Computer algebra independent integration tests
0_Independent_test_suites/Timofeev_Problems

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

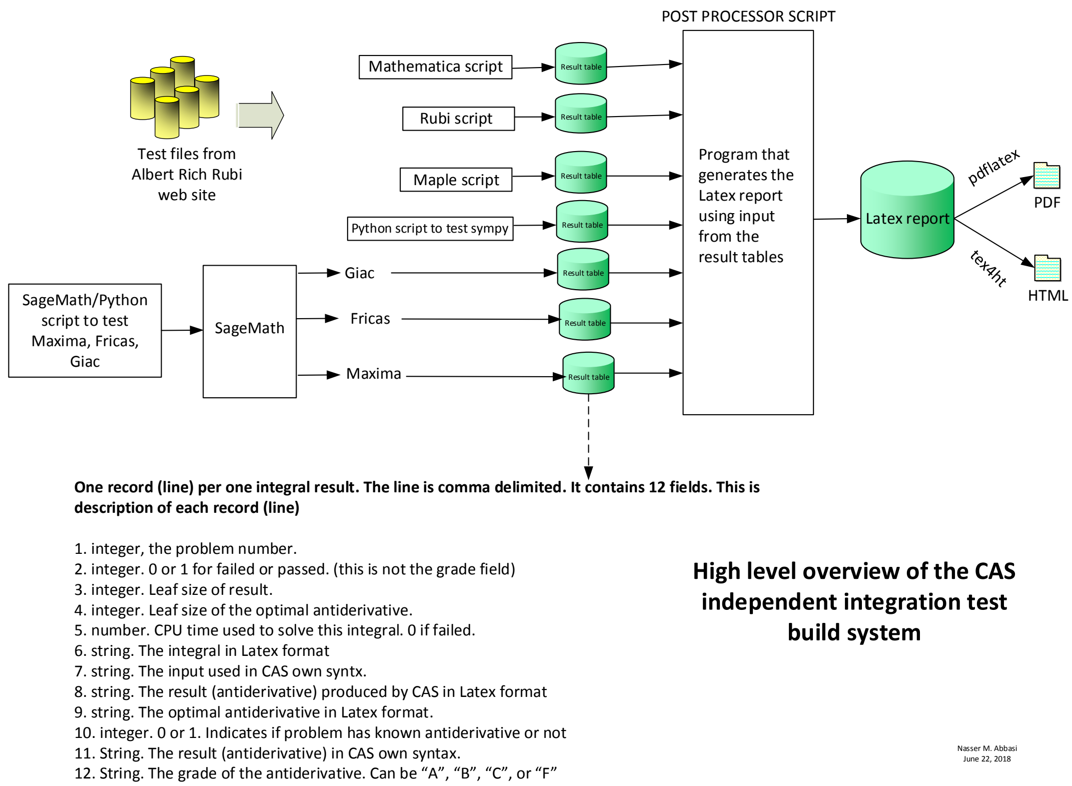
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python’s `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (705)	% 0. (0)
Rubi in Sympy	% 74.33 (524)	% 25.67 (181)
Mathematica	% 99.29 (700)	% 0.71 (5)
Maple	% 91.63 (646)	% 8.37 (59)
Maxima	% 80. (564)	% 20. (141)
Fricas	% 91.77 (647)	% 8.23 (58)
Sympy	% 57.87 (408)	% 42.13 (297)
Giac	% 80. (564)	% 20. (141)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

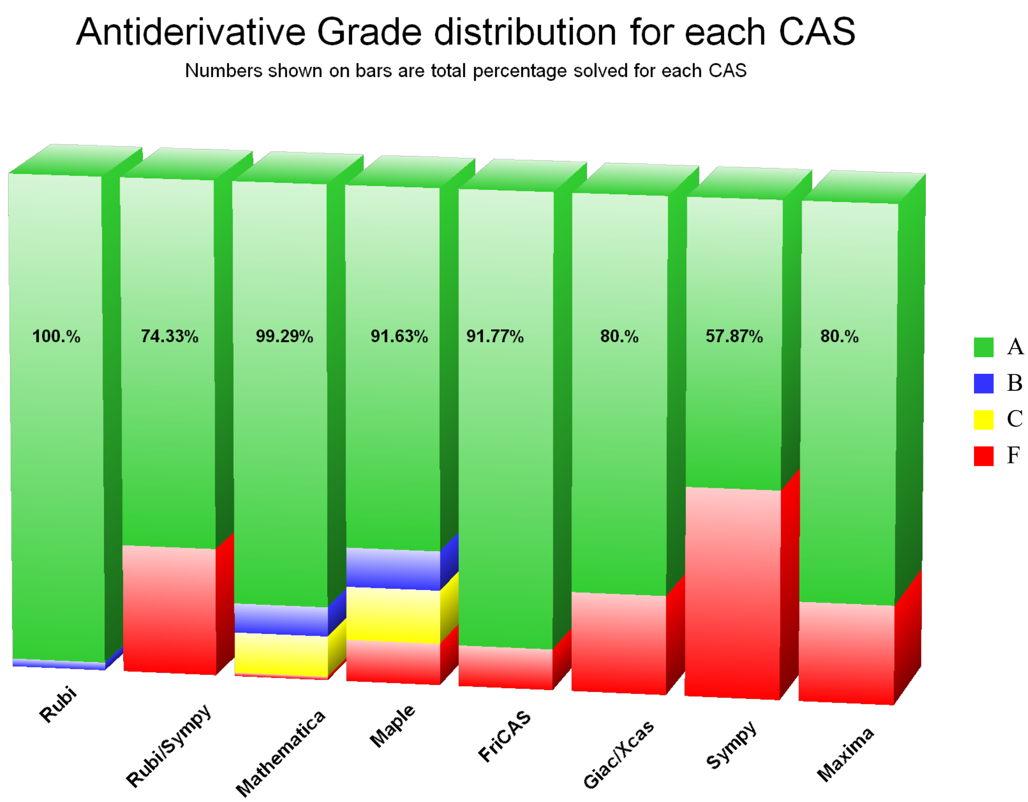
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

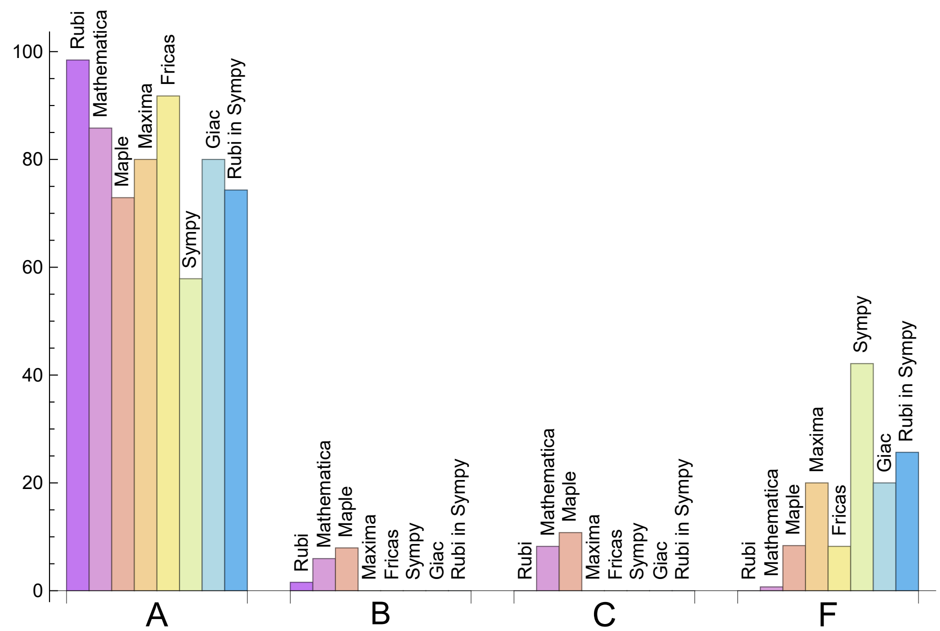
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.44	1.56	0.	0.
Rubi in Sympy	74.33	0.	0.	25.67
Mathematica	85.82	5.96	8.23	0.71
Maple	72.91	7.94	10.78	8.37
Maxima	80.	0.	0.	20.
Fricas	91.77	0.	0.	8.23
Sympy	57.87	0.	0.	42.13
Giac	80.	0.	0.	20.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	51.65	1.05	38.	1.
Rubi in Sympy	7.23	40.52	0.94	32.	0.88
Mathematica	0.41	91.92	1.67	37.	1.
Maple	0.07	638.2	8.23	34.5	0.94
Maxima	1.6	92.89	2.53	46.5	1.23
Fricas	0.45	103.2	2.46	54.	1.5
Sympy	3.73	103.79	2.15	34.	1.
Giac	0.28	66.52	1.76	45.	1.22

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {7, 8, 24, 43, 47, 60, 62, 71, 73, 82, 87, 88, 90, 91, 95, 97, 102, 103, 104, 106, 107, 110, 137, 138, 139, 142, 143, 144, 177, 180, 185, 188, 189, 190, 191, 192, 209, 211, 212, 213, 221, 222, 223, 224, 225, 227, 259, 289, 290, 291, 313, 346, 355, 356, 362, 363, 364, 366, 369, 370, 372, 373, 375, 379, 380, 383, 385, 389, 396, 405, 411, 416, 417, 418, 423, 424, 425, 426, 427, 428, 429, 432, 433, 436, 437, 438, 439, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 479, 480, 487, 488, 490, 492, 493, 495, 499, 503, 504, 505, 508, 509, 510, 513, 514, 515, 518, 519, 520, 526, 542, 550, 552, 553, 554, 555, 565, 566, 567, 568, 569, 570, 571, 574, 580, 581, 582, 583, 584, 586, 587, 588, 589, 590, 591, 592, 595, 596, 597, 598, 599, 611, 612, 614, 629, 630, 631, 632, 634, 635, 636, 639, 643, 644, 649, 650, 651, 652, 654, 657, 659, 660, 665, 668, 683, 684, 689, 699, 703}

Not solved by Mathematica {319, 324, 506, 511, 540}

Not solved by Maple {67, 126, 133, 145, 193, 198, 221, 222, 226, 228, 232, 308, 309, 314, 315, 317, 319, 328, 329, 352, 396, 414, 415, 418, 442, 443, 444, 445, 446, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 632, 699, 703, 705}

Not solved by Maxima {48, 51, 52, 53, 68, 89, 126, 133, 135, 145, 149, 174, 193, 194, 195, 196, 197, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 261, 262, 263, 279, 281, 283, 287, 288, 289, 290, 291, 306, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 395, 402, 403, 406, 407, 411, 427, 432, 434, 435, 436, 438, 440, 441, 443, 446, 447, 452, 455, 457, 463, 473, 487, 494, 495, 500, 506, 511, 516, 521, 523, 527, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 583, 586, 587, 592, 603, 608, 609, 618, 619, 621, 625, 626, 627, 637, 650, 657, 665, 672, 674, 675, 683, 684, 689, 693, 703}

Not solved by Fricas {126, 133, 136, 137, 138, 139, 142, 143, 144, 145, 177, 193, 198, 248, 303, 306, 329, 352, 401, 417, 442, 443, 444, 445, 446, 449, 455, 474, 500, 506, 511, 516, 521, 529, 533, 543, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 615, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698}

Not solved by Sympy {54, 55, 56, 57, 58, 59, 61, 64, 66, 69, 81, 86, 89, 193, 198, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 359, 360, 362, 363, 366, 369, 371, 373, 374, 375, 380, 381, 384, 386, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 469, 473, 474, 477, 478, 481, 482, 490, 491, 492, 495, 500, 505, 506, 510, 511, 516, 521, 528, 529, 532, 533, 534, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 574, 575, 579, 581, 582, 586, 587, 588, 592, 593, 594, 595, 600, 601, 605, 606, 607, 608, 609, 615, 622, 623, 624, 625, 626, 627, 632, 633, 636, 639, 640, 642, 643, 645, 649, 650, 656, 657, 658, 663, 664, 665, 666, 672, 673, 674, 675, 678, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 704, 705}

Not solved by Giac {64, 86, 126, 133, 145, 154, 177, 193, 198, 221, 222, 223, 224, 225, 226, 228, 232, 233, 234, 245, 246, 247, 248, 249, 291, 296, 306, 307, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 352, 390, 391, 392, 393, 395, 396, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 427, 428, 432, 434, 435, 436, 437, 439, 442, 444, 445, 446, 448, 453, 455, 457, 473, 490, 491, 500, 506, 511, 516, 521, 529, 533, 540, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 592, 594, 595, 608, 615, 616, 623, 632, 648, 650, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 695, 698, 699, 703}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {177, 222, 417, 686, 687, 688, 689, 691, 705}

Mathematica {113, 137, 138, 143, 144, 193, 198, 228, 264, 314, 315, 327, 417, 438, 446, 447, 449, 592, 657, 665, 672, 674, 675, 683, 689, 698, 705}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	32	42	34	20	45	8
normalized size	1	1.	1.	2.29	3.	2.43	1.43	3.21	0.57
time (sec)	N/A	0.02	0.005	0.009	1.348	0.227	0.159	0.216	2.832

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	19	26	19	8
normalized size	1	1.	1.	1.07	1.36	1.36	1.86	1.36	0.57
time (sec)	N/A	0.013	0.005	0.006	1.532	0.215	0.145	0.215	1.965

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	37	18	23	35	29	54	10
normalized size	1	1.	2.85	1.38	1.77	2.69	2.23	4.15	0.77
time (sec)	N/A	0.01	0.013	0.006	1.378	0.246	0.109	0.219	0.58

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	15	26	31	22	31	10
normalized size	1	1.	2.09	1.36	2.36	2.82	2.	2.82	0.91
time (sec)	N/A	0.008	0.005	0.01	1.371	0.242	0.1	0.209	0.66

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	55	21	36	39	22	39	12
normalized size	1	1.	3.67	1.4	2.4	2.6	1.47	2.6	0.8
time (sec)	N/A	0.009	0.013	0.01	1.343	0.224	0.251	0.203	0.704

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	5	5	3	5	3
normalized size	1	1.	1.	1.5	2.5	2.5	1.5	2.5	1.5
time (sec)	N/A	0.012	0.002	0.	1.341	0.216	0.048	0.199	0.768

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	8	5	8	0
normalized size	1	1.	1.	1.25	2.	2.	1.25	2.	0.
time (sec)	N/A	0.013	0.003	0.	1.381	0.211	0.053	0.198	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	36	5	7	5	0
normalized size	1	1.	1.	0.83	6.	0.83	1.17	0.83	0.
time (sec)	N/A	0.036	0.01	0.019	1.363	0.209	0.765	0.205	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	12	3	41	7
normalized size	1	1.	0.67	0.56	1.33	1.33	0.33	4.56	0.78
time (sec)	N/A	0.018	0.006	0.002	1.346	0.218	0.21	0.206	0.487

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	14	7	11	8
normalized size	1	1.	0.67	0.75	1.17	1.17	0.58	0.92	0.67
time (sec)	N/A	0.017	0.008	0.	1.345	0.233	0.667	0.21	0.505

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	18	16	15	19	8
normalized size	1	1.	1.	1.08	1.5	1.33	1.25	1.58	0.67
time (sec)	N/A	0.039	0.007	0.013	1.34	0.276	0.701	0.2	2.284

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	20	31	20	10
normalized size	1	1.	1.	1.07	1.33	1.33	2.07	1.33	0.67
time (sec)	N/A	0.047	0.014	0.017	1.496	0.285	1.268	0.202	4.631

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	45	35	44	47	10
normalized size	1	1.	1.	2.27	3.	2.33	2.93	3.13	0.67
time (sec)	N/A	0.051	0.014	0.017	1.386	0.25	1.265	0.202	5.701

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	23	18	23	28	32	104	15
normalized size	1	1.	1.35	1.06	1.35	1.65	1.88	6.12	0.88
time (sec)	N/A	0.063	0.022	0.031	1.352	0.26	3.992	0.238	3.764

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	20	27	31	34	104	17
normalized size	1	1.	1.32	1.05	1.42	1.63	1.79	5.47	0.89
time (sec)	N/A	0.066	0.022	0.032	1.429	0.247	3.742	0.207	4.009

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	22	23	19	24	24	34	130	17
normalized size	1	1.22	1.28	1.06	1.33	1.33	1.89	7.22	0.94
time (sec)	N/A	0.068	0.024	0.024	1.349	0.245	4.027	0.214	3.847

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	22	25	19	26	26	32	165	15
normalized size	1	1.22	1.39	1.06	1.44	1.44	1.78	9.17	0.83
time (sec)	N/A	0.069	0.025	0.028	1.403	0.246	3.747	0.218	4.008

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	19	14	18	42	61	62	19
normalized size	1	1.	0.46	0.34	0.44	1.02	1.49	1.51	0.46
time (sec)	N/A	0.038	0.017	0.023	1.509	0.247	1.346	0.205	0.631

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	6	20	20	15	22	5
normalized size	1	1.	3.83	1.	3.33	3.33	2.5	3.67	0.83
time (sec)	N/A	0.031	0.006	0.	1.339	0.213	0.095	0.199	3.017

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	3	5	3
normalized size	1	1.	1.	1.33	1.33	1.33	1.	1.67	1.
time (sec)	N/A	0.02	0.001	0.	1.355	0.207	0.079	0.204	1.15

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	15	4	3
normalized size	1	1.	1.	1.33	1.33	1.33	5.	1.33	1.
time (sec)	N/A	0.032	0.006	0.001	1.529	0.245	0.14	0.201	3.318

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	7	10	9	9	7	30	7
normalized size	1	1.	0.78	1.11	1.	1.	0.78	3.33	0.78
time (sec)	N/A	0.032	0.003	0.003	1.359	0.219	0.088	0.204	2.325

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	12	7	41	7
normalized size	1	1.	1.	1.11	1.33	1.33	0.78	4.56	0.78
time (sec)	N/A	0.029	0.004	0.002	1.358	0.216	0.099	0.214	2.089

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	30	32	22	31	0
normalized size	1	1.	1.	0.88	1.2	1.28	0.88	1.24	0.
time (sec)	N/A	0.043	0.019	0.01	1.383	0.205	1.017	0.202	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	27	30	27	28	31
normalized size	1	1.	1.	0.7	0.9	1.	0.9	0.93	1.03
time (sec)	N/A	0.153	0.011	0.007	1.36	0.202	0.828	0.205	7.946

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	14	8	15	8
normalized size	1	1.	1.	1.1	1.4	1.4	0.8	1.5	0.8
time (sec)	N/A	0.015	0.003	0.003	1.337	0.212	0.059	0.203	1.657

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	24	49	63	42	50	42
normalized size	1	1.	1.	0.51	1.04	1.34	0.89	1.06	0.89
time (sec)	N/A	0.054	0.048	0.004	1.521	0.203	0.11	0.21	1.846

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	31	36	27	28	27
normalized size	1	1.	1.	0.8	1.03	1.2	0.9	0.93	0.9
time (sec)	N/A	0.024	0.021	0.004	1.509	0.197	0.106	0.201	1.801

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	18	32	22	18	19
normalized size	1	1.	1.	0.67	0.86	1.52	1.05	0.86	0.9
time (sec)	N/A	0.017	0.01	0.042	1.33	0.221	0.752	0.21	1.02

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	32	26	15	10
normalized size	1	1.	1.	0.8	1.	2.13	1.73	1.	0.67
time (sec)	N/A	0.016	0.011	0.	1.333	0.219	0.722	0.203	1.06

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	96	251	120	138	24	19
normalized size	1	1.	1.	4.57	11.95	5.71	6.57	1.14	0.9
time (sec)	N/A	0.06	0.115	0.132	1.501	0.238	1.858	0.217	5.88

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.011	0.003	0.002	1.381	0.217	0.038	0.203	0.502

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.012	0.003	0.	1.346	0.246	0.036	0.204	0.49

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.028	0.002	0.004	1.355	0.213	0.038	0.201	1.122

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	57	32	19	30	14	19	12
normalized size	1	1.	5.18	2.91	1.73	2.73	1.27	1.73	1.09
time (sec)	N/A	0.03	0.008	0.012	1.337	0.21	0.084	0.211	2.414

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	12	24	12	12	15
normalized size	1	1.	0.86	2.14	1.71	3.43	1.71	1.71	2.14
time (sec)	N/A	0.045	0.005	0.	1.36	0.225	0.055	0.202	1.678

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	16	31	19	24	12
normalized size	1	1.	1.	1.	1.14	2.21	1.36	1.71	0.86
time (sec)	N/A	0.012	0.01	0.004	1.505	0.849	0.051	0.216	0.516

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	16	27	17	30	12
normalized size	1	1.	1.	1.19	1.	1.69	1.06	1.88	0.75
time (sec)	N/A	0.024	0.015	0.004	1.497	0.255	0.178	0.213	0.551

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	26	10	16	14
normalized size	1	1.	1.	1.1	1.6	2.6	1.	1.6	1.4
time (sec)	N/A	0.073	0.024	0.02	1.489	0.244	0.574	0.211	68.462

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	15	19	34	10	19	12
normalized size	1	1.	0.86	1.07	1.36	2.43	0.71	1.36	0.86
time (sec)	N/A	0.106	0.011	0.021	1.492	0.226	0.573	0.208	4.055

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	13	38	32	34	16	8
normalized size	1	1.	2.27	1.18	3.45	2.91	3.09	1.45	0.73
time (sec)	N/A	0.036	0.023	0.019	1.504	0.231	0.999	0.212	1.521

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	13	31	19	8	16	10
normalized size	1	1.	1.31	0.81	1.94	1.19	0.5	1.	0.62
time (sec)	N/A	0.041	0.025	0.019	1.498	0.242	0.898	0.204	1.746

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	24	30	19	24	0
normalized size	1	1.	1.	1.16	0.96	1.2	0.76	0.96	0.
time (sec)	N/A	0.046	0.015	0.008	1.339	0.224	0.102	0.208	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	18	18	12	20	12
normalized size	1	1.	1.	0.67	0.86	0.86	0.57	0.95	0.57
time (sec)	N/A	0.011	0.004	0.007	1.335	0.208	0.087	0.203	0.74

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.034	0.008	0.004	1.478	0.207	0.118	0.209	2.501

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	29	47	68	42	59	46
normalized size	1	1.	0.92	0.57	0.92	1.33	0.82	1.16	0.9
time (sec)	N/A	0.046	0.04	0.004	1.486	0.208	0.106	0.204	2.042

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	47	34	34	29	34	0
normalized size	1	1.	1.	1.42	1.03	1.03	0.88	1.03	0.
time (sec)	N/A	0.027	0.012	0.007	1.328	0.212	0.132	0.211	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	0	1	27	24	27
normalized size	1	1.	1.	0.66	0.	0.03	0.93	0.83	0.93
time (sec)	N/A	0.034	0.01	0.009	0.	0.225	0.864	0.206	2.134

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	30	134	37	26	19
normalized size	1	1.	0.74	0.63	1.11	4.96	1.37	0.96	0.7
time (sec)	N/A	0.027	0.008	0.006	1.519	0.213	0.892	0.208	1.804

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	34	29	14	15
normalized size	1	1.	1.	0.86	1.09	1.55	1.32	0.64	0.68
time (sec)	N/A	0.024	0.007	0.016	1.487	0.224	1.744	0.216	1.601

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	35	41	0	35	22	27	14
normalized size	1	1.	1.59	1.86	0.	1.59	1.	1.23	0.64
time (sec)	N/A	0.037	0.017	0.007	0.	0.212	1.762	0.215	2.842

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	37	0	34	22	58	15
normalized size	1	1.	1.39	1.61	0.	1.48	0.96	2.52	0.65
time (sec)	N/A	0.039	0.014	0.006	0.	0.228	1.775	0.21	3.12

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	30	35	0	54	7	50	15
normalized size	1	1.	1.43	1.67	0.	2.57	0.33	2.38	0.71
time (sec)	N/A	0.033	0.011	0.007	0.	0.243	1.736	0.217	2.794

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	11	24	0	8	19
normalized size	1	1.	1.	0.58	0.92	2.	0.	0.67	1.58
time (sec)	N/A	0.014	0.009	0.006	1.528	0.213	0.	0.212	0.666

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	22	55	0	45	32
normalized size	1	1.	0.95	0.79	1.16	2.89	0.	2.37	1.68
time (sec)	N/A	0.028	0.013	0.004	1.541	0.226	0.	0.216	0.732

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	7	8	22	0	8	5
normalized size	1	1.	4.75	0.88	1.	2.75	0.	1.	0.62
time (sec)	N/A	0.007	0.014	0.004	1.576	0.215	0.	0.209	0.573

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	45	0	28	32
normalized size	1	1.	1.	0.81	1.04	1.67	0.	1.04	1.19
time (sec)	N/A	0.028	0.019	0.008	1.526	0.227	0.	0.22	2.214

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	25	45	53	0	96	29
normalized size	1	1.	1.06	0.78	1.41	1.66	0.	3.	0.91
time (sec)	N/A	0.033	0.021	0.006	1.51	0.244	0.	0.231	2.08

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	19	16	23	23	0	38	17
normalized size	1	1.1	0.9	0.76	1.1	1.1	0.	1.81	0.81
time (sec)	N/A	0.028	0.012	0.004	1.567	0.213	0.	0.218	2.019

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	54	23	45	53	22	42	0
normalized size	1	1.	1.93	0.82	1.61	1.89	0.79	1.5	0.
time (sec)	N/A	0.231	0.054	0.07	1.369	0.225	1.5	0.217	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	48	17	14	18	42	0	62	17
normalized size	1	1.3	0.46	0.38	0.49	1.14	0.	1.68	0.46
time (sec)	N/A	0.05	0.021	0.02	1.51	0.232	0.	0.22	0.644

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	21	11	63	43	27	15	0
normalized size	1	1.	1.5	0.79	4.5	3.07	1.93	1.07	0.
time (sec)	N/A	0.06	0.016	0.07	1.407	0.227	1.138	0.215	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	20	61	15	23	3
normalized size	1	1.	2.09	0.36	1.82	5.55	1.36	2.09	0.27
time (sec)	N/A	0.059	0.01	0.009	1.42	0.264	0.211	0.217	5.209

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	19	127	15	19	19	0	0	14
normalized size	1	1.06	7.06	0.83	1.06	1.06	0.	0.	0.78
time (sec)	N/A	0.076	0.264	0.021	1.395	0.258	0.	0.	3.185

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	67	15	19	20	17	35	17
normalized size	1	1.	3.72	0.83	1.06	1.11	0.94	1.94	0.94
time (sec)	N/A	0.083	0.153	0.025	1.332	0.246	3.731	0.261	3.254

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	0	20	14
normalized size	1	1.	1.	0.94	1.18	1.18	0.	1.18	0.82
time (sec)	N/A	0.027	0.012	0.01	1.502	0.251	0.	0.222	1.783

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	24	22	7	22	15
normalized size	1	1.	1.	0.	1.2	1.1	0.35	1.1	0.75
time (sec)	N/A	0.05	0.011	0.103	1.335	0.234	0.551	0.207	2.987

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	16	15	16	8
normalized size	1	1.	1.	1.08	0.	1.33	1.25	1.33	0.67
time (sec)	N/A	0.041	0.005	0.001	0.	0.235	9.22	0.202	3.108

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	486	24	0	16	34
normalized size	1	1.	1.	0.9	11.57	0.57	0.	0.38	0.81
time (sec)	N/A	0.107	0.021	0.101	4.265	0.219	0.	0.311	6.531

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.04	0.004	0.007	1.514	0.22	4.597	0.204	2.959

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	30	22	30	0
normalized size	1	1.	1.	0.82	0.82	1.07	0.79	1.07	0.
time (sec)	N/A	0.021	0.002	0.	1.329	0.216	0.094	0.213	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	15	15	18	15
normalized size	1	1.	1.	0.82	1.06	0.88	0.88	1.06	0.88
time (sec)	N/A	0.017	0.002	0.001	1.352	0.22	0.088	0.233	1.147

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	38	38	53	38	38	26	39	0
normalized size	1	1.06	1.06	1.47	1.06	1.06	0.72	1.08	0.
time (sec)	N/A	0.058	0.005	0.021	1.331	0.218	0.133	0.242	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	17	20	24	17	20	17
normalized size	1	1.	1.21	0.89	1.05	1.26	0.89	1.05	0.89
time (sec)	N/A	0.017	0.003	0.	1.351	0.257	0.044	0.232	0.719

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	24	34	31	30	31
normalized size	1	1.	0.88	0.76	0.71	1.	0.91	0.88	0.91
time (sec)	N/A	0.062	0.014	0.002	1.529	0.228	0.042	0.231	1.751

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	71	26	57	93	46	51	31
normalized size	1	1.	2.73	1.	2.19	3.58	1.77	1.96	1.19
time (sec)	N/A	0.023	0.009	0.056	1.349	0.227	0.172	0.216	0.598

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	18	15	23	17	15	17
normalized size	1	1.	0.61	0.78	0.65	1.	0.74	0.65	0.74
time (sec)	N/A	0.018	0.011	0.009	1.412	0.224	0.8	0.202	1.444

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	28	26	26	26
normalized size	1	1.	0.81	0.81	0.96	1.04	0.96	0.96	0.96
time (sec)	N/A	0.02	0.029	0.003	1.429	0.247	0.372	0.202	1.491

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	71	32	27	107	455	29
normalized size	1	1.	0.65	2.29	1.03	0.87	3.45	14.68	0.94
time (sec)	N/A	0.022	0.018	0.03	1.46	0.217	1.898	0.23	1.565

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	18	15	18	15
normalized size	1	1.	1.	0.82	0.82	1.06	0.88	1.06	0.88
time (sec)	N/A	0.008	0.003	0.	1.455	0.233	0.549	0.211	0.491

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	61	127	30	0	16	15
normalized size	1	1.	1.	5.08	10.58	2.5	0.	1.33	1.25
time (sec)	N/A	0.033	0.011	0.062	1.516	0.224	0.	0.212	1.679

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	144	28	19	31	0
normalized size	1	1.	1.	1.33	9.6	1.87	1.27	2.07	0.
time (sec)	N/A	0.025	0.008	0.	1.551	0.222	0.194	0.215	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	45	53	22	51	15
normalized size	1	1.	1.18	0.95	2.05	2.41	1.	2.32	0.68
time (sec)	N/A	0.04	0.008	0.003	1.53	0.275	2.365	0.212	2.841

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	31	22	31	22
normalized size	1	1.	1.	0.96	1.24	1.24	0.88	1.24	0.88
time (sec)	N/A	0.072	0.01	0.	1.564	0.229	0.235	0.211	2.951

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	32	26	19	28	19
normalized size	1	1.	1.	0.87	1.39	1.13	0.83	1.22	0.83
time (sec)	N/A	0.092	0.006	0.003	1.546	0.226	0.589	0.219	5.438

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	57	49	44	105	43	0	0	53
normalized size	1	1.5	1.29	1.16	2.76	1.13	0.	0.	1.39
time (sec)	N/A	0.064	0.089	0.019	1.538	0.24	0.	0.	3.931

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	1	22	28	0
normalized size	1	1.	1.	0.88	1.12	0.04	0.88	1.12	0.
time (sec)	N/A	0.024	0.002	0.002	1.337	0.171	0.035	0.221	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	39	1	34	39	0
normalized size	1	1.	1.	0.77	1.	0.03	0.87	1.	0.
time (sec)	N/A	0.036	0.002	0.002	1.353	0.178	0.046	0.203	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	29	0	36	0	31	15
normalized size	1	1.	0.96	1.26	0.	1.57	0.	1.35	0.65
time (sec)	N/A	0.025	0.036	0.073	0.	0.221	0.	0.199	1.483

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	23	30	30	20	31	0
normalized size	1	1.	0.9	0.77	1.	1.	0.67	1.03	0.
time (sec)	N/A	0.031	0.009	0.003	1.372	0.196	0.057	0.2	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	27	35	53	34	35	0
normalized size	1	1.	0.85	0.66	0.85	1.29	0.83	0.85	0.
time (sec)	N/A	0.03	0.021	0.004	1.495	0.28	0.091	0.2	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	20	14	23	14
normalized size	1	1.	1.	0.76	0.95	0.95	0.67	1.1	0.67
time (sec)	N/A	0.014	0.005	0.007	1.314	0.201	0.091	0.2	0.819

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	38	45	36	38	32
normalized size	1	1.	0.97	0.88	1.15	1.36	1.09	1.15	0.97
time (sec)	N/A	0.041	0.015	0.005	1.525	0.199	0.11	0.199	2.698

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	28	22	28	22
normalized size	1	1.	1.	0.88	1.12	1.12	0.88	1.12	0.88
time (sec)	N/A	0.044	0.006	0.004	1.501	0.199	0.118	0.2	3.762

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	39	51	68	46	51	0
normalized size	1	1.	1.	0.83	1.09	1.45	0.98	1.09	0.
time (sec)	N/A	0.092	0.027	0.007	1.492	0.201	0.114	0.201	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	23	17	27	17
normalized size	1	1.	1.	0.72	0.92	0.92	0.68	1.08	0.68
time (sec)	N/A	0.047	0.008	0.01	1.342	0.201	0.141	0.2	4.902

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	26	34	34	26	38	0
normalized size	1	1.	0.88	0.79	1.03	1.03	0.79	1.15	0.
time (sec)	N/A	0.089	0.023	0.025	1.368	0.202	0.713	0.2	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	61	37	26	34	34	29	39	42
normalized size	1	1.65	1.	0.7	0.92	0.92	0.78	1.05	1.14
time (sec)	N/A	0.069	0.01	0.013	1.367	0.205	0.252	0.2	9.142

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	69	26	62	82	60	80	34
normalized size	1	1.	2.23	0.84	2.	2.65	1.94	2.58	1.1
time (sec)	N/A	0.028	0.028	0.011	1.535	0.206	0.965	0.201	3.461

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	26	34	34	26	39	26
normalized size	1	1.	1.	0.63	0.83	0.83	0.63	0.95	0.63
time (sec)	N/A	0.067	0.012	0.013	1.373	0.208	0.229	0.201	3.774

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	20	30	39	17	24	15
normalized size	1	1.	0.64	0.8	1.2	1.56	0.68	0.96	0.6
time (sec)	N/A	0.026	0.015	0.009	1.35	0.201	0.096	0.201	1.882

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	43	53	31	61	0
normalized size	1	1.	1.	0.92	1.19	1.47	0.86	1.69	0.
time (sec)	N/A	0.036	0.023	0.009	1.344	0.196	0.078	0.199	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	37	28	41	61	31	36	0
normalized size	1	1.	0.9	0.68	1.	1.49	0.76	0.88	0.
time (sec)	N/A	0.066	0.032	0.011	1.339	0.198	0.151	0.2	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	39	61	26	50	0
normalized size	1	1.	1.	1.04	1.44	2.26	0.96	1.85	0.
time (sec)	N/A	0.046	0.025	0.015	1.333	0.195	0.16	0.203	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	32	39	61	31	47	31
normalized size	1	1.	1.21	0.82	1.	1.56	0.79	1.21	0.79
time (sec)	N/A	0.067	0.028	0.016	1.37	0.202	0.172	0.199	6.451

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	54	88	37	54	0
normalized size	1	1.	0.87	0.76	1.17	1.91	0.8	1.17	0.
time (sec)	N/A	0.042	0.025	0.015	1.386	0.199	0.211	0.199	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	31	31	22	32	0
normalized size	1	1.	1.	0.83	1.07	1.07	0.76	1.1	0.
time (sec)	N/A	0.053	0.011	0.009	1.53	0.21	0.123	0.2	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	28	22	31	22
normalized size	1	1.	1.	0.81	1.04	1.04	0.81	1.15	0.81
time (sec)	N/A	0.062	0.01	0.01	1.536	0.214	0.243	0.199	3.274

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	26	34	34	29	36	20
normalized size	1	1.	1.33	1.08	1.42	1.42	1.21	1.5	0.83
time (sec)	N/A	0.025	0.016	0.01	1.515	0.218	0.198	0.203	2.966

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	59	39	45	0
normalized size	1	1.	1.	0.8	1.05	1.44	0.95	1.1	0.
time (sec)	N/A	0.13	0.034	0.012	1.527	0.216	0.185	0.202	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	37	49	77	37	84	37
normalized size	1	1.	0.87	0.8	1.07	1.67	0.8	1.83	0.8
time (sec)	N/A	0.354	0.031	0.012	1.528	0.215	0.3	0.201	28.029

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	49	80	36	43	36
normalized size	1	1.	0.79	0.72	1.04	1.7	0.77	0.91	0.77
time (sec)	N/A	0.073	0.041	0.01	1.519	0.211	0.199	0.202	4.879

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	72	77	70	72	63
normalized size	1	1.	1.09	0.81	1.07	1.15	1.04	1.07	0.94
time (sec)	N/A	0.077	0.095	0.003	1.495	0.233	0.254	0.201	5.088

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	67	46	73	89	306	86	48
normalized size	1	1.	1.4	0.96	1.52	1.85	6.38	1.79	1.
time (sec)	N/A	0.085	0.03	0.013	1.511	0.24	2.537	0.201	5.947

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	41	58	89	46	53	46
normalized size	1	1.	0.79	0.71	1.	1.53	0.79	0.91	0.79
time (sec)	N/A	0.097	0.04	0.013	1.491	0.217	0.227	0.203	20.543

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	47	73	44	47	44
normalized size	1	1.	0.92	0.71	0.92	1.43	0.86	0.92	0.86
time (sec)	N/A	0.473	0.038	0.016	1.52	0.239	0.775	0.217	43.754

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	45	45	32	45	32
normalized size	1	1.	1.	0.83	1.1	1.1	0.78	1.1	0.78
time (sec)	N/A	0.529	0.013	0.016	1.438	0.234	0.258	0.218	50.561

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	52	66	68	73	68	53
normalized size	1	1.	0.93	0.93	1.18	1.21	1.3	1.21	0.95
time (sec)	N/A	0.066	0.017	0.012	1.507	0.208	0.139	0.218	5.61

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	52	66	68	71	68	48
normalized size	1	1.	0.89	0.93	1.18	1.21	1.27	1.21	0.86
time (sec)	N/A	0.063	0.009	0.009	1.497	0.209	0.129	0.209	5.407

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	15	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.25	0.67
time (sec)	N/A	0.006	0.003	0.002	1.326	0.228	0.096	0.202	0.898

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	31	24	19	30	22
normalized size	1	1.	1.	1.55	1.41	1.09	0.86	1.36	1.
time (sec)	N/A	0.024	0.006	0.01	1.372	0.208	0.254	0.208	2.394

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	77	84	83	78	60
normalized size	1	1.	0.95	0.95	1.22	1.33	1.32	1.24	0.95
time (sec)	N/A	0.08	0.022	0.01	1.505	0.208	0.604	0.207	6.937

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	60	77	95	80	78	63
normalized size	1	1.	1.05	0.92	1.18	1.46	1.23	1.2	0.97
time (sec)	N/A	0.077	0.024	0.01	1.541	0.209	0.653	0.204	7.286

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	43	42	45	29	54	32
normalized size	1	1.	1.	1.3	1.27	1.36	0.88	1.64	0.97
time (sec)	N/A	0.042	0.008	0.013	1.343	0.206	0.778	0.201	3.124

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	67	89	105	90	90	70
normalized size	1	1.	1.01	0.92	1.22	1.44	1.23	1.23	0.96
time (sec)	N/A	0.095	0.02	0.013	1.52	0.212	0.754	0.202	8.533

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	0	0	0	92	0	31
normalized size	1	1.	1.02	0.	0.	0.	2.	0.	0.67
time (sec)	N/A	0.03	0.026	0.079	0.	0.	7.508	0.	2.073

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	33	43	35	37	46	19
normalized size	1	1.	1.41	1.22	1.59	1.3	1.37	1.7	0.7
time (sec)	N/A	0.02	0.007	0.011	1.504	0.203	0.162	0.203	1.806

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	39	35	24	41	12
normalized size	1	1.	1.	2.	2.6	2.33	1.6	2.73	0.8
time (sec)	N/A	0.018	0.005	0.008	1.328	0.199	0.179	0.204	1.789

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	41	34	27	19	35	22
normalized size	1	1.	1.	1.71	1.42	1.12	0.79	1.46	0.92
time (sec)	N/A	0.027	0.009	0.01	1.351	0.198	0.311	0.202	2.596

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	41	54	49	44	57	26
normalized size	1	1.	1.31	1.17	1.54	1.4	1.26	1.63	0.74
time (sec)	N/A	0.039	0.011	0.012	1.523	0.214	0.642	0.201	4.104

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	50	43	50	55	34	51	22
normalized size	1	1.	1.92	1.65	1.92	2.12	1.31	1.96	0.85
time (sec)	N/A	0.033	0.011	0.012	1.348	0.211	0.694	0.205	3.711

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	48	41	54	61	48	57	29
normalized size	1	1.	1.3	1.11	1.46	1.65	1.3	1.54	0.78
time (sec)	N/A	0.032	0.01	0.013	1.513	0.296	0.739	0.202	3.587

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	0	0	0	95	0	29
normalized size	1	1.	1.02	0.	0.	0.	2.11	0.	0.64
time (sec)	N/A	0.027	0.027	0.073	0.	0.	1.516	0.	2.282

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	29	18	12
normalized size	1	1.	1.	0.93	1.2	1.2	1.93	1.2	0.8
time (sec)	N/A	0.017	0.005	0.006	1.483	0.195	0.154	0.201	1.59

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	79	101	0	297	19	154	90
normalized size	1	1.	0.72	0.93	0.	2.72	0.17	1.41	0.83
time (sec)	N/A	0.127	0.046	0.012	0.	0.211	0.151	0.205	11.519

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	101	385	0	39	239	262
normalized size	1	1.	1.01	0.5	1.92	0.	0.19	1.19	1.3
time (sec)	N/A	0.651	0.3	0.046	1.479	0.	0.158	0.216	144.858

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	97	358	0	41	239	0
normalized size	1	1.	1.01	0.48	1.78	0.	0.2	1.19	0.
time (sec)	N/A	0.573	0.133	0.011	1.544	0.	0.156	0.222	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	101	356	0	41	239	0
normalized size	1	1.	1.01	0.5	1.77	0.	0.2	1.19	0.
time (sec)	N/A	0.694	0.251	0.01	1.529	0.	0.175	0.228	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	97	385	0	39	239	0
normalized size	1	1.	1.01	0.48	1.92	0.	0.19	1.19	0.
time (sec)	N/A	0.679	0.19	0.011	1.515	0.	0.168	0.219	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	15	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.25	0.67
time (sec)	N/A	0.006	0.004	0.001	1.343	0.207	0.117	0.204	0.901

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	49	31	24	19	30	22
normalized size	1	1.	1.	2.23	1.41	1.09	0.86	1.36	1.
time (sec)	N/A	0.024	0.006	0.01	1.338	0.207	0.323	0.208	2.396

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	172	109	398	0	48	250	0
normalized size	1	1.	0.82	0.52	1.9	0.	0.23	1.2	0.
time (sec)	N/A	0.685	0.395	0.014	1.564	0.	0.657	0.212	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	174	105	374	0	51	250	0
normalized size	1	1.	0.82	0.5	1.77	0.	0.24	1.18	0.
time (sec)	N/A	0.706	0.357	0.013	1.535	0.	0.728	0.216	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	175	109	374	0	51	250	0
normalized size	1	1.	0.83	0.52	1.77	0.	0.24	1.18	0.
time (sec)	N/A	0.633	0.352	0.012	1.511	0.	0.803	0.22	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	0	0	0	92	0	31
normalized size	1	1.	1.02	0.	0.	0.	2.	0.	0.67
time (sec)	N/A	0.028	0.026	0.069	0.	0.	68.049	0.	2.136

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	21	28	36	12	8	36	29
normalized size	1	1.	0.6	0.8	1.03	0.34	0.23	1.03	0.83
time (sec)	N/A	0.836	0.01	0.07	1.506	0.202	0.131	0.201	77.234

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	52	73	103	63	59	54
normalized size	1	1.	0.85	0.87	1.22	1.72	1.05	0.98	0.9
time (sec)	N/A	0.046	0.046	0.004	1.524	0.202	0.196	0.199	1.14

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	28	51	70	36	38	36
normalized size	1	1.	0.7	0.65	1.19	1.63	0.84	0.88	0.84
time (sec)	N/A	0.035	0.019	0.01	1.531	0.21	0.18	0.2	4.179

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	146	0	1	323	124	76
normalized size	1	1.	0.98	1.62	0.	0.01	3.59	1.38	0.84
time (sec)	N/A	0.149	0.134	0.012	0.	0.248	2.034	0.215	6.724

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	45	68	31	45	32
normalized size	1	1.	1.	0.92	1.18	1.79	0.82	1.18	0.84
time (sec)	N/A	0.047	0.022	0.009	1.52	0.205	0.16	0.206	8.473

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	53	57	90	53	58	49
normalized size	1	1.	0.86	0.93	1.	1.58	0.93	1.02	0.86
time (sec)	N/A	0.053	0.035	0.013	1.549	0.205	0.213	0.208	2.832

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	42	54	32	38	32
normalized size	1	1.	0.92	0.81	1.17	1.5	0.89	1.06	0.89
time (sec)	N/A	0.08	0.024	0.019	1.544	0.197	0.178	0.201	7.106

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	41	54	63	46	54	42
normalized size	1	1.	1.59	0.84	1.1	1.29	0.94	1.1	0.86
time (sec)	N/A	0.073	0.024	0.01	1.559	0.256	0.192	0.201	4.314

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	20	13	17	19	18	14	0	20
normalized size	1	1.54	1.	1.31	1.46	1.38	1.08	0.	1.54
time (sec)	N/A	0.11	0.013	0.029	1.378	0.26	4.718	0.	7.905

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	35	46	51	42	46	37
normalized size	1	1.	0.93	0.85	1.12	1.24	1.02	1.12	0.9
time (sec)	N/A	0.079	0.017	0.008	1.564	0.232	0.123	0.201	4.958

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	32	32	24	34	24
normalized size	1	1.	1.	0.78	1.	1.	0.75	1.06	0.75
time (sec)	N/A	0.055	0.008	0.01	1.356	0.199	0.134	0.202	5.697

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	23	15	26	15
normalized size	1	1.	1.	0.72	0.92	0.92	0.6	1.04	0.6
time (sec)	N/A	0.036	0.006	0.008	1.353	0.194	0.117	0.202	2.661

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	43	62	29	31	24
normalized size	1	1.	0.67	0.75	1.19	1.72	0.81	0.86	0.67
time (sec)	N/A	0.04	0.021	0.008	1.361	0.238	0.131	0.199	3.69

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	24	30	97	97	71	30	32
normalized size	1	1.	0.53	0.67	2.16	2.16	1.58	0.67	0.71
time (sec)	N/A	0.031	0.009	0.009	1.371	0.195	0.235	0.201	2.301

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	41	46	65	90	42	74	44
normalized size	1	1.	0.75	0.84	1.18	1.64	0.76	1.35	0.8
time (sec)	N/A	0.057	0.018	0.009	1.527	0.239	0.146	0.201	5.055

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	51	80	41	58	27
normalized size	1	1.	1.06	0.97	1.42	2.22	1.14	1.61	0.75
time (sec)	N/A	0.035	0.034	0.013	1.729	0.214	0.155	0.199	2.921

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	42	65	29	54	27
normalized size	1	1.	0.89	0.8	1.2	1.86	0.83	1.54	0.77
time (sec)	N/A	0.032	0.028	0.013	1.799	0.214	0.138	0.209	2.394

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	42	68	115	51	57	48
normalized size	1	1.	0.75	0.69	1.11	1.89	0.84	0.93	0.79
time (sec)	N/A	0.029	0.036	0.015	1.637	0.239	0.184	0.214	1.235

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	44	59	84	42	46	41
normalized size	1	1.	0.71	0.86	1.16	1.65	0.82	0.9	0.8
time (sec)	N/A	0.031	0.027	0.006	1.734	0.22	0.193	0.198	1.08

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	81	142	58	63	48
normalized size	1	1.	1.	0.91	1.5	2.63	1.07	1.17	0.89
time (sec)	N/A	0.054	0.023	0.015	1.437	0.236	0.213	0.2	3.425

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	42	53	32	58	72	49	65	36
normalized size	1	1.17	1.47	0.89	1.61	2.	1.36	1.81	1.
time (sec)	N/A	0.037	0.057	0.008	1.62	0.235	0.114	0.213	2.731

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	32	55	80	39	42	49
normalized size	1	1.	0.69	0.55	0.95	1.38	0.67	0.72	0.84
time (sec)	N/A	0.046	0.029	0.011	1.628	0.233	0.186	0.218	4.488

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	49	68	42	49	39
normalized size	1	1.	1.	0.86	1.14	1.58	0.98	1.14	0.91
time (sec)	N/A	0.046	0.059	0.007	1.537	0.2	0.172	0.217	2.397

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	63	99	58	69	37
normalized size	1	1.	1.44	0.86	1.47	2.3	1.35	1.6	0.86
time (sec)	N/A	0.051	0.06	0.004	1.596	0.198	0.15	0.222	3.297

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	28	41	51	24	54	27
normalized size	1	1.	0.68	0.76	1.11	1.38	0.65	1.46	0.73
time (sec)	N/A	0.033	0.016	0.013	1.558	0.221	0.209	0.211	2.37

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	39	26	29	38	51	26	38	31
normalized size	1	1.22	0.81	0.91	1.19	1.59	0.81	1.19	0.97
time (sec)	N/A	0.07	0.02	0.011	1.428	0.205	0.172	0.202	6.523

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	26	20	15	12
normalized size	1	1.	1.	0.92	1.15	2.	1.54	1.15	0.92
time (sec)	N/A	0.006	0.005	0.002	1.422	0.2	1.691	0.201	0.88

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	49	77	109	51	76	51
normalized size	1	1.	0.85	0.91	1.43	2.02	0.94	1.41	0.94
time (sec)	N/A	0.062	0.04	0.023	1.46	0.227	4.931	0.205	4.487

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	152	0	468	65	203	144
normalized size	1	1.	0.85	0.97	0.	2.98	0.41	1.29	0.92
time (sec)	N/A	0.209	0.177	0.018	0.	0.233	9.005	0.204	17.796

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	75	57	81	105	76	68	60
normalized size	1	1.	1.17	0.89	1.27	1.64	1.19	1.06	0.94
time (sec)	N/A	0.07	0.083	0.019	1.525	0.212	16.124	0.227	6.672

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	34	46	61	27	41	29
normalized size	1	1.	0.85	0.87	1.18	1.56	0.69	1.05	0.74
time (sec)	N/A	0.052	0.024	0.013	1.428	0.203	0.258	0.248	3.266

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	319	293	354	452	0	37	0	0
normalized size	1	0.98	0.9	1.09	1.39	0.	0.11	0.	0.
time (sec)	N/A	1.155	0.557	0.164	1.58	0.	0.485	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	65	39	76	109	58	74	46
normalized size	1	1.	1.1	0.66	1.29	1.85	0.98	1.25	0.78
time (sec)	N/A	0.053	0.116	0.013	1.577	0.216	0.211	0.208	3.894

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	42	54	32	38	32
normalized size	1	1.	0.92	0.81	1.17	1.5	0.89	1.06	0.89
time (sec)	N/A	0.078	0.024	0.	1.564	0.212	0.177	0.216	6.767

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	51	88	36	47	0
normalized size	1	1.	0.84	0.82	1.34	2.32	0.95	1.24	0.
time (sec)	N/A	0.106	0.034	0.019	1.414	0.217	0.159	0.218	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	61	68	116	60	81	51
normalized size	1	1.	1.02	0.95	1.06	1.81	0.94	1.27	0.8
time (sec)	N/A	0.13	0.052	0.018	1.597	0.228	0.318	0.217	6.13

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	54	80	147	66	97	58
normalized size	1	1.	1.	0.86	1.27	2.33	1.05	1.54	0.92
time (sec)	N/A	0.143	0.056	0.013	1.591	0.225	0.239	0.214	7.25

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	36	49	74	34	45	36
normalized size	1	1.	1.07	0.84	1.14	1.72	0.79	1.05	0.84
time (sec)	N/A	0.043	0.023	0.013	1.395	0.244	0.158	0.2	3.648

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	24	58	58	37	24	22
normalized size	1	1.	0.74	0.89	2.15	2.15	1.37	0.89	0.81
time (sec)	N/A	0.041	0.011	0.013	1.4	0.194	0.279	0.202	3.23

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	53	74	34	54	0
normalized size	1	1.	0.96	0.85	1.15	1.61	0.74	1.17	0.
time (sec)	N/A	0.061	0.021	0.015	1.435	0.198	0.158	0.202	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	50	39	35	50	76	34	58	34
normalized size	1	1.14	0.89	0.8	1.14	1.73	0.77	1.32	0.77
time (sec)	N/A	0.04	0.034	0.014	1.41	0.2	0.157	0.198	2.624

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	81	142	58	63	48
normalized size	1	1.	1.	0.91	1.5	2.63	1.07	1.17	0.89
time (sec)	N/A	0.054	0.022	0.	1.41	0.198	0.212	0.213	3.361

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	43	53	31	61	0
normalized size	1	1.	1.	0.92	1.19	1.47	0.86	1.69	0.
time (sec)	N/A	0.035	0.022	0.	1.452	0.197	0.078	0.213	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	51	1	39	53	0
normalized size	1	1.	0.93	0.89	1.16	0.02	0.89	1.2	0.
time (sec)	N/A	0.075	0.02	0.003	1.412	0.177	0.045	0.211	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	91	95	123	1	100	132	0
normalized size	1	1.	0.95	0.99	1.28	0.01	1.04	1.38	0.
time (sec)	N/A	0.203	0.045	0.003	1.4	0.178	0.075	0.21	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	167	237	231	1	189	254	0
normalized size	1	1.	1.	1.42	1.38	0.01	1.13	1.52	0.
time (sec)	N/A	0.377	0.057	0.	1.413	0.182	0.098	0.199	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	263	363	369	1	313	414	0
normalized size	1	1.	1.	1.38	1.4	0.	1.19	1.57	0.
time (sec)	N/A	0.666	0.106	0.003	1.432	0.184	0.131	0.199	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	471	0	0	0	0	0	134
normalized size	1	1.	2.96	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.198	3.551	0.201	0.	0.	0.	0.	6.923

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	66	95	0	1	246	81	53
normalized size	1	1.	1.02	1.46	0.	0.02	3.78	1.25	0.82
time (sec)	N/A	0.089	0.075	0.007	0.	0.217	1.02	0.2	7.32

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	88	146	0	1	323	124	76
normalized size	1	1.	0.99	1.64	0.	0.01	3.63	1.39	0.85
time (sec)	N/A	0.106	0.126	0.004	0.	0.214	1.983	0.203	6.493

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	127	274	0	1	622	262	121
normalized size	1	1.	0.98	2.11	0.	0.01	4.78	2.02	0.93
time (sec)	N/A	0.165	0.229	0.007	0.	0.229	3.893	0.204	9.29

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	168	405	0	1	1027	490	165
normalized size	1	1.	0.97	2.34	0.	0.01	5.94	2.83	0.95
time (sec)	N/A	0.254	0.35	0.008	0.	0.228	7.614	0.21	13.272

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	374	0	0	0	0	0	131
normalized size	1	1.	2.21	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.195	2.136	0.119	0.	0.	0.	0.	7.107

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	55	70	46	62	49
normalized size	1	1.	0.9	0.63	1.12	1.43	0.94	1.27	1.
time (sec)	N/A	0.041	0.033	0.003	1.584	0.208	0.1	0.199	2.038

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	70	56	90	138	75	82	53
normalized size	1	1.	1.15	0.92	1.48	2.26	1.23	1.34	0.87
time (sec)	N/A	0.052	0.073	0.003	1.789	0.2	0.19	0.203	2.683

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	28	41	61	31	43	31
normalized size	1	1.	0.92	0.78	1.14	1.69	0.86	1.19	0.86
time (sec)	N/A	0.022	0.026	0.015	1.344	0.197	0.134	0.2	2.003

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	70	122	223	88	84	76
normalized size	1	1.	1.	0.8	1.4	2.56	1.01	0.97	0.87
time (sec)	N/A	0.049	0.04	0.017	1.328	0.198	0.282	0.199	1.715

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	62	93	173	80	90	76
normalized size	1	1.	0.86	0.77	1.15	2.14	0.99	1.11	0.94
time (sec)	N/A	0.124	0.054	0.016	1.507	0.214	0.285	0.203	7.782

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	87	70	122	223	88	84	95
normalized size	1	1.	0.84	0.67	1.17	2.14	0.85	0.81	0.91
time (sec)	N/A	0.198	0.036	0.014	1.341	0.221	0.281	0.205	12.735

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	80	127	234	90	89	90
normalized size	1	1.	0.97	0.78	1.25	2.29	0.88	0.87	0.88
time (sec)	N/A	0.081	0.081	0.016	1.351	0.212	0.327	0.203	5.833

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	53	53	37	63	41
normalized size	1	1.	1.	0.88	1.32	1.32	0.92	1.58	1.02
time (sec)	N/A	0.05	0.008	0.01	1.349	0.218	0.741	0.202	4.599

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	56	112	153	102	78	75
normalized size	1	1.	0.75	0.67	1.35	1.84	1.23	0.94	0.9
time (sec)	N/A	0.087	0.035	0.017	1.51	0.201	92.07	0.204	5.99

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	32	42	50	42	42	42
normalized size	1	1.	1.	0.65	0.86	1.02	0.86	0.86	0.86
time (sec)	N/A	0.095	0.014	0.004	1.351	0.2	1.837	0.201	7.623

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	32	42	42	48	42	0
normalized size	1	1.	1.	0.58	0.76	0.76	0.87	0.76	0.
time (sec)	N/A	0.351	0.022	0.006	1.346	0.198	0.915	0.205	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	31	24	24	19	24	19
normalized size	1	1.	1.	1.41	1.09	1.09	0.86	1.09	0.86
time (sec)	N/A	0.021	0.007	0.003	1.453	0.201	0.13	0.199	1.131

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	16	15	22	16	0
normalized size	1	1.	1.	0.87	1.07	1.	1.47	1.07	0.
time (sec)	N/A	0.017	0.01	0.002	1.365	0.201	1.392	0.207	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	27	26	20	28	0
normalized size	1	1.	0.96	0.84	1.08	1.04	0.8	1.12	0.
time (sec)	N/A	0.042	0.011	0.003	1.336	0.204	0.17	0.201	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	111	34	34	0	35	0
normalized size	1	1.	1.	3.36	1.03	1.03	0.	1.06	0.
time (sec)	N/A	0.038	0.017	0.03	1.38	0.202	0.	0.209	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	20	26	26	134	26	24
normalized size	1	1.	0.83	0.69	0.9	0.9	4.62	0.9	0.83
time (sec)	N/A	0.023	0.01	0.003	1.46	0.206	2.548	0.201	1.601

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	73	74	76	3966	66	48
normalized size	1	1.	0.68	1.38	1.4	1.43	74.83	1.25	0.91
time (sec)	N/A	0.034	0.039	0.021	1.398	0.208	4.331	0.204	2.231

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	61	157	150	155	971	140	104
normalized size	1	1.	0.48	1.24	1.18	1.22	7.65	1.1	0.82
time (sec)	N/A	0.102	0.077	0.026	1.352	0.206	25.901	0.202	5.276

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	63	158	142	128	7998	111	99
normalized size	1	1.	0.61	1.52	1.37	1.23	76.9	1.07	0.95
time (sec)	N/A	0.093	0.025	0.026	1.497	0.216	4.897	0.203	3.234

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	62	39	58	43	0	39	32
normalized size	1	1.	1.63	1.03	1.53	1.13	0.	1.03	0.84
time (sec)	N/A	0.036	0.037	0.007	1.52	0.215	0.	0.205	1.989

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	95	110	208	176	99	0	139	80
normalized size	1	1.03	1.2	2.26	1.91	1.08	0.	1.51	0.87
time (sec)	N/A	0.13	0.194	0.031	1.493	0.209	0.	0.217	7.593

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	64	0	128	0	100	44
normalized size	1	1.	1.19	1.19	0.	2.37	0.	1.85	0.81
time (sec)	N/A	0.841	0.066	0.033	0.	0.233	0.	0.21	42.758

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	319	165	0	0	774	0	0	0
normalized size	1	1.05	0.54	0.	0.	2.55	0.	0.	0.
time (sec)	N/A	1.369	0.397	0.072	0.	0.259	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	522	391	0	0	1841	0	0	0
normalized size	1	1.79	1.34	0.	0.	6.3	0.	0.	0.
time (sec)	N/A	3.298	1.239	0.059	0.	0.432	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	25	22	0	63	0	0	0
normalized size	1	1.16	1.	0.88	0.	2.52	0.	0.	0.
time (sec)	N/A	0.033	0.018	0.004	0.	0.23	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	30	25	22	0	93	0	0	0
normalized size	1	1.2	1.	0.88	0.	3.72	0.	0.	0.
time (sec)	N/A	0.034	0.021	0.006	0.	0.204	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	30	27	0	104	0	0	0
normalized size	1	1.19	0.57	0.51	0.	1.96	0.	0.	0.
time (sec)	N/A	0.052	0.024	0.005	0.	0.198	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	C	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	188	49	0	0	166	0	0	144
normalized size	1	2.81	0.73	0.	0.	2.48	0.	0.	2.15
time (sec)	N/A	0.235	0.023	0.02	0.	0.202	0.	0.	2.885

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	133	112	118	0	185	0	230	0
normalized size	1	1.09	0.92	0.97	0.	1.52	0.	1.89	0.
time (sec)	N/A	0.577	0.096	0.03	0.	0.216	0.	0.267	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	404	145	0	0	387	0	0	301
normalized size	1	2.69	0.97	0.	0.	2.58	0.	0.	2.01
time (sec)	N/A	0.74	0.298	0.051	0.	0.217	0.	0.	12.654

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	27	32	69	123	0	31	36
normalized size	1	1.	0.63	0.74	1.6	2.86	0.	0.72	0.84
time (sec)	N/A	0.018	0.018	0.006	1.383	0.21	0.	0.208	0.765

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	52	45	28	84	0	46	36
normalized size	1	1.	1.24	1.07	0.67	2.	0.	1.1	0.86
time (sec)	N/A	0.08	0.022	0.013	1.55	0.215	0.	0.207	2.029

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	58	144	77	186	0	4	128
normalized size	1	1.	0.42	1.04	0.55	1.34	0.	0.03	0.92
time (sec)	N/A	0.235	0.041	0.025	1.595	0.218	0.	0.528	4.474

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	C	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	188	49	0	0	166	0	0	155
normalized size	1	2.51	0.65	0.	0.	2.21	0.	0.	2.07
time (sec)	N/A	0.233	0.026	0.022	0.	0.21	0.	0.	2.699

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	24	0	30	0	0	26
normalized size	1	1.	0.79	0.83	0.	1.03	0.	0.	0.9
time (sec)	N/A	0.068	0.017	0.005	0.	0.211	0.	0.	1.846

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	32	34	0	43	0	0	85
normalized size	1	1.	0.35	0.37	0.	0.47	0.	0.	0.92
time (sec)	N/A	0.187	0.019	0.006	0.	0.201	0.	0.	3.321

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	22	38	0	22	34
normalized size	1	1.	1.	0.79	1.16	2.	0.	1.16	1.79
time (sec)	N/A	0.027	0.015	0.004	1.527	0.246	0.	0.213	0.74

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	5	11	23	0	5	20
normalized size	1	1.	1.	0.62	1.38	2.88	0.	0.62	2.5
time (sec)	N/A	0.012	0.009	0.004	1.519	0.211	0.	0.206	0.69

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	38	0	15	37
normalized size	1	1.	1.	1.	1.25	3.17	0.	1.25	3.08
time (sec)	N/A	0.011	0.011	0.005	1.58	0.24	0.	0.218	0.741

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	65	0	69	24
normalized size	1	1.	1.06	0.94	0.	2.1	0.	2.23	0.77
time (sec)	N/A	0.026	0.043	0.014	0.	0.212	0.	0.221	2.477

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	56	22	0	119	0	77	26
normalized size	1	1.	1.81	0.71	0.	3.84	0.	2.48	0.84
time (sec)	N/A	0.03	0.072	0.013	0.	0.244	0.	0.217	3.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	100	151	68	61	57	22
normalized size	1	1.	1.	4.17	6.29	2.83	2.54	2.38	0.92
time (sec)	N/A	0.059	0.018	0.047	1.529	0.222	12.504	0.212	4.429

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	100	136	34	20	27	24
normalized size	1	1.	1.	4.	5.44	1.36	0.8	1.08	0.96
time (sec)	N/A	0.058	0.017	0.047	1.52	0.228	4.409	0.213	4.517

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	80	70	0	128	0	100	37
normalized size	1	1.	1.86	1.63	0.	2.98	0.	2.33	0.86
time (sec)	N/A	0.061	0.058	0.034	0.	0.228	0.	0.232	5.631

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	88	49	73	190	0	147	60
normalized size	1	1.	1.42	0.79	1.18	3.06	0.	2.37	0.97
time (sec)	N/A	0.099	0.051	0.017	1.537	0.239	0.	0.245	5.981

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	155	69	0	265	0	221	80
normalized size	1	1.	1.89	0.84	0.	3.23	0.	2.7	0.98
time (sec)	N/A	0.258	0.147	0.032	0.	0.235	0.	0.229	25.566

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	426	53	0	356	0	0	61
normalized size	1	1.	6.76	0.84	0.	5.65	0.	0.	0.97
time (sec)	N/A	0.148	0.489	0.016	0.	0.242	0.	0.	12.526

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	352	128	0	379	0	0	60
normalized size	1	1.	6.29	2.29	0.	6.77	0.	0.	1.07
time (sec)	N/A	0.134	0.338	0.029	0.	0.237	0.	0.	8.217

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	1102	158	0	556	0	0	66
normalized size	1	1.	15.74	2.26	0.	7.94	0.	0.	0.94
time (sec)	N/A	0.179	6.395	0.041	0.	0.254	0.	0.	14.328

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	89	1149	192	0	0	0	0	105
normalized size	1	1.11	14.36	2.4	0.	0.	0.	0.	1.31
time (sec)	N/A	0.289	4.385	0.039	0.	0.	0.	0.	22.736

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	410	94	0	112	0	0	32
normalized size	1	1.	10.79	2.47	0.	2.95	0.	0.	0.84
time (sec)	N/A	0.065	0.597	0.016	0.	0.221	0.	0.	5.852

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	40	49	65	57	155	49	54
normalized size	1	1.	0.62	0.75	1.	0.88	2.38	0.75	0.83
time (sec)	N/A	0.054	0.037	0.009	1.499	0.219	7.688	0.21	3.692

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	50	99	41	69	37
normalized size	1	1.	0.57	0.51	1.02	2.02	0.84	1.41	0.76
time (sec)	N/A	0.036	0.013	0.006	1.502	0.302	8.405	0.209	2.654

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	50	59	0	35	42
normalized size	1	1.	0.57	0.51	1.02	1.2	0.	0.71	0.86
time (sec)	N/A	0.019	0.015	0.005	1.352	0.202	0.	0.22	0.793

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	328	14	84	0	15	10
normalized size	1	1.	1.	27.33	1.17	7.	0.	1.25	0.83
time (sec)	N/A	0.075	0.012	0.062	1.34	0.227	0.	0.217	3.574

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	78	29	30	12
normalized size	1	1.	1.	0.92	1.17	6.5	2.42	2.5	1.
time (sec)	N/A	0.016	0.019	0.009	1.488	0.257	9.91	0.215	2.034

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	143	0	86	27
normalized size	1	1.	1.	0.85	0.	5.3	0.	3.19	1.
time (sec)	N/A	0.035	0.019	0.016	0.	0.253	0.	0.217	4.08

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	46	0	238	0	136	42
normalized size	1	1.	1.	0.96	0.	4.96	0.	2.83	0.88
time (sec)	N/A	0.041	0.033	0.023	0.	0.239	0.	0.215	3.76

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	100	144	149	0	97	36
normalized size	1	1.	1.	2.44	3.51	3.63	0.	2.37	0.88
time (sec)	N/A	0.07	0.037	0.052	1.78	0.252	0.	0.24	5.766

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	70	0	178	0	166	37
normalized size	1	1.	1.	1.49	0.	3.79	0.	3.53	0.79
time (sec)	N/A	0.071	0.071	0.037	0.	0.236	0.	0.213	6.55

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	137	82	0	394	0	182	0
normalized size	1	1.	1.56	0.93	0.	4.48	0.	2.07	0.
time (sec)	N/A	0.415	0.177	0.03	0.	0.22	0.	0.227	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	1046	654	0	439	0	238	112
normalized size	1	1.	7.69	4.81	0.	3.23	0.	1.75	0.82
time (sec)	N/A	2.802	6.348	0.079	0.	0.241	0.	0.22	92.16

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	73	0	69	0	61	53
normalized size	1	1.	1.03	1.14	0.	1.08	0.	0.95	0.83
time (sec)	N/A	0.046	0.081	0.013	0.	0.262	0.	0.215	2.661

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	91	0	81	0	73	78
normalized size	1	1.	0.83	1.02	0.	0.91	0.	0.82	0.88
time (sec)	N/A	0.074	0.08	0.007	0.	0.273	0.	0.211	4.217

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	82	111	0	92	0	85	100
normalized size	1	1.	0.73	0.98	0.	0.81	0.	0.75	0.88
time (sec)	N/A	0.105	0.096	0.007	0.	0.261	0.	0.209	5.777

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	113	42	73	115	0	96	44
normalized size	1	1.	2.31	0.86	1.49	2.35	0.	1.96	0.9
time (sec)	N/A	0.077	0.113	0.019	1.747	0.275	0.	0.222	5.675

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	126	74	89	89	0	198	66
normalized size	1	1.	1.59	0.94	1.13	1.13	0.	2.51	0.84
time (sec)	N/A	0.146	0.144	0.022	1.765	0.233	0.	0.21	6.671

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	15	24	0	24	17
normalized size	1	1.	1.	0.83	1.25	2.	0.	2.	1.42
time (sec)	N/A	0.018	0.007	0.004	1.684	0.2	0.	0.202	0.65

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	47	65	212	0	53	56
normalized size	1	1.	0.77	0.89	1.23	4.	0.	1.	1.06
time (sec)	N/A	0.07	0.026	0.008	1.643	0.203	0.	0.204	3.932

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	30	39	0	20	15
normalized size	1	1.	1.	0.84	1.58	2.05	0.	1.05	0.79
time (sec)	N/A	0.007	0.011	0.005	1.397	0.202	0.	0.203	0.563

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	30	55	0	18	17
normalized size	1	1.	1.	0.82	1.76	3.24	0.	1.06	1.
time (sec)	N/A	0.012	0.011	0.006	1.381	0.2	0.	0.202	1.374

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	41	61	63	185	0	51	58
normalized size	1	1.	0.73	1.09	1.12	3.3	0.	0.91	1.04
time (sec)	N/A	0.068	0.033	0.009	1.597	0.241	0.	0.206	4.175

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	49	76	266	0	59	65
normalized size	1	1.	0.71	0.75	1.17	4.09	0.	0.91	1.
time (sec)	N/A	0.056	0.034	0.007	1.62	0.2	0.	0.206	3.421

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	43	76	266	0	59	56
normalized size	1	1.	0.84	0.78	1.38	4.84	0.	1.07	1.02
time (sec)	N/A	0.031	0.035	0.004	1.568	0.202	0.	0.205	1.018

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	58	104	374	0	73	75
normalized size	1	1.	0.76	0.78	1.41	5.05	0.	0.99	1.01
time (sec)	N/A	0.044	0.043	0.004	1.586	0.204	0.	0.203	1.317

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	31	50	140	0	90	29
normalized size	1	1.	1.11	0.82	1.32	3.68	0.	2.37	0.76
time (sec)	N/A	0.036	0.029	0.007	1.6	0.208	0.	0.209	2.935

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	44	68	234	0	113	48
normalized size	1	1.	0.79	0.77	1.19	4.11	0.	1.98	0.84
time (sec)	N/A	0.074	0.036	0.009	1.589	0.204	0.	0.209	4.615

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	56	78	270	0	108	53
normalized size	1	1.	0.84	0.9	1.26	4.35	0.	1.74	0.85
time (sec)	N/A	0.076	0.052	0.006	1.574	0.209	0.	0.205	4.942

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	69	96	354	0	158	71
normalized size	1	1.	0.72	0.87	1.22	4.48	0.	2.	0.9
time (sec)	N/A	0.116	0.045	0.007	1.594	0.234	0.	0.207	7.019

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	28	22	34	41	0	43	17
normalized size	1	1.	1.27	1.	1.55	1.86	0.	1.95	0.77
time (sec)	N/A	0.025	0.014	0.007	1.598	0.21	0.	0.206	2.359

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	112	69	0	254	0	198	80
normalized size	1	1.	1.3	0.8	0.	2.95	0.	2.3	0.93
time (sec)	N/A	0.494	0.064	0.017	0.	0.233	0.	0.247	21.054

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	72	91	82	247	0	204	68
normalized size	1	1.	1.16	1.47	1.32	3.98	0.	3.29	1.1
time (sec)	N/A	0.122	0.089	0.01	1.581	0.226	0.	0.239	6.876

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	146	123	0	540	0	317	71
normalized size	1	1.	1.92	1.62	0.	7.11	0.	4.17	0.93
time (sec)	N/A	0.189	0.387	0.038	0.	0.228	0.	0.208	31.2

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	21	38	77	0	41	27
normalized size	1	1.	0.67	0.58	1.06	2.14	0.	1.14	0.75
time (sec)	N/A	0.14	0.026	0.01	1.611	0.211	0.	0.208	8.36

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	146	69	0	498	0	200	88
normalized size	1	1.	1.68	0.79	0.	5.72	0.	2.3	1.01
time (sec)	N/A	0.403	0.19	0.027	0.	0.226	0.	0.208	24.027

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	39	38	103	213	0	45	54
normalized size	1	1.	0.67	0.66	1.78	3.67	0.	0.78	0.93
time (sec)	N/A	0.024	0.022	0.005	1.367	0.214	0.	0.203	1.026

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	189	0	36	39
normalized size	1	1.	0.7	0.64	1.7	4.02	0.	0.77	0.83
time (sec)	N/A	0.02	0.02	0.006	1.532	0.212	0.	0.209	0.841

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	69	0	53	41
normalized size	1	1.	0.7	0.64	1.7	1.47	0.	1.13	0.87
time (sec)	N/A	0.02	0.028	0.006	1.434	0.212	0.	0.215	0.847

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	40	0	111	0	81	32
normalized size	1	1.	1.	1.38	0.	3.83	0.	2.79	1.1
time (sec)	N/A	0.067	0.031	0.007	0.	0.201	0.	0.203	4.728

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	56	52	0	209	0	89	54
normalized size	1	1.31	1.24	1.16	0.	4.64	0.	1.98	1.2
time (sec)	N/A	0.067	0.036	0.007	0.	0.215	0.	0.204	3.428

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	59	0	266	0	73	0
normalized size	1	1.	0.71	0.75	0.	3.37	0.	0.92	0.
time (sec)	N/A	0.232	0.062	0.006	0.	0.224	0.	0.212	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	80	0	311	0	142	0
normalized size	1	1.	0.98	1.	0.	3.89	0.	1.78	0.
time (sec)	N/A	0.64	0.041	0.016	0.	0.238	0.	0.224	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	137	140	0	1071	0	0	0
normalized size	1	1.	0.87	0.89	0.	6.78	0.	0.	0.
time (sec)	N/A	1.025	0.172	0.016	0.	0.242	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	51	38	131	39	34
normalized size	1	1.	0.76	0.73	1.24	0.93	3.2	0.95	0.83
time (sec)	N/A	0.027	0.023	0.01	1.632	0.209	4.409	0.201	1.737

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	15	15	10	15	10
normalized size	1	1.	1.	1.38	1.15	1.15	0.77	1.15	0.77
time (sec)	N/A	0.014	0.013	0.004	1.581	0.216	1.621	0.208	0.949

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	109	103	105	418	103	66
normalized size	1	1.	0.83	1.54	1.45	1.48	5.89	1.45	0.93
time (sec)	N/A	0.066	0.036	0.021	1.764	0.222	3.565	0.208	2.712

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	20	38	39	189	66	32
normalized size	1	1.	0.57	0.5	0.95	0.98	4.72	1.65	0.8
time (sec)	N/A	0.023	0.018	0.004	1.534	0.202	3.27	0.2	2.124

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	62	53	70	70	51	0	42
normalized size	1	1.	1.29	1.1	1.46	1.46	1.06	0.	0.88
time (sec)	N/A	0.048	0.047	0.013	1.589	0.223	6.697	0.	3.3

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	36	46	61	34	3305	85	56
normalized size	1	1.	0.52	0.67	0.88	0.49	47.9	1.23	0.81
time (sec)	N/A	0.046	0.019	0.003	1.4	0.217	3.499	0.217	3.334

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	72	130	212	267	44	192	172
normalized size	1	1.	0.37	0.67	1.1	1.38	0.23	0.99	0.89
time (sec)	N/A	0.216	0.045	0.017	1.652	0.229	65.648	0.205	10.044

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	37	12	12	26	12	10
normalized size	1	1.	1.	2.85	0.92	0.92	2.	0.92	0.77
time (sec)	N/A	0.006	0.006	0.037	1.481	0.209	0.818	0.201	0.788

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	37	12	12	12	12	12
normalized size	1	1.	1.	2.85	0.92	0.92	0.92	0.92	0.92
time (sec)	N/A	0.006	0.005	0.009	1.483	0.208	1.023	0.2	0.793

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	74	86	99	42	86	51
normalized size	1	1.	0.73	1.25	1.46	1.68	0.71	1.46	0.86
time (sec)	N/A	0.08	0.027	0.113	1.726	0.226	1.728	0.203	2.343

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	76	89	117	34	90	63
normalized size	1	1.	0.77	1.09	1.27	1.67	0.49	1.29	0.9
time (sec)	N/A	0.078	0.022	0.064	1.621	0.222	3.187	0.208	2.611

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	F(-1)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	35	113	0	41	112	61
normalized size	1	1.	0.68	0.51	1.66	0.	0.6	1.65	0.9
time (sec)	N/A	0.057	0.028	0.066	1.632	0.	1.762	0.214	3.321

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	51	42	177	147	41	149	85
normalized size	1	1.	0.55	0.45	1.9	1.58	0.44	1.6	0.91
time (sec)	N/A	0.086	0.03	0.034	1.631	0.211	4.215	0.215	4.221

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	51	42	176	147	39	147	85
normalized size	1	1.	0.55	0.45	1.89	1.58	0.42	1.58	0.91
time (sec)	N/A	0.082	0.028	0.035	1.691	0.216	2.683	0.211	3.994

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	200	56	15	0	0	0	0	162
normalized size	1	2.15	0.6	0.16	0.	0.	0.	0.	1.74
time (sec)	N/A	0.263	0.028	0.043	0.	0.	0.	0.	8.952

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	114	82	108	212	117	0	0	104
normalized size	1	0.9	0.65	0.86	1.68	0.93	0.	0.	0.83
time (sec)	N/A	0.184	0.171	0.017	1.493	51.65	0.	0.	8.713

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	0	47	113	0	95	32
normalized size	1	1.	1.09	0.	1.38	3.32	0.	2.79	0.94
time (sec)	N/A	0.086	0.038	0.094	1.599	0.212	0.	0.245	4.34

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	242	0	66	58
normalized size	1	1.	0.81	0.	0.	4.17	0.	1.14	1.
time (sec)	N/A	0.078	0.035	0.044	0.	0.206	0.	0.203	3.548

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	60	70	269	0	0	63
normalized size	1	1.	0.83	0.85	0.99	3.79	0.	0.	0.89
time (sec)	N/A	0.127	0.096	0.019	1.544	0.394	0.	0.	7.106

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	59	46	70	80	0	0	22
normalized size	1	1.	2.81	2.19	3.33	3.81	0.	0.	1.05
time (sec)	N/A	0.098	0.086	0.036	1.49	0.206	0.	0.	6.166

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	18	18	36	18	14
normalized size	1	1.	1.	1.29	1.06	1.06	2.12	1.06	0.82
time (sec)	N/A	0.008	0.018	0.007	1.404	0.202	2.137	0.214	2.134

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	116	46	47	226	46	0
normalized size	1	1.	0.87	2.52	1.	1.02	4.91	1.	0.
time (sec)	N/A	0.315	0.028	0.1	1.367	0.203	4.623	0.204	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	112	0	0	274	0	0	102
normalized size	1	1.	1.05	0.	0.	2.56	0.	0.	0.95
time (sec)	N/A	0.15	0.186	0.065	0.	1.926	0.	0.	6.846

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	0	524	0	0	124
normalized size	1	1.	0.94	0.	0.	3.72	0.	0.	0.88
time (sec)	N/A	0.137	0.114	0.05	0.	3.524	0.	0.	7.57

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	29	127	131	71	0	88
normalized size	1	1.	1.44	0.46	2.02	2.08	1.13	0.	1.4
time (sec)	N/A	0.032	0.073	0.044	1.66	0.212	2.785	0.	3.8

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	92	0	0	323	0	0	70
normalized size	1	1.	1.24	0.	0.	4.36	0.	0.	0.95
time (sec)	N/A	0.064	0.161	0.055	0.	4.283	0.	0.	3.686

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	59	26	23	99	54	0	0	49
normalized size	1	1.23	0.54	0.48	2.06	1.12	0.	0.	1.02
time (sec)	N/A	0.038	0.025	0.01	1.41	0.205	0.	0.	3.055

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	489	0	0	116
normalized size	1	1.	0.	0.	0.	3.98	0.	0.	0.94
time (sec)	N/A	0.208	0.13	0.038	0.	6.393	0.	0.	18.569

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	40	112	0	24	0	0	22
normalized size	1	1.	1.74	4.87	0.	1.04	0.	0.	0.96
time (sec)	N/A	0.057	0.06	0.011	0.	0.236	0.	0.	5.596

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	36	112	0	57	0	0	22
normalized size	1	1.	1.57	4.87	0.	2.48	0.	0.	0.96
time (sec)	N/A	0.059	0.058	0.008	0.	0.231	0.	0.	5.654

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	23	23	18	77	66	14	0	17
normalized size	1	1.44	1.44	1.12	4.81	4.12	0.88	0.	1.06
time (sec)	N/A	0.081	0.049	0.013	1.513	0.207	3.015	0.	4.924

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	23	23	18	77	63	14	0	17
normalized size	1	1.44	1.44	1.12	4.81	3.94	0.88	0.	1.06
time (sec)	N/A	0.08	0.046	0.01	1.547	0.203	3.048	0.	4.886

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	0	184	0	63	0	0	26
normalized size	1	1.	0.	7.08	0.	2.42	0.	0.	1.
time (sec)	N/A	0.07	0.241	0.244	0.	0.231	0.	0.	8.178

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	94	188	0	18	0	0	14
normalized size	1	1.	6.27	12.53	0.	1.2	0.	0.	0.93
time (sec)	N/A	0.064	0.104	0.046	0.	0.238	0.	0.	8.014

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	30	19	0	0	12
normalized size	1	1.	1.	1.81	1.88	1.19	0.	0.	0.75
time (sec)	N/A	0.011	0.016	0.009	1.632	0.205	0.	0.	5.126

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	17955	247419	0	1	0	0	76
normalized size	1	1.	242.64	3343.5	0.	0.01	0.	0.	1.03
time (sec)	N/A	0.323	6.296	0.139	0.	0.387	0.	0.	19.633

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	0	0	77	0	0	17
normalized size	1	1.	1.09	0.	0.	3.5	0.	0.	0.77
time (sec)	N/A	0.098	1.384	0.036	0.	0.654	0.	0.	4.061

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	0	0	0	0	0	19
normalized size	1	1.	1.08	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.105	0.052	0.106	0.	0.	0.	0.	3.968

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.012	0.003	0.001	1.404	0.222	0.035	0.216	0.492

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	12	14	8	12	8
normalized size	1	1.	1.36	1.	1.09	1.27	0.73	1.09	0.73
time (sec)	N/A	0.013	0.003	0.003	1.391	0.222	0.056	0.202	0.648

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	26	24	22	24
normalized size	1	1.	0.92	0.75	0.92	1.08	1.	0.92	1.
time (sec)	N/A	0.02	0.003	0.	1.365	0.228	0.038	0.199	0.577

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	34	36	30	36
normalized size	1	1.	0.88	0.71	0.94	1.	1.06	0.88	1.06
time (sec)	N/A	0.033	0.003	0.003	1.447	0.227	0.039	0.2	0.671

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	30	41	42	48	38	48
normalized size	1	1.	0.86	0.68	0.93	0.95	1.09	0.86	1.09
time (sec)	N/A	0.039	0.003	0.053	1.351	0.229	0.042	0.199	0.801

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	64	21	39	31	50	99	19	44
normalized size	1	3.2	1.05	1.95	1.55	2.5	4.95	0.95	2.2
time (sec)	N/A	0.029	0.021	0.01	1.444	0.233	0.947	0.201	0.735

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	31	30	39	31	24
normalized size	1	1.	1.	0.74	1.	0.97	1.26	1.	0.77
time (sec)	N/A	0.021	0.019	0.01	1.368	0.225	0.429	0.2	0.923

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	18	27	50	32	27	32
normalized size	1	1.	1.29	0.86	1.29	2.38	1.52	1.29	1.52
time (sec)	N/A	0.017	0.005	0.043	1.353	0.22	0.041	0.201	0.625

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	95	32	73	126	60	151	42
normalized size	1	1.	2.64	0.89	2.03	3.5	1.67	4.19	1.17
time (sec)	N/A	0.033	0.011	0.054	1.552	0.236	0.214	0.205	0.701

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	57	37	45	54	66	45	66
normalized size	1	1.	1.39	0.9	1.1	1.32	1.61	1.1	1.61
time (sec)	N/A	0.03	0.007	0.052	1.573	0.218	0.043	0.199	1.016

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	69	40	69	95	388	72	31
normalized size	1	1.	1.72	1.	1.72	2.38	9.7	1.8	0.78
time (sec)	N/A	0.022	0.205	0.053	1.463	0.226	2.437	0.204	0.636

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	19	24	24	31	24	17
normalized size	1	1.	1.36	0.86	1.09	1.09	1.41	1.09	0.77
time (sec)	N/A	0.023	0.006	0.004	1.61	0.218	0.061	0.203	0.533

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	16	26	30	54	19	50	20
normalized size	1	1.	0.8	1.3	1.5	2.7	0.95	2.5	1.
time (sec)	N/A	0.024	0.006	0.01	1.446	0.214	0.115	0.208	0.53

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	37	28	41	95	20	72	22
normalized size	1	1.	1.16	0.88	1.28	2.97	0.62	2.25	0.69
time (sec)	N/A	0.025	0.086	0.004	1.544	0.221	0.236	0.209	0.582

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	42	32	50	56	46	58
normalized size	1	1.	0.82	0.75	0.57	0.89	1.	0.82	1.04
time (sec)	N/A	0.098	0.023	0.009	1.365	0.308	0.046	0.2	3.108

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	38	34	34	27	34	27
normalized size	1	1.	1.67	1.15	1.03	1.03	0.82	1.03	0.82
time (sec)	N/A	0.056	0.034	0.012	1.422	0.234	0.053	0.202	3.256

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	54	51	44	51	0
normalized size	1	1.	1.	0.8	1.17	1.11	0.96	1.11	0.
time (sec)	N/A	0.051	0.007	0.016	1.391	0.232	0.101	0.204	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	53	56	50	74	44	35	71
normalized size	1	1.	1.29	1.37	1.22	1.8	1.07	0.85	1.73
time (sec)	N/A	0.059	0.02	0.02	1.417	0.218	0.059	0.205	4.307

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	26	14	14	20
normalized size	1	1.	0.58	0.79	0.58	1.08	0.58	0.58	0.83
time (sec)	N/A	0.045	0.007	0.003	1.588	0.226	0.053	0.202	1.74

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	22	36	22	42	31	22	46
normalized size	1	1.	0.48	0.78	0.48	0.91	0.67	0.48	1.
time (sec)	N/A	0.087	0.01	0.001	1.436	0.218	0.053	0.198	2.988

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	30	52	32	58	46	30	70
normalized size	1	1.	0.44	0.76	0.47	0.85	0.68	0.44	1.03
time (sec)	N/A	0.128	0.016	0.01	1.471	0.231	0.055	0.202	4.293

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	38	68	41	74	61	38	95
normalized size	1	1.	0.42	0.76	0.46	0.82	0.68	0.42	1.06
time (sec)	N/A	0.171	0.022	0.052	1.432	0.268	0.075	0.199	5.705

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	0	0	0	0	0	49
normalized size	1	1.	0.87	0.	0.	0.	0.	0.	0.72
time (sec)	N/A	0.065	0.145	0.397	0.	0.	0.	0.	2.339

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	47	25	55	96	54	178	41
normalized size	1	1.	1.47	0.78	1.72	3.	1.69	5.56	1.28
time (sec)	N/A	0.035	0.027	0.026	1.388	0.226	3.32	0.222	4.056

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	8	19	17	8	5
normalized size	1	1.	1.	1.38	1.	2.38	2.12	1.	0.62
time (sec)	N/A	0.032	0.004	0.013	1.438	0.215	0.048	0.199	1.933

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	57	32	19	30	14	19	0
normalized size	1	1.	5.18	2.91	1.73	2.73	1.27	1.73	0.
time (sec)	N/A	0.025	0.007	0.013	1.454	0.213	0.093	0.2	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.
time (sec)	N/A	0.02	0.003	0.001	1.44	0.25	0.051	0.199	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	71	36	51	92	39	63	31
normalized size	1	1.	2.73	1.38	1.96	3.54	1.5	2.42	1.19
time (sec)	N/A	0.049	0.013	0.017	1.438	0.224	0.174	0.205	3.022

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	19	41	14	24	15
normalized size	1	1.	1.	1.29	1.12	2.41	0.82	1.41	0.88
time (sec)	N/A	0.043	0.008	0.013	1.345	0.216	0.106	0.199	2.496

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	49	26	27	0	31	34
normalized size	1	1.	0.77	1.58	0.84	0.87	0.	1.	1.1
time (sec)	N/A	0.045	0.063	0.125	1.359	0.227	0.	0.211	2.765

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	22	26	18	36	0	18	51
normalized size	1	1.	1.05	1.24	0.86	1.71	0.	0.86	2.43
time (sec)	N/A	0.039	0.042	0.326	1.341	0.227	0.	0.203	5.321

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	95	52	73	126	56	59	46
normalized size	1	1.	2.5	1.37	1.92	3.32	1.47	1.55	1.21
time (sec)	N/A	0.084	0.015	0.018	1.42	0.231	0.209	0.209	4.072

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	76	100	111	0	128	0
normalized size	1	1.	1.08	1.	1.32	1.46	0.	1.68	0.
time (sec)	N/A	0.063	1.255	0.032	1.485	0.239	0.	0.243	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	84	127	186	155	240	0	201	0
normalized size	1	0.95	1.44	2.11	1.76	2.73	0.	2.28	0.
time (sec)	N/A	0.811	3.177	0.253	1.548	0.282	0.	0.207	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	66	73	73	148	73	0
normalized size	1	1.	1.06	0.94	1.04	1.04	2.11	1.04	0.
time (sec)	N/A	0.231	0.024	0.06	1.377	0.221	2.113	0.199	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	33	49	96	39	101	34
normalized size	1	1.	0.88	1.	1.48	2.91	1.18	3.06	1.03
time (sec)	N/A	0.095	0.042	0.005	1.515	0.227	0.98	0.217	8.219

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	20	21	19	35	0	19	0
normalized size	1	1.	1.25	1.31	1.19	2.19	0.	1.19	0.
time (sec)	N/A	0.07	0.015	0.083	1.557	0.229	0.	0.199	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	45	22	28	36	24	31	15
normalized size	1	1.	3.75	1.83	2.33	3.	2.	2.58	1.25
time (sec)	N/A	0.043	0.014	0.027	1.472	0.233	0.843	0.205	1.873

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	24	20	18	12
normalized size	1	1.	1.	0.82	1.06	1.41	1.18	1.06	0.71
time (sec)	N/A	0.016	0.01	0.066	1.377	0.231	0.723	0.198	0.965

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	58	31	73	58	0	51	0
normalized size	1	1.	2.23	1.19	2.81	2.23	0.	1.96	0.
time (sec)	N/A	0.059	0.212	0.032	1.432	0.233	0.	0.209	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	41	42	139	38	0
normalized size	1	1.	1.	0.76	1.08	1.11	3.66	1.	0.
time (sec)	N/A	0.051	0.019	0.07	1.386	0.253	45.846	0.197	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	35	45	58	0	136	29
normalized size	1	1.	1.	1.75	2.25	2.9	0.	6.8	1.45
time (sec)	N/A	0.091	0.013	0.066	1.346	0.23	0.	0.205	23.474

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	19	26	34	14	96	0
normalized size	1	1.	1.	1.58	2.17	2.83	1.17	8.	0.
time (sec)	N/A	0.036	0.011	0.073	1.356	0.235	37.134	0.207	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	39	35	281	96	0	101	0
normalized size	1	1.02	0.61	0.55	4.39	1.5	0.	1.58	0.
time (sec)	N/A	0.137	0.047	0.092	1.596	0.238	0.	0.214	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	17	17	27	124	23	0	32	20
normalized size	1	1.55	1.55	2.45	11.27	2.09	0.	2.91	1.82
time (sec)	N/A	0.059	0.019	0.059	1.349	0.224	0.	0.224	21.456

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	12	12	15	0	12	0
normalized size	1	1.	1.	1.71	1.71	2.14	0.	1.71	0.
time (sec)	N/A	0.063	0.019	0.053	1.521	0.233	0.	0.199	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	83	33	43	36	47	107	105	31
normalized size	1	1.57	0.62	0.81	0.68	0.89	2.02	1.98	0.58
time (sec)	N/A	0.197	0.08	0.075	1.485	0.235	15.03	0.212	1.055

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	94	22	20	53	90	39	50	26
normalized size	1	2.85	0.67	0.61	1.61	2.73	1.18	1.52	0.79
time (sec)	N/A	0.135	0.057	0.052	1.521	0.247	16.543	0.233	0.841

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	9	28	148	27	7
normalized size	1	1.	0.33	0.3	0.33	1.04	5.48	1.	0.26
time (sec)	N/A	0.036	0.016	0.039	1.538	0.223	30.851	0.209	9.273

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	41	29	38	62	102	39	0
normalized size	1	1.	1.46	1.04	1.36	2.21	3.64	1.39	0.
time (sec)	N/A	0.083	0.066	0.14	1.548	0.222	0.928	0.207	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	39	51	90	0	105	0
normalized size	1	1.	0.87	0.58	0.76	1.34	0.	1.57	0.
time (sec)	N/A	0.084	0.169	0.083	1.527	0.228	0.	0.204	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	54	29	38	84	0	82	29
normalized size	1	1.	0.98	0.53	0.69	1.53	0.	1.49	0.53
time (sec)	N/A	0.064	0.121	0.05	1.54	0.221	0.	0.216	21.427

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	70	43	59	104	252	53	39
normalized size	1	1.	1.67	1.02	1.4	2.48	6.	1.26	0.93
time (sec)	N/A	0.15	0.262	0.031	1.579	0.227	0.994	0.212	9.09

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	23	20	0	26	76	28	0
normalized size	1	1.	2.56	2.22	0.	2.89	8.44	3.11	0.
time (sec)	N/A	0.037	0.013	0.059	0.	0.233	11.965	0.222	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	174	46	0	66	17
normalized size	1	1.	11.6	0.87	11.6	3.07	0.	4.4	1.13
time (sec)	N/A	0.027	0.582	0.026	1.51	0.221	0.	0.243	1.877

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	28	19	173	35	22	27	0
normalized size	1	1.	1.65	1.12	10.18	2.06	1.29	1.59	0.
time (sec)	N/A	0.069	0.011	0.051	1.525	0.256	1.807	0.226	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	109	26	0	120	19
normalized size	1	1.	1.	0.86	5.19	1.24	0.	5.71	0.9
time (sec)	N/A	0.084	0.015	0.072	1.62	0.261	0.	0.214	102.15

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	174	26	17	41	19
normalized size	1	1.	1.	1.62	8.29	1.24	0.81	1.95	0.9
time (sec)	N/A	0.047	0.017	0.076	1.523	0.217	1.336	0.205	1.956

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	218	28	231	82	0	65	19
normalized size	1	1.	8.38	1.08	8.88	3.15	0.	2.5	0.73
time (sec)	N/A	0.043	0.519	0.082	1.554	0.23	0.	0.216	2.307

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	218	28	231	82	0	65	0
normalized size	1	1.	8.38	1.08	8.88	3.15	0.	2.5	0.
time (sec)	N/A	0.063	0.494	0.095	1.598	0.234	0.	0.22	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	22	73	46	0	0	15
normalized size	1	1.	1.56	1.38	4.56	2.88	0.	0.	0.94
time (sec)	N/A	0.016	0.025	0.045	1.581	0.212	0.	0.	0.51

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	31	74	47	0	0	14
normalized size	1	1.	1.59	1.82	4.35	2.76	0.	0.	0.82
time (sec)	N/A	0.021	0.025	0.059	1.527	0.217	0.	0.	0.569

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	9	26	55	0	0	29
normalized size	1	1.	1.74	0.33	0.96	2.04	0.	0.	1.07
time (sec)	N/A	0.023	0.026	0.033	1.738	0.217	0.	0.	0.575

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	17	51	72	0	0	31
normalized size	1	1.	1.1	0.57	1.7	2.4	0.	0.	1.03
time (sec)	N/A	0.028	0.026	0.055	1.771	0.214	0.	0.	0.608

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	61	52	103	144	0	80	48
normalized size	1	1.	1.15	0.98	1.94	2.72	0.	1.51	0.91
time (sec)	N/A	0.049	0.186	0.081	1.616	0.216	0.	0.256	0.797

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-1)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	76	47	0	96	0	0	65
normalized size	1	1.	1.04	0.64	0.	1.32	0.	0.	0.89
time (sec)	N/A	0.062	0.132	0.069	0.	0.215	0.	0.	0.871

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	0	58	54	0	0	0
normalized size	1	1.	0.91	0.	1.05	0.98	0.	0.	0.
time (sec)	N/A	0.242	0.206	0.631	6.595	0.223	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	82	49	108	246	0	108	94
normalized size	1	1.	0.84	0.5	1.1	2.51	0.	1.1	0.96
time (sec)	N/A	0.129	0.1	0.004	1.569	0.251	0.	0.206	7.934

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	69	121	53	70	70	0	70	63
normalized size	1	1.21	2.12	0.93	1.23	1.23	0.	1.23	1.11
time (sec)	N/A	0.112	0.133	0.015	1.526	0.227	0.	0.204	5.31

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	130	4338	618	0	0	92
normalized size	1	1.	0.95	1.49	49.86	7.1	0.	0.	1.06
time (sec)	N/A	0.182	0.342	0.072	2.517	0.237	0.	0.	6.778

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	38	227	41	111	0	42	36
normalized size	1	1.	0.95	5.68	1.02	2.78	0.	1.05	0.9
time (sec)	N/A	0.224	1.432	0.564	1.354	0.241	0.	0.218	7.678

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	133	155	97	158	0	0	150	148
normalized size	1	1.58	1.85	1.15	1.88	0.	0.	1.79	1.76
time (sec)	N/A	0.611	0.795	0.028	1.534	0.	0.	0.22	25.611

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	266	0	342	0	0	27
normalized size	1	1.	1.	8.58	0.	11.03	0.	0.	0.87
time (sec)	N/A	0.027	0.05	0.109	0.	0.257	0.	0.	1.41

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	342	0	0	27
normalized size	1	1.	0.94	3.16	0.	11.03	0.	0.	0.87
time (sec)	N/A	0.027	0.034	0.112	0.	0.253	0.	0.	1.383

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	171	1	360	0	0	41
normalized size	1	1.	0.91	3.8	0.02	8.	0.	0.	0.91
time (sec)	N/A	0.048	0.045	0.127	12.227	0.26	0.	0.	2.275

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	442	1	181	0	0	0
normalized size	1	1.	0.91	9.4	0.02	3.85	0.	0.	0.
time (sec)	N/A	0.145	0.067	0.207	20.906	0.248	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	510	0	409	0	0	56
normalized size	1	1.	0.82	8.36	0.	6.7	0.	0.	0.92
time (sec)	N/A	0.124	0.125	0.183	0.	0.27	0.	0.	5.613

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	1108	0	452	0	0	56
normalized size	1	1.	0.92	18.16	0.	7.41	0.	0.	0.92
time (sec)	N/A	0.124	0.102	0.199	0.	0.296	0.	0.	5.741

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	508	2002	38	0	0	15
normalized size	1	1.	1.	31.75	125.12	2.38	0.	0.	0.94
time (sec)	N/A	0.034	0.037	0.167	2.209	0.225	0.	0.	1.909

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	286	2726	32	0	0	29
normalized size	1	1.	0.65	9.23	87.94	1.03	0.	0.	0.94
time (sec)	N/A	0.061	0.033	0.153	3.194	0.225	0.	0.	3.29

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	121	1446	42	0	0	29
normalized size	1	1.	0.83	4.17	49.86	1.45	0.	0.	1.
time (sec)	N/A	0.069	0.039	0.098	2.033	0.224	0.	0.	3.443

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	95	150	761	0	173	0	0	0
normalized size	1	1.4	2.21	11.19	0.	2.54	0.	0.	0.
time (sec)	N/A	1.403	7.452	0.401	0.	0.257	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	29	19	16	8	20	0	0	20
normalized size	1	1.53	1.	0.84	0.42	1.05	0.	0.	1.05
time (sec)	N/A	0.221	0.013	0.201	1.545	0.222	0.	0.	1.652

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	204	66	310	127	1338	0	0	257
normalized size	1	2.22	0.72	3.37	1.38	14.54	0.	0.	2.79
time (sec)	N/A	0.422	0.108	0.207	1.598	0.74	0.	0.	9.695

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	29	0	18	39	0	0	44
normalized size	1	1.	0.62	0.	0.38	0.83	0.	0.	0.94
time (sec)	N/A	0.234	0.035	0.542	1.815	0.214	0.	0.	1.758

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	35	0	104	136	0	0	51
normalized size	1	1.	0.5	0.	1.49	1.94	0.	0.	0.73
time (sec)	N/A	0.317	0.087	0.441	1.601	0.222	0.	0.	1.798

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	C	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	234	105	244	258	761	0	0	0
normalized size	1	2.17	0.97	2.26	2.39	7.05	0.	0.	0.
time (sec)	N/A	2.769	0.364	0.488	1.758	0.531	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	F(-2)	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	364	665	2057	22888	159	0	0	0	0
normalized size	1	1.83	5.65	62.88	0.44	0.	0.	0.	0.
time (sec)	N/A	8.699	20.813	4.821	1.628	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	141	58	0	81	76	0	0	0
normalized size	1	1.13	0.46	0.	0.65	0.61	0.	0.	0.
time (sec)	N/A	1.708	0.454	0.808	1.701	0.231	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	67	61	56	74	147	0	74	73
normalized size	1	0.92	0.84	0.77	1.01	2.01	0.	1.01	1.
time (sec)	N/A	0.076	0.179	0.02	1.532	0.253	0.	0.22	2.845

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	55	103	72	162	0	55	70
normalized size	1	1.	0.8	1.49	1.04	2.35	0.	0.8	1.01
time (sec)	N/A	0.085	0.127	0.136	1.557	0.283	0.	0.212	179.512

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	82	49	528	0	41	61
normalized size	1	1.	0.86	1.41	0.84	9.1	0.	0.71	1.05
time (sec)	N/A	0.083	0.108	0.135	1.596	0.22	0.	0.211	163.063

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	37	44	58	69	0	41	49
normalized size	1	1.	0.67	0.8	1.05	1.25	0.	0.75	0.89
time (sec)	N/A	0.084	0.113	0.043	1.352	0.25	0.	0.213	90.338

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	54	58	967	127	0	51	0
normalized size	1	1.	1.38	1.49	24.79	3.26	0.	1.31	0.
time (sec)	N/A	0.112	0.291	0.164	1.786	0.245	0.	0.217	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	63	53	93	217	0	54	0
normalized size	1	1.	1.31	1.1	1.94	4.52	0.	1.12	0.
time (sec)	N/A	0.124	0.413	0.106	1.53	0.252	0.	0.212	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	46	259	1	0	45	0
normalized size	1	1.	1.2	0.94	5.29	0.02	0.	0.92	0.
time (sec)	N/A	0.178	0.506	0.058	1.444	0.238	0.	0.204	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	338	130	155	1	0	356	0
normalized size	1	1.07	3.05	1.17	1.4	0.01	0.	3.21	0.
time (sec)	N/A	1.	3.772	0.266	1.551	1.422	0.	0.435	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	131	98136	0	340	0	0	0
normalized size	1	1.	1.17	876.21	0.	3.04	0.	0.	0.
time (sec)	N/A	0.819	0.351	4.377	0.	0.277	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	64	38	34	35	0	0	0
normalized size	1	1.	1.94	1.15	1.03	1.06	0.	0.	0.
time (sec)	N/A	0.111	0.159	0.142	1.483	0.233	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	62	659	96	0	36	0
normalized size	1	1.	1.	1.88	19.97	2.91	0.	1.09	0.
time (sec)	N/A	0.038	0.071	0.086	1.951	0.238	0.	0.208	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	1067	142	0	65	65
normalized size	1	1.	0.89	1.	19.4	2.58	0.	1.18	1.18
time (sec)	N/A	0.057	0.121	0.06	2.282	0.243	0.	0.218	112.273

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	39	122	53	0	62	15
normalized size	1	1.	1.	2.44	7.62	3.31	0.	3.88	0.94
time (sec)	N/A	0.024	0.037	0.106	1.993	0.245	0.	0.262	1.318

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	100	0	147	0	0	0
normalized size	1	1.	1.	2.04	0.	3.	0.	0.	0.
time (sec)	N/A	0.144	0.119	0.111	0.	0.248	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	91	62	180	1832	223	0	74	0
normalized size	1	1.05	0.71	2.07	21.06	2.56	0.	0.85	0.
time (sec)	N/A	0.364	0.238	0.233	2.231	5.995	0.	0.225	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-1)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	115	754	0	279	0	0	73
normalized size	1	1.	1.69	11.09	0.	4.1	0.	0.	1.07
time (sec)	N/A	0.129	0.278	0.572	0.	0.24	0.	0.	8.194

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	64	473	0	207	0	0	41
normalized size	1	1.	1.6	11.82	0.	5.18	0.	0.	1.02
time (sec)	N/A	0.064	0.119	0.316	0.	0.257	0.	0.	4.079

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	975	0	131	0	0	0
normalized size	1	1.	0.8	10.37	0.	1.39	0.	0.	0.
time (sec)	N/A	0.353	0.515	0.76	0.	0.296	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	61	85	45	0	0	0
normalized size	1	1.	0.85	1.56	2.18	1.15	0.	0.	0.
time (sec)	N/A	0.267	0.123	0.639	2.945	0.228	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	234	1597	0	358	0	231	0
normalized size	1	1.	3.21	21.88	0.	4.9	0.	3.16	0.
time (sec)	N/A	2.212	2.171	0.345	0.	0.314	0.	0.325	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	116	117	63	113	0	0	0
normalized size	1	1.	2.04	2.05	1.11	1.98	0.	0.	0.
time (sec)	N/A	1.36	1.365	0.52	1.524	0.239	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	82	61	0	68	0	70	58
normalized size	1	1.	1.24	0.92	0.	1.03	0.	1.06	0.88
time (sec)	N/A	0.138	0.593	0.033	0.	0.247	0.	0.205	7.642

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	71	41	0	103	0	49	46
normalized size	1	1.	1.31	0.76	0.	1.91	0.	0.91	0.85
time (sec)	N/A	0.12	0.536	0.026	0.	0.25	0.	0.203	6.855

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	90	0	14	0	0	0	167
normalized size	1	1.	0.68	0.	0.11	0.	0.	0.	1.26
time (sec)	N/A	0.29	1.5	0.152	15.211	0.	0.	0.	14.906

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-1)	F(-1)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	42	0	0	0	0	107	70
normalized size	1	1.	0.61	0.	0.	0.	0.	1.55	1.01
time (sec)	N/A	0.162	0.102	0.066	0.	0.	0.	0.231	6.7

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	84	0	96	0	0	0	0
normalized size	1	1.	1.62	0.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.138	0.229	0.144	1.546	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	85	0	100	0	0	0	0
normalized size	1	1.	1.57	0.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.143	0.228	0.132	1.495	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	174	4397	0	0	0	0	0	0
normalized size	1	1.31	33.06	0.	0.	0.	0.	0.	0.
time (sec)	N/A	8.932	71.146	4.865	0.	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	C	B	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	208	207	559	0	498	0	265	0
normalized size	1	2.08	2.07	5.59	0.	4.98	0.	2.65	0.
time (sec)	N/A	1.927	2.553	0.748	0.	0.337	0.	0.337	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	71	137	184	224	0	0	0
normalized size	1	1.	0.63	1.22	1.64	2.	0.	0.	0.
time (sec)	N/A	0.288	0.321	0.042	1.791	0.267	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	162	924	0	196	0	0	197	0
normalized size	1	1.71	9.73	0.	2.06	0.	0.	2.07	0.
time (sec)	N/A	0.473	25.416	0.145	1.52	0.	0.	0.242	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	0	50	62	0	50	0
normalized size	1	1.	0.61	0.	1.02	1.27	0.	1.02	0.
time (sec)	N/A	0.162	23.791	0.272	1.356	0.29	0.	0.204	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	42	26	116	47	0	34	19
normalized size	1	1.	2.1	1.3	5.8	2.35	0.	1.7	0.95
time (sec)	N/A	0.343	0.179	0.033	1.554	0.229	0.	0.222	11.146

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	35	93	0	0	126	0	55	0
normalized size	1	1.3	3.44	0.	0.	4.67	0.	2.04	0.
time (sec)	N/A	1.632	4.673	0.471	0.	1.291	0.	0.23	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	126	158	0	138	687	0	0	0
normalized size	1	1.25	1.56	0.	1.37	6.8	0.	0.	0.
time (sec)	N/A	2.302	4.498	0.892	1.552	62.177	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	140	0	1392	35	0	34	0
normalized size	1	1.	5.6	0.	55.68	1.4	0.	1.36	0.
time (sec)	N/A	0.098	0.519	0.254	1.965	0.223	0.	0.22	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	154	59	0	0	0	0	0	0
normalized size	1	1.51	0.58	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	2.74	0.352	0.	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	18	45	88	350	85	0	115	0
normalized size	1	1.06	2.65	5.18	20.59	5.	0.	6.76	0.
time (sec)	N/A	0.053	0.079	0.158	1.68	0.241	0.	0.25	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	240	0	107	0	0	0
normalized size	1	1.	1.62	7.5	0.	3.34	0.	0.	0.
time (sec)	N/A	0.079	0.075	0.451	0.	0.234	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	36	41	24	43	27
normalized size	1	1.	1.	0.77	1.16	1.32	0.77	1.39	0.87
time (sec)	N/A	0.034	0.006	0.011	1.429	0.429	0.132	0.202	2.502

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	41	50	34	41	34
normalized size	1	1.	1.	0.74	1.05	1.28	0.87	1.05	0.87
time (sec)	N/A	0.036	0.021	0.008	1.585	0.2	0.16	0.203	3.248

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	60	62	99	41	57	42
normalized size	1	1.	0.69	1.03	1.07	1.71	0.71	0.98	0.72
time (sec)	N/A	0.062	0.026	0.02	1.335	0.198	0.199	0.205	3.242

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	37	45	212	986	47	46
normalized size	1	1.	0.88	0.71	0.87	4.08	18.96	0.9	0.88
time (sec)	N/A	0.053	0.076	0.01	1.532	0.209	10.665	0.204	2.724

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	46	57	217	167	62	54
normalized size	1	1.	0.77	0.75	0.93	3.56	2.74	1.02	0.89
time (sec)	N/A	0.069	0.032	0.009	1.49	0.21	21.991	0.202	3.163

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	0	20	19	19	10
normalized size	1	1.	0.94	0.94	0.	1.25	1.19	1.19	0.62
time (sec)	N/A	0.008	0.004	0.003	0.	0.22	0.038	0.198	0.64

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	33	47	66	32	49	32
normalized size	1	1.	0.78	0.72	1.02	1.43	0.7	1.07	0.7
time (sec)	N/A	0.05	0.02	0.013	1.343	0.198	0.138	0.199	2.839

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	58	49	49	34	28	29
normalized size	1	1.	0.57	1.45	1.22	1.22	0.85	0.7	0.72
time (sec)	N/A	0.077	0.013	0.014	1.36	0.194	0.194	0.201	5.077

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	26	28	41	26	22
normalized size	1	1.	0.74	0.63	0.96	1.04	1.52	0.96	0.81
time (sec)	N/A	0.027	0.008	0.006	1.328	0.203	74.162	0.201	1.799

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	28	38	28	82	35	32
normalized size	1	1.	0.66	0.74	1.	0.74	2.16	0.92	0.84
time (sec)	N/A	0.038	0.015	0.005	1.342	0.202	11.751	0.201	2.269

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	34	112	61	26	27
normalized size	1	1.	0.7	0.61	1.03	3.39	1.85	0.79	0.82
time (sec)	N/A	0.012	0.013	0.004	1.343	0.202	14.553	0.202	0.597

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	26	23	69	123	0	28	36
normalized size	1	1.	0.6	0.53	1.6	2.86	0.	0.65	0.84
time (sec)	N/A	0.016	0.018	0.003	1.397	0.203	0.	0.21	0.741

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	28	23	47	76	153	84	39
normalized size	1	1.	0.6	0.49	1.	1.62	3.26	1.79	0.83
time (sec)	N/A	0.028	0.015	0.005	1.52	0.204	9.773	0.234	2.015

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	16	22	20	24	22	24
normalized size	1	1.	0.68	0.57	0.79	0.71	0.86	0.79	0.86
time (sec)	N/A	0.015	0.008	0.005	1.397	0.199	17.424	0.211	1.482

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	47	72	100	49	78	49
normalized size	1	1.	0.94	0.9	1.38	1.92	0.94	1.5	0.94
time (sec)	N/A	0.057	0.051	0.019	1.361	0.197	0.243	0.229	3.157

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	28	0	0	24
normalized size	1	1.	1.	0.88	0.	1.12	0.	0.	0.96
time (sec)	N/A	0.086	0.02	0.006	0.	0.414	0.	0.	4.459

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	35	28	107	0	0	132	37
normalized size	1	1.	0.7	0.56	2.14	0.	0.	2.64	0.74
time (sec)	N/A	0.062	0.083	0.047	1.501	0.	0.	0.292	2.232

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	28	55	49	39	19
normalized size	1	1.	1.	0.92	1.17	2.29	2.04	1.62	0.79
time (sec)	N/A	0.019	0.033	0.008	1.512	0.212	0.985	0.209	1.62

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	58	61	167	32	104	27
normalized size	1	1.	0.85	1.45	1.52	4.18	0.8	2.6	0.68
time (sec)	N/A	0.027	0.029	0.008	1.552	0.212	4.153	0.239	2.415

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	50	165	0	47	42
normalized size	1	1.	0.57	0.51	1.02	3.37	0.	0.96	0.86
time (sec)	N/A	0.019	0.02	0.005	1.429	0.206	0.	0.208	0.761

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	29	28	80	63	0	47	37
normalized size	1	1.	0.62	0.6	1.7	1.34	0.	1.	0.79
time (sec)	N/A	0.019	0.027	0.005	1.495	0.211	0.	0.244	0.821

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	43	1	34	43	0
normalized size	1	1.	1.	0.92	1.19	0.03	0.94	1.19	0.
time (sec)	N/A	0.025	0.002	0.003	1.767	0.174	0.038	0.204	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	39	1	34	39	0
normalized size	1	1.	1.	0.77	1.	0.03	0.87	1.	0.
time (sec)	N/A	0.035	0.002	0.003	1.767	0.176	0.043	0.2	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	186	0	36	39
normalized size	1	1.	0.7	0.64	1.7	3.96	0.	0.77	0.83
time (sec)	N/A	0.028	0.027	0.006	1.386	0.21	0.	0.21	2.307

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	30	103	186	0	55	58
normalized size	1	1.	0.73	0.67	2.29	4.13	0.	1.22	1.29
time (sec)	N/A	0.042	0.039	0.009	1.351	0.206	0.	0.214	6.49

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	70	74	77	112	74	90
normalized size	1	1.	0.81	0.84	0.89	0.93	1.35	0.89	1.08
time (sec)	N/A	0.137	0.065	0.02	1.444	0.226	5.722	0.237	4.033

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	57	66	70	92	66	78
normalized size	1	1.	0.7	0.78	0.9	0.96	1.26	0.9	1.07
time (sec)	N/A	0.124	0.139	0.037	1.348	0.222	3.179	0.224	3.742

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	96	89	97	192	89	114
normalized size	1	1.	0.67	0.91	0.85	0.92	1.83	0.85	1.09
time (sec)	N/A	0.165	0.116	0.043	1.371	0.229	9.788	0.234	4.215

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	32	47	49	53	47	46
normalized size	1	1.	0.89	0.73	1.07	1.11	1.2	1.07	1.05
time (sec)	N/A	0.064	0.2	0.006	1.36	0.22	1.688	0.197	2.848

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	111	0	61	507	278	0
normalized size	1	1.	1.	3.36	0.	1.85	15.36	8.42	0.
time (sec)	N/A	0.075	0.014	0.16	0.	0.235	4.868	0.229	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	104	30	836	63	551	460	0
normalized size	1	1.	3.47	1.	27.87	2.1	18.37	15.33	0.
time (sec)	N/A	0.061	0.183	0.174	1.584	0.229	3.746	0.437	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	178	20	128	72	15
normalized size	1	1.	1.	0.81	11.12	1.25	8.	4.5	0.94
time (sec)	N/A	0.029	0.008	0.009	1.338	0.237	2.002	0.204	1.402

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	55	89	304	0	0	0
normalized size	1	1.	0.92	0.89	1.44	4.9	0.	0.	0.
time (sec)	N/A	0.11	0.014	0.077	2.521	0.283	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	59	288	338	0	0	48
normalized size	1	1.	0.92	1.	4.88	5.73	0.	0.	0.81
time (sec)	N/A	0.099	0.013	0.116	1.897	0.278	0.	0.	13.223

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	44	105	18	0	14	0
normalized size	1	1.	1.17	3.67	8.75	1.5	0.	1.17	0.
time (sec)	N/A	0.163	0.054	0.879	3.325	0.224	0.	0.232	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	37	93	26	66	53	0
normalized size	1	1.	0.95	1.85	4.65	1.3	3.3	2.65	0.
time (sec)	N/A	0.056	0.063	0.29	1.374	0.211	4.858	0.202	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	30	42	439	22
normalized size	1	1.	1.	1.05	0.	1.36	1.91	19.95	1.
time (sec)	N/A	0.044	0.007	0.009	0.	0.218	1.026	0.224	2.163

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	41	40	65	0	70	0	589	0
normalized size	1	1.21	1.18	1.91	0.	2.06	0.	17.32	0.
time (sec)	N/A	0.319	0.054	0.026	0.	0.211	0.	0.295	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	15	7	9	7
normalized size	1	1.	1.	0.89	1.	1.67	0.78	1.	0.78
time (sec)	N/A	0.007	0.006	0.002	1.363	0.204	0.071	0.222	0.638

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	22	28	17	32	20
normalized size	1	1.	1.	0.86	1.	1.27	0.77	1.45	0.91
time (sec)	N/A	0.039	0.009	0.008	1.384	0.206	0.102	0.219	3.535

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	24	31	32	24	34	24
normalized size	1	1.	0.97	0.77	1.	1.03	0.77	1.1	0.77
time (sec)	N/A	0.038	0.012	0.009	1.355	0.201	0.131	0.2	3.536

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	31	38	42	31	49	0
normalized size	1	1.	0.94	0.86	1.06	1.17	0.86	1.36	0.
time (sec)	N/A	0.052	0.023	0.01	1.352	0.204	0.154	0.198	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	52	52	0	0	0	0	0	48
normalized size	1	1.08	1.08	0.	0.	0.	0.	0.	1.
time (sec)	N/A	0.086	0.038	0.171	0.	0.	0.	0.	5.603

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	56	50	53	54	62	44
normalized size	1	1.	0.77	1.3	1.16	1.23	1.26	1.44	1.02
time (sec)	N/A	0.058	0.046	0.03	1.375	0.217	0.297	0.203	4.755

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	36	35	29	36	19
normalized size	1	1.	1.	1.04	1.33	1.3	1.07	1.33	0.7
time (sec)	N/A	0.024	0.009	0.004	1.342	0.215	0.566	0.195	1.022

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	59	69	84	250	938	0
normalized size	1	1.	1.04	1.11	1.3	1.58	4.72	17.7	0.
time (sec)	N/A	0.135	0.059	0.029	1.389	0.219	3.385	0.241	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	90	104	176	665	1	0
normalized size	1	1.	0.82	1.14	1.32	2.23	8.42	0.01	0.
time (sec)	N/A	0.177	0.135	0.049	1.368	0.219	56.725	0.251	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	134	277	0	1	0
normalized size	1	1.	0.82	1.11	1.37	2.83	0.	0.01	0.
time (sec)	N/A	0.225	0.119	0.027	1.347	0.223	0.	0.248	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	0	0	0	0	0	0	65
normalized size	1	1.11	0.	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.134	0.031	0.077	0.	0.	0.	0.	7.442

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	38	29	38	19
normalized size	1	1.	1.	1.04	1.36	1.36	1.04	1.36	0.68
time (sec)	N/A	0.021	0.011	0.002	1.38	0.216	0.558	0.198	1.139

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	59	69	86	248	938	0
normalized size	1	1.	1.04	1.11	1.3	1.62	4.68	17.7	0.
time (sec)	N/A	0.128	0.057	0.023	1.39	0.218	3.471	0.256	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	90	104	177	663	1	0
normalized size	1	1.	0.84	1.14	1.32	2.24	8.39	0.01	0.
time (sec)	N/A	0.187	0.133	0.04	1.341	0.219	56.931	0.236	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	134	279	0	1	0
normalized size	1	1.	0.82	1.11	1.37	2.85	0.	0.01	0.
time (sec)	N/A	0.208	0.11	0.026	1.347	0.216	0.	0.267	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	82	0	0	0	0	0	0	63
normalized size	1	1.11	0.	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.126	0.037	0.082	0.	0.	0.	0.	7.669

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	26	15	20	10
normalized size	1	1.	1.	1.07	1.33	1.73	1.	1.33	0.67
time (sec)	N/A	0.013	0.007	0.003	1.354	0.211	0.08	0.196	0.804

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	46	42	39	46	41	0
normalized size	1	1.	1.	1.39	1.27	1.18	1.39	1.24	0.
time (sec)	N/A	0.028	0.015	0.013	1.373	0.209	0.117	0.197	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	62	62	53	71	54	0
normalized size	1	1.	0.82	1.24	1.24	1.06	1.42	1.08	0.
time (sec)	N/A	0.036	0.019	0.003	1.377	0.241	0.148	0.2	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	78	82	63	88	65	0
normalized size	1	1.	0.75	1.2	1.26	0.97	1.35	1.	0.
time (sec)	N/A	0.043	0.023	0.004	1.345	0.217	0.174	0.206	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0	31
normalized size	1	1.	1.3	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.042	0.029	0.043	0.	0.	0.	0.	2.333

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	28	19	22	10
normalized size	1	1.	1.	1.06	1.38	1.75	1.19	1.38	0.62
time (sec)	N/A	0.012	0.007	0.001	1.345	0.223	0.085	0.196	0.911

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	46	42	39	46	41	0
normalized size	1	1.	0.76	1.39	1.27	1.18	1.39	1.24	0.
time (sec)	N/A	0.028	0.022	0.003	1.344	0.215	0.126	0.213	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	35	62	62	53	71	54	0
normalized size	1	1.	0.7	1.24	1.24	1.06	1.42	1.08	0.
time (sec)	N/A	0.036	0.041	0.004	1.392	0.214	0.156	0.199	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	78	82	63	88	65	0
normalized size	1	1.	0.66	1.2	1.26	0.97	1.35	1.	0.
time (sec)	N/A	0.044	0.038	0.002	1.399	0.207	0.185	0.231	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	54	0	0	0	0	0	31
normalized size	1	1.	1.23	0.	0.	0.	0.	0.	0.7
time (sec)	N/A	0.04	0.032	0.046	0.	0.	0.	0.	2.618

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	31	30	15	32	22
normalized size	1	1.	1.	1.29	1.29	1.25	0.62	1.33	0.92
time (sec)	N/A	0.032	0.008	0.013	1.335	0.234	0.132	0.2	3.138

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	123	36	95	0	126	22	157	116
normalized size	1	1.23	0.36	0.95	0.	1.26	0.22	1.57	1.16
time (sec)	N/A	0.172	0.015	0.01	0.	0.221	0.199	0.2	14.472

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	15	15	8	15	15
normalized size	1	1.	1.	1.17	1.25	1.25	0.67	1.25	1.25
time (sec)	N/A	0.042	0.005	0.009	1.363	0.219	0.066	0.2	4.748

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	38	50	58	22	50	42
normalized size	1	1.	0.94	0.81	1.06	1.23	0.47	1.06	0.89
time (sec)	N/A	0.1	0.028	0.009	1.537	0.219	0.153	0.198	11.986

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	38	38	48	39	0
normalized size	1	1.	1.	0.74	0.97	0.97	1.23	1.	0.
time (sec)	N/A	0.105	0.032	0.019	1.573	0.231	0.24	0.199	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	50	94	43	20
normalized size	1	1.	1.	1.1	0.	1.67	3.13	1.43	0.67
time (sec)	N/A	0.066	0.048	0.003	0.	0.228	2.05	0.203	5.209

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	70	57	76	76	0	77	42
normalized size	1	1.	1.3	1.06	1.41	1.41	0.	1.43	0.78
time (sec)	N/A	0.051	0.057	0.01	1.568	0.223	0.	0.21	2.533

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	0	0	0	0	0	41
normalized size	1	1.	1.29	0.	0.	0.	0.	0.	0.69
time (sec)	N/A	0.063	0.051	0.04	0.	0.	0.	0.	3.508

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	24	31	24	15
normalized size	1	1.	1.	0.83	0.67	1.33	1.72	1.33	0.83
time (sec)	N/A	0.046	0.017	0.013	1.439	0.217	0.579	0.208	3.55

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	27	27	34	27	15
normalized size	1	1.	1.	0.85	1.35	1.35	1.7	1.35	0.75
time (sec)	N/A	0.048	0.019	0.015	1.355	0.214	0.593	0.229	3.883

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	55	0	53	62	0	65	36
normalized size	1	1.	1.38	0.	1.32	1.55	0.	1.62	0.9
time (sec)	N/A	0.167	0.041	0.029	1.653	0.225	0.	0.245	24.241

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	57	68	0	0	0	0	0	46
normalized size	1	1.54	1.84	0.	0.	0.	0.	0.	1.24
time (sec)	N/A	0.094	0.056	0.033	0.	0.	0.	0.	6.562

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	37	66	41	0	88	49
normalized size	1	1.	0.6	0.51	0.9	0.56	0.	1.21	0.67
time (sec)	N/A	0.081	0.03	0.027	1.483	0.216	0.	0.219	5.634

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	23	22	26	26	20	26	36
normalized size	1	1.	0.52	0.5	0.59	0.59	0.45	0.59	0.82
time (sec)	N/A	0.054	0.005	0.003	1.376	0.214	0.076	0.213	3.041

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	26	31	9	31	32	36	29
normalized size	1	1.	0.67	0.79	0.23	0.79	0.82	0.92	0.74
time (sec)	N/A	0.054	0.02	0.006	2.151	0.202	2.319	0.225	3.297

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	36	36	39	1	41
normalized size	1	1.	0.66	0.64	0.82	0.82	0.89	0.02	0.93
time (sec)	N/A	0.038	0.008	0.01	1.508	0.211	0.109	0.222	2.838

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	24	12	8	24	8
normalized size	1	1.	1.	0.83	2.	1.	0.67	2.	0.67
time (sec)	N/A	0.072	0.004	0.005	1.407	0.216	0.073	0.212	3.653

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	26	34	45	24	54	31
normalized size	1	1.	0.97	0.81	1.06	1.41	0.75	1.69	0.97
time (sec)	N/A	0.087	0.03	0.016	1.576	0.217	0.101	0.22	5.675

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	0	20	28	16	0	0	10
normalized size	1	1.	0.	1.33	1.87	1.07	0.	0.	0.67
time (sec)	N/A	0.091	0.174	0.007	1.544	0.21	0.	0.	5.639

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	28	26	26	26
normalized size	1	1.	0.81	0.81	0.96	1.04	0.96	0.96	0.96
time (sec)	N/A	0.02	0.018	0.01	1.375	0.211	0.81	0.199	1.498

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	32	53	28	29	53	0
normalized size	1	1.	0.74	0.91	1.51	0.8	0.83	1.51	0.
time (sec)	N/A	0.171	0.022	0.079	1.446	0.23	1.476	0.232	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	37	55	39	0	70	51	63
normalized size	1	1.	0.65	0.96	0.68	0.	1.23	0.89	1.11
time (sec)	N/A	0.05	0.064	0.072	1.574	0.	4.746	0.215	2.677

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	45	61	50	269	58	48
normalized size	1	1.	0.72	0.83	1.13	0.93	4.98	1.07	0.89
time (sec)	N/A	0.045	0.037	0.019	1.379	0.216	5.796	0.229	3.276

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	68	99	88	651	85	78
normalized size	1	1.	0.78	0.83	1.21	1.07	7.94	1.04	0.95
time (sec)	N/A	0.069	0.291	0.016	1.412	0.216	25.722	0.217	5.114

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	36	62	36	57	76	45	68
normalized size	1	1.	0.46	0.78	0.46	0.72	0.96	0.57	0.86
time (sec)	N/A	0.075	0.029	0.147	1.388	0.25	4.234	0.233	4.583

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	28	36	54	70	32	29
normalized size	1	1.	0.58	0.78	1.	1.5	1.94	0.89	0.81
time (sec)	N/A	0.071	0.03	0.008	1.345	0.219	8.361	0.209	5.313

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	28	36	68	99	32	29
normalized size	1	1.	0.58	0.78	1.	1.89	2.75	0.89	0.81
time (sec)	N/A	0.071	0.025	0.019	1.379	0.253	8.308	0.204	5.713

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	85	97	0	0	0	0	0	63
normalized size	1	1.47	1.67	0.	0.	0.	0.	0.	1.09
time (sec)	N/A	0.13	0.355	0.095	0.	0.	0.	0.	10.257

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	90	0	0	0	0	0	0
normalized size	1	1.	2.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.261	0.123	0.	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	77	66	0	0	0	0	0	68
normalized size	1	1.51	1.29	0.	0.	0.	0.	0.	1.33
time (sec)	N/A	0.066	0.435	0.143	0.	0.	0.	0.	5.707

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	89	0	0	0	0	0	0
normalized size	1	1.	3.18	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.071	0.048	0.	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	84	0	0	0	0	0	0
normalized size	1	1.	3.23	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.076	0.053	0.	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.513	0.065	0.	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.553	0.082	0.	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	33	30	16	0	14	12
normalized size	1	1.	0.73	2.2	2.	1.07	0.	0.93	0.8
time (sec)	N/A	0.049	0.051	0.079	1.934	0.214	0.	0.214	3.314

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	45	100	0	0	0	0	0	34
normalized size	1	1.1	2.44	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.175	0.112	0.109	0.	0.	0.	0.	14.795

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	8	30	15	0	9	10
normalized size	1	1.	0.83	0.67	2.5	1.25	0.	0.75	0.83
time (sec)	N/A	0.045	0.045	0.069	1.936	0.215	0.	0.223	2.673

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	87	0	0	0	0	0	36
normalized size	1	1.05	2.07	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.174	0.117	0.102	0.	0.	0.	0.	13.077

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	72	0	0	0	0	0	39
normalized size	1	1.04	1.57	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.2	0.665	0.119	0.	0.	0.	0.	19.604

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	53	30	32	0	27	10
normalized size	1	1.	1.64	3.79	2.14	2.29	0.	1.93	0.71
time (sec)	N/A	0.038	0.064	0.106	1.929	0.215	0.	0.222	3.039

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	73	0	0	0	0	0	37
normalized size	1	1.09	1.7	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.198	0.244	0.107	0.	0.	0.	0.	20.021

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	23	51	30	32	0	28	12
normalized size	1	1.	1.77	3.92	2.31	2.46	0.	2.15	0.92
time (sec)	N/A	0.036	0.06	0.091	1.923	0.215	0.	0.218	3.051

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	23	23	27	20	27
normalized size	1	1.	0.6	0.67	0.77	0.77	0.9	0.67	0.9
time (sec)	N/A	0.06	0.029	0.	1.361	0.226	1.109	0.24	2.989

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	25	27	35	35	48	34	0
normalized size	1	1.	0.5	0.54	0.7	0.7	0.96	0.68	0.
time (sec)	N/A	0.167	0.043	0.008	1.393	6.301	3.212	0.198	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	38	36	45	50	80	45	0
normalized size	1	1.	0.51	0.48	0.6	0.67	1.07	0.6	0.
time (sec)	N/A	0.223	0.041	0.007	1.388	0.214	3.666	0.205	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	253	72	78	104	97	202	99	0
normalized size	1	1.35	0.39	0.42	0.56	0.52	1.08	0.53	0.
time (sec)	N/A	0.753	0.209	0.023	1.371	0.252	21.932	0.2	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	40	40	55	55	85	53	0
normalized size	1	1.	0.46	0.46	0.63	0.63	0.98	0.61	0.
time (sec)	N/A	0.247	0.112	0.007	1.37	0.212	3.203	0.237	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	76	78	104	97	202	99	0
normalized size	1	1.	0.41	0.42	0.56	0.52	1.09	0.54	0.
time (sec)	N/A	0.552	0.305	0.009	1.381	0.222	21.847	0.233	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	15	0
normalized size	1	1.	1.	1.5	1.5	1.5	1.	7.5	0.
time (sec)	N/A	0.006	0.003	0.	1.39	0.199	0.103	0.206	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	15	0
normalized size	1	1.	1.	1.5	1.5	1.5	1.	7.5	0.
time (sec)	N/A	0.006	0.003	0.001	1.371	0.199	0.102	0.199	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	24	7	15	10
normalized size	1	1.	1.	1.33	1.33	8.	2.33	5.	3.33
time (sec)	N/A	0.007	0.006	0.002	1.372	0.209	0.121	0.202	22.746

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	24	12	16	17
normalized size	1	1.	1.	1.33	1.33	8.	4.	5.33	5.67
time (sec)	N/A	0.007	0.006	0.001	1.404	0.213	0.424	0.208	36.438

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	4	11	0	7	0
normalized size	1	1.	3.	1.33	1.33	3.67	0.	2.33	0.
time (sec)	N/A	0.006	0.01	0.003	1.329	0.228	0.	0.198	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	6	7	23	0	19	5
normalized size	1	1.	3.4	1.2	1.4	4.6	0.	3.8	1.
time (sec)	N/A	0.007	0.007	0.013	1.335	0.204	0.	0.199	1.656

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	22	14	24	32	17
normalized size	1	1.	1.	0.79	1.57	1.	1.71	2.29	1.21
time (sec)	N/A	0.013	0.003	0.009	1.378	0.203	0.195	0.208	4.149

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	18	47	57	29	50	17
normalized size	1	1.	1.21	0.95	2.47	3.	1.53	2.63	0.89
time (sec)	N/A	0.019	0.003	0.053	1.328	0.198	1.644	0.202	4.79

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	26	51	92	10	35	14
normalized size	1	1.	1.29	1.86	3.64	6.57	0.71	2.5	1.
time (sec)	N/A	0.018	0.006	0.017	1.371	0.203	0.271	0.202	40.009

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	47	11	61	285	0	61	17
normalized size	1	1.	2.94	0.69	3.81	17.81	0.	3.81	1.06
time (sec)	N/A	0.02	0.006	0.059	1.326	0.213	0.	0.207	4.06

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	21	82	622	422	81	0
normalized size	1	1.	1.15	0.81	3.15	23.92	16.23	3.12	0.
time (sec)	N/A	0.029	0.011	0.05	1.497	0.213	8.575	0.23	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	17	47	347	0	58	0
normalized size	1	1.	1.11	0.94	2.61	19.28	0.	3.22	0.
time (sec)	N/A	0.042	0.007	0.018	1.514	0.225	0.	0.207	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	0	0	485	0	0	0
normalized size	1	1.	0.87	0.	0.	15.65	0.	0.	0.
time (sec)	N/A	0.053	0.066	0.069	0.	0.219	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	0	1	126	43	0
normalized size	1	1.02	1.	0.88	0.	0.02	3.07	1.05	0.
time (sec)	N/A	0.084	0.038	0.017	0.	0.226	20.116	0.216	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	16	66	78	14	19	0
normalized size	1	1.	0.64	0.64	2.64	3.12	0.56	0.76	0.
time (sec)	N/A	0.028	0.019	0.01	1.357	0.204	0.624	0.222	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	55	57	146	58	42
normalized size	1	1.	0.74	1.41	1.41	1.46	3.74	1.49	1.08
time (sec)	N/A	0.079	0.062	0.024	1.364	0.224	1.15	0.212	59.349

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	98	0	431	0	107	0
normalized size	1	1.	1.	3.16	0.	13.9	0.	3.45	0.
time (sec)	N/A	0.062	0.042	0.062	0.	0.222	0.	0.237	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	1	0	68	0
normalized size	1	1.	1.	2.11	0.	0.03	0.	1.94	0.
time (sec)	N/A	0.065	0.042	0.034	0.	0.235	0.	0.214	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	99	157	0	65	0
normalized size	1	1.	0.96	2.2	3.96	6.28	0.	2.6	0.
time (sec)	N/A	0.035	0.056	0.032	1.505	0.22	0.	0.202	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	37	78	95	103	102	30	0
normalized size	1	1.	1.12	2.36	2.88	3.12	3.09	0.91	0.
time (sec)	N/A	0.226	0.067	0.116	1.524	0.229	4.667	0.205	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	57	59	114	65	0
normalized size	1	1.	1.	0.77	1.9	1.97	3.8	2.17	0.
time (sec)	N/A	0.056	0.015	0.058	1.357	0.209	21.779	0.2	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	57	150	139	65	0
normalized size	1	1.	1.	0.77	1.9	5.	4.63	2.17	0.
time (sec)	N/A	0.055	0.016	0.138	1.373	0.213	20.412	0.202	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	102	487	609	0	405	0	0	0
normalized size	1	1.48	7.06	8.83	0.	5.87	0.	0.	0.
time (sec)	N/A	1.58	31.186	0.299	0.	0.24	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	26	30	169	3929	0	54	34
normalized size	1	1.	0.7	0.81	4.57	106.19	0.	1.46	0.92
time (sec)	N/A	0.069	0.061	0.019	1.438	0.275	0.	0.224	3.112

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	36	28	239	217	0	0	22
normalized size	1	1.	1.24	0.97	8.24	7.48	0.	0.	0.76
time (sec)	N/A	0.155	0.343	0.062	1.737	0.218	0.	0.	14.704

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	63	138	651	0	0	0
normalized size	1	1.	1.	4.2	9.2	43.4	0.	0.	0.
time (sec)	N/A	0.033	0.067	0.07	1.562	0.221	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	66	126	17	69	0
normalized size	1	1.	1.	1.75	4.12	7.88	1.06	4.31	0.
time (sec)	N/A	0.031	0.008	0.025	1.632	0.215	0.208	0.228	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	72	128	22	72	0
normalized size	1	1.	1.	1.75	4.5	8.	1.38	4.5	0.
time (sec)	N/A	0.033	0.008	0.022	1.476	0.21	1.67	0.219	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	14	18	27	26	15	0
normalized size	1	1.	1.15	0.7	0.9	1.35	1.3	0.75	0.
time (sec)	N/A	0.109	0.064	0.112	1.368	0.215	0.663	0.219	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	20	16	47	27	12	19	0
normalized size	1	1.	1.33	1.07	3.13	1.8	0.8	1.27	0.
time (sec)	N/A	0.2	0.039	0.032	1.359	0.203	0.652	0.216	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	29	20	30	101	0	32	27
normalized size	1	1.	1.45	1.	1.5	5.05	0.	1.6	1.35
time (sec)	N/A	0.038	0.012	0.11	1.366	0.203	0.	0.23	2.144

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	32	52	101	138	0	14	27
normalized size	1	1.	2.46	4.	7.77	10.62	0.	1.08	2.08
time (sec)	N/A	0.028	0.012	0.058	1.353	0.203	0.	0.225	2.19

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	14	8	22	12	8	5
normalized size	1	1.	1.	1.56	0.89	2.44	1.33	0.89	0.56
time (sec)	N/A	0.027	0.002	0.001	1.358	0.205	0.72	0.196	2.58

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	19	18	18	0	34	32	22	10
normalized size	1	1.46	1.38	1.38	0.	2.62	2.46	1.69	0.77
time (sec)	N/A	0.047	0.018	0.004	0.	0.203	0.964	0.203	4.34

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	0
normalized size	1	1.	1.	2.	1.	1.	10.	1.	0.
time (sec)	N/A	0.024	0.001	0.026	1.343	0.188	0.671	0.2	2.319

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	24	22	35	0	23	15
normalized size	1	1.	0.91	1.09	1.	1.59	0.	1.05	0.68
time (sec)	N/A	0.043	0.036	0.022	1.345	0.205	0.	0.197	6.705

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	18	18	15	28	0	15	10
normalized size	1	1.	1.38	1.38	1.15	2.15	0.	1.15	0.77
time (sec)	N/A	0.053	0.024	0.02	1.348	0.204	0.	0.205	2.536

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	20	18	15	30	0	15	12
normalized size	1	1.	1.33	1.2	1.	2.	0.	1.	0.8
time (sec)	N/A	0.059	0.025	0.029	1.382	0.208	0.	0.198	3.213

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	34	0	34	0	0	20
normalized size	1	1.	0.73	1.31	0.	1.31	0.	0.	0.77
time (sec)	N/A	0.021	0.014	0.013	0.	0.214	0.	0.	1.603

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	61	0	61	0	122	39
normalized size	1	1.	0.71	1.45	0.	1.45	0.	2.9	0.93
time (sec)	N/A	0.041	0.019	0.014	0.	0.224	0.	0.207	3.241

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	23	30	23	34	30	34
normalized size	1	1.	0.62	0.68	0.88	0.68	1.	0.88	1.
time (sec)	N/A	0.032	0.008	0.026	1.358	0.209	10.011	0.238	2.037

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	34	34	22	32	0
normalized size	1	1.	0.97	0.86	1.17	1.17	0.76	1.1	0.
time (sec)	N/A	0.023	0.002	0.003	1.336	0.227	0.108	0.214	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	72	93	93	71	96	0
normalized size	1	1.	1.01	1.07	1.39	1.39	1.06	1.43	0.
time (sec)	N/A	0.066	0.021	0.003	1.358	0.207	0.169	0.231	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	49	31	26	31	26
normalized size	1	1.	1.	1.04	2.13	1.35	1.13	1.35	1.13
time (sec)	N/A	0.027	0.004	0.005	1.344	0.205	0.115	0.212	0.996

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	73	60	53	89	65	51	70	0
normalized size	1	1.22	1.	0.88	1.48	1.08	0.85	1.17	0.
time (sec)	N/A	0.143	0.007	0.004	1.353	0.207	0.159	0.204	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	11	0	0	0	44
normalized size	1	1.	1.	0.86	0.26	0.	0.	0.	1.02
time (sec)	N/A	0.091	0.008	0.01	1.794	0.	0.	0.	4.316

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	32	34	0	151	0	22
normalized size	1	1.	0.9	1.1	1.17	0.	5.21	0.	0.76
time (sec)	N/A	0.037	0.011	0.016	1.327	0.	4.688	0.	2.805

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	51	46	24	49	26
normalized size	1	1.	0.93	1.03	1.76	1.59	0.83	1.69	0.9
time (sec)	N/A	0.026	0.019	0.01	1.329	0.247	0.701	0.2	3.395

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	16	15	16	8
normalized size	1	1.	1.	1.08	0.	1.33	1.25	1.33	0.67
time (sec)	N/A	0.027	0.004	0.003	0.	0.218	1.224	0.199	1.674

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	0	30	31	26	14
normalized size	1	1.	0.95	1.05	0.	1.58	1.63	1.37	0.74
time (sec)	N/A	0.047	0.015	0.003	0.	0.221	1.757	0.209	2.354

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	15	8	41	8
normalized size	1	1.	1.	1.09	1.36	1.36	0.73	3.73	0.73
time (sec)	N/A	0.038	0.004	0.003	1.361	0.209	0.132	0.26	2.282

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	0	36	71	30	14
normalized size	1	1.	0.91	1.04	0.	1.57	3.09	1.3	0.61
time (sec)	N/A	0.055	0.018	0.026	0.	0.219	82.078	0.237	2.583

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	12	24	0	24	15
normalized size	1	1.	2.88	0.94	0.75	1.5	0.	1.5	0.94
time (sec)	N/A	0.06	0.007	0.008	1.381	0.204	0.	0.208	3.523

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	50	17	27	27	0	0	15
normalized size	1	1.	2.78	0.94	1.5	1.5	0.	0.	0.83
time (sec)	N/A	0.069	0.007	0.008	1.37	0.208	0.	0.	3.988

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	12	34	0	14	15
normalized size	1	1.	1.	0.94	0.67	1.89	0.	0.78	0.83
time (sec)	N/A	0.066	0.008	0.007	1.528	0.208	0.	0.224	3.993

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	32	37	0	59	0	4	17
normalized size	1	1.	1.45	1.68	0.	2.68	0.	0.18	0.77
time (sec)	N/A	0.149	0.014	0.006	0.	0.239	0.	19.476	9.888

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	34	39	0	36	0	4	17
normalized size	1	1.	1.42	1.62	0.	1.5	0.	0.17	0.71
time (sec)	N/A	0.174	0.017	0.007	0.	0.2	0.	20.317	10.928

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	43	0	36	0	28	15
normalized size	1	1.	1.65	1.87	0.	1.57	0.	1.22	0.65
time (sec)	N/A	0.176	0.021	0.006	0.	0.21	0.	0.208	10.766

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	15	10	15	10
normalized size	1	1.	1.	1.09	1.36	1.36	0.91	1.36	0.91
time (sec)	N/A	0.014	0.002	0.003	1.36	0.21	0.551	0.206	1.306

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	27	24	27	0
normalized size	1	1.	1.	1.05	1.	1.35	1.2	1.35	0.
time (sec)	N/A	0.03	0.002	0.003	1.35	0.225	0.615	0.214	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	30	39	36	39	0
normalized size	1	1.	1.	1.03	1.03	1.34	1.24	1.34	0.
time (sec)	N/A	0.034	0.002	0.003	1.338	0.213	0.688	0.223	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	39	51	48	51	0
normalized size	1	1.	1.	1.03	1.03	1.34	1.26	1.34	0.
time (sec)	N/A	0.04	0.002	0.003	1.371	0.236	0.748	0.211	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	39	19	0	0	0
normalized size	1	1.	1.	0.	1.62	0.79	0.	0.	0.
time (sec)	N/A	0.05	0.018	0.038	1.451	0.276	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	0	7	5
normalized size	1	1.	1.	1.25	1.25	1.25	0.	1.75	1.25
time (sec)	N/A	0.03	0.004	0.012	1.445	0.215	0.	0.222	2.184

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	10	12	16	7	12	0
normalized size	1	1.	1.	1.43	1.71	2.29	1.	1.71	0.
time (sec)	N/A	0.07	0.007	0.016	1.351	0.247	4.068	0.225	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	62	10	43	0
normalized size	1	1.	1.	1.09	1.36	5.64	0.91	3.91	0.
time (sec)	N/A	0.02	0.002	0.005	1.414	0.208	1.396	0.227	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	9	0	22	0
normalized size	1	1.	1.	0.89	1.	1.	0.	2.44	0.
time (sec)	N/A	0.025	0.004	0.011	1.357	0.213	0.	0.223	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	30	20	30	20
normalized size	1	1.	1.	0.88	0.	1.15	0.77	1.15	0.77
time (sec)	N/A	0.012	0.021	0.003	0.	0.217	18.789	0.222	0.867

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	26	34	35	26	36	26
normalized size	1	1.	1.	0.74	0.97	1.	0.74	1.03	0.74
time (sec)	N/A	0.029	0.004	0.01	1.344	0.217	0.148	0.222	1.962

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	32	27	35	0	27	0
normalized size	1	1.	0.75	1.	0.84	1.09	0.	0.84	0.
time (sec)	N/A	0.09	0.033	0.026	1.361	0.213	0.	0.215	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	57	43	57	57	0	58	44
normalized size	1	1.	1.1	0.83	1.1	1.1	0.	1.12	0.85
time (sec)	N/A	0.057	0.054	0.037	1.351	0.226	0.	0.208	5.49

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	126	34	32	42	35	29
normalized size	1	1.	0.83	4.2	1.13	1.07	1.4	1.17	0.97
time (sec)	N/A	0.056	0.035	0.053	1.35	0.222	10.191	0.21	2.881

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	782	34	53	0	35	49
normalized size	1	1.	0.97	26.07	1.13	1.77	0.	1.17	1.63
time (sec)	N/A	0.088	0.09	0.488	1.344	0.229	0.	0.207	4.733

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	164	76	43	0	58	0
normalized size	1	1.	1.14	5.86	2.71	1.54	0.	2.07	0.
time (sec)	N/A	0.057	0.084	0.217	1.37	0.253	0.	0.218	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	106	116	72	107	49	0
normalized size	1	1.	0.93	1.77	1.93	1.2	1.78	0.82	0.
time (sec)	N/A	0.193	0.196	0.141	1.581	0.218	26.604	0.269	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	52	69	59	0	140	53
normalized size	1	1.	0.8	0.8	1.06	0.91	0.	2.15	0.82
time (sec)	N/A	0.173	0.035	0.066	1.847	0.243	0.	0.229	7.982

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	37	63	49	54	77	54
normalized size	1	1.	0.69	0.61	1.03	0.8	0.89	1.26	0.89
time (sec)	N/A	0.147	0.037	0.036	1.574	0.231	0.944	0.213	7.071

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	37	42	59	49	44	58	44
normalized size	1	1.	0.7	0.79	1.11	0.92	0.83	1.09	0.83
time (sec)	N/A	0.18	0.011	0.016	1.537	0.228	0.994	0.212	11.098

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	48	86	72	53	0	53
normalized size	1	1.	0.92	0.79	1.41	1.18	0.87	0.	0.87
time (sec)	N/A	0.201	0.013	0.019	1.503	0.233	1.7	0.	12.627

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	56	74	47	0	1	0
normalized size	1	1.	0.67	0.89	1.17	0.75	0.	0.02	0.
time (sec)	N/A	0.104	0.046	0.073	1.412	0.236	0.	0.248	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	92	165	0	104	0	0	0
normalized size	1	1.	0.62	1.11	0.	0.7	0.	0.	0.
time (sec)	N/A	0.22	0.084	0.099	0.	0.238	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	41	35	48	36	0
normalized size	1	1.	0.88	0.91	1.21	1.03	1.41	1.06	0.
time (sec)	N/A	0.05	0.014	0.065	1.497	0.231	31.761	0.214	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	41	35	48	36	0
normalized size	1	1.	0.88	0.97	1.21	1.03	1.41	1.06	0.
time (sec)	N/A	0.051	0.012	0.065	1.494	0.23	31.937	0.212	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	134	30	36	37	30	20
normalized size	1	1.	0.87	4.47	1.	1.2	1.23	1.	0.67
time (sec)	N/A	0.051	0.025	0.208	1.499	0.234	1.426	0.212	3.026

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	58	68	53	53	68	0
normalized size	1	1.	0.71	0.98	1.15	0.9	0.9	1.15	0.
time (sec)	N/A	0.081	0.029	0.063	1.513	0.236	3.22	0.216	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	37	36	50	63	46	27
normalized size	1	1.	0.89	1.	0.97	1.35	1.7	1.24	0.73
time (sec)	N/A	0.055	0.035	0.068	1.492	0.228	5.542	0.227	3.052

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	430	66	63	0	81	48
normalized size	1	1.	0.77	7.05	1.08	1.03	0.	1.33	0.79
time (sec)	N/A	0.126	0.052	0.378	1.541	0.23	0.	0.231	7.708

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	119	227	0	0	0	0	0
normalized size	1	1.	1.25	2.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.316	0.258	0.	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	201	47	59	0	182	36
normalized size	1	1.	0.88	4.9	1.15	1.44	0.	4.44	0.88
time (sec)	N/A	0.091	0.051	0.664	1.492	0.263	0.	0.222	5.212

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	32	43	35	26	36	0
normalized size	1	1.	0.82	0.94	1.26	1.03	0.76	1.06	0.
time (sec)	N/A	0.098	0.018	0.059	1.491	0.218	0.488	0.221	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	53	70	53	53	68	0
normalized size	1	1.	0.69	0.87	1.15	0.87	0.87	1.11	0.
time (sec)	N/A	0.171	0.063	0.069	1.493	0.223	1.853	0.219	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	32	46	34	59	20	36	14
normalized size	1	1.	1.68	2.42	1.79	3.11	1.05	1.89	0.74
time (sec)	N/A	0.055	0.051	0.067	1.549	0.224	5.406	0.214	2.841

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	32	47	34	59	20	36	14
normalized size	1	1.	1.88	2.76	2.	3.47	1.18	2.12	0.82
time (sec)	N/A	0.051	0.05	0.075	1.493	0.226	7.736	0.215	2.956

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	45	63	65	82	0	73	49
normalized size	1	1.	0.73	1.02	1.05	1.32	0.	1.18	0.79
time (sec)	N/A	0.061	0.093	0.059	1.514	0.225	0.	0.221	4.919

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	102	61	77	0	54	27
normalized size	1	1.	1.11	2.83	1.69	2.14	0.	1.5	0.75
time (sec)	N/A	0.118	0.076	0.214	1.511	0.231	0.	0.215	7.318

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	112	97	0	0	0	0	0
normalized size	1	1.	1.81	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.254	0.222	0.	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	43	57	49	0	128	46
normalized size	1	1.	0.7	0.8	1.06	0.91	0.	2.37	0.85
time (sec)	N/A	0.155	0.037	0.075	1.498	0.227	0.	0.217	6.685

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	158	70	55	78	72	56
normalized size	1	1.	0.76	2.39	1.06	0.83	1.18	1.09	0.85
time (sec)	N/A	0.143	0.052	0.188	1.48	0.225	4.285	0.225	8.305

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	69	194	66	66	81	0
normalized size	1	1.	0.82	0.95	2.66	0.9	0.9	1.11	0.
time (sec)	N/A	0.272	0.038	0.089	10.282	0.254	1.837	0.231	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	27	35	26	31	35	22
normalized size	1	1.	0.66	0.84	1.09	0.81	0.97	1.09	0.69
time (sec)	N/A	0.062	0.012	0.004	1.53	0.215	1.259	0.208	2.696

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	37	53	51	88	46	37
normalized size	1	1.	0.82	0.84	1.2	1.16	2.	1.05	0.84
time (sec)	N/A	0.055	0.016	0.003	1.506	0.213	2.156	0.208	2.77

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	32	26	19	28	19
normalized size	1	1.	1.	0.87	1.39	1.13	0.83	1.22	0.83
time (sec)	N/A	0.077	0.006	0.	1.562	0.216	0.591	0.211	5.425

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	128	0	0	0	0	49
normalized size	1	1.	0.79	1.91	0.	0.	0.	0.	0.73
time (sec)	N/A	0.155	0.054	0.052	0.	0.	0.	0.	9.804

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	54	35	0	0	26
normalized size	1	1.	0.82	0.85	1.59	1.03	0.	0.	0.76
time (sec)	N/A	0.075	0.012	0.01	1.515	0.226	0.	0.	5.121

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	64	139	0	0	0	0	56
normalized size	1	1.	0.81	1.76	0.	0.	0.	0.	0.71
time (sec)	N/A	0.177	0.039	0.045	0.	0.	0.	0.	10.512

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	149	0	0	0	0	68
normalized size	1	1.	0.79	1.67	0.	0.	0.	0.	0.76
time (sec)	N/A	0.372	0.21	0.048	0.	0.	0.	0.	16.995

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	28	35	19	34	22
normalized size	1	1.	1.	1.05	1.27	1.59	0.86	1.55	1.
time (sec)	N/A	0.06	0.007	0.007	1.537	0.231	0.63	0.207	4.004

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	30	42	35	34	42	27
normalized size	1	1.	1.26	0.97	1.35	1.13	1.1	1.35	0.87
time (sec)	N/A	0.039	0.009	0.014	1.49	0.215	1.636	0.21	2.493

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	79	96	0	0	0	51
normalized size	1	1.	0.95	1.25	1.52	0.	0.	0.	0.81
time (sec)	N/A	0.156	0.029	0.029	1.878	0.	0.	0.	8.63

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	36	39	22	0	26
normalized size	1	1.	1.	0.89	1.29	1.39	0.79	0.	0.93
time (sec)	N/A	0.11	0.008	0.012	1.509	0.224	0.993	0.	6.726

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	38	34	49	51	32	0	36
normalized size	1	1.	0.97	0.87	1.26	1.31	0.82	0.	0.92
time (sec)	N/A	0.115	0.018	0.003	1.524	0.219	1.029	0.	7.812

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	57	96	73	61	0	53
normalized size	1	1.	0.93	0.95	1.6	1.22	1.02	0.	0.88
time (sec)	N/A	0.136	0.023	0.026	1.523	0.229	1.752	0.	7.714

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	47	78	127	69	0	0	73
normalized size	1	1.04	0.59	0.99	1.61	0.87	0.	0.	0.92
time (sec)	N/A	0.213	0.023	0.037	1.514	0.219	0.	0.	11.956

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	116	116	199	0	0	0	0	0
normalized size	1	1.08	1.08	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.291	0.213	0.263	0.	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	133	86	305	0	69	0	0	0
normalized size	1	1.25	0.81	2.88	0.	0.65	0.	0.	0.
time (sec)	N/A	0.3	0.373	0.438	0.	0.23	0.	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	198	36	31	0	101	39
normalized size	1	1.	1.17	4.83	0.88	0.76	0.	2.46	0.95
time (sec)	N/A	0.089	0.05	0.405	1.58	0.221	0.	0.223	4.738

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	67	128	65	101	0	78	63
normalized size	1	1.03	1.03	1.97	1.	1.55	0.	1.2	0.97
time (sec)	N/A	0.058	0.154	0.414	1.51	0.229	0.	0.259	4.482

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	61	121	62	92	0	72	46
normalized size	1	1.04	1.2	2.37	1.22	1.8	0.	1.41	0.9
time (sec)	N/A	0.102	0.139	0.442	1.469	0.227	0.	0.256	5.386

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	72	197	86	93	0	86	71
normalized size	1	1.02	0.88	2.4	1.05	1.13	0.	1.05	0.87
time (sec)	N/A	0.158	0.229	0.615	1.433	0.225	0.	0.232	8.657

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F(-1)	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	232	383	240	0	0	0	0	0
normalized size	1	1.33	2.19	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.468	2.775	0.675	0.	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	35	76	23	22	0	68	20
normalized size	1	1.	1.52	3.3	1.	0.96	0.	2.96	0.87
time (sec)	N/A	0.082	0.033	0.223	1.53	0.221	0.	0.22	3.976

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	91	79	203	105	109	0	142	87
normalized size	1	1.3	1.13	2.9	1.5	1.56	0.	2.03	1.24
time (sec)	N/A	0.165	0.177	0.48	1.625	0.256	0.	0.262	10.891

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	101	76	114	78	50	0	0	107
normalized size	1	1.36	1.03	1.54	1.05	0.68	0.	0.	1.45
time (sec)	N/A	0.289	0.062	0.286	1.535	0.225	0.	0.	11.225

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	172	84	327	0	80	0	0	180
normalized size	1	1.29	0.63	2.46	0.	0.6	0.	0.	1.35
time (sec)	N/A	0.309	0.325	0.455	0.	0.226	0.	0.	14.849

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	146	92	250	100	77	0	0	153
normalized size	1	1.33	0.84	2.27	0.91	0.7	0.	0.	1.39
time (sec)	N/A	0.313	0.089	0.48	1.672	0.246	0.	0.	14.246

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	118	99	85	139	69	0	0	80
normalized size	1	2.15	1.8	1.55	2.53	1.25	0.	0.	1.45
time (sec)	N/A	1.347	0.223	0.045	1.527	0.231	0.	0.	78.811

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	75	64	120	78	0	66	26
normalized size	1	1.	1.88	1.6	3.	1.95	0.	1.65	0.65
time (sec)	N/A	0.075	0.095	0.023	1.501	0.233	0.	0.25	2.77

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	32	42	68	153	43	31
normalized size	1	1.	0.9	0.82	1.08	1.74	3.92	1.1	0.79
time (sec)	N/A	0.067	0.026	0.006	1.517	0.226	1.253	0.201	4.5

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	A	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	256	102	159	0	0	0	80
normalized size	1	1.	2.1	0.84	1.3	0.	0.	0.	0.66
time (sec)	N/A	0.161	0.339	0.026	1.63	0.	0.	0.	10.833

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	27	19	22	0	0
normalized size	1	1.	1.	0.	0.96	0.68	0.79	0.	0.
time (sec)	N/A	0.054	0.012	0.086	1.548	0.222	4.287	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	34	26	34	26
normalized size	1	1.	1.	0.84	1.1	1.1	0.84	1.1	0.84
time (sec)	N/A	0.059	0.013	0.012	1.335	0.255	6.258	0.199	3.133

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	70	77	135	0	78	46
normalized size	1	1.	1.07	1.23	1.35	2.37	0.	1.37	0.81
time (sec)	N/A	0.07	0.137	0.01	1.526	0.227	0.	0.217	5.25

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	140	72	76	69	198	0	266	143
normalized size	1	1.71	0.88	0.93	0.84	2.41	0.	3.24	1.74
time (sec)	N/A	0.212	0.105	0.024	1.593	0.234	0.	0.308	5.363

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	66	0	0	566	0	0	0
normalized size	1	1.	1.35	0.	0.	11.55	0.	0.	0.
time (sec)	N/A	0.235	0.325	0.052	0.	0.248	0.	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	144	830	73	629	0	95	36
normalized size	1	1.	4.	23.06	2.03	17.47	0.	2.64	1.
time (sec)	N/A	0.196	0.238	0.556	1.523	0.231	0.	0.26	25.867

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	B	F	A	A	F	A	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	28	67	64	0	22	35	0	39	42
normalized size	1	2.39	2.29	0.	0.79	1.25	0.	1.39	1.5
time (sec)	N/A	0.087	0.851	0.056	1.638	0.227	0.	0.204	9.428

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$

is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [397] had the largest ratio of [1.333]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	14	0.071
2	A	1	1	1.	13	0.077
3	A	1	1	1.	5	0.2
4	A	2	2	1.	10	0.2
5	A	1	1	1.	12	0.083
6	A	2	2	1.	5	0.4
7	A	2	2	1.	5	0.4
8	A	2	1	1.	7	0.143
9	A	1	1	1.	6	0.167
10	A	1	1	1.	8	0.125
11	A	2	2	1.	12	0.167
12	A	2	2	1.	17	0.118
13	A	2	2	1.	18	0.111
14	A	3	2	1.	19	0.105
15	A	3	2	1.	20	0.1
16	A	3	2	1.22	19	0.105
17	A	3	2	1.22	20	0.1
18	A	2	2	1.	10	0.2
19	A	2	2	1.	13	0.154
20	A	2	2	1.	8	0.25
21	A	2	1	1.	12	0.083
22	A	2	2	1.	12	0.167
23	A	2	2	1.	14	0.143
24	A	3	2	1.	16	0.125
25	A	4	2	1.	22	0.091
26	A	2	1	1.	13	0.077
27	A	3	2	1.	15	0.133
28	A	3	3	1.	15	0.2
29	A	1	1	1.	9	0.111
30	A	1	1	1.	9	0.111
31	A	4	3	1.	10	0.3
32	A	2	2	1.	4	0.5
33	A	2	2	1.	4	0.5

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	2	2	1.	7	0.286
35	A	2	1	1.	7	0.143
36	A	3	2	1.	9	0.222
37	A	2	2	1.	8	0.25
38	A	2	2	1.	8	0.25
39	A	4	2	1.	9	0.222
40	A	4	4	1.	9	0.444
41	A	2	2	1.	9	0.222
42	A	2	2	1.	11	0.182
43	A	3	2	1.	19	0.105
44	A	3	2	1.	10	0.2
45	A	3	3	1.	16	0.188
46	A	3	2	1.	14	0.143
47	A	4	3	1.	11	0.273
48	A	2	2	1.	13	0.154
49	A	3	2	1.	13	0.154
50	A	3	3	1.	15	0.2
51	A	3	3	1.	17	0.176
52	A	3	3	1.	17	0.176
53	A	3	3	1.	15	0.2
54	A	2	2	1.	12	0.167
55	A	2	2	1.	14	0.143
56	A	2	2	1.	11	0.182
57	A	3	3	1.	18	0.167
58	A	2	2	1.	16	0.125
59	A	1	1	1.1	18	0.056
60	A	7	5	1.	17	0.294
61	A	2	2	1.3	10	0.2
62	A	3	1	1.	11	0.091
63	A	2	1	1.	17	0.059
64	A	3	2	1.06	19	0.105
65	A	3	2	1.	19	0.105
66	A	3	3	1.	11	0.273
67	A	3	3	1.	17	0.176
68	A	1	1	1.	12	0.083
69	A	2	2	1.	24	0.083

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
70	A	1	1	1.	16	0.062
71	A	2	2	1.	6	0.333
72	A	1	1	1.	6	0.167
73	A	5	4	1.06	12	0.333
74	A	2	1	1.	4	0.25
75	A	4	3	1.	9	0.333
76	A	3	2	1.	4	0.5
77	A	1	1	1.	8	0.125
78	A	1	1	1.	10	0.1
79	A	1	1	1.	6	0.167
80	A	1	1	1.	3	0.333
81	A	3	4	1.	8	0.5
82	A	3	3	1.	6	0.5
83	A	4	4	1.	6	0.667
84	A	3	3	1.	4	0.75
85	A	4	4	1.	13	0.308
86	A	6	6	1.5	12	0.5
87	A	3	2	1.	11	0.182
88	A	2	1	1.	16	0.062
89	A	1	1	1.	15	0.067
90	A	2	1	1.	11	0.091
91	A	3	2	1.	13	0.154
92	A	3	2	1.	12	0.167
93	A	4	4	1.	16	0.25
94	A	5	5	1.	14	0.357
95	A	7	6	1.	29	0.207
96	A	3	2	1.	19	0.105
97	A	2	1	1.	39	0.026
98	A	12	5	1.65	21	0.238
99	A	3	2	1.	20	0.1
100	A	2	1	1.	21	0.048
101	A	2	1	1.	11	0.091
102	A	2	1	1.	9	0.111
103	A	2	1	1.	25	0.04
104	A	2	1	1.	24	0.042
105	A	3	2	1.	19	0.105

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
106	A	2	1	1.	19	0.053
107	A	5	4	1.	19	0.21
108	A	5	4	1.	16	0.25
109	A	3	3	1.	14	0.214
110	A	6	5	1.	33	0.152
111	A	5	4	1.	21	0.19
112	A	5	4	1.	21	0.19
113	A	9	5	1.	10	0.5
114	A	10	9	1.	17	0.529
115	A	6	5	1.	26	0.192
116	A	6	2	1.	29	0.069
117	A	3	2	1.	30	0.067
118	A	6	6	1.	9	0.667
119	A	6	6	1.	11	0.546
120	A	1	1	1.	13	0.077
121	A	4	4	1.	13	0.308
122	A	7	7	1.	13	0.538
123	A	7	7	1.	13	0.538
124	A	3	2	1.	13	0.154
125	A	8	7	1.	13	0.538
126	A	1	1	1.	15	0.067
127	A	3	3	1.	11	0.273
128	A	2	2	1.	13	0.154
129	A	4	4	1.	15	0.267
130	A	4	4	1.	15	0.267
131	A	3	3	1.	15	0.2
132	A	4	4	1.	15	0.267
133	A	1	1	1.	17	0.059
134	A	2	2	1.	11	0.182
135	A	9	6	1.	13	0.462
136	A	6	6	1.	9	0.667
137	A	6	6	1.	11	0.546
138	A	6	6	1.	13	0.462
139	A	6	6	1.	13	0.462
140	A	1	1	1.	13	0.077
141	A	4	4	1.	13	0.308

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
142	A	7	7	1.	13	0.538
143	A	7	7	1.	13	0.538
144	A	7	7	1.	13	0.538
145	A	1	1	1.	15	0.067
146	A	22	8	1.	13	0.615
147	A	4	3	1.	10	0.3
148	A	4	4	1.	16	0.25
149	A	3	3	1.	19	0.158
150	A	5	5	1.	26	0.192
151	A	7	7	1.	7	1.
152	A	4	4	1.	18	0.222
153	A	7	7	1.	9	0.778
154	A	5	4	1.54	18	0.222
155	A	5	5	1.	18	0.278
156	A	5	4	1.	16	0.25
157	A	4	3	1.	16	0.188
158	A	2	1	1.	16	0.062
159	A	2	1	1.	9	0.111
160	A	3	2	1.	15	0.133
161	A	3	2	1.	11	0.182
162	A	2	1	1.	11	0.091
163	A	5	3	1.	10	0.3
164	A	4	3	1.	10	0.3
165	A	2	1	1.	11	0.091
166	A	4	3	1.17	11	0.273
167	A	5	3	1.	11	0.273
168	A	3	3	1.	18	0.167
169	A	3	3	1.	18	0.167
170	A	4	4	1.	11	0.364
171	A	3	3	1.22	21	0.143
172	A	1	1	1.	13	0.077
173	A	3	2	1.	13	0.154
174	A	12	8	1.	13	0.615
175	A	5	4	1.	13	0.308
176	A	3	2	1.	13	0.154
177	A	8	8	0.98	13	0.615

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
178	A	5	4	1.	16	0.25
179	A	4	4	1.	18	0.222
180	A	2	1	1.	44	0.023
181	A	7	6	1.	16	0.375
182	A	7	6	1.	22	0.273
183	A	2	1	1.	15	0.067
184	A	3	2	1.	13	0.154
185	A	3	2	1.	13	0.154
186	A	2	1	1.14	11	0.091
187	A	2	1	1.	11	0.091
188	A	2	1	1.	9	0.111
189	A	2	1	1.	17	0.059
190	A	2	1	1.	19	0.053
191	A	2	1	1.	19	0.053
192	A	2	1	1.	19	0.053
193	A	2	2	1.	19	0.105
194	A	4	4	1.	19	0.21
195	A	3	3	1.	19	0.158
196	A	4	4	1.	19	0.21
197	A	5	4	1.	19	0.21
198	A	2	2	1.	21	0.095
199	A	3	2	1.	14	0.143
200	A	4	4	1.	18	0.222
201	A	4	3	1.	14	0.214
202	A	7	3	1.	10	0.3
203	A	7	6	1.	16	0.375
204	A	8	5	1.	14	0.357
205	A	7	5	1.	20	0.25
206	A	3	2	1.	14	0.143
207	A	6	4	1.	13	0.308
208	A	9	4	1.	24	0.167
209	A	5	2	1.	33	0.061
210	A	4	3	1.	11	0.273
211	A	2	1	1.	13	0.077
212	A	4	3	1.	21	0.143
213	A	5	4	1.	19	0.21

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	2	1.	17	0.118
215	A	5	3	1.	11	0.273
216	A	9	3	1.	13	0.231
217	A	8	5	1.	11	0.454
218	A	3	3	1.	15	0.2
219	A	4	4	1.03	19	0.21
220	A	15	11	1.	27	0.407
221	A	31	14	1.05	52	0.269
222	A	44	19	1.79	56	0.339
223	A	2	2	1.16	15	0.133
224	A	2	2	1.2	15	0.133
225	A	3	3	1.19	15	0.2
226	B	3	3	2.81	13	0.231
227	A	9	9	1.09	19	0.474
228	B	6	6	2.69	17	0.353
229	A	2	2	1.	12	0.167
230	A	4	4	1.	17	0.235
231	A	7	6	1.	17	0.353
232	B	3	3	2.51	17	0.176
233	A	3	3	1.	17	0.176
234	A	5	4	1.	17	0.235
235	A	2	2	1.	14	0.143
236	A	2	2	1.	14	0.143
237	A	2	2	1.	14	0.143
238	A	2	2	1.	19	0.105
239	A	2	2	1.	19	0.105
240	A	3	3	1.	22	0.136
241	A	3	3	1.	22	0.136
242	A	5	4	1.	17	0.235
243	A	5	5	1.	21	0.238
244	A	9	7	1.	20	0.35
245	A	5	5	1.	24	0.208
246	A	5	4	1.	21	0.19
247	A	5	4	1.	30	0.133
248	A	5	4	1.11	32	0.125
249	A	2	2	1.	30	0.067

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
250	A	4	3	1.	15	0.2
251	A	3	2	1.	13	0.154
252	A	3	2	1.	11	0.182
253	A	3	2	1.	20	0.1
254	A	2	2	1.	18	0.111
255	A	4	4	1.	17	0.235
256	A	3	3	1.	17	0.176
257	A	5	5	1.	20	0.25
258	A	4	4	1.	24	0.167
259	A	15	9	1.	33	0.273
260	A	32	14	1.	44	0.318
261	A	4	4	1.	16	0.25
262	A	5	5	1.	18	0.278
263	A	6	5	1.	18	0.278
264	A	5	5	1.	19	0.263
265	A	4	4	1.	24	0.167
266	A	2	2	1.	10	0.2
267	A	4	4	1.	14	0.286
268	A	1	1	1.	10	0.1
269	A	1	1	1.	12	0.083
270	A	4	4	1.	14	0.286
271	A	5	5	1.	14	0.357
272	A	4	3	1.	10	0.3
273	A	5	3	1.	10	0.3
274	A	3	3	1.	14	0.214
275	A	4	4	1.	14	0.286
276	A	4	4	1.	14	0.286
277	A	5	5	1.	14	0.357
278	A	2	2	1.	16	0.125
279	A	10	7	1.	22	0.318
280	A	6	6	1.	18	0.333
281	A	6	6	1.	28	0.214
282	A	4	4	1.	34	0.118
283	A	14	10	1.	24	0.417
284	A	3	2	1.	12	0.167
285	A	2	2	1.	14	0.143

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	A	2	2	1.	14	0.143
287	A	5	4	1.	16	0.25
288	A	3	2	1.31	14	0.143
289	A	7	6	1.	23	0.261
290	A	26	8	1.	29	0.276
291	A	36	8	1.	31	0.258
292	A	4	3	1.	11	0.273
293	A	1	1	1.	15	0.067
294	A	6	6	1.	13	0.462
295	A	2	1	1.	13	0.077
296	A	6	6	1.	17	0.353
297	A	3	2	1.	15	0.133
298	A	13	9	1.	17	0.529
299	A	1	1	1.	13	0.077
300	A	1	1	1.	13	0.077
301	A	5	5	1.	15	0.333
302	A	6	6	1.	13	0.462
303	A	5	5	1.	15	0.333
304	A	6	5	1.	15	0.333
305	A	6	6	1.	15	0.4
306	B	12	12	2.15	13	0.923
307	A	8	6	0.9	13	0.462
308	A	3	3	1.	22	0.136
309	A	5	5	1.	16	0.312
310	A	5	5	1.	18	0.278
311	A	3	3	1.	23	0.13
312	A	1	1	1.	23	0.043
313	A	9	4	1.	39	0.103
314	A	7	7	1.	17	0.412
315	A	10	7	1.	17	0.412
316	A	2	2	1.	15	0.133
317	A	5	5	1.	17	0.294
318	A	3	2	1.23	17	0.118
319	A	10	10	1.	32	0.312
320	A	2	2	1.	24	0.083
321	A	2	2	1.	24	0.083

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
322	A	6	6	1.44	18	0.333
323	A	6	6	1.44	18	0.333
324	A	2	2	1.	27	0.074
325	A	2	2	1.	27	0.074
326	A	1	1	1.	21	0.048
327	A	1	1	1.	44	0.023
328	A	2	2	1.	27	0.074
329	A	2	2	1.	31	0.065
330	A	2	2	1.	4	0.5
331	A	2	1	1.	4	0.25
332	A	3	2	1.	4	0.5
333	A	4	2	1.	4	0.5
334	A	5	2	1.	4	0.5
335	B	3	2	3.2	14	0.143
336	A	2	1	1.	14	0.071
337	A	2	1	1.	4	0.25
338	A	4	2	1.	4	0.5
339	A	2	1	1.	4	0.25
340	A	2	2	1.	12	0.167
341	A	4	2	1.	4	0.5
342	A	3	2	1.	4	0.5
343	A	3	2	1.	14	0.143
344	A	6	3	1.	9	0.333
345	A	3	2	1.	9	0.222
346	A	4	3	1.	7	0.429
347	A	3	2	1.	9	0.222
348	A	3	3	1.	9	0.333
349	A	5	3	1.	9	0.333
350	A	7	3	1.	9	0.333
351	A	9	3	1.	9	0.333
352	A	1	1	1.	13	0.077
353	A	3	2	1.	23	0.087
354	A	2	2	1.	9	0.222
355	A	2	1	1.	7	0.143
356	A	2	2	1.	7	0.286
357	A	3	3	1.	9	0.333

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
358	A	3	2	1.	9	0.222
359	A	3	2	1.	11	0.182
360	A	3	2	1.	11	0.182
361	A	4	3	1.	9	0.333
362	A	3	3	1.	29	0.103
363	A	8	5	0.95	22	0.227
364	A	15	7	1.	17	0.412
365	A	4	3	1.	17	0.176
366	A	4	2	1.	9	0.222
367	A	4	3	1.	7	0.429
368	A	1	1	1.	7	0.143
369	A	4	4	1.	9	0.444
370	A	6	2	1.	9	0.222
371	A	4	3	1.	9	0.333
372	A	3	1	1.	9	0.111
373	A	7	6	1.02	11	0.546
374	A	5	5	1.55	9	0.556
375	A	5	4	1.	16	0.25
376	A	3	3	1.57	14	0.214
377	B	3	3	2.85	12	0.25
378	A	2	1	1.	16	0.062
379	A	6	4	1.	10	0.4
380	A	4	3	1.	9	0.333
381	A	3	2	1.	9	0.222
382	A	4	4	1.	21	0.19
383	A	2	1	1.	9	0.111
384	A	2	2	1.	7	0.286
385	A	4	3	1.	9	0.333
386	A	4	3	1.	9	0.333
387	A	5	5	1.	7	0.714
388	A	4	2	1.	7	0.286
389	A	4	2	1.	9	0.222
390	A	1	1	1.	10	0.1
391	A	1	1	1.	12	0.083
392	A	2	2	1.	10	0.2
393	A	2	2	1.	12	0.167

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
394	A	3	3	1.	12	0.25
395	A	3	2	1.	14	0.143
396	A	4	2	1.	32	0.062
397	A	11	8	1.	6	1.333
398	A	9	9	1.21	8	1.125
399	A	6	5	1.	12	0.417
400	A	4	2	1.	32	0.062
401	A	19	12	1.58	13	0.923
402	A	1	1	1.	11	0.091
403	A	1	1	1.	11	0.091
404	A	2	2	1.	11	0.182
405	A	6	5	1.	16	0.312
406	A	4	3	1.	13	0.231
407	A	4	3	1.	13	0.231
408	A	1	1	1.	13	0.077
409	A	2	2	1.	13	0.154
410	A	3	3	1.	11	0.273
411	A	6	4	1.4	35	0.114
412	A	5	4	1.53	11	0.364
413	B	13	9	2.22	11	0.818
414	A	4	2	1.	13	0.154
415	A	4	2	1.	13	0.154
416	B	27	11	2.17	27	0.407
417	A	66	20	1.83	41	0.488
418	A	13	4	1.13	28	0.143
419	A	5	3	0.92	15	0.2
420	A	5	3	1.	18	0.167
421	A	5	4	1.	20	0.2
422	A	4	3	1.	20	0.15
423	A	3	3	1.	19	0.158
424	A	4	3	1.	22	0.136
425	A	4	3	1.	23	0.13
426	A	18	14	1.07	33	0.424
427	A	27	6	1.	39	0.154
428	A	5	3	1.	17	0.176
429	A	3	3	1.	11	0.273

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
430	A	5	4	1.	11	0.364
431	A	1	1	1.	11	0.091
432	A	6	6	1.	13	0.462
433	A	11	9	1.05	28	0.321
434	A	7	6	1.	12	0.5
435	A	4	4	1.	12	0.333
436	A	10	10	1.	22	0.454
437	A	5	4	1.	23	0.174
438	A	16	12	1.	31	0.387
439	A	10	7	1.	48	0.146
440	A	7	5	1.	15	0.333
441	A	6	5	1.	15	0.333
442	A	6	6	1.	19	0.316
443	A	7	7	1.	15	0.467
444	A	6	6	1.	19	0.316
445	A	6	6	1.	20	0.3
446	A	29	16	1.31	61	0.262
447	B	21	10	2.08	29	0.345
448	A	7	7	1.	20	0.35
449	A	14	10	1.71	15	0.667
450	A	4	3	1.	19	0.158
451	A	2	2	1.	33	0.061
452	A	15	9	1.3	31	0.29
453	A	14	10	1.25	52	0.192
454	A	4	3	1.	15	0.2
455	A	14	10	1.51	15	0.667
456	A	3	3	1.06	11	0.273
457	A	6	5	1.	11	0.454
458	A	3	2	1.	11	0.182
459	A	4	2	1.	11	0.182
460	A	3	2	1.	11	0.182
461	A	5	4	1.	13	0.308
462	A	6	4	1.	13	0.308
463	A	1	1	1.	7	0.143
464	A	3	2	1.	11	0.182
465	A	4	3	1.	19	0.158

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
466	A	3	2	1.	13	0.154
467	A	3	2	1.	13	0.154
468	A	2	2	1.	11	0.182
469	A	2	2	1.	12	0.167
470	A	3	2	1.	13	0.154
471	A	2	1	1.	13	0.077
472	A	3	2	1.	11	0.182
473	A	3	3	1.	23	0.13
474	A	2	2	1.	19	0.105
475	A	2	2	1.	15	0.133
476	A	3	2	1.	15	0.133
477	A	3	2	1.	11	0.182
478	A	2	2	1.	14	0.143
479	A	2	1	1.	12	0.083
480	A	2	1	1.	16	0.062
481	A	2	2	1.	20	0.1
482	A	2	2	1.	25	0.08
483	A	9	4	1.	8	0.5
484	A	8	4	1.	8	0.5
485	A	13	4	1.	8	0.5
486	A	4	4	1.	10	0.4
487	A	6	5	1.	10	0.5
488	A	5	4	1.	8	0.5
489	A	3	3	1.	8	0.375
490	A	8	8	1.	8	1.
491	A	7	7	1.	6	1.167
492	A	2	2	1.	18	0.111
493	A	3	3	1.	15	0.2
494	A	2	2	1.	11	0.182
495	A	9	4	1.21	22	0.182
496	A	3	1	1.	11	0.091
497	A	4	3	1.	13	0.231
498	A	3	2	1.	13	0.154
499	A	4	3	1.	13	0.231
500	A	4	4	1.08	13	0.308
501	A	4	3	1.	15	0.2

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	3	1	1.	11	0.091
503	A	6	2	1.	13	0.154
504	A	7	2	1.	13	0.154
505	A	8	2	1.	13	0.154
506	A	2	2	1.11	13	0.154
507	A	3	1	1.	13	0.077
508	A	6	2	1.	15	0.133
509	A	7	2	1.	15	0.133
510	A	8	2	1.	15	0.133
511	A	2	2	1.11	15	0.133
512	A	2	1	1.	7	0.143
513	A	3	2	1.	9	0.222
514	A	3	2	1.	9	0.222
515	A	3	2	1.	9	0.222
516	A	2	2	1.	9	0.222
517	A	2	1	1.	9	0.111
518	A	3	2	1.	11	0.182
519	A	3	2	1.	11	0.182
520	A	3	2	1.	11	0.182
521	A	2	2	1.	11	0.182
522	A	4	4	1.	11	0.364
523	A	7	7	1.23	15	0.467
524	A	3	2	1.	13	0.154
525	A	5	5	1.	24	0.208
526	A	6	5	1.	29	0.172
527	A	2	2	1.	21	0.095
528	A	6	6	1.	15	0.4
529	A	2	2	1.	15	0.133
530	A	3	3	1.	17	0.176
531	A	3	3	1.	19	0.158
532	A	3	3	1.	39	0.077
533	A	4	4	1.54	17	0.235
534	A	3	2	1.	21	0.095
535	A	4	2	1.	11	0.182
536	A	3	2	1.	11	0.182
537	A	3	2	1.	9	0.222

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
538	A	5	3	1.	12	0.25
539	A	6	6	1.	13	0.462
540	A	1	1	1.	25	0.04
541	A	1	1	1.	10	0.1
542	A	6	4	1.	21	0.19
543	A	2	2	1.	16	0.125
544	A	2	2	1.	10	0.2
545	A	2	2	1.	10	0.2
546	A	3	3	1.	16	0.188
547	A	4	3	1.	14	0.214
548	A	4	3	1.	22	0.136
549	A	5	3	1.47	10	0.3
550	A	1	1	1.	10	0.1
551	A	2	2	1.51	10	0.2
552	A	2	2	1.	10	0.2
553	A	2	2	1.	12	0.167
554	A	2	2	1.	10	0.2
555	A	2	2	1.	12	0.167
556	A	1	1	1.	18	0.056
557	A	7	6	1.1	16	0.375
558	A	1	1	1.	14	0.071
559	A	7	6	1.05	16	0.375
560	A	7	6	1.04	18	0.333
561	A	1	1	1.	16	0.062
562	A	7	6	1.09	14	0.429
563	A	1	1	1.	16	0.062
564	A	4	3	1.	7	0.429
565	A	11	5	1.	9	0.556
566	A	11	5	1.	11	0.454
567	A	31	8	1.35	15	0.533
568	A	11	5	1.	13	0.385
569	A	24	6	1.	17	0.353
570	A	1	1	1.	2	0.5
571	A	1	1	1.	2	0.5
572	A	1	1	1.	2	0.5
573	A	1	1	1.	2	0.5

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
574	A	1	1	1.	2	0.5
575	A	1	1	1.	2	0.5
576	A	2	2	1.	4	0.5
577	A	2	1	1.	4	0.25
578	A	3	2	1.	4	0.5
579	A	2	2	1.	4	0.5
580	A	3	2	1.	4	0.5
581	A	4	3	1.	7	0.429
582	A	3	2	1.	11	0.182
583	A	2	2	1.02	8	0.25
584	A	2	2	1.	6	0.333
585	A	2	2	1.	8	0.25
586	A	2	2	1.	14	0.143
587	A	2	2	1.	15	0.133
588	A	3	3	1.	10	0.3
589	A	5	3	1.	23	0.13
590	A	5	2	1.	11	0.182
591	A	5	2	1.	15	0.133
592	A	8	4	1.48	31	0.129
593	A	3	3	1.	15	0.2
594	A	5	3	1.	21	0.143
595	A	2	2	1.	11	0.182
596	A	3	3	1.	6	0.5
597	A	3	3	1.	6	0.5
598	A	13	7	1.	16	0.438
599	A	8	4	1.	13	0.308
600	A	3	3	1.	10	0.3
601	A	3	3	1.	10	0.3
602	A	2	2	1.	13	0.154
603	A	3	3	1.46	13	0.231
604	A	2	2	1.	11	0.182
605	A	4	3	1.	12	0.25
606	A	3	2	1.	14	0.143
607	A	3	2	1.	18	0.111
608	A	1	1	1.	6	0.167
609	A	2	2	1.	8	0.25

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
610	A	2	2	1.	10	0.2
611	A	2	1	1.	8	0.125
612	A	4	4	1.	10	0.4
613	A	6	2	1.	14	0.143
614	A	13	8	1.22	16	0.5
615	A	5	3	1.	8	0.375
616	A	2	2	1.	10	0.2
617	A	2	2	1.	10	0.2
618	A	2	2	1.	8	0.25
619	A	2	2	1.	12	0.167
620	A	2	2	1.	12	0.167
621	A	2	2	1.	14	0.143
622	A	3	2	1.	16	0.125
623	A	3	2	1.	18	0.111
624	A	3	2	1.	18	0.111
625	A	4	3	1.	20	0.15
626	A	4	3	1.	22	0.136
627	A	4	3	1.	22	0.136
628	A	1	1	1.	7	0.143
629	A	3	2	1.	9	0.222
630	A	4	2	1.	9	0.222
631	A	5	2	1.	9	0.222
632	A	3	2	1.	9	0.222
633	A	3	3	1.	8	0.375
634	A	3	1	1.	8	0.125
635	A	2	2	1.	6	0.333
636	A	2	3	1.	6	0.5
637	A	2	2	1.	14	0.143
638	A	3	2	1.	8	0.25
639	A	8	6	1.	20	0.3
640	A	5	4	1.	14	0.286
641	A	4	4	1.	8	0.5
642	A	4	3	1.	8	0.375
643	A	4	5	1.	14	0.357
644	A	6	7	1.	12	0.583
645	A	5	5	1.	8	0.625

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
646	A	5	5	1.	8	0.625
647	A	10	7	1.	8	0.875
648	A	13	8	1.	8	1.
649	A	5	5	1.	8	0.625
650	A	10	5	1.	8	0.625
651	A	3	3	1.	14	0.214
652	A	3	3	1.	14	0.214
653	A	2	1	1.	15	0.067
654	A	6	5	1.	14	0.357
655	A	3	2	1.	15	0.133
656	A	4	5	1.	17	0.294
657	A	10	7	1.	17	0.412
658	A	4	3	1.	17	0.176
659	A	3	3	1.	17	0.176
660	A	5	3	1.	17	0.176
661	A	2	2	1.	15	0.133
662	A	2	2	1.	15	0.133
663	A	4	4	1.	14	0.286
664	A	3	5	1.	17	0.294
665	A	8	6	1.	17	0.353
666	A	4	4	1.	17	0.235
667	A	5	4	1.	17	0.235
668	A	6	4	1.	19	0.21
669	A	3	3	1.	11	0.273
670	A	4	3	1.	11	0.273
671	A	4	4	1.	13	0.308
672	A	8	8	1.	13	0.615
673	A	2	2	1.	13	0.154
674	A	8	8	1.	13	0.615
675	A	17	11	1.	13	0.846
676	A	8	8	1.	11	0.727
677	A	3	2	1.	11	0.182
678	A	12	6	1.	13	0.462
679	A	7	7	1.	13	0.538
680	A	8	7	1.	8	0.875
681	A	11	8	1.	13	0.615

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
682	A	4	4	1.04	15	0.267
683	A	9	7	1.08	15	0.467
684	A	11	8	1.25	15	0.533
685	A	4	4	1.	15	0.267
686	A	4	6	1.03	12	0.5
687	A	4	5	1.04	15	0.333
688	A	5	6	1.02	15	0.4
689	A	16	11	1.33	15	0.733
690	A	2	3	1.	15	0.2
691	A	5	8	1.3	15	0.533
692	A	6	4	1.36	17	0.235
693	A	11	9	1.29	17	0.529
694	A	8	5	1.33	17	0.294
695	B	8	7	2.15	16	0.438
696	A	4	4	1.	16	0.25
697	A	5	4	1.	8	0.5
698	A	5	5	1.	13	0.385
699	A	2	2	1.	24	0.083
700	A	2	3	1.	21	0.143
701	A	5	5	1.	12	0.417
702	A	7	6	1.71	10	0.6
703	A	5	6	1.	8	0.75
704	A	6	6	1.	10	0.6
705	B	5	5	2.39	7	0.714

3 Listing of integrals

3.1 $\int \frac{1}{a^2-b^2x^2}\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Rubi [A] time = 0.0200329, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 - b^2*x^2)^(-1),x]

[Out] ArcTanh[(b*x)/a]/(a*b)
```

Rubi in Sympy [A] time = 2.83151, size = 8, normalized size = 0.57

$$\frac{\operatorname{atanh}\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b**2*x**2+a**2),x)

[Out] atanh(b*x/a)/(a*b)
```

Mathematica [A] time = 0.00544131, size = 14, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 - b^2*x^2)^(-1),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Maple [B] time = 0.009, size = 32, normalized size = 2.3

$$\frac{\ln(bx+a)}{2\,ab}-\frac{\ln(bx-a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b^2*x^2+a^2),x)
```

```
[Out] 1/2*ln(b*x+a)/a/b-1/2/a/b*ln(b*x-a)
```

Maxima [A] time = 1.34838, size = 42, normalized size = 3.

$$\frac{\log(bx+a)}{2\,ab}-\frac{\log(bx-a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b^2*x^2 - a^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

Fricas [A] time = 0.22663, size = 34, normalized size = 2.43

$$\frac{\log(bx+a)-\log(bx-a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b^2*x^2 - a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

Sympy [A] time = 0.158839, size = 20, normalized size = 1.43

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b**2*x**2+a**2),x)

[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

GIAC/XCAS [A] time = 0.21567, size = 45, normalized size = 3.21

$$\frac{\ln\left(|bx+a|\right)}{2\,ab}-\frac{\ln\left(|bx-a|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b^2*x^2 - a^2),x, algorithm="giac")

[Out] 1/2*ln(abs(b*x + a))/(a*b) - 1/2*ln(abs(b*x - a))/(a*b)
```

3.2 $\int \frac{1}{a^2+b^2x^2}\,dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

Rubi [A] time = 0.012504, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2*x^2)^(-1),x]

[Out] ArcTan[(b*x)/a]/(a*b)
```

Rubi in Sympy [A] time = 1.96461, size = 8, normalized size = 0.57

$$\frac{\operatorname{atan}\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**2+a**2),x)

[Out] atan(b*x/a)/(a*b)
```

Mathematica [A] time = 0.00503461, size = 14, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2*x^2)^(-1),x]

[Out] ArcTan[(b*x)/a]/(a*b)
```

Maple [A] time = 0.006, size = 15, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{bx}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^2+a^2),x)

[Out] arctan(b*x/a)/a/b
```

Maxima [A] time = 1.53156, size = 19, normalized size = 1.36

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*x^2 + a^2),x, algorithm="maxima")

[Out] arctan(b*x/a)/(a*b)
```

Fricas [A] time = 0.215272, size = 19, normalized size = 1.36

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*x^2 + a^2),x, algorithm="fricas")

[Out] arctan(b*x/a)/(a*b)
```

Sympy [A] time = 0.14532, size = 26, normalized size = 1.86

$$\frac{-\frac{i\log\left(-\frac{ia}{b}+x\right)}{2}+\frac{i\log\left(\frac{ia}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/b + x)/2 + I*log(I*a/b + x)/2)/(a*b)
```

GIAC/XCAS [A] time = 0.21507, size = 19, normalized size = 1.36

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*x^2 + a^2),x, algorithm="giac")
```

```
[Out] arctan(b*x/a)/(a*b)
```

3.3 $\int \sec(2ax)\,dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

```
[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Rubi [A] time = 0.00976748, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[2*a*x],x]

[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Rubi in Sympy [A] time = 0.579675, size = 10, normalized size = 0.77

$$\frac{\operatorname{atanh}(\sin(2ax))}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(2*a*x),x)

[Out] atanh(sin(2*a*x))/(2*a)
```

Mathematica [B] time = 0.0133567, size = 37, normalized size = 2.85

$$\frac{\log(\sin(ax)+\cos(ax))}{2a}-\frac{\log(\cos(ax)-\sin(ax))}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[2*a*x],x]
```

```
[Out] -Log[Cos[a*x] - Sin[a*x]]/(2*a) + Log[Cos[a*x] + Sin[a*x]]/(2*a)
```

Maple [A] time = 0.006, size = 18, normalized size = 1.4

$$\frac{\ln\left(\sec\left(2\,ax\right)+\tan\left(2\,ax\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(2*a*x),x)
```

```
[Out] 1/2/a*ln(sec(2*a*x)+tan(2*a*x))
```

Maxima [A] time = 1.37831, size = 23, normalized size = 1.77

$$\frac{\log\left(\sec\left(2\,ax\right)+\tan\left(2\,ax\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(2*a*x) + tan(2*a*x))/a
```

Fricas [A] time = 0.245983, size = 35, normalized size = 2.69

$$\frac{\log\left(\sin\left(2\,ax\right)+1\right)-\log\left(-\sin\left(2\,ax\right)+1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a
```

Sympy [A] time = 0.108791, size = 29, normalized size = 2.23

$$\begin{cases} \frac{-\frac{\log\left(\sin\left(2ax\right)-1\right)}{2}+\frac{\log\left(\sin\left(2ax\right)+1\right)}{2}}{2a} & \text{for}\: 2a\neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*a*x),x)
```

```
[Out] Piecewise(((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a)
, Ne(2*a, 0)), (x, True))
```

GIAC/XCAS [A] time = 0.218672, size = 54, normalized size = 4.15

$$\frac{\ln\left(\left|\frac{1}{\sin(2\,ax)} + \sin\left(2\,ax\right) + 2\right|\right) - \ln\left(\left|\frac{1}{\sin(2\,ax)} + \sin\left(2\,ax\right) - 2\right|\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*a*x),x, algorithm="giac")
```

```
[Out] 1/8*(ln(abs(1/sin(2*a*x) + sin(2*a*x) + 2)) - ln(abs(1/sin(2*a*x)
 + sin(2*a*x) - 2)))/a
```

3.4 $\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx$

Optimal. Leaf size=11

$$-\frac{3}{4}\tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

```
[Out] (-3*ArcTanh[Cos[x/3]])/4
```

Rubi [A] time = 0.00796438, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{3}{4}\tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x/3]/4,x]

[Out] (-3*ArcTanh[Cos[x/3]])/4
```

Rubi in Sympy [A] time = 0.659701, size = 10, normalized size = 0.91

$$-\frac{3\,\operatorname{atanh}\left(\cos\left(\frac{x}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/4/sin(1/3*x),x)

[Out] -3*atanh(cos(x/3))/4
```

Mathematica [B] time = 0.00549987, size = 23, normalized size = 2.09

$$\frac{1}{4}\left(3\log\left(\sin\left(\frac{x}{6}\right)\right)-3\log\left(\cos\left(\frac{x}{6}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x/3]/4,x]
```

```
[Out] (-3*Log[Cos[x/6]] + 3*Log[Sin[x/6]])/4
```

Maple [A] time = 0.01, size = 15, normalized size = 1.4

$$\frac{3}{4}\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/4/sin(1/3*x),x)

[Out] 3/4*ln(csc(1/3*x)-cot(1/3*x))
```

Maxima [A] time = 1.37099, size = 26, normalized size = 2.36

$$-\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right)+\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/4/sin(1/3*x),x, algorithm="maxima")

[Out] -3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)
```

Fricas [A] time = 0.24232, size = 31, normalized size = 2.82

$$-\frac{3}{8}\log\left(\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)+\frac{3}{8}\log\left(-\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/4/sin(1/3*x),x, algorithm="fricas")

[Out] -3/8*log(1/2*cos(1/3*x) + 1/2) + 3/8*log(-1/2*cos(1/3*x) + 1/2)
```

Sympy [A] time = 0.099579, size = 22, normalized size = 2.

$$\frac{3\log\left(\cos\left(\frac{x}{3}\right)-1\right)}{8}-\frac{3\log\left(\cos\left(\frac{x}{3}\right)+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/4/sin(1/3*x),x)
```

```
[Out] 3*log(cos(x/3) - 1)/8 - 3*log(cos(x/3) + 1)/8
```

GIAC/XCAS [A] time = 0.208553, size = 31, normalized size = 2.82

$$-\frac{3}{8}\ln\left(3\cos\left(\frac{1}{3}x\right)+3\right)+\frac{3}{8}\ln\left(-3\cos\left(\frac{1}{3}x\right)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/4/sin(1/3*x),x, algorithm="giac")
```

```
[Out] -3/8*ln(3*cos(1/3*x) + 3) + 3/8*ln(-3*cos(1/3*x) + 3)
```

3.5 $\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

```
[Out] -ArcTanh[Sin[Pi/4 + 2*x]]/2
```

Rubi [A] time = 0.00902192, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[-Sec[Pi/4 + 2*x],x]

[Out] -ArcTanh[Sin[Pi/4 + 2*x]]/2
```

Rubi in Sympy [A] time = 0.703907, size = 12, normalized size = 0.8

$$-\frac{\operatorname{atanh}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(-1/cos(1/4*pi+2*x),x)

[Out] -atanh(sin(2*x + pi/4))/2
```

Mathematica [B] time = 0.0129273, size = 55, normalized size = 3.67

$$\frac{1}{2}\log\left(\cos\left(\frac{1}{8}(8x+\pi)\right)-\sin\left(\frac{1}{8}(8x+\pi)\right)\right)-\frac{1}{2}\log\left(\sin\left(\frac{1}{8}(8x+\pi)\right)+\cos\left(\frac{1}{8}(8x+\pi)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sec[Pi/4 + 2*x],x]
```

```
[Out] Log[Cos[(Pi + 8*x)/8] - Sin[(Pi + 8*x)/8]]/2 - Log[Cos[(Pi + 8*x)
/8] + Sin[(Pi + 8*x)/8]]/2
```

Maple [A] time = 0.01, size = 21, normalized size = 1.4

$$-\frac{1}{2}\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-1/cos(1/4*Pi+2*x),x)
```

```
[Out] -1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))
```

Maxima [A] time = 1.34341, size = 36, normalized size = 2.4

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/cos(1/4*pi + 2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)
```

Fricas [A] time = 0.223654, size = 39, normalized size = 2.6

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/cos(1/4*pi + 2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

Sympy [A] time = 0.251277, size = 22, normalized size = 1.47

$$\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/cos(1/4*pi+2*x),x)
```

```
[Out] log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2
```

GIAC/XCAS [A] time = 0.203297, size = 39, normalized size = 2.6

$$-\frac{1}{4}\ln\left(\sin\left(\frac{1}{4}\pi + 2x\right) + 1\right) + \frac{1}{4}\ln\left(-\sin\left(\frac{1}{4}\pi + 2x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/cos(1/4*pi + 2*x),x, algorithm="giac")
```

```
[Out] -1/4*ln(sin(1/4*pi + 2*x) + 1) + 1/4*ln(-sin(1/4*pi + 2*x) + 1)
```

3.6 $\int \sec(x)\tan(x)\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

```
[Out] Sec[x]
```

Rubi [A] time = 0.0115559, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*Tan[x],x]

[Out] Sec[x]
```

Rubi in Sympy [A] time = 0.768284, size = 3, normalized size = 1.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)*tan(x),x)

[Out] 1/cos(x)
```

Mathematica [A] time = 0.00223956, size = 2, normalized size = 1.

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*Tan[x],x]

[Out] Sec[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)*tan(x),x)
```

```
[Out] sec(x)
```

Maxima [A] time = 1.341, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x)
```

Fricas [A] time = 0.216057, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

Sympy [A] time = 0.04751, size = 3, normalized size = 1.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

GIAC/XCAS [A] time = 0.198644, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x)
```

3.7 $\int \cot(x)\csc(x)\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -Csc[x]
```

Rubi [A] time = 0.0130908, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sin(x)}{\tan^2(x)} - \int \frac{\sin(x)}{\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cot(x)*csc(x),x)
```

```
[Out] -sin(x)/tan(x)**2 - Integral(sin(x)/tan(x), x)
```

Mathematica [A] time = 0.00250003, size = 4, normalized size = 1.

