y),y
```

```
[Out] 9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 -
 y + 1)*sqrt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8
*y^3 + 24*y^2 + 32*y + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2
 + 38*y + 24)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2
 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 + 4*y + 1)) + 1/8*log((35*y^4
+ 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*
y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y\,(y+1)\,(y+2)\sqrt{-y^2-y+1}}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2),y)
```

```
[Out] Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqr
t(-y**2 - y + 1)), y)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-5\,y^2-5\,y+1}(2\,y+1)}{\sqrt{-y^2-y+1}(y+2)(y+1)y}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y + 1)*y),y
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y
 + 2)*(y + 1)*y), y)
```

3.280 $\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$

Optimal. Leaf size=21

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

```
[Out] Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.437358, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 85, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[-
1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]
```

```
[Out] Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

Rubi in Sympy [A] time = 18.5146, size = 19, normalized size = 0.9

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-
1)**(1/2)+x**2*(x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-
1)**(1/2)),x)
```

```
[Out] log(sqrt(x**2 - 4) + sqrt(x**2 - 1) + 1)
```

Mathematica [B] time = 0.0817115, size = 97, normalized size = 4.62

$$\frac{1}{4}\log\left(-5x^2-4\sqrt{x^2-4}\sqrt{x^2-1}+17\right)+\frac{1}{4}\log\left(-2x^2-2\sqrt{x^2-4}\sqrt{x^2-1}+5\right)$$
$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^2-4}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^2-1}}{2}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-
1 + x^2] + x^2*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-
1 + x^2])),x]
```

```
[Out] -ArcTanh[Sqrt[-4 + x^2]]/2 + ArcTanh[Sqrt[-1 + x^2]/2]/2 + Log[17
 - 5*x^2 - 4*Sqrt[-4 + x^2]*Sqrt[-1 + x^2]]/4 + Log[5 - 2*x^2 - 2
*Sqrt[-4 + x^2]*Sqrt[-1 + x^2]]/4
```

Maple [B] time = 0.095, size = 1088, normalized size = 51.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-
1)^(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x)
```

```
[Out] 1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/4*(8
-2*5^(1/2)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(
1/2))-1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x+5^
(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2)+1/(5^(1/2)+1)/(2+5^(1/2))
/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/4*(8+2*5^(1/2)*(x-5^(1/2)))/(
(x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+4)^(1/2))-1/2/(5^(1/2)+1)/(2+
5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x-5^(1/2))^2+2*5^(1/2)*(x-5^(
1/2))+4)^(1/2)-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)
*arctanh(1/2*(2+2*5^(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(
x-5^(1/2))+1)^(1/2))+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1
/2)-1)*((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2)-1/(5^(1/2)+1
)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/2*(2-2*5^(1/2)*(
x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))+1/(5^(
1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x+5^(1/2))^2-2*5^(
1/2)*(x+5^(1/2))+1)^(1/2)+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((-1+x)^2-2
+2*x)^(1/2)+1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((-1+x)^2-2+2*x)^(1/
2))+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((1+x)^2-2-2*x)^(1/2)-1/2/(5^(1/2
)+1)/(5^(1/2)-1)*ln(x+((1+x)^2-2-2*x)^(1/2))-1/4/(2+5^(1/2))/(-2+
5^(1/2))*((2+x)^2-8-4*x)^(1/2)+1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+
((2+x)^2-8-4*x)^(1/2))-1/4/(2+5^(1/2))/(-2+5^(1/2))*((-2+x)^2-8+4
*x)^(1/2)-1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((-2+x)^2-8+4*x)^(1/2
))+1/4*ln(x^2-5)+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-
1)*5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))-1/
(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/2)*ln(x+((x
+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))-1/2/(5^(1/2)+1)/(2+5^
(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(
1/2)*(x-5^(1/2))+4)^(1/2))+1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2
))/(5^(1/2)-1)*5^(1/2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+
4)^(1/2))+7/8*(x^2-4)^(1/2)*(x^2-1)^(1/2)/(x^4-5*x^2+4)^(1/2)*arc
tanh(1/4*(5*x^2-17)/(x^4-5*x^2+4)^(1/2))+1/8*(x^2-4)^(1/2)*(x^2-1
)^(1/2)*(2*ln(-5/2+x^2+(x^4-5*x^2+4)^(1/2))-5*arctanh(1/4*(5*x^2-
```

```
17)/(x^4-5*x^2+4)^(1/2)))/(x^4-5*x^2+4)^(1/2)
```

Maxima [A] time = 1.65515, size = 231, normalized size = 11.

$$\frac{1}{4}\log(x+1)+\frac{3}{8}\log(x-1)+\frac{1}{8}\log(x-2)$$
$$+\frac{1}{4}\log\left(\frac{2x^4+4(x^2-3)\sqrt{x+1}\sqrt{x-1}-7x^2+2\left((x^2-1)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+(2x^2-3)\sqrt{x-2}\right)\sqrt{x+2}+3}{2\left((x^2-1)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+(2x^2-3)\sqrt{x-2}\right)}\right)$$
$$+\frac{1}{4}\log\left(\frac{(x^2-1)\sqrt{x+1}\sqrt{x-1}+2x^2-3}{(x^2-1)\sqrt{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x^2 - 1)*x^2 + sqrt(x^2 - 4)*x^2 - 4*sqrt(x^2 - 1) - sqrt(x^2 - 4))
```

```
[Out] 1/4*log(x + 1) + 3/8*log(x - 1) + 1/8*log(x - 2) + 1/4*log(1/2*(2
*x^4 + 4*(x^2 - 3)*sqrt(x + 1)*sqrt(x - 1) - 7*x^2 + 2*((x^2 - 1)
*sqrt(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))*s
qrt(x + 2) + 3)/((x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)*sqrt(x - 2) +
(2*x^2 - 3)*sqrt(x - 2))) + 1/4*log(((x^2 - 1)*sqrt(x + 1)*sqrt(x
 - 1) + 2*x^2 - 3)/((x^2 - 1)*sqrt(x - 1)))
```