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Maple [A] time = 0., size = 5, normalized size = 1.3

$$-\csc(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)*csc(x),x)
```

```
[Out] -csc(x)
```

Maxima [A] time = 1.38066, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

Fricas [A] time = 0.211163, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [A] time = 0.053472, size = 5, normalized size = 1.25

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x)
```

```
[Out] -1/sin(x)
```

GIAC/XCAS [A] time = 0.197939, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

3.8 $\int \csc(2x)\tan(x)\,dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{2}$$

```
[Out] Tan[x]/2
```

Rubi [A] time = 0.0356592, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*Tan[x],x]
```

```
[Out] Tan[x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(tan(x)/sin(2*x),x)
```

```
[Out] Integral(tan(x)/sin(2*x), x)
```

Mathematica [A] time = 0.010362, size = 6, normalized size = 1.

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Tan[x],x]
```

```
[Out] Tan[x]/2
```

Maple [A] time = 0.019, size = 5, normalized size = 0.8

$$\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/sin(2*x),x)
```

```
[Out] 1/2*tan(x)
```

Maxima [A] time = 1.36323, size = 36, normalized size = 6.

$$\frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="maxima")
```

```
[Out] sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 0.209498, size = 5, normalized size = 0.83

$$\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)
```

Sympy [A] time = 0.764627, size = 7, normalized size = 1.17

$$\frac{\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/sin(2*x),x)
```

```
[Out] sin(x)/(2*cos(x))
```

GIAC/XCAS [A] time = 0.205208, size = 5, normalized size = 0.83

$$\frac{1}{2} \tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*tan(x)
```

3.9 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0176253, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cos[x])^(-1),x]

[Out] Sin[x]/(1 + Cos[x])
```

Rubi in Sympy [A] time = 0.486752, size = 7, normalized size = 0.78

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+cos(x)),x)

[Out] sin(x)/(cos(x) + 1)
```

Mathematica [A] time = 0.00601184, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

Maple [A] time = 0.002, size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+cos(x)),x)
```

```
[Out] tan(1/2*x)
```

Maxima [A] time = 1.34632, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 0.217694, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.210411, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

GIAC/XCAS [A] time = 0.206472, size = 41, normalized size = 4.56

$$-\frac{2 \tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.10 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.0166231, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Cos[x])^(-1),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi in Sympy [A] time = 0.505476, size = 8, normalized size = 0.67

$$-\frac{\sin(x)}{-\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(x)),x)

[Out] -sin(x)/(-cos(x) + 1)
```

Mathematica [A] time = 0.00768407, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0., size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 1.3454, size = 14, normalized size = 1.17

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 0.232885, size = 14, normalized size = 1.17

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.667091, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(x)),x)

[Out] -1/tan(x/2)
```

GIAC/XCAS [A] time = 0.209506, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="giac")

[Out] -1/tan(1/2*x)
```

3.11 $\int \frac{\sin(x)}{a-b\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\log(a-b\cos(x))}{b}$$

```
[Out] Log[a - b*Cos[x]]/b
```

Rubi [A] time = 0.0388799, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\log(a-b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/(a - b*Cos[x]),x]

[Out] Log[a - b*Cos[x]]/b
```

Rubi in Sympy [A] time = 2.2841, size = 8, normalized size = 0.67

$$\frac{\log\left(a-b\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/(a-b*cos(x)),x)

[Out] log(a - b*cos(x))/b
```

Mathematica [A] time = 0.0067702, size = 12, normalized size = 1.

$$\frac{\log(a-b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Log[a - b*Cos[x]]/b
```

Maple [A] time = 0.013, size = 13, normalized size = 1.1

$$\frac{\ln(a - b\cos(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/(a-b*cos(x)),x)
```

```
[Out] ln(a-b*cos(x))/b
```

Maxima [A] time = 1.34029, size = 18, normalized size = 1.5

$$\frac{\log(b\cos(x) - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(x)/(b*cos(x) - a),x, algorithm="maxima")
```

```
[Out] log(b*cos(x) - a)/b
```

Fricas [A] time = 0.27593, size = 16, normalized size = 1.33

$$\frac{\log(-b\cos(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(x)/(b*cos(x) - a),x, algorithm="fricas")
```

```
[Out] log(-b*cos(x) + a)/b
```

Sympy [A] time = 0.701188, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log\left(-\frac{a}{b} + \cos(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(a-b*cos(x)),x)
```

```
[Out] Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

GIAC/XCAS [A] time = 0.199926, size = 19, normalized size = 1.58

$$\frac{\ln\left(|b\cos\left(x\right) - a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(x)/(b*cos(x) - a),x, algorithm="giac")
```

```
[Out] ln(abs(b*cos(x) - a))/b
```

3.12 $\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Rubi [A] time = 0.0473408, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]

[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Rubi in Sympy [A] time = 4.63121, size = 10, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/(a**2+b**2*sin(x)**2),x)

[Out] atan(b*sin(x)/a)/(a*b)
```

Mathematica [A] time = 0.0142117, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Maple [A] time = 0.017, size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{b\sin(x)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/(a^2+b^2*sin(x)^2),x)
```

```
[Out] arctan(b*sin(x)/a)/a/b
```

Maxima [A] time = 1.49584, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(b^2*sin(x)^2 + a^2),x, algorithm="maxima")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

Fricas [A] time = 0.285418, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(b^2*sin(x)^2 + a^2),x, algorithm="fricas")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

Sympy [A] time = 1.26783, size = 31, normalized size = 2.07

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ \frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(a**2+b**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sin(x)), E
q(a, 0)), (sin(x)/a**2, Eq(b, 0)), (atan(b*sin(x)/a)/(a*b), True)
)
```

GIAC/XCAS [A] time = 0.202163, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(b^2*sin(x)^2 + a^2),x, algorithm="giac")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

3.13 $\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Rubi [A] time = 0.0508859, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/(a^2 - b^2*Sin[x]^2),x]

[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Rubi in Sympy [A] time = 5.70068, size = 10, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/(a**2-b**2*sin(x)**2),x)

[Out] atanh(b*sin(x)/a)/(a*b)
```

Mathematica [A] time = 0.013909, size = 15, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]

[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Maple [B] time = 0.017, size = 34, normalized size = 2.3

$$\frac{\ln(a+b\sin(x))}{2ab}-\frac{\ln(b\sin(x)-a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/(a^2-b^2*sin(x)^2),x)

[Out] 1/2/a/b*ln(a+b*sin(x))-1/2/a/b*ln(b*sin(x)-a)
```

Maxima [A] time = 1.38631, size = 45, normalized size = 3.

$$\frac{\log(b\sin(x)+a)}{2ab}-\frac{\log(b\sin(x)-a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(x)/(b^2*sin(x)^2 - a^2),x, algorithm="maxima")

[Out] 1/2*log(b*sin(x) + a)/(a*b) - 1/2*log(b*sin(x) - a)/(a*b)
```

Fricas [A] time = 0.250197, size = 35, normalized size = 2.33

$$\frac{\log(b\sin(x)+a)-\log(-b\sin(x)+a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(x)/(b^2*sin(x)^2 - a^2),x, algorithm="fricas")

[Out] 1/2*(log(b*sin(x) + a) - log(-b*sin(x) + a))/(a*b)
```

Sympy [A] time = 1.26538, size = 44, normalized size = 2.93

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ -\frac{\log\left(-\frac{a}{b}+\sin(x)\right)}{2ab} + \frac{\log\left(\frac{a}{b}+\sin(x)\right)}{2ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(a**2-b**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (1/(b**2*sin(x)), Eq
(a, 0)), (sin(x)/a**2, Eq(b, 0)), (-log(-a/b + sin(x))/(2*a*b) +
log(a/b + sin(x))/(2*a*b), True))
```

GIAC/XCAS [A] time = 0.202103, size = 47, normalized size = 3.13

$$\frac{\ln(|b\sin(x) + a|)}{2\,ab} - \frac{\ln(|b\sin(x) - a|)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(x)/(b^2*sin(x)^2 - a^2),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(b*sin(x) + a))/(a*b) - 1/2*ln(abs(b*sin(x) - a))/(a*b)
```

3.14 $\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Rubi [A] time = 0.0632104, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]

[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Rubi in Sympy [A] time = 3.76368, size = 15, normalized size = 0.88

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)

[Out] log(a**2 + b**2*sin(x)**2)/b**2
```

Mathematica [A] time = 0.0223703, size = 23, normalized size = 1.35

$$\frac{\log\left(2a^2-b^2\cos(2x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[2*a^2 + b^2 - b^2*Cos[2*x]]/b^2
```

Maple [A] time = 0.031, size = 18, normalized size = 1.1

$$\frac{\ln\left(a^2 + b^2 (\sin(x))^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)/(a^2+b^2*sin(x)^2),x)
```

```
[Out] ln(a^2+b^2*sin(x)^2)/b^2
```

Maxima [A] time = 1.3524, size = 23, normalized size = 1.35

$$\frac{\log\left(b^2 \sin(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(b^2*sin(x)^2 + a^2),x, algorithm="maxima")
```

```
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```

Fricas [A] time = 0.260401, size = 28, normalized size = 1.65

$$\frac{\log\left(-b^2 \cos(x)^2 + a^2 + b^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(b^2*sin(x)^2 + a^2),x, algorithm="fricas")
```

```
[Out] log(-b^2*cos(x)^2 + a^2 + b^2)/b^2
```

Sympy [A] time = 3.99181, size = 32, normalized size = 1.88

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log\left(a^2+b^2\sin^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*
sin(x)**2)/(2*b**2), True))
```

GIAC/XCAS [A] time = 0.238326, size = 104, normalized size = 6.12

$$-\frac{2 \ln\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)}{b^2}+\frac{\ln\left(\left|a^2-\frac{2 a^2(\cos(x)-1)}{\cos(x)+1}-\frac{4 b^2(\cos(x)-1)}{\cos(x)+1}+\frac{a^2(\cos(x)-1)^2}{(\cos(x)+1)^2}\right|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(b^2*sin(x)^2 + a^2),x, algorithm="giac")
```

```
[Out] -2*ln(-(cos(x) - 1)/(cos(x) + 1) + 1)/b^2 + ln(abs(a^2 - 2*a^2*(c
os(x) - 1)/(cos(x) + 1) - 4*b^2*(cos(x) - 1)/(cos(x) + 1) + a^2*(
cos(x) - 1)^2/(cos(x) + 1)^2))/b^2
```

3.15 $\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Rubi [A] time = 0.066265, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]

[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Rubi in Sympy [A] time = 4.00925, size = 17, normalized size = 0.89

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)

[Out] -log(a**2 - b**2*sin(x)**2)/b**2
```

Mathematica [A] time = 0.0216296, size = 25, normalized size = 1.32

$$-\frac{\log\left(2a^2+b^2\cos(2x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] -(Log[2*a^2 - b^2 + b^2*Cos[2*x]]/b^2)
```

Maple [A] time = 0.032, size = 20, normalized size = 1.1

$$-\frac{\ln\left(a^2 - b^2 (\sin(x))^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-b^2*sin(x)^2),x)
```

```
[Out] -ln(a^2-b^2*sin(x)^2)/b^2
```

Maxima [A] time = 1.42873, size = 27, normalized size = 1.42

$$-\frac{\log\left(b^2 \sin(x)^2 - a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(2*x)/(b^2*sin(x)^2 - a^2),x, algorithm="maxima")
```

```
[Out] -log(b^2*sin(x)^2 - a^2)/b^2
```

Fricas [A] time = 0.247466, size = 31, normalized size = 1.63

$$-\frac{\log\left(b^2 \cos(x)^2 + a^2 - b^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(2*x)/(b^2*sin(x)^2 - a^2),x, algorithm="fricas")
```

```
[Out] -log(b^2*cos(x)^2 + a^2 - b^2)/b^2
```

Sympy [A] time = 3.74202, size = 34, normalized size = 1.79

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log\left(a^2 - b^2 \sin^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2
*sin(x)**2)/(2*b**2), True))
```

GIAC/XCAS [A] time = 0.20716, size = 104, normalized size = 5.47

$$-\frac{\ln\left(a^2 - \frac{2\,a^2(\cos(x)-1)}{\cos(x)+1} + \frac{4\,b^2(\cos(x)-1)}{\cos(x)+1} + \frac{a^2(\cos(x)-1)^2}{(\cos(x)+1)^2}\right)}{b^2} + \frac{2\ln\left(-\frac{\cos(x)-1}{\cos(x)+1} + 1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(2*x)/(b^2*sin(x)^2 - a^2),x, algorithm="giac")
```

```
[Out] -ln(a^2 - 2*a^2*(cos(x) - 1)/(cos(x) + 1) + 4*b^2*(cos(x) - 1)/(c
os(x) + 1) + a^2*(cos(x) - 1)^2/(cos(x) + 1)^2)/b^2 + 2*ln(-(cos(
x) - 1)/(cos(x) + 1) + 1)/b^2
```

3.16 $\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{\log\left(a^2+b^2\cos^2(x)\right)}{b^2}$$

```
[Out] -(Log[a^2 + b^2*Cos[x]^2]/b^2)
```

Rubi [A] time = 0.0679615, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]

[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

Rubi in Sympy [A] time = 3.84695, size = 17, normalized size = 0.94

$$-\frac{\log\left(a^2+b^2\cos^2\left(x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)

[Out] -log(a**2 + b**2*cos(x)**2)/b**2
```

Mathematica [A] time = 0.0241267, size = 23, normalized size = 1.28

$$-\frac{\log\left(2a^2+b^2\cos(2x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] -(Log[2*a^2 + b^2 + b^2*Cos[2*x]]/b^2)
```

Maple [A] time = 0.024, size = 19, normalized size = 1.1

$$-\frac{\ln\left(a^2 + b^2 (\cos(x))^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2+b^2*cos(x)^2),x)
```

```
[Out] -ln(a^2+b^2*cos(x)^2)/b^2
```

Maxima [A] time = 1.34917, size = 24, normalized size = 1.33

$$-\frac{\log\left(b^2 \cos(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(b^2*cos(x)^2 + a^2),x, algorithm="maxima")
```

```
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Fricas [A] time = 0.245358, size = 24, normalized size = 1.33

$$-\frac{\log\left(b^2 \cos(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(b^2*cos(x)^2 + a^2),x, algorithm="fricas")
```

```
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Sympy [A] time = 4.02715, size = 34, normalized size = 1.89

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log\left(a^2+b^2\cos^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2
*cos(x)**2)/(2*b**2), True))
```

GIAC/XCAS [A] time = 0.214492, size = 130, normalized size = 7.22

$$-\frac{\ln\left(a^2+b^2-\frac{2\,a^2(\cos(x)-1)}{\cos(x)+1}+\frac{2\,b^2(\cos(x)-1)}{\cos(x)+1}+\frac{a^2(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{b^2(\cos(x)-1)^2}{(\cos(x)+1)^2}\right)}{b^2}+\frac{2\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(b^2*cos(x)^2 + a^2),x, algorithm="giac")
```

```
[Out] -ln(a^2 + b^2 - 2*a^2*(cos(x) - 1)/(cos(x) + 1) + 2*b^2*(cos(x) -
 1)/(cos(x) + 1) + a^2*(cos(x) - 1)^2/(cos(x) + 1)^2 + b^2*(cos(x
) - 1)^2/(cos(x) + 1)^2)/b^2 + 2*ln(-(cos(x) - 1)/(cos(x) + 1) +
1)/b^2
```

3.17 $\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

```
[Out] Log[a^2 - b^2*Cos[x]^2]/b^2
```

Rubi [A] time = 0.0691707, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]

[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Rubi in Sympy [A] time = 4.0082, size = 15, normalized size = 0.83

$$\frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)

[Out] log(a**2 - b**2*cos(x)**2)/b**2
```

Mathematica [A] time = 0.0253135, size = 25, normalized size = 1.39

$$\frac{\log\left(2a^2-b^2\cos(2x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[2*a^2 - b^2 - b^2*Cos[2*x]]/b^2
```

Maple [A] time = 0.028, size = 19, normalized size = 1.1

$$\frac{\ln\left(a^{2}-b^{2}\left(\cos\left(x\right)\right)^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)/(a^2-b^2*cos(x)^2),x)
```

```
[Out] ln(a^2-b^2*cos(x)^2)/b^2
```

Maxima [A] time = 1.40321, size = 26, normalized size = 1.44

$$\frac{\log\left(b^{2}\cos\left(x\right)^{2}-a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(2*x)/(b^2*cos(x)^2 - a^2),x, algorithm="maxima")
```

```
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Fricas [A] time = 0.246142, size = 26, normalized size = 1.44

$$\frac{\log\left(b^{2}\cos\left(x\right)^{2}-a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(2*x)/(b^2*cos(x)^2 - a^2),x, algorithm="fricas")
```

```
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Sympy [A] time = 3.74699, size = 32, normalized size = 1.78

$$2\left(\begin{cases}-\frac{\cos^{2}\left(x\right)}{2a^{2}} & \text{for}\: b^{2}=0\\\frac{\log\left(a^{2}-b^{2}\cos^{2}\left(x\right)\right)}{2b^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*
cos(x)**2)/(2*b**2), True))
```

GIAC/XCAS [A] time = 0.217939, size = 165, normalized size = 9.17

$$\frac{(a+b)\ln\left(\left|a-b-\frac{a(\cos(x)-1)}{\cos(x)+1}-\frac{b(\cos(x)-1)}{\cos(x)+1}\right|\right)}{ab^2+b^3}+\frac{(a-b)\ln\left(\left|-a-b+\frac{a(\cos(x)-1)}{\cos(x)+1}-\frac{b(\cos(x)-1)}{\cos(x)+1}\right|\right)}{ab^2-b^3}-\frac{2\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sin(2*x)/(b^2*cos(x)^2 - a^2),x, algorithm="giac")
```

```
[Out] (a + b)*ln(abs(a - b - a*(cos(x) - 1)/(cos(x) + 1) - b*(cos(x) -
1)/(cos(x) + 1)))/(a*b^2 + b^3) + (a - b)*ln(abs(-a - b + a*(cos(
x) - 1)/(cos(x) + 1) - b*(cos(x) - 1)/(cos(x) + 1)))/(a*b^2 - b^3
) - 2*ln(-(cos(x) - 1)/(cos(x) + 1) + 1)/b^2
```

3.18 $\int \frac{1}{4-\cos^2(x)}\,dx$

Optimal. Leaf size=41

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)
]/(2*Sqrt[3])
```

Rubi [A] time = 0.038348, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)
]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 0.630663, size = 19, normalized size = 0.46

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\tan(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4-cos(x)**2),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*tan(x)/3)/6
```

Mathematica [A] time = 0.0169799, size = 19, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.023, size = 14, normalized size = 0.3

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{2\sqrt{3}\tan(x)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4-cos(x)^2),x)
```

```
[Out] 1/6*3^(1/2)*arctan(2/3*3^(1/2)*tan(x))
```

Maxima [A] time = 1.50859, size = 18, normalized size = 0.44

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x)^2 - 4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))
```

Fricas [A] time = 0.247052, size = 42, normalized size = 1.02

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{7\sqrt{3}\cos(x)^2-4\sqrt{3}}{12\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x)^2 - 4),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x
)*sin(x)))
```

Sympy [A] time = 1.34594, size = 61, normalized size = 1.49

$$\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4-cos(x)**2),x)
```

```
[Out] sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6
+ sqrt(3)*(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6
```

GIAC/XCAS [A] time = 0.205371, size = 62, normalized size = 1.51

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x)^2 - 4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)
*cos(2*x) + sqrt(3) - 2*cos(2*x) + 2)))
```

3.19 $\int \frac{e^x}{-1+e^{2x}} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(e^x)$$

```
[Out] -ArcTanh[E^x]
```

Rubi [A] time = 0.0309683, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(-1 + E^(2*x)),x]

[Out] -ArcTanh[E^x]
```

Rubi in Sympy [A] time = 3.017, size = 5, normalized size = 0.83

$$-\operatorname{atanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(-1+exp(2*x)),x)

[Out] -atanh(exp(x))
```

Mathematica [B] time = 0.00554211, size = 23, normalized size = 3.83

$$\frac{1}{2}\log(1-e^x) - \frac{1}{2}\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(-1 + E^(2*x)),x]

[Out] Log[1 - E^x]/2 - Log[1 + E^x]/2
```

Maple [A] time = 0., size = 6, normalized size = 1.

$$-\mathit{Artanh}\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [A] time = 1.33882, size = 20, normalized size = 3.33

$$-\frac{1}{2}\,\log\left(e^{x}+1\right)+\frac{1}{2}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) - 1),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [A] time = 0.212988, size = 20, normalized size = 3.33

$$-\frac{1}{2}\,\log\left(e^{x}+1\right)+\frac{1}{2}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) - 1),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [A] time = 0.095184, size = 15, normalized size = 2.5

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

GIAC/XCAS [A] time = 0.198903, size = 22, normalized size = 3.67

$$-\frac{1}{2}\ln\left(e^{x}+1\right)+\frac{1}{2}\ln\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) - 1),x, algorithm="giac")
```

```
[Out] -1/2*ln(e^x + 1) + 1/2*ln(abs(e^x - 1))
```

3.20 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] Log[Log[x]]
```

Rubi [A] time = 0.020388, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Log[x]),x]

[Out] Log[Log[x]]
```

Rubi in Sympy [A] time = 1.15015, size = 3, normalized size = 1.

$$\log\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x),x)

[Out] log(log(x))
```

Mathematica [A] time = 0.00121082, size = 3, normalized size = 1.

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Log[x]),x]

[Out] Log[Log[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x),x)
```

```
[Out] ln(ln(x))
```

Maxima [A] time = 1.3547, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A] time = 0.207306, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A] time = 0.078751, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

<u>GIAC/XCAS</u> [A] time = 0.203934, size = 5, normalized size = 1.67

$$\ln(|\ln(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="giac")
```

```
[Out] ln(abs(ln(x)))
```

3.21 $\int \frac{1}{x(1+\log^2(x))} \, dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.0316255, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]
```

Rubi in Sympy [A] time = 3.31803, size = 3, normalized size = 1.

$$\operatorname{atan}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+ln(x)**2),x)

[Out] atan(log(x))
```

Mathematica [A] time = 0.00580001, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.52938, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 0.244666, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [A] time = 0.139617, size = 15, normalized size = 5.

$$\mathrm{RootSum}\left(4z^2 + 1, \left(i \mapsto i\log(2i + \log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

GIAC/XCAS [A] time = 0.200576, size = 4, normalized size = 1.33

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="giac")
```

```
[Out] arctan(ln(x))
```

3.22 $\int \frac{1}{x(1-\log(x))}\,dx$

Optimal. Leaf size=9

$$-\log(1-\log(x))$$

```
[Out] -Log[1 - Log[x]]
```

Rubi [A] time = 0.0318175, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\log(1-\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - Log[x])),x]

[Out] -Log[1 - Log[x]]
```

Rubi in Sympy [A] time = 2.32456, size = 7, normalized size = 0.78

$$-\log(-\log(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1-ln(x)),x)

[Out] -log(-log(x) + 1)
```

Mathematica [A] time = 0.00322607, size = 7, normalized size = 0.78

$$-\log(\log(x)-1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 - Log[x])),x]

[Out] -Log[-1 + Log[x]]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$-\ln(1-\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1-ln(x)),x)

[Out] -ln(1-ln(x))
```

Maxima [A] time = 1.35878, size = 9, normalized size = 1.

$$-\log(\log(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x*(log(x) - 1)),x, algorithm="maxima")

[Out] -log(log(x) - 1)
```

Fricas [A] time = 0.219102, size = 9, normalized size = 1.

$$-\log(\log(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x*(log(x) - 1)),x, algorithm="fricas")

[Out] -log(log(x) - 1)
```

Sympy [A] time = 0.088197, size = 7, normalized size = 0.78

$$-\log(\log(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1-ln(x)),x)

[Out] -log(log(x) - 1)
```

GIAC/XCAS [A] time = 0.20366, size = 30, normalized size = 3.33

$$-\frac{1}{2}\ln\left(\frac{1}{4}\pi^2(\operatorname{sign}(x)-1)^2+(\ln(|x|)-1)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x*(log(x) - 1)),x, algorithm="giac")
```

```
[Out] -1/2*ln(1/4*pi^2*(sign(x) - 1)^2 + (ln(abs(x)) - 1)^2)
```

3.23 $\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$

Optimal. Leaf size=9

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[Out] Log[1 + Log[x/a]]
```

Rubi [A] time = 0.0292583, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + Log[x/a])),x]

[Out] Log[1 + Log[x/a]]
```

Rubi in Sympy [A] time = 2.08941, size = 7, normalized size = 0.78

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+ln(x/a)),x)

[Out] log(log(x/a) + 1)
```

Mathematica [A] time = 0.00377644, size = 9, normalized size = 1.

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + Log[x/a])),x]

[Out] Log[1 + Log[x/a]]
```

Maple [A] time = 0.002, size = 10, normalized size = 1.1

$$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+ln(x/a)),x)

[Out] ln(1+ln(x/a))
```

Maxima [A] time = 1.35797, size = 12, normalized size = 1.33

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(log(x/a) + 1)),x, algorithm="maxima")

[Out] log(log(x/a) + 1)
```

Fricas [A] time = 0.215724, size = 12, normalized size = 1.33

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(log(x/a) + 1)),x, algorithm="fricas")

[Out] log(log(x/a) + 1)
```

Sympy [A] time = 0.099098, size = 7, normalized size = 0.78

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+ln(x/a)),x)
```

```
[Out] log(log(x/a) + 1)
```

GIAC/XCAS [A] time = 0.214195, size = 41, normalized size = 4.56

$$\frac{1}{2}\ln\left(\frac{1}{4}\pi^2(\text{sign}(a)\,\text{sign}(x)-1)^2+\left(\ln\left(\frac{|x|}{|a|}\right)+1\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(log(x/a) + 1)),x, algorithm="giac")
```

```
[Out] 1/2*ln(1/4*pi^2*(sign(a)*sign(x) - 1)^2 + (ln(abs(x)/abs(a)) + 1)
^2)
```

3.24 $\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx$

Optimal. Leaf size=25

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Rubi [A] time = 0.0426953, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[x] + x)^2/x^2,x]

[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4\sqrt{x}+6\log\left(\sqrt{x}\right)+2\int^{\sqrt{x}}x\,dx-\frac{1}{x}+\frac{4}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-x**(1/2))**2/x**2,x)

[Out] -4*sqrt(x) + 6*log(sqrt(x)) + 2*Integral(x, (x, sqrt(x))) - 1/x +
 4/sqrt(x)
```

Mathematica [A] time = 0.0185437, size = 25, normalized size = 1.

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[x] + x)^2/x^2,x]
```

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Maple [A] time = 0.01, size = 22, normalized size = 0.9

$$-x^{-1} + x + 3\ln(x) + 4\frac{1}{\sqrt{x}} - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-x^(1/2))^2/x^2,x)

[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

Maxima [A] time = 1.38255, size = 30, normalized size = 1.2

$$x - 4\sqrt{x} + \frac{4\sqrt{x} - 1}{x} + 3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x) + 1)^2/x^2,x, algorithm="maxima")

[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(x)
```

Fricas [A] time = 0.205364, size = 32, normalized size = 1.28

$$\frac{x^2 + 6x\log\left(\sqrt{x}\right) - 4(x-1)\sqrt{x} - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x) + 1)^2/x^2,x, algorithm="fricas")

[Out] (x^2 + 6*x*log(sqrt(x)) - 4*(x - 1)*sqrt(x) - 1)/x
```

Sympy [A] time = 1.01715, size = 22, normalized size = 0.88

$$-4\sqrt{x} + x + 3\log(x) - \frac{1}{x} + \frac{4}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-x**(1/2))**2/x**2,x)
```

```
[Out] -4*sqrt(x) + x + 3*log(x) - 1/x + 4/sqrt(x)
```

GIAC/XCAS [A] time = 0.202278, size = 31, normalized size = 1.24

$$x - 4\sqrt{x} + \frac{4\sqrt{x} - 1}{x} + 3\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x) + 1)^2/x^2,x, algorithm="giac")
```

```
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*ln(abs(x))
```

3.25 $\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

```
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Rubi [A] time = 0.153268, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

```
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Rubi in Sympy [A] time = 7.94552, size = 31, normalized size = 1.03

$$-\frac{6x^{\frac{7}{6}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+4\sqrt{x}+3\log\left(x^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)
```

```
[Out] -6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 3*log(x**(2/3))
```

Mathematica [A] time = 0.0111277, size = 30, normalized size = 1.

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

```
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Maple [A] time = 0.007, size = 21, normalized size = 0.7

$$-\frac{3}{2}x^{\frac{2}{3}}-\frac{6}{7}x^{\frac{7}{6}}+2\ln(x)+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x)
```

```
[Out] -3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

Maxima [A] time = 1.36028, size = 27, normalized size = 0.9

$$-\frac{6}{7}x^{\frac{7}{6}}-\frac{3}{2}x^{\frac{2}{3}}+4\sqrt{x}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(x))*(x^(2/3) - 2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)
```

Fricas [A] time = 0.202086, size = 30, normalized size = 1.

$$-\frac{6}{7}x^{\frac{7}{6}}-\frac{3}{2}x^{\frac{2}{3}}+4\sqrt{x}+12\log\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(x))*(x^(2/3) - 2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))
```

Sympy [A] time = 0.828284, size = 27, normalized size = 0.9

$$-\frac{6x^{\frac{7}{6}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+4\sqrt{x}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)
```

```
[Out] -6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 2*log(x)
```

GIAC/XCAS [A] time = 0.205221, size = 28, normalized size = 0.93

$$-\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 2\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(x))*(x^(2/3) - 2)/x^(3/2),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*ln(abs(x))
```

3.26 $\int \frac{-1+2x}{3+2x}\,dx$

Optimal. Leaf size=10

$$x-2\log(2x+3)$$

```
[Out] x - 2*Log[3 + 2*x]
```

Rubi [A] time = 0.0145224, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$x-2\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*x)/(3 + 2*x),x]

[Out] x - 2*Log[3 + 2*x]
```

Rubi in Sympy [A] time = 1.65684, size = 8, normalized size = 0.8

$$x-2\log\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x-1)/(3+2*x),x)

[Out] x - 2*log(2*x + 3)
```

Mathematica [A] time = 0.00293552, size = 10, normalized size = 1.

$$x-2\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*x)/(3 + 2*x),x]

[Out] x - 2*Log[3 + 2*x]
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$x - 2\ln(3 + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x-1)/(3+2*x),x)

[Out] x-2*ln(3+2*x)
```

Maxima [A] time = 1.33747, size = 14, normalized size = 1.4

$$x - 2\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 1)/(2*x + 3),x, algorithm="maxima")

[Out] x - 2*log(2*x + 3)
```

Fricas [A] time = 0.211592, size = 14, normalized size = 1.4

$$x - 2\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 1)/(2*x + 3),x, algorithm="fricas")

[Out] x - 2*log(2*x + 3)
```

Sympy [A] time = 0.058728, size = 8, normalized size = 0.8

$$x - 2\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x-1)/(3+2*x),x)

[Out] x - 2*log(2*x + 3)
```

GIAC/XCAS [A] time = 0.202613, size = 15, normalized size = 1.5

$$x - 2\ln(|2x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 1)/(2*x + 3),x, algorithm="giac")
```

```
[Out] x - 2*ln(abs(2*x + 3))
```

3.27 $\int \frac{-5+2x}{-2+3x^2}\,dx$

Optimal. Leaf size=47

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sq
rt[6] + 3*x])/12
```

Rubi [A] time = 0.053913, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sq
rt[6] + 3*x])/12
```

Rubi in Sympy [A] time = 1.84634, size = 42, normalized size = 0.89

$$\left(-\frac{5\sqrt{6}}{12}+\frac{1}{3}\right)\log\left(-3x+\sqrt{6}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(3x+\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-5+2*x)/(3*x**2-2),x)
```

```
[Out] (-5*sqrt(6)/12 + 1/3)*log(-3*x + sqrt(6)) + (1/3 + 5*sqrt(6)/12)*
log(3*x + sqrt(6))
```

Mathematica [A] time = 0.0484883, size = 47, normalized size = 1.

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sq
rt[6] + 3*x])/12
```

Maple [A] time = 0.004, size = 24, normalized size = 0.5

$$\frac{\ln \left(3 x^{2}-2\right)}{3}+\frac{5 \sqrt{6}}{6} \mathit{Artanh}\left(\frac{x \sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x-5)/(3*x^2-2),x)
```

```
[Out] 1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))
```

Maxima [A] time = 1.52149, size = 49, normalized size = 1.04

$$-\frac{5}{12} \sqrt{6} \log \left(\frac{3 x-\sqrt{6}}{3 x+\sqrt{6}}\right)+\frac{1}{3} \log \left(3 x^{2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 5)/(3*x^2 - 2),x, algorithm="maxima")
```

```
[Out] -5/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6))) + 1/3*log(3*x^
2 - 2)
```

Fricas [A] time = 0.203088, size = 63, normalized size = 1.34

$$\frac{1}{36} \sqrt{6}\left(2 \sqrt{6} \log \left(3 x^{2}-2\right)+15 \log \left(\frac{\sqrt{6}\left(3 x^{2}+2\right)+12 x}{3 x^{2}-2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 5)/(3*x^2 - 2),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(6)*(2*sqrt(6)*log(3*x^2 - 2) + 15*log((sqrt(6)*(3*x^2 +
 2) + 12*x)/(3*x^2 - 2)))
```

Sympy [A] time = 0.110403, size = 42, normalized size = 0.89

$$\left(-\frac{5\sqrt{6}}{12}+\frac{1}{3}\right)\log\left(x-\frac{\sqrt{6}}{3}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(x+\frac{\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5+2*x)/(3*x**2-2),x)
```

```
[Out] (-5*sqrt(6)/12 + 1/3)*log(x - sqrt(6)/3) + (1/3 + 5*sqrt(6)/12)*l
og(x + sqrt(6)/3)
```

GIAC/XCAS [A] time = 0.210334, size = 50, normalized size = 1.06

$$\frac{1}{12}\left(5\sqrt{6}+4\right)\ln\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\left(5\sqrt{6}-4\right)\ln\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 5)/(3*x^2 - 2),x, algorithm="giac")
```

```
[Out] 1/12*(5*sqrt(6) + 4)*ln(abs(x + 1/3*sqrt(6))) - 1/12*(5*sqrt(6) -
 4)*ln(abs(x - 1/3*sqrt(6)))
```

3.28 $\int \frac{-5+2x}{2+3x^2} \, dx$

Optimal. Leaf size=30

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Rubi [A] time = 0.0239123, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-5 + 2*x)/(2 + 3*x^2),x]

[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Rubi in Sympy [A] time = 1.80086, size = 27, normalized size = 0.9

$$\frac{\log\left(3x^2+2\right)}{3}-\frac{5\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-5+2*x)/(3*x**2+2),x)

[Out] log(3*x**2 + 2)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

Mathematica [A] time = 0.0213083, size = 30, normalized size = 1.

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-5 + 2*x)/(2 + 3*x^2),x]
```

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Maple [A] time = 0.004, size = 24, normalized size = 0.8

$$\frac{\ln\left(3\,x^{2}+2\right)}{3}-\frac{5\,\sqrt{6}}{6}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x-5)/(3*x^2+2),x)
```

```
[Out] 1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.50896, size = 31, normalized size = 1.03

$$-\frac{5}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)+\frac{1}{3}\,\log\left(3\,x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 5)/(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

Fricas [A] time = 0.197123, size = 36, normalized size = 1.2

$$\frac{1}{18}\,\sqrt{6}\left(\sqrt{6}\log\left(3\,x^{2}+2\right)-15\,\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 5)/(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(6)*(sqrt(6)*log(3*x^2 + 2) - 15*arctan(1/2*sqrt(6)*x))
```

Sympy [A] time = 0.106073, size = 27, normalized size = 0.9

$$\frac{\log\left(x^2+\frac{2}{3}\right)}{3}-\frac{5\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5+2*x)/(3*x**2+2),x)
```

```
[Out] log(x**2 + 2/3)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

GIAC/XCAS [A] time = 0.200851, size = 28, normalized size = 0.93

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\ln\left(x^2+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 5)/(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*ln(x^2 + 2/3)
```

3.29 $\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

```
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Rubi [A] time = 0.0169492, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x/4]*Sin[x],x]

[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Rubi in Sympy [A] time = 1.01965, size = 19, normalized size = 0.9

$$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(1/4*x)*sin(x),x)

[Out] 2*sin(3*x/4)/3 - 2*sin(5*x/4)/5
```

Mathematica [A] time = 0.0101627, size = 21, normalized size = 1.

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x/4]*Sin[x],x]
```

```
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Maple [A] time = 0.042, size = 14, normalized size = 0.7

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(1/4*x)*sin(x),x)
```

```
[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

Maxima [A] time = 1.33019, size = 18, normalized size = 0.86

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right)+\frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

Fricas [A] time = 0.220999, size = 32, normalized size = 1.52

$$-\frac{16}{15}\left(6\cos\left(\frac{1}{4}x\right)^{4}-7\cos\left(\frac{1}{4}x\right)^{2}+1\right)\sin\left(\frac{1}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -16/15*(6*cos(1/4*x)^4 - 7*cos(1/4*x)^2 + 1)*sin(1/4*x)
```

Sympy [A] time = 0.752077, size = 22, normalized size = 1.05

$$-\frac{16\sin\left(\frac{x}{4}\right)\cos\left(x\right)}{15}+\frac{4\sin\left(x\right)\cos\left(\frac{x}{4}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/4*x)*sin(x),x)
```

```
[Out] -16*sin(x/4)*cos(x)/15 + 4*sin(x)*cos(x)/4)/15
```

GIAC/XCAS [A] time = 0.209749, size = 18, normalized size = 0.86

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right)+\frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

3.30 $\int \cos(3x)\cos(4x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rubi [A] time = 0.0164814, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[3*x]*Cos[4*x],x]

[Out] Sin[x]/2 + Sin[7*x]/14
```

Rubi in Sympy [A] time = 1.06009, size = 10, normalized size = 0.67

$$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3*x)*cos(4*x),x)

[Out] sin(x)/2 + sin(7*x)/14
```

Mathematica [A] time = 0.0105882, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Maple [A] time = 0., size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3*x)*cos(4*x),x)
```

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Maxima [A] time = 1.33262, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Fricas [A] time = 0.219456, size = 32, normalized size = 2.13

$$\frac{1}{7}\left(32\cos(x)^6 - 40\cos(x)^4 + 12\cos(x)^2 + 3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

Sympy [A] time = 0.722396, size = 26, normalized size = 1.73

$$-\frac{3\sin(3x)\cos(4x)}{7} + \frac{4\sin(4x)\cos(3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(4*x),x)
```

```
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

GIAC/XCAS [A] time = 0.202511, size = 15, normalized size = 1.

$$\frac{1}{14} \sin(7x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

3.31 $\int -\tan(a-x)\tan(x)\,dx$

Optimal. Leaf size=21

$$\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Rubi [A] time = 0.0604058, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

Antiderivative was successfully verified.

```
[In]  Int[-(Tan[a - x]*Tan[x]),x]

[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Rubi in Sympy [A] time = 5.88019, size = 19, normalized size = 0.9

$$-x-\frac{\log\left(\cos\left(x\right)\right)}{\tan\left(a\right)}+\frac{\log\left(\cos\left(a-x\right)\right)}{\tan\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-tan(x)*tan(a-x),x)

[Out] -x - log(cos(x))/tan(a) + log(cos(a - x))/tan(a)
```

Mathematica [A] time = 0.114657, size = 21, normalized size = 1.

$$\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[-(Tan[a - x]*Tan[x]),x]

[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Maple [B] time = 0.132, size = 96, normalized size = 4.6

$$-\frac{(\cos(a))^2 \arctan(\tan(x))}{(\cos(a))^2 + (\sin(a))^2} + \frac{(\cos(a))^3 \ln(\sin(a)\tan(x) + \cos(a))}{\left((\cos(a))^2 + (\sin(a))^2\right)\sin(a)} - \frac{(\sin(a))^2 \arctan(\tan(x))}{(\cos(a))^2 + (\sin(a))^2} + \frac{\cos(a)\sin(a)\ln(\sin(a)\tan(x) + \cos(a))}{(\cos(a))^2 + (\sin(a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-tan(x)*tan(a-x),x)
```

```
[Out] -1/(cos(a)^2+sin(a)^2)*cos(a)^2*arctan(tan(x))+cos(a)^3/(cos(a)^2
+sin(a)^2)/sin(a)*ln(sin(a)*tan(x)+cos(a))-1/(cos(a)^2+sin(a)^2)*
sin(a)^2*arctan(tan(x))+sin(a)*cos(a)/(cos(a)^2+sin(a)^2)*ln(sin(
a)*tan(x)+cos(a))
```

Maxima [A] time = 1.50116, size = 251, normalized size = 11.95

$$-\frac{\left(\cos(2a)^2 + \sin(2a)^2 - 2\cos(2a) + 1\right)x + \left(\cos(2a)^2 + \sin(2a)^2 - 1\right)\arctan(\sin(2a) + \sin(2x), \cos(2a) + \cos(2x)) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-tan(a - x)*tan(x),x, algorithm="maxima")
```

```
[Out] -((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + si
n(2*a)^2 - 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) -
 (cos(2*a)^2 + sin(2*a)^2 - 1)*arctan2(sin(2*x), cos(2*x) + 1) -
log(cos(2*a)^2 + 2*cos(2*a)*cos(2*x) + cos(2*x)^2 + sin(2*a)^2 +
2*sin(2*a)*sin(2*x) + sin(2*x)^2)*sin(2*a) + log(cos(2*x)^2 + sin
(2*x)^2 + 2*cos(2*x) + 1)*sin(2*a))/(cos(2*a)^2 + sin(2*a)^2 - 2*
cos(2*a) + 1)
```

Fricas [A] time = 0.238456, size = 120, normalized size = 5.71

$$\frac{(\cos(2a) + 1)\log\left(-\frac{(\cos(2a)-1)\tan(x)^2 - 2\sin(2a)\tan(x) - \cos(2a) - 1}{(\cos(2a)+1)\tan(x)^2 + \cos(2a) + 1}\right) - (\cos(2a) + 1)\log\left(\frac{1}{\tan(x)^2+1}\right) - 2x\sin(2a)}{2\sin(2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-tan(a - x)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*a) + 1)*log(-((cos(2*a) - 1)*tan(x)^2 - 2*sin(2*a)*ta
n(x) - cos(2*a) - 1)/((cos(2*a) + 1)*tan(x)^2 + cos(2*a) + 1)) -
```

```
(cos(2*a) + 1)*log(1/(tan(x)^2 + 1)) - 2*x*sin(2*a))/sin(2*a)
```

Sympy [A] time = 1.85757, size = 138, normalized size = 6.57

$$-\left(\begin{cases}\frac{2x\tan(a)}{2\tan^2(a)+2}-\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^2(a)+2}+\frac{\log(\tan^2(x)+1)}{2\tan^2(a)+2} & \text{for } a\neq 0\\ \frac{\log(\tan^2(x)+1)}{2} & \text{otherwise}\end{cases}\right)\tan(a)+\begin{cases}-\frac{2x\tan(a)}{2\tan^3(a)+2\tan(a)}+\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^3(a)+2\tan(a)}+\frac{\log(\tan^2(x)+1)\tan^2(a)}{2\tan^3(a)+2\tan(a)} & \text{for } a\neq 0\\ -x+\tan(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-tan(x)*tan(a-x),x)
```

```
[Out] -Piecewise((2*x*tan(a)/(2*tan(a)**2 + 2) - 2*log(tan(x) + 1/tan(a
))/(2*tan(a)**2 + 2) + log(tan(x)**2 + 1)/(2*tan(a)**2 + 2), Ne(a
, 0)), (log(tan(x)**2 + 1)/2, True))*tan(a) + Piecewise((-2*x*tan
(a)/(2*tan(a)**3 + 2*tan(a)) + 2*log(tan(x) + 1/tan(a))/(2*tan(a)
**3 + 2*tan(a)) + log(tan(x)**2 + 1)*tan(a)**2/(2*tan(a)**3 + 2*t
an(a)), Ne(a, 0)), (-x + tan(x), True))
```

GIAC/XCAS [A] time = 0.217077, size = 24, normalized size = 1.14

$$-x+\frac{\ln\left(\left|\tan(a)\tan(x)+1\right|\right)}{\tan(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-tan(a - x)*tan(x),x, algorithm="giac")
```

```
[Out] -x + ln(abs(tan(a)*tan(x) + 1))/tan(a)
```

3.32 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0110247, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.502193, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00279825, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0.002, size = 11, normalized size = 0.8

$$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2,x)

[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.38067, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="maxima")

[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 0.217297, size = 14, normalized size = 1.

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="fricas")

[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.037585, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.202506, size = 14, normalized size = 1.

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.33 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0115699, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.489517, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00281937, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2} + \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.34589, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.245794, size = 14, normalized size = 1.

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.035658, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.204495, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.34 $\int \cos^3(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{4}\cos^4(x)$$

```
[Out] -Cos[x]^4/4
```

Rubi [A] time = 0.0282481, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{1}{4}\cos^4(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3*Sin[x],x]

[Out] -Cos[x]^4/4
```

Rubi in Sympy [A] time = 1.12211, size = 7, normalized size = 0.88

$$-\frac{\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3*sin(x),x)

[Out] -cos(x)**4/4
```

Mathematica [A] time = 0.00187126, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos^4(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3*Sin[x],x]

[Out] -Cos[x]^4/4
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$-\frac{(\cos(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3*sin(x),x)

[Out] -1/4*cos(x)^4
```

Maxima [A] time = 1.35486, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x),x, algorithm="maxima")

[Out] -1/4*cos(x)^4
```

Fricas [A] time = 0.213085, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x),x, algorithm="fricas")

[Out] -1/4*cos(x)^4
```

Sympy [A] time = 0.037774, size = 7, normalized size = 0.88

$$-\frac{\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3*sin(x),x)
```

```
[Out] -cos(x)**4/4
```

GIAC/XCAS [A] time = 0.200644, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/4*cos(x)^4
```

3.35 $\int \cot^3(x)\csc(x)\,dx$

Optimal. Leaf size=11

$$\csc(x)-\frac{\csc^3(x)}{3}$$

```
[Out] Csc[x] - Csc[x]^3/3
```

Rubi [A] time = 0.0297575, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\csc(x)-\frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Rubi in Sympy [A] time = 2.41366, size = 12, normalized size = 1.09

$$\frac{1}{\sin(x)}-\frac{1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3/sin(x)**4,x)
```

```
[Out] 1/sin(x) - 1/(3*sin(x)**3)
```

Mathematica [B] time = 0.00765335, size = 57, normalized size = 5.18

$$\frac{5}{12}\tan\left(\frac{x}{2}\right)+\frac{5}{12}\cot\left(\frac{x}{2}\right)-\frac{1}{24}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)-\frac{1}{24}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^3*Csc[x],x]
```

```
[Out] (5*Cot[x/2])/12 - (Cot[x/2]*Csc[x/2]^2)/24 + (5*Tan[x/2])/12 - (S
ec[x/2]^2*Tan[x/2])/24
```

Maple [B] time = 0.012, size = 32, normalized size = 2.9

$$-\frac{(\cos(x))^4}{3(\sin(x))^3}+\frac{(\cos(x))^4}{3\sin(x)}+\frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^4,x)
```

```
[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x
)
```

Maxima [A] time = 1.33657, size = 19, normalized size = 1.73

$$\frac{3\sin(x)^2-1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Fricas [A] time = 0.210206, size = 30, normalized size = 2.73

$$\frac{3\cos(x)^2-2}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.083591, size = 14, normalized size = 1.27

$$\frac{3\sin^2(x)-1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3/sin(x)**4,x)
```

```
[Out] (3*sin(x)**2 - 1)/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.211272, size = 19, normalized size = 1.73

$$\frac{3\,\sin\left(x\right)^{2} - 1}{3\,\sin\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3/sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.36 $\int \csc^2(x)\sec^2(x)\,dx$

Optimal. Leaf size=7

$$\tan(x) - \cot(x)$$

```
[Out] -Cot[x] + Tan[x]
```

Rubi [A] time = 0.0449599, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^2*Sec[x]^2,x]

[Out] -Cot[x] + Tan[x]
```

Rubi in Sympy [A] time = 1.67775, size = 15, normalized size = 2.14

$$\frac{2\sin(x)}{\cos(x)} - \frac{1}{\sin(x)\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(x)**2/sin(x)**2,x)

[Out] 2*sin(x)/cos(x) - 1/(sin(x)*cos(x))
```

Mathematica [A] time = 0.00525028, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^2*Sec[x]^2,x]

[Out] -2*Cot[2*x]
```

Maple [A] time = 0., size = 15, normalized size = 2.1

$$\frac{1}{\cos(x)\sin(x)} - 2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2/sin(x)^2,x)
```

```
[Out] 1/sin(x)/cos(x)-2*cot(x)
```

Maxima [A] time = 1.36047, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/tan(x) + tan(x)
```

Fricas [A] time = 0.225188, size = 24, normalized size = 3.43

$$-\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

Sympy [A] time = 0.055305, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)
```

```
[Out] -2*cos(2*x)/sin(2*x)
```

GIAC/XCAS [A] time = 0.201517, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/tan(x) + tan(x)
```

3.37 $\int \cot^2\left(\frac{3x}{4}\right)\,dx$

Optimal. Leaf size=14

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

```
[Out] -x - (4*Cot[(3*x)/4])/3
```

Rubi [A] time = 0.0122771, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[(3*x)/4]^2,x]

[Out] -x - (4*Cot[(3*x)/4])/3
```

Rubi in Sympy [A] time = 0.515926, size = 12, normalized size = 0.86

$$-x-\frac{4}{3\tan\left(\frac{3x}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cot(3/4*x)**2,x)

[Out] -x - 4/(3*tan(3*x/4))
```

Mathematica [A] time = 0.00976332, size = 14, normalized size = 1.

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[(3*x)/4]^2,x]
```

```
[Out] -x - (4*Cot[(3*x)/4])/3
```

Maple [A] time = 0.004, size = 14, normalized size = 1.

$$-\frac{4}{3}\cot\left(\frac{3\,x}{4}\right)+\frac{2\,\pi}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(3/4*x)^2,x)
```

```
[Out] -4/3*cot(3/4*x)+2/3*Pi-x
```

Maxima [A] time = 1.50453, size = 16, normalized size = 1.14

$$-x-\frac{4}{3\,\tan\left(\frac{3}{4}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(3/4*x)^2,x, algorithm="maxima")
```

```
[Out] -x - 4/3/tan(3/4*x)
```

Fricas [A] time = 0.84907, size = 31, normalized size = 2.21

$$-\frac{3\,x\sin\left(\frac{3}{2}\,x\right)+4\,\cos\left(\frac{3}{2}\,x\right)+4}{3\,\sin\left(\frac{3}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(3/4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)
```

Sympy [A] time = 0.051077, size = 19, normalized size = 1.36

$$-x-\frac{4\cos\left(\frac{3x}{4}\right)}{3\sin\left(\frac{3x}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(3/4*x)**2,x)
```

```
[Out] -x - 4*cos(3*x/4)/(3*sin(3*x/4))
```

GIAC/XCAS [A] time = 0.21649, size = 24, normalized size = 1.71

$$-x - \frac{2}{3 \tan\left(\frac{3}{8}\, x\right)} + \frac{2}{3} \tan\left(\frac{3}{8}\, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(3/4*x)^2,x, algorithm="giac")
```

```
[Out] -x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)
```

3.38 $\int (1+\tan(2x))^2 \, dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

```
[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Rubi [A] time = 0.0240698, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Tan[2*x])^2,x]

[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Rubi in Sympy [A] time = 0.550574, size = 12, normalized size = 0.75

$$-\log(\cos(2x))+\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+tan(2*x))**2,x)

[Out] -log(cos(2*x)) + tan(2*x)/2
```

Mathematica [A] time = 0.0150738, size = 16, normalized size = 1.

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Tan[2*x])^2,x]

[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Maple [A] time = 0.004, size = 19, normalized size = 1.2

$$\frac{\tan(2x)}{2}+\frac{\ln\left(1+(\tan(2x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+tan(2*x))^2,x)
```

```
[Out] 1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)
```

Maxima [A] time = 1.49691, size = 16, normalized size = 1.

$$\log(\sec(2x))+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(2*x) + 1)^2,x, algorithm="maxima")
```

```
[Out] log(sec(2*x)) + 1/2*tan(2*x)
```

Fricas [A] time = 0.255256, size = 27, normalized size = 1.69

$$-\frac{1}{2}\log\left(\frac{1}{\tan(2x)^2+1}\right)+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(2*x) + 1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

Sympy [A] time = 0.177776, size = 17, normalized size = 1.06

$$\frac{\log\left(\tan^2(2x)+1\right)}{2}+\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+tan(2*x))**2,x)
```

```
[Out] log(tan(2*x)**2 + 1)/2 + tan(2*x)/2
```

GIAC/XCAS [A] time = 0.213304, size = 30, normalized size = 1.88

$$-\frac{1}{2}\ln\left(\frac{4}{\tan\left(2x\right)^{2}+1}\right)+\frac{1}{2}\tan\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(2*x) + 1)^2,x, algorithm="giac")
```

```
[Out] -1/2*ln(4/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

3.39 $\int(-\cot(x)+\tan(x))^2\,dx$

Optimal. Leaf size=10

$$-4x+\tan(x)-\cot(x)$$

```
[Out] -4*x - Cot[x] + Tan[x]
```

Rubi [A] time = 0.0729212, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$

$$-4x+\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-Cot[x] + Tan[x])^2,x]

[Out] -4*x - Cot[x] + Tan[x]
```

Rubi in Sympy [A] time = 68.4622, size = 14, normalized size = 1.4

$$\tan(x)-4\,\text{atan}(\tan(x))-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-cot(x)+tan(x))**2,x)

[Out] tan(x) - 4*atan(tan(x)) - 1/tan(x)
```

Mathematica [A] time = 0.0237431, size = 10, normalized size = 1.

$$-4x+\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Cot[x] + Tan[x])^2,x]

[Out] -4*x - Cot[x] + Tan[x]
```

Maple [A] time = 0.02, size = 11, normalized size = 1.1

$$-4x - \cot(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-cot(x)+tan(x))^2,x)
```

```
[Out] -4*x-cot(x)+tan(x)
```

Maxima [A] time = 1.48934, size = 16, normalized size = 1.6

$$-4x - \frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cot(x) - tan(x))^2,x, algorithm="maxima")
```

```
[Out] -4*x - 1/tan(x) + tan(x)
```

Fricas [A] time = 0.243942, size = 26, normalized size = 2.6

$$-\frac{4x\tan(x) - \tan(x)^2 + 1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cot(x) - tan(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*x*tan(x) - tan(x)^2 + 1)/tan(x)
```

Sympy [A] time = 0.574043, size = 10, normalized size = 1.

$$-4x + \tan(x) - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cot(x)+tan(x))**2,x)
```

```
[Out] -4*x + tan(x) - 1/tan(x)
```

GIAC/XCAS [A] time = 0.210864, size = 16, normalized size = 1.6

$$-4\,x - \frac{1}{\tan\left(x\right)} + \tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cot(x) - tan(x))^2,x, algorithm="giac")
```

```
[Out] -4*x - 1/tan(x) + tan(x)
```

3.40 $\int (-\sec(x)+\tan(x))^2\,dx$

Optimal. Leaf size=14

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

```
[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rubi [A] time = 0.106171, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(-Sec[x] + Tan[x])^2,x]

[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rubi in Sympy [A] time = 4.0545, size = 12, normalized size = 0.86

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-sec(x)+tan(x))**2,x)

[Out] -x - 2*cos(x)/(sin(x) + 1)
```

Mathematica [A] time = 0.0108401, size = 12, normalized size = 0.86

$$-x+2\tan(x)-2\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Sec[x] + Tan[x])^2,x]

[Out] -x - 2*Sec[x] + 2*Tan[x]
```

Maple [A] time = 0.021, size = 15, normalized size = 1.1

$$2\tan(x) - 2(\cos(x))^{-1} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-sec(x)+tan(x))^2,x)
```

```
[Out] 2*tan(x)-2/cos(x)-x
```

Maxima [A] time = 1.49182, size = 19, normalized size = 1.36

$$-x - \frac{2}{\cos(x)} + 2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sec(x) - tan(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 2/cos(x) + 2*tan(x)
```

Fricas [A] time = 0.226354, size = 34, normalized size = 2.43

$$-\frac{(x+2)\cos(x) + (x-2)\sin(x) + x + 2}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sec(x) - tan(x))^2,x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

Sympy [A] time = 0.573023, size = 10, normalized size = 0.71

$$-x + 2\tan(x) - 2\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-sec(x)+tan(x))**2,x)
```

```
[Out] -x + 2*tan(x) - 2*sec(x)
```

GIAC/XCAS [A] time = 0.207797, size = 19, normalized size = 1.36

$$-x-\frac{4}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sec(x) - tan(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4/(tan(1/2*x) + 1)
```

3.41 $\int \frac{\sin(x)}{1+\sin(x)}\,dx$

Optimal. Leaf size=11

$$x + \frac{\cos(x)}{\sin(x) + 1}$$

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rubi [A] time = 0.0356624, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$x + \frac{\cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/(1 + Sin[x]),x]

[Out] x + Cos[x]/(1 + Sin[x])
```

Rubi in Sympy [A] time = 1.52093, size = 8, normalized size = 0.73

$$x + \frac{\cos(x)}{\sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/(1+sin(x)),x)

[Out] x + cos(x)/(sin(x) + 1)
```

Mathematica [B] time = 0.0229655, size = 25, normalized size = 2.27

$$x - \frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/(1 + Sin[x]),x]
```

```
[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.019, size = 13, normalized size = 1.2

$$2\,(1+\tan(x/2))^{-1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/(1+sin(x)),x)
```

```
[Out] 2/(1+tan(1/2*x))+x
```

Maxima [A] time = 1.50407, size = 38, normalized size = 3.45

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}+2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.231073, size = 32, normalized size = 2.91

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A] time = 0.999472, size = 34, normalized size = 3.09

$$\frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}+\frac{x}{\tan\left(\frac{x}{2}\right)+1}-\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(1+sin(x)),x)
```

```
[Out] x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) - 2*tan(x/2)/(tan(x/
2) + 1)
```

GIAC/XCAS [A] time = 0.211878, size = 16, normalized size = 1.45

$$x + \frac{2}{\tan\left(\frac{1}{2}x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(sin(x) + 1),x, algorithm="giac")
```

```
[Out] x + 2/(tan(1/2*x) + 1)
```

3.42 $\int \frac{\cos(x)}{1-\cos(x)} \, dx$

Optimal. Leaf size=16

$$-x - \frac{\sin(x)}{1 - \cos(x)}$$

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rubi [A] time = 0.0409169, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-x - \frac{\sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rubi in Sympy [A] time = 1.74602, size = 10, normalized size = 0.62

$$-x - \frac{\sin(x)}{-\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/(1-cos(x)),x)
```

```
[Out] -x - sin(x)/(-cos(x) + 1)
```

Mathematica [A] time = 0.0249968, size = 21, normalized size = 1.31

$$\frac{2x \sin^2\left(\frac{x}{2}\right) + \sin(x)}{\cos(x) - 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] (2*x*Sin[x/2]^2 + Sin[x])/(-1 + Cos[x])
```

Maple [A] time = 0.019, size = 13, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)-x
```

Maxima [A] time = 1.49827, size = 31, normalized size = 1.94

$$-\frac{\cos(x)+1}{\sin(x)}-2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.241692, size = 19, normalized size = 1.19

$$-\frac{x\sin(x)+\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.898136, size = 8, normalized size = 0.5

$$-x-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(1-cos(x)),x)

[Out] -x - 1/tan(x/2)
```

GIAC/XCAS [A] time = 0.204276, size = 16, normalized size = 1.

$$-x - \frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(x)/(cos(x) - 1),x, algorithm="giac")

[Out] -x - 1/tan(1/2*x)
```

3.43 $\int e^{-x/2} \left(-1+e^{x/2}\right)^3 \, dx$

Optimal. Leaf size=25

$$3x + 2e^{-x/2} - 6e^{x/2} + e^x$$

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Rubi [A] time = 0.0464107, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$3x + 2e^{-x/2} - 6e^{x/2} + e^x$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-6e^{\frac{x}{2}} + 6\log\left(e^{\frac{x}{2}}\right) + 2\int^{e^{\frac{x}{2}}} x\,dx + 2e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
```

```
[Out] -6*exp(x/2) + 6*log(exp(x/2)) + 2*Integral(x, (x, exp(x/2))) + 2*
exp(-x/2)
```

Mathematica [A] time = 0.015117, size = 25, normalized size = 1.

$$3x + 2e^{-x/2} - 6e^{x/2} + e^x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Maple [A] time = 0.008, size = 29, normalized size = 1.2

$$\left(e^{\frac{x}{2}}\right)^{2}-6\,e^{x/2}+2\,\left(e^{x/2}\right)^{-1}+6\,\ln\left(e^{x/2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+exp(1/2*x))^3/exp(1/2*x),x)
```

```
[Out] exp(1/2*x)^2-6*exp(1/2*x)+2/exp(1/2*x)+6*ln(exp(1/2*x))
```

Maxima [A] time = 1.33878, size = 24, normalized size = 0.96

$$3\,x-6\,e^{\left(\frac{1}{2}\,x\right)}+2\,e^{\left(-\frac{1}{2}\,x\right)}+e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(1/2*x) - 1)^3*e^(-1/2*x),x, algorithm="maxima")
```

```
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

Fricas [A] time = 0.224021, size = 30, normalized size = 1.2

$$\left(3\,xe^{\left(\frac{1}{2}\,x\right)}+e^{\left(\frac{3}{2}\,x\right)}-6\,e^{x}+2\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(1/2*x) - 1)^3*e^(-1/2*x),x, algorithm="fricas")
```

```
[Out] (3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)
```

Sympy [A] time = 0.101972, size = 19, normalized size = 0.76

$$3x-6e^{\frac{x}{2}}+e^{x}+2e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
```

```
[Out] 3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)
```

GIAC/XCAS [A] time = 0.20845, size = 24, normalized size = 0.96

$$3x - 6e^{\left(\frac{1}{2}x\right)} + 2e^{\left(-\frac{1}{2}x\right)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(1/2*x) - 1)^3*e^(-1/2*x),x, algorithm="giac")
```

```
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

3.44 $\int \frac{1}{5-6x+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

```
[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Rubi [A] time = 0.0111936, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 6*x + x^2)^(-1),x]

[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Rubi in Sympy [A] time = 0.739674, size = 12, normalized size = 0.57

$$-\frac{\log(-x+1)}{4}+\frac{\log(-x+5)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-6*x+5),x)

[Out] -log(-x + 1)/4 + log(-x + 5)/4
```

Mathematica [A] time = 0.00445992, size = 21, normalized size = 1.

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 6*x + x^2)^(-1),x]

[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Maple [A] time = 0.007, size = 14, normalized size = 0.7

$$\frac{\ln(-5+x)}{4} - \frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-6*x+5),x)
```

```
[Out] 1/4*ln(-5+x)-1/4*ln(-1+x)
```

Maxima [A] time = 1.33546, size = 18, normalized size = 0.86

$$-\frac{1}{4}\log(x-1) + \frac{1}{4}\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 6*x + 5),x, algorithm="maxima")
```

```
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

Fricas [A] time = 0.208437, size = 18, normalized size = 0.86

$$-\frac{1}{4}\log(x-1) + \frac{1}{4}\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 6*x + 5),x, algorithm="fricas")
```

```
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

Sympy [A] time = 0.087385, size = 12, normalized size = 0.57

$$\frac{\log(x-5)}{4} - \frac{\log(x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-6*x+5),x)
```

```
[Out] log(x - 5)/4 - log(x - 1)/4
```

GIAC/XCAS [A] time = 0.203373, size = 20, normalized size = 0.95

$$-\frac{1}{4}\ln\left(|x-1|\right)+\frac{1}{4}\ln\left(|x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 6*x + 5),x, algorithm="giac")
```

```
[Out] -1/4*ln(abs(x - 1)) + 1/4*ln(abs(x - 5))
```

3.45 $\int \frac{x^2}{13-6x^3+x^6} \, dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

```
[Out] ArcTan[(-3 + x^3)/2]/6
```

Rubi [A] time = 0.0337604, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(13 - 6*x^3 + x^6),x]

[Out] ArcTan[(-3 + x^3)/2]/6
```

Rubi in Sympy [A] time = 2.50087, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6-6*x**3+13),x)

[Out] atan(x**3/2 - 3/2)/6
```

Mathematica [A] time = 0.0075148, size = 14, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(13 - 6*x^3 + x^6),x]
```

```
[Out] ArcTan[(-3 + x^3)/2]/6
```

Maple [A] time = 0.004, size = 11, normalized size = 0.8

$$\frac{1}{6}\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^6-6*x^3+13),x)
```

```
[Out] 1/6*arctan(1/2*x^3-3/2)
```

Maxima [A] time = 1.47843, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - 6*x^3 + 13),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Fricas [A] time = 0.206583, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - 6*x^3 + 13),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Sympy [A] time = 0.118414, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6-6*x**3+13),x)
```

```
[Out] atan(x**3/2 - 3/2)/6
```

GIAC/XCAS [A] time = 0.209145, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^{3}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - 6*x^3 + 13),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

3.46 $\int \frac{2+x}{-1-4x+x^2} \, dx$

Optimal. Leaf size=51

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

```
[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[
2 + Sqrt[5] - x])/10
```

Rubi [A] time = 0.0458942, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[
2 + Sqrt[5] - x])/10
```

Rubi in Sympy [A] time = 2.04218, size = 46, normalized size = 0.9

$$\frac{\sqrt{5}\left(\sqrt{5}+4\right)\log\left(-x+2+\sqrt{5}\right)}{10}-\frac{\sqrt{5}\left(-\sqrt{5}+4\right)\log\left(-x-\sqrt{5}+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**2-4*x-1),x)
```

```
[Out] sqrt(5)*(sqrt(5) + 4)*log(-x + 2 + sqrt(5))/10 - sqrt(5)*(-sqrt(5
) + 4)*log(-x - sqrt(5) + 2)/10
```

Mathematica [A] time = 0.0403675, size = 47, normalized size = 0.92

$$\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(x+\sqrt{5}-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[
-2 + Sqrt[5] + x])/10
```

Maple [A] time = 0.004, size = 29, normalized size = 0.6

$$\frac{\ln\left(x^2-4x-1\right)}{2}-\frac{4\sqrt{5}}{5}Artanh\left(\frac{(2x-4)\sqrt{5}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^2-4*x-1),x)
```

```
[Out] 1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(2*x-4)*5^(1/2))
```

Maxima [A] time = 1.48614, size = 47, normalized size = 0.92

$$\frac{2}{5}\sqrt{5}\log\left(\frac{x-\sqrt{5}-2}{x+\sqrt{5}-2}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 - 4*x - 1),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^
2 - 4*x - 1)
```

Fricas [A] time = 0.208359, size = 68, normalized size = 1.33

$$\frac{1}{10}\sqrt{5}\left(\sqrt{5}\log\left(x^2-4x-1\right)+4\log\left(\frac{\sqrt{5}\left(x^2-4x+9\right)-10x+20}{x^2-4x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 - 4*x - 1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*(sqrt(5)*log(x^2 - 4*x - 1) + 4*log((sqrt(5)*(x^2 -
4*x + 9) - 10*x + 20)/(x^2 - 4*x - 1)))
```

Sympy [A] time = 0.106259, size = 42, normalized size = 0.82

$$\left(-\frac{2\sqrt{5}}{5}+\frac{1}{2}\right)\log\left(x-2+\sqrt{5}\right)+\left(\frac{1}{2}+\frac{2\sqrt{5}}{5}\right)\log\left(x-\sqrt{5}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**2-4*x-1),x)
```

```
[Out] (-2*sqrt(5)/5 + 1/2)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*l
og(x - sqrt(5) - 2)
```

GIAC/XCAS [A] time = 0.203616, size = 59, normalized size = 1.16

$$\frac{2}{5}\sqrt{5}\ln\left(\frac{\left|2\,x-2\,\sqrt{5}-4\right|}{\left|2\,x+2\,\sqrt{5}-4\right|}\right)+\frac{1}{2}\ln\left(\left|x^2-4\,x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 - 4*x - 1),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(5)*ln(abs(2*x - 2*sqrt(5) - 4)/abs(2*x + 2*sqrt(5) - 4))
 + 1/2*ln(abs(x^2 - 4*x - 1))
```

3.47 $\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Rubi [A] time = 0.0267663, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (1 + x)^(1/3))^(-1),x]
```

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)+3\int^{\sqrt[3]{x+1}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+(1+x)**(1/3)),x)
```

```
[Out] -3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1) + 3*Integral(x, (x,
 (x + 1)**(1/3)))
```

Mathematica [A] time = 0.0116135, size = 33, normalized size = 1.

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (1 + x)^(1/3))^(-1),x]
```

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Maple [A] time = 0.007, size = 47, normalized size = 1.4

$$\ln(2+x)+\frac{3}{2}(1+x)^{\frac{2}{3}}+2\ln\left(1+\sqrt[3]{1+x}\right)-\ln\left((1+x)^{\frac{2}{3}}-\sqrt[3]{1+x}+1\right)-3\sqrt[3]{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+(1+x)^(1/3)),x)
```

```
[Out] ln(2+x)+3/2*(1+x)^(2/3)+2*ln(1+(1+x)^(1/3))-ln((1+x)^(2/3)-(1+x)^
(1/3)+1)-3*(1+x)^(1/3)
```

Maxima [A] time = 1.32782, size = 34, normalized size = 1.03

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(1/3) + 1),x, algorithm="maxima")
```

```
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

Fricas [A] time = 0.212089, size = 34, normalized size = 1.03

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(1/3) + 1),x, algorithm="fricas")
```

```
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

Sympy [A] time = 0.131618, size = 29, normalized size = 0.88

$$\frac{3(x+1)^{\frac{2}{3}}}{2}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+(1+x)**(1/3)),x)

[Out] 3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1)
```

GIAC/XCAS [A] time = 0.210932, size = 34, normalized size = 1.03

$$\frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\ln\left((x+1)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(1/3) + 1),x, algorithm="giac")

[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*ln((x + 1)^(1/3) + 1)
```

3.48 $\int \frac{1}{\sqrt{x}(b+ax)} \, dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0343649, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(b + a*x)),x]

[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 2.13374, size = 27, normalized size = 0.93

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*x+b)/x**(1/2),x)

[Out] 2*atan(sqrt(a)*sqrt(x)/sqrt(b))/(sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.0100894, size = 29, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(b + a*x)),x]

[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.009, size = 19, normalized size = 0.7

$$2\,{\frac{1}{\sqrt{ab}}\arctan\left({\frac{a\sqrt{x}}{\sqrt{ab}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*x+b)/x^(1/2),x)

[Out] 2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x + b)*sqrt(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.22545, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\,ab\sqrt{x}+\sqrt{-ab}(ax-b)}{ax+b}\right)}{\sqrt{-ab}}, -\frac{2\,\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x + b)*sqrt(x)),x, algorithm="fricas")

[Out] [log((2*a*b*sqrt(x) + sqrt(-a*b)*(a*x - b))/(a*x + b))/sqrt(-a*b)
, -2*arctan(b/(sqrt(a*b)*sqrt(x)))/sqrt(a*b)]
```

Sympy [A] time = 0.863847, size = 27, normalized size = 0.93

$$\frac{2\operatorname{atan}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x+b)/x**(1/2),x)
```

```
[Out] 2*atan(sqrt(a)*sqrt(x)/sqrt(b))/(sqrt(a)*sqrt(b))
```

GIAC/XCAS [A] time = 0.205651, size = 24, normalized size = 0.83

$$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x + b)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.49 $\int x^3\sqrt{1+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] -(1 + x^2)^(3/2)/3 + (1 + x^2)^(5/2)/5
```

Rubi [A] time = 0.0271528, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[1 + x^2],x]

[Out] -(1 + x^2)^(3/2)/3 + (1 + x^2)^(5/2)/5
```

Rubi in Sympy [A] time = 1.80429, size = 19, normalized size = 0.7

$$\frac{\left(x^2+1\right)^{\frac{5}{2}}}{5}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(x**2+1)**(1/2),x)

[Out] (x**2 + 1)**(5/2)/5 - (x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.00782998, size = 20, normalized size = 0.74

$$\frac{1}{15}\left(x^2+1\right)^{3/2}\left(3x^2-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

Maple [A] time = 0.006, size = 17, normalized size = 0.6

$$\frac{3\,x^2-2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(x^2+1)^(1/2),x)
```

```
[Out] 1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

Maxima [A] time = 1.51911, size = 30, normalized size = 1.11

$$\frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2-\frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x^3,x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

Fricas [A] time = 0.212709, size = 134, normalized size = 4.96

$$-\frac{48\,x^{10}+100\,x^8+35\,x^6-40\,x^4-25\,x^2-\left(48\,x^9+76\,x^7+3\,x^5-35\,x^3-10\,x\right)\sqrt{x^2+1}-2}{15\left(16\,x^5+20\,x^3-\left(16\,x^4+12\,x^2+1\right)\sqrt{x^2+1}+5\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/15*(48*x^10 + 100*x^8 + 35*x^6 - 40*x^4 - 25*x^2 - (48*x^9 + 7
6*x^7 + 3*x^5 - 35*x^3 - 10*x)*sqrt(x^2 + 1) - 2)/(16*x^5 + 20*x^
3 - (16*x^4 + 12*x^2 + 1)*sqrt(x^2 + 1) + 5*x)
```

Sympy [A] time = 0.891849, size = 37, normalized size = 1.37

$$\frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(x**2+1)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)
/15
```

GIAC/XCAS [A] time = 0.207732, size = 26, normalized size = 0.96

$$\frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}} - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x^3,x, algorithm="giac")
```

```
[Out] 1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

3.50 $\int \frac{x}{\sqrt{a^4-x^4}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Rubi [A] time = 0.0238282, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a^4 - x^4],x]

[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Rubi in Sympy [A] time = 1.60058, size = 15, normalized size = 0.68

$$\frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**4-x**4)**(1/2),x)

[Out] atan(x**2/sqrt(a**4 - x**4))/2
```

Mathematica [A] time = 0.00739993, size = 22, normalized size = 1.

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a^4 - x^4],x]
```

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Maple [A] time = 0.016, size = 19, normalized size = 0.9

$$\frac{1}{2}\arctan\left(x^{2}\frac{1}{\sqrt{a^{4}-x^{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^4-x^4)^(1/2),x)
```

```
[Out] 1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

Maxima [A] time = 1.48702, size = 24, normalized size = 1.09

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{a^{4}-x^{4}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(a^4 - x^4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(a^4 - x^4)/x^2)
```

Fricas [A] time = 0.223773, size = 34, normalized size = 1.55

$$-\arctan\left(-\frac{a^{2}-\sqrt{a^{4}-x^{4}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(a^4 - x^4),x, algorithm="fricas")
```

```
[Out] -arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)
```

Sympy [A] time = 1.74402, size = 29, normalized size = 1.32

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{x^2}{a^2}\right)}{2} & \text{for } \left|\frac{x^4}{a^4}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{x^2}{a^2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**4-x**4)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4/a**4) > 1), (asin(x**2
/a**2)/2, True))
```

GIAC/XCAS [A] time = 0.216464, size = 14, normalized size = 0.64

$$\frac{1}{2} \arcsin\left(\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(a^4 - x^4),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x^2/a^2)
```

3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

```
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Rubi [A] time = 0.0371878, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-a^2 + x^2]),x]

[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Rubi in Sympy [A] time = 2.84177, size = 14, normalized size = 0.64

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-a**2+x**2)**(1/2),x)

[Out] atan(sqrt(-a**2 + x**2)/a)/a
```

Mathematica [C] time = 0.0169002, size = 35, normalized size = 1.59

$$-\frac{i\log\left(\frac{2\sqrt{x^2-a^2}}{x}-\frac{2ia}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-a^2 + x^2]),x]

[Out] ((-I)*Log[((-2*I)*a)/x + (2*Sqrt[-a^2 + x^2])/x])/a
```

Maple [A] time = 0.007, size = 41, normalized size = 1.9

$$-1\ln\left(\frac{1}{x}\left(-2\,a^2+2\,\sqrt{-a^2}\sqrt{-a^2+x^2}\right)\right)\frac{1}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-a^2+x^2)^(1/2),x)

[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+x^2)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + x^2)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.211591, size = 35, normalized size = 1.59

$$\frac{2\,\arctan\left(-\frac{x-\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + x^2)*x),x, algorithm="fricas")

[Out] 2*arctan(-(x - sqrt(-a^2 + x^2))/a)/a
```

Sympy [A] time = 1.76213, size = 22, normalized size = 1.

$$\begin{cases} \frac{i \operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-a**2+x**2)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(a/x)/a, Abs(a**2/x**2) > 1), (-asin(a/x)/a, Tr
ue))
```

GIAC/XCAS [A] time = 0.21522, size = 27, normalized size = 1.23

$$\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + x^2)*x),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a^2 + x^2)/a)/a
```

3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Rubi [A] time = 0.0391589, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 - x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Rubi in Sympy [A] time = 3.12027, size = 15, normalized size = 0.65

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2-x**2)**(1/2),x)

[Out] -atanh(sqrt(a**2 - x**2)/a)/a
```

Mathematica [A] time = 0.0141429, size = 32, normalized size = 1.39

$$\frac{\log(x)}{a} - \frac{\log\left(a\sqrt{a^2-x^2}+a^2\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 - x^2]),x]

[Out] Log[x]/a - Log[a^2 + a*Sqrt[a^2 - x^2]]/a
```

Maple [A] time = 0.006, size = 37, normalized size = 1.6

$$-1\ln\left(\frac{1}{x}\left(2a^2+2\sqrt{a^2}\sqrt{a^2-x^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2-x^2)^(1/2),x)

[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-x^2)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - x^2)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227966, size = 34, normalized size = 1.48

$$\frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - x^2)*x),x, algorithm="fricas")

[Out] log(-(a - sqrt(a^2 - x^2))/x)/a
```

Sympy [A] time = 1.77494, size = 22, normalized size = 0.96

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-acosh(a/x)/a, Abs(a**2/x**2) > 1), (I*asin(a/x)/a, Tr
ue))
```

GIAC/XCAS [A] time = 0.209626, size = 58, normalized size = 2.52

$$-\frac{\ln\left(\left|a + \sqrt{a^2 - x^2}\right|\right)}{2\,a} + \frac{\ln\left(\left|-a + \sqrt{a^2 - x^2}\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - x^2)*x),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(a + sqrt(a^2 - x^2)))/a + 1/2*ln(abs(-a + sqrt(a^2 -
x^2)))/a
```

3.53 $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Rubi [A] time = 0.0334808, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 + x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Rubi in Sympy [A] time = 2.79408, size = 15, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2+x**2)**(1/2),x)

[Out] -atanh(sqrt(a**2 + x**2)/a)/a
```

Mathematica [A] time = 0.0109799, size = 30, normalized size = 1.43

$$\frac{\log(x)}{a}-\frac{\log\left(a\sqrt{a^2+x^2}+a^2\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 + x^2]),x]
```

```
[Out] Log[x]/a - Log[a^2 + a*Sqrt[a^2 + x^2]]/a
```

Maple [A] time = 0.007, size = 35, normalized size = 1.7

$$-1 \ln \left(\frac{1}{x}\left(2 a^{2}+2 \sqrt{a^{2}} \sqrt{a^{2}+x^{2}}\right)\right) \frac{1}{\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2+x^2)^(1/2),x)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+x^2)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243037, size = 54, normalized size = 2.57

$$-\frac{\log \left(a-x+\sqrt{a^{2}+x^{2}}\right)-\log \left(-a-x+\sqrt{a^{2}+x^{2}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + x^2)*x),x, algorithm="fricas")
```

```
[Out] -(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a
```

Sympy [A] time = 1.7356, size = 7, normalized size = 0.33

$$-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**2+x**2)**(1/2),x)
```

```
[Out] -asinh(a/x)/a
```

GIAC/XCAS [A] time = 0.216509, size = 50, normalized size = 2.38

$$-\frac{\ln\left(a+\sqrt{a^2+x^2}\right)}{2\,a}+\frac{\ln\left(-a+\sqrt{a^2+x^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + x^2)*x),x, algorithm="giac")
```

```
[Out] -1/2*ln(a + sqrt(a^2 + x^2))/a + 1/2*ln(-a + sqrt(a^2 + x^2))/a
```

3.54 $\int \frac{1}{\sqrt{2+x-x^2}} \, dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

```
[Out] -ArcSin[(1 - 2*x)/3]
```

Rubi [A] time = 0.0138181, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + x - x^2],x]

[Out] -ArcSin[(1 - 2*x)/3]
```

Rubi in Sympy [A] time = 0.665824, size = 19, normalized size = 1.58

$$-\operatorname{atan}\left(\frac{-2x+1}{2\sqrt{-x^2+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+x+2)**(1/2),x)

[Out] -atan((-2*x + 1)/(2*sqrt(-x**2 + x + 2)))
```

Mathematica [A] time = 0.0087237, size = 12, normalized size = 1.

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + x - x^2],x]
```

```
[Out] -ArcSin[(1 - 2*x)/3]
```

Maple [A] time = 0.006, size = 7, normalized size = 0.6

$$\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+x+2)^(1/2),x)
```

```
[Out] arcsin(-1/3+2/3*x)
```

Maxima [A] time = 1.52782, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + x + 2),x, algorithm="maxima")
```

```
[Out] -arcsin(-2/3*x + 1/3)
```

Fricas [A] time = 0.213481, size = 24, normalized size = 2.

$$\arctan\left(\frac{2x-1}{2\sqrt{-x^2+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + x + 2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*(2*x - 1)/sqrt(-x^2 + x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^2+x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + x + 2), x)
```

GIAC/XCAS [A] time = 0.212079, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{2}{3}x - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + x + 2),x, algorithm="giac")
```

```
[Out] arcsin(2/3*x - 1/3)
```

3.55 $\int \frac{1}{\sqrt{5-4x+3x^2}} \, dx$

Optimal. Leaf size=19

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])
```

Rubi [A] time = 0.0278955, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[5 - 4*x + 3*x^2],x]

[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])
```

Rubi in Sympy [A] time = 0.732332, size = 32, normalized size = 1.68

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(6x-4)}{6\sqrt{3x^2-4x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-4*x+5)**(1/2),x)

[Out] sqrt(3)*atanh(sqrt(3)*(6*x - 4)/(6*sqrt(3*x**2 - 4*x + 5)))/3
```

Mathematica [A] time = 0.0131011, size = 18, normalized size = 0.95

$$\frac{\sinh^{-1}\left(\frac{3x-2}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(-2 + 3*x)/Sqrt[11]]/Sqrt[3]
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$\frac{\sqrt{3}}{3}Arcsinh\left(\frac{3\sqrt{11}}{11}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-4*x+5)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))
```

Maxima [A] time = 1.54136, size = 22, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\,\text{arsinh}\left(\frac{1}{11}\sqrt{11}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 - 4*x + 5),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/11*sqrt(11)*(3*x - 2))
```

Fricas [A] time = 0.226077, size = 55, normalized size = 2.89

$$\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(18x^2-24x+19\right)-6\sqrt{3x^2-4x+5}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 - 4*x + 5),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-sqrt(3)*(18*x^2 - 24*x + 19) - 6*sqrt(3*x^2 - 4*
x + 5)*(3*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2 - 4x + 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 - 4*x + 5), x)
```

GIAC/XCAS [A] time = 0.216172, size = 45, normalized size = 2.37

$$-\frac{1}{3}\,\sqrt{3}\ln\left(-\sqrt{3}\left(\sqrt{3}x - \sqrt{3\,x^2 - 4\,x + 5}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 - 4*x + 5),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - 4*x + 5)) + 2)
```

3.56 $\int \frac{1}{\sqrt{x-x^2}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] -ArcSin[1 - 2*x]
```

Rubi [A] time = 0.00738297, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x - x^2],x]

[Out] -ArcSin[1 - 2*x]
```

Rubi in Sympy [A] time = 0.573428, size = 5, normalized size = 0.62

$$\operatorname{asin}(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+x)**(1/2),x)

[Out] asin(2*x - 1)
```

Mathematica [B] time = 0.0142764, size = 38, normalized size = 4.75

$$\frac{2\sqrt{x-1}\sqrt{x}\log\left(\sqrt{x-1}+\sqrt{x}\right)}{\sqrt{-(x-1)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x - x^2],x]

[Out] (2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] + Sqrt[x]])/Sqrt[-((-1 +
 x)*x)]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+x)^(1/2),x)

[Out] arcsin(2*x-1)
```

Maxima [A] time = 1.57631, size = 8, normalized size = 1.

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + x),x, algorithm="maxima")

[Out] arcsin(2*x - 1)
```

Fricas [A] time = 0.214582, size = 22, normalized size = 2.75

$$-2\arctan\left(\frac{\sqrt{-x^2 + x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + x),x, algorithm="fricas")

[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+x)**(1/2),x)

[Out] Integral(1/sqrt(-x**2 + x), x)
```

GIAC/XCAS [A] time = 0.208798, size = 8, normalized size = 1.

$$\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + x),x, algorithm="giac")

[Out] arcsin(2*x - 1)
```

3.57 $\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=27

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Rubi [A] time = 0.0275121, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]

[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Rubi in Sympy [A] time = 2.21394, size = 32, normalized size = 1.19

$$-2\sqrt{-x^2+x+2}-2\,\mathrm{atan}\left(\frac{-2x+1}{2\sqrt{-x^2+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(-x**2+x+2)**(1/2),x)

[Out] -2*sqrt(-x**2 + x + 2) - 2*atan((-2*x + 1)/(2*sqrt(-x**2 + x + 2)
))
```

Mathematica [A] time = 0.0186192, size = 27, normalized size = 1.

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Maple [A] time = 0.008, size = 22, normalized size = 0.8

$$2\ \arcsin\left(-1/3+2/3\,x\right)-2\,\sqrt{-x^{2}+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(-x^2+x+2)^(1/2),x)
```

```
[Out] 2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

Maxima [A] time = 1.52555, size = 28, normalized size = 1.04

$$-2\,\sqrt{-x^{2}+x+2}-2\,\arcsin\left(-\frac{2}{3}\,x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/sqrt(-x^2 + x + 2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)
```

Fricas [A] time = 0.226653, size = 45, normalized size = 1.67

$$-2\,\sqrt{-x^{2}+x+2}+2\,\arctan\left(\frac{2\,x-1}{2\,\sqrt{-x^{2}+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/sqrt(-x^2 + x + 2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + x + 2) + 2*arctan(1/2*(2*x - 1)/sqrt(-x^2 + x + 2)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2x+1}{\sqrt{-\left(x-2\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/sqrt(-(x - 2)*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.219947, size = 28, normalized size = 1.04

$$-2\sqrt{-x^2+x+2}+2\arcsin\left(\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/sqrt(-x^2 + x + 2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x^2 + x + 2) + 2*arcsin(2/3*x - 1/3)
```

3.58 $\int \frac{1}{x\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Rubi [A] time = 0.0331915, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[2 + x - x^2]),x]

[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Rubi in Sympy [A] time = 2.08042, size = 29, normalized size = 0.91

$$-\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}(x+4)}{4\sqrt{-x^2+x+2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-x**2+x+2)**(1/2),x)

[Out] -sqrt(2)*atanh(sqrt(2)*(x + 4)/(4*sqrt(-x**2 + x + 2)))/2
```

Mathematica [A] time = 0.0213003, size = 34, normalized size = 1.06

$$\frac{\log(x) - \log\left(2\sqrt{2}\sqrt{-x^2+x+2}+x+4\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] (Log[x] - Log[4 + x + 2*Sqrt[2]*Sqrt[2 + x - x^2]])/Sqrt[2]
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}\mathit{Artanh}\left(\frac{(4+x)\sqrt{2}}{4}\frac{1}{\sqrt{-x^2+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-x^2+x+2)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

Maxima [A] time = 1.50994, size = 45, normalized size = 1.41

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2+x+2}}{|x|}+\frac{4}{|x|}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + x + 2)*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) +
 1)
```

Fricas [A] time = 0.244018, size = 53, normalized size = 1.66

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{-x^2+x+2}(x+4)+7x^2-16x-32}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + x + 2)*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 -
16*x - 32)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(x-2\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)
```

GIAC/XCAS [A] time = 0.231367, size = 96, normalized size = 3.

$$-\frac{1}{2}\,\sqrt{2}\ln\left(\frac{\left|-4\,\sqrt{2}+\frac{2\left(2\,\sqrt{-x^2+x+2}-3\right)}{2\,x-1}-6\right|}{\left|4\,\sqrt{2}+\frac{2\left(2\,\sqrt{-x^2+x+2}-3\right)}{2\,x-1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + x + 2)*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*ln(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*
x - 1) - 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1
) - 6))
```

3.59 $\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

```
[Out] (2*Sqrt[2 + x - x^2])/(3*(-2 + x))
```

Rubi [A] time = 0.0275169, antiderivative size = 23, normalized size of antiderivative = 1.1, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]

[Out] (-2*Sqrt[2 + x - x^2])/(3*(2 - x))
```

Rubi in Sympy [A] time = 2.01858, size = 17, normalized size = 0.81

$$-\frac{2\sqrt{-x^2+x+2}}{3\left(-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)

[Out] -2*sqrt(-x**2 + x + 2)/(3*(-x + 2))
```

Mathematica [A] time = 0.0123312, size = 19, normalized size = 0.9

$$-\frac{2(x+1)}{3\sqrt{-x^2+x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (-2*(1 + x))/(3*Sqrt[2 + x - x^2])
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-\frac{2\,x+2}{3}\frac{1}{\sqrt{-x^{2}+x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(-x^2+x+2)^(1/2),x)
```

```
[Out] -2/3*(1+x)/(-x^2+x+2)^(1/2)
```

Maxima [A] time = 1.56668, size = 23, normalized size = 1.1

$$\frac{2\,\sqrt{-x^{2}+x+2}}{3\,(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + x + 2)*(x - 2)),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

Fricas [A] time = 0.212751, size = 23, normalized size = 1.1

$$\frac{2\,\sqrt{-x^{2}+x+2}}{3\,(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + x + 2)*(x - 2)),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(x-2)(x+1)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)
```

GIAC/XCAS [A] time = 0.217778, size = 38, normalized size = 1.81

$$-\frac{4}{3\left(\frac{2\sqrt{-x^2+x+2}-3}{2x-1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + x + 2)*(x - 2)),x, algorithm="giac")
```

```
[Out] -4/3/((2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) + 1)
```

3.60 $\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

Rubi [A] time = 0.231368, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]

[Out] -ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\sin(x)+2}{(-\cos(x)+1)\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)

[Out] Integral((3*sin(x) + 2)/((-cos(x) + 1)*sin(x)), x)
```

Mathematica [A] time = 0.0543136, size = 54, normalized size = 1.93

$$\frac{1}{2}\csc^2\left(\frac{x}{2}\right)\left(-3\sin(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin
[x/2]]) + Log[Sin[x/2]] - 3*Sin[x]))/2
```

Maple [A] time = 0.07, size = 23, normalized size = 0.8

$$-\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{-2} - 3\ (\tan(x/2))^{-1} + \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

```
[Out] -1/2/tan(1/2*x)^2-3/tan(1/2*x)+ln(tan(1/2*x))
```

Maxima [A] time = 1.369, size = 45, normalized size = 1.61

$$-\frac{(\cos(x)+1)^2}{2\ \sin(x)^2} - \frac{3(\cos(x)+1)}{\sin(x)} + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*sin(x) + 2)/((cos(x) - 1)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)
/(cos(x) + 1))
```

Fricas [A] time = 0.224773, size = 53, normalized size = 1.89

$$-\frac{(\cos(x)-1)\log\left(\frac{1}{2}\ \cos(x)+\frac{1}{2}\right) - (\cos(x)-1)\log\left(-\frac{1}{2}\ \cos(x)+\frac{1}{2}\right) - 6\ \sin(x) - 2}{2(\cos(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*sin(x) + 2)/((cos(x) - 1)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*
cos(x) + 1/2) - 6*sin(x) - 2)/(cos(x) - 1)
```

Sympy [A] time = 1.49997, size = 22, normalized size = 0.79

$$\log\left(\tan\left(\frac{x}{2}\right)\right)-\frac{3}{\tan\left(\frac{x}{2}\right)}-\frac{1}{2\tan^2\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

```
[Out] log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)
```

GIAC/XCAS [A] time = 0.21669, size = 42, normalized size = 1.5

$$-\frac{3\tan\left(\frac{1}{2}x\right)^2+6\tan\left(\frac{1}{2}x\right)+1}{2\tan\left(\frac{1}{2}x\right)^2}+\ln\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*sin(x) + 2)/((cos(x) - 1)*sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + ln(abs(ta
n(1/2*x)))
```

3.61 $\int \frac{1}{2+3\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{3\sin(x)\cos(x)}{3\cos^2(x)+\sqrt{10}+2}\right)}{\sqrt{10}}$$

```
[Out] x/Sqrt[10] - ArcTan[(3*Cos[x]*Sin[x])/(2 + Sqrt[10] + 3*Cos[x]^2)
]/Sqrt[10]
```

Rubi [A] time = 0.0498396, antiderivative size = 48, normalized size of antiderivative = 1.3, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{\left(2-\sqrt{10}\right)\sin(x)\cos(x)}{2-\left(2-\sqrt{10}\right)\cos^2(x)}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[10] + ArcTan[((2 - Sqrt[10])*Cos[x]*Sin[x])/(2 - (2 - Sqrt
[10])*Cos[x]^2)]/Sqrt[10]
```

Rubi in Sympy [A] time = 0.644473, size = 17, normalized size = 0.46

$$\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\tan(x)}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+3*cos(x)**2),x)
```

```
[Out] sqrt(10)*atan(sqrt(10)*tan(x)/5)/10
```

Mathematica [A] time = 0.0213048, size = 17, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5}}\tan(x)\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]
```

Maple [A] time = 0.02, size = 14, normalized size = 0.4

$$\frac{\sqrt{10}}{10}\arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+3*cos(x)^2),x)
```

```
[Out] 1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))
```

Maxima [A] time = 1.51015, size = 18, normalized size = 0.49

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{1}{5}\sqrt{10}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*cos(x)^2 + 2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))
```

Fricas [A] time = 0.231524, size = 42, normalized size = 1.14

$$-\frac{1}{20}\sqrt{10}\arctan\left(\frac{7\sqrt{10}\cos(x)^2-2\sqrt{10}}{20\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*cos(x)^2 + 2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(co
s(x)*sin(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+3*cos(x)**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220057, size = 62, normalized size = 1.68

$$\frac{1}{10}\sqrt{10}\left(x+\arctan\left(-\frac{\sqrt{10}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{10}\cos\left(2x\right)+\sqrt{10}-2\cos\left(2x\right)+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*cos(x)^2 + 2),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt
(10)*cos(2*x) + sqrt(10) - 2*cos(2*x) + 2)))
```

3.62 $\int \csc(2x)(1-\tan(x))\,dx$

Optimal. Leaf size=14

$$\frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}$$

```
[Out] Log[Tan[x]]/2 - Tan[x]/2
```

Rubi [A] time = 0.0598775, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] Log[Tan[x]]/2 - Tan[x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\tan(x)+1}{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-tan(x))/sin(2*x),x)
```

```
[Out] Integral((-tan(x) + 1)/sin(2*x), x)
```

Mathematica [A] time = 0.0156552, size = 21, normalized size = 1.5

$$-\frac{\tan(x)}{2}+\frac{1}{2}\log(\sin(x))-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] -Log[Cos[x]]/2 + Log[Sin[x]]/2 - Tan[x]/2
```

Maple [A] time = 0.07, size = 11, normalized size = 0.8

$$\frac{\ln\left(\tan\left(x\right)\right)}{2}-\frac{\tan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-tan(x))/sin(2*x),x)
```

```
[Out] 1/2*ln(tan(x))-1/2*tan(x)
```

Maxima [A] time = 1.407, size = 63, normalized size = 4.5

$$-\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}-\frac{1}{4}\,\log\left(\cos\left(2\,x\right)+1\right)+\frac{1}{4}\,\log\left(\cos\left(2\,x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x) - 1)/sin(2*x),x, algorithm="maxima")
```

```
[Out] -sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/4*log(co
s(2*x) + 1) + 1/4*log(cos(2*x) - 1)
```

Fricas [A] time = 0.226698, size = 43, normalized size = 3.07

$$\frac{1}{4}\,\log\left(\frac{\tan\left(x\right)^{2}}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{4}\,\log\left(\frac{1}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{2}\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x) - 1)/sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(tan(x)^2/(tan(x)^2 + 1)) - 1/4*log(1/(tan(x)^2 + 1)) - 1/
2*tan(x)
```

Sympy [A] time = 1.13841, size = 27, normalized size = 1.93

$$\frac{\log\left(\cos\left(2x\right)-1\right)}{4}-\frac{\log\left(\cos\left(2x\right)+1\right)}{4}-\frac{\sin\left(x\right)}{2\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-tan(x))/sin(2*x),x)
```

```
[Out] log(cos(2*x) - 1)/4 - log(cos(2*x) + 1)/4 - sin(x)/(2*cos(x))
```

GIAC/XCAS [A] time = 0.21529, size = 15, normalized size = 1.07

$$\frac{1}{2}\ln\left(\left|\tan\left(x\right)\right|\right)-\frac{1}{2}\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x) - 1)/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(tan(x))) - 1/2*tan(x)
```

3.63 $\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rubi [A] time = 0.0585767, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]

[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rubi in Sympy [A] time = 5.20895, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+tan(x)**2)/(1-tan(x)**2),x)

[Out] atanh(tan(x))
```

Mathematica [B] time = 0.0101774, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]

[Out] -Log[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x]]/2
```

Maple [A] time = 0.009, size = 4, normalized size = 0.4

$$\mathit{Artanh}\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+tan(x)^2)/(1-tan(x)^2),x)

[Out] arctanh(tan(x))
```

Maxima [A] time = 1.42001, size = 20, normalized size = 1.82

$$\frac{1}{2}\,\log\left(\tan\left(x\right)+1\right)-\frac{1}{2}\,\log\left(\tan\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x, algorithm="maxima")

[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

Fricas [A] time = 0.263514, size = 61, normalized size = 5.55

$$\frac{1}{4}\,\log\left(\frac{\tan\left(x\right)^{2}+2\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{4}\,\log\left(\frac{\tan\left(x\right)^{2}-2\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x, algorithm="fricas")

[Out] 1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(
x)^2 - 2*tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [A] time = 0.211386, size = 15, normalized size = 1.36

$$-\frac{\log\left(\tan\left(x\right)-1\right)}{2}+\frac{\log\left(\tan\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+tan(x)**2)/(1-tan(x)**2),x)
```

```
[Out] -log(tan(x) - 1)/2 + log(tan(x) + 1)/2
```

GIAC/XCAS [A] time = 0.216861, size = 23, normalized size = 2.09

$$\frac{1}{2}\ln\left(|\tan\left(x\right)+1|\right)-\frac{1}{2}\ln\left(|\tan\left(x\right)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(tan(x) + 1)) - 1/2*ln(abs(tan(x) - 1))
```

3.64 $\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{7}\left(a^2-4\cos^2(x)\right)^{7/4}$$

```
[Out] (a^2 - 4*Cos[x]^2)^(7/4)/7
```

Rubi [A] time = 0.0758641, antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]

[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Rubi in Sympy [A] time = 3.18482, size = 14, normalized size = 0.78

$$\frac{\left(a^2-4\cos^2(x)\right)^{\frac{7}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)

[Out] (a**2 - 4*cos(x)**2)**(7/4)/7
```

Mathematica [B] time = 0.263776, size = 127, normalized size = 7.06

$$\frac{a^4+4a^2\sqrt[4]{\frac{a^2-2\cos(2x)-2}{a^2-2}}-4\left(a^2-2\right)\cos(2x)-4\sqrt[4]{\frac{a^2-2\cos(2x)-2}{a^2-2}}-4a^2+a^4\left(-\sqrt[4]{\frac{a^2-2\cos(2x)-2}{a^2-2}}\right)+2\cos(4x)}{7\sqrt[4]{a^2-2\cos(2x)-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

```
[Out] (6 - 4*a^2 + a^4 - 4*((-2 + a^2 - 2*Cos[2*x])/(-2 + a^2))^(1/4) +
 4*a^2*((-2 + a^2 - 2*Cos[2*x])/(-2 + a^2))^(1/4) - a^4*((-2 + a^
2 - 2*Cos[2*x])/(-2 + a^2))^(1/4) - 4*(-2 + a^2)*Cos[2*x] + 2*Cos
[4*x])/(7*(-2 + a^2 - 2*Cos[2*x])^(1/4))
```

Maple [A] time = 0.021, size = 15, normalized size = 0.8

$$\frac{1}{7}\left(a^{2}-4\left(\cos\left(x\right)\right)^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x)
```

```
[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)
```

Maxima [A] time = 1.39469, size = 19, normalized size = 1.06

$$\frac{1}{7}\left(a^{2}-4\cos\left(x\right)^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 - 4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
```

```
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Fricas [A] time = 0.258358, size = 19, normalized size = 1.06

$$\frac{1}{7}\left(a^{2}-4\cos\left(x\right)^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 - 4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 - 4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.65 $\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx$

Optimal. Leaf size=18

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

```
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Rubi [A] time = 0.0833825, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

```
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Rubi in Sympy [A] time = 3.25389, size = 17, normalized size = 0.94

$$-\frac{3\left(a^2-4\sin^2(x)\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
```

```
[Out] -3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

Mathematica [B] time = 0.153163, size = 67, normalized size = 3.72

$$-\frac{3\left(a^2+2\cos(2x)-2\right)^{2/3}\left(\left(\frac{a^2+2\cos(2x)-2}{a^2-2}\right)^{2/3}-1\right)}{8\left(\frac{a^2+2\cos(2x)-2}{a^2-2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

```
[Out] (-3*(-2 + a^2 + 2*Cos[2*x])^(2/3)*(-1 + ((-2 + a^2 + 2*Cos[2*x])/
(-2 + a^2))^(2/3)))/(8*((-2 + a^2 + 2*Cos[2*x])/(-2 + a^2))^(2/3)
)
```

Maple [A] time = 0.025, size = 15, normalized size = 0.8

$$-\frac{3}{8}\left(a^2-4\,(\sin(x))^2\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x)
```

```
[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)
```

Maxima [A] time = 1.33221, size = 19, normalized size = 1.06

$$-\frac{3}{8}\left(a^2-4\,\sin(x)^2\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

Fricas [A] time = 0.246401, size = 20, normalized size = 1.11

$$-\frac{3}{8}\left(a^2+4\,\cos(x)^2-4\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/8*(a^2 + 4*cos(x)^2 - 4)^(2/3)
```

Sympy [A] time = 3.73106, size = 17, normalized size = 0.94

$$-\frac{3\left(a^2-4\sin^2\left(x\right)\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
```

```
[Out] -3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

GIAC/XCAS [A] time = 0.260595, size = 35, normalized size = 1.94

$$-\frac{3}{8}\left(a^2-\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] -3/8*(a^2 - 16*tan(1/2*x)^2/(tan(1/2*x)^2 + 1)^2)^(2/3)
```

3.66 $\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx$

Optimal. Leaf size=17

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Rubi [A] time = 0.0268088, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-1 + a^(2*x)],x]

[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Rubi in Sympy [A] time = 1.78313, size = 14, normalized size = 0.82

$$\frac{\operatorname{atan}\left(\sqrt{a^{2x}-1}\right)}{\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+a**(2*x))**(1/2),x)

[Out] atan(sqrt(a**(2*x) - 1))/log(a)
```

Mathematica [A] time = 0.0120483, size = 17, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-1 + a^(2*x)],x]
```

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$\frac{1}{\ln(a)} \arctan\left(\sqrt{-1+a^{2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+a^(2*x))^(1/2),x)
```

```
[Out] arctan((-1+a^(2*x))^(1/2))/ln(a)
```

Maxima [A] time = 1.50184, size = 20, normalized size = 1.18

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^(2*x) - 1),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Fricas [A] time = 0.251136, size = 20, normalized size = 1.18

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^(2*x) - 1),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+a**(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**(2*x) - 1), x)
```

GIAC/XCAS [A] time = 0.221577, size = 20, normalized size = 1.18

$$\frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\ln\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^(2*x) - 1),x, algorithm="giac")
```

```
[Out] arctan(sqrt(a^(2*x) - 1))/ln(a)
```

3.67 $\int \frac{e^{x/2}}{\sqrt{-1+e^x}} \, dx$

Optimal. Leaf size=20

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

```
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Rubi [A] time = 0.0503048, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(x/2)/Sqrt[-1 + E^x],x]

[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Rubi in Sympy [A] time = 2.98685, size = 15, normalized size = 0.75

$$2\,\text{atanh}\left(\frac{e^{\frac{x}{2}}}{\sqrt{e^x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)

[Out] 2*atanh(exp(x/2)/sqrt(exp(x) - 1))
```

Mathematica [A] time = 0.011488, size = 20, normalized size = 1.

$$2\log\left(\sqrt{e^x-1}+e^{x/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(x/2)/Sqrt[-1 + E^x],x]
```

```
[Out] 2*Log[E^(x/2) + Sqrt[-1 + E^x]]
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int 1\mathrm{e}^{\frac{x}{2}} \frac{1}{\sqrt{-1+\mathrm{e}^{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

```
[Out] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

Maxima [A] time = 1.33516, size = 24, normalized size = 1.2

$$2\,\log\left(2\,\sqrt{e^{x}-1}+2\,e^{\left(\frac{1}{2}\,x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(1/2*x)/sqrt(e^x - 1),x, algorithm="maxima")
```

```
[Out] 2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))
```

Fricas [A] time = 0.233692, size = 22, normalized size = 1.1

$$-2\,\log\left(\sqrt{e^{x}-1}-e^{\left(\frac{1}{2}\,x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(1/2*x)/sqrt(e^x - 1),x, algorithm="fricas")
```

```
[Out] -2*log(sqrt(e^x - 1) - e^(1/2*x))
```

Sympy [A] time = 0.550696, size = 7, normalized size = 0.35

$$2\,\mathrm{acosh}\left(e^{\frac{x}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)

[Out] 2*acosh(exp(x/2))
```

GIAC/XCAS [A] time = 0.206803, size = 22, normalized size = 1.1

$$-2\ln\left(-\sqrt{e^{x}-1}+e^{\left(\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(1/2*x)/sqrt(e^x - 1),x, algorithm="giac")

[Out] -2*ln(-sqrt(e^x - 1) + e^(1/2*x))
```

3.68 $\int \frac{\tan^{-1}(x)^n}{1+x^2} \, dx$

Optimal. Leaf size=12

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0414055, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[x]^n/(1 + x^2),x]

[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 3.10798, size = 8, normalized size = 0.67

$$\frac{\operatorname{atan}^{n+1}(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x)**n/(x**2+1),x)

[Out] atan(x)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.00460808, size = 12, normalized size = 1.

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$\frac{(\arctan(x))^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x)^n/(x^2+1),x)

[Out] arctan(x)^(1+n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^n/(x^2 + 1),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235106, size = 16, normalized size = 1.33

$$\frac{\arctan(x)^n \arctan(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^n/(x^2 + 1),x, algorithm="fricas")

[Out] arctan(x)^n*arctan(x)/(n + 1)
```

Sympy [A] time = 9.21979, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atan}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{atan}(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x)**n/(x**2+1),x)
```

```
[Out] Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), T
rue))
```

GIAC/XCAS [A] time = 0.202165, size = 16, normalized size = 1.33

$$\frac{\arctan(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^n/(x^2 + 1),x, algorithm="giac")
```

```
[Out] arctan(x)^(n + 1)/(n + 1)
```

3.69 $\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rubi [A] time = 0.106854, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]

[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rubi in Sympy [A] time = 6.53098, size = 34, normalized size = 0.81

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\operatorname{asin}^{\frac{5}{2}}\left(\frac{x}{a}\right)}{5\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)

[Out] 2*a*sqrt(1 - x**2/a**2)*asin(x/a)**(5/2)/(5*sqrt(a**2 - x**2))
```

Mathematica [A] time = 0.0214433, size = 42, normalized size = 1.

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Maple [A] time = 0.101, size = 38, normalized size = 0.9

$$\frac{2a}{5}\left(\arcsin\left(\frac{x}{a}\right)\right)^{\frac{5}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\frac{1}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/5*arcsin(x/a)^(5/2)*a/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)
```

Maxima [A] time = 4.26498, size = 486, normalized size = 11.57

$$\frac{2}{15}\sqrt{2}\left(\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)^2+\frac{1}{4}\log\left(a^2\right)^2-\log\left(a^2\right)\log\left(a\right)+\log\left(a\right)^2\right)^{\frac{5}{4}}\cos\left(\frac{5}{2}\arctan\left(\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right),-\frac{1}{2}\log\left(a^2\right)+\log\left(a\right)\right)\right)$$
$$-\frac{1}{6}\left(\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)^2+\frac{1}{4}\log\left(a^2\right)^2-\log\left(a^2\right)\log\left(a\right)+\log\left(a\right)^2\right)^{\frac{3}{4}}\left(2\sqrt{2}\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)-\sqrt{2}\log\left(a^2\right)-\frac{1}{2}\log\left(a^2\right)+\log\left(a\right)\right)\right)$$
$$+\frac{2}{15}\sqrt{2}\left(\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)^2+\frac{1}{4}\log\left(a^2\right)^2-\log\left(a^2\right)\log\left(a\right)+\log\left(a\right)^2\right)^{\frac{5}{4}}\sin\left(\frac{5}{2}\arctan\left(\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)-\frac{1}{2}\log\left(a^2\right)+\log\left(a\right)\right)\right)$$
$$+\frac{1}{6}\left(\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)^2+\frac{1}{4}\log\left(a^2\right)^2-\log\left(a^2\right)\log\left(a\right)+\log\left(a\right)^2\right)^{\frac{3}{4}}\left(2\sqrt{2}\arctan\left(x,\sqrt{a+x}\sqrt{a-x}\right)+\sqrt{2}\log\left(a^2\right)-\frac{1}{2}\log\left(a^2\right)+\log\left(a\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x/a)^(3/2)/sqrt(a^2 - x^2),x, algorithm="maxima")
```

```
[Out] 2/15*sqrt(2)*(arctan2(x, sqrt(a + x)*sqrt(a - x))^2 + 1/4*log(a^2
)^2 - log(a^2)*log(a) + log(a)^2)^(5/4)*cos(5/2*arctan2(arctan2(x
, sqrt(a + x)*sqrt(a - x)), -1/2*log(a^2) + log(a))) - 1/6*(arcta
```

```
n2(x, sqrt(a + x)*sqrt(a - x))^2 + 1/4*log(a^2)^2 - log(a^2)*log(
a) + log(a)^2)^(3/4)*(2*sqrt(2)*arctan2(x, sqrt(a + x)*sqrt(a - x
)) - sqrt(2)*log(a^2) + 2*sqrt(2)*log(a))*cos(3/2*arctan2(arctan2
(x, sqrt(a + x)*sqrt(a - x)), -1/2*log(a^2) + log(a))) + 2/15*sqr
t(2)*(arctan2(x, sqrt(a + x)*sqrt(a - x))^2 + 1/4*log(a^2)^2 - lo
g(a^2)*log(a) + log(a)^2)^(5/4)*sin(5/2*arctan2(arctan2(x, sqrt(a
 + x)*sqrt(a - x)), -1/2*log(a^2) + log(a))) + 1/6*(arctan2(x, sq
rt(a + x)*sqrt(a - x))^2 + 1/4*log(a^2)^2 - log(a^2)*log(a) + log
(a)^2)^(3/4)*(2*sqrt(2)*arctan2(x, sqrt(a + x)*sqrt(a - x)) + sqr
t(2)*log(a^2) - 2*sqrt(2)*log(a))*sin(3/2*arctan2(arctan2(x, sqrt
(a + x)*sqrt(a - x)), -1/2*log(a^2) + log(a)))
```

Fricas [A] time = 0.218736, size = 24, normalized size = 0.57

$$\frac{2}{5}\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x/a)^(3/2)/sqrt(a^2 - x^2),x, algorithm="fricas")

[Out] 2/5*arctan(x/sqrt(a^2 - x^2))^(5/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.310963, size = 16, normalized size = 0.38

$$\frac{2}{5}\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}\operatorname{sign}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x/a)^(3/2)/sqrt(a^2 - x^2),x, algorithm="giac")

[Out] 2/5*arcsin(x/a)^(5/2)*sign(a)
```

3.70 $\int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)^3} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2 \cos^{-1}(x)^2}$$

```
[Out] 1/(2*ArcCos[x]^2)
```

Rubi [A] time = 0.0402161, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]

[Out] 1/(2*ArcCos[x]^2)
```

Rubi in Sympy [A] time = 2.95948, size = 7, normalized size = 0.88

$$\frac{1}{2 \operatorname{acos}^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)

[Out] 1/(2*acos(x)**2)
```

Mathematica [A] time = 0.003715, size = 8, normalized size = 1.

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] 1/(2*ArcCos[x]^2)
```

Maple [A] time = 0.007, size = 7, normalized size = 0.9

$$\frac{1}{2\,(\arccos(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(x)^3/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2/arccos(x)^2
```

Maxima [A] time = 1.51361, size = 8, normalized size = 1.

$$\frac{1}{2\,\arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + 1)*arccos(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2/arccos(x)^2
```

Fricas [A] time = 0.219914, size = 8, normalized size = 1.

$$\frac{1}{2\,\arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + 1)*arccos(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2/arccos(x)^2
```

Sympy [A] time = 4.59682, size = 7, normalized size = 0.88

$$\frac{1}{2\,\operatorname{acos}^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 1/(2*acos(x)**2)
```

GIAC/XCAS [A] time = 0.204488, size = 8, normalized size = 1.

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*arccos(x)^3),x, algorithm="giac")
```

```
[Out] 1/2/arccos(x)^2
```

3.71 $\int x \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] time = 0.020853, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[x]^2,x]

[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\log(x)^2}{2}-\frac{x^2\log(x)}{2}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x)**2,x)

[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + Integral(x, x)/2
```

Mathematica [A] time = 0.00211829, size = 28, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] time = 0., size = 23, normalized size = 0.8

$$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 (\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A] time = 1.32918, size = 23, normalized size = 0.82

$$\frac{1}{4} \left(2 \log(x)^2 - 2 \log(x) + 1\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A] time = 0.216045, size = 30, normalized size = 1.07

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] time = 0.093562, size = 22, normalized size = 0.79

$$\frac{x^2 \log(x)^2}{2} - \frac{x^2 \log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

GIAC/XCAS [A] time = 0.213172, size = 30, normalized size = 1.07

$$\frac{1}{2}x^2\ln(x)^2 - \frac{1}{2}x^2\ln(x) + \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*ln(x)^2 - 1/2*x^2*ln(x) + 1/4*x^2
```

3.72 $\int \frac{\log(x)}{x^5} dx$

Optimal. Leaf size=17

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

```
[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Rubi [A] time = 0.0174426, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/x^5,x]

[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Rubi in Sympy [A] time = 1.14663, size = 15, normalized size = 0.88

$$-\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(ln(x)/x**5,x)

[Out] -log(x)/(4*x**4) - 1/(16*x**4)
```

Mathematica [A] time = 0.00210101, size = 17, normalized size = 1.

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/x^5,x]
```

```
[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{1}{16\,x^{4}}-\frac{\ln\left(x\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)/x^5,x)
```

```
[Out] -1/16/x^4-1/4*ln(x)/x^4
```

Maxima [A] time = 1.35203, size = 18, normalized size = 1.06

$$-\frac{\log\left(x\right)}{4\,x^{4}}-\frac{1}{16\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

Fricas [A] time = 0.219842, size = 15, normalized size = 0.88

$$-\frac{4\,\log\left(x\right)+1}{16\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(4*log(x) + 1)/x^4
```

Sympy [A] time = 0.088194, size = 15, normalized size = 0.88

$$-\frac{\log\left(x\right)}{4x^{4}}-\frac{1}{16x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/x**5,x)
```

```
[Out] -log(x)/(4*x**4) - 1/(16*x**4)
```

GIAC/XCAS [A] time = 0.232927, size = 18, normalized size = 1.06

$$-\frac{\ln\left(x\right)}{4\,x^{4}}-\frac{1}{16\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*ln(x)/x^4 - 1/16/x^4
```

3.73 $\int x^2 \log\left(\frac{-1+x}{x}\right)\,dx$

Optimal. Leaf size=36

$$\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{x^2}{6}-\frac{x}{3}-\frac{1}{3}\log(x-1)$$

```
[Out] -x/3 - x^2/6 - Log[-1 + x]/3 + (x^3*Log[(-1 + x)/x])/3
```

Rubi [A] time = 0.0579748, antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{x^2}{6}-\frac{x}{3}-\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Log[(-1 + x)/x],x]
```

```
[Out] -x/3 - x^2/6 + (x^3*Log[1 - x^(-1)])/3 - Log[1 - x]/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3\log\left(\frac{x-1}{x}\right)}{3}-\frac{x}{3}-\frac{\log(-x+1)}{3}-\frac{\int x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*ln((-1+x)/x),x)
```

```
[Out] x**3*log((x - 1)/x)/3 - x/3 - log(-x + 1)/3 - Integral(x, x)/3
```

Mathematica [A] time = 0.00505381, size = 38, normalized size = 1.06

$$\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{x^2}{6}-\frac{x}{3}-\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Log[(-1 + x)/x],x]
```

```
[Out] -x/3 - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3
```

Maple [A] time = 0.021, size = 53, normalized size = 1.5

$$-\frac{x^2}{6}+\frac{1}{3}\ln\left(-x^{-1}\right)-\frac{x}{3}+\frac{x^3}{3}\ln\left(1-x^{-1}\right)\left(1-x^{-1}\right)\left(\left(1-x^{-1}\right)^2+3x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*ln((-1+x)/x),x)
```

```
[Out] -1/6*x^2+1/3*ln(-1/x)-1/3*x+1/3*ln(1-1/x)*(1-1/x)*((1-1/x)^2+3/x)
*x^3
```

Maxima [A] time = 1.33148, size = 38, normalized size = 1.06

$$\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{1}{6}x^2-\frac{1}{3}x-\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log((x - 1)/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

Fricas [A] time = 0.218258, size = 38, normalized size = 1.06

$$\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{1}{6}x^2-\frac{1}{3}x-\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log((x - 1)/x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

Sympy [A] time = 0.132544, size = 26, normalized size = 0.72

$$\frac{x^3\log\left(\frac{x-1}{x}\right)}{3}-\frac{x^2}{6}-\frac{x}{3}-\frac{\log\left(x-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*ln((-1+x)/x),x)
```

```
[Out] x**3*log((x - 1)/x)/3 - x**2/6 - x/3 - log(x - 1)/3
```

GIAC/XCAS [A] time = 0.242067, size = 39, normalized size = 1.08

$$\frac{1}{3}\,x^3\ln\left(\frac{x-1}{x}\right)-\frac{1}{6}\,x^2-\frac{1}{3}\,x-\frac{1}{3}\,\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log((x - 1)/x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*ln((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*ln(abs(x - 1))
```

3.74 $\int \cos^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi [A] time = 0.0170276, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi in Sympy [A] time = 0.71909, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

Mathematica [A] time = 0.00340846, size = 23, normalized size = 1.21

$$\frac{5\sin(x)}{8} + \frac{5}{48}\sin(3x) + \frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5,x]
```

```
[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$\frac{\sin(x)}{5}\left(\frac{8}{3}+(\cos(x))^4+\frac{4\,(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^5,x)
```

```
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

Maxima [A] time = 1.35091, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5-\frac{2}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.257404, size = 24, normalized size = 1.26

$$\frac{1}{15}\left(3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 0.043716, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

GIAC/XCAS [A] time = 0.232312, size = 20, normalized size = 1.05

$$\frac{1}{5}\,\sin\left(x\right)^{5} - \frac{2}{3}\,\sin\left(x\right)^{3} + \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

3.75 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[
x])/6
```

Rubi [A] time = 0.0624552, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[
x])/6
```

Rubi in Sympy [A] time = 1.75136, size = 31, normalized size = 0.91

$$\frac{x}{16}-\frac{\sin(x)\cos^5(x)}{6}+\frac{\sin(x)\cos^3(x)}{24}+\frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*sin(x)**2,x)
```

```
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/1
6
```

Mathematica [A] time = 0.0136854, size = 30, normalized size = 0.88

$$\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

Maple [A] time = 0.002, size = 26, normalized size = 0.8

$$-\frac{(\cos(x))^5 \sin(x)}{6} + \frac{\sin(x)}{24}\left((\cos(x))^3 + \frac{3\cos(x)}{2}\right) + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*sin(x)^2,x)
```

```
[Out] -1/6*cos(x)^5*sin(x)+1/24*(cos(x)^3+3/2*cos(x))*sin(x)+1/16*x
```

Maxima [A] time = 1.529, size = 24, normalized size = 0.71

$$\frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Fricas [A] time = 0.227614, size = 34, normalized size = 1.

$$-\frac{1}{48}\left(8\cos(x)^5 - 2\cos(x)^3 - 3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] time = 0.041987, size = 31, normalized size = 0.91

$$\frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*sin(x)**2,x)
```

```
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/1
6
```

GIAC/XCAS [A] time = 0.231015, size = 30, normalized size = 0.88

$$\frac{1}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) - \frac{1}{64}\,\sin\left(4\,x\right) + \frac{1}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

3.76 $\int \csc^5(x)\,dx$

Optimal. Leaf size=26

$$-\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)-\frac{3}{8}\cot(x)\csc(x)$$

```
[Out] (-3*ArcTanh[Cos[x]])/8 - (3*Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/
4
```

Rubi [A] time = 0.0227575, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.5$

$$-\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)-\frac{3}{8}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^5,x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/8 - (3*Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/
4
```

Rubi in Sympy [A] time = 0.597512, size = 31, normalized size = 1.19

$$-\frac{3\operatorname{atanh}(\cos(x))}{8}-\frac{3\cos(x)}{8\sin^2(x)}-\frac{\cos(x)}{4\sin^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sin(x)**5,x)
```

```
[Out] -3*atanh(cos(x))/8 - 3*cos(x)/(8*sin(x)**2) - cos(x)/(4*sin(x)**4
)
```

Mathematica [B] time = 0.00863986, size = 71, normalized size = 2.73

$$-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{3}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{3}{32}\sec^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^5,x]
```

```
[Out] (-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log
[Sin[x/2]])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64
```

Maple [A] time = 0.056, size = 26, normalized size = 1.

$$\left(-\frac{(\csc(x))^3}{4} - \frac{3\csc(x)}{8}\right)\cot(x) + \frac{3\ln(\csc(x) - \cot(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sin(x)^5,x)
```

```
[Out] (-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))
```

Maxima [A] time = 1.34927, size = 57, normalized size = 2.19

$$\frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)} - \frac{3}{16}\log(\cos(x) + 1) + \frac{3}{16}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-5),x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3/16*lo
g(cos(x) + 1) + 3/16*log(cos(x) - 1)
```

Fricas [A] time = 0.226835, size = 93, normalized size = 3.58

$$\frac{6\cos(x)^3 - 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 10\cos(x)}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-5),x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) +
 1/2) + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10
*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [A] time = 0.172121, size = 46, normalized size = 1.77

$$\frac{3\cos^{3}(x) - 5\cos(x)}{8\cos^{4}(x) - 16\cos^{2}(x) + 8} + \frac{3\log(\cos(x) - 1)}{16} - \frac{3\log(\cos(x) + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x)**5,x)
```

```
[Out] (3*cos(x)**3 - 5*cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) + 3*log
(cos(x) - 1)/16 - 3*log(cos(x) + 1)/16
```

GIAC/XCAS [A] time = 0.216434, size = 51, normalized size = 1.96

$$\frac{3\cos(x)^{3} - 5\cos(x)}{8\left(\cos(x)^{2} - 1\right)^{2}} - \frac{3}{16}\ln(\cos(x) + 1) + \frac{3}{16}\ln(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-5),x, algorithm="giac")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^2 - 1)^2 - 3/16*ln(cos(x) + 1
) + 3/16*ln(-cos(x) + 1)
```

3.77 $\int e^{-x} \sin(x) \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

```
[Out] -Cos[x]/(2*E^x) - Sin[x]/(2*E^x)
```

Rubi [A] time = 0.0178771, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/E^x,x]

[Out] -Cos[x]/(2*E^x) - Sin[x]/(2*E^x)
```

Rubi in Sympy [A] time = 1.44417, size = 17, normalized size = 0.74

$$-\frac{e^{-x}\sin(x)}{2}-\frac{e^{-x}\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/exp(x),x)

[Out] -exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

Mathematica [A] time = 0.0114765, size = 14, normalized size = 0.61

$$-\frac{1}{2}e^{-x}(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/E^x,x]

[Out] -(Cos[x] + Sin[x])/(2*E^x)
```

Maple [A] time = 0.009, size = 18, normalized size = 0.8

$$-\frac{\mathrm{e}^{-x}\cos\left(x\right)}{2}-\frac{\mathrm{e}^{-x}\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/exp(x),x)
```

```
[Out] -1/2*exp(-x)*cos(x)-1/2*exp(-x)*sin(x)
```

Maxima [A] time = 1.41194, size = 15, normalized size = 0.65

$$-\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x)
```

Fricas [A] time = 0.22425, size = 23, normalized size = 1.

$$-\frac{1}{2}\cos\left(x\right)e^{(-x)}-\frac{1}{2}e^{(-x)}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^(-x) - 1/2*e^(-x)*sin(x)
```

Sympy [A] time = 0.80014, size = 17, normalized size = 0.74

$$-\frac{e^{-x}\sin\left(x\right)}{2}-\frac{e^{-x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/exp(x),x)
```

```
[Out] -exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.202138, size = 15, normalized size = 0.65

$$-\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{\left(-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x)
```

3.78 $\int e^{2x} \sin(3x)\,dx$

Optimal. Leaf size=27

$$\frac{2}{13}e^{2x}\sin(3x)-\frac{3}{13}e^{2x}\cos(3x)$$

```
[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

Rubi [A] time = 0.0200277, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2}{13}e^{2x}\sin(3x)-\frac{3}{13}e^{2x}\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)*Sin[3*x],x]

[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

Rubi in Sympy [A] time = 1.49142, size = 26, normalized size = 0.96

$$\frac{2e^{2x}\sin(3x)}{13}-\frac{3e^{2x}\cos(3x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)*sin(3*x),x)

[Out] 2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

Mathematica [A] time = 0.0292173, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{2x}(2\sin(3x)-3\cos(3x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)*Sin[3*x],x]
```

```
[Out] (E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13
```

Maple [A] time = 0.003, size = 22, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{2\,x}\cos\left(3\,x\right)}{13}+\frac{2\,\mathrm{e}^{2\,x}\sin\left(3\,x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)*sin(3*x),x)
```

```
[Out] -3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

Maxima [A] time = 1.42945, size = 26, normalized size = 0.96

$$-\frac{1}{13}\,\left(3\,\cos\left(3\,x\right)-2\,\sin\left(3\,x\right)\right)e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

Fricas [A] time = 0.2469, size = 28, normalized size = 1.04

$$-\frac{3}{13}\,\cos\left(3\,x\right)e^{\left(2\,x\right)}+\frac{2}{13}\,e^{\left(2\,x\right)}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -3/13*cos(3*x)*e^(2*x) + 2/13*e^(2*x)*sin(3*x)
```

Sympy [A] time = 0.372339, size = 26, normalized size = 0.96

$$\frac{2e^{2x}\sin\left(3x\right)}{13}-\frac{3e^{2x}\cos\left(3x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)*sin(3*x),x)
```

```
[Out] 2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

GIAC/XCAS [A] time = 0.202422, size = 26, normalized size = 0.96

$$-\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

3.79 $\int a^x \cos(x)\,dx$

Optimal. Leaf size=31

$$\frac{a^x \sin(x)}{\log^2(a)+1}+\frac{a^x \log(a)\cos(x)}{\log^2(a)+1}$$

```
[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

Rubi [A] time = 0.0219576, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{a^x \sin(x)}{\log^2(a)+1}+\frac{a^x \log(a)\cos(x)}{\log^2(a)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[a^x*Cos[x],x]

[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

Rubi in Sympy [A] time = 1.5652, size = 29, normalized size = 0.94

$$\frac{a^x \log(a)\cos(x)}{\log(a)^2+1}+\frac{a^x \sin(x)}{\log(a)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**x*cos(x),x)

[Out] a**x*log(a)*cos(x)/(log(a)**2 + 1) + a**x*sin(x)/(log(a)**2 + 1)
```

Mathematica [A] time = 0.0182058, size = 20, normalized size = 0.65

$$\frac{a^x(\log(a)\cos(x)+\sin(x))}{\log^2(a)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^x*Cos[x],x]
```

```
[Out] (a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)
```

Maple [B] time = 0.03, size = 71, normalized size = 2.3

$$1\left(\frac{\ln(a)\,e^{x\ln(a)}}{1+(\ln(a))^2}+2\,\frac{e^{x\ln(a)}\tan(x/2)}{1+(\ln(a))^2}-\frac{\ln(a)\,e^{x\ln(a)}}{1+(\ln(a))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^x*cos(x),x)
```

```
[Out] (1/(1+ln(a)^2)*ln(a)*exp(x*ln(a))+2/(1+ln(a)^2)*exp(x*ln(a))*tan(
1/2*x)-1/(1+ln(a)^2)*ln(a)*exp(x*ln(a))*tan(1/2*x)^2)/(tan(1/2*x)
^2+1)
```

Maxima [A] time = 1.46046, size = 32, normalized size = 1.03

$$\frac{a^x\cos(x)\log(a)+a^x\sin(x)}{\log(a)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*cos(x),x, algorithm="maxima")
```

```
[Out] (a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)
```

Fricas [A] time = 0.216685, size = 27, normalized size = 0.87

$$\frac{(\cos(x)\log(a)+\sin(x))a^x}{\log(a)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)
```

Sympy [A] time = 1.89758, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for } a = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for } a = e^{i} \\ \frac{a^x\log(a)\cos(x)}{\log(a)^2+1} + \frac{a^x\sin(x)}{\log(a)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(
-I*x)*cos(x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(
I*x)*cos(x)/2 - I*exp(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)
*cos(x)/(log(a)**2 + 1) + a**x*sin(x)/(log(a)**2 + 1), True))
```

GIAC/XCAS [A] time = 0.23007, size = 455, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x*cos(x),x, algorithm="giac")
```

```
[Out] (2*cos(1/2*pi*x*sign(a) - 1/2*pi*x + x)*ln(abs(a))/((pi - pi*sign
(a) - 2)^2 + 4*ln(abs(a))^2) - (pi - pi*sign(a) - 2)*sin(1/2*pi*x
*sign(a) - 1/2*pi*x + x)/((pi - pi*sign(a) - 2)^2 + 4*ln(abs(a))^
2))*e^(x*ln(abs(a))) + (2*cos(1/2*pi*x*sign(a) - 1/2*pi*x - x)*ln
(abs(a))/((pi - pi*sign(a) + 2)^2 + 4*ln(abs(a))^2) - (pi - pi*si
gn(a) + 2)*sin(1/2*pi*x*sign(a) - 1/2*pi*x - x)/((pi - pi*sign(a)
 + 2)^2 + 4*ln(abs(a))^2))*e^(x*ln(abs(a))) - 1/2*I*(-2*I*e^(1/2*
I*pi*x*sign(a) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sign(a) + 4*
ln(abs(a)) + 4*I) + 2*I*e^(-1/2*I*pi*x*sign(a) + 1/2*I*pi*x - I*x
)/(2*I*pi - 2*I*pi*sign(a) + 4*ln(abs(a)) - 4*I))*e^(x*ln(abs(a))
) - 1/2*I*(-2*I*e^(1/2*I*pi*x*sign(a) - 1/2*I*pi*x - I*x)/(-2*I*p
i + 2*I*pi*sign(a) + 4*ln(abs(a)) - 4*I) + 2*I*e^(-1/2*I*pi*x*sig
n(a) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sign(a) + 4*ln(abs(a))
+ 4*I))*e^(x*ln(abs(a)))
```

3.80 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.00780151, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi in Sympy [A] time = 0.491349, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}+\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(ln(x)),x)

[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Mathematica [A] time = 0.00319951, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(ln(x)),x)

[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 1.45498, size = 14, normalized size = 0.82

$$\frac{1}{2}\,x(\cos(\log(x))+\sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="maxima")

[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A] time = 0.233466, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos(\log(x))+\frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="fricas")

[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.549419, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2}+\frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(ln(x)),x)

[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

GIAC/XCAS [A] time = 0.210739, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos\left(\ln\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="giac")

[Out] 1/2*x*cos(ln(x)) + 1/2*x*sin(ln(x))
```

3.81 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rubi [A] time = 0.0333726, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Cos[x]]*Sec[x]^2,x]

[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rubi in Sympy [A] time = 1.67949, size = 15, normalized size = 1.25

$$-x + \frac{\log (\cos (x)) \sin (x)}{\cos (x)} + \tan (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(cos(x))*sec(x)**2,x)

[Out] -x + log(cos(x))*sin(x)/cos(x) + tan(x)
```

Mathematica [A] time = 0.0108513, size = 12, normalized size = 1.

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Cos[x]]*Sec[x]^2,x]

[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [C] time = 0.062, size = 61, normalized size = 5.1

$$\frac{-2\,i\mathrm{e}^{2\,ix}\ln\left(2\,\cos\left(x\right)\right)}{1+\mathrm{e}^{2\,ix}}+\frac{2\,i}{1+\mathrm{e}^{2\,ix}}+i\ln\left(1+\mathrm{e}^{2\,ix}\right)-\frac{2\,i\ln\left(2\right)}{1+\mathrm{e}^{2\,ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(cos(x))*sec(x)^2,x)
```

```
[Out] -2*I/(1+exp(2*I*x))*exp(2*I*x)*ln(2*cos(x))+2*I/(1+exp(2*I*x))+I*
ln(1+exp(2*I*x))-2*I*ln(2)/(1+exp(2*I*x))
```

Maxima [A] time = 1.51629, size = 127, normalized size = 10.58

$$-\frac{2\,\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)\left(\cos\left(x\right)+1\right)}-\frac{2\,\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)\left(\cos\left(x\right)+1\right)}-2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 +
1))*sin(x)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x
)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/
(cos(x) + 1))
```

Fricas [A] time = 0.223666, size = 30, normalized size = 2.5

$$-\frac{x\cos\left(x\right)-\log\left(\cos\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(cos(x))*sec(x)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.212001, size = 16, normalized size = 1.33

$$\ln(\cos(x))\tan(x) - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] ln(cos(x))*tan(x) - x + tan(x)
```

3.82 $\int x\tan^2(x)\,dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rubi [A] time = 0.0253785, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[x*Tan[x]^2,x]

[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x\tan(x)+\log(\cos(x))-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*tan(x)**2,x)

[Out] x*tan(x) + log(cos(x)) - Integral(x, x)
```

Mathematica [A] time = 0.00805365, size = 15, normalized size = 1.

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Maple [A] time = 0., size = 20, normalized size = 1.3

$$x\tan(x) - \frac{x^2}{2} - \frac{\ln\left(1 + (\tan(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*tan(x)^2,x)
```

```
[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

Maxima [A] time = 1.55089, size = 144, normalized size = 9.6

$$-\frac{x^2\cos(2x)^2 + x^2\sin(2x)^2 + 2x^2\cos(2x) + x^2 - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(2x)^2 + \sin(2x)^2 + 2\ \mathrm{co}\right.}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (c
os(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x
)^2 + 2*cos(2*x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 +
2*cos(2*x) + 1)
```

Fricas [A] time = 0.221708, size = 28, normalized size = 1.87

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.194093, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x\tan(x) - \frac{\log(\tan^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)**2,x)
```

```
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

GIAC/XCAS [A] time = 0.215132, size = 31, normalized size = 2.07

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\ln\left(\frac{4}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*ln(4/(tan(x)^2 + 1))
```

3.83 $\int \frac{\sin^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0399035, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rubi in Sympy [A] time = 2.84138, size = 15, normalized size = 0.68

$$-\operatorname{atanh}\left(\sqrt{-x^2+1}\right)-\frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)/x**2,x)

[Out] -atanh(sqrt(-x**2 + 1)) - asin(x)/x
```

Mathematica [A] time = 0.00766711, size = 26, normalized size = 1.18

$$-\log\left(\sqrt{1-x^2}+1\right)+\log(x)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) + Log[x] - Log[1 + Sqrt[1 - x^2]]
```

Maple [A] time = 0.003, size = 21, normalized size = 1.

$$-\frac{\arcsin(x)}{x} - Artanh\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)/x^2,x)
```

```
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.53027, size = 45, normalized size = 2.05

$$-\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

```
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.274934, size = 53, normalized size = 2.41

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right) - x\log\left(\sqrt{-x^2+1}-1\right) + 2\arcsin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*a
rcsin(x))/x
```

Sympy [A] time = 2.36536, size = 22, normalized size = 1.

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)/x**2,x)
```

```
[Out] Piecewise((-acosh(1/x), Abs(x**(-2)) > 1), (I*asin(1/x), True)) -
 asin(x)/x
```

GIAC/XCAS [A] time = 0.211658, size = 51, normalized size = 2.32

$$-\frac{\arcsin(x)}{x} - \frac{1}{2}\ln\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\ln\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/x^2,x, algorithm="giac")
```

```
[Out] -arcsin(x)/x - 1/2*ln(sqrt(-x^2 + 1) + 1) + 1/2*ln(-sqrt(-x^2 + 1
) + 1)
```

3.84 $\int \sin^{-1}(x)^2\,dx$

Optimal. Leaf size=25

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi [A] time = 0.0720576, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x]^2,x]

[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi in Sympy [A] time = 2.95134, size = 22, normalized size = 0.88

$$x\,\mathrm{asin}^2(x)-2x+2\sqrt{-x^2+1}\,\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)**2,x)

[Out] x*asin(x)**2 - 2*x + 2*sqrt(-x**2 + 1)*asin(x)
```

Mathematica [A] time = 0.0101838, size = 25, normalized size = 1.

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x]^2,x]

[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A] time = 0., size = 24, normalized size = 1.

$$-2\,x + x\,(\arcsin(x))^2 + 2\,\arcsin(x)\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)^2,x)
```

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.56445, size = 31, normalized size = 1.24

$$x\arcsin(x)^2 + 2\,\sqrt{-x^2+1}\arcsin(x) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Fricas [A] time = 0.229462, size = 31, normalized size = 1.24

$$x\arcsin(x)^2 + 2\,\sqrt{-x^2+1}\arcsin(x) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] time = 0.234984, size = 22, normalized size = 0.88

$$x\,\mathrm{asin}^2(x) - 2x + 2\sqrt{-x^2+1}\,\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(-x**2 + 1)*asin(x)
```

GIAC/XCAS [A] time = 0.211139, size = 31, normalized size = 1.24

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.85 $\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2} \log \left(x^2+1\right)-\frac{1}{2} \tan^{-1}(x)^2+x \tan^{-1}(x)$$

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0921615, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{1}{2} \log \left(x^2+1\right)-\frac{1}{2} \tan^{-1}(x)^2+x \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*ArcTan[x])/(1 + x^2),x]

[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 5.43782, size = 19, normalized size = 0.83

$$x \operatorname{atan}(x)-\frac{\log \left(x^2+1\right)}{2}-\frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*atan(x)/(x**2+1),x)

[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

Mathematica [A] time = 0.00591425, size = 23, normalized size = 1.

$$-\frac{1}{2} \log \left(x^2+1\right)-\frac{1}{2} \tan^{-1}(x)^2+x \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Maple [A] time = 0.003, size = 20, normalized size = 0.9

$$x\arctan(x) - \frac{(\arctan(x))^2}{2} - \frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arctan(x)/(x^2+1),x)
```

```
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Maxima [A] time = 1.54563, size = 32, normalized size = 1.39

$$(x - \arctan(x))\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.2261, size = 26, normalized size = 1.13

$$x\arctan(x) - \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.589143, size = 19, normalized size = 0.83

$$x\operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*atan(x)/(x**2+1),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

GIAC/XCAS [A] time = 0.218514, size = 28, normalized size = 1.22

$$x \arctan\left(x\right) - \frac{1}{2} \arctan\left(x\right)^{2} - \frac{1}{2} \ln\left(-i x^{2} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*ln(-I*x^2 - I)
```

3.86 $\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx$

Optimal. Leaf size=38

$$(x+1)\left(\sqrt{\frac{1}{x+1}}\sqrt{\frac{x}{x+1}}+\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)\right)$$

```
[Out] (1 + x)*(Sqrt[(1 + x)^(-1)]*Sqrt[x/(1 + x)] + ArcCos[Sqrt[x/(1 +
x)]])
```

Rubi [A] time = 0.0642603, antiderivative size = 57, normalized size of antiderivative = 1.5, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\sqrt{\frac{x}{(x+1)^2}}(x+1)+x\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] Sqrt[x/(1 + x)^2]*(1 + x) + x*ArcCos[Sqrt[x/(1 + x)]] - (Sqrt[x/(
1 + x)^2]*(1 + x)*ArcTan[Sqrt[x]])/Sqrt[x]
```

Rubi in Sympy [A] time = 3.93112, size = 53, normalized size = 1.39

$$\sqrt{x}\sqrt{x+1}\sqrt{\frac{1}{x+1}}+x\operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right)-\sqrt{x+1}\sqrt{\frac{1}{x+1}}\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acos((x/(1+x))**(1/2)),x)
```

```
[Out] sqrt(x)*sqrt(x + 1)*sqrt(1/(x + 1)) + x*acos(sqrt(x/(x + 1))) - s
qrt(x + 1)*sqrt(1/(x + 1))*atan(sqrt(x))
```

Mathematica [A] time = 0.0889415, size = 49, normalized size = 1.29

$$x\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)+\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(\sqrt{x}-\tan^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] -
 ArcTan[Sqrt[x]]))/Sqrt[x]
```

Maple [A] time = 0.019, size = 44, normalized size = 1.2

$$x \arccos\left(\sqrt{\frac{x}{1+x}}\right) + 1\sqrt{x}\sqrt{(1+x)^{-1}}\left(\sqrt{x} - \arctan\left(\sqrt{x}\right)\right)\frac{1}{\sqrt{\frac{x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccos((x/(1+x))^(1/2)),x)
```

```
[Out] x*arccos((x/(1+x))^(1/2))+1/(x/(1+x))^(1/2)*x^(1/2)*(1/(1+x))^(1/
2)*(x^(1/2)-arctan(x^(1/2)))
```

Maxima [A] time = 1.53792, size = 105, normalized size = 2.76

$$-\frac{\arccos\left(\sqrt{\frac{x}{x+1}}\right)}{\frac{x}{x+1}-1} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}+1\right)} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(sqrt(x/(x + 1))),x, algorithm="maxima")
```

```
[Out] -arccos(sqrt(x/(x + 1)))/(x/(x + 1) - 1) - 1/2*sqrt(-x/(x + 1) +
1)/(sqrt(x/(x + 1)) + 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x/(x +
1)) - 1)
```

Fricas [A] time = 0.240061, size = 43, normalized size = 1.13

$$\left(\sqrt{x+1}\arccos\left(\sqrt{\frac{x}{x+1}}\right) + \sqrt{\frac{x}{x+1}}\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(sqrt(x/(x + 1))),x, algorithm="fricas")
```

```
[Out] (sqrt(x + 1)*arccos(sqrt(x/(x + 1))) + sqrt(x/(x + 1)))*sqrt(x +
1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acos((x/(1+x))**(1/2)),x)
```

```
[Out] Integral(acos(sqrt(x/(x + 1))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(sqrt(x/(x + 1))),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.87 $\int (2x + 3x^2)^3 \, dx$

Optimal. Leaf size=25

$$\frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Rubi [A] time = 0.0243206, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + 3*x^2)^3,x]
```

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{8x^3}{105} + \frac{(6x + 2)(3x^2 + 2x)^3}{42} - \frac{(6x + 2)(3x^2 + 2x)^2}{105} + \frac{16 \int x \, dx}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2*x)**3,x)
```

```
[Out] 8*x**3/105 + (6*x + 2)*(3*x**2 + 2*x)**3/42 - (6*x + 2)*(3*x**2 +
 2*x)**2/105 + 16*Integral(x, x)/105
```

Mathematica [A] time = 0.00247379, size = 25, normalized size = 1.

$$\frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + 3*x^2)^3,x]
```

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Maple [A] time = 0.002, size = 22, normalized size = 0.9

$$2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x)^3,x)
```

```
[Out] 2*x^4+36/5*x^5+9*x^6+27/7*x^7
```

Maxima [A] time = 1.33742, size = 28, normalized size = 1.12

$$\frac{27}{7}x^7 + 9x^6 + \frac{36}{5}x^5 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2*x)^3,x, algorithm="maxima")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Fricas [A] time = 0.171303, size = 1, normalized size = 0.04

$$\frac{27}{7}x^7 + 9x^6 + \frac{36}{5}x^5 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2*x)^3,x, algorithm="fricas")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Sympy [A] time = 0.035019, size = 22, normalized size = 0.88

$$\frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2*x)**3,x)
```

```
[Out] 27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4
```

<u>GIAC/XCAS</u> [A] time = 0.220501, size = 28, normalized size = 1.12

$$\frac{27}{7}\,x^{7} + 9\,x^{6} + \frac{36}{5}\,x^{5} + 2\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x)^3,x, algorithm="giac")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

3.88 $\int(-1+x)\left(-1+2x+3x^2\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Rubi [A] time = 0.0356807, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]

[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}-x+5\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)*(3*x**2+2*x-1)**2,x)

[Out] 3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 - x + 5*Integral(x, x)
```

Mathematica [A] time = 0.00198229, size = 39, normalized size = 1.

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Maple [A] time = 0.002, size = 30, normalized size = 0.8

$$-x+\frac{5\,x^{2}}{2}-\frac{2\,x^{3}}{3}-\frac{7\,x^{4}}{2}+\frac{3\,x^{5}}{5}+\frac{3\,x^{6}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)*(3*x^2+2*x-1)^2,x)
```

```
[Out] -x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6
```

Maxima [A] time = 1.35283, size = 39, normalized size = 1.

$$\frac{3}{2}\,x^{6}+\frac{3}{5}\,x^{5}-\frac{7}{2}\,x^{4}-\frac{2}{3}\,x^{3}+\frac{5}{2}\,x^{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 1)^2*(x - 1),x, algorithm="maxima")
```

```
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Fricas [A] time = 0.177777, size = 1, normalized size = 0.03

$$\frac{3}{2}x^{6}+\frac{3}{5}x^{5}-\frac{7}{2}x^{4}-\frac{2}{3}x^{3}+\frac{5}{2}x^{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 1)^2*(x - 1),x, algorithm="fricas")
```

```
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Sympy [A] time = 0.045511, size = 34, normalized size = 0.87

$$\frac{3x^{6}}{2}+\frac{3x^{5}}{5}-\frac{7x^{4}}{2}-\frac{2x^{3}}{3}+\frac{5x^{2}}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)*(3*x**2+2*x-1)**2,x)

[Out] 3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x
```

GIAC/XCAS [A] time = 0.202635, size = 39, normalized size = 1.

$$\frac{3}{2}x^6+\frac{3}{5}x^5-\frac{7}{2}x^4-\frac{2}{3}x^3+\frac{5}{2}x^2-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 1)^2*(x - 1),x, algorithm="giac")

[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

3.89 $\int x^{-1+k} \left(a + bx^k\right)^n \, dx$

Optimal. Leaf size=23

$$\frac{\left(a + bx^k\right)^{n+1}}{bk(n+1)}$$

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Rubi [A] time = 0.0247548, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\left(a + bx^k\right)^{n+1}}{bk(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Rubi in Sympy [A] time = 1.48277, size = 15, normalized size = 0.65

$$\frac{\left(a + bx^k\right)^{n+1}}{bk\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

```
[Out] (a + b*x**k)**(n + 1)/(b*k*(n + 1))
```

Mathematica [A] time = 0.0356439, size = 22, normalized size = 0.96

$$\frac{\left(a + bx^k\right)^{n+1}}{bkn + bk}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k + b*k*n)
```

Maple [A] time = 0.073, size = 29, normalized size = 1.3

$$\frac{\left(a+bx^{k}\right)\left(a+bx^{k}\right)^{n}}{b\left(1+n\right)k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+k)*(a+b*x^k)^n,x)
```

```
[Out] (a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^k + a)^n*x^(k - 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220645, size = 36, normalized size = 1.57

$$\frac{\left(bx^{k}+a\right)\left(bx^{k}+a\right)^{n}}{bkn+bk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^k + a)^n*x^(k - 1),x, algorithm="fricas")
```

```
[Out] (b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.199383, size = 31, normalized size = 1.35

$$\frac{\left(bx^k + a\right)^{n+1}}{bk(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^k + a)^n*x^(k - 1),x, algorithm="giac")
```

```
[Out] (b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

3.90 $\int \frac{x^3}{1+2x}\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{1}{16}\log(2x+1)$$

```
[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

Rubi [A] time = 0.0309251, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{1}{16}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + 2*x),x]

[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{6}-\frac{\log(2x+1)}{16}+\int\frac{1}{8}\,dx-\frac{\int x\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(1+2*x),x)

[Out] x**3/6 - log(2*x + 1)/16 + Integral(1/8, x) - Integral(x, x)/4
```

Mathematica [A] time = 0.00927727, size = 27, normalized size = 0.9

$$\frac{1}{96}\left(16x^3-12x^2+12x-6\log(2x+1)+11\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + 2*x),x]
```

```
[Out] (11 + 12*x - 12*x^2 + 16*x^3 - 6*Log[1 + 2*x])/96
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\frac{x}{8}-\frac{x^{2}}{8}+\frac{x^{3}}{6}-\frac{\ln\left(1+2\,x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(1+2*x),x)
```

```
[Out] 1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)
```

Maxima [A] time = 1.37225, size = 30, normalized size = 1.

$$\frac{1}{6}\,x^{3}-\frac{1}{8}\,x^{2}+\frac{1}{8}\,x-\frac{1}{16}\,\log\left(2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(2*x + 1),x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

Fricas [A] time = 0.195562, size = 30, normalized size = 1.

$$\frac{1}{6}\,x^{3}-\frac{1}{8}\,x^{2}+\frac{1}{8}\,x-\frac{1}{16}\,\log\left(2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(2*x + 1),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

Sympy [A] time = 0.057137, size = 20, normalized size = 0.67

$$\frac{x^{3}}{6}-\frac{x^{2}}{8}+\frac{x}{8}-\frac{\log\left(2x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(1+2*x),x)

[Out] x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16
```

GIAC/XCAS [A] time = 0.200276, size = 31, normalized size = 1.03

$$\frac{1}{6}\,x^{3}-\frac{1}{8}\,x^{2}+\frac{1}{8}\,x-\frac{1}{16}\,\ln\left(\left|2\,x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(2*x + 1),x, algorithm="giac")

[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*ln(abs(2*x + 1))
```

3.91 $\int \frac{x^6}{2+3x^2}\,dx$

Optimal. Leaf size=41

$$\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x]
)/27
```

Rubi [A] time = 0.0303443, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(2 + 3*x^2),x]
```

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x]
)/27
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^5}{15}-\frac{2x^3}{27}-\frac{4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}+\int\frac{4}{27}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(3*x**2+2),x)
```

```
[Out] x**5/15 - 2*x**3/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81 + Integral(4
/27, x)
```

Mathematica [A] time = 0.0212251, size = 35, normalized size = 0.85

$$\frac{1}{405}\left(27x^5-30x^3+60x-20\sqrt{6}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(2 + 3*x^2),x]
```

```
[Out] (60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405
```

Maple [A] time = 0.004, size = 27, normalized size = 0.7

$$\frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4\sqrt{6}}{81}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(3*x^2+2),x)
```

```
[Out] 4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.49517, size = 35, normalized size = 0.85

$$\frac{1}{15}x^5-\frac{2}{27}x^3-\frac{4}{81}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

Fricas [A] time = 0.280028, size = 53, normalized size = 1.29

$$\frac{1}{405}\sqrt{3}\left(\sqrt{3}\left(9x^5-10x^3+20x\right)-20\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/405*sqrt(3)*(sqrt(3)*(9*x^5 - 10*x^3 + 20*x) - 20*sqrt(2)*arcta
n(1/2*sqrt(3)*sqrt(2)*x))
```

Sympy [A] time = 0.091102, size = 34, normalized size = 0.83

$$\frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(3*x**2+2),x)
```

```
[Out] x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81
```

GIAC/XCAS [A] time = 0.200348, size = 35, normalized size = 0.85

$$\frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

3.92 $\int \frac{1}{2-7x+3x^2} dx$

Optimal. Leaf size=21

$$\frac{1}{5}\log(2-x) - \frac{1}{5}\log(1-3x)$$

```
[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Rubi [A] time = 0.0143004, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{5}\log(2-x) - \frac{1}{5}\log(1-3x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 7*x + 3*x^2)^(-1),x]

[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Rubi in Sympy [A] time = 0.819426, size = 14, normalized size = 0.67

$$-\frac{\log(-3x+1)}{5} + \frac{\log(-x+2)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-7*x+2),x)

[Out] -log(-3*x + 1)/5 + log(-x + 2)/5
```

Mathematica [A] time = 0.00455752, size = 21, normalized size = 1.

$$\frac{1}{5}\log(2-x) - \frac{1}{5}\log(1-3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 7*x + 3*x^2)^(-1),x]

[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Maple [A] time = 0.007, size = 16, normalized size = 0.8

$$-\frac{\ln\left(3\,x-1\right)}{5}+\frac{\ln\left(-2+x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-7*x+2),x)

[Out] -1/5*ln(3*x-1)+1/5*ln(-2+x)
```

Maxima [A] time = 1.31405, size = 20, normalized size = 0.95

$$-\frac{1}{5}\,\log\left(3\,x-1\right)+\frac{1}{5}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 - 7*x + 2),x, algorithm="maxima")

[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

Fricas [A] time = 0.201499, size = 20, normalized size = 0.95

$$-\frac{1}{5}\,\log\left(3\,x-1\right)+\frac{1}{5}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 - 7*x + 2),x, algorithm="fricas")

[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

Sympy [A] time = 0.090965, size = 14, normalized size = 0.67

$$\frac{\log\left(x-2\right)}{5}-\frac{\log\left(x-\frac{1}{3}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2-7*x+2),x)

[Out] log(x - 2)/5 - log(x - 1/3)/5
```

GIAC/XCAS [A] time = 0.200035, size = 23, normalized size = 1.1

$$-\frac{1}{5}\ln\left(|3x-1|\right)+\frac{1}{5}\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 - 7*x + 2),x, algorithm="giac")

[Out] -1/5*ln(abs(3*x - 1)) + 1/5*ln(abs(x - 2))
```

3.93 $\int \frac{-1+3x}{1-x+x^2}\,dx$

Optimal. Leaf size=33

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

Rubi [A] time = 0.0413895, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 3*x)/(1 - x + x^2),x]

[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

Rubi in Sympy [A] time = 2.69829, size = 32, normalized size = 0.97

$$\frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+3*x)/(x**2-x+1),x)

[Out] 3*log(x**2 - x + 1)/2 + sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.0149006, size = 32, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 3*x)/(1 - x + x^2),x]

[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

Maple [A] time = 0.005, size = 29, normalized size = 0.9

$$\frac{3\ln\left(x^{2}-x+1\right)}{2}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x-1)/(x^2-x+1),x)

[Out] 3/2*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.52533, size = 38, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{3}{2}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 1)/(x^2 - x + 1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

Fricas [A] time = 0.198617, size = 45, normalized size = 1.36

$$\frac{1}{6}\sqrt{3}\left(3\sqrt{3}\log\left(x^{2}-x+1\right)+2\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 1)/(x^2 - x + 1),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*(3*sqrt(3)*log(x^2 - x + 1) + 2*arctan(1/3*sqrt(3)*(2
*x - 1)))
```

Sympy [A] time = 0.110497, size = 36, normalized size = 1.09

$$\frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+3*x)/(x**2-x+1),x)

[Out] 3*log(x**2 - x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.198693, size = 38, normalized size = 1.15

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{3}{2}\,\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 1)/(x^2 - x + 1),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*ln(x^2 - x + 1)
```

3.94 $\int \frac{x^2}{5+2x+x^2} \, dx$

Optimal. Leaf size=25

$$-\log\left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Rubi [A] time = 0.0443573, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\log\left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(5 + 2*x + x^2),x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Rubi in Sympy [A] time = 3.76183, size = 22, normalized size = 0.88

$$x-\log\left(x^2+2x+5\right)-\frac{3\operatorname{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+2*x+5),x)
```

```
[Out] x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

Mathematica [A] time = 0.00591617, size = 25, normalized size = 1.

$$-\log\left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(5 + 2*x + x^2),x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Maple [A] time = 0.004, size = 22, normalized size = 0.9

$$x - \frac{3}{2}\arctan\left(\frac{1}{2} + \frac{x}{2}\right) - \ln\left(x^2 + 2x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+2*x+5),x)
```

```
[Out] x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

Maxima [A] time = 1.50057, size = 28, normalized size = 1.12

$$x - \frac{3}{2}\arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log\left(x^2 + 2x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 2*x + 5),x, algorithm="maxima")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Fricas [A] time = 0.199437, size = 28, normalized size = 1.12

$$x - \frac{3}{2}\arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log\left(x^2 + 2x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 2*x + 5),x, algorithm="fricas")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Sympy [A] time = 0.118235, size = 22, normalized size = 0.88

$$x - \log\left(x^2 + 2x + 5\right) - \frac{3\operatorname{atan}\left(\frac{x}{2} + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2+2*x+5),x)
```

```
[Out] x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

GIAC/XCAS [A] time = 0.20023, size = 28, normalized size = 1.12

$$x - \frac{3}{2} \arctan\left(\frac{1}{2} x + \frac{1}{2}\right) - \ln\left(x^2 + 2 x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 2*x + 5),x, algorithm="giac")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - ln(x^2 + 2*x + 5)
```

3.95 $\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx$

Optimal. Leaf size=47

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

```
[Out] -x^2/2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x
+ 2*x^2]/4
```

Rubi [A] time = 0.0918473, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -x^2/2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x
+ 2*x^2]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x^3+\frac{\log\left(2x^2-x+1\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\sqrt{7}\left(\frac{4x}{7}-\frac{1}{7}\right)\right)}{14}-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
```

```
[Out] x**3 + log(2*x**2 - x + 1)/4 + sqrt(7)*atan(sqrt(7)*(4*x/7 - 1/7)
)/14 - Integral(x, x)
```

Mathematica [A] time = 0.0265621, size = 47, normalized size = 1.

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{4x-1}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -x^2/2 + x^3 + ArcTan[(-1 + 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x
 + 2*x^2]/4
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$x^{3}-\frac{x^{2}}{2}+\frac{\ln\left(2\,x^{2}-x+1\right)}{4}+\frac{\sqrt{7}}{14}\arctan\left(\frac{(-1+4\,x)\,\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x)
```

```
[Out] x^3-1/2*x^2+1/4*ln(2*x^2-x+1)+1/14*7^(1/2)*arctan(1/7*(-1+4*x)*7^
(1/2))
```

Maxima [A] time = 1.49189, size = 51, normalized size = 1.09

$$x^{3}-\frac{1}{2}\,x^{2}+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4 - 5*x^3 + 4*x^2)/(2*x^2 - x + 1),x, algorithm="maxima")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*
log(2*x^2 - x + 1)
```

Fricas [A] time = 0.201234, size = 68, normalized size = 1.45

$$\frac{1}{28}\,\sqrt{7}\left(2\,\sqrt{7}\left(2\,x^{3}-x^{2}\right)+\sqrt{7}\log\left(2\,x^{2}-x+1\right)+2\,\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4 - 5*x^3 + 4*x^2)/(2*x^2 - x + 1),x, algorithm="fricas")
```

```
[Out] 1/28*sqrt(7)*(2*sqrt(7)*(2*x^3 - x^2) + sqrt(7)*log(2*x^2 - x + 1
) + 2*arctan(1/7*sqrt(7)*(4*x - 1)))
```

Sympy [A] time = 0.114343, size = 46, normalized size = 0.98

$$x^3 - \frac{x^2}{2} + \frac{\log\left(x^2 - \frac{x}{2} + \frac{1}{2}\right)}{4} + \frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\sqrt{7}x}{7} - \frac{\sqrt{7}}{7}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
```

```
[Out] x**3 - x**2/2 + log(x**2 - x/2 + 1/2)/4 + sqrt(7)*atan(4*sqrt(7)*
x/7 - sqrt(7)/7)/14
```

GIAC/XCAS [A] time = 0.201327, size = 51, normalized size = 1.09

$$x^3 - \frac{1}{2}x^2 + \frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x - 1)\right) + \frac{1}{4}\ln\left(2x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4 - 5*x^3 + 4*x^2)/(2*x^2 - x + 1),x, algorithm="giac")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*
ln(2*x^2 - x + 1)
```

3.96 $\int \frac{-1+x+x^2}{-6x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Rubi [A] time = 0.0468113, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]

[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Rubi in Sympy [A] time = 4.90234, size = 17, normalized size = 0.68

$$\frac{\log(x)}{6}+\frac{\log(-x+2)}{2}+\frac{\log(x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x-1)/(x**3+x**2-6*x),x)

[Out] log(x)/6 + log(-x + 2)/2 + log(x + 3)/3
```

Mathematica [A] time = 0.00845203, size = 25, normalized size = 1.

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Maple [A] time = 0.01, size = 18, normalized size = 0.7

$$\frac{\ln(x)}{6} + \frac{\ln(-2+x)}{2} + \frac{\ln(3+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x-1)/(x^3+x^2-6*x),x)
```

```
[Out] 1/6*ln(x)+1/2*ln(-2+x)+1/3*ln(3+x)
```

Maxima [A] time = 1.34207, size = 23, normalized size = 0.92

$$\frac{1}{3}\log(x+3) + \frac{1}{2}\log(x-2) + \frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(x^3 + x^2 - 6*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

Fricas [A] time = 0.201388, size = 23, normalized size = 0.92

$$\frac{1}{3}\log(x+3) + \frac{1}{2}\log(x-2) + \frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(x^3 + x^2 - 6*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

Sympy [A] time = 0.140896, size = 17, normalized size = 0.68

$$\frac{\log(x)}{6} + \frac{\log(x-2)}{2} + \frac{\log(x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x-1)/(x**3+x**2-6*x),x)
```

```
[Out] log(x)/6 + log(x - 2)/2 + log(x + 3)/3
```

GIAC/XCAS [A] time = 0.200219, size = 27, normalized size = 1.08

$$\frac{1}{3}\ln\left(|x+3|\right)+\frac{1}{2}\ln\left(|x-2|\right)+\frac{1}{6}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(x^3 + x^2 - 6*x),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(x + 3)) + 1/2*ln(abs(x - 2)) + 1/6*ln(abs(x))
```

3.97 $\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3} \, dx$

Optimal. Leaf size=33

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

```
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

Rubi [A] time = 0.0892791, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

```
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
```

```
[Out] Integral((11*a**2 - 7*a*x + 5*x**2)/(-6*a**3 + 11*a**2*x - 6*a*x*
*2 + x**3), x)
```

Mathematica [A] time = 0.0227617, size = 29, normalized size = 0.88

$$\frac{35}{2}\log(x-3a)-17\log(x-2a)+\frac{9}{2}\log(x-a)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

```
[Out] (35*Log[-3*a + x])/2 - 17*Log[-2*a + x] + (9*Log[-a + x])/2
```

Maple [A] time = 0.025, size = 26, normalized size = 0.8

$$-17\ \ln\left(-2\,a+x\right)+\frac{35\ \ln\left(x-3\,a\right)}{2}+\frac{9\ \ln\left(-a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x)

[Out] -17*ln(-2*a+x)+35/2*ln(x-3*a)+9/2*ln(-a+x)
```

Maxima [A] time = 1.36806, size = 34, normalized size = 1.03

$$\frac{9}{2}\ \log\left(-a+x\right)-17\ \log\left(-2\,a+x\right)+\frac{35}{2}\ \log\left(-3\,a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(11*a^2 - 7*a*x + 5*x^2)/(6*a^3 - 11*a^2*x + 6*a*x^2 - x^3),x, algorith

[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

Fricas [A] time = 0.202146, size = 34, normalized size = 1.03

$$\frac{9}{2}\ \log\left(-a+x\right)-17\ \log\left(-2\,a+x\right)+\frac{35}{2}\ \log\left(-3\,a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(11*a^2 - 7*a*x + 5*x^2)/(6*a^3 - 11*a^2*x + 6*a*x^2 - x^3),x, algorith

[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

Sympy [A] time = 0.712835, size = 26, normalized size = 0.79

$$\frac{35\log\left(-3a+x\right)}{2}-17\log\left(-2a+x\right)+\frac{9\log\left(-a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
```

```
[Out] 35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2
```

GIAC/XCAS [A] time = 0.200097, size = 38, normalized size = 1.15

$$\frac{9}{2}\ln\left(|-a+x|\right) - 17\ln\left(|-2\,a+x|\right) + \frac{35}{2}\ln\left(|-3\,a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(11*a^2 - 7*a*x + 5*x^2)/(6*a^3 - 11*a^2*x + 6*a*x^2 - x^3),x, algorith
```

```
[Out] 9/2*ln(abs(-a + x)) - 17*ln(abs(-2*a + x)) + 35/2*ln(abs(-3*a + x
))
```

3.98 $\int \frac{2-x+x^2}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=37

$$-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(x+2)$$

```
[Out] -Log[1 - x]/3 + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/
3
```

Rubi [A] time = 0.0686088, antiderivative size = 61, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{1}{6}\log\left(1-x^2\right)-\frac{1}{6}\log\left(4-x^2\right)-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(2-x)+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[2 - x]/2 + Log[1 + x]/2 - Log[2 + x]/2 + Log[
1 - x^2]/6 - Log[4 - x^2]/6
```

Rubi in Sympy [A] time = 9.14246, size = 42, normalized size = 1.14

$$-\frac{\log\left(-x+1\right)}{2}+\frac{\log\left(-x+2\right)}{2}+\frac{\log\left(x+1\right)}{2}-\frac{\log\left(x+2\right)}{2}+\frac{\log\left(-x^2+1\right)}{6}-\frac{\log\left(-x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] -log(-x + 1)/2 + log(-x + 2)/2 + log(x + 1)/2 - log(x + 2)/2 + lo
g(-x**2 + 1)/6 - log(-x**2 + 4)/6
```

Mathematica [A] time = 0.0104017, size = 37, normalized size = 1.

$$-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]

[Out] -Log[1 - x]/3 + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/
3
```

Maple [A] time = 0.013, size = 26, normalized size = 0.7

$$-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}-\frac{\ln(-1+x)}{3}+\frac{\ln(-2+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-x+2)/(x^4-5*x^2+4),x)

[Out] -2/3*ln(2+x)+2/3*ln(1+x)-1/3*ln(-1+x)+1/3*ln(-2+x)
```

Maxima [A] time = 1.36662, size = 34, normalized size = 0.92

$$-\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")

[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2
)
```

Fricas [A] time = 0.205317, size = 34, normalized size = 0.92

$$-\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")

[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2
)
```

Sympy [A] time = 0.252275, size = 29, normalized size = 0.78

$$\frac{\log\left(x-2\right)}{3}-\frac{\log\left(x-1\right)}{3}+\frac{2\log\left(x+1\right)}{3}-\frac{2\log\left(x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

GIAC/XCAS [A] time = 0.199863, size = 39, normalized size = 1.05

$$-\frac{2}{3}\ln\left(|x+2|\right)+\frac{2}{3}\ln\left(|x+1|\right)-\frac{1}{3}\ln\left(|x-1|\right)+\frac{1}{3}\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -2/3*ln(abs(x + 2)) + 2/3*ln(abs(x + 1)) - 1/3*ln(abs(x - 1)) + 1
/3*ln(abs(x - 2))
```

3.99 $\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0275989, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]

[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

Rubi in Sympy [A] time = 3.46065, size = 34, normalized size = 1.1

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{2}\right)}{2}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2-5)/(x**4-5*x**2+6),x)

[Out] -sqrt(2)*atanh(sqrt(2)*x/2)/2 - sqrt(3)*atanh(sqrt(3)*x/3)/3
```

Mathematica [B] time = 0.0280497, size = 69, normalized size = 2.23

$$\frac{1}{12}\left(3\sqrt{2}\log\left(\sqrt{2}-x\right)+2\sqrt{3}\log\left(\sqrt{3}-x\right)-3\sqrt{2}\log\left(x+\sqrt{2}\right)-2\sqrt{3}\log\left(x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

```
[Out] (3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt
[2]*Log[Sqrt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12
```

Maple [A] time = 0.011, size = 26, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}Artanh\left(\frac{x\sqrt{2}}{2}\right)-\frac{\sqrt{3}}{3}Artanh\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2-5)/(x^4-5*x^2+6),x)
```

```
[Out] -1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^
(1/2)
```

Maxima [A] time = 1.53518, size = 62, normalized size = 2.

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}\right)}{2x+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log(2*
(x - sqrt(2))/((2*sqrt(2)) + 2*x))
```

Fricas [A] time = 0.205935, size = 82, normalized size = 2.65

$$\frac{1}{12}\sqrt{3}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{3}\left(x^2+3\right)-6x}{x^2-3}\right)+\sqrt{3}\log\left(\frac{\sqrt{2}\left(x^2+2\right)-4x}{x^2-2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*(sqrt(2)*log((sqrt(3)*(x^2 + 3) - 6*x)/(x^2
- 3)) + sqrt(3)*log((sqrt(2)*(x^2 + 2) - 4*x)/(x^2 - 2)))
```

Sympy [A] time = 0.965049, size = 60, normalized size = 1.94

$$\frac{\sqrt{2}\log\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\log\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{3}\log\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\log\left(x+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2-5)/(x**4-5*x**2+6),x)
```

```
[Out] sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)
*log(x - sqrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6
```

GIAC/XCAS [A] time = 0.201396, size = 80, normalized size = 2.58

$$\frac{1}{6}\sqrt{3}\ln\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+\frac{1}{4}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}\right|}{\left|2x+2\sqrt{2}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*ln(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + 1/4*s
qrt(2)*ln(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)))
```

3.100 $\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

```
[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Rubi [A] time = 0.0666397, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Rubi in Sympy [A] time = 3.77353, size = 26, normalized size = 0.63

$$-\frac{\log(-x+1)}{6}+\frac{\log(-x+2)}{2}-\frac{\log(-x+3)}{2}+\frac{\log(-x+4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] -log(-x + 1)/6 + log(-x + 2)/2 - log(-x + 3)/2 + log(-x + 4)/6
```

Mathematica [A] time = 0.0118567, size = 41, normalized size = 1.

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Maple [A] time = 0.013, size = 26, normalized size = 0.6

$$-\frac{\ln\left(-3+x\right)}{2}+\frac{\ln\left(x-4\right)}{6}-\frac{\ln\left(-1+x\right)}{6}+\frac{\ln\left(-2+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-4)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] -1/2*ln(-3+x)+1/6*ln(x-4)-1/6*ln(-1+x)+1/2*ln(-2+x)
```

Maxima [A] time = 1.37337, size = 34, normalized size = 0.83

$$-\frac{1}{6}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(x-2\right)-\frac{1}{2}\,\log\left(x-3\right)+\frac{1}{6}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x, algorithm="maxima")
```

```
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4
)
```

Fricas [A] time = 0.207978, size = 34, normalized size = 0.83

$$-\frac{1}{6}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(x-2\right)-\frac{1}{2}\,\log\left(x-3\right)+\frac{1}{6}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x, algorithm="fricas")
```

```
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4
)
```

Sympy [A] time = 0.228868, size = 26, normalized size = 0.63

$$\frac{\log\left(x-4\right)}{6}-\frac{\log\left(x-3\right)}{2}+\frac{\log\left(x-2\right)}{2}-\frac{\log\left(x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] log(x - 4)/6 - log(x - 3)/2 + log(x - 2)/2 - log(x - 1)/6
```

GIAC/XCAS [A] time = 0.201241, size = 39, normalized size = 0.95

$$-\frac{1}{6}\ln\left(|x-1|\right)+\frac{1}{2}\ln\left(|x-2|\right)-\frac{1}{2}\ln\left(|x-3|\right)+\frac{1}{6}\ln\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x, algorithm="giac")
```

```
[Out] -1/6*ln(abs(x - 1)) + 1/2*ln(abs(x - 2)) - 1/2*ln(abs(x - 3)) + 1
/6*ln(abs(x - 4))
```

3.101 $\int \frac{1+x^2}{(-1+x)^3} \, dx$

Optimal. Leaf size=25

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

```
[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

Rubi [A] time = 0.0257369, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(-1 + x)^3,x]

[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

Rubi in Sympy [A] time = 1.88216, size = 15, normalized size = 0.6

$$\log(-x+1) + \frac{2}{-x+1} - \frac{1}{(-x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(-1+x)**3,x)

[Out] log(-x + 1) + 2/(-x + 1) - 1/(-x + 1)**2
```

Mathematica [A] time = 0.014837, size = 16, normalized size = 0.64

$$\frac{1-2x}{(x-1)^2} + \log(x-1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] (1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

Maple [A] time = 0.009, size = 20, normalized size = 0.8

$$-(-1+x)^{-2}+\ln(-1+x)-2\,(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(-1+x)^3,x)
```

```
[Out] -1/(-1+x)^2+ln(-1+x)-2/(-1+x)
```

Maxima [A] time = 1.35041, size = 30, normalized size = 1.2

$$-\frac{2\,x-1}{x^2-2\,x+1}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x - 1)^3,x, algorithm="maxima")
```

```
[Out] -(2*x - 1)/(x^2 - 2*x + 1) + log(x - 1)
```

Fricas [A] time = 0.201252, size = 39, normalized size = 1.56

$$\frac{\left(x^2-2\,x+1\right)\log(x-1)-2\,x+1}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x - 1)^3,x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.09555, size = 17, normalized size = 0.68

$$-\frac{2x-1}{x^2-2x+1}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(-1+x)**3,x)
```

```
[Out] -(2*x - 1)/(x**2 - 2*x + 1) + log(x - 1)
```

GIAC/XCAS [A] time = 0.200817, size = 24, normalized size = 0.96

$$-\frac{2\,x-1}{\left(x-1\right)^{2}}+\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x - 1)^3,x, algorithm="giac")
```

```
[Out] -(2*x - 1)/(x - 1)^2 + ln(abs(x - 1))
```

3.102 $\int \frac{x^5}{(3+x)^2} dx$

Optimal. Leaf size=36

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x
]
```

Rubi [A] time = 0.0361862, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(3 + x)^2,x]
```

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4} - 2x^3 - 108x + 405\log(x+3) + 27\int x\,dx + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 - 108*x + 405*log(x + 3) + 27*Integral(x, x) + 24
3/(x + 3)
```

Mathematica [A] time = 0.022626, size = 36, normalized size = 1.

$$\frac{1}{4}\left(x^4 - 8x^3 + 54x^2 - 432x + \frac{972}{x+3} - 2079\right) + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(3 + x)^2,x]
```

```
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[
3 + x]
```

Maple [A] time = 0.009, size = 33, normalized size = 0.9

$$-108\,x+\frac{27\,x^{2}}{2}-2\,x^{3}+\frac{x^{4}}{4}+243\,\left(3+x\right)^{-1}+405\,\ln\left(3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(3+x)^2,x)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Maxima [A] time = 1.34434, size = 43, normalized size = 1.19

$$\frac{1}{4}\,x^{4}-2\,x^{3}+\frac{27}{2}\,x^{2}-108\,x+\frac{243}{x+3}+405\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x + 3)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

Fricas [A] time = 0.196458, size = 53, normalized size = 1.47

$$\frac{x^{5}-5\,x^{4}+30\,x^{3}-270\,x^{2}+1620\,(x+3)\log\left(x+3\right)-1296\,x+972}{4\,(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1
296*x + 972)/(x + 3)
```

Sympy [A] time = 0.078265, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log(x+3) + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3
)
```

GIAC/XCAS [A] time = 0.199435, size = 61, normalized size = 1.69

$$-\frac{1}{4}(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\ln(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x + 3)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1)
+ 243/(x + 3) + 405*ln(abs(x + 3))
```

3.103 $\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx$

Optimal. Leaf size=41

$$\frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

```
[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*
Log[1 + x])/64
```

Rubi [A] time = 0.06588, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*
Log[1 + x])/64
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^3-2}{x^4-8x^3+18x^2-27}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
```

```
[Out] Integral((5*x**3 - 2)/(x**4 - 8*x**3 + 18*x**2 - 27), x)
```

Mathematica [A] time = 0.0318054, size = 37, normalized size = 0.9

$$-\frac{407}{16(x-3)}-\frac{133}{8(x-3)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (
7*Log[1 + x])/64
```

Maple [A] time = 0.011, size = 28, normalized size = 0.7

$$-\frac{133}{8\,(-3+x)^2}-\frac{407}{-48+16\,x}+\frac{313\,\ln(-3+x)}{64}+\frac{7\,\ln(1+x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x)
```

```
[Out] -133/8/(-3+x)^2-407/16/(-3+x)+313/64*ln(-3+x)+7/64*ln(1+x)
```

Maxima [A] time = 1.33897, size = 41, normalized size = 1.

$$-\frac{407\,x-955}{16\,(x^2-6\,x+9)}+\frac{7}{64}\,\log(x+1)+\frac{313}{64}\,\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3 - 2)/(x^4 - 8*x^3 + 18*x^2 - 27),x, algorithm="maxima")
```

```
[Out] -1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*lo
g(x - 3)
```

Fricas [A] time = 0.198308, size = 61, normalized size = 1.49

$$\frac{7\,\left(x^2-6\,x+9\right)\log(x+1)+313\,\left(x^2-6\,x+9\right)\log(x-3)-1628\,x+3820}{64\,(x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3 - 2)/(x^4 - 8*x^3 + 18*x^2 - 27),x, algorithm="fricas")
```

```
[Out] 1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x -
3) - 1628*x + 3820)/(x^2 - 6*x + 9)
```

Sympy [A] time = 0.150699, size = 31, normalized size = 0.76

$$-\frac{407x-955}{16x^2-96x+144}+\frac{313\log(x-3)}{64}+\frac{7\log(x+1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
```

```
[Out] -(407*x - 955)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log
(x + 1)/64
```

GIAC/XCAS [A] time = 0.1998, size = 36, normalized size = 0.88

$$-\frac{407\,x-955}{16\,(x-3)^2}+\frac{7}{64}\ln(|x+1|)+\frac{313}{64}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3 - 2)/(x^4 - 8*x^3 + 18*x^2 - 27),x, algorithm="giac")
```

```
[Out] -1/16*(407*x - 955)/(x - 3)^2 + 7/64*ln(abs(x + 1)) + 313/64*ln(a
bs(x - 3))
```

3.104 $\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} \, dx$

Optimal. Leaf size=27

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Rubi [A] time = 0.04577, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]

[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3-6x^2+3x-9}{(x+3)^2(x+4)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)

[Out] Integral((x**3 - 6*x**2 + 3*x - 9)/((x + 3)**2*(x + 4)**2), x)
```

Mathematica [A] time = 0.0250112, size = 27, normalized size = 1.

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Maple [A] time = 0.015, size = 28, normalized size = 1.

$$99\ (3+x)^{-1}+181\ (4+x)^{-1}+264\ \ln(3+x)-263\ \ln(4+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x)
```

```
[Out] 99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

Maxima [A] time = 1.33281, size = 39, normalized size = 1.44

$$\frac{280\,x+939}{x^2+7\,x+12}-263\ \log(x+4)+264\ \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 6*x^2 + 3*x - 9)/((x + 4)^2*(x + 3)^2),x, algorithm="maxima")
```

```
[Out] (280*x + 939)/(x^2 + 7*x + 12) - 263*log(x + 4) + 264*log(x + 3)
```

Fricas [A] time = 0.194637, size = 61, normalized size = 2.26

$$-\frac{263\left(x^2+7\,x+12\right)\log(x+4)-264\left(x^2+7\,x+12\right)\log(x+3)-280\,x-939}{x^2+7\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 6*x^2 + 3*x - 9)/((x + 4)^2*(x + 3)^2),x, algorithm="fricas")
```

```
[Out] -(263*(x^2 + 7*x + 12)*log(x + 4) - 264*(x^2 + 7*x + 12)*log(x +
3) - 280*x - 939)/(x^2 + 7*x + 12)
```

Sympy [A] time = 0.159509, size = 26, normalized size = 0.96

$$\frac{280x+939}{x^2+7x+12}+264\log(x+3)-263\log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)
```

```
[Out] (280*x + 939)/(x**2 + 7*x + 12) + 264*log(x + 3) - 263*log(x + 4)
```

GIAC/XCAS [A] time = 0.20252, size = 50, normalized size = 1.85

$$\frac{181}{x+4} - \frac{99}{\frac{1}{x+4} - 1} + \ln\left(|x+4|\right) + 264\ln\left(\left|-\frac{1}{x+4} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 6*x^2 + 3*x - 9)/((x + 4)^2*(x + 3)^2),x, algorithm="giac")
```

```
[Out] 181/(x + 4) - 99/(1/(x + 4) - 1) + ln(abs(x + 4)) + 264*ln(abs(-1
/(x + 4) + 1))
```

3.105 $\int \frac{2+x^2+x^3}{x(-1+x^2)^2} \, dx$

Optimal. Leaf size=39

$$\frac{x+3}{2(1-x^2)} - \frac{3}{4}\log(1-x) + 2\log(x) - \frac{5}{4}\log(x+1)$$

```
[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 +
x])/4
```

Rubi [A] time = 0.0671782, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x+3}{2(1-x^2)} - \frac{3}{4}\log(1-x) + 2\log(x) - \frac{5}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 +
x])/4
```

Rubi in Sympy [A] time = 6.45137, size = 31, normalized size = 0.79

$$\frac{x\left(1+\frac{3}{x}\right)}{2(-x^2+1)} - \log(x) + \frac{3\log(-x+1)}{4} + \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
```

```
[Out] x*(1 + 3/x)/(2*(-x**2 + 1)) - log(x) + 3*log(-x + 1)/4 + log(x +
1)/4
```

Mathematica [A] time = 0.0280024, size = 47, normalized size = 1.21

$$\frac{1}{4}\left(-\frac{4}{x^2-1} - 4\log\left(1-x^2\right) - \frac{2}{x-1} + \log(1-x) + 8\log(x) - \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x]
- 4*Log[1 - x^2])/4
```

Maple [A] time = 0.016, size = 32, normalized size = 0.8

$$\frac{1}{2x+2}-\frac{5\ln(1+x)}{4}+2\ln(x)-(-1+x)^{-1}-\frac{3\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+2)/x/(x^2-1)^2,x)
```

```
[Out] 1/2/(1+x)-5/4*ln(1+x)+2*ln(x)-1/(-1+x)-3/4*ln(-1+x)
```

Maxima [A] time = 1.3695, size = 39, normalized size = 1.

$$-\frac{x+3}{2(x^2-1)}-\frac{5}{4}\log(x+1)-\frac{3}{4}\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 2)/((x^2 - 1)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(
x)
```

Fricas [A] time = 0.202227, size = 61, normalized size = 1.56

$$-\frac{5\left(x^2-1\right)\log(x+1)+3\left(x^2-1\right)\log(x-1)-8\left(x^2-1\right)\log(x)+2x+6}{4(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 2)/((x^2 - 1)^2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 -
1)*log(x) + 2*x + 6)/(x^2 - 1)
```

Sympy [A] time = 0.172215, size = 31, normalized size = 0.79

$$-\frac{x+3}{2x^2-2}+2\log(x)-\frac{3\log(x-1)}{4}-\frac{5\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
```

```
[Out] -(x + 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/
4
```

GIAC/XCAS [A] time = 0.19948, size = 47, normalized size = 1.21

$$-\frac{x+3}{2(x+1)(x-1)}-\frac{5}{4}\ln(|x+1|)-\frac{3}{4}\ln(|x-1|)+2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 2)/((x^2 - 1)^2*x),x, algorithm="giac")
```

```
[Out] -1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*ln(abs(x + 1)) - 3/4*ln(abs(
x - 1)) + 2*ln(abs(x))
```

3.106 $\int \frac{1}{x^3-x^4-x^5+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

```
[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x]
- Log[1 + x]/4
```

Rubi [A] time = 0.0424125, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x]
- Log[1 + x]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6-x^5-x^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-x**5-x**4+x**3),x)
```

```
[Out] Integral(1/(x**6 - x**5 - x**4 + x**3), x)
```

Mathematica [A] time = 0.0253667, size = 40, normalized size = 0.87

$$\frac{1}{4}\left(-\frac{2}{x^2}-\frac{2}{x-1}-\frac{4}{x}-7\log(1-x)+8\log(x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] (-2/(-1 + x) - 2/x^2 - 4/x - 7*Log[1 - x] + 8*Log[x] - Log[1 + x]
)/4
```

Maple [A] time = 0.015, size = 35, normalized size = 0.8

$$-\frac{\ln(1+x)}{4}-\frac{1}{2\,x^2}-x^{-1}+2\,\ln(x)-\frac{1}{-2+2\,x}-\frac{7\,\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-x^5-x^4+x^3),x)
```

```
[Out] -1/4*ln(1+x)-1/2/x^2-1/x+2*ln(x)-1/2/(-1+x)-7/4*ln(-1+x)
```

Maxima [A] time = 1.3856, size = 54, normalized size = 1.17

$$-\frac{3\,x^2-x-1}{2\,(x^3-x^2)}-\frac{1}{4}\,\log(x+1)-\frac{7}{4}\,\log(x-1)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^5 - x^4 + x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*log(x + 1) - 7/4*log(x - 1
) + 2*log(x)
```

Fricas [A] time = 0.19938, size = 88, normalized size = 1.91

$$-\frac{6\,x^2+\left(x^3-x^2\right)\,\log(x+1)+7\,\left(x^3-x^2\right)\,\log(x-1)-8\,\left(x^3-x^2\right)\,\log(x)-2\,x-2}{4\,(x^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^5 - x^4 + x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x^2 + (x^3 - x^2)*log(x + 1) + 7*(x^3 - x^2)*log(x - 1) -
 8*(x^3 - x^2)*log(x) - 2*x - 2)/(x^3 - x^2)
```

Sympy [A] time = 0.211312, size = 37, normalized size = 0.8

$$2\log(x) - \frac{7\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{3x^2 - x - 1}{2x^3 - 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-x**5-x**4+x**3),x)
```

```
[Out] 2*log(x) - 7*log(x - 1)/4 - log(x + 1)/4 - (3*x**2 - x - 1)/(2*x*
*3 - 2*x**2)
```

GIAC/XCAS [A] time = 0.19884, size = 54, normalized size = 1.17

$$-\frac{3\,x^2 - x - 1}{2\,(x-1)x^2} - \frac{1}{4}\ln(|x+1|) - \frac{7}{4}\ln(|x-1|) + 2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^5 - x^4 + x^3),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*ln(abs(x + 1)) - 7/4*ln(
abs(x - 1)) + 2*ln(abs(x))
```

3.107 $\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0534352, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(-1 + x - x^2 + x^3),x]

[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^3-x^2+x-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**3-x**2+x-1),x)

[Out] Integral((x**4 + 1)/(x**3 - x**2 + x - 1), x)
```

Mathematica [A] time = 0.0111239, size = 29, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Maple [A] time = 0.009, size = 24, normalized size = 0.8

$$x+\frac{x^2}{2}-\frac{\ln\left(x^2+1\right)}{2}-\arctan\left(x\right)+\ln\left(-1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^3-x^2+x-1),x)
```

```
[Out] x+1/2*x^2-1/2*ln(x^2+1)-arctan(x)+ln(-1+x)
```

Maxima [A] time = 1.53022, size = 31, normalized size = 1.07

$$\frac{1}{2}\,x^2+x-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^3 - x^2 + x - 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [A] time = 0.209742, size = 31, normalized size = 1.07

$$\frac{1}{2}\,x^2+x-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^3 - x^2 + x - 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Sympy [A] time = 0.123373, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+x+\log\left(x-1\right)-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**3-x**2+x-1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x**2 + 1)/2 - atan(x)
```

GIAC/XCAS [A] time = 0.200354, size = 32, normalized size = 1.1

$$\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\ln\left(x^2+1\right) + \ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^3 - x^2 + x - 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - arctan(x) - 1/2*ln(x^2 + 1) + ln(abs(x - 1))
```

3.108 $\int \frac{1}{x(1+x)(1+x^2)} dx$

Optimal. Leaf size=27

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.061534, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x)*(1 + x^2)),x]

[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi in Sympy [A] time = 3.27409, size = 22, normalized size = 0.81

$$\log(x)-\frac{\log(x+1)}{2}-\frac{\log\left(x^2+1\right)}{4}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+x)/(x**2+1),x)

[Out] log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

Mathematica [A] time = 0.0104167, size = 27, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.01, size = 22, normalized size = 0.8

$$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+x)/(x^2+1),x)
```

```
[Out] -1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.53609, size = 28, normalized size = 1.04

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

Fricas [A] time = 0.213579, size = 28, normalized size = 1.04

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

Sympy [A] time = 0.242739, size = 22, normalized size = 0.81

$$\log(x)-\frac{\log(x+1)}{2}-\frac{\log(x^2+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+x)/(x**2+1),x)
```

```
[Out] log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

GIAC/XCAS [A] time = 0.199496, size = 31, normalized size = 1.15

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\ln\left(x^2+1\right) - \frac{1}{2}\ln\left(|x+1|\right) + \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 1)*x),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*ln(x^2 + 1) - 1/2*ln(abs(x + 1)) + ln(abs(x)
)
```

3.109 $\int \frac{x^2}{-2+x^2+x^4}\,dx$

Optimal. Leaf size=24

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

Rubi [A] time = 0.0252483, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(-2 + x^2 + x^4),x]

[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

Rubi in Sympy [A] time = 2.96615, size = 20, normalized size = 0.83

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}-\frac{\mathrm{atanh}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4+x**2-2),x)

[Out] sqrt(2)*atan(sqrt(2)*x/2)/3 - atanh(x)/3
```

Mathematica [A] time = 0.0159883, size = 32, normalized size = 1.33

$$\frac{1}{6}\left(\log(1-x)-\log(x+1)+2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(-2 + x^2 + x^4),x]
```

```
[Out] (2*Sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6
```

Maple [A] time = 0.01, size = 26, normalized size = 1.1

$$-\frac{\ln(1+x)}{6}+\frac{\ln(-1+x)}{6}+\frac{\sqrt{2}}{3}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4+x^2-2),x)
```

```
[Out] -1/6*ln(1+x)+1/6*ln(-1+x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.51545, size = 34, normalized size = 1.42

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log(x+1)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + x^2 - 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x -
1)
```

Fricas [A] time = 0.217796, size = 34, normalized size = 1.42

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log(x+1)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + x^2 - 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x -
1)
```

Sympy [A] time = 0.197665, size = 29, normalized size = 1.21

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4+x**2-2),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3
```

GIAC/XCAS [A] time = 0.20265, size = 36, normalized size = 1.5

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\ln(|x+1|) + \frac{1}{6}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + x^2 - 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*ln(abs(x + 1)) + 1/6*ln(a
bs(x - 1))
```

3.110 $\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} \, dx$

Optimal. Leaf size=41

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (
2*Log[2 + x^2])/3
```

Rubi [A] time = 0.129699, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (
2*Log[2 + x^2])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0342219, size = 41, normalized size = 1.

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (
2*Log[2 + x^2])/3
```

Maple [A] time = 0.012, size = 33, normalized size = 0.8

$$(1+x)^{-1} - \frac{\ln(1+x)}{3} + \frac{2\,\ln\left(x^2+2\right)}{3} + \frac{4\sqrt{2}}{3}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x)
```

```
[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/
2)
```

Maxima [A] time = 1.52682, size = 43, normalized size = 1.05

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right) - \frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 4*x^2 + 6*x)/(x^4 + 2*x^3 + 3*x^2 + 4*x + 2),x, algorithm="maxima
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2)
- 1/3*log(x + 1)
```

Fricas [A] time = 0.21553, size = 59, normalized size = 1.44

$$\frac{4\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 2(x+1)\log\left(x^2+2\right) - (x+1)\log(x+1) + 3}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 4*x^2 + 6*x)/(x^4 + 2*x^3 + 3*x^2 + 4*x + 2),x, algorithm="fricas
```

```
[Out] 1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2
+ 2) - (x + 1)*log(x + 1) + 3)/(x + 1)
```

Sympy [A] time = 0.184711, size = 39, normalized size = 0.95

$$-\frac{\log(x+1)}{3}+\frac{2\log(x^2+2)}{3}+\frac{4\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
```

```
[Out] -log(x + 1)/3 + 2*log(x**2 + 2)/3 + 4*sqrt(2)*atan(sqrt(2)*x/2)/3
 + 1/(x + 1)
```

GIAC/XCAS [A] time = 0.201937, size = 45, normalized size = 1.1

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1}+\frac{2}{3}\ln\left(x^2+2\right)-\frac{1}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 4*x^2 + 6*x)/(x^4 + 2*x^3 + 3*x^2 + 4*x + 2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*ln(x^2 + 2) -
 1/3*ln(abs(x + 1))
```

3.111 $\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} \, dx$

Optimal. Leaf size=46

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])
/25 - (7*Log[1 + x^2])/100
```

Rubi [A] time = 0.353672, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])
/25 - (7*Log[1 + x^2])/100
```

Rubi in Sympy [A] time = 28.029, size = 37, normalized size = 0.8

$$-\frac{\log(x+1)}{2}+\frac{16\log(2x+1)}{25}-\frac{7\log\left(x^2+1\right)}{100}+\frac{\operatorname{atan}(x)}{50}+\frac{2}{5(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

```
[Out] -log(x + 1)/2 + 16*log(2*x + 1)/25 - 7*log(x**2 + 1)/100 + atan(x
)/50 + 2/(5*(2*x + 1))
```

Mathematica [A] time = 0.0314044, size = 40, normalized size = 0.87

$$\frac{1}{100}\left(-7\log\left(x^2+1\right)+\frac{40}{2x+1}-50\log(x+1)+64\log(2x+1)+2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] (40/(1 + 2*x) + 2*ArcTan[x] - 50*Log[1 + x] + 64*Log[1 + 2*x] - 7
*Log[1 + x^2])/100
```

Maple [A] time = 0.012, size = 37, normalized size = 0.8

$$\frac{2}{5+10\,x}+\frac{\arctan\left(x\right)}{50}-\frac{\ln\left(1+x\right)}{2}+\frac{16\,\ln\left(1+2\,x\right)}{25}-\frac{7\,\ln\left(x^{2}+1\right)}{100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/(1+2*x)^2/(x^2+1),x)
```

```
[Out] 2/5/(1+2*x)+1/50*arctan(x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x
^2+1)
```

Maxima [A] time = 1.52773, size = 49, normalized size = 1.07

$$\frac{2}{5\left(2\,x+1\right)}+\frac{1}{50}\,\arctan\left(x\right)-\frac{7}{100}\,\log\left(x^{2}+1\right)+\frac{16}{25}\,\log\left(2\,x+1\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(2*x + 1)^2*(x + 1)),x, algorithm="maxima")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(2
*x + 1) - 1/2*log(x + 1)
```

Fricas [A] time = 0.215212, size = 77, normalized size = 1.67

$$\frac{2\left(2\,x+1\right)\arctan\left(x\right)-7\left(2\,x+1\right)\log\left(x^{2}+1\right)+64\left(2\,x+1\right)\log\left(2\,x+1\right)-50\left(2\,x+1\right)\log\left(x+1\right)+40}{100\left(2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(2*x + 1)^2*(x + 1)),x, algorithm="fricas")
```

```
[Out] 1/100*(2*(2*x + 1)*arctan(x) - 7*(2*x + 1)*log(x^2 + 1) + 64*(2*x
 + 1)*log(2*x + 1) - 50*(2*x + 1)*log(x + 1) + 40)/(2*x + 1)
```

Sympy [A] time = 0.300442, size = 37, normalized size = 0.8

$$\frac{16\log\left(x+\frac{1}{2}\right)}{25}-\frac{\log\left(x+1\right)}{2}-\frac{7\log\left(x^2+1\right)}{100}+\frac{\operatorname{atan}\left(x\right)}{50}+\frac{2}{10x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

```
[Out] 16*log(x + 1/2)/25 - log(x + 1)/2 - 7*log(x**2 + 1)/100 + atan(x)
/50 + 2/(10*x + 5)
```

GIAC/XCAS [A] time = 0.201311, size = 84, normalized size = 1.83

$$\frac{2}{5\,(2\,x+1)}+\frac{1}{50}\,\arctan\left(-\frac{5}{2\,(2\,x+1)}+\frac{1}{2}\right)-\frac{7}{100}\ln\left(-\frac{2}{2\,x+1}+\frac{5}{(2\,x+1)^2}+1\right)-\frac{1}{2}\ln\left(\left|-\frac{1}{2\,x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(2*x + 1)^2*(x + 1)),x, algorithm="giac")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(-5/2/(2*x + 1) + 1/2) - 7/100*ln(-2/(
2*x + 1) + 5/(2*x + 1)^2 + 1) - 1/2*ln(abs(-1/(2*x + 1) - 1))
```

3.112 $\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=47

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)-\tan^{-1}(x)$$

```
[Out] -1/(2*(1 - x)^2) + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 +
 (3*Log[1 + x^2])/4
```

Rubi [A] time = 0.0732499, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -1/(2*(1 - x)^2) + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 +
 (3*Log[1 + x^2])/4
```

Rubi in Sympy [A] time = 4.8787, size = 36, normalized size = 0.77

$$-\frac{3\log\left(-x+1\right)}{2}+\frac{3\log\left(x^2+1\right)}{4}-\operatorname{atan}\left(x\right)+\frac{5}{2\left(-x+1\right)}-\frac{1}{2\left(-x+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
```

```
[Out] -3*log(-x + 1)/2 + 3*log(x**2 + 1)/4 - atan(x) + 5/(2*(-x + 1)) -
 1/(2*(-x + 1)**2)
```

Mathematica [A] time = 0.0406449, size = 37, normalized size = 0.79

$$\frac{1}{4}\left(3\log\left(x^2+1\right)-\frac{10}{x-1}-\frac{2}{(x-1)^2}-6\log(x-1)-4\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] (-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Lo
g[1 + x^2])/4
```

Maple [A] time = 0.01, size = 34, normalized size = 0.7

$$\frac{3\ln\left(x^2+1\right)}{4}-\arctan\left(x\right)-\frac{1}{2\left(-1+x\right)^2}-\frac{5}{-2+2x}-\frac{3\ln\left(-1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x)
```

```
[Out] 3/4*ln(x^2+1)-arctan(x)-1/2/(-1+x)^2-5/2/(-1+x)-3/2*ln(-1+x)
```

Maxima [A] time = 1.51853, size = 49, normalized size = 1.04

$$-\frac{5x-4}{2\left(x^2-2x+1\right)}-\arctan\left(x\right)+\frac{3}{4}\log\left(x^2+1\right)-\frac{3}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x - 2)/((x^2 + 1)*(x - 1)^3),x, algorithm="maxima")
```

```
[Out] -1/2*(5*x - 4)/(x^2 - 2*x + 1) - arctan(x) + 3/4*log(x^2 + 1) - 3
/2*log(x - 1)
```

Fricas [A] time = 0.211384, size = 80, normalized size = 1.7

$$-\frac{4\left(x^2-2x+1\right)\arctan\left(x\right)-3\left(x^2-2x+1\right)\log\left(x^2+1\right)+6\left(x^2-2x+1\right)\log\left(x-1\right)+10x-8}{4\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x - 2)/((x^2 + 1)*(x - 1)^3),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(x^2 - 2*x + 1)*arctan(x) - 3*(x^2 - 2*x + 1)*log(x^2 + 1
) + 6*(x^2 - 2*x + 1)*log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.1988, size = 36, normalized size = 0.77

$$-\frac{5x-4}{2x^2-4x+2}-\frac{3\log(x-1)}{2}+\frac{3\log(x^2+1)}{4}-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
```

```
[Out] -(5*x - 4)/(2*x**2 - 4*x + 2) - 3*log(x - 1)/2 + 3*log(x**2 + 1)/
4 - atan(x)
```

GIAC/XCAS [A] time = 0.201523, size = 43, normalized size = 0.91

$$-\frac{5\,x-4}{2\,(x-1)^2}-\arctan(x)+\frac{3}{4}\ln\left(x^2+1\right)-\frac{3}{2}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x - 2)/((x^2 + 1)*(x - 1)^3),x, algorithm="giac")
```

```
[Out] -1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*ln(x^2 + 1) - 3/2*ln(a
bs(x - 1))
```

3.113 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rubi [A] time = 0.0768999, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rubi in Sympy [A] time = 5.08814, size = 63, normalized size = 0.94

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(sqrt(3)
*(2*x/3 - 1/3))/6 + sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/6
```

Mathematica [C] time = 0.094802, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)-\sqrt{1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I
*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] time = 0.003, size = 54, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^
2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.49503, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1
)
```

Fricas [A] time = 0.233014, size = 77, normalized size = 1.15

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-\sqrt{3}\log\left(x^2-x+1\right)+2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - sqrt(3)*log(x^2 - x + 1)
 + 2*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*arctan(1/3*sqrt(3)*(2*x -
1)))
```

Sympy [A] time = 0.254245, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2 - x + 1\right)}{4} + \frac{\log\left(x^2 + x + 1\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(
3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.200695, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\ln\left(x^2 + x + 1\right) - \frac{1}{4}\ln\left(x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/4*ln(x^2 + x + 1) - 1/4*ln(x^2 - x + 1)
```

3.114 $\int \frac{3+2x^3}{-9x+x^5}\,dx$

Optimal. Leaf size=48

$$\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3
 + Log[9 - x^4]/12
```

Rubi [A] time = 0.0851059, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3
 + Log[9 - x^4]/12
```

Rubi in Sympy [A] time = 5.94699, size = 48, normalized size = 1.

$$-\frac{\log\left(x^4\right)}{12}+\frac{\log\left(-x^4+9\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3)/(x**5-9*x),x)
```

```
[Out] -log(x**4)/12 + log(-x**4 + 9)/12 + sqrt(3)*atan(sqrt(3)*x/3)/3 -
 sqrt(3)*atanh(sqrt(3)*x/3)/3
```

Mathematica [A] time = 0.0303987, size = 67, normalized size = 1.4

$$\frac{1}{12}\left(\log\left(9-x^4\right)-4\log(x)+2\sqrt{3}\log\left(3-\sqrt{3}x\right)-2\sqrt{3}\log\left(\sqrt{3}x+3\right)+4\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[
3]*x] - 2*Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

Maple [A] time = 0.013, size = 46, normalized size = 1.

$$\frac{\ln\left(x^{2}-3\right)}{12}-\frac{\sqrt{3}}{3}Artanh\left(\frac{x\sqrt{3}}{3}\right)-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x^{2}+3\right)}{12}+\frac{\sqrt{3}}{3}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3)/(x^5-9*x),x)
```

```
[Out] 1/12*ln(x^2-3)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/3*ln(x)+1/12*
ln(x^2+3)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.51054, size = 73, normalized size = 1.52

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{12}\log\left(x^{2}+3\right)+\frac{1}{12}\log\left(x^{2}-3\right)-\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3)/(x^5 - 9*x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x - sqrt(3))
/(x + sqrt(3))) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log
(x)
```

Fricas [A] time = 0.239664, size = 89, normalized size = 1.85

$$\frac{1}{36}\sqrt{3}\left(\sqrt{3}\log\left(x^{2}+3\right)+\sqrt{3}\log\left(x^{2}-3\right)-4\sqrt{3}\log\left(x\right)+12\arctan\left(\frac{1}{3}\sqrt{3}x\right)+6\log\left(\frac{\sqrt{3}\left(x^{2}+3\right)-6x}{x^{2}-3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3)/(x^5 - 9*x),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(sqrt(3)*log(x^2 + 3) + sqrt(3)*log(x^2 - 3) - 4*sqr
t(3)*log(x) + 12*arctan(1/3*sqrt(3)*x) + 6*log((sqrt(3)*(x^2 + 3)
 - 6*x)/(x^2 - 3)))
```

Sympy [A] time = 2.53721, size = 306, normalized size = 6.38

$$-\frac{\log(x)}{3} + \left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)\log\left(x+\frac{17413}{11544}-\frac{943\sqrt{3}i}{5772}+\frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^3}{481}+\frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^2}{481}-\frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^4}{481}\right) + \left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)\log\left(x+\frac{17413}{11544}-\frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^4}{481}+\frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^2}{481}+\frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^3}{481}+\frac{943\sqrt{3}i}{5772}\right) + \left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)\log\left(x-\frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^4}{481}-\frac{943\sqrt{3}}{5772}+\frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^3}{481}+\frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^2}{481}+\frac{17413}{11544}\right) + \left(-\frac{\sqrt{3}}{6}+\frac{1}{12}\right)\log\left(x-\frac{108000\left(-\frac{\sqrt{3}}{6}+\frac{1}{12}\right)^4}{481}+\frac{1368\left(-\frac{\sqrt{3}}{6}+\frac{1}{12}\right)^3}{481}+\frac{943\sqrt{3}}{5772}+\frac{4158\left(-\frac{\sqrt{3}}{6}+\frac{1}{12}\right)^2}{481}+\frac{17413}{11544}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3)/(x**5-9*x),x)
```

```
[Out] -log(x)/3 + (1/12 + sqrt(3)*I/6)*log(x + 17413/11544 - 943*sqrt(3
)*I/5772 + 1368*(1/12 + sqrt(3)*I/6)**3/481 + 4158*(1/12 + sqrt(3
)*I/6)**2/481 - 108000*(1/12 + sqrt(3)*I/6)**4/481) + (1/12 - sqr
t(3)*I/6)*log(x + 17413/11544 - 108000*(1/12 - sqrt(3)*I/6)**4/48
1 + 4158*(1/12 - sqrt(3)*I/6)**2/481 + 1368*(1/12 - sqrt(3)*I/6)*
*3/481 + 943*sqrt(3)*I/5772) + (1/12 + sqrt(3)/6)*log(x - 108000*
(1/12 + sqrt(3)/6)**4/481 - 943*sqrt(3)/5772 + 1368*(1/12 + sqrt(
3)/6)**3/481 + 4158*(1/12 + sqrt(3)/6)**2/481 + 17413/11544) + (-
sqrt(3)/6 + 1/12)*log(x - 108000*(-sqrt(3)/6 + 1/12)**4/481 + 136
8*(-sqrt(3)/6 + 1/12)**3/481 + 943*sqrt(3)/5772 + 4158*(-sqrt(3)/
6 + 1/12)**2/481 + 17413/11544)
```

GIAC/XCAS [A] time = 0.201174, size = 86, normalized size = 1.79

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\ln\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+\frac{1}{12}\ln\left(x^2+3\right)+\frac{1}{12}\ln\left(\left|x^2-3\right|\right)-\frac{1}{3}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3)/(x^5 - 9*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*ln(abs(2*x - 2*sq
rt(3))/abs(2*x + 2*sqrt(3))) + 1/12*ln(x^2 + 3) + 1/12*ln(abs(x^2
 - 3)) - 1/3*ln(abs(x))
```

3.115 $\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$

Optimal. Leaf size=58

$$\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45
*Log[4 - x])/16 + (45*Log[8 - 4*x + x^2])/32
```

Rubi [A] time = 0.0966032, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45
*Log[4 - x])/16 + (45*Log[8 - 4*x + x^2])/32
```

Rubi in Sympy [A] time = 20.5429, size = 46, normalized size = 0.79

$$-\frac{45\log\left(-x+4\right)}{16}+\frac{45\log\left(x^2-4x+8\right)}{32}+\frac{3\,\mathrm{atan}\left(\frac{x}{2}-1\right)}{16}+\frac{41}{4\left(-x+4\right)}-\frac{83}{4\left(-x+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
```

```
[Out] -45*log(-x + 4)/16 + 45*log(x**2 - 4*x + 8)/32 + 3*atan(x/2 - 1)/
16 + 41/(4*(-x + 4)) - 83/(4*(-x + 4)**2)
```

Mathematica [A] time = 0.0400513, size = 46, normalized size = 0.79

$$\frac{1}{32}\left(45\log\left(x^2-4x+8\right)-\frac{328}{x-4}-\frac{664}{(x-4)^2}-90\log(x-4)+6\tan^{-1}\left(\frac{x-2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] (-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-
4 + x] + 45*Log[8 - 4*x + x^2])/32
```

Maple [A] time = 0.013, size = 41, normalized size = 0.7

$$\frac{45\ln\left(x^2-4x+8\right)}{32}+\frac{3}{16}\arctan\left(-1+\frac{x}{2}\right)-\frac{83}{4\left(x-4\right)^2}-\frac{41}{4x-16}-\frac{45\ln\left(x-4\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^3+8*x-20)/(x-4)^3/(x^2-4*x+8),x)
```

```
[Out] 45/32*ln(x^2-4*x+8)+3/16*arctan(-1+1/2*x)-83/4/(x-4)^2-41/4/(x-4)
-45/16*ln(x-4)
```

Maxima [A] time = 1.49062, size = 58, normalized size = 1.

$$-\frac{41x-81}{4\left(x^2-8x+16\right)}+\frac{3}{16}\arctan\left(\frac{1}{2}x-1\right)+\frac{45}{32}\log\left(x^2-4x+8\right)-\frac{45}{16}\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^3 + 8*x - 20)/((x^2 - 4*x + 8)*(x - 4)^3),x, algorithm="maxima")
```

```
[Out] -1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*arctan(1/2*x - 1) + 45/3
2*log(x^2 - 4*x + 8) - 45/16*log(x - 4)
```

Fricas [A] time = 0.217226, size = 89, normalized size = 1.53

$$\frac{6\left(x^2-8x+16\right)\arctan\left(\frac{1}{2}x-1\right)+45\left(x^2-8x+16\right)\log\left(x^2-4x+8\right)-90\left(x^2-8x+16\right)\log\left(x-4\right)-328x+648}{32\left(x^2-8x+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^3 + 8*x - 20)/((x^2 - 4*x + 8)*(x - 4)^3),x, algorithm="fricas")
```

```
[Out] 1/32*(6*(x^2 - 8*x + 16)*arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*
log(x^2 - 4*x + 8) - 90*(x^2 - 8*x + 16)*log(x - 4) - 328*x + 648
```

```
)/(x^2 - 8*x + 16)
```

Sympy [A] time = 0.226978, size = 46, normalized size = 0.79

$$-\frac{41x-81}{4x^2-32x+64}-\frac{45\log(x-4)}{16}+\frac{45\log(x^2-4x+8)}{32}+\frac{3\,\mathrm{atan}\left(\frac{x}{2}-1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
```

```
[Out] -(41*x - 81)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**
2 - 4*x + 8)/32 + 3*atan(x/2 - 1)/16
```

GIAC/XCAS [A] time = 0.202825, size = 53, normalized size = 0.91

$$-\frac{41\,x-81}{4\,(x-4)^2}+\frac{3}{16}\,\arctan\left(\frac{1}{2}\,x-1\right)+\frac{45}{32}\,\ln\left(x^2-4\,x+8\right)-\frac{45}{16}\,\ln\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^3 + 8*x - 20)/((x^2 - 4*x + 8)*(x - 4)^3),x, algorithm="giac")
```

```
[Out] -1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*ln(x^
2 - 4*x + 8) - 45/16*ln(abs(x - 4))
```

3.116 $\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[x/2]/12 + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + A
rcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.473394, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/12 + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + A
rcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 43.7541, size = 44, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12}+\frac{\operatorname{atan}(x)}{6}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] -atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)
*atan(sqrt(3)*x/3)/6
```

Mathematica [A] time = 0.0377071, size = 47, normalized size = 0.92

$$\frac{1}{12}\left(-\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-3\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] (-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqr
t[3]*ArcTan[x/Sqrt[3]])/12
```

Maple [A] time = 0.016, size = 36, normalized size = 0.7

$$-\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6}-\frac{\sqrt{2}}{4}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{\sqrt{3}}{6}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)
```

```
[Out] -1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/
2)+1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.52032, size = 47, normalized size = 0.92

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*(x^2 + 3)*(x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2
)*x) - 1/12*arctan(1/2*x) + 1/6*arctan(x)
```

Fricas [A] time = 0.239113, size = 73, normalized size = 1.43

$$-\frac{1}{72}\sqrt{3}\sqrt{2}\left(\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}x\right)-2\sqrt{3}\sqrt{2}\arctan(x)-6\sqrt{2}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+6\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*(x^2 + 3)*(x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(3)*sqrt(2)*(sqrt(3)*sqrt(2)*arctan(1/2*x) - 2*sqrt(3)*
sqrt(2)*arctan(x) - 6*sqrt(2)*arctan(1/3*sqrt(3)*x) + 6*sqrt(3)*a
rctan(1/2*sqrt(2)*x))
```

Sympy [A] time = 0.774853, size = 44, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12}+\frac{\operatorname{atan}(x)}{6}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] -atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)
*atan(sqrt(3)*x/3)/6
```

GIAC/XCAS [A] time = 0.217276, size = 47, normalized size = 0.92

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 4)*(x^2 + 3)*(x^2 + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2
)*x) - 1/12*arctan(1/2*x) + 1/6*arctan(x)
```

3.117 $\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=41

$$\frac{1}{12}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{12}\log\left(x^2+4\right)$$

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/
12
```

Rubi [A] time = 0.529343, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{1}{12}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{12}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/
12
```

Rubi in Sympy [A] time = 50.5609, size = 32, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{12}-\frac{\log\left(x^2+2\right)}{4}+\frac{\log\left(x^2+3\right)}{4}-\frac{\log\left(x^2+4\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 +
 4)/12
```

Mathematica [A] time = 0.0132908, size = 41, normalized size = 1.

$$\frac{1}{12}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{12}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/
12
```

Maple [A] time = 0.016, size = 34, normalized size = 0.8

$$\frac{\ln\left(x^2+1\right)}{12}-\frac{\ln\left(x^2+2\right)}{4}+\frac{\ln\left(x^2+3\right)}{4}-\frac{\ln\left(x^2+4\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)
```

```
[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

Maxima [A] time = 1.43785, size = 45, normalized size = 1.1

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^2 + 4)*(x^2 + 3)*(x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*l
og(x^2 + 1)
```

Fricas [A] time = 0.233671, size = 45, normalized size = 1.1

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^2 + 4)*(x^2 + 3)*(x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*l
og(x^2 + 1)
```

Sympy [A] time = 0.257805, size = 32, normalized size = 0.78

$$\frac{\log(x^2+1)}{12} - \frac{\log(x^2+2)}{4} + \frac{\log(x^2+3)}{4} - \frac{\log(x^2+4)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 +
 4)/12
```

GIAC/XCAS [A] time = 0.217703, size = 45, normalized size = 1.1

$$-\frac{1}{12}\ln\left(x^2+4\right) + \frac{1}{4}\ln\left(x^2+3\right) - \frac{1}{4}\ln\left(x^2+2\right) + \frac{1}{12}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 3)*(x^2 + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/12*ln(x^2 + 4) + 1/4*ln(x^2 + 3) - 1/4*ln(x^2 + 2) + 1/12*ln(x
^2 + 1)
```

3.118 $\int \frac{1}{a^3+x^3}\,dx$

Optimal. Leaf size=56

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}$$

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^
2) - Log[a^2 - a*x + x^2]/(6*a^2)
```

Rubi [A] time = 0.0663533, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^3 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^
2) - Log[a^2 - a*x + x^2]/(6*a^2)
```

Rubi in Sympy [A] time = 5.61049, size = 53, normalized size = 0.95

$$\frac{\log\left(a+x\right)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**3+x**3),x)
```

```
[Out] log(a + x)/(3*a**2) - log(a**2 - a*x + x**2)/(6*a**2) - sqrt(3)*a
tan(sqrt(3)*(a/3 - 2*x/3)/a)/(3*a**2)
```

Mathematica [A] time = 0.0165434, size = 52, normalized size = 0.93

$$\frac{-\log\left(a^2-ax+x^2\right)+2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^3 + x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^
2 - a*x + x^2])/(6*a^2)
```

Maple [A] time = 0.012, size = 52, normalized size = 0.9

$$\frac{\ln\left(a+x\right)}{3\,a^{2}}-\frac{\ln\left(a^{2}-ax+x^{2}\right)}{6\,a^{2}}+\frac{\sqrt{3}}{3\,a^{2}}\arctan\left(\frac{\left(2\,x-a\right)\sqrt{3}}{3\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^3+x^3),x)
```

```
[Out] 1/3*ln(a+x)/a^2-1/6*ln(a^2-a*x+x^2)/a^2+1/3/a^2*3^(1/2)*arctan(1/
3*(2*x-a)*3^(1/2)/a)
```

Maxima [A] time = 1.50671, size = 66, normalized size = 1.18

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^{2}}-\frac{\log\left(a^{2}-ax+x^{2}\right)}{6\,a^{2}}+\frac{\log\left(a+x\right)}{3\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^3 + x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 -
a*x + x^2)/a^2 + 1/3*log(a + x)/a^2
```

Fricas [A] time = 0.20756, size = 68, normalized size = 1.21

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(a^{2}-ax+x^{2}\right)-2\,\sqrt{3}\log\left(a+x\right)-6\,\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)\right)}{18\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^3 + x^3),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(a^2 - a*x + x^2) - 2*sqrt(3)*log(a + x
) - 6*arctan(-1/3*sqrt(3)*(a - 2*x)/a))/a^2
```

Sympy [A] time = 0.13902, size = 73, normalized size = 1.3

$$\frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**3+x**3),x)
```

```
[Out] (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(3*a*(-1/6 - sqrt(3)*I/6)
 + x) + (-1/6 + sqrt(3)*I/6)*log(3*a*(-1/6 + sqrt(3)*I/6) + x))/a
**2
```

GIAC/XCAS [A] time = 0.217697, size = 68, normalized size = 1.21

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\ln\left(a^2 - ax + x^2\right)}{6a^2} + \frac{\ln\left(|a+x|\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^3 + x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*ln(a^2 - a
*x + x^2)/a^2 + 1/3*ln(abs(a + x))/a^2
```

3.119 $\int \frac{x}{a^3+x^3} \, dx$

Optimal. Leaf size=56

$$\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log(a+x)}{3a}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}$$

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) +
 Log[a^2 - a*x + x^2]/(6*a)
```

Rubi [A] time = 0.06278, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546

$$\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log(a+x)}{3a}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^3 + x^3),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) +
 Log[a^2 - a*x + x^2]/(6*a)
```

Rubi in Sympy [A] time = 5.40662, size = 48, normalized size = 0.86

$$-\frac{\log\left(a+x\right)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**3+x**3),x)
```

```
[Out] -log(a + x)/(3*a) + log(a**2 - a*x + x**2)/(6*a) - sqrt(3)*atan(s
qrt(3)*(a/3 - 2*x/3)/a)/(3*a)
```

Mathematica [A] time = 0.00929327, size = 50, normalized size = 0.89

$$\frac{\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{6a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^
2 - a*x + x^2])/(6*a)
```

Maple [A] time = 0.009, size = 52, normalized size = 0.9

$$-\frac{\ln(a+x)}{3a}+\frac{\ln(a^2-ax+x^2)}{6a}+\frac{\sqrt{3}}{3a}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^3+x^3),x)
```

```
[Out] -1/3*ln(a+x)/a+1/6*ln(a^2-a*x+x^2)/a+1/3*3^(1/2)/a*arctan(1/3*(2*
x-a)*3^(1/2)/a)
```

Maxima [A] time = 1.49684, size = 66, normalized size = 1.18

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a}+\frac{\log(a^2-ax+x^2)}{6a}-\frac{\log(a+x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^3 + x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*
x + x^2)/a - 1/3*log(a + x)/a
```

Fricas [A] time = 0.208996, size = 68, normalized size = 1.21

$$\frac{\sqrt{3}\left(\sqrt{3}\log\left(a^2-ax+x^2\right)-2\sqrt{3}\log(a+x)+6\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)\right)}{18a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^3 + x^3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*log(a^2 - a*x + x^2) - 2*sqrt(3)*log(a + x)
 + 6*arctan(-1/3*sqrt(3)*(a - 2*x)/a))/a
```

Sympy [A] time = 0.12903, size = 71, normalized size = 1.27

$$\frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**3+x**3),x)
```

```
[Out] (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)*
*2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x)
)/a
```

GIAC/XCAS [A] time = 0.208759, size = 68, normalized size = 1.21

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a} + \frac{\ln\left(a^2 - ax + x^2\right)}{6a} - \frac{\ln\left(|a + x|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^3 + x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*ln(a^2 - a*x
 + x^2)/a - 1/3*ln(abs(a + x))/a
```

3.120 $\int \frac{x^2}{a^3+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3} \log \left(a^3 + x^3\right)$$

```
[Out] Log[a^3 + x^3]/3
```

Rubi [A] time = 0.00599168, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{1}{3} \log \left(a^3 + x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^3 + x^3),x]
```

```
[Out] Log[a^3 + x^3]/3
```

Rubi in Sympy [A] time = 0.898281, size = 8, normalized size = 0.67

$$\frac{\log \left(a^3 + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**3+x**3),x)
```

```
[Out] log(a**3 + x**3)/3
```

Mathematica [A] time = 0.0033611, size = 12, normalized size = 1.

$$\frac{1}{3} \log \left(a^3 + x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^3 + x^3),x]
```

```
[Out] Log[a^3 + x^3]/3
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$\frac{\ln(a^3 + x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^3+x^3),x)
```

```
[Out] 1/3*ln(a^3+x^3)
```

Maxima [A] time = 1.32586, size = 14, normalized size = 1.17

$$\frac{1}{3}\log(a^3 + x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^3 + x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(a^3 + x^3)
```

Fricas [A] time = 0.227863, size = 14, normalized size = 1.17

$$\frac{1}{3}\log(a^3 + x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^3 + x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(a^3 + x^3)
```

Sympy [A] time = 0.096228, size = 8, normalized size = 0.67

$$\frac{\log(a^3 + x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**3+x**3),x)

[Out] log(a**3 + x**3)/3
```

GIAC/XCAS [A] time = 0.202165, size = 15, normalized size = 1.25

$$\frac{1}{3}\ln\left(\left|a^3 + x^3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^3 + x^3),x, algorithm="giac")

[Out] 1/3*ln(abs(a^3 + x^3))
```

3.121 $\int \frac{1}{x(a^3+x^3)} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Rubi [A] time = 0.0236263, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^3 + x^3)),x]
```

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Rubi in Sympy [A] time = 2.39399, size = 22, normalized size = 1.

$$\frac{\log\left(x^3\right)}{3a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**3+x**3),x)
```

```
[Out] log(x**3)/(3*a**3) - log(a**3 + x**3)/(3*a**3)
```

Mathematica [A] time = 0.00625151, size = 22, normalized size = 1.

$$\frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^3 + x^3)),x]
```

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Maple [A] time = 0.01, size = 34, normalized size = 1.6

$$-\frac{\ln\left(a+x\right)}{3\,a^{3}}-\frac{\ln\left(a^{2}-ax+x^{2}\right)}{3\,a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^3+x^3),x)
```

```
[Out] -1/3*ln(a+x)/a^3-1/3/a^3*ln(a^2-a*x+x^2)+ln(x)/a^3
```

Maxima [A] time = 1.37183, size = 31, normalized size = 1.41

$$-\frac{\log\left(a^{3}+x^{3}\right)}{3\,a^{3}}+\frac{\log\left(x^{3}\right)}{3\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x),x, algorithm="maxima")
```

```
[Out] -1/3*log(a^3 + x^3)/a^3 + 1/3*log(x^3)/a^3
```

Fricas [A] time = 0.2078, size = 24, normalized size = 1.09

$$-\frac{\log\left(a^{3}+x^{3}\right)-3\,\log\left(x\right)}{3\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x),x, algorithm="fricas")
```

```
[Out] -1/3*(log(a^3 + x^3) - 3*log(x))/a^3
```

Sympy [A] time = 0.254326, size = 19, normalized size = 0.86

$$\frac{\log\left(x\right)}{a^{3}}-\frac{\log\left(a^{3}+x^{3}\right)}{3a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**3+x**3),x)
```

```
[Out] log(x)/a**3 - log(a**3 + x**3)/(3*a**3)
```

GIAC/XCAS [A] time = 0.207743, size = 30, normalized size = 1.36

$$-\frac{\ln\left(\left|a^{3}+x^{3}\right|\right)}{3\,a^{3}}+\frac{\ln\left(\left|x\right|\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(a^3 + x^3))/a^3 + ln(abs(x))/a^3
```

3.122 $\int \frac{1}{x^2(a^3+x^3)}\,dx$

Optimal. Leaf size=63

$$\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}-\frac{1}{a^3x}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}$$

```
[Out] -(1/(a^3*x)) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[
a + x]/(3*a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

Rubi [A] time = 0.0799318, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}-\frac{1}{a^3x}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(1/(a^3*x)) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[
a + x]/(3*a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

Rubi in Sympy [A] time = 6.93739, size = 60, normalized size = 0.95

$$-\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a**3+x**3),x)
```

```
[Out] -1/(a**3*x) + log(a + x)/(3*a**4) - log(a**2 - a*x + x**2)/(6*a**
4) + sqrt(3)*atan(sqrt(3)*(a/3 - 2*x/3)/a)/(3*a**4)
```

Mathematica [A] time = 0.0217771, size = 60, normalized size = 0.95

$$-\frac{x\log\left(a^2-ax+x^2\right)-2x\log(a+x)+2\sqrt{3}x\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)+6a}{6a^4x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(6*a + 2*Sqrt[3]*x*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*x*Log[a +
x] + x*Log[a^2 - a*x + x^2])/(6*a^4*x)
```

Maple [A] time = 0.01, size = 60, normalized size = 1.

$$\frac{\ln(a+x)}{3a^4} - \frac{\ln(a^2-ax+x^2)}{6a^4} - \frac{\sqrt{3}}{3a^4}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right) - \frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a^3+x^3),x)
```

```
[Out] 1/3*ln(a+x)/a^4-1/6*ln(a^2-a*x+x^2)/a^4-1/3/a^4*3^(1/2)*arctan(1/
3*(2*x-a)*3^(1/2)/a)-1/a^3/x
```

Maxima [A] time = 1.50526, size = 77, normalized size = 1.22

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4} + \frac{\log(a+x)}{3a^4} - \frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 -
 a*x + x^2)/a^4 + 1/3*log(a + x)/a^4 - 1/(a^3*x)
```

Fricas [A] time = 0.207766, size = 84, normalized size = 1.33

$$-\frac{\sqrt{3}\left(\sqrt{3}x\log(a^2-ax+x^2) - 2\sqrt{3}x\log(a+x) + 6x\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) + 6\sqrt{3}a\right)}{18a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x*log(a^2 - a*x + x^2) - 2*sqrt(3)*x*log(a
 + x) + 6*x*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 6*sqrt(3)*a)/(a^4*
x)
```

Sympy [A] time = 0.603833, size = 83, normalized size = 1.32

$$-\frac{1}{a^3x}+\frac{\frac{\log(a+x)}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a**3+x**3),x)
```

```
[Out] -1/(a**3*x) + (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(9*a*(-1/6
- sqrt(3)*I/6)**2 + x) + (-1/6 + sqrt(3)*I/6)*log(9*a*(-1/6 + sqr
t(3)*I/6)**2 + x))/a**4
```

GIAC/XCAS [A] time = 0.207033, size = 78, normalized size = 1.24

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^4}-\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^4}+\frac{\ln\left(|a+x|\right)}{3\,a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*ln(a^2 -
a*x + x^2)/a^4 + 1/3*ln(abs(a + x))/a^4 - 1/(a^3*x)
```

3.123 $\int \frac{1}{x^3(a^3+x^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

```
[Out] -1/(2*a^3*x^2) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Lo
g[a + x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Rubi [A] time = 0.0767767, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/(2*a^3*x^2) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Lo
g[a + x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Rubi in Sympy [A] time = 7.28626, size = 63, normalized size = 0.97

$$-\frac{1}{2a^3x^2}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a**3+x**3),x)
```

```
[Out] -1/(2*a**3*x**2) - log(a + x)/(3*a**5) + log(a**2 - a*x + x**2)/(
6*a**5) + sqrt(3)*atan(sqrt(3)*(a/3 - 2*x/3)/a)/(3*a**5)
```

Mathematica [A] time = 0.024095, size = 68, normalized size = 1.05

$$-\frac{\log(a+x)}{3a^5}-\frac{\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/(2*a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - L
og[a + x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Maple [A] time = 0.01, size = 60, normalized size = 0.9

$$-\frac{\ln\left(a+x\right)}{3\,a^{5}}-\frac{1}{2\,a^{3}x^{2}}+\frac{\ln\left(a^{2}-ax+x^{2}\right)}{6\,a^{5}}-\frac{\sqrt{3}}{3\,a^{5}}\arctan\left(\frac{\left(2\,x-a\right)\sqrt{3}}{3\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a^3+x^3),x)
```

```
[Out] -1/3*ln(a+x)/a^5-1/2/a^3/x^2+1/6*ln(a^2-a*x+x^2)/a^5-1/3/a^5*3^(1
/2)*arctan(1/3*(2*x-a)*3^(1/2)/a)
```

Maxima [A] time = 1.54124, size = 77, normalized size = 1.18

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\,x\right)}{3\,a}\right)}{3\,a^{5}}+\frac{\log\left(a^{2}-ax+x^{2}\right)}{6\,a^{5}}-\frac{\log\left(a+x\right)}{3\,a^{5}}-\frac{1}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 -
 a*x + x^2)/a^5 - 1/3*log(a + x)/a^5 - 1/2/(a^3*x^2)
```

Fricas [A] time = 0.209283, size = 95, normalized size = 1.46

$$\frac{\sqrt{3}\left(\sqrt{3}x^{2}\log\left(a^{2}-ax+x^{2}\right)-2\,\sqrt{3}x^{2}\log\left(a+x\right)-6\,x^{2}\arctan\left(-\frac{\sqrt{3}\left(a-2\,x\right)}{3\,a}\right)-3\,\sqrt{3}a^{2}\right)}{18\,a^{5}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*x^2*log(a^2 - a*x + x^2) - 2*sqrt(3)*x^2*lo
g(a + x) - 6*x^2*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - 3*sqrt(3)*a^2
)/(a^5*x^2)
```

Sympy [A] time = 0.652751, size = 80, normalized size = 1.23

$$-\frac{1}{2a^3x^2}+\frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)+x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a**3+x**3),x)
```

```
[Out] -1/(2*a**3*x**2) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(-3*a*
(1/6 - sqrt(3)*I/6) + x) + (1/6 + sqrt(3)*I/6)*log(-3*a*(1/6 + sq
rt(3)*I/6) + x))/a**5
```

GIAC/XCAS [A] time = 0.203779, size = 78, normalized size = 1.2

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^5}+\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^5}-\frac{\ln\left(|a+x|\right)}{3\,a^5}-\frac{1}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*ln(a^2 -
a*x + x^2)/a^5 - 1/3*ln(abs(a + x))/a^5 - 1/2/(a^3*x^2)
```

3.124 $\int \frac{1}{x^4(a^3+x^3)}\,dx$

Optimal. Leaf size=33

$$-\frac{\log(x)}{a^6}-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Rubi [A] time = 0.0417475, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{\log(x)}{a^6}-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Rubi in Sympy [A] time = 3.12443, size = 32, normalized size = 0.97

$$-\frac{1}{3a^3x^3}-\frac{\log\left(x^3\right)}{3a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(a**3+x**3),x)
```

```
[Out] -1/(3*a**3*x**3) - log(x**3)/(3*a**6) + log(a**3 + x**3)/(3*a**6)
```

Mathematica [A] time = 0.00817301, size = 33, normalized size = 1.

$$-\frac{\log(x)}{a^6}-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Maple [A] time = 0.013, size = 43, normalized size = 1.3

$$\frac{\ln\left(a+x\right)}{3\,a^{6}}+\frac{\ln\left(a^{2}-ax+x^{2}\right)}{3\,a^{6}}-\frac{1}{3\,a^{3}x^{3}}-\frac{\ln\left(x\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(a^3+x^3),x)
```

```
[Out] 1/3*ln(a+x)/a^6+1/3/a^6*ln(a^2-a*x+x^2)-1/3/a^3/x^3-ln(x)/a^6
```

Maxima [A] time = 1.34345, size = 42, normalized size = 1.27

$$\frac{\log\left(a^{3}+x^{3}\right)}{3\,a^{6}}-\frac{\log\left(x^{3}\right)}{3\,a^{6}}-\frac{1}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^4),x, algorithm="maxima")
```

```
[Out] 1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)
```

Fricas [A] time = 0.206498, size = 45, normalized size = 1.36

$$\frac{x^{3}\log\left(a^{3}+x^{3}\right)-3\,x^{3}\log\left(x\right)-a^{3}}{3\,a^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)
```

Sympy [A] time = 0.777857, size = 29, normalized size = 0.88

$$-\frac{1}{3a^{3}x^{3}}-\frac{\log\left(x\right)}{a^{6}}+\frac{\log\left(a^{3}+x^{3}\right)}{3a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(a**3+x**3),x)
```

```
[Out] -1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)
```

GIAC/XCAS [A] time = 0.201435, size = 54, normalized size = 1.64

$$\frac{\ln\left(\left|a^3+x^3\right|\right)}{3\,a^6}-\frac{\ln\left(|x|\right)}{a^6}-\frac{a^3-x^3}{3\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(a^3 + x^3))/a^6 - ln(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^
6*x^3)
```

3.125 $\int \frac{1}{x^5(a^3+x^3)}\,dx$

Optimal. Leaf size=73

$$-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}+\frac{1}{a^6x}-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) - ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[
3]*a^7) - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Rubi [A] time = 0.0952836, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}+\frac{1}{a^6x}-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) - ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[
3]*a^7) - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Rubi in Sympy [A] time = 8.53334, size = 70, normalized size = 0.96

$$-\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(a**3+x**3),x)
```

```
[Out] -1/(4*a**3*x**4) + 1/(a**6*x) - log(a + x)/(3*a**7) + log(a**2 -
a*x + x**2)/(6*a**7) - sqrt(3)*atan(sqrt(3)*(a/3 - 2*x/3)/a)/(3*a
**7)
```

Mathematica [A] time = 0.0200911, size = 74, normalized size = 1.01

$$-\frac{\log(a+x)}{3a^7}+\frac{\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{\sqrt{3}a^7}+\frac{1}{a^6x}-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt
[3]*a^7) - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Maple [A] time = 0.013, size = 67, normalized size = 0.9

$$-\frac{\ln\left(a+x\right)}{3\,a^{7}}+\frac{\ln\left(a^{2}-ax+x^{2}\right)}{6\,a^{7}}+\frac{\sqrt{3}}{3\,a^{7}}\arctan\left(\frac{\left(2\,x-a\right)\sqrt{3}}{3\,a}\right)-\frac{1}{4\,a^{3}x^{4}}+\frac{1}{a^{6}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(a^3+x^3),x)
```

```
[Out] -1/3*ln(a+x)/a^7+1/6*ln(a^2-a*x+x^2)/a^7+1/3/a^7*3^(1/2)*arctan(1
/3*(2*x-a)*3^(1/2)/a)-1/4/a^3/x^4+1/a^6/x
```

Maxima [A] time = 1.52047, size = 89, normalized size = 1.22

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^{7}}+\frac{\log\left(a^{2}-ax+x^{2}\right)}{6\,a^{7}}-\frac{\log\left(a+x\right)}{3\,a^{7}}-\frac{a^{3}-4\,x^{3}}{4\,a^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^5),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 -
a*x + x^2)/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

Fricas [A] time = 0.211786, size = 105, normalized size = 1.44

$$\frac{\sqrt{3}\left(2\,\sqrt{3}x^{4}\log\left(a^{2}-ax+x^{2}\right)-4\,\sqrt{3}x^{4}\log\left(a+x\right)+12\,x^{4}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-3\,\sqrt{3}\left(a^{4}-4\,ax^{3}\right)\right)}{36\,a^{7}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^5),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(2*sqrt(3)*x^4*log(a^2 - a*x + x^2) - 4*sqrt(3)*x^4*
log(a + x) + 12*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - 3*sqrt(3)*
(a^4 - 4*a*x^3))/(a^7*x^4)
```

Sympy [A] time = 0.754398, size = 90, normalized size = 1.23

$$\frac{-a^3+4x^3}{4a^6x^4}+\frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(a**3+x**3),x)
```

```
[Out] (-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*
I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*lo
g(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a**7
```

GIAC/XCAS [A] time = 0.202021, size = 90, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^7}+\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^7}-\frac{\ln\left(|a+x|\right)}{3\,a^7}-\frac{a^3-4\,x^3}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^5),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*ln(a^2 - a
*x + x^2)/a^7 - 1/3*ln(abs(a + x))/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x
^4)
```

3.126 $\int \frac{x^{-m}}{a^3+x^3}\,dx$

Optimal. Leaf size=46

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)]
)/(a^3*(1 - m))
```

Rubi [A] time = 0.0295284, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{3};\frac{4-m}{3};-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)]
)/(a^3*(1 - m))
```

Rubi in Sympy [A] time = 2.07274, size = 31, normalized size = 0.67

$$\frac{x^{-m+1}{}_2F_1\left(\begin{matrix}1,-\frac{m}{3}+\frac{1}{3}\\-\frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{x^3}{a^3}\right)}{a^3\left(-m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**m)/(a**3+x**3),x)
```

```
[Out] x**(-m + 1)*hyper((1, -m/3 + 1/3), (-m/3 + 4/3,), -x**3/a**3)/(a*
*3*(-m + 1))
```

Mathematica [A] time = 0.0257698, size = 47, normalized size = 1.02

$$-\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{1-m}{3}+1,-\frac{x^3}{a^3}\right)}{a^3(m-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, 1 + (1 - m)/3, -(x^3
/a^3)])/(a^3*(-1 + m)))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{x^m (a^3 + x^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^m)/(a^3+x^3),x)
```

```
[Out] int(1/(x^m)/(a^3+x^3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-m}}{a^3 + x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^m),x, algorithm="maxima")
```

```
[Out] integrate(x^(-m)/(a^3 + x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a^3 + x^3)x^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^m),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3 + x^3)*x^m), x)
```

Sympy [A] time = 7.5076, size = 92, normalized size = 2.

$$-\frac{mxx^{-m}\left(\frac{x^3e^{i\pi}}{a^3},1,-\frac{m}{3}+\frac{1}{3}\right)\left(-\frac{m}{3}+\frac{1}{3}\right)}{9a^3\left(-\frac{m}{3}+\frac{4}{3}\right)}+\frac{xx^{-m}\left(\frac{x^3e^{i\pi}}{a^3},1,-\frac{m}{3}+\frac{1}{3}\right)\left(-\frac{m}{3}+\frac{1}{3}\right)}{9a^3\left(-\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**m)/(a**3+x**3),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, -m/3 + 1/3)*g
amma(-m/3 + 1/3)/(9*a**3*gamma(-m/3 + 4/3)) + x*x**(-m)*lerchphi(
x**3*exp_polar(I*pi)/a**3, 1, -m/3 + 1/3)*gamma(-m/3 + 1/3)/(9*a*
*3*gamma(-m/3 + 4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^3+x^3)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^3 + x^3)*x^m),x, algorithm="giac")
```

```
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

3.127 $\int \frac{1}{a^4-x^4} \, dx$

Optimal. Leaf size=27

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

```
[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

Rubi [A] time = 0.0195542, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^4 - x^4)^(-1),x]

[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

Rubi in Sympy [A] time = 1.80589, size = 19, normalized size = 0.7

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2a^3}+\frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**4-x**4),x)

[Out] atan(x/a)/(2*a**3) + atanh(x/a)/(2*a**3)
```

Mathematica [A] time = 0.00701467, size = 38, normalized size = 1.41

$$-\frac{\log(a-x)}{4a^3}+\frac{\log(a+x)}{4a^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^4 - x^4)^(-1),x]
```

```
[Out] ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)
```

Maple [A] time = 0.011, size = 33, normalized size = 1.2

$$\frac{\ln(a+x)}{4a^3}+\frac{1}{2a^3}\arctan\left(\frac{x}{a}\right)-\frac{\ln(-a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^4-x^4),x)
```

```
[Out] 1/4*ln(a+x)/a^3+1/2*arctan(x/a)/a^3-1/4/a^3*ln(-a+x)
```

Maxima [A] time = 1.50392, size = 43, normalized size = 1.59

$$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\log(a+x)}{4a^3}-\frac{\log(-a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^4 - x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(a + x)/a^3 - 1/4*log(-a + x)/a^3
```

Fricas [A] time = 0.203108, size = 35, normalized size = 1.3

$$\frac{2\arctan\left(\frac{x}{a}\right)+\log(a+x)-\log(-a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^4 - x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(x/a) + log(a + x) - log(-a + x))/a^3
```

Sympy [A] time = 0.16196, size = 37, normalized size = 1.37

$$-\frac{\frac{\log(-a+x)}{4}-\frac{\log(a+x)}{4}+\frac{i\log(-ia+x)}{4}-\frac{i\log(ia+x)}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**4-x**4),x)
```

```
[Out] -(log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*log(I*a +
x)/4)/a**3
```

GIAC/XCAS [A] time = 0.202625, size = 46, normalized size = 1.7

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\ln\left(|a + x|\right)}{4\,a^3} - \frac{\ln\left(|-a + x|\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^4 - x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x/a)/a^3 + 1/4*ln(abs(a + x))/a^3 - 1/4*ln(abs(-a + x)
)/a^3
```

3.128 $\int \frac{x}{a^4-x^4}\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Rubi [A] time = 0.018065, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^4 - x^4),x]
```

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Rubi in Sympy [A] time = 1.78894, size = 12, normalized size = 0.8

$$\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**4-x**4),x)
```

```
[Out] atanh(x**2/a**2)/(2*a**2)
```

Mathematica [A] time = 0.00490054, size = 15, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^4 - x^4),x]

[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Maple [B] time = 0.008, size = 30, normalized size = 2.

$$\frac{\ln\left(a^2+x^2\right)}{4\,a^2}-\frac{\ln\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^4-x^4),x)

[Out] 1/4/a^2*ln(a^2+x^2)-1/4/a^2*ln(-a^2+x^2)
```

Maxima [A] time = 1.32774, size = 39, normalized size = 2.6

$$\frac{\log\left(a^2+x^2\right)}{4\,a^2}-\frac{\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^4 - x^4),x, algorithm="maxima")

[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2
```

Fricas [A] time = 0.19856, size = 35, normalized size = 2.33

$$\frac{\log\left(a^2+x^2\right)-\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^4 - x^4),x, algorithm="fricas")

[Out] 1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

Sympy [A] time = 0.178762, size = 24, normalized size = 1.6

$$-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**4-x**4),x)
```

```
[Out] -(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

GIAC/XCAS [A] time = 0.203549, size = 41, normalized size = 2.73

$$\frac{\ln\left(a^2 + x^2\right)}{4\,a^2} - \frac{\ln\left(\left|-a^2 + x^2\right|\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^4 - x^4),x, algorithm="giac")
```

```
[Out] 1/4*ln(a^2 + x^2)/a^2 - 1/4*ln(abs(-a^2 + x^2))/a^2
```

3.129 $\int \frac{1}{x(a^4-x^4)} \, dx$

Optimal. Leaf size=24

$$\frac{\log(x)}{a^4} - \frac{\log(a^4 - x^4)}{4a^4}$$

```
[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

Rubi [A] time = 0.0272014, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{\log(x)}{a^4} - \frac{\log(a^4 - x^4)}{4a^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^4 - x^4)),x]

[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

Rubi in Sympy [A] time = 2.59572, size = 22, normalized size = 0.92

$$\frac{\log(x^4)}{4a^4} - \frac{\log(a^4 - x^4)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**4-x**4),x)

[Out] log(x**4)/(4*a**4) - log(a**4 - x**4)/(4*a**4)
```

Mathematica [A] time = 0.00884881, size = 24, normalized size = 1.

$$\frac{\log(x)}{a^4} - \frac{\log(x^4 - a^4)}{4a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)
```

Maple [A] time = 0.01, size = 41, normalized size = 1.7

$$-\frac{\ln(a+x)}{4\,a^4}+\frac{\ln(x)}{a^4}-\frac{\ln\left(a^2+x^2\right)}{4\,a^4}-\frac{\ln(-a+x)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^4-x^4),x)
```

```
[Out] -1/4*ln(a+x)/a^4+ln(x)/a^4-1/4/a^4*ln(a^2+x^2)-1/4/a^4*ln(-a+x)
```

Maxima [A] time = 1.35116, size = 34, normalized size = 1.42

$$-\frac{\log\left(-a^4+x^4\right)}{4\,a^4}+\frac{\log\left(x^4\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(-a^4 + x^4)/a^4 + 1/4*log(x^4)/a^4
```

Fricas [A] time = 0.198396, size = 27, normalized size = 1.12

$$-\frac{\log\left(-a^4+x^4\right)-4\,\log(x)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x),x, algorithm="fricas")
```

```
[Out] -1/4*(log(-a^4 + x^4) - 4*log(x))/a^4
```

Sympy [A] time = 0.311318, size = 19, normalized size = 0.79

$$\frac{\log(x)}{a^4}-\frac{\log\left(-a^4+x^4\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**4-x**4),x)

[Out] log(x)/a**4 - log(-a**4 + x**4)/(4*a**4)
```

GIAC/XCAS [A] time = 0.202234, size = 35, normalized size = 1.46

$$\frac{\ln\left(x^4\right)}{4\,a^4} - \frac{\ln\left(\left|-a^4 + x^4\right|\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x),x, algorithm="giac")

[Out] 1/4*ln(x^4)/a^4 - 1/4*ln(abs(-a^4 + x^4))/a^4
```

3.130 $\int \frac{1}{x^2(a^4-x^4)}\,dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

Rubi [A] time = 0.0391307, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^4 - x^4)),x]
```

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

Rubi in Sympy [A] time = 4.10381, size = 26, normalized size = 0.74

$$-\frac{1}{a^4x}-\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2a^5}+\frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a**4-x**4),x)
```

```
[Out] -1/(a**4*x) - atan(x/a)/(2*a**5) + atanh(x/a)/(2*a**5)
```

Mathematica [A] time = 0.0107853, size = 46, normalized size = 1.31

$$-\frac{\log(a-x)}{4a^5}+\frac{\log(a+x)}{4a^5}-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^4 - x^4)),x]
```

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a +
 x]/(4*a^5)
```

Maple [A] time = 0.012, size = 41, normalized size = 1.2

$$\frac{\ln(a+x)}{4\,a^5}-\frac{1}{a^4x}-\frac{1}{2\,a^5}\arctan\left(\frac{x}{a}\right)-\frac{\ln(-a+x)}{4\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a^4-x^4),x)
```

```
[Out] 1/4*ln(a+x)/a^5-1/a^4/x-1/2*arctan(x/a)/a^5-1/4/a^5*ln(-a+x)
```

Maxima [A] time = 1.52333, size = 54, normalized size = 1.54

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5}+\frac{\log(a+x)}{4\,a^5}-\frac{\log(-a+x)}{4\,a^5}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(a + x)/a^5 - 1/4*log(-a + x)/a^5 -
 1/(a^4*x)
```

Fricas [A] time = 0.21403, size = 49, normalized size = 1.4

$$-\frac{2\,x\arctan\left(\frac{x}{a}\right)-x\log(a+x)+x\log(-a+x)+4\,a}{4\,a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*arctan(x/a) - x*log(a + x) + x*log(-a + x) + 4*a)/(a^5*
x)
```

Sympy [A] time = 0.642451, size = 44, normalized size = 1.26

$$-\frac{1}{a^4x}-\frac{\frac{\log(-a+x)}{4}-\frac{\log(a+x)}{4}-\frac{i\log(-ia+x)}{4}+\frac{i\log(ia+x)}{4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a**4-x**4),x)
```

```
[Out] -1/(a**4*x) - (log(-a + x)/4 - log(a + x)/4 - I*log(-I*a + x)/4 +
 I*log(I*a + x)/4)/a**5
```

GIAC/XCAS [A] time = 0.201303, size = 57, normalized size = 1.63

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5}+\frac{\ln\left(|a+x|\right)}{4\,a^5}-\frac{\ln\left(|-a+x|\right)}{4\,a^5}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/a)/a^5 + 1/4*ln(abs(a + x))/a^5 - 1/4*ln(abs(-a + x
))/a^5 - 1/(a^4*x)
```

3.131 $\int \frac{1}{x^3(a^4-x^4)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}-\frac{1}{2a^4x^2}$$

```
[Out] -1/(2*a^4*x^2) + ArcTanh[x^2/a^2]/(2*a^6)
```

Rubi [A] time = 0.03274, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/(2*a^4*x^2) + ArcTanh[x^2/a^2]/(2*a^6)
```

Rubi in Sympy [A] time = 3.71084, size = 22, normalized size = 0.85

$$-\frac{1}{2a^4x^2}+\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a**4-x**4),x)
```

```
[Out] -1/(2*a**4*x**2) + atanh(x**2/a**2)/(2*a**6)
```

Mathematica [A] time = 0.0108749, size = 50, normalized size = 1.92

$$-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}-\frac{1}{2a^4x^2}+\frac{\log\left(a^2+x^2\right)}{4a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/(2*a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^
2 + x^2]/(4*a^6)
```

Maple [A] time = 0.012, size = 43, normalized size = 1.7

$$-\frac{\ln(a+x)}{4\,a^6}-\frac{1}{2\,a^4x^2}+\frac{\ln\left(a^2+x^2\right)}{4\,a^6}-\frac{\ln(-a+x)}{4\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a^4-x^4),x)
```

```
[Out] -1/4*ln(a+x)/a^6-1/2/a^4/x^2+1/4/a^6*ln(a^2+x^2)-1/4/a^6*ln(-a+x)
```

Maxima [A] time = 1.34828, size = 50, normalized size = 1.92

$$\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(-a^2+x^2\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)
```

Fricas [A] time = 0.211081, size = 55, normalized size = 2.12

$$\frac{x^2\log\left(a^2+x^2\right)-x^2\log\left(-a^2+x^2\right)-2\,a^2}{4\,a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^3),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2*log(a^2 + x^2) - x^2*log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)
```

Sympy [A] time = 0.693625, size = 34, normalized size = 1.31

$$-\frac{1}{2a^4x^2}-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a**4-x**4),x)
```

```
[Out] -1/(2*a**4*x**2) - (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**
6
```

GIAC/XCAS [A] time = 0.204585, size = 51, normalized size = 1.96

$$\frac{\ln\left(a^{2}+x^{2}\right)}{4\,a^{6}}-\frac{\ln\left(\left|-a^{2}+x^{2}\right|\right)}{4\,a^{6}}-\frac{1}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*ln(a^2 + x^2)/a^6 - 1/4*ln(abs(-a^2 + x^2))/a^6 - 1/2/(a^4*x^
2)
```

3.132 $\int \frac{1}{x^4(a^4-x^4)} \, dx$

Optimal. Leaf size=37

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

Rubi [A] time = 0.0321356, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

Rubi in Sympy [A] time = 3.58675, size = 29, normalized size = 0.78

$$-\frac{1}{3a^4x^3}+\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2a^7}+\frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(a**4-x**4),x)
```

```
[Out] -1/(3*a**4*x**3) + atan(x/a)/(2*a**7) + atanh(x/a)/(2*a**7)
```

Mathematica [A] time = 0.010457, size = 48, normalized size = 1.3

$$-\frac{\log(a-x)}{4a^7}+\frac{\log(a+x)}{4a^7}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a
 + x]/(4*a^7)
```

Maple [A] time = 0.013, size = 41, normalized size = 1.1

$$\frac{\ln(a+x)}{4\,a^7} - \frac{1}{3\,a^4x^3} + \frac{1}{2\,a^7}\arctan\left(\frac{x}{a}\right) - \frac{\ln(-a+x)}{4\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(a^4-x^4),x)
```

```
[Out] 1/4*ln(a+x)/a^7-1/3/a^4/x^3+1/2*arctan(x/a)/a^7-1/4/a^7*ln(-a+x)
```

Maxima [A] time = 1.51284, size = 54, normalized size = 1.46

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\log(a+x)}{4\,a^7} - \frac{\log(-a+x)}{4\,a^7} - \frac{1}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(a + x)/a^7 - 1/4*log(-a + x)/a^7 -
1/3/(a^4*x^3)
```

Fricas [A] time = 0.296172, size = 61, normalized size = 1.65

$$\frac{6\,x^3\arctan\left(\frac{x}{a}\right) + 3\,x^3\log(a+x) - 3\,x^3\log(-a+x) - 4\,a^3}{12\,a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(6*x^3*arctan(x/a) + 3*x^3*log(a + x) - 3*x^3*log(-a + x) -
4*a^3)/(a^7*x^3)
```

Sympy [A] time = 0.738934, size = 48, normalized size = 1.3

$$-\frac{1}{3a^4x^3} - \frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} + \frac{i\log(-ia+x)}{4} - \frac{i\log(ia+x)}{4}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(a**4-x**4),x)
```

```
[Out] -1/(3*a**4*x**3) - (log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x
)/4 - I*log(I*a + x)/4)/a**7
```

GIAC/XCAS [A] time = 0.202227, size = 57, normalized size = 1.54

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^{7}}+\frac{\ln\left(|a+x|\right)}{4\,a^{7}}-\frac{\ln\left(|-a+x|\right)}{4\,a^{7}}-\frac{1}{3\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*ln(abs(a + x))/a^7 - 1/4*ln(abs(-a + x)
)/a^7 - 1/3/(a^4*x^3)
```

3.133 $\int \frac{x^{-m}}{a^4-x^4}\,dx$

Optimal. Leaf size=45

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(
a^4*(1 - m))
```

Rubi [A] time = 0.026629, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(
a^4*(1 - m))
```

Rubi in Sympy [A] time = 2.28221, size = 29, normalized size = 0.64

$$\frac{x^{-m+1}{}_2F_1\left(\begin{matrix}1,-\frac{m}{4}+\frac{1}{4}\\-\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{x^4}{a^4}\right)}{a^4\left(-m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**m)/(a**4-x**4),x)
```

```
[Out] x**(-m + 1)*hyper((1, -m/4 + 1/4), (-m/4 + 5/4,), x**4/a**4)/(a**
4*(-m + 1))
```

Mathematica [A] time = 0.0271733, size = 46, normalized size = 1.02

$$-\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{1-m}{4}+1,\frac{x^4}{a^4}\right)}{a^4(m-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, 1 + (1 - m)/4, x^4/a
^4])/(a^4*(-1 + m)))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x^m (a^4 - x^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^m)/(a^4-x^4),x)
```

```
[Out] int(1/(x^m)/(a^4-x^4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-m}}{a^4 - x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^m),x, algorithm="maxima")
```

```
[Out] integrate(x^(-m)/(a^4 - x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a^4 - x^4)x^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^m),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4 - x^4)*x^m), x)
```

Sympy [A] time = 1.51561, size = 95, normalized size = 2.11

$$-\frac{mxx^{-m}\left(\frac{x^{4}e^{2i\pi}}{a^{4}},1,-\frac{m}{4}+\frac{1}{4}\right)\left(-\frac{m}{4}+\frac{1}{4}\right)}{16a^{4}\left(-\frac{m}{4}+\frac{5}{4}\right)}+\frac{xx^{-m}\left(\frac{x^{4}e^{2i\pi}}{a^{4}},1,-\frac{m}{4}+\frac{1}{4}\right)\left(-\frac{m}{4}+\frac{1}{4}\right)}{16a^{4}\left(-\frac{m}{4}+\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**m)/(a**4-x**4),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, -m/4 + 1/4)
*gamma(-m/4 + 1/4)/(16*a**4*gamma(-m/4 + 5/4)) + x*x**(-m)*lerchp
hi(x**4*exp_polar(2*I*pi)/a**4, 1, -m/4 + 1/4)*gamma(-m/4 + 1/4)/
(16*a**4*gamma(-m/4 + 5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^4 - x^4)x^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 - x^4)*x^m),x, algorithm="giac")
```

```
[Out] integrate(1/((a^4 - x^4)*x^m), x)
```

3.134 $\int \frac{x}{a^4+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Rubi [A] time = 0.0165114, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^4 + x^4),x]

[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Rubi in Sympy [A] time = 1.59015, size = 12, normalized size = 0.8

$$\frac{\operatorname{atan}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**4+x**4),x)

[Out] atan(x**2/a**2)/(2*a**2)
```

Mathematica [A] time = 0.00483174, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Maple [A]   time = 0.006, size = 14, normalized size = 0.9

$$\frac{1}{2\,a^{2}}\arctan\left(\frac{x^{2}}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^4+x^4),x)
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Maxima [A]   time = 1.48336, size = 18, normalized size = 1.2

$$\frac{\arctan\left(\frac{x^{2}}{a^{2}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^4 + x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Fricas [A]   time = 0.195442, size = 18, normalized size = 1.2

$$\frac{\arctan\left(\frac{x^{2}}{a^{2}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^4 + x^4),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Sympy [A] time = 0.154332, size = 29, normalized size = 1.93

$$\frac{-\frac{i\log\left(-ia^2+x^2\right)}{4}+\frac{i\log\left(ia^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**4+x**4),x)
```

```
[Out] (-I*log(-I*a**2 + x**2)/4 + I*log(I*a**2 + x**2)/4)/a**2
```

GIAC/XCAS [A] time = 0.201055, size = 18, normalized size = 1.2

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^4 + x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

3.135 $\int \frac{x^2}{a^4+x^4}\,dx$

Optimal. Leaf size=109

$$\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{2\sqrt{2}a}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/a]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*x)
/a]/(2*Sqrt[2]*a) + Log[a^2 - Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a) -
Log[a^2 + Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a)
```

Rubi [A] time = 0.126976, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{2\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^4 + x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/a]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*x)
/a]/(2*Sqrt[2]*a) + Log[a^2 - Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a) -
Log[a^2 + Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a)
```

Rubi in Sympy [A] time = 11.5185, size = 90, normalized size = 0.83

$$\frac{\sqrt{2}\log\left(a^2-\sqrt{2}ax+x^2\right)}{8a}-\frac{\sqrt{2}\log\left(a^2+\sqrt{2}ax+x^2\right)}{8a}-\frac{\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}x}{a}\right)}{4a}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}x}{a}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**4+x**4),x)
```

```
[Out] sqrt(2)*log(a**2 - sqrt(2)*a*x + x**2)/(8*a) - sqrt(2)*log(a**2 +
 sqrt(2)*a*x + x**2)/(8*a) - sqrt(2)*atan(1 - sqrt(2)*x/a)/(4*a)
+ sqrt(2)*atan(1 + sqrt(2)*x/a)/(4*a)
```

Mathematica [A] time = 0.0464372, size = 79, normalized size = 0.72

$$\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)-\log\left(a^2+\sqrt{2}ax+x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)+2\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{4\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^4 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/a] + 2*ArcTan[1 + (Sqrt[2]*x)/a] + Log
[a^2 - Sqrt[2]*a*x + x^2] - Log[a^2 + Sqrt[2]*a*x + x^2])/(4*Sqrt
[2]*a)
```

Maple [A] time = 0.012, size = 101, normalized size = 0.9

$$\frac{\sqrt{2}}{8}\ln\left(1\left(x^2-\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)\left(x^2+\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)^{-1}\right)\frac{1}{\sqrt[4]{a^4}}$$
$$+\frac{\sqrt{2}}{4}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{a^4}}+1\right)\frac{1}{\sqrt[4]{a^4}}+\frac{\sqrt{2}}{4}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{a^4}}-1\right)\frac{1}{\sqrt[4]{a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^4+x^4),x)
```

```
[Out] 1/8*2^(1/2)/(a^4)^(1/4)*ln((x^2-(a^4)^(1/4)*x*2^(1/2)+(a^4)^(1/2)
)/(x^2+(a^4)^(1/4)*x*2^(1/2)+(a^4)^(1/2)))+1/4*2^(1/2)/(a^4)^(1/4
)*arctan(2^(1/2)/(a^4)^(1/4)*x+1)+1/4*2^(1/2)/(a^4)^(1/4)*arctan(
2^(1/2)/(a^4)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^4 + x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210741, size = 297, normalized size = 2.72

$$-\frac{1}{2}\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}}{\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}+2x+2\sqrt{\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^4}}+x^2}}\right)-\frac{1}{2}\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}}{\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}-2x-2\sqrt{-\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^4}}+x^2}}\right)-\frac{1}{8}\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\log\left(\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^4}}+x^2\right)+\frac{1}{8}\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\log\left(-\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^4}}+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^4 + x^4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(a^(-4))^(1/4)*arctan(sqrt(2)*a^4*(a^(-4))^(3/4)/(sq
rt(2)*a^4*(a^(-4))^(3/4) + 2*x + 2*sqrt(sqrt(2)*a^4*(a^(-4))^(3/4
)*x + a^4*sqrt(a^(-4)) + x^2))) - 1/2*sqrt(2)*(a^(-4))^(1/4)*arct
an(-sqrt(2)*a^4*(a^(-4))^(3/4)/(sqrt(2)*a^4*(a^(-4))^(3/4) - 2*x
- 2*sqrt(-sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqrt(a^(-4)) + x^2))
) - 1/8*sqrt(2)*(a^(-4))^(1/4)*log(sqrt(2)*a^4*(a^(-4))^(3/4)*x +
 a^4*sqrt(a^(-4)) + x^2) + 1/8*sqrt(2)*(a^(-4))^(1/4)*log(-sqrt(2
)*a^4*(a^(-4))^(3/4)*x + a^4*sqrt(a^(-4)) + x^2)
```

Sympy [A] time = 0.150857, size = 19, normalized size = 0.17

$$\frac{\operatorname{RootSum}\left(256t^4+1,\left(t\mapsto t\log\left(64t^3a+x\right)\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**4+x**4),x)
```

```
[Out] RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a
```

GIAC/XCAS [A] time = 0.204535, size = 154, normalized size = 1.41

$$\frac{\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{4a^2}+\frac{\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{4a^2}-\frac{\sqrt{2}|a|\ln\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{8a^2}+\frac{\sqrt{2}|a|\ln\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^4 + x^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(
a))/a^2 + 1/4*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a)
- 2*x)/abs(a))/a^2 - 1/8*sqrt(2)*abs(a)*ln(sqrt(2)*x*abs(a) + x^2
 + abs(a)^2)/a^2 + 1/8*sqrt(2)*abs(a)*ln(-sqrt(2)*x*abs(a) + x^2
+ abs(a)^2)/a^2
```

3.136 $\int \frac{1}{a^5+x^5}\,dx$

Optimal. Leaf size=201

$$\frac{\log(a+x)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
 + Sqrt[5])]*a)])/(5*a^4) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(
5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^4) + Log[a
 + x]/(5*a^4) - ((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 +
x^2])/(20*a^4) - ((1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 +
 x^2])/(20*a^4)
```

Rubi [A] time = 0.650836, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$\frac{\log(a+x)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^5 + x^5)^(-1),x]
```


```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
 + Sqrt[5])]*a)])/(5*a^4) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(
5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^4) + Log[a
 + x]/(5*a^4) - ((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 +
x^2])/(20*a^4) - ((1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 +
 x^2])/(20*a^4)
```

Rubi in Sympy [A] time = 144.858, size = 262, normalized size = 1.3

$$\frac{\log(a+x)}{5a^4} - \frac{\left(-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\log\left(a^2+ax\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+x^2\right)}{a^4} - \frac{\left(\frac{1}{20}+\frac{\sqrt{5}}{20}\right)\log\left(a^2+ax\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+x^2\right)}{a^4} + \frac{2\left(-\left(\frac{1}{4}+\frac{\sqrt{5}}{4}\right)^2+1\right)\operatorname{atan}\left(\frac{-\frac{a\left(1+\sqrt{5}\right)}{4}+x}{a\sqrt{-\frac{\sqrt{5}}{4}+\frac{3}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{5}{4}}}\right)}{5a^4\sqrt{-\frac{\sqrt{5}}{4}+\frac{3}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{5}{4}}} + \frac{2\left(-\left(-\frac{\sqrt{5}}{4}+\frac{1}{4}\right)^2+1\right)\operatorname{atan}\left(\frac{\frac{a\left(-1+\sqrt{5}\right)}{4}+x}{a\sqrt{-\frac{\sqrt{5}}{4}+\frac{5}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{3}{4}}}\right)}{5a^4\sqrt{-\frac{\sqrt{5}}{4}+\frac{5}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**5+x**5),x)
```

```
[Out] log(a + x)/(5*a**4) - (-sqrt(5)/20 + 1/20)*log(a**2 + a*x*(-1/2 +
 sqrt(5)/2) + x**2)/a**4 - (1/20 + sqrt(5)/20)*log(a**2 + a*x*(-s
qrt(5)/2 - 1/2) + x**2)/a**4 + 2*(-(1/4 + sqrt(5)/4)**2 + 1)*atan
((-a*(1 + sqrt(5))/4 + x)/(a*sqrt(-sqrt(5)/4 + 3/4)*sqrt(sqrt(5)/
4 + 5/4)))/(5*a**4*sqrt(-sqrt(5)/4 + 3/4)*sqrt(sqrt(5)/4 + 5/4))
+ 2*(-(-sqrt(5)/4 + 1/4)**2 + 1)*atan((a*(-1 + sqrt(5))/4 + x)/(a
*sqrt(-sqrt(5)/4 + 5/4)*sqrt(sqrt(5)/4 + 3/4)))/(5*a**4*sqrt(-sqr
t(5)/4 + 5/4)*sqrt(sqrt(5)/4 + 3/4))
```

Mathematica [A] time = 0.29973, size = 204, normalized size = 1.01

$$-\frac{-\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\left(1\right.\right.}{20a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^5 + x^5)^(-1),x]
```

```
[Out] -(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[
2*(5 + Sqrt[5])]*a)] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt
[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2
 + ((-1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[
5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt
[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^4)
```

Maple [C] time = 0.046, size = 101, normalized size = 0.5

$$\frac{\ln(a+x)}{5\,a^4}+\frac{1}{5\,a^4}\sum_{\_R=RootOf\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3\_R\,a^2+4\,a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^5+x^5),x)
```

```
[Out] 1/5*ln(a+x)/a^4+1/5/a^4*sum((-_R^3+2*_R^2*a-3*_R*a^2+4*a^3)/(4*_R
^3-3*_R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2
-_Z*a^3+a^4))
```

Maxima [A] time = 1.4789, size = 385, normalized size = 1.92

$$\frac{\sqrt{5}\left(\sqrt{5}-1\right)\log\left(\frac{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{10\left(a^{5}\right)^{\frac{4}{5}}\sqrt{2\sqrt{5}-10}}+\frac{\sqrt{5}\left(\sqrt{5}+1\right)\log\left(\frac{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{10\left(a^{5}\right)^{\frac{4}{5}}\sqrt{-2\sqrt{5}-10}}-\frac{\left(\sqrt{5}+3\right)\log\left(-\left(a^{5}\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^{2}+2\left(a^{5}\right)^{\frac{2}{5}}\right)}{10\left(a^{5}\right)^{\frac{4}{5}}\left(\sqrt{5}+1\right)}-\frac{\left(\sqrt{5}-3\right)\log\left(\left(a^{5}\right)^{\frac{1}{5}}x\left(\sqrt{5}-1\right)+2x^{2}+2\left(a^{5}\right)^{\frac{2}{5}}\right)}{10\left(a^{5}\right)^{\frac{4}{5}}\left(\sqrt{5}-1\right)}+\frac{\log\left(x+\left(a^{5}\right)^{\frac{1}{5}}\right)}{5\left(a^{5}\right)^{\frac{4}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^5 + x^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*(sqrt(5) - 1)*log(((a^5)^(1/5)*(sqrt(5) + 1) - 4*x +
 (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) - 4
*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(4/5)*sqrt(2*sqrt(
5) - 10)) + 1/10*sqrt(5)*(sqrt(5) + 1)*log(((a^5)^(1/5)*(sqrt(5)
- 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^(1/5)*(sqr
t(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/((a^5)^(4/5
)*sqrt(-2*sqrt(5) - 10)) - 1/10*(sqrt(5) + 3)*log(-(a^5)^(1/5)*x*
(sqrt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(4/5)*(sqrt(5) + 1)
) - 1/10*(sqrt(5) - 3)*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 +
2*(a^5)^(2/5))/((a^5)^(4/5)*(sqrt(5) - 1)) + 1/5*log(x + (a^5)^(1
/5))/(a^5)^(4/5)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^5 + x^5),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.158108, size = 39, normalized size = 0.19

$$\frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t\log(5ta + x)\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**5+x**5),x)
```

```
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t +
 1, Lambda(_t, _t*log(5*_t*a + x))))/a**4
```

GIAC/XCAS [A] time = 0.216174, size = 239, normalized size = 1.19

$$\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^4} + \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^4} - \frac{\sqrt{5}\ln\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^4} + \frac{\sqrt{5}\ln\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^4} - \frac{\ln\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20a^4} + \frac{\ln\left(|a+x|\right)}{5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^5 + x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(
2*sqrt(5) + 10)))/a^4 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sq
rt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^4 - 1/20*sqrt(5)*l
n(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^4 + 1/20*sqrt(5)*ln(a^2 +
1/2*(sqrt(5)*a - a)*x + x^2)/a^4 - 1/20*ln(abs(a^4 - a^3*x + a^2*
x^2 - a*x^3 + x^4))/a^4 + 1/5*ln(abs(a + x))/a^4
```

3.137 $\int \frac{x}{a^5+x^5}\,dx$

Optimal. Leaf size=201

$$-\frac{\log(a+x)}{5a^3}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}$$

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
+ Sqrt[5])]*a)])/(5*a^3) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5
 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^3) - Log[a
+ x]/(5*a^3) + ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x
^2])/(20*a^3) + ((1 - Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 +
x^2])/(20*a^3)
```

Rubi [A] time = 0.572741, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$

$$-\frac{\log(a+x)}{5a^3}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^5 + x^5),x]
```


```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
+ Sqrt[5])]*a)])/(5*a^3) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5
 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^3) - Log[a
+ x]/(5*a^3) + ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x
^2])/(20*a^3) + ((1 - Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 +
x^2])/(20*a^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**5+x**5),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.132639, size = 204, normalized size = 1.01

$$\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)-\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\left(1+\right.\right.}{20a^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(a^5 + x^5),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*
(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[
5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2
+ ((-1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5
])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[
5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3)
```

Maple [C] time = 0.011, size = 97, normalized size = 0.5

$$-\frac{\ln(a+x)}{5\,a^3}+\frac{1}{5\,a^3}\sum_{\_R=RootOf\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\,\_R^2a+3\,\_R\,a^2+a^3\right)\ln(x-\_R)}{4\,\_R^3-3\,\_R^2a+2\,\_R\,a^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^5+x^5),x)
```

```
[Out] -1/5*ln(a+x)/a^3+1/5/a^3*sum((_R^3-2*_R^2*a+3*_R*a^2+a^3)/(4*_R^3
-3*_R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_
Z*a^3+a^4))
```

Maxima [A] time = 1.54449, size = 358, normalized size = 1.78

$$-\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{5\left(a^5\right)^{\frac{3}{5}}}+\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{3}{5}}\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{3}{5}}\sqrt{-2\sqrt{5}-10}}-\frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{5\left(a^5\right)^{\frac{3}{5}}\left(\sqrt{5}+1\right)}+\frac{\log\left(\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}-1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{5\left(a^5\right)^{\frac{3}{5}}\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^5 + x^5),x, algorithm="maxima")
```

```
[Out] -1/5*log(x + (a^5)^(1/5))/(a^5)^(3/5) + 1/5*sqrt(5)*log(((a^5)^(1
/5)*(sqrt(5) + 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5
)^(1/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/
((a^5)^(3/5)*sqrt(2*sqrt(5) - 10)) - 1/5*sqrt(5)*log(((a^5)^(1/5)
*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^
(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/(
(a^5)^(3/5)*sqrt(-2*sqrt(5) - 10)) - log(-(a^5)^(1/5)*x*(sqrt(5)
+ 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*((5*sqrt(5)) + 5)) + 1
/5*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5
)^(3/5)*(sqrt(5) - 1))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^5 + x^5),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.156183, size = 41, normalized size = 0.2

$$\frac{-\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(-125t^3a+x\right)\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t
+ 1, Lambda(_t, _t*log(-125*_t**3*a + x))))/a**3
```

GIAC/XCAS [A] time = 0.222489, size = 239, normalized size = 1.19

$$-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^3}+\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^3}-\frac{\sqrt{5}\ln\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^3}+\frac{\sqrt{5}\ln\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^3}+\frac{\ln\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20a^3}-\frac{\ln\left(\left|a+x\right|\right)}{5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a^5 + x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqr
t(2*sqrt(5) + 10)))/a^3 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(s
qrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^3 - 1/20*sqrt(5)*
ln(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^3 + 1/20*sqrt(5)*ln(a^2 +
 1/2*(sqrt(5)*a - a)*x + x^2)/a^3 + 1/20*ln(abs(a^4 - a^3*x + a^2
*x^2 - a*x^3 + x^4))/a^3 - 1/5*ln(abs(a + x))/a^3
```

3.138 $\int \frac{x^2}{a^5+x^5}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^2}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2}$$

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
+ Sqrt[5])]*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5
 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a
+ x]/(5*a^2) - ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x
^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 +
x^2])/(20*a^2)
```

Rubi [A] time = 0.693589, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^2}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
+ Sqrt[5])]*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5
 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a
+ x]/(5*a^2) - ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x
^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 +
x^2])/(20*a^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**5+x**5),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.251289, size = 204, normalized size = 1.01

$$-\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)-\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\left(1+\right.\right.}{20a^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/(a^5 + x^5),x]
```

```
[Out] -(2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*
(5 + Sqrt[5])]*a)] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[
5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2
+ ((-1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5
])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[
5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2)
```

Maple [C] time = 0.01, size = 101, normalized size = 0.5

$$\frac{\ln(a+x)}{5\,a^2}+\frac{1}{5\,a^2}\sum_{\_R=RootOf\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a+2\_R\,a^2-a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^5+x^5),x)
```

```
[Out] 1/5*ln(a+x)/a^2+1/5/a^2*sum((-_R^3+2*_R^2*a+2*_R*a^2-a^3)/(4*_R^3
-3*_R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_
Z*a^3+a^4))
```

Maxima [A] time = 1.52918, size = 356, normalized size = 1.77

$$\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{5\left(a^5\right)^{\frac{2}{5}}}+\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{5\left(a^5\right)^{\frac{2}{5}}\left(\sqrt{5}+1\right)}-\frac{\log\left(\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}-1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{5\left(a^5\right)^{\frac{2}{5}}\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^5 + x^5),x, algorithm="maxima")
```

```
[Out] 1/5*log(x + (a^5)^(1/5))/(a^5)^(2/5) + 1/5*sqrt(5)*log(((a^5)^(1/
5)*(sqrt(5) + 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)
^(1/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/(
(a^5)^(2/5)*sqrt(2*sqrt(5) - 10)) - 1/5*sqrt(5)*log(((a^5)^(1/5)*
(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^(
1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/((
a^5)^(2/5)*sqrt(-2*sqrt(5) - 10)) + log(-(a^5)^(1/5)*x*(sqrt(5) +
 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*((5*sqrt(5)) + 5)) - 1/
5*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)
^(2/5)*(sqrt(5) - 1))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^5 + x^5),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.174824, size = 41, normalized size = 0.2

$$\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**5+x**5),x)
```

```
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t +
 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**2
```

GIAC/XCAS [A] time = 0.228394, size = 239, normalized size = 1.19

$$-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^2}+\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^2}+\frac{\sqrt{5}\ln\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^2}-\frac{\sqrt{5}\ln\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^2}-\frac{\ln\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20a^2}+\frac{\ln\left(\left|a+x\right|\right)}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^5 + x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqr
t(2*sqrt(5) + 10)))/a^2 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(s
qrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^2 + 1/20*sqrt(5)*
ln(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^2 - 1/20*sqrt(5)*ln(a^2 +
 1/2*(sqrt(5)*a - a)*x + x^2)/a^2 - 1/20*ln(abs(a^4 - a^3*x + a^2
*x^2 - a*x^3 + x^4))/a^2 + 1/5*ln(abs(a + x))/a^2
```

3.139 $\int \frac{x^3}{a^5+x^5}\,dx$

Optimal. Leaf size=201

$$\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}-\frac{\log(a+x)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a}$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
 + Sqrt[5])]*a)])/(5*a) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5
+ Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a) - Log[a + x
]/(5*a) + ((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/
(20*a) + ((1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(
20*a)
```

Rubi [A] time = 0.678689, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}-\frac{\log(a+x)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^5 + x^5),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5
 + Sqrt[5])]*a)])/(5*a) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5
+ Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a) - Log[a + x
]/(5*a) + ((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/
(20*a) + ((1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(
20*a)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a**5+x**5),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.18976, size = 204, normalized size = 1.01

$$\frac{-\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\left(1+\right.\right.}{20a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^5 + x^5),x]
```

```
[Out] (2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*
(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5
])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 +
 ((-1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5]
)*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5
]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

Maple [C] time = 0.011, size = 97, normalized size = 0.5

$$-\frac{\ln(a+x)}{5\,a}+\frac{1}{5\,a}\sum_{\_R=RootOf\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3+3\,\_R^2a-2\,\_R\,a^2+a^3\right)\ln(x-\_R)}{4\,\_R^3-3\,\_R^2a+2\,\_R\,a^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a^5+x^5),x)
```

```
[Out] -1/5*ln(a+x)/a+1/5/a*sum((_R^3+3*_R^2*a-2*_R*a^2+a^3)/(4*_R^3-3*_
R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^
3+a^4))
```

Maxima [A] time = 1.51521, size = 385, normalized size = 1.92

$$\frac{\sqrt{5}\left(\sqrt{5}-1\right)\log\left(\frac{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4\,x+\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}}{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4\,x-\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}}\right)}{10\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}}+\frac{\sqrt{5}\left(\sqrt{5}+1\right)\log\left(\frac{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4\,x-\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}}{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4\,x+\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}}\right)}{10\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-\left(a^{5}\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2\,x^{2}+2\left(a^{5}\right)^{\frac{2}{5}}\right)}{10\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)}+\frac{\left(\sqrt{5}-3\right)\log\left(\left(a^{5}\right)^{\frac{1}{5}}x\left(\sqrt{5}-1\right)+2\,x^{2}+2\left(a^{5}\right)^{\frac{2}{5}}\right)}{10\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)}-\frac{\log\left(x+\left(a^{5}\right)^{\frac{1}{5}}\right)}{5\left(a^{5}\right)^{\frac{1}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a^5 + x^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*(sqrt(5) - 1)*log(((a^5)^(1/5)*(sqrt(5) + 1) - 4*x +
 (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) - 4
*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(1/5)*sqrt(2*sqrt(
5) - 10)) + 1/10*sqrt(5)*(sqrt(5) + 1)*log(((a^5)^(1/5)*(sqrt(5)
- 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^(1/5)*(sqr
t(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/((a^5)^(1/5
)*sqrt(-2*sqrt(5) - 10)) + 1/10*(sqrt(5) + 3)*log(-(a^5)^(1/5)*x*
(sqrt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(1/5)*(sqrt(5) + 1)
) + 1/10*(sqrt(5) - 3)*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 +
2*(a^5)^(2/5))/((a^5)^(1/5)*(sqrt(5) - 1)) - 1/5*log(x + (a^5)^(1
/5))/(a^5)^(1/5)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a^5 + x^5),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.168115, size = 39, normalized size = 0.19

$$\frac{-\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 - 125t^3 + 25t^2 - 5t + 1, \left(t \mapsto t\log\left(625t^4a + x\right)\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t
+ 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a
```

GIAC/XCAS [A] time = 0.219072, size = 239, normalized size = 1.19

$$\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a} + \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a} + \frac{\sqrt{5}\ln\left(a^2 - \frac{1}{2}\left(\sqrt{5}a + a\right)x + x^2\right)}{20a} - \frac{\sqrt{5}\ln\left(a^2 + \frac{1}{2}\left(\sqrt{5}a - a\right)x + x^2\right)}{20a} + \frac{\ln\left(\left|a^4 - a^3x + a^2x^2 - ax^3 + x^4\right|\right)}{20a} - \frac{\ln\left(\left|a + x\right|\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a^5 + x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(
2*sqrt(5) + 10)))/a + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt
(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a + 1/20*sqrt(5)*ln(a^
2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a - 1/20*sqrt(5)*ln(a^2 + 1/2*(s
qrt(5)*a - a)*x + x^2)/a + 1/20*ln(abs(a^4 - a^3*x + a^2*x^2 - a*
x^3 + x^4))/a - 1/5*ln(abs(a + x))/a
```

3.140 $\int \frac{x^4}{a^5+x^5} \, dx$

Optimal. Leaf size=12

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

```
[Out] Log[a^5 + x^5]/5
```

Rubi [A] time = 0.00592801, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^5 + x^5),x]

[Out] Log[a^5 + x^5]/5
```

Rubi in Sympy [A] time = 0.901328, size = 8, normalized size = 0.67

$$\frac{\log\left(a^5+x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(a**5+x**5),x)

[Out] log(a**5 + x**5)/5
```

Mathematica [A] time = 0.00356109, size = 12, normalized size = 1.

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^5 + x^5),x]
```

```
[Out] Log[a^5 + x^5]/5
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$\frac{\ln\left(a^5+x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(a^5+x^5),x)
```

```
[Out] 1/5*ln(a^5+x^5)
```

Maxima [A] time = 1.34269, size = 14, normalized size = 1.17

$$\frac{1}{5}\,\log\left(a^5+x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a^5 + x^5),x, algorithm="maxima")
```

```
[Out] 1/5*log(a^5 + x^5)
```

Fricas [A] time = 0.206842, size = 14, normalized size = 1.17

$$\frac{1}{5}\,\log\left(a^5+x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a^5 + x^5),x, algorithm="fricas")
```

```
[Out] 1/5*log(a^5 + x^5)
```

Sympy [A] time = 0.117478, size = 8, normalized size = 0.67

$$\frac{\log\left(a^5+x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(a**5+x**5),x)
```

```
[Out] log(a**5 + x**5)/5
```

GIAC/XCAS [A] time = 0.203886, size = 15, normalized size = 1.25

$$\frac{1}{5}\ln\left(\left|a^{5}+x^{5}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a^5 + x^5),x, algorithm="giac")
```

```
[Out] 1/5*ln(abs(a^5 + x^5))
```

3.141 $\int \frac{1}{x(a^5+x^5)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Rubi [A] time = 0.0236483, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^5 + x^5)),x]
```

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Rubi in Sympy [A] time = 2.3961, size = 22, normalized size = 1.

$$\frac{\log\left(x^5\right)}{5a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**5+x**5),x)
```

```
[Out] log(x**5)/(5*a**5) - log(a**5 + x**5)/(5*a**5)
```

Mathematica [A] time = 0.00616191, size = 22, normalized size = 1.

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^5 + x^5)),x]
```

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Maple [B] time = 0.01, size = 49, normalized size = 2.2

$$-\frac{\ln(a+x)}{5\,a^5}+\frac{\ln(x)}{a^5}-\frac{\ln\left(a^4-a^3x+a^2x^2-ax^3+x^4\right)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^5+x^5),x)
```

```
[Out] -1/5*ln(a+x)/a^5+ln(x)/a^5-1/5/a^5*ln(a^4-a^3*x+a^2*x^2-a*x^3+x^4
)
```

Maxima [A] time = 1.33773, size = 31, normalized size = 1.41

$$-\frac{\log\left(a^5+x^5\right)}{5\,a^5}+\frac{\log\left(x^5\right)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x),x, algorithm="maxima")
```

```
[Out] -1/5*log(a^5 + x^5)/a^5 + 1/5*log(x^5)/a^5
```

Fricas [A] time = 0.206647, size = 24, normalized size = 1.09

$$-\frac{\log\left(a^5+x^5\right)-5\,\log(x)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x),x, algorithm="fricas")
```

```
[Out] -1/5*(log(a^5 + x^5) - 5*log(x))/a^5
```

Sympy [A] time = 0.322804, size = 19, normalized size = 0.86

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**5+x**5),x)
```

```
[Out] log(x)/a**5 - log(a**5 + x**5)/(5*a**5)
```

GIAC/XCAS [A] time = 0.207787, size = 30, normalized size = 1.36

$$-\frac{\ln\left(\left|a^5 + x^5\right|\right)}{5\,a^5} + \frac{\ln\left(|x|\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x),x, algorithm="giac")
```

```
[Out] -1/5*ln(abs(a^5 + x^5))/a^5 + ln(abs(x))/a^5
```

3.142 $\int \frac{1}{x^2(a^5+x^5)}\,dx$

Optimal. Leaf size=209

$$\frac{\log(a+x)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}-\frac{1}{a^5x}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}$$

```
[Out] -(1/(a^5*x)) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4
*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^6) + (Sqrt[(5 - Sqrt[5])/2]*
ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(
5*a^6) + Log[a + x]/(5*a^6) - ((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt
[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + Sqrt[5])*Log[a^2 - ((1 + Sqr
t[5])*a*x)/2 + x^2])/(20*a^6)
```

Rubi [A] time = 0.685276, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$\frac{\log(a+x)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}-\frac{1}{a^5x}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^5 + x^5)),x]
```


```
[Out] -(1/(a^5*x)) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4
*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^6) + (Sqrt[(5 - Sqrt[5])/2]*
ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(
5*a^6) + Log[a + x]/(5*a^6) - ((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt
[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + Sqrt[5])*Log[a^2 - ((1 + Sqr
t[5])*a*x)/2 + x^2])/(20*a^6)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a**5+x**5),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.394745, size = 172, normalized size = 0.82

$$-\frac{-\left(\sqrt{5}-1\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\frac{20a}{x}-4\log(a+x)+2\sqrt{2\left(5+\sqrt{5}\right.}}{20a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4
*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-
((1 + Sqrt[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x]
 - (-1 + Sqrt[5])*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (1 +
Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^6)
```

Maple [C] time = 0.014, size = 109, normalized size = 0.5

$$\frac{\ln(a+x)}{5a^6}-\frac{1}{a^5x}+\frac{1}{5a^6}\sum_{\_R=RootOf\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3-3\_R^2a+2\_Ra^2-a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a^5+x^5),x)
```

```
[Out] 1/5*ln(a+x)/a^6-1/a^5/x+1/5/a^6*sum((-_R^3-3*_R^2*a+2*_R*a^2-a^3)
/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z
^2*a^2-_Z*a^3+a^4))
```

Maxima [A] time = 1.56438, size = 398, normalized size = 1.9

$$-\frac{\frac{\sqrt{5}\left(\sqrt{5}-1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}+\frac{\sqrt{5}\left(\sqrt{5}+1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)}+\frac{\left(\sqrt{5}\right.}{}}{10a^5}$$

$$-\frac{1}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^2),x, algorithm="maxima")
```

```
[Out] -1/10*(sqrt(5)*(sqrt(5) - 1)*log(((a^5)^(1/5)*(sqrt(5) + 1) - 4*x
 + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) -
 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(1/5)*sqrt(2*sqr
t(5) - 10)) + sqrt(5)*(sqrt(5) + 1)*log(((a^5)^(1/5)*(sqrt(5) - 1
) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5
) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/((a^5)^(1/5)*s
qrt(-2*sqrt(5) - 10)) + (sqrt(5) + 3)*log(-(a^5)^(1/5)*x*(sqrt(5)
 + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(1/5)*(sqrt(5) + 1)) + (sqr
t(5) - 3)*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5)
)/((a^5)^(1/5)*(sqrt(5) - 1)) - 2*log(x + (a^5)^(1/5))/(a^5)^(1/5
))/a^5 - 1/(a^5*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.656655, size = 48, normalized size = 0.23

$$-\frac{1}{a^5 x} + \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t \log\left(625t^4 a + x\right)\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a**5+x**5),x)
```

```
[Out] -1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*
_t**2 + 5*_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6
```

GIAC/XCAS [A] time = 0.211933, size = 250, normalized size = 1.2

$$-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^6}-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^6}-\frac{\sqrt{5}\ln\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^6}+\frac{\sqrt{5}\ln\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^6}-\frac{\ln\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20\,a^6}+\frac{\ln\left(\left|a+x\right|\right)}{5\,a^6}-\frac{1}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt
(2*sqrt(5) + 10)))/a^6 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(s
qrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*
ln(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^6 + 1/20*sqrt(5)*ln(a^2 +
 1/2*(sqrt(5)*a - a)*x + x^2)/a^6 - 1/20*ln(abs(a^4 - a^3*x + a^2
*x^2 - a*x^3 + x^4))/a^6 + 1/5*ln(abs(a + x))/a^6 - 1/(a^5*x)
```

3.143 $\int \frac{1}{x^3(a^5+x^5)}\,dx$

Optimal. Leaf size=211

$$-\frac{\log(a+x)}{5a^7}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^7}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7}-\frac{1}{2a^5x^2}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}$$

```
[Out] -1/(2*a^5*x^2) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a -
 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^7) + (Sqrt[(5 + Sqrt[5])/2
]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])
/(5*a^7) - Log[a + x]/(5*a^7) + ((1 + Sqrt[5])*Log[a^2 - ((1 - Sq
rt[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - Sqrt[5])*Log[a^2 - ((1 + S
qrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Rubi [A] time = 0.706152, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$-\frac{\log(a+x)}{5a^7}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^7}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7}-\frac{1}{2a^5x^2}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/(2*a^5*x^2) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a -
 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^7) + (Sqrt[(5 + Sqrt[5])/2
]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])
/(5*a^7) - Log[a + x]/(5*a^7) + ((1 + Sqrt[5])*Log[a^2 - ((1 - Sq
rt[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - Sqrt[5])*Log[a^2 - ((1 + S
qrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a**5+x**5),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.357117, size = 174, normalized size = 0.82

$$-\frac{\frac{10a^2}{x^2}-\left(1+\sqrt{5}\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\left(\sqrt{5}-1\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+4\log(a+x)-2\sqrt{10-2\sqrt{5}}\,\mathrm{ta}}{20a^7}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -((10*a^2)/x^2 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a
+ 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTa
n[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 4*Log[a
+ x] - (1 + Sqrt[5])*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (-
1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Maple [C] time = 0.013, size = 105, normalized size = 0.5

$$-\frac{\ln(a+x)}{5\,a^7}-\frac{1}{2\,a^5x^2}+\frac{1}{5\,a^7}\sum_{\_R=RootOf\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a-2\_R\,a^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a^5+x^5),x)
```

```
[Out] -1/5*ln(a+x)/a^7-1/2/a^5/x^2+1/5/a^7*sum((_R^3-2*_R^2*a-2*_R*a^2+
a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*
a+_Z^2*a^2-_Z*a^3+a^4))
```

Maxima [A] time = 1.5351, size = 374, normalized size = 1.77

$$-\frac{\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{\left(a^5\right)^{\frac{2}{5}}}+\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{\left(a^5\right)^{\frac{2}{5}}\left(\sqrt{5}+1\right)}-\frac{\log\left(\left(a^5\right)\right.}{}}{5\,a^5}$$

$$-\frac{1}{2\,a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^3),x, algorithm="maxima")
```

```
[Out] -1/5*(log(x + (a^5)^(1/5))/(a^5)^(2/5) + sqrt(5)*log(((a^5)^(1/5)
*(sqrt(5) + 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(
1/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a
^5)^(2/5)*sqrt(2*sqrt(5) - 10)) - sqrt(5)*log(((a^5)^(1/5)*(sqrt(
5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^(1/5)*(
sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/((a^5)^(
2/5)*sqrt(-2*sqrt(5) - 10)) + 5*log(-(a^5)^(1/5)*x*(sqrt(5) + 1)
+ 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*((5*sqrt(5)) + 5)) - log((a
^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*(
sqrt(5) - 1)))/a^5 - 1/2/(a^5*x^2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.728488, size = 51, normalized size = 0.24

$$-\frac{1}{2a^5x^2}+\frac{-\frac{\log{(a+x)}}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log{\left(25t^2a+x\right)}\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a**5+x**5),x)
```

```
[Out] -1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3
 + 25*_t**2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7
```

GIAC/XCAS [A] time = 0.215921, size = 250, normalized size = 1.18

$$\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^7}-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^7}-\frac{\sqrt{5}\ln\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^7}+\frac{\sqrt{5}\ln\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^7}+\frac{\ln\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20\,a^7}-\frac{\ln\left(|a+x|\right)}{5\,a^7}-\frac{1}{2\,a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^3),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt
(2*sqrt(5) + 10)))/a^7 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sq
rt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*l
n(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^7 + 1/20*sqrt(5)*ln(a^2 +
1/2*(sqrt(5)*a - a)*x + x^2)/a^7 + 1/20*ln(abs(a^4 - a^3*x + a^2*
x^2 - a*x^3 + x^4))/a^7 - 1/5*ln(abs(a + x))/a^7 - 1/2/(a^5*x^2)
```

3.144 $\int \frac{1}{x^4(a^5+x^5)}\,dx$

Optimal. Leaf size=211

$$\frac{\log(a+x)}{5a^8}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}$$

```
[Out] -1/(3*a^5*x^3) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a -
 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^8) + (Sqrt[(5 + Sqrt[5])/2
]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])
/(5*a^8) + Log[a + x]/(5*a^8) - ((1 + Sqrt[5])*Log[a^2 - ((1 - Sq
rt[5])*a*x)/2 + x^2])/(20*a^8) - ((1 - Sqrt[5])*Log[a^2 - ((1 + S
qrt[5])*a*x)/2 + x^2])/(20*a^8)
```

Rubi [A] time = 0.633135, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$\frac{\log(a+x)}{5a^8}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] -1/(3*a^5*x^3) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a -
 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^8) + (Sqrt[(5 + Sqrt[5])/2
]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])
/(5*a^8) + Log[a + x]/(5*a^8) - ((1 + Sqrt[5])*Log[a^2 - ((1 - Sq
rt[5])*a*x)/2 + x^2])/(20*a^8) - ((1 - Sqrt[5])*Log[a^2 - ((1 + S
qrt[5])*a*x)/2 + x^2])/(20*a^8)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(a**5+x**5),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.351812, size = 175, normalized size = 0.83

$$\frac{-\frac{20a^3}{x^3} - 3\left(1+\sqrt{5}\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right) + 3\left(\sqrt{5}-1\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right) + 12\log(a+x) + 6\sqrt{10-2\sqrt{\ldots}}}{60a^8}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] ((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a
+ 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTa
n[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a
 + x] - 3*(1 + Sqrt[5])*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] +
 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(60*a^8
)
```

Maple [C] time = 0.012, size = 109, normalized size = 0.5

$$\frac{\ln(a+x)}{5a^8} - \frac{1}{3a^5x^3} + \frac{1}{5a^8}\sum_{\_R=RootOf(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4)} \frac{\left(-\_R^3+2\_R^2a-3\_Ra^2-a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(a^5+x^5),x)
```

```
[Out] 1/5*ln(a+x)/a^8-1/3/a^5/x^3+1/5/a^8*sum((-_R^3+2*_R^2*a-3*_R*a^2-
a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln(x-_R),_R=RootOf(_Z^4-_Z^3*
a+_Z^2*a^2-_Z*a^3+a^4))
```

Maxima [A] time = 1.51066, size = 374, normalized size = 1.77

$$\frac{\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{\left(a^5\right)^{\frac{3}{5}}} - \frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{3}{5}}\sqrt{2\sqrt{5}-10}} + \frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{3}{5}}\sqrt{-2\sqrt{5}-10}} + \frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{\left(a^5\right)^{\frac{3}{5}}\left(\sqrt{5}+1\right)} - \frac{\log\left(\left(a^5\right)^{\frac{1}{5}}\ldots\right.}{\ldots}}{5a^5}$$

$$-\frac{1}{3a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^4),x, algorithm="maxima")
```

```
[Out] 1/5*(log(x + (a^5)^(1/5))/(a^5)^(3/5) - sqrt(5)*log(((a^5)^(1/5)*
(sqrt(5) + 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1
/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^
5)^(3/5)*sqrt(2*sqrt(5) - 10)) + sqrt(5)*log(((a^5)^(1/5)*(sqrt(5
) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10))/((a^5)^(1/5)*(s
qrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))/((a^5)^(3
/5)*sqrt(-2*sqrt(5) - 10)) + 5*log(-(a^5)^(1/5)*x*(sqrt(5) + 1) +
 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*((5*sqrt(5)) + 5)) - log((a^
5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*(s
qrt(5) - 1)))/a^5 - 1/3/(a^5*x^3)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^4),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.80284, size = 51, normalized size = 0.24

$$-\frac{1}{3a^5x^3}+\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(125t^3a+x\right)\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(a**5+x**5),x)
```

```
[Out] -1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3
+ 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8
```

GIAC/XCAS [A] time = 0.220392, size = 250, normalized size = 1.18

$$\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^8}-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^8}+\frac{\sqrt{5}\ln\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^8}-\frac{\sqrt{5}\ln\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^8}-\frac{\ln\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20a^8}+\frac{\ln\left(\left|a+x\right|\right)}{5a^8}-\frac{1}{3a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^4),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt
(2*sqrt(5) + 10)))/a^8 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sq
rt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*l
n(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^8 - 1/20*sqrt(5)*ln(a^2 +
1/2*(sqrt(5)*a - a)*x + x^2)/a^8 - 1/20*ln(abs(a^4 - a^3*x + a^2*
x^2 - a*x^3 + x^4))/a^8 + 1/5*ln(abs(a + x))/a^8 - 1/3/(a^5*x^3)
```

3.145 $\int \frac{x^{-m}}{a^5+x^5}\,dx$

Optimal. Leaf size=46

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)]
)/(a^5*(1 - m))
```

Rubi [A] time = 0.0280382, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{5};\frac{6-m}{5};-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)]
)/(a^5*(1 - m))
```

Rubi in Sympy [A] time = 2.13645, size = 31, normalized size = 0.67

$$\frac{x^{-m+1}{}_2F_1\left(\begin{matrix}1,-\frac{m}{5}+\frac{1}{5}\\-\frac{m}{5}+\frac{6}{5}\end{matrix}\middle|-\frac{x^5}{a^5}\right)}{a^5\left(-m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**m)/(a**5+x**5),x)
```

```
[Out] x**(-m + 1)*hyper((1, -m/5 + 1/5), (-m/5 + 6/5,), -x**5/a**5)/(a*
*5*(-m + 1))
```

Mathematica [A] time = 0.0262242, size = 47, normalized size = 1.02

$$-\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{1-m}{5}+1,-\frac{x^5}{a^5}\right)}{a^5(m-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, 1 + (1 - m)/5, -(x^5
/a^5)])/(a^5*(-1 + m)))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x^m (a^5 + x^5)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^m)/(a^5+x^5),x)
```

```
[Out] int(1/(x^m)/(a^5+x^5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-m}}{a^5 + x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^m),x, algorithm="maxima")
```

```
[Out] integrate(x^(-m)/(a^5 + x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a^5 + x^5)x^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^m),x, algorithm="fricas")
```

```
[Out] integral(1/((a^5 + x^5)*x^m), x)
```

Sympy [A] time = 68.0489, size = 92, normalized size = 2.

$$-\frac{mxx^{-m}\left(\frac{x^5e^{i\pi}}{a^5},1,-\frac{m}{5}+\frac{1}{5}\right)\left(-\frac{m}{5}+\frac{1}{5}\right)}{25a^5\left(-\frac{m}{5}+\frac{6}{5}\right)}+\frac{xx^{-m}\left(\frac{x^5e^{i\pi}}{a^5},1,-\frac{m}{5}+\frac{1}{5}\right)\left(-\frac{m}{5}+\frac{1}{5}\right)}{25a^5\left(-\frac{m}{5}+\frac{6}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**m)/(a**5+x**5),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, -m/5 + 1/5)*g
amma(-m/5 + 1/5)/(25*a**5*gamma(-m/5 + 6/5)) + x*x**(-m)*lerchphi
(x**5*exp_polar(I*pi)/a**5, 1, -m/5 + 1/5)*gamma(-m/5 + 1/5)/(25*
a**5*gamma(-m/5 + 6/5))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^5+x^5)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^5 + x^5)*x^m),x, algorithm="giac")
```

```
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

3.146 $\int \frac{1+x^4}{1+x^6}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x]/3 + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x
]/3
```

Rubi [A] time = 0.835775, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 + x^6),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/3 + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x
]/3
```

Rubi in Sympy [A] time = 77.2345, size = 29, normalized size = 0.83

$$\frac{2\,\mathrm{atan}\,(x)}{3}+\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{3}+\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**6+1),x)
```

```
[Out] 2*atan(x)/3 + atan(2*x - sqrt(3))/3 + atan(2*x + sqrt(3))/3
```

Mathematica [A] time = 0.0103454, size = 21, normalized size = 0.6

$$\frac{2}{3}\tan^{-1}(x)-\frac{1}{3}\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 + x^6),x]
```

```
[Out] (2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3
```

Maple [A] time = 0.07, size = 28, normalized size = 0.8

$$\frac{2\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{3}+\frac{\arctan\left(2x+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^6+1),x)
```

```
[Out] 2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))
```

Maxima [A] time = 1.50584, size = 36, normalized size = 1.03

$$\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2x-\sqrt{3}\right)+\frac{2}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^6 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arcta
n(x)
```

Fricas [A] time = 0.201957, size = 12, normalized size = 0.34

$$\frac{1}{3}\arctan\left(x^{3}\right)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^6 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(x^3) + arctan(x)
```

Sympy [A] time = 0.131135, size = 8, normalized size = 0.23

$$\operatorname{atan}(x) + \frac{\operatorname{atan}(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**6+1),x)
```

```
[Out] atan(x) + atan(x**3)/3
```

GIAC/XCAS [A] time = 0.200594, size = 36, normalized size = 1.03

$$\frac{1}{3} \arctan\left(2\,x + \sqrt{3}\right) + \frac{1}{3} \arctan\left(2\,x - \sqrt{3}\right) + \frac{2}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^6 + 1),x, algorithm="giac")
```

```
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arcta
n(x)
```

3.147 $\int \frac{1}{(5+3x+x^2)^3}\,dx$

Optimal. Leaf size=60

$$\frac{3(2x+3)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

```
[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x +
x^2)) + (12*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])
```

Rubi [A] time = 0.0459499, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3(2x+3)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x +
x^2)) + (12*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])
```

Rubi in Sympy [A] time = 1.14035, size = 54, normalized size = 0.9

$$\frac{3\left(2x+3\right)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\sqrt{11}\,\mathrm{atan}\left(\sqrt{11}\left(\frac{2x}{11}+\frac{3}{11}\right)\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+3*x+5)**3,x)
```

```
[Out] 3*(2*x + 3)/(121*(x**2 + 3*x + 5)) + (2*x + 3)/(22*(x**2 + 3*x +
5)**2) + 12*sqrt(11)*atan(sqrt(11)*(2*x/11 + 3/11))/1331
```

Mathematica [A] time = 0.0463191, size = 51, normalized size = 0.85

$$\frac{\frac{11(2x+3)\left(6x^2+18x+41\right)}{\left(x^2+3x+5\right)^2}+24\sqrt{11}\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{2662}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] ((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*Sqrt[1
1]*ArcTan[(3 + 2*x)/Sqrt[11]])/2662
```

Maple [A] time = 0.004, size = 52, normalized size = 0.9

$$\frac{3+2x}{22\left(x^{2}+3x+5\right)^{2}}+\frac{9+6x}{121x^{2}+363x+605}+\frac{12\sqrt{11}}{1331}\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+3*x+5)^3,x)
```

```
[Out] 1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*arct
an(1/11*(3+2*x)*11^(1/2))*11^(1/2)
```

Maxima [A] time = 1.52378, size = 73, normalized size = 1.22

$$\frac{12}{1331}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+3)\right)+\frac{12x^{3}+54x^{2}+136x+123}{242\left(x^{4}+6x^{3}+19x^{2}+30x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 5)^(-3),x, algorithm="maxima")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3
+ 54*x^2 + 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

Fricas [A] time = 0.202243, size = 103, normalized size = 1.72

$$\frac{\sqrt{11}\left(24\left(x^{4}+6x^{3}+19x^{2}+30x+25\right)\arctan\left(\frac{1}{11}\sqrt{11}(2x+3)\right)+\sqrt{11}\left(12x^{3}+54x^{2}+136x+123\right)\right)}{2662\left(x^{4}+6x^{3}+19x^{2}+30x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 5)^(-3),x, algorithm="fricas")
```

```
[Out] 1/2662*sqrt(11)*(24*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*arctan(1/1
1*sqrt(11)*(2*x + 3)) + sqrt(11)*(12*x^3 + 54*x^2 + 136*x + 123))
/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

Sympy [A] time = 0.196179, size = 63, normalized size = 1.05

$$\frac{12x^3 + 54x^2 + 136x + 123}{242x^4 + 1452x^3 + 4598x^2 + 7260x + 6050} + \frac{12\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11} + \frac{3\sqrt{11}}{11}\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+3*x+5)**3,x)
```

```
[Out] (12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x*
*2 + 7260*x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(1
1)/11)/1331
```

GIAC/XCAS [A] time = 0.198692, size = 59, normalized size = 0.98

$$\frac{12}{1331}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x + 3)\right) + \frac{12x^3 + 54x^2 + 136x + 123}{242(x^2 + 3x + 5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 5)^(-3),x, algorithm="giac")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3
+ 54*x^2 + 136*x + 123)/(x^2 + 3*x + 5)^2
```

3.148 $\int \frac{1+x^2+x^4}{(1+x^2)^4} \, dx$

Optimal. Leaf size=43

$$\frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3} + \frac{7}{16}\tan^{-1}(x)$$

```
[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (
7*ArcTan[x])/16
```

Rubi [A] time = 0.035257, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3} + \frac{7}{16}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

```
[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (
7*ArcTan[x])/16
```

Rubi in Sympy [A] time = 4.17884, size = 36, normalized size = 0.84

$$\frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3} + \frac{7\operatorname{atan}(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)/(x**2+1)**4,x)
```

```
[Out] 7*x/(16*(x**2 + 1)) - x/(24*(x**2 + 1)**2) + x/(6*(x**2 + 1)**3)
+ 7*atan(x)/16
```

Mathematica [A] time = 0.0194242, size = 30, normalized size = 0.7

$$\frac{1}{48}\left(\frac{x(21x^4+40x^2+27)}{(x^2+1)^3} + 21\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

```
[Out] ((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

Maple [A] time = 0.01, size = 28, normalized size = 0.7

$$\frac{1}{(x^2+1)^3}\left(\frac{7x^5}{16}+\frac{5x^3}{6}+\frac{9x}{16}\right)+\frac{7\arctan(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)/(x^2+1)^4,x)
```

```
[Out] (7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)
```

Maxima [A] time = 1.53079, size = 51, normalized size = 1.19

$$\frac{21x^5+40x^3+27x}{48(x^6+3x^4+3x^2+1)}+\frac{7}{16}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^2 + 1)/(x^2 + 1)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*ar
ctan(x)
```

Fricas [A] time = 0.20977, size = 70, normalized size = 1.63

$$\frac{21x^5+40x^3+21(x^6+3x^4+3x^2+1)\arctan(x)+27x}{48(x^6+3x^4+3x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^2 + 1)/(x^2 + 1)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) +
27*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A] time = 0.180481, size = 36, normalized size = 0.84

$$\frac{21x^5 + 40x^3 + 27x}{48x^6 + 144x^4 + 144x^2 + 48} + \frac{7\,\mathrm{atan}\,(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)/(x**2+1)**4,x)
```

```
[Out] (21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) +
 7*atan(x)/16
```

GIAC/XCAS [A] time = 0.200456, size = 38, normalized size = 0.88

$$\frac{21\,x^5 + 40\,x^3 + 27\,x}{48\,(x^2 + 1)^3} + \frac{7}{16}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^2 + 1)/(x^2 + 1)^4,x, algorithm="giac")
```

```
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)
```

3.149 $\int \frac{B+Ax}{(c+2bx+ax^2)^2} dx$

Optimal. Leaf size=90

$$-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

```
[Out] -(b*B - A*c - (A*b - a*B)*x)/(2*(b^2 - a*c)*(c + 2*b*x + a*x^2))
- ((A*b - a*B)*ArcTanh[(b + a*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)
^(3/2))
```

Rubi [A] time = 0.149021, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] -(b*B - A*c - (A*b - a*B)*x)/(2*(b^2 - a*c)*(c + 2*b*x + a*x^2))
- ((A*b - a*B)*ArcTanh[(b + a*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)
^(3/2))
```

Rubi in Sympy [A] time = 6.72386, size = 76, normalized size = 0.84

$$-\frac{(Ab-Ba)\operatorname{atanh}\left(\frac{ax+b}{\sqrt{-ac+b^2}}\right)}{2(-ac+b^2)^{\frac{3}{2}}}+\frac{2Ac-2Bb+x(2Ab-2Ba)}{4(-ac+b^2)(ax^2+2bx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

```
[Out] -(A*b - B*a)*atanh((a*x + b)/sqrt(-a*c + b**2))/(2*(-a*c + b**2)*
*(3/2)) + (2*A*c - 2*B*b + x*(2*A*b - 2*B*a))/(4*(-a*c + b**2)*(a
*x**2 + 2*b*x + c))
```

Mathematica [A] time = 0.134121, size = 88, normalized size = 0.98

$$\frac{\frac{(Ab-aB)\tan^{-1}\left(\frac{ax+b}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}+\frac{-aBx+Abx+Ac-bB}{x(ax+2b)+c}}{2\left(b^2-ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] ((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B
)*ArcTan[(b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 -
 a*c))
```

Maple [A] time = 0.012, size = 146, normalized size = 1.6

$$\frac{(-2\,Ab+2\,Ba)\,x+2\,bB-2\,Ac}{(4\,ac-4\,b^2)\,(ax^2+2\,bx+c)}-2\,\frac{Ab}{(4\,ac-4\,b^2)\,\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,ax+2\,b}{\sqrt{ac-b^2}}\right)$$
$$+2\,\frac{Ba}{(4\,ac-4\,b^2)\,\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,ax+2\,b}{\sqrt{ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((A*x+B)/(a*x^2+2*b*x+c)^2,x)
```

```
[Out] ((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)-2/(4
*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2
))*A*b+2/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*
c-b^2)^(1/2))*B*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x + B)/(a*x^2 + 2*b*x + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248016, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(Ba^2-Aab\right)x^2+(Ba-Ab)c+2\left(Bab-Ab^2\right)x\right)\log\left(\frac{2\,b^3-2\,abc+2\,(ab^2-a^2c)\,x+\left(a^2x^2+2\,abx+2\,b^2-ac\right)\sqrt{b^2-ac}}{ax^2+2\,bx+c}\right)-2\,(Bb-Ac+(Ba\right.}{4\,(b^2c-ac^2+(ab^2-a^2c)x^2+2\,(b^3-abc)x)\sqrt{b^2-ac}}\right.$$

$$\left.-\frac{\left(\left(Ba^2-Aab\right)x^2+(Ba-Ab)c+2\left(Bab-Ab^2\right)x\right)\arctan\left(-\frac{\sqrt{-b^2+ac}(ax+b)}{b^2-ac}\right)+(Bb-Ac+(Ba-Ab)x)\sqrt{-b^2+ac}}{2\,(b^2c-ac^2+(ab^2-a^2c)x^2+2\,(b^3-abc)x)\sqrt{-b^2+ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x + B)/(a*x^2 + 2*b*x + c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x)
*log((2*b^3 - 2*a*b*c + 2*(a*b^2 - a^2*c)*x + (a^2*x^2 + 2*a*b*x
+ 2*b^2 - a*c)*sqrt(b^2 - a*c))/(a*x^2 + 2*b*x + c)) - 2*(B*b - A
*c + (B*a - A*b)*x)*sqrt(b^2 - a*c))/((b^2*c - a*c^2 + (a*b^2 - a
^2*c)*x^2 + 2*(b^3 - a*b*c)*x)*sqrt(b^2 - a*c)), -1/2*(((B*a^2 -
A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x)*arctan(-sqrt(-b
^2 + a*c)*(a*x + b)/(b^2 - a*c)) + (B*b - A*c + (B*a - A*b)*x)*sq
rt(-b^2 + a*c))/((b^2*c - a*c^2 + (a*b^2 - a^2*c)*x^2 + 2*(b^3 -
a*b*c)*x)*sqrt(-b^2 + a*c))]
```

Sympy [A] time = 2.03425, size = 323, normalized size = 3.59

$$-\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}\,(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab-a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)+2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)-b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)}{-Aab+Ba^2}\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}\,(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab+a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)-2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)+b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)}{-Aab+Ba^2}\right)}{4}$$
$$+\frac{-Ac+Bb+x\,(-Ab+Ba)}{2ac^2-2b^2c+x^2\,(2a^2c-2ab^2)+x\,(4abc-4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

```
[Out] -sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b -
 a**2*c**2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqr
t(-1/(a*c - b**2)**3)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3
)*(-A*b + B*a))/(-A*a*b + B*a**2))/4 + sqrt(-1/(a*c - b**2)**3)*(
-A*b + B*a)*log(x + (-A*b**2 + B*a*b + a**2*c**2*sqrt(-1/(a*c - b
```

```
**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(-A*b
 + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B
*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c +
x**2*(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))
```

GIAC/XCAS [A] time = 0.214814, size = 124, normalized size = 1.38

$$-\frac{(Ba - Ab)\arctan\left(\frac{ax+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2 - ac)\sqrt{-b^2 + ac}} - \frac{Bax - Abx + Bb - Ac}{2\,(ax^2 + 2\,bx + c)(b^2 - ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x + B)/(a*x^2 + 2*b*x + c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*
sqrt(-b^2 + a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x^2 + 2*b
*x + c)*(b^2 - a*c))
```

3.150 $\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx$

Optimal. Leaf size=38

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2]
)/2
```

Rubi [A] time = 0.0472333, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2]
)/2
```

Rubi in Sympy [A] time = 8.47311, size = 32, normalized size = 0.84

$$\frac{0.125\,(-8x+8)}{x^2-4x+5}+\frac{5\log\left(x^2-4x+5\right)}{2}+2.0\,\mathrm{atan}\,(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
```

```
[Out] 0.125*(-8*x + 8)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2.0
*atan(x - 2)
```

Mathematica [A] time = 0.0219512, size = 38, normalized size = 1.

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2]
)/2
```

Maple [A]   time = 0.009, size = 35, normalized size = 0.9

$$\frac{1-x}{x^2-4x+5}+2\arctan(-2+x)+\frac{5\ln(x^2-4x+5)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x)
```

```
[Out] (1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

Maxima [A]   time = 1.52015, size = 45, normalized size = 1.18

$$-\frac{x-1}{x^2-4x+5}+2\arctan(x-2)+\frac{5}{2}\log(x^2-4x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3 - 27*x^2 + 55*x - 41)/(x^2 - 4*x + 5)^2,x, algorithm="maxima")
```

```
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x +
5)
```

Fricas [A]   time = 0.204527, size = 68, normalized size = 1.79

$$\frac{4(x^2-4x+5)\arctan(x-2)+5(x^2-4x+5)\log(x^2-4x+5)-2x+2}{2(x^2-4x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3 - 27*x^2 + 55*x - 41)/(x^2 - 4*x + 5)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(x^2 - 4*x + 5)*arctan(x - 2) + 5*(x^2 - 4*x + 5)*log(x^2
- 4*x + 5) - 2*x + 2)/(x^2 - 4*x + 5)
```

Sympy [A] time = 0.159657, size = 31, normalized size = 0.82

$$-\frac{x-1}{x^2-4x+5}+\frac{5\log\left(x^2-4x+5\right)}{2}+2\,\mathrm{atan}\,(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
```

```
[Out] -(x - 1)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x -
2)
```

GIAC/XCAS [A] time = 0.206152, size = 45, normalized size = 1.18

$$-\frac{x-1}{x^2-4\,x+5}+2\,\arctan\left(x-2\right)+\frac{5}{2}\,\ln\left(x^2-4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^3 - 27*x^2 + 55*x - 41)/(x^2 - 4*x + 5)^2,x, algorithm="giac")
```

```
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*ln(x^2 - 4*x + 5
)
```

3.151 $\int \frac{1}{(-1+x^3)^2} \, dx$

Optimal. Leaf size=57

$$\frac{x}{3(1-x^3)} + \frac{1}{9}\log\left(x^2+x+1\right) - \frac{2}{9}\log(1-x) + \frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*
Log[1 - x])/9 + Log[1 + x + x^2]/9
```

Rubi [A] time = 0.0525124, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.

$$\frac{x}{3(1-x^3)} + \frac{1}{9}\log\left(x^2+x+1\right) - \frac{2}{9}\log(1-x) + \frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^3)^(-2),x]
```

```
[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*
Log[1 - x])/9 + Log[1 + x + x^2]/9
```

Rubi in Sympy [A] time = 2.83207, size = 49, normalized size = 0.86

$$\frac{x}{3(-x^3+1)} - \frac{2\log(-x+1)}{9} + \frac{\log\left(x^2+x+1\right)}{9} + \frac{2\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-1)**2,x)
```

```
[Out] x/(3*(-x**3 + 1)) - 2*log(-x + 1)/9 + log(x**2 + x + 1)/9 + 2*sqr
t(3)*atan(sqrt(3)*(2*x/3 + 1/3))/9
```

Mathematica [A] time = 0.035352, size = 49, normalized size = 0.86

$$\frac{1}{9}\left(-\frac{3x}{x^3-1} + \log\left(x^2+x+1\right) - 2\log(1-x) + 2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^3)^(-2),x]
```

```
[Out] ((-3*x)/(-1 + x^3) + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[
1 - x] + Log[1 + x + x^2])/9
```

Maple [A] time = 0.013, size = 53, normalized size = 0.9

$$-\frac{1}{-9+9\,x}-\frac{2\,\ln\left(-1+x\right)}{9}+\frac{-1+x}{9\,x^{2}+9\,x+9}+\frac{\ln\left(x^{2}+x+1\right)}{9}+\frac{2\,\sqrt{3}}{9}\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-1)^2,x)
```

```
[Out] -1/9/(-1+x)-2/9*ln(-1+x)+1/9*(-1+x)/(x^2+x+1)+1/9*ln(x^2+x+1)+2/9
*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.54858, size = 57, normalized size = 1.

$$\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{x}{3\,(x^{3}-1)}+\frac{1}{9}\,\log\left(x^{2}+x+1\right)-\frac{2}{9}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)^(-2),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9
*log(x^2 + x + 1) - 2/9*log(x - 1)
```

Fricas [A] time = 0.204845, size = 90, normalized size = 1.58

$$\frac{\sqrt{3}\left(\sqrt{3}(x^{3}-1)\,\log\left(x^{2}+x+1\right)-2\,\sqrt{3}(x^{3}-1)\,\log\left(x-1\right)+6\,\left(x^{3}-1\right)\,\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-3\,\sqrt{3}x\right)}{27\,(x^{3}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(3)*(sqrt(3)*(x^3 - 1)*log(x^2 + x + 1) - 2*sqrt(3)*(x^3
 - 1)*log(x - 1) + 6*(x^3 - 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*
sqrt(3)*x)/(x^3 - 1)
```

Sympy [A] time = 0.21292, size = 53, normalized size = 0.93

$$-\frac{x}{3x^3-3}-\frac{2\log(x-1)}{9}+\frac{\log\left(x^2+x+1\right)}{9}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-1)**2,x)
```

```
[Out] -x/(3*x**3 - 3) - 2*log(x - 1)/9 + log(x**2 + x + 1)/9 + 2*sqrt(3
)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.208405, size = 58, normalized size = 1.02

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3\left(x^3-1\right)}+\frac{1}{9}\ln\left(x^2+x+1\right)-\frac{2}{9}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)^(-2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9
*ln(x^2 + x + 1) - 2/9*ln(abs(x - 1))
```

3.152 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[
x])/8
```

Rubi [A] time = 0.0797231, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[
x])/8
```

Rubi in Sympy [A] time = 7.10579, size = 32, normalized size = 0.89

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{57\,\mathrm{atan}(x)}{8}-\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] -25*x/(8*(x**2 + 1)) - 7*x/(4*(x**2 + 1)**2) - 57*atan(x)/8 - 4/x
```

Mathematica [A] time = 0.0240458, size = 33, normalized size = 0.92

$$-\frac{57x^4+103x^2+32}{8x(x^2+1)^2}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]

[Out] -(32 + 103*x^2 + 57*x^4)/(8*x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

Maple [A] time = 0.019, size = 29, normalized size = 0.8

$$-\frac{1}{(x^2+1)^2}\left(\frac{25\,x^3}{8}+\frac{39\,x}{8}\right)-\frac{57\,\arctan(x)}{8}-4\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^4+4)/x^2/(x^2+1)^3,x)

[Out] -(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

Maxima [A] time = 1.54384, size = 42, normalized size = 1.17

$$-\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)}-\frac{57}{8}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 4)/((x^2 + 1)^3*x^2),x, algorithm="maxima")

[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

Fricas [A] time = 0.196872, size = 54, normalized size = 1.5

$$-\frac{57\,x^4+103\,x^2+57\,\left(x^5+2\,x^3+x\right)\arctan(x)+32}{8\,(x^5+2\,x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 4)/((x^2 + 1)^3*x^2),x, algorithm="fricas")

[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^
5 + 2*x^3 + x)
```

Sympy [A] time = 0.177832, size = 32, normalized size = 0.89

$$-\frac{57x^4+103x^2+32}{8x^5+16x^3+8x}-\frac{57\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] -(57*x**4 + 103*x**2 + 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/
8
```

GIAC/XCAS [A] time = 0.201239, size = 38, normalized size = 1.06

$$-\frac{25\,x^3+39\,x}{8\,(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 4)/((x^2 + 1)^3*x^2),x, algorithm="giac")
```

```
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

3.153 $\int \frac{x}{1+x^6} \, dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log[1
 - x^2 + x^4]/12
```

Rubi [A] time = 0.0730563, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log[1
 - x^2 + x^4]/12
```

Rubi in Sympy [A] time = 4.31447, size = 42, normalized size = 0.86

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}-\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(sqrt(3)*
(2*x**2/3 - 1/3))/6
```

Mathematica [A] time = 0.0242909, size = 78, normalized size = 1.59

$$\frac{1}{12}\left(2\log\left(x^2+1\right)-\log\left(x^2-\sqrt{3}x+1\right)-\log\left(x^2+\sqrt{3}x+1\right)\right.$$
$$\left.-2\sqrt{3}\tan^{-1}\left(\sqrt{3}-2x\right)-2\sqrt{3}\tan^{-1}\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*
x] + 2*Log[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*
x + x^2])/12
```

Maple [A] time = 0.01, size = 41, normalized size = 0.8

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^6+1),x)
```

```
[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(1/3*(2*x^2-1)
*3^(1/2))
```

Maxima [A] time = 1.55928, size = 54, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2
+ 1) + 1/6*log(x^2 + 1)
```

Fricas [A] time = 0.256307, size = 63, normalized size = 1.29

$$-\frac{1}{36}\sqrt{3}\left(\sqrt{3}\log\left(x^4-x^2+1\right)-2\sqrt{3}\log\left(x^2+1\right)-6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(sqrt(3)*log(x^4 - x^2 + 1) - 2*sqrt(3)*log(x^2 + 1
) - 6*arctan(1/3*sqrt(3)*(2*x^2 - 1)))
```

Sympy [A] time = 0.192293, size = 46, normalized size = 0.94

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3
)*x**2/3 - sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.200641, size = 54, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{12}\ln\left(x^4-x^2+1\right)+\frac{1}{6}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*ln(x^4 - x^2 +
 1) + 1/6*ln(x^2 + 1)
```

3.154 $\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx$

Optimal. Leaf size=13

$$\frac{\log(x^n - nx)}{n}$$

```
[Out] Log[-(n*x) + x^n]/n
```

Rubi [A] time = 0.110383, antiderivative size = 20, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\log\left(1 - nx^{1-n}\right)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]

[Out] Log[x] + Log[1 - n*x^(1 - n)]/n
```

Rubi in Sympy [A] time = 7.9054, size = 20, normalized size = 1.54

$$\frac{\log\left(x^{-n+1}\right)}{-n+1} + \frac{\log\left(-nx^{-n+1} + 1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x**(-1+n))/(-n*x+x**n),x)

[Out] log(x**(-n + 1))/(-n + 1) + log(-n*x**(-n + 1) + 1)/n
```

Mathematica [A] time = 0.0127593, size = 13, normalized size = 1.

$$\frac{\log(x^n - nx)}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

```
[Out] Log[-(n*x) + x^n]/n
```

Maple [A] time = 0.029, size = 17, normalized size = 1.3

$$\frac{\ln\left(nx - e^{n\ln(x)}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x^(-1+n))/(-n*x+x^n),x)
```

```
[Out] 1/n*ln(n*x-exp(n*ln(x)))
```

Maxima [A] time = 1.37792, size = 19, normalized size = 1.46

$$\frac{\log\left(nx - x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^(n - 1) - 1)/(n*x - x^n),x, algorithm="maxima")
```

```
[Out] log(n*x - x^n)/n
```

Fricas [A] time = 0.259542, size = 18, normalized size = 1.38

$$\frac{\log\left(-nx + x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^(n - 1) - 1)/(n*x - x^n),x, algorithm="fricas")
```

```
[Out] log(-n*x + x^n)/n
```

Sympy [A] time = 4.71774, size = 14, normalized size = 1.08

$$\begin{cases} \frac{\log\left(x - \frac{x^n}{n}\right)}{n} & \text{for } n \neq 0 \\ -x + \log\left(x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x**(-1+n))/(-n*x+x**n),x)
```

```
[Out] Piecewise((log(x - x**n/n)/n, Ne(n, 0)), (-x + log(x), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^{n-1}-1}{nx-x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^(n - 1) - 1)/(n*x - x^n),x, algorithm="giac")
```

```
[Out] integrate(-(x^(n - 1) - 1)/(n*x - x^n), x)
```

3.155 $\int \frac{x^3}{1-2x^2+3x^4}\,dx$

Optimal. Leaf size=41

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[Out] -ArcTan[(1 - 3*x^2)/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]
/12
```

Rubi [A] time = 0.079102, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] -ArcTan[(1 - 3*x^2)/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]
/12
```

Rubi in Sympy [A] time = 4.95828, size = 37, normalized size = 0.9

$$\frac{\log\left(3x^4-2x^2+1\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x^2}{2}-\frac{1}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(3*x**4-2*x**2+1),x)
```

```
[Out] log(3*x**4 - 2*x**2 + 1)/12 + sqrt(2)*atan(sqrt(2)*(3*x**2/2 - 1/
2))/12
```

Mathematica [A] time = 0.0167207, size = 38, normalized size = 0.93

$$\frac{1}{12}\left(\sqrt{2}\tan^{-1}\left(\frac{3x^2-1}{\sqrt{2}}\right)+\log\left(3x^4-2x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/1
2
```

Maple [A] time = 0.008, size = 35, normalized size = 0.9

$$\frac{\ln\left(3x^4-2x^2+1\right)}{12}+\frac{\sqrt{2}}{12}\arctan\left(\frac{\left(6x^2-2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(3*x^4-2*x^2+1),x)
```

```
[Out] 1/12*ln(3*x^4-2*x^2+1)+1/12*2^(1/2)*arctan(1/4*(6*x^2-2)*2^(1/2))
```

Maxima [A] time = 1.56406, size = 46, normalized size = 1.12

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right)+\frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(3*x^4 - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2
*x^2 + 1)
```

Fricas [A] time = 0.231784, size = 51, normalized size = 1.24

$$\frac{1}{24}\sqrt{2}\left(\sqrt{2}\log\left(3x^4-2x^2+1\right)+2\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(3*x^4 - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*(sqrt(2)*log(3*x^4 - 2*x^2 + 1) + 2*arctan(1/2*sqrt(
2)*(3*x^2 - 1)))
```

Sympy [A] time = 0.122649, size = 42, normalized size = 1.02

$$\frac{\log\left(x^4 - \frac{2x^2}{3} + \frac{1}{3}\right)}{12} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x^2}{2} - \frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(3*x**4-2*x**2+1),x)
```

```
[Out] log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - s
qrt(2)/2)/12
```

GIAC/XCAS [A] time = 0.200715, size = 46, normalized size = 1.12

$$\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(3\,x^2 - 1\right)\right) + \frac{1}{12}\ln\left(3\,x^4 - 2\,x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(3*x^4 - 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*ln(3*x^4 - 2*
x^2 + 1)
```

3.156 $\int \frac{x^5}{-4+x^2+3x^4} \, dx$

Optimal. Leaf size=32

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Rubi [A] time = 0.0551289, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(-4 + x^2 + 3*x^4),x]
```

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Rubi in Sympy [A] time = 5.69744, size = 24, normalized size = 0.75

$$\frac{x^2}{6}+\frac{\log\left(-x^2+1\right)}{14}-\frac{8\log\left(3x^2+4\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(3*x**4+x**2-4),x)
```

```
[Out] x**2/6 + log(-x**2 + 1)/14 - 8*log(3*x**2 + 4)/63
```

Mathematica [A] time = 0.0079071, size = 32, normalized size = 1.

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(-4 + x^2 + 3*x^4),x]
```

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$\frac{x^2}{6} + \frac{\ln\left(x^2-1\right)}{14} - \frac{8\ \ln\left(3\,x^2+4\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(3*x^4+x^2-4),x)
```

```
[Out] 1/6*x^2+1/14*ln(x^2-1)-8/63*ln(3*x^2+4)
```

Maxima [A] time = 1.35628, size = 32, normalized size = 1.

$$\frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(3*x^4 + x^2 - 4),x, algorithm="maxima")
```

```
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

Fricas [A] time = 0.198794, size = 32, normalized size = 1.

$$\frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(3*x^4 + x^2 - 4),x, algorithm="fricas")
```

```
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

Sympy [A] time = 0.13361, size = 24, normalized size = 0.75

$$\frac{x^2}{6} + \frac{\log\left(x^2-1\right)}{14} - \frac{8\log\left(x^2+\frac{4}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(3*x**4+x**2-4),x)
```

```
[Out] x**2/6 + log(x**2 - 1)/14 - 8*log(x**2 + 4/3)/63
```

GIAC/XCAS [A] time = 0.201976, size = 34, normalized size = 1.06

$$\frac{1}{6}\,x^2 - \frac{8}{63}\ln\left(3\,x^2 + 4\right) + \frac{1}{14}\ln\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(3*x^4 + x^2 - 4),x, algorithm="giac")
```

```
[Out] 1/6*x^2 - 8/63*ln(3*x^2 + 4) + 1/14*ln(abs(x^2 - 1))
```

3.157 $\int \frac{x^2}{9-10x^3+x^6}\,dx$

Optimal. Leaf size=25

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

```
[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Rubi [A] time = 0.0356861, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(9 - 10*x^3 + x^6),x]

[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Rubi in Sympy [A] time = 2.66058, size = 15, normalized size = 0.6

$$-\frac{\log\left(-x^3+1\right)}{24}+\frac{\log\left(-x^3+9\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6-10*x**3+9),x)

[Out] -log(-x**3 + 1)/24 + log(-x**3 + 9)/24
```

Mathematica [A] time = 0.00649757, size = 25, normalized size = 1.

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(9 - 10*x^3 + x^6),x]
```

```
[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Maple [A] time = 0.008, size = 18, normalized size = 0.7

$$\frac{\ln\left(x^3-9\right)}{24}-\frac{\ln\left(x^3-1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-10*x^3+9),x)
```

```
[Out] 1/24*ln(x^3-9)-1/24*ln(x^3-1)
```

Maxima [A] time = 1.35314, size = 23, normalized size = 0.92

$$-\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 - 10*x^3 + 9),x, algorithm="maxima")
```

```
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

Fricas [A] time = 0.194433, size = 23, normalized size = 0.92

$$-\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 - 10*x^3 + 9),x, algorithm="fricas")
```

```
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

Sympy [A] time = 0.11688, size = 15, normalized size = 0.6

$$\frac{\log\left(x^3-9\right)}{24}-\frac{\log\left(x^3-1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6-10*x**3+9),x)
```

```
[Out] log(x**3 - 9)/24 - log(x**3 - 1)/24
```

GIAC/XCAS [A] time = 0.201907, size = 26, normalized size = 1.04

$$-\frac{1}{24}\ln\left(\left|x^3-1\right|\right)+\frac{1}{24}\ln\left(\left|x^3-9\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - 10*x^3 + 9),x, algorithm="giac")
```

```
[Out] -1/24*ln(abs(x^3 - 1)) + 1/24*ln(abs(x^3 - 9))
```

3.158 $\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx$

Optimal. Leaf size=36

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

```
[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

Rubi [A] time = 0.0397896, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

```
[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

Rubi in Sympy [A] time = 3.68987, size = 24, normalized size = 0.67

$$\log(-x+2)+\frac{2}{-x+2}+\frac{2}{(-x+2)^2}-\frac{7}{3(-x+2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-4*x**2+1)/(-2+x)**4,x)
```

```
[Out] log(-x + 2) + 2/(-x + 2) + 2/(-x + 2)**2 - 7/(3*(-x + 2)**3)
```

Mathematica [A] time = 0.020773, size = 24, normalized size = 0.67

$$\frac{-6x^2+30x-29}{3(x-2)^3}+\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

```
[Out] (-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

Maple [A] time = 0.008, size = 27, normalized size = 0.8

$$2\,(-2+x)^{-2}+\frac{7}{3\,(-2+x)^{3}}+\ln(-2+x)-2\,(-2+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-4*x^2+1)/(-2+x)^4,x)
```

```
[Out] 2/(-2+x)^2+7/3/(-2+x)^3+ln(-2+x)-2/(-2+x)
```

Maxima [A] time = 1.36118, size = 43, normalized size = 1.19

$$-\frac{6\,x^{2}-30\,x+29}{3\,(x^{3}-6\,x^{2}+12\,x-8)}+\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 4*x^2 + 1)/(x - 2)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + log(x - 2)
```

Fricas [A] time = 0.237896, size = 62, normalized size = 1.72

$$-\frac{6\,x^{2}-3\,\left(x^{3}-6\,x^{2}+12\,x-8\right)\log(x-2)-30\,x+29}{3\,(x^{3}-6\,x^{2}+12\,x-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 4*x^2 + 1)/(x - 2)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/
(x^3 - 6*x^2 + 12*x - 8)
```

Sympy [A] time = 0.131074, size = 29, normalized size = 0.81

$$-\frac{6x^{2}-30x+29}{3x^{3}-18x^{2}+36x-24}+\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-4*x**2+1)/(-2+x)**4,x)
```

```
[Out] -(6*x**2 - 30*x + 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)
```

GIAC/XCAS [A] time = 0.199311, size = 31, normalized size = 0.86

$$-\frac{6\,x^2-30\,x+29}{3\,(x-2)^3}+\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 4*x^2 + 1)/(x - 2)^4,x, algorithm="giac")
```

```
[Out] -1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + ln(abs(x - 2))
```

3.159 $\int \frac{x^3}{(-1+x)^{12}} \, dx$

Optimal. Leaf size=45

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

```
[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1
 - x)^8)
```

Rubi [A] time = 0.0305824, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(-1 + x)^12,x]
```

```
[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1
 - x)^8)
```

Rubi in Sympy [A] time = 2.3012, size = 32, normalized size = 0.71

$$-\frac{1}{8\left(-x+1\right)^{8}}+\frac{1}{3\left(-x+1\right)^{9}}-\frac{3}{10\left(-x+1\right)^{10}}+\frac{1}{11\left(-x+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-1+x)**12,x)
```

```
[Out] -1/(8*(-x + 1)**8) + 1/(3*(-x + 1)**9) - 3/(10*(-x + 1)**10) + 1/
(11*(-x + 1)**11)
```

Mathematica [A] time = 0.00928015, size = 24, normalized size = 0.53

$$\frac{-165x^3+55x^2-11x+1}{1320(x-1)^{11}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(-1 + x)^12,x]
```

```
[Out] (1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)
```

Maple [A] time = 0.009, size = 30, normalized size = 0.7

$$-\frac{3}{10\,\left(-1+x\right)^{10}}-\frac{1}{8\,\left(-1+x\right)^{8}}-\frac{1}{11\,\left(-1+x\right)^{11}}-\frac{1}{3\,\left(-1+x\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-1+x)^12,x)
```

```
[Out] -3/10/(-1+x)^10-1/8/(-1+x)^8-1/11/(-1+x)^11-1/3/(-1+x)^9
```

Maxima [A] time = 1.37124, size = 97, normalized size = 2.16

$$-\frac{165\,x^{3}-55\,x^{2}+11\,x-1}{1320\,\left(x^{11}-11\,x^{10}+55\,x^{9}-165\,x^{8}+330\,x^{7}-462\,x^{6}+462\,x^{5}-330\,x^{4}+165\,x^{3}-55\,x^{2}+11\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x - 1)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 -
165*x^8 + 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^
2 + 11*x - 1)
```

Fricas [A] time = 0.195251, size = 97, normalized size = 2.16

$$-\frac{165\,x^{3}-55\,x^{2}+11\,x-1}{1320\,\left(x^{11}-11\,x^{10}+55\,x^{9}-165\,x^{8}+330\,x^{7}-462\,x^{6}+462\,x^{5}-330\,x^{4}+165\,x^{3}-55\,x^{2}+11\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x - 1)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 -
165*x^8 + 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^
2 + 11*x - 1)
```

Sympy [A] time = 0.235022, size = 71, normalized size = 1.58

$$-\frac{165x^3 - 55x^2 + 11x - 1}{1320x^{11} - 14520x^{10} + 72600x^9 - 217800x^8 + 435600x^7 - 609840x^6 + 609840x^5 - 435600x^4 + 217800x^3 - 72600x^2 + 1452}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-1+x)**12,x)
```

```
[Out] -(165*x**3 - 55*x**2 + 11*x - 1)/(1320*x**11 - 14520*x**10 + 7260
0*x**9 - 217800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 -
435600*x**4 + 217800*x**3 - 72600*x**2 + 14520*x - 1320)
```

GIAC/XCAS [A] time = 0.201214, size = 30, normalized size = 0.67

$$-\frac{165\,x^3 - 55\,x^2 + 11\,x - 1}{1320\,(x-1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x - 1)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11
```

3.160 $\int \frac{-3x+x^4}{(1+2x)^5}\,dx$

Optimal. Leaf size=55

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} + \frac{1}{32}\log(2x+1)$$

```
[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) +
 1/(8*(1 + 2*x)) + Log[1 + 2*x]/32
```

Rubi [A] time = 0.0572651, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} + \frac{1}{32}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) +
 1/(8*(1 + 2*x)) + Log[1 + 2*x]/32
```

Rubi in Sympy [A] time = 5.05529, size = 44, normalized size = 0.8

$$\frac{\log(2x+1)}{32} + \frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-3*x)/(1+2*x)**5,x)
```

```
[Out] log(2*x + 1)/32 + 1/(8*(2*x + 1)) - 3/(32*(2*x + 1)**2) + 7/(24*(
2*x + 1)**3) - 25/(128*(2*x + 1)**4)
```

Mathematica [A] time = 0.018089, size = 41, normalized size = 0.75

$$\frac{384x^3 + 432x^2 + 368x + 12(2x+1)^4\log(2x+1) + 49}{384(2x+1)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] (49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(3
84*(1 + 2*x)^4)
```

Maple [A] time = 0.009, size = 46, normalized size = 0.8

$$-\frac{25}{128\,(1+2\,x)^4}+\frac{7}{24\,(1+2\,x)^3}-\frac{3}{32\,(1+2\,x)^2}+\frac{1}{8+16\,x}+\frac{\ln(1+2\,x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-3*x)/(1+2*x)^5,x)
```

```
[Out] -25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*
ln(1+2*x)
```

Maxima [A] time = 1.52722, size = 65, normalized size = 1.18

$$\frac{384\,x^3+432\,x^2+368\,x+49}{384\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)}+\frac{1}{32}\,\log(2\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 3*x)/(2*x + 1)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2
+ 8*x + 1) + 1/32*log(2*x + 1)
```

Fricas [A] time = 0.239141, size = 90, normalized size = 1.64

$$\frac{384\,x^3+432\,x^2+12\,\left(16\,x^4+32\,x^3+24\,x^2+8\,x+1\right)\log(2\,x+1)+368\,x+49}{384\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 3*x)/(2*x + 1)^5,x, algorithm="fricas")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1
)*log(2*x + 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
```

Sympy [A] time = 0.145639, size = 42, normalized size = 0.76

$$\frac{384x^3 + 432x^2 + 368x + 49}{6144x^4 + 12288x^3 + 9216x^2 + 3072x + 384} + \frac{\log(2x+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-3*x)/(1+2*x)**5,x)
```

```
[Out] (384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216
*x**2 + 3072*x + 384) + log(2*x + 1)/32
```