Fricas [A] time = 0.214008, size = 219, normalized size = 10.43

$$-\frac{1}{4}\log\left(4x^4-(4x^2-11)\sqrt{x^2-1}\sqrt{x^2-4}-21x^2+23\right)-\frac{1}{4}\log\left(x^2-\sqrt{x^2-1}(x+2)+2x-1\right)$$
$$+\frac{1}{4}\log\left(x^2-\sqrt{x^2-4}(x+1)+x-4\right)-\frac{1}{4}\log\left(x^2-\sqrt{x^2-4}(x-1)-x-4\right)$$
$$+\frac{1}{4}\log\left(x^2-\sqrt{x^2-1}(x-2)-2x-1\right)+\frac{1}{4}\log\left(x^2-5\right)+\frac{1}{4}\log\left(-x^2+\sqrt{x^2-1}\sqrt{x^2-4}+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x^2 - 1)*x^2 + sqrt(x^2 - 4)*x^2 - 4*sqrt(x^2 - 1) - sqrt(x^2 - 4))
```

```
[Out] -1/4*log(4*x^4 - (4*x^2 - 11)*sqrt(x^2 - 1)*sqrt(x^2 - 4) - 21*x^
2 + 23) - 1/4*log(x^2 - sqrt(x^2 - 1)*(x + 2) + 2*x - 1) + 1/4*lo
g(x^2 - sqrt(x^2 - 4)*(x + 1) + x - 4) - 1/4*log(x^2 - sqrt(x^2 -
 4)*(x - 1) - x - 4) + 1/4*log(x^2 - sqrt(x^2 - 1)*(x - 2) - 2*x
- 1) + 1/4*log(x^2 - 5) + 1/4*log(-x^2 + sqrt(x^2 - 1)*sqrt(x^2 -
 4) + 7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*(x**2-
1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.349827, size = 105, normalized size = 5.

$$\frac{1}{2}\ln\left(\sqrt{x^2-1}+2\right)-\frac{1}{2}\ln\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}\right|\right)-\frac{1}{2}\ln\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}-1\right|\right)+\frac{1}{2}\ln\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x^2 - 1)*x^2 + sqrt(x^2 - 4)*x^2 - 4*sqrt(x^2 - 1) - sqrt(x^2 - 4))
```

```
[Out] 1/2*ln(sqrt(x^2 - 1) + 2) - 1/2*ln(abs(-sqrt(x^2 - 1) + sqrt(x^2
- 4))) - 1/2*ln(abs(-sqrt(x^2 - 1) + sqrt(x^2 - 4) - 1)) + 1/2*ln
(abs(-sqrt(x^2 - 1) + sqrt(x^2 - 4) - 3))
```

3.281 $\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}x^2-\sqrt{2}\text{Int}\left(\sqrt{x^4+2x^2+4x+1},x\right)$$

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*(-Sqrt[1 + 4*x + 2*x^2 + x^
4]/3 + ((1 + x)*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 + ((4*I)*(-13 + 3*
Sqrt[33])^(1/3)*Sqrt[1 + 4*x + 2*x^2 + x^4])/(4*2^(2/3)*(-I + Sqr
t[3]) - (2*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(I + Sqrt[3])*(-
13 + 3*Sqrt[33])^(2/3) + (6*I)*(-13 + 3*Sqrt[33])^(1/3)*x) - (8*2
^(2/3)*Sqrt[3/(-13 + 3*Sqrt[33] + 4*(-26 + 6*Sqrt[33])^(1/3))]*Sq
rt[(I*(-19899 + 3445*Sqrt[33] + (-26 + 6*Sqrt[33])^(2/3)*(-2574 +
 466*Sqrt[33]) + (-26 + 6*Sqrt[33])^(1/3)*(-19899 + 3445*Sqrt[33]
) + (59697 - 10335*Sqrt[33])*x))/((-39 - (13*I)*Sqrt[3] + (9*I)*S
qrt[11] + 9*Sqrt[33] + (4*I)*(3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(
1/3))*(26 - 6*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] +
 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*(-26
 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x))]*Sqrt[1 + 4*x + 2
*x^2 + x^4]*EllipticE[ArcSin[Sqrt[26 - 6*Sqrt[33] + (-13 - (13*I)
*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[
33])*x]/(Sqrt[(39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] - 9*Sqrt[33]
+ 4*(3 - I*Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))/(39 - (13*I)*Sqrt[3
] + (9*I)*Sqrt[11] - 9*Sqrt[33] + 4*(3 + I*Sqrt[3])*(-26 + 6*Sqrt
[33])^(1/3))]*Sqrt[26 - 6*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I
)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*S
qrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x])], (4*
(21 + (7*I)*Sqrt[3] - (3*I)*Sqrt[11] - 3*Sqrt[33]) + (3 - I*Sqrt[
3] - (3*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3))/(4*(2
1 - (7*I)*Sqrt[3] + (3*I)*Sqrt[11] - 3*Sqrt[33]) + (3 + I*Sqrt[3]
 + (3*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3))])/((4*2
^(2/3) - (-13 + 3*Sqrt[33])^(1/3) - 2^(1/3)*(-13 + 3*Sqrt[33])^(2
/3) + 3*(-13 + 3*Sqrt[33])^(1/3)*x)*Sqrt[(I*(1 + x))/((104 - 24*S
qrt[33] + (-13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-
26 + 6*Sqrt[33])^(1/3) + (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(
2/3))*(26 - 6*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] +
 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*(-26
 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x))]*Sqrt[26 - 6*Sqrt
[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26
+ 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*(-26 + 6*Sqrt[33])^(2/
3) + 6*(-13 + 3*Sqrt[33])*x]*Sqrt[26 - 6*Sqrt[33] + (-13 - (13*I)
*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[
33])*x]) + ((2^(1/3)*(13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] - 3*Sq
rt[33]) + 4*2^(2/3)*(1 + I*Sqrt[3])*(-13 + 3*Sqrt[33])^(1/3) + 20
*(-13 + 3*Sqrt[33])^(2/3))*(4*2^(2/3)*(I + Sqrt[3]) + (8*I)*(-13
+ 3*Sqrt[33])^(1/3) + 2^(1/3)*(-I + Sqrt[3])*(-13 + 3*Sqrt[33])^(
2/3))*Sqrt[(52 - 12*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) +
 4*(-26 + 6*Sqrt[33])^(2/3))/(-13 + 3*Sqrt[33] + 4*(-26 + 6*Sqrt[
33])^(1/3))]*Sqrt[((-8*I)*(-13 + 3*Sqrt[33]) + (-43*I - 13*Sqrt[3
] + 9*Sqrt[11] + (5*I)*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (2*I
+ 4*Sqrt[3] - (2*I)*Sqrt[33])*(-26 + 6*Sqrt[33])^(2/3) + ((8*I)*(
-13 + 3*Sqrt[33]) + (13*I - 13*Sqrt[3] + 9*Sqrt[11] - (3*I)*Sqrt[
```

```
33])*(-26 + 6*Sqrt[33])^(1/3) + 4*(I + Sqrt[3])*(-26 + 6*Sqrt[33]
)^(2/3))*x)/(1 + x)]*Sqrt[1 + 4*x + 2*x^2 + x^4]*EllipticF[ArcSin
[(Sqrt[52 - 12*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) + 4*(-
26 + 6*Sqrt[33])^(2/3)]*Sqrt[26 - 6*Sqrt[33] + (-13 - (13*I)*Sqrt
[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (4*
I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*
x])/(2^(1/6)*Sqrt[3]*(-13 + 3*Sqrt[33])^(2/3)*Sqrt[39 + (13*I)*Sq
rt[3] - (9*I)*Sqrt[11] - 9*Sqrt[33] + 4*(3 - I*Sqrt[3])*(-26 + 6*
Sqrt[33])^(1/3)]*Sqrt[1 + x])], (4*(21*I - 7*Sqrt[3] + 3*Sqrt[11]
 - (3*I)*Sqrt[33]) + (3*I + Sqrt[3] + 3*Sqrt[11] + (3*I)*Sqrt[33]
)*(-26 + 6*Sqrt[33])^(1/3))/(-56*Sqrt[3] + 24*Sqrt[11] + 2*(Sqrt[
3] + 3*Sqrt[11])*(-26 + 6*Sqrt[33])^(1/3))])/(3*2^(2/3)*3^(3/4)*(
-13 + 3*Sqrt[33])^(1/3)*Sqrt[39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11]
 - 9*Sqrt[33] + 4*(3 - I*Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3)]*Sqrt[
1 + x]*(4*2^(2/3)*(-I + Sqrt[3]) - (2*I)*(-13 + 3*Sqrt[33])^(1/3)
 + 2^(1/3)*(I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3) + (6*I)*(-13 +
3*Sqrt[33])^(1/3)*x)*Sqrt[26 - 6*Sqrt[33] + (-13 - (13*I)*Sqrt[3]
 + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (4*I)*
(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x]*
Sqrt[(8*(-13 + 3*Sqrt[33]) - (5 - (3*I)*Sqrt[3] + (3*I)*Sqrt[11]
+ Sqrt[33])*(-26 + 6*Sqrt[33])^(2/3) + (-26 + 6*Sqrt[33])^(1/3)*(
-41 + (15*I)*Sqrt[3] - (3*I)*Sqrt[11] + 7*Sqrt[33]) + (104 - 24*S
qrt[33] + (-13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-
26 + 6*Sqrt[33])^(1/3) + (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(
2/3))*x)/((-39 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 9*Sqrt[33] + (
4*I)*(3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))*(1 + x))]) + ((4*2
^(2/3) + 2*(-13 + 3*Sqrt[33])^(1/3) - 2^(1/3)*(-13 + 3*Sqrt[33])^
(2/3))*(4*2^(2/3)*(I + Sqrt[3]) - (4*I)*(-13 + 3*Sqrt[33])^(1/3)
+ 2^(1/3)*(-I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3))*(4*2^(2/3)*(-I
 + Sqrt[3]) + (4*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(I + Sqrt[
3])*(-13 + 3*Sqrt[33])^(2/3))*Sqrt[(-39 + (13*I)*Sqrt[3] - (9*I)*
Sqrt[11] + 9*Sqrt[33] - (4*I)*(-3*I + Sqrt[3])*(-26 + 6*Sqrt[33])
^(1/3))/(104 - 24*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[1
1] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*
(-26 + 6*Sqrt[33])^(2/3))]*Sqrt[1 + x]*Sqrt[(104 - 24*Sqrt[33] +
2*(1 + (14*I)*Sqrt[3] - (6*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[
33])^(1/3) + (-7 - I*Sqrt[3] - (3*I)*Sqrt[11] + Sqrt[33])*(-26 +
6*Sqrt[33])^(2/3) + 2*(-52 + 12*Sqrt[33] + 2^(1/3)*(-13 + 3*Sqrt[
33])^(4/3) - 4*(-26 + 6*Sqrt[33])^(2/3))*x)/((-39 + (13*I)*Sqrt[3
] - (9*I)*Sqrt[11] + 9*Sqrt[33] - (4*I)*(-3*I + Sqrt[3])*(-26 + 6
*Sqrt[33])^(1/3))*(1 + x))]*Sqrt[(104 - 24*Sqrt[33] + 2*(1 - (14*
I)*Sqrt[3] + (6*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ (-7 + I*Sqrt[3] + (3*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[33])
^(2/3) + 2*(-52 + 12*Sqrt[33] + 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3)
- 4*(-26 + 6*Sqrt[33])^(2/3))*x)/((-39 - (13*I)*Sqrt[3] + (9*I)*S
qrt[11] + 9*Sqrt[33] + (4*I)*(3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(
1/3))*(1 + x))]*Sqrt[1 + 4*x + 2*x^2 + x^4]*EllipticPi[(2^(1/3)*(
4*2^(1/3)*(-3*I + Sqrt[3]) + (3*I + Sqrt[3])*(-13 + 3*Sqrt[33])^(
2/3)))/(4*2^(2/3)*(-I + Sqrt[3]) - (8*I)*(-13 + 3*Sqrt[33])^(1/3)
 + 2^(1/3)*(I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3)), ArcSin[Sqrt[1
3 - 3*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) + 4*(-26 + 6*Sq
rt[33])^(2/3) + (-39 + 9*Sqrt[33])*x]/(2^(1/6)*Sqrt[3]*(-13 + 3*S
qrt[33])^(2/3)*Sqrt[(-39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 9*Sq
rt[33] - (4*I)*(-3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))/(104 -
24*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 3*Sqrt[33]
)*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*(-26 + 6*Sqrt[3
```

```
3])^(2/3))]*Sqrt[1 + x])], (4*(21 - (7*I)*Sqrt[3] + (3*I)*Sqrt[11
] - 3*Sqrt[33]) + (3 + I*Sqrt[3] + (3*I)*Sqrt[11] + 3*Sqrt[33])*(
-26 + 6*Sqrt[33])^(1/3))/(4*(21 + (7*I)*Sqrt[3] - (3*I)*Sqrt[11]
- 3*Sqrt[33]) + (3 - I*Sqrt[3] - (3*I)*Sqrt[11] + 3*Sqrt[33])*(-2
6 + 6*Sqrt[33])^(1/3))])/(2^(1/6)*Sqrt[3]*(4*2^(2/3)*(I + Sqrt[3]
) + (2*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(-I + Sqrt[3])*(-13
+ 3*Sqrt[33])^(2/3) - (6*I)*(-13 + 3*Sqrt[33])^(1/3)*x)*(4*2^(2/3
)*(-I + Sqrt[3]) - (2*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(I +
Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3) + (6*I)*(-13 + 3*Sqrt[33])^(1/3
)*x)*Sqrt[13 - 3*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) + 4*
(-26 + 6*Sqrt[33])^(2/3) + (-39 + 9*Sqrt[33])*x]))
```