GIAC/XCAS [A] time = 0.201381, size = 74, normalized size = 1.35

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} - \frac{1}{32}\ln\left(\frac{|2x+1|}{2(2x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 3*x)/(2*x + 1)^5,x, algorithm="giac")
```

```
[Out] 1/8/(2*x + 1) - 3/32/(2*x + 1)^2 + 7/24/(2*x + 1)^3 - 25/128/(2*x
 + 1)^4 - 1/32*ln(1/2*abs(2*x + 1)/(2*x + 1)^2)
```

3.161 $\int \frac{1}{(-1+x)^2(1+x)^3} \, dx$

Optimal. Leaf size=36

$$\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/
8
```

Rubi [A] time = 0.0348106, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/
8
```

Rubi in Sympy [A] time = 2.92106, size = 27, normalized size = 0.75

$$\frac{3\,\text{atanh}\,(x)}{8}-\frac{1}{4\,(x+1)}-\frac{1}{8\,(x+1)^2}+\frac{1}{8\,(-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**2/(1+x)**3,x)
```

```
[Out] 3*atanh(x)/8 - 1/(4*(x + 1)) - 1/(8*(x + 1)**2) + 1/(8*(-x + 1))
```

Mathematica [A] time = 0.033627, size = 38, normalized size = 1.06

$$\frac{1}{16}\left(\frac{-6x^2-6x+4}{(x-1)(x+1)^2}-3\log(x-1)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] ((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1
 + x])/16
```

Maple [A] time = 0.013, size = 35, normalized size = 1.

$$-\frac{1}{8\,(1+x)^2}-\frac{1}{4+4\,x}+\frac{3\,\ln(1+x)}{16}-\frac{1}{-8+8\,x}-\frac{3\,\ln(-1+x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^2/(1+x)^3,x)
```

```
[Out] -1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)-1/8/(-1+x)-3/16*ln(-1+x)
```

Maxima [A] time = 1.72926, size = 51, normalized size = 1.42

$$-\frac{3\,x^2+3\,x-2}{8\,(x^3+x^2-x-1)}+\frac{3}{16}\,\log(x+1)-\frac{3}{16}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^3*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*log(x + 1) - 3/
16*log(x - 1)
```

Fricas [A] time = 0.213652, size = 80, normalized size = 2.22

$$-\frac{6\,x^2-3\,\left(x^3+x^2-x-1\right)\,\log(x+1)+3\,\left(x^3+x^2-x-1\right)\,\log(x-1)+6\,x-4}{16\,(x^3+x^2-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^3*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*log(x + 1) + 3*(x^3 + x^2 -
x - 1)*log(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)
```

Sympy [A] time = 0.154671, size = 41, normalized size = 1.14

$$-\frac{3x^2+3x-2}{8x^3+8x^2-8x-8}-\frac{3\log(x-1)}{16}+\frac{3\log(x+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**2/(1+x)**3,x)
```

```
[Out] -(3*x**2 + 3*x - 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*log(x - 1)/16
 + 3*log(x + 1)/16
```

GIAC/XCAS [A] time = 0.199435, size = 58, normalized size = 1.61

$$-\frac{1}{8(x-1)}+\frac{\frac{12}{x-1}+5}{32\left(\frac{2}{x-1}+1\right)^2}+\frac{3}{16}\ln\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^3*(x - 1)^2),x, algorithm="giac")
```

```
[Out] -1/8/(x - 1) + 1/32*(12/(x - 1) + 5)/(2/(x - 1) + 1)^2 + 3/16*ln(
abs(-2/(x - 1) - 1))
```

3.162 $\int \frac{1}{(5-6x)^2x^2}\,dx$

Optimal. Leaf size=35

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

```
[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])
/125
```

Rubi [A] time = 0.0324588, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((5 - 6*x)^2*x^2),x]
```

```
[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])
/125
```

Rubi in Sympy [A] time = 2.39384, size = 27, normalized size = 0.77

$$\frac{12\log(x)}{125}-\frac{12\log(-6x+5)}{125}+\frac{6}{25(-6x+5)}-\frac{1}{25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5-6*x)**2/x**2,x)
```

```
[Out] 12*log(x)/125 - 12*log(-6*x + 5)/125 + 6/(25*(-6*x + 5)) - 1/(25*
x)
```

Mathematica [A] time = 0.0280564, size = 31, normalized size = 0.89

$$\frac{1}{125}\left(\frac{30}{5-6x}-\frac{5}{x}-12\log(5-6x)+12\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((5 - 6*x)^2*x^2),x]

[Out] (30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125
```

Maple [A] time = 0.013, size = 28, normalized size = 0.8

$$-\frac{6}{-125+150\,x}-\frac{12\,\ln\left(-5+6\,x\right)}{125}-\frac{1}{25\,x}+\frac{12\,\ln\left(x\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5-6*x)^2/x^2,x)

[Out] -6/25/(-5+6*x)-12/125*ln(-5+6*x)-1/25/x+12/125*ln(x)
```

Maxima [A] time = 1.79919, size = 42, normalized size = 1.2

$$-\frac{12\,x-5}{25\,(6\,x^2-5\,x)}-\frac{12}{125}\,\log\left(6\,x-5\right)+\frac{12}{125}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((6*x - 5)^2*x^2),x, algorithm="maxima")

[Out] -1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log
(x)
```

Fricas [A] time = 0.214233, size = 65, normalized size = 1.86

$$-\frac{12\left(6\,x^2-5\,x\right)\log\left(6\,x-5\right)-12\left(6\,x^2-5\,x\right)\log\left(x\right)+60\,x-25}{125\,(6\,x^2-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((6*x - 5)^2*x^2),x, algorithm="fricas")

[Out] -1/125*(12*(6*x^2 - 5*x)*log(6*x - 5) - 12*(6*x^2 - 5*x)*log(x) +
 60*x - 25)/(6*x^2 - 5*x)
```

Sympy [A] time = 0.138363, size = 29, normalized size = 0.83

$$-\frac{12x-5}{150x^2-125x}+\frac{12\log(x)}{125}-\frac{12\log\left(x-\frac{5}{6}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(5-6*x)**2/x**2,x)
```

```
[Out] -(12*x - 5)/(150*x**2 - 125*x) + 12*log(x)/125 - 12*log(x - 5/6)/
125
```

GIAC/XCAS [A] time = 0.209292, size = 54, normalized size = 1.54

$$-\frac{6}{25\,(6\,x-5)}+\frac{6}{125\left(\frac{5}{6\,x-5}+1\right)}+\frac{12}{125}\ln\left(\left|-\frac{5}{6\,x-5}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((6*x - 5)^2*x^2),x, algorithm="giac")
```

```
[Out] -6/25/(6*x - 5) + 6/125/(5/(6*x - 5) + 1) + 12/125*ln(abs(-5/(6*x
 - 5) - 1))
```

3.163 $\int \frac{1}{(-3-2x+x^2)^3} \, dx$

Optimal. Leaf size=61

$$\frac{3(1-x)}{128\left(-x^2+2x+3\right)}+\frac{1-x}{16\left(-x^2+2x+3\right)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

```
[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)
) + (3*Log[3 - x])/512 - (3*Log[1 + x])/512
```

Rubi [A] time = 0.0286666, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3(1-x)}{128\left(-x^2+2x+3\right)}+\frac{1-x}{16\left(-x^2+2x+3\right)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)
) + (3*Log[3 - x])/512 - (3*Log[1 + x])/512
```

Rubi in Sympy [A] time = 1.23511, size = 48, normalized size = 0.79

$$\frac{3\left(-2x+2\right)}{256\left(-x^2+2x+3\right)}+\frac{-2x+2}{32\left(-x^2+2x+3\right)^2}+\frac{3\log\left(-x+3\right)}{512}-\frac{3\log\left(x+1\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x-3)**3,x)
```

```
[Out] 3*(-2*x + 2)/(256*(-x**2 + 2*x + 3)) + (-2*x + 2)/(32*(-x**2 + 2*
x + 3)**2) + 3*log(-x + 3)/512 - 3*log(x + 1)/512
```

Mathematica [A] time = 0.0358042, size = 46, normalized size = 0.75

$$\frac{1}{512}\left(\frac{4\left(3x^3-9x^2-11x+17\right)}{\left(x^2-2x-3\right)^2}+3\log(3-x)-3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] ((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x
] - 3*Log[1 + x])/512
```

Maple [A] time = 0.015, size = 42, normalized size = 0.7

$$-\frac{1}{128\left(-3+x\right)^{2}}+\frac{3}{-768+256\,x}+\frac{3\ln\left(-3+x\right)}{512}+\frac{1}{128\left(1+x\right)^{2}}+\frac{3}{256+256\,x}-\frac{3\ln\left(1+x\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x-3)^3,x)
```

```
[Out] -1/128/(-3+x)^2+3/256/(-3+x)+3/512*ln(-3+x)+1/128/(1+x)^2+3/256/(
1+x)-3/512*ln(1+x)
```

Maxima [A] time = 1.63688, size = 68, normalized size = 1.11

$$\frac{3\,x^{3}-9\,x^{2}-11\,x+17}{128\left(x^{4}-4\,x^{3}-2\,x^{2}+12\,x+9\right)}-\frac{3}{512}\log\left(x+1\right)+\frac{3}{512}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 3)^(-3),x, algorithm="maxima")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9
) - 3/512*log(x + 1) + 3/512*log(x - 3)
```

Fricas [A] time = 0.239488, size = 115, normalized size = 1.89

$$\frac{12\,x^{3}-36\,x^{2}-3\left(x^{4}-4\,x^{3}-2\,x^{2}+12\,x+9\right)\log\left(x+1\right)+3\left(x^{4}-4\,x^{3}-2\,x^{2}+12\,x+9\right)\log\left(x-3\right)-44\,x+68}{512\left(x^{4}-4\,x^{3}-2\,x^{2}+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 3)^(-3),x, algorithm="fricas")
```

```
[Out] 1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x
 + 1) + 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x - 3) - 44*x + 68
)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)
```

Sympy [A] time = 0.184265, size = 51, normalized size = 0.84

$$\frac{3x^3 - 9x^2 - 11x + 17}{128x^4 - 512x^3 - 256x^2 + 1536x + 1152} + \frac{3\log(x-3)}{512} - \frac{3\log(x+1)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x-3)**3,x)
```

```
[Out] (3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1
536*x + 1152) + 3*log(x - 3)/512 - 3*log(x + 1)/512
```

GIAC/XCAS [A] time = 0.213562, size = 57, normalized size = 0.93

$$\frac{3\,x^3 - 9\,x^2 - 11\,x + 17}{128\,(x^2 - 2\,x - 3)^2} - \frac{3}{512}\ln(|x+1|) + \frac{3}{512}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 3)^(-3),x, algorithm="giac")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*ln(ab
s(x + 1)) + 3/512*ln(abs(x - 3))
```

3.164 $\int \frac{1}{(13-4x+x^2)^3} dx$

Optimal. Leaf size=51

$$-\frac{2-x}{216(x^2-4x+13)}-\frac{2-x}{36(x^2-4x+13)^2}+\frac{1}{648}\tan^{-1}\left(\frac{x-2}{3}\right)$$

```
[Out] -(2 - x)/(36*(13 - 4*x + x^2)^2) - (2 - x)/(216*(13 - 4*x + x^2))
 + ArcTan[(-2 + x)/3]/648
```

Rubi [A] time = 0.0313865, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{2-x}{216(x^2-4x+13)}-\frac{2-x}{36(x^2-4x+13)^2}+\frac{1}{648}\tan^{-1}\left(\frac{x-2}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] -(2 - x)/(36*(13 - 4*x + x^2)^2) - (2 - x)/(216*(13 - 4*x + x^2))
 + ArcTan[(-2 + x)/3]/648
```

Rubi in Sympy [A] time = 1.0803, size = 41, normalized size = 0.8

$$-\frac{-2x+4}{432(x^2-4x+13)}-\frac{-2x+4}{72(x^2-4x+13)^2}+\frac{\operatorname{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-4*x+13)**3,x)
```

```
[Out] -(-2*x + 4)/(432*(x**2 - 4*x + 13)) - (-2*x + 4)/(72*(x**2 - 4*x
+ 13)**2) + atan(x/3 - 2/3)/648
```

Mathematica [A] time = 0.0267499, size = 36, normalized size = 0.71

$$\frac{1}{648}\left(\frac{3(x-2)(x^2-4x+19)}{(x^2-4x+13)^2}+\tan^{-1}\left(\frac{x-2}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] ((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 +
x)/3])/648
```

Maple [A] time = 0.006, size = 44, normalized size = 0.9

$$\frac{2x-4}{72\left(x^2-4x+13\right)^2}+\frac{2x-4}{432x^2-1728x+5616}+\frac{1}{648}\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-4*x+13)^3,x)
```

```
[Out] 1/72*(2*x-4)/(x^2-4*x+13)^2+1/432*(2*x-4)/(x^2-4*x+13)+1/648*arct
an(-2/3+1/3*x)
```

Maxima [A] time = 1.73425, size = 59, normalized size = 1.16

$$\frac{x^3-6x^2+27x-38}{216\left(x^4-8x^3+42x^2-104x+169\right)}+\frac{1}{648}\arctan\left(\frac{1}{3}x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4*x + 13)^(-3),x, algorithm="maxima")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 1
69) + 1/648*arctan(1/3*x - 2/3)
```

Fricas [A] time = 0.219573, size = 84, normalized size = 1.65

$$\frac{3x^3-18x^2+\left(x^4-8x^3+42x^2-104x+169\right)\arctan\left(\frac{1}{3}x-\frac{2}{3}\right)+81x-114}{648\left(x^4-8x^3+42x^2-104x+169\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4*x + 13)^(-3),x, algorithm="fricas")
```

```
[Out] 1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*arct
an(1/3*x - 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169
```

)

Sympy [A] time = 0.192971, size = 42, normalized size = 0.82

$$\frac{x^3 - 6x^2 + 27x - 38}{216x^4 - 1728x^3 + 9072x^2 - 22464x + 36504} + \frac{\operatorname{atan}\left(\frac{x}{3} - \frac{2}{3}\right)}{648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-4*x+13)**3,x)
```

```
[Out] (x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 2
2464*x + 36504) + atan(x/3 - 2/3)/648
```

GIAC/XCAS [A] time = 0.197977, size = 46, normalized size = 0.9

$$\frac{x^3 - 6\,x^2 + 27\,x - 38}{216\,(x^2 - 4\,x + 13)^2} + \frac{1}{648}\,\arctan\left(\frac{1}{3}\,x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4*x + 13)^(-3),x, algorithm="giac")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan
(1/3*x - 2/3)
```

3.165 $\int \frac{1}{(2+x)^3(3+x)^4}\,dx$

Optimal. Leaf size=54

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2)
+ 6/(3 + x) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rubi [A] time = 0.0537357, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2)
+ 6/(3 + x) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rubi in Sympy [A] time = 3.42466, size = 48, normalized size = 0.89

$$10\log(x+2)-10\log(x+3)+\frac{6}{x+3}+\frac{3}{2\left(x+3\right)^2}+\frac{1}{3\left(x+3\right)^3}+\frac{4}{x+2}-\frac{1}{2\left(x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] 10*log(x + 2) - 10*log(x + 3) + 6/(x + 3) + 3/(2*(x + 3)**2) + 1/
(3*(x + 3)**3) + 4/(x + 2) - 1/(2*(x + 2)**2)
```

Mathematica [A] time = 0.0230714, size = 54, normalized size = 1.

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2)
+ 6/(3 + x) + 10*Log[2 + x] - 10*Log[3 + x]
```

Maple [A] time = 0.015, size = 49, normalized size = 0.9

$$-\frac{1}{2\,(2+x)^2}+4\,(2+x)^{-1}+\frac{1}{3\,(3+x)^3}+\frac{3}{2\,(3+x)^2}+6\,(3+x)^{-1}+10\,\ln(2+x)-10\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+x)^3/(3+x)^4,x)
```

```
[Out] -1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-1
0*ln(3+x)
```

Maxima [A] time = 1.43668, size = 81, normalized size = 1.5

$$\frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}-10\,\log(x+3)+10\,\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^4*(x + 2)^3),x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 +
 67*x^3 + 171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

Fricas [A] time = 0.236183, size = 142, normalized size = 2.63

$$\frac{60\,x^4+630\,x^3+2450\,x^2-60\,\left(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108\right)\log(x+3)+60\,\left(x^5+13\,x^4+67\,x^3+171\,x^2+216}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^4*(x + 2)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 17
1*x^2 + 216*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171
*x^2 + 216*x + 108)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 6
```

```
7*x^3 + 171*x^2 + 216*x + 108)
```

Sympy [A] time = 0.212659, size = 58, normalized size = 1.07

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10\log(x + 2) - 10\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**
4 + 402*x**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log
(x + 3)
```

GIAC/XCAS [A] time = 0.200404, size = 63, normalized size = 1.17

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x + 3)^3(x + 2)^2} - 10\ln(|x + 3|) + 10\ln(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^4*(x + 2)^3),x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x +
 2)^2) - 10*ln(abs(x + 3)) + 10*ln(abs(x + 2))
```

3.166 $\int \frac{x^6}{(-2+x^2)^2}\,dx$

Optimal. Leaf size=36

$$\frac{x^3}{3}-\frac{2x}{x^2-2}+4x-5\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]
```

Rubi [A] time = 0.0369001, antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{5x^3}{6}+\frac{x^5}{2(2-x^2)}+5x-5\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(-2 + x^2)^2,x]
```

```
[Out] 5*x + (5*x^3)/6 + x^5/(2*(2 - x^2)) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]
]
```

Rubi in Sympy [A] time = 2.73084, size = 36, normalized size = 1.

$$\frac{x^5}{2(-x^2+2)}+\frac{5x^3}{6}+5x-5\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**2-2)**2,x)
```

```
[Out] x**5/(2*(-x**2 + 2)) + 5*x**3/6 + 5*x - 5*sqrt(2)*atanh(sqrt(2)*x
/2)
```

Mathematica [A] time = 0.057379, size = 53, normalized size = 1.47

$$\frac{x^3}{3}-\frac{2x}{x^2-2}+4x+\frac{5\log\left(\sqrt{2}-x\right)}{\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(-2 + x^2)^2,x]
```

```
[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (
5*Log[Sqrt[2] + x])/Sqrt[2]
```

Maple [A] time = 0.008, size = 32, normalized size = 0.9

$$4x + \frac{x^3}{3} - 2\frac{x}{x^2-2} - 5\,Artanh\left(1/2\,x\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^2-2)^2,x)
```

```
[Out] 4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.6202, size = 58, normalized size = 1.61

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}\right)}{2x+2\sqrt{2}}\right) + 4x - \frac{2x}{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^2 - 2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*log(2*(x - sqrt(2))/((2*sqrt(2)) + 2*x)) +
4*x - 2*x/(x^2 - 2)
```

Fricas [A] time = 0.234546, size = 72, normalized size = 2.

$$\frac{2x^5 + 20x^3 + 15\sqrt{2}\left(x^2-2\right)\log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right) - 60x}{6\left(x^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^2 - 2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x
 + 2)/(x^2 - 2)) - 60*x)/(x^2 - 2)
```

Sympy [A] time = 0.113764, size = 49, normalized size = 1.36

$$\frac{x^3}{3} + 4x - \frac{2x}{x^2 - 2} + \frac{5\sqrt{2}\log\left(x - \sqrt{2}\right)}{2} - \frac{5\sqrt{2}\log\left(x + \sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**2-2)**2,x)
```

```
[Out] x**3/3 + 4*x - 2*x/(x**2 - 2) + 5*sqrt(2)*log(x - sqrt(2))/2 - 5*
sqrt(2)*log(x + sqrt(2))/2
```

GIAC/XCAS [A] time = 0.213028, size = 65, normalized size = 1.81

$$\frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\ln\left(\frac{\left|2\,x - 2\,\sqrt{2}\right|}{\left|2\,x + 2\,\sqrt{2}\right|}\right) + 4\,x - \frac{2\,x}{x^2 - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^2 - 2)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*ln(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)
)) + 4*x - 2*x/(x^2 - 2)
```

3.167 $\int \frac{x^8}{(4+x^2)^4}\,dx$

Optimal. Leaf size=58

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*
x^3)/(48*(4 + x^2)) - (35*ArcTan[x/2])/8
```

Rubi [A] time = 0.0460804, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(4 + x^2)^4,x]
```

```
[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*
x^3)/(48*(4 + x^2)) - (35*ArcTan[x/2])/8
```

Rubi in Sympy [A] time = 4.48775, size = 49, normalized size = 0.84

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**2+4)**4,x)
```

```
[Out] -x**7/(6*(x**2 + 4)**3) - 7*x**5/(24*(x**2 + 4)**2) - 35*x**3/(48
*(x**2 + 4)) + 35*x/16 - 35*atan(x/2)/8
```

Mathematica [A] time = 0.0286586, size = 40, normalized size = 0.69

$$\frac{x\left(12x^6+231x^4+1120x^2+1680\right)}{12\left(x^2+4\right)^3}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(4 + x^2)^4,x]
```

```
[Out] (x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*A
rcTan[x/2])/8
```

Maple [A] time = 0.011, size = 32, normalized size = 0.6

$$x-16\,\frac{1}{\left(x^{2}+4\right)^{3}}\left(-\frac{29\,x^{5}}{64}-\frac{17\,x^{3}}{6}-\frac{19\,x}{4}\right)-\frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^2+4)^4,x)
```

```
[Out] x-16*(-29/64*x^5-17/6*x^3-19/4*x)/(x^2+4)^3-35/8*arctan(1/2*x)
```

Maxima [A] time = 1.62779, size = 55, normalized size = 0.95

$$x+\frac{87\,x^{5}+544\,x^{3}+912\,x}{12\left(x^{6}+12\,x^{4}+48\,x^{2}+64\right)}-\frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^2 + 4)^4,x, algorithm="maxima")
```

```
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
- 35/8*arctan(1/2*x)
```

Fricas [A] time = 0.232662, size = 80, normalized size = 1.38

$$\frac{24\,x^{7}+462\,x^{5}+2240\,x^{3}-105\left(x^{6}+12\,x^{4}+48\,x^{2}+64\right)\arctan\left(\frac{1}{2}\,x\right)+3360\,x}{24\left(x^{6}+12\,x^{4}+48\,x^{2}+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^2 + 4)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 +
64)*arctan(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
```

Sympy [A] time = 0.185658, size = 39, normalized size = 0.67

$$x + \frac{87x^5 + 544x^3 + 912x}{12x^6 + 144x^4 + 576x^2 + 768} - \frac{35 \operatorname{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**2+4)**4,x)
```

```
[Out] x + (87*x**5 + 544*x**3 + 912*x)/(12*x**6 + 144*x**4 + 576*x**2 +
 768) - 35*atan(x/2)/8
```

GIAC/XCAS [A] time = 0.218448, size = 42, normalized size = 0.72

$$x + \frac{87\,x^5 + 544\,x^3 + 912\,x}{12\,(x^2 + 4)^3} - \frac{35}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^2 + 4)^4,x, algorithm="giac")
```

```
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*arctan(1/2
*x)
```

3.168 $\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{19x+39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

```
[Out] -(39 + 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[1
4]])/(28*Sqrt[14])
```

Rubi [A] time = 0.0456069, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{19x+39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] -(39 + 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[1
4]])/(28*Sqrt[14])
```

Rubi in Sympy [A] time = 2.39726, size = 39, normalized size = 0.91

$$-\frac{38x+78}{56\left(3x^2+2x+5\right)}-\frac{19\sqrt{14}\,\mathrm{atan}\left(\sqrt{14}\left(\frac{3x}{14}+\frac{1}{14}\right)\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

```
[Out] -(38*x + 78)/(56*(3*x**2 + 2*x + 5)) - 19*sqrt(14)*atan(sqrt(14)*
(3*x/14 + 1/14))/392
```

Mathematica [A] time = 0.0586574, size = 43, normalized size = 1.

$$\frac{-19x-39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] (-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[1
4]])/(28*Sqrt[14])
```

Maple [A] time = 0.007, size = 37, normalized size = 0.9

$$\frac{-38\,x-78}{168\,x^2+112\,x+280}-\frac{19\,\sqrt{14}}{392}\arctan\left(\frac{(6\,x+2)\,\sqrt{14}}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4+7*x)/(3*x^2+2*x+5)^2,x)
```

```
[Out] 1/56*(-38*x-78)/(3*x^2+2*x+5)-19/392*14^(1/2)*arctan(1/28*(6*x+2)
*14^(1/2))
```

Maxima [A] time = 1.53723, size = 49, normalized size = 1.14

$$-\frac{19}{392}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(3\,x+1)\right)-\frac{19\,x+39}{28\,(3\,x^2+2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*x - 4)/(3*x^2 + 2*x + 5)^2,x, algorithm="maxima")
```

```
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 3
9)/(3*x^2 + 2*x + 5)
```

Fricas [A] time = 0.200073, size = 68, normalized size = 1.58

$$-\frac{\sqrt{14}\left(19\,\left(3\,x^2+2\,x+5\right)\,\arctan\left(\frac{1}{14}\,\sqrt{14}(3\,x+1)\right)+\sqrt{14}(19\,x+39)\right)}{392\,(3\,x^2+2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*x - 4)/(3*x^2 + 2*x + 5)^2,x, algorithm="fricas")
```

```
[Out] -1/392*sqrt(14)*(19*(3*x^2 + 2*x + 5)*arctan(1/14*sqrt(14)*(3*x +
 1)) + sqrt(14)*(19*x + 39))/(3*x^2 + 2*x + 5)
```

Sympy [A] time = 0.171697, size = 42, normalized size = 0.98

$$-\frac{19x+39}{84x^2+56x+140}-\frac{19\sqrt{14}\operatorname{atan}\left(\frac{3\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

```
[Out] -(19*x + 39)/(84*x**2 + 56*x + 140) - 19*sqrt(14)*atan(3*sqrt(14)
*x/14 + sqrt(14)/14)/392
```

GIAC/XCAS [A] time = 0.21723, size = 49, normalized size = 1.14

$$-\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right)-\frac{19x+39}{28(3x^2+2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*x - 4)/(3*x^2 + 2*x + 5)^2,x, algorithm="giac")
```

```
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 3
9)/(3*x^2 + 2*x + 5)
```

3.169 $\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{18-7x}{20\left(-3x^2+4x+2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

```
[Out] -(18 - 7*x)/(20*(2 + 4*x - 3*x^2)) - (7*ArcTanh[(2 - 3*x)/Sqrt[10
]])/(20*Sqrt[10])
```

Rubi [A] time = 0.0507928, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{18-7x}{20\left(-3x^2+4x+2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] -(18 - 7*x)/(20*(2 + 4*x - 3*x^2)) - (7*ArcTanh[(2 - 3*x)/Sqrt[10
]])/(20*Sqrt[10])
```

Rubi in Sympy [A] time = 3.29662, size = 37, normalized size = 0.86

$$-\frac{-14x+36}{40\left(-3x^2+4x+2\right)}+\frac{7\sqrt{10}\,\operatorname{atanh}\left(\sqrt{10}\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

```
[Out] -(-14*x + 36)/(40*(-3*x**2 + 4*x + 2)) + 7*sqrt(10)*atanh(sqrt(10
)*(3*x/10 - 1/5))/200
```

Mathematica [A] time = 0.0604995, size = 62, normalized size = 1.44

$$\frac{18-7x}{20\left(3x^2-4x-2\right)}-\frac{7\log\left(-3x+\sqrt{10}+2\right)}{40\sqrt{10}}+\frac{7\log\left(3x+\sqrt{10}-2\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] (18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/
(40*Sqrt[10]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Maple [A] time = 0.004, size = 37, normalized size = 0.9

$$-\frac{14\,x-36}{120\,x^2-160\,x-80}+\frac{7\,\sqrt{10}}{200}Artanh\left(\frac{(6\,x-4)\,\sqrt{10}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5-4*x)/(3*x^2-4*x-2)^2,x)
```

```
[Out] -1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^(1/2)*arctanh(1/20*(6*x-4)
*10^(1/2))
```

Maxima [A] time = 1.59646, size = 63, normalized size = 1.47

$$-\frac{7}{400}\,\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right)-\frac{7\,x-18}{20\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x - 5)/(3*x^2 - 4*x - 2)^2,x, algorithm="maxima")
```

```
[Out] -7/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) -
1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

Fricas [A] time = 0.197833, size = 99, normalized size = 2.3

$$\frac{\sqrt{10}\left(7\,\left(3\,x^2-4\,x-2\right)\,\log\left(\frac{\sqrt{10}(9\,x^2-12\,x+14)+60\,x-40}{3\,x^2-4\,x-2}\right)-2\,\sqrt{10}(7\,x-18)\right)}{400\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x - 5)/(3*x^2 - 4*x - 2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*sqrt(10)*(7*(3*x^2 - 4*x - 2)*log((sqrt(10)*(9*x^2 - 12*x +
 14) + 60*x - 40)/(3*x^2 - 4*x - 2)) - 2*sqrt(10)*(7*x - 18))/(3*
x^2 - 4*x - 2)
```

Sympy [A] time = 0.149857, size = 58, normalized size = 1.35

$$-\frac{7x-18}{60x^2-80x-40}+\frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

```
[Out] -(7*x - 18)/(60*x**2 - 80*x - 40) + 7*sqrt(10)*log(x - 2/3 + sqrt
(10)/3)/400 - 7*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

GIAC/XCAS [A] time = 0.221801, size = 69, normalized size = 1.6

$$-\frac{7}{400}\sqrt{10}\ln\left(\frac{\left|6\,x-2\,\sqrt{10}-4\right|}{\left|6\,x+2\,\sqrt{10}-4\right|}\right)-\frac{7\,x-18}{20\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x - 5)/(3*x^2 - 4*x - 2)^2,x, algorithm="giac")
```

```
[Out] -7/400*sqrt(10)*ln(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10)
 - 4)) - 1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

3.170 $\int \frac{x^5}{(1+x^4)^3} \, dx$

Optimal. Leaf size=37

$$\frac{1}{16}\tan^{-1}\left(x^2\right)+\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}$$

```
[Out] -x^2/(8*(1 + x^4)^2) + x^2/(16*(1 + x^4)) + ArcTan[x^2]/16
```

Rubi [A] time = 0.0329375, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{1}{16}\tan^{-1}\left(x^2\right)+\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 + x^4)^3,x]

[Out] -x^2/(8*(1 + x^4)^2) + x^2/(16*(1 + x^4)) + ArcTan[x^2]/16
```

Rubi in Sympy [A] time = 2.3697, size = 27, normalized size = 0.73

$$\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}+\frac{\operatorname{atan}\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**4+1)**3,x)

[Out] x**2/(16*(x**4 + 1)) - x**2/(8*(x**4 + 1)**2) + atan(x**2)/16
```

Mathematica [A] time = 0.0156113, size = 25, normalized size = 0.68

$$\frac{1}{16}\left(\tan^{-1}\left(x^2\right)+\frac{\left(x^4-1\right)x^2}{\left(x^4+1\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 + x^4)^3,x]
```

```
[Out] ((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16
```

Maple [A] time = 0.013, size = 28, normalized size = 0.8

$$\frac{1}{2\left(x^4+1\right)^2}\left(\frac{x^6}{8}-\frac{x^2}{8}\right)+\frac{\arctan\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^4+1)^3,x)
```

```
[Out] 1/2*(1/8*x^6-1/8*x^2)/(x^4+1)^2+1/16*arctan(x^2)
```

Maxima [A] time = 1.55799, size = 41, normalized size = 1.11

$$\frac{x^6-x^2}{16\left(x^8+2x^4+1\right)}+\frac{1}{16}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^4 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*arctan(x^2)
```

Fricas [A] time = 0.221489, size = 51, normalized size = 1.38

$$\frac{x^6-x^2+\left(x^8+2x^4+1\right)\arctan\left(x^2\right)}{16\left(x^8+2x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^4 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*arctan(x^2))/(x^8 + 2*x^4 + 1
)
```

Sympy [A] time = 0.209027, size = 24, normalized size = 0.65

$$\frac{x^6-x^2}{16x^8+32x^4+16}+\frac{\operatorname{atan}\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**4+1)**3,x)
```

```
[Out] (x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + atan(x**2)/16
```

GIAC/XCAS [A] time = 0.210997, size = 54, normalized size = 1.46

$$\frac{x^2 - \frac{1}{x^2}}{16\left(\left(x^2 - \frac{1}{x^2}\right)^2 + 4\right)} + \frac{1}{32}\arctan\left(\frac{x^4 - 1}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^4 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 -
 1)/x^2)
```

3.171 $\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx$

Optimal. Leaf size=32

$$\frac{1}{4(x^4+2x^2+2)} + \frac{1}{4}\log\left(x^4+2x^2+2\right)$$

```
[Out] 1/(4*(2 + 2*x^2 + x^4)) + Log[2 + 2*x^2 + x^4]/4
```

Rubi [A] time = 0.0698568, antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{4}\log\left(x^4+2x^2+2\right) - \frac{\left(x^2+1\right)^2}{4(x^4+2x^2+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]

[Out] -(1 + x^2)^2/(4*(2 + 2*x^2 + x^4)) + Log[2 + 2*x^2 + x^4]/4
```

Rubi in Sympy [A] time = 6.52288, size = 31, normalized size = 0.97

$$-\frac{\left(x^2+1\right)^2}{4(x^4+2x^2+2)} + \frac{\log\left(x^4+2x^2+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)

[Out] -(x**2 + 1)**2/(4*(x**4 + 2*x**2 + 2)) + log(x**4 + 2*x**2 + 2)/4
```

Mathematica [A] time = 0.0195718, size = 26, normalized size = 0.81

$$\frac{1}{4}\left(\frac{1}{(x^2+1)^2+1} + \log\left(\left(x^2+1\right)^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

```
[Out] ((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4
```

Maple [A] time = 0.011, size = 29, normalized size = 0.9

$$\frac{1}{4\,x^{4}+8\,x^{2}+8}+\frac{\ln\left(x^{4}+2\,x^{2}+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x)
```

```
[Out] 1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

Maxima [A] time = 1.42767, size = 38, normalized size = 1.19

$$\frac{1}{4\,(x^{4}+2\,x^{2}+2)}+\frac{1}{4}\,\log\left(x^{4}+2\,x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3*x/(x^4 + 2*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

Fricas [A] time = 0.204666, size = 51, normalized size = 1.59

$$\frac{\left(x^{4}+2\,x^{2}+2\right)\log\left(x^{4}+2\,x^{2}+2\right)+1}{4\,(x^{4}+2\,x^{2}+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3*x/(x^4 + 2*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 + 2*x^2 + 2)*log(x^4 + 2*x^2 + 2) + 1)/(x^4 + 2*x^2 + 2
)
```

Sympy [A] time = 0.172494, size = 26, normalized size = 0.81

$$\frac{\log\left(x^{4}+2x^{2}+2\right)}{4}+\frac{1}{4x^{4}+8x^{2}+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)
```

```
[Out] log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)
```

GIAC/XCAS [A] time = 0.201695, size = 38, normalized size = 1.19

$$\frac{1}{4\left(x^{4}+2\,x^{2}+2\right)}+\frac{1}{4}\,\ln\left(x^{4}+2\,x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^3*x/(x^4 + 2*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 1/4/(x^4 + 2*x^2 + 2) + 1/4*ln(x^4 + 2*x^2 + 2)
```

3.172 $\int \frac{x^3}{(a^4+x^4)^3} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{8 \left(a^4+x^4\right)^2}$$

```
[Out] -1/(8*(a^4 + x^4)^2)
```

Rubi [A] time = 0.00623231, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{1}{8 \left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^4 + x^4)^3,x]

[Out] -1/(8*(a^4 + x^4)^2)
```

Rubi in Sympy [A] time = 0.879579, size = 12, normalized size = 0.92

$$-\frac{1}{8 \left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a**4+x**4)**3,x)

[Out] -1/(8*(a**4 + x**4)**2)
```

Mathematica [A] time = 0.00514981, size = 13, normalized size = 1.

$$-\frac{1}{8 \left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^4 + x^4)^3,x]
```

```
[Out] -1/(8*(a^4 + x^4)^2)
```

Maple [A] time = 0.002, size = 12, normalized size = 0.9

$$-\frac{1}{8\left(a^{4}+x^{4}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a^4+x^4)^3,x)
```

```
[Out] -1/8/(a^4+x^4)^2
```

Maxima [A] time = 1.42243, size = 15, normalized size = 1.15

$$-\frac{1}{8\left(a^{4}+x^{4}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a^4 + x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

Fricas [A] time = 0.200288, size = 26, normalized size = 2.

$$-\frac{1}{8\left(a^{8}+2\,a^{4}x^{4}+x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a^4 + x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(a^8 + 2*a^4*x^4 + x^8)
```

Sympy [A] time = 1.69079, size = 20, normalized size = 1.54

$$-\frac{1}{8a^{8}+16a^{4}x^{4}+8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a**4+x**4)**3,x)
```

```
[Out] -1/(8*a**8 + 16*a**4*x**4 + 8*x**8)
```

GIAC/XCAS [A] time = 0.200912, size = 15, normalized size = 1.15

$$-\frac{1}{8\,(a^4+x^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a^4 + x^4)^3,x, algorithm="giac")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

3.173 $\int \frac{1}{x(a^4+x^4)^3} \, dx$

Optimal. Leaf size=54

$$\frac{\log(x)}{a^{12}}+\frac{1}{8a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{1}{4a^8\left(a^4+x^4\right)}$$

```
[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - L
og[a^4 + x^4]/(4*a^12)
```

Rubi [A] time = 0.0616274, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\log(x)}{a^{12}}+\frac{1}{8a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{1}{4a^8\left(a^4+x^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - L
og[a^4 + x^4]/(4*a^12)
```

Rubi in Sympy [A] time = 4.48663, size = 51, normalized size = 0.94

$$\frac{1}{8a^4\left(a^4+x^4\right)^2}+\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{\log\left(x^4\right)}{4a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x/(a**4+x**4)**3,x)
```

```
[Out] 1/(8*a**4*(a**4 + x**4)**2) + 1/(4*a**8*(a**4 + x**4)) + log(x**4
)/(4*a**12) - log(a**4 + x**4)/(4*a**12)
```

Mathematica [A] time = 0.0402737, size = 46, normalized size = 0.85

$$\frac{-2\log\left(a^4+x^4\right)+\frac{3a^8+2a^4x^4}{\left(a^4+x^4\right)^2}+8\log(x)}{8a^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] ((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])
/(8*a^12)
```

Maple [A] time = 0.023, size = 49, normalized size = 0.9

$$\frac{1}{8\,a^{4}\left(a^{4}+x^{4}\right)^{2}}+\frac{1}{4\,a^{8}\left(a^{4}+x^{4}\right)}+\frac{\ln\left(x\right)}{a^{12}}-\frac{\ln\left(a^{4}+x^{4}\right)}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^4+x^4)^3,x)
```

```
[Out] 1/8/a^4/(a^4+x^4)^2+1/4/a^8/(a^4+x^4)+ln(x)/a^12-1/4*ln(a^4+x^4)/
a^12
```

Maxima [A] time = 1.4597, size = 77, normalized size = 1.43

$$\frac{3\,a^{4}+2\,x^{4}}{8\left(a^{16}+2\,a^{12}x^{4}+a^{8}x^{8}\right)}-\frac{\log\left(a^{4}+x^{4}\right)}{4\,a^{12}}+\frac{\log\left(x^{4}\right)}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x),x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^4 + 2*x^4)/(a^16 + 2*a^12*x^4 + a^8*x^8) - 1/4*log(a^4 +
 x^4)/a^12 + 1/4*log(x^4)/a^12
```

Fricas [A] time = 0.227378, size = 109, normalized size = 2.02

$$\frac{3\,a^{8}+2\,a^{4}x^{4}-2\left(a^{8}+2\,a^{4}x^{4}+x^{8}\right)\log\left(a^{4}+x^{4}\right)+8\left(a^{8}+2\,a^{4}x^{4}+x^{8}\right)\log\left(x\right)}{8\left(a^{20}+2\,a^{16}x^{4}+a^{12}x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^8 + 2*a^4*x^4 - 2*(a^8 + 2*a^4*x^4 + x^8)*log(a^4 + x^4)
 + 8*(a^8 + 2*a^4*x^4 + x^8)*log(x))/(a^20 + 2*a^16*x^4 + a^12*x^
8)
```

Sympy [A] time = 4.93131, size = 51, normalized size = 0.94

$$\frac{3a^4 + 2x^4}{8a^{16} + 16a^{12}x^4 + 8a^8x^8} + \frac{\log(x)}{a^{12}} - \frac{\log\left(a^4 + x^4\right)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**4+x**4)**3,x)
```

```
[Out] (3*a**4 + 2*x**4)/(8*a**16 + 16*a**12*x**4 + 8*a**8*x**8) + log(x
)/a**12 - log(a**4 + x**4)/(4*a**12)
```

GIAC/XCAS [A] time = 0.204906, size = 76, normalized size = 1.41

$$-\frac{\ln\left(a^4 + x^4\right)}{4\,a^{12}} + \frac{\ln\left(x^4\right)}{4\,a^{12}} + \frac{6\,a^8 + 8\,a^4x^4 + 3\,x^8}{8\,(a^4 + x^4)^2a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x),x, algorithm="giac")
```

```
[Out] -1/4*ln(a^4 + x^4)/a^12 + 1/4*ln(x^4)/a^12 + 1/8*(6*a^8 + 8*a^4*x
^4 + 3*x^8)/((a^4 + x^4)^2*a^12)
```

3.174 $\int \frac{1}{x^2(a^4+x^4)^3}\,dx$

Optimal. Leaf size=157

$$\frac{45\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{64\sqrt{2}a^{13}}-\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{9}{32a^8x\left(a^4+x^4\right)}$$

```
[Out] -45/(32*a^12*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 +
x^4)) + (45*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*Ar
cTan[1 + (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*Log[a^2 - Sqrt[2
]*a*x + x^2])/(128*Sqrt[2]*a^13) + (45*Log[a^2 + Sqrt[2]*a*x + x^
2])/(128*Sqrt[2]*a^13)
```

Rubi [A] time = 0.209406, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$\frac{45\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{64\sqrt{2}a^{13}}-\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{9}{32a^8x\left(a^4+x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -45/(32*a^12*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 +
x^4)) + (45*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*Ar
cTan[1 + (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*Log[a^2 - Sqrt[2
]*a*x + x^2])/(128*Sqrt[2]*a^13) + (45*Log[a^2 + Sqrt[2]*a*x + x^
2])/(128*Sqrt[2]*a^13)
```

Rubi in Sympy [A] time = 17.7957, size = 144, normalized size = 0.92

$$\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45}{32a^{12}x}-\frac{45\sqrt{2}\log\left(a^2-\sqrt{2}ax+x^2\right)}{256a^{13}}+\frac{45\sqrt{2}\log\left(a^2+\sqrt{2}ax+x^2\right)}{256a^{13}}+\frac{45\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}x}{a}\right)}{128a^{13}}-\frac{45\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}x}{a}\right)}{128a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a**4+x**4)**3,x)
```

```
[Out] 1/(8*a**4*x*(a**4 + x**4)**2) + 9/(32*a**8*x*(a**4 + x**4)) - 45/
(32*a**12*x) - 45*sqrt(2)*log(a**2 - sqrt(2)*a*x + x**2)/(256*a**
13) + 45*sqrt(2)*log(a**2 + sqrt(2)*a*x + x**2)/(256*a**13) + 45*
sqrt(2)*atan(1 - sqrt(2)*x/a)/(128*a**13) - 45*sqrt(2)*atan(1 + s
qrt(2)*x/a)/(128*a**13)
```

Mathematica [A] time = 0.177288, size = 134, normalized size = 0.85

$$-\frac{\frac{104ax^3}{a^4+x^4}+45\sqrt{2}\log\left(a^2-\sqrt{2}ax+x^2\right)-45\sqrt{2}\log\left(a^2+\sqrt{2}ax+x^2\right)+\frac{32a^5x^3}{\left(a^4+x^4\right)^2}+\frac{256a}{x}-90\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)+90\sqrt{2}\tan^{-1}}{256a^{13}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4
) - 90*Sqrt[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 +
(Sqrt[2]*x)/a] + 45*Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqr
t[2]*Log[a^2 + Sqrt[2]*a*x + x^2])/(256*a^13)
```

Maple [A] time = 0.018, size = 152, normalized size = 1.

$$-\frac{1}{a^{12}x}-\frac{13\,x^7}{32\,a^{12}\left(a^4+x^4\right)^2}-\frac{17\,x^3}{32\,a^8\left(a^4+x^4\right)^2}-\frac{45\,\sqrt{2}}{256\,a^{12}}\ln\left(1\left(x^2-\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)\left(x^2+\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)^{-1}\right)\frac{1}{\sqrt[4]{a^4}}-\frac{45\,\sqrt{2}}{128\,a^{12}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{a^4}}+1\right)\frac{1}{\sqrt[4]{a^4}}-\frac{45\,\sqrt{2}}{128\,a^{12}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{a^4}}-1\right)\frac{1}{\sqrt[4]{a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a^4+x^4)^3,x)
```

```
[Out] -1/a^12/x-13/32/a^12/(a^4+x^4)^2*x^7-17/32/a^8/(a^4+x^4)^2*x^3-45
/256/a^12/(a^4)^(1/4)*2^(1/2)*ln((x^2-(a^4)^(1/4)*x*2^(1/2)+(a^4)
^(1/2))/(x^2+(a^4)^(1/4)*x*2^(1/2)+(a^4)^(1/2)))-45/128/a^12/(a^4
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a^4)^(1/4)*x+1)-45/128/a^12/(a^4)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a^4)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233237, size = 468, normalized size = 2.98

$$-\frac{256\,a^{8}+648\,a^{4}x^{4}+360\,x^{8}-180\,\sqrt{2}\left(a^{20}x+2\,a^{16}x^{5}+a^{12}x^{9}\right)\frac{1}{a^{52}}^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}a^{40}\frac{1}{a^{52}}^{\frac{3}{4}}}{\sqrt{2}a^{40}\frac{1}{a^{52}}^{\frac{3}{4}}+2\,x+2\,\sqrt{\sqrt{2}a^{40}\frac{1}{a^{52}}^{\frac{3}{4}}x+a^{28}\sqrt{\frac{1}{a^{52}}}+x^{2}}}\right)-180\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x^2),x, algorithm="fricas")
```

```
[Out] -1/256*(256*a^8 + 648*a^4*x^4 + 360*x^8 - 180*sqrt(2)*(a^20*x + 2
*a^16*x^5 + a^12*x^9)*(a^(-52))^(1/4)*arctan(sqrt(2)*a^40*(a^(-52
))^(3/4)/(sqrt(2)*a^40*(a^(-52))^(3/4) + 2*x + 2*sqrt(sqrt(2)*a^4
0*(a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2))) - 180*sqrt(2)*(
a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(1/4)*arctan(-sqrt(2)*a
^40*(a^(-52))^(3/4)/(sqrt(2)*a^40*(a^(-52))^(3/4) - 2*x - 2*sqrt(
-sqrt(2)*a^40*(a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2))) - 4
5*sqrt(2)*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(1/4)*log(sq
rt(2)*a^40*(a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2) + 45*sqr
t(2)*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(1/4)*log(-sqrt(2
)*a^40*(a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2))/(a^20*x + 2
*a^16*x^5 + a^12*x^9)
```

Sympy [A] time = 9.00492, size = 65, normalized size = 0.41

$$-\frac{32a^{8}+81a^{4}x^{4}+45x^{8}}{32a^{20}x+64a^{16}x^{5}+32a^{12}x^{9}}+\frac{\operatorname{RootSum}\left(268435456t^{4}+4100625,\left(t\mapsto t\log\left(-\frac{2097152t^{3}a}{91125}+x\right)\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a**4+x**4)**3,x)
```

```
[Out] -(32*a**8 + 81*a**4*x**4 + 45*x**8)/(32*a**20*x + 64*a**16*x**5 +
 32*a**12*x**9) + RootSum(268435456*_t**4 + 4100625, Lambda(_t, _
t*log(-2097152*_t**3*a/91125 + x)))/a**13
```

GIAC/XCAS [A] time = 0.203752, size = 203, normalized size = 1.29

$$-\frac{45\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{128a^{14}}-\frac{45\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{128a^{14}}+\frac{45\sqrt{2}|a|\ln\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{256a^{14}}-\frac{45\sqrt{2}|a|\ln\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{256a^{14}}-\frac{17a^4x^3+13x^7}{32\left(a^4+x^4\right)^2a^{12}}-\frac{1}{a^{12}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x^2),x, algorithm="giac")
```

```
[Out] -45/128*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/
abs(a))/a^14 - 45/128*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)
*abs(a) - 2*x)/abs(a))/a^14 + 45/256*sqrt(2)*abs(a)*ln(sqrt(2)*x*
abs(a) + x^2 + abs(a)^2)/a^14 - 45/256*sqrt(2)*abs(a)*ln(-sqrt(2)
*x*abs(a) + x^2 + abs(a)^2)/a^14 - 1/32*(17*a^4*x^3 + 13*x^7)/((a
^4 + x^4)^2*a^12) - 1/(a^12*x)
```

3.175 $\int \frac{1}{x^3(a^4+x^4)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}$$

```
[Out] -15/(16*a^12*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(
a^4 + x^4)) - (15*ArcTan[x^2/a^2])/(16*a^14)
```

Rubi [A] time = 0.0701617, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] -15/(16*a^12*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(
a^4 + x^4)) - (15*ArcTan[x^2/a^2])/(16*a^14)
```

Rubi in Sympy [A] time = 6.67207, size = 60, normalized size = 0.94

$$\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15}{16a^{12}x^2}-\frac{15\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a**4+x**4)**3,x)
```

```
[Out] 1/(8*a**4*x**2*(a**4 + x**4)**2) + 5/(16*a**8*x**2*(a**4 + x**4))
 - 15/(16*a**12*x**2) - 15*atan(x**2/a**2)/(16*a**14)
```

Mathematica [A] time = 0.0831857, size = 75, normalized size = 1.17

$$-\frac{\frac{a^2\left(8a^8+25a^4x^4+15x^8\right)}{x^2\left(a^4+x^4\right)^2}+15\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)+15\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{16a^{14}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] (-((a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*
ArcTan[1 - (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a])/(16*a^1
4)
```

Maple [A] time = 0.019, size = 57, normalized size = 0.9

$$-\frac{1}{2\,a^{12}x^{2}}-\frac{9\,x^{2}}{16\,a^{8}\,(a^{4}+x^{4})^{2}}-\frac{7\,x^{6}}{16\,a^{12}\,(a^{4}+x^{4})^{2}}-\frac{15}{16\,a^{14}}\arctan\left(\frac{x^{2}}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a^4+x^4)^3,x)
```

```
[Out] -1/2/a^12/x^2-9/16/a^8/(a^4+x^4)^2*x^2-7/16/a^12/(a^4+x^4)^2*x^6-
15/16*arctan(x^2/a^2)/a^14
```

Maxima [A] time = 1.52541, size = 81, normalized size = 1.27

$$-\frac{8\,a^{8}+25\,a^{4}x^{4}+15\,x^{8}}{16\,(a^{20}x^{2}+2\,a^{16}x^{6}+a^{12}x^{10})}-\frac{15\,\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x^3),x, algorithm="maxima")
```

```
[Out] -1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^20*x^2 + 2*a^16*x^6 + a^12
*x^10) - 15/16*arctan(x^2/a^2)/a^14
```

Fricas [A] time = 0.211853, size = 105, normalized size = 1.64

$$-\frac{8\,a^{10}+25\,a^{6}x^{4}+15\,a^{2}x^{8}+15\,\left(a^{8}x^{2}+2\,a^{4}x^{6}+x^{10}\right)\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\,(a^{22}x^{2}+2\,a^{18}x^{6}+a^{14}x^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x^3),x, algorithm="fricas")
```

```
[Out] -1/16*(8*a^10 + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6
 + x^10)*arctan(x^2/a^2))/(a^22*x^2 + 2*a^18*x^6 + a^14*x^10)
```

Sympy [A] time = 16.1242, size = 76, normalized size = 1.19

$$-\frac{8a^8 + 25a^4x^4 + 15x^8}{16a^{20}x^2 + 32a^{16}x^6 + 16a^{12}x^{10}} + \frac{\frac{15i\log\left(-ia^2+x^2\right)}{32} - \frac{15i\log\left(ia^2+x^2\right)}{32}}{a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a**4+x**4)**3,x)
```

```
[Out] -(8*a**8 + 25*a**4*x**4 + 15*x**8)/(16*a**20*x**2 + 32*a**16*x**6
 + 16*a**12*x**10) + (15*I*log(-I*a**2 + x**2)/32 - 15*I*log(I*a*
*2 + x**2)/32)/a**14
```

GIAC/XCAS [A] time = 0.226912, size = 68, normalized size = 1.06

$$-\frac{9\,a^4x^2 + 7\,x^6}{16\,(a^4 + x^4)^2a^{12}} - \frac{15\,\arctan\left(\frac{x^2}{a^2}\right)}{16\,a^{14}} - \frac{1}{2\,a^{12}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a^4 + x^4)^3*x^3),x, algorithm="giac")
```

```
[Out] -1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^12) - 15/16*arctan(x^2
/a^2)/a^14 - 1/2/(a^12*x^2)
```

3.176 $\int \frac{x^{14}}{(3+2x^5)^3} \, dx$

Optimal. Leaf size=39

$$\frac{3}{20\left(2x^5+3\right)}-\frac{9}{80\left(2x^5+3\right)^2}+\frac{1}{40}\log\left(2x^5+3\right)$$

```
[Out] -9/(80*(3 + 2*x^5)^2) + 3/(20*(3 + 2*x^5)) + Log[3 + 2*x^5]/40
```

Rubi [A] time = 0.0519595, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3}{20\left(2x^5+3\right)}-\frac{9}{80\left(2x^5+3\right)^2}+\frac{1}{40}\log\left(2x^5+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^14/(3 + 2*x^5)^3,x]

[Out] -9/(80*(3 + 2*x^5)^2) + 3/(20*(3 + 2*x^5)) + Log[3 + 2*x^5]/40
```

Rubi in Sympy [A] time = 3.26626, size = 29, normalized size = 0.74

$$\frac{\log\left(2x^5+3\right)}{40}+\frac{3}{20\left(2x^5+3\right)}-\frac{9}{80\left(2x^5+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14/(2*x**5+3)**3,x)

[Out] log(2*x**5 + 3)/40 + 3/(20*(2*x**5 + 3)) - 9/(80*(2*x**5 + 3)**2)
```

Mathematica [A] time = 0.0235591, size = 33, normalized size = 0.85

$$\frac{1}{80}\left(\frac{3\left(8x^5+9\right)}{\left(2x^5+3\right)^2}+2\log\left(2x^5+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14/(3 + 2*x^5)^3,x]
```

```
[Out] ((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80
```

Maple [A] time = 0.013, size = 34, normalized size = 0.9

$$-\frac{9}{80\left(2x^5+3\right)^2}+\frac{3}{40x^5+60}+\frac{\ln\left(2x^5+3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14/(2*x^5+3)^3,x)
```

```
[Out] -9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*ln(2*x^5+3)
```

Maxima [A] time = 1.42791, size = 46, normalized size = 1.18

$$\frac{3\left(8x^5+9\right)}{80\left(4x^{10}+12x^5+9\right)}+\frac{1}{40}\log\left(2x^5+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(2*x^5 + 3)^3,x, algorithm="maxima")
```

```
[Out] 3/80*(8*x^5 + 9)/(4*x^10 + 12*x^5 + 9) + 1/40*log(2*x^5 + 3)
```

Fricas [A] time = 0.203124, size = 61, normalized size = 1.56

$$\frac{24x^5+2\left(4x^{10}+12x^5+9\right)\log\left(2x^5+3\right)+27}{80\left(4x^{10}+12x^5+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(2*x^5 + 3)^3,x, algorithm="fricas")
```

```
[Out] 1/80*(24*x^5 + 2*(4*x^10 + 12*x^5 + 9)*log(2*x^5 + 3) + 27)/(4*x^
10 + 12*x^5 + 9)
```

Sympy [A] time = 0.257725, size = 27, normalized size = 0.69

$$\frac{24x^5+27}{320x^{10}+960x^5+720}+\frac{\log\left(2x^5+3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14/(2*x**5+3)**3,x)
```

```
[Out] (24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40
```

GIAC/XCAS [A] time = 0.248318, size = 41, normalized size = 1.05

$$-\frac{3\left(x^{10}+x^{5}\right)}{20\left(2x^{5}+3\right)^{2}}+\frac{1}{40}\ln\left(\left|2x^{5}+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(2*x^5 + 3)^3,x, algorithm="giac")
```

```
[Out] -3/20*(x^10 + x^5)/(2*x^5 + 3)^2 + 1/40*ln(abs(2*x^5 + 3))
```

3.177 $\int \frac{x^6}{(3+2x^5)^3}\,dx$

Optimal. Leaf size=325

$$\frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{x^2}{150\left(2x^5+3\right)}-\frac{x^2}{20\left(2x^5+3\right)^2}-\frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{250\ 2^{2/5}3^{3/5}}+\frac{\sqrt{5-\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2^{7/10}\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{250\ 2^{9/10}3^{3/5}}-\frac{\sqrt{5+\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[5]{6}\left(1+\sqrt{5}\right)-4\ 2^{2/5}x}{2^{7/10}\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250\ 2^{9/10}3^{3/5}}$$

```
[Out] -x^2/(20*(3 + 2*x^5)^2) + x^2/(150*(3 + 2*x^5)) + (Sqrt[5 - Sqrt[
5]]*ArcTan[(6^(1/5)*(1 - Sqrt[5]) - 4*2^(2/5)*x)/(2^(7/10)*3^(1/5
)*Sqrt[5 + Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - (Sqrt[5 + Sqrt[5]
]*ArcTan[(6^(1/5)*(1 + Sqrt[5]) - 4*2^(2/5)*x)/(2^(7/10)*3^(1/5)*
Sqrt[5 - Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - Log[3^(1/5) + 2^(1/
5)*x]/(250*2^(2/5)*3^(3/5)) + ((1 + Sqrt[5])*Log[3^(2/5) - (3^(1/
5)*(1 - Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5)
) + ((1 - Sqrt[5])*Log[3^(2/5) - (3^(1/5)*(1 + Sqrt[5])*x)/2^(4/5
) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5))
```

Rubi [A] time = 1.15452, antiderivative size = 319, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$\frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{x^2}{150\left(2x^5+3\right)}-\frac{x^2}{20\left(2x^5+3\right)^2}-\frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{250\ 2^{2/5}3^{3/5}}-\frac{\sqrt{5+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{2\ 2^{7/10}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250\ 2^{9/10}3^{3/5}}-\frac{\sqrt{5-\sqrt{5}}\tan^{-1}\left(\frac{2\ 2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}+\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\right)}{250\ 2^{9/10}3^{3/5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] -x^2/(20*(3 + 2*x^5)^2) + x^2/(150*(3 + 2*x^5)) - (Sqrt[5 + Sqrt[
5]]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - (2*2^(7/10)*x)/(3^(1/5)*Sqrt
[5 - Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - (Sqrt[5 - Sqrt[5]]*ArcT
an[Sqrt[(5 - 2*Sqrt[5])/5] + (2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 + Sqr
```

```
t[5]])])/(250*2^(9/10)*3^(3/5)) - Log[3^(1/5) + 2^(1/5)*x]/(250*2
^(2/5)*3^(3/5)) + ((1 + Sqrt[5])*Log[3^(2/5) - (3^(1/5)*(1 - Sqrt
[5])*x)/2^(4/5) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5)) + ((1 - Sq
rt[5])*Log[3^(2/5) - (3^(1/5)*(1 + Sqrt[5])*x)/2^(4/5) + 2^(2/5)*
x^2])/(1000*2^(2/5)*3^(3/5))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(2*x**5+3)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.556505, size = 293, normalized size = 0.9

$$2^{3/5}3^{2/5}\left(1+\sqrt{5}\right)\log\left(2^{2/5}3^{3/5}x^2+\left(\tfrac{3}{2}\right)^{4/5}\left(\sqrt{5}-1\right)x+3\right)-2^{3/5}3^{2/5}\left(\sqrt{5}-1\right)\log\left(2^{2/5}3^{3/5}x^2-\left(\tfrac{3}{2}\right)^{4/5}\left(1+\sqrt{5}\right)x+3\right)+$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] ((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(
2/5)*Sqrt[5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)
*x)/(3*Sqrt[2*(5 + Sqrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt
[5]]*ArcTan[(-3*(1 + Sqrt[5]) + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 -
 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Log[3 + 2^(1/5)*3^(4/5)*x] + 2^
(3/5)*3^(2/5)*(1 + Sqrt[5])*Log[3 + (3/2)^(4/5)*(-1 + Sqrt[5])*x
+ 2^(2/5)*3^(3/5)*x^2] - 2^(3/5)*3^(2/5)*(-1 + Sqrt[5])*Log[3 - (
3/2)^(4/5)*(1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2])/6000
```

Maple [A] time = 0.164, size = 354, normalized size = 1.1

$$4\frac{1}{(2x^5+3)^2}\left(\frac{x^7}{300}-\frac{3x^2}{400}\right)+\frac{48^{\frac{2}{5}}\ln\left(\sqrt[5]{48}+2x\right)}{\left(150\sqrt{5}-750\right)\left(5+\sqrt{5}\right)}$$
$$+\frac{48^{\frac{2}{5}}\ln\left(x\sqrt{5}\sqrt[5]{48}-x\sqrt[5]{48}+48^{\frac{2}{5}}+4x^2\right)\sqrt{5}}{12000}+\frac{48^{\frac{2}{5}}\ln\left(x\sqrt{5}\sqrt[5]{48}-x\sqrt[5]{48}+48^{\frac{2}{5}}+4x^2\right)}{12000}$$
$$-\frac{\sqrt{5}48^{\frac{3}{5}}}{1500\sqrt{10\,48^{2/5}+2\sqrt{5}48^{2/5}}}\arctan\left(\frac{\sqrt{5}\sqrt[5]{48}}{\sqrt{10\,48^{2/5}+2\sqrt{5}48^{2/5}}}-\frac{\sqrt[5]{48}}{\sqrt{10\,48^{2/5}+2\sqrt{5}48^{2/5}}}+8\frac{x}{\sqrt{10\,48^{2/5}+2\sqrt{5}48^{2/5}}}\right)$$
$$-\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\sqrt[5]{48}-x\sqrt[5]{48}+48^{\frac{2}{5}}+4x^2\right)\sqrt{5}}{12000}+\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\sqrt[5]{48}-x\sqrt[5]{48}+48^{\frac{2}{5}}+4x^2\right)}{12000}$$
$$+\frac{\sqrt{5}48^{\frac{3}{5}}}{1500\sqrt{10\,48^{2/5}-2\sqrt{5}48^{2/5}}}\arctan\left(-\frac{\sqrt{5}\sqrt[5]{48}}{\sqrt{10\,48^{2/5}-2\sqrt{5}48^{2/5}}}-\frac{\sqrt[5]{48}}{\sqrt{10\,48^{2/5}-2\sqrt{5}48^{2/5}}}+8\frac{x}{\sqrt{10\,48^{2/5}-2\sqrt{5}48^{2/5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6/(2*x^5+3)^3,x)
```

```
[Out] 4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/150*48^(2/5)/(5^(1/2)-5)/(5
+5^(1/2))*ln(48^(1/5)+2*x)+1/12000*48^(2/5)*ln(x*5^(1/2)*48^(1/5)
-x*48^(1/5)+48^(2/5)+4*x^2)*5^(1/2)+1/12000*48^(2/5)*ln(x*5^(1/2)
*48^(1/5)-x*48^(1/5)+48^(2/5)+4*x^2)-1/1500*48^(3/5)*5^(1/2)/(10*
48^(2/5)+2*5^(1/2)*48^(2/5))^(1/2)*arctan(1/(10*48^(2/5)+2*5^(1/2
)*48^(2/5))^(1/2)*5^(1/2)*48^(1/5)-1/(10*48^(2/5)+2*5^(1/2)*48^(2
/5))^(1/2)*48^(1/5)+8/(10*48^(2/5)+2*5^(1/2)*48^(2/5))^(1/2)*x)-1
/12000*48^(2/5)*ln(-x*5^(1/2)*48^(1/5)-x*48^(1/5)+48^(2/5)+4*x^2)
*5^(1/2)+1/12000*48^(2/5)*ln(-x*5^(1/2)*48^(1/5)-x*48^(1/5)+48^(2
/5)+4*x^2)+1/1500*48^(3/5)*5^(1/2)/(10*48^(2/5)-2*5^(1/2)*48^(2/5
))^(1/2)*arctan(-1/(10*48^(2/5)-2*5^(1/2)*48^(2/5))^(1/2)*5^(1/2)
*48^(1/5)-1/(10*48^(2/5)-2*5^(1/2)*48^(2/5))^(1/2)*48^(1/5)+8/(10
*48^(2/5)-2*5^(1/2)*48^(2/5))^(1/2)*x)
```

Maxima [A] time = 1.58024, size = 452, normalized size = 1.39

$$\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}-5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot2^{\frac{2}{5}}x+\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}-3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{2\sqrt{5}+10}} + \frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}+5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot2^{\frac{2}{5}}x-\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{-2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}+3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{-2\sqrt{5}+10}} - \frac{1}{1500}\cdot3^{\frac{2}{5}}2^{\frac{3}{5}}\log\left(2^{\frac{1}{5}}x+3^{\frac{1}{5}}\right) + \frac{4x^7-9x^2}{300\left(4x^{10}+12x^5+9\right)} - \frac{\log\left(2\cdot2^{\frac{2}{5}}x^2-x\left(\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}+3^{\frac{1}{5}}2^{\frac{1}{5}}\right)+2\cdot3^{\frac{2}{5}}\right)}{250\left(\sqrt{5}3^{\frac{3}{5}}2^{\frac{2}{5}}+3^{\frac{3}{5}}2^{\frac{2}{5}}\right)} + \frac{\log\left(2\cdot2^{\frac{2}{5}}x^2+x\left(\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)+2\cdot3^{\frac{2}{5}}\right)}{250\left(\sqrt{5}3^{\frac{3}{5}}2^{\frac{2}{5}}-3^{\frac{3}{5}}2^{\frac{2}{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(2*x^5 + 3)^3,x, algorithm="maxima")
```

```
[Out] 1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4
*2^(2/5)*x + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sq
rt(5) + 10))/((sqrt(5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*
sqrt(5) + 10)) + 1/750*3^(4/5)*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3
^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2
^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2/5)*2^(1/5) + 3^(2/5
)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(3/5)*log(2^
(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9)
- 1/250*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*
2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5))
 + 1/250*log(2*2^(2/5)*x^2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)
*2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) - 3^(3/5)*2^(2/5)
)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(2*x^5 + 3)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.484555, size = 37, normalized size = 0.11

$$\frac{4x^7 - 9x^2}{1200x^{10} + 3600x^5 + 2700} + \text{RootSum}\left(105468750000000t^5 + 1, \left(t \mapsto t\log\left(-281250000t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(2*x**5+3)**3,x)
```

```
[Out] (4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(10546
8750000000*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(2*x^5 + 3)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.178 $\int \frac{9}{5x^2(3-2x^2)^3}\,dx$

Optimal. Leaf size=59

$$\frac{1}{8x\,(3-2x^2)}+\frac{3}{20x\,(3-2x^2)^2}-\frac{1}{8x}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

```
[Out] -1/(8*x) + 3/(20*x*(3 - 2*x^2)^2) + 1/(8*x*(3 - 2*x^2)) + ArcTanh
[Sqrt[2/3]*x]/(4*Sqrt[6])
```

Rubi [A] time = 0.0528567, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{8x\,(3-2x^2)}+\frac{3}{20x\,(3-2x^2)^2}-\frac{1}{8x}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] -1/(8*x) + 3/(20*x*(3 - 2*x^2)^2) + 1/(8*x*(3 - 2*x^2)) + ArcTanh
[Sqrt[2/3]*x]/(4*Sqrt[6])
```

Rubi in Sympy [A] time = 3.89367, size = 46, normalized size = 0.78

$$\frac{\sqrt{6}\,\operatorname{atanh}\left(\frac{\sqrt{6}x}{3}\right)}{24}-\frac{1}{8x}+\frac{1}{8x\,(-2x^2+3)}+\frac{3}{20x\,(-2x^2+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

```
[Out] sqrt(6)*atanh(sqrt(6)*x/3)/24 - 1/(8*x) + 1/(8*x*(-2*x**2 + 3)) +
 3/(20*x*(-2*x**2 + 3)**2)
```

Mathematica [A] time = 0.115907, size = 65, normalized size = 1.1

$$\frac{1}{240}\left(-\frac{12\left(10x^4-25x^2+12\right)}{x\,(3-2x^2)^2}-5\sqrt{6}\log\left(\sqrt{6}-2x\right)+5\sqrt{6}\log\left(2x+\sqrt{6}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] ((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[S
qrt[6] - 2*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240
```

Maple [A] time = 0.013, size = 39, normalized size = 0.7

$$-\frac{1}{15\,x}-\frac{8}{15\,(2\,x^2-3)^2}\left(\frac{7\,x^3}{16}-\frac{27\,x}{32}\right)+\frac{\sqrt{6}}{24}\mathit{Artanh}\left(\frac{x\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(9/5/x^2/(-2*x^2+3)^3,x)
```

```
[Out] -1/15/x-8/15*(7/16*x^3-27/32*x)/(2*x^2-3)^2+1/24*arctanh(1/3*x*6^
(1/2))*6^(1/2)
```

Maxima [A] time = 1.57735, size = 76, normalized size = 1.29

$$-\frac{1}{48}\,\sqrt{6}\log\left(\frac{2\,x-\sqrt{6}}{2\,x+\sqrt{6}}\right)-\frac{10\,x^4-25\,x^2+12}{20\,(4\,x^5-12\,x^3+9\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-9/5/((2*x^2 - 3)^3*x^2),x, algorithm="maxima")
```

```
[Out] -1/48*sqrt(6)*log((2*x - sqrt(6))/(2*x + sqrt(6))) - 1/20*(10*x^4
 - 25*x^2 + 12)/(4*x^5 - 12*x^3 + 9*x)
```

Fricas [A] time = 0.216251, size = 109, normalized size = 1.85

$$\frac{\sqrt{6}\left(5\,\left(4\,x^5-12\,x^3+9\,x\right)\,\log\left(\frac{\sqrt{6}\left(2\,x^2+3\right)+12\,x}{2\,x^2-3}\right)-2\,\sqrt{6}\left(10\,x^4-25\,x^2+12\right)\right)}{240\,(4\,x^5-12\,x^3+9\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-9/5/((2*x^2 - 3)^3*x^2),x, algorithm="fricas")
```

```
[Out] 1/240*sqrt(6)*(5*(4*x^5 - 12*x^3 + 9*x)*log((sqrt(6)*(2*x^2 + 3)
+ 12*x)/(2*x^2 - 3)) - 2*sqrt(6)*(10*x^4 - 25*x^2 + 12))/(4*x^5 -
 12*x^3 + 9*x)
```

Sympy [A] time = 0.210724, size = 58, normalized size = 0.98

$$-\frac{9\left(10x^4-25x^2+12\right)}{720x^5-2160x^3+1620x}-\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{2}\right)}{48}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{2}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

```
[Out] -9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqr
t(6)*log(x - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48
```

GIAC/XCAS [A] time = 0.207875, size = 74, normalized size = 1.25

$$-\frac{1}{48}\sqrt{6}\ln\left(\frac{\left|4x-2\sqrt{6}\right|}{\left|4x+2\sqrt{6}\right|}\right)-\frac{14x^3-27x}{60\left(2x^2-3\right)^2}-\frac{1}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-9/5/((2*x^2 - 3)^3*x^2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(6)*ln(abs(4*x - 2*sqrt(6))/abs(4*x + 2*sqrt(6))) - 1/6
0*(14*x^3 - 27*x)/(2*x^2 - 3)^2 - 1/15/x
```

3.179 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[
x])/8
```

Rubi [A] time = 0.0779338, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[
x])/8
```

Rubi in Sympy [A] time = 6.76685, size = 32, normalized size = 0.89

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{57\,\mathrm{atan}(x)}{8}-\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] -25*x/(8*(x**2 + 1)) - 7*x/(4*(x**2 + 1)**2) - 57*atan(x)/8 - 4/x
```

Mathematica [A] time = 0.0235331, size = 33, normalized size = 0.92

$$-\frac{57x^4+103x^2+32}{8x(x^2+1)^2}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]

[Out] -(32 + 103*x^2 + 57*x^4)/(8*x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

Maple [A] time = 0., size = 29, normalized size = 0.8

$$-\frac{1}{\left(x^{2}+1\right)^{2}}\left(\frac{25\,x^{3}}{8}+\frac{39\,x}{8}\right)-\frac{57\,\arctan\left(x\right)}{8}-4\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^4+4)/x^2/(x^2+1)^3,x)

[Out] -(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

Maxima [A] time = 1.56388, size = 42, normalized size = 1.17

$$-\frac{57\,x^{4}+103\,x^{2}+32}{8\,\left(x^{5}+2\,x^{3}+x\right)}-\frac{57}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 4)/((x^2 + 1)^3*x^2),x, algorithm="maxima")

[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

Fricas [A] time = 0.211723, size = 54, normalized size = 1.5

$$-\frac{57\,x^{4}+103\,x^{2}+57\,\left(x^{5}+2\,x^{3}+x\right)\arctan\left(x\right)+32}{8\,\left(x^{5}+2\,x^{3}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 4)/((x^2 + 1)^3*x^2),x, algorithm="fricas")

[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^
5 + 2*x^3 + x)
```

Sympy [A] time = 0.177497, size = 32, normalized size = 0.89

$$-\frac{57x^4 + 103x^2 + 32}{8x^5 + 16x^3 + 8x} - \frac{57\,\mathrm{atan}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] -(57*x**4 + 103*x**2 + 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/
8
```

GIAC/XCAS [A] time = 0.216204, size = 38, normalized size = 1.06

$$-\frac{25\,x^3 + 39\,x}{8\,(x^2 + 1)^2} - \frac{4}{x} - \frac{57}{8}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 4)/((x^2 + 1)^3*x^2),x, algorithm="giac")
```

```
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

3.180 $\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx$

Optimal. Leaf size=38

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[
1 + x]
```

Rubi [A] time = 0.106258, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x]
```

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[
1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-x^4+5x^3+6x^2-3x+5}{x^5-x^4-2x^3+2x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-
1),x)
```

```
[Out] Integral((-x**4 + 5*x**3 + 6*x**2 - 3*x + 5)/(x**5 - x**4 - 2*x**
3 + 2*x**2 + x - 1), x)
```

Mathematica [A] time = 0.033907, size = 32, normalized size = 0.84

$$-\frac{2}{x-1}+\frac{1}{x+1}-\frac{3}{2(x-1)^2}+\log(x-1)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x]
```

```
[Out] -3/(2*(-1 + x)^2) - 2/(-1 + x) + (1 + x)^(-1) + Log[-1 + x] - 2*L
og[1 + x]
```

Maple [A] time = 0.019, size = 31, normalized size = 0.8

$$(1+x)^{-1} - 2\,\ln(1+x) + \ln(-1+x) - \frac{3}{2\,(-1+x)^2} - 2\,(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x)
```

```
[Out] 1/(1+x)-2*ln(1+x)+ln(-1+x)-3/2/(-1+x)^2-2/(-1+x)
```

Maxima [A] time = 1.41395, size = 51, normalized size = 1.34

$$-\frac{2\,x^2 + 7\,x - 3}{2\,(x^3 - x^2 - x + 1)} - 2\,\log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^4 - 5*x^3 - 6*x^2 + 3*x - 5)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1),x, al
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*log(x + 1) + log(x
 - 1)
```

Fricas [A] time = 0.21676, size = 88, normalized size = 2.32

$$-\frac{2\,x^2 + 4\,\left(x^3 - x^2 - x + 1\right)\,\log(x+1) - 2\,\left(x^3 - x^2 - x + 1\right)\,\log(x-1) + 7\,x - 3}{2\,(x^3 - x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^4 - 5*x^3 - 6*x^2 + 3*x - 5)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1),x, al
```

```
[Out] -1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x
 + 1)*log(x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)
```

Sympy [A] time = 0.159428, size = 36, normalized size = 0.95

$$-\frac{2x^2+7x-3}{2x^3-2x^2-2x+2}+\log(x-1)-2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-
1),x)
```

```
[Out] -(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*
log(x + 1)
```

GIAC/XCAS [A] time = 0.217615, size = 47, normalized size = 1.24

$$-\frac{2\,x^2+7\,x-3}{2\,(x+1)(x-1)^2}-2\ln(|x+1|)+\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 5*x^3 - 6*x^2 + 3*x - 5)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1),x, al
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*ln(abs(x + 1)) + l
n(abs(x - 1))
```

3.181 $\int \frac{1+x^2}{x(1+x^3)^2} \, dx$

Optimal. Leaf size=64

$$-\frac{5}{18}\log\left(x^2-x+1\right)+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}+\log(x)-\frac{4}{9}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3
]) + Log[x] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18
```

Rubi [A] time = 0.12952, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{5}{18}\log\left(x^2-x+1\right)+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}+\log(x)-\frac{4}{9}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3
]) + Log[x] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18
```

Rubi in Sympy [A] time = 6.12965, size = 51, normalized size = 0.8

$$\frac{x\left(x+\frac{1}{x}\right)}{3\left(x^3+1\right)}-\frac{\log(x+1)}{9}+\frac{\log\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/x/(x**3+1)**2,x)
```

```
[Out] x*(x + 1/x)/(3*(x**3 + 1)) - log(x + 1)/9 + log(x**2 - x + 1)/18
+ sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/9
```

Mathematica [A] time = 0.0515672, size = 65, normalized size = 1.02

$$\frac{1}{18}\left(-6\log\left(x^3+1\right)+\log\left(x^2-x+1\right)+\frac{6\left(x^2+1\right)}{x^3+1}+18\log(x)-2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] ((6*(1 + x^2))/(1 + x^3) + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] +
 18*Log[x] - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18
```

Maple [A] time = 0.018, size = 61, normalized size = 1.

$$\frac{2}{9+9x}-\frac{4\ln(1+x)}{9}+\ln(x)-\frac{-1-x}{9x^2-9x+9}-\frac{5\ln\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/x/(x^3+1)^2,x)
```

```
[Out] 2/9/(1+x)-4/9*ln(1+x)+ln(x)-1/9*(-1-x)/(x^2-x+1)-5/18*ln(x^2-x+1)
+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.59685, size = 68, normalized size = 1.06

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{x^2+1}{3\left(x^3+1\right)}-\frac{5}{18}\log\left(x^2-x+1\right)-\frac{4}{9}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/((x^3 + 1)^2*x),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/(x^3 +
1) - 5/18*log(x^2 - x + 1) - 4/9*log(x + 1) + log(x)
```

Fricas [A] time = 0.2285, size = 116, normalized size = 1.81

$$-\frac{\sqrt{3}\left(5\sqrt{3}\left(x^3+1\right)\log\left(x^2-x+1\right)+8\sqrt{3}\left(x^3+1\right)\log(x+1)-18\sqrt{3}\left(x^3+1\right)\log(x)-6\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\right.}{54\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/((x^3 + 1)^2*x),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(5*sqrt(3)*(x^3 + 1)*log(x^2 - x + 1) + 8*sqrt(3)*(
x^3 + 1)*log(x + 1) - 18*sqrt(3)*(x^3 + 1)*log(x) - 6*(x^3 + 1)*a
rctan(1/3*sqrt(3)*(2*x - 1)) - 6*sqrt(3)*(x^2 + 1))/(x^3 + 1)
```

Sympy [A] time = 0.318244, size = 60, normalized size = 0.94

$$\frac{x^2+1}{3x^3+3}+\log(x)-\frac{4\log(x+1)}{9}-\frac{5\log\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/x/(x**3+1)**2,x)
```

```
[Out] (x**2 + 1)/(3*x**3 + 3) + log(x) - 4*log(x + 1)/9 - 5*log(x**2 -
x + 1)/18 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.216559, size = 81, normalized size = 1.27

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{x^2+1}{3\,(x^2-x+1)(x+1)}-\frac{5}{18}\ln\left(x^2-x+1\right)-\frac{4}{9}\ln\left(|x+1|\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/((x^3 + 1)^2*x),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/((x^2 -
 x + 1)*(x + 1)) - 5/18*ln(x^2 - x + 1) - 4/9*ln(abs(x + 1)) + ln
(abs(x))
```

3.182 $\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Rubi [A] time = 0.142791, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Rubi in Sympy [A] time = 7.24958, size = 58, normalized size = 0.92

$$-\frac{5x+7}{3\left(x^2+x+1\right)}-\log(x+1)+\frac{\log\left(x^2+x+1\right)}{2}-\frac{25\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{9}-\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

```
[Out] -(5*x + 7)/(3*(x**2 + x + 1)) - log(x + 1) + log(x**2 + x + 1)/2
- 25*sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/9 - 2/(x + 1)
```

Mathematica [A] time = 0.0558649, size = 63, normalized size = 1.

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Maple [A] time = 0.013, size = 54, normalized size = 0.9

$$-2\,(1+x)^{-1}-\ln(1+x)+\frac{1}{x^2+x+1}\left(-\frac{5\,x}{3}-\frac{7}{3}\right)+\frac{\ln\left(x^2+x+1\right)}{2}-\frac{25\,\sqrt{3}}{9}\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x)
```

```
[Out] -2/(1+x)-ln(1+x)+(-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arct
an(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.59144, size = 80, normalized size = 1.27

$$-\frac{25}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{11\,x^2+18\,x+13}{3\,(x^3+2\,x^2+2\,x+1)}+\frac{1}{2}\,\log\left(x^2+x+1\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 3*x - 2)/((x^2 + x + 1)^2*(x + 1)^2),x, algorithm="maxima")
```

```
[Out] -25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x
+ 13)/(x^3 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)
```

Fricas [A] time = 0.224771, size = 147, normalized size = 2.33

$$\frac{\sqrt{3}\Big(3\,\sqrt{3}\left(x^3+2\,x^2+2\,x+1\right)\,\log\left(x^2+x+1\right)-6\,\sqrt{3}\left(x^3+2\,x^2+2\,x+1\right)\,\log\left(x+1\right)-50\,\left(x^3+2\,x^2+2\,x+1\right)\,\arctan\left(\frac{1}{3}\,\sqrt{3}\right.}{18\,(x^3+2\,x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 3*x - 2)/((x^2 + x + 1)^2*(x + 1)^2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(3*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1)
- 6*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*log(x + 1) - 50*(x^3 + 2*x^2
+ 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(11*x^2 + 18
*x + 13))/(x^3 + 2*x^2 + 2*x + 1)
```

Sympy [A] time = 0.238796, size = 66, normalized size = 1.05

$$-\frac{11x^2+18x+13}{3x^3+6x^2+6x+3}-\log\left(x+1\right)+\frac{\log\left(x^2+x+1\right)}{2}-\frac{25\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

```
[Out] -(11*x**2 + 18*x + 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) +
 log(x**2 + x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/9
```

GIAC/XCAS [A] time = 0.214229, size = 97, normalized size = 1.54

$$-\frac{25}{9}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(\frac{2}{x+1}-1\right)\right)+\frac{\frac{7}{x+1}-2}{3\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}-1\right)}-\frac{2}{x+1}+\frac{1}{2}\ln\left(-\frac{1}{x+1}+\frac{1}{(x+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 3*x - 2)/((x^2 + x + 1)^2*(x + 1)^2),x, algorithm="giac")
```

```
[Out] -25/9*sqrt(3)*arctan(-1/3*sqrt(3)*(2/(x + 1) - 1)) + 1/3*(7/(x +
1) - 2)/(1/(x + 1) - 1/(x + 1)^2 - 1) - 2/(x + 1) + 1/2*ln(-1/(x
+ 1) + 1/(x + 1)^2 + 1)
```

3.183 $\int \frac{1}{(1-4x)^3(2-3x)}\,dx$

Optimal. Leaf size=43

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

```
[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9
*Log[2 - 3*x])/125
```

Rubi [A] time = 0.0434537, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9
*Log[2 - 3*x])/125
```

Rubi in Sympy [A] time = 3.64796, size = 36, normalized size = 0.84

$$-\frac{9\log(-4x+1)}{125}+\frac{9\log(-3x+2)}{125}-\frac{3}{25(-4x+1)}+\frac{1}{10(-4x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-4*x)**3/(2-3*x),x)
```

```
[Out] -9*log(-4*x + 1)/125 + 9*log(-3*x + 2)/125 - 3/(25*(-4*x + 1)) +
1/(10*(-4*x + 1)**2)
```

Mathematica [A] time = 0.0226849, size = 46, normalized size = 1.07

$$\frac{120x+18(1-4x)^2\log(8-12x)-18(1-4x)^2\log(4x-1)-5}{250(1-4x)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] (-5 + 120*x + 18*(1 - 4*x)^2*Log[8 - 12*x] - 18*(1 - 4*x)^2*Log[-
1 + 4*x])/(250*(1 - 4*x)^2)
```

Maple [A] time = 0.013, size = 36, normalized size = 0.8

$$\frac{9\,\ln\left(-2+3\,x\right)}{125}+\frac{1}{10\,\left(-1+4\,x\right)^{2}}+\frac{3}{-25+100\,x}-\frac{9\,\ln\left(-1+4\,x\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-4*x)^3/(2-3*x),x)
```

```
[Out] 9/125*ln(-2+3*x)+1/10/(-1+4*x)^2+3/25/(-1+4*x)-9/125*ln(-1+4*x)
```

Maxima [A] time = 1.39507, size = 49, normalized size = 1.14

$$\frac{24\,x-1}{50\,(16\,x^{2}-8\,x+1)}-\frac{9}{125}\,\log\left(4\,x-1\right)+\frac{9}{125}\,\log\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((4*x - 1)^3*(3*x - 2)),x, algorithm="maxima")
```

```
[Out] 1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*log(4*x - 1) + 9/125*l
og(3*x - 2)
```

Fricas [A] time = 0.243584, size = 74, normalized size = 1.72

$$-\frac{18\,\left(16\,x^{2}-8\,x+1\right)\,\log\left(4\,x-1\right)-18\,\left(16\,x^{2}-8\,x+1\right)\,\log\left(3\,x-2\right)-120\,x+5}{250\,(16\,x^{2}-8\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((4*x - 1)^3*(3*x - 2)),x, algorithm="fricas")
```

```
[Out] -1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1
)*log(3*x - 2) - 120*x + 5)/(16*x^2 - 8*x + 1)
```

Sympy [A] time = 0.157614, size = 34, normalized size = 0.79

$$\frac{24x-1}{800x^2-400x+50}+\frac{9\log\left(x-\frac{2}{3}\right)}{125}-\frac{9\log\left(x-\frac{1}{4}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-4*x)**3/(2-3*x),x)
```

```
[Out] (24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x
 - 1/4)/125
```

GIAC/XCAS [A] time = 0.199659, size = 45, normalized size = 1.05

$$\frac{24\,x-1}{50\,(4\,x-1)^2}-\frac{9}{125}\ln\left(|4\,x-1|\right)+\frac{9}{125}\ln\left(|3\,x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((4*x - 1)^3*(3*x - 2)),x, algorithm="giac")
```

```
[Out] 1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*ln(abs(4*x - 1)) + 9/125*ln(a
bs(3*x - 2))
```

3.184 $\int \frac{x^3}{(2-5x^2)^7} \, dx$

Optimal. Leaf size=27

$$\frac{1}{150\left(2-5x^2\right)^6}-\frac{1}{250\left(2-5x^2\right)^5}$$

```
[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)
```

Rubi [A] time = 0.0410365, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{150\left(2-5x^2\right)^6}-\frac{1}{250\left(2-5x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(2 - 5*x^2)^7,x]

[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)
```

Rubi in Sympy [A] time = 3.22999, size = 22, normalized size = 0.81

$$-\frac{1}{250\left(-5x^2+2\right)^5}+\frac{1}{150\left(-5x^2+2\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-5*x**2+2)**7,x)

[Out] -1/(250*(-5*x**2 + 2)**5) + 1/(150*(-5*x**2 + 2)**6)
```

Mathematica [A] time = 0.0108385, size = 20, normalized size = 0.74

$$\frac{15x^2-1}{750\left(2-5x^2\right)^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(2 - 5*x^2)^7,x]
```

```
[Out] (-1 + 15*x^2)/(750*(2 - 5*x^2)^6)
```

Maple [A] time = 0.013, size = 24, normalized size = 0.9

$$\frac{1}{150\,\left(5\,x^{2}-2\right)^{6}}+\frac{1}{250\,\left(5\,x^{2}-2\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-5*x^2+2)^7,x)
```

```
[Out] 1/150/(5*x^2-2)^6+1/250/(5*x^2-2)^5
```

Maxima [A] time = 1.39956, size = 58, normalized size = 2.15

$$\frac{15\,x^{2}-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^{8}-20000\,x^{6}+6000\,x^{4}-960\,x^{2}+64)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/(5*x^2 - 2)^7,x, algorithm="maxima")
```

```
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x
^6 + 6000*x^4 - 960*x^2 + 64)
```

Fricas [A] time = 0.194346, size = 58, normalized size = 2.15

$$\frac{15\,x^{2}-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^{8}-20000\,x^{6}+6000\,x^{4}-960\,x^{2}+64)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/(5*x^2 - 2)^7,x, algorithm="fricas")
```

```
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x
^6 + 6000*x^4 - 960*x^2 + 64)
```

Sympy [A] time = 0.279183, size = 37, normalized size = 1.37

$$\frac{15x^{2}-1}{11718750x^{12}-28125000x^{10}+28125000x^{8}-15000000x^{6}+4500000x^{4}-720000x^{2}+48000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-5*x**2+2)**7,x)
```

```
[Out] (15*x**2 - 1)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 -
15000000*x**6 + 4500000*x**4 - 720000*x**2 + 48000)
```

GIAC/XCAS [A] time = 0.202066, size = 24, normalized size = 0.89

$$\frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/(5*x^2 - 2)^7,x, algorithm="giac")
```

```
[Out] 1/750*(15*x^2 - 1)/(5*x^2 - 2)^6
```

3.185 $\int \frac{x^7}{(2-5x^2)^3} \, dx$

Optimal. Leaf size=46

$$-\frac{x^2}{250}-\frac{6}{625\left(2-5x^2\right)}+\frac{2}{625\left(2-5x^2\right)^2}-\frac{3}{625}\log\left(2-5x^2\right)$$

```
[Out] -x^2/250 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2
 - 5*x^2])/625
```

Rubi [A] time = 0.0608665, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{x^2}{250}-\frac{6}{625\left(2-5x^2\right)}+\frac{2}{625\left(2-5x^2\right)^2}-\frac{3}{625}\log\left(2-5x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -x^2/250 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2
 - 5*x^2])/625
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\log\left(-5x^2+2\right)}{625}+\frac{\int^{x^2}\left(-\frac{1}{125}\right)\,dx}{2}-\frac{6}{625\left(-5x^2+2\right)}+\frac{2}{625\left(-5x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(-5*x**2+2)**3,x)
```

```
[Out] -3*log(-5*x**2 + 2)/625 + Integral(-1/125, (x, x**2))/2 - 6/(625*
(-5*x**2 + 2)) + 2/(625*(-5*x**2 + 2)**2)
```

Mathematica [A] time = 0.0210744, size = 44, normalized size = 0.96

$$-\frac{125x^6-150x^4+6\left(2-5x^2\right)^2\log\left(5x^2-2\right)+12}{1250\left(2-5x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(1250
*(2 - 5*x^2)^2)
```

Maple [A] time = 0.015, size = 39, normalized size = 0.9

$$-\frac{x^2}{250} - \frac{3\ln\left(5x^2-2\right)}{625} + \frac{2}{625\left(5x^2-2\right)^2} + \frac{6}{3125x^2-1250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(-5*x^2+2)^3,x)
```

```
[Out] -1/250*x^2-3/625*ln(5*x^2-2)+2/625/(5*x^2-2)^2+6/625/(5*x^2-2)
```

Maxima [A] time = 1.43499, size = 53, normalized size = 1.15

$$-\frac{1}{250}x^2 + \frac{2\left(3x^2-1\right)}{125\left(25x^4-20x^2+4\right)} - \frac{3}{625}\log\left(5x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/(5*x^2 - 2)^3,x, algorithm="maxima")
```

```
[Out] -1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(
5*x^2 - 2)
```

Fricas [A] time = 0.198086, size = 74, normalized size = 1.61

$$-\frac{125x^6-100x^4-40x^2+6\left(25x^4-20x^2+4\right)\log\left(5x^2-2\right)+20}{1250\left(25x^4-20x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/(5*x^2 - 2)^3,x, algorithm="fricas")
```

```
[Out] -1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log
(5*x^2 - 2) + 20)/(25*x^4 - 20*x^2 + 4)
```

Sympy [A] time = 0.158108, size = 34, normalized size = 0.74

$$-\frac{x^2}{250} + \frac{6x^2 - 2}{3125x^4 - 2500x^2 + 500} - \frac{3\log\left(5x^2 - 2\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(-5*x**2+2)**3,x)
```

```
[Out] -x**2/250 + (6*x**2 - 2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*
x**2 - 2)/625
```

GIAC/XCAS [A] time = 0.202203, size = 54, normalized size = 1.17

$$-\frac{1}{250}\,x^2 + \frac{225\,x^4 - 120\,x^2 + 16}{1250\,(5\,x^2 - 2)^2} - \frac{3}{625}\ln\left(\left|5\,x^2 - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/(5*x^2 - 2)^3,x, algorithm="giac")
```

```
[Out] -1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/62
5*ln(abs(5*x^2 - 2))
```

3.186 $\int \frac{1}{(-2+x)^3(1+x)^2}\,dx$

Optimal. Leaf size=44

$$\frac{2}{27(x-2)}+\frac{1}{27(x+1)}-\frac{1}{18(x-2)^2}+\frac{1}{27}\log(x-2)-\frac{1}{27}\log(x+1)$$

```
[Out] -1/(18*(-2 + x)^2) + 2/(27*(-2 + x)) + 1/(27*(1 + x)) + Log[-2 +
x]/27 - Log[1 + x]/27
```

Rubi [A] time = 0.0395534, antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2}{27(2-x)}+\frac{1}{27(x+1)}-\frac{1}{18(2-x)^2}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] -1/(18*(2 - x)^2) - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/
27 - Log[1 + x]/27
```

Rubi in Sympy [A] time = 2.62354, size = 34, normalized size = 0.77

$$\frac{\log(-x+2)}{27}-\frac{\log(x+1)}{27}+\frac{1}{27(x+1)}-\frac{2}{27(-x+2)}-\frac{1}{18(-x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2+x)**3/(1+x)**2,x)
```

```
[Out] log(-x + 2)/27 - log(x + 1)/27 + 1/(27*(x + 1)) - 2/(27*(-x + 2))
 - 1/(18*(-x + 2)**2)
```

Mathematica [A] time = 0.0338507, size = 39, normalized size = 0.89

$$\frac{1}{54}\left(\frac{3\left(2x^2-5x-1\right)}{(x-2)^2(x+1)}+2\log(x-2)-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] ((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*
Log[1 + x])/54
```

Maple [A] time = 0.014, size = 35, normalized size = 0.8

$$-\frac{1}{18\ (-2+x)^{2}}+\frac{2}{-54+27\,x}+\frac{1}{27+27\,x}+\frac{\ln\left(-2+x\right)}{27}-\frac{\ln\left(1+x\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2+x)^3/(1+x)^2,x)
```

```
[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

Maxima [A] time = 1.40987, size = 50, normalized size = 1.14

$$\frac{2\,x^{2}-5\,x-1}{18\,(x^{3}-3\,x^{2}+4)}-\frac{1}{27}\,\log\left(x+1\right)+\frac{1}{27}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^2*(x - 2)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27
*log(x - 2)
```

Fricas [A] time = 0.200311, size = 76, normalized size = 1.73

$$\frac{6\,x^{2}-2\,\left(x^{3}-3\,x^{2}+4\right)\,\log\left(x+1\right)+2\,\left(x^{3}-3\,x^{2}+4\right)\,\log\left(x-2\right)-15\,x-3}{54\,(x^{3}-3\,x^{2}+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^2*(x - 2)^3),x, algorithm="fricas")
```

```
[Out] 1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4
)*log(x - 2) - 15*x - 3)/(x^3 - 3*x^2 + 4)
```

Sympy [A] time = 0.15745, size = 34, normalized size = 0.77

$$\frac{2x^2 - 5x - 1}{18x^3 - 54x^2 + 72} + \frac{\log(x-2)}{27} - \frac{\log(x+1)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2+x)**3/(1+x)**2,x)
```

```
[Out] (2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log
(x + 1)/27
```

GIAC/XCAS [A] time = 0.198398, size = 58, normalized size = 1.32

$$\frac{1}{27(x+1)} - \frac{\frac{18}{x+1} - 5}{162\left(\frac{3}{x+1} - 1\right)^2} + \frac{1}{27}\ln\left(\left|-\frac{3}{x+1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^2*(x - 2)^3),x, algorithm="giac")
```

```
[Out] 1/27/(x + 1) - 1/162*(18/(x + 1) - 5)/(3/(x + 1) - 1)^2 + 1/27*ln
(abs(-3/(x + 1) + 1))
```

3.187 $\int \frac{1}{(2+x)^3(3+x)^4}\,dx$

Optimal. Leaf size=54

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2)
+ 6/(3 + x) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rubi [A] time = 0.0536938, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2)
+ 6/(3 + x) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rubi in Sympy [A] time = 3.36141, size = 48, normalized size = 0.89

$$10\log(x+2)-10\log(x+3)+\frac{6}{x+3}+\frac{3}{2\left(x+3\right)^2}+\frac{1}{3\left(x+3\right)^3}+\frac{4}{x+2}-\frac{1}{2\left(x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] 10*log(x + 2) - 10*log(x + 3) + 6/(x + 3) + 3/(2*(x + 3)**2) + 1/
(3*(x + 3)**3) + 4/(x + 2) - 1/(2*(x + 2)**2)
```

Mathematica [A] time = 0.022466, size = 54, normalized size = 1.

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2)
+ 6/(3 + x) + 10*Log[2 + x] - 10*Log[3 + x]
```

Maple [A] time = 0., size = 49, normalized size = 0.9

$$-\frac{1}{2\,(2+x)^{2}}+4\,(2+x)^{-1}+\frac{1}{3\,(3+x)^{3}}+\frac{3}{2\,(3+x)^{2}}+6\,(3+x)^{-1}+10\,\ln(2+x)-10\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+x)^3/(3+x)^4,x)
```

```
[Out] -1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-1
0*ln(3+x)
```

Maxima [A] time = 1.41008, size = 81, normalized size = 1.5

$$\frac{60\,x^{4}+630\,x^{3}+2450\,x^{2}+4175\,x+2627}{6\,(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108)}-10\,\log(x+3)+10\,\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^4*(x + 2)^3),x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 +
 67*x^3 + 171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

Fricas [A] time = 0.197954, size = 142, normalized size = 2.63

$$\frac{60\,x^{4}+630\,x^{3}+2450\,x^{2}-60\,\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)\log(x+3)+60\,\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216}{6\,(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^4*(x + 2)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 17
1*x^2 + 216*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171
*x^2 + 216*x + 108)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 6
```

```
7*x^3 + 171*x^2 + 216*x + 108)
```

Sympy [A] time = 0.211755, size = 58, normalized size = 1.07

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10\log(x + 2) - 10\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**
4 + 402*x**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log
(x + 3)
```

GIAC/XCAS [A] time = 0.212501, size = 63, normalized size = 1.17

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x + 3)^3(x + 2)^2} - 10\ln(|x + 3|) + 10\ln(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^4*(x + 2)^3),x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x +
 2)^2) - 10*ln(abs(x + 3)) + 10*ln(abs(x + 2))
```

3.188 $\int \frac{x^5}{(3+x)^2} dx$

Optimal. Leaf size=36

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x
]
```

Rubi [A] time = 0.0349828, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(3 + x)^2,x]
```

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4} - 2x^3 - 108x + 405\log(x+3) + 27\int x\,dx + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 - 108*x + 405*log(x + 3) + 27*Integral(x, x) + 24
3/(x + 3)
```

Mathematica [A] time = 0.0222017, size = 36, normalized size = 1.

$$\frac{1}{4}\left(x^4 - 8x^3 + 54x^2 - 432x + \frac{972}{x+3} - 2079\right) + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(3 + x)^2,x]
```

```
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[
3 + x]
```

Maple [A] time = 0., size = 33, normalized size = 0.9

$$-108\,x+\frac{27\,x^{2}}{2}-2\,x^{3}+\frac{x^{4}}{4}+243\,\left(3+x\right)^{-1}+405\,\ln\left(3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(3+x)^2,x)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Maxima [A] time = 1.45179, size = 43, normalized size = 1.19

$$\frac{1}{4}\,x^{4}-2\,x^{3}+\frac{27}{2}\,x^{2}-108\,x+\frac{243}{x+3}+405\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x + 3)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

Fricas [A] time = 0.196623, size = 53, normalized size = 1.47

$$\frac{x^{5}-5\,x^{4}+30\,x^{3}-270\,x^{2}+1620\,\left(x+3\right)\log\left(x+3\right)-1296\,x+972}{4\,\left(x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1
296*x + 972)/(x + 3)
```

Sympy [A] time = 0.078203, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log(x+3) + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3
)
```

GIAC/XCAS [A] time = 0.21336, size = 61, normalized size = 1.69

$$-\frac{1}{4}(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\ln(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x + 3)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1)
+ 243/(x + 3) + 405*ln(abs(x + 3))
```

3.189 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)\, dx$

Optimal. Leaf size=44

$$\frac{1}{2}x^2(ac1 + 2bb1) + ab1x + \frac{1}{3}x^3(2bc1 + b1c) + \frac{1}{4}cc1x^4$$

```
[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c
1*x^4)/4
```

Rubi [A] time = 0.0747276, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{2}x^2(ac1 + 2bb1) + ab1x + \frac{1}{3}x^3(2bc1 + b1c) + \frac{1}{4}cc1x^4$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

```
[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c
1*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$b_1 \int a\,dx + \frac{cc_1x^4}{4} + x^3\left(\frac{2bc_1}{3} + \frac{b_1c}{3}\right) + (ac_1 + 2bb_1)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)
```

```
[Out] b1*Integral(a, x) + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + (a*c
1 + 2*b*b1)*Integral(x, x)
```

Mathematica [A] time = 0.0196038, size = 41, normalized size = 0.93

$$\frac{1}{12}x(6a(2b1 + c1x) + x(4b(3b1 + 2c1x) + cx(4b1 + 3c1x)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

```
[Out] (x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1
*x))))/12
```

Maple [A] time = 0.003, size = 39, normalized size = 0.9

$$ab1x + \frac{(ac1 + 2\,bb1)\,x^2}{2} + \frac{(2\,bc1 + b1c)\,x^3}{3} + \frac{cc1\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)*(c*x^2+2*b*x+a),x)
```

```
[Out] a*b1*x+1/2*(a*c1+2*b*b1)*x^2+1/3*(2*b*c1+b1*c)*x^3+1/4*c*c1*x^4
```

Maxima [A] time = 1.41228, size = 51, normalized size = 1.16

$$\frac{1}{4}\,cc_1x^4 + \frac{1}{3}\,(b_1c + 2\,bc_1)x^3 + ab_1x + \frac{1}{2}\,(2\,bb_1 + ac_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)*(c1*x + b1),x, algorithm="maxima")
```

```
[Out] 1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a
*c1)*x^2
```

Fricas [A] time = 0.17669, size = 1, normalized size = 0.02

$$\frac{1}{4}x^4c_1c + \frac{1}{3}x^3cb_1 + \frac{2}{3}x^3c_1b + x^2b_1b + \frac{1}{2}x^2c_1a + xb_1a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)*(c1*x + b1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c1*c + 1/3*x^3*c*b1 + 2/3*x^3*c1*b + x^2*b1*b + 1/2*x^2*c
1*a + x*b1*a
```

Sympy [A] time = 0.044589, size = 39, normalized size = 0.89

$$ab_1x + \frac{cc_1x^4}{4} + x^3\left(\frac{2bc_1}{3} + \frac{b_1c}{3}\right) + x^2\left(\frac{ac_1}{2} + bb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)
```

```
[Out] a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 +
b*b1)
```

GIAC/XCAS [A] time = 0.210533, size = 53, normalized size = 1.2

$$\frac{1}{4}\,cc_1x^4 + \frac{1}{3}\,b_1cx^3 + \frac{2}{3}\,bc_1x^3 + bb_1x^2 + \frac{1}{2}\,ac_1x^2 + ab_1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)*(c1*x + b1),x, algorithm="giac")
```

```
[Out] 1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*
x^2 + a*b1*x
```

3.190 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^2 \, dx$

Optimal. Leaf size=96

$$a^2\text{b1}x + \frac{1}{2}x^4\left(ac\text{c1} + 2b^2\text{c1} + 2b\text{b1}c\right) + \frac{2}{3}x^3\left(2ab\text{c1} + a\text{b1}c + 2b^2\text{b1}\right) + \frac{1}{2}ax^2(a\text{c1} + 4b\text{b1}) + \frac{1}{5}cx^5(4b\text{c1} + \text{b1}c) + \frac{1}{6}c^2\text{c1}x^6$$

```
[Out] a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*
a*b*c1)*x^3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*
c + 4*b*c1)*x^5)/5 + (c^2*c1*x^6)/6
```

Rubi [A] time = 0.202996, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$a^2\text{b1}x + \frac{1}{2}x^4\left(ac\text{c1} + 2b^2\text{c1} + 2b\text{b1}c\right) + \frac{2}{3}x^3\left(2ab\text{c1} + a\text{b1}c + 2b^2\text{b1}\right) + \frac{1}{2}ax^2(a\text{c1} + 4b\text{b1}) + \frac{1}{5}cx^5(4b\text{c1} + \text{b1}c) + \frac{1}{6}c^2\text{c1}x^6$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*
a*b*c1)*x^3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*
c + 4*b*c1)*x^5)/5 + (c^2*c1*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \int b_1 \, dx + a\left(ac_1 + 4bb_1\right) \int x \, dx + \frac{c^2 c_1 x^6}{6} + \frac{cx^5\left(4bc_1 + b_1 c\right)}{5} + x^4\left(\frac{ac c_1}{2} + b^2 c_1 + b b_1 c\right) + x^3\left(\frac{4abc_1}{3} + \frac{2ab_1 c}{3} + \frac{4b^2 b_1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

```
[Out] a**2*Integral(b1, x) + a*(a*c1 + 4*b*b1)*Integral(x, x) + c**2*c1
*x**6/6 + c*x**5*(4*b*c1 + b1*c)/5 + x**4*(a*c*c1/2 + b**2*c1 + b
*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3)
```

Mathematica [A] time = 0.045292, size = 91, normalized size = 0.95

$$\frac{1}{30}x\left(15a^2(2\text{b1}+\text{c1}x)+5ax(4b(3\text{b1}+2\text{c1}x)+cx(4\text{b1}+3\text{c1}x))+x^2\left(10b^2(4\text{b1}+3\text{c1}x)+6bcx(5\text{b1}+4\text{c1}x)+c^2x^2(6\text{b1}+5\text{c1}x)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] (x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1
 + 3*c1*x)) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*
x) + c^2*x^2*(6*b1 + 5*c1*x))))/30
```

Maple [A] time = 0.003, size = 95, normalized size = 1.

$$\frac{c^2c1x^6}{6}+\frac{(4c1bc+b1c^2)x^5}{5}+\frac{(4bb1c+c1(2ac+4b^2))x^4}{4}+\frac{(b1(2ac+4b^2)+4abc1)x^3}{3}+\frac{(c1a^2+4b1ab)x^2}{2}+a^2b1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x)
```

```
[Out] 1/6*c^2*c1*x^6+1/5*(4*b*c*c1+b1*c^2)*x^5+1/4*(4*b*b1*c+c1*(2*a*c+
4*b^2))*x^4+1/3*(b1*(2*a*c+4*b^2)+4*a*b*c1)*x^3+1/2*(a^2*c1+4*a*b
*b1)*x^2+a^2*b1*x
```

Maxima [A] time = 1.39976, size = 123, normalized size = 1.28

$$\frac{1}{6}c^2c_1x^6+\frac{1}{5}\left(b_1c^2+4bcc_1\right)x^5+\frac{1}{2}\left(2bb_1c+\left(2b^2+ac\right)c_1\right)x^4+a^2b_1x+\frac{2}{3}\left(2b^2b_1+ab_1c+2abc_1\right)x^3+\frac{1}{2}\left(4abb_1+a^2c_1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^2*(c1*x + b1),x, algorithm="maxima")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2
*b^2 + a*c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c
```

```
1)*x^3 + 1/2*(4*a*b*b1 + a^2*c1)*x^2
```

Fricas [A] time = 0.177925, size = 1, normalized size = 0.01

$$\frac{1}{6}x^6c_1c^2 + \frac{1}{5}x^5c^2b_1 + \frac{4}{5}x^5c_1cb + x^4cb_1b + x^4c_1b^2 + \frac{1}{2}x^4c_1ca + \frac{4}{3}x^3b_1b^2 + \frac{2}{3}x^3cb_1a + \frac{4}{3}x^3c_1ba + 2x^2b_1ba + \frac{1}{2}x^2c_1a^2 + xb_1a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^2*(c1*x + b1),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c1*c^2 + 1/5*x^5*c^2*b1 + 4/5*x^5*c1*c*b + x^4*c*b1*b + x
^4*c1*b^2 + 1/2*x^4*c1*c*a + 4/3*x^3*b1*b^2 + 2/3*x^3*c*b1*a + 4/
3*x^3*c1*b*a + 2*x^2*b1*b*a + 1/2*x^2*c1*a^2 + x*b1*a^2
```

Sympy [A] time = 0.074569, size = 100, normalized size = 1.04

$$a^2b_1x + \frac{c^2c_1x^6}{6} + x^5\left(\frac{4bcc_1}{5} + \frac{b_1c^2}{5}\right) + x^4\left(\frac{acc_1}{2} + b^2c_1 + bb_1c\right) + x^3\left(\frac{4abc_1}{3} + \frac{2ab_1c}{3} + \frac{4b^2b_1}{3}\right) + x^2\left(\frac{a^2c_1}{2} + 2abb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

```
[Out] a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4
*(a*c*c1/2 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 +
4*b**2*b1/3) + x**2*(a**2*c1/2 + 2*a*b*b1)
```

GIAC/XCAS [A] time = 0.210321, size = 132, normalized size = 1.38

$$\frac{1}{6}\,c^2c_1x^6 + \frac{1}{5}\,b_1c^2x^5 + \frac{4}{5}\,bcc_1x^5 + bb_1cx^4 + b^2c_1x^4 + \frac{1}{2}\,acc_1x^4 + \frac{4}{3}\,b^2b_1x^3 + \frac{2}{3}\,ab_1cx^3 + \frac{4}{3}\,abc_1x^3 + 2\,abb_1x^2 + \frac{1}{2}\,a^2c_1x^2 + a^2b_1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^2*(c1*x + b1),x, algorithm="giac")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b
^2*c1*x^4 + 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/
3*a*b*c1*x^3 + 2*a*b*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x
```

3.191 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^3 \, dx$

Optimal. Leaf size=167

$$a^3\text{b1}x + \frac{1}{4}x^4\left(3a^2c\text{c1} + 12ab^2\text{c1} + 12ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{2}a^2x^2(a\text{c1} + 6b\text{b1}) + \frac{1}{2}cx^6\left(ac\text{c1} + 4b^2\text{c1} + 2b\text{b1}c\right) + ax^3\left(2ab\text{c1} + a\text{b1}c + 4b^2\text{b1}\right) + \frac{1}{5}x^5\left(12abc\text{c1} + 3a\text{b1}c^2 + 8b^3\text{c1} + 12b^2\text{b1}c\right) + \frac{1}{7}c^2x^7(6b\text{c1} + \text{b1}c) + \frac{1}{8}c^3\text{c1}x^8$$

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2
*a*b*c1)*x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c
1)*x^4)/4 + ((12*b^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*
x^5)/5 + (c*(2*b*b1*c + 4*b^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c +
6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Rubi [A] time = 0.376939, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$a^3\text{b1}x + \frac{1}{4}x^4\left(3a^2c\text{c1} + 12ab^2\text{c1} + 12ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{2}a^2x^2(a\text{c1} + 6b\text{b1}) + \frac{1}{2}cx^6\left(ac\text{c1} + 4b^2\text{c1} + 2b\text{b1}c\right) + ax^3\left(2ab\text{c1} + a\text{b1}c + 4b^2\text{b1}\right) + \frac{1}{5}x^5\left(12abc\text{c1} + 3a\text{b1}c^2 + 8b^3\text{c1} + 12b^2\text{b1}c\right) + \frac{1}{7}c^2x^7(6b\text{c1} + \text{b1}c) + \frac{1}{8}c^3\text{c1}x^8$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2
*a*b*c1)*x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c
1)*x^4)/4 + ((12*b^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*
x^5)/5 + (c*(2*b*b1*c + 4*b^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c +
6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\int b_1\,dx + a^2(ac_1 + 6bb_1)\int x\,dx + ax^3\left(2abc_1 + ab_1c + 4b^2b_1\right) + \frac{c^3c_1x^8}{8} + \frac{c^2x^7(6bc_1 + b_1c)}{7} + \frac{cx^6\left(acc_1 + 4b^2c_1 + 2bb_1c\right)}{2} + x^5\left(\frac{12abcc_1}{5} + \frac{3ab_1c^2}{5} + \frac{8b^3c_1}{5} + \frac{12b^2b_1c}{5}\right) + x^4\left(\frac{3a^2cc_1}{4} + 3ab^2c_1 + 3abb_1c + 2b^3b_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

```
[Out] a**3*Integral(b1, x) + a**2*(a*c1 + 6*b*b1)*Integral(x, x) + a*x*
*3*(2*a*b*c1 + a*b1*c + 4*b**2*b1) + c**3*c1*x**8/8 + c**2*x**7*(
6*b*c1 + b1*c)/7 + c*x**6*(a*c*c1 + 4*b**2*c1 + 2*b*b1*c)/2 + x**
5*(12*a*b*c*c1/5 + 3*a*b1*c**2/5 + 8*b**3*c1/5 + 12*b**2*b1*c/5)
+ x**4*(3*a**2*c*c1/4 + 3*a*b**2*c1 + 3*a*b*b1*c + 2*b**3*b1)
```

Mathematica [A] time = 0.0572232, size = 167, normalized size = 1.

$$a^3\text{b1}x+\frac{1}{4}x^4\left(3a^2c\text{c1}+12ab^2\text{c1}+12ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{2}a^2x^2(a\text{c1}+6b\text{b1})+\frac{1}{2}cx^6\left(ac\text{c1}+4b^2\text{c1}+2b\text{b1}c\right)+ax^3\left(2ab\text{c1}+a\text{b1}c+4b^2\text{b1}\right)+\frac{1}{5}x^5\left(12abc\text{c1}+3a\text{b1}c^2+8b^3\text{c1}+12b^2\text{b1}c\right)+\frac{1}{7}c^2x^7(6b\text{c1}+\text{b1}c)+\frac{1}{8}c^3\text{c1}x^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2
*a*b*c1)*x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c
1)*x^4)/4 + ((12*b^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*
x^5)/5 + (c*(2*b*b1*c + 4*b^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c +
6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Maple [A] time = 0., size = 237, normalized size = 1.4

$$\frac{c^3c1x^8}{8}+\frac{\left(6\,c1\,bc^2+b1\,c^3\right)x^7}{7}+\frac{\left(6\,b1\,bc^2+c1\,\left(ac^2+8\,b^2c+c\left(2\,ac+4\,b^2\right)\right)\right)x^6}{6}+\frac{\left(b1\,\left(ac^2+8\,b^2c+c\left(2\,ac+4\,b^2\right)\right)+c1\,\left(8\,abc+2\,b\left(2\,ac+4\,b^2\right)\right)\right)x^5}{5}+\frac{\left(b1\,\left(8\,abc+2\,b\left(2\,ac+4\,b^2\right)\right)+c1\,\left(a\left(2\,ac+4\,b^2\right)+8\,b^2a+ca^2\right)\right)x^4}{4}+\frac{\left(b1\,\left(a\left(2\,ac+4\,b^2\right)+8\,b^2a+ca^2\right)+6\,c1\,a^2b\right)x^3}{3}+\frac{\left(c1\,a^3+6\,b1\,a^2b\right)x^2}{2}+a^3b1\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x)
```

```
[Out] 1/8*c^3*c1*x^8+1/7*(6*b*c^2*c1+b1*c^3)*x^7+1/6*(6*b1*b*c^2+c1*(a*
c^2+8*b^2*c+c*(2*a*c+4*b^2)))*x^6+1/5*(b1*(a*c^2+8*b^2*c+c*(2*a*c
```

```
+4*b^2))+c1*(8*a*b*c+2*b*(2*a*c+4*b^2)))*x^5+1/4*(b1*(8*a*b*c+2*b
*(2*a*c+4*b^2))+c1*(a*(2*a*c+4*b^2)+8*b^2*a+c*a^2))*x^4+1/3*(b1*(
a*(2*a*c+4*b^2)+8*b^2*a+c*a^2)+6*c1*a^2*b)*x^3+1/2*(a^3*c1+6*a^2*
b*b1)*x^2+a^3*b1*x
```

Maxima [A] time = 1.41289, size = 231, normalized size = 1.38

$$\frac{1}{8}c^3c_1x^8 + \frac{1}{7}\left(b_1c^3 + 6\,bc^2c_1\right)x^7 + \frac{1}{2}\left(2\,bb_1c^2 + \left(4\,b^2c + ac^2\right)c_1\right)x^6 + \frac{1}{5}\left(12\,b^2b_1c + 3\,ab_1c^2 + 4\left(2\,b^3 + 3\,abc\right)c_1\right)x^5 + a^3b_1x + \frac{1}{4}\left(8\,b^3b_1 + 12\,abb_1c + 3\left(4\,ab^2 + a^2c\right)c_1\right)x^4 + \left(4\,ab^2b_1 + a^2b_1c + 2\,a^2bc_1\right)x^3 + \frac{1}{2}\left(6\,a^2bb_1 + a^3c_1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^3*(c1*x + b1),x, algorithm="maxima")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2
+ (4*b^2*c + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(
2*b^3 + 3*a*b*c)*c1)*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c
 + 3*(4*a*b^2 + a^2*c)*c1)*x^4 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b
*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)*x^2
```

Fricas [A] time = 0.182254, size = 1, normalized size = 0.01

$$\frac{1}{8}x^8c_1c^3 + \frac{1}{7}x^7c^3b_1 + \frac{6}{7}x^7c_1c^2b + x^6c^2b_1b + 2x^6c_1cb^2 + \frac{1}{2}x^6c_1c^2a + \frac{12}{5}x^5cb_1b^2 + \frac{8}{5}x^5c_1b^3 + \frac{3}{5}x^5c^2b_1a + \frac{12}{5}x^5c_1cba + 2x^4b_1b^3 + 3x^4cb_1ba + 3x^4c_1b^2a + \frac{3}{4}x^4c_1ca^2 + 4x^3b_1b^2a + x^3cb_1a^2 + 2x^3c_1ba^2 + 3x^2b_1ba^2 + \frac{1}{2}x^2c_1a^3 + xb_1a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^3*(c1*x + b1),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c1*c^3 + 1/7*x^7*c^3*b1 + 6/7*x^7*c1*c^2*b + x^6*c^2*b1*b
 + 2*x^6*c1*c*b^2 + 1/2*x^6*c1*c^2*a + 12/5*x^5*c*b1*b^2 + 8/5*x^
5*c1*b^3 + 3/5*x^5*c^2*b1*a + 12/5*x^5*c1*c*b*a + 2*x^4*b1*b^3 +
3*x^4*c*b1*b*a + 3*x^4*c1*b^2*a + 3/4*x^4*c1*c*a^2 + 4*x^3*b1*b^2
*a + x^3*c*b1*a^2 + 2*x^3*c1*b*a^2 + 3*x^2*b1*b*a^2 + 1/2*x^2*c1*
a^3 + x*b1*a^3
```

Sympy [A] time = 0.097552, size = 189, normalized size = 1.13

$$a^{3}b_{1}x+\frac{c^{3}c_{1}x^{8}}{8}+x^{7}\left(\frac{6bc^{2}c_{1}}{7}+\frac{b_{1}c^{3}}{7}\right)+x^{6}\left(\frac{ac^{2}c_{1}}{2}+2b^{2}cc_{1}+bb_{1}c^{2}\right)+x^{5}\left(\frac{12abcc_{1}}{5}+\frac{3ab_{1}c^{2}}{5}+\frac{8b^{3}c_{1}}{5}+\frac{12b^{2}b_{1}c}{5}\right)+x^{4}\left(\frac{3a^{2}cc_{1}}{4}+3ab^{2}c_{1}+3abb_{1}c+2b^{3}b_{1}\right)+x^{3}\left(2a^{2}bc_{1}+a^{2}b_{1}c+4ab^{2}b_{1}\right)+x^{2}\left(\frac{a^{3}c_{1}}{2}+3a^{2}bb_{1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

```
[Out] a**3*b1*x + c**3*c1*x**8/8 + x**7*(6*b*c**2*c1/7 + b1*c**3/7) + x
**6*(a*c**2*c1/2 + 2*b**2*c*c1 + b*b1*c**2) + x**5*(12*a*b*c*c1/5
 + 3*a*b1*c**2/5 + 8*b**3*c1/5 + 12*b**2*b1*c/5) + x**4*(3*a**2*c
*c1/4 + 3*a*b**2*c1 + 3*a*b*b1*c + 2*b**3*b1) + x**3*(2*a**2*b*c1
 + a**2*b1*c + 4*a*b**2*b1) + x**2*(a**3*c1/2 + 3*a**2*b*b1)
```

GIAC/XCAS [A] time = 0.198786, size = 254, normalized size = 1.52

$$\frac{1}{8}c^{3}c_{1}x^{8}+\frac{1}{7}b_{1}c^{3}x^{7}+\frac{6}{7}bc^{2}c_{1}x^{7}+bb_{1}c^{2}x^{6}+2b^{2}cc_{1}x^{6}+\frac{1}{2}ac^{2}c_{1}x^{6}+\frac{12}{5}b^{2}b_{1}cx^{5}+\frac{3}{5}ab_{1}c^{2}x^{5}+\frac{8}{5}b^{3}c_{1}x^{5}+\frac{12}{5}abcc_{1}x^{5}+2b^{3}b_{1}x^{4}+3abb_{1}cx^{4}+3ab^{2}c_{1}x^{4}+\frac{3}{4}a^{2}cc_{1}x^{4}+4ab^{2}b_{1}x^{3}+a^{2}b_{1}cx^{3}+2a^{2}bc_{1}x^{3}+3a^{2}bb_{1}x^{2}+\frac{1}{2}a^{3}c_{1}x^{2}+a^{3}b_{1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^3*(c1*x + b1),x, algorithm="giac")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*b1*c^3*x^7 + 6/7*b*c^2*c1*x^7 + b*b1*c^2*x^6
 + 2*b^2*c*c1*x^6 + 1/2*a*c^2*c1*x^6 + 12/5*b^2*b1*c*x^5 + 3/5*a*
b1*c^2*x^5 + 8/5*b^3*c1*x^5 + 12/5*a*b*c*c1*x^5 + 2*b^3*b1*x^4 +
3*a*b*b1*c*x^4 + 3*a*b^2*c1*x^4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x
^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 + 3*a^2*b*b1*x^2 + 1/2*a^3*c1*
x^2 + a^3*b1*x
```

3.192 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^4 \, dx$

Optimal. Leaf size=263

$$a^4\text{b1}x+\frac{1}{2}a^3x^2(a\text{c1}+8b\text{b1})+\frac{4}{3}a^2x^3\left(2abc\text{c1}+a\text{b1}c+6b^2\text{b1}\right)$$
$$+ax^4\left(a^2c\text{c1}+6ab^2\text{c1}+6ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+8b^4\text{c1}+16b^3\text{b1}c\right)$$
$$+\frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+16ab^3\text{c1}+24ab^2\text{b1}c+8b^4\text{b1}\right)+\frac{1}{2}c^2x^8\left(ac\text{c1}+6b^2\text{c1}+2b\text{b1}c\right)$$
$$+\frac{4}{7}cx^7\left(6abc\text{c1}+a\text{b1}c^2+8b^3\text{c1}+6b^2\text{b1}c\right)+\frac{1}{9}c^3x^9(8b\text{c1}+\text{b1}c)+\frac{1}{10}c^4\text{c1}x^{10}$$

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*
c + 2*a*b*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^
2*c*c1)*x^4 + (2*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*
b^3*c1 + 12*a^2*b*c*c1)*x^5)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 +
8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)*x^6)/3 + (4*c*(6*b^2*b1*
c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^2*(2*b*b1*c + 6
*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^4*c1*
x^10)/10
```

Rubi [A] time = 0.666408, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$a^4\text{b1}x+\frac{1}{2}a^3x^2(a\text{c1}+8b\text{b1})+\frac{4}{3}a^2x^3\left(2abc\text{c1}+a\text{b1}c+6b^2\text{b1}\right)$$
$$+ax^4\left(a^2c\text{c1}+6ab^2\text{c1}+6ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+8b^4\text{c1}+16b^3\text{b1}c\right)$$
$$+\frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+16ab^3\text{c1}+24ab^2\text{b1}c+8b^4\text{b1}\right)+\frac{1}{2}c^2x^8\left(ac\text{c1}+6b^2\text{c1}+2b\text{b1}c\right)$$
$$+\frac{4}{7}cx^7\left(6abc\text{c1}+a\text{b1}c^2+8b^3\text{c1}+6b^2\text{b1}c\right)+\frac{1}{9}c^3x^9(8b\text{c1}+\text{b1}c)+\frac{1}{10}c^4\text{c1}x^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```


```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*
c + 2*a*b*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^
2*c*c1)*x^4 + (2*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*
b^3*c1 + 12*a^2*b*c*c1)*x^5)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 +
8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)*x^6)/3 + (4*c*(6*b^2*b1*
c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^2*(2*b*b1*c + 6
*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^4*c1*
x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\int b_1\,dx + a^3\left(ac_1 + 8bb_1\right)\int x\,dx + \frac{4a^2x^3\left(2abc_1 + ab_1c + 6b^2b_1\right)}{3} + ax^4\left(a^2cc_1 + 6ab^2c_1 + 6abb_1c + 8b^3b_1\right) + \frac{c^4c_1x^{10}}{10} + \frac{c^3x^9\left(8bc_1 + b_1c\right)}{9} + \frac{c^2x^8\left(acc_1 + 6b^2c_1 + 2bb_1c\right)}{2} + \frac{4cx^7\left(6abcc_1 + ab_1c^2 + 8b^3c_1 + 6b^2b_1c\right)}{7} + x^6\left(a^2c^2c_1 + 8ab^2cc_1 + 4abb_1c^2 + \frac{8b^4c_1}{3} + \frac{16b^3b_1c}{3}\right) + x^5\left(\frac{24a^2bcc_1}{5} + \frac{6a^2b_1c^2}{5} + \frac{32ab^3c_1}{5} + \frac{48ab^2b_1c}{5} + \frac{16b^4b_1}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

```
[Out] a**4*Integral(b1, x) + a**3*(a*c1 + 8*b*b1)*Integral(x, x) + 4*a*
*2*x**3*(2*a*b*c1 + a*b1*c + 6*b**2*b1)/3 + a*x**4*(a**2*c*c1 + 6
*a*b**2*c1 + 6*a*b*b1*c + 8*b**3*b1) + c**4*c1*x**10/10 + c**3*x*
*9*(8*b*c1 + b1*c)/9 + c**2*x**8*(a*c*c1 + 6*b**2*c1 + 2*b*b1*c)/
2 + 4*c*x**7*(6*a*b*c*c1 + a*b1*c**2 + 8*b**3*c1 + 6*b**2*b1*c)/7
 + x**6*(a**2*c**2*c1 + 8*a*b**2*c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1
/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*c*c1/5 + 6*a**2*b1*c**2/5
+ 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1/5)
```

Mathematica [A] time = 0.106323, size = 263, normalized size = 1.

$$a^4\mathrm{b1}x + \frac{1}{2}a^3x^2(a\mathrm{c1} + 8b\mathrm{b1}) + \frac{4}{3}a^2x^3\left(2ab\mathrm{c1} + a\mathrm{b1}c + 6b^2\mathrm{b1}\right) + ax^4\left(a^2c\mathrm{c1} + 6ab^2\mathrm{c1} + 6ab\mathrm{b1}c + 8b^3\mathrm{b1}\right) + \frac{1}{3}x^6\left(3a^2c^2\mathrm{c1} + 24ab^2c\mathrm{c1} + 12ab\mathrm{b1}c^2 + 8b^4\mathrm{c1} + 16b^3\mathrm{b1}c\right) + \frac{2}{5}x^5\left(12a^2bc\mathrm{c1} + 3a^2\mathrm{b1}c^2 + 16ab^3\mathrm{c1} + 24ab^2\mathrm{b1}c + 8b^4\mathrm{b1}\right) + \frac{1}{2}c^2x^8\left(ac\mathrm{c1} + 6b^2\mathrm{c1} + 2b\mathrm{b1}c\right) + \frac{4}{7}cx^7\left(6abc\mathrm{c1} + a\mathrm{b1}c^2 + 8b^3\mathrm{c1} + 6b^2\mathrm{b1}c\right) + \frac{1}{9}c^3x^9(8b\mathrm{c1} + \mathrm{b1}c) + \frac{1}{10}c^4\mathrm{c1}x^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*
c + 2*a*b*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^
2*c*c1)*x^4 + (2*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*
b^3*c1 + 12*a^2*b*c*c1)*x^5)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 +
8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)*x^6)/3 + (4*c*(6*b^2*b1*
c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^2*(2*b*b1*c + 6
*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^4*c1*
x^10)/10
```

Maple [A] time = 0.003, size = 363, normalized size = 1.4

$$\frac{c^4 c1 x^{10}}{10}+\frac{\left(8 c1 b c^3+b1 c^4\right) x^9}{9}+\frac{\left(8 b1 b c^3+c1 \left(2 \left(2 a c+4 b^2\right) c^2+16 b^2 c^2\right)\right) x^8}{8} +\frac{\left(b1 \left(2 \left(2 a c+4 b^2\right) c^2+16 b^2 c^2\right)+c1 \left(8 a b c^2+8 \left(2 a c+4 b^2\right) b c\right)\right) x^7}{7} +\frac{\left(b1 \left(8 a b c^2+8 \left(2 a c+4 b^2\right) b c\right)+c1 \left(2 a^2 c^2+32 a b^2 c+\left(2 a c+4 b^2\right)^2\right)\right) x^6}{6} +\frac{\left(b1 \left(2 a^2 c^2+32 a b^2 c+\left(2 a c+4 b^2\right)^2\right)+c1 \left(8 a^2 b c+8 a b \left(2 a c+4 b^2\right)\right)\right) x^5}{5} +\frac{\left(b1 \left(8 a^2 b c+8 a b \left(2 a c+4 b^2\right)\right)+c1 \left(2 a^2 \left(2 a c+4 b^2\right)+16 b^2 a^2\right)\right) x^4}{4} +\frac{\left(b1 \left(2 a^2 \left(2 a c+4 b^2\right)+16 b^2 a^2\right)+8 c1 a^3 b\right) x^3}{3}+\frac{\left(c1 a^4+8 b1 a^3 b\right) x^2}{2}+a^4 b1 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x)
```

```
[Out] 1/10*c^4*c1*x^10+1/9*(8*b*c^3*c1+b1*c^4)*x^9+1/8*(8*b1*b*c^3+c1*(
2*(2*a*c+4*b^2)*c^2+16*b^2*c^2))*x^8+1/7*(b1*(2*(2*a*c+4*b^2)*c^2
+16*b^2*c^2)+c1*(8*a*b*c^2+8*(2*a*c+4*b^2)*b*c))*x^7+1/6*(b1*(8*a
*b*c^2+8*(2*a*c+4*b^2)*b*c)+c1*(2*a^2*c^2+32*a*b^2*c+(2*a*c+4*b^2
)^2))*x^6+1/5*(b1*(2*a^2*c^2+32*a*b^2*c+(2*a*c+4*b^2)^2)+c1*(8*a^
2*b*c+8*a*b*(2*a*c+4*b^2)))*x^5+1/4*(b1*(8*a^2*b*c+8*a*b*(2*a*c+4
*b^2))+c1*(2*a^2*(2*a*c+4*b^2)+16*b^2*a^2))*x^4+1/3*(b1*(2*a^2*(2
*a*c+4*b^2)+16*b^2*a^2)+8*c1*a^3*b)*x^3+1/2*(a^4*c1+8*a^3*b*b1)*x
^2+a^4*b1*x
```

Maxima [A] time = 1.43196, size = 369, normalized size = 1.4

$$\frac{1}{10} c^4 c_1 x^{10}+\frac{1}{9}\left(b_1 c^4+8 b c^3 c_1\right) x^9+\frac{1}{2}\left(2 b b_1 c^3+\left(6 b^2 c^2+a c^3\right) c_1\right) x^8 +\frac{4}{7}\left(6 b^2 b_1 c^2+a b_1 c^3+2\left(4 b^3 c+3 a b c^2\right) c_1\right) x^7 +\frac{1}{3}\left(16 b^3 b_1 c+12 a b b_1 c^2+\left(8 b^4+24 a b^2 c+3 a^2 c^2\right) c_1\right) x^6+a^4 b_1 x +\frac{2}{5}\left(8 b^4 b_1+24 a b^2 b_1 c+3 a^2 b_1 c^2+4\left(4 a b^3+3 a^2 b c\right) c_1\right) x^5 +\left(8 a b^3 b_1+6 a^2 b b_1 c+\left(6 a^2 b^2+a^3 c\right) c_1\right) x^4 +\frac{4}{3}\left(6 a^2 b^2 b_1+a^3 b_1 c+2 a^3 b c_1\right) x^3+\frac{1}{2}\left(8 a^3 b b_1+a^4 c_1\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^4*(c1*x + b1),x, algorithm="maxima")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^
3 + (6*b^2*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 +
2*(4*b^3*c + 3*a*b*c^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^
2 + (8*b^4 + 24*a*b^2*c + 3*a^2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 4*(4*a*b^3 + 3*a^2*b*c)*c
1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*b^2 + a^3*c)*c1)*x^4
 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/2*(8*a^3*b*
b1 + a^4*c1)*x^2
```

Fricas [A] time = 0.18359, size = 1, normalized size = 0.

$$\frac{1}{10}x^{10}c_1c^4 + \frac{1}{9}x^9c^4b_1 + \frac{8}{9}x^9c_1c^3b + x^8c^3b_1b + 3x^8c_1c^2b^2 + \frac{1}{2}x^8c_1c^3a + \frac{24}{7}x^7c^2b_1b^2 + \frac{32}{7}x^7c_1cb^3 + \frac{4}{7}x^7c^3b_1a + \frac{24}{7}x^7c_1c^2ba + \frac{16}{3}x^6cb_1b^3 + \frac{8}{3}x^6c_1b^4 + 4x^6c^2b_1ba + 8x^6c_1cb^2a + x^6c_1c^2a^2 + \frac{16}{5}x^5b_1b^4 + \frac{48}{5}x^5cb_1b^2a + \frac{32}{5}x^5c_1b^3a + \frac{6}{5}x^5c^2b_1a^2 + \frac{24}{5}x^5c_1cba^2 + 8x^4b_1b^3a + 6x^4cb_1ba^2 + 6x^4c_1b^2a^2 + x^4c_1ca^3 + 8x^3b_1b^2a^2 + \frac{4}{3}x^3cb_1a^3 + \frac{8}{3}x^3c_1ba^3 + 4x^2b_1ba^3 + \frac{1}{2}x^2c_1a^4 + xb_1a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^4*(c1*x + b1),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c1*c^4 + 1/9*x^9*c^4*b1 + 8/9*x^9*c1*c^3*b + x^8*c^3*b1
*b + 3*x^8*c1*c^2*b^2 + 1/2*x^8*c1*c^3*a + 24/7*x^7*c^2*b1*b^2 +
32/7*x^7*c1*c*b^3 + 4/7*x^7*c^3*b1*a + 24/7*x^7*c1*c^2*b*a + 16/3
*x^6*c*b1*b^3 + 8/3*x^6*c1*b^4 + 4*x^6*c^2*b1*b*a + 8*x^6*c1*c*b^
2*a + x^6*c1*c^2*a^2 + 16/5*x^5*b1*b^4 + 48/5*x^5*c*b1*b^2*a + 32
/5*x^5*c1*b^3*a + 6/5*x^5*c^2*b1*a^2 + 24/5*x^5*c1*c*b*a^2 + 8*x^
4*b1*b^3*a + 6*x^4*c*b1*b*a^2 + 6*x^4*c1*b^2*a^2 + x^4*c1*c*a^3 +
 8*x^3*b1*b^2*a^2 + 4/3*x^3*c*b1*a^3 + 8/3*x^3*c1*b*a^3 + 4*x^2*b
1*b*a^3 + 1/2*x^2*c1*a^4 + x*b1*a^4
```

Sympy [A] time = 0.131373, size = 313, normalized size = 1.19

$$a^4b_1x + \frac{c^4c_1x^{10}}{10} + x^9\left(\frac{8bc^3c_1}{9} + \frac{b_1c^4}{9}\right) + x^8\left(\frac{ac^3c_1}{2} + 3b^2c^2c_1 + bb_1c^3\right) + x^7\left(\frac{24abc^2c_1}{7} + \frac{4ab_1c^3}{7} + \frac{32b^3cc_1}{7} + \frac{24b^2b_1c^2}{7}\right) + x^6\left(a^2c^2c_1 + 8ab^2cc_1 + 4abb_1c^2 + \frac{8b^4c_1}{3} + \frac{16b^3b_1c}{3}\right) + x^5\left(\frac{24a^2bcc_1}{5} + \frac{6a^2b_1c^2}{5} + \frac{32ab^3c_1}{5} + \frac{48ab^2b_1c}{5} + \frac{16b^4b_1}{5}\right) + x^4\left(a^3cc_1 + 6a^2b^2c_1 + 6a^2bb_1c + 8ab^3b_1\right) + x^3\left(\frac{8a^3bc_1}{3} + \frac{4a^3b_1c}{3} + 8a^2b^2b_1\right) + x^2\left(\frac{a^4c_1}{2} + 4a^3bb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

```
[Out] a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) +
 x**8*(a*c**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c
**2*c1/7 + 4*a*b1*c**3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) +
x**6*(a**2*c**2*c1 + 8*a*b**2*c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3
+ 16*b**3*b1*c/3) + x**5*(24*a**2*b*c*c1/5 + 6*a**2*b1*c**2/5 + 3
2*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1/5) + x**4*(a**3*c*c
1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x**3*(8*a**3*
b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*a*
*3*b*b1)
```

GIAC/XCAS [A] time = 0.19904, size = 414, normalized size = 1.57

$$\frac{1}{10}c^4c_1x^{10} + \frac{1}{9}b_1c^4x^9 + \frac{8}{9}bc^3c_1x^9 + bb_1c^3x^8 + 3b^2c^2c_1x^8 + \frac{1}{2}ac^3c_1x^8 + \frac{24}{7}b^2b_1c^2x^7 + \frac{4}{7}ab_1c^3x^7 + \frac{32}{7}b^3cc_1x^7 + \frac{24}{7}abc^2c_1x^7 + \frac{16}{3}b^3b_1cx^6 + 4abb_1c^2x^6 + \frac{8}{3}b^4c_1x^6 + 8ab^2cc_1x^6 + a^2c^2c_1x^6 + \frac{16}{5}b^4b_1x^5 + \frac{48}{5}ab^2b_1cx^5 + \frac{6}{5}a^2b_1c^2x^5 + \frac{32}{5}ab^3c_1x^5 + \frac{24}{5}a^2bcc_1x^5 + 8ab^3b_1x^4 + 6a^2bb_1cx^4 + 6a^2b^2c_1x^4 + a^3cc_1x^4 + 8a^2b^2b_1x^3 + \frac{4}{3}a^3b_1cx^3 + \frac{8}{3}a^3bc_1x^3 + 4a^3bb_1x^2 + \frac{1}{2}a^4c_1x^2 + a^4b_1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + 2*b*x + a)^4*(c1*x + b1),x, algorithm="giac")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x
^8 + 3*b^2*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 +
4/7*a*b1*c^3*x^7 + 32/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3
*b^3*b1*c*x^6 + 4*a*b*b1*c^2*x^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*
x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x^5 + 48/5*a*b^2*b1*c*x^5 + 6/
5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a^2*b*c*c1*x^5 + 8*a*
b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3*c*c1*x^4 +
 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3*b
*b1*x^2 + 1/2*a^4*c1*x^2 + a^4*b1*x
```

3.193 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^n\,dx$

Optimal. Leaf size=159

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)} - \frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}{}_2F_1\left(-n,n+1;n+2;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(n+1)\sqrt{b^2-ac}}$$

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c
1)*(-((b - Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a +
 2*b*x + c*x^2)^(1 + n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b +
Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(
1 + n))
```

Rubi [A] time = 0.197746, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)} - \frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}{}_2F_1\left(-n,n+1;n+2;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(n+1)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c
1)*(-((b - Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a +
 2*b*x + c*x^2)^(1 + n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b +
Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(
1 + n))
```

Rubi in Sympy [A] time = 6.92274, size = 134, normalized size = 0.84

$$\frac{c_1\left(a+2bx+cx^2\right)^{n+1}}{2c\left(n+1\right)} + \frac{\left(\frac{-\frac{b}{2}-\frac{cx}{2}+\frac{\sqrt{-ac+b^2}}{2}}{\sqrt{-ac+b^2}}\right)^{-n-1}\left(bc_1-b_1c\right)\left(a+2bx+cx^2\right)^{n+1}{}_2F_1\left(\left.\begin{matrix}-n,n+1\\n+2\end{matrix}\right|\frac{\frac{b}{2}+\frac{cx}{2}+\frac{\sqrt{-ac+b^2}}{2}}{\sqrt{-ac+b^2}}\right)}{2c\left(n+1\right)\sqrt{-ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
```

```
[Out] c1*(a + 2*b*x + c*x**2)**(n + 1)/(2*c*(n + 1)) + ((-b/2 - c*x/2 +
 sqrt(-a*c + b**2)/2)/sqrt(-a*c + b**2))**(-n - 1)*(b*c1 - b1*c)*
(a + 2*b*x + c*x**2)**(n + 1)*hyper((-n, n + 1), (n + 2,), (b/2 +
 c*x/2 + sqrt(-a*c + b**2)/2)/sqrt(-a*c + b**2))/(2*c*(n + 1)*sqr
t(-a*c + b**2))
```

Mathematica [C] time = 3.55119, size = 471, normalized size = 2.96

$$\frac{1}{2}\left(-\sqrt{b^2-ac}+b+cx\right)(a+x(2b+cx))^n\left(\frac{\text{b1}2^{n+1}\left(\frac{\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n}\text{Hypergeometric2F1}\left(-n,n+1,n+2,\frac{\sqrt{b^2-ac}-b-cx}{2\sqrt{b^2-ac}}\right)}{c(n+1)}+\frac{3\text{c1}x^2\left(\sqrt{b^2-ac}+b\right)\left(x\left(b-\sqrt{b^2-ac}\right)+a\right)^2F_1\left(2;\cdots\right.}{\left(\sqrt{b^2-ac}-b\right)\left(\sqrt{b^2-ac}+b+cx\right)(a+x(2b+cx))\left(nx\left(\left(\sqrt{b^2-ac}-b\right)F_1\left(3;1-n,-n;4;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-ac}-b}\right)-\left(\sqrt{\cdots}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] ((b - Sqrt[b^2 - a*c] + c*x)*(a + x*(2*b + c*x))^n*((3*(b + Sqrt[
b^2 - a*c])*c1*x^2*(a + (b - Sqrt[b^2 - a*c])*x)^2*AppellF1[2, -n
, -n, 3, -((c*x)/(b + Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a
*c])])/((-b + Sqrt[b^2 - a*c])*(b + Sqrt[b^2 - a*c] + c*x)*(a + x
*(2*b + c*x))*(-3*a*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b^2
- a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] + n*x*((-b + Sqrt[b^2 - a
*c])*AppellF1[3, 1 - n, -n, 4, -((c*x)/(b + Sqrt[b^2 - a*c])), (c
*x)/(-b + Sqrt[b^2 - a*c])] - (b + Sqrt[b^2 - a*c])*AppellF1[3, -
n, 1 - n, 4, -((c*x)/(b + Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2
 - a*c])]))) + (2^(1 + n)*b1*Hypergeometric2F1[-n, 1 + n, 2 + n,
(-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*((b
 + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n)))/2
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int (c1\,x+b1)\left(cx^2+2\,bx+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

```
[Out] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c_1x + b_1)(cx^2 + 2\,bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c_1x + b_1)(cx^2 + 2\,bx + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n,x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (c_1x + b_1)(cx^2 + 2\,bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n,x, algorithm="giac")

[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

3.194 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{a+2bx+cx^2}\,dx$

Optimal. Leaf size=65

$$\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

```
[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(c*Sqrt[b^2
- a*c])) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Rubi [A] time = 0.0886123, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(c*Sqrt[b^2
- a*c])) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Rubi in Sympy [A] time = 7.32027, size = 53, normalized size = 0.82

$$\frac{c_1\log\left(a+2bx+cx^2\right)}{2c}+\frac{(bc_1-b_1c)\operatorname{atanh}\left(\frac{b+cx}{\sqrt{-ac+b^2}}\right)}{c\sqrt{-ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

```
[Out] c1*log(a + 2*b*x + c*x**2)/(2*c) + (b*c1 - b1*c)*atanh((b + c*x)/
sqrt(-a*c + b**2))/(c*sqrt(-a*c + b**2))
```

Mathematica [A] time = 0.0754987, size = 66, normalized size = 1.02

$$\frac{(\mathrm{b1}c-b\mathrm{c1})\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{c\sqrt{ac-b^2}}+\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] ((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 +
 a*c]) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Maple [A] time = 0.007, size = 95, normalized size = 1.5

$$\frac{c1\ln\left(cx^2+2\,bx+a\right)}{2\,c}+b1\arctan\left(\frac{2\,cx+2\,b}{2}\frac{1}{\sqrt{ac-b^2}}\right)\frac{1}{\sqrt{ac-b^2}}-\frac{bc1}{c}\arctan\left(\frac{2\,cx+2\,b}{2}\frac{1}{\sqrt{ac-b^2}}\right)\frac{1}{\sqrt{ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)/(c*x^2+2*b*x+a),x)
```

```
[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c+1/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*
b)/(a*c-b^2)^(1/2))*b1-1/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(
a*c-b^2)^(1/2))*c1*b/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217302, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{b^2-ac}c_1\log\left(cx^2+2\,bx+a\right)-\left(b_1c-bc_1\right)\log\left(\frac{2\,b^3-2\,abc+2\left(b^2c-ac^2\right)x+\left(c^2x^2+2\,bcx+2\,b^2-ac\right)\sqrt{b^2-ac}}{cx^2+2\,bx+a}\right)}{2\sqrt{b^2-ac}c},\frac{\sqrt{-b^2+ac}c_1\log\left(cx^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - (b1*c - b*c1)*l
og((2*b^3 - 2*a*b*c + 2*(b^2*c - a*c^2)*x + (c^2*x^2 + 2*b*c*x +
2*b^2 - a*c)*sqrt(b^2 - a*c))/(c*x^2 + 2*b*x + a)))/(sqrt(b^2 - a
*c)*c), 1/2*(sqrt(-b^2 + a*c)*c1*log(c*x^2 + 2*b*x + a) + 2*(b1*c
 - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)))/(sqrt(-
b^2 + a*c)*c)]
```

Sympy [A] time = 1.02035, size = 246, normalized size = 3.78

$$\left(\frac{c_1}{2c} - \frac{\sqrt{-ac+b^2}\,(bc_1-b_1c)}{2c\,(ac-b^2)}\right)\log\left(x+\frac{-2ac\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)+ac_1+2b^2\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)-bb_1}{bc_1-b_1c}\right)$$
$$+\left(\frac{c_1}{2c} + \frac{\sqrt{-ac+b^2}\,(bc_1-b_1c)}{2c\,(ac-b^2)}\right)\log\left(x+\frac{-2ac\left(\frac{c_1}{2c}+\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)+ac_1+2b^2\left(\frac{c_1}{2c}+\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)-bb_1}{bc_1-b_1c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

```
[Out] (c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*l
og(x + (-2*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(
a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1
 - b1*c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c)) + (c1/(2*c) +
 sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-2*
a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)
)) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2
*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))
```

GIAC/XCAS [A] time = 0.199813, size = 81, normalized size = 1.25

$$\frac{c_1\ln\left(cx^2+2\,bx+a\right)}{2\,c}+\frac{(b_1c-bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a),x, algorithm="giac")
```

```
[Out] 1/2*c1*ln(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/s
qrt(-b^2 + a*c))/(sqrt(-b^2 + a*c)*c)
```

3.195 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{(a+2bx+cx^2)^2}\,dx$

Optimal. Leaf size=89

$$\frac{(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}$$

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(2*(b^2 - a*c)*(a + 2*b*x + c*x^
2)) + ((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2
- a*c)^(3/2))
```

Rubi [A] time = 0.106163, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(2*(b^2 - a*c)*(a + 2*b*x + c*x^
2)) + ((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2
- a*c)^(3/2))
```

Rubi in Sympy [A] time = 6.49314, size = 76, normalized size = 0.85

$$\frac{2ac_1-2bb_1+x\left(2bc_1-2b_1c\right)}{4\left(-ac+b^2\right)\left(a+2bx+cx^2\right)}-\frac{\left(bc_1-b_1c\right)\operatorname{atanh}\left(\frac{b+cx}{\sqrt{-ac+b^2}}\right)}{2\left(-ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

```
[Out] (2*a*c1 - 2*b*b1 + x*(2*b*c1 - 2*b1*c))/(4*(-a*c + b**2)*(a + 2*b
*x + c*x**2)) - (b*c1 - b1*c)*atanh((b + c*x)/sqrt(-a*c + b**2))/
(2*(-a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.126222, size = 88, normalized size = 0.99

$$\frac{\frac{(bc1-b1c)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}+\frac{ac1-bb1+bc1x-b1cx}{a+x(2b+cx)}}{2(b^2-ac)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] ((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*
c) + b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/
(2*(b^2 - a*c))
```

Maple [A] time = 0.004, size = 146, normalized size = 1.6

$$\frac{(-2\,bc1+2\,b1c)\,x+2\,bb1-2\,ac1}{(4\,ac-4\,b^2)\,(cx^2+2\,bx+a)}-2\,\frac{bc1}{(4\,ac-4\,b^2)\,\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right)+2\,\frac{b1c}{(4\,ac-4\,b^2)\,\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x)
```

```
[Out] ((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)-
2/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^
(1/2))*b*c1+2/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b
)/(a*c-b^2)^(1/2))*b1*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.214481, size = 1, normalized size = 0.01

$$\left[\frac{\left(ab_1c-abc_1+\left(b_1c^2-bcc_1\right)x^2+2\left(bb_1c-b^2c_1\right)x\right)\log\left(\frac{2b^3-2abc+2\left(b^2c-ac^2\right)x+\left(c^2x^2+2bcx+2b^2-ac\right)\sqrt{b^2-ac}}{cx^2+2bx+a}\right)-2\sqrt{b^2-ac}(bb_1 ...}{4\left(ab^2-a^2c+(b^2c-ac^2)x^2+2(b^3-abc)x\right)\sqrt{b^2-ac}}\right.$$
$$\left.-\frac{\left(ab_1c-abc_1+\left(b_1c^2-bcc_1\right)x^2+2\left(bb_1c-b^2c_1\right)x\right)\arctan\left(-\frac{\sqrt{-b^2+ac}(cx+b)}{b^2-ac}\right)+\sqrt{-b^2+ac}(bb_1-ac_1+(b_1c-bc_1)x)}{2\left(ab^2-a^2c+(b^2c-ac^2)x^2+2(b^3-abc)x\right)\sqrt{-b^2+ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2*(b*b1*c - b^2*
c1)*x)*log((2*b^3 - 2*a*b*c + 2*(b^2*c - a*c^2)*x + (c^2*x^2 + 2*
b*c*x + 2*b^2 - a*c)*sqrt(b^2 - a*c))/(c*x^2 + 2*b*x + a)) - 2*sq
rt(b^2 - a*c)*(b*b1 - a*c1 + (b1*c - b*c1)*x))/((a*b^2 - a^2*c +
(b^2*c - a*c^2)*x^2 + 2*(b^3 - a*b*c)*x)*sqrt(b^2 - a*c)), -1/2*(
(a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2*(b*b1*c - b^2*c1)*x)
*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) + sqrt(-b^2 + a*
c)*(b*b1 - a*c1 + (b1*c - b*c1)*x))/((a*b^2 - a^2*c + (b^2*c - a*
c^2)*x^2 + 2*(b^3 - a*b*c)*x)*sqrt(-b^2 + a*c))]
```

Sympy [A] time = 1.98327, size = 323, normalized size = 3.63

$$\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)\log\left(x+\frac{-a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)+2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)-b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)+b^2c_1-bb_1c}{bcc_1-b_1c^2}\right)}{4}$$
$$-\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)\log\left(x+\frac{a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)-2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)+b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)+b^2c_1-bb_1c}{bcc_1-b_1c^2}\right)}{4}$$
$$-\frac{ac_1-bb_1+x(bc_1-b_1c)}{2a^2c-2ab^2+x^2(2ac^2-2b^2c)+x(4abc-4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

```
[Out] sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (-a**2*c**2*sqrt(-
1/(a*c - b**2)**3)*(b*c1 - b1*c) + 2*a*b**2*c*sqrt(-1/(a*c - b**2
)**3)*(b*c1 - b1*c) - b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)
 + b**2*c1 - b*b1*c)/(b*c*c1 - b1*c**2))/4 - sqrt(-1/(a*c - b**2)
**3)*(b*c1 - b1*c)*log(x + (a**2*c**2*sqrt(-1/(a*c - b**2)**3)*(b
```

```
*c1 - b1*c) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) +
 b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/
(b*c*c1 - b1*c**2))/4 - (a*c1 - b*b1 + x*(b*c1 - b1*c))/(2*a**2*c
 - 2*a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))
```

GIAC/XCAS [A] time = 0.20287, size = 124, normalized size = 1.39

$$-\frac{(b_1c - bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2 - ac)\sqrt{-b^2 + ac}} - \frac{b_1cx - bc_1x + bb_1 - ac_1}{2\,(cx^2 + 2\,bx + a)(b^2 - ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c
)*sqrt(-b^2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2
 + 2*b*x + a)*(b^2 - a*c))
```

3.196 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{(a+2bx+cx^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{3(b+cx)(\mathrm{b1}c-b\mathrm{c1})}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{3c(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2-ac\right)^{5/2}}$$

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(4*(b^2 - a*c)*(a + 2*b*x + c*x^
2)^2) + (3*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x +
 c*x^2)) - (3*c*(b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])
/(8*(b^2 - a*c)^(5/2))
```

Rubi [A] time = 0.164698, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{3(b+cx)(\mathrm{b1}c-b\mathrm{c1})}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{3c(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2-ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(4*(b^2 - a*c)*(a + 2*b*x + c*x^
2)^2) + (3*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x +
 c*x^2)) - (3*c*(b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])
/(8*(b^2 - a*c)^(5/2))
```

Rubi in Sympy [A] time = 9.28953, size = 121, normalized size = 0.93

$$\frac{3c\left(bc_1-b_1c\right)\operatorname{atanh}\left(\frac{b+cx}{\sqrt{-ac+b^2}}\right)}{8\left(-ac+b^2\right)^{\frac{5}{2}}}-\frac{3\left(2b+2cx\right)\left(bc_1-b_1c\right)}{16\left(-ac+b^2\right)^2\left(a+2bx+cx^2\right)}+\frac{2ac_1-2bb_1+x\left(2bc_1-2b_1c\right)}{8\left(-ac+b^2\right)\left(a+2bx+cx^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

```
[Out] 3*c*(b*c1 - b1*c)*atanh((b + c*x)/sqrt(-a*c + b**2))/(8*(-a*c + b
**2)**(5/2)) - 3*(2*b + 2*c*x)*(b*c1 - b1*c)/(16*(-a*c + b**2)**2
*(a + 2*b*x + c*x**2)) + (2*a*c1 - 2*b*b1 + x*(2*b*c1 - 2*b1*c))/
(8*(-a*c + b**2)*(a + 2*b*x + c*x**2)**2)
```

Mathematica [A] time = 0.228586, size = 127, normalized size = 0.98

$$\frac{\frac{2(b^2-ac)(ac1-bb1+bc1x-b1cx)}{(a+x(2b+cx))^2}+\frac{3c(b1c-bc1)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}+\frac{3(b+cx)(b1c-bc1)}{a+x(2b+cx)}}{8\left(b^2-ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] ((2*(b^2 - a*c)*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b +
 c*x))^2 + (3*(b1*c - b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (3*c
*(b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*
c])/(8*(b^2 - a*c)^2)
```

Maple [B] time = 0.007, size = 274, normalized size = 2.1

$$\frac{(-2\,bc1+2\,b1c)\,x+2\,bb1-2\,ac1}{(8\,ac-8\,b^2)\,(cx^2+2\,bx+a)^2}-6\,\frac{cxbc1}{(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)}+6\,\frac{xc^2b1}{(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)}-6\,\frac{b^2c1}{(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)}+6\,\frac{bb1c}{(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)}-6\,\frac{c1bc}{(4\,ac-4\,b^2)^2\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right)+6\,\frac{b1c^2}{(4\,ac-4\,b^2)^2\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x)
```

```
[Out] 1/2*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x
+a)^2-6/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)*x*c*b*c1+6/(4*a*c-4*b^2)^
2/(c*x^2+2*b*x+a)*x*c^2*b1-6/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)*b^2*
c1+6/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)*b*b1*c-6/(4*a*c-4*b^2)^2*c/(
a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*b*c1+6/(4*
a*c-4*b^2)^2*c^2/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)
^(1/2))*b1
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229314, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*
(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3
*c + a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*log((2*b^3
 - 2*a*b*c + 2*(b^2*c - a*c^2)*x + (c^2*x^2 + 2*b*c*x + 2*b^2 - a
*c)*sqrt(b^2 - a*c))/(c*x^2 + 2*b*x + a)) + 2*(2*b^3*b1 - 5*a*b*b
1*c - 3*(b1*c^3 - b*c^2*c1)*x^3 - 9*(b*b1*c^2 - b^2*c*c1)*x^2 + (
a*b^2 + 2*a^2*c)*c1 - (4*b^2*b1*c + 5*a*b1*c^2 - (4*b^3 + 5*a*b*c
)*c1)*x)*sqrt(b^2 - a*c))/((a^2*b^4 - 2*a^3*b^2*c + a^4*c^2 + (b^
4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^4 + 4*(b^5*c - 2*a*b^3*c^2 + a^2
*b*c^3)*x^3 + 2*(2*b^6 - 3*a*b^4*c + a^3*c^3)*x^2 + 4*(a*b^5 - 2*
a^2*b^3*c + a^3*b*c^2)*x)*sqrt(b^2 - a*c)), 1/8*(3*(a^2*b1*c^2 -
a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c1)*
x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 +
4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)
/(b^2 - a*c)) - (2*b^3*b1 - 5*a*b*b1*c - 3*(b1*c^3 - b*c^2*c1)*x^
3 - 9*(b*b1*c^2 - b^2*c*c1)*x^2 + (a*b^2 + 2*a^2*c)*c1 - (4*b^2*b
1*c + 5*a*b1*c^2 - (4*b^3 + 5*a*b*c)*c1)*x)*sqrt(-b^2 + a*c))/((a
^2*b^4 - 2*a^3*b^2*c + a^4*c^2 + (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4
)*x^4 + 4*(b^5*c - 2*a*b^3*c^2 + a^2*b*c^3)*x^3 + 2*(2*b^6 - 3*a*
b^4*c + a^3*c^3)*x^2 + 4*(a*b^5 - 2*a^2*b^3*c + a^3*b*c^2)*x)*sqr
t(-b^2 + a*c))]
```

Sympy [A] time = 3.89344, size = 622, normalized size = 4.78

$$\frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}\,(bc_1-b_1c)\log\left(x+\frac{-3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+3b^6c\sqrt{-\frac{1}{(ac-b^2)^5}}\cdots}{3bc^2c_1-3b_1c^3}\right.}{16}$$

$$-\frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}\,(bc_1-b_1c)\log\left(x+\frac{3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-3b^6c\sqrt{-\frac{1}{(ac-b^2)^5}}\cdots}{3bc^2c_1-3b_1c^3}\right.}{16}$$

$$-\frac{2a^2cc_1+ab^2c_1-5abb_1c+2b^3b_1+x^3\left(3bc^2c_1-3b_1c^3\right)+x^2\left(9b^2cc_1-9bb_1c^2\right)+x\left(5abcc_1-5ab_1c^2+4b\cdots\right.}{8a^4c^2-16a^3b^2c+8a^2b^4+x^4\left(8a^2c^4-16ab^2c^3+8b^4c^2\right)+x^3\left(32a^2bc^3-64ab^3c^2+32b^5c\right)+x^2\left(16a^3c^3-48ab^4c+32b^6\right)+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (-3*a**3*c**4*
sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a**2*b**2*c**3*sqrt(-1
/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a*b**4*c**2*sqrt(-1/(a*c - b*
*2)**5)*(b*c1 - b1*c) + 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 -
 b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16
 - 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (3*a**3*c**
4*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a**2*b**2*c**3*sqrt(
-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a*b**4*c**2*sqrt(-1/(a*c -
b**2)**5)*(b*c1 - b1*c) - 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1
 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/
16 - (2*a**2*c*c1 + a*b**2*c1 - 5*a*b*b1*c + 2*b**3*b1 + x**3*(3*
b*c**2*c1 - 3*b1*c**3) + x**2*(9*b**2*c*c1 - 9*b*b1*c**2) + x*(5*
a*b*c*c1 - 5*a*b1*c**2 + 4*b**3*c1 - 4*b**2*b1*c))/(8*a**4*c**2 -
 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8*a**2*c**4 - 16*a*b**2*c**
3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b**3*c**2 + 32*b**
5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(32*a**3*b
*c**2 - 64*a**2*b**3*c + 32*a*b**5))
```

GIAC/XCAS [A] time = 0.204036, size = 262, normalized size = 2.02

$$\frac{3\left(b_1c^2-bcc_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{8\left(b^4-2\,ab^2c+a^2c^2\right)\sqrt{-b^2+ac}}+\frac{3\,b_1c^3x^3-3\,bc^2c_1x^3+9\,bb_1c^2x^2-9\,b^2cc_1x^2+4\,b^2b_1cx+5\,ab_1c^2x-4\,b^3c_1x-5\,abcc_1x-2\,b^3b_1+5\,abb_1c-ab^2c_1-2\,a^2c\cdots}{8\left(b^4-2\,ab^2c+a^2c^2\right)\left(cx^2+2\,bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b1*c^2 - b*c*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^4 -
2*a*b^2*c + a^2*c^2)*sqrt(-b^2 + a*c)) + 1/8*(3*b1*c^3*x^3 - 3*b*
c^2*c1*x^3 + 9*b*b1*c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a
*b1*c^2*x - 4*b^3*c1*x - 5*a*b*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a
*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^2*c + a^2*c^2)*(c*x^2 + 2*b*x
 + a)^2)
```

3.197 $\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^4}\,dx$

Optimal. Leaf size=173

$$\frac{5c^2(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2-ac\right)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}+\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(6*(b^2 - a*c)*(a + 2*b*x + c*x^
2)^3) + (5*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x
+ c*x^2)^2) - (5*c*(b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a
+ 2*b*x + c*x^2)) + (5*c^2*(b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b
^2 - a*c]])/(16*(b^2 - a*c)^(7/2))
```

Rubi [A] time = 0.254358, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{5c^2(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2-ac\right)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}+\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(6*(b^2 - a*c)*(a + 2*b*x + c*x^
2)^3) + (5*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x
+ c*x^2)^2) - (5*c*(b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a
+ 2*b*x + c*x^2)) + (5*c^2*(b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b
^2 - a*c]])/(16*(b^2 - a*c)^(7/2))
```

Rubi in Sympy [A] time = 13.2722, size = 165, normalized size = 0.95

$$-\frac{5c^2\left(bc_1-b_1c\right)\operatorname{atanh}\left(\frac{b+cx}{\sqrt{-ac+b^2}}\right)}{16\left(-ac+b^2\right)^{\frac{7}{2}}}+\frac{5c\left(2b+2cx\right)\left(bc_1-b_1c\right)}{32\left(-ac+b^2\right)^3\left(a+2bx+cx^2\right)}-\frac{5\left(2b+2cx\right)\left(bc_1-b_1c\right)}{48\left(-ac+b^2\right)^2\left(a+2bx+cx^2\right)^2}+\frac{2ac_1-2bb_1+x\left(2bc_1-2b_1c\right)}{12\left(-ac+b^2\right)\left(a+2bx+cx^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

```
[Out] -5*c**2*(b*c1 - b1*c)*atanh((b + c*x)/sqrt(-a*c + b**2))/(16*(-a*
c + b**2)**(7/2)) + 5*c*(2*b + 2*c*x)*(b*c1 - b1*c)/(32*(-a*c + b
**2)**3*(a + 2*b*x + c*x**2)) - 5*(2*b + 2*c*x)*(b*c1 - b1*c)/(48
*(-a*c + b**2)**2*(a + 2*b*x + c*x**2)**2) + (2*a*c1 - 2*b*b1 + x
*(2*b*c1 - 2*b1*c))/(12*(-a*c + b**2)*(a + 2*b*x + c*x**2)**3)
```

Mathematica [A] time = 0.350171, size = 168, normalized size = 0.97

$$\frac{\frac{15c^2(bc1-b1c)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}-\frac{10(b^2-ac)(b+cx)(bc1-b1c)}{(a+x(2b+cx))^2}+\frac{8(b^2-ac)^2(ac1-bb1+bc1x-b1cx)}{(a+x(2b+cx))^3}+\frac{15c(b+cx)(bc1-b1c)}{a+x(2b+cx)}}{48(b^2-ac)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] ((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b
 + c*x))^3 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(
2*b + c*x))^2 + (15*c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c
*x)) + (15*c^2*(-(b1*c) + b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]
])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)^3)
```

Maple [B] time = 0.008, size = 405, normalized size = 2.3

$$\frac{(-2\,bc1+2\,b1\,c)\,x+2\,bb1-2\,ac1}{(12\,ac-12\,b^2)\,(cx^2+2\,bx+a)^3}-\frac{10\,cxbc1}{3\,(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)^2}+\frac{10\,xc^2b1}{3\,(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)^2}-\frac{10\,b^2c1}{3\,(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)^2}+\frac{10\,bb1\,c}{3\,(4\,ac-4\,b^2)^2\,(cx^2+2\,bx+a)^2}-20\,\frac{xc^2bc1}{(4\,ac-4\,b^2)^3\,(cx^2+2\,bx+a)}+20\,\frac{c^3xb1}{(4\,ac-4\,b^2)^3\,(cx^2+2\,bx+a)}-20\,\frac{b^2cc1}{(4\,ac-4\,b^2)^3\,(cx^2+2\,bx+a)}+20\,\frac{b1\,bc^2}{(4\,ac-4\,b^2)^3\,(cx^2+2\,bx+a)}-20\,\frac{c1\,bc^2}{(4\,ac-4\,b^2)^3\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right)+20\,\frac{b1\,c^3}{(4\,ac-4\,b^2)^3\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x)
```

```
[Out] 1/3*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x
+a)^3-10/3/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)^2*x*c*b*c1+10/3/(4*a*c
-4*b^2)^2/(c*x^2+2*b*x+a)^2*x*c^2*b1-10/3/(4*a*c-4*b^2)^2/(c*x^2+
2*b*x+a)^2*b^2*c1+10/3/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)^2*b*b1*c-2
0/(4*a*c-4*b^2)^3*c^2/(c*x^2+2*b*x+a)*x*b*c1+20/(4*a*c-4*b^2)^3*c
^3/(c*x^2+2*b*x+a)*x*b1-20/(4*a*c-4*b^2)^3*c/(c*x^2+2*b*x+a)*b^2*
c1+20/(4*a*c-4*b^2)^3*c^2/(c*x^2+2*b*x+a)*b*b1-20/(4*a*c-4*b^2)^3
*c^2/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*b*c1
+20/(4*a*c-4*b^2)^3*c^3/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a
*c-b^2)^(1/2))*b1
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228335, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*x^6 +
6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^5 - (4*b
^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^4 - (2*b
^4*c^2 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b1*c^4 -
(4*a*b^3*c^2 + a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^2
*c1)*x)*log((2*b^3 - 2*a*b*c + 2*(b^2*c - a*c^2)*x + (c^2*x^2 + 2
*b*c*x + 2*b^2 - a*c)*sqrt(b^2 - a*c))/(c*x^2 + 2*b*x + a)) - 2*(
8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 15*(b1*c^5 - b*c^4*c
1)*x^5 + 75*(b*b1*c^4 - b^2*c^3*c1)*x^4 + 10*(11*b^2*b1*c^3 + 4*a
*b1*c^4 - (11*b^3*c^2 + 4*a*b*c^3)*c1)*x^3 + 30*(b^3*b1*c^2 + 4*a
*b*b1*c^3 - (b^4*c + 4*a*b^2*c^2)*c1)*x^2 + (2*a*b^4 - 9*a^2*b^2*
c - 8*a^3*c^2)*c1 - 3*(4*b^4*b1*c - 18*a*b^2*b1*c^2 - 11*a^2*b1*c
^3 - (4*b^5 - 18*a*b^3*c - 11*a^2*b*c^2)*c1)*x)*sqrt(b^2 - a*c))/
((a^3*b^6 - 3*a^4*b^4*c + 3*a^5*b^2*c^2 - a^6*c^3 + (b^6*c^3 - 3*
a*b^4*c^4 + 3*a^2*b^2*c^5 - a^3*c^6)*x^6 + 6*(b^7*c^2 - 3*a*b^5*c
^3 + 3*a^2*b^3*c^4 - a^3*b*c^5)*x^5 + 3*(4*b^8*c - 11*a*b^6*c^2 +
 9*a^2*b^4*c^3 - a^3*b^2*c^4 - a^4*c^5)*x^4 + 4*(2*b^9 - 3*a*b^7*
c - 3*a^2*b^5*c^2 + 7*a^3*b^3*c^3 - 3*a^4*b*c^4)*x^3 + 3*(4*a*b^8
```

```
- 11*a^2*b^6*c + 9*a^3*b^4*c^2 - a^4*b^2*c^3 - a^5*c^4)*x^2 + 6*
(a^2*b^7 - 3*a^3*b^5*c + 3*a^4*b^3*c^2 - a^5*b*c^3)*x)*sqrt(b^2 -
 a*c)), -1/48*(15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1
)*x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^
5 - (4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^
4 - (2*b^4*c^2 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b
1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2
*b^2*c^2*c1)*x)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 15*(b1*c^5 - b*c^4
*c1)*x^5 + 75*(b*b1*c^4 - b^2*c^3*c1)*x^4 + 10*(11*b^2*b1*c^3 + 4
*a*b1*c^4 - (11*b^3*c^2 + 4*a*b*c^3)*c1)*x^3 + 30*(b^3*b1*c^2 + 4
*a*b*b1*c^3 - (b^4*c + 4*a*b^2*c^2)*c1)*x^2 + (2*a*b^4 - 9*a^2*b^
2*c - 8*a^3*c^2)*c1 - 3*(4*b^4*b1*c - 18*a*b^2*b1*c^2 - 11*a^2*b1
*c^3 - (4*b^5 - 18*a*b^3*c - 11*a^2*b*c^2)*c1)*x)*sqrt(-b^2 + a*c
))/((a^3*b^6 - 3*a^4*b^4*c + 3*a^5*b^2*c^2 - a^6*c^3 + (b^6*c^3 -
 3*a*b^4*c^4 + 3*a^2*b^2*c^5 - a^3*c^6)*x^6 + 6*(b^7*c^2 - 3*a*b^
5*c^3 + 3*a^2*b^3*c^4 - a^3*b*c^5)*x^5 + 3*(4*b^8*c - 11*a*b^6*c^
2 + 9*a^2*b^4*c^3 - a^3*b^2*c^4 - a^4*c^5)*x^4 + 4*(2*b^9 - 3*a*b
^7*c - 3*a^2*b^5*c^2 + 7*a^3*b^3*c^3 - 3*a^4*b*c^4)*x^3 + 3*(4*a*
b^8 - 11*a^2*b^6*c + 9*a^3*b^4*c^2 - a^4*b^2*c^3 - a^5*c^4)*x^2 +
 6*(a^2*b^7 - 3*a^3*b^5*c + 3*a^4*b^3*c^2 - a^5*b*c^3)*x)*sqrt(-b
^2 + a*c))]
```

Sympy [A] time = 7.61434, size = 1027, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

```
[Out] 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (-5*a**4*c*
*6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 20*a**3*b**2*c**5*sqr
t(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 30*a**2*b**4*c**4*sqrt(-1/(
a*c - b**2)**7)*(b*c1 - b1*c) + 20*a*b**6*c**3*sqrt(-1/(a*c - b**
2)**7)*(b*c1 - b1*c) - 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1
 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4
))/32 - 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (5*
a**4*c**6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a**3*b**2*c
**5*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 30*a**2*b**4*c**4*sq
rt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a*b**6*c**3*sqrt(-1/(a*
c - b**2)**7)*(b*c1 - b1*c) + 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7
)*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3*c1 - 5*
b1*c**4))/32 - (8*a**3*c**2*c1 + 9*a**2*b**2*c*c1 - 33*a**2*b*b1*
c**2 - 2*a*b**4*c1 + 26*a*b**3*b1*c - 8*b**5*b1 + x**5*(15*b*c**4
*c1 - 15*b1*c**5) + x**4*(75*b**2*c**3*c1 - 75*b*b1*c**4) + x**3*
(40*a*b*c**3*c1 - 40*a*b1*c**4 + 110*b**3*c**2*c1 - 110*b**2*b1*c
**3) + x**2*(120*a*b**2*c**2*c1 - 120*a*b*b1*c**3 + 30*b**4*c*c1
- 30*b**3*b1*c**2) + x*(33*a**2*b*c**2*c1 - 33*a**2*b1*c**3 + 54*
a*b**3*c*c1 - 54*a*b**2*b1*c**2 - 12*b**5*c1 + 12*b**4*b1*c))/(48
*a**6*c**3 - 144*a**5*b**2*c**2 + 144*a**4*b**4*c - 48*a**3*b**6
+ x**6*(48*a**3*c**6 - 144*a**2*b**2*c**5 + 144*a*b**4*c**4 - 48*
```

```
b**6*c**3) + x**5*(288*a**3*b*c**5 - 864*a**2*b**3*c**4 + 864*a*b
**5*c**3 - 288*b**7*c**2) + x**4*(144*a**4*c**5 + 144*a**3*b**2*c
**4 - 1296*a**2*b**4*c**3 + 1584*a*b**6*c**2 - 576*b**8*c) + x**3
*(576*a**4*b*c**4 - 1344*a**3*b**3*c**3 + 576*a**2*b**5*c**2 + 57
6*a*b**7*c - 384*b**9) + x**2*(144*a**5*c**4 + 144*a**4*b**2*c**3
 - 1296*a**3*b**4*c**2 + 1584*a**2*b**6*c - 576*a*b**8) + x*(288*
a**5*b*c**3 - 864*a**4*b**3*c**2 + 864*a**3*b**5*c - 288*a**2*b**
7))
```

GIAC/XCAS [A] time = 0.210147, size = 490, normalized size = 2.83

$$-\frac{5\,\left(b_1c^3 - bc^2c_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{16\,(b^6 - 3\,ab^4c + 3\,a^2b^2c^2 - a^3c^3)\sqrt{-b^2+ac}} - \frac{15\,b_1c^5x^5 - 15\,bc^4c_1x^5 + 75\,bb_1c^4x^4 - 75\,b^2c^3c_1x^4 + 110\,b^2b_1c^3x^3 + 40\,ab_1c^4x^3 - 110\,b^3c^2c_1x^3 - 40\,abc^3c_1x^3 + 30\,b^3b_1c^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^4,x, algorithm="giac")
```

```
[Out] -5/16*(b1*c^3 - b*c^2*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^
6 - 3*a*b^4*c + 3*a^2*b^2*c^2 - a^3*c^3)*sqrt(-b^2 + a*c)) - 1/48
*(15*b1*c^5*x^5 - 15*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*
c1*x^4 + 110*b^2*b1*c^3*x^3 + 40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^
3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^2*x^2 + 120*a*b*b1*c^3*x^2 -
30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b^4*b1*c*x + 54*a*b^2
*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c*c1*x - 33*
a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2*a
*b^4*c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^
2*b^2*c^2 - a^3*c^3)*(c*x^2 + 2*b*x + a)^3)
```

3.198 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx$

Optimal. Leaf size=169

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)} - \frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}\,{}_2F_1\left(1-n,n;2-n;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(1-n)\sqrt{b^2-ac}}$$

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(
-((b - Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b
*x + c*x^2)^(1 - n)*Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[
b^2 - a*c] + c*x)/(2*Sqrt[b^2 - a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1
 - n))
```

Rubi [A] time = 0.19529, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)} - \frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}\,{}_2F_1\left(1-n,n;2-n;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(1-n)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(
-((b - Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b
*x + c*x^2)^(1 - n)*Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[
b^2 - a*c] + c*x)/(2*Sqrt[b^2 - a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1
 - n))
```

Rubi in Sympy [A] time = 7.10722, size = 131, normalized size = 0.78

$$\frac{c_1\left(a+2bx+cx^2\right)^{-n+1}}{2c\left(-n+1\right)} + \frac{\left(\frac{-\frac{b}{2}-\frac{cx}{2}+\frac{\sqrt{-ac+b^2}}{2}}{\sqrt{-ac+b^2}}\right)^{n-1}\left(bc_1-b_1c\right)\left(a+2bx+cx^2\right)^{-n+1}\,{}_2F_1\left(\begin{matrix} n, -n+1 \\ -n+2 \end{matrix}\middle| \frac{\frac{b}{2}+\frac{cx}{2}+\frac{\sqrt{-ac+b^2}}{2}}{\sqrt{-ac+b^2}}\right)}{2c\left(-n+1\right)\sqrt{-ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
```

```
[Out] c1*(a + 2*b*x + c*x**2)**(-n + 1)/(2*c*(-n + 1)) + ((-b/2 - c*x/2
 + sqrt(-a*c + b**2)/2)/sqrt(-a*c + b**2))**(n - 1)*(b*c1 - b1*c)
*(a + 2*b*x + c*x**2)**(-n + 1)*hyper((n, -n + 1), (-n + 2,), (b/
2 + c*x/2 + sqrt(-a*c + b**2)/2)/sqrt(-a*c + b**2))/(2*c*(-n + 1)
*sqrt(-a*c + b**2))
```

Mathematica [C] time = 2.13625, size = 374, normalized size = 2.21

$$\frac{1}{2}(a+x(2b+cx))^{-n}\left(-\frac{3ac1x^2F_1\left(2;n,n;3;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-ac}-b}\right)}{nx\left(\left(\sqrt{b^2-ac}+b\right)F_1\left(3;n,n+1;4;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-ac}-b}\right)+\left(b-\sqrt{b^2-ac}\right)F_1\left(3;n+1,n;4;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-a}}\right.\right.}-\frac{b12^{1-n}\left(-\sqrt{b^2-ac}+b+cx\right)\left(\frac{\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^n{}_2F_1\left(1-n,n;2-n;\frac{-b-cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(n-1)}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] ((-3*a*c1*x^2*AppellF1[2, n, n, 3, -((c*x)/(b + Sqrt[b^2 - a*c]))
, (c*x)/(-b + Sqrt[b^2 - a*c])])/(-3*a*AppellF1[2, n, n, 3, -((c*
x)/(b + Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] + n*x*((
b + Sqrt[b^2 - a*c])*AppellF1[3, n, 1 + n, 4, -((c*x)/(b + Sqrt[b
^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] + (b - Sqrt[b^2 - a*c]
)*AppellF1[3, 1 + n, n, 4, -((c*x)/(b + Sqrt[b^2 - a*c])), (c*x)/
(-b + Sqrt[b^2 - a*c])])) - (2^(1 - n)*b1*(b - Sqrt[b^2 - a*c] +
c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hypergeometr
ic2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 -
 a*c])])/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \frac{c1x+b1}{(cx^2+2\,bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

```
[Out] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^(-n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{c_1x + b_1}{(cx^2 + 2\,bx + a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n,x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c_1x + b_1}{(cx^2 + 2\,bx + a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n,x, algorithm="giac")

[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

3.199 $\int \frac{x}{3+6x+2x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

```
[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 +
 Sqrt[3] + 2*x])/4
```

Rubi [A] time = 0.0414951, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 +
 Sqrt[3] + 2*x])/4
```

Rubi in Sympy [A] time = 2.03755, size = 49, normalized size = 1.

$$-\frac{\sqrt{3}\left(-\sqrt{3}+3\right)\log\left(2x-\sqrt{3}+3\right)}{12}+\frac{\sqrt{3}\left(\sqrt{3}+3\right)\log\left(2x+\sqrt{3}+3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2*x**2+6*x+3),x)
```

```
[Out] -sqrt(3)*(-sqrt(3) + 3)*log(2*x - sqrt(3) + 3)/12 + sqrt(3)*(sqrt
(3) + 3)*log(2*x + sqrt(3) + 3)/12
```