Rubi [A] time = 0.0487657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*Defer[Int][Sqrt[1 + 4*x + 2
*x^2 + x^4], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\left(-2\sqrt{2}+1\right)\int x\,dx-\sqrt{2}\int\sqrt{x^4+2x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)
```

```
[Out] -(-2*sqrt(2) + 1)*Integral(x, x) - sqrt(2)*Integral(sqrt(x**4 + 2
*x**2 + 4*x + 1), x)
```

Mathematica [A] time = 6.07292, size = 3168, normalized size = 66.

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x
^4])/3 - (2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0
])^2*(-(EllipticF[ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 +
#1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 +
#1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1^2 +
#1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3
 & , 3, 0]))]*Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) + EllipticPi
[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-Root[1 + 3*#1 - #1
^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]), Arc
Sin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root
[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 +
#1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]],
((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 +
#1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1
+ Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 +
#1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*(1 + Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]))*Sqrt[(x - Root[1 + 3*#1 - #
1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]
)*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*(-1 - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3
 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0])))])/(Sqrt[1 + 4*x + 2*x^2 + x^4]*(Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3
 & , 3, 0])) + (2*EllipticF[ArcSin[Sqrt[((1 + x)*(-Root[1 + 3*#1
- #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))
/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1
- #1^2 + #1^3 & , 3, 0]))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(-1 - Root[1 + 3*#1
- #1^2 + #1^3 & , 3, 0]))/((-1 - Root[1 + 3*#1 - #1^2 + #1^3 & ,
2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #
1^2 + #1^3 & , 3, 0]))]*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0
])^2*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^
3 & , 2, 0]))]*(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[
(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 -
 #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0
]))]*Sqrt[((1 + x)*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root
[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 +
#1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))])/(S
qrt[1 + 4*x + 2*x^2 + x^4]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0
] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])) + ((1 + x)*(x - Root[
1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(x - Root[1 + 3*#1 - #1^2 + #1^
3 & , 3, 0]) + (x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])^2*(1 +
 Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*Sqrt[(x - Root[1 + 3*#1 -
 #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*Sqrt[(x - Root[
1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1
```

```
^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[
-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]*(1 + Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])*((EllipticE[ArcSin[Sqrt[-(((1 +
x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2
+ #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 -
 #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 -
 #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] -
Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*(1 + Root[1 + 3*#1 - #1^
2 + #1^3 & , 2, 0]))/(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])
- (EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 &
, 3, 0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & ,
 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & ,
 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3
*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & ,
 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]))]*(1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
- #1^2 + #1^3 & , 2, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))
/(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 +
 #1^3 & , 3, 0]) + (EllipticF[ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*
#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0
]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3 &
, 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 &
, 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 -
 #1^2 + #1^3 & , 3, 0]))]*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]
+ Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]*(-Root[1 + 3*#1 - #1^2 +
#1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) - Root[1
+ 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^
3 & , 1, 0])*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0]))))/Sqrt[1 + 4*x + 2*x^2 + x^4]))/3
```

Maple [A] time = 0.504, size = 4640, normalized size = 96.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x)
```

```
[Out] 1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(1/3*x*(x^4+2*x^2+4*x+1)^(1/2)+4/3
*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*
3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(
(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*
(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/
```

```
6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3
*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1
/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*
(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6
*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33
^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3+1/2*I*3^(1/2)*(-1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/
2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6
*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))
^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33
^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)/(1/2*(26+6*33^(1/2))^
(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))
^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)+8/
3/(26+6*33^(1/2))^(1/3)+4/3)/((1+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-
8/3/(26+6*33^(1/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(
26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3
)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(2
6+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)
-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*EllipticF(((1/2*(26+6*33^(1/2
))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/
2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^
(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(
1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((-1/2*(26+6*33^(1/2))
^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2)
)^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+6*33^(1/2))^(1/
3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2))^(1/3)
+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26
+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/
(26+6*33^(1/2))^(1/3))))^(1/2))+4*(-4/3-1/6*(26+6*33^(1/2))^(1/3)
+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-4/(26
+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/
(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+
6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8
/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6
*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*
33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1
/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+
6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2)
)^(1/3)+4/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))
^(1/3)-1/3))^(1/2)*((-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2
))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1
/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1
/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)
+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-
```

1/3))^(1/2)/(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/
2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)/((1
+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)*(
x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3
^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x
-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1
/2)*((-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)*E
llipticF(((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*
I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))
(1+x)/(1/6(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1
/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3
)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^
(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*
3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(
-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4
/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1
/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2
*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+(-4/3
+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))*EllipticPi(
((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)
(-1/3(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1
/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1
/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),(1/
6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/2*(
26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*
(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))),((-1/2*(26+6*33
^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*3
3^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+6*33^(1/
2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2)
)^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1
/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3
)+4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/
3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+2/3*((1+x)*(x-1/6*(26+6*3
3^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*
(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/6*(26+6*33
^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))+(-4/3-1/6*(26+6*
33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26
+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/
2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1
/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))
^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33
^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(26+6*33^(1/2)
)^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))^(
1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))
^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3
/(26+6*33^(1/2))^(1/3)+4/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1

```
/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/
(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*33^(1/2))^(1/3)+8/
3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26
+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-
8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*
33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)-1/3))^(1/2)*((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33
^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6
*33^(1/2))^(1/3))+(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^
(1/3)+1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^
(1/2))^(1/3)))*(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1
/3)+1/3)+(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/
3)^2)/(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1
/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*EllipticF
(((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2
)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(
1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(
1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1
/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((
-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*
(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6
*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-
1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(
26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*3
3^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*
33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+(-4/3-1/6*(26
+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*
(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*EllipticE(((1/2
*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(2
6+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(
-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26
+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((-1/2*(
26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*
(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+6
*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*3
3^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+
6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2
))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/
2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))/(-1/3*(26+6*33^(1/2
))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)))/((1+x)*(x+1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2)
)^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*3
3^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/6*(26+6*33^(1/2))
^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33
^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(\left(2\sqrt{2}\right)-1\right)x^2-\sqrt{2}\int\sqrt{x^3-x^2+3x+1}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1),x, algorithm="ma
```

```
[Out] 1/2*((2*sqrt(2)) - 1)*x^2 - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*
x + 1)*sqrt(x + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(2\sqrt{2}x-\sqrt{2}\sqrt{x^4+2x^2+4x+1}-x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1),x, algorithm="fr
```

```
[Out] integral(2*sqrt(2)*x - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1) - x, x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\left(x\left(-1+2\sqrt{2}\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)
```

```
[Out] Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x +
1), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1),x, algorithm="gi

[Out] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1)
, x)
```

3.282 $\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\mathbf{mc}^8+4\mathbf{mc}^9+24\mathbf{mc}^6x-48\mathbf{mc}^7x-144\mathbf{mc}^5x^2-24\mathbf{mc}^2x^3+176\mathbf{mc}^3x^3+3x\right.\right.}{384x^2}$

Optimal. Leaf size=330

$$\frac{\pi^2(3-4\text{mc})\text{mc}^8\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\text{mc})\text{mc}^6\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\text{mc}^3y\left(-12\text{mc}^2+3\text{mc}-8y\right)\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{\pi^2(3-4\text{mc})\text{mc}^8e^{-\frac{x}{y}}}{384x}+\frac{3}{8}\pi^2\text{mc}^5ye^{-\frac{x}{y}}+\frac{1}{4}\pi^2\text{mc}^3y^2e^{-\frac{x}{y}}+\frac{1}{48}\pi^2(3-22\text{mc})\text{mc}^2y^2e^{-\frac{x}{y}}+\frac{1}{48}\pi^2(3-22\text{mc})\text{mc}^2xye^{-\frac{x}{y}}+\frac{1}{4}\pi^2\text{mc}^3y^2e^{-\frac{x}{y}}\log\left(\frac{x}{\text{mc}^2}\right)-\frac{1}{32}\pi^2\text{mc}^3y(3(1-4\text{mc})\text{mc}-8x)e^{-\frac{x}{y}}\log\left(\frac{x}{\text{mc}^2}\right)-\frac{1}{128}\pi^2(4\text{mc}+1)x^2ye^{-\frac{x}{y}}-\frac{1}{64}\pi^2(4\text{mc}+1)y^3e^{-\frac{x}{y}}-\frac{1}{64}\pi^2(4\text{mc}+1)xy^2e^{-\frac{x}{y}}$$

```
[Out] ((3 - 4*mc)*mc^8*Pi^2)/(384*E^(x/y)*x) + (3*mc^5*Pi^2*y)/(8*E^(x/
y)) + ((3 - 22*mc)*mc^2*Pi^2*x*y)/(48*E^(x/y)) - ((1 + 4*mc)*Pi^2
*x^2*y)/(128*E^(x/y)) + ((3 - 22*mc)*mc^2*Pi^2*y^2)/(48*E^(x/y))
+ (mc^3*Pi^2*y^2)/(4*E^(x/y)) - ((1 + 4*mc)*Pi^2*x*y^2)/(64*E^(x/
y)) - ((1 + 4*mc)*Pi^2*y^3)/(64*E^(x/y)) + ((1 - 2*mc)*mc^6*Pi^2*
ExpIntegralEi[-(x/y)])/16 + ((3 - 4*mc)*mc^8*Pi^2*ExpIntegralEi[-
(x/y)])/(384*y) + (mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpIntegral
Ei[-(x/y)])/32 - (mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2]
)/(32*E^(x/y)) + (mc^3*Pi^2*y^2*Log[x/mc^2])/(4*E^(x/y))
```

Rubi [A] time = 1.41925, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 107, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$

$$\frac{\pi^2(3-4\text{mc})\text{mc}^8\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\text{mc})\text{mc}^6\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\text{mc}^3y\left(-12\text{mc}^2+3\text{mc}-8y\right)\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{\pi^2(3-4\text{mc})\text{mc}^8e^{-\frac{x}{y}}}{384x}+\frac{3}{8}\pi^2\text{mc}^5ye^{-\frac{x}{y}}+\frac{1}{4}\pi^2\text{mc}^3y^2e^{-\frac{x}{y}}+\frac{1}{48}\pi^2(3-22\text{mc})\text{mc}^2y^2e^{-\frac{x}{y}}+\frac{1}{48}\pi^2(3-22\text{mc})\text{mc}^2xye^{-\frac{x}{y}}+\frac{1}{4}\pi^2\text{mc}^3y^2e^{-\frac{x}{y}}\log\left(\frac{x}{\text{mc}^2}\right)-\frac{1}{32}\pi^2\text{mc}^3y(3(1-4\text{mc})\text{mc}-8x)e^{-\frac{x}{y}}\log\left(\frac{x}{\text{mc}^2}\right)-\frac{1}{128}\pi^2(4\text{mc}+1)x^2ye^{-\frac{x}{y}}-\frac{1}{64}\pi^2(4\text{mc}+1)y^3e^{-\frac{x}{y}}-\frac{1}{64}\pi^2(4\text{mc}+1)xy^2e^{-\frac{x}{y}}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^2*x^3 +
```


```
[Out] ((3 - 4*mc)*mc^8*Pi^2)/(384*E^(x/y)*x) + (3*mc^5*Pi^2*y)/(8*E^(x/
y)) + ((3 - 22*mc)*mc^2*Pi^2*x*y)/(48*E^(x/y)) - ((1 + 4*mc)*Pi^2
```

```
*x^2*y)/(128*E^(x/y)) + ((3 - 22*mc)*mc^2*Pi^2*y^2)/(48*E^(x/y))
+ (mc^3*Pi^2*y^2)/(4*E^(x/y)) - ((1 + 4*mc)*Pi^2*x*y^2)/(64*E^(x/
y)) - ((1 + 4*mc)*Pi^2*y^3)/(64*E^(x/y)) + ((1 - 2*mc)*mc^6*Pi^2*
ExpIntegralEi[-(x/y)])/16 + ((3 - 4*mc)*mc^8*Pi^2*ExpIntegralEi[-
(x/y)])/(384*y) + (mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpIntegral
Ei[-(x/y)])/32 - (mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2]
)/(32*E^(x/y)) + (mc^3*Pi^2*y^2*Log[x/mc^2])/(4*E^(x/y))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-
144*mc**5*x**2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-
12*mc**2+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.219004, size = 181, normalized size = 0.55

$$\frac{1}{384}\pi^2\left(\frac{e^{-\frac{x}{y}}\left(-4\text{mc}^9+3\text{mc}^8+144\text{mc}^5xy-16\text{mc}^3xy(11x+5y)+24\text{mc}^2xy(x+y)+12\text{mc}^3xy\left(12\text{mc}^2-3\text{mc}+8(x+y)\right)\text{lo}\right.}{x}\right.$$
$$\left.-\frac{\text{mc}^3\left(4\text{mc}^6-3\text{mc}^5+48\text{mc}^4y-24\text{mc}^3y+144\text{mc}^2y^2-36\text{mc}y^2+96y^3\right)\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{y}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^
```

```
[Out] (Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*
y^2 + 144*mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)])/y) + (3*mc^8
- 4*mc^9 + 144*mc^5*x*y + 24*mc^2*x*y*(x + y) - 16*mc^3*x*y*(11*x
 + 5*y) - 3*x*y*(x^2 + 2*x*y + 2*y^2) - 12*mc*x*y*(x^2 + 2*x*y +
2*y^2) + 12*mc^3*x*y*(-3*mc + 12*mc^2 + 8*(x + y))*Log[x/mc^2])/(
E^(x/y)*x)))/384
```