Mathematica [A] time = 0.0332802, size = 44, normalized size = 0.9

$$\frac{1}{4}\left(\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)-\left(\sqrt{3}-1\right)\log\left(-2x+\sqrt{3}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] (-((-1 + Sqrt[3])*Log[-3 + Sqrt[3] - 2*x]) + (1 + Sqrt[3])*Log[3
+ Sqrt[3] + 2*x])/4
```

Maple [A] time = 0.003, size = 31, normalized size = 0.6

$$\frac{\ln\left(2x^2+6x+3\right)}{4}+\frac{\sqrt{3}}{2}\mathit{Artanh}\left(\frac{(6+4x)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2*x^2+6*x+3),x)
```

```
[Out] 1/4*ln(2*x^2+6*x+3)+1/2*3^(1/2)*arctanh(1/6*(6+4*x)*3^(1/2))
```

Maxima [A] time = 1.58415, size = 55, normalized size = 1.12

$$-\frac{1}{4}\sqrt{3}\log\left(\frac{2x-\sqrt{3}+3}{2x+\sqrt{3}+3}\right)+\frac{1}{4}\log\left(2x^2+6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*x^2 + 6*x + 3),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) + 1/4*l
og(2*x^2 + 6*x + 3)
```

Fricas [A] time = 0.208354, size = 70, normalized size = 1.43

$$\frac{1}{4}\sqrt{3}\log\left(\frac{2x^2+\sqrt{3}(2x+3)+6x+6}{2x^2+6x+3}\right)+\frac{1}{4}\log\left(2x^2+6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*x^2 + 6*x + 3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*
x + 3)) + 1/4*log(2*x^2 + 6*x + 3)
```

Sympy [A] time = 0.099945, size = 46, normalized size = 0.94

$$\left(-\frac{\sqrt{3}}{4}+\frac{1}{4}\right)\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)+\left(\frac{1}{4}+\frac{\sqrt{3}}{4}\right)\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*x**2+6*x+3),x)
```

```
[Out] (-sqrt(3)/4 + 1/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*l
og(x + sqrt(3)/2 + 3/2)
```

GIAC/XCAS [A] time = 0.199466, size = 62, normalized size = 1.27

$$-\frac{1}{4}\sqrt{3}\ln\left(\frac{\left|4x-2\sqrt{3}+6\right|}{\left|4x+2\sqrt{3}+6\right|}\right)+\frac{1}{4}\ln\left(\left|2x^2+6x+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*x^2 + 6*x + 3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*ln(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)
) + 1/4*ln(abs(2*x^2 + 6*x + 3))
```

3.200 $\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx$

Optimal. Leaf size=61

$$-\frac{2x+3}{2(2x^2+6x+3)}+\frac{4x+5}{4(2x^2+6x+3)^2}+\frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2
)) + ArcTanh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.051684, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{2x+3}{2(2x^2+6x+3)}+\frac{4x+5}{4(2x^2+6x+3)^2}+\frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2
)) + ArcTanh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi in Sympy [A] time = 2.68301, size = 53, normalized size = 0.87

$$-\frac{4x+6}{4(2x^2+6x+3)}+\frac{24x+30}{24(2x^2+6x+3)^2}+\frac{\sqrt{3}\,\text{atanh}\left(\sqrt{3}\left(\frac{2x}{3}+1\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
```

```
[Out] -(4*x + 6)/(4*(2*x**2 + 6*x + 3)) + (24*x + 30)/(24*(2*x**2 + 6*x
 + 3)**2) + sqrt(3)*atanh(sqrt(3)*(2*x/3 + 1))/3
```

Mathematica [A] time = 0.0733273, size = 70, normalized size = 1.15

$$\frac{1}{12}\left(-\frac{3(8x^3+36x^2+44x+13)}{(2x^2+6x+3)^2}-2\sqrt{3}\log\left(-2x+\sqrt{3}-3\right)+2\sqrt{3}\log\left(2x+\sqrt{3}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] ((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3
]*Log[-3 + Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

Maple [A] time = 0.003, size = 56, normalized size = 0.9

$$-\frac{-24\,x-30}{24\,\left(2\,x^{2}+6\,x+3\right)^{2}}-\frac{6+4\,x}{8\,x^{2}+24\,x+12}+\frac{\sqrt{3}}{3}\mathit{Artanh}\left(\frac{\left(6+4\,x\right)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+2*x)/(2*x^2+6*x+3)^3,x)
```

```
[Out] -1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(6+4*x)/(2*x^2+6*x+3)+1/3*3^
(1/2)*arctanh(1/6*(6+4*x)*3^(1/2))
```

Maxima [A] time = 1.78898, size = 90, normalized size = 1.48

$$-\frac{1}{6}\,\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right)-\frac{8\,x^{3}+36\,x^{2}+44\,x+13}{4\,\left(4\,x^{4}+24\,x^{3}+48\,x^{2}+36\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 3)/(2*x^2 + 6*x + 3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) - 1/4*(
8*x^3 + 36*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

Fricas [A] time = 0.199676, size = 138, normalized size = 2.26

$$\frac{\sqrt{3}\left(2\,\left(4\,x^{4}+24\,x^{3}+48\,x^{2}+36\,x+9\right)\,\log\left(\frac{2\,\sqrt{3}\left(x^{2}+3\,x+3\right)+6\,x+9}{2\,x^{2}+6\,x+3}\right)-\sqrt{3}\left(8\,x^{3}+36\,x^{2}+44\,x+13\right)\right)}{12\,\left(4\,x^{4}+24\,x^{3}+48\,x^{2}+36\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 3)/(2*x^2 + 6*x + 3)^3,x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(2*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*log((2*sqrt(
3)*(x^2 + 3*x + 3) + 6*x + 9)/(2*x^2 + 6*x + 3)) - sqrt(3)*(8*x^3
 + 36*x^2 + 44*x + 13))/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

Sympy [A] time = 0.190143, size = 75, normalized size = 1.23

$$-\frac{8x^3+36x^2+44x+13}{16x^4+96x^3+192x^2+144x+36}-\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
```

```
[Out] -(8*x**3 + 36*x**2 + 44*x + 13)/(16*x**4 + 96*x**3 + 192*x**2 + 1
44*x + 36) - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x +
 sqrt(3)/2 + 3/2)/6
```

GIAC/XCAS [A] time = 0.202728, size = 82, normalized size = 1.34

$$-\frac{1}{6}\sqrt{3}\ln\left(\frac{\left|4x-2\sqrt{3}+6\right|}{\left|4x+2\sqrt{3}+6\right|}\right)-\frac{8x^3+36x^2+44x+13}{4\left(2x^2+6x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 3)/(2*x^2 + 6*x + 3)^3,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*ln(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)
) - 1/4*(8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2
```

3.201 $\int \frac{-1+x}{(4+5x+x^2)^2} \, dx$

Optimal. Leaf size=36

$$\frac{7x+13}{9(x^2+5x+4)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

```
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x
])/27
```

Rubi [A] time = 0.0220206, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{7x+13}{9(x^2+5x+4)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x
])/27
```

Rubi in Sympy [A] time = 2.00344, size = 31, normalized size = 0.86

$$\frac{7x+13}{9(x^2+5x+4)}+\frac{7\log(x+1)}{27}-\frac{7\log(x+4)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)/(x**2+5*x+4)**2,x)
```

```
[Out] (7*x + 13)/(9*(x**2 + 5*x + 4)) + 7*log(x + 1)/27 - 7*log(x + 4)/
27
```

Mathematica [A] time = 0.025965, size = 33, normalized size = 0.92

$$\frac{1}{27}\left(\frac{21x+39}{x^2+5x+4}+7\log(x+1)-7\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] ((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27
```

Maple [A] time = 0.015, size = 28, normalized size = 0.8

$$\frac{2}{9+9x}+\frac{7\ln(1+x)}{27}+\frac{5}{36+9x}-\frac{7\ln(4+x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(x^2+5*x+4)^2,x)
```

```
[Out] 2/9/(1+x)+7/27*ln(1+x)+5/9/(4+x)-7/27*ln(4+x)
```

Maxima [A] time = 1.34371, size = 41, normalized size = 1.14

$$\frac{7x+13}{9(x^2+5x+4)}-\frac{7}{27}\log(x+4)+\frac{7}{27}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 + 5*x + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(x + 4) + 7/27*log(x + 1
)
```

Fricas [A] time = 0.196879, size = 61, normalized size = 1.69

$$-\frac{7\left(x^2+5x+4\right)\log(x+4)-7\left(x^2+5x+4\right)\log(x+1)-21x-39}{27(x^2+5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 + 5*x + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(7*(x^2 + 5*x + 4)*log(x + 4) - 7*(x^2 + 5*x + 4)*log(x + 1
) - 21*x - 39)/(x^2 + 5*x + 4)
```

Sympy [A] time = 0.134272, size = 31, normalized size = 0.86

$$\frac{7x+13}{9x^2+45x+36}+\frac{7\log(x+1)}{27}-\frac{7\log(x+4)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(x**2+5*x+4)**2,x)
```

```
[Out] (7*x + 13)/(9*x**2 + 45*x + 36) + 7*log(x + 1)/27 - 7*log(x + 4)/
27
```

GIAC/XCAS [A] time = 0.200359, size = 43, normalized size = 1.19

$$\frac{7\,x+13}{9\,(x^2+5\,x+4)}-\frac{7}{27}\ln(|x+4|)+\frac{7}{27}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 + 5*x + 4)^2,x, algorithm="giac")
```

```
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*ln(abs(x + 4)) + 7/27*ln(ab
s(x + 1))
```

3.202 $\int \frac{1}{(2+3x+x^2)^5}\,dx$

Optimal. Leaf size=87

$$\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}-\frac{2x+3}{4\left(x^2+3x+2\right)^4}+70\log(x+1)-70\log(x+2)$$

```
[Out] -(3 + 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^
2)^3) - (35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2
+ 3*x + x^2) + 70*Log[1 + x] - 70*Log[2 + x]
```

Rubi [A] time = 0.0488873, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}-\frac{2x+3}{4\left(x^2+3x+2\right)^4}+70\log(x+1)-70\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] -(3 + 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^
2)^3) - (35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2
+ 3*x + x^2) + 70*Log[1 + x] - 70*Log[2 + x]
```

Rubi in Sympy [A] time = 1.71516, size = 76, normalized size = 0.87

$$\frac{35\,(2x+3)}{x^2+3x+2}-\frac{35\,(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7\,(2x+3)}{6\left(x^2+3x+2\right)^3}-\frac{2x+3}{4\left(x^2+3x+2\right)^4}+70\log\left(x+1\right)-70\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+3*x+2)**5,x)
```

```
[Out] 35*(2*x + 3)/(x**2 + 3*x + 2) - 35*(2*x + 3)/(6*(x**2 + 3*x + 2)*
*2) + 7*(2*x + 3)/(6*(x**2 + 3*x + 2)**3) - (2*x + 3)/(4*(x**2 +
3*x + 2)**4) + 70*log(x + 1) - 70*log(x + 2)
```

Mathematica [A] time = 0.0401726, size = 87, normalized size = 1.

$$\frac{-2x-3}{4\left(x^2+3x+2\right)^4}+\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}+70\log(x+1)-70\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] (-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^
2)^3) - (35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2
+ 3*x + x^2) + 70*Log[1 + x] - 70*Log[2 + x]
```

Maple [A] time = 0.017, size = 70, normalized size = 0.8

$$\frac{1}{4\ (2+x)^{4}}+\frac{5}{3\ (2+x)^{3}}+\frac{15}{2\ (2+x)^{2}}+35\ (2+x)^{-1}-70\ \ln(2+x) -\frac{1}{4\ (1+x)^{4}}+\frac{5}{3\ (1+x)^{3}}-\frac{15}{2\ (1+x)^{2}}+35\ (1+x)^{-1}+70\ \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+3*x+2)^5,x)
```

```
[Out] 1/4/(2+x)^4+5/3/(2+x)^3+15/2/(2+x)^2+35/(2+x)-70*ln(2+x)-1/4/(1+x
)^4+5/3/(1+x)^3-15/2/(1+x)^2+35/(1+x)+70*ln(1+x)
```

Maxima [A] time = 1.32784, size = 122, normalized size = 1.4

$$\frac{840\,x^{7}+8820\,x^{6}+38920\,x^{5}+93450\,x^{4}+131768\,x^{3}+109116\,x^{2}+49176\,x+9315}{12\,(x^{8}+12\,x^{7}+62\,x^{6}+180\,x^{5}+321\,x^{4}+360\,x^{3}+248\,x^{2}+96\,x+16)} -70\ \log(x+2)+70\ \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 2)^(-5),x, algorithm="maxima")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 1
09116*x^2 + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 32
1*x^4 + 360*x^3 + 248*x^2 + 96*x + 16) - 70*log(x + 2) + 70*log(x
 + 1)
```

Fricas [A] time = 0.197988, size = 223, normalized size = 2.56

$$\frac{840\,x^{7}+8820\,x^{6}+38920\,x^{5}+93450\,x^{4}+131768\,x^{3}+109116\,x^{2}-840\,\left(x^{8}+12\,x^{7}+62\,x^{6}+180\,x^{5}+321\,x^{4}+360\,x^{3}+248\,\right.}{12\,(x^{8}+12\,x^{7}+62\,x^{6}+180\,x^{5}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 2)^(-5),x, algorithm="fricas")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 1
09116*x^2 - 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*
x^3 + 248*x^2 + 96*x + 16)*log(x + 2) + 840*(x^8 + 12*x^7 + 62*x^
6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1)
 + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 3
60*x^3 + 248*x^2 + 96*x + 16)
```

Sympy [A] time = 0.281554, size = 88, normalized size = 1.01

$$\frac{840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315}{12x^8 + 144x^7 + 744x^6 + 2160x^5 + 3852x^4 + 4320x^3 + 2976x^2 + 1152x + 192} + 70\log(x+1) - 70\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+3*x+2)**5,x)
```

```
[Out] (840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 1
09116*x**2 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 216
0*x**5 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*l
og(x + 1) - 70*log(x + 2)
```

GIAC/XCAS [A] time = 0.198758, size = 84, normalized size = 0.97

$$\frac{840\,x^7 + 8820\,x^6 + 38920\,x^5 + 93450\,x^4 + 131768\,x^3 + 109116\,x^2 + 49176\,x + 9315}{12\,(x^2 + 3\,x + 2)^4} - 70\ln(|x+2|) + 70\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 2)^(-5),x, algorithm="giac")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 1
09116*x^2 + 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*ln(abs(x + 2))
 + 70*ln(abs(x + 1))
```

3.203 $\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{2-3x}{35x^2(2x^2-6x+7)}-\frac{1}{490x^2}-\frac{40\log(2x^2-6x+7)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

```
[Out] -1/(490*x^2) - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2))
 - (234*ArcTan[(3 - 2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/
2401 - (40*Log[7 - 6*x + 2*x^2])/2401
```

Rubi [A] time = 0.124158, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{2-3x}{35x^2(2x^2-6x+7)}-\frac{1}{490x^2}-\frac{40\log(2x^2-6x+7)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] -1/(490*x^2) - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2))
 - (234*ArcTan[(3 - 2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/
2401 - (40*Log[7 - 6*x + 2*x^2])/2401
```

Rubi in Sympy [A] time = 7.7816, size = 76, normalized size = 0.94

$$\frac{80\log(x)}{2401}-\frac{40\log(2x^2-6x+7)}{2401}+\frac{234\sqrt{5}\,\text{atan}\left(\sqrt{5}\left(\frac{2x}{5}-\frac{3}{5}\right)\right)}{60025}-\frac{69}{1715x}-\frac{-12x+8}{140x^2(2x^2-6x+7)}-\frac{1}{490x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

```
[Out] 80*log(x)/2401 - 40*log(2*x**2 - 6*x + 7)/2401 + 234*sqrt(5)*atan
(sqrt(5)*(2*x/5 - 3/5))/60025 - 69/(1715*x) - (-12*x + 8)/(140*x*
*2*(2*x**2 - 6*x + 7)) - 1/(490*x**2)
```

Mathematica [A] time = 0.0536967, size = 70, normalized size = 0.86

$$\frac{-\frac{140(9x-41)}{2x^2-6x+7}-\frac{1225}{x^2}-2000\log\left(2x^2-6x+7\right)-\frac{4200}{x}+4000\log(x)+468\sqrt{5}\tan^{-1}\left(\frac{2x-3}{\sqrt{5}}\right)}{120050}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] (-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*S
qrt[5]*ArcTan[(-3 + 2*x)/Sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*
x + 2*x^2])/120050
```

Maple [A] time = 0.016, size = 62, normalized size = 0.8

$$-\frac{1}{98\,x^2}-\frac{12}{343\,x}+\frac{80\,\ln(x)}{2401}-\frac{4}{2401}\left(\frac{63\,x}{20}-\frac{287}{20}\right)\left(x^2-3\,x+\frac{7}{2}\right)^{-1}-\frac{40\,\ln\left(4\,x^2-12\,x+14\right)}{2401}+\frac{234\,\sqrt{5}}{60025}\arctan\left(\frac{(-12+8\,x)\sqrt{5}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(2*x^2-6*x+7)^2,x)
```

```
[Out] -1/98/x^2-12/343/x+80/2401*ln(x)-4/2401*(63/20*x-287/20)/(x^2-3*x
+7/2)-40/2401*ln(4*x^2-12*x+14)+234/60025*5^(1/2)*arctan(1/20*(-1
2+8*x)*5^(1/2))
```

Maxima [A] time = 1.50741, size = 93, normalized size = 1.15

$$\frac{234}{60025}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(2\,x-3)\right)-\frac{276\,x^3-814\,x^2+630\,x+245}{3430\,(2\,x^4-6\,x^3+7\,x^2)}-\frac{40}{2401}\log\left(2\,x^2-6\,x+7\right)+\frac{80}{2401}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 - 6*x + 7)^2*x^3),x, algorithm="maxima")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3
 - 814*x^2 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2
*x^2 - 6*x + 7) + 80/2401*log(x)
```

Fricas [A] time = 0.21385, size = 173, normalized size = 2.14

$$-\frac{\sqrt{5}\left(400\,\sqrt{5}\left(2\,x^{4}-6\,x^{3}+7\,x^{2}\right)\,\log\left(2\,x^{2}-6\,x+7\right)-800\,\sqrt{5}\left(2\,x^{4}-6\,x^{3}+7\,x^{2}\right)\,\log\left(x\right)-468\,\left(2\,x^{4}-6\,x^{3}+7\,x^{2}\right)\,\arctan\left(\right.\right.}{120050\,\left(2\,x^{4}-6\,x^{3}+7\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 - 6*x + 7)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/120050*sqrt(5)*(400*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2
- 6*x + 7) - 800*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*log(x) - 468*(2*
x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(5)*(2*x - 3)) + 7*sqrt(5)*(2
76*x^3 - 814*x^2 + 630*x + 245))/(2*x^4 - 6*x^3 + 7*x^2)
```

Sympy [A] time = 0.284866, size = 80, normalized size = 0.99

$$\frac{80\log\left(x\right)}{2401}-\frac{40\log\left(x^{2}-3x+\frac{7}{2}\right)}{2401}+\frac{234\sqrt{5}\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}-\frac{3\sqrt{5}}{5}\right)}{60025}-\frac{276x^{3}-814x^{2}+630x+245}{6860x^{4}-20580x^{3}+24010x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

```
[Out] 80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan
(2*sqrt(5)*x/5 - 3*sqrt(5)/5)/60025 - (276*x**3 - 814*x**2 + 630*
x + 245)/(6860*x**4 - 20580*x**3 + 24010*x**2)
```

GIAC/XCAS [A] time = 0.202639, size = 90, normalized size = 1.11

$$\frac{234}{60025}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(2\,x-3)\right)-\frac{276\,x^{3}-814\,x^{2}+630\,x+245}{3430\,(2\,x^{2}-6\,x+7)x^{2}}-\frac{40}{2401}\ln\left(2\,x^{2}-6\,x+7\right)+\frac{80}{2401}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 - 6*x + 7)^2*x^3),x, algorithm="giac")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3
 - 814*x^2 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*ln(2*
x^2 - 6*x + 7) + 80/2401*ln(abs(x))
```

3.204 $\int \frac{x^9}{(2+3x+x^2)^5}\,dx$

Optimal. Leaf size=104

$$-\frac{(1593x+2206)x^2}{2(x^2+3x+2)}+\frac{(3x+4)x^8}{4(x^2+3x+2)^4}-\frac{(81x+110)x^6}{12(x^2+3x+2)^3}+\frac{(135x+184)x^4}{2(x^2+3x+2)^2}+735x-1471\log(x+1)+1472\log(x+2)$$

```
[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x)
)/(12*(2 + 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)
^2) - (x^2*(2206 + 1593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x]
 + 1472*Log[2 + x]
```

Rubi [A] time = 0.198087, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{(1593x+2206)x^2}{2(x^2+3x+2)}+\frac{(3x+4)x^8}{4(x^2+3x+2)^4}-\frac{(81x+110)x^6}{12(x^2+3x+2)^3}+\frac{(135x+184)x^4}{2(x^2+3x+2)^2}+735x-1471\log(x+1)+1472\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x)
)/(12*(2 + 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)
^2) - (x^2*(2206 + 1593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x]
 + 1472*Log[2 + x]
```

Rubi in Sympy [A] time = 12.735, size = 95, normalized size = 0.91

$$\frac{x^8(3x+4)}{4(x^2+3x+2)^4}-\frac{x^6(81x+110)}{12(x^2+3x+2)^3}+\frac{x^4(1620x+2208)}{24(x^2+3x+2)^2}-\frac{x^2(19116x+26472)}{24(x^2+3x+2)}+735x-1471\log(x+1)+1472\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**2+3*x+2)**5,x)
```

```
[Out] x**8*(3*x + 4)/(4*(x**2 + 3*x + 2)**4) - x**6*(81*x + 110)/(12*(x
**2 + 3*x + 2)**3) + x**4*(1620*x + 2208)/(24*(x**2 + 3*x + 2)**2
) - x**2*(19116*x + 26472)/(24*(x**2 + 3*x + 2)) + 735*x - 1471*l
og(x + 1) + 1472*log(x + 2)
```

Mathematica [A] time = 0.036145, size = 87, normalized size = 0.84

$$\frac{3(456x+451)}{4\left(x^2+3x+2\right)^2}-\frac{2(729x+1114)}{x^2+3x+2}+\frac{1998x+415}{12\left(x^2+3x+2\right)^3}+\frac{513x+514}{4\left(x^2+3x+2\right)^4}-1471\log(x+1)+1472\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] (514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x
 + x^2)^3) + (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 +
 729*x))/(2 + 3*x + x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Maple [A] time = 0.014, size = 70, normalized size = 0.7

$$-128\,(2+x)^{-4}-\frac{256}{3\,(2+x)^3}-384\,(2+x)^{-2}-1024\,(2+x)^{-1}+1472\,\ln(2+x)$$
$$+\frac{1}{4\,(1+x)^4}-\frac{14}{3\,(1+x)^3}+48\,(1+x)^{-2}-434\,(1+x)^{-1}-1471\,\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^2+3*x+2)^5,x)
```

```
[Out] -128/(2+x)^4-256/3/(2+x)^3-384/(2+x)^2-1024/(2+x)+1472*ln(2+x)+1/
4/(1+x)^4-14/3/(1+x)^3+48/(1+x)^2-434/(1+x)-1471*ln(1+x)
```

Maxima [A] time = 1.34094, size = 122, normalized size = 1.17

$$-\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2+1030560\,x+195280}{12\left(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16\right)}$$
$$+1472\,\log(x+2)-1471\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^2 + 3*x + 2)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 275940
0*x^3 + 2286008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6
+ 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x
 + 2) - 1471*log(x + 1)
```

Fricas [A] time = 0.22114, size = 223, normalized size = 2.14

$$-\frac{17496\,x^{7} + 184200\,x^{6} + 813888\,x^{5} + 1955853\,x^{4} + 2759400\,x^{3} + 2286008\,x^{2} - 17664\,\left(x^{8} + 12\,x^{7} + 62\,x^{6} + 180\,x^{5} + 321\,x^{4}}{12\,(x^{8} + 12\,x^{7} + 62\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^2 + 3*x + 2)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 275940
0*x^3 + 2286008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 32
1*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 +
12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x +
16)*log(x + 1) + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180
*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

Sympy [A] time = 0.281353, size = 88, normalized size = 0.85

$$-\frac{17496x^{7} + 184200x^{6} + 813888x^{5} + 1955853x^{4} + 2759400x^{3} + 2286008x^{2} + 1030560x + 195280}{12x^{8} + 144x^{7} + 744x^{6} + 2160x^{5} + 3852x^{4} + 4320x^{3} + 2976x^{2} + 1152x + 192} - 1471\log{(x + 1)} + 1472\log{(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**2+3*x+2)**5,x)
```

```
[Out] -(17496*x**7 + 184200*x**6 + 813888*x**5 + 1955853*x**4 + 2759400
*x**3 + 2286008*x**2 + 1030560*x + 195280)/(12*x**8 + 144*x**7 +
744*x**6 + 2160*x**5 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x
 + 192) - 1471*log(x + 1) + 1472*log(x + 2)
```

GIAC/XCAS [A] time = 0.20519, size = 84, normalized size = 0.81

$$-\frac{17496\,x^{7} + 184200\,x^{6} + 813888\,x^{5} + 1955853\,x^{4} + 2759400\,x^{3} + 2286008\,x^{2} + 1030560\,x + 195280}{12\,(x + 2)^{4}(x + 1)^{4}} + 1472\,\ln\left(|x + 2|\right) - 1471\,\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^2 + 3*x + 2)^5,x, algorithm="giac")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 275940
0*x^3 + 2286008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) +
 1472*ln(abs(x + 2)) - 1471*ln(abs(x + 1))
```

3.205 $\int \frac{(1+2x)^2}{(3+5x+2x^2)^5} \, dx$

Optimal. Leaf size=102

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3(2x^2+5x+3)^2}+\frac{62x+73}{3(2x^2+5x+3)^3}+\frac{(2x+1)(6x+7)}{4(2x^2+5x+3)^4}+2480\log(x+1)-2480\log(2x+3)$$

```
[Out] ((1 + 2*x)*(7 + 6*x))/(4*(3 + 5*x + 2*x^2)^4) + (73 + 62*x)/(3*(3
 + 5*x + 2*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (6
20*(5 + 4*x))/(3 + 5*x + 2*x^2) + 2480*Log[1 + x] - 2480*Log[3 +
2*x]
```

Rubi [A] time = 0.0813586, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3(2x^2+5x+3)^2}+\frac{62x+73}{3(2x^2+5x+3)^3}+\frac{(2x+1)(6x+7)}{4(2x^2+5x+3)^4}+2480\log(x+1)-2480\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] ((1 + 2*x)*(7 + 6*x))/(4*(3 + 5*x + 2*x^2)^4) + (73 + 62*x)/(3*(3
 + 5*x + 2*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (6
20*(5 + 4*x))/(3 + 5*x + 2*x^2) + 2480*Log[1 + x] - 2480*Log[3 +
2*x]
```

Rubi in Sympy [A] time = 5.83344, size = 90, normalized size = 0.88

$$\frac{(2x+1)(6x+7)}{4(2x^2+5x+3)^4}+\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3(2x^2+5x+3)^2}+\frac{248x+292}{12(2x^2+5x+3)^3}+2480\log(x+1)-2480\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

```
[Out] (2*x + 1)*(6*x + 7)/(4*(2*x**2 + 5*x + 3)**4) + 620*(4*x + 5)/(2*
x**2 + 5*x + 3) - 155*(4*x + 5)/(3*(2*x**2 + 5*x + 3)**2) + (248*
x + 292)/(12*(2*x**2 + 5*x + 3)**3) + 2480*log(x + 1) - 2480*log(
2*x + 3)
```

Mathematica [A] time = 0.0807416, size = 99, normalized size = 0.97

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3(2x^2+5x+3)^2}+\frac{31(4x+5)}{6(2x^2+5x+3)^3}-\frac{10x+11}{4(2x^2+5x+3)^4}+2480\log(2(x+1))-2480\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] -(11 + 10*x)/(4*(3 + 5*x + 2*x^2)^4) + (31*(5 + 4*x))/(6*(3 + 5*x
 + 2*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5
+ 4*x))/(3 + 5*x + 2*x^2) + 2480*Log[2*(1 + x)] - 2480*Log[3 + 2*
x]
```

Maple [A] time = 0.016, size = 80, normalized size = 0.8

$$16\,(3+2\,x)^{-4}+\frac{256}{3\,(3+2\,x)^3}+328\,(3+2\,x)^{-2}+1360\,(3+2\,x)^{-1}-2480\,\ln(3+2\,x)-\frac{1}{4\,(1+x)^4}+\frac{14}{3\,(1+x)^3}-52\,(1+x)^{-2}+560\,(1+x)^{-1}+2480\,\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)^2/(2*x^2+5*x+3)^5,x)
```

```
[Out] 16/(3+2*x)^4+256/3/(3+2*x)^3+328/(3+2*x)^2+1360/(3+2*x)-2480*ln(3
+2*x)-1/4/(1+x)^4+14/3/(1+x)^3-52/(1+x)^2+560/(1+x)+2480*ln(1+x)
```

Maxima [A] time = 1.3506, size = 127, normalized size = 1.25

$$\frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\,(16\,x^8+160\,x^7+696\,x^6+1720\,x^5+2641\,x^4+2580\,x^3+1566\,x^2+540\,x+81)}-2480\,\log(2\,x+3)+2480\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)^2/(2*x^2 + 5*x + 3)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 194
95776*x^3 + 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7
+ 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 8
```

```
1) - 2480*log(2*x + 3) + 2480*log(x + 1)
```

Fricas [A] time = 0.212014, size = 234, normalized size = 2.29

$$\frac{238080\,x^7 + 2083200\,x^6 + 7757440\,x^5 + 15934000\,x^4 + 19495776\,x^3 + 14209160\,x^2 - 29760\,\left(16\,x^8 + 160\,x^7 + 696\,x^6 + 1720\,x^5 + \cdots\right)}{12\,(16\,x^8 + 1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)^2/(2*x^2 + 5*x + 3)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 194
95776*x^3 + 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 17
20*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3
) + 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 + 258
0*x^3 + 1566*x^2 + 540*x + 81)*log(x + 1) + 5712464*x + 977397)/(
16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 156
6*x^2 + 540*x + 81)
```

Sympy [A] time = 0.327257, size = 90, normalized size = 0.88

$$\frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{192x^8 + 1920x^7 + 8352x^6 + 20640x^5 + 31692x^4 + 30960x^3 + 18792x^2 + 6480x + 972} + 2480\log\left(x+1\right) - 2480\log\left(x+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

```
[Out] (238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 1949
5776*x**3 + 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*
x**7 + 8352*x**6 + 20640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x
**2 + 6480*x + 972) + 2480*log(x + 1) - 2480*log(x + 3/2)
```

GIAC/XCAS [A] time = 0.202539, size = 89, normalized size = 0.87

$$\frac{238080\,x^7 + 2083200\,x^6 + 7757440\,x^5 + 15934000\,x^4 + 19495776\,x^3 + 14209160\,x^2 + 5712464\,x + 977397}{12\,(2\,x^2 + 5\,x + 3)^4} - 2480\ln\left(|2\,x + 3|\right) + 2480\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)^2/(2*x^2 + 5*x + 3)^5,x, algorithm="giac")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 194
95776*x^3 + 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^
4 - 2480*ln(abs(2*x + 3)) + 2480*ln(abs(x + 1))
```

3.206 $\int \frac{(a-bx^2)^3}{x^7} dx$

Optimal. Leaf size=40

$$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Rubi [A] time = 0.049783, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Rubi in Sympy [A] time = 4.59869, size = 41, normalized size = 1.02

$$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-\frac{b^3\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**3/x**7,x)
```

```
[Out] -a**3/(6*x**6) + 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) - b**3*log
(x**2)/2
```

Mathematica [A] time = 0.00776439, size = 40, normalized size = 1.

$$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Maple [A] time = 0.01, size = 35, normalized size = 0.9

$$-\frac{a^{3}}{6\,x^{6}}+\frac{3\,a^{2}b}{4\,x^{4}}-\frac{3\,b^{2}a}{2\,x^{2}}-b^{3}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

Maxima [A] time = 1.34949, size = 53, normalized size = 1.32

$$-\frac{1}{2}\,b^{3}\log\left(x^{2}\right)-\frac{18\,ab^{2}x^{4}-9\,a^{2}bx^{2}+2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - a)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

Fricas [A] time = 0.217696, size = 53, normalized size = 1.32

$$-\frac{12\,b^{3}x^{6}\log\left(x\right)+18\,ab^{2}x^{4}-9\,a^{2}bx^{2}+2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - a)^3/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^3*x^6*log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^
6
```

Sympy [A] time = 0.740808, size = 37, normalized size = 0.92

$$-b^{3}\log\left(x\right)-\frac{2a^{3}-9a^{2}bx^{2}+18ab^{2}x^{4}}{12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**3/x**7,x)
```

```
[Out] -b**3*log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6
)
```

GIAC/XCAS [A] time = 0.201529, size = 63, normalized size = 1.58

$$-\frac{1}{2}\, b^3 \ln\left(x^2\right) + \frac{11\, b^3 x^6 - 18\, a b^2 x^4 + 9\, a^2 b x^2 - 2\, a^3}{12\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 - a)^3/x^7,x, algorithm="giac")
```

```
[Out] -1/2*b^3*ln(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2
- 2*a^3)/x^6
```

3.207 $\int \frac{x^{13}}{(a^4+x^4)^5}\,dx$

Optimal. Leaf size=83

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

```
[Out] -x^10/(16*(a^4 + x^4)^4) - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(
128*(a^4 + x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^
2/a^2])/(256*a^6)
```

Rubi [A] time = 0.0869423, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^13/(a^4 + x^4)^5,x]
```

```
[Out] -x^10/(16*(a^4 + x^4)^4) - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(
128*(a^4 + x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^
2/a^2])/(256*a^6)
```

Rubi in Sympy [A] time = 5.98969, size = 75, normalized size = 0.9

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}+\frac{5\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**13/(a**4+x**4)**5,x)
```

```
[Out] -x**10/(16*(a**4 + x**4)**4) - 5*x**6/(96*(a**4 + x**4)**3) - 5*x
**2/(128*(a**4 + x**4)**2) + 5*x**2/(256*a**4*(a**4 + x**4)) + 5*
atan(x**2/a**2)/(256*a**6)
```

Mathematica [A] time = 0.0351716, size = 62, normalized size = 0.75

$$\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)-\frac{a^2x^2\left(15a^{12}+55a^8x^4+73a^4x^8-15x^{12}\right)}{\left(a^4+x^4\right)^4}}{768a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^13/(a^4 + x^4)^5,x]
```

```
[Out] (-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 +
 x^4)^4) + 15*ArcTan[x^2/a^2])/(768*a^6)
```

Maple [A] time = 0.017, size = 56, normalized size = 0.7

$$\frac{1}{2\,\left(a^4+x^4\right)^4}\left(\frac{5\,x^{14}}{128\,a^4}-\frac{73\,x^{10}}{384}-\frac{55\,x^6a^4}{384}-\frac{5\,a^8x^2}{128}\right)+\frac{5}{256\,a^6}\arctan\left(\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^13/(a^4+x^4)^5,x)
```

```
[Out] 1/2*(5/128/a^4*x^14-73/384*x^10-55/384*x^6*a^4-5/128*a^8*x^2)/(a^
4+x^4)^4+5/256*arctan(x^2/a^2)/a^6
```

Maxima [A] time = 1.51007, size = 112, normalized size = 1.35

$$-\frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^{20}+4\,a^{16}x^4+6\,a^{12}x^8+4\,a^8x^{12}+a^4x^{16})}+\frac{5\,\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(a^4 + x^4)^5,x, algorithm="maxima")
```

```
[Out] -1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 - 15*x^14)/(a^20 +
 4*a^16*x^4 + 6*a^12*x^8 + 4*a^8*x^12 + a^4*x^16) + 5/256*arctan(
x^2/a^2)/a^6
```

Fricas [A] time = 0.200782, size = 153, normalized size = 1.84

$$-\frac{15\,a^{14}x^2+55\,a^{10}x^6+73\,a^6x^{10}-15\,a^2x^{14}-15\,\left(a^{16}+4\,a^{12}x^4+6\,a^8x^8+4\,a^4x^{12}+x^{16}\right)\arctan\left(\frac{x^2}{a^2}\right)}{768\,(a^{22}+4\,a^{18}x^4+6\,a^{14}x^8+4\,a^{10}x^{12}+a^6x^{16})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(a^4 + x^4)^5,x, algorithm="fricas")
```

```
[Out] -1/768*(15*a^14*x^2 + 55*a^10*x^6 + 73*a^6*x^10 - 15*a^2*x^14 - 1
5*(a^16 + 4*a^12*x^4 + 6*a^8*x^8 + 4*a^4*x^12 + x^16)*arctan(x^2/
a^2))/(a^22 + 4*a^18*x^4 + 6*a^14*x^8 + 4*a^10*x^12 + a^6*x^16)
```

Sympy [A] time = 92.0704, size = 102, normalized size = 1.23

$$\frac{-15a^{12}x^2 - 55a^8x^6 - 73a^4x^{10} + 15x^{14}}{768a^{20} + 3072a^{16}x^4 + 4608a^{12}x^8 + 3072a^8x^{12} + 768a^4x^{16}} + \frac{-\frac{5i\log\left(-ia^2+x^2\right)}{512} + \frac{5i\log\left(ia^2+x^2\right)}{512}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**13/(a**4+x**4)**5,x)
```

```
[Out] (-15*a**12*x**2 - 55*a**8*x**6 - 73*a**4*x**10 + 15*x**14)/(768*a
**20 + 3072*a**16*x**4 + 4608*a**12*x**8 + 3072*a**8*x**12 + 768*
a**4*x**16) + (-5*I*log(-I*a**2 + x**2)/512 + 5*I*log(I*a**2 + x*
*2)/512)/a**6
```

GIAC/XCAS [A] time = 0.203804, size = 78, normalized size = 0.94

$$\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{15\,a^{12}x^2 + 55\,a^8x^6 + 73\,a^4x^{10} - 15\,x^{14}}{768\,(a^4 + x^4)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(a^4 + x^4)^5,x, algorithm="giac")
```

```
[Out] 5/256*arctan(x^2/a^2)/a^6 - 1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*
a^4*x^10 - 15*x^14)/((a^4 + x^4)^4*a^4)
```

3.208 $\int \left(2\sqrt{x}-x\right)^2 x^{3/2} \left(1+x^2\right) \, dx$

Optimal. Leaf size=49

$$\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3
 + (2*x^(13/2))/13
```

Rubi [A] time = 0.0953821, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3
 + (2*x^(13/2))/13
```

Rubi in Sympy [A] time = 7.62265, size = 42, normalized size = 0.86

$$\frac{2x^{\frac{13}{2}}}{13}+\frac{8x^{\frac{11}{2}}}{11}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{7}{2}}}{7}-\frac{2x^6}{3}-x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
```

```
[Out] 2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2
*x**6/3 - x**4
```

Mathematica [A] time = 0.0135958, size = 49, normalized size = 1.

$$\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3
 + (2*x^(13/2))/13
```

Maple [A] time = 0.004, size = 32, normalized size = 0.7

$$\frac{8}{7}x^{\frac{7}{2}} - x^4 + \frac{2}{9}x^{\frac{9}{2}} + \frac{8}{11}x^{\frac{11}{2}} - \frac{2x^6}{3} + \frac{2}{13}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x)
```

```
[Out] 8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

Maxima [A] time = 1.35094, size = 42, normalized size = 0.86

$$\frac{2}{13}x^{\frac{13}{2}} - \frac{2}{3}x^6 + \frac{8}{11}x^{\frac{11}{2}} + \frac{2}{9}x^{\frac{9}{2}} - x^4 + \frac{8}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*(x - 2*sqrt(x))^2*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7
*x^(7/2)
```

Fricas [A] time = 0.199842, size = 50, normalized size = 1.02

$$-\frac{2}{3}x^6 - x^4 + \frac{2}{9009}\left(693x^6 + 3276x^5 + 1001x^4 + 5148x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*(x - 2*sqrt(x))^2*x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3
)*sqrt(x)
```

Sympy [A] time = 1.83699, size = 42, normalized size = 0.86

$$\frac{2x^{\frac{13}{2}}}{13}+\frac{8x^{\frac{11}{2}}}{11}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{7}{2}}}{7}-\frac{2x^6}{3}-x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
```

```
[Out] 2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2
*x**6/3 - x**4
```

GIAC/XCAS [A] time = 0.200711, size = 42, normalized size = 0.86

$$\frac{2}{13}\,x^{\frac{13}{2}}-\frac{2}{3}\,x^6+\frac{8}{11}\,x^{\frac{11}{2}}+\frac{2}{9}\,x^{\frac{9}{2}}-x^4+\frac{8}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*(x - 2*sqrt(x))^2*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7
*x^(7/2)
```

3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx$

Optimal. Leaf size=55

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (
120*x^(23/5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Rubi [A] time = 0.350986, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3*x^(3/5) + x^(3/2))^2*(-x^(2/3)/3 + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (
120*x^(23/5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$15\int^{\sqrt[15]{x}} x^{42}\left(4\left(x^{15}\right)^{\frac{5}{6}}-\frac{1}{3}\right)\left(\left(x^{15}\right)^{\frac{9}{10}}-3\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

```
[Out] 15*Integral(x**42*(4*(x**15)**(5/6) - 1/3)*((x**15)**(9/10) - 3)*
*2, (x, x**(1/15)))
```

Mathematica [A] time = 0.0221227, size = 55, normalized size = 1.

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3*x^(3/5) + x^(3/2))^2*(-x^(2/3)/3 + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (
120*x^(23/5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Maple [A] time = 0.006, size = 32, normalized size = 0.6

$$-\frac{45}{43}x^{\frac{43}{15}}+\frac{360}{37}x^{\frac{37}{10}}+\frac{60}{113}x^{\frac{113}{30}}-\frac{120}{23}x^{\frac{23}{5}}-\frac{1}{14}x^{\frac{14}{3}}+\frac{8}{11}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x)
```

```
[Out] -45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/
5)-1/14*x^(14/3)+8/11*x^(11/2)
```

Maxima [A] time = 1.34606, size = 42, normalized size = 0.76

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*(12*x^(3/2) - x^(2/3))*(x^(3/2) - 3*x^(3/5))^2,x, algorithm="maxim
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/3
0) + 360/37*x^(37/10) - 45/43*x^(43/15)
```

Fricas [A] time = 0.198419, size = 42, normalized size = 0.76

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*(12*x^(3/2) - x^(2/3))*(x^(3/2) - 3*x^(3/5))^2,x, algorithm="frica
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/3
0) + 360/37*x^(37/10) - 45/43*x^(43/15)
```

Sympy [A] time = 0.91534, size = 48, normalized size = 0.87

$$\frac{60x^{\frac{113}{30}}}{113} - \frac{45x^{\frac{43}{15}}}{43} + \frac{360x^{\frac{37}{10}}}{37} - \frac{120x^{\frac{23}{5}}}{23} - \frac{x^{\frac{14}{3}}}{14} + \frac{8x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

```
[Out] 60*x**(113/30)/113 - 45*x**(43/15)/43 + 360*x**(37/10)/37 - 120*x
**(23/5)/23 - x**(14/3)/14 + 8*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.205134, size = 42, normalized size = 0.76

$$\frac{8}{11}\,x^{\frac{11}{2}} - \frac{1}{14}\,x^{\frac{14}{3}} - \frac{120}{23}\,x^{\frac{23}{5}} + \frac{60}{113}\,x^{\frac{113}{30}} + \frac{360}{37}\,x^{\frac{37}{10}} - \frac{45}{43}\,x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/3*(12*x^(3/2) - x^(2/3))*(x^(3/2) - 3*x^(3/5))^2,x, algorithm="giac"
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/3
0) + 360/37*x^(37/10) - 45/43*x^(43/15)
```

3.210 $\int \frac{1}{1+\sqrt{1+x}} \, dx$

Optimal. Leaf size=22

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

```
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Rubi [A] time = 0.0209189, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[1 + x])^(-1),x]

[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Rubi in Sympy [A] time = 1.13092, size = 19, normalized size = 0.86

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+(1+x)**(1/2)),x)

[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Mathematica [A] time = 0.00710266, size = 22, normalized size = 1.

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[1 + x])^(-1),x]

[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Maple [A] time = 0.003, size = 31, normalized size = 1.4

$$2\sqrt{1+x}+\ln\left(-1+\sqrt{1+x}\right)-\ln\left(1+\sqrt{1+x}\right)-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+(1+x)^(1/2)),x)
```

```
[Out] 2*(1+x)^(1/2)+ln(-1+(1+x)^(1/2))-ln(1+(1+x)^(1/2))-ln(x)
```

Maxima [A] time = 1.45334, size = 24, normalized size = 1.09

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Fricas [A] time = 0.201436, size = 24, normalized size = 1.09

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Sympy [A] time = 0.129975, size = 19, normalized size = 0.86

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

GIAC/XCAS [A] time = 0.199365, size = 24, normalized size = 1.09

$$2\sqrt{x+1} - 2\ln\left(\sqrt{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 1) - 2*ln(sqrt(x + 1) + 1)
```

3.211 $\int \frac{x}{1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=15

$$\frac{2}{3}(x+1)^{3/2}-x$$

```
[Out] -x + (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0171658, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2}{3}(x+1)^{3/2}-x$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + Sqrt[1 + x]),x]
```

```
[Out] -x + (2*(1 + x)^(3/2))/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(x+1)^{\frac{3}{2}}}{3}-2\int^{\sqrt{x+1}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+(1+x)**(1/2)),x)
```

```
[Out] 2*(x + 1)**(3/2)/3 - 2*Integral(x, (x, sqrt(x + 1)))
```

Mathematica [A] time = 0.00966893, size = 15, normalized size = 1.

$$\frac{2}{3}(x+1)^{3/2}-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + Sqrt[1 + x]),x]
```

```
[Out] -x + (2*(1 + x)^(3/2))/3
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$\frac{2}{3}(1+x)^{\frac{3}{2}}-1-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+(1+x)^(1/2)),x)
```

```
[Out] 2/3*(1+x)^(3/2)-1-x
```

Maxima [A] time = 1.36507, size = 16, normalized size = 1.07

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x + 1) + 1),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

Fricas [A] time = 0.200508, size = 15, normalized size = 1.

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x + 1) + 1),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - x
```

Sympy [A] time = 1.3924, size = 22, normalized size = 1.47

$$\frac{2x\sqrt{x+1}}{3}-x+\frac{2\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+(1+x)**(1/2)),x)
```

```
[Out] 2*x*sqrt(x + 1)/3 - x + 2*sqrt(x + 1)/3
```

GIAC/XCAS [A] time = 0.207377, size = 16, normalized size = 1.07

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x + 1) + 1),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=25

$$x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

```
[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]
```

Rubi [A] time = 0.042009, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

```
[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\sqrt{x+1}+4\log\left(-\sqrt{x+1}+1\right)+2\int^{\sqrt{x+1}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)
```

```
[Out] 4*sqrt(x + 1) + 4*log(-sqrt(x + 1) + 1) + 2*Integral(x, (x, sqrt(
x + 1)))
```

Mathematica [A] time = 0.0111498, size = 24, normalized size = 0.96

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)-4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

```
[Out] -4 + x + 4*Sqrt[1 + x] + 4*Log[-1 + Sqrt[1 + x]]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$1 + x + 4\sqrt{1+x} + 4\ln\left(-1 + \sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x)

[Out] 1+x+4*(1+x)^(1/2)+4*ln(-1+(1+x)^(1/2))
```

Maxima [A] time = 1.33571, size = 27, normalized size = 1.08

$$x + 4\sqrt{x+1} + 4\log\left(\sqrt{x+1} - 1\right) + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + 1)/(sqrt(x + 1) - 1),x, algorithm="maxima")

[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + 1
```

Fricas [A] time = 0.204492, size = 26, normalized size = 1.04

$$x + 4\sqrt{x+1} + 4\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + 1)/(sqrt(x + 1) - 1),x, algorithm="fricas")

[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

Sympy [A] time = 0.170294, size = 20, normalized size = 0.8

$$x + 4\sqrt{x+1} + 4\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)

[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

GIAC/XCAS [A] time = 0.201074, size = 28, normalized size = 1.12

$$x + 4\sqrt{x+1} + 4\ln\left(\left|\sqrt{x+1} - 1\right|\right) + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + 1)/(sqrt(x + 1) - 1),x, algorithm="giac")

[Out] x + 4*sqrt(x + 1) + 4*ln(abs(sqrt(x + 1) - 1)) + 1
```

3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx$

Optimal. Leaf size=33

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

```
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]
```

Rubi [A] time = 0.0377612, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

```
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$6\sqrt[6]{x+1}+6\log\left(-\sqrt[6]{x+1}+1\right)+6\int^{\sqrt[6]{x+1}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
```

```
[Out] 6*(x + 1)**(1/6) + 6*log(-(x + 1)**(1/6) + 1) + 6*Integral(x, (x,
 (x + 1)**(1/6)))
```

Mathematica [A] time = 0.0169437, size = 33, normalized size = 1.

$$3\left(\sqrt[3]{x+1}+2\sqrt[6]{x+1}+2\log\left(1-\sqrt[6]{x+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

```
[Out] 3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[1 - (1 + x)^(1/6)])
```

Maple [B] time = 0.03, size = 111, normalized size = 3.4

$$6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+\ln(x)+2\ln\left(-1+\sqrt[6]{1+x}\right)-\ln\left(\sqrt[3]{1+x}+\sqrt[6]{1+x}+1\right)$$
$$-2\ln\left(1+\sqrt[6]{1+x}\right)+\ln\left(\sqrt[3]{1+x}-\sqrt[6]{1+x}+1\right)-\ln\left(1+\sqrt{1+x}\right)$$
$$+\ln\left(-1+\sqrt{1+x}\right)+2\ln\left(-1+\sqrt[3]{1+x}\right)-\ln\left((1+x)^{\frac{2}{3}}+\sqrt[3]{1+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((1+x)^(2/3)-(1+x)^(1/2)),x)
```

```
[Out] 6*(1+x)^(1/6)+3*(1+x)^(1/3)+ln(x)+2*ln(-1+(1+x)^(1/6))-ln((1+x)^(
1/3)+(1+x)^(1/6)+1)-2*ln(1+(1+x)^(1/6))+ln((1+x)^(1/3)-(1+x)^(1/6
)+1)-ln(1+(1+x)^(1/2))+ln(-1+(1+x)^(1/2))+2*ln(-1+(1+x)^(1/3))-ln
((1+x)^(2/3)+(1+x)^(1/3)+1)
```

Maxima [A] time = 1.38007, size = 34, normalized size = 1.03

$$3(x+1)^{\frac{1}{3}}+6(x+1)^{\frac{1}{6}}+6\log\left((x+1)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(2/3) - sqrt(x + 1)),x, algorithm="maxima")
```

```
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

Fricas [A] time = 0.202291, size = 34, normalized size = 1.03

$$3(x+1)^{\frac{1}{3}}+6(x+1)^{\frac{1}{6}}+6\log\left((x+1)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(2/3) - sqrt(x + 1)),x, algorithm="fricas")
```

```
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)^{\frac{2}{3}} - \sqrt{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
```

```
[Out] Integral(1/((x + 1)**(2/3) - sqrt(x + 1)), x)
```

GIAC/XCAS [A] time = 0.209296, size = 35, normalized size = 1.06

$$3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\ln\left(\left|(x+1)^{\frac{1}{6}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(2/3) - sqrt(x + 1)),x, algorithm="giac")
```

```
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*ln(abs((x + 1)^(1/6) - 1))
```

3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$\frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-3\left(\sqrt[4]{x}+1\right)^{4/3}$$

```
[Out] -3*(1 + x^(1/4))^(4/3) + (12*(1 + x^(1/4))^(7/3))/7
```

Rubi [A] time = 0.022731, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-3\left(\sqrt[4]{x}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

```
[Out] -3*(1 + x^(1/4))^(4/3) + (12*(1 + x^(1/4))^(7/3))/7
```

Rubi in Sympy [A] time = 1.6011, size = 24, normalized size = 0.83

$$\frac{12\left(\sqrt[4]{x}+1\right)^{\frac{7}{3}}}{7}-3\left(\sqrt[4]{x}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
```

```
[Out] 12*(x**(1/4) + 1)**(7/3)/7 - 3*(x**(1/4) + 1)**(4/3)
```

Mathematica [A] time = 0.0100897, size = 24, normalized size = 0.83

$$\frac{3}{7}\left(\sqrt[4]{x}+1\right)^{4/3}\left(4\sqrt[4]{x}-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

```
[Out] (3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7
```

Maple [A] time = 0.003, size = 20, normalized size = 0.7

$$-3\left(1+\sqrt[4]{x}\right)^{4/3}+\frac{12}{7}\left(1+\sqrt[4]{x}\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/4))^(1/3)/x^(1/2),x)
```

```
[Out] -3*(1+x^(1/4))^(4/3)+12/7*(1+x^(1/4))^(7/3)
```

Maxima [A] time = 1.46018, size = 26, normalized size = 0.9

$$\frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}}-3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/4) + 1)^(1/3)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

Fricas [A] time = 0.206433, size = 26, normalized size = 0.9

$$\frac{3}{7}\left(4\sqrt{x}+x^{\frac{1}{4}}-3\right)\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/4) + 1)^(1/3)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 3/7*(4*sqrt(x) + x^(1/4) - 3)*(x^(1/4) + 1)^(1/3)
```

Sympy [A] time = 2.54791, size = 134, normalized size = 4.62

$$\frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}-\frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}+\frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}}+7x}+\frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}-\frac{9x\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}+\frac{9x}{7x^{\frac{5}{4}}+7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
```

```
[Out] 12*x**(7/4)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 6*x**(5/4)
*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x**(5/4)/(7*x**(5/4
) + 7*x) + 15*x**(3/2)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) -
 9*x*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x/(7*x**(5/4) +
 7*x)
```

GIAC/XCAS [A] time = 0.200745, size = 26, normalized size = 0.9

$$\frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}}-3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/4) + 1)^(1/3)/sqrt(x),x, algorithm="giac")
```

```
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

3.215 $\int \frac{1}{x^3(1+x)^{3/2}} \, dx$

Optimal. Leaf size=53

$$-\frac{5\sqrt{x+1}}{2x^2}+\frac{2}{x^2\sqrt{x+1}}+\frac{15\sqrt{x+1}}{4x}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] 2/(x^2*Sqrt[1 + x]) - (5*Sqrt[1 + x])/(2*x^2) + (15*Sqrt[1 + x])/
(4*x) - (15*ArcTanh[Sqrt[1 + x]])/4
```

Rubi [A] time = 0.0340558, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{5\sqrt{x+1}}{2x^2}+\frac{2}{x^2\sqrt{x+1}}+\frac{15\sqrt{x+1}}{4x}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 + x)^(3/2)),x]
```

```
[Out] 2/(x^2*Sqrt[1 + x]) - (5*Sqrt[1 + x])/(2*x^2) + (15*Sqrt[1 + x])/
(4*x) - (15*ArcTanh[Sqrt[1 + x]])/4
```

Rubi in Sympy [A] time = 2.23075, size = 48, normalized size = 0.91

$$-\frac{15\,\operatorname{atanh}\left(\sqrt{x+1}\right)}{4}+\frac{15\sqrt{x+1}}{4x}-\frac{5\sqrt{x+1}}{2x^2}+\frac{2}{x^2\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(1+x)**(3/2),x)
```

```
[Out] -15*atanh(sqrt(x + 1))/4 + 15*sqrt(x + 1)/(4*x) - 5*sqrt(x + 1)/(
2*x**2) + 2/(x**2*sqrt(x + 1))
```

Mathematica [A] time = 0.0394597, size = 36, normalized size = 0.68

$$\frac{1}{4}\left(\frac{15x^2+5x-2}{x^2\sqrt{x+1}}-15\tanh^{-1}\left(\sqrt{x+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[1/(x^3*(1 + x)^(3/2)),x]
```

```
[Out] ((-2 + 5*x + 15*x^2)/(x^2*Sqrt[1 + x]) - 15*ArcTanh[Sqrt[1 + x]])
/4
```

Maple [A] time = 0.021, size = 73, normalized size = 1.4

$$\frac{1}{8}\left(1+\sqrt{1+x}\right)^{-2}+\frac{7}{8}\left(1+\sqrt{1+x}\right)^{-1}-\frac{15}{8}\ln\left(1+\sqrt{1+x}\right)+2\frac{1}{\sqrt{1+x}}-\frac{1}{8}\left(-1+\sqrt{1+x}\right)^{-2}+\frac{7}{8}\left(-1+\sqrt{1+x}\right)^{-1}+\frac{15}{8}\ln\left(-1+\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x^3/(1+x)^(3/2),x)
```

```
[Out] 1/8/(1+(1+x)^(1/2))^2+7/8/(1+(1+x)^(1/2))-15/8*ln(1+(1+x)^(1/2))+
2/(1+x)^(1/2)-1/8/(-1+(1+x)^(1/2))^2+7/8/(-1+(1+x)^(1/2))+15/8*ln
(-1+(1+x)^(1/2))
```

Maxima [A] time = 1.39804, size = 74, normalized size = 1.4

$$\frac{15(x+1)^2-25x-17}{4\left((x+1)^{\frac{5}{2}}-2(x+1)^{\frac{3}{2}}+\sqrt{x+1}\right)}-\frac{15}{8}\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x + 1)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) +
 sqrt(x + 1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1)
- 1)
```

Fricas [A] time = 0.207916, size = 76, normalized size = 1.43

$$-\frac{15\sqrt{x+1}x^2\log\left(\sqrt{x+1}+1\right)-15\sqrt{x+1}x^2\log\left(\sqrt{x+1}-1\right)-30x^2-10x+4}{8\sqrt{x+1}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/8*(15*sqrt(x + 1)*x^2*log(sqrt(x + 1) + 1) - 15*sqrt(x + 1)*x^
2*log(sqrt(x + 1) - 1) - 30*x^2 - 10*x + 4)/(sqrt(x + 1)*x^2)
```

Sympy [A] time = 4.3313, size = 3966, normalized size = 74.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17
/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11
/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2
) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 15*I*pi*(x + 1)**(17/2)/
(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 4
48*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 22
4*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*(x +
1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(1
5/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(
9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2
) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2)
- 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2)
+ 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) -
64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x + 1)**(13/2)*acoth(sq
rt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)*
*(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1))
- 420*I*pi*(x + 1)**(13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2
) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2
) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) +
 8*sqrt(x + 1)) + 1680*(x + 1)**(11/2)*acoth(sqrt(x + 1))/(8*(x +
 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x +
 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x +
1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 840*I*pi*(x + 1)
**(11/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(
13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(
7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
2100*(x + 1)**(9/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x
 + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(
x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 1050*I*pi*(x + 1)**(9/2)/(8*(x + 1)
**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)
**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)*
*(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**(7/2)
*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224
*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448
*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt
(x + 1)) + 840*I*pi*(x + 1)**(7/2)/(8*(x + 1)**(17/2) - 64*(x + 1
```

```
)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)*
*(3/2) + 8*sqrt(x + 1)) - 840*(x + 1)**(5/2)*acoth(sqrt(x + 1))/(
8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 44
8*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224
*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420*I*pi*(
x + 1)**(5/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x +
1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1
)) + 240*(x + 1)**(3/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 6
4*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 5
60*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*
(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1)**(3/2)/(8*(x +
 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x +
 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x +
1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 30*sqrt(x + 1)*a
coth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(
x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(
x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x
 + 1)) - 15*I*pi*sqrt(x + 1)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15
/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9
/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2)
 + 8*sqrt(x + 1)) + 30*(x + 1)**8/(8*(x + 1)**(17/2) - 64*(x + 1)
**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1
)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**
(3/2) + 8*sqrt(x + 1)) - 230*(x + 1)**7/(8*(x + 1)**(17/2) - 64*(
x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*
(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x
+ 1)**(3/2) + 8*sqrt(x + 1)) + 766*(x + 1)**6/(8*(x + 1)**(17/2)
- 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2)
+ 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) -
64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 1446*(x + 1)**5/(8*(x + 1)**
(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**
(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1690*(x + 1)**4/(8*(x
 + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x
 + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
+ 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 1250*(x + 1)**
3/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) -
 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) +
224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 570*(x
+ 1)**2/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(1
3/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7
/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 1
46*(x + 1)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)*
*(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1))
+ 16/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2
) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2)
 + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)), Abs(x
 + 1) > 1), (-15*(x + 1)**(17/2)*atanh(sqrt(x + 1))/(4*(x + 1)**(
17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5
/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(15/2)*at
anh(sqrt(x + 1))/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x
 + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x
```

```
 + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x
+ 1)) - 420*(x + 1)**(13/2)*atanh(sqrt(x + 1))/(4*(x + 1)**(17/2)
 - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2)
 + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) -
 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(11/2)*atanh(s
qrt(x + 1))/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)
**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)
**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1))
 - 1050*(x + 1)**(9/2)*atanh(sqrt(x + 1))/(4*(x + 1)**(17/2) - 32
*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 28
0*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(7/2)*atanh(sqrt(x
+ 1))/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/
2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2
) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 420
*(x + 1)**(5/2)*atanh(sqrt(x + 1))/(4*(x + 1)**(17/2) - 32*(x + 1
)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x +
1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)*
*(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(3/2)*atanh(sqrt(x + 1))/(
4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 22
4*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112
*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 15*sqrt(x
+ 1)*atanh(sqrt(x + 1))/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) +
 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) -
 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*
sqrt(x + 1)) + 15*(x + 1)**8/(4*(x + 1)**(17/2) - 32*(x + 1)**(15
/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9
/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2)
 + 4*sqrt(x + 1)) - 115*(x + 1)**7/(4*(x + 1)**(17/2) - 32*(x + 1
)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x +
1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)*
*(3/2) + 4*sqrt(x + 1)) + 383*(x + 1)**6/(4*(x + 1)**(17/2) - 32*
(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280
*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
 + 1)**(3/2) + 4*sqrt(x + 1)) - 723*(x + 1)**5/(4*(x + 1)**(17/2)
 - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2)
 + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) -
 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 845*(x + 1)**4/(4*(x + 1)**
(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**
(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(
5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 625*(x + 1)**3/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x
+ 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x +
 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 285*(x + 1)**2/
(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 2
24*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 11
2*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 73*(x + 1
)/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) -
 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) +
112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 8/(4*(x
 + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x
 + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x
+ 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)), True))
```

GIAC/XCAS [A] time = 0.203944, size = 66, normalized size = 1.25

$$\frac{2}{\sqrt{x+1}} + \frac{7\,(x+1)^{\frac{3}{2}} - 9\,\sqrt{x+1}}{4\,x^2} - \frac{15}{8}\ln\left(\sqrt{x+1}+1\right) + \frac{15}{8}\ln\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 1)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] 2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*
ln(sqrt(x + 1) + 1) + 15/8*ln(abs(sqrt(x + 1) - 1))
```

3.216 $\int \frac{1}{(1-x)^{7/2}x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{429\sqrt{1-x}}{20x^4}+\frac{286}{15\sqrt{1-x}\,x^4}+\frac{26}{15(1-x)^{3/2}x^4}+\frac{2}{5(1-x)^{5/2}x^4}-\frac{1001\sqrt{1-x}}{40x^3}-\frac{1001\sqrt{1-x}}{32x^2}-\frac{3003\sqrt{1-x}}{64x}-\frac{3003}{64}\tanh^{-1}\left(\sqrt{1-x}\right)$$

```
[Out] 2/(5*(1 - x)^(5/2)*x^4) + 26/(15*(1 - x)^(3/2)*x^4) + 286/(15*Sqr
t[1 - x]*x^4) - (429*Sqrt[1 - x])/(20*x^4) - (1001*Sqrt[1 - x])/(
40*x^3) - (1001*Sqrt[1 - x])/(32*x^2) - (3003*Sqrt[1 - x])/(64*x)
 - (3003*ArcTanh[Sqrt[1 - x]])/64
```

Rubi [A] time = 0.101853, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{429\sqrt{1-x}}{20x^4}+\frac{286}{15\sqrt{1-x}\,x^4}+\frac{26}{15(1-x)^{3/2}x^4}+\frac{2}{5(1-x)^{5/2}x^4}-\frac{1001\sqrt{1-x}}{40x^3}-\frac{1001\sqrt{1-x}}{32x^2}-\frac{3003\sqrt{1-x}}{64x}-\frac{3003}{64}\tanh^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] 2/(5*(1 - x)^(5/2)*x^4) + 26/(15*(1 - x)^(3/2)*x^4) + 286/(15*Sqr
t[1 - x]*x^4) - (429*Sqrt[1 - x])/(20*x^4) - (1001*Sqrt[1 - x])/(
40*x^3) - (1001*Sqrt[1 - x])/(32*x^2) - (3003*Sqrt[1 - x])/(64*x)
 - (3003*ArcTanh[Sqrt[1 - x]])/64
```

Rubi in Sympy [A] time = 5.27613, size = 104, normalized size = 0.82

$$-\frac{3003\,\operatorname{atanh}\left(\sqrt{-x+1}\right)}{64}-\frac{3003\sqrt{-x+1}}{64x}-\frac{1001\sqrt{-x+1}}{32x^2}-\frac{1001\sqrt{-x+1}}{40x^3}-\frac{429\sqrt{-x+1}}{20x^4}+\frac{286}{15x^4\sqrt{-x+1}}+\frac{26}{15x^4\left(-x+1\right)^{\frac{3}{2}}}+\frac{2}{5x^4\left(-x+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-x)**(7/2)/x**5,x)
```

```
[Out] -3003*atanh(sqrt(-x + 1))/64 - 3003*sqrt(-x + 1)/(64*x) - 1001*sq
rt(-x + 1)/(32*x**2) - 1001*sqrt(-x + 1)/(40*x**3) - 429*sqrt(-x
+ 1)/(20*x**4) + 286/(15*x**4*sqrt(-x + 1)) + 26/(15*x**4*(-x + 1
)**(3/2)) + 2/(5*x**4*(-x + 1)**(5/2))
```

Mathematica [A] time = 0.0770529, size = 61, normalized size = 0.48

$$-\frac{-45045x^6+105105x^5-69069x^4+6435x^3+1430x^2+520x+240}{960(1-x)^{5/2}x^4}-\frac{3003}{64}\tanh^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] -(240 + 520*x + 1430*x^2 + 6435*x^3 - 69069*x^4 + 105105*x^5 - 45
045*x^6)/(960*(1 - x)^(5/2)*x^4) - (3003*ArcTanh[Sqrt[1 - x]])/64
```

Maple [A] time = 0.026, size = 157, normalized size = 1.2

$$\frac{1}{64}\left(1+\sqrt{1-x}\right)^{-4}+\frac{17}{96}\left(1+\sqrt{1-x}\right)^{-3}+\frac{159}{128}\left(1+\sqrt{1-x}\right)^{-2}+\frac{1083}{128}\left(1+\sqrt{1-x}\right)^{-1}$$
$$-\frac{3003}{128}\ln\left(1+\sqrt{1-x}\right)+\frac{2}{5}(1-x)^{-\frac{5}{2}}+\frac{10}{3}(1-x)^{-\frac{3}{2}}+30\frac{1}{\sqrt{1-x}}-\frac{1}{64}\left(-1+\sqrt{1-x}\right)^{-4}$$
$$+\frac{17}{96}\left(-1+\sqrt{1-x}\right)^{-3}-\frac{159}{128}\left(-1+\sqrt{1-x}\right)^{-2}+\frac{1083}{128}\left(-1+\sqrt{1-x}\right)^{-1}+\frac{3003}{128}\ln\left(-1+\sqrt{1-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-x)^(7/2)/x^5,x)
```

```
[Out] 1/64/(1+(1-x)^(1/2))^4+17/96/(1+(1-x)^(1/2))^3+159/128/(1+(1-x)^(
1/2))^2+1083/128/(1+(1-x)^(1/2))-3003/128*ln(1+(1-x)^(1/2))+2/5/(
1-x)^(5/2)+10/3/(1-x)^(3/2)+30/(1-x)^(1/2)-1/64/(-1+(1-x)^(1/2))^
4+17/96/(-1+(1-x)^(1/2))^3-159/128/(-1+(1-x)^(1/2))^2+1083/128/(-
1+(1-x)^(1/2))+3003/128*ln(-1+(1-x)^(1/2))
```

Maxima [A] time = 1.35221, size = 150, normalized size = 1.18

$$\frac{45045(x-1)^6+165165(x-1)^5+219219(x-1)^4+119691(x-1)^3+18304(x-1)^2-1664x+2048}{960\left((-x+1)^{\frac{13}{2}}-4(-x+1)^{\frac{11}{2}}+6(-x+1)^{\frac{9}{2}}-4(-x+1)^{\frac{7}{2}}+(-x+1)^{\frac{5}{2}}\right)}$$
$$-\frac{3003}{128}\log\left(\sqrt{-x+1}+1\right)+\frac{3003}{128}\log\left(\sqrt{-x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5*(-x + 1)^(7/2)),x, algorithm="maxima")
```

```
[Out] 1/960*(45045*(x - 1)^6 + 165165*(x - 1)^5 + 219219*(x - 1)^4 + 11
9691*(x - 1)^3 + 18304*(x - 1)^2 - 1664*x + 2048)/((-x + 1)^(13/2
) - 4*(-x + 1)^(11/2) + 6*(-x + 1)^(9/2) - 4*(-x + 1)^(7/2) + (-x
 + 1)^(5/2)) - 3003/128*log(sqrt(-x + 1) + 1) + 3003/128*log(sqrt
(-x + 1) - 1)
```

Fricas [A]   time = 0.206486, size = 155, normalized size = 1.22

$$\frac{90090\,x^{6} - 210210\,x^{5} + 138138\,x^{4} - 12870\,x^{3} - 45045\,\left(x^{6} - 2\,x^{5} + x^{4}\right)\sqrt{-x + 1}\log\left(\sqrt{-x + 1} + 1\right) + 45045\,\left(x^{6} - 2\,x^{5} + x^{4}\right)}{1920\,\left(x^{6} - 2\,x^{5} + x^{4}\right)\sqrt{-x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5*(-x + 1)^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/1920*(90090*x^6 - 210210*x^5 + 138138*x^4 - 12870*x^3 - 45045*(
x^6 - 2*x^5 + x^4)*sqrt(-x + 1)*log(sqrt(-x + 1) + 1) + 45045*(x^
6 - 2*x^5 + x^4)*sqrt(-x + 1)*log(sqrt(-x + 1) - 1) - 2860*x^2 -
1040*x - 480)/((x^6 - 2*x^5 + x^4)*sqrt(-x + 1))
```

Sympy [A]   time = 25.9014, size = 971, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-x)**(7/2)/x**5,x)
```

```
[Out] Piecewise((45045*I*x**7*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 2
880*x**5 - 960*x**4) - 45045*I*x**6*sqrt(x - 1)/(960*x**7 - 2880*
x**6 + 2880*x**5 - 960*x**4) - 135135*I*x**6*asin(1/sqrt(x))/(960
*x**7 - 2880*x**6 + 2880*x**5 - 960*x**4) + 105105*I*x**5*sqrt(x
- 1)/(960*x**7 - 2880*x**6 + 2880*x**5 - 960*x**4) + 135135*I*x**
5*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 2880*x**5 - 960*x**4) -
 69069*I*x**4*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 - 960
*x**4) - 45045*I*x**4*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 288
0*x**5 - 960*x**4) + 6435*I*x**3*sqrt(x - 1)/(960*x**7 - 2880*x**
6 + 2880*x**5 - 960*x**4) + 1430*I*x**2*sqrt(x - 1)/(960*x**7 - 2
880*x**6 + 2880*x**5 - 960*x**4) + 520*I*x*sqrt(x - 1)/(960*x**7
- 2880*x**6 + 2880*x**5 - 960*x**4) + 240*I*sqrt(x - 1)/(960*x**7
 - 2880*x**6 + 2880*x**5 - 960*x**4), Abs(x) > 1), (45045*x**7*lo
```

```
g(x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 90090*x**7
*log(sqrt(-x + 1) + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*
x**4) + 45045*I*pi*x**7/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920
*x**4) - 90090*x**6*sqrt(-x + 1)/(1920*x**7 - 5760*x**6 + 5760*x*
*5 - 1920*x**4) - 135135*x**6*log(x)/(1920*x**7 - 5760*x**6 + 576
0*x**5 - 1920*x**4) + 270270*x**6*log(sqrt(-x + 1) + 1)/(1920*x**
7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 135135*I*pi*x**6/(1920*x
**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 210210*x**5*sqrt(-x +
1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 135135*x**5*
log(x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 270270*x
**5*log(sqrt(-x + 1) + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 19
20*x**4) + 135135*I*pi*x**5/(1920*x**7 - 5760*x**6 + 5760*x**5 -
1920*x**4) - 138138*x**4*sqrt(-x + 1)/(1920*x**7 - 5760*x**6 + 57
60*x**5 - 1920*x**4) - 45045*x**4*log(x)/(1920*x**7 - 5760*x**6 +
 5760*x**5 - 1920*x**4) + 90090*x**4*log(sqrt(-x + 1) + 1)/(1920*
x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 45045*I*pi*x**4/(1920
*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 12870*x**3*sqrt(-x +
 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 2860*x**2*s
qrt(-x + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 104
0*x*sqrt(-x + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4)
+ 480*sqrt(-x + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4
), True))
```

GIAC/XCAS [A] time = 0.201556, size = 140, normalized size = 1.1

$$\frac{2\left(225\,(x-1)^2-25\,x+28\right)}{15\,(x-1)^2\sqrt{-x+1}}-\frac{3249\,(x-1)^3\sqrt{-x+1}+10633\,(x-1)^2\sqrt{-x+1}-11767\,(-x+1)^{\frac{3}{2}}+4431\,\sqrt{-x+1}}{192\,x^4}-\frac{3003}{128}\ln\left(\sqrt{-x+1}+1\right)+\frac{3003}{128}\ln\left(\left|\sqrt{-x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5*(-x + 1)^(7/2)),x, algorithm="giac")
```

```
[Out] 2/15*(225*(x - 1)^2 - 25*x + 28)/((x - 1)^2*sqrt(-x + 1)) - 1/192
*(3249*(x - 1)^3*sqrt(-x + 1) + 10633*(x - 1)^2*sqrt(-x + 1) - 11
767*(-x + 1)^(3/2) + 4431*sqrt(-x + 1))/x^4 - 3003/128*ln(sqrt(-x
 + 1) + 1) + 3003/128*ln(abs(sqrt(-x + 1) - 1))
```

3.217 $\int \frac{1}{(-1+x)^{2/3}x^5}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt[3]{x-1}}{4x^4}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55\log(x)}{243}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

```
[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 +
 x)^(1/3))/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1
 - 2*(-1 + x)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x
)^(1/3)])/81 - (55*Log[x])/243
```

Rubi [A] time = 0.0933074, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\frac{\sqrt[3]{x-1}}{4x^4}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55\log(x)}{243}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 +
 x)^(1/3))/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1
 - 2*(-1 + x)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x
)^(1/3)])/81 - (55*Log[x])/243
```

Rubi in Sympy [A] time = 3.23367, size = 99, normalized size = 0.95

$$-\frac{55\log(x)}{243}+\frac{55\log\left(\sqrt[3]{x-1}+1\right)}{81}+\frac{110\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{x-1}}{3}-\frac{1}{3}\right)\right)}{243}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{\sqrt[3]{x-1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**(2/3)/x**5,x)
```

```
[Out] -55*log(x)/243 + 55*log((x - 1)**(1/3) + 1)/81 + 110*sqrt(3)*atan
(sqrt(3)*(2*(x - 1)**(1/3)/3 - 1/3))/243 + 55*(x - 1)**(1/3)/(81*
```

```
x) + 11*(x - 1)**(1/3)/(27*x**2) + 11*(x - 1)**(1/3)/(36*x**3) +
(x - 1)**(1/3)/(4*x**4)
```

Mathematica [C] time = 0.0247523, size = 63, normalized size = 0.61

$$\frac{-220\left(\frac{x-1}{x}\right)^{2/3} x^4\, {}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{x}\right)+220x^4-88x^3-33x^2-18x-81}{324(x-1)^{2/3}x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] (-81 - 18*x - 33*x^2 - 88*x^3 + 220*x^4 - 220*((-1 + x)/x)^(2/3)*
x^4*Hypergeometric2F1[2/3, 2/3, 5/3, x^(-1)])/(324*(-1 + x)^(2/3)
*x^4)
```

Maple [B] time = 0.026, size = 158, normalized size = 1.5

$$-\frac{1}{324}\left(1+\sqrt[3]{-1+x}\right)^{-4}-\frac{5}{243}\left(1+\sqrt[3]{-1+x}\right)^{-3}$$
$$-\frac{20}{243}\left(1+\sqrt[3]{-1+x}\right)^{-2}-\frac{25}{81}\left(1+\sqrt[3]{-1+x}\right)^{-1}+\frac{110}{243}\ln\left(1+\sqrt[3]{-1+x}\right)$$
$$-\frac{1}{243}\left(-75\,(-1+x)^{7/3}+190\,(-1+x)^2-350\,(-1+x)^{5/3}+\frac{1157}{4}(-1+x)^{\frac{4}{3}}+\frac{149}{4}-138\,x-116\,(-1+x)^{2/3}+137\sqrt[3]{-1+x}\right.$$
$$\left.-\frac{55}{243}\ln\left((-1+x)^{\frac{2}{3}}-\sqrt[3]{-1+x}+1\right)+\frac{110\sqrt{3}}{243}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{-1+x}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^(2/3)/x^5,x)
```

```
[Out] -1/324/(1+(-1+x)^(1/3))^4-5/243/(1+(-1+x)^(1/3))^3-20/243/(1+(-1+
x)^(1/3))^2-25/81/(1+(-1+x)^(1/3))+110/243*ln(1+(-1+x)^(1/3))-1/2
43*(-75*(-1+x)^(7/3)+190*(-1+x)^2-350*(-1+x)^(5/3)+1157/4*(-1+x)^
(4/3)+149/4-138*x-116*(-1+x)^(2/3)+137*(-1+x)^(1/3))/((-1+x)^(2/3
)-(-1+x)^(1/3)+1)^4-55/243*ln((-1+x)^(2/3)-(-1+x)^(1/3)+1)+110/24
3*3^(1/2)*arctan(1/3*(2*(-1+x)^(1/3)-1)*3^(1/2))
```

Maxima [A] time = 1.49667, size = 142, normalized size = 1.37

$$\frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right)+\frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\left((x-1)^4+4\,(x-1)^3+6\,(x-1)^2+4\,x-3\right)}$$
$$-\frac{55}{243}\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+\frac{110}{243}\log\left((x-1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)^(2/3)*x^5),x, algorithm="maxima")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324
*(220*(x - 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 5
32*(x - 1)^(1/3))/((x - 1)^4 + 4*(x - 1)^3 + 6*(x - 1)^2 + 4*x -
3) - 55/243*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 110/243*log(
(x - 1)^(1/3) + 1)
```

Fricas [A] time = 0.215965, size = 128, normalized size = 1.23

$$-\frac{\sqrt{3}\left(220\,\sqrt{3}x^4\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)-440\,\sqrt{3}x^4\log\left((x-1)^{\frac{1}{3}}+1\right)-1320\,x^4\arctan\left(\frac{2}{3}\,\sqrt{3}(x-1)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-3\,\sqrt{3}\right.}{2916\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)^(2/3)*x^5),x, algorithm="fricas")
```

```
[Out] -1/2916*sqrt(3)*(220*sqrt(3)*x^4*log((x - 1)^(2/3) - (x - 1)^(1/3
) + 1) - 440*sqrt(3)*x^4*log((x - 1)^(1/3) + 1) - 1320*x^4*arctan
(2/3*sqrt(3)*(x - 1)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(3)*(220*x^3 +
132*x^2 + 99*x + 81)*(x - 1)^(1/3))/x^4
```

Sympy [A] time = 4.89729, size = 7998, normalized size = 76.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**(2/3)/x**5,x)
```

```
[Out] 660*(x - 1)**(34/3)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 320
76*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 48114
0*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*
(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x
 - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x -
1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gam
ma(4/3)) + 6996*(x - 1)**(31/3)*gamma(1/3)/(2916*(x - 1)**12*gamm
a(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(
4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3
) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3)
 + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) +
160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916
*(x - 1)*gamma(4/3)) + 33561*(x - 1)**(28/3)*gamma(1/3)/(2916*(x
```

```
- 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x -
1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)
**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)*
*6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*
gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamm
a(4/3) + 2916*(x - 1)*gamma(4/3)) + 96075*(x - 1)**(25/3)*gamma(1
/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) +
 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 9
62280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 134
7192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 48114
0*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x
 - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 181881*(x - 1)**
(22/3)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**1
1*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*
gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*ga
mma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamm
a(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4
/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 23
7951*(x - 1)**(19/3)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32
076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 4811
40*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192
*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(
x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x -
 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*ga
mma(4/3)) + 217791*(x - 1)**(16/3)*gamma(1/3)/(2916*(x - 1)**12*g
amma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gam
ma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(
4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4
/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3)
 + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2
916*(x - 1)*gamma(4/3)) + 137601*(x - 1)**(13/3)*gamma(1/3)/(2916
*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(
x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x
- 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x -
 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)
**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*
gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 57375*(x - 1)**(10/3)*gam
ma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/
3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3)
 + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) +
 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 4
81140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 3207
6*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 14241*(x - 1
)**(7/3)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)*
*11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**
9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*
gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*ga
mma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma
(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) +
1596*(x - 1)**(4/3)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 320
76*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 48114
0*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*
(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x
 - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x -
1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gam
ma(4/3)) - 440*(x - 1)**12*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*exp_
```

```
polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 3207
6*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140
*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(
x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x
- 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1
)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamm
a(4/3)) + 440*(x - 1)**12*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1
)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gam
ma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma
(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4
/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3
) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) +
 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 440*(x
- 1)**12*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)
*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamm
a(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(
4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/
3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3)
 + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) +
32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 4840*(x
- 1)**11*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)
*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamm
a(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(
4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/
3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3)
 + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) +
32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 4840*(x
- 1)**11*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(291
6*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*
(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x
 - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x
- 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1
)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2
*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 4840*(x - 1)**11*exp(I*p
i/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(291
6*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*
(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x
 - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x
- 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1
)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2
*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 24200*(x - 1)**10*exp(5*
I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(29
16*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380
*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(
x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x
 - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x -
1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**
2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 24200*(x - 1)**10*log(-
(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**12*
gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*ga
mma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma
(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(
4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3
) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) +
2916*(x - 1)*gamma(4/3)) - 24200*(x - 1)**10*exp(I*pi/3)*log(-(x
- 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12
```

```
*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*g
amma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamm
a(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma
(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/
3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) +
 2916*(x - 1)*gamma(4/3)) - 72600*(x - 1)**9*exp(5*I*pi/3)*log(-(
x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12
*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*g
amma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamm
a(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma
(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/
3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) +
 2916*(x - 1)*gamma(4/3)) + 72600*(x - 1)**9*log(-(x - 1)**(1/3)*
exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32
076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 4811
40*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192
*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(
x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x -
 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*ga
mma(4/3)) - 72600*(x - 1)**9*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_
polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32
076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 4811
40*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192
*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(
x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x -
 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*ga
mma(4/3)) - 145200*(x - 1)**8*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*e
xp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 3
2076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481
140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 134719
2*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*
(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x
- 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*g
amma(4/3)) + 145200*(x - 1)**8*log(-(x - 1)**(1/3)*exp_polar(I*pi
) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**1
1*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*
gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*ga
mma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamm
a(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4
/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 14
5200*(x - 1)**8*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/
3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**
11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9
*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*g
amma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gam
ma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(
4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 2
03280*(x - 1)**7*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi
/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)*
*11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**
9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*
gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*ga
mma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma
(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) +
203280*(x - 1)**7*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(
1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3)
```

```
+ 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) +
962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 13
47192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 4811
40*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(
x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 203280*(x - 1)*
*7*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma
(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3)
 + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) +
 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1
347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481
140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*
(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 203280*(x - 1)
**6*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamm
a(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3
) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3)
+ 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) +
1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 48
1140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076
*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 203280*(x - 1
)**6*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x
 - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x -
 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1
)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)
**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4
*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gam
ma(4/3) + 2916*(x - 1)*gamma(4/3)) - 203280*(x - 1)**6*exp(I*pi/3
)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(
x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x
- 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x -
1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1
)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**
4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*ga
mma(4/3) + 2916*(x - 1)*gamma(4/3)) - 145200*(x - 1)**5*exp(5*I*p
i/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*
(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x
 - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x -
 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x -
1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)*
*4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*g
amma(4/3) + 2916*(x - 1)*gamma(4/3)) + 145200*(x - 1)**5*log(-(x
- 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**12*gam
ma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma
(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/
3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3
) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) +
 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 291
6*(x - 1)*gamma(4/3)) - 145200*(x - 1)**5*exp(I*pi/3)*log(-(x - 1
)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*ga
mma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamm
a(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4
/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/
3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3)
+ 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 29
16*(x - 1)*gamma(4/3)) - 72600*(x - 1)**4*exp(5*I*pi/3)*log(-(x -
 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*ga
mma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamm
```

a(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4
/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/
3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3)
+ 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 29
16*(x - 1)*gamma(4/3)) + 72600*(x - 1)**4*log(-(x - 1)**(1/3)*exp
_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076
*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*
(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x
 - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x -
 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)
3*gamma(4/3) + 32076*(x - 1)2*gamma(4/3) + 2916*(x - 1)*gamma
(4/3)) - 72600*(x - 1)**4*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_pol
ar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076
*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*
(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x
 - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x -
 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)
3*gamma(4/3) + 32076*(x - 1)2*gamma(4/3) + 2916*(x - 1)*gamma
(4/3)) - 24200*(x - 1)**3*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*exp_p
olar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076
*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*
(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x
 - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x -
 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)
3*gamma(4/3) + 32076*(x - 1)2*gamma(4/3) + 2916*(x - 1)*gamma
(4/3)) + 24200*(x - 1)**3*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1
)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gam
ma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma
(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4
/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3
) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) +
 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 24200*(
x - 1)**3*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1
)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gam
ma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma
(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4
/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3
) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) +
 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 4840*(x
 - 1)**2*exp(5*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)
gamma(1/3)/(2916(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamm
a(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(
4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/
3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3)
 + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) +
32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(4/3)) + 4840*(x
- 1)**2*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916
*(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(
x - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x
- 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x -
 1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)
4*gamma(4/3) + 160380*(x - 1)3*gamma(4/3) + 32076*(x - 1)**2*
gamma(4/3) + 2916*(x - 1)*gamma(4/3)) - 4840*(x - 1)**2*exp(I*pi/
3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*
(x - 1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x
 - 1)**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x -

```
 1)**8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x -
1)**6*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)*
*4*gamma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*g
amma(4/3) + 2916*(x - 1)*gamma(4/3)) - 440*(x - 1)*exp(5*I*pi/3)*
log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x -
1)**12*gamma(4/3) + 32076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)
**10*gamma(4/3) + 481140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**
8*gamma(4/3) + 1347192*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6
*gamma(4/3) + 962280*(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*ga
mma(4/3) + 160380*(x - 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(
4/3) + 2916*(x - 1)*gamma(4/3)) + 440*(x - 1)*log(-(x - 1)**(1/3)
*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 3
2076*(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481
140*(x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 134719
2*(x - 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*
(x - 1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x
- 1)**3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*g
amma(4/3)) - 440*(x - 1)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_pola
r(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**12*gamma(4/3) + 32076*
(x - 1)**11*gamma(4/3) + 160380*(x - 1)**10*gamma(4/3) + 481140*(
x - 1)**9*gamma(4/3) + 962280*(x - 1)**8*gamma(4/3) + 1347192*(x
- 1)**7*gamma(4/3) + 1347192*(x - 1)**6*gamma(4/3) + 962280*(x -
1)**5*gamma(4/3) + 481140*(x - 1)**4*gamma(4/3) + 160380*(x - 1)*
*3*gamma(4/3) + 32076*(x - 1)**2*gamma(4/3) + 2916*(x - 1)*gamma(
4/3))
```

GIAC/XCAS [A] time = 0.203377, size = 111, normalized size = 1.07

$$\frac{110}{243}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right)+\frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\,x^4}-\frac{55}{243}\ln\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+\frac{110}{243}\ln\left((x-1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)^(2/3)*x^5),x, algorithm="giac")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324
*(220*(x - 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 5
32*(x - 1)^(1/3))/x^4 - 55/243*ln((x - 1)^(2/3) - (x - 1)^(1/3) +
 1) + 110/243*ln((x - 1)^(1/3) + 1)
```

3.218 $\int \sqrt{\frac{1-x}{1+x}}\, dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi [A] time = 0.0360083, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi in Sympy [A] time = 1.98876, size = 32, normalized size = 0.84

$$\frac{2\sqrt{\frac{-x+1}{x+1}}}{\frac{-x+1}{x+1}+1}-2\,\mathrm{atan}\left(\sqrt{\frac{-x+1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] 2*sqrt((-x + 1)/(x + 1))/((-x + 1)/(x + 1) + 1) - 2*atan(sqrt((-x
 + 1)/(x + 1)))
```

Mathematica [A] time = 0.0374179, size = 62, normalized size = 1.63

$$\frac{\sqrt{\frac{1-x}{x+1}}\left(\sqrt{1-x}(x+1)+2\sqrt{x+1}\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*(Sqrt[1 - x]*(1 + x) + 2*Sqrt[1 + x]*ArcSi
n[Sqrt[1 + x]/Sqrt[2]]))/Sqrt[1 - x]
```

Maple [A] time = 0.007, size = 39, normalized size = 1.

$$(1+x)\sqrt{-\frac{-1+x}{1+x}}\left(\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-(-1+x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)/(1+x))^(1/2),x)
```

```
[Out] (-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)
+arcsin(x))
```

Maxima [A] time = 1.5196, size = 58, normalized size = 1.53

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-
(x - 1)/(x + 1)))
```

Fricas [A] time = 0.214837, size = 43, normalized size = 1.13

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1)),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-x+1}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-x + 1)/(x + 1)), x)
```

GIAC/XCAS [A] time = 0.2053, size = 39, normalized size = 1.03

$$\frac{1}{2}\,\pi \operatorname{sign}\left(x+1\right) + \arcsin\left(x\right) \operatorname{sign}\left(x+1\right) + \sqrt{-x^{2}+1}\operatorname{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sign(x + 1) + arcsin(x)*sign(x + 1) + sqrt(-x^2 + 1)*sign(
x + 1)
```

3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}(b-x)^2\sqrt{\frac{x-a}{b-x}}+\frac{1}{4}(a-5b)(b-x)\sqrt{\frac{x-a}{b-x}}-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{\frac{x-a}{b-x}}\right)$$

```
[Out] ((a - 5*b)*(b - x)*Sqrt[(-a + x)/(b - x)])/4 + ((b - x)^2*Sqrt[(-
a + x)/(b - x)])/2 - ((a - b)*(a + 3*b)*ArcTan[Sqrt[(-a + x)/(b -
 x)]])/4
```

Rubi [A] time = 0.129557, antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{1}{2}(b-x)^2\sqrt{-\frac{a-x}{b-x}}+\frac{1}{4}(a-5b)(b-x)\sqrt{-\frac{a-x}{b-x}}-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{-\frac{a-x}{b-x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] ((a - 5*b)*Sqrt[-((a - x)/(b - x))]*(b - x))/4 + (Sqrt[-((a - x)/
(b - x))]*(b - x)^2)/2 - ((a - b)*(a + 3*b)*ArcTan[Sqrt[-((a - x)
/(b - x))]])/4
```

Rubi in Sympy [A] time = 7.59273, size = 80, normalized size = 0.87

$$-\frac{\sqrt{\frac{-a+x}{b-x}}\,(a-5b)\,(a-b)}{4\left(\frac{-a+x}{b-x}+1\right)}+\frac{\sqrt{\frac{-a+x}{b-x}}\,(a-b)^2}{2\left(\frac{-a+x}{b-x}+1\right)^2}-\left(\frac{a}{4}-\frac{b}{4}\right)(a+3b)\,\mathrm{atan}\left(\sqrt{\frac{-a+x}{b-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((-a+x)/(b-x))**(1/2),x)
```

```
[Out] -sqrt((-a + x)/(b - x))*(a - 5*b)*(a - b)/(4*((-a + x)/(b - x) +
1)) + sqrt((-a + x)/(b - x))*(a - b)**2/(2*((-a + x)/(b - x) + 1)
**2) - (a/4 - b/4)*(a + 3*b)*atan(sqrt((-a + x)/(b - x)))
```

Mathematica [A] time = 0.193515, size = 110, normalized size = 1.2

$$\frac{\sqrt{\frac{x-a}{b-x}}\left(2\sqrt{x-a}(b-x)(a-3b-2x)-\left(-a^2-2ab+3b^2\right)\sqrt{b-x}\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x-a}\sqrt{b-x}}\right)\right)}{8\sqrt{x-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] (Sqrt[(-a + x)/(b - x)]*(2*(a - 3*b - 2*x)*(b - x)*Sqrt[-a + x] -
 (-a^2 - 2*a*b + 3*b^2)*Sqrt[b - x]*ArcTan[(a + b - 2*x)/(2*Sqrt[
b - x]*Sqrt[-a + x])]))/(8*Sqrt[-a + x])
```

Maple [B] time = 0.031, size = 208, normalized size = 2.3

$$\frac{-b+x}{8}\sqrt{-\frac{-a+x}{-b+x}}\left(\arctan\left(\frac{-a+2x-b}{2}\frac{1}{\sqrt{-ab+ax+bx-x^2}}\right)a^2+2\arctan\left(1/2\frac{-a+2x-b}{\sqrt{-ab+ax+bx-x^2}}\right)ab-3\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((-a+x)/(b-x))^(1/2),x)
```

```
[Out] 1/8*(-(-a+x)/(-b+x))^(1/2)*(-b+x)*(arctan(1/2*(-a+2*x-b)/(-a*b+a*
x+b*x-x^2)^(1/2))*a^2+2*arctan(1/2*(-a+2*x-b)/(-a*b+a*x+b*x-x^2)^
(1/2))*a*b-3*arctan(1/2*(-a+2*x-b)/(-a*b+a*x+b*x-x^2)^(1/2))*b^2+
4*(-a*b+a*x+b*x-x^2)^(1/2)*x-2*(-a*b+a*x+b*x-x^2)^(1/2)*a+6*b*(-a
*b+a*x+b*x-x^2)^(1/2))/(-(-b+x)*(-a+x))^(1/2)
```

Maxima [A] time = 1.49273, size = 176, normalized size = 1.91

$$-\frac{1}{4}\left(a^2+2ab-3b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)-\frac{\left(a^2-6ab+5b^2\right)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}}-\left(a^2+2ab-3b^2\right)\sqrt{-\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2}-\frac{2(a-x)}{b-x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(a - x)/(b - x)),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) - 1/4*(
(a^2 - 6*a*b + 5*b^2)*(-(a - x)/(b - x))^(3/2) - (a^2 + 2*a*b - 3
*b^2)*sqrt(-(a - x)/(b - x)))/((a - x)^2/(b - x)^2 - 2*(a - x)/(b
 - x) + 1)
```

Fricas [A] time = 0.208526, size = 99, normalized size = 1.08

$$-\frac{1}{4}\left(a^2+2ab-3b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)+\frac{1}{4}\left(ab-3b^2-(a-b)x+2x^2\right)\sqrt{-\frac{a-x}{b-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(a - x)/(b - x)),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) + 1/4*(
a*b - 3*b^2 - (a - b)*x + 2*x^2)*sqrt(-(a - x)/(b - x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((-a+x)/(b-x))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216916, size = 139, normalized size = 1.51

$$\frac{1}{8}\left(a^2\mathrm{sign}\left(-b+x\right)+2\,ab\mathrm{sign}\left(-b+x\right)-3\,b^2\mathrm{sign}\left(-b+x\right)\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sign}\left(-a+b\right)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}\left(a\mathrm{sign}\left(-b+x\right)-3\,b\mathrm{sign}\left(-b+x\right)-2\,x\mathrm{sign}\left(-b+x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(a - x)/(b - x)),x, algorithm="giac")
```

```
[Out] 1/8*(a^2*sign(-b + x) + 2*a*b*sign(-b + x) - 3*b^2*sign(-b + x))*
arcsin((a + b - 2*x)/(a - b))*sign(-a + b) - 1/4*sqrt(-a*b + a*x
+ b*x - x^2)*(a*sign(-b + x) - 3*b*sign(-b + x) - 2*x*sign(-b + x
))
```

3.220 $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx$

Optimal. Leaf size=54

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3
+ x])/Sqrt[-5 + x]]/(3*Sqrt[5])
```

Rubi [A] time = 0.841077, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3
+ x])/Sqrt[-5 + x]]/(3*Sqrt[5])
```

Rubi in Sympy [A] time = 42.7577, size = 44, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x-5}\sqrt{x+3}}{4}\right)}{6}+\frac{\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
```

```
[Out] atan(sqrt(x - 5)*sqrt(x + 3)/4)/6 + sqrt(5)*atanh(sqrt(5)*sqrt(x
+ 3)/sqrt(x - 5))/15
```

Mathematica [A] time = 0.0664899, size = 64, normalized size = 1.19

$$\frac{1}{30}\left(\sqrt{5}\left(\log(x+5)-\log\left(3x-\sqrt{5}\sqrt{x-5}\sqrt{x+3}+5\right)\right)-5\tan^{-1}\left(\frac{4}{\sqrt{x-5}\sqrt{x+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] (-5*ArcTan[4/(Sqrt[-5 + x]*Sqrt[3 + x])] + Sqrt[5]*(Log[5 + x] -
Log[5 + 3*x - Sqrt[5]*Sqrt[-5 + x]*Sqrt[3 + x]]))/30
```

Maple [A] time = 0.033, size = 64, normalized size = 1.2

$$\frac{1}{30}\sqrt{-5+x}\sqrt{3+x}\left(\sqrt{5}Artanh\left(\frac{(5+3\,x)\sqrt{5}}{5}\frac{1}{\sqrt{x^2-2\,x-15}}\right)-5\,\arctan\left(4\,\frac{1}{\sqrt{x^2-2\,x-15}}\right)\right)\frac{1}{\sqrt{x^2-2\,x-15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x)
```

```
[Out] 1/30*(-5+x)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2
)/(x^2-2*x-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+3}\sqrt{x-5}}{(x^2-25)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)
```

Fricas [A] time = 0.233124, size = 128, normalized size = 2.37

$$\frac{1}{30}\sqrt{5}\left(2\sqrt{5}\arctan\left(\frac{1}{4}\sqrt{x+3}\sqrt{x-5}-\frac{1}{4}\,x+\frac{1}{4}\right)+\log\left(\frac{\left(\sqrt{5}(x+5)+10\right)\sqrt{x+3}\sqrt{x-5}-\sqrt{5}\left(x^2+4\,x+15\right)-10\,x-50}{(x+5)\sqrt{x+3}\sqrt{x-5}-x^2-4\,x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(5)*(2*sqrt(5)*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*
x + 1/4) + log(((sqrt(5)*(x + 5) + 10)*sqrt(x + 3)*sqrt(x - 5) -
sqrt(5)*(x^2 + 4*x + 15) - 10*x - 50)/((x + 5)*sqrt(x + 3)*sqrt(x
 - 5) - x^2 - 4*x + 5)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+3}}{\sqrt{x-5}\,(x-1)\,(x+5)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
```

```
[Out] Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

GIAC/XCAS [A] time = 0.210029, size = 100, normalized size = 1.85

$$-\frac{1}{30}\,\sqrt{5}\ln\left(\frac{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2-4\,\sqrt{5}+12}{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2+4\,\sqrt{5}+12}\right)-\frac{1}{3}\,\arctan\left(\frac{1}{8}\,\left(\sqrt{x+3}-\sqrt{x-5}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(5)*ln(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)
/((sqrt(x + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1
/8*(sqrt(x + 3) - sqrt(x - 5))^2)
```

3.221 $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

Optimal. Leaf size=304

$$\frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4}+\frac{x\left(1-x^2\right)^{5/4}}{6\sqrt{1-x}}+\frac{7\left(1-x^2\right)^{5/4}}{24\sqrt{1-x}}$$
$$+\frac{1}{24}(x+1)^{3/4}(1-x)^{5/4}+\frac{5}{16}\sqrt[4]{x+1}(1-x)^{3/4}-\frac{1}{16}(x+1)^{3/4}\sqrt[4]{1-x}+\frac{\log\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{8\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{8\sqrt{2}}$$

```
[Out] (5*(1 - x)^(3/4)*(1 + x)^(1/4))/16 - ((1 - x)^(1/4)*(1 + x)^(3/4)
)/16 + ((1 - x)^(5/4)*(1 + x)^(3/4))/24 + (7*(1 - x^2)^(5/4))/(24
*Sqrt[1 - x]) + (x*(1 - x^2)^(5/4))/(6*Sqrt[1 - x]) + (Sqrt[1 + x
]*(1 - x^2)^(5/4))/6 - (3*ArcTan[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 +
 x)^(1/4)])/(8*Sqrt[2]) + (3*ArcTan[1 + (Sqrt[2]*(1 - x)^(1/4))/(
1 + x)^(1/4)])/(8*Sqrt[2]) + Log[1 + Sqrt[1 - x]/Sqrt[1 + x] - (S
qrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/(8*Sqrt[2]) - Log[1 + Sqrt[1
 - x]/Sqrt[1 + x] + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/(8*Sqr
t[2])
```

Rubi [A] time = 1.36897, antiderivative size = 319, normalized size of antiderivative = 1.05, number of steps used = 31, number of rules used = 14, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}+\frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4}$$
$$+\frac{1}{6}(1-x)^{7/4}(x+1)^{5/4}+\frac{1}{24}(1-x)^{5/4}(x+1)^{3/4}-\frac{1}{16}\sqrt[4]{1-x}(x+1)^{3/4}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{x+1}-\frac{5}{48}(1-x)^{3/4}\sqrt[4]{x+1}+\frac{\log\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 + x])),x]
```

```
[Out] (-5*(1 - x)^(3/4)*(1 + x)^(1/4))/48 + (5*(1 - x)^(7/4)*(1 + x)^(1
/4))/24 - ((1 - x)^(1/4)*(1 + x)^(3/4))/16 + ((1 - x)^(5/4)*(1 +
x)^(3/4))/24 + ((1 - x)^(7/4)*(1 + x)^(5/4))/6 + (Sqrt[1 + x]*(1
- x^2)^(5/4))/6 + (1 - x^2)^(9/4)/(3*(1 - x)^(3/2)) - (3*ArcTan[1
 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*Sqrt[2]) + (3*ArcTa
n[1 + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*Sqrt[2]) + Log[1
 + Sqrt[1 - x]/Sqrt[1 + x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4
)]/(8*Sqrt[2]) - Log[1 + Sqrt[1 - x]/Sqrt[1 + x] + (Sqrt[2]*(1 -
x)^(1/4))/(1 + x)^(1/4)]/(8*Sqrt[2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(-x+1)^{\frac{5}{4}}(x+1)^{\frac{3}{4}}}{24}-\frac{\sqrt[4]{-x+1}\,(x+1)^{\frac{3}{4}}}{16}+\frac{\sqrt{x+1}\,(-x^2+1)^{\frac{5}{4}}}{6}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt[4]{-x+1}}{\sqrt[4]{x+1}}+\frac{\sqrt{-x+1}}{\sqrt{x+1}}+1\right)}{64}+\frac{\sqrt{2}\log\left(\frac{\sqrt{2}\sqrt[4]{-x+1}}{\sqrt[4]{x+1}}+\frac{\sqrt{-x+1}}{\sqrt{x+1}}+1\right)}{64}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt[4]{-x+1}}{\sqrt[4]{x+1}}-1\right)}{32}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt[4]{-x+1}}{\sqrt[4]{x+1}}+1\right)}{32}+\int^{\sqrt{-x+1}}(x^2-2)(x^2-1)\sqrt[4]{-(-x^2+1)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-
x)**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] (-x + 1)**(5/4)*(x + 1)**(3/4)/24 - (-x + 1)**(1/4)*(x + 1)**(3/4
)/16 + sqrt(x + 1)*(-x**2 + 1)**(5/4)/6 - sqrt(2)*log(-sqrt(2)*(-
x + 1)**(1/4)/(x + 1)**(1/4) + sqrt(-x + 1)/sqrt(x + 1) + 1)/64 +
 sqrt(2)*log(sqrt(2)*(-x + 1)**(1/4)/(x + 1)**(1/4) + sqrt(-x + 1
)/sqrt(x + 1) + 1)/64 + sqrt(2)*atan(sqrt(2)*(-x + 1)**(1/4)/(x +
 1)**(1/4) - 1)/32 + sqrt(2)*atan(sqrt(2)*(-x + 1)**(1/4)/(x + 1)
**(1/4) + 1)/32 + Integral((x**2 - 2)*(x**2 - 1)*(-(-x**2 + 1)**2
 + 1)**(1/4), (x, sqrt(-x + 1)))
```

Mathematica [C] time = 0.39659, size = 165, normalized size = 0.54

$$\frac{\sqrt[4]{-(x-1)^2-2(x-1)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1-x}{2}\right)}{8\sqrt[4]{2}\sqrt[4]{x+1}}+\frac{5\left(-(x-1)^2-2(x-1)\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1-x}{2}\right)}{24\,2^{3/4}(x+1)^{3/4}}-\frac{1}{48}\sqrt{x+1}\sqrt[4]{1-x^2}\left(8x^2-\frac{\sqrt{1-x^2}\,(8x^2+22x+29)}{x+1}+2x-7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 + x
```

```
[Out] -(Sqrt[1 + x]*(1 - x^2)^(1/4)*(-7 + 2*x + 8*x^2 - (Sqrt[1 - x^2]*
(29 + 22*x + 8*x^2))/(1 + x)))/48 + ((-2*(-1 + x) - (-1 + x)^2)^(
1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - x)/2])/(8*2^(1/4)*(1 +
 x)^(1/4)) + (5*(-2*(-1 + x) - (-1 + x)^2)^(3/4)*Hypergeometric2F
1[3/4, 3/4, 7/4, (1 - x)/2])/(24*2^(3/4)*(1 + x)^(3/4))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x^{2}\sqrt[4]{-x^{2}+1}\,\sqrt{1+x}\,\frac{1}{\sqrt{1-x}}\left(\sqrt{1-x}-\sqrt{1+x}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
```

```
[Out] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)
^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-x^{2}+1\right)^{\frac{1}{4}}\sqrt{x+1}\,x^{2}}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sqrt(-
x + 1))),x, algorithm="maxima")
```

```
[Out] -integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x
 + 1) - sqrt(-x + 1))), x)
```

Fricas [A] time = 0.259284, size = 774, normalized size = 2.55

$$-\frac{1}{48}\left(8x^2+2x-7\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}+\frac{1}{48}\left(8x^2+22x+29\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}$$
$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}(x+1)\sqrt{\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}+x+\sqrt{-x^2+1}+1}{x+1}}+\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}+x+1}\right)$$
$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}(x+1)\sqrt{-\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}-x-\sqrt{-x^2+1}-1}{x+1}}+\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}-x-1}\right)$$
$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{x-1}{\sqrt{2}(x-1)\sqrt{\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}+x-\sqrt{-x^2+1}-1}{x-1}}+\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}+x-1}\right)$$
$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{x-1}{\sqrt{2}(x-1)\sqrt{-\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}-x+\sqrt{-x^2+1}+1}{x-1}}+\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}-x+1}\right)$$
$$+\frac{1}{64}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}+x+\sqrt{-x^2+1}+1\right)}{x+1}\right)$$
$$-\frac{1}{64}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}-x-\sqrt{-x^2+1}-1\right)}{x+1}\right)$$
$$+\frac{5}{64}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}+x-\sqrt{-x^2+1}-1\right)}{x-1}\right)$$
$$-\frac{5}{64}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}-x+\sqrt{-x^2+1}+1\right)}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sqrt(-
x + 1))),x, algorithm="fricas")
```

```
[Out] -1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + 1/48*(8*x^
2 + 22*x + 29)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - 1/16*sqrt(2)*arcta
n((x + 1)/(sqrt(2)*(x + 1)*sqrt((sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x
+ 1) + x + sqrt(-x^2 + 1) + 1)/(x + 1)) + sqrt(2)*(-x^2 + 1)^(1/4
)*sqrt(x + 1) + x + 1)) - 1/16*sqrt(2)*arctan((x + 1)/(sqrt(2)*(x
 + 1)*sqrt(-(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sqrt(-x^2
 + 1) - 1)/(x + 1)) + sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x -
1)) - 5/16*sqrt(2)*arctan((x - 1)/(sqrt(2)*(x - 1)*sqrt((sqrt(2)*
```

```
(-x^2 + 1)^(1/4)*sqrt(-x + 1) + x - sqrt(-x^2 + 1) - 1)/(x - 1))
+ sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + x - 1)) - 5/16*sqrt(2)*
arctan((x - 1)/(sqrt(2)*(x - 1)*sqrt(-(sqrt(2)*(-x^2 + 1)^(1/4)*s
qrt(-x + 1) - x + sqrt(-x^2 + 1) + 1)/(x - 1)) + sqrt(2)*(-x^2 +
1)^(1/4)*sqrt(-x + 1) - x + 1)) + 1/64*sqrt(2)*log(2*(sqrt(2)*(-x
^2 + 1)^(1/4)*sqrt(x + 1) + x + sqrt(-x^2 + 1) + 1)/(x + 1)) - 1/
64*sqrt(2)*log(-2*(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sqr
t(-x^2 + 1) - 1)/(x + 1)) + 5/64*sqrt(2)*log(2*(sqrt(2)*(-x^2 + 1
)^(1/4)*sqrt(-x + 1) + x - sqrt(-x^2 + 1) - 1)/(x - 1)) - 5/64*sq
rt(2)*log(-2*(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - x + sqrt(-x
^2 + 1) + 1)/(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-
(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sqrt(-
x + 1))),x, algorithm="giac")
```

```
[Out] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x
 + 1) - sqrt(-x + 1))), x)
```

3.222 $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$

Optimal. Leaf size=292

$$-\frac{1}{12}(1-x)^{2/3}\sqrt[3]{x+1}(1-3x)-\frac{1}{4}(1-x)(x+3)+\frac{1}{12}\sqrt[3]{1-x}(x+1)^{2/3}(3x+1)$$

$$+\frac{1}{12}\sqrt[6]{1-x}(x+1)^{5/6}(3x+2)-\frac{1}{12}(1-x)^{5/6}\sqrt[6]{x+1}(3x+10)+\frac{1}{4}\sqrt{1-x}x\sqrt{x+1}+\frac{1}{6}\tan^{-1}\left(\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}\right)-\frac{4\tan^{-1}\left(\frac{\sqrt[3]{1-x}-2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{3\sqrt{3}}$$

```
[Out] -((1 - 3*x)*(1 - x)^(2/3)*(1 + x)^(1/3))/12 + (Sqrt[1 - x]*x*Sqrt
[1 + x])/4 - ((1 - x)*(3 + x))/4 + ((1 - x)^(1/3)*(1 + x)^(2/3)*(
1 + 3*x))/12 + ((1 - x)^(1/6)*(1 + x)^(5/6)*(2 + 3*x))/12 - ((1 -
 x)^(5/6)*(1 + x)^(1/6)*(10 + 3*x))/12 + ArcTan[(1 + x)^(1/6)/(1
- x)^(1/6)]/6 - (4*ArcTan[((1 - x)^(1/3) - 2*(1 + x)^(1/3))/(Sqrt
[3]*(1 - x)^(1/3))])/(3*Sqrt[3]) - (5*ArcTan[((1 - x)^(1/3) - (1
+ x)^(1/3))/((1 - x)^(1/6)*(1 + x)^(1/6))])/6 + ArcTanh[(Sqrt[3]*
(1 - x)^(1/6)*(1 + x)^(1/6))/((1 - x)^(1/3) + (1 + x)^(1/3))]/(6*
Sqrt[3])
```

Rubi [A] time = 3.29845, antiderivative size = 522, normalized size of antiderivative = 1.79, number of steps used = 44, number of rules used = 19, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.339

$$\frac{x^2}{4}+\frac{1}{4}\sqrt{1-x^2}x+\frac{x}{2}-\frac{1}{4}(1-x)^{5/6}(x+1)^{7/6}$$

$$-\frac{1}{4}(1-x)^{7/6}(x+1)^{5/6}+\frac{5}{12}\sqrt[6]{1-x}(x+1)^{5/6}-\frac{1}{4}(1-x)^{4/3}(x+1)^{2/3}+\frac{1}{3}\sqrt[3]{1-x}(x+1)^{2/3}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{x+1}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{x+1}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)^(2/3)*
```

```
[Out] x/2 + x^2/4 - (7*(1 - x)^(5/6)*(1 + x)^(1/6))/12 + ((1 - x)^(2/3)
*(1 + x)^(1/3))/6 - ((1 - x)^(5/3)*(1 + x)^(1/3))/4 + ((1 - x)^(1
/3)*(1 + x)^(2/3))/3 - ((1 - x)^(4/3)*(1 + x)^(2/3))/4 + (5*(1 -
x)^(1/6)*(1 + x)^(5/6))/12 - ((1 - x)^(7/6)*(1 + x)^(5/6))/4 - ((
1 - x)^(5/6)*(1 + x)^(7/6))/4 + (x*Sqrt[1 - x^2])/4 + ArcSin[x]/4
 - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3 + (2*ArcTan[1/Sqrt[3
] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3]) + Arc
Tan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3]
 + (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - (2*ArcTan[1/Sqrt[3] - (2*
(1 + x)^(1/3))/(Sqrt[3]*(1 - x)^(1/3))])/(3*Sqrt[3]) - Log[1 - x]
/9 + Log[1 + x]/9 + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3 - Log[
1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)
^(1/6)]/(12*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqr
t[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) - Log[1 + (1 + x)
```

```
^(1/3)/(1 - x)^(1/3)]/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(2/3)*(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.23889, size = 391, normalized size = 1.34

$$-\frac{1}{12}\sqrt[3]{x+1}\left(-4\ 2^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x+1}{2}\right)+\frac{(3x+10)\left(1-x^2\right)^{5/6}}{x+1}-\frac{(3x+2)\sqrt{1-x^2}}{\sqrt[3]{1-x}}-(3x+1)\sqrt[3]{1-x^2}-3\sqrt[3]{1-x}x\sqrt[6]{1-x^2}-\frac{3\sqrt[3]{1-x}x(x+2)}{\sqrt[3]{1-x^2}}+(1-x)^{2/3}(1-3x)\right)-\frac{2^{2/3}\sqrt[3]{-(x-1)^2-2(x-1)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1-x}{2}\right)}{3\sqrt[3]{x+1}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)^
```

```
[Out] -(2^(2/3)*(-2*(-1 + x) - (-1 + x)^2)^(1/3)*Hypergeometric2F1[1/3,
 1/3, 4/3, (1 - x)/2])/(3*(1 + x)^(1/3)) - ((1 + x)^(1/3)*((1 - 3
*x)*(1 - x)^(2/3) - (3*(1 - x)^(1/3)*x*(2 + x))/(1 - x^2)^(1/3) -
 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6) - (1 + 3*x)*(1 - x^2)^(1/3) -
((2 + 3*x)*Sqrt[1 - x^2])/(1 - x)^(1/3) + ((10 + 3*x)*(1 - x^2)^(
5/6))/(1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x
)/2]))/12 - (7*(-2*(-1 + x) - (-1 + x)^2)^(5/6)*Hypergeometric2F1
[5/6, 5/6, 11/6, (1 - x)/2])/(30*2^(5/6)*(1 + x)^(5/6)) + ((1 + x
)^(1/3)*Sqrt[2*(1 + x) - (1 + x)^2]*Hypergeometric2F1[5/6, 5/6, 1
1/6, (1 + x)/2])/(6*2^(5/6)*Sqrt[1 - x]) + ((1 - x)^(1/3)*Sqrt[-1
 + x]*(1 + x)^(5/6)*Log[Sqrt[-1 + x] + Sqrt[1 + x]])/(2*(2*(1 + x
) - (1 + x)^2)^(5/6))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x\left(1+x\right)^{\frac{2}{3}}\sqrt{1-x}\left(-\left(1-x\right)^{\frac{5}{6}}\sqrt[3]{1+x}+\left(1-x\right)^{\frac{2}{3}}\sqrt{1+x}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(1/2
```

```
[Out] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/
3)*(1+x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{2}{3}} x \sqrt{-x+1}}{\sqrt{x+1}(-x+1)^{\frac{2}{3}} - (x+1)^{\frac{1}{3}}(-x+1)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1)^(1/3)
x + 1)^(5/6)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3
) - (x + 1)^(1/3)*(-x + 1)^(5/6)), x)
```

Fricas [A] time = 0.431658, size = 1841, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1)^(1/3)
x + 1)^(5/6)),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(3)*(3*x^3 - 4*x^2 + 2*x - 6)*(x + 1)^(5/6)*(-x + 1)^
(1/6) + 6*sqrt(3)*(3*x^3 + 7*x^2 + 8*x - 22)*(x + 1)^(2/3)*(-x +
1)^(1/3) + 18*sqrt(3)*(x^3 + 2*x^2 + 2*x)*sqrt(x + 1)*sqrt(-x + 1
) + 6*sqrt(3)*(3*x^3 + 5*x^2 + 4*x + 6)*(x + 1)^(1/3)*(-x + 1)^(2
/3) - 6*sqrt(3)*(3*x^3 + 4*x^2 - 14*x - 22)*(x + 1)^(1/6)*(-x + 1
)^(5/6) - 48*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*arctan(-1/3*(
sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(-x + 1)^(1/3))/(x + 1)
) + 20*(sqrt(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1))*a
rctan((x + 1)/(sqrt(3)*(x + 1) + 2*(x + 1)*sqrt((sqrt(3)*(x + 1)^
(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x +
 1)) + 2*(x + 1)^(5/6)*(-x + 1)^(1/6))) + 20*(sqrt(3)*(x^2 - 2) +
 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1))*arctan(-(x + 1)/(sqrt(3)*(x
+ 1) - 2*(x + 1)*sqrt(-(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x
- (x + 1)^(2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 2*(x + 1)^(5/6)*(-
x + 1)^(1/6))) - 48*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*arctan
(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x + 1)^(1/3)*(-x + 1)^(2/3))/(
```

```
x - 1)) + 28*(sqrt(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x +
 1))*arctan((x - 1)/(sqrt(3)*(x - 1) + 2*(x - 1)*sqrt((sqrt(3)*(x
 + 1)^(1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1
)/(x - 1)) + 2*(x + 1)^(1/6)*(-x + 1)^(5/6))) + 28*(sqrt(3)*(x^2
- 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1))*arctan(-(x - 1)/(sqrt(
3)*(x - 1) - 2*(x - 1)*sqrt(-(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6
) - x + (x + 1)^(1/3)*(-x + 1)^(2/3) + 1)/(x - 1)) - 2*(x + 1)^(1
/6)*(-x + 1)^(5/6))) - 20*(sqrt(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x +
 1)*sqrt(-x + 1))*arctan((-x + 1)^(1/6)/(x + 1)^(1/6)) - 28*(sqrt
(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1))*arctan((x + 1
)^(1/6)*(-x + 1)^(5/6)/(x - 1)) - 36*(sqrt(3)*(x^2 - 2) + 2*sqrt(
3)*sqrt(x + 1)*sqrt(-x + 1))*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x) - 15*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*log(25*(sqrt(3)*
(x + 1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) +
 1)/(x + 1)) + 15*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*log(-25*
(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x - (x + 1)^(2/3)*(-x + 1
)^(1/3) - 1)/(x + 1)) - 16*(sqrt(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x
+ 1)*sqrt(-x + 1))*log((x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x
+ 1)) + 8*(sqrt(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1)
)*log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*(-x + 1)^
(2/3) + 1)/(x + 1)) - 21*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*l
og(49*(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(
-x + 1)^(2/3) - 1)/(x - 1)) + 21*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1
) - 2)*log(-49*(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1
)^(1/3)*(-x + 1)^(2/3) + 1)/(x - 1)) - 8*(sqrt(3)*(x^2 - 2) + 2*s
qrt(3)*sqrt(x + 1)*sqrt(-x + 1))*log((x - (x + 1)^(2/3)*(-x + 1)^
(1/3) + (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 16*(sqrt(3)*
(x^2 - 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1))*log(-(x - (x + 1)
^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 18*sqrt(3)*(x^4 - 2*x^2 - 2
*x))/(sqrt(3)*(x^2 - 2) + 2*sqrt(3)*sqrt(x + 1)*sqrt(-x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-x+1}\,(x+1)^{\frac{2}{3}}}{-(-x+1)^{\frac{5}{6}}\,\sqrt[3]{x+1}+(-x+1)^{\frac{2}{3}}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-
x)**(2/3)*(1+x)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(-x + 1)*(x + 1)**(2/3)/(-(-x + 1)**(5/6)*(x + 1)*
*(1/3) + (-x + 1)**(2/3)*sqrt(x + 1)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1)^(1/3)
x + 1)^(5/6)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.223 $\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(x-1)^4(x+1)^2}}$$

```
[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

Rubi [A] time = 0.0327394, antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3(1-x)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(2*((1 - x)^4*(1 + x)^2)^(1/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^4\,(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

Mathematica [A] time = 0.0175971, size = 25, normalized size = 1.

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(x-1)^4(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

Maple [A] time = 0.004, size = 22, normalized size = 0.9

$$-\frac{(-3+3x)(1+x)}{2}\frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^4*(1+x)^2)^(1/3),x)
```

```
[Out] -3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2*(x - 1)^4)^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

Fricas [A] time = 0.229943, size = 63, normalized size = 2.52

$$-\frac{3\left(x^6-2x^5-x^4+4x^3-x^2-2x+1\right)^{\frac{2}{3}}}{2\left(x^4-2x^3+2x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2*(x - 1)^4)^(-1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x
^3 + 2*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^4 (x+1)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2*(x - 1)^4)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

3.224 $\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx$

Optimal. Leaf size=25

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

```
[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

Rubi [A] time = 0.0343195, antiderivative size = 30, normalized size of antiderivative = 1.2, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{4(1-x)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)^3*(2 + x)^5)^(-1/4),x]

[Out] (-4*(1 - x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{(x-1)^3\,(x+2)^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)

[Out] Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

Mathematica [A] time = 0.0207509, size = 25, normalized size = 1.

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x)^3*(2 + x)^5)^(-1/4),x]

[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

Maple [A] time = 0.006, size = 22, normalized size = 0.9

$$\frac{(-4+4x)(2+x)}{3}\frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^3*(2+x)^5)^(1/4),x)

[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 2)^5*(x - 1)^3)^(-1/4),x, algorithm="maxima")

[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

Fricas [A] time = 0.204119, size = 93, normalized size = 3.72

$$\frac{4\left(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32\right)^{\frac{3}{4}}}{3\,(x^6+6x^5+9x^4-8x^3-24x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 2)^5*(x - 1)^3)^(-1/4),x, algorithm="fricas")

[Out] 4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16
*x - 32)^(3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{(x-1)^3 (x+2)^5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)
```

```
[Out] Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 2)^5*(x - 1)^3)^(-1/4),x, algorithm="giac")
```

```
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$

Optimal. Leaf size=53

$$\frac{9(x-1)^2(x+1)}{16\sqrt[3]{(x-1)^7(x+1)^2}}-\frac{3(x-1)(x+1)}{8\sqrt[3]{(x-1)^7(x+1)^2}}$$

```
[Out] (-3*(-1 + x)*(1 + x))/(8*((-1 + x)^7*(1 + x)^2)^(1/3)) + (9*(-1 +
 x)^2*(1 + x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

Rubi [A] time = 0.0518936, antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{9(x+1)(1-x)^2}{16\sqrt[3]{-(1-x)^7(x+1)^2}}+\frac{3(x+1)(1-x)}{8\sqrt[3]{-(1-x)^7(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(8*(-((1 - x)^7*(1 + x)^2))^(1/3)) + (9*(1 -
x)^2*(1 + x))/(16*(-((1 - x)^7*(1 + x)^2))^(1/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^7\,(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

Mathematica [A] time = 0.0242048, size = 30, normalized size = 0.57

$$\frac{3(x-1)(x+1)(3x-5)}{16\sqrt[3]{(x-1)^7(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

Maple [A] time = 0.005, size = 27, normalized size = 0.5

$$\frac{(-3+3x)(1+x)(3x-5)}{16}\frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^7*(1+x)^2)^(1/3),x)
```

```
[Out] 3/16*(-1+x)*(1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2*(x - 1)^7)^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

Fricas [A] time = 0.197622, size = 104, normalized size = 1.96

$$\frac{3\left(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1\right)^{\frac{2}{3}}(3x-5)}{16\left(x^7-5x^6+9x^5-5x^4-5x^3+9x^2-5x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2*(x - 1)^7)^(-1/3),x, algorithm="fricas")
```

```
[Out] 3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2
/3)*(3*x - 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x
+ 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^7\,(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2*(x - 1)^7)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\log(x+1)-\frac{3}{2}\log\left(1-\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)}{\sqrt[3]{(x-1)^2(x+1)}}+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(-1 + x))/((-1 + x)^2*(1 + x))^(1/3))/Sqrt
[3]] - Log[1 + x]/2 - (3*Log[1 - (-1 + x)/((-1 + x)^2*(1 + x))^(1
/3)])/2
```

Rubi [B] time = 0.235168, antiderivative size = 188, normalized size of antiderivative = 2.81, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{(1-x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{8(1-x)}{3}\right)}{2\sqrt[3]{x^3-x^2-x+1}}-\frac{3(1-x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+1\right)}{2\sqrt[3]{x^3-x^2-x+1}}-\frac{\sqrt{3}(1-x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{\sqrt[3]{x^3-x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] -((Sqrt[3]*(1 - x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 +
 x)^(1/3))/(Sqrt[3]*(1 - x)^(1/3))])/(1 - x - x^2 + x^3)^(1/3)) -
 ((1 - x)^(2/3)*(1 + x)^(1/3)*Log[(8*(1 - x))/3])/(2*(1 - x - x^2
 + x^3)^(1/3)) - (3*(1 - x)^(2/3)*(1 + x)^(1/3)*Log[1 + (1 + x)^(
1/3)/(1 - x)^(1/3)])/(2*(1 - x - x^2 + x^3)^(1/3))
```

Rubi in Sympy [A] time = 2.88529, size = 144, normalized size = 2.15

$$-\frac{3\,(x-1)^{\frac{2}{3}}\,\sqrt[3]{x+1}\log\left(-1+\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}\right)}{2\sqrt[3]{x^3-x^2-x+1}}-\frac{(x-1)^{\frac{2}{3}}\,\sqrt[3]{x+1}\log(x-1)}{2\sqrt[3]{x^3-x^2-x+1}}-\frac{\sqrt{3}\,(x-1)^{\frac{2}{3}}\,\sqrt[3]{x+1}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt[3]{x+1}}{3\sqrt[3]{x-1}}\right)}{\sqrt[3]{x^3-x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

```
[Out] -3*(x - 1)**(2/3)*(x + 1)**(1/3)*log(-1 + (x + 1)**(1/3)/(x - 1)*
*(1/3))/(2*(x**3 - x**2 - x + 1)**(1/3)) - (x - 1)**(2/3)*(x + 1)
**(1/3)*log(x - 1)/(2*(x**3 - x**2 - x + 1)**(1/3)) - sqrt(3)*(x
- 1)**(2/3)*(x + 1)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*(x + 1)**(1
/3)/(3*(x - 1)**(1/3)))/(x**3 - x**2 - x + 1)**(1/3)
```

Mathematica [C] time = 0.0228561, size = 49, normalized size = 0.73

$$\frac{3(x-1)\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1-x}{2}\right)}{\sqrt[3]{2}\sqrt[3]{(x-1)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - x
)/2])/(2^(1/3)*((-1 + x)^2*(1 + x))^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^2*(1+x))^(1/3),x)
```

```
[Out] int(1/((-1+x)^2*(1+x))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)*(x - 1)^2)^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

Fricas [A] time = 0.202478, size = 166, normalized size = 2.48

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x+2\left(x^3-x^2-x+1\right)^{\frac{1}{3}}-1\right)}{3\left(x-1\right)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-1)-2x+\left(x^3-x^2-x+1\right)^{\frac{2}{3}}+1}{x^2-2x+1}\right)-\log\left(-\frac{x-\left(x^3-x^2-x+1\right)^{\frac{1}{3}}-1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)*(x - 1)^2)^(-1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(x + 2*(x^3 - x^2 - x + 1)^(1/3) - 1)
/(x - 1)) + 1/2*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*
x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - log(-(x - (
x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[3]{\left(x-1\right)^2\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)*(x - 1)^2)^(-1/3),x, algorithm="giac")

[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

3.227 $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

Optimal. Leaf size=122

$$-\frac{4(x-2)(x+1)}{3\sqrt{(x-2)(x+1)^3}}-\frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{(x-2)(x+1)^3}}+\frac{2\sqrt{x-2}(x+1)^{3/2}\sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{(x-2)(x+1)^3}}$$

```
[Out] (-4*(-2 + x)*(1 + x))/(3*Sqrt[(-2 + x)*(1 + x)^3]) + (2*Sqrt[-2 +
 x]*(1 + x)^(3/2)*ArcSinh[Sqrt[-2 + x]/Sqrt[3]])/Sqrt[(-2 + x)*(1
 + x)^3] - (Sqrt[2]*Sqrt[-2 + x]*(1 + x)^(3/2)*ArcTan[(Sqrt[2]*Sq
rt[1 + x])/Sqrt[-2 + x]])/Sqrt[(-2 + x)*(1 + x)^3]
```

Rubi [A] time = 0.576834, antiderivative size = 133, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\frac{4(2-x)(x+1)}{3\sqrt{-(2-x)(x+1)^3}}-\frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{-(2-x)(x+1)^3}}+\frac{2\sqrt{x-2}(x+1)^{3/2}\sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{-(2-x)(x+1)^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] (4*(2 - x)*(1 + x))/(3*Sqrt[-((2 - x)*(1 + x)^3)]) + (2*Sqrt[-2 +
 x]*(1 + x)^(3/2)*ArcSinh[Sqrt[-2 + x]/Sqrt[3]])/Sqrt[-((2 - x)*(
1 + x)^3)] - (Sqrt[2]*Sqrt[-2 + x]*(1 + x)^(3/2)*ArcTan[(Sqrt[2]*
Sqrt[1 + x])/Sqrt[-2 + x]])/Sqrt[-((2 - x)*(1 + x)^3)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x\sqrt{(x-2)\,(x+1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)
```

Mathematica [A] time = 0.0964413, size = 112, normalized size = 0.92

$$-\frac{(x+1)\left(8(x-2)-6\sqrt{x+1}\sqrt{x-2}\log\left(-2x-2\sqrt{x-2}\sqrt{x+1}+1\right)+3\sqrt{2}\sqrt{x+1}\sqrt{x-2}\tan^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{x-2}\sqrt{x+1}}\right)\right)}{6\sqrt{(x-2)(x+1)^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] -((1 + x)*(8*(-2 + x) + 3*Sqrt[2]*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTan
[(4 + x)/(2*Sqrt[2]*Sqrt[-2 + x]*Sqrt[1 + x])] - 6*Sqrt[-2 + x]*S
qrt[1 + x]*Log[1 - 2*x - 2*Sqrt[-2 + x]*Sqrt[1 + x]]))/(6*Sqrt[(-
2 + x)*(1 + x)^3])
```

Maple [A] time = 0.03, size = 118, normalized size = 1.

$$\frac{1}{6}\left(-3\sqrt{2}\arctan\left(1/4\frac{\sqrt{2}(4+x)}{\sqrt{x^2-x-2}}\right)x+6\ln\left(x-1/2+\sqrt{x^2-x-2}\right)x-3\sqrt{2}\arctan\left(1/4\frac{\sqrt{2}(4+x)}{\sqrt{x^2-x-2}}\right)+6\ln\left(x-1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x)
```

```
[Out] 1/6*(-3*2^(1/2)*arctan(1/4*2^(1/2)*(4+x)/(x^2-x-2)^(1/2))*x+6*ln(
x-1/2+(x^2-x-2)^(1/2))*x-3*2^(1/2)*arctan(1/4*2^(1/2)*(4+x)/(x^2-
x-2)^(1/2))+6*ln(x-1/2+(x^2-x-2)^(1/2))-8*(x^2-x-2)^(1/2))*((1+x)
*(-2+x))^(1/2)/((-2+x)*(1+x)^3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)
```

Fricas [A] time = 0.215842, size = 185, normalized size = 1.52

$$\frac{3\sqrt{2}\left(x^2+2x+1\right)\arctan\left(-\frac{\sqrt{2}\left(x^2+x-\sqrt{x^4+x^3-3x^2-5x-2}\right)}{2(x+1)}\right)-4x^2-3\left(x^2+2x+1\right)\log\left(-\frac{2x^2+x-2\sqrt{x^4+x^3-3x^2-5x-2}-1}{x+1}\right)-8x}{3\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*sqrt(2)*(x^2 + x - sqr
t(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x +
 1)*log(-(2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x
 + 1)) - 8*x - 4*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*
x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x\sqrt{(x-2)(x+1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)
```

GIAC/XCAS [A] time = 0.266843, size = 230, normalized size = 1.89

$$-\frac{\sqrt{2}\arcsin\left(\frac{4}{3x}+\frac{1}{3}\right)}{2\,\mathrm{sign}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}-\frac{\ln\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3\right)}{\frac{4}{x}+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3\right)}{\frac{4}{x}+1}+6\right|}\right)}{\mathrm{sign}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}+\frac{8\sqrt{2}}{3\left(\frac{2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3}{\frac{4}{x}+1}-1\right)\mathrm{sign}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arcsin(4/3/x + 1/3)/sign(1/x^2 + 1/x^3) - ln(abs(-4*
sqrt(2) + 2*(2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x + 1) + 6)
```

```
/abs(4*sqrt(2) + 2*(2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x +
1) + 6))/sign(1/x^2 + 1/x^3) + 8/3*sqrt(2)/(((2*sqrt(2)*sqrt(-1/x
 - 2/x^2 + 1) - 3)/(4/x + 1) - 1)*sign(1/x^2 + 1/x^3))
```

3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{(x-1)^2(x+1)}}{x}+\frac{\log(x)}{6}-\frac{2}{3}\log(x+1)-\frac{3}{2}\log\left(1-\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\right)-\frac{1}{2}\log\left(\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2(x-1)}{\sqrt[3]{(x-1)^2(x+1)}}}{\sqrt{3}}\right)}{\sqrt{3}}-\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)}{\sqrt[3]{(x-1)^2(x+1)}}+1}{\sqrt{3}}\right)$$

```
[Out] -(((-1 + x)^2*(1 + x))^(1/3)/x) - ArcTan[(1 - (2*(-1 + x))/((-1 +
 x)^2*(1 + x))^(1/3))/Sqrt[3]]/Sqrt[3] - Sqrt[3]*ArcTan[(1 + (2*(
-1 + x))/((-1 + x)^2*(1 + x))^(1/3))/Sqrt[3]] + Log[x]/6 - (2*Log
[1 + x])/3 - (3*Log[1 - (-1 + x)/((-1 + x)^2*(1 + x))^(1/3)])/2 -
 Log[1 + (-1 + x)/((-1 + x)^2*(1 + x))^(1/3)]/2
```

Rubi [B] time = 0.740025, antiderivative size = 404, normalized size of antiderivative = 2.69, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{\sqrt[3]{x^3-x^2-x+1}}{x}+\frac{\sqrt[3]{x^3-x^2-x+1}\log(x)}{6(1-x)^{2/3}\sqrt[3]{x+1}}-\frac{\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{4(x+1)}{3}\right)}{2(1-x)^{2/3}\sqrt[3]{x+1}}-\frac{3\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)}{2(1-x)^{2/3}\sqrt[3]{x+1}}-\frac{\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{2^{2/3}\sqrt[3]{1-x}}{\sqrt[3]{3}}-\frac{2^{2/3}\sqrt[3]{x+1}}{\sqrt[3]{3}}\right)}{2(1-x)^{2/3}\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[3]{x^3-x^2-x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{(1-x)^{2/3}\sqrt[3]{x+1}}-\frac{\sqrt[3]{x^3-x^2-x+1}\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}(1-x)^{2/3}\sqrt[3]{x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] -((1 - x - x^2 + x^3)^(1/3)/x) - (Sqrt[3]*(1 - x - x^2 + x^3)^(1/
3)*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])
/((1 - x)^(2/3)*(1 + x)^(1/3)) - ((1 - x - x^2 + x^3)^(1/3)*ArcTa
n[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(Sqrt[3
]*(1 - x)^(2/3)*(1 + x)^(1/3)) + ((1 - x - x^2 + x^3)^(1/3)*Log[x
])/(6*(1 - x)^(2/3)*(1 + x)^(1/3)) - ((1 - x - x^2 + x^3)^(1/3)*L
og[(4*(1 + x))/3])/(2*(1 - x)^(2/3)*(1 + x)^(1/3)) - (3*(1 - x -
x^2 + x^3)^(1/3)*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)])/(2*(1 - x)
^(2/3)*(1 + x)^(1/3)) - ((1 - x - x^2 + x^3)^(1/3)*Log[(2^(2/3)*(
1 - x)^(1/3))/3^(1/3) - (2^(2/3)*(1 + x)^(1/3))/3^(1/3)])/(2*(1 -
```

```
x)^(2/3)*(1 + x)^(1/3))
```

Rubi in Sympy [A] time = 12.6542, size = 301, normalized size = 2.01

$$\frac{\sqrt[3]{x^3-x^2-x+1}\log(x)}{6(x-1)^{\frac{2}{3}}\sqrt[3]{x+1}}-\frac{\sqrt[3]{x^3-x^2-x+1}\log(x+1)}{2(x-1)^{\frac{2}{3}}\sqrt[3]{x+1}}-\frac{3\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x+1}}-1\right)}{2(x-1)^{\frac{2}{3}}\sqrt[3]{x+1}}-\frac{\sqrt[3]{x^3-x^2-x+1}\log\left(-\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)}{2(x-1)^{\frac{2}{3}}\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[3]{x^3-x^2-x+1}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x-1}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{3(x-1)^{\frac{2}{3}}\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[3]{x^3-x^2-x+1}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x-1}}{3\sqrt[3]{x+1}}+\frac{\sqrt{3}}{3}\right)}{(x-1)^{\frac{2}{3}}\sqrt[3]{x+1}}-\frac{\sqrt[3]{x^3-x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
```

```
[Out] (x**3 - x**2 - x + 1)**(1/3)*log(x)/(6*(x - 1)**(2/3)*(x + 1)**(1
/3)) - (x**3 - x**2 - x + 1)**(1/3)*log(x + 1)/(2*(x - 1)**(2/3)*
(x + 1)**(1/3)) - 3*(x**3 - x**2 - x + 1)**(1/3)*log((x - 1)**(1/
3)/(x + 1)**(1/3) - 1)/(2*(x - 1)**(2/3)*(x + 1)**(1/3)) - (x**3
- x**2 - x + 1)**(1/3)*log(-(x - 1)**(1/3) - (x + 1)**(1/3))/(2*(
x - 1)**(2/3)*(x + 1)**(1/3)) + sqrt(3)*(x**3 - x**2 - x + 1)**(1
/3)*atan(2*sqrt(3)*(x - 1)**(1/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)
/(3*(x - 1)**(2/3)*(x + 1)**(1/3)) - sqrt(3)*(x**3 - x**2 - x + 1
)**(1/3)*atan(2*sqrt(3)*(x - 1)**(1/3)/(3*(x + 1)**(1/3)) + sqrt(
3)/3)/((x - 1)**(2/3)*(x + 1)**(1/3)) - (x**3 - x**2 - x + 1)**(1
/3)/x
```

Mathematica [C] time = 0.298082, size = 145, normalized size = 0.97

$$\frac{1}{2}\sqrt[3]{(x-1)^2(x+1)}\left(-\frac{4xF_1\left(1;\frac{1}{3},\frac{2}{3};2;\frac{1}{x},-\frac{1}{x}\right)}{(x-1)(x+1)\left(6xF_1\left(1;\frac{1}{3},\frac{2}{3};2;\frac{1}{x},-\frac{1}{x}\right)-2F_1\left(2;\frac{1}{3},\frac{5}{3};3;\frac{1}{x},-\frac{1}{x}\right)+F_1\left(2;\frac{4}{3},\frac{2}{3};3;\frac{1}{x},-\frac{1}{x}\right)\right)}-\frac{3\,2^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x+1}{2}\right)}{(1-x)^{2/3}}-\frac{2}{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] (((-1 + x)^2*(1 + x))^(1/3)*(-2/x - (4*x*AppellF1[1, 1/3, 2/3, 2,
 x^(-1), -x^(-1)])/((-1 + x)*(1 + x)*(6*x*AppellF1[1, 1/3, 2/3, 2
```

```
, x^(-1), -x^(-1)] - 2*AppellF1[2, 1/3, 5/3, 3, x^(-1), -x^(-1)]
+ AppellF1[2, 4/3, 2/3, 3, x^(-1), -x^(-1)])) - (3*2^(2/3)*Hyperg
eometric2F1[1/3, 1/3, 4/3, (1 + x)/2])/(1 - x)^(2/3)))/2
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[3]{(-1+x)^2\,(1+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-1+x)^2*(1+x))^(1/3)/x^2,x)
```

```
[Out] int(((-1+x)^2*(1+x))^(1/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

Fricas [A] time = 0.216828, size = 387, normalized size = 2.58

$$\sqrt{3}\left(3\sqrt{3}x\log\left(\frac{x^2+\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-1)-2\,x+\left(x^3-x^2-x+1\right)^{\frac{2}{3}}+1}{x^2-2\,x+1}\right)+\sqrt{3}x\log\left(\frac{x^2-\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-1)-2\,x+\left(x^3-x^2-x+1\right)^{\frac{2}{3}}+1}{x^2-2\,x+1}\right)-2\sqrt{3}x\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(3*sqrt(3)*x*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x
 - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + s
qrt(3)*x*log((x^2 - (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^
3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - 2*sqrt(3)*x*log((x
```

```
 + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*sqrt(3)*x*log(-(x
- (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) + 18*x*arctan(1/3*sqrt(
3)*(x + 2*(x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*x*arctan(-1
/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3))/(x - 1
)) - 6*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{(x-1)^2(x+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

3.229 $\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{1-x}{12\left(x^2-2x-3\right)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

```
[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x +
 x^2])
```

Rubi [A] time = 0.0175085, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1-x}{12\left(x^2-2x-3\right)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x +
 x^2])
```

Rubi in Sympy [A] time = 0.765092, size = 36, normalized size = 0.84

$$-\frac{-4x+4}{96\sqrt{x^2-2x-3}}+\frac{-2x+2}{24\left(x^2-2x-3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x-3)**(5/2),x)
```

```
[Out] -(-4*x + 4)/(96*sqrt(x**2 - 2*x - 3)) + (-2*x + 2)/(24*(x**2 - 2*
x - 3)**(3/2))
```

Mathematica [A] time = 0.0176384, size = 27, normalized size = 0.63

$$\frac{(x-1)\left(x^2-2x-5\right)}{24\left(x^2-2x-3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] ((-1 + x)*(-5 - 2*x + x^2))/(24*(-3 - 2*x + x^2)^(3/2))
```

Maple [A] time = 0.006, size = 32, normalized size = 0.7

$$\frac{(1+x)(-3+x)\left(x^3-3x^2-3x+5\right)}{24}\left(x^2-2x-3\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x-3)^(5/2),x)
```

```
[Out] 1/24*(1+x)*(-3+x)*(x^3-3*x^2-3*x+5)/(x^2-2*x-3)^(5/2)
```

Maxima [A] time = 1.38311, size = 69, normalized size = 1.6

$$\frac{x}{24\sqrt{x^2-2x-3}}-\frac{1}{24\sqrt{x^2-2x-3}}-\frac{x}{12\left(x^2-2x-3\right)^{\frac{3}{2}}}+\frac{1}{12\left(x^2-2x-3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 3)^(-5/2),x, algorithm="maxima")
```

```
[Out] 1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x
^2 - 2*x - 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)
```

Fricas [A] time = 0.209939, size = 123, normalized size = 2.86

$$\frac{3x^2-3\sqrt{x^2-2x-3}(x-1)-6x-5}{12\left(x^6-6x^5+6x^4+16x^3-15x^2-\left(x^5-5x^4+3x^3+11x^2-4x-6\right)\sqrt{x^2-2x-3}-18x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 3)^(-5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*x^2 - 3*sqrt(x^2 - 2*x - 3)*(x - 1) - 6*x - 5)/(x^6 - 6*x
^5 + 6*x^4 + 16*x^3 - 15*x^2 - (x^5 - 5*x^4 + 3*x^3 + 11*x^2 - 4*
x - 6)*sqrt(x^2 - 2*x - 3) - 18*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - 2x - 3)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x-3)**(5/2),x)
```

```
[Out] Integral((x**2 - 2*x - 3)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.20813, size = 31, normalized size = 0.72

$$\frac{((x-3)x-3)x+5}{24\,(x^2-2\,x-3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 3)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)
```

3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx$

Optimal. Leaf size=42

$$\frac{(3-x)\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

```
[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2
 + x^3]
```

Rubi [A] time = 0.0797846, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{(3-x)\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

```
[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2
 + x^3]
```

Rubi in Sympy [A] time = 2.02922, size = 36, normalized size = 0.86

$$\frac{(-x+3)\sqrt{x+1}\,\operatorname{atanh}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
```

```
[Out] (-x + 3)*sqrt(x + 1)*atanh(sqrt(x + 1)/2)/sqrt(x**3 - 5*x**2 + 3*
x + 9)
```

Mathematica [A] time = 0.021747, size = 52, normalized size = 1.24

$$\frac{(x-3)\sqrt{x+1}\left(\log\left(2-\sqrt{x+1}\right)-\log\left(\sqrt{x+1}+2\right)\right)}{2\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

```
[Out] ((-3 + x)*Sqrt[1 + x]*(Log[2 - Sqrt[1 + x]] - Log[2 + Sqrt[1 + x]
]))/(2*Sqrt[(-3 + x)^2*(1 + x)])
```

Maple [A] time = 0.013, size = 45, normalized size = 1.1

$$-\frac{-3+x}{2}\sqrt{1+x}\left(\ln\left(\sqrt{1+x}+2\right)-\ln\left(\sqrt{1+x}-2\right)\right)\frac{1}{\sqrt{x^3-5x^2+3x+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-5*x^2+3*x+9)^(1/2),x)
```

```
[Out] -1/2*(-3+x)*(1+x)^(1/2)*(ln((1+x)^(1/2)+2)-ln((1+x)^(1/2)-2))/(x^
3-5*x^2+3*x+9)^(1/2)
```

Maxima [A] time = 1.54953, size = 28, normalized size = 0.67

$$-\frac{1}{2}\log\left(\sqrt{x+1}+2\right)+\frac{1}{2}\log\left(\sqrt{x+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9),x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(x + 1) + 2) + 1/2*log(sqrt(x + 1) - 2)
```

Fricas [A] time = 0.214519, size = 84, normalized size = 2.

$$-\frac{1}{2}\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)+\frac{1}{2}\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9),x, algorithm="fricas")
```

```
[Out] -1/2*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + 1/2*l
og(-(2*x - sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-5x^2+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)
```

GIAC/XCAS [A] time = 0.206842, size = 46, normalized size = 1.1

$$-\frac{\ln\left(\sqrt{x+1}+2\right)}{2\,\mathrm{sign}\left(x-3\right)}+\frac{\ln\left(\left|\sqrt{x+1}-2\right|\right)}{2\,\mathrm{sign}\left(x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9),x, algorithm="giac")
```

```
[Out] -1/2*ln(sqrt(x + 1) + 2)/sign(x - 3) + 1/2*ln(abs(sqrt(x + 1) - 2
))/sign(x - 3)
```

3.231 $\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{15(x+1)^{3/2}(3-x)^3\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{512\left(x^3-5x^2+3x+9\right)^{3/2}}$$

```
[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^
2*(1 + x))/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1
+ x))/(256*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)
^(3/2)*ArcTanh[Sqrt[1 + x]/2])/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2)
)
```

Rubi [A] time = 0.235337, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{15(x+1)^{3/2}(3-x)^3\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{512\left(x^3-5x^2+3x+9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^
2*(1 + x))/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1
+ x))/(256*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)
^(3/2)*ArcTanh[Sqrt[1 + x]/2])/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2)
)
```

Rubi in Sympy [A] time = 4.47437, size = 128, normalized size = 0.92

$$\frac{15\left(-x+3\right)^{3}\left(x+1\right)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{x+1}}{2}\right)}{512\left(x^{3}-5x^{2}+3x+9\right)^{\frac{3}{2}}}+\frac{15\left(-x+3\right)^{2}\left(x+1\right)^{2}}{256\left(x^{3}-5x^{2}+3x+9\right)^{\frac{3}{2}}}+\frac{5\left(-x+3\right)\left(x+1\right)^{2}}{32\left(x^{3}-5x^{2}+3x+9\right)^{\frac{3}{2}}}-\frac{\left(-\frac{x}{2}+\frac{3}{2}\right)\left(x+1\right)}{\left(x^{3}-5x^{2}+3x+9\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
```

```
[Out] 15*(-x + 3)**3*(x + 1)**(3/2)*atanh(sqrt(x + 1)/2)/(512*(x**3 - 5
*x**2 + 3*x + 9)**(3/2)) + 15*(-x + 3)**2*(x + 1)**2/(256*(x**3 -
 5*x**2 + 3*x + 9)**(3/2)) + 5*(-x + 3)*(x + 1)**2/(32*(x**3 - 5*
x**2 + 3*x + 9)**(3/2)) - (-x/2 + 3/2)*(x + 1)/(x**3 - 5*x**2 + 3
*x + 9)**(3/2)
```

Mathematica [A] time = 0.0406292, size = 58, normalized size = 0.42

$$\frac{30x^2 - 140x - 15\sqrt{x+1}(x-3)^2 \tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right) + 86}{512(x-3)\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] (86 - 140*x + 30*x^2 - 15*(-3 + x)^2*Sqrt[1 + x]*ArcTanh[Sqrt[1 +
 x]/2])/(512*(-3 + x)*Sqrt[(-3 + x)^2*(1 + x)])
```

Maple [A] time = 0.025, size = 144, normalized size = 1.

$$-\frac{(-3+x)^3(1+x)}{1024}\left(15\,(1+x)^{5/2}\ln\left(\sqrt{1+x}+2\right) - 15\,(1+x)^{5/2}\ln\left(\sqrt{1+x}-2\right) - 120\,(1+x)^{3/2}\ln\left(\sqrt{1+x}+2\right) + 120\,(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-5*x^2+3*x+9)^(3/2),x)
```

```
[Out] -1/1024*(-3+x)^3*(1+x)*(15*(1+x)^(5/2)*ln((1+x)^(1/2)+2)-15*(1+x)
^(5/2)*ln((1+x)^(1/2)-2)-120*(1+x)^(3/2)*ln((1+x)^(1/2)+2)+120*(1
+x)^(3/2)*ln((1+x)^(1/2)-2)+240*ln((1+x)^(1/2)+2)*(1+x)^(1/2)-240
*ln((1+x)^(1/2)-2)*(1+x)^(1/2)-60*x^2+280*x-172)/(x^3-5*x^2+3*x+9
)^(3/2)/((1+x)^(1/2)+2)^2/((1+x)^(1/2)-2)^2
```

Maxima [A] time = 1.59453, size = 77, normalized size = 0.55

$$\frac{15\,(x+1)^2 - 100\,x + 28}{256\left((x+1)^{\frac{5}{2}} - 8\,(x+1)^{\frac{3}{2}} + 16\,\sqrt{x+1}\right)} - \frac{15}{1024}\,\log\left(\sqrt{x+1}+2\right) + \frac{15}{1024}\,\log\left(\sqrt{x+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2),x, algorithm="maxima")
```

```
[Out] 1/256*(15*(x + 1)^2 - 100*x + 28)/((x + 1)^(5/2) - 8*(x + 1)^(3/2
) + 16*sqrt(x + 1)) - 15/1024*log(sqrt(x + 1) + 2) + 15/1024*log(
sqrt(x + 1) - 2)
```

Fricas [A] time = 0.218368, size = 186, normalized size = 1.34

$$-\frac{15\left(x^4-8x^3+18x^2-27\right)\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)-15\left(x^4-8x^3+18x^2-27\right)\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)-4\sqrt{x^3-5}}{1024\left(x^4-8x^3+18x^2-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x
^2 + 3*x + 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(
-(2*x - sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 -
5*x^2 + 3*x + 9)*(15*x^2 - 70*x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.527824, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$

Optimal. Leaf size=75

$$-\frac{3}{2}\log\left(1-\frac{x-3}{\sqrt[3]{x^3-5x^2+3x+9}}\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-3)}{\sqrt[3]{x^3-5x^2+3x+9}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(-3 + x))/(9 + 3*x - 5*x^2 + x^3)^(1/3))/S
qrt[3]] - Log[1 + x]/2 - (3*Log[1 - (-3 + x)/(9 + 3*x - 5*x^2 + x
^3)^(1/3)])/2
```

Rubi [B] time = 0.233295, antiderivative size = 188, normalized size of antiderivative = 2.51, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{(3-x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{32(3-x)}{3}\right)}{2\sqrt[3]{x^3-5x^2+3x+9}}-\frac{3(3-x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{3-x}}+1\right)}{2\sqrt[3]{x^3-5x^2+3x+9}}-\frac{\sqrt{3}(3-x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{3-x}}\right)}{\sqrt[3]{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] -((Sqrt[3]*(3 - x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 +
 x)^(1/3))/(Sqrt[3]*(3 - x)^(1/3))])/(9 + 3*x - 5*x^2 + x^3)^(1/3
)) - ((3 - x)^(2/3)*(1 + x)^(1/3)*Log[(32*(3 - x))/3])/(2*(9 + 3*
x - 5*x^2 + x^3)^(1/3)) - (3*(3 - x)^(2/3)*(1 + x)^(1/3)*Log[1 +
(1 + x)^(1/3)/(3 - x)^(1/3)])/(2*(9 + 3*x - 5*x^2 + x^3)^(1/3))
```

Rubi in Sympy [A] time = 2.69883, size = 155, normalized size = 2.07

$$-\frac{3(x-3)^{\frac{2}{3}}\sqrt[3]{x+1}\log\left(-1+\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-3}}\right)}{2\sqrt[3]{x^3-5x^2+3x+9}}-\frac{(x-3)^{\frac{2}{3}}\sqrt[3]{x+1}\log(x-3)}{2\sqrt[3]{x^3-5x^2+3x+9}}-\frac{\sqrt{3}(x-3)^{\frac{2}{3}}\sqrt[3]{x+1}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt[3]{x+1}}{3\sqrt[3]{x-3}}\right)}{\sqrt[3]{x^3-5x^2+3x+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

```
[Out] -3*(x - 3)**(2/3)*(x + 1)**(1/3)*log(-1 + (x + 1)**(1/3)/(x - 3)*
*(1/3))/(2*(x**3 - 5*x**2 + 3*x + 9)**(1/3)) - (x - 3)**(2/3)*(x
+ 1)**(1/3)*log(x - 3)/(2*(x**3 - 5*x**2 + 3*x + 9)**(1/3)) - sqr
t(3)*(x - 3)**(2/3)*(x + 1)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*(x
+ 1)**(1/3)/(3*(x - 3)**(1/3)))/(x**3 - 5*x**2 + 3*x + 9)**(1/3)
```

Mathematica [C] time = 0.0262072, size = 49, normalized size = 0.65

$$\frac{3(x-3)\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{3-x}{4}\right)}{2^{2/3}\sqrt[3]{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*(-3 + x)*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 - x
)/4])/(2^(2/3)*((-3 + x)^2*(1 + x))^(1/3))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-5\,x^2+3\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-5*x^2+3*x+9)^(1/3),x)
```

```
[Out] int(1/(x^3-5*x^2+3*x+9)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

Fricas [A] time = 0.210028, size = 166, normalized size = 2.21

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x+2\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}-3\right)}{3(x-3)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}(x-3)-6x+\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}+9}{x^2-6x+9}\right)-\log\left(-\frac{x-\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}-3}{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(x + 2*(x^3 - 5*x^2 + 3*x + 9)^(1/3)
- 3)/(x - 3)) + 1/2*log((x^2 + (x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x -
 3) - 6*x + (x^3 - 5*x^2 + 3*x + 9)^(2/3) + 9)/(x^2 - 6*x + 9)) -
 log(-(x - (x^3 - 5*x^2 + 3*x + 9)^(1/3) - 3)/(x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

3.233 $\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx$

Optimal. Leaf size=29

$$\frac{3(3-x)(x+1)}{4\left(x^3-5x^2+3x+9\right)^{2/3}}$$

```
[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

Rubi [A] time = 0.0678521, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{3(3-x)(x+1)}{4\left(x^3-5x^2+3x+9\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]

[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

Rubi in Sympy [A] time = 1.8465, size = 26, normalized size = 0.9

$$\frac{3\left(-x+3\right)\left(x+1\right)}{4\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)

[Out] 3*(-x + 3)*(x + 1)/(4*(x**3 - 5*x**2 + 3*x + 9)**(2/3))
```

Mathematica [A] time = 0.0170167, size = 23, normalized size = 0.79

$$-\frac{3(x-3)(x+1)}{4\left((x-3)^2(x+1)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

```
[Out] (-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))
```

Maple [A] time = 0.005, size = 24, normalized size = 0.8

$$-\frac{(3+3x)(-3+x)}{4}\left(x^3-5x^2+3x+9\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-5*x^2+3*x+9)^(2/3),x)
```

```
[Out] -3/4*(1+x)*(-3+x)/(x^3-5*x^2+3*x+9)^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

Fricas [A] time = 0.210786, size = 30, normalized size = 1.03

$$-\frac{3\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}{4(x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3),x, algorithm="fricas")
```

```
[Out] -3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 5\,x^2 + 3\,x + 9)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

3.234 $\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx$

Optimal. Leaf size=92

$$-\frac{27(x+1)(3-x)^3}{320\,(x^3-5x^2+3x+9)^{4/3}}+\frac{9(x+1)(3-x)^2}{80\,(x^3-5x^2+3x+9)^{4/3}}+\frac{3(x+1)(3-x)}{20\,(x^3-5x^2+3x+9)^{4/3}}$$

```
[Out] (3*(3 - x)*(1 + x))/(20*(9 + 3*x - 5*x^2 + x^3)^(4/3)) + (9*(3 -
x)^2*(1 + x))/(80*(9 + 3*x - 5*x^2 + x^3)^(4/3)) - (27*(3 - x)^3*
(1 + x))/(320*(9 + 3*x - 5*x^2 + x^3)^(4/3))
```

Rubi [A] time = 0.187232, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{27(x+1)(3-x)^3}{320\,(x^3-5x^2+3x+9)^{4/3}}+\frac{9(x+1)(3-x)^2}{80\,(x^3-5x^2+3x+9)^{4/3}}+\frac{3(x+1)(3-x)}{20\,(x^3-5x^2+3x+9)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(3 - x)*(1 + x))/(20*(9 + 3*x - 5*x^2 + x^3)^(4/3)) + (9*(3 -
x)^2*(1 + x))/(80*(9 + 3*x - 5*x^2 + x^3)^(4/3)) - (27*(3 - x)^3*
(1 + x))/(320*(9 + 3*x - 5*x^2 + x^3)^(4/3))
```

Rubi in Sympy [A] time = 3.32098, size = 85, normalized size = 0.92

$$\frac{27\,(-x+3)^2\,(x+1)^2}{320\,(x^3-5x^2+3x+9)^{\frac{4}{3}}}+\frac{9\,(-x+3)\,(x+1)^2}{40\,(x^3-5x^2+3x+9)^{\frac{4}{3}}}-\frac{3\,(-x+3)\,(x+1)}{4\,(x^3-5x^2+3x+9)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
```

```
[Out] 27*(-x + 3)**2*(x + 1)**2/(320*(x**3 - 5*x**2 + 3*x + 9)**(4/3))
+ 9*(-x + 3)*(x + 1)**2/(40*(x**3 - 5*x**2 + 3*x + 9)**(4/3)) - 3
*(-x + 3)*(x + 1)/(4*(x**3 - 5*x**2 + 3*x + 9)**(4/3))
```

Mathematica [A] time = 0.0192313, size = 32, normalized size = 0.35

$$\frac{3\left(9x^2-42x+29\right)}{320(x-3)\sqrt[3]{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(29 - 42*x + 9*x^2))/(320*(-3 + x)*((-3 + x)^2*(1 + x))^(1/3))
```

Maple [A] time = 0.006, size = 34, normalized size = 0.4

$$\frac{(3+3x)(-3+x)\left(9x^2-42x+29\right)}{320}\left(x^3-5x^2+3x+9\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-5*x^2+3*x+9)^(4/3),x)
```

```
[Out] 3/320*(1+x)*(-3+x)*(9*x^2-42*x+29)/(x^3-5*x^2+3*x+9)^(4/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

Fricas [A] time = 0.200946, size = 43, normalized size = 0.47

$$\frac{3\left(9x^2-42x+29\right)}{320\,(x^3-5x^2+3x+9)^{\frac{1}{3}}(x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3),x, algorithm="fricas")
```

```
[Out] 3/320*(9*x^2 - 42*x + 29)/((x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 5\,x^2 + 3\,x + 9)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

3.235 $\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Rubi [A] time = 0.0272357, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4 + 3*x - 2*x^2],x]

[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Rubi in Sympy [A] time = 0.740419, size = 34, normalized size = 1.79

$$-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}(-4x+3)}{4\sqrt{-2x^2+3x+4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**2+3*x+4)**(1/2),x)

[Out] -sqrt(2)*atan(sqrt(2)*(-4*x + 3)/(4*sqrt(-2*x**2 + 3*x + 4)))/2
```

Mathematica [A] time = 0.0148946, size = 19, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4 + 3*x - 2*x^2],x]

[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\arcsin\left(\frac{4\sqrt{41}}{41}\left(x-\frac{3}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^2+3*x+4)^(1/2),x)

[Out] 1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))
```

Maxima [A] time = 1.52664, size = 22, normalized size = 1.16

$$-\frac{1}{2}\sqrt{2}\arcsin\left(-\frac{1}{41}\sqrt{41}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^2 + 3*x + 4),x, algorithm="maxima")

[Out] -1/2*sqrt(2)*arcsin(-1/41*sqrt(41)*(4*x - 3))
```

Fricas [A] time = 0.245629, size = 38, normalized size = 2.

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{-2x^2+3x+4}-2\right)}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^2 + 3*x + 4),x, algorithm="fricas")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(-2*x^2 + 3*x + 4) - 2)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^2 + 3x + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**2+3*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**2 + 3*x + 4), x)
```

GIAC/XCAS [A] time = 0.213498, size = 22, normalized size = 1.16

$$\frac{1}{2}\sqrt{2}\arcsin\left(\frac{1}{41}\sqrt{41}(4\,x - 3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^2 + 3*x + 4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arcsin(1/41*sqrt(41)*(4*x - 3))
```

3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(2-x)$$

```
[Out] -ArcSin[2 - x]
```

Rubi [A] time = 0.0122102, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\sin^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 4*x - x^2],x]

[Out] -ArcSin[2 - x]
```

Rubi in Sympy [A] time = 0.689936, size = 20, normalized size = 2.5

$$-\operatorname{atan}\left(\frac{-2x+4}{2\sqrt{-x^2+4x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+4*x-3)**(1/2),x)

[Out] -atan((-2*x + 4)/(2*sqrt(-x**2 + 4*x - 3)))
```

Mathematica [A] time = 0.00939278, size = 8, normalized size = 1.

$$-\sin^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 4*x - x^2],x]

[Out] -ArcSin[2 - x]
```

Maple [A] time = 0.004, size = 5, normalized size = 0.6

$$\arcsin\left(-2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+4*x-3)^(1/2),x)
```

```
[Out] arcsin(-2+x)
```

Maxima [A] time = 1.51884, size = 11, normalized size = 1.38

$$-\arcsin\left(-x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 4*x - 3),x, algorithm="maxima")
```

```
[Out] -arcsin(-x + 2)
```

Fricas [A] time = 0.210584, size = 23, normalized size = 2.88

$$\arctan\left(\frac{x-2}{\sqrt{-x^2+4\,x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 4*x - 3),x, algorithm="fricas")
```

```
[Out] arctan((x - 2)/sqrt(-x^2 + 4*x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+4*x-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + 4*x - 3), x)
```

GIAC/XCAS [A] time = 0.206316, size = 5, normalized size = 0.62

$$\arcsin(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 4*x - 3),x, algorithm="giac")
```

```
[Out] arcsin(x - 2)
```

3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Rubi [A] time = 0.0110829, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 5*x - 3*x^2],x]

[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Rubi in Sympy [A] time = 0.740902, size = 37, normalized size = 3.08

$$\frac{\sqrt{3}\,\operatorname{atan}\left(-\frac{\sqrt{3}(-6x-5)}{6\sqrt{-3x^2-5x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2-5*x-2)**(1/2),x)

[Out] sqrt(3)*atan(-sqrt(3)*(-6*x - 5)/(6*sqrt(-3*x**2 - 5*x - 2)))/3
```

Mathematica [A] time = 0.0105297, size = 12, normalized size = 1.

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Maple [A] time = 0.005, size = 12, normalized size = 1.

$$\frac{\arcsin(6x+5)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2-5*x-2)^(1/2),x)
```

```
[Out] 1/3*arcsin(6*x+5)*3^(1/2)
```

Maxima [A] time = 1.58009, size = 15, normalized size = 1.25

$$\frac{1}{3}\sqrt{3}\arcsin(6x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 - 5*x - 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

Fricas [A] time = 0.240111, size = 38, normalized size = 3.17

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(6x+5)}{6\sqrt{-3x^2-5x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 - 5*x - 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/6*sqrt(3)*(6*x + 5)/sqrt(-3*x^2 - 5*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2-5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2-5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 - 5*x - 2), x)
```

GIAC/XCAS [A] time = 0.218086, size = 15, normalized size = 1.25

$$\frac{1}{3}\sqrt{3}\arcsin\left(6\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 - 5*x - 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

3.238 $\int \frac{1}{\sqrt{1-x^2}(4+x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Rubi [A] time = 0.0258338, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]

[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 2.47719, size = 24, normalized size = 0.77

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{2\sqrt{-x^2+1}}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)

[Out] sqrt(5)*atan(sqrt(5)*x/(2*sqrt(-x**2 + 1)))/10
```

Mathematica [A] time = 0.0426013, size = 33, normalized size = 1.06

$$-\frac{\tan^{-1}\left(\frac{x\sqrt{5-5x^2}}{2(x^2-1)}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

```
[Out] -ArcTan[(x*Sqrt[5 - 5*x^2])/(2*(-1 + x^2))]/(2*Sqrt[5])
```

Maple [A] time = 0.014, size = 29, normalized size = 0.9

$$-\frac{\sqrt{5}}{10}\arctan\left(\frac{x\sqrt{5}}{2\,x^2-2}\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/10*5^(1/2)*arctan(1/2*5^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+4)\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 0.211857, size = 65, normalized size = 2.1

$$\frac{1}{10}\sqrt{5}\arctan\left(\frac{2\left(\sqrt{5}(x^2-1)+\sqrt{5}\sqrt{-x^2+1}\right)}{5\left(\sqrt{-x^2+1}x-x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*arctan(2/5*(sqrt(5)*(x^2 - 1) + sqrt(5)*sqrt(-x^2 +
1))/(sqrt(-x^2 + 1)*x - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}(x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)
```

GIAC/XCAS [A] time = 0.221259, size = 69, normalized size = 2.23

$$\frac{1}{20}\sqrt{5}\left(\pi\operatorname{sign}(x)+2\arctan\left(-\frac{\sqrt{5}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{5\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*(pi*sign(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 +
1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

3.239 $\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

```
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Rubi [A] time = 0.0298029, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]

[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Rubi in Sympy [A] time = 2.99983, size = 26, normalized size = 0.84

$$\frac{\sqrt{15}\,\mathrm{atanh}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)

[Out] sqrt(15)*atanh(sqrt(15)*x/(2*sqrt(4*x**2 + 1)))/30
```

Mathematica [A] time = 0.0717559, size = 56, normalized size = 1.81

$$\frac{\log\left(31x^2+4\sqrt{60x^2+15}x+4\right)-\log\left(31x^2-4\sqrt{60x^2+15}x+4\right)}{8\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

```
[Out] (-Log[4 + 31*x^2 - 4*x*Sqrt[15 + 60*x^2]] + Log[4 + 31*x^2 + 4*x*
Sqrt[15 + 60*x^2]])/(8*Sqrt[15])
```

Maple [A] time = 0.013, size = 22, normalized size = 0.7

$$\frac{\sqrt{15}}{30}\mathit{Artanh}\left(\frac{x\sqrt{15}}{2}\frac{1}{\sqrt{4\,x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4)/(4*x^2+1)^(1/2),x)
```

```
[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+1}(x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)
```

Fricas [A] time = 0.244482, size = 119, normalized size = 3.84

$$\frac{1}{60}\sqrt{15}\log\left(\frac{120\,x^2+\sqrt{15}\left(8\,x^4+33\,x^2+124\right)-4\,\sqrt{4\,x^2+1}\left(\sqrt{15}\left(x^3+4\,x\right)+15\,x\right)+480}{8\,x^4+33\,x^2-4\,(x^3+4\,x)\sqrt{4\,x^2+1}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*log((120*x^2 + sqrt(15)*(8*x^4 + 33*x^2 + 124) - 4*
sqrt(4*x^2 + 1)*(sqrt(15)*(x^3 + 4*x) + 15*x) + 480)/(8*x^4 + 33*
x^2 - 4*(x^3 + 4*x)*sqrt(4*x^2 + 1) + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+4)\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.217359, size = 77, normalized size = 2.48

$$-\frac{1}{60}\,\sqrt{15}\ln\left(\frac{\left(2\,x-\sqrt{4\,x^2+1}\right)^2-8\,\sqrt{15}+31}{\left(2\,x-\sqrt{4\,x^2+1}\right)^2+8\,\sqrt{15}+31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(15)*ln(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/(
(2*x - sqrt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))
```

3.240 $\int \frac{x}{(3-x^2)\sqrt{5-x^2}} \, dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0588942, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.42922, size = 22, normalized size = 0.92

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{-x^2+5}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
```

```
[Out] sqrt(2)*atanh(sqrt(2)*sqrt(-x**2 + 5)/2)/2
```

Mathematica [A] time = 0.017568, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Maple [B] time = 0.047, size = 100, normalized size = 4.2

$$\frac{\sqrt{2}}{4}\operatorname{Artanh}\left(\frac{\left(4-2\sqrt{3}\left(x-\sqrt{3}\right)\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(x-\sqrt{3}\right)^{2}-2\sqrt{3}\left(x-\sqrt{3}\right)+2}}\right)+\frac{\sqrt{2}}{4}\operatorname{Artanh}\left(\frac{\left(4+2\sqrt{3}\left(x+\sqrt{3}\right)\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(x+\sqrt{3}\right)^{2}+2\sqrt{3}\left(x+\sqrt{3}\right)+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+3)/(-x^2+5)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)*arctanh(1/4*(4-2*3^(1/2)*(x-3^(1/2)))*2^(1/2)/(-(x-3^
(1/2))^2-2*3^(1/2)*(x-3^(1/2))+2)^(1/2))+1/4*2^(1/2)*arctanh(1/4*
(4+2*3^(1/2)*(x+3^(1/2)))*2^(1/2)/(-(x+3^(1/2))^2+2*3^(1/2)*(x+3^
(1/2))+2)^(1/2))
```

Maxima [A] time = 1.52851, size = 151, normalized size = 6.29

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}\sqrt{2}\log\left(\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^{2}+5}}{\left|2x+2\sqrt{3}\right|}+\frac{4}{\left|2x+2\sqrt{3}\right|}\right)+\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^{2}+5}}{\left|2x-2\sqrt{3}\right|}+\frac{4}{\left|2x-2\sqrt{3}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^2 - 3)*sqrt(-x^2 + 5)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*sqrt(2)*log(sqrt(3) + 2*sqrt(2)*sqrt(-x^2 +
 5)/abs(2*x + 2*sqrt(3)) + 4/abs(2*x + 2*sqrt(3))) + sqrt(3)*sqrt
(2)*log(-sqrt(3) + 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2*x - 2*sqrt(3))
+ 4/abs(2*x - 2*sqrt(3))))
```

Fricas [A] time = 0.222103, size = 68, normalized size = 2.83

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\sqrt{2}\left(x^{4}-22x^{2}+89\right)-8\left(x^{2}-7\right)\sqrt{-x^{2}+5}}{x^{4}-6x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^2 - 3)*sqrt(-x^2 + 5)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((sqrt(2)*(x^4 - 22*x^2 + 89) - 8*(x^2 - 7)*sqrt(-
x^2 + 5))/(x^4 - 6*x^2 + 9))
```

Sympy [A] time = 12.5044, size = 61, normalized size = 2.54

$$-\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{-x^2+5}}\right)}{2} & \text{for } \frac{1}{-x^2+5} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{-x^2+5}}\right)}{2} & \text{for } \frac{1}{-x^2+5} < \frac{1}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
```

```
[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(-x**2 + 5))/2, 1/(-x**2 +
 5) > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(-x**2 + 5))/2, 1/(-x**2
+ 5) < 1/2))
```

GIAC/XCAS [A] time = 0.21188, size = 57, normalized size = 2.38

$$\frac{1}{4}\sqrt{2}\ln\left(\sqrt{2}+\sqrt{-x^2+5}\right)-\frac{1}{4}\sqrt{2}\ln\left(\left|-\sqrt{2}+\sqrt{-x^2+5}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^2 - 3)*sqrt(-x^2 + 5)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*ln(abs(-sq
rt(2) + sqrt(-x^2 + 5)))
```

3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)} \, dx$

Optimal. Leaf size=25

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0577787, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]

[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Rubi in Sympy [A] time = 4.51676, size = 24, normalized size = 0.96

$$-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-x^2+3}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)

[Out] -sqrt(2)*atan(sqrt(2)*sqrt(-x**2 + 3)/2)/2
```

Mathematica [A] time = 0.0171354, size = 25, normalized size = 1.