Maple [C] time = 0.086, size = 1356, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-
24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/mc^2))/exp(x/
```

```
[Out] -3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*x)*csgn(I/mc^2*x)^2+3/16*I*y
*Pi^3*exp(-x/y)*mc^5*csgn(I*x)*csgn(I/mc^2*x)^2-3/64*I*y*Pi^3*exp
(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)^2-3/64*I*y*Pi^3*exp(-x/y)
*mc^4*csgn(I*mc)^2*csgn(I*mc^2)+3/32*I*y*Pi^3*exp(-x/y)*mc^4*csgn
(I*mc)*csgn(I*mc^2)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*c
sgn(I/mc^2*x)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*
mc^2)-3/8*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)*csgn(I*mc^2)^2+1/8*I
*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)^2*exp(-x/y)+1/8*I*y^2*
Pi^3*mc^3*csgn(I*x)*csgn(I/mc^2*x)^2*exp(-x/y)-1/8*I*y*Pi^3*mc^3*
csgn(I/mc^2*x)^3*x*exp(-x/y)-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc)*csgn(
I*mc^2)^2*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)
*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*mc^2)^3*x*exp(-x/y)-1/128*y*P
i^2*exp(-x/y)*x^2-1/64*y^2*Pi^2*x*exp(-x/y)-1/16*y^3*Pi^2*mc*exp(
-x/y)+1/16*y^2*Pi^2*mc^2*exp(-x/y)+3/8*y*Pi^2*exp(-x/y)*mc^5-1/12
8/y*Pi^2*mc^8*Ei(1,x/y)+1/96/y*Pi^2*mc^9*Ei(1,x/y)+1/128*Pi^2*mc^
8/x*exp(-x/y)-1/96*Pi^2*mc^9/x*exp(-x/y)-1/4*I*y*Pi^3*mc^3*csgn(I
*mc)*csgn(I*mc^2)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*cs
gn(I*mc^2)*x*exp(-x/y)+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*
csgn(I*x)*csgn(I/mc^2*x)-3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2
)*csgn(I*x)*csgn(I/mc^2*x)+1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/
mc^2*x)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*x)*csgn(I/mc^2*x)^
2*x*exp(-x/y)-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I*x)*csgn(I/m
c^2*x)*exp(-x/y)-5/24*mc^3*Pi^2*y^2*exp(-x/y)-1/16*Pi^2*mc^6*Ei(1
,x/y)+1/8*Pi^2*mc^7*Ei(1,x/y)-1/64*y^3*Pi^2*exp(-x/y)-1/2*y*Pi^2*
ln(mc)*mc^3*x*exp(-x/y)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc^2)
^3-3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2*x)^3-3/64*I*y*Pi^3*ex
p(-x/y)*mc^4*csgn(I*mc^2)^3+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/m
c^2*x)^3+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc^2)^3*exp(-x/y)-1/8*I*y^2*P
i^3*mc^3*csgn(I/mc^2*x)^3*exp(-x/y)+1/4*Pi^2*mc^3*y^2*Ei(1,x/y)-3
/32*Pi^2*mc^4*y*Ei(1,x/y)+3/8*Pi^2*mc^5*y*Ei(1,x/y)+1/384*(144*Pi
^2*mc^5*y-36*Pi^2*mc^4*y+96*Pi^2*mc^3*x*y+96*Pi^2*mc^3*y^2)*exp(-
x/y)*ln(x)-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x
)*x*exp(-x/y)-1/2*y^2*Pi^2*ln(mc)*mc^3*exp(-x/y)-1/32*y*Pi^2*mc*e
xp(-x/y)*x^2-1/16*y^2*Pi^2*mc*x*exp(-x/y)+1/16*y*Pi^2*mc^2*x*exp(
-x/y)-11/24*y*Pi^2*mc^3*x*exp(-x/y)-3/4*y*Pi^2*exp(-x/y)*ln(mc)*m
c^5+3/16*y*Pi^2*exp(-x/y)*ln(mc)*mc^4
```

Maxima [A] time = 1.59409, size = 435, normalized size = 1.32

$$-\frac{\pi^2 mc^9 \left(-1, \frac{x}{y}\right)}{96\,y} - \frac{1}{8}\,\pi^2 mc^7 \mathrm{Ei}\left(-\frac{x}{y}\right) + \frac{\pi^2 mc^8 \left(-1, \frac{x}{y}\right)}{128\,y} + \frac{3}{8}\,\pi^2 mc^5 y e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) + \frac{1}{16}\,\pi^2 mc^6 \mathrm{Ei}\left(-\frac{x}{y}\right) - \frac{3}{8}\,\pi^2 mc^5 y \mathrm{Ei}\left(-\frac{x}{y}\right) + \frac{3}{8}\,\pi^2 mc^5 y e^{\left(-\frac{x}{y}\right)} - \frac{3}{32}\,\pi^2 mc^4 y e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) + \frac{3}{32}\,\pi^2 mc^4 y \mathrm{Ei}\left(-\frac{x}{y}\right) + \frac{1}{4}\,\pi^2 \left(xy + y^2\right) mc^3 e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) - \frac{11}{24}\,\pi^2 \left(xy + y^2\right) mc^3 e^{\left(-\frac{x}{y}\right)} - \frac{1}{4}\,\pi^2 \left(y^2 \mathrm{Ei}\left(-\frac{x}{y}\right) - y^2 e^{\left(-\frac{x}{y}\right)}\right) mc^3 + \frac{1}{16}\,\pi^2 \left(xy + y^2\right) mc^2 e^{\left(-\frac{x}{y}\right)} - \frac{1}{32}\,\pi^2 \left(x^2 y + 2\,xy^2 + 2\,y^3\right) mce^{\left(-\frac{x}{y}\right)} - \frac{1}{128}\,\pi^2 \left(x^2 y + 2\,xy^2 + 2\,y^3\right) e^{\left(-\frac{x}{y}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/384*(12*pi^2*(12*mc^2 - 3*mc + 8*x)*mc^3*x^2*log(x/mc^2) - pi^2*(4*
x/y)/x^2,x, algorithm="maxima")
```

```
[Out] -1/96*pi^2*mc^9*gamma(-1, x/y)/y - 1/8*pi^2*mc^7*Ei(-x/y) + 1/128
*pi^2*mc^8*gamma(-1, x/y)/y + 3/8*pi^2*mc^5*y*e^(-x/y)*log(x/mc^2
) + 1/16*pi^2*mc^6*Ei(-x/y) - 3/8*pi^2*mc^5*y*Ei(-x/y) + 3/8*pi^2
*mc^5*y*e^(-x/y) - 3/32*pi^2*mc^4*y*e^(-x/y)*log(x/mc^2) + 3/32*p
i^2*mc^4*y*Ei(-x/y) + 1/4*pi^2*(x*y + y^2)*mc^3*e^(-x/y)*log(x/mc
^2) - 11/24*pi^2*(x*y + y^2)*mc^3*e^(-x/y) - 1/4*pi^2*(y^2*Ei(-x/
y) - y^2*e^(-x/y))*mc^3 + 1/16*pi^2*(x*y + y^2)*mc^2*e^(-x/y) - 1
/32*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^(-x/y) - 1/128*pi^2*(x^2*
y + 2*x*y^2 + 2*y^3)*e^(-x/y)
```