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

```
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Maple [B]   time = 0.047, size = 100, normalized size = 4.

$$-\frac{\sqrt{2}}{4}\arctan\left(\frac{\left(-4-2\sqrt{5}\left(x-\sqrt{5}\right)\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(x-\sqrt{5}\right)^{2}-2\sqrt{5}\left(x-\sqrt{5}\right)-2}}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{\left(-4+2\sqrt{5}\left(x+\sqrt{5}\right)\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(x+\sqrt{5}\right)^{2}+2\sqrt{5}\left(x+\sqrt{5}\right)-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+5)/(-x^2+3)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(-4-2*5^(1/2)*(x-5^(1/2)))*2^(1/2)/(-(x-5
^(1/2))^2-2*5^(1/2)*(x-5^(1/2))-2)^(1/2))-1/4*2^(1/2)*arctan(1/4*
(-4+2*5^(1/2)*(x+5^(1/2)))*2^(1/2)/(-(x+5^(1/2))^2+2*5^(1/2)*(x+5
^(1/2))-2)^(1/2))
```

Maxima [A]   time = 1.52042, size = 136, normalized size = 5.44

$$-\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\left|2x+2\sqrt{5}\right|}+\frac{2\sqrt{3}}{\left|2x+2\sqrt{5}\right|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\left|2x-2\sqrt{5}\right|}-\frac{2\sqrt{3}}{\left|2x-2\sqrt{5}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^2 - 5)*sqrt(-x^2 + 3)),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*(sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2
*x + 2*sqrt(5)) + 2*sqrt(3)/abs(2*x + 2*sqrt(5))) - sqrt(5)*sqrt(
2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2*x - 2*sqrt(5)) - 2*sqrt(3)/
abs(2*x - 2*sqrt(5))))
```

Fricas [A]   time = 0.228137, size = 34, normalized size = 1.36

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}-1\right)}{4\sqrt{-x^{2}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^2 - 5)*sqrt(-x^2 + 3)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 1)/sqrt(-x^2 + 3))
```

Sympy [A] time = 4.40852, size = 20, normalized size = 0.8

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}}{\sqrt{-x^2+3}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)/sqrt(-x**2 + 3))/2
```

GIAC/XCAS [A] time = 0.213471, size = 27, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^2 - 5)*sqrt(-x^2 + 3)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))
```

3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[x/Sqrt[2 + x^2]]/2 - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(
2*Sqrt[3])
```

Rubi [A] time = 0.0610182, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] -ArcTan[x/Sqrt[2 + x^2]]/2 - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(
2*Sqrt[3])
```

Rubi in Sympy [A] time = 5.63085, size = 37, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x}{\sqrt{x^2+2}}\right)}{2}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-1)/(x**2+2)**(1/2),x)
```

```
[Out] -atan(x/sqrt(x**2 + 2))/2 - sqrt(3)*atanh(sqrt(3)*x/sqrt(x**2 + 2
))/6
```

Mathematica [A] time = 0.0579566, size = 80, normalized size = 1.86

$$\frac{1}{12}\left(\sqrt{3}\left(\log\left(\sqrt{3}\sqrt{x^2+2}-x+2\right)-\log\left(\sqrt{3}\sqrt{x^2+2}+x+2\right)+\log(1-x)-\log(x+1)\right)-6\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] (-6*ArcTan[x/Sqrt[2 + x^2]] + Sqrt[3]*(Log[1 - x] - Log[1 + x] +
Log[2 - x + Sqrt[3]*Sqrt[2 + x^2]] - Log[2 + x + Sqrt[3]*Sqrt[2 +
 x^2]]))/12
```

Maple [B] time = 0.034, size = 70, normalized size = 1.6

$$-\frac{\sqrt{3}}{12}Artanh\left(\frac{(2x+4)\sqrt{3}}{6}\frac{1}{\sqrt{(-1+x)^2+1+2x}}\right)+\frac{\sqrt{3}}{12}Artanh\left(\frac{(4-2x)\sqrt{3}}{6}\frac{1}{\sqrt{(1+x)^2+1-2x}}\right)-\frac{1}{2}\arctan\left(x\frac{1}{\sqrt{x^2+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-1)/(x^2+2)^(1/2),x)
```

```
[Out] -1/12*3^(1/2)*arctanh(1/6*(2*x+4)*3^(1/2)/((-1+x)^2+1+2*x)^(1/2))
+1/12*3^(1/2)*arctanh(1/6*(4-2*x)*3^(1/2)/((1+x)^2+1-2*x)^(1/2))-
1/2*arctan(x/(x^2+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4-1)\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 - 1)*sqrt(x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)
```

Fricas [A] time = 0.228427, size = 128, normalized size = 2.98

$$-\frac{1}{12}\sqrt{3}\left(2\sqrt{3}\arctan\left(-x^2+\sqrt{x^2+2}x-1\right)-\log\left(\frac{3x^2+\sqrt{3}\left(x^4+2\right)-\sqrt{x^2+2}\left(\sqrt{3}\left(x^3-x\right)+3x\right)-3}{x^4-(x^3-x)\sqrt{x^2+2}-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 - 1)*sqrt(x^2 + 2)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*(2*sqrt(3)*arctan(-x^2 + sqrt(x^2 + 2)*x - 1) - log
((3*x^2 + sqrt(3)*(x^4 + 2) - sqrt(x^2 + 2)*(sqrt(3)*(x^3 - x) +
3*x) - 3)/(x^4 - (x^3 - x)*sqrt(x^2 + 2) - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-1)(x+1)(x^2+1)\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-1)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)
```

GIAC/XCAS [A] time = 0.232083, size = 100, normalized size = 2.33

$$-\frac{1}{12}\sqrt{3}\ln\left(\frac{\left|2\left(x-\sqrt{x^2+2}\right)^2-4\sqrt{3}-8\right|}{\left|2\left(x-\sqrt{x^2+2}\right)^2+4\sqrt{3}-8\right|}\right)+\frac{1}{2}\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 - 1)*sqrt(x^2 + 2)),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*ln(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs
(2*(x - sqrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x -
sqrt(x^2 + 2))^2)
```

3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}} \, dx$

Optimal. Leaf size=62

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - A
rcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.09866, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - A
rcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 5.98143, size = 60, normalized size = 0.97

$$-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\sqrt{x^2+2x+4}}{3}\right)}{6}-\frac{\sqrt{7}\,\operatorname{atanh}\left(\frac{\sqrt{7}(4x+10)}{14\sqrt{x^2+2x+4}}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] -sqrt(3)*atanh(sqrt(3)*sqrt(x**2 + 2*x + 4)/3)/6 - sqrt(7)*atanh(
sqrt(7)*(4*x + 10)/(14*sqrt(x**2 + 2*x + 4)))/14
```

Mathematica [A] time = 0.0508107, size = 88, normalized size = 1.42

$$\frac{1}{42}\left(-7\sqrt{3}\log\left(\sqrt{3}\sqrt{x^2+2x+4}+3\right)-3\sqrt{7}\log\left(\sqrt{7}\sqrt{x^2+2x+4}+2x+5\right)+3\sqrt{7}\log(1-x)+7\sqrt{3}\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (3*Sqrt[7]*Log[1 - x] + 7*Sqrt[3]*Log[1 + x] - 7*Sqrt[3]*Log[3 +
Sqrt[3]*Sqrt[4 + 2*x + x^2]] - 3*Sqrt[7]*Log[5 + 2*x + Sqrt[7]*Sq
rt[4 + 2*x + x^2]])/42
```

Maple [A] time = 0.017, size = 49, normalized size = 0.8

$$-\frac{\sqrt{7}}{14} Artanh\left(\frac{(10+4x)\sqrt{7}}{14}\frac{1}{\sqrt{(-1+x)^2+3+4x}}\right)-\frac{\sqrt{3}}{6} Artanh\left(\sqrt{3}\frac{1}{\sqrt{(1+x)^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x)
```

```
[Out] -1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2
))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))
```

Maxima [A] time = 1.53733, size = 73, normalized size = 1.18

$$-\frac{1}{14}\sqrt{7}\,\mathrm{arsinh}\left(\frac{4\sqrt{3}x}{3|2x-2|}+\frac{10\sqrt{3}}{3|2x-2|}\right)-\frac{1}{6}\sqrt{3}\,\mathrm{arsinh}\left(\frac{2\sqrt{3}}{|2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^2 + 2*x + 4)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] -1/14*sqrt(7)*arcsinh(4/3*sqrt(3)*x/abs(2*x - 2) + 10/3*sqrt(3)/a
bs(2*x - 2)) - 1/6*sqrt(3)*arcsinh(2*sqrt(3)/abs(2*x + 2))
```

Fricas [A] time = 0.238609, size = 190, normalized size = 3.06

$$\frac{1}{42}\sqrt{7}\sqrt{3}\left(\sqrt{7}\log\left(\frac{\sqrt{3}\left(x^2+2x+4\right)-\sqrt{x^2+2x+4}\left(\sqrt{3}(x+1)+3\right)+3x+3}{x^2-\sqrt{x^2+2x+4}(x+1)+2x+1}\right)+\sqrt{3}\log\left(\frac{\sqrt{7}\left(x^2+6\right)-\sqrt{x^2+2x+4}\left(\sqrt{7}(x\right.}{x^2-\sqrt{x^2+2x+4}(x\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^2 + 2*x + 4)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/42*sqrt(7)*sqrt(3)*(sqrt(7)*log((sqrt(3)*(x^2 + 2*x + 4) - sqrt
(x^2 + 2*x + 4)*(sqrt(3)*(x + 1) + 3) + 3*x + 3)/(x^2 - sqrt(x^2
+ 2*x + 4)*(x + 1) + 2*x + 1)) + sqrt(3)*log((sqrt(7)*(x^2 + 6) -
 sqrt(x^2 + 2*x + 4)*(sqrt(7)*(x - 1) + 7) + 7*x - 7)/(x^2 - sqrt
(x^2 + 2*x + 4)*(x - 1) - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

GIAC/XCAS [A] time = 0.245366, size = 147, normalized size = 2.37

$$\frac{1}{14}\sqrt{7}\ln\left(\frac{\left|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}{\left|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}\right)+\frac{1}{6}\sqrt{3}\ln\left(-\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^2 + 2*x + 4)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*ln(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)
/abs(-2*x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)
*ln(-1/2*abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - s
qrt(3) - sqrt(x^2 + 2*x + 4) + 1))
```

3.244 $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} \, dx$

Optimal. Leaf size=82

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] -ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/(4*Sqrt[3]) - ArcT
anh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + Arc
Tanh[Sqrt[5 + 2*x + x^2]]/12
```

Rubi [A] time = 0.258057, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] -ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/(4*Sqrt[3]) - ArcT
anh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + Arc
Tanh[Sqrt[5 + 2*x + x^2]]/12
```

Rubi in Sympy [A] time = 25.5657, size = 80, normalized size = 0.98

$$-\frac{\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{12}+\frac{\sqrt{13}\,\text{atanh}\left(\frac{\sqrt{13}(-6x-14)}{26\sqrt{x^2+2x+5}}\right)}{156}+\frac{\text{atanh}\left(\sqrt{x^2+2x+5}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] -sqrt(3)*atan(sqrt(3)*(2*x + 2)/(6*sqrt(x**2 + 2*x + 5)))/12 + sq
rt(13)*atanh(sqrt(13)*(-6*x - 14)/(26*sqrt(x**2 + 2*x + 5)))/156
+ atanh(sqrt(x**2 + 2*x + 5))/12
```

Mathematica [A] time = 0.146666, size = 155, normalized size = 1.89

$$\frac{1}{312}\left(-13\log\left(\left(x^2+2x+4\right)^2\right)+13\log\left(\left(x^2+2x+4\right)\left(x^2+2\sqrt{x^2+2x+5}+2x+6\right)\right)\right.$$
$$-2\sqrt{13}\log\left(\sqrt{13}\sqrt{x^2+2x+5}+3x+7\right)$$
$$\left.-26\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x^2+\left(\sqrt{x^2+2x+5}+2\right)x+\sqrt{x^2+2x+5}+4\right)}{2x^2+4x+11}\right)+2\sqrt{13}\log(2-x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] (-26*Sqrt[3]*ArcTan[(Sqrt[3]*(4 + x^2 + Sqrt[5 + 2*x + x^2] + x*(
2 + Sqrt[5 + 2*x + x^2])))/(11 + 4*x + 2*x^2)] + 2*Sqrt[13]*Log[2
 - x] - 13*Log[(4 + 2*x + x^2)^2] + 13*Log[(4 + 2*x + x^2)*(6 + 2
*x + x^2 + 2*Sqrt[5 + 2*x + x^2])] - 2*Sqrt[13]*Log[7 + 3*x + Sqr
t[13]*Sqrt[5 + 2*x + x^2]])/312
```

Maple [A] time = 0.032, size = 69, normalized size = 0.8

$$-\frac{\sqrt{13}}{156}\mathit{Artanh}\left(\frac{(14+6x)\sqrt{13}}{26}\frac{1}{\sqrt{(-2+x)^2+6x+1}}\right)$$
$$+\frac{1}{12}\mathit{Artanh}\left(\sqrt{x^2+2x+5}\right)-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\frac{1}{\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -1/156*13^(1/2)*arctanh(1/26*(14+6*x)*13^(1/2)/((-2+x)^2+6*x+1)^(
1/2))+1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*3^(1/2)*arctan(1/6*3^(
1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-8)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)
```

Fricas [A] time = 0.235276, size = 265, normalized size = 3.23

$$-\frac{1}{936}\sqrt{13}\sqrt{3}\left(\sqrt{13}\sqrt{3}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right)-\sqrt{13}\sqrt{3}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)-6\sqrt{13}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)),x, algorithm="fricas")
```

```
[Out] -1/936*sqrt(13)*sqrt(3)*(sqrt(13)*sqrt(3)*log(x^2 - sqrt(x^2 + 2*
x + 5)*(x + 2) + 3*x + 6) - sqrt(13)*sqrt(3)*log(x^2 - sqrt(x^2 +
 2*x + 5)*x + x + 4) - 6*sqrt(13)*arctan(-1/3*sqrt(3)*(x + 2) + 1
/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 6*sqrt(13)*arctan(-1/3*sqrt(3)*
x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) - 2*sqrt(3)*log((sqrt(13)*(x
^2 - x + 11) - sqrt(x^2 + 2*x + 5)*(sqrt(13)*(x - 2) + 13) + 13*x
 - 26)/(x^2 - sqrt(x^2 + 2*x + 5)*(x - 2) - x - 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x-2)(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

GIAC/XCAS [A] time = 0.228852, size = 221, normalized size = 2.7

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$
$$+\frac{1}{156}\sqrt{13}\ln\left(\frac{\left|-2x-2\sqrt{13}+2\sqrt{x^2+2x+5}+4\right|}{\left|-2x+2\sqrt{13}+2\sqrt{x^2+2x+5}+4\right|}\right)$$
$$-\frac{1}{24}\ln\left(\left(x-\sqrt{x^2+2x+5}\right)^2+4x-4\sqrt{x^2+2x+5}+7\right)+\frac{1}{24}\ln\left(\left(x-\sqrt{x^2+2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) -
 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/
156*sqrt(13)*ln(abs(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4
)/abs(-2*x + 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)) - 1/24*ln((
x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) + 1
/24*ln((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

```
[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[1
1/15]*(1 + 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

Rubi [A] time = 0.147867, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[1
1/15]*(1 + 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

Rubi in Sympy [A] time = 12.5264, size = 61, normalized size = 0.97

$$\frac{\sqrt{11}\operatorname{atan}\left(\frac{\sqrt{11}\sqrt{4x^2+4x+5}}{11}\right)}{11}-\frac{\sqrt{165}\operatorname{atanh}\left(\frac{\sqrt{165}(8x+4)}{60\sqrt{4x^2+4x+5}}\right)}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
```

```
[Out] sqrt(11)*atan(sqrt(11)*sqrt(4*x**2 + 4*x + 5)/11)/11 - sqrt(165)*
atanh(sqrt(165)*(8*x + 4)/(60*sqrt(4*x**2 + 4*x + 5)))/165
```

Mathematica [C] time = 0.489383, size = 426, normalized size = 6.76

$$-\frac{i\left(\sqrt{15}+i\right)\log\left(\left(x^2+x+4\right)\left(52x^2-2\sqrt{165}\sqrt{4x^2+4x+5}x-\sqrt{165}\sqrt{4x^2+4x+5}+52x+43\right)\right)}{4\sqrt{165}}-\frac{i\left(\sqrt{15}-i\right)\log\left(\left(x^2+x+4\right)\left(52x^2+2\sqrt{165}\sqrt{4x^2+4x+5}x+\sqrt{165}\sqrt{4x^2+4x+5}+52x+43\right)\right)}{4\sqrt{165}}+\frac{\left(\sqrt{15}+i\right)\tan^{-1}\left(\frac{-\sqrt{15}x^2-4\sqrt{11}\sqrt{4x^2+4x+5}-\sqrt{15}x-4\sqrt{15}}{15x^2+15x+16}\right)}{2\sqrt{165}}+\frac{\left(\sqrt{15}-i\right)\tan^{-1}\left(\frac{\sqrt{15}x^2-4\sqrt{11}\sqrt{4x^2+4x+5}+\sqrt{15}x+4\sqrt{15}}{15x^2+15x+16}\right)}{2\sqrt{165}}+\frac{i\left(\sqrt{15}+i\right)\log\left(\left(-2ix+\sqrt{15}-i\right)^2\left(2ix+\sqrt{15}+i\right)^2\right)}{4\sqrt{165}}+\frac{i\left(\sqrt{15}-i\right)\log\left(\left(-2ix+\sqrt{15}-i\right)^2\left(2ix+\sqrt{15}+i\right)^2\right)}{4\sqrt{165}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ((I + Sqrt[15])*ArcTan[(-4*Sqrt[15] - Sqrt[15]*x - Sqrt[15]*x^2 -
 4*Sqrt[11]*Sqrt[5 + 4*x + 4*x^2])/(16 + 15*x + 15*x^2)])/(2*Sqrt
[165]) + ((-I + Sqrt[15])*ArcTan[(4*Sqrt[15] + Sqrt[15]*x + Sqrt[
15]*x^2 - 4*Sqrt[11]*Sqrt[5 + 4*x + 4*x^2])/(16 + 15*x + 15*x^2)]
)/(2*Sqrt[165]) + ((I/4)*(-I + Sqrt[15])*Log[(-I + Sqrt[15] - (2*
I)*x)^2*(I + Sqrt[15] + (2*I)*x)^2])/Sqrt[165] + ((I/4)*(I + Sqrt
[15])*Log[(-I + Sqrt[15] - (2*I)*x)^2*(I + Sqrt[15] + (2*I)*x)^2]
)/Sqrt[165] - ((I/4)*(I + Sqrt[15])*Log[(4 + x + x^2)*(43 + 52*x
+ 52*x^2 - Sqrt[165]*Sqrt[5 + 4*x + 4*x^2] - 2*Sqrt[165]*x*Sqrt[5
 + 4*x + 4*x^2])])/Sqrt[165] - ((I/4)*(-I + Sqrt[15])*Log[(4 + x
+ x^2)*(43 + 52*x + 52*x^2 + Sqrt[165]*Sqrt[5 + 4*x + 4*x^2] + 2*
Sqrt[165]*x*Sqrt[5 + 4*x + 4*x^2])])/Sqrt[165]
```

Maple [A] time = 0.016, size = 53, normalized size = 0.8

$$\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\sqrt{4\,x^2+4\,x+5}\right)-\frac{\sqrt{165}}{165}\mathit{Artanh}\left(\frac{\sqrt{165}\,(8\,x+4)}{60}\frac{1}{\sqrt{4\,x^2+4\,x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x)
```

```
[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165
^(1/2)*arctanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,x^2+4\,x+5}(x^2+x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

Fricas [A] time = 0.24151, size = 356, normalized size = 5.65

$$-\frac{1}{4950}\,\sqrt{165}\sqrt{15}\left(\sqrt{15}\log\left(30720\,x^2-7680\,\sqrt{4\,x^2+4\,x+5}(2\,x+1)+30720\,x+7680\,\sqrt{165}+122880\right)-\sqrt{15}\log\left(30720\,x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)),x, algorithm="fricas")
```

```
[Out] -1/4950*sqrt(165)*sqrt(15)*(sqrt(15)*log(30720*x^2 - 7680*sqrt(4*
x^2 + 4*x + 5)*(2*x + 1) + 30720*x + 7680*sqrt(165) + 122880) - s
qrt(15)*log(30720*x^2 - 7680*sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 30
720*x - 7680*sqrt(165) + 122880) + 60*arctan(-15*(sqrt(165) + 11)
/(sqrt(165)*sqrt(15)*(2*x + 1) - sqrt(165)*sqrt(30)*sqrt(4*x^2 -
sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x + sqrt(165) + 16) - sqrt(16
5)*sqrt(15)*sqrt(4*x^2 + 4*x + 5))) - 60*arctan(-15*(sqrt(165) -
11)/(sqrt(165)*sqrt(15)*(2*x + 1) - sqrt(165)*sqrt(30)*sqrt(4*x^2
 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x - sqrt(165) + 16) - sqrt
(165)*sqrt(15)*sqrt(4*x^2 + 4*x + 5))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2+x+4)\sqrt{4x^2+4x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,x^2+4\,x+5}(x^2+x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

3.246 $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=56

$$\sqrt{2}\tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

```
[Out] -2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*
ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

Rubi [A] time = 0.134256, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\sqrt{2}\tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*
ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

Rubi in Sympy [A] time = 8.21748, size = 60, normalized size = 1.07

$$2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(4x-4)}{8\sqrt{x^2+x+1}}\right)-\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,(-2x-2)}{4\sqrt{x^2+x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

```
[Out] 2*sqrt(2)*atan(sqrt(2)*(4*x - 4)/(8*sqrt(x**2 + x + 1))) - sqrt(2
)*atanh(sqrt(2)*(-2*x - 2)/(4*sqrt(x**2 + x + 1)))
```

Mathematica [C] time = 0.337649, size = 352, normalized size = 6.29

$$\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{3/4}\left(2\log\left(x^2+1\right)\right.$$
$$-(1+2i)\log\left((5+4i)x^2+4\sqrt[4]{-1}\sqrt{x^2+x+1}x+8\sqrt[4]{-1}\sqrt{x^2+x+1}+(8+4i)x+(5+4i)\right)$$
$$-(1-2i)\log\left((5+4i)x^2+8\sqrt[4]{-1}\sqrt{x^2+x+1}x+4\sqrt[4]{-1}\sqrt{x^2+x+1}+(8+4i)x+(5+4i)\right)$$
$$+(4+2i)\tan^{-1}\left(\frac{(12+25i)x^3+\left(20(-1)^{3/4}\sqrt{x^2+x+1}+(5+28i)\right)x^2+\left((-10+30i)\sqrt{2}\sqrt{x^2+x+1}-(4-37i)\right)x+40\sqrt[4]{-1}\sqrt{}}{(4+25i)x^3+(5+16i)x^2+(32-11i)x+(1-36i)}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] (1/4 + I/4)*(-1)^(3/4)*((4 + 2*I)*ArcTan[((-7 + 12*I) + (12 + 25*
I)*x^3 + 40*(-1)^(1/4)*Sqrt[1 + x + x^2] + x^2*((5 + 28*I) + 20*(
-1)^(3/4)*Sqrt[1 + x + x^2]) + x*((-4 + 37*I) - (10 - 30*I)*Sqrt[
2]*Sqrt[1 + x + x^2]))/((1 - 36*I) + (32 - 11*I)*x + (5 + 16*I)*x
^2 + (4 + 25*I)*x^3)] + (4 - 2*I)*ArcTan[((-7 - 12*I) + (12 - 25*
I)*x^3 + 20*(-1)^(1/4)*Sqrt[1 + x + x^2] + x^2*((5 - 28*I) - 40*(
-1)^(3/4)*Sqrt[1 + x + x^2]) + x*((-4 - 37*I) + (30 + 10*I)*Sqrt[
2]*Sqrt[1 + x + x^2]))/((-49 + 36*I) - (48 - 61*I)*x - (45 - 64*I
)*x^2 + (4 + 25*I)*x^3)] + 2*Log[1 + x^2] - (1 + 2*I)*Log[(5 + 4*
I) + (8 + 4*I)*x + (5 + 4*I)*x^2 + 8*(-1)^(1/4)*Sqrt[1 + x + x^2]
 + 4*(-1)^(1/4)*x*Sqrt[1 + x + x^2]] - (1 - 2*I)*Log[(5 + 4*I) +
(8 + 4*I)*x + (5 + 4*I)*x^2 + 4*(-1)^(1/4)*Sqrt[1 + x + x^2] + 8*
(-1)^(1/4)*x*Sqrt[1 + x + x^2]])
```

Maple [B] time = 0.029, size = 128, normalized size = 2.3

$$\sqrt{2}\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\left(Artanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\right)-2\arctan\left(\frac{\sqrt{2}(-1+x)}{-1-x}\frac{1}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}}\right)\right)\frac{1}{\sqrt{1\left(\frac{(-1+x)^2}{(-1-x)^2}+3\right)\left(1+\frac{-1+x}{-1-x}\right)^{-2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x)
```

```
[Out] ((-1+x)^2/(-1-x)^2+3)^(1/2)*2^(1/2)*(arctanh(1/2*((-1+x)^2/(-1-x)
^2+3)^(1/2)*2^(1/2))-2*arctan(2^(1/2)/((-1+x)^2/(-1-x)^2+3)^(1/2)
*(-1+x)/(-1-x)))/(((-1+x)^2/(-1-x)^2+3)/(1+(-1+x)/(-1-x))^2)^(1/2
)/(1+(-1+x)/(-1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\sqrt{x^2+x+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

Fricas [A] time = 0.23664, size = 379, normalized size = 6.77

$$-\frac{1}{10}\sqrt{5}\left(8\sqrt{10}\arctan\left(-\frac{5\left(\sqrt{10}+2\sqrt{5}\right)}{10\sqrt{5}x-\sqrt{5}\sqrt{-20\sqrt{10}\sqrt{5}(x-1)+200x^2+20\sqrt{x^2+x+1}\left(\sqrt{10}\sqrt{5}-10x\right)+100x+300}-10\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(5)*(8*sqrt(10)*arctan(-5*(sqrt(10) + 2*sqrt(5))/(10*sq
rt(5)*x - sqrt(5)*sqrt(-20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 + 2
0*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 10*x) + 100*x + 300) - 10
*sqrt(5)*sqrt(x^2 + x + 1) - 5*sqrt(10))) + 8*sqrt(10)*arctan(-(s
qrt(10) - 2*sqrt(5))/(2*sqrt(5)*x - 2*sqrt(5)*sqrt(x^2 + x + 1) +
 sqrt(10) - 2*sqrt(sqrt(10)*sqrt(5)*(x - 1) + 10*x^2 - sqrt(x^2 +
 x + 1)*(sqrt(10)*sqrt(5) + 10*x) + 5*x + 15))) + sqrt(10)*log(20
*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 - 20*sqrt(x^2 + x + 1)*(sqrt(
10)*sqrt(5) + 10*x) + 100*x + 300) - sqrt(10)*log(-20*sqrt(10)*sq
rt(5)*(x - 1) + 200*x^2 + 20*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5)
- 10*x) + 100*x + 300))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\sqrt{x^2+x+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

```
[Out] (-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt
[14]) - ArcTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7
])
```

Rubi [A] time = 0.179451, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] (-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt
[14]) - ArcTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7
])
```

Rubi in Sympy [A] time = 14.3278, size = 66, normalized size = 0.94

$$-\frac{5\sqrt{14}\,\mathrm{atan}\left(\frac{\sqrt{14}(-140x+280)}{560\sqrt{x^2+6x-1}}\right)}{84}+\frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}(-224x-224)}{224\sqrt{x^2+6x-1}}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
```

```
[Out] -5*sqrt(14)*atan(sqrt(14)*(-140*x + 280)/(560*sqrt(x**2 + 6*x - 1
)))/84 + sqrt(7)*atanh(sqrt(7)*(-224*x - 224)/(224*sqrt(x**2 + 6*
x - 1)))/21
```

Mathematica [C] time = 6.39459, size = 1102, normalized size = 15.74

$$\frac{i\left(i+4\sqrt{2}\right)\tan^{-1}\left(\frac{4\left(1008\sqrt{2}x^4+3789ix^4+7728\sqrt{2}x^3+11364ix^3+9772\sqrt{2}x^2+9847ix^2\cdots\right.}{333i\sqrt{2}x^4-1008x^4-297i\sqrt{14\left(-7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}x^3+11346i\sqrt{2}x^3-2184x^3-1287i\sqrt{14\left(-7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}x^2+14563i\sqrt{2}x^2+28112x^2\cdots}\right.}{4\sqrt{14\left(-7+4i\sqrt{2}\right)}}$$

$$+\frac{\left(-i+4\sqrt{2}\right)\tan^{-1}\left(\frac{666i\sqrt{2}x^4+2016x^4-2079i\sqrt{7\left(7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}x^3+22692i\sqrt{2}x^3+4368x^3-99i\sqrt{7\left(7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}x^2+29126i\sqrt{2}x^2-56224x^2+792\cdots}{8064\sqrt{2}x^4-13383ix^4+61824\sqrt{2}x^3-91506ix^3+78176\sqrt{2}x^2-41651ix^2+201\cdots}\right.}{4\sqrt{14\left(7+4i\sqrt{2}\right)}}$$

$$+\frac{\left(i+4\sqrt{2}\right)\log\left(\left(-3ix+2\sqrt{2}-2i\right)^2\left(3ix+2\sqrt{2}+2i\right)^2\right)}{8\sqrt{14\left(-7+4i\sqrt{2}\right)}}$$

$$+\frac{i\left(-i+4\sqrt{2}\right)\log\left(\left(-3ix+2\sqrt{2}-2i\right)^2\left(3ix+2\sqrt{2}+2i\right)^2\right)}{8\sqrt{14\left(7+4i\sqrt{2}\right)}}$$

$$-\frac{\left(i+4\sqrt{2}\right)\log\left(\left(3x^2+4x+4\right)\left(14\sqrt{2}x^2-4ix^2-7i\sqrt{7\left(-7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}x+84\sqrt{2}x+186ix+9i\sqrt{7\left(-7+4i\sqrt{2}\right)}\cdots\right.\right.}{8\sqrt{14\left(-7+4i\sqrt{2}\right)}}$$

$$-\frac{i\left(-i+4\sqrt{2}\right)\log\left(\left(3x^2+4x+4\right)\left(14\sqrt{2}x^2-53ix^2-2i\sqrt{14\left(7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}x+84\sqrt{2}x-108ix-6i\sqrt{14\left(7+4i\cdots\right.}\right.\right.}{8\sqrt{14\left(7+4i\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] ((I/4)*(I + 4*Sqrt[2])*ArcTan[(4*(6344*I - 700*Sqrt[2] + (8586*I)
*x + 2520*Sqrt[2]*x + (9847*I)*x^2 + 9772*Sqrt[2]*x^2 + (11364*I)
*x^3 + 7728*Sqrt[2]*x^3 + (3789*I)*x^4 + 1008*Sqrt[2]*x^4))/(-392
0 - (2908*I)*Sqrt[2] + 19656*x - (9540*I)*Sqrt[2]*x + 28112*x^2 +
 (14563*I)*Sqrt[2]*x^2 - 2184*x^3 + (11346*I)*Sqrt[2]*x^3 - 1008*
x^4 + (333*I)*Sqrt[2]*x^4 - (1188*I)*Sqrt[14*(-7 + (4*I)*Sqrt[2])
]*Sqrt[-1 + 6*x + x^2] - (1584*I)*Sqrt[14*(-7 + (4*I)*Sqrt[2])]*x
*Sqrt[-1 + 6*x + x^2] - (1287*I)*Sqrt[14*(-7 + (4*I)*Sqrt[2])]*x^
2*Sqrt[-1 + 6*x + x^2] - (297*I)*Sqrt[14*(-7 + (4*I)*Sqrt[2])]*x^
3*Sqrt[-1 + 6*x + x^2])])/Sqrt[14*(-7 + (4*I)*Sqrt[2])] + ((-I +
4*Sqrt[2])*ArcTan[(7840 - (5816*I)*Sqrt[2] - 39312*x - (19080*I)*
Sqrt[2]*x - 56224*x^2 + (29126*I)*Sqrt[2]*x^2 + 4368*x^3 + (22692
*I)*Sqrt[2]*x^3 + 2016*x^4 + (666*I)*Sqrt[2]*x^4 + (3564*I)*Sqrt[
7*(7 + (4*I)*Sqrt[2])]*Sqrt[-1 + 6*x + x^2] + (792*I)*Sqrt[7*(7 +
 (4*I)*Sqrt[2])]*x*Sqrt[-1 + 6*x + x^2] - (99*I)*Sqrt[7*(7 + (4*I
```

```
)*Sqrt[2])]*x^2*Sqrt[-1 + 6*x + x^2] - (2079*I)*Sqrt[7*(7 + (4*I)
*Sqrt[2])]*x^3*Sqrt[-1 + 6*x + x^2])/(9836*I - 5600*Sqrt[2] - (38
988*I)*x + 20160*Sqrt[2]*x - (41651*I)*x^2 + 78176*Sqrt[2]*x^2 -
(91506*I)*x^3 + 61824*Sqrt[2]*x^3 - (13383*I)*x^4 + 8064*Sqrt[2]*
x^4)])/(4*Sqrt[14*(7 + (4*I)*Sqrt[2])]) + ((I/8)*(-I + 4*Sqrt[2])
*Log[(-2*I + 2*Sqrt[2] - (3*I)*x)^2*(2*I + 2*Sqrt[2] + (3*I)*x)^2
])/Sqrt[14*(7 + (4*I)*Sqrt[2])] + ((I + 4*Sqrt[2])*Log[(-2*I + 2*
Sqrt[2] - (3*I)*x)^2*(2*I + 2*Sqrt[2] + (3*I)*x)^2])/(8*Sqrt[14*(
-7 + (4*I)*Sqrt[2])]) - ((I + 4*Sqrt[2])*Log[(4 + 4*x + 3*x^2)*(-
101*I - 14*Sqrt[2] + (186*I)*x + 84*Sqrt[2]*x - (4*I)*x^2 + 14*Sq
rt[2]*x^2 + (9*I)*Sqrt[7*(-7 + (4*I)*Sqrt[2])]*Sqrt[-1 + 6*x + x^
2] - (7*I)*Sqrt[7*(-7 + (4*I)*Sqrt[2])]*x*Sqrt[-1 + 6*x + x^2])])
/(8*Sqrt[14*(-7 + (4*I)*Sqrt[2])]) - ((I/8)*(-I + 4*Sqrt[2])*Log[
(4 + 4*x + 3*x^2)*(-52*I - 14*Sqrt[2] - (108*I)*x + 84*Sqrt[2]*x
- (53*I)*x^2 + 14*Sqrt[2]*x^2 - (6*I)*Sqrt[14*(7 + (4*I)*Sqrt[2])
]*Sqrt[-1 + 6*x + x^2] - (2*I)*Sqrt[14*(7 + (4*I)*Sqrt[2])]*x*Sqr
t[-1 + 6*x + x^2])])/Sqrt[14*(7 + (4*I)*Sqrt[2])]
```

Maple [B] time = 0.041, size = 158, normalized size = 2.3

$$-\frac{1}{84}\sqrt{-6\,\frac{(-2+x)^2}{(-1-x)^2}+15}\left(4\sqrt{7}\mathit{Artanh}\left(1/21\sqrt{-6\,\frac{(-2+x)^2}{(-1-x)^2}+15}\sqrt{7}\right)-5\sqrt{14}\arctan\left(1/4\,\frac{\sqrt{14}(-2+x)}{-1-x}\sqrt{-6\,\frac{(-2+x)^2}{(-1-x)^2}+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x)
```

```
[Out] -1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(4*7^(1/2)*arctanh(1/21*(-6
*(-2+x)^2/(-1-x)^2+15)^(1/2)*7^(1/2))-5*14^(1/2)*arctan(1/4*14^(1
/2)*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x
)/(-1-x)))/(-3*(2*(-2+x)^2/(-1-x)^2-5)/(1+(-2+x)/(-1-x))^2)^(1/2)
/(1+(-2+x)/(-1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x+1}{(3\,x^2+4\,x+4)\sqrt{x^2+6\,x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

Fricas [A] time = 0.254243, size = 556, normalized size = 7.94

$$\frac{1}{2772}\sqrt{42}\sqrt{33}\left(\sqrt{11}\sqrt{2}\log\left(22\sqrt{42}\sqrt{33}\sqrt{11}\sqrt{2}(x-2)+4356x^2-22\left(\sqrt{42}\sqrt{33}\sqrt{11}\sqrt{2}+198x+132\right)\sqrt{x^2+6x-1}+15972\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)),x, algorithm="fricas")
```

```
[Out] 1/2772*sqrt(42)*sqrt(33)*(sqrt(11)*sqrt(2)*log(22*sqrt(42)*sqrt(3
3)*sqrt(11)*sqrt(2)*(x - 2) + 4356*x^2 - 22*(sqrt(42)*sqrt(33)*sq
rt(11)*sqrt(2) + 198*x + 132)*sqrt(x^2 + 6*x - 1) + 15972*x + 159
72) - sqrt(11)*sqrt(2)*log(-22*sqrt(42)*sqrt(33)*sqrt(11)*sqrt(2)
*(x - 2) + 4356*x^2 + 22*(sqrt(42)*sqrt(33)*sqrt(11)*sqrt(2) - 19
8*x - 132)*sqrt(x^2 + 6*x - 1) + 15972*x + 15972) - 10*sqrt(11)*a
rctan(-44*(sqrt(42)*sqrt(33) + 21*sqrt(11)*sqrt(2))/(11*sqrt(42)*
sqrt(33)*sqrt(2)*(3*x + 2) - 33*sqrt(42)*sqrt(33)*sqrt(2)*sqrt(x^
2 + 6*x - 1) - sqrt(42)*sqrt(33)*sqrt(-22*sqrt(42)*sqrt(33)*sqrt(
11)*sqrt(2)*(x - 2) + 4356*x^2 + 22*(sqrt(42)*sqrt(33)*sqrt(11)*s
qrt(2) - 198*x - 132)*sqrt(x^2 + 6*x - 1) + 15972*x + 15972) - 46
2*sqrt(11))) + 10*sqrt(11)*arctan(-44*(sqrt(42)*sqrt(33) - 21*sqr
t(11)*sqrt(2))/(11*sqrt(42)*sqrt(33)*sqrt(2)*(3*x + 2) - 33*sqrt(
42)*sqrt(33)*sqrt(2)*sqrt(x^2 + 6*x - 1) - sqrt(42)*sqrt(33)*sqrt
(22*sqrt(42)*sqrt(33)*sqrt(11)*sqrt(2)*(x - 2) + 4356*x^2 - 22*(s
qrt(42)*sqrt(33)*sqrt(11)*sqrt(2) + 198*x + 132)*sqrt(x^2 + 6*x -
 1) + 15972*x + 15972) + 462*sqrt(11))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{(3x^2+4x+4)\sqrt{x^2+6x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)),x, algorithm="giac")

[Out] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

```
[Out] -(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])
/Sqrt[35]) - ((A + B)*ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*
x + 10*x^2])])/(2*Sqrt[35])
```

Rubi [A] time = 0.288736, antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}(-x(A+B)+A+B)}{2\sqrt{10x^2-22x+13}(A+B)}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In]  Int[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] -(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])
/Sqrt[35]) - ((A + B)*ArcTanh[(Sqrt[35]*(A + B - (A + B)*x))/(2*(
A + B)*Sqrt[13 - 22*x + 10*x^2])])/(2*Sqrt[35])
```

Rubi in Sympy [A] time = 22.7356, size = 105, normalized size = 1.31

$$\frac{\sqrt{35}\,(A+B)\,\text{atanh}\left(\frac{\sqrt{35}(-140A-140B+x(140A+140B))}{280(A+B)\sqrt{10x^2-22x+13}}\right)}{70}-\frac{\sqrt{35}\,(2A+B)\,\text{atan}\left(\frac{\sqrt{35}(2240A+1120B+x(-1120A-560B))}{560(2A+B)\sqrt{10x^2-22x+13}}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] sqrt(35)*(A + B)*atanh(sqrt(35)*(-140*A - 140*B + x*(140*A + 140*
B))/(280*(A + B)*sqrt(10*x**2 - 22*x + 13)))/70 - sqrt(35)*(2*A +
 B)*atan(sqrt(35)*(2240*A + 1120*B + x*(-1120*A - 560*B))/(560*(2
*A + B)*sqrt(10*x**2 - 22*x + 13)))/35
```

Mathematica [C] time = 4.38478, size = 1149, normalized size = 14.36

$$\frac{((8-2i)A+(4-2i)B)\tan^{-1}\left(\frac{}{\left((-4800+2800i)x^3+10\left((34+17i)\sqrt{35}\sqrt{10x^2-22x+13}+(1969-892i)\right)x^2-\left((1054+357i)\sqrt{35}\sqrt{10x^2-22x+13}+(25633-9460i)\right)x+\cdots\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] (((8 - 2*I)*A + (4 - 2*I)*B)*ArcTan[(4*A^2*((-2494 - 6746*I) + (3
811 + 15444*I)*x - (1900 + 11640*I)*x^2 + (300 + 2800*I)*x^3) + (
2 + 4*I)*B^2*((-1843 + 92*I) + (3955 + 186*I)*x - (2827 + 336*I)*
x^2 + (645 + 110*I)*x^3) + (4 + 8*I)*A*B*((-3439 - 76*I) + (7427
+ 942*I)*x - (5354 + 1092*I)*x^2 + (1240 + 320*I)*x^3))/((1 + 2*I
)*B^2*((-608 - 1208*I) + (395 + 610*I)*x^3 + (66 - 77*I)*Sqrt[35]
*Sqrt[13 - 22*x + 10*x^2] + x*((1540 + 3036*I) - (104 - 103*I)*Sq
rt[35]*Sqrt[13 - 22*x + 10*x^2]) + (4 - 3*I)*x^2*((80 - 549*I) +
10*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2])) + A^2*((10987 - 3210*I) -
(4800 - 2800*I)*x^3 + (748 + 187*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*
x^2] + 10*x^2*((1969 - 892*I) + (34 + 17*I)*Sqrt[35]*Sqrt[13 - 22
*x + 10*x^2]) - x*((25633 - 9460*I) + (1054 + 357*I)*Sqrt[35]*Sqr
t[13 - 22*x + 10*x^2])) + A*B*((9519 - 6362*I) - (4225 - 4200*I)*
x^3 + (792 + 198*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2] + (10 + 5*I
)*x^2*((828 - 1871*I) + 36*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2]) - x
*((22801 - 16808*I) + (1116 + 378*I)*Sqrt[35]*Sqrt[13 - 22*x + 10
*x^2])))] + ((-2 + 8*I)*A - (2 - 4*I)*B)*ArcTanh[(A^2*((3594 - 15
991*I) - (8096 - 43289*I)*x + (5990 - 39425*I)*x^2 - (1400 - 1217
5*I)*x^3) + (2 + I)*B^2*((-367 - 3288*I) + (1085 + 8506*I)*x - (1
073 + 7336*I)*x^2 + (355 + 2110*I)*x^3) + (4 + 2*I)*A*B*((-1147 -
 4952*I) + (3185 + 12882*I)*x - (2993 + 11256*I)*x^2 + (955 + 331
0*I)*x^3))/(2*((2 + I)*B^2*((-1208 - 608*I) + (610 + 395*I)*x^3 +
 (77 - 66*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2] + x*((3036 + 1540*
I) - (103 - 104*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2]) + (3 - 4*I)
*x^2*((-80 - 549*I) + 10*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2])) + A*
B*((-9519 - 6362*I) + (4225 + 4200*I)*x^3 + (792 - 198*I)*Sqrt[35
]*Sqrt[13 - 22*x + 10*x^2] + x*((22801 + 16808*I) - (1116 - 378*I
)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2]) + (10 - 5*I)*x^2*((-828 - 18
71*I) + 36*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2])) + A^2*((4800 + 280
0*I)*x^3 + x*((25633 + 9460*I) - (1054 - 357*I)*Sqrt[35]*Sqrt[13
- 22*x + 10*x^2]) - (10*I)*x^2*((892 - 1969*I) + (17 + 34*I)*Sqrt
[35]*Sqrt[13 - 22*x + 10*x^2]) + (1 - 4*I)*((109 - 2774*I) + (88
+ 165*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2]))))] - 2*A*Log[I*(17 -
 18*x + 5*x^2)] - 2*B*Log[I*(17 - 18*x + 5*x^2)] + (1 - 4*I)*A*Lo
g[(1 + 2*I)*((-127 - 1566*I) + (118 + 2844*I)*x - (25 + 1350*I)*x
^2 + (68*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2] - (70*I)*Sqrt[35]*x
*Sqrt[13 - 22*x + 10*x^2])] + (1 - 2*I)*B*Log[(1 + 2*I)*((-127 -
1566*I) + (118 + 2844*I)*x - (25 + 1350*I)*x^2 + (68*I)*Sqrt[35]*
Sqrt[13 - 22*x + 10*x^2] - (70*I)*Sqrt[35]*x*Sqrt[13 - 22*x + 10*
x^2])] + (1 + 4*I)*A*Log[(2 + I)*((1566 + 127*I) - (2844 + 118*I)
*x + (1350 + 25*I)*x^2 - 68*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2] + 7
0*Sqrt[35]*x*Sqrt[13 - 22*x + 10*x^2])] + (1 + 2*I)*B*Log[(2 + I)
*((1566 + 127*I) - (2844 + 118*I)*x + (1350 + 25*I)*x^2 - 68*Sqrt
[35]*Sqrt[13 - 22*x + 10*x^2] + 70*Sqrt[35]*x*Sqrt[13 - 22*x + 10
```

```
*x^2])])/(8*Sqrt[35])
```

Maple [B] time = 0.039, size = 192, normalized size = 2.4

$$\frac{\sqrt{35}}{70}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\left(Artanh\left(\frac{2\sqrt{35}}{35}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\right)A-4\arctan\left(\frac{\sqrt{35}\,(-2+x)}{1-x}\frac{1}{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}}\right)A+Artanh\left(\frac{2\sqrt{35}}{35}\sqrt{\frac{(-2}{(1-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x)
```

```
[Out] 1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/
(1-x)^2+9)^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+
9)^(1/2)*(-2+x)/(1-x))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*
35^(1/2))*B-2*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(
1-x))*B)/((((-2+x)^2/(1-x)^2+9)/(1+(-2+x)/(1-x))^2)^(1/2)/(1+(-2+x
)/(1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Ax+B}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),x, algorithm="m
```

```
[Out] integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),x, algorithm="f
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Ax+B}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13
)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),x, algorithm="g
```

```
[Out] Exception raised: RuntimeError
```

3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

```
[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[
35])
```

Rubi [A] time = 0.0649665, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[
35])
```

Rubi in Sympy [A] time = 5.85201, size = 32, normalized size = 0.84

$$\frac{\sqrt{35}\,\operatorname{atanh}\left(\frac{\sqrt{35}(-2x+2)}{4\sqrt{10x^2-22x+13}}\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] sqrt(35)*atanh(sqrt(35)*(-2*x + 2)/(4*sqrt(10*x**2 - 22*x + 13)))
/70
```

Mathematica [C] time = 0.597147, size = 410, normalized size = 10.79

$$2\log\left(i\left(5x^2-18x+17\right)\right)-\log\left((1+2i)\left((1350-25i)x^2+70\sqrt{35}\sqrt{10x^2-22x+13}\,x-68\sqrt{35}\sqrt{10x^2-22x+13}-(2844-\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] ((-2*I)*ArcTan[(4*((-2 + 2*I) + 5*x)*(13 - 22*x + 10*x^2))/((-819
 - 182*I) + 350*x^3 + (44 + 11*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^
2] + x*((1841 + 308*I) - (62 + 21*I)*Sqrt[35]*Sqrt[13 - 22*x + 10
*x^2]) + 10*x^2*((-140 - 14*I) + (2 + I)*Sqrt[35]*Sqrt[13 - 22*x
+ 10*x^2]))] - (2*I)*ArcTan[((7 + 14*I)*((-169 - 116*I) + (419 +
218*I)*x - (335 + 140*I)*x^2 + (85 + 30*I)*x^3))/((-1638 + 364*I)
 + 700*x^3 - (88 - 22*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2] + (20*
I)*x^2*((14 + 140*I) + (1 + 2*I)*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2
]) + (4 - 2*I)*x*((798 + 245*I) + (29 + 4*I)*Sqrt[35]*Sqrt[13 - 2
2*x + 10*x^2]))] + 2*Log[I*(17 - 18*x + 5*x^2)] - Log[(1 + 2*I)*(
(1566 - 127*I) - (2844 - 118*I)*x + (1350 - 25*I)*x^2 - 68*Sqrt[3
5]*Sqrt[13 - 22*x + 10*x^2] + 70*Sqrt[35]*x*Sqrt[13 - 22*x + 10*x
^2])] - Log[(2 + I)*((1566 + 127*I) - (2844 + 118*I)*x + (1350 +
25*I)*x^2 - 68*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2] + 70*Sqrt[35]*x*
Sqrt[13 - 22*x + 10*x^2])])/(8*Sqrt[35])
```

Maple [B] time = 0.016, size = 94, normalized size = 2.5

$$-\frac{\sqrt{35}}{70}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}Artanh\left(\frac{2\sqrt{35}}{35}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\right)\frac{1}{\sqrt{1\left(\frac{(-2+x)^2}{(1-x)^2}+9\right)\left(1+\frac{-2+x}{1-x}\right)^{-2}}}\left(1+\frac{-2+x}{1-x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x)
```

```
[Out] -1/70/((((-2+x)^2/(1-x)^2+9)/(1+(-2+x)/(1-x))^2)^(1/2)/(1+(-2+x)/(
1-x))*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*arctanh(2/35*((-2+x)^2/
(1-x)^2+9)^(1/2)*35^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-2}{\sqrt{10x^2-22x+13}(5x^2-18x+17)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),x, algorithm="max
```

```
[Out] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),
 x)
```

Fricas [A] time = 0.220966, size = 112, normalized size = 2.95

$$\frac{1}{280}\sqrt{35}\log\left(\frac{\sqrt{35}\left(11225\,x^4-47220\,x^3+75534\,x^2-54372\,x+14849\right)-280\left(75\,x^3-233\,x^2+245\,x-87\right)\sqrt{10\,x^2-22\,x+}}{25\,x^4-180\,x^3+494\,x^2-612\,x+289}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),x, algorithm="fri
```

```
[Out] 1/280*sqrt(35)*log((sqrt(35)*(11225*x^4 - 47220*x^3 + 75534*x^2 -
 54372*x + 14849) - 280*(75*x^3 - 233*x^2 + 245*x - 87)*sqrt(10*x
^2 - 22*x + 13))/(25*x^4 - 180*x^3 + 494*x^2 - 612*x + 289))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13))
, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)),x, algorithm="gia
```

```
[Out] Exception raised: RuntimeError
```

3.250 $\int x^4\sqrt{5-x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{25}{16}\sqrt{5-x^2}x+\frac{1}{6}\sqrt{5-x^2}x^5-\frac{5}{24}\sqrt{5-x^2}x^3+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5
 - x^2])/6 + (125*ArcSin[x/Sqrt[5]])/16
```

Rubi [A] time = 0.0544259, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{25}{16}\sqrt{5-x^2}x+\frac{1}{6}\sqrt{5-x^2}x^5-\frac{5}{24}\sqrt{5-x^2}x^3+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5
 - x^2])/6 + (125*ArcSin[x/Sqrt[5]])/16
```

Rubi in Sympy [A] time = 3.6922, size = 54, normalized size = 0.83

$$\frac{x^5\sqrt{-x^2+5}}{6}-\frac{5x^3\sqrt{-x^2+5}}{24}-\frac{25x\sqrt{-x^2+5}}{16}+\frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(-x**2+5)**(1/2),x)
```

```
[Out] x**5*sqrt(-x**2 + 5)/6 - 5*x**3*sqrt(-x**2 + 5)/24 - 25*x*sqrt(-x
**2 + 5)/16 + 125*asin(sqrt(5)*x/5)/16
```

Mathematica [A] time = 0.0368336, size = 40, normalized size = 0.62

$$\frac{1}{48}\left(x\sqrt{5-x^2}\left(8x^4-10x^2-75\right)+375\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (x*Sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4) + 375*ArcSin[x/Sqrt[5]])/
48
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$-\frac{x^3}{6}\left(-x^2+5\right)^{\frac{3}{2}}-\frac{5\,x}{8}\left(-x^2+5\right)^{\frac{3}{2}}+\frac{25\,x}{16}\sqrt{-x^2+5}+\frac{125}{16}\arcsin\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(-x^2+5)^(1/2),x)
```

```
[Out] -1/6*x^3*(-x^2+5)^(3/2)-5/8*x*(-x^2+5)^(3/2)+25/16*x*(-x^2+5)^(1/
2)+125/16*arcsin(1/5*x*5^(1/2))
```

Maxima [A] time = 1.49936, size = 65, normalized size = 1.

$$-\frac{1}{6}\left(-x^2+5\right)^{\frac{3}{2}}x^3-\frac{5}{8}\left(-x^2+5\right)^{\frac{3}{2}}x+\frac{25}{16}\sqrt{-x^2+5}x+\frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 5)*x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 5)^(3/2)*x^3 - 5/8*(-x^2 + 5)^(3/2)*x + 25/16*sqrt(-
x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)*x)
```

Fricas [A] time = 0.219146, size = 57, normalized size = 0.88

$$\frac{1}{48}\left(8\,x^5-10\,x^3-75\,x\right)\sqrt{-x^2+5}-\frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 5)*x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2 + 5) - 125/16*arctan(sqrt(
-x^2 + 5)/x)
```

Sympy [A] time = 7.68795, size = 155, normalized size = 2.38

$$\begin{cases} \frac{ix^7}{6\sqrt{x^2-5}} - \frac{25ix^5}{24\sqrt{x^2-5}} - \frac{25ix^3}{48\sqrt{x^2-5}} + \frac{125ix}{16\sqrt{x^2-5}} - \frac{125i\operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{for } \frac{|x^2|}{5} > 1 \\ -\frac{x^7}{6\sqrt{-x^2+5}} + \frac{25x^5}{24\sqrt{-x^2+5}} + \frac{25x^3}{48\sqrt{-x^2+5}} - \frac{125x}{16\sqrt{-x^2+5}} + \frac{125\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 -
5)) - 25*I*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5))
 - 125*I*acosh(sqrt(5)*x/5)/16, Abs(x**2)/5 > 1), (-x**7/(6*sqrt(
-x**2 + 5)) + 25*x**5/(24*sqrt(-x**2 + 5)) + 25*x**3/(48*sqrt(-x*
*2 + 5)) - 125*x/(16*sqrt(-x**2 + 5)) + 125*asin(sqrt(5)*x/5)/16,
 True))
```

GIAC/XCAS [A] time = 0.210067, size = 49, normalized size = 0.75

$$\frac{1}{48}\left(2\left(4x^2-5\right)x^2-75\right)\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 5)*x^4,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/
5*sqrt(5)*x)
```

3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{x^2+2}}{15x}-\frac{\sqrt{x^2+2}}{10x^5}+\frac{\sqrt{x^2+2}}{15x^3}$$

```
[Out] -Sqrt[2 + x^2]/(10*x^5) + Sqrt[2 + x^2]/(15*x^3) - Sqrt[2 + x^2]/
(15*x)
```

Rubi [A] time = 0.0355364, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{\sqrt{x^2+2}}{15x}-\frac{\sqrt{x^2+2}}{10x^5}+\frac{\sqrt{x^2+2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -Sqrt[2 + x^2]/(10*x^5) + Sqrt[2 + x^2]/(15*x^3) - Sqrt[2 + x^2]/
(15*x)
```

Rubi in Sympy [A] time = 2.65416, size = 37, normalized size = 0.76

$$-\frac{\sqrt{x^2+2}}{15x}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**2+2)**(1/2),x)
```

```
[Out] -sqrt(x**2 + 2)/(15*x) + sqrt(x**2 + 2)/(15*x**3) - sqrt(x**2 + 2
)/(10*x**5)
```

Mathematica [A] time = 0.0134748, size = 28, normalized size = 0.57

$$-\frac{\sqrt{x^2+2}\left(2x^4-2x^2+3\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -(Sqrt[2 + x^2]*(3 - 2*x^2 + 2*x^4))/(30*x^5)
```

Maple [A] time = 0.006, size = 25, normalized size = 0.5

$$-\frac{2x^4-2x^2+3}{30x^5}\sqrt{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^2+2)^(1/2),x)
```

```
[Out] -1/30*(x^2+2)^(1/2)*(2*x^4-2*x^2+3)/x^5
```

Maxima [A] time = 1.50153, size = 50, normalized size = 1.02

$$-\frac{\sqrt{x^2+2}}{15x}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*x^6),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 +
2)/x^5
```

Fricas [A] time = 0.30163, size = 99, normalized size = 2.02

$$\frac{20x^4+35x^2-5\left(4x^3+3x\right)\sqrt{x^2+2}+6}{30\left(4x^{10}+10x^8+5x^6-(4x^9+6x^7+x^5)\sqrt{x^2+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*x^6),x, algorithm="fricas")
```

```
[Out] 1/30*(20*x^4 + 35*x^2 - 5*(4*x^3 + 3*x)*sqrt(x^2 + 2) + 6)/(4*x^1
0 + 10*x^8 + 5*x^6 - (4*x^9 + 6*x^7 + x^5)*sqrt(x^2 + 2))
```

Sympy [A] time = 8.40544, size = 41, normalized size = 0.84

$$-\frac{\sqrt{1+\frac{2}{x^2}}}{15}+\frac{\sqrt{1+\frac{2}{x^2}}}{15x^2}-\frac{\sqrt{1+\frac{2}{x^2}}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**2+2)**(1/2),x)
```

```
[Out] -sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x*
*2)/(10*x**4)
```

GIAC/XCAS [A] time = 0.2088, size = 69, normalized size = 1.41

$$\frac{32\left(5\left(x-\sqrt{x^2+2}\right)^4-5\left(x-\sqrt{x^2+2}\right)^2+2\right)}{15\left(\left(x-\sqrt{x^2+2}\right)^2-2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*x^6),x, algorithm="giac")
```

```
[Out] 32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x
 - sqrt(x^2 + 2))^2 - 2)^5
```

3.252 $\int \frac{1}{(3+2x^2)^{7/2}}\,dx$

Optimal. Leaf size=49

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\,(2x^2+3)^{3/2}}+\frac{x}{15\,(2x^2+3)^{5/2}}$$

```
[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/
(405*Sqrt[3 + 2*x^2])
```

Rubi [A] time = 0.0190796, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\,(2x^2+3)^{3/2}}+\frac{x}{15\,(2x^2+3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/
(405*Sqrt[3 + 2*x^2])
```

Rubi in Sympy [A] time = 0.792919, size = 42, normalized size = 0.86

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\,(2x^2+3)^{\frac{3}{2}}}+\frac{x}{15\,(2x^2+3)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**2+3)**(7/2),x)
```

```
[Out] 8*x/(405*sqrt(2*x**2 + 3)) + 4*x/(135*(2*x**2 + 3)**(3/2)) + x/(1
5*(2*x**2 + 3)**(5/2))
```

Mathematica [A] time = 0.0149045, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4+120x^2+135\right)}{405\,(2x^2+3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] (x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

Maple [A] time = 0.005, size = 25, normalized size = 0.5

$$\frac{x\left(32x^4+120x^2+135\right)}{405}\left(2x^2+3\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^2+3)^(7/2),x)
```

```
[Out] 1/405*x*(32*x^4+120*x^2+135)/(2*x^2+3)^(5/2)
```

Maxima [A] time = 1.35213, size = 50, normalized size = 1.02

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\left(2x^2+3\right)^{\frac{3}{2}}}+\frac{x}{15\left(2x^2+3\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)^(-7/2),x, algorithm="maxima")
```

```
[Out] 8/405*x/sqrt(2*x^2 + 3) + 4/135*x/(2*x^2 + 3)^(3/2) + 1/15*x/(2*x
^2 + 3)^(5/2)
```

Fricas [A] time = 0.201533, size = 59, normalized size = 1.2

$$\frac{\left(32x^5+120x^3+135x\right)\sqrt{2x^2+3}}{405\left(8x^6+36x^4+54x^2+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)^(-7/2),x, algorithm="fricas")
```

```
[Out] 1/405*(32*x^5 + 120*x^3 + 135*x)*sqrt(2*x^2 + 3)/(8*x^6 + 36*x^4
+ 54*x^2 + 27)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**2+3)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219546, size = 35, normalized size = 0.71

$$\frac{\left(8\left(4x^2+15\right)x^2+135\right)x}{405\left(2x^2+3\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)^(-7/2),x, algorithm="giac")
```

```
[Out] 1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^(5/2)
```

3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=12

$$\log\left(a+\sqrt{x^2+1}\right)$$

```
[Out] Log[a + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0748354, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\log\left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]

[Out] Log[a + Sqrt[1 + x^2]]
```

Rubi in Sympy [A] time = 3.57427, size = 10, normalized size = 0.83

$$\log\left(a+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)

[Out] log(a + sqrt(x**2 + 1))
```

Mathematica [A] time = 0.0124393, size = 12, normalized size = 1.

$$\log\left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]

[Out] Log[a + Sqrt[1 + x^2]]
```

Maple [B] time = 0.062, size = 328, normalized size = 27.3

$$\frac{1}{a}\sqrt{x^2+1}-\frac{1}{2\,a}\sqrt{\left(x-\sqrt{(1+a)\,(a-1)}\right)^2+2\,\sqrt{(1+a)\,(a-1)}\,\left(x-\sqrt{(1+a)\,(a-1)}\right)+a^2}$$
$$+\frac{a}{2}\ln\left(1\left(2\,a^2+2\,\sqrt{(1+a)\,(a-1)}\,\left(x-\sqrt{(1+a)\,(a-1)}\right)+2\,\sqrt{a^2}\sqrt{\left(x-\sqrt{(1+a)\,(a-1)}\right)^2+2\,\sqrt{(1+a)\,(a-1)}\,\left(x-\sqrt{(1+a\right.\right.\right.$$
$$-\frac{1}{2\,a}\sqrt{\left(x+\sqrt{(1+a)\,(a-1)}\right)^2-2\,\sqrt{(1+a)\,(a-1)}\,\left(x+\sqrt{(1+a)\,(a-1)}\right)+a^2}$$
$$+\frac{a}{2}\ln\left(1\left(2\,a^2-2\,\sqrt{(1+a)\,(a-1)}\,\left(x+\sqrt{(1+a)\,(a-1)}\right)+2\,\sqrt{a^2}\sqrt{\left(x+\sqrt{(1+a)\,(a-1)}\right)^2-2\,\sqrt{(1+a)\,(a-1)}\,\left(x+\sqrt{(1+a\right.\right.\right.$$
$$+\frac{\ln\left(-a^2+x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x^2+a*(x^2+1)^(1/2)),x)
```

```
[Out] 1/a*(x^2+1)^(1/2)-1/2/a*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1)
)^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((
2*a^2+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)
*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1)
)^(1/2))+a^2)^(1/2))/(x-((1+a)*(a-1))^(1/2)))-1/2/a*((x+((1+a)*(a
-1))^(1/2))^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^
(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)
*(a-1))^(1/2))+2*(a^2)^(1/2)*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*
(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x+((1+a)*(a-1))
^(1/2)))+1/2*ln(-a^2+x^2+1)
```

Maxima [A] time = 1.33992, size = 14, normalized size = 1.17

$$\log\left(a+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + sqrt(x^2 + 1)*a + 1),x, algorithm="maxima")
```

```
[Out] log(a + sqrt(x^2 + 1))
```

Fricas [A] time = 0.227411, size = 84, normalized size = 7.

$$\frac{1}{2}\log\left(-a^2+x^2+1\right)-\frac{1}{2}\log\left(ax+x^2-\sqrt{x^2+1}(a+x)+1\right)+\frac{1}{2}\log\left(-ax+x^2+\sqrt{x^2+1}(a-x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + sqrt(x^2 + 1)*a + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a +
x) + 1) + 1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a\sqrt{x^2+1}+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(a*sqrt(x**2 + 1) + x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.217181, size = 15, normalized size = 1.25

$$\ln\left(\left|a+\sqrt{x^2+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + sqrt(x^2 + 1)*a + 1),x, algorithm="giac")
```

```
[Out] ln(abs(a + sqrt(x^2 + 1)))
```

3.254 $\int \frac{1-x+x^2}{(1+x^2)^{3/2}} \, dx$

Optimal. Leaf size=12

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

```
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Rubi [A] time = 0.0161735, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x + x^2)/(1 + x^2)^(3/2),x]

[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Rubi in Sympy [A] time = 2.03418, size = 12, normalized size = 1.

$$\operatorname{asinh}(x)+\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-x+1)/(x**2+1)**(3/2),x)

[Out] asinh(x) + 1/sqrt(x**2 + 1)
```

Mathematica [A] time = 0.0193417, size = 12, normalized size = 1.

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Maple [A] time = 0.009, size = 11, normalized size = 0.9

$$Arcsinh(x) + \frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-x+1)/(x^2+1)^(3/2),x)
```

```
[Out] arcsinh(x)+1/(x^2+1)^(1/2)
```

Maxima [A] time = 1.48769, size = 14, normalized size = 1.17

$$\frac{1}{\sqrt{x^2+1}} + \operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x + 1)/(x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/sqrt(x^2 + 1) + arcsinh(x)
```

Fricas [A] time = 0.25672, size = 78, normalized size = 6.5

$$-\frac{\left(x^2 - \sqrt{x^2+1}x + 1\right)\log\left(-x + \sqrt{x^2+1}\right) + x - \sqrt{x^2+1}}{x^2 - \sqrt{x^2+1}x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x + 1)/(x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x^2 - sqrt(x^2 + 1)*x + 1)*log(-x + sqrt(x^2 + 1)) + x - sqrt(
x^2 + 1))/(x^2 - sqrt(x^2 + 1)*x + 1)
```

Sympy [A] time = 9.90964, size = 29, normalized size = 2.42

$$\frac{x^2 \operatorname{asinh}(x)}{x^2+1} + \frac{\operatorname{asinh}(x)}{x^2+1} + \frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-x+1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)
```

GIAC/XCAS [A] time = 0.215245, size = 30, normalized size = 2.5

$$\frac{1}{\sqrt{x^2+1}} - \ln\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x + 1)/(x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/sqrt(x^2 + 1) - ln(-x + sqrt(x^2 + 1))
```

3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2} \, dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Rubi [A] time = 0.0350852, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.0803, size = 27, normalized size = 1.

$$\operatorname{asinh}(x)-\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}x}{2\sqrt{x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(x**2+2),x)
```

```
[Out] asinh(x) - sqrt(2)*atanh(sqrt(2)*x/(2*sqrt(x**2 + 1)))/2
```

Mathematica [A] time = 0.0193743, size = 27, normalized size = 1.

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Maple [A] time = 0.016, size = 23, normalized size = 0.9

$$Arcsinh(x) - \frac{\sqrt{2}}{2} Artanh\left(\frac{x\sqrt{2}}{2}\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(x^2+2),x)
```

```
[Out] arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{x^2+1}x}{x^2+2} + \int \frac{\sqrt{x^2+1}x^4}{x^6+5x^4+8x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/(x^2 + 2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*
x^4 + 8*x^2 + 4), x)
```

Fricas [A] time = 0.25256, size = 143, normalized size = 5.3

$$-\frac{1}{4}\sqrt{2}\left(2\sqrt{2}\log\left(-x+\sqrt{x^2+1}\right) - \log\left(-\frac{4x^2 - \sqrt{2}\left(2x^4+5x^2+6\right) + 2\sqrt{x^2+1}\left(\sqrt{2}\left(x^3+2x\right) - 2x\right) + 8}{2x^4+5x^2-2\left(x^3+2x\right)\sqrt{x^2+1}+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/(x^2 + 2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(2)*log(-x + sqrt(x^2 + 1)) - log(-(4*x^2 - s
qrt(2)*(2*x^4 + 5*x^2 + 6) + 2*sqrt(x^2 + 1)*(sqrt(2)*(x^3 + 2*x)
```

```
- 2*x) + 8)/(2*x^4 + 5*x^2 - 2*(x^3 + 2*x)*sqrt(x^2 + 1) + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/(x**2 + 2), x)
```

GIAC/XCAS [A] time = 0.217259, size = 86, normalized size = 3.19

$$\frac{1}{4}\,\sqrt{2}\ln\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\,\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\,\sqrt{2}+3}\right)-\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/(x^2 + 2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt
(x^2 + 1))^2 + 2*sqrt(2) + 3)) - ln(-x + sqrt(x^2 + 1))
```

3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4(x^2+2)}$$

```
[Out] -(x*Sqrt[1 + x^2])/(4*(2 + x^2)) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 +
 x^2])])/(4*Sqrt[2])
```

Rubi [A] time = 0.0411108, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4(x^2+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[1 + x^2])/(4*(2 + x^2)) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 +
 x^2])])/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 3.75993, size = 42, normalized size = 0.88

$$-\frac{x\sqrt{x^2+1}}{4(x^2+2)}+\frac{3\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}x}{2\sqrt{x^2+1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(x**2 + 1)/(4*(x**2 + 2)) + 3*sqrt(2)*atanh(sqrt(2)*x/(2*s
qrt(x**2 + 1)))/8
```

Mathematica [A] time = 0.0333742, size = 48, normalized size = 1.

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4(x^2+2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[1 + x^2])/(4*(2 + x^2)) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 +
 x^2])])/(4*Sqrt[2])
```

Maple [A] time = 0.023, size = 46, normalized size = 1.

$$\frac{x}{4}\frac{1}{\sqrt{x^2+1}}\left(\frac{x^2}{x^2+1}-2\right)^{-1}+\frac{3\sqrt{2}}{8}\mathit{Artanh}\left(\frac{x\sqrt{2}}{2}\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+2)^2/(x^2+1)^(1/2),x)
```

```
[Out] 1/4/(x^2+1)^(1/2)*x/(x^2/(x^2+1)-2)+3/8*arctanh(1/2*x*2^(1/2)/(x^
2+1)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+2)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)
```

Fricas [A] time = 0.238703, size = 238, normalized size = 4.96

$$-\frac{6\sqrt{2}\sqrt{x^2+1}x+3\left(2x^4+5x^2-2\left(x^3+2x\right)\sqrt{x^2+1}+2\right)\log\left(\frac{4x^2+\sqrt{2}(2x^4+5x^2+6)-2\sqrt{x^2+1}\left(\sqrt{2}(x^3+2x)+2x\right)+8}{2x^4+5x^2-2(x^3+2x)\sqrt{x^2+1}+2}\right)-2\sqrt{2}(3x^2+2}{8\left(2\sqrt{2}(x^3+2x)\sqrt{x^2+1}-\sqrt{2}(2x^4+5x^2+2)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/8*(6*sqrt(2)*sqrt(x^2 + 1)*x + 3*(2*x^4 + 5*x^2 - 2*(x^3 + 2*x
)*sqrt(x^2 + 1) + 2)*log((4*x^2 + sqrt(2)*(2*x^4 + 5*x^2 + 6) - 2
*sqrt(x^2 + 1)*(sqrt(2)*(x^3 + 2*x) + 2*x) + 8)/(2*x^4 + 5*x^2 -
2*(x^3 + 2*x)*sqrt(x^2 + 1) + 2)) - 2*sqrt(2)*(3*x^2 + 2))/(2*sqr
t(2)*(x^3 + 2*x)*sqrt(x^2 + 1) - sqrt(2)*(2*x^4 + 5*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)
```

GIAC/XCAS [A] time = 0.215476, size = 136, normalized size = 2.83

$$-\frac{3}{16}\,\sqrt{2}\ln\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\,\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\,\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2+1}\right)^2+1}{2\left(\left(x-\sqrt{x^2+1}\right)^4+6\left(x-\sqrt{x^2+1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*ln(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sq
rt(x^2 + 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 +
 1)/((x - sqrt(x^2 + 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)
```

3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=41

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[
-2 + x^2]]
```

Rubi [A] time = 0.0701995, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[
-2 + x^2]]
```

Rubi in Sympy [A] time = 5.7656, size = 36, normalized size = 0.88

$$\operatorname{atanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-\frac{\sqrt{6}\operatorname{atanh}\left(\frac{\sqrt{6}x}{3\sqrt{x^2-2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
```

```
[Out] atanh(x/sqrt(x**2 - 2)) - sqrt(6)*atanh(sqrt(6)*x/(3*sqrt(x**2 -
2)))/2
```

Mathematica [A] time = 0.0373171, size = 41, normalized size = 1.

$$\log \left(\sqrt{x^2-2}+x\right)-\sqrt{\frac{3}{2}} \tanh ^{-1}\left(\frac{\sqrt{\frac{2}{3}} x}{\sqrt{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] -(Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]) + Log[x + Sqrt
[-2 + x^2]]
```

Maple [B] time = 0.052, size = 100, normalized size = 2.4

$$\ln \left(x+\sqrt{x^2-2}\right)-\frac{\sqrt{6}}{4} \mathit{Artanh}\left(\frac{8+2\left(x-\sqrt{6}\right)\sqrt{6}}{4} \frac{1}{\sqrt{\left(x-\sqrt{6}\right)^2+2\left(x-\sqrt{6}\right)\sqrt{6}+4}}\right)$$
$$+\frac{\sqrt{6}}{4} \mathit{Artanh}\left(\frac{8-2\left(x+\sqrt{6}\right)\sqrt{6}}{4} \frac{1}{\sqrt{\left(x+\sqrt{6}\right)^2-2\left(x+\sqrt{6}\right)\sqrt{6}+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2-6)/(x^2-2)^(1/2),x)
```

```
[Out] ln(x+(x^2-2)^(1/2))-1/4*6^(1/2)*arctanh(1/4*(8+2*(x-6^(1/2))*6^(1
/2))/((x-6^(1/2))^2+2*(x-6^(1/2))*6^(1/2)+4)^(1/2))+1/4*6^(1/2)*a
rctanh(1/4*(8-2*(x+6^(1/2))*6^(1/2))/((x+6^(1/2))^2-2*(x+6^(1/2))
*6^(1/2)+4)^(1/2))
```

Maxima [A] time = 1.77965, size = 144, normalized size = 3.51

$$\frac{1}{12} \sqrt{6}\left(2 \sqrt{6} \log \left(x+\sqrt{x^2-2}\right)-3 \log \left(\sqrt{6}+\frac{4 \sqrt{x^2-2}}{\left|2 x-2 \sqrt{6}\right|}+\frac{8}{\left|2 x-2 \sqrt{6}\right|}\right)+3 \log \left(-\sqrt{6}+\frac{4 \sqrt{x^2-2}}{\left|2 x+2 \sqrt{6}\right|}+\frac{8}{\left|2 x+2 \sqrt{6}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 - 2)*(x^2 - 6)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) +
4*sqrt(x^2 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) +
3*log(-sqrt(6) + 4*sqrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x
 + 2*sqrt(6))))
```

Fricas [A] time = 0.252485, size = 149, normalized size = 3.63

$$-\frac{1}{4}\sqrt{2}\left(2\sqrt{2}\log\left(-x+\sqrt{x^2-2}\right)-\sqrt{3}\log\left(\frac{\sqrt{2}\left(x^4-7x^2+30\right)+4\sqrt{3}\left(x^2-6\right)-\sqrt{x^2-2}\left(\sqrt{2}\left(x^3-6x\right)+4\sqrt{3}x\right)}{x^4-7x^2-\left(x^3-6x\right)\sqrt{x^2-2}+6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 - 2)*(x^2 - 6)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(2)*log(-x + sqrt(x^2 - 2)) - sqrt(3)*log((sq
rt(2)*(x^4 - 7*x^2 + 30) + 4*sqrt(3)*(x^2 - 6) - sqrt(x^2 - 2)*(s
qrt(2)*(x^3 - 6*x) + 4*sqrt(3)*x))/(x^4 - 7*x^2 - (x^3 - 6*x)*sqr
t(x^2 - 2) + 6)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2-6)\sqrt{x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)
```

GIAC/XCAS [A] time = 0.240321, size = 97, normalized size = 2.37

$$-\frac{1}{4}\sqrt{6}\ln\left(\frac{\left|2\left(x-\sqrt{x^2-2}\right)^2-8\sqrt{6}-20\right|}{\left|2\left(x-\sqrt{x^2-2}\right)^2+8\sqrt{6}-20\right|}\right)-\frac{1}{2}\ln\left(\left(x-\sqrt{x^2-2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 - 2)*(x^2 - 6)),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(6)*ln(abs(2*(x - sqrt(x^2 - 2))^2 - 8*sqrt(6) - 20)/abs
(2*(x - sqrt(x^2 - 2))^2 + 8*sqrt(6) - 20)) - 1/2*ln((x - sqrt(x^
2 - 2))^2)
```

3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2} \, dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1-x^2}x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1
 - x^2]]
```

Rubi [A] time = 0.0707482, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sqrt{1-x^2}x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1
 - x^2]]
```

Rubi in Sympy [A] time = 6.55003, size = 37, normalized size = 0.79

$$\frac{x\sqrt{-x^2+1}}{x^2+1}+2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 1)/(x**2 + 1) + 2*sqrt(2)*atan(sqrt(2)*x/sqrt(-x**
2 + 1))
```

Mathematica [A] time = 0.0713594, size = 47, normalized size = 1.

$$\frac{\sqrt{1-x^2}x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1
 - x^2]]
```

Maple [A] time = 0.037, size = 70, normalized size = 1.5

$$-2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}x}{x^2-1}\right)-\frac{x}{2x^2-2}\sqrt{-x^2+1}\left(\frac{\left(-x^2+1\right)x^2}{\left(x^2-1\right)^2}+\frac{1}{2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-1/2*(-x^2+1)^
(1/2)/(x^2-1)*x/((-x^2+1)/(x^2-1)^2*x^2+1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+5}{\left(x^2+1\right)^2\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 0.236125, size = 178, normalized size = 3.79

$$-\frac{2x^3-2\left(2\sqrt{2}\left(x^2+1\right)\sqrt{-x^2+1}+\sqrt{2}\left(x^4-x^2-2\right)\right)\arctan\left(\frac{x^2+\sqrt{-x^2+1}-1}{\sqrt{2}\sqrt{-x^2+1}x-\sqrt{2}x}\right)-\left(x^3-2x\right)\sqrt{-x^2+1}-2x}{x^4-x^2+2\left(x^2+1\right)\sqrt{-x^2+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -(2*x^3 - 2*(2*sqrt(2)*(x^2 + 1)*sqrt(-x^2 + 1) + sqrt(2)*(x^4 -
x^2 - 2))*arctan((x^2 + sqrt(-x^2 + 1) - 1)/(sqrt(2)*sqrt(-x^2 +
1)*x - sqrt(2)*x)) - (x^3 - 2*x)*sqrt(-x^2 + 1) - 2*x)/(x^4 - x^2
 + 2*(x^2 + 1)*sqrt(-x^2 + 1) - 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.212859, size = 166, normalized size = 3.53

$$\sqrt{2}\left(\pi \operatorname{sign}(x) + 2 \arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{2\left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x}\right)}{\left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x}\right)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] sqrt(2)*(pi*sign(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) -
1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1)
- (sqrt(-x^2 + 1) - 1)/x)/((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 +
 1) - 1)/x)^2 + 8)
```

3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=88

$$-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(
2*Sqrt[6]) - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt
[6]) + 20*Log[5 + Sqrt[1 - x^2]]
```

Rubi [A] time = 0.415065, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(
2*Sqrt[6]) - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt
[6]) + 20*Log[5 + Sqrt[1 - x^2]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x-\sqrt{-x^2+1}}{\sqrt{-x^2+1}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral((4*x - sqrt(-x**2 + 1))/(sqrt(-x**2 + 1) + 5), x)
```

Mathematica [A] time = 0.177381, size = 137, normalized size = 1.56

$$-4\sqrt{1-x^2}+10\log\left(x^2+24\right)-10\log\left(\left(x^2+24\right)^2\right)+10\log\left(\left(x^2+24\right)\left(-x^2+10\sqrt{1-x^2}+26\right)\right)+\frac{25\tan^{-1}\left(\frac{4x^2+409\sqrt{1-x^2}x+96}{10\sqrt{6}(17x^2-1)}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(
2*Sqrt[6]) + (25*ArcTan[(96 + 4*x^2 + 409*x*Sqrt[1 - x^2])/(10*Sq
rt[6]*(-1 + 17*x^2))])/(2*Sqrt[6]) + 10*Log[24 + x^2] - 10*Log[(2
4 + x^2)^2] + 10*Log[(24 + x^2)*(26 - x^2 + 10*Sqrt[1 - x^2])]
```

Maple [A] time = 0.03, size = 82, normalized size = 0.9

$$\frac{25\sqrt{6}}{12}\arctan\left(\frac{x\sqrt{6}}{12}\right)+10\ln\left(x^2+24\right)-x+5\arcsin(x)+\frac{25\sqrt{6}}{12}\arctan\left(\frac{5x\sqrt{6}}{12x^2-12}\sqrt{-x^2+1}\right)-4\sqrt{-x^2+1}+20\,\mathit{Artanh}\left(1/5\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x)
```

```
[Out] 25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+
25/12*6^(1/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x
^2+1)^(1/2)+20*arctanh(1/5*(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x-4\sqrt{-x^2+1}+5\int\frac{1}{\sqrt{x+1}\sqrt{-x+1}+5}\,dx+20\log\left(\sqrt{-x^2+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x - sqrt(-x^2 + 1))/(sqrt(-x^2 + 1) + 5),x, algorithm="maxima")
```

```
[Out] -x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) +
 5), x) + 20*log(sqrt(-x^2 + 1) + 5)
```

Fricas [A] time = 0.220115, size = 394, normalized size = 4.48

$$\frac{25\left(\sqrt{-x^2+1}-1\right)\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2x}\right)+25\left(\sqrt{-x^2+1}-1\right)\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3x}\right)-20\left(\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}\right)\arctan\left(\frac{\sqrt{-}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x - sqrt(-x^2 + 1))/(sqrt(-x^2 + 1) + 5),x, algorithm="fricas")
```

```
[Out] 1/2*(25*(sqrt(-x^2 + 1) - 1)*arctan(1/2*(sqrt(6)*sqrt(-x^2 + 1) -
 sqrt(6))/x) + 25*(sqrt(-x^2 + 1) - 1)*arctan(1/3*(sqrt(6)*sqrt(-
x^2 + 1) - sqrt(6))/x) - 20*(sqrt(6)*sqrt(-x^2 + 1) - sqrt(6))*ar
ctan((sqrt(-x^2 + 1) - 1)/x) - 20*(sqrt(6)*sqrt(-x^2 + 1) - sqrt(
6))*log(-(x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2) + 20*(sqrt(6)*sqrt(-x^
2 + 1) - sqrt(6))*log((x^2 - 4*sqrt(-x^2 + 1) + 4)/x^2) + 2*sqrt(
6)*(4*x^2 + x) - sqrt(-x^2 + 1)*(2*sqrt(6)*x - 20*sqrt(6)*log(x^2
 + 24) - 25*arctan(1/12*sqrt(6)*x)) - 20*sqrt(6)*log(x^2 + 24) -
25*arctan(1/12*sqrt(6)*x))/(sqrt(6)*sqrt(-x^2 + 1) - sqrt(6))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x-\sqrt{-x^2+1}}{\sqrt{-x^2+1}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral((4*x - sqrt(-x**2 + 1))/(sqrt(-x**2 + 1) + 5), x)
```

GIAC/XCAS [A] time = 0.226963, size = 182, normalized size = 2.07

$$\frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)-\frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{3x}\right)-\frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{2x}\right)-x-4\sqrt{-x^2+1}+5\arcsin(x)+10\ln\left(x^2+24\right)-10\ln\left(\frac{3\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\right)+10\ln\left(\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x - sqrt(-x^2 + 1))/(sqrt(-x^2 + 1) + 5),x, algorithm="giac")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*
sqrt(6)*(sqrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(
6)*(sqrt(-x^2 + 1) - 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) +
 10*ln(x^2 + 24) - 10*ln(3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*l
n(2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3)
```

3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx$

Optimal. Leaf size=136

$$-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)+\frac{8x}{9}+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{3x+1}{2\sqrt{2}}\right)-\frac{41}{54}\sinh^{-1}(x)$$

```
[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41
*ArcSinh[x])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 +
(4*Sqrt[2]*ArcTan[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTa
nh[(1 - x)/(2*Sqrt[1 + x^2])])/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

Rubi [A] time = 2.80218, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 14, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)+\frac{8x}{9}+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{3x+1}{2\sqrt{2}}\right)-\frac{41}{54}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x]
```

```
[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41
*ArcSinh[x])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 +
(4*Sqrt[2]*ArcTan[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTa
nh[(1 - x)/(2*Sqrt[1 + x^2])])/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

Rubi in Sympy [A] time = 92.1603, size = 112, normalized size = 0.82

$$\frac{8x}{9}-\frac{\left(x+\sqrt{x^2+1}\right)^2}{12}+\frac{8\sqrt{x^2+1}}{9}-\frac{\log\left(x+\sqrt{x^2+1}\right)}{2}-\frac{7\log\left(-2x+3\left(x+\sqrt{x^2+1}\right)^2-2\sqrt{x^2+1}+1\right)}{27}+\frac{8\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x}{2}+\frac{3\sqrt{x^2+1}}{2}-\frac{1}{2}\right)\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/
```

```
[Out] 8*x/9 - (x + sqrt(x**2 + 1))**2/12 + 8*sqrt(x**2 + 1)/9 - log(x +
 sqrt(x**2 + 1))/2 - 7*log(-2*x + 3*(x + sqrt(x**2 + 1))**2 - 2*s
qrt(x**2 + 1) + 1)/27 + 8*sqrt(2)*atan(sqrt(2)*(3*x/2 + 3*sqrt(x*
*2 + 1)/2 - 1/2))/27
```

Mathematica [C] time = 6.34784, size = 1046, normalized size = 7.69

$$-\frac{x^2}{6}+\frac{8x}{9}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{3x+1}{2\sqrt{2}}\right)$$
$$+\frac{\left(i+11\sqrt{2}\right)\tan^{-1}\left(\frac{3447i\sqrt{2}x^4-792x^4-2187i\sqrt{2\left(-1+2i\sqrt{2}\right)}\sqrt{x^2+1}x^3+2760i\sqrt{2}x^3+11040x^3+5103i\sqrt{2\left(-1+2i\sqrt{2}\right)}\sqrt{x^2+1}x^2-862i\sqrt{2}x^2+3680x^2+2187i\sqrt{2\left(\cdots\right.}}{8712\sqrt{2}x^4+22581ix^4+6864\sqrt{2}x^3+3432ix^3+10064\sqrt{2}x^2+39862ix^2+6864\sqrt{2}x+\cdots}\right.}{54\sqrt{-1+2i\sqrt{2}}}$$
$$-\frac{i\left(-i+11\sqrt{2}\right)\tan^{-1}\left(\frac{2\left(4356\sqrt{2}x^4-1449ix^4+3432\sqrt{2}x^3-1716ix^3+5032\sqrt{2}x^2-4622ix^2+3432\sqrt{2}x-1716ix+676\sqrt{2}+11\cdots\right.}{3447i\sqrt{2}x^4+792x^4+8748i\sqrt{1+2i\sqrt{2}}\sqrt{x^2+1}x^3+2760i\sqrt{2}x^3-11040x^3+5832i\sqrt{1+2i\sqrt{2}}\sqrt{x^2+1}x^2-862i\sqrt{2}x^2-3680x^2+8748i\sqrt{1+2i\sqrt{2}}\sqrt{x\cdots}}\right.}{54\sqrt{1+2i\sqrt{2}}}$$
$$+\frac{i\left(i+11\sqrt{2}\right)\log\left(\left(-3ix+2\sqrt{2}-i\right)^2\left(3ix+2\sqrt{2}+i\right)^2\right)}{108\sqrt{-1+2i\sqrt{2}}}$$
$$-\frac{\left(-i+11\sqrt{2}\right)\log\left(\left(-3ix+2\sqrt{2}-i\right)^2\left(3ix+2\sqrt{2}+i\right)^2\right)}{108\sqrt{1+2i\sqrt{2}}}-\frac{7}{54}\log\left(3x^2+2x+3\right)$$
$$-\frac{i\left(i+11\sqrt{2}\right)\log\left(\left(3x^2+2x+3\right)\left(4\sqrt{2}x^2-7ix^2-8i\sqrt{-1+2i\sqrt{2}}\sqrt{x^2+1}x+6ix+4\sqrt{2}-7i\right)\right)}{108\sqrt{-1+2i\sqrt{2}}}$$
$$+\frac{\left(-i+11\sqrt{2}\right)\log\left(\left(3x^2+2x+3\right)\left(4\sqrt{2}x^2-11ix^2+2i\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{x^2+1}x+6ix-6i\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{x^2+1}+4\sqrt{2}-\cdots\right.\right.}{108\sqrt{1+2i\sqrt{2}}}$$
$$+\left(\frac{8}{9}-\frac{x}{6}\right)\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x]
```

```
[Out] (8*x)/9 - x^2/6 + (8/9 - x/6)*Sqrt[1 + x^2] - (41*ArcSinh[x])/54
+ (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + ((I + 11*Sqrt[2]
)*ArcTan[(4472 - (3913*I)*Sqrt[2] + 11040*x + (2760*I)*Sqrt[2]*x
+ 3680*x^2 - (862*I)*Sqrt[2]*x^2 + 11040*x^3 + (2760*I)*Sqrt[2]*x
^3 - 792*x^4 + (3447*I)*Sqrt[2]*x^4 + (6561*I)*Sqrt[2*(-1 + (2*I)
*Sqrt[2])]*Sqrt[1 + x^2] + (2187*I)*Sqrt[2*(-1 + (2*I)*Sqrt[2])]*
x*Sqrt[1 + x^2] + (5103*I)*Sqrt[2*(-1 + (2*I)*Sqrt[2])]*x^2*Sqrt[
```

```
1 + x^2] - (2187*I)*Sqrt[2*(-1 + (2*I)*Sqrt[2])]*x^3*Sqrt[1 + x^2
])/(17317*I + 1352*Sqrt[2] + (3432*I)*x + 6864*Sqrt[2]*x + (39862
*I)*x^2 + 10064*Sqrt[2]*x^2 + (3432*I)*x^3 + 6864*Sqrt[2]*x^3 + (
22581*I)*x^4 + 8712*Sqrt[2]*x^4)])/(54*Sqrt[-1 + (2*I)*Sqrt[2]])
- ((I/54)*(-I + 11*Sqrt[2])*ArcTan[(2*(1183*I + 676*Sqrt[2] - (17
16*I)*x + 3432*Sqrt[2]*x - (4622*I)*x^2 + 5032*Sqrt[2]*x^2 - (171
6*I)*x^3 + 3432*Sqrt[2]*x^3 - (1449*I)*x^4 + 4356*Sqrt[2]*x^4))/(
-4472 - (3913*I)*Sqrt[2] - 11040*x + (2760*I)*Sqrt[2]*x - 3680*x^
2 - (862*I)*Sqrt[2]*x^2 - 11040*x^3 + (2760*I)*Sqrt[2]*x^3 + 792*
x^4 + (3447*I)*Sqrt[2]*x^4 + (8748*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x*S
qrt[1 + x^2] + (5832*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[1 + x^2]
 + (8748*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqrt[1 + x^2])])/Sqrt[1 +
 (2*I)*Sqrt[2]] - ((-I + 11*Sqrt[2])*Log[(-I + 2*Sqrt[2] - (3*I)*
x)^2*(I + 2*Sqrt[2] + (3*I)*x)^2])/(108*Sqrt[1 + (2*I)*Sqrt[2]])
+ ((I/108)*(I + 11*Sqrt[2])*Log[(-I + 2*Sqrt[2] - (3*I)*x)^2*(I +
 2*Sqrt[2] + (3*I)*x)^2])/Sqrt[-1 + (2*I)*Sqrt[2]] - (7*Log[3 + 2
*x + 3*x^2])/54 - ((I/108)*(I + 11*Sqrt[2])*Log[(3 + 2*x + 3*x^2)
*(-7*I + 4*Sqrt[2] + (6*I)*x - (7*I)*x^2 + 4*Sqrt[2]*x^2 - (8*I)*
Sqrt[-1 + (2*I)*Sqrt[2]]*x*Sqrt[1 + x^2])])/Sqrt[-1 + (2*I)*Sqrt[
2]] + ((-I + 11*Sqrt[2])*Log[(3 + 2*x + 3*x^2)*(-11*I + 4*Sqrt[2]
 + (6*I)*x - (11*I)*x^2 + 4*Sqrt[2]*x^2 - (6*I)*Sqrt[2*(1 + (2*I)
*Sqrt[2])]*Sqrt[1 + x^2] + (2*I)*Sqrt[2*(1 + (2*I)*Sqrt[2])]*x*Sq
rt[1 + x^2])])/(108*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.079, size = 654, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

```
[Out] -1/6*x^2+8/9*x-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+
2)*2^(1/2))-41/54*arcsinh(x)-1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(
1/2)*(-2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1
+x)^2/(1-x)^2+1)*(1+x)/(1-x))+5*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/
2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))^2)^(1/2)/(1+(1+x)/(1-x)
)+3/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arctan(1/2*2^
(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x)
)+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(
1+x)/(1-x))^2)^(1/2)/(1+(1+x)/(1-x))-1/6*x*(x^2+1)^(1/2)+8/9*(x^2
+1)^(1/2)+1/216*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*a
rctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)
*(1+x)/(1-x))+43*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/
(1-x)^2+1)/(1+(1+x)/(1-x))^2)^(1/2)/(1+(1+x)/(1-x))-1/36*2^(1/2)*
(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-11*2^(1/2)*arctan(1/2*2^(1/2)*(2*(1
+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+arctanh((
2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))
^2)^(1/2)/(1+(1+x)/(1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)+\int -\frac{3x^{10}-4x^9+5x^8-2x^7+15x^6+6x^5+9x^4}{2\left(2x^{13}+7x^{11}-4x^{10}+11x^9-11x^8+13x^7-13x^6+11x^5-11x^4+4x^3-7x^2-2\left(x^{12}+3x^{10}-2x^9+3x^8-6x^7+\right.\right.}$$
$$+\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(sqrt(x^2 + 1) - 2)/((x^3 - (x^2 + 1)^(3/2) - 1)*sqrt(x^2 + 1)),x, a
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9
 + 5*x^8 - 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x
^10 + 11*x^9 - 11*x^8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3
 - 7*x^2 - 2*(x^12 + 3*x^10 - 2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x
^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt(x^2 + 1) - 2), x) + 1/6*log(
x^2 + x + 1) + 1/6*log(x - 1)
```

Fricas [A] time = 0.241463, size = 439, normalized size = 3.23

$$\frac{8\sqrt{2}\left(2x^2+1\right)\arctan\left(\frac{1}{4}\sqrt{2}(3x+1)\right)+9x^2-8\left(2\sqrt{2}\sqrt{x^2+1}x-\sqrt{2}\left(2x^2+1\right)\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(3x-3\sqrt{x^2+1}-1\right)\right)+8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(sqrt(x^2 + 1) - 2)/((x^3 - (x^2 + 1)^(3/2) - 1)*sqrt(x^2 + 1)),x, a
```

```
[Out] 1/54*(8*sqrt(2)*(2*x^2 + 1)*arctan(1/4*sqrt(2)*(3*x + 1)) + 9*x^2
 - 8*(2*sqrt(2)*sqrt(x^2 + 1)*x - sqrt(2)*(2*x^2 + 1))*arctan(-1/
2*sqrt(2)*(3*x - 3*sqrt(x^2 + 1) - 1)) + 8*(2*sqrt(2)*sqrt(x^2 +
1)*x - sqrt(2)*(2*x^2 + 1))*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 1
) + 1)) + 7*(2*x^2 - 2*sqrt(x^2 + 1)*x + 1)*log(3*x^2 - sqrt(x^2
+ 1)*(3*x - 1) - x + 2) - 7*(2*x^2 + 1)*log(3*x^2 + 2*x + 3) - 7*
(2*x^2 - 2*sqrt(x^2 + 1)*x + 1)*log(x^2 - sqrt(x^2 + 1)*(x + 1) +
 x + 2) + 41*(2*x^2 - 2*sqrt(x^2 + 1)*x + 1)*log(-x + sqrt(x^2 +
1)) - (16*sqrt(2)*x*arctan(1/4*sqrt(2)*(3*x + 1)) - 14*x*log(3*x^
2 + 2*x + 3) + 9*x - 48)*sqrt(x^2 + 1) - 48*x)/(2*x^2 - 2*sqrt(x^
2 + 1)*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22043, size = 238, normalized size = 1.75

$$-\frac{1}{6}x^2 - \frac{1}{18}\sqrt{x^2+1}(3x-16) + \frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(3x-3\sqrt{x^2+1}-1\right)\right) + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3x+1)\right) - \frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+1}+1\right)\right) + \frac{8}{9}x + \frac{7}{54}\ln\left(3\left(x-\sqrt{x^2+1}\right)^2 - 2x + 2\sqrt{x^2+1} + 1\right) - \frac{7}{54}\ln\left(\left(x-\sqrt{x^2+1}\right)^2 + 2x - 2\sqrt{x^2+1} + 3\right) - \frac{7}{54}\ln\left(3x^2+2x+3\right) + \frac{41}{54}\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(sqrt(x^2 + 1) - 2)/((x^3 - (x^2 + 1)^(3/2) - 1)*sqrt(x^2 + 1)),x, a
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(-1
/2*sqrt(2)*(3*x - 3*sqrt(x^2 + 1) - 1)) + 4/27*sqrt(2)*arctan(1/4
*sqrt(2)*(3*x + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(
x^2 + 1) + 1)) + 8/9*x + 7/54*ln(3*(x - sqrt(x^2 + 1))^2 - 2*x +
2*sqrt(x^2 + 1) + 1) - 7/54*ln((x - sqrt(x^2 + 1))^2 + 2*x - 2*sq
rt(x^2 + 1) + 3) - 7/54*ln(3*x^2 + 2*x + 3) + 41/54*ln(-x + sqrt(
x^2 + 1))
```

3.261 $\int x\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=64

$$r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

```
[Out] -(r*(r - x)*Sqrt[2*r*x - x^2])/2 - (2*r*x - x^2)^(3/2)/3 + r^3*Ar
cTan[x/Sqrt[2*r*x - x^2]]
```

Rubi [A] time = 0.0459022, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] -(r*(r - x)*Sqrt[2*r*x - x^2])/2 - (2*r*x - x^2)^(3/2)/3 + r^3*Ar
cTan[x/Sqrt[2*r*x - x^2]]
```

Rubi in Sympy [A] time = 2.66127, size = 53, normalized size = 0.83

$$r^3\operatorname{atan}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{r\left(2r-2x\right)\sqrt{2rx-x^2}}{4}-\frac{\left(2rx-x^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*r*x-x**2)**(1/2),x)
```

```
[Out] r**3*atan(x/sqrt(2*r*x - x**2)) - r*(2*r - 2*x)*sqrt(2*r*x - x**2
)/4 - (2*r*x - x**2)**(3/2)/3
```

Mathematica [A] time = 0.0805324, size = 66, normalized size = 1.03

$$\frac{1}{6}\sqrt{-x(x-2r)}\left(-\frac{6r^3\log\left(\sqrt{x-2r}+\sqrt{x}\right)}{\sqrt{x}\sqrt{x-2r}}-3r^2-rx+2x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 - (6*r^3*Log[Sqrt[x]
 + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/6
```

Maple [A] time = 0.013, size = 73, normalized size = 1.1

$$-\frac{1}{3}\left(2\,rx-x^{2}\right)^{\frac{3}{2}}+\frac{rx}{2}\sqrt{2\,rx-x^{2}}-\frac{r^{2}}{2}\sqrt{2\,rx-x^{2}}+\frac{r^{3}}{2}\arctan\left((x-r)\frac{1}{\sqrt{2\,rx-x^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*r*x-x^2)^(1/2),x)
```

```
[Out] -1/3*(2*r*x-x^2)^(3/2)+1/2*r*x*(2*r*x-x^2)^(1/2)-1/2*(2*r*x-x^2)^
(1/2)*r^2+1/2*r^3*arctan((x-r)/(2*r*x-x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262489, size = 69, normalized size = 1.08

$$-r^{3}\arctan\left(\frac{\sqrt{2\,rx-x^{2}}}{x}\right)-\frac{1}{6}\left(3\,r^{2}+rx-2\,x^{2}\right)\sqrt{2\,rx-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x,x, algorithm="fricas")
```

```
[Out] -r^3*arctan(sqrt(2*r*x - x^2)/x) - 1/6*(3*r^2 + r*x - 2*x^2)*sqrt
(2*r*x - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-x\left(-2r+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-x*(-2*r + x)), x)
```

GIAC/XCAS [A] time = 0.214533, size = 61, normalized size = 0.95

$$-\frac{1}{2}\,r^{3}\arcsin\left(\frac{r-x}{r}\right)\mathrm{sign}\left(r\right)-\frac{1}{6}\,\left(3\,r^{2}+\left(r-2\,x\right)x\right)\sqrt{2\,rx-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x,x, algorithm="giac")
```

```
[Out] -1/2*r^3*arcsin((r - x)/r)*sign(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sq
rt(2*r*x - x^2)
```

3.262 $\int x^2\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=89

$$\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

```
[Out] (-5*r^2*(r - x)*Sqrt[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^(3/2))/
12 - (x*(2*r*x - x^2)^(3/2))/4 + (5*r^4*ArcTan[x/Sqrt[2*r*x - x^2
]])/4
```

Rubi [A] time = 0.0737487, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-5*r^2*(r - x)*Sqrt[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^(3/2))/
12 - (x*(2*r*x - x^2)^(3/2))/4 + (5*r^4*ArcTan[x/Sqrt[2*r*x - x^2
]])/4
```

Rubi in Sympy [A] time = 4.21713, size = 78, normalized size = 0.88

$$\frac{5r^4\operatorname{atan}\left(\frac{x}{\sqrt{2rx-x^2}}\right)}{4}-\frac{5r^2\left(2r-2x\right)\sqrt{2rx-x^2}}{16}-\frac{5r\left(2rx-x^2\right)^{\frac{3}{2}}}{12}-\frac{x\left(2rx-x^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*r*x-x**2)**(1/2),x)
```

```
[Out] 5*r**4*atan(x/sqrt(2*r*x - x**2))/4 - 5*r**2*(2*r - 2*x)*sqrt(2*r
*x - x**2)/16 - 5*r*(2*r*x - x**2)**(3/2)/12 - x*(2*r*x - x**2)**
(3/2)/4
```

Mathematica [A] time = 0.0799129, size = 74, normalized size = 0.83

$$\frac{1}{24}\sqrt{-x(x-2r)}\left(-\frac{30r^4\log\left(\sqrt{x-2r}+\sqrt{x}\right)}{\sqrt{x}\sqrt{x-2r}}-15r^3-5r^2x-2rx^2+6x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 - (30
*r^4*Log[Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/24
```

Maple [A] time = 0.007, size = 91, normalized size = 1.

$$-\frac{x}{4}\left(2\,rx-x^{2}\right)^{\frac{3}{2}}-\frac{5\,r}{12}\left(2\,rx-x^{2}\right)^{\frac{3}{2}}+\frac{5\,r^{2}x}{8}\sqrt{2\,rx-x^{2}}-\frac{5\,r^{3}}{8}\sqrt{2\,rx-x^{2}}+\frac{5\,r^{4}}{8}\arctan\left((x-r)\frac{1}{\sqrt{2\,rx-x^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*r*x-x^2)^(1/2),x)
```

```
[Out] -1/4*x*(2*r*x-x^2)^(3/2)-5/12*r*(2*r*x-x^2)^(3/2)+5/8*r^2*(2*r*x-
x^2)^(1/2)*x-5/8*(2*r*x-x^2)^(1/2)*r^3+5/8*r^4*arctan((x-r)/(2*r*
x-x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272533, size = 81, normalized size = 0.91

$$-\frac{5}{4}\,r^{4}\arctan\left(\frac{\sqrt{2\,rx-x^{2}}}{x}\right)-\frac{1}{24}\left(15\,r^{3}+5\,r^{2}x+2\,rx^{2}-6\,x^{3}\right)\sqrt{2\,rx-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x^2,x, algorithm="fricas")
```

```
[Out] -5/4*r^4*arctan(sqrt(2*r*x - x^2)/x) - 1/24*(15*r^3 + 5*r^2*x + 2
*r*x^2 - 6*x^3)*sqrt(2*r*x - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-x\left(-2r+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-x*(-2*r + x)), x)
```

GIAC/XCAS [A] time = 0.210951, size = 73, normalized size = 0.82

$$-\frac{5}{8}\,r^4\arcsin\left(\frac{r-x}{r}\right)\mathrm{sign}\left(r\right)-\frac{1}{24}\left(15\,r^3+\left(5\,r^2+2\left(r-3\,x\right)x\right)x\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x^2,x, algorithm="giac")
```

```
[Out] -5/8*r^4*arcsin((r - x)/r)*sign(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r
 - 3*x)*x)*x)*sqrt(2*r*x - x^2)
```

3.263 $\int x^3\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=113

$$\frac{7}{4}r^5\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

```
[Out] (-7*r^3*(r - x)*Sqrt[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^(3/2)
)/12 - (7*r*x*(2*r*x - x^2)^(3/2))/20 - (x^2*(2*r*x - x^2)^(3/2))
/5 + (7*r^5*ArcTan[x/Sqrt[2*r*x - x^2]])/4
```

Rubi [A] time = 0.105325, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{7}{4}r^5\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-7*r^3*(r - x)*Sqrt[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^(3/2)
)/12 - (7*r*x*(2*r*x - x^2)^(3/2))/20 - (x^2*(2*r*x - x^2)^(3/2))
/5 + (7*r^5*ArcTan[x/Sqrt[2*r*x - x^2]])/4
```

Rubi in Sympy [A] time = 5.77736, size = 100, normalized size = 0.88

$$\frac{7r^5\operatorname{atan}\left(\frac{x}{\sqrt{2rx-x^2}}\right)}{4}-\frac{7r^3\left(2r-2x\right)\sqrt{2rx-x^2}}{16}-\frac{7r^2\left(2rx-x^2\right)^{\frac{3}{2}}}{12}-\frac{7rx\left(2rx-x^2\right)^{\frac{3}{2}}}{20}-\frac{x^2\left(2rx-x^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

```
[Out] 7*r**5*atan(x/sqrt(2*r*x - x**2))/4 - 7*r**3*(2*r - 2*x)*sqrt(2*r
*x - x**2)/16 - 7*r**2*(2*r*x - x**2)**(3/2)/12 - 7*r*x*(2*r*x -
x**2)**(3/2)/20 - x**2*(2*r*x - x**2)**(3/2)/5
```

Mathematica [A] time = 0.0963619, size = 82, normalized size = 0.73

$$\frac{1}{120}\sqrt{-x(x-2r)}\left(-\frac{210r^5\log\left(\sqrt{x-2r}+\sqrt{x}\right)}{\sqrt{x}\sqrt{x-2r}}-105r^4-35r^3x-14r^2x^2-6rx^3+24x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^
3 + 24*x^4 - (210*r^5*Log[Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqr
t[-2*r + x])))/120
```

Maple [A] time = 0.007, size = 111, normalized size = 1.

$$-\frac{x^2}{5}\left(2rx-x^2\right)^{\frac{3}{2}}-\frac{7rx}{20}\left(2rx-x^2\right)^{\frac{3}{2}}-\frac{7r^2}{12}\left(2rx-x^2\right)^{\frac{3}{2}}+\frac{7r^3x}{8}\sqrt{2rx-x^2}-\frac{7r^4}{8}\sqrt{2rx-x^2}+\frac{7r^5}{8}\arctan\left((x-r)\frac{1}{\sqrt{2rx-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(2*r*x-x^2)^(1/2),x)
```

```
[Out] -1/5*x^2*(2*r*x-x^2)^(3/2)-7/20*r*x*(2*r*x-x^2)^(3/2)-7/12*r^2*(2
*r*x-x^2)^(3/2)+7/8*r^3*(2*r*x-x^2)^(1/2)*x-7/8*(2*r*x-x^2)^(1/2)
*r^4+7/8*r^5*arctan((x-r)/(2*r*x-x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261184, size = 92, normalized size = 0.81

$$-\frac{7}{4}r^5\arctan\left(\frac{\sqrt{2rx-x^2}}{x}\right)-\frac{1}{120}\left(105r^4+35r^3x+14r^2x^2+6rx^3-24x^4\right)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x^3,x, algorithm="fricas")
```

```
[Out] -7/4*r^5*arctan(sqrt(2*r*x - x^2)/x) - 1/120*(105*r^4 + 35*r^3*x
+ 14*r^2*x^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{-x(-2r+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-x*(-2*r + x)), x)
```

GIAC/XCAS [A] time = 0.208697, size = 85, normalized size = 0.75

$$-\frac{7}{8}\,r^5 \arcsin\left(\frac{r-x}{r}\right)\operatorname{sign}(r) - \frac{1}{120}\left(105\,r^4 + \left(35\,r^3 + 2\left(7\,r^2 + 3\,(r-4\,x)x\right)x\right)x\right)\sqrt{2\,rx - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*r*x - x^2)*x^3,x, algorithm="giac")
```

```
[Out] -7/8*r^5*arcsin((r - x)/r)*sign(r) - 1/120*(105*r^4 + (35*r^3 + 2
*(7*r^2 + 3*(r - 4*x)*x)*x)*x)*sqrt(2*r*x - x^2)
```

3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{x^2+2x}\right)-\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[Sqrt[2*x + x^2]]/2 - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x
+ x^2])]/(2*Sqrt[3])
```

Rubi [A] time = 0.0765489, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{x^2+2x}\right)-\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] -ArcTan[Sqrt[2*x + x^2]]/2 - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x
+ x^2])]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 5.6748, size = 44, normalized size = 0.9

$$-\frac{\operatorname{atan}\left(\sqrt{x^2+2x}\right)}{2}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(4x+2)}{6\sqrt{x^2+2x}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
```

```
[Out] -atan(sqrt(x**2 + 2*x))/2 - sqrt(3)*atanh(sqrt(3)*(4*x + 2)/(6*sq
rt(x**2 + 2*x)))/6
```

Mathematica [B] time = 0.113248, size = 113, normalized size = 2.31

$$\frac{\sqrt{x}\sqrt{x+2}\left(\sqrt{3}\left(\log\left(1-\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)+\log\left(-\sqrt{x}+\sqrt{3}\sqrt{x+2}+2\right)-\log\left(\sqrt{x}+\sqrt{3}\sqrt{x+2}+2\right)\right)-6\tan^{-1}\left(\sqrt{\frac{x}{x+2}}\right.}{6\sqrt{x(x+2)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + x]*(-6*ArcTan[Sqrt[x/(2 + x)]] + Sqrt[3]*(Log[1
 - Sqrt[x]] - Log[1 + Sqrt[x]] + Log[2 - Sqrt[x] + Sqrt[3]*Sqrt[2
 + x]] - Log[2 + Sqrt[x] + Sqrt[3]*Sqrt[2 + x]])))/(6*Sqrt[x*(2 +
 x)])
```

Maple [A] time = 0.019, size = 42, normalized size = 0.9

$$-\frac{\sqrt{3}}{6}Artanh\left(\frac{(4x+2)\sqrt{3}}{6}\frac{1}{\sqrt{(-1+x)^2-1+4x}}\right)+\frac{1}{2}\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)/(x^2+2*x)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*arctanh(1/6*(4*x+2)*3^(1/2)/((-1+x)^2-1+4*x)^(1/2))+
1/2*arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A] time = 1.74745, size = 73, normalized size = 1.49

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^2+2x}}{|2x-2|}+\frac{6}{|2x-2|}+2\right)+\frac{1}{2}\arcsin\left(\frac{2}{|2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2 + 2*x)/abs(2*x - 2) + 6/abs(2
*x - 2) + 2) + 1/2*arcsin(2/abs(2*x + 2))
```

Fricas [A] time = 0.274888, size = 115, normalized size = 2.35

$$-\frac{1}{6}\sqrt{3}\left(2\sqrt{3}\arctan\left(-x+\sqrt{x^2+2x}-1\right)-\log\left(\frac{\sqrt{3}(x^2+2)-\sqrt{x^2+2x}\left(\sqrt{3}(x-1)+3\right)+3x-3}{x^2-\sqrt{x^2+2x}(x-1)-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(2*sqrt(3)*arctan(-x + sqrt(x^2 + 2*x) - 1) - log((s
qrt(3)*(x^2 + 2) - sqrt(x^2 + 2*x)*(sqrt(3)*(x - 1) + 3) + 3*x -
3)/(x^2 - sqrt(x^2 + 2*x)*(x - 1) - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\,(x+2)}\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.222128, size = 96, normalized size = 1.96

$$\frac{1}{6}\,\sqrt{3}\ln\left(\frac{\left|-2\,x-2\,\sqrt{3}+2\,\sqrt{x^2+2\,x}+2\right|}{\left|-2\,x+2\,\sqrt{3}+2\,\sqrt{x^2+2\,x}+2\right|}\right)-\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*ln(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(
-2*x + 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2
 + 2*x) - 1)
```

3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

```
[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 +
x)) + ArcTan[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])
```

Rubi [A] time = 0.14551, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 +
x)) + ArcTan[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 6.67142, size = 66, normalized size = 0.84

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}(4x-2)}{6\sqrt{-x^2+2x}}\right)}{6}-\frac{2\sqrt{-x^2+2x}}{3\,(x+1)}-\frac{5\sqrt{-x^2+2x}}{6\,(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
```

```
[Out] -sqrt(3)*atan(sqrt(3)*(4*x - 2)/(6*sqrt(-x**2 + 2*x)))/6 - 2*sqrt
(-x**2 + 2*x)/(3*(x + 1)) - 5*sqrt(-x**2 + 2*x)/(6*(x + 1)**2)
```

Mathematica [A] time = 0.143895, size = 126, normalized size = 1.59

$$\frac{\sqrt{x}\left(2\sqrt{x}\left(4x^2+x-18\right)-\sqrt{3}\sqrt{x-2}(x+1)^2\log\left(-2x-\sqrt{3}\sqrt{x-2}\sqrt{x}+1\right)+\sqrt{3}\sqrt{x-2}(x+1)^2\log\left(-2x+\sqrt{3}\sqrt{x-2}\sqrt{x}+\right.\right.}{12\sqrt{-(x-2)x}(x+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (Sqrt[x]*(2*Sqrt[x]*(-18 + x + 4*x^2) - Sqrt[3]*Sqrt[-2 + x]*(1 +
 x)^2*Log[1 - Sqrt[3]*Sqrt[-2 + x]*Sqrt[x] - 2*x] + Sqrt[3]*Sqrt[
-2 + x]*(1 + x)^2*Log[1 + Sqrt[3]*Sqrt[-2 + x]*Sqrt[x] - 2*x]))/(
12*Sqrt[-((-2 + x)*x)]*(1 + x)^2)
```

Maple [A] time = 0.022, size = 74, normalized size = 0.9

$$-\frac{5}{6\,(1+x)^2}\sqrt{-(1+x)^2+1+4x}-\frac{2}{3+3x}\sqrt{-(1+x)^2+1+4x}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(-2+4x)\sqrt{3}}{6}\frac{1}{\sqrt{-(1+x)^2+1+4x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x)
```

```
[Out] -5/6/(1+x)^2*(-(1+x)^2+1+4*x)^(1/2)-2/3/(1+x)*(-(1+x)^2+1+4*x)^(1
/2)-1/6*3^(1/2)*arctan(1/6*(-2+4*x)*3^(1/2)/(-(1+x)^2+1+4*x)^(1/2
))
```

Maxima [A] time = 1.76523, size = 89, normalized size = 1.13

$$-\frac{1}{6}\sqrt{3}\arcsin\left(\frac{2x}{|x+1|}-\frac{1}{|x+1|}\right)-\frac{5\sqrt{-x^2+2x}}{6\,(x^2+2x+1)}-\frac{2\sqrt{-x^2+2x}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 2)/(sqrt(-x^2 + 2*x)*(x + 1)^3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsin(2*x/abs(x + 1) - 1/abs(x + 1)) - 5/6*sqrt(-x^
2 + 2*x)/(x^2 + 2*x + 1) - 2/3*sqrt(-x^2 + 2*x)/(x + 1)
```

Fricas [A] time = 0.233458, size = 89, normalized size = 1.13

$$-\frac{\sqrt{3}\left(\sqrt{3}\sqrt{-x^2+2x}(4x+9)-6\left(x^2+2x+1\right)\arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{3x}\right)\right)}{18\,(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x - 2)/(sqrt(-x^2 + 2*x)*(x + 1)^3),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*sqrt(-x^2 + 2*x)*(4*x + 9) - 6*(x^2 + 2*x
+ 1)*arctan(1/3*sqrt(3)*sqrt(-x^2 + 2*x)/x))/(x^2 + 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x-2}{\sqrt{-x(x-2)}\,(x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
```

```
[Out] Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)
```

GIAC/XCAS [A] time = 0.210128, size = 198, normalized size = 2.51

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2+2x}-1\right)}{x-1}-1\right)\right)+\frac{\frac{34\left(\sqrt{-x^2+2x}-1\right)}{x-1}-\frac{39\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}+\frac{18\left(\sqrt{-x^2+2x}-1\right)^3}{(x-1)^3}-26}{24\left(\frac{\sqrt{-x^2+2x}-1}{x-1}-\frac{\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x - 2)/(sqrt(-x^2 + 2*x)*(x + 1)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1)
- 1)) + 1/24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 +
 2*x) - 1)^2/(x - 1)^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 -
26)/((sqrt(-x^2 + 2*x) - 1)/(x - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x
 - 1)^2 - 1)^2
```

3.266 $\int \frac{1}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=12

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Rubi [A] time = 0.0183536, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Rubi in Sympy [A] time = 0.649856, size = 17, normalized size = 1.42

$$\operatorname{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+x+1)**(1/2),x)
```

```
[Out] atanh((2*x + 1)/(2*sqrt(x**2 + x + 1)))
```

Mathematica [A] time = 0.00727097, size = 12, normalized size = 1.

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Maple [A] time = 0.004, size = 10, normalized size = 0.8

$$Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+x+1)^(1/2),x)
```

```
[Out] arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.6841, size = 15, normalized size = 1.25

$$\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 0.199638, size = 24, normalized size = 2.

$$-\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+x+1)**(1/2),x)

[Out] Integral(1/sqrt(x**2 + x + 1), x)
```

GIAC/XCAS [A] time = 0.202128, size = 24, normalized size = 2.

$$-\ln\left(-2\,x + 2\,\sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + x + 1),x, algorithm="giac")

[Out] -ln(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (
7*ArcSinh[(1 + 2*x)/Sqrt[3]])/16
```

Rubi [A] time = 0.0695073, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (
7*ArcSinh[(1 + 2*x)/Sqrt[3]])/16
```

Rubi in Sympy [A] time = 3.93157, size = 56, normalized size = 1.06

$$\frac{x^2\sqrt{x^2+x+1}}{3}-\frac{\left(\frac{5x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{6}+\frac{7\,\mathrm{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + x + 1)/3 - (5*x/2 + 1/4)*sqrt(x**2 + x + 1)/6 +
7*atanh((2*x + 1)/(2*sqrt(x**2 + x + 1)))/16
```

Mathematica [A] time = 0.0264348, size = 41, normalized size = 0.77

$$\frac{1}{24}\sqrt{x^2+x+1}\left(8x^2-10x-1\right)+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2))/24 + (7*ArcSinh[(1 + 2*x)
/Sqrt[3]])/16
```

Maple [A] time = 0.008, size = 47, normalized size = 0.9

$$\frac{x^2}{3}\sqrt{x^2+x+1}-\frac{5x}{12}\sqrt{x^2+x+1}-\frac{1}{24}\sqrt{x^2+x+1}+\frac{7}{16}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/3*x^2*(x^2+x+1)^(1/2)-5/12*x*(x^2+x+1)^(1/2)-1/24*(x^2+x+1)^(1/
2)+7/16*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.64278, size = 65, normalized size = 1.23

$$\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{5}{12}\sqrt{x^2+x+1}x-\frac{1}{24}\sqrt{x^2+x+1}+\frac{7}{16}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 + x + 1)*x^2 - 5/12*sqrt(x^2 + x + 1)*x - 1/24*sqrt(
x^2 + x + 1) + 7/16*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 0.20298, size = 212, normalized size = 4.

$$-\frac{2048x^6+1536x^5-384x^4-3840x^3-3456x^2+84\left(32x^3+48x^2-2\left(16x^2+16x+7\right)\sqrt{x^2+x+1}+42x+13\right)\log\left(-2\right.}{192\left(32x^3+48x^2-2\left(16x^2+16x+7\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] -1/192*(2048*x^6 + 1536*x^5 - 384*x^4 - 3840*x^3 - 3456*x^2 + 84*
(32*x^3 + 48*x^2 - 2*(16*x^2 + 16*x + 7)*sqrt(x^2 + x + 1) + 42*x
 + 13)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 2*(1024*x^5 + 256*x^
4 - 704*x^3 - 1472*x^2 - 704*x - 59)*sqrt(x^2 + x + 1) - 1530*x -
 125)/(32*x^3 + 48*x^2 - 2*(16*x^2 + 16*x + 7)*sqrt(x^2 + x + 1)
+ 42*x + 13)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2 + x + 1), x)
```

GIAC/XCAS [A] time = 0.203827, size = 53, normalized size = 1.

$$\frac{1}{24}\left(2\left(4\,x-5\right)x-1\right)\sqrt{x^2+x+1}-\frac{7}{16}\ln\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(x^2 + x + 1),x, algorithm="giac")
```

```
[Out] 1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*ln(-2*x + 2*sqr
t(x^2 + x + 1) - 1)
```

3.268 $\int \frac{1}{(1+x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Rubi [A] time = 0.0070777, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2)^(-3/2),x]

[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Rubi in Sympy [A] time = 0.563205, size = 15, normalized size = 0.79

$$\frac{4x+2}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+x+1)**(3/2),x)

[Out] (4*x + 2)/(3*sqrt(x**2 + x + 1))
```

Mathematica [A] time = 0.010649, size = 19, normalized size = 1.

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$\frac{4x+2}{3}\frac{1}{\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x+1)^(3/2),x)
```

```
[Out] 2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

Maxima [A] time = 1.39681, size = 30, normalized size = 1.58

$$\frac{4x}{3\sqrt{x^2+x+1}}+\frac{2}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + x + 1)^(-3/2),x, algorithm="maxima")
```

```
[Out] 4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)
```

Fricas [A] time = 0.201689, size = 39, normalized size = 2.05

$$\frac{2}{2x^2-\sqrt{x^2+x+1}(2x+1)+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + x + 1)^(-3/2),x, algorithm="fricas")
```

```
[Out] 2/(2*x^2 - sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+x+1)**(3/2),x)

[Out] Integral((x**2 + x + 1)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.2029, size = 20, normalized size = 1.05

$$\frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(-3/2),x, algorithm="giac")

[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1)
```

3.269 $\int \frac{x}{(1+x+x^2)^{3/2}} \, dx$

Optimal. Leaf size=17

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

```
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

Rubi [A] time = 0.0123177, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x + x^2)^(3/2),x]

[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

Rubi in Sympy [A] time = 1.37436, size = 17, normalized size = 1.

$$-\frac{2x+4}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(x**2+x+1)**(3/2),x)

[Out] -(2*x + 4)/(3*sqrt(x**2 + x + 1))
```

Mathematica [A] time = 0.0111808, size = 17, normalized size = 1.

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

Maple [A] time = 0.006, size = 14, normalized size = 0.8

$$-\frac{2\,x+4}{3}\frac{1}{\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^(3/2),x)
```

```
[Out] -2/3*(2+x)/(x^2+x+1)^(1/2)
```

Maxima [A] time = 1.38131, size = 30, normalized size = 1.76

$$-\frac{2\,x}{3\,\sqrt{x^2+x+1}}-\frac{4}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2 + x + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)
```

Fricas [A] time = 0.200396, size = 55, normalized size = 3.24

$$\frac{2\left(x-\sqrt{x^2+x+1}\right)}{2\,x^2-\sqrt{x^2+x+1}(2\,x+1)+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2 + x + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(x - sqrt(x^2 + x + 1))/(2*x^2 - sqrt(x^2 + x + 1)*(2*x + 1) +
2*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+x+1)**(3/2),x)

[Out] Integral(x/(x**2 + x + 1)**(3/2), x)
```

GIAC/XCAS [A] time = 0.201742, size = 18, normalized size = 1.06

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x + 1)^(3/2),x, algorithm="giac")

[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1)
```

3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}} \, dx$

Optimal. Leaf size=56

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}}+\frac{1}{3}(2x+5)\sqrt{x^2+x+1}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] (-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + ((5 + 2*x)*Sqrt[1 + x +
x^2])/3 - (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2
```

Rubi [A] time = 0.068452, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}}+\frac{1}{3}(2x+5)\sqrt{x^2+x+1}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + ((5 + 2*x)*Sqrt[1 + x +
x^2])/3 - (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2
```

Rubi in Sympy [A] time = 4.17458, size = 58, normalized size = 1.04

$$-\frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}+\frac{(2x+5)\sqrt{x^2+x+1}}{3}-\frac{3\,\operatorname{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] -2*x**2*(x + 2)/(3*sqrt(x**2 + x + 1)) + (2*x + 5)*sqrt(x**2 + x
+ 1)/3 - 3*atanh((2*x + 1)/(2*sqrt(x**2 + x + 1)))/2
```

Mathematica [A] time = 0.0327183, size = 41, normalized size = 0.73

$$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + x + x^2)^(3/2),x]
```

```
[Out] (5 + 7*x + 3*x^2)/(3*Sqrt[1 + x + x^2]) - (3*ArcSinh[(1 + 2*x)/Sq
rt[3]])/2
```

Maple [A] time = 0.009, size = 61, normalized size = 1.1

$$x^2\frac{1}{\sqrt{x^2+x+1}}+\frac{3x}{2}\frac{1}{\sqrt{x^2+x+1}}+\frac{5}{4}\frac{1}{\sqrt{x^2+x+1}}+\frac{5+10x}{12}\frac{1}{\sqrt{x^2+x+1}}-\frac{3}{2}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^2+x+1)^(3/2),x)
```

```
[Out] x^2/(x^2+x+1)^(1/2)+3/2*x/(x^2+x+1)^(1/2)+5/4/(x^2+x+1)^(1/2)+5/1
2*(1+2*x)/(x^2+x+1)^(1/2)-3/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.59714, size = 63, normalized size = 1.12

$$\frac{x^2}{\sqrt{x^2+x+1}}+\frac{7x}{3\sqrt{x^2+x+1}}+\frac{5}{3\sqrt{x^2+x+1}}-\frac{3}{2}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + x + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/sqrt(x^2 + x + 1) + 7/3*x/sqrt(x^2 + x + 1) + 5/3/sqrt(x^2 +
x + 1) - 3/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 0.241266, size = 185, normalized size = 3.3

$$-\frac{32x^4+56x^3+72x^2-6\left(8x^3+12x^2-\left(8x^2+8x+5\right)\sqrt{x^2+x+1}+12x+4\right)\log\left(-2x+2\sqrt{x^2+x+1}-1\right)-\left(32x^3+\right.}{4\left(8x^3+12x^2-(8x^2+8x+5)\sqrt{x^2+x+1}+12x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + x + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(32*x^4 + 56*x^3 + 72*x^2 - 6*(8*x^3 + 12*x^2 - (8*x^2 + 8*x
 + 5)*sqrt(x^2 + x + 1) + 12*x + 4)*log(-2*x + 2*sqrt(x^2 + x + 1
) - 1) - (32*x^3 + 40*x^2 + 40*x - 5)*sqrt(x^2 + x + 1) + 24*x +
8)/(8*x^3 + 12*x^2 - (8*x^2 + 8*x + 5)*sqrt(x^2 + x + 1) + 12*x +
 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2 + x + 1)**(3/2), x)
```

GIAC/XCAS [A] time = 0.205979, size = 51, normalized size = 0.91

$$\frac{(3\,x+7)x+5}{3\,\sqrt{x^2+x+1}}+\frac{3}{2}\ln\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + x + 1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*ln(-2*x + 2*sqrt(x^
2 + x + 1) - 1)
```

3.271 $\int x^2\sqrt{1+x+x^2}\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{5}{24}\left(x^2+x+1\right)^{3/2}+\frac{1}{64}(2x+1)\sqrt{x^2+x+1}+\frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] ((1 + 2*x)*Sqrt[1 + x + x^2])/64 - (5*(1 + x + x^2)^(3/2))/24 + (
x*(1 + x + x^2)^(3/2))/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rubi [A] time = 0.0562837, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{5}{24}\left(x^2+x+1\right)^{3/2}+\frac{1}{64}(2x+1)\sqrt{x^2+x+1}+\frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[1 + x + x^2])/64 - (5*(1 + x + x^2)^(3/2))/24 + (
x*(1 + x + x^2)^(3/2))/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rubi in Sympy [A] time = 3.42105, size = 65, normalized size = 1.

$$\frac{x\left(x^2+x+1\right)^{\frac{3}{2}}}{4}+\frac{(2x+1)\sqrt{x^2+x+1}}{64}-\frac{5\left(x^2+x+1\right)^{\frac{3}{2}}}{24}+\frac{3\,\mathrm{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(x**2+x+1)**(1/2),x)
```

```
[Out] x*(x**2 + x + 1)**(3/2)/4 + (2*x + 1)*sqrt(x**2 + x + 1)/64 - 5*(
x**2 + x + 1)**(3/2)/24 + 3*atanh((2*x + 1)/(2*sqrt(x**2 + x + 1)
))/128
```

Mathematica [A] time = 0.0343313, size = 46, normalized size = 0.71

$$\frac{1}{384}\left(2\sqrt{x^2+x+1}\left(48x^3+8x^2+14x-37\right)+9\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3) + 9*ArcSinh[(1
 + 2*x)/Sqrt[3]])/384
```

Maple [A] time = 0.007, size = 49, normalized size = 0.8

$$\frac{x}{4}\left(x^{2}+x+1\right)^{\frac{3}{2}}-\frac{5}{24}\left(x^{2}+x+1\right)^{\frac{3}{2}}+\frac{1+2x}{64}\sqrt{x^{2}+x+1}+\frac{3}{128}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(x^2+x+1)^(1/2),x)
```

```
[Out] 1/4*x*(x^2+x+1)^(3/2)-5/24*(x^2+x+1)^(3/2)+1/64*(1+2*x)*(x^2+x+1)
^(1/2)+3/128*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.62022, size = 76, normalized size = 1.17

$$\frac{1}{4}\left(x^{2}+x+1\right)^{\frac{3}{2}}x-\frac{5}{24}\left(x^{2}+x+1\right)^{\frac{3}{2}}+\frac{1}{32}\sqrt{x^{2}+x+1}x+\frac{1}{64}\sqrt{x^{2}+x+1}+\frac{3}{128}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + x + 1)*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x - 5/24*(x^2 + x + 1)^(3/2) + 1/32*sqrt(
x^2 + x + 1)*x + 1/64*sqrt(x^2 + x + 1) + 3/128*arcsinh(1/3*sqrt(
3)*(2*x + 1))
```

Fricas [A] time = 0.199672, size = 266, normalized size = 4.09

$$-\frac{98304\,x^{8}+262144\,x^{7}+425984\,x^{6}+344064\,x^{5}+120960\,x^{4}-102144\,x^{3}-137952\,x^{2}+72\left(128\,x^{4}+256\,x^{3}+288\,x^{2}-8\,(1\right.}{307}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + x + 1)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3072*(98304*x^8 + 262144*x^7 + 425984*x^6 + 344064*x^5 + 12096
0*x^4 - 102144*x^3 - 137952*x^2 + 72*(128*x^4 + 256*x^3 + 288*x^2
 - 8*(16*x^3 + 24*x^2 + 18*x + 5)*sqrt(x^2 + x + 1) + 160*x + 41)
*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 8*(12288*x^7 + 26624*x^6 +
 35328*x^5 + 17664*x^4 - 2256*x^3 - 12984*x^2 - 8298*x - 2369)*sq
rt(x^2 + x + 1) - 78688*x - 18227)/(128*x^4 + 256*x^3 + 288*x^2 -
 8*(16*x^3 + 24*x^2 + 18*x + 5)*sqrt(x^2 + x + 1) + 160*x + 41)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x^2 + x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2 + x + 1), x)
```

GIAC/XCAS [A] time = 0.206108, size = 59, normalized size = 0.91

$$\frac{1}{192}\left(2\left(4\left(6\,x+1\right)x+7\right)x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\ln\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + x + 1)*x^2,x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 3/128*ln
(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.272 $\int \left(1+x+x^2\right)^{3/2} dx$

Optimal. Leaf size=55

$$\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{9}{64}(2x+1)\sqrt{x^2+x+1}+\frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/
2))/8 + (27*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rubi [A] time = 0.0306179, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{9}{64}(2x+1)\sqrt{x^2+x+1}+\frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2)^(3/2),x]
```

```
[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/
2))/8 + (27*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rubi in Sympy [A] time = 1.01764, size = 56, normalized size = 1.02

$$\frac{(2x+1)\left(x^2+x+1\right)^{\frac{3}{2}}}{8}+\frac{9\left(2x+1\right)\sqrt{x^2+x+1}}{64}+\frac{27\,\text{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x+1)**(3/2),x)
```

```
[Out] (2*x + 1)*(x**2 + x + 1)**(3/2)/8 + 9*(2*x + 1)*sqrt(x**2 + x + 1
)/64 + 27*atanh((2*x + 1)/(2*sqrt(x**2 + x + 1)))/128
```

Mathematica [A] time = 0.0346881, size = 46, normalized size = 0.84

$$\frac{1}{128}\left(2\sqrt{x^2+x+1}\left(16x^3+24x^2+42x+17\right)+27\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3) + 27*ArcSinh[(
1 + 2*x)/Sqrt[3]])/128
```

Maple [A] time = 0.004, size = 43, normalized size = 0.8

$$\frac{1+2x}{8}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{9+18x}{64}\sqrt{x^2+x+1}+\frac{27}{128}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x+1)^(3/2),x)
```

```
[Out] 1/8*(1+2*x)*(x^2+x+1)^(3/2)+9/64*(1+2*x)*(x^2+x+1)^(1/2)+27/128*a
rcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.56755, size = 76, normalized size = 1.38

$$\frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{1}{8}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{9}{32}\sqrt{x^2+x+1}x+\frac{9}{64}\sqrt{x^2+x+1}+\frac{27}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x + 1/8*(x^2 + x + 1)^(3/2) + 9/32*sqrt(x
^2 + x + 1)*x + 9/64*sqrt(x^2 + x + 1) + 27/128*arcsinh(1/3*sqrt(
3)*(2*x + 1))
```

Fricas [A] time = 0.202184, size = 266, normalized size = 4.84

$$-\frac{32768\,x^8+131072\,x^7+327680\,x^6+524288\,x^5+587136\,x^4+453376\,x^3+234080\,x^2+216\left(128\,x^4+256\,x^3+288\,x^2-8\,(1\right.}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(32768*x^8 + 131072*x^7 + 327680*x^6 + 524288*x^5 + 58713
6*x^4 + 453376*x^3 + 234080*x^2 + 216*(128*x^4 + 256*x^3 + 288*x^
2 - 8*(16*x^3 + 24*x^2 + 18*x + 5)*sqrt(x^2 + x + 1) + 160*x + 41
)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 8*(4096*x^7 + 14336*x^6 +
 32256*x^5 + 44800*x^4 + 41488*x^3 + 24600*x^2 + 8434*x + 1269)*s
qrt(x^2 + x + 1) + 72928*x + 9855)/(128*x^4 + 256*x^3 + 288*x^2 -
 8*(16*x^3 + 24*x^2 + 18*x + 5)*sqrt(x^2 + x + 1) + 160*x + 41)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + x + 1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x+1)**(3/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(3/2), x)
```

GIAC/XCAS [A] time = 0.205444, size = 59, normalized size = 1.07

$$\frac{1}{64}\,(2\,(4\,(2\,x+3)x+21)x+17)\sqrt{x^2+x+1}-\frac{27}{128}\ln\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*l
n(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.273 $\int \left(1+x+x^2\right)^{5/2} dx$

Optimal. Leaf size=74

$$\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}+\frac{135\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

```
[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)
^(3/2))/64 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1
 + 2*x)/Sqrt[3]])/1024
```

Rubi [A] time = 0.0437919, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}+\frac{135\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2)^(5/2),x]
```

```
[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)
^(3/2))/64 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1
 + 2*x)/Sqrt[3]])/1024
```

Rubi in Sympy [A] time = 1.31677, size = 75, normalized size = 1.01

$$\frac{(2x+1)\left(x^2+x+1\right)^{\frac{5}{2}}}{12}+\frac{5\,(2x+1)\left(x^2+x+1\right)^{\frac{3}{2}}}{64}+\frac{45\,(2x+1)\sqrt{x^2+x+1}}{512}+\frac{135\,\operatorname{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x+1)**(5/2),x)
```

```
[Out] (2*x + 1)*(x**2 + x + 1)**(5/2)/12 + 5*(2*x + 1)*(x**2 + x + 1)**
(3/2)/64 + 45*(2*x + 1)*sqrt(x**2 + x + 1)/512 + 135*atanh((2*x +
 1)/(2*sqrt(x**2 + x + 1)))/1024
```

Mathematica [A] time = 0.0429609, size = 56, normalized size = 0.76

$$\frac{2\sqrt{x^2+x+1}\left(256x^5+640x^4+1264x^3+1256x^2+1142x+383\right)+405\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3072}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^
4 + 256*x^5) + 405*ArcSinh[(1 + 2*x)/Sqrt[3]])/3072
```

Maple [A] time = 0.004, size = 58, normalized size = 0.8

$$\frac{1+2x}{12}\left(x^2+x+1\right)^{\frac{5}{2}}+\frac{5+10x}{64}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{45+90x}{512}\sqrt{x^2+x+1}+\frac{135}{1024}\mathit{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x+1)^(5/2),x)
```

```
[Out] 1/12*(1+2*x)*(x^2+x+1)^(5/2)+5/64*(1+2*x)*(x^2+x+1)^(3/2)+45/512*
(1+2*x)*(x^2+x+1)^(1/2)+135/1024*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.58612, size = 104, normalized size = 1.41

$$\frac{1}{6}\left(x^2+x+1\right)^{\frac{5}{2}}x+\frac{1}{12}\left(x^2+x+1\right)^{\frac{5}{2}}+\frac{5}{32}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{5}{64}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{45}{256}\sqrt{x^2+x+1}x+\frac{45}{512}\sqrt{x^2+x+1}+\frac{135}{1024}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(x^2 + x + 1)^(5/2)*x + 1/12*(x^2 + x + 1)^(5/2) + 5/32*(x^2
+ x + 1)^(3/2)*x + 5/64*(x^2 + x + 1)^(3/2) + 45/256*sqrt(x^2 + x
 + 1)*x + 45/512*sqrt(x^2 + x + 1) + 135/1024*arcsinh(1/3*sqrt(3)
*(2*x + 1))
```

Fricas [A] time = 0.204212, size = 374, normalized size = 5.05

$$-\frac{8388608x^{12}+50331648x^{11}+171442176x^{10}+395837440x^{9}+686850048x^{8}+926023680x^{7}+987430912x^{6}+83007897\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24576*(8388608*x^12 + 50331648*x^11 + 171442176*x^10 + 3958374
40*x^9 + 686850048*x^8 + 926023680*x^7 + 987430912*x^6 + 83007897
6*x^5 + 541943808*x^4 + 265932800*x^3 + 92389944*x^2 + 3240*(2048
*x^6 + 6144*x^5 + 9984*x^4 + 9728*x^3 + 6024*x^2 - 4*(512*x^5 + 1
280*x^4 + 1664*x^3 + 1216*x^2 + 502*x + 91)*sqrt(x^2 + x + 1) + 2
184*x + 365)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*(2097152*x^1
1 + 11534336*x^10 + 36306944*x^9 + 76873728*x^8 + 121774080*x^7 +
 148205568*x^6 + 140093440*x^5 + 101420032*x^4 + 54518272*x^3 + 2
0512512*x^2 + 4795842*x + 519777)*sqrt(x^2 + x + 1) + 20240952*x
+ 2072547)/(2048*x^6 + 6144*x^5 + 9984*x^4 + 9728*x^3 + 6024*x^2
- 4*(512*x^5 + 1280*x^4 + 1664*x^3 + 1216*x^2 + 502*x + 91)*sqrt(
x^2 + x + 1) + 2184*x + 365)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + x + 1\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x+1)**(5/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(5/2), x)
```

GIAC/XCAS [A] time = 0.203149, size = 73, normalized size = 0.99

$$\frac{1}{1536} \left(2 \left(4 \left(2 \left(8 \left(2 x + 5\right)x + 79\right)x + 157\right)x + 571\right)x + 383\right)\sqrt{x^2 + x + 1} - \frac{135}{1024} \ln\left(-2 x + 2 \sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*s
qrt(x^2 + x + 1) - 135/1024*ln(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

```
[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Rubi [A] time = 0.0359264, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[1 + x + x^2]),x]

[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Rubi in Sympy [A] time = 2.9348, size = 29, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)}{2}-\frac{\sqrt{x^2+x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**2+x+1)**(1/2),x)

[Out] atanh((x + 2)/(2*sqrt(x**2 + x + 1)))/2 - sqrt(x**2 + x + 1)/x
```

Mathematica [A] time = 0.0289863, size = 42, normalized size = 1.11

$$-\frac{\sqrt{x^2+x+1}}{x}+\frac{1}{2}\log\left(2\sqrt{x^2+x+1}+x+2\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -(Sqrt[1 + x + x^2]/x) - Log[x]/2 + Log[2 + x + 2*Sqrt[1 + x + x^
2]]/2
```

Maple [A] time = 0.007, size = 31, normalized size = 0.8

$$\frac{1}{2}Artanh\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-\frac{1}{x}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

Maxima [A] time = 1.59974, size = 50, normalized size = 1.32

$$-\frac{\sqrt{x^2+x+1}}{x}+\frac{1}{2}\,\text{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 + x + 1)/x + 1/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqr
t(3)/abs(x))
```

Fricas [A] time = 0.207621, size = 140, normalized size = 3.68

$$\frac{\left(2\,x^2-2\,\sqrt{x^2+x+1}x+x\right)\log\left(-x+\sqrt{x^2+x+1}+1\right)-\left(2\,x^2-2\,\sqrt{x^2+x+1}x+x\right)\log\left(-x+\sqrt{x^2+x+1}-1\right)+2\,x-}{2\left(2\,x^2-2\,\sqrt{x^2+x+1}x+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*x^2 - 2*sqrt(x^2 + x + 1)*x + x)*log(-x + sqrt(x^2 + x +
1) + 1) - (2*x^2 - 2*sqrt(x^2 + x + 1)*x + x)*log(-x + sqrt(x^2 +
 x + 1) - 1) + 2*x - 2*sqrt(x^2 + x + 1) + 4)/(2*x^2 - 2*sqrt(x^2
 + x + 1)*x + x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2 + x + 1)), x)
```

GIAC/XCAS [A] time = 0.208744, size = 90, normalized size = 2.37

$$\frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1}+\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*x^2),x, algorithm="giac")
```

```
[Out] (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2 + x + 1))^2 - 1) + 1/2
*ln(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/2*ln(abs(-x + sqrt(x^2 +
 x + 1) - 1))
```

3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTan
h[(2 + x)/(2*Sqrt[1 + x + x^2])]/8
```

Rubi [A] time = 0.0737049, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTan
h[(2 + x)/(2*Sqrt[1 + x + x^2])]/8
```

Rubi in Sympy [A] time = 4.6148, size = 48, normalized size = 0.84

$$\frac{\operatorname{atanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)}{8}+\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] atanh((x + 2)/(2*sqrt(x**2 + x + 1)))/8 + 3*sqrt(x**2 + x + 1)/(4
*x) - sqrt(x**2 + x + 1)/(2*x**2)
```

Mathematica [A] time = 0.0362938, size = 45, normalized size = 0.79

$$\frac{1}{8}\left(\frac{2\sqrt{x^2+x+1}(3x-2)}{x^2}+\log\left(2\sqrt{x^2+x+1}+x+2\right)-\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] ((2*(-2 + 3*x)*Sqrt[1 + x + x^2])/x^2 - Log[x] + Log[2 + x + 2*Sq
rt[1 + x + x^2]])/8
```

Maple [A] time = 0.009, size = 44, normalized size = 0.8

$$\frac{1}{8}Artanh\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-\frac{1}{2\,x^2}\sqrt{x^2+x+1}+\frac{3}{4\,x}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/
4*(x^2+x+1)^(1/2)/x
```

Maxima [A] time = 1.58918, size = 68, normalized size = 1.19

$$\frac{3\sqrt{x^2+x+1}}{4\,x}-\frac{\sqrt{x^2+x+1}}{2\,x^2}+\frac{1}{8}\,\text{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*x^3),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh
(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Fricas [A] time = 0.203862, size = 234, normalized size = 4.11

$$\frac{8\,x^3+6\,x^2+\left(8\,x^4+8\,x^3+5\,x^2-4\left(2\,x^3+x^2\right)\sqrt{x^2+x+1}\right)\log\left(-x+\sqrt{x^2+x+1}+1\right)-\left(8\,x^4+8\,x^3+5\,x^2-4\left(2\,x^3+x^2\right.\right.}{8\left(8\,x^4+8\,x^3+5\,x^2-4\left(2\,x^3+x^2\right)\sqrt{x^2+x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^3 + 6*x^2 + (8*x^4 + 8*x^3 + 5*x^2 - 4*(2*x^3 + x^2)*sqr
t(x^2 + x + 1))*log(-x + sqrt(x^2 + x + 1) + 1) - (8*x^4 + 8*x^3
+ 5*x^2 - 4*(2*x^3 + x^2)*sqrt(x^2 + x + 1))*log(-x + sqrt(x^2 +
x + 1) - 1) - 2*(4*x^2 + x + 10)*sqrt(x^2 + x + 1) + 24*x + 16)/(
8*x^4 + 8*x^3 + 5*x^2 - 4*(2*x^3 + x^2)*sqrt(x^2 + x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

GIAC/XCAS [A] time = 0.208975, size = 113, normalized size = 1.98

$$\frac{\left(x-\sqrt{x^2+x+1}\right)^3+9\,x-9\,\sqrt{x^2+x+1}+8}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2}+\frac{1}{8}\ln\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{1}{8}\ln\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/(
(x - sqrt(x^2 + x + 1))^2 - 1)^2 + 1/8*ln(abs(-x + sqrt(x^2 + x +
 1) + 1)) - 1/8*ln(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] (2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) - (5*Sqrt[1 + x + x^2])/(3*x)
 + (3*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2
```

Rubi [A] time = 0.0756296, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) - (5*Sqrt[1 + x + x^2])/(3*x)
 + (3*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2
```

Rubi in Sympy [A] time = 4.94227, size = 53, normalized size = 0.85

$$\frac{3\operatorname{atanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)}{2}+\frac{2(-x+1)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**2+x+1)**(3/2),x)
```

```
[Out] 3*atanh((x + 2)/(2*sqrt(x**2 + x + 1)))/2 + 2*(-x + 1)/(3*x*sqrt(
x**2 + x + 1)) - 5*sqrt(x**2 + x + 1)/(3*x)
```

Mathematica [A] time = 0.0519758, size = 52, normalized size = 0.84

$$\frac{1}{6}\left(-\frac{2\left(5x^2+7x+3\right)}{x\sqrt{x^2+x+1}}+9\log\left(2\sqrt{x^2+x+1}+x+2\right)-9\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] ((-2*(3 + 7*x + 5*x^2))/(x*Sqrt[1 + x + x^2]) - 9*Log[x] + 9*Log[
2 + x + 2*Sqrt[1 + x + x^2]])/6
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{1}{x}\frac{1}{\sqrt{x^2+x+1}}-\frac{3}{2}\frac{1}{\sqrt{x^2+x+1}}-\frac{5+10\,x}{6}\frac{1}{\sqrt{x^2+x+1}}+\frac{3}{2}Artanh\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^2+x+1)^(3/2),x)
```

```
[Out] -1/x/(x^2+x+1)^(1/2)-3/2/(x^2+x+1)^(1/2)-5/6*(1+2*x)/(x^2+x+1)^(1
/2)+3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))
```

Maxima [A] time = 1.57382, size = 78, normalized size = 1.26

$$-\frac{5\,x}{3\sqrt{x^2+x+1}}-\frac{7}{3\sqrt{x^2+x+1}}-\frac{1}{\sqrt{x^2+x+1}x}+\frac{3}{2}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + x + 1)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] -5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 +
x + 1)*x) + 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x)
)
```

Fricas [A] time = 0.209398, size = 270, normalized size = 4.35

$$\frac{24\,x^3+30\,x^2+3\left(8\,x^4+12\,x^3+12\,x^2-\left(8\,x^3+8\,x^2+5\,x\right)\sqrt{x^2+x+1}+4\,x\right)\log\left(-x+\sqrt{x^2+x+1}+1\right)-3\left(8\,x^4+12\,x^3\right.}{2\left(8\,x^4+12\,x^3+12\,x^2-(8\,x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + x + 1)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(24*x^3 + 30*x^2 + 3*(8*x^4 + 12*x^3 + 12*x^2 - (8*x^3 + 8*x^
2 + 5*x)*sqrt(x^2 + x + 1) + 4*x)*log(-x + sqrt(x^2 + x + 1) + 1)
 - 3*(8*x^4 + 12*x^3 + 12*x^2 - (8*x^3 + 8*x^2 + 5*x)*sqrt(x^2 +
x + 1) + 4*x)*log(-x + sqrt(x^2 + x + 1) - 1) - 2*(12*x^2 + 9*x +
 4)*sqrt(x^2 + x + 1) + 26*x + 10)/(8*x^4 + 12*x^3 + 12*x^2 - (8*
x^3 + 8*x^2 + 5*x)*sqrt(x^2 + x + 1) + 4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (x^2 + x + 1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.204623, size = 108, normalized size = 1.74

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}} + \frac{x - \sqrt{x^2+x+1} + 2}{\left(x - \sqrt{x^2+x+1}\right)^2 - 1} + \frac{3}{2}\ln\left(\left|-x + \sqrt{x^2+x+1} + 1\right|\right) - \frac{3}{2}\ln\left(\left|-x + \sqrt{x^2+x+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + x + 1)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1) + (x - sqrt(x^2 + x + 1) + 2)/((x
- sqrt(x^2 + x + 1))^2 - 1) + 3/2*ln(abs(-x + sqrt(x^2 + x + 1) +
 1)) - 3/2*ln(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}+\frac{37\sqrt{x^2+x+1}}{12x}-\frac{7\sqrt{x^2+x+1}}{6x^2}-\frac{3}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] (2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) - (7*Sqrt[1 + x + x^2])/(6*
x^2) + (37*Sqrt[1 + x + x^2])/(12*x) - (3*ArcTanh[(2 + x)/(2*Sqrt
[1 + x + x^2])])/8
```

Rubi [A] time = 0.115994, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}+\frac{37\sqrt{x^2+x+1}}{12x}-\frac{7\sqrt{x^2+x+1}}{6x^2}-\frac{3}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) - (7*Sqrt[1 + x + x^2])/(6*
x^2) + (37*Sqrt[1 + x + x^2])/(12*x) - (3*ArcTanh[(2 + x)/(2*Sqrt
[1 + x + x^2])])/8
```

Rubi in Sympy [A] time = 7.01911, size = 71, normalized size = 0.9

$$-\frac{3\,\text{atanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)}{8}+\frac{37\sqrt{x^2+x+1}}{12x}+\frac{2\,(-x+1)}{3x^2\sqrt{x^2+x+1}}-\frac{7\sqrt{x^2+x+1}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] -3*atanh((x + 2)/(2*sqrt(x**2 + x + 1)))/8 + 37*sqrt(x**2 + x + 1
)/(12*x) + 2*(-x + 1)/(3*x**2*sqrt(x**2 + x + 1)) - 7*sqrt(x**2 +
 x + 1)/(6*x**2)
```

Mathematica [A] time = 0.0453761, size = 57, normalized size = 0.72

$$\frac{1}{24}\left(-9\log\left(2\sqrt{x^2+x+1}+x+2\right)+\frac{2\left(37x^3+23x^2+15x-6\right)}{x^2\sqrt{x^2+x+1}}+9\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] ((2*(-6 + 15*x + 23*x^2 + 37*x^3))/(x^2*Sqrt[1 + x + x^2]) + 9*Lo
g[x] - 9*Log[2 + x + 2*Sqrt[1 + x + x^2]])/24
```

Maple [A] time = 0.007, size = 69, normalized size = 0.9

$$-\frac{1}{2x^2}\frac{1}{\sqrt{x^2+x+1}}+\frac{5}{4x}\frac{1}{\sqrt{x^2+x+1}}+\frac{3}{8}\frac{1}{\sqrt{x^2+x+1}}+\frac{37+74x}{24}\frac{1}{\sqrt{x^2+x+1}}-\frac{3}{8}Artanh\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^2+x+1)^(3/2),x)
```

```
[Out] -1/2/x^2/(x^2+x+1)^(1/2)+5/4/x/(x^2+x+1)^(1/2)+3/8/(x^2+x+1)^(1/2
)+37/24*(1+2*x)/(x^2+x+1)^(1/2)-3/8*arctanh(1/2*(2+x)/(x^2+x+1)^(
1/2))
```

Maxima [A] time = 1.59365, size = 96, normalized size = 1.22

$$\frac{37\,x}{12\,\sqrt{x^2+x+1}}+\frac{23}{12\,\sqrt{x^2+x+1}}+\frac{5}{4\,\sqrt{x^2+x+1}x}-\frac{1}{2\,\sqrt{x^2+x+1}x^2}-\frac{3}{8}\,\text{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + x + 1)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] 37/12*x/sqrt(x^2 + x + 1) + 23/12/sqrt(x^2 + x + 1) + 5/4/(sqrt(x
^2 + x + 1)*x) - 1/2/(sqrt(x^2 + x + 1)*x^2) - 3/8*arcsinh(1/3*sq
rt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Fricas [A] time = 0.234071, size = 354, normalized size = 4.48

$$-\frac{96\,x^5+168\,x^4+182\,x^3+82\,x^2+3\left(32\,x^6+64\,x^5+78\,x^4+46\,x^3+14\,x^2-\left(32\,x^5+48\,x^4+42\,x^3+13\,x^2\right)\sqrt{x^2+x+1}\right)\log}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + x + 1)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/8*(96*x^5 + 168*x^4 + 182*x^3 + 82*x^2 + 3*(32*x^6 + 64*x^5 +
78*x^4 + 46*x^3 + 14*x^2 - (32*x^5 + 48*x^4 + 42*x^3 + 13*x^2)*sq
rt(x^2 + x + 1))*log(-x + sqrt(x^2 + x + 1) + 1) - 3*(32*x^6 + 64
*x^5 + 78*x^4 + 46*x^3 + 14*x^2 - (32*x^5 + 48*x^4 + 42*x^3 + 13*
x^2)*sqrt(x^2 + x + 1))*log(-x + sqrt(x^2 + x + 1) - 1) - 2*(48*x
^4 + 60*x^3 + 43*x^2 + 6*x - 28)*sqrt(x^2 + x + 1) - 38*x - 52)/(
32*x^6 + 64*x^5 + 78*x^4 + 46*x^3 + 14*x^2 - (32*x^5 + 48*x^4 + 4
2*x^3 + 13*x^2)*sqrt(x^2 + x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (x^2 + x + 1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.207246, size = 158, normalized size = 2.

$$\frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}} - \frac{3\left(x-\sqrt{x^2+x+1}\right)^3 + 8\left(x-\sqrt{x^2+x+1}\right)^2 - 13\,x + 13\,\sqrt{x^2+x+1} - 16}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2 - 1\right)^2} - \frac{3}{8}\ln\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) + \frac{3}{8}\ln\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + x + 1)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1) - 1/4*(3*(x - sqrt(x^2 + x + 1))^
3 + 8*(x - sqrt(x^2 + x + 1))^2 - 13*x + 13*sqrt(x^2 + x + 1) - 1
6)/((x - sqrt(x^2 + x + 1))^2 - 1)^2 - 3/8*ln(abs(-x + sqrt(x^2 +
 x + 1) + 1)) + 3/8*ln(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Rubi [A] time = 0.0247833, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*Sqrt[1 + x + x^2]),x]

[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Rubi in Sympy [A] time = 2.35862, size = 17, normalized size = 0.77

$$-\operatorname{atanh}\left(\frac{-x+1}{2\sqrt{x^2+x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(x**2+x+1)**(1/2),x)

[Out] -atanh((-x + 1)/(2*sqrt(x**2 + x + 1)))
```

Mathematica [A] time = 0.014181, size = 28, normalized size = 1.27

$$\log(x+1)-\log\left(-x+2\sqrt{(x+1)^2-x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

```
[Out] Log[1 + x] - Log[1 - x + 2*Sqrt[-x + (1 + x)^2]]
```

Maple [A] time = 0.007, size = 22, normalized size = 1.

$$-Artanh\left(\frac{1-x}{2}\frac{1}{\sqrt{(1+x)^2-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(x^2+x+1)^(1/2),x)
```

```
[Out] -arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

Maxima [A] time = 1.59763, size = 34, normalized size = 1.55

$$\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x+1|}-\frac{\sqrt{3}}{3|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*(x + 1)),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))
```

Fricas [A] time = 0.209753, size = 41, normalized size = 1.86

$$-\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*(x + 1)),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)
```

GIAC/XCAS [A] time = 0.20579, size = 43, normalized size = 1.95

$$-\ln\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\ln\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x + 1)*(x + 1)),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(x^2 + x + 1))) + ln(abs(-x + sqrt(x^2 + x + 1)
- 2))
```

3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(S
qrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x +
 x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.493948, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(S
qrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x +
 x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 21.054, size = 80, normalized size = 0.93

$$-\frac{\sqrt{3}\,\text{atanh}\left(\frac{\sqrt{3}\sqrt{x^2+2x+4}}{3}\right)}{6}+\frac{\text{atanh}\left(\frac{2x+8}{4\sqrt{x^2+2x+4}}\right)}{2}-\frac{\sqrt{7}\,\text{atanh}\left(\frac{\sqrt{7}(4x+10)}{14\sqrt{x^2+2x+4}}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] -sqrt(3)*atanh(sqrt(3)*sqrt(x**2 + 2*x + 4)/3)/6 + atanh((2*x + 8
)/(4*sqrt(x**2 + 2*x + 4)))/2 - sqrt(7)*atanh(sqrt(7)*(4*x + 10)/
(14*sqrt(x**2 + 2*x + 4)))/14
```

Mathematica [A] time = 0.0635976, size = 112, normalized size = 1.3

$$\frac{1}{42}\left(21\log\left(2\sqrt{x^2+2x+4}+x+4\right)-7\sqrt{3}\log\left(\sqrt{3}\sqrt{x^2+2x+4}+3\right)\right.$$
$$\left.-3\sqrt{7}\log\left(\sqrt{7}\sqrt{x^2+2x+4}+2x+5\right)+3\sqrt{7}\log(1-x)-21\log(x)+7\sqrt{3}\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] (3*Sqrt[7]*Log[1 - x] - 21*Log[x] + 7*Sqrt[3]*Log[1 + x] + 21*Log
[4 + x + 2*Sqrt[4 + 2*x + x^2]] - 7*Sqrt[3]*Log[3 + Sqrt[3]*Sqrt[
4 + 2*x + x^2]] - 3*Sqrt[7]*Log[5 + 2*x + Sqrt[7]*Sqrt[4 + 2*x +
x^2]])/42
```

Maple [A] time = 0.017, size = 69, normalized size = 0.8

$$\frac{1}{2}\mathrm{Artanh}\left(\frac{8+2x}{4}\frac{1}{\sqrt{x^2+2x+4}}\right)-\frac{\sqrt{7}}{14}\mathrm{Artanh}\left(\frac{(10+4x)\sqrt{7}}{14}\frac{1}{\sqrt{(-1+x)^2+3+4x}}\right)$$
$$-\frac{\sqrt{3}}{6}\mathrm{Artanh}\left(\sqrt{3}\frac{1}{\sqrt{(1+x)^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/4*(8+2*x)/(x^2+2*x+4)^(1/2))-1/14*7^(1/2)*arctanh(1
/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(1/2)*arctanh(
3^(1/2)/((1+x)^2+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-x)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)
```

Fricas [A] time = 0.232557, size = 254, normalized size = 2.95

$$\frac{1}{42}\sqrt{7}\sqrt{3}\left(\sqrt{7}\sqrt{3}\log\left(-x+\sqrt{x^2+2x+4}+2\right)-\sqrt{7}\sqrt{3}\log\left(-x+\sqrt{x^2+2x+4}-2\right)+\sqrt{7}\log\left(\frac{\sqrt{3}\left(x^2+2x+4\right)-\sqrt{x^2+2}}{x^2-\sqrt{x^2+2x+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)),x, algorithm="fricas")
```

```
[Out] 1/42*sqrt(7)*sqrt(3)*(sqrt(7)*sqrt(3)*log(-x + sqrt(x^2 + 2*x + 4
) + 2) - sqrt(7)*sqrt(3)*log(-x + sqrt(x^2 + 2*x + 4) - 2) + sqrt
(7)*log((sqrt(3)*(x^2 + 2*x + 4) - sqrt(x^2 + 2*x + 4)*(sqrt(3)*(
x + 1) + 3) + 3*x + 3)/(x^2 - sqrt(x^2 + 2*x + 4)*(x + 1) + 2*x +
 1)) + sqrt(3)*log((sqrt(7)*(x^2 + 6) - sqrt(x^2 + 2*x + 4)*(sqrt
(7)*(x - 1) + 7) + 7*x - 7)/(x^2 - sqrt(x^2 + 2*x + 4)*(x - 1) -
1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(x-1\right)\left(x+1\right)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

GIAC/XCAS [A] time = 0.247236, size = 198, normalized size = 2.3

$$\frac{1}{14}\sqrt{7}\ln\left(\frac{\left|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}{\left|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}\right)+\frac{1}{6}\sqrt{3}\ln\left(-\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)+\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2+2x+4}+2\right|\right)-\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2+2x+4}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*ln(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)
/abs(-2*x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)
```

```
*ln(-1/2*abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - s
qrt(3) - sqrt(x^2 + 2*x + 4) + 1)) + 1/2*ln(abs(-x + sqrt(x^2 + 2
*x + 4) + 2)) - 1/2*ln(abs(-x + sqrt(x^2 + 2*x + 4) - 2))
```

3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTa
nh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Rubi [A] time = 0.122442, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTa
nh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Rubi in Sympy [A] time = 6.87574, size = 68, normalized size = 1.1

$$\operatorname{atanh}\left(\frac{2x+2}{2\sqrt{x^2+2x+4}}\right)-\frac{2\sqrt{7}\operatorname{atanh}\left(\frac{\sqrt{7}(4x+10)}{14\sqrt{x^2+2x+4}}\right)}{7}+\frac{\sqrt{x^2+2x+4}}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

```
[Out] atanh((2*x + 2)/(2*sqrt(x**2 + 2*x + 4))) - 2*sqrt(7)*atanh(sqrt(
7)*(4*x + 10)/(14*sqrt(x**2 + 2*x + 4)))/7 + sqrt(x**2 + 2*x + 4)
/(-x + 1)
```

Mathematica [A] time = 0.0890938, size = 72, normalized size = 1.16

$$-\frac{\sqrt{x^2+2x+4}}{x-1}-\frac{2\log\left(\sqrt{7}\sqrt{x^2+2x+4}+2x+5\right)}{\sqrt{7}}+\frac{2\log(x-1)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] -(Sqrt[4 + 2*x + x^2]/(-1 + x)) + ArcSinh[(1 + x)/Sqrt[3]] + (2*L
og[-1 + x])/Sqrt[7] - (2*Log[5 + 2*x + Sqrt[7]*Sqrt[4 + 2*x + x^2
]])/Sqrt[7]
```

Maple [A] time = 0.01, size = 91, normalized size = 1.5

$$-\frac{1}{-7+7x}\left((-1+x)^2+3+4x\right)^{\frac{3}{2}}+\frac{2}{7}\sqrt{(-1+x)^2+3+4x}+\mathit{Arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}}{7}\mathit{Artanh}\left(\frac{(10+4x)\sqrt{7}}{14}\frac{1}{\sqrt{(-1+x)^2+3+4x}}\right)+\frac{2x+2}{14}\sqrt{(-1+x)^2+3+4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x+4)^(1/2)/(-1+x)^2,x)
```

```
[Out] -1/7/(-1+x)*((-1+x)^2+3+4*x)^(3/2)+2/7*((-1+x)^2+3+4*x)^(1/2)+arc
sinh(1/3*(1+x)*3^(1/2))-2/7*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)
/((-1+x)^2+3+4*x)^(1/2))+1/14*(2*x+2)*((-1+x)^2+3+4*x)^(1/2)
```

Maxima [A] time = 1.58115, size = 82, normalized size = 1.32

$$-\frac{2}{7}\sqrt{7}\,\mathrm{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right)-\frac{\sqrt{x^2+2x+4}}{x-1}+\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2*x + 4)/(x - 1)^2,x, algorithm="maxima")
```

```
[Out] -2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x
 - 1)) - sqrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/
3*sqrt(3))
```

Fricas [A] time = 0.226483, size = 247, normalized size = 3.98

$$-\frac{\left(\sqrt{7}\sqrt{x^2+2x+4}(x-1)-\sqrt{7}\left(x^2-1\right)\right)\log\left(-x+\sqrt{x^2+2x+4}-1\right)+2\left(x^2-\sqrt{x^2+2x+4}(x-1)-1\right)\log\left(\frac{\sqrt{7}\left(x^2+6\right)-\cdots}{x^2\cdots}\right.}{\sqrt{7}\sqrt{x^2+2x+4}(x-1)-\sqrt{7}\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2*x + 4)/(x - 1)^2,x, algorithm="fricas")
```

```
[Out] -((sqrt(7)*sqrt(x^2 + 2*x + 4)*(x - 1) - sqrt(7)*(x^2 - 1))*log(-
x + sqrt(x^2 + 2*x + 4) - 1) + 2*(x^2 - sqrt(x^2 + 2*x + 4)*(x -
1) - 1)*log((sqrt(7)*(x^2 + 6) - sqrt(x^2 + 2*x + 4)*(sqrt(7)*(x
- 1) + 7) + 7*x - 7)/(x^2 - sqrt(x^2 + 2*x + 4)*(x - 1) - 1)) + s
qrt(7)*(2*x + 5) - 2*sqrt(7)*sqrt(x^2 + 2*x + 4))/(sqrt(7)*sqrt(x
^2 + 2*x + 4)*(x - 1) - sqrt(7)*(x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

```
[Out] Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)
```

GIAC/XCAS [A] time = 0.23889, size = 204, normalized size = 3.29

$$-\frac{2}{7}\sqrt{7}\ln\left(2\sqrt{7}+7\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{7\sqrt{7}}{x-1}\right)\operatorname{sign}\left(\frac{1}{x-1}\right)$$
$$+\ln\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}+1\right)\operatorname{sign}\left(\frac{1}{x-1}\right)$$
$$-\ln\left(\left|\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}-1\right|\right)\operatorname{sign}\left(\frac{1}{x-1}\right)-\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}\operatorname{sign}\left(\frac{1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2*x + 4)/(x - 1)^2,x, algorithm="giac")
```

```
[Out] -2/7*sqrt(7)*ln(2*sqrt(7) + 7*sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) +
 7*sqrt(7)/(x - 1))*sign(1/(x - 1)) + ln(sqrt(4/(x - 1) + 7/(x -
1)^2 + 1) + sqrt(7)/(x - 1) + 1)*sign(1/(x - 1)) - ln(abs(sqrt(4/
(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)/(x - 1) - 1))*sign(1/(x - 1)
) - sqrt(4/(x - 1) + 7/(x - 1)^2 + 1)*sign(1/(x - 1))
```

3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{x^2+2x+4}(3-x)}{4(x^2+2x+3)}-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

```
[Out] -((3 - x)*Sqrt[4 + 2*x + x^2])/(4*(3 + 2*x + x^2)) - ArcTan[(1 +
x)/(Sqrt[2]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 +
2*x + x^2]]
```

Rubi [A] time = 0.18859, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{\sqrt{x^2+2x+4}(3-x)}{4(x^2+2x+3)}-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -((3 - x)*Sqrt[4 + 2*x + x^2])/(4*(3 + 2*x + x^2)) - ArcTan[(1 +
x)/(Sqrt[2]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 +
2*x + x^2]]
```

Rubi in Sympy [A] time = 31.1996, size = 71, normalized size = 0.93

$$-\frac{(-2x+6)\sqrt{x^2+2x+4}}{8(x^2+2x+3)}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}(2x+2)}{4\sqrt{x^2+2x+4}}\right)}{8}+\operatorname{atanh}\left(\sqrt{x^2+2x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
```

```
[Out] -(-2*x + 6)*sqrt(x**2 + 2*x + 4)/(8*(x**2 + 2*x + 3)) - sqrt(2)*a
tan(sqrt(2)*(2*x + 2)/(4*sqrt(x**2 + 2*x + 4)))/8 + atanh(sqrt(x*
*2 + 2*x + 4))
```

Mathematica [A] time = 0.38744, size = 146, normalized size = 1.92

$$\frac{1}{32}\left(8\left(\frac{\sqrt{x^2+2x+4}(x-3)}{x^2+2x+3}-2\log\left(\left(x^2+2x+3\right)^2\right)\right.\right.$$
$$\left.+2\log\left(\left(x^2+2x+3\right)\left(x^2+2\sqrt{x^2+2x+4}+2x+5\right)\right)\right)$$
$$\left.-4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\left(5x^2+10x+4\right)}{4x^2+\left(11\sqrt{x^2+2x+4}+8\right)x+11\sqrt{x^2+2x+4}+12}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (-4*Sqrt[2]*ArcTan[(Sqrt[2]*(4 + 10*x + 5*x^2))/(12 + 4*x^2 + 11*
Sqrt[4 + 2*x + x^2] + x*(8 + 11*Sqrt[4 + 2*x + x^2]))] + 8*(((-3
+ x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) - 2*Log[(3 + 2*x + x^2)
^2] + 2*Log[(3 + 2*x + x^2)*(5 + 2*x + x^2 + 2*Sqrt[4 + 2*x + x^2
])]))/32
```

Maple [A] time = 0.038, size = 123, normalized size = 1.6

$$-\frac{1}{2}\left(1+\sqrt{x^2+2x+4}\right)^{-1}+\frac{1}{2}\ln\left(1+\sqrt{x^2+2x+4}\right)$$
$$-\frac{1}{2}\left(-1+\sqrt{x^2+2x+4}\right)^{-1}-\frac{1}{2}\ln\left(-1+\sqrt{x^2+2x+4}\right)$$
$$+\frac{3+3x}{4}\frac{1}{\sqrt{x^2+2x+4}}\left(\frac{(1+x)^2}{x^2+2x+4}+2\right)^{-1}-\frac{\sqrt{2}}{8}\arctan\left(\frac{(1+x)\sqrt{2}}{2}\frac{1}{\sqrt{x^2+2x+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x)
```

```
[Out] -1/2/(1+(x^2+2*x+4)^(1/2))+1/2*ln(1+(x^2+2*x+4)^(1/2))-1/2/(-1+(x
^2+2*x+4)^(1/2))-1/2*ln(-1+(x^2+2*x+4)^(1/2))+3/4*(1+x)/(x^2+2*x+
4)^(1/2)/((1+x)^2/(x^2+2*x+4)+2)-1/8*arctan(1/2*(1+x)*2^(1/2)/(x^
2+2*x+4)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2x+3}{\sqrt{x^2+2x+4}\left(x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)
```

Fricas [A] time = 0.228227, size = 540, normalized size = 7.11

$$\frac{\sqrt{2}\left(8\,x^{2} + 17\,x + 21\right)\sqrt{x^{2} + 2\,x + 4} - \left(2\,x^{4} + 8\,x^{3} + 19\,x^{2} - 2\,\left(x^{3} + 3\,x^{2} + 5\,x + 3\right)\sqrt{x^{2} + 2\,x + 4} + 22\,x + 15\right)\arctan\left(-\frac{1}{2}\,\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(8*x^2 + 17*x + 21)*sqrt(x^2 + 2*x + 4) - (2*x^4 + 8
*x^3 + 19*x^2 - 2*(x^3 + 3*x^2 + 5*x + 3)*sqrt(x^2 + 2*x + 4) + 2
2*x + 15)*arctan(-1/2*sqrt(2)*(x + 2) + 1/2*sqrt(2)*sqrt(x^2 + 2*
x + 4)) + (2*x^4 + 8*x^3 + 19*x^2 - 2*(x^3 + 3*x^2 + 5*x + 3)*sqr
t(x^2 + 2*x + 4) + 22*x + 15)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)
*sqrt(x^2 + 2*x + 4)) - 2*(2*sqrt(2)*(x^3 + 3*x^2 + 5*x + 3)*sqrt
(x^2 + 2*x + 4) - sqrt(2)*(2*x^4 + 8*x^3 + 19*x^2 + 22*x + 15))*l
og(x^2 - sqrt(x^2 + 2*x + 4)*(x + 2) + 3*x + 5) + 2*(2*sqrt(2)*(x
^3 + 3*x^2 + 5*x + 3)*sqrt(x^2 + 2*x + 4) - sqrt(2)*(2*x^4 + 8*x^
3 + 19*x^2 + 22*x + 15))*log(x^2 - sqrt(x^2 + 2*x + 4)*x + x + 3)
 - sqrt(2)*(8*x^3 + 25*x^2 + 50*x + 39))/(2*sqrt(2)*(x^3 + 3*x^2
+ 5*x + 3)*sqrt(x^2 + 2*x + 4) - sqrt(2)*(2*x^4 + 8*x^3 + 19*x^2
+ 22*x + 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + 3}{\left(x^{2} + 2x + 3\right)^{2}\sqrt{x^{2} + 2x + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)
```

GIAC/XCAS [A] time = 0.207833, size = 317, normalized size = 4.17

$$\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}+2\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}\right)\right)+\frac{4\left(x-\sqrt{x^2+2x+4}\right)^3+13\left(x-\sqrt{x^2+2x+4}\right)^2+26x-26\sqrt{x^2+2x+4}+26}{2\left(\left(x-\sqrt{x^2+2x+4}\right)^4+4\left(x-\sqrt{x^2+2x+4}\right)^3+8\left(x-\sqrt{x^2+2x+4}\right)^2+8x-8\sqrt{x^2+2x+4}+12\right)}-\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+2x+4}\right)^2+4x-4\sqrt{x^2+2x+4}+6\right)+\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+2x+4}\right)^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) -
1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*
(4*(x - sqrt(x^2 + 2*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 +
 26*x - 26*sqrt(x^2 + 2*x + 4) + 26)/((x - sqrt(x^2 + 2*x + 4))^4
 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8*(x - sqrt(x^2 + 2*x + 4))^2
+ 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*ln((x - sqrt(x^2 + 2*x
+ 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*ln((x - sqrt(x^2
 + 2*x + 4))^2 + 2)
```

3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

```
[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Rubi [A] time = 0.14043, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Rubi in Sympy [A] time = 8.35967, size = 27, normalized size = 0.75

$$\sqrt{x^2+2x-3}+\frac{\sqrt{x^2+2x-3}}{2\left(-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
```

```
[Out] sqrt(x**2 + 2*x - 3) + sqrt(x**2 + 2*x - 3)/(2*(-x + 1))
```

Mathematica [A] time = 0.0258345, size = 24, normalized size = 0.67

$$\frac{(x+3)(2x-3)}{2\sqrt{x^2+2x-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] ((3 + x)*(-3 + 2*x))/(2*Sqrt[-3 + 2*x + x^2])
```

Maple [A] time = 0.01, size = 21, normalized size = 0.6

$$\frac{(-3+2x)(3+x)}{2}\frac{1}{\sqrt{x^2+2x-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x)
```

```
[Out] 1/2*(-3+2*x)*(3+x)/(x^2+2*x-3)^(1/2)
```

Maxima [A] time = 1.61093, size = 38, normalized size = 1.06

$$\sqrt{x^2+2x-3}-\frac{\sqrt{x^2+2x-3}}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2)/((2*x^2 + x - 3)*sqrt(x^2 + 2*x - 3)),x, algorithm="maxi
```

```
[Out] sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

Fricas [A] time = 0.211069, size = 77, normalized size = 2.14

$$-\frac{2x^3+3x^2-\left(2x^2+x-6\right)\sqrt{x^2+2x-3}-9x}{2\left(x^2-\sqrt{x^2+2x-3}x+x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2)/((2*x^2 + x - 3)*sqrt(x^2 + 2*x - 3)),x, algorithm="fric
```

```
[Out] -1/2*(2*x^3 + 3*x^2 - (2*x^2 + x - 6)*sqrt(x^2 + 2*x - 3) - 9*x)/
(x^2 - sqrt(x^2 + 2*x - 3)*x + x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{(x-1)(x+3)}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

GIAC/XCAS [A] time = 0.208305, size = 41, normalized size = 1.14

$$\sqrt{x^2+2x-3}+\frac{2}{x-\sqrt{x^2+2x-3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2)/((2*x^2 + x - 3)*sqrt(x^2 + 2*x - 3)),x, algorithm="giac
```

```
[Out] sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{1}{2}\sqrt{x^2+x+2}x-\frac{7}{4}\sqrt{x^2+x+2}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\tanh^{-1}\left(\sqrt{x^2+x+2}\right)-\frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

```
[Out] (-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 +
 2*x)/Sqrt[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/
Sqrt[3] - ArcTanh[Sqrt[2 + x + x^2]]
```

Rubi [A] time = 0.402581, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{1}{2}\sqrt{x^2+x+2}x-\frac{7}{4}\sqrt{x^2+x+2}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\tanh^{-1}\left(\sqrt{x^2+x+2}\right)-\frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 +
 2*x)/Sqrt[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/
Sqrt[3] - ArcTanh[Sqrt[2 + x + x^2]]
```

Rubi in Sympy [A] time = 24.0269, size = 88, normalized size = 1.01

$$\frac{x\sqrt{x^2+x+2}}{2}-\frac{7\sqrt{x^2+x+2}}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}(2x+1)}{3\sqrt{x^2+x+2}}\right)}{3}-\frac{\mathrm{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+2}}\right)}{8}-\mathrm{atanh}\left(\sqrt{x^2+x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + x + 2)/2 - 7*sqrt(x**2 + x + 2)/4 + sqrt(3)*atan(sq
rt(3)*(2*x + 1)/(3*sqrt(x**2 + x + 2)))/3 - atanh((2*x + 1)/(2*sq
rt(x**2 + x + 2)))/8 - atanh(sqrt(x**2 + x + 2))
```

Mathematica [A] time = 0.189655, size = 146, normalized size = 1.68

$$\frac{1}{24}\left(12\sqrt{x^2+x+2}x-42\sqrt{x^2+x+2}+12\log\left(\left(x^2+x+1\right)^2\right)\right.$$
$$-12\log\left(\left(x^2+x+1\right)\left(x^2+2\sqrt{x^2+x+2}+x+3\right)\right)$$
$$-8\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(-3x^2-3x+1\right)}{7x^2+8\sqrt{x^2+x+2}x+4\sqrt{x^2+x+2}+7x+7}\right)$$
$$\left.-3\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)+\log(281474976710656)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (-42*Sqrt[2 + x + x^2] + 12*x*Sqrt[2 + x + x^2] - 3*ArcSinh[(1 +
2*x)/Sqrt[7]] - 8*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 3*x - 3*x^2))/(7 +
 7*x + 7*x^2 + 4*Sqrt[2 + x + x^2] + 8*x*Sqrt[2 + x + x^2])] + Lo
g[281474976710656] + 12*Log[(1 + x + x^2)^2] - 12*Log[(1 + x + x^
2)*(3 + x + x^2 + 2*Sqrt[2 + x + x^2])])/24
```

Maple [A] time = 0.027, size = 69, normalized size = 0.8

$$\frac{x}{2}\sqrt{x^2+x+2}-\frac{7}{4}\sqrt{x^2+x+2}-\frac{1}{8}Arcsinh\left(\frac{2\sqrt{7}}{7}\left(x+\frac{1}{2}\right)\right)$$
$$-Artanh\left(\sqrt{x^2+x+2}\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\frac{1}{\sqrt{x^2+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x)
```

```
[Out] 1/2*x*(x^2+x+2)^(1/2)-7/4*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*7^(1/2)
*(x+1/2))-arctanh((x^2+x+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)
/(x^2+x+2)^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4+1}{\sqrt{x^2+x+2}(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

Fricas [A] time = 0.226175, size = 498, normalized size = 5.72

$$-\frac{4\sqrt{3}\left(32x^3-80x^2-46x-111\right)\sqrt{x^2+x+2}-32\left(8x^2-4\sqrt{x^2+x+2}(2x+1)+8x+9\right)\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3)+\frac{2}{3}\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)),x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(3)*(32*x^3 - 80*x^2 - 46*x - 111)*sqrt(x^2 + x + 2)
 - 32*(8*x^2 - 4*sqrt(x^2 + x + 2)*(2*x + 1) + 8*x + 9)*arctan(-1
/3*sqrt(3)*(2*x + 3) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 32*(8*x^2
 - 4*sqrt(x^2 + x + 2)*(2*x + 1) + 8*x + 9)*arctan(-1/3*sqrt(3)*(
2*x - 1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) - 16*(4*sqrt(3)*sqrt(x^
2 + x + 2)*(2*x + 1) - sqrt(3)*(8*x^2 + 8*x + 9))*log(2*x^2 - sqr
t(x^2 + x + 2)*(2*x + 3) + 4*x + 5) + 16*(4*sqrt(3)*sqrt(x^2 + x
+ 2)*(2*x + 1) - sqrt(3)*(8*x^2 + 8*x + 9))*log(2*x^2 - sqrt(x^2
+ x + 2)*(2*x - 1) + 3) - 4*(4*sqrt(3)*sqrt(x^2 + x + 2)*(2*x + 1
) - sqrt(3)*(8*x^2 + 8*x + 9))*log(-2*x + 2*sqrt(x^2 + x + 2) - 1
) - sqrt(3)*(128*x^4 - 256*x^3 - 232*x^2 - 872*x - 313))/(4*sqrt(
3)*sqrt(x^2 + x + 2)*(2*x + 1) - sqrt(3)*(8*x^2 + 8*x + 9))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{\left(x^2+x+1\right)\sqrt{x^2+x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

```
[Out] Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)
```

GIAC/XCAS [A] time = 0.207571, size = 200, normalized size = 2.3

$$\frac{1}{4}\sqrt{x^2+x+2}(2x-7)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}+3\right)\right)$$
$$+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}-1\right)\right)$$
$$+\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+x+2}\right)^2+3x-3\sqrt{x^2+x+2}+3\right)$$
$$-\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+x+2}\right)^2-x+\sqrt{x^2+x+2}+1\right)+\frac{1}{8}\ln\left(-2x+2\sqrt{x^2+x+2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)
*(2*x - 2*sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(
3)*(2*x - 2*sqrt(x^2 + x + 2) - 1)) + 1/2*ln((x - sqrt(x^2 + x +
2))^2 + 3*x - 3*sqrt(x^2 + x + 2) + 3) - 1/2*ln((x - sqrt(x^2 + x
 + 2))^2 - x + sqrt(x^2 + x + 2) + 1) + 1/8*ln(-2*x + 2*sqrt(x^2
+ x + 2) - 1)
```

3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx$

Optimal. Leaf size=58

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\left(x^2+2x+4\right)^{3/2}}+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

```
[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x +
x^2)^(3/2)) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])
```

Rubi [A] time = 0.0239542, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\left(x^2+2x+4\right)^{3/2}}+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x +
x^2)^(3/2)) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])
```

Rubi in Sympy [A] time = 1.0255, size = 54, normalized size = 0.93

$$\frac{2\left(2x+2\right)}{135\left(x^2+2x+4\right)^{\frac{3}{2}}}+\frac{2x+2}{30\left(x^2+2x+4\right)^{\frac{5}{2}}}+\frac{2\left(4x+4\right)}{405\sqrt{x^2+2x+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+2*x+4)**(7/2),x)
```

```
[Out] 2*(2*x + 2)/(135*(x**2 + 2*x + 4)**(3/2)) + (2*x + 2)/(30*(x**2 +
 2*x + 4)**(5/2)) + 2*(4*x + 4)/(405*sqrt(x**2 + 2*x + 4))
```

Mathematica [A] time = 0.021684, size = 39, normalized size = 0.67

$$\frac{(x+1)\left(8x^4+32x^3+108x^2+152x+203\right)}{405\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] ((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x
+ x^2)^(5/2))
```

Maple [A] time = 0.005, size = 38, normalized size = 0.7

$$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405}\left(x^2+2x+4\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+2*x+4)^(7/2),x)
```

```
[Out] 1/405*(8*x^5+40*x^4+140*x^3+260*x^2+355*x+203)/(x^2+2*x+4)^(5/2)
```

Maxima [A] time = 1.36689, size = 103, normalized size = 1.78

$$\frac{8x}{405\sqrt{x^2+2x+4}}+\frac{8}{405\sqrt{x^2+2x+4}}+\frac{4x}{135\left(x^2+2x+4\right)^{\frac{3}{2}}}+\frac{4}{135\left(x^2+2x+4\right)^{\frac{3}{2}}}+\frac{x}{15\left(x^2+2x+4\right)^{\frac{5}{2}}}+\frac{1}{15\left(x^2+2x+4\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 4)^(-7/2),x, algorithm="maxima")
```

```
[Out] 8/405*x/sqrt(x^2 + 2*x + 4) + 8/405/sqrt(x^2 + 2*x + 4) + 4/135*x
/(x^2 + 2*x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^
2 + 2*x + 4)^(5/2) + 1/15/(x^2 + 2*x + 4)^(5/2)
```

Fricas [A] time = 0.213773, size = 213, normalized size = 3.67

$$\frac{40x^4+160x^3+375x^2-5\left(8x^3+24x^2+39x+23\right.}{15\left(16x^{10}+160x^9+900x^8+3360x^7+9165x^6+18702x^5+28920x^4+33240x^3+27360x^2-\left(16x^9+144x^8+732x^7+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 4)^(-7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(40*x^4 + 160*x^3 + 375*x^2 - 5*(8*x^3 + 24*x^2 + 39*x + 23)
*sqrt(x^2 + 2*x + 4) + 430*x + 247)/(16*x^10 + 160*x^9 + 900*x^8
+ 3360*x^7 + 9165*x^6 + 18702*x^5 + 28920*x^4 + 33240*x^3 + 27360
*x^2 - (16*x^9 + 144*x^8 + 732*x^7 + 2436*x^6 + 5841*x^5 + 10221*
x^4 + 13104*x^3 + 11772*x^2 + 6816*x + 1936)*sqrt(x^2 + 2*x + 4)
+ 14560*x + 3904)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{2}+2x+4\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+2*x+4)**(7/2),x)
```

```
[Out] Integral((x**2 + 2*x + 4)**(-7/2), x)
```

GIAC/XCAS [A] time = 0.202849, size = 45, normalized size = 0.78

$$\frac{(4\,((2\,(x+5)x+35)x+65)x+355)x+203}{405\,(x^{2}+2\,x+4)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 4)^(-7/2),x, algorithm="giac")
```

```
[Out] 1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*
x + 4)^(5/2)
```

3.285 $\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\,(3x^2+8x+1)^{3/2}}$$

```
[Out] -(4 + 3*x)/(39*(1 + 8*x + 3*x^2)^(3/2)) + (2*(4 + 3*x))/(169*Sqrt
[1 + 8*x + 3*x^2])
```

Rubi [A] time = 0.0196662, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\,(3x^2+8x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] -(4 + 3*x)/(39*(1 + 8*x + 3*x^2)^(3/2)) + (2*(4 + 3*x))/(169*Sqrt
[1 + 8*x + 3*x^2])
```

Rubi in Sympy [A] time = 0.840868, size = 39, normalized size = 0.83

$$-\frac{6x+8}{78\,(3x^2+8x+1)^{\frac{3}{2}}}+\frac{12x+16}{338\sqrt{3x^2+8x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+8*x+1)**(5/2),x)
```

```
[Out] -(6*x + 8)/(78*(3*x**2 + 8*x + 1)**(3/2)) + (12*x + 16)/(338*sqrt
(3*x**2 + 8*x + 1))
```

Mathematica [A] time = 0.0198991, size = 33, normalized size = 0.7

$$\frac{(3x+4)\left(18x^2+48x-7\right)}{507\,(3x^2+8x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] ((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))
```

Maple [A] time = 0.006, size = 30, normalized size = 0.6

$$\frac{54\,x^{3}+216\,x^{2}+171\,x-28}{507}\left(3\,x^{2}+8\,x+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+8*x+1)^(5/2),x)
```

```
[Out] 1/507*(54*x^3+216*x^2+171*x-28)/(3*x^2+8*x+1)^(3/2)
```

Maxima [A] time = 1.53215, size = 80, normalized size = 1.7

$$\frac{6\,x}{169\,\sqrt{3\,x^{2}+8\,x+1}}+\frac{8}{169\,\sqrt{3\,x^{2}+8\,x+1}}-\frac{x}{13\,(3\,x^{2}+8\,x+1)^{\frac{3}{2}}}-\frac{4}{39\,(3\,x^{2}+8\,x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 8*x + 1)^(-5/2),x, algorithm="maxima")
```

```
[Out] 6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/1
3*x/(3*x^2 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)
```

Fricas [A] time = 0.211984, size = 189, normalized size = 4.02

$$-\frac{108\,x^{6}+630\,x^{5}+1164\,x^{4}+739\,x^{3}+168\,x^{2}-\left(66\,x^{5}+284\,x^{4}+337\,x^{3}+120\,x^{2}+12\,x\right)\sqrt{3\,x^{2}+8\,x+1}+12\,x}{3\left(459\,x^{6}+2736\,x^{5}+5142\,x^{4}+3248\,x^{3}+843\,x^{2}-(300\,x^{5}+1259\,x^{4}+1468\,x^{3}+549\,x^{2}+80\,x+4)\sqrt{3\,x^{2}+8\,x+1}+96\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 8*x + 1)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(108*x^6 + 630*x^5 + 1164*x^4 + 739*x^3 + 168*x^2 - (66*x^5
+ 284*x^4 + 337*x^3 + 120*x^2 + 12*x)*sqrt(3*x^2 + 8*x + 1) + 12*
x)/(459*x^6 + 2736*x^5 + 5142*x^4 + 3248*x^3 + 843*x^2 - (300*x^5
 + 1259*x^4 + 1468*x^3 + 549*x^2 + 80*x + 4)*sqrt(3*x^2 + 8*x + 1
```

```
) + 96*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+8x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+8*x+1)**(5/2),x)
```

```
[Out] Integral((3*x**2 + 8*x + 1)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.208953, size = 36, normalized size = 0.77

$$\frac{9\,(6\,(x+4)x+19)x-28}{507\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 8*x + 1)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)
```

3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\left(-3x^2+4x+5\right)^{3/2}}$$

```
[Out] -(2 - 3*x)/(57*(5 + 4*x - 3*x^2)^(3/2)) - (2*(2 - 3*x))/(361*Sqrt
[5 + 4*x - 3*x^2])
```

Rubi [A] time = 0.0199695, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\left(-3x^2+4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -(2 - 3*x)/(57*(5 + 4*x - 3*x^2)^(3/2)) - (2*(2 - 3*x))/(361*Sqrt
[5 + 4*x - 3*x^2])
```

Rubi in Sympy [A] time = 0.847117, size = 41, normalized size = 0.87

$$-\frac{-12x+8}{722\sqrt{-3x^2+4x+5}}-\frac{-6x+4}{114\left(-3x^2+4x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

```
[Out] -(-12*x + 8)/(722*sqrt(-3*x**2 + 4*x + 5)) - (-6*x + 4)/(114*(-3*
x**2 + 4*x + 5)**(3/2))
```

Mathematica [A] time = 0.0276974, size = 33, normalized size = 0.7

$$-\frac{(3x-2)\left(18x^2-24x-49\right)}{1083\left(-3x^2+4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -((-2 + 3*x)*(-49 - 24*x + 18*x^2))/(1083*(5 + 4*x - 3*x^2)^(3/2)
)
```

Maple [A] time = 0.006, size = 30, normalized size = 0.6

$$-\frac{54\,x^3-108\,x^2-99\,x+98}{1083}\left(-3\,x^2+4\,x+5\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+4*x+5)^(5/2),x)
```

```
[Out] -1/1083*(54*x^3-108*x^2-99*x+98)/(-3*x^2+4*x+5)^(3/2)
```

Maxima [A] time = 1.43435, size = 80, normalized size = 1.7

$$\frac{6\,x}{361\,\sqrt{-3\,x^2+4\,x+5}}-\frac{4}{361\,\sqrt{-3\,x^2+4\,x+5}}+\frac{x}{19\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}}-\frac{2}{57\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x^2 + 4*x + 5)^(-5/2),x, algorithm="maxima")
```

```
[Out] 6/361*x/sqrt(-3*x^2 + 4*x + 5) - 4/361/sqrt(-3*x^2 + 4*x + 5) + 1
/19*x/(-3*x^2 + 4*x + 5)^(3/2) - 2/57/(-3*x^2 + 4*x + 5)^(3/2)
```

Fricas [A] time = 0.211885, size = 69, normalized size = 1.47

$$-\frac{\left(54\,x^3-108\,x^2-99\,x+98\right)\sqrt{-3\,x^2+4\,x+5}}{1083\,(9\,x^4-24\,x^3-14\,x^2+40\,x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x^2 + 4*x + 5)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/1083*(54*x^3 - 108*x^2 - 99*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(9*
x^4 - 24*x^3 - 14*x^2 + 40*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 4x + 5)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

```
[Out] Integral((-3*x**2 + 4*x + 5)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.215196, size = 53, normalized size = 1.13

$$-\frac{(9\,(6\,(x-2)x-11)x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\,(3\,x^2-4\,x-5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x^2 + 4*x + 5)^(-5/2),x, algorithm="giac")
```

```
[Out] -1/1083*(9*(6*(x - 2)*x - 11)*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(3*x
^2 - 4*x - 5)^2
```

3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Rubi [A] time = 0.0667507, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Rubi in Sympy [A] time = 4.72814, size = 32, normalized size = 1.1

$$\log\left(x+\sqrt{x^2+2x+2}+1\right)+\frac{2}{x+\sqrt{x^2+2x+2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
```

```
[Out] log(x + sqrt(x**2 + 2*x + 2) + 1) + 2/(x + sqrt(x**2 + 2*x + 2) +
 2)
```

Mathematica [A] time = 0.0308931, size = 29, normalized size = 1.