Fricas [A] time = 0.225735, size = 363, normalized size = 1.1

$$12\left(8\,\pi^2 mc^3 xy^3 + \left(8\,\pi^2 mc^3 x^2 + 3\,\pi^2\left(4\,mc^5 - mc^4\right)x\right)y^2\right)e^{\left(-\frac{x}{y}\right)}\log\left(\frac{x}{mc^2}\right) - \left(96\,\pi^2 mc^3 xy^3 + 36\,\pi^2\left(4\,mc^5 - mc^4\right)xy^2 + 24\,\pi$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/384*(12*pi^2*(12*mc^2 - 3*mc + 8*x)*mc^3*x^2*log(x/mc^2) - pi^2*(4*
x/y)/x^2,x, algorithm="fricas")
```

```
[Out] 1/384*(12*(8*pi^2*mc^3*x*y^3 + (8*pi^2*mc^3*x^2 + 3*pi^2*(4*mc^5
- mc^4)*x)*y^2)*e^(-x/y)*log(x/mc^2) - (96*pi^2*mc^3*x*y^3 + 36*p
i^2*(4*mc^5 - mc^4)*x*y^2 + 24*pi^2*(2*mc^7 - mc^6)*x*y + pi^2*(4
*mc^9 - 3*mc^8)*x)*Ei(-x/y) - (6*pi^2*(4*mc + 1)*x*y^4 + pi^2*(4*
mc^9 - 3*mc^8)*y + 2*(3*pi^2*(4*mc + 1)*x^2 + 4*pi^2*(10*mc^3 - 3
*mc^2)*x)*y^3 - (144*pi^2*mc^5*x - 3*pi^2*(4*mc + 1)*x^3 - 8*pi^2
*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^(-x/y))/(x*y)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\pi^2\left(\int\left(-144mc^5e^{-\frac{x}{y}}\right)\,dx+\int 3x^2e^{-\frac{x}{y}}\,dx+\int 12mcx^2e^{-\frac{x}{y}}\,dx+\int\left(-24mc^2xe^{-\frac{x}{y}}\right)\,dx+\int 176mc^3xe^{-\frac{x}{y}}\,dx+\int 36mc^4e^{-\frac{x}{y}}\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x**2+1
24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-12*mc**2+3*mc-8*x)*x**2*ln(x/mc**2
```

```
[Out] pi**2*(Integral(-144*mc**5*exp(-x/y), x) + Integral(3*x**2*exp(-x
/y), x) + Integral(12*mc*x**2*exp(-x/y), x) + Integral(-24*mc**2*
x*exp(-x/y), x) + Integral(176*mc**3*x*exp(-x/y), x) + Integral(3
6*mc**4*exp(-x/y)*log(x/mc**2), x) + Integral(-144*mc**5*exp(-x/y
)*log(x/mc**2), x) + Integral(24*mc**6*exp(-x/y)/x, x) + Integral
(-48*mc**7*exp(-x/y)/x, x) + Integral(-3*mc**8*exp(-x/y)/x**2, x)
 + Integral(4*mc**9*exp(-x/y)/x**2, x) + Integral(-96*mc**3*x*exp
(-x/y)*log(x/mc**2), x))/384
```

GIAC/XCAS [A] time = 0.215655, size = 637, normalized size = 1.93

$$-\frac{4\,\pi^2mc^9x\mathrm{Ei}\left(-\frac{x}{y}\right)+4\,\pi^2mc^9ye^{\left(-\frac{x}{y}\right)}-3\,\pi^2mc^8x\mathrm{Ei}\left(-\frac{x}{y}\right)+48\,\pi^2mc^7xy\mathrm{Ei}\left(-\frac{x}{y}\right)-3\,\pi^2mc^8ye^{\left(-\frac{x}{y}\right)}-144\,\pi^2mc^5xy^2e^{\left(-\frac{x}{y}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/384*(12*pi^2*(12*mc^2 - 3*mc + 8*x)*mc^3*x^2*log(x/mc^2) - pi^2*(4*
x/y)/x^2,x, algorithm="giac")
```

```
[Out] -1/384*(4*pi^2*mc^9*x*Ei(-x/y) + 4*pi^2*mc^9*y*e^(-x/y) - 3*pi^2*
mc^8*x*Ei(-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) - 3*pi^2*mc^8*y*e^(-x
/y) - 144*pi^2*mc^5*x*y^2*e^(-x/y)*ln(x/mc^2) - 24*pi^2*mc^6*x*y*
Ei(-x/y) + 144*pi^2*mc^5*x*y^2*Ei(-x/y) - 144*pi^2*mc^5*x*y^2*e^(
-x/y) + 36*pi^2*mc^4*x*y^2*e^(-x/y)*ln(x/mc^2) - 96*pi^2*mc^3*x^2
*y^2*e^(-x/y)*ln(x/mc^2) - 96*pi^2*mc^3*x*y^3*e^(-x/y)*ln(x/mc^2)
 - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x*y^3*Ei(-x/y) + 17
6*pi^2*mc^3*x^2*y^2*e^(-x/y) + 80*pi^2*mc^3*x*y^3*e^(-x/y) - 24*p
i^2*mc^2*x^2*y^2*e^(-x/y) + 12*pi^2*mc*x^3*y^2*e^(-x/y) - 24*pi^2
*mc^2*x*y^3*e^(-x/y) + 24*pi^2*mc*x^2*y^3*e^(-x/y) + 24*pi^2*mc*x
*y^4*e^(-x/y) + 3*pi^2*x^3*y^2*e^(-x/y) + 6*pi^2*x^2*y^3*e^(-x/y)
 + 6*pi^2*x*y^4*e^(-x/y))/(x*y)
```