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Maple [A] time = 0.007, size = 40, normalized size = 1.4

$$-\frac{1}{1+x}\left((1+x)^2+1\right)^{\frac{3}{2}}+(1+x)\sqrt{(1+x)^2+1}+\mathit{Arcsinh}\,(1+x)+(1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+(x^2+2*x+2)^(1/2)),x)
```

```
[Out] -1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1
/(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2\,x+2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x + 2) + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)
```

Fricas [A] time = 0.200665, size = 111, normalized size = 3.83

$$-\frac{\left(x^2-\sqrt{x^2+2\,x+2}(x+1)+2\,x+1\right)\log\left(-x+\sqrt{x^2+2\,x+2}-1\right)-x+\sqrt{x^2+2\,x+2}-2}{x^2-\sqrt{x^2+2\,x+2}(x+1)+2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x + 2) + 1),x, algorithm="fricas")
```

```
[Out] -((x^2 - sqrt(x^2 + 2*x + 2)*(x + 1) + 2*x + 1)*log(-x + sqrt(x^2
 + 2*x + 2) - 1) - x + sqrt(x^2 + 2*x + 2) - 2)/(x^2 - sqrt(x^2 +
 2*x + 2)*(x + 1) + 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 + 2x + 2} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)
```

GIAC/XCAS [A] time = 0.202553, size = 81, normalized size = 2.79

$$\frac{2}{\left(x - \sqrt{x^2 + 2\,x + 2}\right)^2 + 2\,x - 2\,\sqrt{x^2 + 2\,x + 2}} + \frac{1}{x + 1} - \ln\left(-x + \sqrt{x^2 + 2\,x + 2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x + 2) + 1),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1
/(x + 1) - ln(-x + sqrt(x^2 + 2*x + 2) - 1)
```

3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=45

$$\sqrt{x^2+x+1}+2\log\left(\sqrt{x^2+x+1}+x\right)-x-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -x + Sqrt[1 + x + x^2] - (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 2*Log
[x + Sqrt[1 + x + x^2]]
```

Rubi [A] time = 0.0670844, antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{3}{2\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+2\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{2}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] 3/(2*(1 + 2*(x + Sqrt[1 + x + x^2]))) + 2*Log[x + Sqrt[1 + x + x^
2]] - (3*Log[1 + 2*(x + Sqrt[1 + x + x^2])])/2
```

Rubi in Sympy [A] time = 3.4279, size = 54, normalized size = 1.2

$$2\log\left(x+\sqrt{x^2+x+1}\right)-\frac{3\log\left(2x+2\sqrt{x^2+x+1}+1\right)}{2}+\frac{3}{2\left(2x+2\sqrt{x^2+x+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(x**2+x+1)**(1/2)),x)
```

```
[Out] 2*log(x + sqrt(x**2 + x + 1)) - 3*log(2*x + 2*sqrt(x**2 + x + 1)
+ 1)/2 + 3/(2*(2*x + 2*sqrt(x**2 + x + 1) + 1))
```

Mathematica [A] time = 0.0364221, size = 56, normalized size = 1.24

$$\sqrt{x^2+x+1}-\log\left(2\sqrt{x^2+x+1}-x+1\right)-x+2\log(x+1)-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] -x + Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2 + 2*Log[1 +
 x] - Log[1 - x + 2*Sqrt[1 + x + x^2]]
```

Maple [A] time = 0.007, size = 52, normalized size = 1.2

$$\sqrt{(1+x)^2-x}-\frac{1}{2}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)-Artanh\left(\frac{1-x}{2}\frac{1}{\sqrt{(1+x)^2-x}}\right)-x+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(x^2+x+1)^(1/2)),x)
```

```
[Out] ((1+x)^2-x)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(1/2*(1
-x)/((1+x)^2-x)^(1/2))-x+ln(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x^2 + x + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 + x + 1)), x)
```

Fricas [A] time = 0.214635, size = 209, normalized size = 4.64

$$-\frac{16\,x^2-4\,(2\,x+1)\log(x+1)+4\left(2\,x-2\,\sqrt{x^2+x+1}+1\right)\log\left(-x+\sqrt{x^2+x+1}\right)-4\left(2\,x-2\,\sqrt{x^2+x+1}+1\right)\log\left(-x\right.}{4\left(2\,x-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x^2 + x + 1)),x, algorithm="fricas")
```

```
[Out] -1/4*(16*x^2 - 4*(2*x + 1)*log(x + 1) + 4*(2*x - 2*sqrt(x^2 + x +
 1) + 1)*log(-x + sqrt(x^2 + x + 1)) - 4*(2*x - 2*sqrt(x^2 + x +
1) + 1)*log(-x + sqrt(x^2 + x + 1) - 2) - 2*(2*x - 2*sqrt(x^2 + x
 + 1) + 1)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 2*sqrt(x^2 + x +
 1)*(8*x - 4*log(x + 1) + 1) + 10*x + 7)/(2*x - 2*sqrt(x^2 + x +
1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + \sqrt{x^2 + x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 + x + 1)), x)
```

GIAC/XCAS [A] time = 0.203884, size = 89, normalized size = 1.98

$$-x + \sqrt{x^2 + x + 1} + \frac{1}{2}\ln\left(-2\,x + 2\,\sqrt{x^2 + x + 1} - 1\right) + \ln\left(|x + 1|\right) - \ln\left(\left|-x + \sqrt{x^2 + x + 1}\right|\right) + \ln\left(\left|-x + \sqrt{x^2 + x + 1} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x^2 + x + 1)),x, algorithm="giac")
```

```
[Out] -x + sqrt(x^2 + x + 1) + 1/2*ln(-2*x + 2*sqrt(x^2 + x + 1) - 1) +
 ln(abs(x + 1)) - ln(abs(-x + sqrt(x^2 + x + 1))) + ln(abs(-x + s
qrt(x^2 + x + 1) - 2))
```

3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}+\frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -x^3/9 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x
^2)^(3/2))/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqr
t[3]]/64
```

Rubi [A] time = 0.231676, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}+\frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -x^3/9 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x
^2)^(3/2))/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqr
t[3]]/64
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(2x+2\sqrt{x^2+x+1}+1\right)}{64}+2\int^{x+\sqrt{x^2+x+1}}\left(-\frac{3}{64}\right)dx+\frac{\int^{x+\sqrt{x^2+x+1}}x\,dx}{16}-\frac{3}{128\left(2x+2\sqrt{x^2+x+1}+1\right)^2}-\frac{3}{16\left(2x+2\sqrt{x^2+x+1}+1\right)^3}-\frac{27}{256\left(2x+2\sqrt{x^2+x+1}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

```
[Out] log(2*x + 2*sqrt(x**2 + x + 1) + 1)/64 + 2*Integral(-3/64, (x, x
+ sqrt(x**2 + x + 1))) + Integral(x, (x, x + sqrt(x**2 + x + 1)))
/16 - 3/(128*(2*x + 2*sqrt(x**2 + x + 1) + 1)**2) - 3/(16*(2*x +
2*sqrt(x**2 + x + 1) + 1)**3) - 27/(256*(2*x + 2*sqrt(x**2 + x +
1) + 1)**4)
```

Mathematica [A] time = 0.0616879, size = 56, normalized size = 0.71

$$\frac{1}{576}\left(-96x^4-64x^3+2\sqrt{x^2+x+1}\left(48x^3+8x^2+14x-37\right)+9\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] (-64*x^3 - 96*x^4 + 2*Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*
x^3) + 9*ArcSinh[(1 + 2*x)/Sqrt[3]])/576
```

Maple [A] time = 0.006, size = 59, normalized size = 0.8

$$-\frac{x^3}{9}-\frac{x^4}{6}+\frac{x}{6}\left(x^2+x+1\right)^{\frac{3}{2}}-\frac{5}{36}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1+2x}{96}\sqrt{x^2+x+1}+\frac{1}{64}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x)
```

```
[Out] -1/9*x^3-1/6*x^4+1/6*x*(x^2+x+1)^(3/2)-5/36*(x^2+x+1)^(3/2)+1/96*
(1+2*x)*(x^2+x+1)^(1/2)+1/64*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)
```

Fricas [A] time = 0.223613, size = 266, normalized size = 3.37

$$-\frac{196608\,x^8+524288\,x^7+778240\,x^6+614400\,x^5+234368\,x^4-81152\,x^3-137952\,x^2+72\left(128\,x^4+256\,x^3+288\,x^2-8\left(1\right.\right.}{4608}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1),x, algorithm="fricas")
```

```
[Out] -1/4608*(196608*x^8 + 524288*x^7 + 778240*x^6 + 614400*x^5 + 2343
68*x^4 - 81152*x^3 - 137952*x^2 + 72*(128*x^4 + 256*x^3 + 288*x^2
 - 8*(16*x^3 + 24*x^2 + 18*x + 5)*sqrt(x^2 + x + 1) + 160*x + 41)
*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 8*(24576*x^7 + 53248*x^6 +
 61440*x^5 + 30720*x^4 + 304*x^3 - 12984*x^2 - 8298*x - 2369)*sqr
t(x^2 + x + 1) - 78688*x - 18227)/(128*x^4 + 256*x^3 + 288*x^2 -
8*(16*x^3 + 24*x^2 + 18*x + 5)*sqrt(x^2 + x + 1) + 160*x + 41)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{2x + 2\sqrt{x^2 + x + 1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)
```

GIAC/XCAS [A] time = 0.211508, size = 73, normalized size = 0.92

$$-\frac{1}{6}\,x^4 - \frac{1}{9}\,x^3 + \frac{1}{288}\,(2\,(4\,(6\,x + 1)x + 7)x - 37)\sqrt{x^2 + x + 1} - \frac{1}{64}\ln\left(-2\,x + 2\,\sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1),x, algorithm="giac")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^
2 + x + 1) - 1/64*ln(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=80

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*Ar
cTanh[(1 - x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1
+ x + x^2])] + Log[x] - 4*Log[1 + x]
```

Rubi [A] time = 0.640126, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*Ar
cTanh[(1 - x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1
+ x + x^2])] + Log[x] - 4*Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-3x+\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral((-3*x + sqrt(x**2 + x + 1))/(sqrt(x**2 + x + 1) - 1), x)
```

Mathematica [A] time = 0.040754, size = 78, normalized size = 0.98

$$-3\sqrt{x^2+x+1}+4\log\left(2\sqrt{x^2+x+1}-x+1\right)-\log\left(2\sqrt{x^2+x+1}+x+2\right)+x+2\log(x)-8\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 2*Lo
g[x] - 8*Log[1 + x] + 4*Log[1 - x + 2*Sqrt[1 + x + x^2]] - Log[2
+ x + 2*Sqrt[1 + x + x^2]]
```

Maple [A] time = 0.016, size = 80, normalized size = 1.

$$x-4\ln(1+x)+\ln(x)+\sqrt{x^2+x+1}+\frac{5}{2}\mathit{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)-\mathit{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-4\sqrt{(1+x)^2-x}+4\,\mathit{Artanh}\left(1/2\,\frac{1-x}{\sqrt{(1+x)^2-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x)
```

```
[Out] x-4*ln(1+x)+ln(x)+(x^2+x+1)^(1/2)+5/2*arcsinh(2/3*3^(1/2)*(x+1/2)
)-arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-4*((1+x)^2-x)^(1/2)+4*arctan
h(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{4}x^2+\frac{1}{2}x+\int-\frac{3\,x^3+2\,x^2-x}{2\left(x^2+x-2\sqrt{x^2+x+1}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x - sqrt(x^2 + x + 1))/(sqrt(x^2 + x + 1) - 1),x, algorithm="maxima"
```

```
[Out] 3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2
*sqrt(x^2 + x + 1) + 2), x)
```

Fricas [A] time = 0.238473, size = 311, normalized size = 3.89

$$\frac{32\,x^2 - 16\,(2\,x+1)\log(x+1) + 4\,(2\,x+1)\log(x) - 4\left(2\,x - 2\,\sqrt{x^2+x+1}+1\right)\log\left(-x+\sqrt{x^2+x+1}+1\right) + 16\left(2\,x - 2\,\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x - sqrt(x^2 + x + 1))/(sqrt(x^2 + x + 1) - 1),x, algorithm="fricas"
```

```
[Out] 1/4*(32*x^2 - 16*(2*x + 1)*log(x + 1) + 4*(2*x + 1)*log(x) - 4*(2
*x - 2*sqrt(x^2 + x + 1) + 1)*log(-x + sqrt(x^2 + x + 1) + 1) + 1
6*(2*x - 2*sqrt(x^2 + x + 1) + 1)*log(-x + sqrt(x^2 + x + 1)) + 4
*(2*x - 2*sqrt(x^2 + x + 1) + 1)*log(-x + sqrt(x^2 + x + 1) - 1)
- 16*(2*x - 2*sqrt(x^2 + x + 1) + 1)*log(-x + sqrt(x^2 + x + 1) -
 2) - 10*(2*x - 2*sqrt(x^2 + x + 1) + 1)*log(-2*x + 2*sqrt(x^2 +
x + 1) - 1) - 2*sqrt(x^2 + x + 1)*(16*x - 16*log(x + 1) + 4*log(x
) + 3) + 22*x + 21)/(2*x - 2*sqrt(x^2 + x + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3x}{\sqrt{x^2+x+1}-1}\,dx - \int \left(-\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

```
[Out] -Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2
+ x + 1)/(sqrt(x**2 + x + 1) - 1), x)
```

GIAC/XCAS [A] time = 0.22439, size = 142, normalized size = 1.78

$$x - 3\,\sqrt{x^2+x+1} - \frac{5}{2}\ln\left(-2\,x + 2\,\sqrt{x^2+x+1} - 1\right) - 4\ln\left(|x+1|\right) + \ln\left(|x|\right) - \ln\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) + 4\ln\left(\left|-x+\sqrt{x^2+x+1}\right|\right) + \ln\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right) - 4\ln\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x - sqrt(x^2 + x + 1))/(sqrt(x^2 + x + 1) - 1),x, algorithm="giac")
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 5/2*ln(-2*x + 2*sqrt(x^2 + x + 1) - 1)
- 4*ln(abs(x + 1)) + ln(abs(x)) - ln(abs(-x + sqrt(x^2 + x + 1) +
 1)) + 4*ln(abs(-x + sqrt(x^2 + x + 1))) + ln(abs(-x + sqrt(x^2 +
 x + 1) - 1)) - 4*ln(abs(-x + sqrt(x^2 + x + 1) - 2))
```

3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=158

$$\frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}-2\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2\sqrt{7}\tanh^{-1}\left(\frac{1-2x}{\sqrt{7}\sqrt{x^2+2x+4}}\right)+\frac{11}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)+\frac{43}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4
 + 2*x + x^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1
+ x)/Sqrt[3]])/2 + (43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*
ArcTanh[(1 + 5*x)/(2*Sqrt[7]*Sqrt[1 + x + x^2])] + 2*Sqrt[7]*ArcT
anh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]
```

Rubi [A] time = 1.02453, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$

$$\frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}-2\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2\sqrt{7}\tanh^{-1}\left(\frac{1-2x}{\sqrt{7}\sqrt{x^2+2x+4}}\right)+\frac{11}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)+\frac{43}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4
 + 2*x + x^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1
+ x)/Sqrt[3]])/2 + (43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*
ArcTanh[(1 + 5*x)/(2*Sqrt[7]*Sqrt[1 + x + x^2])] + 2*Sqrt[7]*ArcT
anh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
```

```
[Out] Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

Mathematica [A] time = 0.171707, size = 137, normalized size = 0.87

$$\frac{1}{8}\left(4\sqrt{x^2+2x+4}(x-3)+2(2x-7)\sqrt{x^2+x+1}+16\sqrt{7}\log\left(-2\sqrt{7}\sqrt{x^2+x+1}+5x+1\right)+16\sqrt{7}\log\left(\sqrt{7}\sqrt{x^2+2x+4}-2x+1\right)-32\sqrt{7}\log(x+3)+44\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)+43\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (2*(-7 + 2*x)*Sqrt[1 + x + x^2] + 4*(-3 + x)*Sqrt[4 + 2*x + x^2]
+ 44*ArcSinh[(1 + x)/Sqrt[3]] + 43*ArcSinh[(1 + 2*x)/Sqrt[3]] - 3
2*Sqrt[7]*Log[3 + x] + 16*Sqrt[7]*Log[1 + 5*x - 2*Sqrt[7]*Sqrt[1
+ x + x^2]] + 16*Sqrt[7]*Log[1 - 2*x + Sqrt[7]*Sqrt[4 + 2*x + x^2
]])/8
```

Maple [A] time = 0.016, size = 140, normalized size = 0.9

$$-2\sqrt{(3+x)^2-5x-8}+\frac{43}{8}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)+2\sqrt{7}Artanh\left(1/14\frac{(-1-5x)\sqrt{7}}{\sqrt{(3+x)^2-5x-8}}\right)-2\sqrt{(3+x)^2-4x-5}+\frac{11}{2}Arcsinh\left(\frac{(1+x)\sqrt{3}}{3}\right)+2\sqrt{7}Artanh\left(1/14\frac{(2-4x)\sqrt{7}}{\sqrt{(3+x)^2-4x-5}}\right)+\frac{1+2x}{4}\sqrt{x^2+x+1}+\frac{2x+2}{4}\sqrt{x^2+2x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x)
```

```
[Out] -2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*3^(1/2)*(x+1/2))+2*7^(1
/2)*arctanh(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)
^2-4*x-5)^(1/2)+11/2*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh
(1/14*(2-4*x)*7^(1/2)/((3+x)^2-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1
)^(1/2)+1/4*(2*x+2)*(x^2+2*x+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\sqrt{x^2+x+1}-\frac{1}{4}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\int\frac{2x^3-2\left(x^2+x+1\right)(x+1)+5x^2+8x+5}{2\left((2x^2+3x+5)\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}(x^2+x+1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x^2 + x + 1) - 1/4*arcsinh(1/3*sqrt(3)*(2*x + 1)) + int
egrate(1/2*(2*x^3 - 2*(x^2 + x + 1)*(x + 1) + 5*x^2 + 8*x + 5)/((
2*x^2 + 3*x + 5)*sqrt(x^2 + x + 1) - 2*sqrt(x^2 + 2*x + 4)*(x^2 +
 x + 1)), x)
```

Fricas [A] time = 0.242253, size = 1071, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)),x, algorithm="fricas")
```

```
[Out] -1/32*(512*x^6 + 256*x^5 - 1264*x^4 - 6704*x^3 - 8890*x^2 + 176*(
16*x^4 + 48*x^3 + 82*x^2 - 2*(8*x^3 + 16*x^2 - 4*(2*x^2 + 3*x + 1
)*sqrt(x^2 + x + 1) + 13*x + 5)*sqrt(x^2 + 2*x + 4) - 4*(4*x^3 +
10*x^2 + 14*x + 5)*sqrt(x^2 + x + 1) + 60*x + 25)*log(-x + sqrt(x
^2 + 2*x + 4) - 1) + 172*(16*x^4 + 48*x^3 + 82*x^2 - 2*(8*x^3 + 1
6*x^2 - 4*(2*x^2 + 3*x + 1)*sqrt(x^2 + x + 1) + 13*x + 5)*sqrt(x^
2 + 2*x + 4) - 4*(4*x^3 + 10*x^2 + 14*x + 5)*sqrt(x^2 + x + 1) +
60*x + 25)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + 64*(4*sqrt(7)*(4
*x^3 + 10*x^2 + 14*x + 5)*sqrt(x^2 + x + 1) - sqrt(7)*(16*x^4 + 4
8*x^3 + 82*x^2 + 60*x + 25) - 2*(4*sqrt(7)*(2*x^2 + 3*x + 1)*sqrt
(x^2 + x + 1) - sqrt(7)*(8*x^3 + 16*x^2 + 13*x + 5))*sqrt(x^2 + 2
*x + 4))*log((2*x^2 - 2*sqrt(x^2 + x + 1)*(x - sqrt(7) + 3) - 2*s
qrt(7)*(x + 3) + 7*x + 17)/(2*x^2 - 2*sqrt(x^2 + x + 1)*(x + 3) +
 7*x + 3)) + 64*(4*sqrt(7)*(4*x^3 + 10*x^2 + 14*x + 5)*sqrt(x^2 +
 x + 1) - sqrt(7)*(16*x^4 + 48*x^3 + 82*x^2 + 60*x + 25) - 2*(4*s
qrt(7)*(2*x^2 + 3*x + 1)*sqrt(x^2 + x + 1) - sqrt(7)*(8*x^3 + 16*
x^2 + 13*x + 5))*sqrt(x^2 + 2*x + 4))*log((x^2 - sqrt(x^2 + 2*x +
 4)*(x - sqrt(7) + 3) - sqrt(7)*(x + 3) + 4*x + 10)/(x^2 - sqrt(x
^2 + 2*x + 4)*(x + 3) + 4*x + 3)) - 2*(256*x^5 - 128*x^4 - 888*x^
3 - 1888*x^2 - 4*(64*x^4 - 64*x^3 - 214*x^2 - 329*x - 147)*sqrt(x
^2 + x + 1) - 1513*x - 609)*sqrt(x^2 + 2*x + 4) - 4*(128*x^5 - 36
4*x^3 - 1470*x^2 - 1354*x - 519)*sqrt(x^2 + x + 1) - 6668*x - 196
5)/(16*x^4 + 48*x^3 + 82*x^2 - 2*(8*x^3 + 16*x^2 - 4*(2*x^2 + 3*x
 + 1)*sqrt(x^2 + x + 1) + 13*x + 5)*sqrt(x^2 + 2*x + 4) - 4*(4*x^
```

```
3 + 10*x^2 + 14*x + 5)*sqrt(x^2 + x + 1) + 60*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
```

```
[Out] Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

3.292 $\int \frac{1}{\sqrt{-1+x}x^3} dx$

Optimal. Leaf size=41

$$\frac{\sqrt{x-1}}{2x^2}+\frac{3\sqrt{x-1}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1
 + x]])/4
```

Rubi [A] time = 0.0266168, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sqrt{x-1}}{2x^2}+\frac{3\sqrt{x-1}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x]*x^3),x]
```

```
[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1
 + x]])/4
```

Rubi in Sympy [A] time = 1.73682, size = 34, normalized size = 0.83

$$\frac{3\operatorname{atan}\left(\sqrt{x-1}\right)}{4}+\frac{3\sqrt{x-1}}{4x}+\frac{\sqrt{x-1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(-1+x)**(1/2),x)
```

```
[Out] 3*atan(sqrt(x - 1))/4 + 3*sqrt(x - 1)/(4*x) + sqrt(x - 1)/(2*x**2
)
```

Mathematica [A] time = 0.0225726, size = 31, normalized size = 0.76

$$\frac{1}{4}\left(\frac{\sqrt{x-1}(3x+2)}{x^2}+3\tan^{-1}\left(\sqrt{x-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x]*x^3),x]

[Out] ((Sqrt[-1 + x]*(2 + 3*x))/x^2 + 3*ArcTan[Sqrt[-1 + x]])/4
```

Maple [A] time = 0.01, size = 30, normalized size = 0.7

$$\frac{3}{4}\arctan\left(\sqrt{-1+x}\right)+\frac{1}{2x^{2}}\sqrt{-1+x}+\frac{3}{4x}\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(-1+x)^(1/2),x)

[Out] 3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

Maxima [A] time = 1.63153, size = 51, normalized size = 1.24

$$\frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\left((x-1)^{2}+2\,x-1\right)}+\frac{3}{4}\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - 1)*x^3),x, algorithm="maxima")

[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/((x - 1)^2 + 2*x - 1) + 3/4
*arctan(sqrt(x - 1))
```

Fricas [A] time = 0.209398, size = 38, normalized size = 0.93

$$\frac{3\,x^{2}\arctan\left(\sqrt{x-1}\right)+(3\,x+2)\sqrt{x-1}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - 1)*x^3),x, algorithm="fricas")

[Out] 1/4*(3*x^2*arctan(sqrt(x - 1)) + (3*x + 2)*sqrt(x - 1))/x^2
```

Sympy [A] time = 4.40891, size = 131, normalized size = 3.2

$$\begin{cases} \frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \left|\frac{1}{x}\right| > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(-1+x)**(1/2),x)
```

```
[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)
) + I/(4*x**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x))
, Abs(1/x) > 1), (-3*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/
x)) - 1/(4*x**(3/2)*sqrt(1 - 1/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x))
, True))
```

GIAC/XCAS [A] time = 0.200848, size = 39, normalized size = 0.95

$$\frac{3\,(x-1)^{\frac{3}{2}} + 5\,\sqrt{x-1}}{4\,x^2} + \frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1
))
```

3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

```
[Out] -(1 - 3/x)^(-1/3)
```

Rubi [A] time = 0.0140873, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - 3/x)^(4/3)*x^2),x]

[Out] -(1 - 3/x)^(-1/3)
```

Rubi in Sympy [A] time = 0.949123, size = 10, normalized size = 0.77

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-3/x)**(4/3)/x**2,x)

[Out] -1/(1 - 3/x)**(1/3)
```

Mathematica [A] time = 0.0130915, size = 13, normalized size = 1.

$$-\frac{1}{\sqrt[3]{\frac{x-3}{x}}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[1/((1 - 3/x)^(4/3)*x^2),x]
```

```
[Out] -((-3 + x)/x)^(-1/3)
```

Maple [A] time = 0.004, size = 18, normalized size = 1.4

$$-\frac{-3+x}{x}\left(\frac{-3+x}{x}\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(1-3/x)^(4/3)/x^2,x)
```

```
[Out] -(-3+x)/x/((-3+x)/x)^(4/3)
```

Maxima [A] time = 1.58058, size = 15, normalized size = 1.15

$$-\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(x^2*(-3/x + 1)^(4/3)),x, algorithm="maxima")
```

```
[Out] -1/(-3/x + 1)^(1/3)
```

Fricas [A] time = 0.215903, size = 15, normalized size = 1.15

$$-\frac{1}{\left(\frac{x-3}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(x^2*(-3/x + 1)^(4/3)),x, algorithm="fricas")
```

```
[Out] -1/((x - 3)/x)^(1/3)
```

Sympy [A] time = 1.62116, size = 10, normalized size = 0.77

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-3/x)**(4/3)/x**2,x)
```

```
[Out] -1/(1 - 3/x)**(1/3)
```

GIAC/XCAS [A] time = 0.207521, size = 15, normalized size = 1.15

$$-\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(-3/x + 1)^(4/3)),x, algorithm="giac")
```

```
[Out] -1/(-3/x + 1)^(1/3)
```

3.294 $\int \frac{(-1+3x)^{4/3}}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{(3x-1)^{4/3}}{x}+12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

```
[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 -
2*(-1 + 3*x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1
/3)]
```

Rubi [A] time = 0.0657274, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$-\frac{(3x-1)^{4/3}}{x}+12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 3*x)^(4/3)/x^2,x]
```

```
[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 -
2*(-1 + 3*x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1
/3)]
```

Rubi in Sympy [A] time = 2.71189, size = 66, normalized size = 0.93

$$12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)-4\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{3x-1}}{3}-\frac{1}{3}\right)\right)-\frac{(3x-1)^{\frac{4}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+3*x)**(4/3)/x**2,x)
```

```
[Out] 12*(3*x - 1)**(1/3) + 2*log(x) - 6*log((3*x - 1)**(1/3) + 1) - 4*
sqrt(3)*atan(sqrt(3)*(2*(3*x - 1)**(1/3)/3 - 1/3)) - (3*x - 1)**(
4/3)/x
```

Mathematica [C] time = 0.0363561, size = 59, normalized size = 0.83

$$\frac{2\sqrt[3]{3}\left(3-\frac{1}{x}\right)^{2/3}x\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{3x}\right)+27x^2-6x-1}{x(3x-1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 3*x)^(4/3)/x^2,x]
```

```
[Out] (-1 - 6*x + 27*x^2 + 2*3^(1/3)*(3 - x^(-1))^(2/3)*x*Hypergeometri
c2F1[2/3, 2/3, 5/3, 1/(3*x)])/(x*(-1 + 3*x)^(2/3))
```

Maple [A] time = 0.021, size = 109, normalized size = 1.5

$$9\sqrt[3]{3x-1}-\left(1+\sqrt[3]{3x-1}\right)^{-1}-4\ln\left(1+\sqrt[3]{3x-1}\right)+1\left(1+\sqrt[3]{3x-1}\right)\left((3x-1)^{\frac{2}{3}}-\sqrt[3]{3x-1}+1\right)^{-1}$$
$$+2\ln\left((3x-1)^{2/3}-\sqrt[3]{3x-1}+1\right)-4\sqrt{3}\arctan\left(1/3\left(2\sqrt[3]{3x-1}-1\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x-1)^(4/3)/x^2,x)
```

```
[Out] 9*(3*x-1)^(1/3)-1/(1+(3*x-1)^(1/3))-4*ln(1+(3*x-1)^(1/3))+(1+(3*x
-1)^(1/3))/((3*x-1)^(2/3)-(3*x-1)^(1/3)+1)+2*ln((3*x-1)^(2/3)-(3*
x-1)^(1/3)+1)-4*3^(1/2)*arctan(1/3*(2*(3*x-1)^(1/3)-1)*3^(1/2))
```

Maxima [A] time = 1.76369, size = 103, normalized size = 1.45

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right)+9(3x-1)^{\frac{1}{3}}+\frac{(3x-1)^{\frac{1}{3}}}{x}$$
$$+2\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\log\left((3x-1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 1)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x -
 1)^(1/3) + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)
^(1/3) + 1) - 4*log((3*x - 1)^(1/3) + 1)
```

Fricas [A] time = 0.221992, size = 105, normalized size = 1.48

$$-\frac{4\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right)-2x\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)+4x\log\left((3x-1)^{\frac{1}{3}}+1\right)-(9x+1)(3x-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 1)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -(4*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) - 2*x*l
og((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/
3) + 1) - (9*x + 1)*(3*x - 1)^(1/3))/x
```

Sympy [A] time = 3.56537, size = 418, normalized size = 5.89

$$\frac{189\sqrt[3]{3}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}+\frac{84\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}+\frac{84\left(x-\frac{1}{3}\right)e^{\frac{5i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}-\frac{84\left(x-\frac{1}{3}\right)\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{i\pi}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}+\frac{84\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}+\frac{28e^{\frac{5i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}-\frac{28\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{i\pi}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}+\frac{28e^{\frac{i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)+3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+3*x)**(4/3)/x**2,x)
```

```
[Out] 189*3**(1/3)*(x - 1/3)**(4/3)*gamma(7/3)/(9*(x - 1/3)*gamma(10/3)
 + 3*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3)*gamma(7/3)/(9*(x
 - 1/3)*gamma(10/3) + 3*gamma(10/3)) + 84*(x - 1/3)*exp(5*I*pi/3)
*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(7/3)
/(9*(x - 1/3)*gamma(10/3) + 3*gamma(10/3)) - 84*(x - 1/3)*log(-3*
*(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1
/3)*gamma(10/3) + 3*gamma(10/3)) + 84*(x - 1/3)*exp(I*pi/3)*log(-
3**(1/3)*(x - 1/3)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*
(x - 1/3)*gamma(10/3) + 3*gamma(10/3)) + 28*exp(5*I*pi/3)*log(-3*
*(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x -
 1/3)*gamma(10/3) + 3*gamma(10/3)) - 28*log(-3**(1/3)*(x - 1/3)**
(1/3)*exp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*gamma(10/3) +
3*gamma(10/3)) + 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*ex
p_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*gamma(10/3) + 3*ga
mma(10/3))
```

GIAC/XCAS [A] time = 0.207637, size = 103, normalized size = 1.45

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right)+9(3x-1)^{\frac{1}{3}}+\frac{(3x-1)^{\frac{1}{3}}}{x}+2\ln\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\ln\left((3x-1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 1)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x -
 1)^(1/3) + (3*x - 1)^(1/3)/x + 2*ln((3*x - 1)^(2/3) - (3*x - 1)^
(1/3) + 1) - 4*ln((3*x - 1)^(1/3) + 1)
```

3.295 $\int (4-3x)^{4/3}x^2\,dx$

Optimal. Leaf size=40

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

```
[Out] (-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(1
3/3)/117
```

Rubi [A] time = 0.0234212, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*x)^(4/3)*x^2,x]
```

```
[Out] (-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(1
3/3)/117
```

Rubi in Sympy [A] time = 2.12378, size = 32, normalized size = 0.8

$$-\frac{(-3x+4)^{\frac{13}{3}}}{117}+\frac{4(-3x+4)^{\frac{10}{3}}}{45}-\frac{16(-3x+4)^{\frac{7}{3}}}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4-3*x)**(4/3)*x**2,x)
```

```
[Out] -(-3*x + 4)**(13/3)/117 + 4*(-3*x + 4)**(10/3)/45 - 16*(-3*x + 4)
**(7/3)/63
```

Mathematica [A] time = 0.0177789, size = 23, normalized size = 0.57

$$-\frac{1}{455}(4-3x)^{7/3}\left(35x^2+28x+16\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 3*x)^(4/3)*x^2,x]

[Out] -((4 - 3*x)^(7/3)*(16 + 28*x + 35*x^2))/455
```

Maple [A] time = 0.004, size = 20, normalized size = 0.5

$$-\frac{35\,x^2+28\,x+16}{455}\,(4-3\,x)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4-3*x)^(4/3)*x^2,x)

[Out] -1/455*(35*x^2+28*x+16)*(4-3*x)^(7/3)
```

Maxima [A] time = 1.53388, size = 38, normalized size = 0.95

$$-\frac{1}{117}\,(-3\,x+4)^{\frac{13}{3}}+\frac{4}{45}\,(-3\,x+4)^{\frac{10}{3}}-\frac{16}{63}\,(-3\,x+4)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(-3*x + 4)^(4/3),x, algorithm="maxima")

[Out] -1/117*(-3*x + 4)^(13/3) + 4/45*(-3*x + 4)^(10/3) - 16/63*(-3*x +
 4)^(7/3)
```

Fricas [A] time = 0.201517, size = 39, normalized size = 0.98

$$-\frac{1}{455}\,\left(315\,x^4-588\,x^3+32\,x^2+64\,x+256\right)(-3\,x+4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(-3*x + 4)^(4/3),x, algorithm="fricas")

[Out] -1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^(1/3)
```

Sympy [A] time = 3.27038, size = 189, normalized size = 4.72

$$\begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{13i\pi}{3}}}{13} + \frac{84x^3\sqrt[3]{3x-4}e^{\frac{13i\pi}{3}}}{65} - \frac{32x^2\sqrt[3]{3x-4}e^{\frac{13i\pi}{3}}}{455} - \frac{64x\sqrt[3]{3x-4}e^{\frac{13i\pi}{3}}}{455} - \frac{256\sqrt[3]{3x-4}e^{\frac{13i\pi}{3}}}{455} & \text{for } \frac{3|x|}{4} > 1 \\ -\frac{9x^4\sqrt[3]{-3x+4}}{13} + \frac{84x^3\sqrt[3]{-3x+4}}{65} - \frac{32x^2\sqrt[3]{-3x+4}}{455} - \frac{64x\sqrt[3]{-3x+4}}{455} - \frac{256\sqrt[3]{-3x+4}}{455} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4-3*x)**(4/3)*x**2,x)
```

```
[Out] Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(13*I*pi/3)/13 + 84*x**3*(
3*x - 4)**(1/3)*exp(13*I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(
13*I*pi/3)/455 - 64*x*(3*x - 4)**(1/3)*exp(13*I*pi/3)/455 - 256*(
3*x - 4)**(1/3)*exp(13*I*pi/3)/455, 3*Abs(x)/4 > 1), (-9*x**4*(-3
*x + 4)**(1/3)/13 + 84*x**3*(-3*x + 4)**(1/3)/65 - 32*x**2*(-3*x
+ 4)**(1/3)/455 - 64*x*(-3*x + 4)**(1/3)/455 - 256*(-3*x + 4)**(1
/3)/455, True))
```

GIAC/XCAS [A] time = 0.199665, size = 66, normalized size = 1.65

$$-\frac{1}{117}(3x-4)^4(-3x+4)^{\frac{1}{3}} - \frac{4}{45}(3x-4)^3(-3x+4)^{\frac{1}{3}} - \frac{16}{63}(3x-4)^2(-3x+4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(-3*x + 4)^(4/3),x, algorithm="giac")
```

```
[Out] -1/117*(3*x - 4)^4*(-3*x + 4)^(1/3) - 4/45*(3*x - 4)^3*(-3*x + 4)
^(1/3) - 16/63*(3*x - 4)^2*(-3*x + 4)^(1/3)
```

3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

Optimal. Leaf size=48

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*Arc
Tanh[(1 - 2*x^(1/3))^(1/4)]
```

Rubi [A] time = 0.0483133, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*Arc
Tanh[(1 - 2*x^(1/3))^(1/4)]
```

Rubi in Sympy [A] time = 3.30034, size = 42, normalized size = 0.88

$$4\left(-2\sqrt[3]{x}+1\right)^{\frac{3}{4}}+6\,\mathrm{atan}\left(\sqrt[4]{-2\sqrt[3]{x}+1}\right)-6\,\mathrm{atanh}\left(\sqrt[4]{-2\sqrt[3]{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-2*x**(1/3))**(3/4)/x,x)
```

```
[Out] 4*(-2*x**(1/3) + 1)**(3/4) + 6*atan((-2*x**(1/3) + 1)**(1/4)) - 6
*atanh((-2*x**(1/3) + 1)**(1/4))
```

Mathematica [C] time = 0.0467015, size = 62, normalized size = 1.29

$$\frac{-6\,2^{3/4}\sqrt[4]{2-\frac{1}{\sqrt[3]{x}}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2\sqrt[3]{x}}\right)-8\sqrt[3]{x}+4}{\sqrt[4]{1-2\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] (4 - 8*x^(1/3) - 6*2^(3/4)*(2 - x^(-1/3))^(1/4)*Hypergeometric2F1
[1/4, 1/4, 5/4, 1/(2*x^(1/3))])/(1 - 2*x^(1/3))^(1/4)
```

Maple [A] time = 0.013, size = 53, normalized size = 1.1

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+3\ln\left(-1+\sqrt[4]{1-2\sqrt[3]{x}}\right)-3\ln\left(1+\sqrt[4]{1-2\sqrt[3]{x}}\right)+6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-2*x^(1/3))^(3/4)/x,x)
```

```
[Out] 4*(1-2*x^(1/3))^(3/4)+3*ln(-1+(1-2*x^(1/3))^(1/4))-3*ln(1+(1-2*x^
(1/3))^(1/4))+6*arctan((1-2*x^(1/3))^(1/4))
```

Maxima [A] time = 1.58913, size = 70, normalized size = 1.46

$$4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^(1/3) + 1)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*l
og((-2*x^(1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1
)
```

Fricas [A] time = 0.22315, size = 70, normalized size = 1.46

$$4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^(1/3) + 1)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*l
og((-2*x^(1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1
)
```

Sympy [A] time = 6.69654, size = 51, normalized size = 1.06

$$-\frac{3\cdot 2^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-2*x**(1/3))**(3/4)/x,x)
```

```
[Out] -3*2**(3/4)*x**(1/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4)
, (1/4,), 1/(2*x**(1/3))))/gamma(1/4)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^(1/3) + 1)^(3/4)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.297 $\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\, dx$

Optimal. Leaf size=69

$$\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

```
[Out] (-27*(3 - 2*Sqrt[x])^(1/4))/2 + (27*(3 - 2*Sqrt[x])^(5/4))/10 - (
3 - 2*Sqrt[x])^(9/4)/2 + (3 - 2*Sqrt[x])^(13/4)/26
```

Rubi [A] time = 0.0455742, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-27*(3 - 2*Sqrt[x])^(1/4))/2 + (27*(3 - 2*Sqrt[x])^(5/4))/10 - (
3 - 2*Sqrt[x])^(9/4)/2 + (3 - 2*Sqrt[x])^(13/4)/26
```

Rubi in Sympy [A] time = 3.33405, size = 56, normalized size = 0.81

$$\frac{\left(-2\sqrt{x}+3\right)^{\frac{13}{4}}}{26}-\frac{\left(-2\sqrt{x}+3\right)^{\frac{9}{4}}}{2}+\frac{27\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}}{10}-\frac{27\sqrt[4]{-2\sqrt{x}+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(3-2*x**(1/2))**(3/4),x)
```

```
[Out] (-2*sqrt(x) + 3)**(13/4)/26 - (-2*sqrt(x) + 3)**(9/4)/2 + 27*(-2*
sqrt(x) + 3)**(5/4)/10 - 27*(-2*sqrt(x) + 3)**(1/4)/2
```

Mathematica [A] time = 0.0186742, size = 36, normalized size = 0.52

$$-\frac{4}{65}\sqrt[4]{3-2\sqrt{x}}\left(5x^{3/2}+10x+24\sqrt{x}+144\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/
65
```

Maple [A] time = 0.003, size = 46, normalized size = 0.7

$$-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{\frac{5}{4}}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{\frac{9}{4}}+\frac{1}{26}\left(3-2\sqrt{x}\right)^{\frac{13}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(3-2*x^(1/2))^(3/4),x)
```

```
[Out] -27/2*(3-2*x^(1/2))^(1/4)+27/10*(3-2*x^(1/2))^(5/4)-1/2*(3-2*x^(1
/2))^(9/4)+1/26*(3-2*x^(1/2))^(13/4)
```

Maxima [A] time = 1.40031, size = 61, normalized size = 0.88

$$\frac{1}{26}\left(-2\sqrt{x}+3\right)^{\frac{13}{4}}-\frac{1}{2}\left(-2\sqrt{x}+3\right)^{\frac{9}{4}}+\frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}-\frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-2*sqrt(x) + 3)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10
*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

Fricas [A] time = 0.216549, size = 34, normalized size = 0.49

$$-\frac{4}{65}\left((5x+24)\sqrt{x}+10x+144\right)\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-2*sqrt(x) + 3)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

Sympy [A] time = 3.49906, size = 3305, normalized size = 47.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3-2*x**(1/2))**(3/4),x)
```

```
[Out] Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(13*
I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)
*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 26304*3**(
1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4)*exp(13*I*pi/4)/(-37440*3**(
1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(
17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(
1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2*s
qrt(x) - 3)**(1/4)*exp(13*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 28
0800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/
4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3
**(1/4)*x**8) - 331776*sqrt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/
2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160
*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2*sqrt(x) - 3)
**(1/4)*exp(13*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/
4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 +
140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**
8) - 2488320*sqrt(3)*x**(19/2)/(-37440*3**(1/4)*x**(21/2) - 28080
0*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*
x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(
1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(2*sqrt(x) - 3)**(1/4)*ex
p(13*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/
2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**
(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 16796
16*sqrt(3)*x**(17/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)
*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14
0400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8)
 - 8960*3**(1/4)*x**12*(2*sqrt(x) - 3)**(1/4)*exp(13*I*pi/4)/(-37
440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315
900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 18432*3**(1/4)*x**11*(
2*sqrt(x) - 3)**(1/4)*exp(13*I*pi/4)/(-37440*3**(1/4)*x**(21/2) -
 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**
(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 4738
5*3**(1/4)*x**8) + 36864*sqrt(3)*x**11/(-37440*3**(1/4)*x**(21/2)
 - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3
**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47
385*3**(1/4)*x**8) + 965520*3**(1/4)*x**10*(2*sqrt(x) - 3)**(1/4)
*exp(13*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(
19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*
3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 12
44160*sqrt(3)*x**10/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*
x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140
400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8)
+ 2548584*3**(1/4)*x**9*(2*sqrt(x) - 3)**(1/4)*exp(13*I*pi/4)/(-3
```

```
7440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(
1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 2799360*sqrt(3)*x**9/
(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3
**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*3**(1/4)*x*
*8*(2*sqrt(x) - 3)**(1/4)*exp(13*I*pi/4)/(-37440*3**(1/4)*x**(21/
2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160
*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
47385*3**(1/4)*x**8) + 419904*sqrt(3)*x**8/(-37440*3**(1/4)*x**(2
1/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 41
60*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
+ 47385*3**(1/4)*x**8), 2*Abs(sqrt(x))/3 > 1), (-1280*3**(1/4)*x*
*(25/2)*(-2*sqrt(x) + 3)**(1/4)/(-37440*3**(1/4)*x**(21/2) - 2808
00*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)
*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**
(1/4)*x**8) - 26304*3**(1/4)*x**(23/2)*(-2*sqrt(x) + 3)**(1/4)/(-
37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**
(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 200016*3**(1/4)*x**(
21/2)*(-2*sqrt(x) + 3)**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800
*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x
**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1
/4)*x**8) - 331776*sqrt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) -
 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**
(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 4738
5*3**(1/4)*x**8) + 2123820*3**(1/4)*x**(19/2)*(-2*sqrt(x) + 3)**(
1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189
540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x*
*10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(
3)*x**(19/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/
2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**
(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 16096
32*3**(1/4)*x**(17/2)*(-2*sqrt(x) + 3)**(1/4)/(-37440*3**(1/4)*x*
*(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) +
 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37440*3**
(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**
(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**
(1/4)*x**9 + 47385*3**(1/4)*x**8) + 8960*3**(1/4)*x**12*(-2*sqrt(
x) + 3)**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(1
9/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3
**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 184
32*3**(1/4)*x**11*(-2*sqrt(x) + 3)**(1/4)/(-37440*3**(1/4)*x**(21
/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 416
0*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
 47385*3**(1/4)*x**8) + 36864*sqrt(3)*x**11/(-37440*3**(1/4)*x**(
21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4
160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
 + 47385*3**(1/4)*x**8) - 965520*3**(1/4)*x**10*(-2*sqrt(x) + 3)*
*(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*
x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 1244160*sqr
t(3)*x**10/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2)
 - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1
/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2548584
```

```
*3**(1/4)*x**9*(-2*sqrt(x) + 3)**(1/4)/(-37440*3**(1/4)*x**(21/2)
 - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3
**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47
385*3**(1/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21
/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 416
0*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
 47385*3**(1/4)*x**8) - 419904*3**(1/4)*x**8*(-2*sqrt(x) + 3)**(1
/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1895
40*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**
10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*sqrt(3)
*x**8/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 18
9540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x
**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8), True))
```

GIAC/XCAS [A] time = 0.216701, size = 85, normalized size = 1.23

$$-\frac{1}{26}\left(2\sqrt{x}-3\right)^{3}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}-\frac{1}{2}\left(2\sqrt{x}-3\right)^{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}+\frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}-\frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-2*sqrt(x) + 3)^(3/4),x, algorithm="giac")
```

```
[Out] -1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) -
 3)^2*(-2*sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/
2*(-2*sqrt(x) + 3)^(1/4)
```

3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}-\frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}+\frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{2\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{2\sqrt{2}}$$

```
[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[
x]) - (5*ArcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2])
+ (5*ArcTan[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5
*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/
(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + Sqrt[-1
 + 2*Sqrt[x]]])/(4*Sqrt[2])
```

Rubi [A] time = 0.216225, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}-\frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}+\frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{2\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```



```
[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[
x]) - (5*ArcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2])
+ (5*ArcTan[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5
*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/
(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + Sqrt[-1
 + 2*Sqrt[x]]])/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 10.0442, size = 172, normalized size = 0.89

$$-\frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+\sqrt{2\sqrt{x}-1}+1\right)}{8}+\frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+\sqrt{2\sqrt{x}-1}+1\right)}{8}+\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}-1\right)}{4}+\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4}-\frac{\left(2\sqrt{x}-1\right)^{\frac{5}{4}}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

```
[Out] -5*sqrt(2)*log(-sqrt(2)*(2*sqrt(x) - 1)**(1/4) + sqrt(2*sqrt(x) -
 1) + 1)/8 + 5*sqrt(2)*log(sqrt(2)*(2*sqrt(x) - 1)**(1/4) + sqrt(
2*sqrt(x) - 1) + 1)/8 + 5*sqrt(2)*atan(sqrt(2)*(2*sqrt(x) - 1)**(
1/4) - 1)/4 + 5*sqrt(2)*atan(sqrt(2)*(2*sqrt(x) - 1)**(1/4) + 1)/
4 - (2*sqrt(x) - 1)**(5/4)/x - 5*(2*sqrt(x) - 1)**(1/4)/(2*sqrt(x
))
```

Mathematica [C] time = 0.0451998, size = 72, normalized size = 0.37

$$\frac{-5\sqrt[4]{2}\left(2-\frac{1}{\sqrt{x}}\right)^{3/4}x\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2\sqrt{x}}\right)-54x+39\sqrt{x}-6}{6\left(2\sqrt{x}-1\right)^{3/4}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] (-6 + 39*Sqrt[x] - 54*x - 5*2^(1/4)*(2 - 1/Sqrt[x])^(3/4)*x*Hyper
geometric2F1[3/4, 3/4, 7/4, 1/(2*Sqrt[x])])/(6*(-1 + 2*Sqrt[x])^(
3/4)*x)
```

Maple [A] time = 0.017, size = 130, normalized size = 0.7

$$8\,\frac{1}{x}\left(-\frac{9\,\left(-1+2\,\sqrt{x}\right)^{5/4}}{32}-\frac{5\,\sqrt[4]{-1+2\,\sqrt{x}}}{32}\right)+\frac{5\,\sqrt{2}}{4}\arctan\left(\sqrt{2}\sqrt[4]{-1+2\,\sqrt{x}}-1\right)+\frac{5\,\sqrt{2}}{8}\ln\left(1\left(1+\sqrt{2}\sqrt[4]{-1+2\,\sqrt{x}}+\sqrt{-1+2\,\sqrt{x}}\right)\left(1-\sqrt{2}\sqrt[4]{-1+2\,\sqrt{x}}+\sqrt{-1+2\,\sqrt{x}}\right)^{-1}\right)+\frac{5\,\sqrt{2}}{4}\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\,\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+2*x^(1/2))^(5/4)/x^2,x)
```

```
[Out] 8*(-9/32*(-1+2*x^(1/2))^(5/4)-5/32*(-1+2*x^(1/2))^(1/4))/x+5/4*ar
ctan(2^(1/2)*(-1+2*x^(1/2))^(1/4)-1)*2^(1/2)+5/8*2^(1/2)*ln((1+2^
(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2))/(1-2^(1/2)*(-1+2
*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2)))+5/4*arctan(1+2^(1/2)*(-1+2
*x^(1/2))^(1/4))*2^(1/2)
```

Maxima [A] time = 1.65154, size = 212, normalized size = 1.1

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)-\frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)-\frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{\left(2\sqrt{x}-1\right)^{2}+4\sqrt{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sqrt(x) - 1)^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)
)) + 5/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)
^(1/4))) + 5/8*sqrt(2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2
*sqrt(x) - 1) + 1) - 5/8*sqrt(2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/
4) + sqrt(2*sqrt(x) - 1) + 1) - (9*(2*sqrt(x) - 1)^(5/4) + 5*(2*s
qrt(x) - 1)^(1/4))/((2*sqrt(x) - 1)^2 + 4*sqrt(x) - 1)
```

Fricas [A] time = 0.228809, size = 267, normalized size = 1.38

$$-\frac{20\sqrt{2}x\arctan\left(\frac{1}{\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+2\sqrt{2\sqrt{x}-1}+2}+1}\right)+20\sqrt{2}x\arctan\left(\frac{1}{\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{-2\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+2\sqrt{2\sqrt{x}-1}+2}-1}\right)-5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sqrt(x) - 1)^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*sqrt(2)*x*arctan(1/(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt
(2*sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 2*sqrt(2*sqrt(x) - 1) + 2) + 1
)) + 20*sqrt(2)*x*arctan(1/(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(
-2*sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 2*sqrt(2*sqrt(x) - 1) + 2) - 1
```

```
)) - 5*sqrt(2)*x*log(2*sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 2*sqrt(2*s
qrt(x) - 1) + 2) + 5*sqrt(2)*x*log(-2*sqrt(2)*(2*sqrt(x) - 1)^(1/
4) + 2*sqrt(2*sqrt(x) - 1) + 2) + 4*(9*sqrt(x) - 2)*(2*sqrt(x) -
1)^(1/4))/x
```

Sympy [A] time = 65.6479, size = 44, normalized size = 0.23

$$-\frac{4\sqrt[4]{2}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

```
[Out] -4*2**(1/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), exp_polar(2*I*p
i)/(2*sqrt(x)))/(x**(3/8)*gamma(7/4))
```

GIAC/XCAS [A] time = 0.204753, size = 192, normalized size = 0.99

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)$$
$$+\frac{5}{8}\sqrt{2}\ln\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)$$
$$-\frac{5}{8}\sqrt{2}\ln\left(-\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)-\frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sqrt(x) - 1)^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)
)) + 5/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)
^(1/4))) + 5/8*sqrt(2)*ln(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*
sqrt(x) - 1) + 1) - 5/8*sqrt(2)*ln(-sqrt(2)*(2*sqrt(x) - 1)^(1/4)
 + sqrt(2*sqrt(x) - 1) + 1) - 1/4*(9*(2*sqrt(x) - 1)^(5/4) + 5*(2
*sqrt(x) - 1)^(1/4))/x
```

3.299 $\int x^6\sqrt[3]{1+x^7}\,dx$

Optimal. Leaf size=13

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

```
[Out] (3*(1 + x^7)^(4/3))/28
```

Rubi [A] time = 0.00632446, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(1 + x^7)^(1/3),x]

[Out] (3*(1 + x^7)^(4/3))/28
```

Rubi in Sympy [A] time = 0.787928, size = 10, normalized size = 0.77

$$\frac{3\left(x^7+1\right)^{\frac{4}{3}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(x**7+1)**(1/3),x)

[Out] 3*(x**7 + 1)**(4/3)/28
```

Mathematica [A] time = 0.00600928, size = 13, normalized size = 1.

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(1 + x^7)^(1/3),x]
```

```
[Out] (3*(1 + x^7)^(4/3))/28
```

Maple [B] time = 0.037, size = 37, normalized size = 2.9

$$\frac{(3+3x)\left(x^{6}-x^{5}+x^{4}-x^{3}+x^{2}-x+1\right)}{28}\sqrt[3]{x^{7}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^7+1)^(1/3),x)
```

```
[Out] 3/28*(1+x)*(x^6-x^5+x^4-x^3+x^2-x+1)*(x^7+1)^(1/3)
```

Maxima [A] time = 1.48073, size = 12, normalized size = 0.92

$$\frac{3}{28}\left(x^{7}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7 + 1)^(1/3)*x^6,x, algorithm="maxima")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

Fricas [A] time = 0.208712, size = 12, normalized size = 0.92

$$\frac{3}{28}\left(x^{7}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7 + 1)^(1/3)*x^6,x, algorithm="fricas")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

Sympy [A] time = 0.81843, size = 26, normalized size = 2.

$$\frac{3x^{7}\sqrt[3]{x^{7}+1}}{28}+\frac{3\sqrt[3]{x^{7}+1}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(x**7+1)**(1/3),x)
```

```
[Out] 3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28
```

GIAC/XCAS [A] time = 0.200746, size = 12, normalized size = 0.92

$$\frac{3}{28}\left(x^{7}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^7 + 1)^(1/3)*x^6,x, algorithm="giac")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

3.300 $\int \frac{x^6}{(1+x^7)^{5/3}}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

```
[Out] -3/(14*(1 + x^7)^(2/3))
```

Rubi [A] time = 0.0061152, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 + x^7)^(5/3),x]

[Out] -3/(14*(1 + x^7)^(2/3))
```

Rubi in Sympy [A] time = 0.792509, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**7+1)**(5/3),x)

[Out] -3/(14*(x**7 + 1)**(2/3))
```

Mathematica [A] time = 0.00528452, size = 13, normalized size = 1.

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 + x^7)^(5/3),x]
```

```
[Out] -3/(14*(1 + x^7)^(2/3))
```

Maple [B] time = 0.009, size = 37, normalized size = 2.9

$$-\frac{(3+3x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{14}\left(x^7+1\right)^{-\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^7+1)^(5/3),x)
```

```
[Out] -3/14*(1+x)*(x^6-x^5+x^4-x^3+x^2-x+1)/(x^7+1)^(5/3)
```

Maxima [A] time = 1.48308, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7 + 1)^(5/3),x, algorithm="maxima")
```

```
[Out] -3/14/(x^7 + 1)^(2/3)
```

Fricas [A] time = 0.208365, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7 + 1)^(5/3),x, algorithm="fricas")
```

```
[Out] -3/14/(x^7 + 1)^(2/3)
```

Sympy [A] time = 1.02295, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**7+1)**(5/3),x)

[Out] -3/(14*(x**7 + 1)**(2/3))
```

GIAC/XCAS [A] time = 0.199936, size = 12, normalized size = 0.92

$$-\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^7 + 1)^(5/3),x, algorithm="giac")

[Out] -3/14/(x^7 + 1)^(2/3)
```

3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$

Optimal. Leaf size=59

$$\frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right)-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}$$

```
[Out] -ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) - L
og[x]/18 + Log[3 + (-27 + 2*x^7)^(1/3)]/42
```

Rubi [A] time = 0.0804597, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right)-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] -ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) - L
og[x]/18 + Log[3 + (-27 + 2*x^7)^(1/3)]/42
```

Rubi in Sympy [A] time = 2.34342, size = 51, normalized size = 0.86

$$-\frac{\log\left(x^7\right)}{126}+\frac{\log\left(\sqrt[3]{2x^7-27}+3\right)}{42}+\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2x^7-27}}{9}-\frac{1}{3}\right)\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x/(2*x**7-27)**(2/3),x)
```

```
[Out] -log(x**7)/126 + log((2*x**7 - 27)**(1/3) + 3)/42 + sqrt(3)*atan(
sqrt(3)*(2*(2*x**7 - 27)**(1/3)/9 - 1/3))/63
```

Mathematica [C] time = 0.0272546, size = 43, normalized size = 0.73

$$-\frac{3\left(2-\frac{27}{x^7}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{27}{2x^7}\right)}{14\left(4x^7-54\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] (-3*(2 - 27/x^7)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, 27/(2*x^7
)])/(14*(-54 + 4*x^7)^(2/3))
```

Maple [C] time = 0.113, size = 74, normalized size = 1.3

$$\frac{1}{63\ (2/3)}\left(-signum\left(-1+\frac{2\,x^{7}}{27}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\,\sqrt{3}}{6}-\frac{9\ \ln(3)}{2}+7\ \ln(x)+\ln(2)+i\pi\right)\left(\frac{2}{3}\right)+\frac{4\ (2/3)\,x^{7}}{81}{}_{3}F_{2}(1,1,\frac{5}{3};\ 2,2;\ \frac{2\,x^{7}}{27})\right)\Big(si$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(2*x^7-27)^(2/3),x)
```

```
[Out] 1/63/signum(-1+2/27*x^7)^(2/3)*(-signum(-1+2/27*x^7))^(2/3)*((1/6
*Pi*3^(1/2)-9/2*ln(3)+7*ln(x)+ln(2)+I*Pi)*GAMMA(2/3)+4/81*GAMMA(2
/3)*x^7*hypergeom([1,1,5/3],[2,2],2/27*x^7))/GAMMA(2/3)
```

Maxima [A] time = 1.72589, size = 86, normalized size = 1.46

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2\,x^{7}-27\right)^{\frac{1}{3}}-3\right)\right)-\frac{1}{126}\log\left(\left(2\,x^{7}-27\right)^{\frac{2}{3}}-3\left(2\,x^{7}-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2\,x^{7}-27\right)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^7 - 27)^(2/3)*x),x, algorithm="maxima")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/1
26*log((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log(
(2*x^7 - 27)^(1/3) + 3)
```

Fricas [A] time = 0.225627, size = 99, normalized size = 1.68

$$-\frac{1}{378}\sqrt{3}\Bigg(\sqrt{3}\log\left(\left(2\,x^{7}-27\right)^{\frac{2}{3}}-3\left(2\,x^{7}-27\right)^{\frac{1}{3}}+9\right)-2\sqrt{3}\log\left(\left(2\,x^{7}-27\right)^{\frac{1}{3}}+3\right)-6\ \arctan\Bigg(\frac{2}{9}\sqrt{3}\left(2\,x^{7}-27\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^7 - 27)^(2/3)*x),x, algorithm="fricas")
```

```
[Out] -1/378*sqrt(3)*(sqrt(3)*log((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(
1/3) + 9) - 2*sqrt(3)*log((2*x^7 - 27)^(1/3) + 3) - 6*arctan(2/9*
sqrt(3)*(2*x^7 - 27)^(1/3) - 1/3*sqrt(3)))
```

Sympy [A] time = 1.72816, size = 42, normalized size = 0.71

$$-\frac{\sqrt[3]{2}\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{27e^{2i\pi}}{2x^7}\right)}{14x^{\frac{14}{3}}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(2*x**7-27)**(2/3),x)
```

```
[Out] -2**(1/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), 27*exp_polar(2*I*p
i)/(2*x**7))/(14*x**(14/3)*gamma(5/3))
```

GIAC/XCAS [A] time = 0.203055, size = 86, normalized size = 1.46

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\ln\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\ln\left(\left(2x^7-27\right)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^7 - 27)^(2/3)*x),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/1
26*ln((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*ln((2
*x^7 - 27)^(1/3) + 3)
```

3.302 $\int \frac{(1+x^7)^{2/3}}{x^8}\,dx$

Optimal. Leaf size=70

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}$$

```
[Out] -(1 + x^7)^(2/3)/(7*x^7) + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt
[3]])/(7*Sqrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7
```

Rubi [A] time = 0.0784704, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] -(1 + x^7)^(2/3)/(7*x^7) + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt
[3]])/(7*Sqrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7
```

Rubi in Sympy [A] time = 2.61101, size = 63, normalized size = 0.9

$$-\frac{\log(x^7)}{21}+\frac{\log\left(-\sqrt[3]{x^7+1}+1\right)}{7}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{x^7+1}}{3}+\frac{1}{3}\right)\right)}{21}-\frac{(x^7+1)^{\frac{2}{3}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**7+1)**(2/3)/x**8,x)
```

```
[Out] -log(x**7)/21 + log(-(x**7 + 1)**(1/3) + 1)/7 + 2*sqrt(3)*atan(sq
rt(3)*(2*(x**7 + 1)**(1/3)/3 + 1/3))/21 - (x**7 + 1)**(2/3)/(7*x*
*7)
```

Mathematica [C] time = 0.0223726, size = 54, normalized size = 0.77

$$-\frac{2\sqrt[3]{\frac{1}{x^7}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{1}{x^7}\right)}{7\sqrt[3]{x^7+1}}-\frac{\left(x^7+1\right)^{2/3}}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] -(1 + x^7)^(2/3)/(7*x^7) - (2*(1 + x^(-7))^(1/3)*Hypergeometric2F
1[1/3, 1/3, 4/3, -x^(-7)])/(7*(1 + x^7)^(1/3))
```

Maple [C] time = 0.064, size = 76, normalized size = 1.1

$$-\frac{1}{7\,x^7}\left(x^7+1\right)^{\frac{2}{3}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)}{21\,\pi}\left(\frac{2\,\pi\,\sqrt{3}}{3\,\Gamma\left(2/3\right)}\left(-\frac{\pi\,\sqrt{3}}{6}-\frac{3\,\ln(3)}{2}+7\,\ln(x)\right)-\frac{2\,\pi\,\sqrt{3}x^7}{9\,\Gamma\left(2/3\right)}{}_3\mathrm{F}_2(1,1,\frac{4}{3};\,2,2;\,-x^7)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^7+1)^(2/3)/x^8,x)
```

```
[Out] -1/7*(x^7+1)^(2/3)/x^7+1/21/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3
^(1/2)-3/2*ln(3)+7*ln(x))*Pi*3^(1/2)/GAMMA(2/3)-2/9*Pi*3^(1/2)/GA
MMA(2/3)*x^7*hypergeom([1,1,4/3],[2,2],-x^7))
```

Maxima [A] time = 1.6205, size = 89, normalized size = 1.27

$$\frac{2}{21}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7}-\frac{1}{21}\,\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\,\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^7 + 1)^(2/3)/x^8,x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x
^7 + 1)^(2/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) +
1) + 2/21*log((x^7 + 1)^(1/3) - 1)
```

Fricas [A] time = 0.221927, size = 117, normalized size = 1.67

$$-\frac{\sqrt{3}\left(\sqrt{3}x^7\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)-2\sqrt{3}x^7\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)-6\,x^7\arctan\left(\frac{2}{3}\sqrt{3}\left(x^7+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+3\sqrt{3}\left(x^7\right.}{63\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^7 + 1)^(2/3)/x^8,x, algorithm="fricas")
```

```
[Out] -1/63*sqrt(3)*(sqrt(3)*x^7*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3)
+ 1) - 2*sqrt(3)*x^7*log((x^7 + 1)^(1/3) - 1) - 6*x^7*arctan(2/3*
sqrt(3)*(x^7 + 1)^(1/3) + 1/3*sqrt(3)) + 3*sqrt(3)*(x^7 + 1)^(2/3
))/x^7
```

Sympy [A] time = 3.18736, size = 34, normalized size = 0.49

$$-\frac{\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^7}\right)}{7x^{\frac{7}{3}}\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**7+1)**(2/3)/x**8,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**7)/(7*x
**(7/3)*gamma(4/3))
```

GIAC/XCAS [A] time = 0.207936, size = 90, normalized size = 1.29

$$\frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7}-\frac{1}{21}\ln\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\ln\left(\left|\left(x^7+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^7 + 1)^(2/3)/x^8,x, algorithm="giac")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x
^7 + 1)^(2/3)/x^7 - 1/21*ln((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1
) + 2/21*ln(abs((x^7 + 1)^(1/3) - 1))
```

3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sq
rt[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.0572341, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sq
rt[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Rubi in Sympy [A] time = 3.32102, size = 61, normalized size = 0.9

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{2}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{2}-\frac{\sqrt[4]{4x^4+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**4+3)**(1/4)/x**2,x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*x/(4*x**4 + 3)**(1/4))/2 + sqrt(2)*atanh(sq
rt(2)*x/(4*x**4 + 3)**(1/4))/2 - (4*x**4 + 3)**(1/4)/x
```

Mathematica [C] time = 0.0281943, size = 46, normalized size = 0.68

$$\frac{4x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)}{3\ 3^{3/4}}-\frac{\sqrt[4]{4x^4+3}}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) + (4*x^3*Hypergeometric2F1[3/4, 3/4, 7/4,
(-4*x^4)/3])/(3*3^(3/4))
```

Maple [C] time = 0.066, size = 35, normalized size = 0.5

$$-\frac{1}{x}\sqrt[4]{4x^4+3}+\frac{4\sqrt[4]{3}x^3}{9}{}_2F_1(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^4+3)^(1/4)/x^2,x)
```

```
[Out] -(4*x^4+3)^(1/4)/x+4/9*3^(1/4)*x^3*hypergeom([3/4,3/4],[7/4],-4/3
*x^4)
```

Maxima [A] time = 1.63216, size = 113, normalized size = 1.66

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right)-\frac{1}{4}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}\right)}{2\sqrt{2}+\frac{2(4x^4+3)^{\frac{1}{4}}}{x}}\right)-\frac{(4x^4+3)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)
*log(-2*(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/((2*sqrt(2)) + 2*(4*x^4 +
 3)^(1/4)/x)) - (4*x^4 + 3)^(1/4)/x
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.76157, size = 41, normalized size = 0.6

$$\frac{\sqrt[4]{3}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**4+3)**(1/4)/x**2,x)
```

```
[Out] 3**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), 4*x**4*exp_polar
(I*pi)/3)/(4*x*gamma(3/4))
```

GIAC/XCAS [A] time = 0.214045, size = 112, normalized size = 1.65

$$\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{4}\,\sqrt{2}\ln\left(-\frac{\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)
*ln(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)
/x)) - (4*x^4 + 3)^(1/4)/x
```

3.304 $\int x^2 \left(3+4x^4\right)^{5/4} dx$

Optimal. Leaf size=93

$$-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}x^3$$

```
[Out] (15*x^3*(3 + 4*x^4)^(1/4))/32 + (x^3*(3 + 4*x^4)^(5/4))/8 - (45*A
rcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh
[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2])
```

Rubi [A] time = 0.0860697, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] (15*x^3*(3 + 4*x^4)^(1/4))/32 + (x^3*(3 + 4*x^4)^(5/4))/8 - (45*A
rcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh
[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2])
```

Rubi in Sympy [A] time = 4.22124, size = 85, normalized size = 0.91

$$\frac{x^3\left(4x^4+3\right)^{\frac{5}{4}}}{8}+\frac{15x^3\sqrt[4]{4x^4+3}}{32}-\frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{256}+\frac{45\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(4*x**4+3)**(5/4),x)
```

```
[Out] x**3*(4*x**4 + 3)**(5/4)/8 + 15*x**3*(4*x**4 + 3)**(1/4)/32 - 45*
sqrt(2)*atan(sqrt(2)*x/(4*x**4 + 3)**(1/4))/256 + 45*sqrt(2)*atan
h(sqrt(2)*x/(4*x**4 + 3)**(1/4))/256
```

Mathematica [C] time = 0.0301574, size = 51, normalized size = 0.55

$$\frac{1}{32}x^3\left(5\sqrt[4]{3}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)+\sqrt[4]{4x^4+3}\left(16x^4+27\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] (x^3*((3 + 4*x^4)^(1/4)*(27 + 16*x^4) + 5*3^(1/4)*Hypergeometric2
F1[3/4, 3/4, 7/4, (-4*x^4)/3]))/32
```

Maple [C] time = 0.034, size = 42, normalized size = 0.5

$$\frac{x^3\left(16\,x^4+27\right)}{32}\sqrt[4]{4\,x^4+3}+\frac{5\,\sqrt[4]{3}x^3}{32}{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{4};\;\frac{7}{4};\;-\frac{4\,x^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(4*x^4+3)^(5/4),x)
```

```
[Out] 1/32*x^3*(16*x^4+27)*(4*x^4+3)^(1/4)+5/32*3^(1/4)*x^3*hypergeom([
3/4,3/4],[7/4],-4/3*x^4)
```

Maxima [A] time = 1.63117, size = 177, normalized size = 1.9

$$\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{45}{512}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}\right)}{2\sqrt{2}+\frac{2\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)+\frac{9\left(\frac{20\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}-\frac{9\left(4\,x^4+3\right)^{\frac{5}{4}}}{x^5}\right)}{32\left(\frac{8\left(4\,x^4+3\right)}{x^4}-\frac{\left(4\,x^4+3\right)^2}{x^8}-16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(5/4)*x^2,x, algorithm="maxima")
```

```
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*s
qrt(2)*log(-2*(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/((2*sqrt(2)) + 2*(4
*x^4 + 3)^(1/4)/x)) + 9/32*(20*(4*x^4 + 3)^(1/4)/x - 9*(4*x^4 + 3
)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4*x^4 + 3)^2/x^8 - 16)
```

Fricas [A] time = 0.211196, size = 147, normalized size = 1.58

$$\frac{1}{512}\sqrt{2}\left(8\sqrt{2}\left(16x^7+27x^3\right)\left(4x^4+3\right)^{\frac{1}{4}}+90\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+45\log\left(-\frac{2\sqrt{2}x^2+4\left(4x^4+3\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{4x^4+3}}{2x^2-\sqrt{4x^4+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(5/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/512*sqrt(2)*(8*sqrt(2)*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4) + 90
*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 45*log(-(2*sqrt(2)*x^2
 + 4*(4*x^4 + 3)^(1/4)*x + sqrt(2)*sqrt(4*x^4 + 3))/(2*x^2 - sqrt
(4*x^4 + 3))))
```

Sympy [A] time = 4.21478, size = 41, normalized size = 0.44

$$\frac{3\sqrt[4]{3}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(4*x**4+3)**(5/4),x)
```

```
[Out] 3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_
polar(I*pi)/3)/(4*gamma(7/4))
```

GIAC/XCAS [A] time = 0.215104, size = 149, normalized size = 1.6

$$\frac{1}{32}x^8\left(\frac{9\left(4x^4+3\right)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}-\frac{20\left(4x^4+3\right)^{\frac{1}{4}}}{x}\right)+\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)-\frac{45}{512}\sqrt{2}\ln\left(-\frac{\sqrt{2}-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(5/4)*x^2,x, algorithm="giac")
```

```
[Out] 1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4
)/x) + 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 4
```

```
5/512*sqrt(2)*ln(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x
^4 + 3)^(1/4)/x))
```

3.305 $\int x^6 \sqrt[4]{3+4x^4}\,dx$

Optimal. Leaf size=93

$$\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}+\frac{1}{8}\sqrt[4]{4x^4+3}x^7+\frac{3}{128}\sqrt[4]{4x^4+3}x^3$$

```
[Out] (3*x^3*(3 + 4*x^4)^(1/4))/128 + (x^7*(3 + 4*x^4)^(1/4))/8 + (27*A
rcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh
[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2])
```

Rubi [A] time = 0.0819841, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}+\frac{1}{8}\sqrt[4]{4x^4+3}x^7+\frac{3}{128}\sqrt[4]{4x^4+3}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (3*x^3*(3 + 4*x^4)^(1/4))/128 + (x^7*(3 + 4*x^4)^(1/4))/8 + (27*A
rcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh
[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2])
```

Rubi in Sympy [A] time = 3.99397, size = 85, normalized size = 0.91

$$\frac{x^7\sqrt[4]{4x^4+3}}{8}+\frac{3x^3\sqrt[4]{4x^4+3}}{128}+\frac{27\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{1024}-\frac{27\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(4*x**4+3)**(1/4),x)
```

```
[Out] x**7*(4*x**4 + 3)**(1/4)/8 + 3*x**3*(4*x**4 + 3)**(1/4)/128 + 27*
sqrt(2)*atan(sqrt(2)*x/(4*x**4 + 3)**(1/4))/1024 - 27*sqrt(2)*ata
nh(sqrt(2)*x/(4*x**4 + 3)**(1/4))/1024
```

Mathematica [C] time = 0.0275489, size = 51, normalized size = 0.55

$$\frac{1}{128}x^3\left(\sqrt[4]{4x^4+3}\left(16x^4+3\right)-3\sqrt[4]{3}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (x^3*((3 + 4*x^4)^(1/4)*(3 + 16*x^4) - 3*3^(1/4)*Hypergeometric2F
1[3/4, 3/4, 7/4, (-4*x^4)/3]))/128
```

Maple [C] time = 0.035, size = 42, normalized size = 0.5

$$\frac{x^3\left(16x^4+3\right)}{128}\sqrt[4]{4x^4+3}-\frac{3\sqrt[4]{3}x^3}{128}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(4*x^4+3)^(1/4),x)
```

```
[Out] 1/128*x^3*(16*x^4+3)*(4*x^4+3)^(1/4)-3/128*3^(1/4)*x^3*hypergeom(
[3/4,3/4],[7/4],-4/3*x^4)
```

Maxima [A] time = 1.69122, size = 176, normalized size = 1.89

$$-\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{27}{2048}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}\right)}{2\sqrt{2}+\frac{2\left(4x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{9\left(\frac{12\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{\left(4x^4+3\right)^{\frac{5}{4}}}{x^5}\right)}{128\left(\frac{8\left(4x^4+3\right)}{x^4}-\frac{\left(4x^4+3\right)^2}{x^8}-16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(1/4)*x^6,x, algorithm="maxima")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/204
8*sqrt(2)*log(-2*(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/((2*sqrt(2)) + 2
*(4*x^4 + 3)^(1/4)/x)) - 9/128*(12*(4*x^4 + 3)^(1/4)/x + (4*x^4 +
 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4*x^4 + 3)^2/x^8 - 16)
```

Fricas [A] time = 0.215886, size = 147, normalized size = 1.58

$$\frac{1}{2048}\sqrt{2}\left(8\sqrt{2}\left(16x^{7}+3x^{3}\right)\left(4x^{4}+3\right)^{\frac{1}{4}}-54\arctan\left(\frac{\sqrt{2}\left(4x^{4}+3\right)^{\frac{1}{4}}}{2x}\right)+27\log\left(-\frac{2\sqrt{2}x^{2}-4\left(4x^{4}+3\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{4x^{4}+3}}{2x^{2}-\sqrt{4x^{4}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(1/4)*x^6,x, algorithm="fricas")
```

```
[Out] 1/2048*sqrt(2)*(8*sqrt(2)*(16*x^7 + 3*x^3)*(4*x^4 + 3)^(1/4) - 54
*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27*log(-(2*sqrt(2)*x^2
 - 4*(4*x^4 + 3)^(1/4)*x + sqrt(2)*sqrt(4*x^4 + 3))/(2*x^2 - sqrt
(4*x^4 + 3))))
```

Sympy [A] time = 2.68289, size = 39, normalized size = 0.42

$$\frac{\sqrt[4]{3}x^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{4x^{4}e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(4*x**4+3)**(1/4),x)
```

```
[Out] 3**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), 4*x**4*exp_p
olar(I*pi)/3)/(4*gamma(11/4))
```

GIAC/XCAS [A] time = 0.210533, size = 147, normalized size = 1.58

$$\frac{1}{128}x^{8}\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\left(\frac{3}{x^{4}}+4\right)}{x}+\frac{12\left(4x^{4}+3\right)^{\frac{1}{4}}}{x}\right)-\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^{4}+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{27}{2048}\sqrt{2}\ln\left(-\frac{\sqrt{2}-\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 3)^(1/4)*x^6,x, algorithm="giac")
```

```
[Out] 1/128*x^8*((4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^(1/4)
/x) - 27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 2
```

```
7/2048*sqrt(2)*ln(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*
x^4 + 3)^(1/4)/x))
```

3.306 $\int \sqrt[3]{x\left(1-x^2\right)}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}\sqrt[3]{x\left(1-x^2\right)}x-\frac{1}{4}\log\left(\sqrt[3]{x\left(1-x^2\right)}+x\right)+\frac{\tan^{-1}\left(\frac{2x-\sqrt[3]{x\left(1-x^2\right)}}{\sqrt{3}\sqrt[3]{x\left(1-x^2\right)}}\right)}{2\sqrt{3}}+\frac{\log(x)}{12}$$

```
[Out] (x*(x*(1 - x^2))^(1/3))/2 + ArcTan[(2*x - (x*(1 - x^2))^(1/3))/(S
qrt[3]*(x*(1 - x^2))^(1/3))]/(2*Sqrt[3]) + Log[x]/12 - Log[x + (x
*(1 - x^2))^(1/3)]/4
```

Rubi [B] time = 0.263093, antiderivative size = 200, normalized size of antiderivative = 2.15, number of steps used = 12, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923

$$\frac{1}{2}\sqrt[3]{x-x^3}x+\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{(1-x^2)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}+1\right)}{12\left(x-x^3\right)^{2/3}}-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(\frac{x^{2/3}}{\sqrt[3]{1-x^2}}+1\right)}{6\left(x-x^3\right)^{2/3}}-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (x*(x - x^3)^(1/3))/2 - (x^(2/3)*(1 - x^2)^(2/3)*ArcTan[(1 - (2*x
^(2/3))/(1 - x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(x - x^3)^(2/3)) +
(x^(2/3)*(1 - x^2)^(2/3)*Log[1 + x^(4/3)/(1 - x^2)^(2/3) - x^(2/3
)/(1 - x^2)^(1/3)])/(12*(x - x^3)^(2/3)) - (x^(2/3)*(1 - x^2)^(2/
3)*Log[1 + x^(2/3)/(1 - x^2)^(1/3)])/(6*(x - x^3)^(2/3))
```

Rubi in Sympy [A] time = 8.95156, size = 162, normalized size = 1.74

$$\frac{x\sqrt[3]{-x^3+x}}{2}-\frac{\sqrt[3]{-x^3+x}\log\left(\frac{x^{\frac{2}{3}}}{\sqrt[3]{-x^2+1}}+1\right)}{6\sqrt[3]{x}\sqrt[3]{-x^2+1}}+\frac{\sqrt[3]{-x^3+x}\log\left(\frac{x^{\frac{4}{3}}}{(-x^2+1)^{\frac{2}{3}}}-\frac{x^{\frac{2}{3}}}{\sqrt[3]{-x^2+1}}+1\right)}{12\sqrt[3]{x}\sqrt[3]{-x^2+1}}+\frac{\sqrt{3}\sqrt[3]{-x^3+x}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^{\frac{2}{3}}}{3\sqrt[3]{-x^2+1}}-\frac{1}{3}\right)\right)}{6\sqrt[3]{x}\sqrt[3]{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x*(-x**2+1))**(1/3),x)
```

```
[Out] x*(-x**3 + x)**(1/3)/2 - (-x**3 + x)**(1/3)*log(x**(2/3)/(-x**2 +
 1)**(1/3) + 1)/(6*x**(1/3)*(-x**2 + 1)**(1/3)) + (-x**3 + x)**(1
/3)*log(x**(4/3)/(-x**2 + 1)**(2/3) - x**(2/3)/(-x**2 + 1)**(1/3)
 + 1)/(12*x**(1/3)*(-x**2 + 1)**(1/3)) + sqrt(3)*(-x**3 + x)**(1/
3)*atan(sqrt(3)*(2*x**(2/3)/(3*(-x**2 + 1)**(1/3)) - 1/3))/(6*x**
(1/3)*(-x**2 + 1)**(1/3))
```

Mathematica [C] time = 0.028443, size = 56, normalized size = 0.6

$$\frac{x\sqrt[3]{x-x^3}\left(-\left(1-x^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};x^2\right)+2x^2-2\right)}{4\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (x*(x - x^3)^(1/3)*(-2 + 2*x^2 - (1 - x^2)^(2/3)*Hypergeometric2F
1[2/3, 2/3, 5/3, x^2]))/(4*(-1 + x^2))
```

Maple [C] time = 0.043, size = 15, normalized size = 0.2

$$\frac{3}{4}x^{\frac{4}{3}}{}_2F_1(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x*(-x^2+1))^(1/3),x)
```

```
[Out] 3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 1)*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 1)*x)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 1)*x)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{x\left(-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x*(-x**2+1))**(1/3),x)
```

```
[Out] Integral((x*(-x**2 + 1))**(1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 1)*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 1)*x)^(1/3), x)
```

3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$

Optimal. Leaf size=126

$$\frac{3}{40}\sqrt{\left(\sqrt[3]{x}+1\right)x}x^{2/3}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{\left(\sqrt[3]{x}+1\right)x}}\right)+\frac{3}{5}\sqrt{\left(\sqrt[3]{x}+1\right)x}x-\frac{7}{80}\sqrt{\left(\sqrt[3]{x}+1\right)x}\sqrt[3]{x}+\frac{7}{64}\sqrt{\left(\sqrt[3]{x}+1\right)x}-\frac{21\sqrt{\left(\sqrt[3]{x}+1\right)x}}{128\sqrt[3]{x}}$$

```
[Out] (7*Sqrt[(1 + x^(1/3))*x])/64 - (21*Sqrt[(1 + x^(1/3))*x])/(128*x^
(1/3)) - (7*x^(1/3)*Sqrt[(1 + x^(1/3))*x])/80 + (3*x^(2/3)*Sqrt[(
1 + x^(1/3))*x])/40 + (3*x*Sqrt[(1 + x^(1/3))*x])/5 + (21*ArcTanh
[x^(2/3)/Sqrt[(1 + x^(1/3))*x]])/128
```

Rubi [A] time = 0.183601, antiderivative size = 114, normalized size of antiderivative = 0.9, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$\frac{3}{5}\sqrt{x^{4/3}+x}x+\frac{3}{40}\sqrt{x^{4/3}+x}x^{2/3}-\frac{7}{80}\sqrt{x^{4/3}+x}\sqrt[3]{x}+\frac{7}{64}\sqrt{x^{4/3}+x}-\frac{21\sqrt{x^{4/3}+x}}{128\sqrt[3]{x}}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (7*Sqrt[x + x^(4/3)])/64 - (21*Sqrt[x + x^(4/3)])/(128*x^(1/3)) -
 (7*x^(1/3)*Sqrt[x + x^(4/3)])/80 + (3*x^(2/3)*Sqrt[x + x^(4/3)])
/40 + (3*x*Sqrt[x + x^(4/3)])/5 + (21*ArcTanh[x^(2/3)/Sqrt[x + x^
(4/3)]])/128
```

Rubi in Sympy [A] time = 8.71257, size = 104, normalized size = 0.83

$$\frac{3x^{\frac{2}{3}}\sqrt{x^{\frac{4}{3}}+x}}{40}-\frac{7\sqrt[3]{x}\sqrt{x^{\frac{4}{3}}+x}}{80}+\frac{3x\sqrt{x^{\frac{4}{3}}+x}}{5}+\frac{7\sqrt{x^{\frac{4}{3}}+x}}{64}+\frac{21\operatorname{atanh}\left(\frac{x^{\frac{2}{3}}}{\sqrt{x^{\frac{4}{3}}+x}}\right)}{128}-\frac{21\sqrt{x^{\frac{4}{3}}+x}}{128\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(((1+x**(1/3))*x)**(1/2),x)
```

```
[Out] 3*x**(2/3)*sqrt(x**(4/3) + x)/40 - 7*x**(1/3)*sqrt(x**(4/3) + x)/
80 + 3*x*sqrt(x**(4/3) + x)/5 + 7*sqrt(x**(4/3) + x)/64 + 21*atan
h(x**(2/3)/sqrt(x**(4/3) + x))/128 - 21*sqrt(x**(4/3) + x)/(128*x
**(1/3))
```

Mathematica [A] time = 0.171119, size = 82, normalized size = 0.65

$$\frac{\frac{2\sqrt{x^{4/3}+x}\left(384x^{4/3}-56x^{2/3}+48x+70\sqrt[3]{x}-105\right)}{\sqrt[3]{x}}+105\log\left(2x^{2/3}+2\sqrt{x^{4/3}+x}+\sqrt[3]{x}\right)-35\log(x)}{1280}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] ((2*Sqrt[x + x^(4/3)]*(-105 + 70*x^(1/3) - 56*x^(2/3) + 48*x + 38
4*x^(4/3)))/x^(1/3) - 35*Log[x] + 105*Log[x^(1/3) + 2*x^(2/3) + 2
*Sqrt[x + x^(4/3)]])/1280
```

Maple [A] time = 0.017, size = 108, normalized size = 0.9

$$\frac{1}{1280}\sqrt{\left(\sqrt[3]{x}+1\right)x}\left(768\,x^{2/3}\left(x^{2/3}+\sqrt[3]{x}\right)^{3/2}-672\sqrt[3]{x}\left(x^{2/3}+\sqrt[3]{x}\right)^{3/2}+560\left(x^{2/3}+\sqrt[3]{x}\right)^{3/2}-420\sqrt{x^{2/3}+\sqrt[3]{x}}\sqrt[3]{x}-210\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^(1/3)+1)*x)^(1/2),x)
```

```
[Out] 1/1280*((x^(1/3)+1)*x)^(1/2)*(768*x^(2/3)*(x^(2/3)+x^(1/3))^(3/2)
-672*x^(1/3)*(x^(2/3)+x^(1/3))^(3/2)+560*(x^(2/3)+x^(1/3))^(3/2)-
420*(x^(2/3)+x^(1/3))^(1/2)*x^(1/3)-210*(x^(2/3)+x^(1/3))^(1/2)+1
05*ln(1/2+x^(1/3)+(x^(2/3)+x^(1/3))^(1/2)))/x^(1/3)/((x^(1/3)+1)*
x^(1/3))^(1/2)
```

Maxima [A] time = 1.49326, size = 212, normalized size = 1.68

$$-\frac{\frac{105\left(x^{\frac{1}{3}}+1\right)^{\frac{9}{2}}}{x^{\frac{3}{2}}}-\frac{490\left(x^{\frac{1}{3}}+1\right)^{\frac{7}{2}}}{x^{\frac{7}{6}}}+\frac{896\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}}}{x^{\frac{5}{6}}}-\frac{790\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}}}{\sqrt{x}}-\frac{105\sqrt{x^{\frac{1}{3}}+1}}{x^{\frac{1}{6}}}}{640\left(\frac{\left(x^{\frac{1}{3}}+1\right)^{5}}{x^{\frac{5}{3}}}-\frac{5\left(x^{\frac{1}{3}}+1\right)^{4}}{x^{\frac{4}{3}}}+\frac{10\left(x^{\frac{1}{3}}+1\right)^{3}}{x}-\frac{10\left(x^{\frac{1}{3}}+1\right)^{2}}{x^{\frac{2}{3}}}+\frac{5\left(x^{\frac{1}{3}}+1\right)}{x^{\frac{1}{3}}}-1\right)}$$
$$+\frac{21}{256}\log\left(\frac{\sqrt{x^{\frac{1}{3}}+1}}{x^{\frac{1}{6}}}+1\right)-\frac{21}{256}\log\left(\frac{\sqrt{x^{\frac{1}{3}}+1}}{x^{\frac{1}{6}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x*(x^(1/3) + 1)),x, algorithm="maxima")
```

```
[Out] -1/640*(105*(x^(1/3) + 1)^(9/2)/x^(3/2) - 490*(x^(1/3) + 1)^(7/2)
/x^(7/6) + 896*(x^(1/3) + 1)^(5/2)/x^(5/6) - 790*(x^(1/3) + 1)^(3
/2)/sqrt(x) - 105*sqrt(x^(1/3) + 1)/x^(1/6))/((x^(1/3) + 1)^5/x^(
5/3) - 5*(x^(1/3) + 1)^4/x^(4/3) + 10*(x^(1/3) + 1)^3/x - 10*(x^(
1/3) + 1)^2/x^(2/3) + 5*(x^(1/3) + 1)/x^(1/3) - 1) + 21/256*log(s
qrt(x^(1/3) + 1)/x^(1/6) + 1) - 21/256*log(sqrt(x^(1/3) + 1)/x^(1
/6) - 1)
```

Fricas [A] time = 51.6498, size = 117, normalized size = 0.93

$$\frac{35\,x\log\left(\frac{32\,x^2+48\,x^{\frac{5}{3}}+2\left(16\,x^{\frac{4}{3}}+16\,x+3\,x^{\frac{2}{3}}\right)\sqrt{x^{\frac{4}{3}}+x}+18\,x^{\frac{4}{3}}+x}{x}\right)+2\left(384\,x^2+3\,(16\,x-35)x^{\frac{2}{3}}-56\,x^{\frac{4}{3}}+70\,x\right)\sqrt{x^{\frac{4}{3}}+x}}{1280\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x*(x^(1/3) + 1)),x, algorithm="fricas")
```

```
[Out] 1/1280*(35*x*log((32*x^2 + 48*x^(5/3) + 2*(16*x^(4/3) + 16*x + 3*
x^(2/3))*sqrt(x^(4/3) + x) + 18*x^(4/3) + x)/x) + 2*(384*x^2 + 3*
(16*x - 35)*x^(2/3) - 56*x^(4/3) + 70*x)*sqrt(x^(4/3) + x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x\left(\sqrt[3]{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1+x**(1/3))*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(x**(1/3) + 1)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x*(x^(1/3) + 1)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

Rubi [A] time = 0.085941, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]

[Out] -ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 4.34022, size = 32, normalized size = 0.94

$$-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(2x^4+1)}{3\sqrt{2x^8+1}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)

[Out] -sqrt(3)*atanh(sqrt(3)*(2*x**4 + 1)/(3*sqrt(2*x**8 + 1)))/12
```

Mathematica [A] time = 0.0384946, size = 37, normalized size = 1.09

$$\frac{\log\left(x^4-1\right)-\log\left(\sqrt{6x^8+3}+2x^4+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]

[Out] (Log[-1 + x^4] - Log[1 + 2*x^4 + Sqrt[3 + 6*x^8]])/(4*Sqrt[3])
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^4-1}\frac{1}{\sqrt{2\,x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x)

[Out] int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x)
```

Maxima [A] time = 1.59858, size = 47, normalized size = 1.38

$$-\frac{1}{12}\sqrt{3}\,\mathrm{arsinh}\left(\frac{\sqrt{2}x^4}{|x^4-1|}+\frac{\sqrt{2}}{2\,|x^4-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)),x, algorithm="maxima")

[Out] -1/12*sqrt(3)*arcsinh(sqrt(2)*x^4/abs(x^4 - 1) + 1/2*sqrt(2)/abs(
x^4 - 1))
```

Fricas [A] time = 0.212497, size = 113, normalized size = 3.32

$$\frac{1}{12}\sqrt{3}\log\left(\frac{3\,x^8-3\,x^4-\sqrt{3}\left(3\,x^8-x^4+1\right)+\sqrt{2\,x^8+1}\left(3\,x^4-\sqrt{3}\left(x^4-1\right)\right)}{x^4-\sqrt{2\,x^8+1}(x^4-1)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log((3*x^8 - 3*x^4 - sqrt(3)*(3*x^8 - x^4 + 1) + sqr
t(2*x^8 + 1)*(3*x^4 - sqrt(3)*(x^4 - 1)))/(x^4 - sqrt(2*x^8 + 1)*
(x^4 - 1) - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x-1)(x+1)(x^2+1)\sqrt{2x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)
```

```
[Out] Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)
```

GIAC/XCAS [A] time = 0.245081, size = 95, normalized size = 2.79

$$\frac{1}{12}\sqrt{3}\ln\left(-\frac{\left|-2\sqrt{2}x^4-2\sqrt{3}+2\sqrt{2}+2\sqrt{2x^8+1}\right|}{2\left(\sqrt{2}x^4-\sqrt{3}-\sqrt{2}-\sqrt{2x^8+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*ln(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) +
 2*sqrt(2*x^8 + 1))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8
 + 1)))
```

3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$

Optimal. Leaf size=58

$$-\frac{3}{80}\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)+\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}$$

```
[Out] -((1 + 2*x^5)*Sqrt[1 + x^5 + x^10])/40 + (1 + x^5 + x^10)^(3/2)/1
5 - (3*ArcSinh[(1 + 2*x^5)/Sqrt[3]])/80
```

Rubi [A] time = 0.0776813, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{3}{80}\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)+\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] -((1 + 2*x^5)*Sqrt[1 + x^5 + x^10])/40 + (1 + x^5 + x^10)^(3/2)/1
5 - (3*ArcSinh[(1 + 2*x^5)/Sqrt[3]])/80
```

Rubi in Sympy [A] time = 3.54834, size = 58, normalized size = 1.

$$-\frac{\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}}{40}+\frac{\left(x^{10}+x^5+1\right)^{\frac{3}{2}}}{15}-\frac{3\,\mathrm{atanh}\left(\frac{2x^5+1}{2\sqrt{x^{10}+x^5+1}}\right)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(x**10+x**5+1)**(1/2),x)
```

```
[Out] -(2*x**5 + 1)*sqrt(x**10 + x**5 + 1)/40 + (x**10 + x**5 + 1)**(3/
2)/15 - 3*atanh((2*x**5 + 1)/(2*sqrt(x**10 + x**5 + 1)))/80
```

Mathematica [A] time = 0.0345934, size = 47, normalized size = 0.81

$$\frac{1}{240}\left(2\sqrt{x^{10}+x^5+1}\left(8x^{10}+2x^5+5\right)-9\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] (2*Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10) - 9*ArcSinh[(1 + 2*x
^5)/Sqrt[3]])/240
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int x^{9}\sqrt{x^{10}+x^{5}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(x^10+x^5+1)^(1/2),x)
```

```
[Out] int(x^9*(x^10+x^5+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{10}+x^{5}+1}x^{9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^10 + x^5 + 1)*x^9,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^10 + x^5 + 1)*x^9, x)
```

Fricas [A] time = 0.205588, size = 242, normalized size = 4.17

$$-\frac{2048\,x^{30}+4608\,x^{25}+7296\,x^{20}+6528\,x^{15}+4416\,x^{10}+1770\,x^{5}-36\left(32\,x^{15}+48\,x^{10}+42\,x^{5}-2\left(16\,x^{10}+16\,x^{5}+7\right)\sqrt{x^{10}}\right.}{960\left(32\,x^{15}+48\,x^{10}+42\,x^{5}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^10 + x^5 + 1)*x^9,x, algorithm="fricas")
```

```
[Out] -1/960*(2048*x^30 + 4608*x^25 + 7296*x^20 + 6528*x^15 + 4416*x^10
 + 1770*x^5 - 36*(32*x^15 + 48*x^10 + 42*x^5 - 2*(16*x^10 + 16*x^
5 + 7)*sqrt(x^10 + x^5 + 1) + 13)*log(-2*x^5 + 2*sqrt(x^10 + x^5
+ 1) - 1) - 2*(1024*x^25 + 1792*x^20 + 2368*x^15 + 1600*x^10 + 83
```

```
2*x^5 + 211)*sqrt(x^10 + x^5 + 1) + 469)/(32*x^15 + 48*x^10 + 42*
x^5 - 2*(16*x^10 + 16*x^5 + 7)*sqrt(x^10 + x^5 + 1) + 13)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\sqrt{(x^2+x+1)\,(x^8-x^7+x^5-x^4+x^3-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(x**10+x**5+1)**(1/2),x)
```

```
[Out] Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x*
*3 - x + 1)), x)
```

GIAC/XCAS [A] time = 0.202591, size = 66, normalized size = 1.14

$$\frac{1}{120}\sqrt{x^{10}+x^5+1}\left(2\left(4\,x^5+1\right)x^5+5\right)+\frac{3}{80}\ln\left(-2\,x^5+2\sqrt{x^{10}+x^5+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^10 + x^5 + 1)*x^9,x, algorithm="giac")
```

```
[Out] 1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*ln(-2*x
^5 + 2*sqrt(x^10 + x^5 + 1) - 1)
```

3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$

Optimal. Leaf size=71

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)$$

```
[Out] -Sqrt[4 + 2*x^2 + x^4]/(16*x^4) + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x
^2) + ArcTanh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128
```

Rubi [A] time = 0.127249, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

```
[Out] -Sqrt[4 + 2*x^2 + x^4]/(16*x^4) + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x
^2) + ArcTanh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128
```

Rubi in Sympy [A] time = 7.10581, size = 63, normalized size = 0.89

$$\frac{\operatorname{atanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

```
[Out] atanh((2*x**2 + 8)/(4*sqrt(x**4 + 2*x**2 + 4)))/128 + 3*sqrt(x**4
 + 2*x**2 + 4)/(64*x**2) - sqrt(x**4 + 2*x**2 + 4)/(16*x**4)
```

Mathematica [A] time = 0.0957421, size = 59, normalized size = 0.83

$$\frac{1}{128}\left(\frac{2\sqrt{x^4+2x^2+4}\left(3x^2-4\right)}{x^4}+\log\left(2\left(x^2+2\sqrt{x^4+2x^2+4}+4\right)\right)-2\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

```
[Out] ((2*(-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/x^4 - 2*Log[x] + Log[2*(4
 + x^2 + 2*Sqrt[4 + 2*x^2 + x^4])])/128
```

Maple [A] time = 0.019, size = 60, normalized size = 0.9

$$-\frac{1}{16\,x^4}\sqrt{x^4+2\,x^2+4}+\frac{3}{64\,x^2}\sqrt{x^4+2\,x^2+4}+\frac{1}{128}\mathit{Artanh}\left(\frac{2\,x^2+8}{4}\frac{1}{\sqrt{x^4+2\,x^2+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^4+2*x^2+4)^(1/2),x)
```

```
[Out] -1/16*(x^4+2*x^2+4)^(1/2)/x^4+3/64*(x^4+2*x^2+4)^(1/2)/x^2+1/128*
arctanh(1/4*(2*x^2+8)/(x^4+2*x^2+4)^(1/2))
```

Maxima [A] time = 1.54376, size = 70, normalized size = 0.99

$$\frac{3\,\sqrt{x^4+2\,x^2+4}}{64\,x^2}-\frac{\sqrt{x^4+2\,x^2+4}}{16\,x^4}+\frac{1}{128}\ \mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}+\frac{4\,\sqrt{3}}{3\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 2*x^2 + 4)*x^5),x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(x^4 + 2*x^2 + 4)/x^2 - 1/16*sqrt(x^4 + 2*x^2 + 4)/x^4 +
 1/128*arcsinh(1/3*sqrt(3) + 4/3*sqrt(3)/x^2)
```

Fricas [A] time = 0.394308, size = 269, normalized size = 3.79

$$\frac{4\,x^6+6\,x^4+48\,x^2+\left(2\,x^8+4\,x^6+5\,x^4-2\,\left(x^6+x^4\right)\sqrt{x^4+2\,x^2+4}\right)\log\left(-x^2+\sqrt{x^4+2\,x^2+4}+2\right)-\left(2\,x^8+4\,x^6+5\,x^4-\right.}{128\left(2\,x^8+4\,x^6+5\,x^4-2\,(x^6+x^4)\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 2*x^2 + 4)*x^5),x, algorithm="fricas")
```

```
[Out] 1/128*(4*x^6 + 6*x^4 + 48*x^2 + (2*x^8 + 4*x^6 + 5*x^4 - 2*(x^6 +
 x^4)*sqrt(x^4 + 2*x^2 + 4))*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2
) - (2*x^8 + 4*x^6 + 5*x^4 - 2*(x^6 + x^4)*sqrt(x^4 + 2*x^2 + 4))
*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) - 2) - 2*(2*x^4 + x^2 + 20)*sqr
t(x^4 + 2*x^2 + 4) + 64)/(2*x^8 + 4*x^6 + 5*x^4 - 2*(x^6 + x^4)*s
qrt(x^4 + 2*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{x^4+2x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+2\,x^2+4}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 + 2*x^2 + 4)*x^5),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 2*x^2 + 4)*x^5), x)
```

3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

Rubi [A] time = 0.0980367, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]

[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

Rubi in Sympy [A] time = 6.16567, size = 22, normalized size = 1.05

$$\operatorname{atanh}\left(\frac{2x^2+2}{2\sqrt{x^4+3x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)

[Out] atanh((2*x**2 + 2)/(2*sqrt(x**4 + 3*x**2 + 1)))
```

Mathematica [B] time = 0.0856934, size = 59, normalized size = 2.81

$$\frac{1}{2}\left(-\log\left(x^2\right)+\log\left(2x^2+2\sqrt{x^4+3x^2+1}+3\right)+\log\left(3x^2+2\sqrt{x^4+3x^2+1}+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] (-Log[x^2] + Log[3 + 2*x^2 + 2*Sqrt[1 + 3*x^2 + x^4]] + Log[2 + 3
*x^2 + 2*Sqrt[1 + 3*x^2 + x^4]])/2
```

Maple [B] time = 0.036, size = 46, normalized size = 2.2

$$\frac{1}{2}\ln\left(x^{2}+\frac{3}{2}+\sqrt{x^{4}+3x^{2}+1}\right)+\frac{1}{2}Artanh\left(\frac{3x^{2}+2}{2}\frac{1}{\sqrt{x^{4}+3x^{2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x)
```

```
[Out] 1/2*ln(x^2+3/2+(x^4+3*x^2+1)^(1/2))+1/2*arctanh(1/2*(3*x^2+2)/(x^
4+3*x^2+1)^(1/2))
```

Maxima [A] time = 1.48992, size = 70, normalized size = 3.33

$$\frac{1}{2}\log\left(2x^{2}+2\sqrt{x^{4}+3x^{2}+1}+3\right)+\frac{1}{2}\log\left(\frac{2\sqrt{x^{4}+3x^{2}+1}}{x^{2}}+\frac{2}{x^{2}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(2*sqrt(x^4
 + 3*x^2 + 1)/x^2 + 2/x^2 + 3)
```

Fricas [A] time = 0.20587, size = 80, normalized size = 3.81

$$-\frac{1}{2}\log\left(4x^{4}+11x^{2}-\sqrt{x^{4}+3x^{2}+1}\left(4x^{2}+5\right)+5\right)+\frac{1}{2}\log\left(-x^{2}+\sqrt{x^{4}+3x^{2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(4*x^4 + 11*x^2 - sqrt(x^4 + 3*x^2 + 1)*(4*x^2 + 5) + 5)
+ 1/2*log(-x^2 + sqrt(x^4 + 3*x^2 + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)(x+1)}{x\sqrt{x^4+3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{x^4+3\,x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*x),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*x), x)
```

3.312 $\int \left(-3x+2x^3\right) \left(-3x^2+x^4\right)^{3/5} \, dx$

Optimal. Leaf size=17

$$\frac{5}{16} \left(x^4-3x^2\right)^{8/5}$$

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

Rubi [A] time = 0.00790294, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{5}{16} \left(x^4-3x^2\right)^{8/5}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

Rubi in Sympy [A] time = 2.13438, size = 14, normalized size = 0.82

$$\frac{5 \left(x^4-3x^2\right)^{\frac{8}{5}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
```

```
[Out] 5*(x**4 - 3*x**2)**(8/5)/16
```

Mathematica [A] time = 0.0183971, size = 17, normalized size = 1.

$$\frac{5}{16} \left(x^2 \left(x^2-3\right)\right)^{8/5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(x^2*(-3 + x^2))^(8/5))/16
```

Maple [A] time = 0.007, size = 22, normalized size = 1.3

$$\frac{5\,x^{2}\left(x^{2}-3\right)}{16}\left(x^{4}-3\,x^{2}\right)^{\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x)

[Out] 5/16*(x^4-3*x^2)^(3/5)*x^2*(x^2-3)
```

Maxima [A] time = 1.40429, size = 18, normalized size = 1.06

$$\frac{5}{16}\left(x^{4}-3\,x^{2}\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 3*x^2)^(3/5)*(2*x^3 - 3*x),x, algorithm="maxima")

[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Fricas [A] time = 0.202221, size = 18, normalized size = 1.06

$$\frac{5}{16}\left(x^{4}-3\,x^{2}\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 3*x^2)^(3/5)*(2*x^3 - 3*x),x, algorithm="fricas")

[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Sympy [A] time = 2.13713, size = 36, normalized size = 2.12

$$\frac{5x^{4}\left(x^{4}-3x^{2}\right)^{\frac{3}{5}}}{16}-\frac{15x^{2}\left(x^{4}-3x^{2}\right)^{\frac{3}{5}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
```

```
[Out] 5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)
/16
```

GIAC/XCAS [A] time = 0.213954, size = 18, normalized size = 1.06

$$\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 3*x^2)^(3/5)*(2*x^3 - 3*x),x, algorithm="giac")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx$

Optimal. Leaf size=46

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

```
[Out] (-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1
 + 3*x^3)^(9/4))/243
```

Rubi [A] time = 0.314525, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

```
[Out] (-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1
 + 3*x^3)^(9/4))/243
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^8-2x^5-x^2\left(3x^3-1\right)^{\frac{2}{3}}}{\left(3x^3-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

```
[Out] Integral((3*x**8 - 2*x**5 - x**2*(3*x**3 - 1)**(2/3))/(3*x**3 - 1
)**(3/4), x)
```

Mathematica [A] time = 0.0284164, size = 40, normalized size = 0.87

$$-\frac{4\sqrt[4]{3x^3-1}\left(-99x^6+66x^3+81\left(3x^3-1\right)^{2/3}+88\right)}{2673}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

```
[Out] (-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2
/3)))/2673
```

Maple [C] time = 0.1, size = 116, normalized size = 2.5

$$-\frac{x^6}{3}\left(-signum\left(3x^3-1\right)\right)^{\frac{3}{4}}{}_2F_1(\frac{3}{4},2;\,3;\,3x^3)\left(signum\left(3x^3-1\right)\right)^{-\frac{3}{4}} + \frac{x^9}{3}\left(-signum\left(3x^3-1\right)\right)^{\frac{3}{4}}{}_2F_1(\frac{3}{4},3;\,4;\,3x^3)\left(signum\left(3x^3-1\right)\right)^{-\frac{3}{4}} - \frac{x^3}{3}\left(-signum\left(3x^3-1\right)\right)^{\frac{1}{12}}{}_2F_1(\frac{1}{12},1;\,2;\,3x^3)\left(signum\left(3x^3-1\right)\right)^{-\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x)
```

```
[Out] -1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^6*hypergeom
([3/4,2],[3],3*x^3)+1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^
(3/4)*x^9*hypergeom([3/4,3],[4],3*x^3)-1/3/signum(3*x^3-1)^(1/12)
*(-signum(3*x^3-1))^(1/12)*x^3*hypergeom([1/12,1],[2],3*x^3)
```

Maxima [A] time = 1.36701, size = 46, normalized size = 1.

$$\frac{4}{243}\left(3x^3-1\right)^{\frac{9}{4}} - \frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}} - \frac{4}{27}\left(3x^3-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^8 - 2*x^5 - (3*x^3 - 1)^(2/3)*x^2)/(3*x^3 - 1)^(3/4),x, algorithm=
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3
- 1)^(1/4)
```

Fricas [A] time = 0.203172, size = 47, normalized size = 1.02

$$\frac{4}{243}\left(9x^6-6x^3-8\right)\left(3x^3-1\right)^{\frac{1}{4}} - \frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^8 - 2*x^5 - (3*x^3 - 1)^(2/3)*x^2)/(3*x^3 - 1)^(3/4),x, algorithm=
```

```
[Out] 4/243*(9*x^6 - 6*x^3 - 8)*(3*x^3 - 1)^(1/4) - 4/33*(3*x^3 - 1)^(1
1/12)
```

Sympy [A] time = 4.62344, size = 226, normalized size = 4.91

$$-\frac{4\left(3x^3-1\right)^{\frac{11}{12}}}{33}-2\left(\begin{cases}\frac{4x^3\sqrt[4]{3x^3-1}}{45}+\frac{16\sqrt[4]{3x^3-1}}{135} & \text{for } 3\left|x^3\right|>1\\ -\frac{4x^3\sqrt[4]{-3x^3+1}e^{\frac{5i\pi}{4}}}{45}-\frac{16\sqrt[4]{-3x^3+1}e^{\frac{5i\pi}{4}}}{135} & \text{otherwise}\end{cases}\right)+3\left(\begin{cases}\frac{4x^6\sqrt[4]{3x^3-1}}{81}+\frac{32x^3\sqrt[4]{3x^3-1}}{1215}+\frac{128\sqrt[4]{3x^3-1}}{3645} & \text{for } 3\left|x^3\right|>1\\ \frac{4x^6\sqrt[4]{-3x^3+1}e^{\frac{9i\pi}{4}}}{81}+\frac{32x^3\sqrt[4]{-3x^3+1}e^{\frac{9i\pi}{4}}}{1215}+\frac{128\sqrt[4]{-3x^3+1}e^{\frac{9i\pi}{4}}}{3645} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

```
[Out] -4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(
1/4)/45 + 16*(3*x**3 - 1)**(1/4)/135, 3*Abs(x**3) > 1), (-4*x**3*
(-3*x**3 + 1)**(1/4)*exp(5*I*pi/4)/45 - 16*(-3*x**3 + 1)**(1/4)*e
xp(5*I*pi/4)/135, True)) + 3*Piecewise((4*x**6*(3*x**3 - 1)**(1/4
)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1215 + 128*(3*x**3 - 1)**(1/4)
/3645, 3*Abs(x**3) > 1), (4*x**6*(-3*x**3 + 1)**(1/4)*exp(9*I*pi/
4)/81 + 32*x**3*(-3*x**3 + 1)**(1/4)*exp(9*I*pi/4)/1215 + 128*(-3
*x**3 + 1)**(1/4)*exp(9*I*pi/4)/3645, True))
```

GIAC/XCAS [A] time = 0.204289, size = 46, normalized size = 1.

$$\frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^8 - 2*x^5 - (3*x^3 - 1)^(2/3)*x^2)/(3*x^3 - 1)^(3/4),x, algorithm=
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3
- 1)^(1/4)
```

3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=107

$$\frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+\frac{3^{2/3}x^2}{(x^3+2)^{2/3}}+1\right)}{6\sqrt[3]{3}}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(2 + x^3)^(1/3))]/3^(5/6)) +
Log[1 - (3^(1/3)*x)/(2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3^(2/
3)*x^2)/(2 + x^3)^(2/3) + (3^(1/3)*x)/(2 + x^3)^(1/3)]/(6*3^(1/3)
)
```

Rubi [A] time = 0.149767, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$\frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+\frac{3^{2/3}x^2}{(x^3+2)^{2/3}}+1\right)}{6\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(2 + x^3)^(1/3))]/3^(5/6)) +
Log[1 - (3^(1/3)*x)/(2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3^(2/
3)*x^2)/(2 + x^3)^(2/3) + (3^(1/3)*x)/(2 + x^3)^(1/3)]/(6*3^(1/3)
)
```

Rubi in Sympy [A] time = 6.84566, size = 102, normalized size = 0.95

$$\frac{3^{\frac{2}{3}}\log\left(-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1\right)}{9}-\frac{3^{\frac{2}{3}}\log\left(\frac{3^{\frac{2}{3}}x^2}{(x^3+2)^{\frac{2}{3}}}+\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1\right)}{18}-\frac{\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{3}x}{3\sqrt[3]{x^3+2}}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] 3**(2/3)*log(-3**(1/3)*x/(x**3 + 2)**(1/3) + 1)/9 - 3**(2/3)*log(
3**(2/3)*x**2/(x**3 + 2)**(2/3) + 3**(1/3)*x/(x**3 + 2)**(1/3) +
1)/18 - 3**(1/6)*atan(sqrt(3)*(2*3**(1/3)*x/(3*(x**3 + 2)**(1/3))
```

```
+ 1/3))/3
```

Mathematica [A] time = 0.186317, size = 112, normalized size = 1.05

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{2x^3+1}}\right)-\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{2x^3+1}}+\frac{3^{2/3}x^2}{(2x^3+1)^{2/3}}+1\right)\right)-6\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{2x^3+1}}+\frac{1}{\sqrt{3}}\right)}{6\ 3^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(1 + 2*x^3)^(1/3))] + Sqrt[
3]*(2*Log[1 - (3^(1/3)*x)/(1 + 2*x^3)^(1/3)] - Log[1 + (3^(2/3)*x
^2)/(1 + 2*x^3)^(2/3) + (3^(1/3)*x)/(1 + 2*x^3)^(1/3)]))/(6*3^(5/
6))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^3-1}\frac{1}{\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-1)/(x^3+2)^(1/3),x)
```

```
[Out] int(1/(x^3-1)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

Fricas [A] time = 1.92608, size = 274, normalized size = 2.56

$$\frac{1}{54} \cdot 3^{\frac{1}{6}} \left(2\sqrt{3}\log\left(-\frac{3\cdot 3^{\frac{2}{3}}\left(x^3+2\right)^{\frac{2}{3}}x-9\left(x^3+2\right)^{\frac{1}{3}}x^2+2\cdot 3^{\frac{1}{3}}\left(x^3-1\right)}{x^3-1}\right)-\sqrt{3}\log\left(\frac{3^{\frac{2}{3}}\left(31\,x^6+46\,x^3+4\right)+9\cdot 3^{\frac{1}{3}}\left(5\,x^5+4\,x^2\right)\left(x\right.}{x^6-2\,x^3+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] 1/54*3^(1/6)*(2*sqrt(3)*log(-(3*3^(2/3)*(x^3 + 2)^(2/3)*x - 9*(x^
3 + 2)^(1/3)*x^2 + 2*3^(1/3)*(x^3 - 1))/(x^3 - 1)) - sqrt(3)*log(
(3^(2/3)*(31*x^6 + 46*x^3 + 4) + 9*3^(1/3)*(5*x^5 + 4*x^2)*(x^3 +
 2)^(1/3) + 9*(7*x^4 + 2*x)*(x^3 + 2)^(2/3))/(x^6 - 2*x^3 + 1)) +
 6*arctan(1/3*(9*sqrt(3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(5/6)*(x^3 - 1
) + 18*3^(1/6)*(x^3 + 2)^(2/3)*x)/(9*(x^3 + 2)^(1/3)*x^2 + 2*3^(1
/3)*(x^3 - 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-1)\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}+\frac{\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTan[1 +
 (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 +
x^4] - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqr
t[2 + x^4] + (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(4*Sqrt[2])
```

Rubi [A] time = 0.137354, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}+\frac{\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTan[1 +
 (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 +
x^4] - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqr
t[2 + x^4] + (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 7.5697, size = 124, normalized size = 0.88

$$-\frac{\sqrt{2}\log\left(\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{8}+\frac{\sqrt{2}\log\left(\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

```
[Out] -sqrt(2)*log(x**2/sqrt(x**4 + 2) - sqrt(2)*x/(x**4 + 2)**(1/4) +
1)/8 + sqrt(2)*log(x**2/sqrt(x**4 + 2) + sqrt(2)*x/(x**4 + 2)**(1
/4) + 1)/8 + sqrt(2)*atan(sqrt(2)*x/(x**4 + 2)**(1/4) - 1)/4 + sq
rt(2)*atan(sqrt(2)*x/(x**4 + 2)**(1/4) + 1)/4
```

Mathematica [A] time = 0.113781, size = 132, normalized size = 0.94

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2x^4+1}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{2x^4+1}}+1\right)-\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{2x^4+1}}+\frac{x^2}{\sqrt{2x^4+1}}+1\right)+\log\left(\frac{\sqrt{2}x}{\sqrt[4]{2x^4+1}}+\frac{x^2}{\sqrt{2x^4+1}}+1\right)}{4\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/(1 + 2*x^4)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*x)/(1 + 2*x^4)^(1/4)] - Log[1 + x^2/Sqrt[1 + 2*x^4] - (Sqrt[
2]*x)/(1 + 2*x^4)^(1/4)] + Log[1 + x^2/Sqrt[1 + 2*x^4] + (Sqrt[2]
*x)/(1 + 2*x^4)^(1/4)])/(4*Sqrt[2])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+1}\frac{1}{\sqrt[4]{x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+1)/(x^4+2)^(1/4),x)
```

```
[Out] int(1/(x^4+1)/(x^4+2)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

Fricas [A] time = 3.52353, size = 524, normalized size = 3.72

$$\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(x^4+2\right)^{\frac{1}{4}}x^3+\sqrt{2}\sqrt{x^4+2}x^2+\left(x^4+2\right)^{\frac{3}{4}}x}{(x^4+2)^{\frac{1}{4}}x^3-\sqrt{2}(x^4+1)\sqrt{\frac{x^4+\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3+2\sqrt{x^4+2}x^2+\sqrt{2}(x^4+2)^{\frac{3}{4}}x+1}{x^4+1}}-(x^4+2)^{\frac{3}{4}}x-\sqrt{2}}\right)$$
$$+\frac{1}{4}\sqrt{2}\arctan\left(\frac{\left(x^4+2\right)^{\frac{1}{4}}x^3-\sqrt{2}\sqrt{x^4+2}x^2+\left(x^4+2\right)^{\frac{3}{4}}x}{(x^4+2)^{\frac{1}{4}}x^3+\sqrt{2}(x^4+1)\sqrt{\frac{x^4-\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3+2\sqrt{x^4+2}x^2-\sqrt{2}(x^4+2)^{\frac{3}{4}}x+1}{x^4+1}}-(x^4+2)^{\frac{3}{4}}x+\sqrt{2}}\right)$$
$$+\frac{1}{16}\sqrt{2}\log\left(\frac{2\left(x^4+\sqrt{2}\left(x^4+2\right)^{\frac{1}{4}}x^3+2\sqrt{x^4+2}x^2+\sqrt{2}\left(x^4+2\right)^{\frac{3}{4}}x+1\right)}{x^4+1}\right)$$
$$-\frac{1}{16}\sqrt{2}\log\left(\frac{2\left(x^4-\sqrt{2}\left(x^4+2\right)^{\frac{1}{4}}x^3+2\sqrt{x^4+2}x^2-\sqrt{2}\left(x^4+2\right)^{\frac{3}{4}}x+1\right)}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-((x^4 + 2)^(1/4)*x^3 + sqrt(2)*sqrt(x^4 + 2)*
x^2 + (x^4 + 2)^(3/4)*x)/((x^4 + 2)^(1/4)*x^3 - sqrt(2)*(x^4 + 1)
*sqrt((x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 +
sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1)) - (x^4 + 2)^(3/4)*x - s
qrt(2))) + 1/4*sqrt(2)*arctan(((x^4 + 2)^(1/4)*x^3 - sqrt(2)*sqrt
(x^4 + 2)*x^2 + (x^4 + 2)^(3/4)*x)/((x^4 + 2)^(1/4)*x^3 + sqrt(2)
*(x^4 + 1)*sqrt((x^4 - sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 +
 2)*x^2 - sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1)) - (x^4 + 2)^(
3/4)*x + sqrt(2))) + 1/16*sqrt(2)*log(2*(x^4 + sqrt(2)*(x^4 + 2)^
(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/
(x^4 + 1)) - 1/16*sqrt(2)*log(2*(x^4 - sqrt(2)*(x^4 + 2)^(1/4)*x^
3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+1)\sqrt[4]{x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

```
[Out] Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rubi [A] time = 0.0315039, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rubi in Sympy [A] time = 3.80031, size = 88, normalized size = 1.4

$$\frac{x\left(x^3+2\right)^{\frac{2}{3}}}{3}+\frac{5\log\left(-\frac{x}{\sqrt[3]{x^3+2}}+1\right)}{9}-\frac{5\log\left(\frac{x^2}{\left(x^3+2\right)^{\frac{2}{3}}}+\frac{x}{\sqrt[3]{x^3+2}}+1\right)}{18}-\frac{5\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{x^3+2}}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] x*(x**3 + 2)**(2/3)/3 + 5*log(-x/(x**3 + 2)**(1/3) + 1)/9 - 5*log
(x**2/(x**3 + 2)**(2/3) + x/(x**3 + 2)**(1/3) + 1)/18 - 5*sqrt(3)
*atan(sqrt(3)*(2*x/(3*(x**3 + 2)**(1/3)) + 1/3))/9
```

Mathematica [A] time = 0.0732227, size = 91, normalized size = 1.44

$$\frac{1}{18}\left(6\left(x^3+2\right)^{2/3}x+10\log\left(1-\frac{x}{\sqrt[3]{x^3+2}}\right)-10\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-5\log\left(\frac{x}{\sqrt[3]{x^3+2}}+\frac{x^2}{(x^3+2)^{2/3}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(1 + (2*x)/(2 + x^3)^(1/
3))/Sqrt[3]] + 10*Log[1 - x/(2 + x^3)^(1/3)] - 5*Log[1 + x^2/(2 +
 x^3)^(2/3) + x/(2 + x^3)^(1/3)])/18
```

Maple [C] time = 0.044, size = 29, normalized size = 0.5

$$\frac{x}{3}\left(x^3+2\right)^{\frac{2}{3}}-\frac{5\,2^{2/3}x}{6}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\,\frac{4}{3};\,-\frac{x^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-1)/(x^3+2)^(1/3),x)
```

```
[Out] 1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*
x^3)
```

Maxima [A] time = 1.66005, size = 127, normalized size = 2.02

$$\frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+2\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\left(x^3+2\right)^{\frac{2}{3}}}{3\,x^2\left(\frac{x^3+2}{x^3}-1\right)}-\frac{5}{18}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+2\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{9}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^3 + 2)^(1/3),x, algorithm="maxima")
```

```
[Out] 5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(
x^3 + 2)^(2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/
3)/x + (x^3 + 2)^(2/3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)
```

Fricas [A] time = 0.212255, size = 131, normalized size = 2.08

$$\frac{1}{54}\sqrt{3}\left(6\sqrt{3}\left(x^3+2\right)^{\frac{2}{3}}x+10\sqrt{3}\log\left(-\frac{x-\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)-5\sqrt{3}\log\left(\frac{x^2+\left(x^3+2\right)^{\frac{1}{3}}x+\left(x^3+2\right)^{\frac{2}{3}}}{x^2}\right)+30\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}}{3x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^3 + 2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*(6*sqrt(3)*(x^3 + 2)^(2/3)*x + 10*sqrt(3)*log(-(x -
(x^3 + 2)^(1/3))/x) - 5*sqrt(3)*log((x^2 + (x^3 + 2)^(1/3)*x + (x
^3 + 2)^(2/3))/x^2) + 30*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 +
 2)^(1/3))/x))
```

Sympy [A] time = 2.78536, size = 71, normalized size = 1.13

$$\frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)}-\frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] 2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar
(I*pi)/2)/(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3)
, (4/3,), x**3*exp_polar(I*pi)/2)/(6*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3-1}{(x^3+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^3 + 2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} \, dx$

Optimal. Leaf size=74

$$\frac{(x^4+1)^{3/4}\, x}{8\,(x^4+2)} + \frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}$$

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4
)^(1/4))])/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(16*2^(3/4))
```

Rubi [A] time = 0.0641195, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$\frac{(x^4+1)^{3/4}\, x}{8\,(x^4+2)} + \frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4
)^(1/4))])/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(16*2^(3/4))
```

Rubi in Sympy [A] time = 3.68551, size = 70, normalized size = 0.95

$$\frac{x\,(x^4+1)^{\frac{3}{4}}}{8\,(x^4+2)} + \frac{3\sqrt[4]{2}\,\operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2\sqrt[4]{x^4+1}}\right)}{32} + \frac{3\sqrt[4]{2}\,\operatorname{atanh}\left(\frac{2^{\frac{3}{4}}x}{2\sqrt[4]{x^4+1}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
```

```
[Out] x*(x**4 + 1)**(3/4)/(8*(x**4 + 2)) + 3*2**(1/4)*atan(2**(3/4)*x/(
2*(x**4 + 1)**(1/4)))/32 + 3*2**(1/4)*atanh(2**(3/4)*x/(2*(x**4 +
 1)**(1/4)))/32
```

Mathematica [A] time = 0.160765, size = 92, normalized size = 1.24

$$\frac{\left(x^4+1\right)^{3/4}x}{8\left(x^4+2\right)}+\frac{3\left(-\log\left(2-\frac{2^{3/4}x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{2^{3/4}x}{\sqrt[4]{x^4+1}}+2\right)+2\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)\right)}{32\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*(2*ArcTan[x/(2^(1/4)*(1 + 
x^4)^(1/4))] - Log[2 - (2^(3/4)*x)/(1 + x^4)^(1/4)] + Log[2 + (2^
(3/4)*x)/(1 + x^4)^(1/4)]))/(32*2^(3/4))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int\frac{1}{\left(x^4+2\right)^2}\left(x^4+1\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)^(3/4)/(x^4+2)^2,x)
```

```
[Out] int((x^4+1)^(3/4)/(x^4+2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

Fricas [A] time = 4.28261, size = 323, normalized size = 4.36

$$\frac{8^{\frac{3}{4}}\left(8\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+12\left(x^4+2\right)\arctan\left(-\frac{4\sqrt{x^4+1}x^2-\sqrt{2}\left(3x^4+2\right)}{2\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3-8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{3}{4}}x-\sqrt{2}\left(x^4+2\right)}\right)+3\left(x^4+2\right)\log\left(\frac{2\left(2\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+4\right)}{x^4+2}\right)\right)}{512\left(x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/512*8^(3/4)*(8*8^(1/4)*(x^4 + 1)^(3/4)*x + 12*(x^4 + 2)*arctan(
-(4*sqrt(x^4 + 1)*x^2 - sqrt(2)*(3*x^4 + 2))/(2*8^(1/4)*(x^4 + 1)
^(1/4)*x^3 - 8^(3/4)*(x^4 + 1)^(3/4)*x - sqrt(2)*(x^4 + 2))) + 3*
(x^4 + 2)*log(2*(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 + 8^(3/4)*(x^4 + 1
)^(3/4)*x + 4*sqrt(x^4 + 1)*x^2 + sqrt(2)*(3*x^4 + 2))/(x^4 + 2))
 - 3*(x^4 + 2)*log(2*(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 + 8^(3/4)*(x^
4 + 1)^(3/4)*x - 4*sqrt(x^4 + 1)*x^2 - sqrt(2)*(3*x^4 + 2))/(x^4
+ 2)))/(x^4 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
```

```
[Out] Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}} \, dx$

Optimal. Leaf size=48

$$\frac{97x}{891\sqrt[5]{x^5+3}}+\frac{5x}{297\,(x^5+3)^{6/5}}-\frac{5\,(x^5-2)\,x}{33\,(x^5+3)^{11/5}}$$

```
[Out] (-5*x*(-2 + x^5))/(33*(3 + x^5)^(11/5)) + (5*x)/(297*(3 + x^5)^(6
/5)) + (97*x)/(891*(3 + x^5)^(1/5))
```

Rubi [A] time = 0.0375836, antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{x\,(2-x^5)^2}{33\,(x^5+3)^{11/5}}+\frac{10x\,(2-x^5)}{297\,(x^5+3)^{6/5}}+\frac{100x}{891\sqrt[5]{x^5+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

```
[Out] (x*(2 - x^5)^2)/(33*(3 + x^5)^(11/5)) + (10*x*(2 - x^5))/(297*(3
+ x^5)^(6/5)) + (100*x)/(891*(3 + x^5)^(1/5))
```

Rubi in Sympy [A] time = 3.05508, size = 49, normalized size = 1.02

$$\frac{x\,(-x^5+2)^2}{33\,(x^5+3)^{\frac{11}{5}}}+\frac{10x\,(-x^5+2)}{297\,(x^5+3)^{\frac{6}{5}}}+\frac{100x}{891\sqrt[5]{x^5+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5-2)**2/(x**5+3)**(16/5),x)
```

```
[Out] x*(-x**5 + 2)**2/(33*(x**5 + 3)**(11/5)) + 10*x*(-x**5 + 2)/(297*
(x**5 + 3)**(6/5)) + 100*x/(891*(x**5 + 3)**(1/5))
```

Mathematica [A] time = 0.0249177, size = 26, normalized size = 0.54

$$\frac{x\,(97x^{10}+462x^5+1188)}{891\,(x^5+3)^{11/5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

```
[Out] (x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))
```

Maple [A] time = 0.01, size = 23, normalized size = 0.5

$$\frac{x\left(97\,x^{10}+462\,x^{5}+1188\right)}{891}\left(x^{5}+3\right)^{-\frac{11}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5-2)^2/(x^5+3)^(16/5),x)
```

```
[Out] 1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)
```

Maxima [A] time = 1.40955, size = 99, normalized size = 2.06

$$-\frac{4\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-\frac{33\,(x^5+3)^2}{x^{10}}-3\right)}{891\,(x^5+3)^{\frac{11}{5}}}-\frac{2\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-6\right)}{297\,(x^5+3)^{\frac{11}{5}}}+\frac{x^{11}}{33\,(x^5+3)^{\frac{11}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 2)^2/(x^5 + 3)^(16/5),x, algorithm="maxima")
```

```
[Out] -4/891*x^11*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^10 - 3)/(x^5 + 3
)^(11/5) - 2/297*x^11*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^(11/5) + 1
/33*x^11/(x^5 + 3)^(11/5)
```

Fricas [A] time = 0.205232, size = 54, normalized size = 1.12

$$\frac{\left(97\,x^{11}+462\,x^{6}+1188\,x\right)\left(x^{5}+3\right)^{\frac{4}{5}}}{891\,(x^{15}+9\,x^{10}+27\,x^{5}+27)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 2)^2/(x^5 + 3)^(16/5),x, algorithm="fricas")
```

```
[Out] 1/891*(97*x^11 + 462*x^6 + 1188*x)*(x^5 + 3)^(4/5)/(x^15 + 9*x^10
 + 27*x^5 + 27)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5-2)**2/(x**5+3)**(16/5),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^5-2\right)^2}{\left(x^5+3\right)^{\frac{16}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 2)^2/(x^5 + 3)^(16/5),x, algorithm="giac")
```

```
[Out] integrate((x^5 - 2)^2/(x^5 + 3)^(16/5), x)
```

3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{((x+1)^3+2)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]
/3^(5/6)) + Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3
^(1/3)) - Log[1 + (3^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^
(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(6*3^(1/3))
```

Rubi [A] time = 0.208128, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{((x+1)^3+2)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]
/3^(5/6)) + Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3
^(1/3)) - Log[1 + (3^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^
(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(6*3^(1/3))
```

Rubi in Sympy [A] time = 18.5694, size = 116, normalized size = 0.94

$$\frac{3^{\frac{2}{3}}\log\left(-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{9}-\frac{3^{\frac{2}{3}}\log\left(\frac{3^{\frac{2}{3}}(x+1)^2}{\left((x+1)^3+2\right)^{\frac{2}{3}}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{18}-\frac{\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{3}(x+1)}{3\sqrt[3]{(x+1)^3+2}}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] 3**(2/3)*log(-3**(1/3)*(x + 1)/((x + 1)**3 + 2)**(1/3) + 1)/9 - 3
**(2/3)*log(3**(2/3)*(x + 1)**2/((x + 1)**3 + 2)**(2/3) + 3**(1/3
)*(x + 1)/((x + 1)**3 + 2)**(1/3) + 1)/18 - 3**(1/6)*atan(sqrt(3)
*(2*3**(1/3)*(x + 1)/(3*((x + 1)**3 + 2)**(1/3)) + 1/3))/3
```

Mathematica [A] time = 0.13037, size = 0, normalized size = 0.

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),
x]
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+3\,x^2+3\,x}\frac{1}{\sqrt[3]{x^3+3\,x^2+3\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

```
[Out] int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+3\,x^2+3\,x+3)^{\frac{1}{3}}(x^3+3\,x^2+3\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)),x, algorithm="maxi
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)),
x)
```

Fricas [A] time = 6.39309, size = 489, normalized size = 3.98

$$\frac{1}{54} \cdot 3^{\frac{1}{6}} \left(2\sqrt{3}\log\left(\frac{3 \cdot 3^{\frac{2}{3}} \left(x^3 + 3x^2 + 3x + 3\right)^{\frac{2}{3}}(x+1) + 2 \cdot 3^{\frac{1}{3}}\left(x^3 + 3x^2 + 3x\right) - 9\left(x^3 + 3x^2 + 3x + 3\right)^{\frac{1}{3}}\left(x^2 + 2x + 1\right)}{x^3 + 3x^2 + 3x} \right) - \sqrt{3}\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)),x, algorithm="fric
```

```
[Out] 1/54*3^(1/6)*(2*sqrt(3)*log((3*3^(2/3)*(x^3 + 3*x^2 + 3*x + 3)^(2
/3)*(x + 1) + 2*3^(1/3)*(x^3 + 3*x^2 + 3*x) - 9*(x^3 + 3*x^2 + 3*
x + 3)^(1/3)*(x^2 + 2*x + 1))/(x^3 + 3*x^2 + 3*x)) - sqrt(3)*log(
(3^(2/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*x^2 + 324*x
+ 81) + 9*3^(1/3)*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(
x^3 + 3*x^2 + 3*x + 3)^(1/3) + 9*(7*x^4 + 28*x^3 + 42*x^2 + 30*x
+ 9)*(x^3 + 3*x^2 + 3*x + 3)^(2/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^
3 + 9*x^2)) + 6*arctan(-1/3*(2*3^(5/6)*(x^3 + 3*x^2 + 3*x) - 9*sq
rt(3)*(x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 2*x + 1) - 18*3^(1/6)*
(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(2*3^(1/3)*(x^3 + 3*x^2 +
3*x) + 9*(x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 2*x + 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2 + 3x + 3\right)\sqrt[3]{x^3 + 3x^2 + 3x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3 + 3x^2 + 3x + 3\right)^{\frac{1}{3}}\left(x^3 + 3x^2 + 3x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)),x, algorithm="giac

[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)),
x)
```

3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rubi [A] time = 0.0567477, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.59571, size = 22, normalized size = 0.96

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(x**4 + 1))/2
```

Mathematica [C] time = 0.0600707, size = 40, normalized size = 1.74

$$\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-2\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi
[-I, I*ArcSinh[(-1)^(1/4)*x], -1])
```

Maple [C] time = 0.011, size = 112, normalized size = 4.9

$$-\frac{EllipticF\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}} - 2\,\frac{(-1)^{3/4}\sqrt{1-ix^2}\sqrt{1+ix^2}EllipticPi\left(\sqrt[4]{-1}x,i,\sqrt{-i}-(-1)^{3/4}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x
^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/
4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^
(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Fricas [A] time = 0.236382, size = 24, normalized size = 1.04

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx-\int \left(-\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integ
ral(-1/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rubi [A] time = 0.0593245, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]

[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.65391, size = 22, normalized size = 0.96

$$\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)

[Out] sqrt(2)*atanh(sqrt(2)*x/sqrt(x**4 + 1))/2
```

Mathematica [C] time = 0.0579364, size = 36, normalized size = 1.57

$$\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-2\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi
[I, ArcSin[(-1)^(3/4)*x], -1])
```

Maple [C] time = 0.008, size = 112, normalized size = 4.9

$$-\frac{EllipticF\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}} - 2\frac{(-1)^{3/4}\sqrt{1-ix^2}\sqrt{1+ix^2}EllipticPi\left(\sqrt[4]{-1}x,-i,\sqrt{-i}-(-1)^{3/4}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x
^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/
4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^
(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Fricas [A] time = 0.231113, size = 57, normalized size = 2.48

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^
4 - 2*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integ
ral(1/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x^2-1}{\sqrt{x^4+1}}\right)$$

```
[Out] ArcTanh[(-1 + x^2)/Sqrt[1 + x^4]]
```

Rubi [A] time = 0.0807468, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

Rubi in Sympy [A] time = 4.92422, size = 17, normalized size = 1.06

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/x/(x**4+1)**(1/2),x)

[Out] asinh(x**2)/2 - atanh(sqrt(x**4 + 1))/2
```

Mathematica [A] time = 0.0492745, size = 23, normalized size = 1.44

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

Maple [A] time = 0.013, size = 18, normalized size = 1.1

$$-\frac{1}{2}Artanh\left(\frac{1}{\sqrt{x^4+1}}\right)+\frac{Arcsinh\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x/(x^4+1)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))+1/2*arcsinh(x^2)
```

Maxima [A] time = 1.51317, size = 77, normalized size = 4.81

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)/(sqrt(x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*lo
g(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Fricas [A] time = 0.207123, size = 66, normalized size = 4.12

$$-\frac{1}{2}\log\left(2x^4-x^2-\sqrt{x^4+1}\left(2x^2-1\right)+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)/(sqrt(x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^4 - x^2 - sqrt(x^4 + 1)*(2*x^2 - 1) + 1) + 1/2*log(-
x^2 + sqrt(x^4 + 1) - 1)
```

Sympy [A] time = 3.01548, size = 14, normalized size = 0.88

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/x/(x**4+1)**(1/2),x)

[Out] -asinh(x**(-2))/2 + asinh(x**2)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{x^4+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(sqrt(x^4 + 1)*x),x, algorithm="giac")

[Out] integrate((x^2 + 1)/(sqrt(x^4 + 1)*x), x)
```

3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+1}}\right)$$

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + x^4]]
```

Rubi [A] time = 0.0798617, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)+\frac{1}{2}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4]]/2
```

Rubi in Sympy [A] time = 4.8858, size = 17, normalized size = 1.06

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/x/(x**4+1)**(1/2),x)

[Out] asinh(x**2)/2 + atanh(sqrt(x**4 + 1))/2
```

Mathematica [A] time = 0.0457774, size = 23, normalized size = 1.44

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)+\frac{1}{2}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4]]/2
```

Maple [A] time = 0.01, size = 18, normalized size = 1.1

$$\frac{\mathit{Arcsinh}\left(x^{2}\right)}{2}+\frac{1}{2}\mathit{Artanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/x/(x^4+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(x^2)+1/2*arctanh(1/(x^4+1)^(1/2))
```

Maxima [A] time = 1.54676, size = 77, normalized size = 4.81

$$\frac{1}{4}\,\log\left(\sqrt{x^{4}+1}+1\right)-\frac{1}{4}\,\log\left(\sqrt{x^{4}+1}-1\right)+\frac{1}{4}\,\log\left(\frac{\sqrt{x^{4}+1}}{x^{2}}+1\right)-\frac{1}{4}\,\log\left(\frac{\sqrt{x^{4}+1}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log
(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Fricas [A] time = 0.202686, size = 63, normalized size = 3.94

$$-\frac{1}{2}\,\log\left(2\,x^{4}+x^{2}-\sqrt{x^{4}+1}\left(2\,x^{2}+1\right)+1\right)+\frac{1}{2}\,\log\left(-x^{2}+\sqrt{x^{4}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) + 1/2*log(-
x^2 + sqrt(x^4 + 1) + 1)
```

Sympy [A] time = 3.04785, size = 14, normalized size = 0.88

$$\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/x/(x**4+1)**(1/2),x)
```

```
[Out] asinh(x**(-2))/2 + asinh(x**2)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{x^4+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^4 + 1)*x), x)
```

3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Rubi [A] time = 0.0702955, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Rubi in Sympy [A] time = 8.17769, size = 26, normalized size = 1.

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] sqrt(3)*atanh(sqrt(3)*x/sqrt(x**4 + x**2 + 1))/3
```

Mathematica [A] time = 0.241202, size = 0, normalized size = 0.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]), x]
```

Maple [C] time = 0.244, size = 184, normalized size = 7.1

$$-2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\,EllipticF\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}}+2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}}{\sqrt{-1/2+i/2\sqrt{3}}\sqrt{x^4+x^2+1}}EllipticPi\left(\sqrt{-1/2+i/2\sqrt{3}}x,\left(-1/2+i/2\sqrt{3}\right)^{-1},\frac{\sqrt{-1/2-i/2\sqrt{3}}}{\sqrt{-1/2+i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] -2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(
-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*
(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*
3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1
/2))^(1/2)*x,1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-
1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

Fricas [A] time = 0.231322, size = 63, normalized size = 2.42

$$\frac{1}{6}\,\sqrt{3}\log\left(\frac{\sqrt{3}\left(x^4+4\,x^2+1\right)+6\,\sqrt{x^4+x^2+1}x}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((sqrt(3)*(x^4 + 4*x^2 + 1) + 6*sqrt(x^4 + x^2 + 1
)*x)/(x^4 - 2*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1
)), x) - Integral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**
2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=15

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Rubi [A] time = 0.0641825, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Rubi in Sympy [A] time = 8.01422, size = 14, normalized size = 0.93

$$\operatorname{atan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] atan(x/sqrt(x**4 + x**2 + 1))
```

Mathematica [C] time = 0.103621, size = 94, normalized size = 6.27

$$-\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\left(\sqrt[3]{-1};-i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(
EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*EllipticPi[(-1
)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 +
 x^4])
```

Maple [C] time = 0.046, size = 188, normalized size = 12.5

$$-2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\,EllipticF\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}}+2\,\frac{\sqrt{1+1/2\,x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2\,x^2+i/2x^2\sqrt{3}}}{\sqrt{-1/2+i/2\sqrt{3}}\sqrt{x^4+x^2+1}}EllipticPi\left(\sqrt{-1/2+i/2\sqrt{3}}x,-\left(-1/2+i/2\sqrt{3}\right)^{-1},\frac{\sqrt{-1/2-i/2\sqrt{3}}}{\sqrt{-1/2+i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] -2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(
-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*
(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*
3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2
*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^
(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)
/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Fricas [A] time = 0.237536, size = 18, normalized size = 1.2

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] arctan(x/sqrt(x^4 + x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx-\int \left(-\frac{1}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1
)), x) - Integral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x*
*2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

Rubi [A] time = 0.0112432, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]

[Out] Sqrt[1 + x^2 + x^4]/x
```

Rubi in Sympy [A] time = 5.12579, size = 12, normalized size = 0.75

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)

[Out] sqrt(x**4 + x**2 + 1)/x
```

Mathematica [A] time = 0.0158107, size = 16, normalized size = 1.

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

Maple [A] time = 0.009, size = 29, normalized size = 1.8

$$\frac{(x^2+x+1)\,(x^2-x+1)}{x}\frac{1}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x)

[Out] (x^2+x+1)*(x^2-x+1)/(x^4+x^2+1)^(1/2)/x
```

Maxima [A] time = 1.63205, size = 30, normalized size = 1.88

$$\frac{\sqrt{x^2+x+1}\sqrt{x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2),x, algorithm="maxima")

[Out] sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x
```

Fricas [A] time = 0.205142, size = 19, normalized size = 1.19

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2),x, algorithm="fricas")

[Out] sqrt(x^4 + x^2 + 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)\,(x+1)\,(x^2+1)}{x^2\sqrt{(x^2-x+1)\,(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x*
*2 + x + 1))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 - 1}{\sqrt{x^4 + x^2 + 1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2), x)
```

3.327 $\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{2x(a^2-b+1)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

```
[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[
1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

Rubi [A] time = 0.322856, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023

$$\frac{\tan^{-1}\left(\frac{2x(a^2-b+1)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]),x]
```

```
[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[
1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

Rubi in Sympy [A] time = 19.6328, size = 76, normalized size = 1.03

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(2ax^2+2a+x\left(4a^2-4b+4\right)\right)}{4\sqrt{-b+1}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{2\sqrt{-b+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2
```

```
[Out] sqrt(2)*atan(sqrt(2)*(2*a*x**2 + 2*a + x*(4*a**2 - 4*b + 4))/(4*s
qrt(-b + 1)*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)))/(2*sqr
t(-b + 1))
```

Mathematica [C] time = 6.29635, size = 17955, normalized size = 242.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]),x
```

```
[Out] Result too large to show
```

Maple [C] time = 0.139, size = 247419, normalized size = 3343.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{\sqrt{2\,ax^3 + x^4 + 2\,bx^2 + 2\,ax + 1}(2\,ax + x^2 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^2 + 1)),
```

```
[Out] -integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(
2*a*x + x^2 + 1)), x)
```

Fricas [A] time = 0.387437, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\log\left(\frac{4\,a^3x^3+\left(a^2+2\,b-2\right)x^4+4\,a^3x+2\left(2\,a^4+5\,a^2-2\left(2\,a^2+3\right)b+4\,b^2+2\right)x^2+a^2-\frac{2\sqrt{2}\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}\left((ab-a)x^2+ab-2\left(a^2-\left(a^2+2\right)b+b^2+1\right)x-a\right)}{\sqrt{b-1}}+2\,b-2}{4\,ax^3+x^4+2\,(2\,a^2+1)x^2+4\,ax+1}\right)}{4\sqrt{b-1}},\right.$$

$$\left.-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{b-1}}\arctan\left(\frac{\sqrt{2}\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}}{(ax^2+2\,(a^2-b+1)x+a)\sqrt{-\frac{1}{b-1}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^2 + 1)),
```

```
[Out] [1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(
2*a^4 + 5*a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2
)*sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b
- 2*(a^2 - (a^2 + 2)*b + b^2 + 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(
4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 + 4*a*x + 1))/sqrt(b - 1), -1/2
*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(2*a*x^3 + x^4 + 2*b
*x^2 + 2*a*x + 1)/((a*x^2 + 2*(a^2 - b + 1)*x + a)*sqrt(-1/(b - 1
))))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\,dx$$
$$-\int \left(-\frac{1}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1
) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*
x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqr
t(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*sqrt(2*a*x**3 +
2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 +
 x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^2 + 1)),
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(
2*a*x + x^2 + 1)), x)
```

3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] time = 0.0976636, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rubi in Sympy [A] time = 4.06068, size = 17, normalized size = 0.77

$$\operatorname{atan}\left(\frac{x}{\sqrt{-x^2+\sqrt{x^4+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] atan(x/sqrt(-x**2 + sqrt(x**4 + 1)))
```

Mathematica [A] time = 1.38382, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+1}\frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Fricas [A] time = 0.654353, size = 77, normalized size = 3.5

$$-\frac{1}{4}\arctan\left(\frac{4\left(2x^3-\sqrt{x^4+1}x\right)\sqrt{-x^2+\sqrt{x^4+1}}}{9x^4-8\sqrt{x^4+1}x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(2*x^3 - sqrt(x^4 + 1)*x)*sqrt(-x^2 + sqrt(x^4 + 1)
)/(9*x^4 - 8*sqrt(x^4 + 1)*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.329 $\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx$

Optimal. Leaf size=24

$$\tan^{-1}\left(\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

Rubi [A] time = 0.10464, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\tan^{-1}\left(\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]

[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

Rubi in Sympy [A] time = 3.96785, size = 19, normalized size = 0.79

$$\operatorname{atan}\left(\frac{x}{\sqrt{-x^2+(x^{2n}+1)^{\frac{1}{n}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)

[Out] atan(x/sqrt(-x**2 + (x**(2*n) + 1)**(1/n)))
```

Mathematica [A] time = 0.0517336, size = 26, normalized size = 1.08

$$\cot^{-1}\left(\frac{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{1}{1+x^{2n}}\frac{1}{\sqrt{-x^{2}+\sqrt[n]{1+x^{2n}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

```
[Out] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^{2}+\left(x^{2n}+1\right)^{\left(\frac{1}{n}\right)}}\left(x^{2n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)),x, algorithm="maxima
```

```
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)),x, algorithm="fricas
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + (x^{2n} + 1)^{\frac{1}{n}}}\,(x^{2n} + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + (x^{2n} + 1)^{\left(\frac{1}{n}\right)}}\,(x^{2n} + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)),x, algorithm="giac"
```

```
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

3.330 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.012055, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.492407, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00288945, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.001, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2,x)

[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.40409, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.222455, size = 14, normalized size = 1.

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.034867, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.215665, size = 14, normalized size = 1.