3.283 $\int \sec(x)\sin(2x)\,dx$

Optimal. Leaf size=4

$$-2\cos(x)$$

```
[Out] -2*Cos[x]
```

Rubi [A] time = 0.0172042, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*Sin[2*x],x]

[Out] -2*Cos[x]
```

Rubi in Sympy [A] time = 1.36283, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)/cos(x),x)

[Out] -2*cos(x)
```

Mathematica [A] time = 0.00205237, size = 4, normalized size = 1.

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*Sin[2*x],x]

[Out] -2*Cos[x]
```

Maple [A] time = 0.007, size = 5, normalized size = 1.3

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

Maxima [A] time = 1.37056, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/cos(x),x, algorithm="maxima")
```

```
[Out] -2*cos(x)
```

Fricas [A] time = 0.226277, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/cos(x),x, algorithm="fricas")
```

```
[Out] -2*cos(x)
```

Sympy [A] time = 0.622873, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

GIAC/XCAS [A] time = 0.202089, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/cos(x),x, algorithm="giac")
```

```
[Out] -2*cos(x)
```

3.284 $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$

Optimal. Leaf size=71

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(-x^7+\sqrt{2}x^2+\sqrt{2}x+x+1\right)-\left(\sqrt{2}-1\right)\log\left(x^7+\sqrt{2}x^2+\left(\sqrt{2}-1\right)x-1\right)\right)$$

```
[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 +
 Sqrt[2])*Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

Rubi [F] time = 1.21349, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x - x^2 - 4*x^
```

```
[Out] Log[1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14]/2 + 2*D
efer[Int][(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14)^
(-1), x] + 4*Defer[Int][x/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7
- 2*x^8 + x^14), x] + 2*Defer[Int][x^2/(1 + 2*x - x^2 - 4*x^3 - 2
*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 12*Defer[Int][x^7/(1 + 2*x - x
^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 10*Defer[Int][x^
8/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-
2*x**8-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0609612, size = 71, normalized size = 1.

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(-x^7+\sqrt{2}x^2+\sqrt{2}x+x+1\right)-\left(\sqrt{2}-1\right)\log\left(x^7+\sqrt{2}x^2+\left(\sqrt{2}-1\right)x-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x - x^2
```

```
[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 +
 Sqrt[2])*Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

Maple [A] time = 0.017, size = 102, normalized size = 1.4

$$\frac{\ln\left(x^7 - x^2\sqrt{2} + \left(-\sqrt{2}-1\right)x - 1\right)}{2} + \frac{\ln\left(x^7 - x^2\sqrt{2} + \left(-\sqrt{2}-1\right)x - 1\right)\sqrt{2}}{2} + \frac{\ln\left(-1 + x^7 + x\left(\sqrt{2}-1\right) + x^2\sqrt{2}\right)}{2} - \frac{\ln\left(-1 + x^7 + x\left(\sqrt{2}-1\right) + x^2\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-
2*x^4-4*x^3-x^2+2*x+1),x)
```

```
[Out] 1/2*ln(x^7-x^2*2^(1/2)+(-2^(1/2)-1)*x-1)+1/2*ln(x^7-x^2*2^(1/2)+(
-2^(1/2)-1)*x-1)*2^(1/2)+1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))
-1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{7\,x^{13} + 10\,x^8 + 4\,x^7 - 7\,x^6 - 4\,x^3 - 4\,x^2 + 3\,x + 3}{x^{14} - 2\,x^8 - 2\,x^7 - 2\,x^4 - 4\,x^3 - x^2 + 2\,x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14 - 2*x^8
```

```
[Out] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x
+ 3)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)
```

Fricas [A] time = 0.203009, size = 185, normalized size = 2.61

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^{14} - 2\,x^8 - 2\,x^7 + 2\,x^4 + 4\,x^3 + 3\,x^2 - 2\,\sqrt{2}\left(x^9 + x^8 - x^3 - 2\,x^2 - x\right) + 2\,x + 1}{x^{14} - 2\,x^8 - 2\,x^7 - 2\,x^4 - 4\,x^3 - x^2 + 2\,x + 1}\right) + \frac{1}{2}\log\left(x^{14} - 2\,x^8 - 2\,x^7 - 2\,x^4 - 4\,x^3 - x^2 + 2\,x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14 - 2*x^8
```

```
[Out] 1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2
*sqrt(2)*(x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 -
 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 -
 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1)
```

Sympy [A] time = 0.31017, size = 76, normalized size = 1.07

$$\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^{7}-\sqrt{2}x^{2}-2x\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)-1\right)+\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^{7}+\sqrt{2}x^{2}-2x\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-
2*x**8-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)
```

```
[Out] (1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2)
 - 1) + (-sqrt(2)/2 + 1/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(-sqrt(2
)/2 + 1/2) - 1)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{7\,x^{13}+10\,x^{8}+4\,x^{7}-7\,x^{6}-4\,x^{3}-4\,x^{2}+3\,x+3}{x^{14}-2\,x^{8}-2\,x^{7}-2\,x^{4}-4\,x^{3}-x^{2}+2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14 - 2*x^8
```

```
[Out] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x
+ 3)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```