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.331 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi [A] time = 0.01264, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi in Sympy [A] time = 0.648194, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

Mathematica [A] time = 0.00307696, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4} + \frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A] time = 0.003, size = 11, normalized size = 1.

$$\frac{\left(2+\left(\cos\left(x\right)\right)^{2}\right)\sin\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 1.3914, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin\left(x\right)^{3}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.222471, size = 14, normalized size = 1.27

$$\frac{1}{3}\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.056423, size = 8, normalized size = 0.73

$$-\frac{\sin^{3}\left(x\right)}{3}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

GIAC/XCAS [A] time = 0.202286, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin\left(x\right)^{3} + \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

3.332 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi [A] time = 0.0196434, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^4,x]

[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi in Sympy [A] time = 0.576694, size = 24, normalized size = 1.

$$\frac{3x}{8}-\frac{\sin^3(x)\cos(x)}{4}-\frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**4,x)

[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00306096, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0., size = 18, normalized size = 0.8

$$-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A] time = 1.36453, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.227922, size = 26, normalized size = 1.08

$$\frac{1}{8}\left(2\cos(x)^3-5\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.038342, size = 24, normalized size = 1.

$$\frac{3x}{8}-\frac{\sin^3(x)\cos(x)}{4}-\frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

GIAC/XCAS [A] time = 0.198787, size = 22, normalized size = 0.92

$$\frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.333 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x
]^5*Sin[x])/6
```

Rubi [A] time = 0.0326187, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x
]^5*Sin[x])/6
```

Rubi in Sympy [A] time = 0.671031, size = 36, normalized size = 1.06

$$\frac{5x}{16}+\frac{\sin(x)\cos^5(x)}{6}+\frac{5\sin(x)\cos^3(x)}{24}+\frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*co
s(x)/16
```

Mathematica [A] time = 0.00325647, size = 30, normalized size = 0.88

$$\frac{5x}{16}+\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)+\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]   Integrate[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A] time = 0.003, size = 24, normalized size = 0.7

$$\frac{\sin(x)}{6}\left((\cos(x))^5+\frac{5\ (\cos(x))^3}{4}+\frac{15\ \cos(x)}{8}\right)+\frac{5\,x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(cos(x)^6,x)
```

```
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x
```

Maxima [A] time = 1.44664, size = 32, normalized size = 0.94

$$-\frac{1}{48}\ \sin(2\,x)^3+\frac{5}{16}\,x+\frac{3}{64}\ \sin(4\,x)+\frac{1}{4}\ \sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(cos(x)^6,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.227172, size = 34, normalized size = 1.

$$\frac{1}{48}\left(8\ \cos(x)^5+10\ \cos(x)^3+15\ \cos(x)\right)\ \sin(x)+\frac{5}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(cos(x)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.038952, size = 36, normalized size = 1.06

$$\frac{5x}{16}+\frac{\sin(x)\cos^5(x)}{6}+\frac{5\sin(x)\cos^3(x)}{24}+\frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*co
s(x)/16
```

GIAC/XCAS [A] time = 0.199635, size = 30, normalized size = 0.88

$$\frac{5}{16}\,x + \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) + \frac{15}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

3.334 $\int \sin^8(x)\,dx$

Optimal. Leaf size=44

$$\frac{35x}{128}-\frac{1}{8}\sin^7(x)\cos(x)-\frac{7}{48}\sin^5(x)\cos(x)-\frac{35}{192}\sin^3(x)\cos(x)-\frac{35}{128}\sin(x)\cos(x)$$

```
[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 -
(7*Cos[x]*Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8
```

Rubi [A] time = 0.0390427, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{35x}{128}-\frac{1}{8}\sin^7(x)\cos(x)-\frac{7}{48}\sin^5(x)\cos(x)-\frac{35}{192}\sin^3(x)\cos(x)-\frac{35}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 -
(7*Cos[x]*Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8
```

Rubi in Sympy [A] time = 0.801034, size = 48, normalized size = 1.09

$$\frac{35x}{128}-\frac{\sin^7(x)\cos(x)}{8}-\frac{7\sin^5(x)\cos(x)}{48}-\frac{35\sin^3(x)\cos(x)}{192}-\frac{35\sin(x)\cos(x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**8,x)
```

```
[Out] 35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)
**3*cos(x)/192 - 35*sin(x)*cos(x)/128
```

Mathematica [A] time = 0.00329582, size = 38, normalized size = 0.86

$$\frac{35x}{128}-\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)-\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + S
in[8*x]/1024
```

Maple [A] time = 0.053, size = 30, normalized size = 0.7

$$-\frac{\cos(x)}{8}\left((\sin(x))^7+\frac{7\,(\sin(x))^5}{6}+\frac{35\,(\sin(x))^3}{24}+\frac{35\,\sin(x)}{16}\right)+\frac{35\,x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^8,x)
```

```
[Out] -1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+3
5/128*x
```

Maxima [A] time = 1.35111, size = 41, normalized size = 0.93

$$\frac{1}{24}\,\sin(2\,x)^3+\frac{35}{128}\,x+\frac{1}{1024}\,\sin(8\,x)+\frac{7}{128}\,\sin(4\,x)-\frac{1}{4}\,\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1
/4*sin(2*x)
```

Fricas [A] time = 0.228707, size = 42, normalized size = 0.95

$$\frac{1}{384}\left(48\,\cos(x)^7-200\,\cos(x)^5+326\,\cos(x)^3-279\,\cos(x)\right)\sin(x)+\frac{35}{128}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*si
n(x) + 35/128*x
```

Sympy [A] time = 0.041714, size = 48, normalized size = 1.09

$$\frac{35x}{128} - \frac{\sin^7(x)\cos(x)}{8} - \frac{7\sin^5(x)\cos(x)}{48} - \frac{35\sin^3(x)\cos(x)}{192} - \frac{35\sin(x)\cos(x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**8,x)
```

```
[Out] 35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)
**3*cos(x)/192 - 35*sin(x)*cos(x)/128
```

GIAC/XCAS [A] time = 0.199166, size = 38, normalized size = 0.86

$$\frac{35}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{96}\sin(6x) + \frac{7}{128}\sin(4x) - \frac{7}{32}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/3
2*sin(2*x)
```

3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=20

$$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 + Cos[x]/2 - (Cos[x]*Sin[x])/8
```

Rubi [B] time = 0.0289181, antiderivative size = 64, normalized size of antiderivative = 3.2, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{3x}{8}+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)+\frac{3}{4}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[Pi/4 + x/2]*Sin[Pi/4 + x/2])/4 + (Cos[Pi/4 + x/2
]^3*Sin[Pi/4 + x/2])/2
```

Rubi in Sympy [A] time = 0.735055, size = 44, normalized size = 2.2

$$\frac{3x}{8}+\frac{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)}{2}+\frac{3\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(1/4*pi+1/2*x)**4,x)
```

```
[Out] 3*x/8 + sin(x/2 + pi/4)*cos(x/2 + pi/4)**3/2 + 3*sin(x/2 + pi/4)*
cos(x/2 + pi/4)/4
```

Mathematica [A] time = 0.0211051, size = 21, normalized size = 1.05

$$\frac{1}{16}(6x+8\cos(x)-2\sin(x)\cos(x)+3\pi)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16
```

Maple [B] time = 0.01, size = 39, normalized size = 2.

$$\frac{1}{2}\left(\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^3+\frac{3}{2}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\,\pi}{16}+\frac{3\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(1/4*Pi+1/2*x)^4,x)
```

```
[Out] 1/2*(cos(1/4*Pi+1/2*x)^3+3/2*cos(1/4*Pi+1/2*x))*sin(1/4*Pi+1/2*x)
+3/16*Pi+3/8*x
```

Maxima [A] time = 1.44371, size = 31, normalized size = 1.55

$$\frac{3}{16}\,\pi+\frac{3}{8}\,x+\frac{1}{16}\,\sin\left(\pi+2\,x\right)+\frac{1}{2}\,\sin\left(\frac{1}{2}\,\pi+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi + 1/2*x)^4,x, algorithm="maxima")
```

```
[Out] 3/16*pi + 3/8*x + 1/16*sin(pi + 2*x) + 1/2*sin(1/2*pi + x)
```

Fricas [A] time = 0.23287, size = 50, normalized size = 2.5

$$\frac{1}{4}\left(2\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)^3+3\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)+\frac{3}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi + 1/2*x)^4,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(1/4*pi + 1/2*x)^3 + 3*cos(1/4*pi + 1/2*x))*sin(1/4*pi
+ 1/2*x) + 3/8*x
```

Sympy [A] time = 0.946761, size = 99, normalized size = 4.95

$$\frac{3x\sin^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3x\sin^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{3x\cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3\sin^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{5\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi+1/2*x)**4,x)
```

```
[Out] 3*x*sin(x/2 + pi/4)**4/8 + 3*x*sin(x/2 + pi/4)**2*cos(x/2 + pi/4)
**2/4 + 3*x*cos(x/2 + pi/4)**4/8 + 3*sin(x/2 + pi/4)**3*cos(x/2 +
 pi/4)/4 + 5*sin(x/2 + pi/4)*cos(x/2 + pi/4)**3/4
```

GIAC/XCAS [A] time = 0.201193, size = 19, normalized size = 0.95

$$\frac{3}{8}\,x+\frac{1}{2}\,\cos\left(x\right)-\frac{1}{16}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi + 1/2*x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/2*cos(x) - 1/16*sin(2*x)
```

3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

```
[Out] -Cos[Pi/12 - 3*x]/3 + Cos[Pi/12 - 3*x]^3/9
```

Rubi [A] time = 0.0205327, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[-Sin[Pi/12 - 3*x]^3,x]

[Out] -Cos[Pi/12 - 3*x]/3 + Cos[Pi/12 - 3*x]^3/9
```

Rubi in Sympy [A] time = 0.923016, size = 24, normalized size = 0.77

$$\frac{\sin^3\left(3x+\frac{5\pi}{12}\right)}{9}-\frac{\sin\left(3x+\frac{5\pi}{12}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-cos(5/12*pi+3*x)**3,x)

[Out] sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)/3
```

Mathematica [A] time = 0.018904, size = 31, normalized size = 1.

$$\frac{1}{36}\cos\left(3\left(\frac{\pi}{12}-3x\right)\right)-\frac{1}{4}\cos\left(\frac{\pi}{12}-3x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] -Cos[Pi/12 - 3*x]/4 + Cos[3*(Pi/12 - 3*x)]/36
```

Maple [A] time = 0.01, size = 23, normalized size = 0.7

$$-\frac{1}{9}\left(2+\left(\cos\left(\frac{5\,\pi}{12}+3\,x\right)\right)^{2}\right)\sin\left(\frac{5\,\pi}{12}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-cos(5/12*Pi+3*x)^3,x)
```

```
[Out] -1/9*(2+cos(5/12*Pi+3*x)^2)*sin(5/12*Pi+3*x)
```

Maxima [A] time = 1.36815, size = 31, normalized size = 1.

$$\frac{1}{9}\,\sin\left(\frac{5}{12}\,\pi+3\,x\right)^{3}-\frac{1}{3}\,\sin\left(\frac{5}{12}\,\pi+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(5/12*pi + 3*x)^3,x, algorithm="maxima")
```

```
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

Fricas [A] time = 0.224658, size = 30, normalized size = 0.97

$$-\frac{1}{9}\left(\cos\left(\frac{5}{12}\,\pi+3\,x\right)^{2}+2\right)\sin\left(\frac{5}{12}\,\pi+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(5/12*pi + 3*x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)
```

Sympy [A] time = 0.428717, size = 39, normalized size = 1.26

$$-\frac{2\sin^{3}\left(3x+\frac{5\pi}{12}\right)}{9}-\frac{\sin\left(3x+\frac{5\pi}{12}\right)\cos^{2}\left(3x+\frac{5\pi}{12}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(5/12*pi+3*x)**3,x)
```

```
[Out] -2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12
)**2/3
```

GIAC/XCAS [A] time = 0.199541, size = 31, normalized size = 1.

$$\frac{1}{9}\,\sin\left(\frac{5}{12}\,\pi + 3\,x\right)^{3} - \frac{1}{3}\,\sin\left(\frac{5}{12}\,\pi + 3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-cos(5/12*pi + 3*x)^3,x, algorithm="giac")
```

```
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

3.337 $\int \csc^6(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rubi [A] time = 0.016691, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^6,x]

[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rubi in Sympy [A] time = 0.624918, size = 32, normalized size = 1.52

$$-\frac{8\cos(x)}{15\sin(x)}-\frac{4\cos(x)}{15\sin^3(x)}-\frac{\cos(x)}{5\sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sin(x)**6,x)

[Out] -8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x
)**5)
```

Mathematica [A] time = 0.00484198, size = 27, normalized size = 1.29

$$-\frac{8\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)-\frac{4}{15}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^6,x]
```

```
[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$\left(-\frac{8}{15}-\frac{(\csc(x))^4}{5}-\frac{4\,(\csc(x))^2}{15}\right)\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sin(x)^6,x)
```

```
[Out] (-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)
```

Maxima [A] time = 1.3529, size = 27, normalized size = 1.29

$$-\frac{15\,\tan(x)^4+10\,\tan(x)^2+3}{15\,\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-6),x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

Fricas [A] time = 0.21989, size = 50, normalized size = 2.38

$$-\frac{8\,\cos(x)^5-20\,\cos(x)^3+15\,\cos(x)}{15\left(\cos(x)^4-2\,\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-6),x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x
)^2 + 1)*sin(x))
```

Sympy [A] time = 0.041453, size = 32, normalized size = 1.52

$$-\frac{8\cos(x)}{15\sin(x)}-\frac{4\cos(x)}{15\sin^3(x)}-\frac{\cos(x)}{5\sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x)**6,x)
```

```
[Out] -8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x
)**5)
```

GIAC/XCAS [A] time = 0.200622, size = 27, normalized size = 1.29

$$-\frac{15 \tan(x)^4 + 10 \tan(x)^2 + 3}{15 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-6),x, algorithm="giac")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

3.338 $\int \csc^7(x)\,dx$

Optimal. Leaf size=36

$$-\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot(x)\csc^5(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{5}{16}\cot(x)\csc(x)$$

```
[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]
^3)/24 - (Cot[x]*Csc[x]^5)/6
```

Rubi [A] time = 0.0329247, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot(x)\csc^5(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{5}{16}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^7,x]
```

```
[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]
^3)/24 - (Cot[x]*Csc[x]^5)/6
```

Rubi in Sympy [A] time = 0.700672, size = 42, normalized size = 1.17

$$-\frac{5\,\mathrm{atanh}\,(\cos(x))}{16}-\frac{5\cos(x)}{16\sin^2(x)}-\frac{5\cos(x)}{24\sin^4(x)}-\frac{\cos(x)}{6\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(csc(x)**7,x)
```

```
[Out] -5*atanh(cos(x))/16 - 5*cos(x)/(16*sin(x)**2) - 5*cos(x)/(24*sin(
x)**4) - cos(x)/(6*sin(x)**6)
```

Mathematica [B] time = 0.0106596, size = 95, normalized size = 2.64

$$-\frac{1}{384}\csc^6\left(\frac{x}{2}\right)-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{5}{64}\csc^2\left(\frac{x}{2}\right)+\frac{1}{384}\sec^6\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{5}{64}\sec^2\left(\frac{x}{2}\right)+\frac{5}{16}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{5}{16}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^7,x]
```

```
[Out] (-5*Csc[x/2]^2)/64 - Csc[x/2]^4/64 - Csc[x/2]^6/384 - (5*Log[Cos[
x/2]])/16 + (5*Log[Sin[x/2]])/16 + (5*Sec[x/2]^2)/64 + Sec[x/2]^4
/64 + Sec[x/2]^6/384
```

Maple [A] time = 0.054, size = 32, normalized size = 0.9

$$\left(-\frac{(\csc(x))^5}{6} - \frac{5\,(\csc(x))^3}{24} - \frac{5\,\csc(x)}{16}\right)\cot(x) + \frac{5\,\ln(\csc(x) - \cot(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csc(x)^7,x)
```

```
[Out] (-1/6*csc(x)^5-5/24*csc(x)^3-5/16*csc(x))*cot(x)+5/16*ln(csc(x)-c
ot(x))
```

Maxima [A] time = 1.55237, size = 73, normalized size = 2.03

$$\frac{15\,\cos(x)^5 - 40\,\cos(x)^3 + 33\,\cos(x)}{48\left(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1\right)} - \frac{5}{32}\,\log(\cos(x) + 1) + \frac{5}{32}\,\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^7,x, algorithm="maxima")
```

```
[Out] 1/48*(15*cos(x)^5 - 40*cos(x)^3 + 33*cos(x))/(cos(x)^6 - 3*cos(x)
^4 + 3*cos(x)^2 - 1) - 5/32*log(cos(x) + 1) + 5/32*log(cos(x) - 1
)
```

Fricas [A] time = 0.235632, size = 126, normalized size = 3.5

$$\frac{30\,\cos(x)^5 - 80\,\cos(x)^3 - 15\left(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\,\cos(x) + \frac{1}{2}\right) + 15\left(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos\right.}{96\left(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(30*cos(x)^5 - 80*cos(x)^3 - 15*(cos(x)^6 - 3*cos(x)^4 + 3*c
os(x)^2 - 1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^6 - 3*cos(x)^4 +
3*cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2) + 66*cos(x))/(cos(x)^6 - 3
*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.213763, size = 60, normalized size = 1.67

$$\frac{15\cos^5(x) - 40\cos^3(x) + 33\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{5\log(\cos(x) - 1)}{32} - \frac{5\log(\cos(x) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)**7,x)
```

```
[Out] (15*cos(x)**5 - 40*cos(x)**3 + 33*cos(x))/(48*cos(x)**6 - 144*cos
(x)**4 + 144*cos(x)**2 - 48) + 5*log(cos(x) - 1)/32 - 5*log(cos(x
) + 1)/32
```

GIAC/XCAS [A] time = 0.204668, size = 151, normalized size = 4.19

$$-\frac{\left(\frac{9(\cos(x)-1)}{\cos(x)+1} - \frac{45(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{110(\cos(x)-1)^3}{(\cos(x)+1)^3} - 1\right)(\cos(x)+1)^3}{384(\cos(x)-1)^3} - \frac{15(\cos(x)-1)}{128(\cos(x)+1)} + \frac{3(\cos(x)-1)^2}{128(\cos(x)+1)^2} - \frac{(\cos(x)-1)^3}{384(\cos(x)+1)^3} + \frac{5}{32}\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^7,x, algorithm="giac")
```

```
[Out] -1/384*(9*(cos(x) - 1)/(cos(x) + 1) - 45*(cos(x) - 1)^2/(cos(x) +
 1)^2 + 110*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1)*(cos(x) + 1)^3/(co
s(x) - 1)^3 - 15/128*(cos(x) - 1)/(cos(x) + 1) + 3/128*(cos(x) -
1)^2/(cos(x) + 1)^2 - 1/384*(cos(x) - 1)^3/(cos(x) + 1)^3 + 5/32*
ln(-(cos(x) - 1)/(cos(x) + 1))
```

3.339 $\int \sec^{12}(x)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x
]^9)/9 + Tan[x]^11/11
```

Rubi [A] time = 0.0298864, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^12,x]
```

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x
]^9)/9 + Tan[x]^11/11
```

Rubi in Sympy [A] time = 1.01575, size = 66, normalized size = 1.61

$$\frac{256\sin(x)}{693\cos(x)}+\frac{128\sin(x)}{693\cos^3(x)}+\frac{32\sin(x)}{231\cos^5(x)}+\frac{80\sin(x)}{693\cos^7(x)}+\frac{10\sin(x)}{99\cos^9(x)}+\frac{\sin(x)}{11\cos^{11}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/cos(x)**12,x)
```

```
[Out] 256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/
(231*cos(x)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x
)**9) + sin(x)/(11*cos(x)**11)
```

Mathematica [A] time = 0.00694843, size = 57, normalized size = 1.39

$$\frac{256\tan(x)}{693}+\frac{1}{11}\tan(x)\sec^{10}(x)+\frac{10}{99}\tan(x)\sec^8(x)+\frac{80}{693}\tan(x)\sec^6(x)+\frac{32}{231}\tan(x)\sec^4(x)+\frac{128}{693}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^12,x]
```

```
[Out] (256*Tan[x])/693 + (128*Sec[x]^2*Tan[x])/693 + (32*Sec[x]^4*Tan[x
])/231 + (80*Sec[x]^6*Tan[x])/693 + (10*Sec[x]^8*Tan[x])/99 + (Se
c[x]^10*Tan[x])/11
```

Maple [A] time = 0.052, size = 37, normalized size = 0.9

$$-\left(-\frac{256}{693}-\frac{(\sec(x))^{10}}{11}-\frac{10\ (\sec(x))^{8}}{99}-\frac{80\ (\sec(x))^{6}}{693}-\frac{32\ (\sec(x))^{4}}{231}-\frac{128\ (\sec(x))^{2}}{693}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(x)^12,x)
```

```
[Out] -(-256/693-1/11*sec(x)^10-10/99*sec(x)^8-80/693*sec(x)^6-32/231*s
ec(x)^4-128/693*sec(x)^2)*tan(x)
```

Maxima [A] time = 1.57325, size = 45, normalized size = 1.1

$$\frac{1}{11}\tan(x)^{11}+\frac{5}{9}\tan(x)^{9}+\frac{10}{7}\tan(x)^{7}+2\tan(x)^{5}+\frac{5}{3}\tan(x)^{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(-12),x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*
tan(x)^3 + tan(x)
```

Fricas [A] time = 0.217848, size = 54, normalized size = 1.32

$$\frac{\left(256\cos(x)^{10}+128\cos(x)^{8}+96\cos(x)^{6}+80\cos(x)^{4}+70\cos(x)^{2}+63\right)\sin(x)}{693\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(-12),x, algorithm="fricas")
```

```
[Out] 1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 +
 70*cos(x)^2 + 63)*sin(x)/cos(x)^11
```

Sympy [A] time = 0.043031, size = 66, normalized size = 1.61

$$\frac{256\sin(x)}{693\cos(x)} + \frac{128\sin(x)}{693\cos^3(x)} + \frac{32\sin(x)}{231\cos^5(x)} + \frac{80\sin(x)}{693\cos^7(x)} + \frac{10\sin(x)}{99\cos^9(x)} + \frac{\sin(x)}{11\cos^{11}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x)**12,x)
```

```
[Out] 256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/
(231*cos(x)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x
)**9) + sin(x)/(11*cos(x)**11)
```

GIAC/XCAS [A] time = 0.199023, size = 45, normalized size = 1.1

$$\frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(-12),x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*
tan(x)^3 + tan(x)
```

3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$

Optimal. Leaf size=40

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Rubi [A] time = 0.0218456, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[Pi/4 + 3*x]^3,x]

[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Rubi in Sympy [A] time = 0.636444, size = 31, normalized size = 0.78

$$\frac{\sin\left(3x+\frac{\pi}{4}\right)}{6\cos^2\left(3x+\frac{\pi}{4}\right)}+\frac{\operatorname{atanh}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(1/4*pi+3*x)**3,x)

[Out] sin(3*x + pi/4)/(6*cos(3*x + pi/4)**2) + atanh(sin(3*x + pi/4))/6
```

Mathematica [A] time = 0.205425, size = 69, normalized size = 1.72

$$\frac{1}{24}\left(-\csc^2\left(\frac{3}{8}(4x+\pi)\right)+\sec^2\left(\frac{3}{8}(4x+\pi)\right)+4\log\left(\sqrt{2}\sin\left(\frac{3}{8}(4x+\pi)\right)\right)-4\log\left(\sqrt{2}\cos\left(\frac{3}{8}(4x+\pi)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] (-Csc[(3*(Pi + 4*x))/8]^2 - 4*Log[Sqrt[2]*Cos[(3*(Pi + 4*x))/8]]
+ 4*Log[Sqrt[2]*Sin[(3*(Pi + 4*x))/8]] + Sec[(3*(Pi + 4*x))/8]^2)
/24
```

Maple [A] time = 0.053, size = 40, normalized size = 1.

$$\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)+\frac{1}{6}\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(1/4*Pi+3*x)^3,x)
```

```
[Out] 1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/
4*Pi+3*x))
```

Maxima [A] time = 1.46276, size = 69, normalized size = 1.72

$$-\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi + 3*x)^(-3),x, algorithm="maxima")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1
/4*pi + 3*x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)
```

Fricas [A] time = 0.226466, size = 95, normalized size = 2.38

$$\frac{\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)+2\sin\left(\frac{1}{4}\pi+3x\right)}{12\cos\left(\frac{1}{4}\pi+3x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi + 3*x)^(-3),x, algorithm="fricas")
```

```
[Out] 1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi
 + 3*x)^2*log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(
1/4*pi + 3*x)^2
```

Sympy [A] time = 2.4366, size = 388, normalized size = 9.7

$$-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}-\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{2\tan^3\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{2\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(1/4*pi+3*x)**3,x)
```

```
[Out] -log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + p
i/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8)
- 1)*tan(3*x/2 + pi/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2
+ pi/8)**2 + 6) - log(tan(3*x/2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)
**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x/2 + pi/8) + 1)*t
an(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8
)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**2/(6*
tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x
/2 + pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2
 + 6) + 2*tan(3*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3
*x/2 + pi/8)**2 + 6) + 2*tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**
4 - 12*tan(3*x/2 + pi/8)**2 + 6)
```

GIAC/XCAS [A] time = 0.204239, size = 72, normalized size = 1.8

$$-\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\ln\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\ln\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/4*pi + 3*x)^(-3),x, algorithm="giac")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*ln(sin(1/
4*pi + 3*x) + 1) - 1/12*ln(-sin(1/4*pi + 3*x) + 1)
```

3.341 $\int \tan^6(x)\,dx$

Optimal. Leaf size=22

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0230807, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rubi in Sympy [A] time = 0.53301, size = 17, normalized size = 0.77

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**6,x)
```

```
[Out] -x + tan(x)**5/5 - tan(x)**3/3 + tan(x)
```

Mathematica [A] time = 0.00637214, size = 30, normalized size = 1.36

$$-x+\frac{23\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{11}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^6,x]
```

```
[Out] -x + (23*Tan[x])/15 - (11*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])
/5
```

Maple [A] time = 0.004, size = 19, normalized size = 0.9

$$-x + \tan(x) - \frac{(\tan(x))^3}{3} + \frac{(\tan(x))^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x)
```

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Maxima [A] time = 1.61045, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Fricas [A] time = 0.218354, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Sympy [A] time = 0.060743, size = 31, normalized size = 1.41

$$-x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**6,x)
```

```
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/c
os(x)
```

GIAC/XCAS [A] time = 0.20299, size = 24, normalized size = 1.09

$$\frac{1}{5} \tan(x)^5 - \frac{1}{3} \tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^6,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.342 $\int \cot^5(x)\, dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\cot^4(x) + \frac{\cot^2(x)}{2} + \log(\sin(x))$$

```
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

Rubi [A] time = 0.024199, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{4}\cot^4(x) + \frac{\cot^2(x)}{2} + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^5,x]
```

```
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.529923, size = 20, normalized size = 1.

$$\log(\sin(x)) + \frac{1}{2\tan^2(x)} - \frac{1}{4\tan^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/tan(x)**5,x)
```

```
[Out] log(sin(x)) + 1/(2*tan(x)**2) - 1/(4*tan(x)**4)
```

Mathematica [A] time = 0.00550819, size = 16, normalized size = 0.8

$$-\frac{1}{4}\csc^4(x) + \csc^2(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^5,x]
```

```
[Out] Csc[x]^2 - Csc[x]^4/4 + Log[Sin[x]]
```

Maple [A] time = 0.01, size = 26, normalized size = 1.3

$$-\frac{\ln\left(1+(\tan(x))^2\right)}{2}-\frac{1}{4\,(\tan(x))^4}+\ln(\tan(x))+\frac{1}{2\,(\tan(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/tan(x)^5,x)
```

```
[Out] -1/2*ln(1+tan(x)^2)-1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2
```

Maxima [A] time = 1.44574, size = 30, normalized size = 1.5

$$\frac{4\,\sin(x)^2-1}{4\,\sin(x)^4}+\frac{1}{2}\,\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^(-5),x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)
```

Fricas [A] time = 0.214313, size = 54, normalized size = 2.7

$$\frac{2\,\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^4+3\,\tan(x)^4+2\,\tan(x)^2-1}{4\,\tan(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^(-5),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan
(x)^2 - 1)/tan(x)^4
```

Sympy [A] time = 0.114924, size = 19, normalized size = 0.95

$$\frac{4\sin^2(x)-1}{4\sin^4(x)}+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(x)**5,x)
```

```
[Out] (4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))
```

GIAC/XCAS [A] time = 0.208124, size = 50, normalized size = 2.5

$$-\frac{3\tan(x)^4 - 2\tan(x)^2 + 1}{4\tan(x)^4} - \frac{1}{2}\ln\left(\tan(x)^2 + 1\right) + \frac{1}{2}\ln\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^(-5),x, algorithm="giac")
```

```
[Out] -1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*ln(tan(x)^2 + 1
) + 1/2*ln(tan(x)^2)
```

3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$

Optimal. Leaf size=32

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

```
[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

Rubi [A] time = 0.0252111, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[Pi/4 + x/3]^4,x]

[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

Rubi in Sympy [A] time = 0.581904, size = 22, normalized size = 0.69

$$x+\frac{3}{\tan\left(\frac{x}{3}+\frac{\pi}{4}\right)}-\frac{1}{\tan^3\left(\frac{x}{3}+\frac{\pi}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(1/4*pi+1/3*x)**4,x)

[Out] x + 3/tan(x/3 + pi/4) - 1/tan(x/3 + pi/4)**3
```

Mathematica [A] time = 0.0857138, size = 37, normalized size = 1.16

$$x-\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)\left(\csc^2\left(\frac{x}{3}+\frac{\pi}{4}\right)-4\right)+\frac{3\pi}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[Pi/4 + x/3]^4,x]

[Out] (3*Pi)/4 + x - Cot[Pi/4 + x/3]*(-4 + Csc[Pi/4 + x/3]^2)
```

Maple [A] time = 0.004, size = 28, normalized size = 0.9

$$-\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)^{3}+3\cot\left(\pi/4+x/3\right)-\frac{3\pi}{4}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(1/4*Pi+1/3*x)^4,x)
```

```
[Out] -cot(1/4*Pi+1/3*x)^3+3*cot(1/4*Pi+1/3*x)-3/4*Pi+x
```

Maxima [A] time = 1.54385, size = 41, normalized size = 1.28

$$\frac{3}{4}\pi+x+\frac{3\tan\left(\frac{1}{4}\pi+\frac{1}{3}x\right)^{2}-1}{\tan\left(\frac{1}{4}\pi+\frac{1}{3}x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(1/4*pi + 1/3*x)^4,x, algorithm="maxima")
```

```
[Out] 3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3
```

Fricas [A] time = 0.22082, size = 95, normalized size = 2.97

$$\frac{4\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)^{2}+\left(x\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-x\right)\sin\left(\frac{1}{2}\pi+\frac{2}{3}x\right)+2\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-2}{\left(\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-1\right)\sin\left(\frac{1}{2}\pi+\frac{2}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(1/4*pi + 1/3*x)^4,x, algorithm="fricas")
```

```
[Out] (4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi
 + 2/3*x) + 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)
*sin(1/2*pi + 2/3*x))
```

Sympy [A] time = 0.236, size = 20, normalized size = 0.62

$$x-\cot^{3}\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(1/4*pi+1/3*x)**4,x)
```

```
[Out] x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)
```

GIAC/XCAS [A] time = 0.20905, size = 72, normalized size = 2.25

$$\frac{3}{4}\,\pi + \frac{1}{8}\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^{3} + x + \frac{15\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^{2} - 1}{8\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^{3}} - \frac{15}{8}\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(1/4*pi + 1/3*x)^4,x, algorithm="giac")
```

```
[Out] 3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6
*x)^2 - 1)/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)
```

3.344 $\int \cos^6(x)\sin^4(x)\,dx$

Optimal. Leaf size=56

$$\frac{3x}{256}-\frac{1}{10}\sin^3(x)\cos^7(x)-\frac{3}{80}\sin(x)\cos^7(x)+\frac{1}{160}\sin(x)\cos^5(x)+\frac{1}{128}\sin(x)\cos^3(x)+\frac{3}{256}\sin(x)\cos(x)$$

```
[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[
x]^5*Sin[x])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/1
0
```

Rubi [A] time = 0.0979612, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{3x}{256}-\frac{1}{10}\sin^3(x)\cos^7(x)-\frac{3}{80}\sin(x)\cos^7(x)+\frac{1}{160}\sin(x)\cos^5(x)+\frac{1}{128}\sin(x)\cos^3(x)+\frac{3}{256}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[
x]^5*Sin[x])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/1
0
```

Rubi in Sympy [A] time = 3.10782, size = 58, normalized size = 1.04

$$\frac{3x}{256}-\frac{\sin^3(x)\cos^7(x)}{10}-\frac{3\sin(x)\cos^7(x)}{80}+\frac{\sin(x)\cos^5(x)}{160}+\frac{\sin(x)\cos^3(x)}{128}+\frac{3\sin(x)\cos(x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**6*sin(x)**4,x)
```

```
[Out] 3*x/256 - sin(x)**3*cos(x)**7/10 - 3*sin(x)*cos(x)**7/80 + sin(x)
*cos(x)**5/160 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

Mathematica [A] time = 0.0234356, size = 46, normalized size = 0.82

$$\frac{3x}{256}+\frac{1}{512}\sin(2x)-\frac{1}{256}\sin(4x)-\frac{\sin(6x)}{1024}+\frac{\sin(8x)}{2048}+\frac{\sin(10x)}{5120}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x
]/2048 + Sin[10*x]/5120
```

Maple [A] time = 0.009, size = 42, normalized size = 0.8

$$-\frac{(\cos(x))^7(\sin(x))^3}{10}-\frac{3(\cos(x))^7\sin(x)}{80}+\frac{\sin(x)}{160}\left((\cos(x))^5+\frac{5(\cos(x))^3}{4}+\frac{15\cos(x)}{8}\right)+\frac{3x}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^6*sin(x)^4,x)
```

```
[Out] -1/10*cos(x)^7*sin(x)^3-3/80*cos(x)^7*sin(x)+1/160*(cos(x)^5+5/4*
cos(x)^3+15/8*cos(x))*sin(x)+3/256*x
```

Maxima [A] time = 1.36461, size = 32, normalized size = 0.57

$$\frac{1}{320}\sin(2x)^5+\frac{3}{256}x+\frac{1}{2048}\sin(8x)-\frac{1}{256}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/320*sin(2*x)^5 + 3/256*x + 1/2048*sin(8*x) - 1/256*sin(4*x)
```

Fricas [A] time = 0.307835, size = 50, normalized size = 0.89

$$\frac{1}{1280}\left(128\cos(x)^9-176\cos(x)^7+8\cos(x)^5+10\cos(x)^3+15\cos(x)\right)\sin(x)+\frac{3}{256}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(128*cos(x)^9 - 176*cos(x)^7 + 8*cos(x)^5 + 10*cos(x)^3 +
15*cos(x))*sin(x) + 3/256*x
```

Sympy [A] time = 0.046293, size = 56, normalized size = 1.

$$\frac{3x}{256} + \frac{\sin(x)\cos^9(x)}{10} - \frac{11\sin(x)\cos^7(x)}{80} + \frac{\sin(x)\cos^5(x)}{160} + \frac{\sin(x)\cos^3(x)}{128} + \frac{3\sin(x)\cos(x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**6*sin(x)**4,x)
```

```
[Out] 3*x/256 + sin(x)*cos(x)**9/10 - 11*sin(x)*cos(x)**7/80 + sin(x)*c
os(x)**5/160 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

GIAC/XCAS [A] time = 0.199761, size = 46, normalized size = 0.82

$$\frac{3}{256}\,x + \frac{1}{5120}\,\sin(10\,x) + \frac{1}{2048}\,\sin(8\,x) - \frac{1}{1024}\,\sin(6\,x) - \frac{1}{256}\,\sin(4\,x) + \frac{1}{512}\,\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) -
1/256*sin(4*x) + 1/512*sin(2*x)
```

3.345 $\int \cos^6(x)\sin^7(x)\,dx$

Optimal. Leaf size=33

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

```
[Out] -Cos[x]^7/7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

Rubi [A] time = 0.0555727, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^6*Sin[x]^7,x]

[Out] -Cos[x]^7/7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

Rubi in Sympy [A] time = 3.256, size = 27, normalized size = 0.82

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**6*sin(x)**7,x)

[Out] cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

Mathematica [A] time = 0.034187, size = 55, normalized size = 1.67

$$-\frac{5\cos(x)}{1024}-\frac{5\cos(3x)}{4096}+\frac{3\cos(5x)}{4096}+\frac{3\cos(7x)}{14336}-\frac{\cos(9x)}{6144}-\frac{\cos(11x)}{45056}+\frac{\cos(13x)}{53248}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^6*Sin[x]^7,x]
```

```
[Out] (-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos
[7*x])/14336 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

Maple [A] time = 0.012, size = 38, normalized size = 1.2

$$-\frac{(\cos(x))^7 (\sin(x))^6}{13} - \frac{6\ (\sin(x))^4 (\cos(x))^7}{143} - \frac{8\ (\sin(x))^2 (\cos(x))^7}{429} - \frac{16\ (\cos(x))^7}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^6*sin(x)^7,x)
```

```
[Out] -1/13*cos(x)^7*sin(x)^6-6/143*sin(x)^4*cos(x)^7-8/429*sin(x)^2*co
s(x)^7-16/3003*cos(x)^7
```

Maxima [A] time = 1.42244, size = 34, normalized size = 1.03

$$\frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")
```

```
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Fricas [A] time = 0.234401, size = 34, normalized size = 1.03

$$\frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")
```

```
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Sympy [A] time = 0.052812, size = 27, normalized size = 0.82

$$\frac{\cos^{13}(x)}{13} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^9(x)}{3} - \frac{\cos^7(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**6*sin(x)**7,x)
```

```
[Out] cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

GIAC/XCAS [A] time = 0.202166, size = 34, normalized size = 1.03

$$\frac{1}{13} \cos\left(x\right)^{13} - \frac{3}{11} \cos\left(x\right)^{11} + \frac{1}{3} \cos\left(x\right)^{9} - \frac{1}{7} \cos\left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^7,x, algorithm="giac")
```

```
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

3.346 $\int \sin^{10}(x)\tan(x)\,dx$

Optimal. Leaf size=46

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8
 + Cos[x]^10/10 - Log[Cos[x]]
```

Rubi [A] time = 0.0514238, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^10*Tan[x],x]
```

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8
 + Cos[x]^10/10 - Log[Cos[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(-\sin^2(x)+1\right)}{2}-\frac{\sin^{10}(x)}{10}-\frac{\sin^8(x)}{8}-\frac{\sin^6(x)}{6}-\frac{\sin^2(x)}{2}-\frac{\int^{\sin^2(x)} x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**11/cos(x),x)
```

```
[Out] -log(-sin(x)**2 + 1)/2 - sin(x)**10/10 - sin(x)**8/8 - sin(x)**6/
6 - sin(x)**2/2 - Integral(x, (x, sin(x)**2))/2
```

Mathematica [A] time = 0.00744216, size = 46, normalized size = 1.

$$\frac{281}{512}\cos(2x)-\frac{29}{256}\cos(4x)+\frac{67\cos(6x)}{3072}-\frac{3\cos(8x)}{1024}+\frac{\cos(10x)}{5120}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^10*Tan[x],x]
```

```
[Out] (281*Cos[2*x])/512 - (29*Cos[4*x])/256 + (67*Cos[6*x])/3072 - (3*
Cos[8*x])/1024 + Cos[10*x]/5120 - Log[Cos[x]]
```

Maple [A] time = 0.016, size = 37, normalized size = 0.8

$$-\frac{(\sin(x))^{10}}{10}-\frac{(\sin(x))^{8}}{8}-\frac{(\sin(x))^{6}}{6}-\frac{(\sin(x))^{4}}{4}-\frac{(\sin(x))^{2}}{2}-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^11/cos(x),x)
```

```
[Out] -1/10*sin(x)^10-1/8*sin(x)^8-1/6*sin(x)^6-1/4*sin(x)^4-1/2*sin(x)
^2-ln(cos(x))
```

Maxima [A] time = 1.39123, size = 54, normalized size = 1.17

$$-\frac{1}{10}\sin(x)^{10}-\frac{1}{8}\sin(x)^{8}-\frac{1}{6}\sin(x)^{6}-\frac{1}{4}\sin(x)^{4}-\frac{1}{2}\sin(x)^{2}-\frac{1}{2}\log\left(\sin(x)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^11/cos(x),x, algorithm="maxima")
```

```
[Out] -1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/
2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 0.232479, size = 51, normalized size = 1.11

$$\frac{1}{10}\cos(x)^{10}-\frac{5}{8}\cos(x)^{8}+\frac{5}{3}\cos(x)^{6}-\frac{5}{2}\cos(x)^{4}+\frac{5}{2}\cos(x)^{2}-\log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^11/cos(x),x, algorithm="fricas")
```

```
[Out] 1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2
*cos(x)^2 - log(-cos(x))
```

Sympy [A] time = 0.100526, size = 44, normalized size = 0.96

$$-\log(\cos(x)) + \frac{\cos^{10}(x)}{10} - \frac{5\cos^{8}(x)}{8} + \frac{5\cos^{6}(x)}{3} - \frac{5\cos^{4}(x)}{2} + \frac{5\cos^{2}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**11/cos(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**10/10 - 5*cos(x)**8/8 + 5*cos(x)**6/3 - 5*
cos(x)**4/2 + 5*cos(x)**2/2
```

GIAC/XCAS [A] time = 0.204104, size = 51, normalized size = 1.11

$$\frac{1}{10}\cos(x)^{10} - \frac{5}{8}\cos(x)^{8} + \frac{5}{3}\cos(x)^{6} - \frac{5}{2}\cos(x)^{4} + \frac{5}{2}\cos(x)^{2} - \frac{1}{2}\ln\left(\cos(x)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^11/cos(x),x, algorithm="giac")
```

```
[Out] 1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2
*cos(x)^2 - 1/2*ln(cos(x)^2)
```

3.347 $\int \csc^6(x)\sec^6(x)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

```
[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^
3)/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0587911, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^
3)/3 + Tan[x]^5/5
```

Rubi in Sympy [A] time = 4.30726, size = 71, normalized size = 1.73

$$\frac{256\sin(x)}{15\cos(x)}+\frac{128\sin(x)}{15\cos^3(x)}+\frac{32\sin(x)}{5\cos^5(x)}-\frac{16}{3\sin(x)\cos^5(x)}-\frac{2}{3\sin^3(x)\cos^5(x)}-\frac{1}{5\sin^5(x)\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(x)**6/sin(x)**6,x)
```

```
[Out] 256*sin(x)/(15*cos(x)) + 128*sin(x)/(15*cos(x)**3) + 32*sin(x)/(5
*cos(x)**5) - 16/(3*sin(x)*cos(x)**5) - 2/(3*sin(x)**3*cos(x)**5)
 - 1/(5*sin(x)**5*cos(x)**5)
```

Mathematica [A] time = 0.0196556, size = 53, normalized size = 1.29

$$\frac{128\tan(x)}{15}-\frac{128\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)-\frac{19}{15}\cot(x)\csc^2(x)+\frac{1}{5}\tan(x)\sec^4(x)+\frac{19}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] (-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
+ (128*Tan[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.02, size = 56, normalized size = 1.4

$$\frac{1}{5\left(\cos\left(x\right)\right)^{5}\left(\sin\left(x\right)\right)^{5}}-\frac{2}{5\left(\sin\left(x\right)\right)^{5}\left(\cos\left(x\right)\right)^{3}}+\frac{16}{15\left(\cos\left(x\right)\right)^{3}\left(\sin\left(x\right)\right)^{3}}-\frac{32}{15\cos\left(x\right)\left(\sin\left(x\right)\right)^{3}}+\frac{128}{15\cos\left(x\right)\sin\left(x\right)}-\frac{256\cot\left(x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(x)^6/sin(x)^6,x)
```

```
[Out] 1/5/sin(x)^5/cos(x)^5-2/5/sin(x)^5/cos(x)^3+16/15/sin(x)^3/cos(x)
^3-32/15/sin(x)^3/cos(x)+128/15/sin(x)/cos(x)-256/15*cot(x)
```

Maxima [A] time = 1.41687, size = 50, normalized size = 1.22

$$\frac{1}{5}\tan\left(x\right)^{5}+\frac{5}{3}\tan\left(x\right)^{3}-\frac{150\tan\left(x\right)^{4}+25\tan\left(x\right)^{2}+3}{15\tan\left(x\right)^{5}}+10\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^6*sin(x)^6),x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 5/3*tan(x)^3 - 1/15*(150*tan(x)^4 + 25*tan(x)^2 +
3)/tan(x)^5 + 10*tan(x)
```

Fricas [A] time = 0.218421, size = 74, normalized size = 1.8

$$-\frac{256\cos\left(x\right)^{10}-640\cos\left(x\right)^{8}+480\cos\left(x\right)^{6}-80\cos\left(x\right)^{4}-10\cos\left(x\right)^{2}-3}{15\left(\cos\left(x\right)^{9}-2\cos\left(x\right)^{7}+\cos\left(x\right)^{5}\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^6*sin(x)^6),x, algorithm="fricas")
```

```
[Out] -1/15*(256*cos(x)^10 - 640*cos(x)^8 + 480*cos(x)^6 - 80*cos(x)^4
- 10*cos(x)^2 - 3)/((cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x))
```

Sympy [A] time = 0.058768, size = 44, normalized size = 1.07

$$-\frac{256\cos(2x)}{15\sin(2x)}-\frac{128\cos(2x)}{15\sin^3(2x)}-\frac{32\cos(2x)}{5\sin^5(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x)**6/sin(x)**6,x)
```

```
[Out] -256*cos(2*x)/(15*sin(2*x)) - 128*cos(2*x)/(15*sin(2*x)**3) - 32*
cos(2*x)/(5*sin(2*x)**5)
```

GIAC/XCAS [A] time = 0.204656, size = 35, normalized size = 0.85

$$-\frac{32\left(15\tan(2x)^4+10\tan(2x)^2+3\right)}{15\tan(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^6*sin(x)^6),x, algorithm="giac")
```

```
[Out] -32/15*(15*tan(2*x)^4 + 10*tan(2*x)^2 + 3)/tan(2*x)^5
```

3.348 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0449727, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[x]^2,x]

[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi in Sympy [A] time = 1.73988, size = 20, normalized size = 0.83

$$\frac{x}{8}-\frac{\sin(x)\cos^3(x)}{4}+\frac{\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)**2,x)

[Out] x/8 - sin(x)*cos(x)**3/4 + sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00684828, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0.003, size = 19, normalized size = 0.8

$$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos(x))^3\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 1.58757, size = 14, normalized size = 0.58

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 0.225895, size = 26, normalized size = 1.08

$$-\frac{1}{8}\left(2\,\cos(x)^3-\cos(x)\right)\,\sin(x)+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.052567, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

GIAC/XCAS [A] time = 0.201809, size = 14, normalized size = 0.58

$$\frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.349 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x
]^5*Sin[x])/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rubi [A] time = 0.08663, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x
]^5*Sin[x])/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rubi in Sympy [A] time = 2.98813, size = 46, normalized size = 1.

$$\frac{3x}{128}-\frac{\sin^3(x)\cos^5(x)}{8}-\frac{\sin(x)\cos^5(x)}{16}+\frac{\sin(x)\cos^3(x)}{64}+\frac{3\sin(x)\cos(x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(x)**3*cos(x)**5/8 - sin(x)*cos(x)**5/16 + sin(x)*co
s(x)**3/64 + 3*sin(x)*cos(x)/128
```

Mathematica [A] time = 0.00957197, size = 22, normalized size = 0.48

$$\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$-\frac{(\cos(x))^5(\sin(x))^3}{8}-\frac{(\cos(x))^5\sin(x)}{16}+\frac{\sin(x)}{64}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*sin(x)^4,x)
```

```
[Out] -1/8*cos(x)^5*sin(x)^3-1/16*cos(x)^5*sin(x)+1/64*(cos(x)^3+3/2*co
s(x))*sin(x)+3/128*x
```

Maxima [A] time = 1.43621, size = 22, normalized size = 0.48

$$\frac{3}{128}x+\frac{1}{1024}\sin(8x)-\frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Fricas [A] time = 0.217911, size = 42, normalized size = 0.91

$$\frac{1}{128}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x)
+ 3/128*x
```

Sympy [A] time = 0.053401, size = 31, normalized size = 0.67

$$\frac{3x}{128} - \frac{\sin^3(2x)\cos(2x)}{128} - \frac{3\sin(2x)\cos(2x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

GIAC/XCAS [A] time = 0.197948, size = 22, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{1}{1024}\,\sin(8\,x) - \frac{1}{128}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

3.350 $\int \cos^6(x)\sin^6(x)\,dx$

Optimal. Leaf size=68

$$\frac{5x}{1024}-\frac{1}{12}\sin^5(x)\cos^7(x)-\frac{1}{24}\sin^3(x)\cos^7(x)-\frac{1}{64}\sin(x)\cos^7(x)+\frac{1}{384}\sin(x)\cos^5(x)+\frac{5\sin(x)\cos^3(x)}{1536}+\frac{5\sin(x)\cos(x)}{1024}$$

```
[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 +
(Cos[x]^5*Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3
)/24 - (Cos[x]^7*Sin[x]^5)/12
```

Rubi [A] time = 0.128312, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{5x}{1024}-\frac{1}{12}\sin^5(x)\cos^7(x)-\frac{1}{24}\sin^3(x)\cos^7(x)-\frac{1}{64}\sin(x)\cos^7(x)+\frac{1}{384}\sin(x)\cos^5(x)+\frac{5\sin(x)\cos^3(x)}{1536}+\frac{5\sin(x)\cos(x)}{1024}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^6*Sin[x]^6,x]
```

```
[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 +
(Cos[x]^5*Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3
)/24 - (Cos[x]^7*Sin[x]^5)/12
```

Rubi in Sympy [A] time = 4.2927, size = 70, normalized size = 1.03

$$\frac{5x}{1024}-\frac{\sin^5(x)\cos^7(x)}{12}-\frac{\sin^3(x)\cos^7(x)}{24}-\frac{\sin(x)\cos^7(x)}{64}+\frac{\sin(x)\cos^5(x)}{384}+\frac{5\sin(x)\cos^3(x)}{1536}+\frac{5\sin(x)\cos(x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**6*sin(x)**6,x)
```

```
[Out] 5*x/1024 - sin(x)**5*cos(x)**7/12 - sin(x)**3*cos(x)**7/24 - sin(
x)*cos(x)**7/64 + sin(x)*cos(x)**5/384 + 5*sin(x)*cos(x)**3/1536
+ 5*sin(x)*cos(x)/1024
```

Mathematica [A] time = 0.0161367, size = 30, normalized size = 0.44

$$\frac{5x}{1024}-\frac{15\sin(4x)}{8192}+\frac{3\sin(8x)}{8192}-\frac{\sin(12x)}{24576}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^6*Sin[x]^6,x]
```

```
[Out] (5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/2
4576
```

Maple [A] time = 0.01, size = 52, normalized size = 0.8

$$-\frac{(\cos(x))^7(\sin(x))^5}{12}-\frac{(\cos(x))^7(\sin(x))^3}{24}-\frac{(\cos(x))^7\sin(x)}{64}+\frac{\sin(x)}{384}\left((\cos(x))^5+\frac{5(\cos(x))^3}{4}+\frac{15\cos(x)}{8}\right)+\frac{5x}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^6*sin(x)^6,x)
```

```
[Out] -1/12*cos(x)^7*sin(x)^5-1/24*cos(x)^7*sin(x)^3-1/64*cos(x)^7*sin(
x)+1/384*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/1024*x
```

Maxima [A] time = 1.47053, size = 32, normalized size = 0.47

$$\frac{1}{6144}\sin(4x)^3+\frac{5}{1024}x+\frac{3}{8192}\sin(8x)-\frac{1}{512}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/6144*sin(4*x)^3 + 5/1024*x + 3/8192*sin(8*x) - 1/512*sin(4*x)
```

Fricas [A] time = 0.230682, size = 58, normalized size = 0.85

$$-\frac{1}{3072}\left(256\cos(x)^{11}-640\cos(x)^9+432\cos(x)^7-8\cos(x)^5-10\cos(x)^3-15\cos(x)\right)\sin(x)+\frac{5}{1024}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/3072*(256*cos(x)^11 - 640*cos(x)^9 + 432*cos(x)^7 - 8*cos(x)^5
 - 10*cos(x)^3 - 15*cos(x))*sin(x) + 5/1024*x
```

Sympy [A] time = 0.055292, size = 46, normalized size = 0.68

$$\frac{5x}{1024} - \frac{\sin^5(2x)\cos(2x)}{768} - \frac{5\sin^3(2x)\cos(2x)}{3072} - \frac{5\sin(2x)\cos(2x)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**6*sin(x)**6,x)
```

```
[Out] 5*x/1024 - sin(2*x)**5*cos(2*x)/768 - 5*sin(2*x)**3*cos(2*x)/3072
 - 5*sin(2*x)*cos(2*x)/2048
```

GIAC/XCAS [A] time = 0.2024, size = 30, normalized size = 0.44

$$\frac{5}{1024}\,x - \frac{1}{24576}\,\sin(12\,x) + \frac{3}{8192}\,\sin(8\,x) - \frac{15}{8192}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6*sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)
```

3.351 $\int \cos^8(x)\sin^8(x)\,dx$

Optimal. Leaf size=90

$$\frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin(x)\cos^5(x)}{12288}+\frac{35\sin(x)\cos^3(x)}{49152}+\frac{35\sin(x)\cos(x)}{32768}$$

```
[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49
152 + (7*Cos[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x
]^9*Sin[x])/256 - (5*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)
/32 - (Cos[x]^9*Sin[x]^7)/16
```

Rubi [A] time = 0.170995, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin(x)\cos^5(x)}{12288}+\frac{35\sin(x)\cos^3(x)}{49152}+\frac{35\sin(x)\cos(x)}{32768}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49
152 + (7*Cos[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x
]^9*Sin[x])/256 - (5*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)
/32 - (Cos[x]^9*Sin[x]^7)/16
```

Rubi in Sympy [A] time = 5.7053, size = 95, normalized size = 1.06

$$\frac{35x}{32768}-\frac{\sin^7(x)\cos^9(x)}{16}-\frac{\sin^5(x)\cos^9(x)}{32}-\frac{5\sin^3(x)\cos^9(x)}{384}-\frac{\sin(x)\cos^9(x)}{256}+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin(x)\cos^5(x)}{12288}+\frac{35\sin(x)\cos^3(x)}{49152}+\frac{35\sin(x)\cos(x)}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**8*sin(x)**8,x)
```

```
[Out] 35*x/32768 - sin(x)**7*cos(x)**9/16 - sin(x)**5*cos(x)**9/32 - 5*
sin(x)**3*cos(x)**9/384 - sin(x)*cos(x)**9/256 + sin(x)*cos(x)**7
```

```
/2048 + 7*sin(x)*cos(x)**5/12288 + 35*sin(x)*cos(x)**3/49152 + 35
*sin(x)*cos(x)/32768
```

Mathematica [A] time = 0.0224001, size = 38, normalized size = 0.42

$$\frac{35x}{32768} - \frac{7\sin(4x)}{16384} + \frac{7\sin(8x)}{65536} - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x
]/49152 + Sin[16*x]/524288
```

Maple [A] time = 0.052, size = 68, normalized size = 0.8

$$-\frac{(\cos(x))^9(\sin(x))^7}{16} - \frac{(\cos(x))^9(\sin(x))^5}{32} - \frac{5\,(\cos(x))^9(\sin(x))^3}{384} - \frac{(\cos(x))^9\sin(x)}{256} + \frac{\sin(x)}{2048}\left((\cos(x))^7 + \frac{7\,(\cos(x))^5}{6} + \frac{35\,(\cos(x))^3}{24} + \frac{35\,\cos(x)}{16}\right) + \frac{35\,x}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^8*sin(x)^8,x)
```

```
[Out] -1/16*cos(x)^9*sin(x)^7-1/32*cos(x)^9*sin(x)^5-5/384*cos(x)^9*sin
(x)^3-1/256*cos(x)^9*sin(x)+1/2048*(cos(x)^7+7/6*cos(x)^5+35/24*c
os(x)^3+35/16*cos(x))*sin(x)+35/32768*x
```

Maxima [A] time = 1.43209, size = 41, normalized size = 0.46

$$\frac{1}{12288}\sin(4x)^3 + \frac{35}{32768}x + \frac{1}{524288}\sin(16x) + \frac{7}{65536}\sin(8x) - \frac{1}{2048}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*si
n(8*x) - 1/2048*sin(4*x)
```

Fricas [A] time = 0.268089, size = 74, normalized size = 0.82

$$\frac{1}{98304}\left(6144\cos\left(x\right)^{15}-21504\cos\left(x\right)^{13}+25856\cos\left(x\right)^{11}-10880\cos\left(x\right)^{9}+48\cos\left(x\right)^{7}+56\cos\left(x\right)^{5}+70\cos\left(x\right)^{3}+105\cos\left(x\right)\right)\sin\left(x\right)+\frac{35}{32768}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 108
80*cos(x)^9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x
))*sin(x) + 35/32768*x
```

Sympy [A] time = 0.075374, size = 61, normalized size = 0.68

$$\frac{35x}{32768}-\frac{\sin^{7}\left(2x\right)\cos\left(2x\right)}{4096}-\frac{7\sin^{5}\left(2x\right)\cos\left(2x\right)}{24576}-\frac{35\sin^{3}\left(2x\right)\cos\left(2x\right)}{98304}-\frac{35\sin\left(2x\right)\cos\left(2x\right)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**8*sin(x)**8,x)
```

```
[Out] 35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/2
4576 - 35*sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536
```

GIAC/XCAS [A] time = 0.199247, size = 38, normalized size = 0.42

$$\frac{35}{32768}x+\frac{1}{524288}\sin\left(16x\right)-\frac{1}{49152}\sin\left(12x\right)+\frac{7}{65536}\sin\left(8x\right)-\frac{7}{16384}\sin\left(4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^8*sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin
(8*x) - 7/16384*sin(4*x)
```

3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$

Optimal. Leaf size=68

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\sin^2(x)\right)}{2m+1}$$

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*
m)/2, (1 + 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 +
2*m)
```

Rubi [A] time = 0.0647825, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\sin^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*
m)/2, (1 + 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 +
2*m)
```

Rubi in Sympy [A] time = 2.33874, size = 49, normalized size = 0.72

$$\frac{\left(\cos^2(x)\right)^{-m-\frac{1}{2}}\sin^{2m+1}(x)\cos^{2m+1}(x)\,{}_2F_1\left(\begin{matrix}m+\frac{1}{2},-m+\frac{1}{2}\\ m+\frac{3}{2}\end{matrix}\middle|\sin^2(x)\right)}{2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
```

```
[Out] (cos(x)**2)**(-m - 1/2)*sin(x)**(2*m + 1)*cos(x)**(2*m + 1)*hyper
((m + 1/2, -m + 1/2), (m + 3/2,), sin(x)**2)/(2*m + 1)
```

Mathematica [A] time = 0.145133, size = 59, normalized size = 0.87

$$-\frac{\sin^{2m+1}(x)\sin^2(x)^{-m-\frac{1}{2}}\cos^{2m+1}(x)\,{}_2F_1\left(\frac{1}{2}-m,m+\frac{1}{2};m+\frac{3}{2};\cos^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] -((Cos[x]^(1 + 2*m)*Hypergeometric2F1[1/2 - m, 1/2 + m, 3/2 + m,
Cos[x]^2]*Sin[x]^(1 + 2*m)*(Sin[x]^2)^(-1/2 - m))/(1 + 2*m))
```

Maple [F] time = 0.397, size = 0, normalized size = 0.

$$\int (\cos(x))^{2m} (\sin(x))^{2m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

```
[Out] int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)^{2m} \sin(x)^{2m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos(x)^{2m} \sin(x)^{2m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")
```

```
[Out] integral(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{2m}(x)\cos^{2m}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
```

```
[Out] Integral(sin(x)**(2*m)*cos(x)**(2*m), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

```
[Out] -Cot[Pi/4 + 2*x]^2/4 + Log[Tan[Pi/4 + 2*x]]/2
```

Rubi [A] time = 0.0345857, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] -Cot[Pi/4 + 2*x]^2/4 + Log[Tan[Pi/4 + 2*x]]/2
```

Rubi in Sympy [A] time = 4.05648, size = 41, normalized size = 1.28

$$-\frac{\log\left(-\sin^2\left(2x+\frac{\pi}{4}\right)+1\right)}{4}+\frac{\log\left(\sin^2\left(2x+\frac{\pi}{4}\right)\right)}{4}-\frac{1}{4\sin^2\left(2x+\frac{\pi}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
```

```
[Out] -log(-sin(2*x + pi/4)**2 + 1)/4 + log(sin(2*x + pi/4)**2)/4 - 1/(
4*sin(2*x + pi/4)**2)
```

Mathematica [A] time = 0.0267842, size = 47, normalized size = 1.47

$$-\frac{1}{4}\csc^2\left(\frac{1}{4}(8x+\pi)\right)+\frac{1}{2}\log\left(\sin\left(\frac{1}{4}(8x+\pi)\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{1}{4}(8x+\pi)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] -Csc[(Pi + 8*x)/4]^2/4 - Log[Cos[(Pi + 8*x)/4]]/2 + Log[Sin[(Pi +
 8*x)/4]]/2
```

Maple [A] time = 0.026, size = 25, normalized size = 0.8

$$-\frac{1}{4}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)^{-2}+\frac{1}{2}\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x)
```

```
[Out] -1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

Maxima [A] time = 1.38782, size = 55, normalized size = 1.72

$$-\frac{1}{4\sin\left(\frac{1}{4}\pi+2x\right)^{2}}-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(1/4*pi + 2*x)*sin(1/4*pi + 2*x)^3),x, algorithm="maxima")
```

```
[Out] -1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4
*log(sin(1/4*pi + 2*x)^2)
```

Fricas [A] time = 0.225618, size = 96, normalized size = 3.

$$-\frac{\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)\log\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}\right)-\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)\log\left(-\frac{1}{4}\cos\left(\frac{1}{4}\pi+2x\right)^{2}+\frac{1}{4}\right)-1}{4\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(1/4*pi + 2*x)*sin(1/4*pi + 2*x)^3),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1
/4*pi + 2*x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos
(1/4*pi + 2*x)^2 - 1)
```

Sympy [A] time = 3.32024, size = 54, normalized size = 1.69

$$-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}+\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)\right)}{2}-\frac{\tan^2\left(x+\frac{\pi}{8}\right)}{16}-\frac{1}{16\tan^2\left(x+\frac{\pi}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
```

```
[Out] -log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x
+ pi/8))/2 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)
```

GIAC/XCAS [A] time = 0.222349, size = 178, normalized size = 5.56

$$-\frac{\left(\frac{4\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}{\cos\left(\frac{1}{4}\pi+2x\right)+1}-1\right)\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}+\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}+\frac{1}{4}\ln\left(-\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{\cos\left(\frac{1}{4}\pi+2x\right)+1}\right)-\frac{1}{2}\ln\left(\left|-\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{\cos\left(\frac{1}{4}\pi+2x\right)+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(1/4*pi + 2*x)*sin(1/4*pi + 2*x)^3),x, algorithm="giac")
```

```
[Out] -1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(co
s(1/4*pi + 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi +
 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) + 1/4*ln(-(cos(1/4*pi + 2*x) -
 1)/(cos(1/4*pi + 2*x) + 1)) - 1/2*ln(abs(-(cos(1/4*pi + 2*x) - 1
)/(cos(1/4*pi + 2*x) + 1) - 1))
```

3.354 $\int \sec^2(x)\tan^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3
```

Rubi [A] time = 0.0324581, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^2*Tan[x]^2,x]

[Out] Tan[x]^3/3
```

Rubi in Sympy [A] time = 1.93293, size = 5, normalized size = 0.62

$$\frac{\tan^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**2*tan(x)**2,x)

[Out] tan(x)**3/3
```

Mathematica [A] time = 0.00380236, size = 8, normalized size = 1.

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3
```

Maple [A] time = 0.013, size = 11, normalized size = 1.4

$$\frac{(\sin(x))^3}{3\ (\cos(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^2*tan(x)^2,x)
```

```
[Out] 1/3*sin(x)^3/cos(x)^3
```

Maxima [A] time = 1.43849, size = 8, normalized size = 1.

$$\frac{1}{3}\ \tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3
```

Fricas [A] time = 0.215282, size = 19, normalized size = 2.38

$$-\frac{\left(\cos(x)^2 - 1\right)\ \sin(x)}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

Sympy [A] time = 0.048051, size = 17, normalized size = 2.12

$$-\frac{\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2*tan(x)**2,x)
```

```
[Out] -sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

GIAC/XCAS [A] time = 0.199104, size = 8, normalized size = 1.

$$\frac{1}{3}\tan\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3
```

3.355 $\int \cot^3(x)\csc(x)\,dx$

Optimal. Leaf size=11

$$\csc(x)-\frac{\csc^3(x)}{3}$$

```
[Out] Csc[x] - Csc[x]^3/3
```

Rubi [A] time = 0.0247094, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\csc(x)-\frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\int \frac{1}{\sin(x)\tan(x)}\,dx}{3}-\frac{1}{3\sin(x)\tan^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cot(x)**3*csc(x),x)
```

```
[Out] -2*Integral(1/(sin(x)*tan(x)), x)/3 - 1/(3*sin(x)*tan(x)**2)
```

Mathematica [B] time = 0.00745944, size = 57, normalized size = 5.18

$$\frac{5}{12}\tan\left(\frac{x}{2}\right)+\frac{5}{12}\cot\left(\frac{x}{2}\right)-\frac{1}{24}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)-\frac{1}{24}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x],x]
```

```
[Out] (5*Cot[x/2])/12 - (Cot[x/2]*Csc[x/2]^2)/24 + (5*Tan[x/2])/12 - (S
ec[x/2]^2*Tan[x/2])/24
```

Maple [B] time = 0.013, size = 32, normalized size = 2.9

$$-\frac{(\cos(x))^4}{3(\sin(x))^3}+\frac{(\cos(x))^4}{3\sin(x)}+\frac{(2+(\cos(x))^2)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x),x)
```

```
[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x
)
```

Maxima [A] time = 1.45372, size = 19, normalized size = 1.73

$$\frac{3\sin(x)^2-1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Fricas [A] time = 0.212615, size = 30, normalized size = 2.73

$$\frac{3\cos(x)^2-2}{3(\cos(x)^2-1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.093162, size = 14, normalized size = 1.27

$$\frac{3\sin^2(x)-1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**3*csc(x),x)

[Out] (3*sin(x)**2 - 1)/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.199707, size = 19, normalized size = 1.73

$$\frac{3 \sin(x)^2 - 1}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3*csc(x),x, algorithm="giac")

[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.356 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3
```

Rubi [A] time = 0.0200002, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x),x)
```

```
[Out] Integral(tan(x)/cos(x)**3, x)
```

Mathematica [A] time = 0.00297648, size = 8, normalized size = 1.

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x),x)
```

```
[Out] 1/3*sec(x)^3
```

Maxima [A] time = 1.44046, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3/cos(x)^3
```

Fricas [A] time = 0.249965, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3/cos(x)^3
```

Sympy [A] time = 0.050763, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

GIAC/XCAS [A] time = 0.199054, size = 8, normalized size = 1.

$$\frac{1}{3\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

```
[Out] 1/3/cos(x)^3
```

3.357 $\int \cot^2(x)\csc^3(x)\,dx$

Optimal. Leaf size=26

$$\frac{1}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)$$

```
[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rubi [A] time = 0.0489788, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rubi in Sympy [A] time = 3.02223, size = 31, normalized size = 1.19

$$\frac{\operatorname{atanh}(\cos(x))}{8}+\frac{\cos(x)}{8(-\cos^2(x)+1)}-\frac{\cos(x)}{4(-\cos^2(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**2*csc(x)**3,x)
```

```
[Out] atanh(cos(x))/8 + cos(x)/(8*(-cos(x)**2 + 1)) - cos(x)/(4*(-cos(x
)**2 + 1)**2)
```

Mathematica [B] time = 0.0130118, size = 71, normalized size = 2.73

$$-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)+\frac{1}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)-\frac{1}{32}\sec^2\left(\frac{x}{2}\right)-\frac{1}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8
 - Sec[x/2]^2/32 + Sec[x/2]^4/64
```

Maple [A] time = 0.017, size = 36, normalized size = 1.4

$$-\frac{(\cos(x))^3}{4(\sin(x))^4}-\frac{(\cos(x))^3}{8(\sin(x))^2}-\frac{\cos(x)}{8}-\frac{\ln(\csc(x)-\cot(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^2*csc(x)^3,x)
```

```
[Out] -1/4*cos(x)^3/sin(x)^4-1/8/sin(x)^2*cos(x)^3-1/8*cos(x)-1/8*ln(cs
c(x)-cot(x))
```

Maxima [A] time = 1.43799, size = 51, normalized size = 1.96

$$-\frac{\cos(x)^3+\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{1}{16}\log(\cos(x)+1)-\frac{1}{16}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/16*log(c
os(x) + 1) - 1/16*log(cos(x) - 1)
```

Fricas [A] time = 0.223595, size = 92, normalized size = 3.54

$$-\frac{2\cos(x)^3-\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2\cos(x)}{16\left(\cos(x)^4-2\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cos(x)^3 - (cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) +
1/2) + (cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) + 2*cos
(x))/(cos(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [A] time = 0.173895, size = 39, normalized size = 1.5

$$-\frac{\cos^3(x)+\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{\log(\cos(x)-1)}{16}+\frac{\log(\cos(x)+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**2*csc(x)**3,x)
```

```
[Out] -(cos(x)**3 + cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) - log(cos(
x) - 1)/16 + log(cos(x) + 1)/16
```

GIAC/XCAS [A] time = 0.20536, size = 63, normalized size = 2.42

$$-\frac{\frac{1}{\cos(x)}+\cos(x)}{8\left(\left(\frac{1}{\cos(x)}+\cos(x)\right)^2-4\right)}+\frac{1}{32}\ln\left(\left|\frac{1}{\cos(x)}+\cos(x)+2\right|\right)-\frac{1}{32}\ln\left(\left|\frac{1}{\cos(x)}+\cos(x)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2*csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(1/cos(x) + cos(x))/((1/cos(x) + cos(x))^2 - 4) + 1/32*ln(ab
s(1/cos(x) + cos(x) + 2)) - 1/32*ln(abs(1/cos(x) + cos(x) - 2))
```

3.358 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi [A] time = 0.0429606, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^3*Csc[x]^4,x]

[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi in Sympy [A] time = 2.49643, size = 15, normalized size = 0.88

$$\frac{1}{4\sin^4(x)}-\frac{1}{6\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3/sin(x)**7,x)

[Out] 1/(4*sin(x)**4) - 1/(6*sin(x)**6)
```

Mathematica [A] time = 0.00818421, size = 17, normalized size = 1.

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] time = 0.013, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^4}{6(\sin(x))^6}-\frac{(\cos(x))^4}{12(\sin(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

Maxima [A] time = 1.34547, size = 19, normalized size = 1.12

$$\frac{3\sin(x)^2-2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Fricas [A] time = 0.215632, size = 41, normalized size = 2.41

$$\frac{3\cos(x)^2-1}{12\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.105976, size = 14, normalized size = 0.82

$$\frac{3\sin^2(x)-2}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3/sin(x)**7,x)
```

```
[Out] (3*sin(x)**2 - 2)/(12*sin(x)**6)
```

GIAC/XCAS [A] time = 0.199059, size = 24, normalized size = 1.41

$$\frac{3\cos\left(x\right)^{2} - 1}{12\left(\cos\left(x\right)^{2} - 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^2 - 1)^3
```

3.359 $\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx$

Optimal. Leaf size=31

$$\frac{2}{11} \sec^{\frac{11}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{3} \sec^{\frac{3}{2}}(x)$$

```
[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

Rubi [A] time = 0.0450667, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2}{11} \sec^{\frac{11}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{3} \sec^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^(13/2)*Sin[x]^5,x]
```

```
[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

Rubi in Sympy [A] time = 2.76493, size = 34, normalized size = 1.1

$$\frac{2 \left(\frac{1}{\cos(x)}\right)^{\frac{11}{2}}}{11} - \frac{4 \left(\frac{1}{\cos(x)}\right)^{\frac{7}{2}}}{7} + \frac{2 \left(\frac{1}{\cos(x)}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**(3/2)*tan(x)**5,x)
```

```
[Out] 2*(1/cos(x))**(11/2)/11 - 4*(1/cos(x))**(7/2)/7 + 2*(1/cos(x))**(
3/2)/3
```

Mathematica [A] time = 0.0630408, size = 24, normalized size = 0.77

$$\frac{1}{924}(44 \cos(2x) + 77 \cos(4x) + 135) \sec^{\frac{11}{2}}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^(13/2)*Sin[x]^5,x]
```

```
[Out] ((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924
```

Maple [B] time = 0.125, size = 49, normalized size = 1.6

$$\frac{32}{231}\left(77\left(\sin\left(x/2\right)\right)^{8}-154\left(\sin\left(x/2\right)\right)^{6}+99\left(\sin\left(x/2\right)\right)^{4}-22\left(\sin\left(x/2\right)\right)^{2}+2\right)\left(-2\left(\sin\left(x/2\right)\right)^{2}+1\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^(3/2)*tan(x)^5,x)
```

```
[Out] 32/231/(-2*sin(1/2*x)^2+1)^(11/2)*(77*sin(1/2*x)^8-154*sin(1/2*x)
^6+99*sin(1/2*x)^4-22*sin(1/2*x)^2+2)
```

Maxima [A] time = 1.35897, size = 26, normalized size = 0.84

$$\frac{2}{3\cos\left(x\right)^{\frac{3}{2}}}-\frac{4}{7\cos\left(x\right)^{\frac{7}{2}}}+\frac{2}{11\cos\left(x\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)
```

Fricas [A] time = 0.226871, size = 27, normalized size = 0.87

$$\frac{2\left(77\cos\left(x\right)^{4}-66\cos\left(x\right)^{2}+21\right)}{231\cos\left(x\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**(3/2)*tan(x)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.210575, size = 31, normalized size = 1.

$$\frac{2\left(77 \cos(x)^4 - 66 \cos(x)^2 + 21\right)\operatorname{sign}(\cos(x))}{231 \cos(x)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)*sign(cos(x))/cos(x)^(11/2)
```

3.360 $\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

```
[Out] (2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

Rubi [A] time = 0.0391774, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

Rubi in Sympy [A] time = 5.32071, size = 51, normalized size = 2.43

$$-\frac{8\cos(x)\tan^{\frac{3}{2}}(x)}{45\sin(x)}-\frac{2\tan^{\frac{3}{2}}(x)}{45\sin(x)\cos(x)}+\frac{2\tan^{\frac{3}{2}}(x)}{9\sin(x)\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**4*tan(x)**(3/2),x)
```

```
[Out] -8*cos(x)*tan(x)**(3/2)/(45*sin(x)) - 2*tan(x)**(3/2)/(45*sin(x)*
cos(x)) + 2*tan(x)**(3/2)/(9*sin(x)*cos(x)**3)
```

Mathematica [A] time = 0.0422605, size = 22, normalized size = 1.05

$$\frac{2}{45}(2\cos(2x)+7)\tan^{\frac{5}{2}}(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45
```

Maple [A] time = 0.326, size = 26, normalized size = 1.2

$$\frac{\left(8 \left(\cos \left(x\right)\right)^{2}+10\right) \sin \left(x\right)}{45 \left(\cos \left(x\right)\right)^{3}}\left(\frac{\sin \left(x\right)}{\cos \left(x\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^4*tan(x)^(3/2),x)
```

```
[Out] 2/45*(4*cos(x)^2+5)*sin(x)*(sin(x)/cos(x))^(3/2)/cos(x)^3
```

Maxima [A] time = 1.34053, size = 18, normalized size = 0.86

$$\frac{2}{9} \tan \left(x\right)^{\frac{9}{2}} + \frac{2}{5} \tan \left(x\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

Fricas [A] time = 0.226855, size = 36, normalized size = 1.71

$$-\frac{2 \left(4 \cos \left(x\right)^{4} + \cos \left(x\right)^{2} - 5\right) \sqrt{\frac{\sin(x)}{\cos(x)}}}{45 \cos \left(x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**4*tan(x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.203383, size = 18, normalized size = 0.86

$$\frac{2}{9}\tan\left(x\right)^{\frac{9}{2}} + \frac{2}{5}\tan\left(x\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

3.361 $\int \cot^4(x)\csc^3(x)\,dx$

Optimal. Leaf size=38

$$-\frac{1}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot^3(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{16}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/16 - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 -
(Cot[x]^3*Csc[x]^3)/6
```

Rubi [A] time = 0.083856, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{1}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot^3(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{16}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] -ArcTanh[Cos[x]]/16 - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 -
(Cot[x]^3*Csc[x]^3)/6
```

Rubi in Sympy [A] time = 4.07219, size = 46, normalized size = 1.21

$$-\frac{\operatorname{atanh}(\cos(x))}{16}-\frac{\cos(x)}{16(-\cos^2(x)+1)}+\frac{\cos(x)}{8(-\cos^2(x)+1)^2}-\frac{\cos^3(x)}{6(-\cos^2(x)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**4*csc(x)**3,x)
```

```
[Out] -atanh(cos(x))/16 - cos(x)/(16*(-cos(x)**2 + 1)) + cos(x)/(8*(-co
s(x)**2 + 1)**2) - cos(x)**3/(6*(-cos(x)**2 + 1)**3)
```

Mathematica [B] time = 0.0147586, size = 95, normalized size = 2.5

$$-\frac{1}{384}\csc^6\left(\frac{x}{2}\right)+\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{1}{64}\csc^2\left(\frac{x}{2}\right)+\frac{1}{384}\sec^6\left(\frac{x}{2}\right)-\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{1}{64}\sec^2\left(\frac{x}{2}\right)+\frac{1}{16}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{16}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] -Csc[x/2]^2/64 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/1
6 + Log[Sin[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6
/384
```

Maple [A] time = 0.018, size = 52, normalized size = 1.4

$$-\frac{(\cos(x))^5}{6(\sin(x))^6}-\frac{(\cos(x))^5}{24(\sin(x))^4}+\frac{(\cos(x))^5}{48(\sin(x))^2}+\frac{(\cos(x))^3}{48}+\frac{\cos(x)}{16}+\frac{\ln(\csc(x)-\cot(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^4*csc(x)^3,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^5-1/24/sin(x)^4*cos(x)^5+1/48/sin(x)^2*cos(x
)^5+1/48*cos(x)^3+1/16*cos(x)+1/16*ln(csc(x)-cot(x))
```

Maxima [A] time = 1.41991, size = 73, normalized size = 1.92

$$\frac{3\cos(x)^5+8\cos(x)^3-3\cos(x)}{48\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}-\frac{1}{32}\log(\cos(x)+1)+\frac{1}{32}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^6 - 3*cos(x)^4
+ 3*cos(x)^2 - 1) - 1/32*log(cos(x) + 1) + 1/32*log(cos(x) - 1)
```

Fricas [A] time = 0.231298, size = 126, normalized size = 3.32

$$\frac{6\cos(x)^5+16\cos(x)^3-3\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2\right.}{96\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(x)^5 + 16*cos(x)^3 - 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos
(x)^2 - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^6 - 3*cos(x)^4 + 3*c
os(x)^2 - 1)*log(-1/2*cos(x) + 1/2) - 6*cos(x))/(cos(x)^6 - 3*cos
(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.209411, size = 56, normalized size = 1.47

$$\frac{3\cos^5(x) + 8\cos^3(x) - 3\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{\log(\cos(x) - 1)}{32} - \frac{\log(\cos(x) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**4*csc(x)**3,x)
```

```
[Out] (3*cos(x)**5 + 8*cos(x)**3 - 3*cos(x))/(48*cos(x)**6 - 144*cos(x)
**4 + 144*cos(x)**2 - 48) + log(cos(x) - 1)/32 - log(cos(x) + 1)/
32
```

GIAC/XCAS [A] time = 0.208855, size = 59, normalized size = 1.55

$$\frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^2 - 1\right)^3} - \frac{1}{32}\ln(\cos(x) + 1) + \frac{1}{32}\ln(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4*csc(x)^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32
*ln(cos(x) + 1) + 1/32*ln(-cos(x) + 1)
```

3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=76

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

```
[Out] -ArcTanh[Sin[Pi/4 + x/2]]/4 - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4
 + (Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2])/2
```

Rubi [A] time = 0.0631573, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + x/2]]/4 - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4
 + (Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.25496, size = 82, normalized size = 1.08

$$\frac{1}{8}\left(2\log\left(\cos\left(\frac{1}{8}(2x+\pi)\right)-\sin\left(\frac{1}{8}(2x+\pi)\right)\right)-2\log\left(\sin\left(\frac{1}{8}(2x+\pi)\right)+\cos\left(\frac{1}{8}(2x+\pi)\right)\right)+(\sin(x)+3)\tan\left(\frac{1}{4}(2x+\pi)\right)\sec^3\left(\frac{1}{4}(2x+\pi)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] (2*Log[Cos[(Pi + 2*x)/8] - Sin[(Pi + 2*x)/8]] - 2*Log[Cos[(Pi + 2
*x)/8] + Sin[(Pi + 2*x)/8]] + Sec[(Pi + 2*x)/4]^3*(3 + Sin[x])*Ta
n[(Pi + 2*x)/4])/8
```

Maple [A] time = 0.032, size = 76, normalized size = 1.

$$\frac{1}{2}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{3}\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{-4}+\frac{1}{4}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{3}\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{-2}+\frac{1}{4}\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{4}\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x)
```

```
[Out] 1/2*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^4+1/4*sin(1/4*Pi+1/2*x)
^3/cos(1/4*Pi+1/2*x)^2+1/4*sin(1/4*Pi+1/2*x)-1/4*ln(sec(1/4*Pi+1/
2*x)+tan(1/4*Pi+1/2*x))
```

Maxima [A] time = 1.48451, size = 100, normalized size = 1.32

$$\frac{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{3}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}-2\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{2}+1\right)}-\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(1/4*pi + 1/2*x)^3*tan(1/4*pi + 1/2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(1/4*pi + 1/2*x)^3 + sin(1/4*pi + 1/2*x))/(sin(1/4*pi + 1
/2*x)^4 - 2*sin(1/4*pi + 1/2*x)^2 + 1) - 1/8*log(sin(1/4*pi + 1/2
*x) + 1) + 1/8*log(sin(1/4*pi + 1/2*x) - 1)
```

Fricas [A] time = 0.239254, size = 111, normalized size = 1.46

$$-\frac{\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)-\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}\log\left(-\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+2\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{2}-2\right)\sin\left(\frac{1}{4}\right.}{8\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi + 1/2*x)^3*tan(1/4*pi + 1/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cos(1/4*pi + 1/2*x)^4*log(sin(1/4*pi + 1/2*x) + 1) - cos(1/
4*pi + 1/2*x)^4*log(-sin(1/4*pi + 1/2*x) + 1) + 2*(cos(1/4*pi + 1
/2*x)^2 - 2)*sin(1/4*pi + 1/2*x))/cos(1/4*pi + 1/2*x)^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^2\left(\frac{x}{2} + \frac{\pi}{4}\right)\sec^3\left(\frac{x}{2} + \frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)
```

```
[Out] Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)
```

GIAC/XCAS [A] time = 0.24285, size = 128, normalized size = 1.68

$$\frac{\frac{1}{\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)} + \sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)}{4\left(\left(\frac{1}{\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)} + \sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)\right)^2 - 4\right)} - \frac{1}{16}\ln\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)} + \sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right) + 2\right|\right) + \frac{1}{16}\ln\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)} + \sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi + 1/2*x)^3*tan(1/4*pi + 1/2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi
+ 1/2*x) + sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*ln(abs(1/sin(1/4*pi
 + 1/2*x) + sin(1/4*pi + 1/2*x) + 2)) + 1/16*ln(abs(1/sin(1/4*pi
+ 1/2*x) + sin(1/4*pi + 1/2*x) - 2))
```

3.363 $\int \left(1+\cot^3(x)\right) \left(a\sec^2(x)-\sin(2x)\right)^2 \, dx$

Optimal. Leaf size=88

$$\frac{1}{3}a^2\tan^3(x)+a^2\tan(x)-\frac{1}{2}a^2\cot^2(x)+\left(a^2+4\right)\log(\sin(x))+4ax+4a\cot(x)+(4-a)a\log(\cos(x))+\frac{x}{2}+\cos^4(x)+2\cos^2(x)-\sin(x)\cos^3(x)+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + 4*a*x + 2*Cos[x]^2 + Cos[x]^4 + 4*a*Cot[x] - (a^2*Cot[x]^2)
/2 + (4 - a)*a*Log[Cos[x]] + (4 + a^2)*Log[Sin[x]] + (Cos[x]*Sin[
x])/2 - Cos[x]^3*Sin[x] + a^2*Tan[x] + (a^2*Tan[x]^3)/3
```

Rubi [A] time = 0.810965, antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{1}{3}a^2\tan^3(x)+a^2\tan(x)-\frac{1}{2}a^2\cot^2(x)+\left(a^2+4\right)\log(\tan(x))+\frac{1}{2}(8a+1)x+4a\cot(x)+4(a+1)\log(\cos(x))+\cos^4(x)(1-\tan(x))+\frac{1}{2}\cos^2(x)(\tan(x)+4)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] ((1 + 8*a)*x)/2 + 4*a*Cot[x] - (a^2*Cot[x]^2)/2 + 4*(1 + a)*Log[C
os[x]] + (4 + a^2)*Log[Tan[x]] + Cos[x]^4*(1 - Tan[x]) + a^2*Tan[
x] + (a^2*Tan[x]^3)/3 + (Cos[x]^2*(4 + Tan[x]))/2
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

```
[Out] Exception raised: CoercionFailed
```

Mathematica [A] time = 3.17657, size = 127, normalized size = 1.44

$$-\frac{2\sin(x)\cos^3(x)\left(\sin(2x)-a\sec^2(x)\right)^2\left(-8a^2(\cos(2x)+2)\sec^2(x)-3\cot(x)\left(-4a^2\csc^2(x)+8a^2\log(\sin(x))-8a^2\log(\cos(x\right.\right.}{3(-4a+2\sin(2x)+\sin(4x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] (-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2
- 8*a^2*(2 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos
[2*x] + Cos[4*x] - 4*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[
Cos[x]] + 32*Log[Sin[x]] + 8*a^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4
*a + 2*Sin[2*x] + Sin[4*x])^2)
```

Maple [B] time = 0.253, size = 186, normalized size = 2.1

$$4\,a\cot(x)+2\,a(\cot(x))^2+4\,a\ln(\sin(x))-\frac{2\,a^2\cot(x)}{3}-\frac{a^2}{2\,(\sin(x))^2}+a^2\ln(\tan(x))-4\,a\ln(\tan(x))-2\,\frac{a}{(\sin(x))^2}-4\,\frac{(\cos(x))^7}{\sin(x)}+2\,\frac{(\cos(x))^8}{(\sin(x))^2}+8\,\frac{(\cos(x))^5}{\sin(x)}+2\,(\cos(x))^2+8\,\left((\cos(x))^3+3/2\,\cos(x)\right)\sin(x)+4\,\ln(\sin(x))-4\,\cot(x)+4\,ax+(\cos(x))^4+\frac{a^2}{3\,\cos(x)\sin(x)}+\frac{a^2}{3\,(\cos(x))^3\sin(x)}-4\left((\cos(x))^5+5/4\,(\cos(x))^3+\frac{15\,\cos(x)}{8}\right)\sin(x)+2\,(\cos(x))^6-2\,\frac{(\cos(x))^6}{(\sin(x))^2}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x)
```

```
[Out] 4*a*cot(x)+2*a*cot(x)^2+4*a*ln(sin(x))-2/3*a^2*cot(x)-1/2*a^2/sin
(x)^2+a^2*ln(tan(x))-4*a*ln(tan(x))-2*a/sin(x)^2-4/sin(x)*cos(x)^
7+2/sin(x)^2*cos(x)^8+8/sin(x)*cos(x)^5+2*cos(x)^2+8*(cos(x)^3+3/
2*cos(x))*sin(x)+4*ln(sin(x))-4*cot(x)+4*a*x+cos(x)^4+1/3*a^2/sin
(x)/cos(x)+1/3*a^2/sin(x)/cos(x)^3-4*(cos(x)^5+5/4*cos(x)^3+15/8*
cos(x))*sin(x)+2*cos(x)^6-2/sin(x)^2*cos(x)^6+1/2*x
```

Maxima [A] time = 1.54819, size = 155, normalized size = 1.76

$$\frac{1}{3}\left(\tan(x)^3+3\,\tan(x)\right)a^2-\frac{1}{2}\,a^2\left(\frac{1}{\sin(x)^2}+\log\left(\sin(x)^2-1\right)-\log\left(\sin(x)^2\right)\right)+4\,a\left(x+\frac{1}{\tan(x)}\right)+2\,a\log\left(-\sin(x)^2+1\right)+\frac{1}{2}\,x+\frac{1}{8}\,\cos(4\,x)+\frac{3}{2}\,\cos(2\,x)+2\,\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)+2\,\log\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)-\frac{1}{8}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)^3 + 1)*(a*sec(x)^2 - sin(2*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^
2 - 1) - log(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2
+ 1) + 1/2*x + 1/8*cos(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin
(x)^2 + 2*cos(x) + 1) + 2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
 - 1/8*sin(4*x)
```

Fricas [A] time = 0.282081, size = 240, normalized size = 2.73

$$\frac{24\cos(x)^9 + 24\cos(x)^7 + 3(4(8a+1)x - 27)\cos(x)^5 + 3\left(4a^2 - 4(8a+1)x + 11\right)\cos(x)^3 - 12\left(\left(a^2-4a\right)\cos(x)^5 - \left(a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)^3 + 1)*(a*sec(x)^2 - sin(2*x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(24*cos(x)^9 + 24*cos(x)^7 + 3*(4*(8*a + 1)*x - 27)*cos(x)^5
 + 3*(4*a^2 - 4*(8*a + 1)*x + 11)*cos(x)^3 - 12*((a^2 - 4*a)*cos(
x)^5 - (a^2 - 4*a)*cos(x)^3)*log(cos(x)^2) + 12*((a^2 + 4)*cos(x)
^5 - (a^2 + 4)*cos(x)^3)*log(-1/4*cos(x)^2 + 1/4) - 4*(6*cos(x)^8
 - 9*cos(x)^6 - (4*a^2 - 24*a - 3)*cos(x)^4 + 2*a^2*cos(x)^2 + 2*
a^2)*sin(x))/(cos(x)^5 - cos(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.207036, size = 201, normalized size = 2.28

$$\frac{1}{3}a^2\tan(x)^3 + a^2\tan(x) + \frac{1}{2}(8a+1)x - 2(a+1)\ln\left(\tan(x)^2+1\right) + \left(a^2+4\right)\ln\left(|\tan(x)|\right) - \frac{a^2\tan(x)^6 - 4a\tan(x)^6 + 3a^2\tan(x)^4 - 8a\tan(x)^5 - 8a\tan(x)^4 - \tan(x)^5 + 3a^2\tan(x)^2 - 16a\tan(x)^3 - 4\tan(x)^4 - }{2\left(\tan(x)^3+\tan(x)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)^3 + 1)*(a*sec(x)^2 - sin(2*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*tan(x)^3 + a^2*tan(x) + 1/2*(8*a + 1)*x - 2*(a + 1)*ln(ta
n(x)^2 + 1) + (a^2 + 4)*ln(abs(tan(x))) - 1/2*(a^2*tan(x)^6 - 4*a
*tan(x)^6 + 3*a^2*tan(x)^4 - 8*a*tan(x)^5 - 8*a*tan(x)^4 - tan(x)
^5 + 3*a^2*tan(x)^2 - 16*a*tan(x)^3 - 4*tan(x)^4 - 4*a*tan(x)^2 +
 tan(x)^3 + a^2 - 8*a*tan(x) - 6*tan(x)^2)/(tan(x)^3 + tan(x))^2
```

3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx$

Optimal. Leaf size=70

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-3\cos^2(x)+10\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin(x)\cos(x)$$

```
[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] -
 (99*Cos[x]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 +
(3*Sin[x]^4)/8 - (3*Sin[x]^5)/80
```

Rubi [A] time = 0.231081, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-3\cos^2(x)+10\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] -
 (99*Cos[x]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 +
(3*Sin[x]^4)/8 - (3*Sin[x]^5)/80
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\frac{\sin(x)}{2}+1\right)^4 (-3\cos(x)+4)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

```
[Out] Integral((-sin(x)/2 + 1)**4*(-3*cos(x) + 4), x)
```

Mathematica [A] time = 0.0235411, size = 74, normalized size = 1.06

$$\frac{227x}{32}-\frac{531\sin(x)}{128}-\frac{25}{16}\sin(2x)+\frac{99}{256}\sin(3x)+\frac{1}{128}\sin(4x)-\frac{3\sin(5x)}{1280}+\frac{19\cos(x)}{2}-\frac{27}{16}\cos(2x)-\frac{1}{6}\cos(3x)+\frac{3}{64}\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + (19*Cos[x])/2 - (27*Cos[2*x])/16 - Cos[3*x]/6 + (3*C
os[4*x])/64 - (531*Sin[x])/128 - (25*Sin[2*x])/16 + (99*Sin[3*x])
/256 + Sin[4*x]/128 - (3*Sin[5*x])/1280
```

Maple [A] time = 0.06, size = 66, normalized size = 0.9

$$\frac{227\,x}{32}+8\,\cos(x)-3\,\cos(x)\sin(x)+\frac{\left(4+2\,(\sin(x))^2\right)\cos(x)}{3}-\frac{\cos(x)}{16}\left((\sin(x))^3+\frac{3\,\sin(x)}{2}\right)-3\,\sin(x)-3\,(\cos(x))^2-\frac{3\,(\sin(x))^3}{2}+\frac{3\,(\sin(x))^4}{8}-\frac{3\,(\sin(x))^5}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4-3*cos(x))*(1-1/2*sin(x))^4,x)
```

```
[Out] 227/32*x+8*cos(x)-3*cos(x)*sin(x)+2/3*(2+sin(x)^2)*cos(x)-1/16*(s
in(x)^3+3/2*sin(x))*cos(x)-3*sin(x)-3*cos(x)^2-3/2*sin(x)^3+3/8*s
in(x)^4-3/80*sin(x)^5
```

Maxima [A] time = 1.37653, size = 73, normalized size = 1.04

$$-\frac{3}{80}\,\sin(x)^5+\frac{3}{8}\,\sin(x)^4-\frac{2}{3}\,\cos(x)^3-\frac{3}{2}\,\sin(x)^3-3\,\cos(x)^2+\frac{227}{32}\,x+10\,\cos(x)+\frac{1}{128}\,\sin(4\,x)-\frac{25}{16}\,\sin(2\,x)-3\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/16*(3*cos(x) - 4)*(sin(x) - 2)^4,x, algorithm="maxima")
```

```
[Out] -3/80*sin(x)^5 + 3/8*sin(x)^4 - 2/3*cos(x)^3 - 3/2*sin(x)^3 - 3*c
os(x)^2 + 227/32*x + 10*cos(x) + 1/128*sin(4*x) - 25/16*sin(2*x)
- 3*sin(x)
```

Fricas [A] time = 0.221279, size = 73, normalized size = 1.04

$$\frac{3}{8}\cos(x)^4 - \frac{2}{3}\cos(x)^3 - \frac{15}{4}\cos(x)^2 - \frac{1}{160}\left(6\cos(x)^4 - 10\cos(x)^3 - 252\cos(x)^2 + 505\cos(x) + 726\right)\sin(x) + \frac{227}{32}x + 10\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/16*(3*cos(x) - 4)*(sin(x) - 2)^4,x, algorithm="fricas")
```

```
[Out] 3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 -
 10*cos(x)^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x
 + 10*cos(x)
```

Sympy [A] time = 2.113, size = 148, normalized size = 2.11

$$\frac{3x\sin^4(x)}{32} + \frac{3x\sin^2(x)\cos^2(x)}{16} + 3x\sin^2(x) + \frac{3x\cos^4(x)}{32} + 3x\cos^2(x) + 4x - \frac{3\sin^5(x)}{80} + \frac{3\sin^4(x)}{8} - \frac{5\sin^3(x)\cos(x)}{32} - \frac{3\sin^3(x)}{2} + 2\sin^2(x)\cos(x) + 3\sin^2(x) - \frac{3\sin(x)\cos^3(x)}{32} - 3\sin(x)\cos(x) - 3\sin(x) + \frac{4\cos^3(x)}{3} + 8\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

```
[Out] 3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3
*x*cos(x)**4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 + 3*sin(x)
**4/8 - 5*sin(x)**3*cos(x)/32 - 3*sin(x)**3/2 + 2*sin(x)**2*cos(x
) + 3*sin(x)**2 - 3*sin(x)*cos(x)**3/32 - 3*sin(x)*cos(x) - 3*sin
(x) + 4*cos(x)**3/3 + 8*cos(x)
```

GIAC/XCAS [A] time = 0.198921, size = 73, normalized size = 1.04

$$\frac{227}{32}x + \frac{3}{64}\cos(4x) - \frac{1}{6}\cos(3x) - \frac{27}{16}\cos(2x) + \frac{19}{2}\cos(x) - \frac{3}{1280}\sin(5x) + \frac{1}{128}\sin(4x) + \frac{99}{256}\sin(3x) - \frac{25}{16}\sin(2x) - \frac{531}{128}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/16*(3*cos(x) - 4)*(sin(x) - 2)^4,x, algorithm="giac")
```

```
[Out] 227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*c
os(x) - 3/1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/1
6*sin(2*x) - 531/128*sin(x)
```

3.365 $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$

Optimal. Leaf size=33

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

```
[Out] (-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] +
4*Log[Sin[x]]
```

Rubi [A] time = 0.0945191, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] +
4*Log[Sin[x]]
```

Rubi in Sympy [A] time = 8.21935, size = 34, normalized size = 1.03

$$-\frac{285x}{2}+\left(3-\frac{2}{\tan(x)}\right)^3+5\left(3-\frac{2}{\tan(x)}\right)^2+4\log(\sin(x))-\frac{42}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
```

```
[Out] -285*x/2 + (3 - 2/tan(x))**3 + 5*(3 - 2/tan(x))**2 + 4*log(sin(x)
) - 42/tan(x)
```

Mathematica [A] time = 0.0417091, size = 29, normalized size = 0.88

$$-\frac{285x}{2}-148\cot(x)+56\csc^2(x)+4\log(\sin(x))-8\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 - 148*Cot[x] + 56*Csc[x]^2 - 8*Cot[x]*Csc[x]^2 + 4*Log
[Sin[x]]
```

Maple [A] time = 0.005, size = 33, normalized size = 1.

$$-8\left(\cot(x)\right)^3+56\left(\cot(x)\right)^2-156\cot(x)-2\ln\left(\left(\cot(x)\right)^2+1\right)+\frac{285\pi}{4}-\frac{285x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/2-3*cot(x))*(3-2*cot(x))^3,x)
```

```
[Out] -8*cot(x)^3+56*cot(x)^2-156*cot(x)-2*ln(cot(x)^2+1)+285/4*Pi-285/
2*x
```

Maxima [A] time = 1.51454, size = 49, normalized size = 1.48

$$-\frac{285}{2}x-\frac{4\left(39\tan(x)^2-14\tan(x)+2\right)}{\tan(x)^3}-2\log\left(\tan(x)^2+1\right)+4\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(6*cot(x) - 1)*(2*cot(x) - 3)^3,x, algorithm="maxima")
```

```
[Out] -285/2*x - 4*(39*tan(x)^2 - 14*tan(x) + 2)/tan(x)^3 - 2*log(tan(x
)^2 + 1) + 4*log(tan(x))
```

Fricas [A] time = 0.227121, size = 96, normalized size = 2.91

$$\frac{4(\cos(2x)-1)\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)\sin(2x)-296\cos(2x)^2-(285x\cos(2x)-285x+224)\sin(2x)+32\cos(2x)+32}{2(\cos(2x)-1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(6*cot(x) - 1)*(2*cot(x) - 3)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2)*sin(2*x) - 296*cos
(2*x)^2 - (285*x*cos(2*x) - 285*x + 224)*sin(2*x) + 32*cos(2*x) +
 328)/((cos(2*x) - 1)*sin(2*x))
```

Sympy [A] time = 0.980392, size = 39, normalized size = 1.18

$$-\frac{285x}{2} - 2\log\left(\tan^2(x) + 1\right) + 4\log(\tan(x)) - \frac{156}{\tan(x)} + \frac{56}{\tan^2(x)} - \frac{8}{\tan^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
```

```
[Out] -285*x/2 - 2*log(tan(x)**2 + 1) + 4*log(tan(x)) - 156/tan(x) + 56
/tan(x)**2 - 8/tan(x)**3
```

GIAC/XCAS [A] time = 0.21699, size = 101, normalized size = 3.06

$$\tan\left(\frac{1}{2}x\right)^3 + 14\tan\left(\frac{1}{2}x\right)^2 - \frac{285}{2}x - \frac{22\tan\left(\frac{1}{2}x\right)^3 + 225\tan\left(\frac{1}{2}x\right)^2 - 42\tan\left(\frac{1}{2}x\right) + 3}{3\tan\left(\frac{1}{2}x\right)^3} - 4\ln\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 4\ln\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right) + 75\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(6*cot(x) - 1)*(2*cot(x) - 3)^3,x, algorithm="giac")
```

```
[Out] tan(1/2*x)^3 + 14*tan(1/2*x)^2 - 285/2*x - 1/3*(22*tan(1/2*x)^3 +
 225*tan(1/2*x)^2 - 42*tan(1/2*x) + 3)/tan(1/2*x)^3 - 4*ln(tan(1/
2*x)^2 + 1) + 4*ln(abs(tan(1/2*x))) + 75*tan(1/2*x)
```

3.366 $\int \cos(5x)\sec^5(x)\,dx$

Optimal. Leaf size=16

$$16x+\frac{5\tan^3(x)}{3}-15\tan(x)$$

```
[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3
```

Rubi [A] time = 0.0702948, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$

$$16x+\frac{5\tan^3(x)}{3}-15\tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(5x)}{\cos^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(5*x)/cos(x)**5,x)
```

```
[Out] Integral(cos(5*x)/cos(x)**5, x)
```

Mathematica [A] time = 0.0147957, size = 20, normalized size = 1.25

$$16x-\frac{50\tan(x)}{3}+\frac{5}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.083, size = 21, normalized size = 1.3

$$16\,x-5\,\left(-2/3-1/3\,\left(\sec\left(x\right)\right)^{2}\right)\tan\left(x\right)-20\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(5*x)/cos(x)^5,x)
```

```
[Out] 16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)
```

Maxima [A] time = 1.55745, size = 19, normalized size = 1.19

$$\frac{5}{3}\,\tan\left(x\right)^{3}+16\,x-15\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")
```

```
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

Fricas [A] time = 0.228851, size = 35, normalized size = 2.19

$$\frac{48\,x\cos\left(x\right)^{3}-5\,\left(10\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}{3\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)/cos(x)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.198598, size = 19, normalized size = 1.19

$$\frac{5}{3}\tan(x)^3 + 16x - 15\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)/cos(x)^5,x, algorithm="giac")
```

```
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

3.367 $\int \cos(4x)\sec(x)\,dx$

Optimal. Leaf size=12

$$\tanh^{-1}(\sin(x))-\frac{8\sin^3(x)}{3}$$

```
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Rubi [A] time = 0.0431068, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\tanh^{-1}(\sin(x))-\frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[4*x]*Sec[x],x]

[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Rubi in Sympy [A] time = 1.8727, size = 15, normalized size = 1.25

$$x\cos(3x)+\log(\cos(x))\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(4*x)/cos(x),x)

[Out] x*cos(3*x) + log(cos(x))*sin(3*x)
```

Mathematica [B] time = 0.0141269, size = 45, normalized size = 3.75

$$-2\sin(x)+\frac{2}{3}\sin(3x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[4*x]*Sec[x],x]

[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x] +
 (2*Sin[3*x])/3
```

Maple [B] time = 0.027, size = 22, normalized size = 1.8

$$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+\frac{\left(16+8\ \left(\cos\left(x\right)\right)^{2}\right)\ \sin\left(x\right)}{3}-8\ \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(4*x)/cos(x),x)
```

```
[Out] ln(sec(x)+tan(x))+8/3*(2+cos(x)^2)*sin(x)-8*sin(x)
```

Maxima [A] time = 1.47201, size = 28, normalized size = 2.33

$$-\frac{8}{3}\ \sin\left(x\right)^{3}+\frac{1}{2}\ \log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\ \log\left(\sin\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x),x, algorithm="maxima")
```

```
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Fricas [A] time = 0.233364, size = 36, normalized size = 3.

$$\frac{8}{3}\left(\cos\left(x\right)^{2}-1\right)\ \sin\left(x\right)+\frac{1}{2}\ \log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\ \log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x),x, algorithm="fricas")
```

```
[Out] 8/3*(cos(x)^2 - 1)*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x)
 + 1)
```

Sympy [A] time = 0.842551, size = 24, normalized size = 2.

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}-\frac{8\sin^{3}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x),x)

[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - 8*sin(x)**3/3
```

GIAC/XCAS [A] time = 0.204612, size = 31, normalized size = 2.58

$$-\frac{8}{3}\sin(x)^3+\frac{1}{2}\ln(\sin(x)+1)-\frac{1}{2}\ln(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x),x, algorithm="giac")

[Out] -8/3*sin(x)^3 + 1/2*ln(sin(x) + 1) - 1/2*ln(-sin(x) + 1)
```

3.368 $\int \cos(x)\cos(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rubi [A] time = 0.0163079, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rubi in Sympy [A] time = 0.965101, size = 12, normalized size = 0.71

$$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(4*x),x)

[Out] sin(3*x)/6 + sin(5*x)/10
```

Mathematica [A] time = 0.00964717, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Maple [A] time = 0.066, size = 14, normalized size = 0.8

$$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(4*x),x)
```

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Maxima [A] time = 1.37692, size = 18, normalized size = 1.06

$$\frac{1}{10}\sin(5x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Fricas [A] time = 0.231394, size = 24, normalized size = 1.41

$$\frac{1}{15}\left(24\cos(x)^4-8\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.723057, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(4x)}{15}+\frac{4\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*cos(4*x),x)
```

```
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

GIAC/XCAS [A] time = 0.198041, size = 18, normalized size = 1.06

$$\frac{1}{10}\sin\left(5\,x\right)+\frac{1}{6}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

3.369 $\int \cos(4x)\sec^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{35}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)-\frac{29}{8}\tan(x)\sec(x)$$

```
[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])
/4
```

Rubi [A] time = 0.0591457, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{35}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)-\frac{29}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])
/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(4x)}{\cos^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(4*x)/cos(x)**5,x)
```

```
[Out] Integral(cos(4*x)/cos(x)**5, x)
```

Mathematica [B] time = 0.211743, size = 58, normalized size = 2.23

$$\frac{1}{16}\left(-\frac{1}{2}(21\sin(x)+29\sin(3x))\sec^4(x)-70\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+70\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (-70*Log[Cos[x/2] - Sin[x/2]] + 70*Log[Cos[x/2] + Sin[x/2]] - (Se
c[x]^4*(21*Sin[x] + 29*Sin[3*x]))/2)/16
```

Maple [A] time = 0.032, size = 31, normalized size = 1.2

$$-\left(-\frac{(\sec(x))^3}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\frac{35\ln(\sec(x)+\tan(x))}{8}-4\sec(x)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(4*x)/cos(x)^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+35/8*ln(sec(x)+tan(x))-4*sec(x
)*tan(x)
```

Maxima [A] time = 1.4318, size = 73, normalized size = 2.81

$$\frac{5\sin(x)^3-3\sin(x)}{8\left(\sin(x)^4-2\sin(x)^2+1\right)}+\frac{3\sin(x)}{\sin(x)^2-1}+\frac{35}{16}\log(\sin(x)+1)-\frac{35}{16}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*sin(x
)/(sin(x)^2 - 1) + 35/16*log(sin(x) + 1) - 35/16*log(sin(x) - 1)
```

Fricas [A] time = 0.233192, size = 58, normalized size = 2.23

$$\frac{35\cos(x)^4\log(\sin(x)+1)-35\cos(x)^4\log(-\sin(x)+1)-2\left(29\cos(x)^2-2\right)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(35*cos(x)^4*log(sin(x) + 1) - 35*cos(x)^4*log(-sin(x) + 1)
- 2*(29*cos(x)^2 - 2)*sin(x))/cos(x)^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.208823, size = 51, normalized size = 1.96

$$\frac{29\ \sin\left(x\right)^{3} - 27\ \sin\left(x\right)}{8\left(\sin\left(x\right)^{2} - 1\right)^{2}} + \frac{35}{16}\ln\left(\sin\left(x\right) + 1\right) - \frac{35}{16}\ln\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(29*sin(x)^3 - 27*sin(x))/(sin(x)^2 - 1)^2 + 35/16*ln(sin(x)
+ 1) - 35/16*ln(-sin(x) + 1)
```

3.370 $\int \cos^4(x) \cos(4x) \, dx$

Optimal. Leaf size=38

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

```
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Rubi [A] time = 0.0512354, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4*Cos[4*x],x]
```

```
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}+\int\frac{1}{16}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*cos(4*x),x)
```

```
[Out] sin(2*x)/8 + 3*sin(4*x)/32 + sin(6*x)/24 + sin(8*x)/128 + Integra
l(1/16, x)
```

Mathematica [A] time = 0.0187952, size = 38, normalized size = 1.

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4*Cos[4*x],x]
```

```
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Maple [A] time = 0.07, size = 29, normalized size = 0.8

$$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*cos(4*x),x)
```

```
[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

Maxima [A] time = 1.38624, size = 41, normalized size = 1.08

$$-\frac{1}{6}\sin(2x)^3+\frac{1}{16}x+\frac{1}{128}\sin(8x)+\frac{3}{32}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*s
in(2*x)
```

Fricas [A] time = 0.252802, size = 42, normalized size = 1.11

$$\frac{1}{48}\left(48\cos(x)^7-8\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) +
1/16*x
```

Sympy [A] time = 45.8458, size = 139, normalized size = 3.66

$$\frac{x\sin^4(x)\cos(4x)}{16} - \frac{x\sin^3(x)\sin(4x)\cos(x)}{4} - \frac{3x\sin^2(x)\cos^2(x)\cos(4x)}{8} + \frac{x\sin(x)\sin(4x)\cos^3(x)}{4} + \frac{x\cos^4(x)\cos(4x)}{16} - \frac{\sin^4(x)\sin(4x)}{64} - \frac{5\sin^2(x)\sin(4x)\cos^2(x)}{32} - \frac{\sin(x)\cos^3(x)\cos(4x)}{3} + \frac{61\sin(4x)\cos^4(x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*cos(4*x),x)
```

```
[Out] x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin
(x)**2*cos(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*c
os(x)**4*cos(4*x)/16 - sin(x)**4*sin(4*x)/64 - 5*sin(x)**2*sin(4*
x)*cos(x)**2/32 - sin(x)*cos(x)**3*cos(4*x)/3 + 61*sin(4*x)*cos(x
)**4/192
```

GIAC/XCAS [A] time = 0.196763, size = 38, normalized size = 1.

$$\frac{1}{16}x + \frac{1}{128}\sin(8x) + \frac{1}{24}\sin(6x) + \frac{3}{32}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin
(2*x)
```

3.371 $\int \cos(5x)\csc^5(x)\,dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

```
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Rubi [A] time = 0.0908678, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[5*x]*Csc[x]^5,x]

[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Rubi in Sympy [A] time = 23.4735, size = 29, normalized size = 1.45

$$8\log\left(-\cos^2(x)+1\right)+\frac{6}{-\cos^2(x)+1}-\frac{1}{4\left(-\cos^2(x)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(5*x)/sin(x)**5,x)

[Out] 8*log(-cos(x)**2 + 1) + 6/(-cos(x)**2 + 1) - 1/(4*(-cos(x)**2 + 1
)**2)
```

Mathematica [A] time = 0.0125984, size = 20, normalized size = 1.

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[5*x]*Csc[x]^5,x]
```

```
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Maple [A] time = 0.066, size = 35, normalized size = 1.8

$$-\frac{5}{4\ (\sin(x))^4}+5\,\frac{(\cos(x))^4}{(\sin(x))^4}-4\ (\cot(x))^4+8\ (\cot(x))^2+16\ \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/sin(x)^5,x)
```

```
[Out] -5/4/sin(x)^4+5/sin(x)^4*cos(x)^4-4*cot(x)^4+8*cot(x)^2+16*ln(sin
(x))
```

Maxima [A] time = 1.34561, size = 45, normalized size = 2.25

$$\frac{5}{\sin(x)^2}+\frac{4\ \sin(x)^2-1}{4\ \sin(x)^4}+\frac{11}{2}\ \log\left(\sin(x)^2\right)+5\ \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 5/sin(x)^2 + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 11/2*log(sin(x)^2) +
 5*log(sin(x))
```

Fricas [A] time = 0.230294, size = 58, normalized size = 2.9

$$-\frac{24\ \cos(x)^2-64\left(\cos(x)^4-2\ \cos(x)^2+1\right)\log\left(\frac{1}{2}\ \sin(x)\right)-23}{4\left(\cos(x)^4-2\ \cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(24*cos(x)^2 - 64*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*sin(x)
) - 23)/(cos(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)/sin(x)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.204975, size = 136, normalized size = 6.8

$$-\frac{\left(\frac{92\,(\cos(x)-1)}{\cos(x)+1}+\frac{768\,(\cos(x)-1)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)^2}{64\,(\cos(x)-1)^2}-\frac{23\,(\cos(x)-1)}{16\,(\cos(x)+1)}-\frac{(\cos(x)-1)^2}{64\,(\cos(x)+1)^2}-16\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)+8\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)/sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/64*(92*(cos(x) - 1)/(cos(x) + 1) + 768*(cos(x) - 1)^2/(cos(x)
+ 1)^2 + 1)*(cos(x) + 1)^2/(cos(x) - 1)^2 - 23/16*(cos(x) - 1)/(c
os(x) + 1) - 1/64*(cos(x) - 1)^2/(cos(x) + 1)^2 - 16*ln(-(cos(x)
- 1)/(cos(x) + 1) + 1) + 8*ln(-(cos(x) - 1)/(cos(x) + 1))
```

3.372 $\int \csc^4(x)\sin(4x)\,dx$

Optimal. Leaf size=12

$$-2\csc^2(x) - 8\log(\sin(x))$$

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Rubi [A] time = 0.0360659, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-2\csc^2(x) - 8\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^4*Sin[4*x],x]

[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(4x)}{\sin^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(4*x)/sin(x)**4,x)

[Out] Integral(sin(4*x)/sin(x)**4, x)
```

Mathematica [A] time = 0.0112359, size = 12, normalized size = 1.

$$-2\csc^2(x) - 8\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^4*Sin[4*x],x]

[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Maple [A] time = 0.073, size = 19, normalized size = 1.6

$$2 \left(\sin \left(x\right)\right)^{-2}-4 \left(\cot \left(x\right)\right)^{2}-8 \ln \left(\sin \left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(4*x)/sin(x)^4,x)

[Out] 2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

Maxima [A] time = 1.35628, size = 26, normalized size = 2.17

$$-\frac{2}{\sin \left(x\right)^{2}}-2 \log \left(\sin \left(x\right)^{2}\right)-4 \log \left(\sin \left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")

[Out] -2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

Fricas [A] time = 0.234589, size = 34, normalized size = 2.83

$$-\frac{2 \left(4 \left(\cos \left(x\right)^{2}-1\right) \log \left(\frac{1}{2} \sin \left(x\right)\right)-1\right)}{\cos \left(x\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")

[Out] -2*(4*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)
```

Sympy [A] time = 37.1343, size = 14, normalized size = 1.17

$$-8 \log \left(\sin \left(x\right)\right)-\frac{2}{\sin^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)/sin(x)**4,x)
```

```
[Out] -8*log(sin(x)) - 2/sin(x)**2
```

GIAC/XCAS [A] time = 0.206748, size = 96, normalized size = 8.

$$\frac{\left(\frac{8\,(\cos(x)-1)}{\cos(x)+1}+1\right)(\cos(x)+1)}{2\,(\cos(x)-1)}+\frac{\cos(x)-1}{2\,(\cos(x)+1)}+8\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)-4\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)/sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) +
 1/2*(cos(x) - 1)/(cos(x) + 1) + 8*ln(-(cos(x) - 1)/(cos(x) + 1)
+ 1) - 4*ln(-(cos(x) - 1)/(cos(x) + 1))
```

3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

Optimal. Leaf size=64

$$-\frac{x}{2\sqrt{3}}+\frac{1}{2}\log(\sin(x))+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}-\frac{1}{4}\log(\sin(x)\cos(x)+1)$$

```
[Out] -x/(2*Sqrt[3]) + ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] + 2*Cos[x]*
Sin[x])]/(2*Sqrt[3]) + Log[Sin[x]]/2 - Log[1 + Cos[x]*Sin[x]]/4
```

Rubi [A] time = 0.137103, antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546

$$-\frac{x}{2\sqrt{3}}-\frac{1}{4}\log\left(\tan^2(x)+\tan(x)+1\right)+\frac{1}{2}\log(\tan(x))+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] -x/(2*Sqrt[3]) + ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] + 2*Cos[x]*
Sin[x])]/(2*Sqrt[3]) + Log[Tan[x]]/2 - Log[1 + Tan[x] + Tan[x]^2]
/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0474634, size = 39, normalized size = 0.61

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)+6\log(\sin(x))-3\log(\sin(2x)+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Lo
g[2 + Sin[2*x]])/12
```

Maple [A] time = 0.092, size = 35, normalized size = 0.6

$$\frac{\ln(\tan(x))}{2} - \frac{\ln\left(1+\tan(x)+(\tan(x))^2\right)}{4} - \frac{\sqrt{3}}{6}\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/sin(x)/(2+sin(2*x)),x)
```

```
[Out] 1/2*ln(tan(x))-1/4*ln(1+tan(x)+tan(x)^2)-1/6*3^(1/2)*arctan(1/3*(
2*tan(x)+1)*3^(1/2))
```

Maxima [A] time = 1.59571, size = 281, normalized size = 4.39

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(-2(4\sin(2x)+1)\cos(4x)+\cos(4x)^2+16\cos(2x)^2+8\cos(2x)\sin(4x)+\sin(4x)^2+16\sin(2x)^2+8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/((sin(2*x) + 2)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x
)^2 + 16*cos(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2
*x)^2 + 8*sin(2*x) + 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) -
2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*
x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2 + sin(2*x)^2 + 4*si
n(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 -
 4*sqrt(3) + 7)))
```

Fricas [A] time = 0.237682, size = 96, normalized size = 1.5

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right)-2\sqrt{3}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)+2\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)}{3\left(2\cos(x)^2-1\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/((sin(2*x) + 2)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x)
 + 1) - 2*sqrt(3)*log(-1/4*cos(x)^2 + 1/4) + 2*arctan(1/3*(4*sqrt
(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{(\sin(2x)+2)\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
```

```
[Out] Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)
```

GIAC/XCAS [A] time = 0.214449, size = 101, normalized size = 1.58

$$-\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right)-\frac{1}{4}\ln\left(\tan(x)^2+\tan(x)+1\right)+\frac{1}{2}\ln\left(|\tan(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/((sin(2*x) + 2)*sin(x)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*
x) - 1)/(sqrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))
) - 1/4*ln(tan(x)^2 + tan(x) + 1) + 1/2*ln(abs(tan(x)))
```

3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\csc^2(x)-4\right)$$

```
[Out] -Log[-4 + Csc[x]^2]/2
```

Rubi [A] time = 0.0591418, antiderivative size = 17, normalized size of antiderivative = 1.55, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$

$$\log(\sin(x))-\frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Cot[x]*Sec[3*x],x]

[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

Rubi in Sympy [A] time = 21.4559, size = 20, normalized size = 1.82

$$-\frac{\log\left(-4\cos^2(x)+3\right)}{2}+\frac{\log\left(-\cos^2(x)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2/cos(3*x)/sin(x),x)

[Out] -log(-4*cos(x)**2 + 3)/2 + log(-cos(x)**2 + 1)/2
```

Mathematica [A] time = 0.0188572, size = 17, normalized size = 1.55

$$\log(\sin(x))-\frac{1}{2}\log(1-2\cos(2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Cot[x]*Sec[3*x],x]
```

```
[Out] -Log[1 - 2*Cos[2*x]]/2 + Log[Sin[x]]
```

Maple [B] time = 0.059, size = 27, normalized size = 2.5

$$\frac{\ln\left(1+\cos\left(x\right)\right)}{2}+\frac{\ln\left(\cos\left(x\right)-1\right)}{2}-\frac{\ln\left(4\left(\cos\left(x\right)\right)^{2}-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2/cos(3*x)/sin(x),x)
```

```
[Out] 1/2*ln(1+cos(x))+1/2*ln(cos(x)-1)-1/2*ln(4*cos(x)^2-3)
```

Maxima [A] time = 1.34871, size = 124, normalized size = 11.27

$$-\frac{1}{4}\log\left(-2\left(\cos\left(2x\right)-1\right)\cos\left(4x\right)+\cos\left(4x\right)^{2}+\cos\left(2x\right)^{2}+\sin\left(4x\right)^{2}-2\sin\left(4x\right)\sin\left(2x\right)+\sin\left(2x\right)^{2}-2\cos\left(2x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/(cos(3*x)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + s
in(4*x)^2 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) +
1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 0.22399, size = 23, normalized size = 2.09

$$-\frac{1}{2}\log\left(4\cos\left(x\right)^{2}-3\right)+\log\left(\frac{1}{2}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/(cos(3*x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(4*cos(x)^2 - 3) + log(1/2*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(x)}{\sin(x)\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2/cos(3*x)/sin(x),x)
```

```
[Out] Integral(cos(x)**2/(sin(x)*cos(3*x)), x)
```

GIAC/XCAS [A] time = 0.224393, size = 32, normalized size = 2.91

$$\frac{1}{2}\ln\left(-\cos(x)^2+1\right)-\frac{1}{2}\ln\left(\left|4\cos(x)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/(cos(3*x)*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*ln(-cos(x)^2 + 1) - 1/2*ln(abs(4*cos(x)^2 - 3))
```

3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal. Leaf size=7

$$-\tan^{-1}(\cos(2x))$$

```
[Out] -ArcTan[Cos[2*x]]
```

Rubi [A] time = 0.0627147, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]

[Out] -ArcTan[Cos[2*x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)}{\sin^4(x)+\cos^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)

[Out] Integral(sin(2*x)/(sin(x)**4 + cos(x)**4), x)
```

Mathematica [A] time = 0.0188649, size = 7, normalized size = 1.

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]

[Out] -ArcTan[Cos[2*x]]
```

Maple [A] time = 0.053, size = 12, normalized size = 1.7

$$-\arctan\left(2\ (\cos(x))^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)/(cos(x)^4+sin(x)^4),x)
```

```
[Out] -arctan(2*cos(x)^2-1)
```

Maxima [A] time = 1.52137, size = 12, normalized size = 1.71

$$\arctan\left(2\ \sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(cos(x)^4 + sin(x)^4),x, algorithm="maxima")
```

```
[Out] arctan(2*sin(x)^2 - 1)
```

Fricas [A] time = 0.233173, size = 15, normalized size = 2.14

$$-\arctan\left(2\ \cos(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(cos(x)^4 + sin(x)^4),x, algorithm="fricas")
```

```
[Out] -arctan(2*cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.199004, size = 12, normalized size = 1.71

$$\arctan\left(2\,\sin\left(x\right)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(cos(x)^4 + sin(x)^4),x, algorithm="giac")
```

```
[Out] arctan(2*sin(x)^2 - 1)
```

3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\cos(x)-\sqrt{3}\sin(x)}{\sin(x)+\sqrt{3}\cos(x)+2\left(2+\sqrt{3}\right)}\right)}{\sqrt{3}}$$

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x] - Sqrt[3]*Sin[x])/(2*(2 + Sqrt[3])
 + Sqrt[3]*Cos[x] + Sin[x])]/Sqrt[3]
```

Rubi [A] time = 0.196873, antiderivative size = 83, normalized size of antiderivative = 1.57, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(3-4\sqrt{3}\right)\sin(x)+\left(4-\sqrt{3}\right)\cos(x)}{\left(4-\sqrt{3}\right)\sin(x)-\left(3-4\sqrt{3}\right)\cos(x)+2\left(5+2\sqrt{3}\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[((4 - Sqrt[3])*Cos[x] + (3 - 4*Sqrt[3])*Si
n[x])/(2*(5 + 2*Sqrt[3]) - (3 - 4*Sqrt[3])*Cos[x] + (4 - Sqrt[3])
*Sin[x])]/Sqrt[3]
```

Rubi in Sympy [A] time = 1.05476, size = 31, normalized size = 0.58

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\left(-\frac{\sqrt{3}}{6}+\frac{2}{3}\right)\tan\left(\frac{x}{2}\right)+\frac{1}{6}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*((-sqrt(3)/6 + 2/3)*tan(x/2) + 1/6))/3
```

Mathematica [A] time = 0.0795651, size = 33, normalized size = 0.62

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt{3}-4\right)\tan\left(\frac{x}{2}\right)-1}{2\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])
```

Maple [A] time = 0.075, size = 43, normalized size = 0.8

$$-52\frac{1}{\left(\sqrt{3}-4\right)\left(16\sqrt{3}+12\right)}\arctan\left(\frac{26\tan(x/2)+2\sqrt{3}+8}{16\sqrt{3}+12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4+sin(x)+cos(x)*3^(1/2)),x)
```

```
[Out] -52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8
)/(16*3^(1/2)+12))
```

Maxima [A] time = 1.48518, size = 36, normalized size = 0.68

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{\left(\sqrt{3}-4\right)\sin(x)}{\cos(x)+1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3)*cos(x) + sin(x) + 4),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3) - 4)*sin(x)/(cos(x) + 1
) - 1))
```

Fricas [A] time = 0.235278, size = 47, normalized size = 0.89

$$-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{2\,\left(\sqrt{3}\sin\left(x\right)+\sqrt{3}+3\,\cos\left(x\right)\right)}{3\,\left(\sqrt{3}\sin\left(x\right)-\cos\left(x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3)*cos(x) + sin(x) + 4),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(2/3*(sqrt(3)*sin(x) + sqrt(3) + 3*cos(x))/(sq
rt(3)*sin(x) - cos(x)))
```

Sympy [A] time = 15.0299, size = 107, normalized size = 2.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

```
[Out] -566281207851278924670873301782883361333136358814569713466828545 9
880259664341133289449385432355252040810381906529297855036821637 99
903899723386341595971933164152351579111667531570238858548215783 74
471173305508724204664925117118228716789710429927603063675376140 17
285687614239315634861973066641854236409602777 9*sqrt(3)*(atan(-tan
(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/
2)/pi))/(-169884362355383677401261990534865008399940907644370914 0
400485637964077899302339986834815629706575612243114571958789356 51
104649139971169917015902478791579949245705473733500259471071657 56
446473512341351991652617261399477535135468615036913128978280919 10
2612842051857062842717946904585919199925562709228808333 7 + 980827
823369886918356167164347198540607174082401187506216982171503302 38
384126593851159394996450499088535783529278726660262335067772200 86
279046181356124232790191632005066672566913862506830135059195461 21
567417172540892074704740205814595088770662901497095894439087244 54
027554596549165362939031865731186980004*sqrt(3)) + 98082782336988
691835616716434719854060717408240118750621698217150330238384126 59
385115939499645049908853578352927872666026233506777220086279046 18
135612423279019163200506667256691386250683013505919546121567417 17
254089207470474020581459508877066290149709589443908724454027554 59
654916536293903186573118698000 4*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan
(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-169884362355 3
836774012619905348650083999409076443709140400485637964077899302 33
998683481562970657561224311457195878935651104649139971169917015 90
247879157994924570547373350025947107165756446473512341351991652 61
726139947753513546861503691312897828091910261284205185706284271 79
4690458591919992556270922880833 37 + 98082782336988691835616716434
719854060717408240118750621698217150330238384126593851159394996 45
```

```
0499088535783529278726660262335067772200862790461813561242327901 9
1632005066672566913862506830135059195461215674171725408920747047 4
0205814595088770662901497095894439087244540275545965491653629390 3
1865731186980004*sqrt(3))
```

GIAC/XCAS [A] time = 0.211906, size = 105, normalized size = 1.98

$$\frac{\left(x + 2\,\arctan\left(\frac{\sqrt{3}\cos(x) - 8\,\sqrt{3}\sin(x) + \sqrt{3} + 4\,\cos(x) + 7\,\sin(x) + 4}{8\,\sqrt{3}\cos(x) + \sqrt{3}\sin(x) + 8\,\sqrt{3} - 7\,\cos(x) + 4\,\sin(x) + 19}\right)\right)\left(\sqrt{3} + 4\right)}{2\left(4\,\sqrt{3} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3)*cos(x) + sin(x) + 4),x, algorithm="giac")
```

```
[Out] 1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) +
4*cos(x) + 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*s
qrt(3) - 7*cos(x) + 4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3
)
```

3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{3\sin(x)+3\cos(x)+8}\right)}{\sqrt{23}}$$

```
[Out] -(ArcTanh[(Sqrt[23]*(Cos[x] - Sin[x]))/(8 + 3*Cos[x] + 3*Sin[x])]
/Sqrt[23])
```

Rubi [B] time = 0.134708, antiderivative size = 94, normalized size of antiderivative = 2.85, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\log\left(\sqrt{23}\sin(x)-4\sin(x)-4\sqrt{23}\cos(x)+19\cos(x)+4\left(5-\sqrt{23}\right)\right)}{2\sqrt{23}}-\frac{\log\left(-\sqrt{23}\sin(x)-4\sin(x)+4\sqrt{23}\cos(x)+19\cos(x)+4\left(5+\sqrt{23}\right)\right)}{2\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] -Log[4*(5 + Sqrt[23]) + 19*Cos[x] + 4*Sqrt[23]*Cos[x] - 4*Sin[x]
- Sqrt[23]*Sin[x]]/(2*Sqrt[23]) + Log[4*(5 - Sqrt[23]) + 19*Cos[x
] - 4*Sqrt[23]*Cos[x] - 4*Sin[x] + Sqrt[23]*Sin[x]]/(2*Sqrt[23])
```

Rubi in Sympy [A] time = 0.841031, size = 26, normalized size = 0.79

$$-\frac{2\sqrt{23}\,\mathrm{atanh}\left(\sqrt{23}\left(-\frac{\tan\left(\frac{x}{2}\right)}{23}+\frac{4}{23}\right)\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3+4*cos(x)+4*sin(x)),x)
```

```
[Out] -2*sqrt(23)*atanh(sqrt(23)*(-tan(x/2)/23 + 4/23))/23
```

Mathematica [A] time = 0.0570898, size = 22, normalized size = 0.67

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-4}{\sqrt{23}}\right)}{\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]
```

Maple [A] time = 0.052, size = 20, normalized size = 0.6

$$\frac{2\sqrt{23}}{23}Artanh\left(\frac{\sqrt{23}}{46}\left(-8+2\tan(x/2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3+4*cos(x)+4*sin(x)),x)
```

```
[Out] 2/23*23^(1/2)*arctanh(1/46*(-8+2*tan(1/2*x))*23^(1/2))
```

Maxima [A] time = 1.52082, size = 53, normalized size = 1.61

$$-\frac{1}{23}\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x) + 4*sin(x) + 3),x, algorithm="maxima")
```

```
[Out] -1/23*sqrt(23)*log(-(sqrt(23) - sin(x)/(cos(x) + 1) + 4)/(sqrt(23
) + sin(x)/(cos(x) + 1) - 4))
```

Fricas [A] time = 0.246933, size = 90, normalized size = 2.73

$$\frac{1}{46}\sqrt{23}\log\left(\frac{8\left(3\sqrt{23}-23\right)\cos(x)-138\cos(x)^2-2\left(7\sqrt{23}\cos(x)-12\sqrt{23}-92\right)\sin(x)+48\sqrt{23}+69}{8\left(4\cos(x)+3\right)\sin(x)+24\cos(x)+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x) + 4*sin(x) + 3),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*log((8*(3*sqrt(23) - 23)*cos(x) - 138*cos(x)^2 - 2*
(7*sqrt(23)*cos(x) - 12*sqrt(23) - 92)*sin(x) + 48*sqrt(23) + 69)
/(8*(4*cos(x) + 3)*sin(x) + 24*cos(x) + 25))
```

Sympy [A] time = 16.5426, size = 39, normalized size = 1.18

$$\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23}-\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3+4*cos(x)+4*sin(x)),x)
```

```
[Out] sqrt(23)*log(tan(x/2) - 4 + sqrt(23))/23 - sqrt(23)*log(tan(x/2)
- sqrt(23) - 4)/23
```

GIAC/XCAS [A] time = 0.233252, size = 50, normalized size = 1.52

$$-\frac{1}{23}\sqrt{23}\ln\left(\frac{\left|-2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}{\left|2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x) + 4*sin(x) + 3),x, algorithm="giac")
```

```
[Out] -1/23*sqrt(23)*ln(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(
23) + 2*tan(1/2*x) - 8))
```

3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)$$

```
[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3
```

Rubi [A] time = 0.0364624, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]

[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3
```

Rubi in Sympy [A] time = 9.27266, size = 7, normalized size = 0.26

$$\frac{\operatorname{atan}(3\tan(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)

[Out] atan(3*tan(x))/3
```

Mathematica [A] time = 0.0159438, size = 9, normalized size = 0.33

$$\frac{1}{3}\tan^{-1}(3\tan(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[3*Tan[x]]/3
```

Maple [A] time = 0.039, size = 8, normalized size = 0.3

$$\frac{\arctan(3\tan(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-3*cos(x)^2+5*sin(x)^2),x)
```

```
[Out] 1/3*arctan(3*tan(x))
```

Maxima [A] time = 1.53778, size = 9, normalized size = 0.33

$$\frac{1}{3}\arctan(3\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(3*cos(x)^2 - 5*sin(x)^2 - 4),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(3*tan(x))
```

Fricas [A] time = 0.222552, size = 28, normalized size = 1.04

$$-\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-9}{6\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(3*cos(x)^2 - 5*sin(x)^2 - 4),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))
```

Sympy [A] time = 30.8507, size = 148, normalized size = 5.48

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{6\sqrt{2}\sqrt{-12\sqrt{2}+17}+9\sqrt{-12\sqrt{2}+17}}+\frac{\sqrt{-12\sqrt{2}+17}\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6\sqrt{2}\sqrt{-12\sqrt{2}+17}+9\sqrt{-12\sqrt{2}+17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)
```

```
[Out] (atan(tan(x/2)/sqrt(-12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/pi
))/(6*sqrt(2)*sqrt(-12*sqrt(2) + 17) + 9*sqrt(-12*sqrt(2) + 17))
+ sqrt(-12*sqrt(2) + 17)*sqrt(12*sqrt(2) + 17)*(atan(tan(x/2)/sqr
t(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/pi))/(6*sqrt(2)*sqrt(
-12*sqrt(2) + 17) + 9*sqrt(-12*sqrt(2) + 17))
```

GIAC/XCAS [A] time = 0.208632, size = 27, normalized size = 1.

$$\frac{1}{3}\,x - \frac{1}{3}\,\arctan\left(\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right) - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*cos(x)^2 - 5*sin(x)^2 - 4),x, algorithm="giac")
```

```
[Out] 1/3*x - 1/3*arctan(sin(2*x)/(cos(2*x) - 2))
```

3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

Optimal. Leaf size=28

$$\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

```
[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))
```

Rubi [A] time = 0.0830554, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\tan(x)+4+\frac{4}{\tan(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] Integral(1/(tan(x) + 4 + 4/tan(x)), x)
```

Mathematica [A] time = 0.0660362, size = 41, normalized size = 1.46

$$\frac{4x-3\log(\sin(x)+2\cos(x))+\cot(x)(8x-6\log(\sin(x)+2\cos(x)))-5}{50\cot(x)+25}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos
[x] + Sin[x]])/(25 + 50*Cot[x])
```

Maple [A] time = 0.14, size = 29, normalized size = 1.

$$\frac{3 \ln \left(1+(\tan (x))^{2}\right)}{50}+\frac{2}{10+5 \tan (x)}-\frac{3 \ln (2+\tan (x))}{25}+\frac{4 x}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] 3/50*ln(1+tan(x)^2)+2/5/(2+tan(x))-3/25*ln(2+tan(x))+4/25*x
```

Maxima [A] time = 1.54814, size = 38, normalized size = 1.36

$$\frac{4}{25} x+\frac{2}{5 (\tan (x)+2)}+\frac{3}{50} \log \left(\tan (x)^{2}+1\right)-\frac{3}{25} \log (\tan (x)+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cot(x) + tan(x) + 4),x, algorithm="maxima")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(tan
(x) + 2)
```

Fricas [A] time = 0.221816, size = 62, normalized size = 2.21

$$-\frac{3 (\tan (x)+2) \log \left(\frac{\tan (x)^{2}+4 \tan (x)+4}{\tan (x)^{2}+1}\right)-8 (x-1) \tan (x)-16 x-4}{50 (\tan (x)+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cot(x) + tan(x) + 4),x, algorithm="fricas")
```

```
[Out] -1/50*(3*(tan(x) + 2)*log((tan(x)^2 + 4*tan(x) + 4)/(tan(x)^2 + 1
)) - 8*(x - 1)*tan(x) - 16*x - 4)/(tan(x) + 2)
```

Sympy [A] time = 0.927537, size = 102, normalized size = 3.64

$$\frac{8x\tan(x)}{50\tan(x)+100}+\frac{16x}{50\tan(x)+100}-\frac{6\log(\tan(x)+2)\tan(x)}{50\tan(x)+100}-\frac{12\log(\tan(x)+2)}{50\tan(x)+100}+\frac{3\log\left(\tan^2(x)+1\right)\tan(x)}{50\tan(x)+100}+\frac{6\log\left(\tan^2(x)+1\right)}{50\tan(x)+100}+\frac{20}{50\tan(x)+100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] 8*x*tan(x)/(50*tan(x) + 100) + 16*x/(50*tan(x) + 100) - 6*log(tan
(x) + 2)*tan(x)/(50*tan(x) + 100) - 12*log(tan(x) + 2)/(50*tan(x)
 + 100) + 3*log(tan(x)**2 + 1)*tan(x)/(50*tan(x) + 100) + 6*log(t
an(x)**2 + 1)/(50*tan(x) + 100) + 20/(50*tan(x) + 100)
```

GIAC/XCAS [A] time = 0.206758, size = 39, normalized size = 1.39

$$\frac{4}{25}x+\frac{2}{5(\tan(x)+2)}+\frac{3}{50}\ln\left(\tan(x)^2+1\right)-\frac{3}{25}\ln(|\tan(x)+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cot(x) + tan(x) + 4),x, algorithm="giac")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*ln(tan(x)^2 + 1) - 3/25*ln(abs(t
an(x) + 2))
```

3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{8x}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}}$$

```
[Out] (8*x)/(15*Sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + Sqrt[15] +
2*Cos[x]*Sin[x])])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x]
 + 2*Tan[x]^2))
```

Rubi [A] time = 0.0840653, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{8x}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sec[x] + Sin[x])^(-2),x]
```



```
[Out] (8*x)/(15*Sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + Sqrt[15] +
2*Cos[x]*Sin[x])])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x]
 + 2*Tan[x]^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sin(x)+\frac{2}{\cos(x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*sec(x)+sin(x))**2,x)
```

```
[Out] Integral((sin(x) + 2/cos(x))**(-2), x)
```

Mathematica [A] time = 0.168501, size = 58, normalized size = 0.87

$$\frac{(\sin(2x)+4)\sec^2(x)\left(15(\cos(2x)-15)+8\sqrt{15}(\sin(2x)+4)\tan^{-1}\left(\frac{4\tan(x)+1}{\sqrt{15}}\right)\right)}{900(\sin(x)+2\sec(x))^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan
[(1 + 4*Tan[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x
])^2)
```

Maple [A] time = 0.083, size = 39, normalized size = 0.6

$$\frac{1+4\,\tan(x)}{30+15\,\tan(x)+30\,(\tan(x))^2}+\frac{8\sqrt{15}}{225}\arctan\left(\frac{(1+4\,\tan(x))\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*sec(x)+sin(x))^2,x)
```

```
[Out] 1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)+8/225*15^(1/2)*arctan(1/1
5*(1+4*tan(x))*15^(1/2))
```

Maxima [A] time = 1.52742, size = 51, normalized size = 0.76

$$\frac{8}{225}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\,\tan(x)+1)\right)+\frac{4\,\tan(x)+1}{15\left(2\,\tan(x)^2+\tan(x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sec(x) + sin(x))^(-2),x, algorithm="maxima")
```

```
[Out] 8/225*sqrt(15)*arctan(1/15*sqrt(15)*(4*tan(x) + 1)) + 1/15*(4*tan
(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

Fricas [A] time = 0.227777, size = 90, normalized size = 1.34

$$\frac{\sqrt{15}\cos(x)^2+4\left(\cos(x)\sin(x)+2\right)\arctan\left(\frac{8\sqrt{15}\cos(x)\sin(x)+\sqrt{15}}{15\left(2\cos(x)^2-1\right)}\right)-8\sqrt{15}}{15\left(\sqrt{15}\cos(x)\sin(x)+2\sqrt{15}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sec(x) + sin(x))^(-2),x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(15)*cos(x)^2 + 4*(cos(x)*sin(x) + 2)*arctan(1/15*(8*sq
rt(15)*cos(x)*sin(x) + sqrt(15))/(2*cos(x)^2 - 1)) - 8*sqrt(15))/
(sqrt(15)*cos(x)*sin(x) + 2*sqrt(15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sin(x) + 2\sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*sec(x)+sin(x))**2,x)
```

```
[Out] Integral((sin(x) + 2*sec(x))**(-2), x)
```

GIAC/XCAS [A] time = 0.204369, size = 105, normalized size = 1.57

$$\frac{8}{225}\sqrt{15}\left(x + \arctan\left(-\frac{\sqrt{15}\sin(2x) - \cos(2x) - 4\sin(2x) - 1}{\sqrt{15}\cos(2x) + \sqrt{15} - 4\cos(2x) + \sin(2x) + 4}\right)\right) + \frac{4\tan(x) + 1}{15\left(2\tan(x)^2 + \tan(x) + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sec(x) + sin(x))^(-2),x, algorithm="giac")
```

```
[Out] 8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin
(2*x) - 1)/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x)
+ 4))) + 1/15*(4*tan(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$

Optimal. Leaf size=55

$$\frac{x}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}}$$

```
[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/
(6*Sqrt[6]) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

Rubi [A] time = 0.064034, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{x}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/
(6*Sqrt[6]) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

Rubi in Sympy [A] time = 21.4266, size = 29, normalized size = 0.53

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\tan(x)}{3}\right)}{36}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(cos(x)+2*sec(x))**2,x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*tan(x)/3)/36 + tan(x)/(6*(2*tan(x)**2 + 3))
```

Mathematica [A] time = 0.120515, size = 54, normalized size = 0.98

$$\frac{(\cos(2x)+5)\sec^4(x)\left(6\sin(2x)+\sqrt{6}(\cos(2x)+5)\tan^{-1}\left(\sqrt{\frac{2}{3}}\tan(x)\right)\right)}{144\left(2\sec^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] ((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + C
os[2*x]) + 6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

Maple [A] time = 0.05, size = 29, normalized size = 0.5

$$\frac{\tan(x)}{18 + 12 \, (\tan(x))^2} + \frac{\sqrt{6}}{36} \arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(cos(x)+2*sec(x))^2,x)
```

```
[Out] 1/6*tan(x)/(3+2*tan(x)^2)+1/36*6^(1/2)*arctan(1/3*tan(x)*6^(1/2))
```

Maxima [A] time = 1.54043, size = 38, normalized size = 0.69

$$\frac{1}{36}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}\tan(x)\right) + \frac{\tan(x)}{6\left(2\,\tan(x)^2 + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 2*sec(x))^(-2),x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(6)*arctan(1/3*sqrt(6)*tan(x)) + 1/6*tan(x)/(2*tan(x)^2
+ 3)
```

Fricas [A] time = 0.220624, size = 84, normalized size = 1.53

$$\frac{2\sqrt{6}\cos(x)\sin(x) - \left(\cos(x)^2 + 2\right)\arctan\left(\frac{5\sqrt{6}\cos(x)^2 - 2\sqrt{6}}{12\,\cos(x)\sin(x)}\right)}{12\left(\sqrt{6}\cos(x)^2 + 2\sqrt{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 2*sec(x))^(-2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*sqrt(6)*cos(x)*sin(x) - (cos(x)^2 + 2)*arctan(1/12*(5*sqr
t(6)*cos(x)^2 - 2*sqrt(6))/(cos(x)*sin(x))))/(sqrt(6)*cos(x)^2 +
2*sqrt(6))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\cos(x) + 2\sec(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)+2*sec(x))**2,x)
```

```
[Out] Integral((cos(x) + 2*sec(x))**(-2), x)
```

GIAC/XCAS [A] time = 0.215598, size = 82, normalized size = 1.49

$$\frac{1}{36}\sqrt{6}\left(x + \arctan\left(-\frac{\sqrt{6}\sin(2x) - 2\sin(2x)}{\sqrt{6}\cos(2x) + \sqrt{6} - 2\cos(2x) + 2}\right)\right) + \frac{\tan(x)}{6\left(2\tan(x)^2 + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 2*sec(x))^(-2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6
)*cos(2*x) + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2
 + 3)
```

3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

Optimal. Leaf size=42

$$-\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2}-\frac{28}{125}\log(3\sin(x)+\cos(x))$$

```
[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[
x])^2) - 29/(50*(1 + 3*Tan[x]))
```

Rubi [A] time = 0.149512, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2}-\frac{28}{125}\log(3\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[
x])^2) - 29/(50*(1 + 3*Tan[x]))
```

Rubi in Sympy [A] time = 9.09042, size = 39, normalized size = 0.93

$$-\frac{67x}{250}-\frac{28\log\left(3\sin\left(x\right)+\cos\left(x\right)\right)}{125}-\frac{29}{50\left(3\tan\left(x\right)+1\right)}-\frac{7}{10\left(3\tan\left(x\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

```
[Out] -67*x/250 - 28*log(3*sin(x) + cos(x))/125 - 29/(50*(3*tan(x) + 1)
) - 7/(10*(3*tan(x) + 1)**2)
```

Mathematica [A] time = 0.26211, size = 70, normalized size = 1.67

$$-\frac{670x+560\log(3\sin(x)+\cos(x))-4\cos(2x)(134x+112\log(3\sin(x)+\cos(x))-405)+6\sin(2x)(67x+56\log(3\sin(x)+c}{500(3\sin(x)+\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] -(-1305 + 670*x + 560*Log[Cos[x] + 3*Sin[x]] - 4*Cos[2*x]*(-405 +
 134*x + 112*Log[Cos[x] + 3*Sin[x]]) + 6*(-90 + 67*x + 56*Log[Cos
[x] + 3*Sin[x]])*Sin[2*x])/(500*(Cos[x] + 3*Sin[x])^2)
```

Maple [A] time = 0.031, size = 43, normalized size = 1.

$$\frac{14\,\ln\left(1+(\tan(x))^2\right)}{125}-\frac{7}{10\,(1+3\,\tan(x))^2}-\frac{29}{50+150\,\tan(x)}-\frac{28\,\ln(1+3\,\tan(x))}{125}-\frac{67\,x}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x)
```

```
[Out] 14/125*ln(1+tan(x)^2)-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x))-28/1
25*ln(1+3*tan(x))-67/250*x
```

Maxima [A] time = 1.57893, size = 59, normalized size = 1.4

$$-\frac{67}{250}\,x-\frac{87\,\tan(x)+64}{50\,\left(9\,\tan(x)^2+6\,\tan(x)+1\right)}+\frac{14}{125}\,\log\left(\tan(x)^2+1\right)-\frac{28}{125}\,\log(3\,\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(6*tan(x)^2 + tan(x) - 5)/(3*tan(x) + 1)^3,x, algorithm="maxima")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 1
4/125*log(tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)
```

Fricas [A] time = 0.226912, size = 104, normalized size = 2.48

$$-\frac{9\,(134\,x-1)\tan(x)^2+56\,\left(9\,\tan(x)^2+6\,\tan(x)+1\right)\log\left(\frac{9\,\tan(x)^2+6\,\tan(x)+1}{\tan(x)^2+1}\right)+12\,(67\,x+72)\tan(x)+134\,x+639}{500\,\left(9\,\tan(x)^2+6\,\tan(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(6*tan(x)^2 + tan(x) - 5)/(3*tan(x) + 1)^3,x, algorithm="fricas")
```

```
[Out] -1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*l
og((9*tan(x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*t
```

```
an(x) + 134*x + 639)/(9*tan(x)^2 + 6*tan(x) + 1)
```

Sympy [A] time = 0.994077, size = 252, normalized size = 6.

$$-\frac{603x\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{402x\tan(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{67x}{2250\tan^2(x)+1500\tan(x)+250}-\frac{504\log\left(\tan(x)+\frac{1}{3}\right)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{336\log\left(\tan(x)+\frac{1}{3}\right)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{56\log\left(\tan(x)+\frac{1}{3}\right)}{2250\tan^2(x)+1500\tan(x)+250}+\frac{252\log\left(\tan^2(x)+1\right)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250}+\frac{168\log\left(\tan^2(x)+1\right)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250}+\frac{28\log\left(\tan^2(x)+1\right)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{435\tan(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{320}{2250\tan^2(x)+1500\tan(x)+250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

```
[Out] -603*x*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 402*x*tan
(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) - 67*x/(2250*tan(x)**2 +
 1500*tan(x) + 250) - 504*log(tan(x) + 1/3)*tan(x)**2/(2250*tan(x
)**2 + 1500*tan(x) + 250) - 336*log(tan(x) + 1/3)*tan(x)/(2250*ta
n(x)**2 + 1500*tan(x) + 250) - 56*log(tan(x) + 1/3)/(2250*tan(x)*
*2 + 1500*tan(x) + 250) + 252*log(tan(x)**2 + 1)*tan(x)**2/(2250*
tan(x)**2 + 1500*tan(x) + 250) + 168*log(tan(x)**2 + 1)*tan(x)/(2
250*tan(x)**2 + 1500*tan(x) + 250) + 28*log(tan(x)**2 + 1)/(2250*
tan(x)**2 + 1500*tan(x) + 250) - 435*tan(x)/(2250*tan(x)**2 + 150
0*tan(x) + 250) - 320/(2250*tan(x)**2 + 1500*tan(x) + 250)
```

GIAC/XCAS [A] time = 0.211515, size = 53, normalized size = 1.26

$$-\frac{67}{250}x-\frac{87\tan(x)+64}{50(3\tan(x)+1)^2}+\frac{14}{125}\ln\left(\tan(x)^2+1\right)-\frac{28}{125}\ln\left(|3\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(6*tan(x)^2 + tan(x) - 5)/(3*tan(x) + 1)^3,x, algorithm="giac")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(3*tan(x) + 1)^2 + 14/125*ln(ta
n(x)^2 + 1) - 28/125*ln(abs(3*tan(x) + 1))
```

3.383 $\int \cos^2(x) \sec(3x) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2}\tanh^{-1}(2\sin(x))$$

```
[Out] ArcTanh[2*Sin[x]]/2
```

Rubi [A] time = 0.0370851, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{2}\tanh^{-1}(2\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sec[3*x],x]

[Out] ArcTanh[2*Sin[x]]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(x)}{\cos(3x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2/cos(3*x),x)

[Out] Integral(cos(x)**2/cos(3*x), x)
```

Mathematica [B] time = 0.0126828, size = 23, normalized size = 2.56

$$\frac{1}{4}\log(2\sin(x)+1)-\frac{1}{4}\log(1-2\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sec[3*x],x]
```

```
[Out] -Log[1 - 2*Sin[x]]/4 + Log[1 + 2*Sin[x]]/4
```

Maple [B] time = 0.059, size = 20, normalized size = 2.2

$$-\frac{\ln\left(2\ \sin\left(x\right)-1\right)}{4}+\frac{\ln\left(1+2\ \sin\left(x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2/cos(3*x),x)
```

```
[Out] -1/4*ln(2*sin(x)-1)+1/4*ln(1+2*sin(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(x\right)^2}{\cos\left(3\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^2/cos(3*x), x)
```

Fricas [A] time = 0.232863, size = 26, normalized size = 2.89

$$\frac{1}{4}\,\log\left(2\ \sin\left(x\right)+1\right)-\frac{1}{4}\,\log\left(-2\ \sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*sin(x) + 1) - 1/4*log(-2*sin(x) + 1)
```

Sympy [A] time = 11.965, size = 76, normalized size = 8.44

$$-\frac{\log\left(\sin\left(3x\right)-1\right)}{12}+\frac{\log\left(\sin\left(3x\right)+1\right)}{12}-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6}-\frac{\log\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}{12}+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2/cos(3*x),x)
```

```
[Out] -log(sin(3*x) - 1)/12 + log(sin(3*x) + 1)/12 - log(tan(x/2) - 1)/
6 + log(tan(x/2) + 1)/6 - log(tan(x/2)**2 - 4*tan(x/2) + 1)/12 +
log(tan(x/2)**2 + 4*tan(x/2) + 1)/12
```

GIAC/XCAS [A] time = 0.221887, size = 28, normalized size = 3.11

$$\frac{1}{4}\ln\left(|2\sin\left(x\right)+1|\right)-\frac{1}{4}\ln\left(|2\sin\left(x\right)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(2*sin(x) + 1)) - 1/4*ln(abs(2*sin(x) - 1))
```

3.384 $\int \sec(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rubi [A] time = 0.0270533, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rubi in Sympy [A] time = 1.87678, size = 17, normalized size = 1.13

$$-x\sin(x) - \frac{\log(\cos(2x))\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/cos(2*x),x)
```

```
[Out] -x*sin(x) - log(cos(2*x))*cos(x)/2
```

Mathematica [C] time = 0.582307, size = 174, normalized size = 11.6

$$\frac{4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)+\log\left(-\sqrt{2}\sin(x)+\sqrt{2}\cos(x)+2\right)+2i\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(\sqrt{2}-1\right)\text{s}}{\left(1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\text{s}}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[2*x]*Sin[x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])
*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*S
in[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2
] + Tan[x/2]] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2
+ Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*Sqrt[2])
```

Maple [A] time = 0.026, size = 13, normalized size = 0.9

$$\frac{\mathit{Artanh}\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/cos(2*x),x)
```

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.5105, size = 174, normalized size = 11.6

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin\left(2x\right)\sin\left(x\right)+2\left(\sqrt{2}\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+2\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}+2\sin\left(x\right)^{2}+2\sqrt{2}\cos\left(x\right)+1\right)-\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin\left(2x\right)\sin\left(x\right)-2\left(\sqrt{2}\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+2\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}+2\sin\left(x\right)^{2}-2\sqrt{2}\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1
)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 +
2*sqrt(2)*cos(x) + 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x
) - 2*(sqrt(2)*cos(x) - 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + s
in(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 1)
```

Fricas [A] time = 0.220754, size = 46, normalized size = 3.07

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\cos\left(x\right)^{2}+\sqrt{2}+4\cos\left(x\right)}{2\cos\left(x\right)^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*sqrt(2)*cos(x)^2 + sqrt(2) + 4*cos(x))/(2*cos(
x)^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)}{\cos(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x),x)
```

```
[Out] Integral(sin(x)/cos(2*x), x)
```

GIAC/XCAS [A] time = 0.242778, size = 66, normalized size = 4.4

$$\frac{1}{4}\sqrt{2}\ln\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/
abs(4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

3.385 $\int \sec(2x)\sin^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))-\frac{x}{2}$$

```
[Out] -x/2 + ArcTanh[2*Cos[x]*Sin[x]]/4
```

Rubi [A] time = 0.0685826, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x]^2,x]

[Out] -x/2 + ArcTanh[2*Cos[x]*Sin[x]]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(x)}{\cos(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x)**2/cos(2*x),x)

[Out] Integral(sin(x)**2/cos(2*x), x)
```

Mathematica [A] time = 0.0111216, size = 28, normalized size = 1.65

$$-\frac{x}{2}-\frac{1}{4}\log(\cos(x)-\sin(x))+\frac{1}{4}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x]^2,x]
```

```
[Out] -x/2 - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4
```

Maple [A] time = 0.051, size = 19, normalized size = 1.1

$$\frac{\ln(1+\tan(x))}{4}-\frac{\ln(-1+\tan(x))}{4}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2/cos(2*x),x)
```

```
[Out] 1/4*ln(1+tan(x))-1/4*ln(-1+tan(x))-1/2*x
```

Maxima [A] time = 1.52528, size = 173, normalized size = 10.18

$$-\frac{1}{2}x-\frac{1}{8}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)$$
$$+\frac{1}{8}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)$$
$$+\frac{1}{8}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)$$
$$-\frac{1}{8}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*s
qrt(2)*sin(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*
cos(x) - 2*sqrt(2)*sin(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*log(2*cos(x)^2 +
 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)
```

Fricas [A] time = 0.256314, size = 35, normalized size = 2.06

$$-\frac{1}{2}x+\frac{1}{8}\log(2\cos(x)\sin(x)+1)-\frac{1}{8}\log(-2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x + 1/8*log(2*cos(x)*sin(x) + 1) - 1/8*log(-2*cos(x)*sin(x)
+ 1)
```

Sympy [A] time = 1.80675, size = 22, normalized size = 1.29

$$-\frac{x}{2}-\frac{\log\left(\sin\left(2x\right)-1\right)}{8}+\frac{\log\left(\sin\left(2x\right)+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2/cos(2*x),x)
```

```
[Out] -x/2 - log(sin(2*x) - 1)/8 + log(sin(2*x) + 1)/8
```

GIAC/XCAS [A] time = 0.226357, size = 27, normalized size = 1.59

$$-\frac{1}{2}\,x+\frac{1}{4}\,\ln\left(\left|\tan\left(x\right)+1\right|\right)-\frac{1}{4}\,\ln\left(\left|\tan\left(x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/cos(2*x),x, algorithm="giac")
```

```
[Out] -1/2*x + 1/4*ln(abs(tan(x) + 1)) - 1/4*ln(abs(tan(x) - 1))
```

3.386 $\int \sec(3x)\sin^3(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24
```

Rubi [A] time = 0.0842454, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[3*x]*Sin[x]^3,x]

[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24
```

Rubi in Sympy [A] time = 102.15, size = 19, normalized size = 0.9

$$-\frac{\log\left(-4\cos^2(x)+3\right)}{24}+\frac{\log\left(\cos^2(x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**3/cos(3*x),x)

[Out] -log(-4*cos(x)**2 + 3)/24 + log(cos(x)**2)/6
```

Mathematica [A] time = 0.015396, size = 21, normalized size = 1.

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log(1-2\cos(2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[3*x]*Sin[x]^3,x]
```

```
[Out] Log[Cos[x]]/3 - Log[1 - 2*Cos[2*x]]/24
```

Maple [A] time = 0.072, size = 18, normalized size = 0.9

$$\frac{\ln(\cos(x))}{3} - \frac{\ln\left(4\,(\cos(x))^2 - 3\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^3/cos(3*x),x)
```

```
[Out] 1/3*ln(cos(x))-1/24*ln(4*cos(x)^2-3)
```

Maxima [A] time = 1.62035, size = 109, normalized size = 5.19

$$-\frac{1}{48}\log\left(-2\left(\cos(2x)-1\right)\cos(4x)+\cos(4x)^2+\cos(2x)^2+\sin(4x)^2-2\sin(4x)\sin(2x)+\sin(2x)^2-2\cos(2x)+1\right)+\frac{1}{6}\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")
```

```
[Out] -1/48*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 +
sin(4*x)^2 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) +
 1/6*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 0.261277, size = 26, normalized size = 1.24

$$-\frac{1}{24}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log\left(-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")
```

```
[Out] -1/24*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(x)}{\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3/cos(3*x),x)
```

```
[Out] Integral(sin(x)**3/cos(3*x), x)
```

GIAC/XCAS [A] time = 0.213627, size = 120, normalized size = 5.71

$$-\frac{1}{8}\ln\left(-\frac{\cos(x)+1}{\cos(x)-1}-\frac{\cos(x)-1}{\cos(x)+1}+2\right)+\frac{1}{6}\ln\left(-\frac{\cos(x)+1}{\cos(x)-1}-\frac{\cos(x)-1}{\cos(x)+1}-2\right)-\frac{1}{24}\ln\left(\left|-\frac{\cos(x)+1}{\cos(x)-1}-\frac{\cos(x)-1}{\cos(x)+1}-14\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3/cos(3*x),x, algorithm="giac")
```

```
[Out] -1/8*ln(-(cos(x) + 1)/(cos(x) - 1) - (cos(x) - 1)/(cos(x) + 1) +
2) + 1/6*ln(-(cos(x) + 1)/(cos(x) - 1) - (cos(x) - 1)/(cos(x) + 1
) - 2) - 1/24*ln(abs(-(cos(x) + 1)/(cos(x) - 1) - (cos(x) - 1)/(c
os(x) + 1) - 14))
```

3.387 $\int \cos(x)\csc(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rubi [A] time = 0.047265, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Csc[3*x],x]

[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rubi in Sympy [A] time = 1.95558, size = 19, normalized size = 0.9

$$x\sin(2x) + \frac{\log(\sin(3x))\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/sin(3*x),x)

[Out] x*sin(2*x) + log(sin(3*x))*cos(2*x)/3
```

Mathematica [A] time = 0.0170433, size = 21, normalized size = 1.

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log(2\cos(2x) + 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Csc[3*x],x]

[Out] -Log[1 + 2*Cos[2*x]]/6 + Log[Sin[x]]/3
```

Maple [A] time = 0.076, size = 34, normalized size = 1.6

$$\frac{\ln\left(1+\cos\left(x\right)\right)}{6}+\frac{\ln\left(\cos\left(x\right)-1\right)}{6}-\frac{\ln\left(2\cos\left(x\right)-1\right)}{6}-\frac{\ln\left(1+2\cos\left(x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/sin(3*x),x)
```

```
[Out] 1/6*ln(1+cos(x))+1/6*ln(cos(x)-1)-1/6*ln(2*cos(x)-1)-1/6*ln(1+2*c
os(x))
```

Maxima [A] time = 1.52344, size = 174, normalized size = 8.29

$$-\frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}+2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)-\frac{1}{12}\log\left(-2\left(\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}-2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)+\frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)+\frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2
*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-
2*(cos(x) - 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*
sin(2*x)*sin(x) + sin(x)^2 - 2*cos(x) + 1) + 1/6*log(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

Fricas [A] time = 0.217301, size = 26, normalized size = 1.24

$$-\frac{1}{6}\log\left(4\cos\left(x\right)^{2}-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

Sympy [A] time = 1.33635, size = 17, normalized size = 0.81

$$-\frac{\log\left(4\sin^{2}(x)-3\right)}{6}+\frac{\log(\sin(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(3*x),x)
```

```
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

GIAC/XCAS [A] time = 0.204712, size = 41, normalized size = 1.95

$$-\frac{1}{6}\ln\left(\left|-\frac{3(\cos(x)+1)}{\cos(x)-1}-\frac{3(\cos(x)-1)}{\cos(x)+1}-10\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(3*x),x, algorithm="giac")
```

```
[Out] -1/6*ln(abs(-3*(cos(x) + 1)/(cos(x) - 1) - 3*(cos(x) - 1)/(cos(x)
 + 1) - 10))
```

3.388 $\int \csc(4x)\sin(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0431286, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[4*x]*Sin[x],x]

[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 2.30653, size = 19, normalized size = 0.73

$$x\cos(3x)-\frac{\log(\sin(4x))\sin(3x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/sin(4*x),x)

[Out] x*cos(3*x) - log(sin(4*x))*sin(3*x)/4
```

Mathematica [C] time = 0.519368, size = 218, normalized size = 8.38

$$\frac{2\log\left(2\sin(x)+\sqrt{2}\right)+2\sqrt{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\sqrt{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)-1}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[4*x]*Sin[x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2]
)*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*
Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 2*Sqrt[2]*Log[C
os[x/2] - Sin[x/2]] - 2*Sqrt[2]*Log[Cos[x/2] + Sin[x/2]] + 2*Log[
Sqrt[2] + 2*Sin[x]] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] -
Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(8*Sqrt[2])
```

Maple [A] time = 0.082, size = 28, normalized size = 1.1

$$-\frac{\ln(1+\sin(x))}{8}+\frac{\ln(-1+\sin(x))}{8}+\frac{\mathit{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/sin(4*x),x)
```

```
[Out] -1/8*ln(1+sin(x))+1/8*ln(-1+sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2
^(1/2)
```

Maxima [A] time = 1.55444, size = 231, normalized size = 8.88

$$\begin{aligned}&\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)+\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*s
qrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2
*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(
x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/
16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqr
t(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) +
1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [A] time = 0.229545, size = 82, normalized size = 3.15

$$-\frac{1}{16}\sqrt{2}\left(\sqrt{2}\log\left(\sin\left(x\right)+1\right)-\sqrt{2}\log\left(-\sin\left(x\right)+1\right)-2\log\left(-\frac{2\sqrt{2}\cos\left(x\right)^{2}-3\sqrt{2}-4\sin\left(x\right)}{2\cos\left(x\right)^{2}-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(sqrt(2)*log(sin(x) + 1) - sqrt(2)*log(-sin(x) + 1)
 - 2*log(-(2*sqrt(2)*cos(x)^2 - 3*sqrt(2) - 4*sin(x))/(2*cos(x)^2
 - 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sin(4*x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21593, size = 65, normalized size = 2.5

$$-\frac{1}{8}\sqrt{2}\ln\left(\frac{\left|-2\sqrt{2}+4\sin\left(x\right)\right|}{\left|2\sqrt{2}+4\sin\left(x\right)\right|}\right)-\frac{1}{8}\ln\left(\sin\left(x\right)+1\right)+\frac{1}{8}\ln\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*ln(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(
x))) - 1/8*ln(sin(x) + 1) + 1/8*ln(-sin(x) + 1)
```

3.389 $\int \csc(4x)\sin^3(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(4*Sqrt[2])
```

Rubi [A] time = 0.0627791, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[4*x]*Sin[x]^3,x]

[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(4*Sqrt[2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(x)}{\sin(4x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**3/sin(4*x),x)

[Out] Integral(sin(x)**3/sin(4*x), x)
```

Mathematica [C] time = 0.493763, size = 218, normalized size = 8.38

$$\frac{2\log\left(2\sin(x)+\sqrt{2}\right)+4\sqrt{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-4\sqrt{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)-l}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] ((-2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2]
)*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*
Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 4*Sqrt[2]*Log[C
os[x/2] - Sin[x/2]] - 4*Sqrt[2]*Log[Cos[x/2] + Sin[x/2]] + 2*Log[
Sqrt[2] + 2*Sin[x]] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] -
Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(16*Sqrt[2])
```

Maple [A] time = 0.095, size = 28, normalized size = 1.1

$$-\frac{\ln(1+\sin(x))}{8}+\frac{\ln(-1+\sin(x))}{8}+\frac{\mathit{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^3/sin(4*x),x)
```

```
[Out] -1/8*ln(1+sin(x))+1/8*ln(-1+sin(x))+1/8*arctanh(sin(x)*2^(1/2))*2
^(1/2)
```

Maxima [A] time = 1.59756, size = 231, normalized size = 8.88

$$\begin{aligned}&\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&+\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)+\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*s
qrt(2)*sin(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2
*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + 1/32*sqrt(2)*log(2*cos(
x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/
32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqr
t(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) +
1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [A] time = 0.233936, size = 82, normalized size = 3.15

$$-\frac{1}{16}\sqrt{2}\left(\sqrt{2}\log\left(\sin\left(x\right)+1\right)-\sqrt{2}\log\left(-\sin\left(x\right)+1\right)-\log\left(-\frac{2\sqrt{2}\cos\left(x\right)^{2}-3\sqrt{2}-4\sin\left(x\right)}{2\cos\left(x\right)^{2}-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(sqrt(2)*log(sin(x) + 1) - sqrt(2)*log(-sin(x) + 1)
 - log(-(2*sqrt(2)*cos(x)^2 - 3*sqrt(2) - 4*sin(x))/(2*cos(x)^2 -
 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3/sin(4*x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219861, size = 65, normalized size = 2.5

$$-\frac{1}{16}\sqrt{2}\ln\left(\frac{\left|-2\sqrt{2}+4\sin\left(x\right)\right|}{\left|2\sqrt{2}+4\sin\left(x\right)\right|}\right)-\frac{1}{8}\ln\left(\sin\left(x\right)+1\right)+\frac{1}{8}\ln\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*ln(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin
(x))) - 1/8*ln(sin(x) + 1) + 1/8*ln(-sin(x) + 1)
```

3.390 $\int \sqrt{1+\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

```
[Out] -(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

Rubi [A] time = 0.0163047, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[2*x]],x]

[Out] -(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

Rubi in Sympy [A] time = 0.509548, size = 15, normalized size = 0.94

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+sin(2*x))**(1/2),x)

[Out] -cos(2*x)/sqrt(sin(2*x) + 1)
```

Mathematica [A] time = 0.024504, size = 25, normalized size = 1.56

$$\frac{\sqrt{\sin(2x)+1}(\sin(x)-\cos(x))}{\sin(x)+\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[2*x]],x]
```

```
[Out] ((-Cos[x] + Sin[x])*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

Maple [A] time = 0.045, size = 22, normalized size = 1.4

$$\frac{-1+\sin(2x)}{\cos(2x)}\sqrt{1+\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+sin(2*x))^(1/2),x)
```

```
[Out] (-1+sin(2*x))*(1+sin(2*x))^(1/2)/cos(2*x)
```

Maxima [A] time = 1.58069, size = 73, normalized size = 4.56

$$-\frac{1}{\sqrt{\frac{\sin(2x)^2}{(\cos(2x)+1)^2}+1}}+\frac{\sin(2x)}{\sqrt{\frac{\sin(2x)^2}{(\cos(2x)+1)^2}+1}(\cos(2x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(2*x) + 1),x, algorithm="maxima")
```

```
[Out] -1/sqrt(sin(2*x)^2/(cos(2*x) + 1)^2 + 1) + sin(2*x)/(sqrt(sin(2*x
)^2/(cos(2*x) + 1)^2 + 1)*(cos(2*x) + 1))
```

Fricas [A] time = 0.211955, size = 46, normalized size = 2.88

$$-\frac{(\cos(2x)-\sin(2x)+1)\sqrt{\sin(2x)+1}}{\cos(2x)+\sin(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(2*x) + 1),x, algorithm="fricas")
```

```
[Out] -(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x
) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin (2x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+sin(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(2*x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin (2\, x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(2*x) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*x) + 1), x)
```

3.391 $\int \sqrt{1-\sin(2x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

```
[Out] Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

Rubi [A] time = 0.0214891, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - Sin[2*x]],x]

[Out] Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

Rubi in Sympy [A] time = 0.569023, size = 14, normalized size = 0.82

$$\frac{\cos(2x)}{\sqrt{-\sin(2x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-sin(2*x))**(1/2),x)

[Out] cos(2*x)/sqrt(-sin(2*x) + 1)
```

Mathematica [A] time = 0.0252099, size = 27, normalized size = 1.59

$$\frac{\sqrt{1-\sin(2x)}(\sin(x)+\cos(x))}{\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - Sin[2*x]],x]
```

```
[Out] ((Cos[x] + Sin[x])*Sqrt[1 - Sin[2*x]])/(Cos[x] - Sin[x])
```

Maple [A] time = 0.059, size = 31, normalized size = 1.8

$$-\frac{(-1+\sin(2x))(1+\sin(2x))}{\cos(2x)}\frac{1}{\sqrt{1-\sin(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-sin(2*x))^(1/2),x)
```

```
[Out] -(-1+sin(2*x))*(1+sin(2*x))/cos(2*x)/(1-sin(2*x))^(1/2)
```

Maxima [A] time = 1.52717, size = 74, normalized size = 4.35

$$-\frac{1}{\sqrt{\frac{\sin(2x)^2}{(\cos(2x)+1)^2}+1}}-\frac{\sin(2x)}{\sqrt{\frac{\sin(2x)^2}{(\cos(2x)+1)^2}+1}(\cos(2x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(2*x) + 1),x, algorithm="maxima")
```

```
[Out] -1/sqrt(sin(2*x)^2/(cos(2*x) + 1)^2 + 1) - sin(2*x)/(sqrt(sin(2*x
)^2/(cos(2*x) + 1)^2 + 1)*(cos(2*x) + 1))
```

Fricas [A] time = 0.217482, size = 47, normalized size = 2.76

$$\frac{(\cos(2x)+\sin(2x)+1)\sqrt{-\sin(2x)+1}}{\cos(2x)-\sin(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(2*x) + 1),x, algorithm="fricas")
```

```
[Out] (cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x
) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(2x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-sin(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(-sin(2*x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(2x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(2*x) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sin(2*x) + 1), x)
```

3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

Rubi [A] time = 0.0234976, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + Cos[2*x]],x]

[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

Rubi in Sympy [A] time = 0.574884, size = 29, normalized size = 1.07

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sin(2x)}{2\sqrt{\cos(2x)+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+cos(2*x))**(1/2),x)

[Out] sqrt(2)*atanh(sqrt(2)*sin(2*x)/(2*sqrt(cos(2*x) + 1)))/2
```

Mathematica [A] time = 0.0257318, size = 47, normalized size = 1.74

$$-\frac{\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{\cos(2x)+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] -((Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]))/
Sqrt[1 + Cos[2*x]])
```

Maple [C] time = 0.033, size = 9, normalized size = 0.3

$$\frac{\sqrt{2}\,InverseJacobiAM\left(x,1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+cos(2*x))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*InverseJacobiAM(x,1)
```

Maxima [A] time = 1.73778, size = 26, normalized size = 0.96

$$\frac{1}{2}\,\sqrt{2}\operatorname{arsinh}\left(\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(cos(2*x) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arcsinh(sin(2*x)/(cos(2*x) + 1))
```

Fricas [A] time = 0.216912, size = 55, normalized size = 2.04

$$\frac{1}{4}\,\sqrt{2}\log\left(-\frac{\sqrt{\cos\left(2\,x\right)+1}(\cos\left(2\,x\right)-3)-2\,\sqrt{2}\sin\left(2\,x\right)}{(\cos\left(2\,x\right)+1)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(cos(2*x) + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(cos(2*x) + 1)*(cos(2*x) - 3) - 2*sqrt(2)*s
in(2*x))/(cos(2*x) + 1)^(3/2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(2x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(2*x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(2\,x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(cos(2*x) + 1),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(2*x) + 1), x)
```

3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

Rubi [A] time = 0.0277617, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 - Cos[2*x]],x]

[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

Rubi in Sympy [A] time = 0.607716, size = 31, normalized size = 1.03

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sin(2x)}{2\sqrt{-\cos(2x)+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(2*x))**(1/2),x)

[Out] -sqrt(2)*atanh(sqrt(2)*sin(2*x)/(2*sqrt(-cos(2*x) + 1)))/2
```

Mathematica [A] time = 0.0261692, size = 33, normalized size = 1.1

$$-\frac{\sin(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{1-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] -(((Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])/Sqrt[1 - Cos[2*x]])
```

Maple [A] time = 0.055, size = 17, normalized size = 0.6

$$-\frac{\sin(x)\,Artanh(\cos(x))\,\sqrt{2}}{2}\frac{1}{\sqrt{(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(2*x))^(1/2),x)
```

```
[Out] -1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)
```

Maxima [A] time = 1.7707, size = 51, normalized size = 1.7

$$-\frac{1}{4}\sqrt{2}\left(\log\left(\frac{\sqrt{2}}{\sqrt{\cos(2x)+1}}+1\right)-\log\left(\frac{\sqrt{2}}{\sqrt{\cos(2x)+1}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-cos(2*x) + 1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(log(sqrt(2)/sqrt(cos(2*x) + 1) + 1) - log(sqrt(2)/s
qrt(cos(2*x) + 1) - 1))
```

Fricas [A] time = 0.214218, size = 72, normalized size = 2.4

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{(\cos(2x)+3)\sqrt{-\cos(2x)+1}-2\sqrt{2}\sin(2x)}{(\cos(2x)-1)\sqrt{-\cos(2x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-cos(2*x) + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-((cos(2*x) + 3)*sqrt(-cos(2*x) + 1) - 2*sqrt(2)*
sin(2*x))/((cos(2*x) - 1)*sqrt(-cos(2*x) + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(2x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cos(2*x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(2x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-cos(2*x) + 1),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-cos(2*x) + 1), x)
```

3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

```
[Out] -ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/(6*Sqrt[2]) - Sin
[3*x]/(6*(1 - Cos[3*x])^(3/2))
```

Rubi [A] time = 0.0494847, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/(6*Sqrt[2]) - Sin
[3*x]/(6*(1 - Cos[3*x])^(3/2))
```

Rubi in Sympy [A] time = 0.79686, size = 48, normalized size = 0.91

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sin(3x)}{2\sqrt{-\cos(3x)+1}}\right)}{12}-\frac{\sin(3x)}{6\left(-\cos(3x)+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(3*x))**(3/2),x)
```

```
[Out] -sqrt(2)*atanh(sqrt(2)*sin(3*x)/(2*sqrt(-cos(3*x) + 1)))/12 - sin
(3*x)/(6*(-cos(3*x) + 1)**(3/2))
```

Mathematica [A] time = 0.185542, size = 61, normalized size = 1.15

$$-\frac{\sin^3\left(\frac{3x}{2}\right)\left(\csc^2\left(\frac{3x}{4}\right)-\sec^2\left(\frac{3x}{4}\right)-4\log\left(\sin\left(\frac{3x}{4}\right)\right)+4\log\left(\cos\left(\frac{3x}{4}\right)\right)\right)}{12(1-\cos(3x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - S
ec[(3*x)/4]^2)*Sin[(3*x)/2]^3)/(12*(1 - Cos[3*x])^(3/2))
```

Maple [A] time = 0.081, size = 52, normalized size = 1.

$$-\frac{\sqrt{2}}{6}\left(\frac{1}{2}\cos\left(\frac{3x}{2}\right)+\frac{1}{4}\left(\ln\left(1+\cos\left(\frac{3x}{2}\right)\right)-\ln\left(\cos\left(\frac{3x}{2}\right)-1\right)\right)\left(\sin\left(\frac{3x}{2}\right)\right)^{2}\right)\left(\sin\left(\frac{3x}{2}\right)\right)^{-1}\frac{1}{\sqrt{\left(\sin\left(\frac{3x}{2}\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(3*x))^(3/2),x)
```

```
[Out] -1/6*(1/2*cos(3/2*x)+1/4*(ln(1+cos(3/2*x))-ln(cos(3/2*x)-1))*sin(
3/2*x)^2)/sin(3/2*x)*2^(1/2)/(sin(3/2*x)^2)^(1/2)
```

Maxima [A] time = 1.6159, size = 103, normalized size = 1.94

$$-\frac{1}{12}\sqrt{2}\left(\frac{\left(\frac{\sin(3x)^{2}}{(\cos(3x)+1)^{2}}+1\right)^{\frac{3}{2}}(\cos(3x)+1)^{2}}{\sin(3x)^{2}}-\sqrt{\frac{\sin(3x)^{2}}{(\cos(3x)+1)^{2}}+1}+\operatorname{arsinh}\left(\frac{\cos(3x)+1}{|\sin(3x)|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cos(3*x) + 1)^(-3/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*((sin(3*x)^2/(cos(3*x) + 1)^2 + 1)^(3/2)*(cos(3*x)
+ 1)^2/sin(3*x)^2 - sqrt(sin(3*x)^2/(cos(3*x) + 1)^2 + 1) + arcsi
nh((cos(3*x) + 1)/abs(sin(3*x))))
```

Fricas [A] time = 0.215972, size = 144, normalized size = 2.72

$$\frac{\left(\sqrt{2}\cos(3x)-\sqrt{2}\right)\log\left(-\frac{\left(\sqrt{2}\cos(3x)+3\sqrt{2}\right)\sqrt{-\cos(3x)+1}-4\sin(3x)}{(\cos(3x)-1)\sqrt{-\cos(3x)+1}}\right)\sin(3x)+4(\cos(3x)+1)\sqrt{-\cos(3x)+1}}{24(\cos(3x)-1)\sin(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cos(3*x) + 1)^(-3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((sqrt(2)*cos(3*x) - sqrt(2))*log(-((sqrt(2)*cos(3*x) + 3*sq
rt(2))*sqrt(-cos(3*x) + 1) - 4*sin(3*x))/((cos(3*x) - 1)*sqrt(-co
s(3*x) + 1)))*sin(3*x) + 4*(cos(3*x) + 1)*sqrt(-cos(3*x) + 1))/((
cos(3*x) - 1)*sin(3*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(3x) + 1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(3*x))**(3/2),x)
```

```
[Out] Integral((-cos(3*x) + 1)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.256494, size = 80, normalized size = 1.51

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{\tan\left(\frac{3}{2}\,x\right)^2+1}}{\tan\left(\frac{3}{2}\,x\right)^2} + \ln\left(\sqrt{\tan\left(\frac{3}{2}\,x\right)^2 + 1} + 1\right) - \ln\left(\sqrt{\tan\left(\frac{3}{2}\,x\right)^2 + 1} - 1\right)\right)}{24\,\mathrm{sign}\left(\tan\left(\frac{3}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cos(3*x) + 1)^(-3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*sqrt(tan(3/2*x)^2 + 1)/tan(3/2*x)^2 + ln(sqrt(ta
n(3/2*x)^2 + 1) + 1) - ln(sqrt(tan(3/2*x)^2 + 1) - 1))/sign(tan(3
/2*x))
```

3.395 $\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx$

Optimal. Leaf size=73

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

```
[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sq
rt[1 - Sin[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2
))/5
```

Rubi [A] time = 0.061678, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sq
rt[1 - Sin[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2
))/5
```

Rubi in Sympy [A] time = 0.871285, size = 65, normalized size = 0.89

$$\frac{3\left(-\sin\left(\frac{2x}{3}\right)+1\right)^{\frac{3}{2}}\cos\left(\frac{2x}{3}\right)}{5}+\frac{8\sqrt{-\sin\left(\frac{2x}{3}\right)+1}\cos\left(\frac{2x}{3}\right)}{5}+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{-\sin\left(\frac{2x}{3}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-sin(2/3*x))**(5/2),x)
```

```
[Out] 3*(-sin(2*x/3) + 1)**(3/2)*cos(2*x/3)/5 + 8*sqrt(-sin(2*x/3) + 1)
*cos(2*x/3)/5 + 32*cos(2*x/3)/(5*sqrt(-sin(2*x/3) + 1))
```

Mathematica [A] time = 0.132038, size = 76, normalized size = 1.04

$$\frac{\left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\left(150\sin\left(\frac{x}{3}\right)-25\sin(x)-3\sin\left(\frac{5x}{3}\right)+150\cos\left(\frac{x}{3}\right)+25\cos(x)-3\cos\left(\frac{5x}{3}\right)\right)}{20\left(\cos\left(\frac{x}{3}\right)-\sin\left(\frac{x}{3}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] ((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)
/3] + 150*Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] -
 Sin[x/3])^5)
```

Maple [A] time = 0.069, size = 47, normalized size = 0.6

$$-\frac{1}{5}\left(-1+\sin\left(\frac{2x}{3}\right)\right)\left(1+\sin\left(\frac{2x}{3}\right)\right)\left(3\left(\sin(2/3x)\right)^2-14\sin(2/3x)+43\right)\left(\cos\left(\frac{2x}{3}\right)\right)^{-1}\frac{1}{\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-sin(2/3*x))^(5/2),x)
```

```
[Out] -1/5*(-1+sin(2/3*x))*(1+sin(2/3*x))*(3*sin(2/3*x)^2-14*sin(2/3*x)
+43)/cos(2/3*x)/(1-sin(2/3*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-sin(2/3*x) + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.215377, size = 96, normalized size = 1.32

$$-\frac{\left(3\cos\left(\frac{2}{3}x\right)^3-11\cos\left(\frac{2}{3}x\right)^2+\left(3\cos\left(\frac{2}{3}x\right)^2+14\cos\left(\frac{2}{3}x\right)-32\right)\sin\left(\frac{2}{3}x\right)-46\cos\left(\frac{2}{3}x\right)-32\right)\sqrt{-\sin\left(\frac{2}{3}x\right)+1}}{5\left(\cos\left(\frac{2}{3}x\right)-\sin\left(\frac{2}{3}x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-sin(2/3*x) + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*cos(2/3*x)^3 - 11*cos(2/3*x)^2 + (3*cos(2/3*x)^2 + 14*cos
(2/3*x) - 32)*sin(2/3*x) - 46*cos(2/3*x) - 32)*sqrt(-sin(2/3*x) +
 1)/(cos(2/3*x) - sin(2/3*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-sin(2/3*x))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sin\left(\frac{2}{3}x\right) + 1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-sin(2/3*x) + 1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-sin(2/3*x) + 1)^(5/2), x)
```

3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{12}(2\sin(x)+1)^{3/2}-\frac{1}{2}\sqrt{2\sin(x)+1}-\frac{4}{\sqrt[4]{2\sin(x)+1}}+\frac{3}{4\sqrt{2\sin(x)+1}}$$

```
[Out] 3/(4*Sqrt[1 + 2*Sin[x]]) - 4/(1 + 2*Sin[x])^(1/4) - Sqrt[1 + 2*Si
n[x]]/2 + (1 + 2*Sin[x])^(3/2)/12
```

Rubi [A] time = 0.241763, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{12}(2\sin(x)+1)^{3/2}-\frac{1}{2}\sqrt{2\sin(x)+1}-\frac{4}{\sqrt[4]{2\sin(x)+1}}+\frac{3}{4\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

```
[Out] 3/(4*Sqrt[1 + 2*Sin[x]]) - 4/(1 + 2*Sin[x])^(1/4) - Sqrt[1 + 2*Si
n[x]]/2 + (1 + 2*Sin[x])^(3/2)/12
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x
```

```
[Out] Timed out
```

Mathematica [A] time = 0.206393, size = 50, normalized size = 0.91

$$\frac{4\sin^2(x)-8\sin(x)+5\sqrt{2\sin(x)+1}-48\sqrt[4]{2\sin(x)+1}+4}{12\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

```
[Out] (4 - 8*Sin[x] + 4*Sin[x]^2 - 48*(1 + 2*Sin[x])^(1/4) + 5*Sqrt[1 +
 2*Sin[x]])/(12*Sqrt[1 + 2*Sin[x]])
```

Maple [F] time = 0.631, size = 0, normalized size = 0.

$$\int \cos(x)\left(-(\cos(x))^2 + 2\sqrt[4]{1 + 2\sin(x)}\right)(1 + 2\sin(x))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x)
```

```
[Out] int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x)
```

Maxima [A] time = 6.59508, size = 58, normalized size = 1.05

$$\frac{1}{12}(2\sin(x) + 1)^{\frac{3}{2}} - \frac{16(2\sin(x) + 1)^{\frac{1}{4}} - 3}{4\sqrt{2\sin(x) + 1}} - \frac{1}{2}\sqrt{2\sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x)^2 - 2*(2*sin(x) + 1)^(1/4))*cos(x)/(2*sin(x) + 1)^(3/2),x, al
```

```
[Out] 1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqr
t(2*sin(x) + 1) - 1/2*sqrt(2*sin(x) + 1)
```

Fricas [A] time = 0.223371, size = 54, normalized size = 0.98

$$-\frac{\left(\cos(x)^2 + 2\sin(x) - 2\right)\sqrt{2\sin(x) + 1} + 12(2\sin(x) + 1)^{\frac{3}{4}}}{3(2\sin(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x)^2 - 2*(2*sin(x) + 1)^(1/4))*cos(x)/(2*sin(x) + 1)^(3/2),x, al
```

```
[Out] -1/3*((cos(x)^2 + 2*sin(x) - 2)*sqrt(2*sin(x) + 1) + 12*(2*sin(x)
 + 1)^(3/4))/(2*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\cos\left(x\right)^{2} - 2\left(2\,\sin\left(x\right) + 1\right)^{\frac{1}{4}}\right)\cos\left(x\right)}{\left(2\,\sin\left(x\right) + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(cos(x)^2 - 2*(2*sin(x) + 1)^(1/4))*cos(x)/(2*sin(x) + 1)^(3/2),x, al
```

```
[Out] integrate(-(cos(x)^2 - 2*(2*sin(x) + 1)^(1/4))*cos(x)/(2*sin(x) +
 1)^(3/2), x)
```

3.397 $\int \sqrt{\tan(x)}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*
Sqrt[Tan[x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2
*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.128825, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Tan[x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*
Sqrt[Tan[x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2
*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 7.93392, size = 94, normalized size = 0.96

$$\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)}{4}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(x)}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**(1/2),x)
```

```
[Out] sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1)/4 - sqrt(2)*log(s
qrt(2)*sqrt(tan(x)) + tan(x) + 1)/4 + sqrt(2)*atan(sqrt(2)*sqrt(t
an(x)) - 1)/2 + sqrt(2)*atan(sqrt(2)*sqrt(tan(x)) + 1)/2
```

Mathematica [A] time = 0.0997435, size = 82, normalized size = 0.84

$$\frac{-2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)+\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)-\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Tan[x]],x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]] - Log[1 + Sqrt[
2]*Sqrt[Tan[x]] + Tan[x]])/(2*Sqrt[2])
```

Maple [A] time = 0.004, size = 49, normalized size = 0.5

$$\frac{\cos(x)\sqrt{2}\arccos(\cos(x)-\sin(x))}{2}\sqrt{\tan(x)}\frac{1}{\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}}{2}\ln\left(\cos(x)+\sqrt{2}\sqrt{\tan(x)}\cos(x)+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^(1/2),x)
```

```
[Out] 1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(
x)-sin(x))-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(
x))
```

Maxima [A] time = 1.56905, size = 108, normalized size = 1.1

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqr
t(2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sq
rt(2)*sqrt(tan(x)) + tan(x) + 1)
```

Fricas [A] time = 0.251358, size = 246, normalized size = 2.51

$$-\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}\sqrt{\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)+\cos(x)+\sin(x)}{\cos(x)}}+\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}+1}\right)-\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}\sqrt{-\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)-\cos(x)-\sin(x)}{\cos(x)}}+\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}-1}\right)-\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)+\cos(x)+\sin(x)\right)}{\cos(x)}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)-\cos(x)-\sin(x)\right)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(tan(x)),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/(sqrt(2)*sqrt((sqrt(2)*sqrt(sin(x)/cos(x))*cos(
x) + cos(x) + sin(x))/cos(x)) + sqrt(2)*sqrt(sin(x)/cos(x)) + 1))
 - sqrt(2)*arctan(1/(sqrt(2)*sqrt(-(sqrt(2)*sqrt(sin(x)/cos(x))*c
os(x) - cos(x) - sin(x))/cos(x)) + sqrt(2)*sqrt(sin(x)/cos(x)) -
1)) - 1/4*sqrt(2)*log(2*(sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) + cos
(x) + sin(x))/cos(x)) + 1/4*sqrt(2)*log(-2*(sqrt(2)*sqrt(sin(x)/c
os(x))*cos(x) - cos(x) - sin(x))/cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)), x)
```

GIAC/XCAS [A] time = 0.205685, size = 108, normalized size = 1.1

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\ln\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+\frac{1}{4}\sqrt{2}\ln\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqr
t(2)*ln(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*ln(-sqrt
(2)*sqrt(tan(x)) + tan(x) + 1)
```

3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{3}{20}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{2}(5x)+1\right)$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + (3*Log[1 +
 Tan[5*x]^(2/3)])/20 - Log[1 + Tan[5*x]^2]/20
```

Rubi [A] time = 0.111656, antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125

$$-\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{1}{10}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[5*x]^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Ta
n[5*x]^(2/3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Rubi in Sympy [A] time = 5.31044, size = 63, normalized size = 1.11

$$\frac{\log\left(\tan^{\frac{2}{3}}(5x)+1\right)}{10}-\frac{\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)}{20}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\tan^{\frac{2}{3}}(5x)}{3}-\frac{1}{3}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/tan(5*x)**(1/3),x)
```

```
[Out] log(tan(5*x)**(2/3) + 1)/10 - log(tan(5*x)**(4/3) - tan(5*x)**(2/
3) + 1)/20 + sqrt(3)*atan(sqrt(3)*(2*tan(5*x)**(2/3)/3 - 1/3))/10
```

Mathematica [B] time = 0.132768, size = 121, normalized size = 2.12

$$\frac{1}{20}\left(-2\sqrt{3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{\tan(5x)}\right)-2\sqrt{3}\tan^{-1}\left(2\sqrt[3]{\tan(5x)}+\sqrt{3}\right)+2\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right.$$
$$\left.-\log\left(\tan^{\frac{2}{3}}(5x)-\sqrt{3}\sqrt[3]{\tan(5x)}+1\right)-\log\left(\tan^{\frac{2}{3}}(5x)+\sqrt{3}\sqrt[3]{\tan(5x)}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[5*x]^(-1/3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*Tan[5*x]^(1/3)] - 2*Sqrt[3]*ArcTan
[Sqrt[3] + 2*Tan[5*x]^(1/3)] + 2*Log[1 + Tan[5*x]^(2/3)] - Log[1
- Sqrt[3]*Tan[5*x]^(1/3) + Tan[5*x]^(2/3)] - Log[1 + Sqrt[3]*Tan[
5*x]^(1/3) + Tan[5*x]^(2/3)])/20
```

Maple [A] time = 0.015, size = 53, normalized size = 0.9

$$\frac{1}{10}\ln\left(1+(\tan(5x))^{\frac{2}{3}}\right)-\frac{1}{20}\ln\left(1-(\tan(5x))^{\frac{2}{3}}+(\tan(5x))^{\frac{4}{3}}\right)+\frac{\sqrt{3}}{10}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,(\tan(5x))^{2/3}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/tan(5*x)^(1/3),x)
```

```
[Out] 1/10*ln(1+tan(5*x)^(2/3))-1/20*ln(1-tan(5*x)^(2/3)+tan(5*x)^(4/3)
)+1/10*3^(1/2)*arctan(1/3*(2*tan(5*x)^(2/3)-1)*3^(1/2))
```

Maxima [A] time = 1.52597, size = 70, normalized size = 1.23

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(5x)^{\frac{2}{3}}-1\right)\right)-\frac{1}{20}\log\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(5*x)^(-1/3),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*lo
g(tan(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3)
+ 1)
```

Fricas [A] time = 0.227265, size = 70, normalized size = 1.23

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(5x)^{\frac{2}{3}}-1\right)\right)-\frac{1}{20}\log\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(5*x)^(-1/3),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*lo
g(tan(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3)
+ 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/tan(5*x)**(1/3),x)
```

```
[Out] Integral(tan(5*x)**(-1/3), x)
```

GIAC/XCAS [A] time = 0.204016, size = 70, normalized size = 1.23

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(5x)^{\frac{2}{3}}-1\right)\right)-\frac{1}{20}\ln\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)+\frac{1}{10}\ln\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(5*x)^(-1/3),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*ln
(tan(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*ln(tan(5*x)^(2/3) +
1)
```

3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{3\tan(2x)+4}}+\frac{13\tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}$$

```
[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250
*Sqrt[2]) + (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*
x]])])/(250*Sqrt[2]) - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

Rubi [A] time = 0.181672, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{3\tan(2x)+4}}+\frac{13\tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250
*Sqrt[2]) + (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*
x]])])/(250*Sqrt[2]) - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

Rubi in Sympy [A] time = 6.77787, size = 92, normalized size = 1.06

$$-\frac{9\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}(-27\tan(2x)+9)}{18\sqrt{3\tan(2x)+4}}\right)}{500}-\frac{13\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}(-117\tan(2x)-351)}{234\sqrt{3\tan(2x)+4}}\right)}{500}-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4+3*tan(2*x))**(3/2),x)
```

```
[Out] -9*sqrt(2)*atan(sqrt(2)*(-27*tan(2*x) + 9)/(18*sqrt(3*tan(2*x) +
4)))/500 - 13*sqrt(2)*atanh(sqrt(2)*(-117*tan(2*x) - 351)/(234*sq
rt(3*tan(2*x) + 4)))/500 - 3/(25*sqrt(3*tan(2*x) + 4))
```

Mathematica [C] time = 0.342002, size = 83, normalized size = 0.95

$$\frac{-\frac{150}{\sqrt{3\tan(2x)+4}}+(24-7i)\sqrt{4-3i}\tanh^{-1}\left(\frac{\sqrt{3\tan(2x)+4}}{\sqrt{4-3i}}\right)+(24+7i)\sqrt{4+3i}\tanh^{-1}\left(\frac{\sqrt{3\tan(2x)+4}}{\sqrt{4+3i}}\right)}{1250}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] ((24 - 7*I)*Sqrt[4 - 3*I]*ArcTanh[Sqrt[4 + 3*Tan[2*x]]/Sqrt[4 - 3
*I]] + (24 + 7*I)*Sqrt[4 + 3*I]*ArcTanh[Sqrt[4 + 3*Tan[2*x]]/Sqrt
[4 + 3*I]] - 150/Sqrt[4 + 3*Tan[2*x]])/1250
```

Maple [A] time = 0.072, size = 130, normalized size = 1.5

$$-\frac{13\sqrt{2}}{1000}\ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\sqrt{2}\right)+\frac{9\sqrt{2}}{500}\arctan\left(\frac{\sqrt{2}}{2}\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\right)+\frac{13\sqrt{2}}{1000}\ln\left(9+3\tan(2x)+3\sqrt{4+3\tan(2x)}\sqrt{2}\right)+\frac{9\sqrt{2}}{500}\arctan\left(\frac{\sqrt{2}}{2}\left(2\sqrt{4+3\tan(2x)}+3\sqrt{2}\right)\right)-\frac{3}{25}\frac{1}{\sqrt{4+3\tan(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4+3*tan(2*x))^(3/2),x)
```

```
[Out] -13/1000*2^(1/2)*ln(9+3*tan(2*x)-3*(4+3*tan(2*x))^(1/2)*2^(1/2))+
9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/
2))+13/1000*2^(1/2)*ln(9+3*tan(2*x)+3*(4+3*tan(2*x))^(1/2)*2^(1/2
))+9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)+3*2^(1/2))*2^
(1/2))-3/25/(4+3*tan(2*x))^(1/2)
```

Maxima [A] time = 2.51699, size = 4338, normalized size = 49.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*tan(2*x) + 4)^(-3/2),x, algorithm="maxima")
```

```
[Out] -1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x
) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(
```

8*x) + 4))^3 + 2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*co
s(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3
*sin(8*x) + 4))*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(
4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 2000*(co
s(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x)
 + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 -
 80*(48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x)
+ 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*
x) + 4)) - 80*(25*(cos(4*x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*
cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) +
 3*sin(8*x) + 4))^2 - 25*cos(4*x) + 48*sin(4*x) + 75)*sin(1/2*arc
tan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*co
s(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*arctan2(-3*cos
(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*
sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*
x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2
)*arctan2(1/3*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*
x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4
*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x)
 + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*
(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*c
os(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9
*cos(4*x) + 25/9)) + cos(4*x) - 4/3*sin(4*x), 1/3*25^(1/4)*(25*co
s(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*
cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(c
os(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*co
s(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) +
8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) +
 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9)) - 4/3*cos(
4*x) - sin(4*x) - 4/3) + 18*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x)
+ 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*
x) + 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8
*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arct
an2(2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8
*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*c
os(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*
x) + 13)^(1/4)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3
*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x
) - 8/3*sin(4*x) + 16/9)) + 4/3*sin(4*x) + 1, 2/3*4^(1/4)*(4*cos(
4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*
x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x
)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4)*cos(1/2*
arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*c
os(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9
)) + 4/3*cos(4*x) + 4/3) + 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x
) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(
8*x) + 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*lo
g(-2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 +
2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 +
 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*c
os(4*x) + 25)^(1/4)*(4*cos(4*x) + 3*sin(4*x) + 4)*cos(1/2*arctan2
(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^
2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x)
- 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9)) + 5/9*sqrt(25*cos(4*x)^
4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x

```
) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)
^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*cos(1/2*arctan2
(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^
2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x)
- 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9))^2 + 2/9*25^(1/4)*(25*co
s(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*
cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(c
os(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*(3
*cos(4*x) - 4*sin(4*x))*sin(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*
cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(
4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*co
s(4*x) + 25/9)) + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos
(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*si
n(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4
*x) + 64*cos(4*x) + 25)*sin(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*
cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(
4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*co
s(4*x) + 25/9))^2 + 25/9*cos(4*x)^2 + 25/9*sin(4*x)^2 + 32/9*cos(
4*x) + 8/3*sin(4*x) + 16/9) - 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(
8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*s
in(8*x) + 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x
) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)
*log(16/9*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 +
(8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33
*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(
4*x) + 13)^(1/4)*(cos(4*x) + 1)*cos(1/2*arctan2(32/9*(cos(4*x) +
1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)
^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^
4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) +
17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 +
 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)*cos(1/2*arctan2(32/
9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 -
 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))^2 + 4/9*
4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x
)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^
2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)
^(1/4)*(4*sin(4*x) + 3)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4
*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/
9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 + 4*si
n(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(
4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4
*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)*sin(1/2*arctan2(32/9*(cos(4
*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*si
n(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))^2 + 16/9*cos(4*x
)^2 + 16/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3*sin(4*x) + 25/9))*(2*
(32*cos(4*x) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^2 + 64*cos
(4*x)^2 + 16*(3*cos(4*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)
^2 + 64*sin(4*x)^2 + 64*cos(4*x) + 48*sin(4*x) + 25)^(1/4))/((2*(
32*cos(4*x) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^2 + 64*cos(
4*x)^2 + 16*(3*cos(4*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)^
2 + 64*sin(4*x)^2 + 64*cos(4*x) + 48*sin(4*x) + 25)^(1/4)*(cos(1/
2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) +
 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sin(1/2*arctan2(-3*cos(8*x) +
4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x)
 + 4))^2))
```

Fricas [A] time = 0.236857, size = 618, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*tan(2*x) + 4)^(-3/2),x, algorithm="fricas")
```

```
[Out] -1/5000*sqrt(10)*(36*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2
*x))*arctan(5*sqrt(5)/(sqrt(15)*sqrt(10)*sqrt((sqrt(10)*sqrt(5)*s
qrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) + 15*cos(2*x) +
5*sin(2*x))/cos(2*x)) + 5*sqrt(10)*sqrt((4*cos(2*x) + 3*sin(2*x))
/cos(2*x)) + 15*sqrt(5))) + 36*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2
*x))/cos(2*x))*arctan(5*sqrt(5)/(sqrt(15)*sqrt(10)*sqrt(-(sqrt(10
)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) - 15*
cos(2*x) - 5*sin(2*x))/cos(2*x)) + 5*sqrt(10)*sqrt((4*cos(2*x) +
3*sin(2*x))/cos(2*x)) - 15*sqrt(5))) - 13*sqrt(5)*sqrt((4*cos(2*x
) + 3*sin(2*x))/cos(2*x))*log(9375*(sqrt(10)*sqrt(5)*sqrt((4*cos(
2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) + 15*cos(2*x) + 5*sin(2*x))
/cos(2*x)) + 13*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*
log(-9375*(sqrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*
x))*cos(2*x) - 15*cos(2*x) - 5*sin(2*x))/cos(2*x)) + 60*sqrt(10))
/sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4+3*tan(2*x))**(3/2),x)
```

```
[Out] Integral((3*tan(2*x) + 4)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*tan(2*x) + 4)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((3*tan(2*x) + 4)^(-3/2), x)
```

3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

```
[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Ta
n[x]])/3
```

Rubi [A] time = 0.223963, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
```

```
[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Ta
n[x]])/3
```

Rubi in Sympy [A] time = 7.678, size = 36, normalized size = 0.9

$$\frac{2\sqrt{-3\tan(x)+4}}{3}+\frac{\log(-3\tan(x)+4)}{3}+\frac{8}{3\sqrt{-3\tan(x)+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x
```

```
[Out] 2*sqrt(-3*tan(x) + 4)/3 + log(-3*tan(x) + 4)/3 + 8/(3*sqrt(-3*tan
(x) + 4))
```

Mathematica [A] time = 1.43235, size = 38, normalized size = 0.95

$$\frac{-6\tan(x)+\sqrt{4-3\tan(x)}\log(4-3\tan(x))+16}{3\sqrt{4-3\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
```

```
[Out] (16 + Log[4 - 3*Tan[x]]*Sqrt[4 - 3*Tan[x]] - 6*Tan[x])/(3*Sqrt[4
- 3*Tan[x]])
```

Maple [B] time = 0.564, size = 227, normalized size = 5.7

$$\frac{(\cos(x)-1)^2(1+\cos(x))^2}{(-12\cos(x)+9\sin(x))(\sin(x))^4}\left(3\sin(x)\sqrt{-2\frac{-4\cos(x)+3\sin(x)}{\cos(x)}}\sqrt{2}-8\cos(x)\sqrt{-2\frac{-4\cos(x)+3\sin(x)}{\cos(x)}}\sqrt{2}+3\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x)
```

```
[Out] 1/3*(cos(x)-1)^2*(1+cos(x))^2*(3*sin(x)*(-2*(-4*cos(x)+3*sin(x))/
cos(x))^(1/2)*2^(1/2)-8*cos(x)*(-2*(-4*cos(x)+3*sin(x))/cos(x))^(
1/2)*2^(1/2)+3*sin(x)*ln((-cos(x)+1+2*sin(x))/sin(x))-3*sin(x)*ln
((1-cos(x)+sin(x))/sin(x))-3*sin(x)*ln(-(sin(x)-1+cos(x))/sin(x))
+3*sin(x)*ln(-(2*cos(x)-2+sin(x))/sin(x))-4*cos(x)*ln((-cos(x)+1+
2*sin(x))/sin(x))+4*cos(x)*ln((1-cos(x)+sin(x))/sin(x))+4*cos(x)*
ln(-(sin(x)-1+cos(x))/sin(x))-4*cos(x)*ln(-(2*cos(x)-2+sin(x))/si
n(x)))/(-4*cos(x)+3*sin(x))/sin(x)^4
```

Maxima [A] time = 1.35364, size = 41, normalized size = 1.02

$$\frac{2}{3}\sqrt{-3\tan(x)+4}+\frac{8}{3\sqrt{-3\tan(x)+4}}+\frac{1}{3}\log(-3\tan(x)+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(-3*tan(x) + 4) - 3*tan(x))/((-3*tan(x) + 4)^(3/2)*cos(x)^2),x, a
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(-3*ta
n(x) + 4)
```

Fricas [A] time = 0.241023, size = 111, normalized size = 2.78

$$\frac{(4\cos(x)-3\sin(x))\log\left(\frac{7}{4}\cos(x)^2-6\cos(x)\sin(x)+\frac{9}{4}\right)-(4\cos(x)-3\sin(x))\log\left(\cos(x)^2\right)+4\sqrt{\frac{4\cos(x)-3\sin(x)}{\cos(x)}}(8\cos}{6(4\cos(x)-3\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(-3*tan(x) + 4) - 3*tan(x))/((-3*tan(x) + 4)^(3/2)*cos(x)^2),x, a
```

```
[Out] 1/6*((4*cos(x) - 3*sin(x))*log(7/4*cos(x)^2 - 6*cos(x)*sin(x) + 9
/4) - (4*cos(x) - 3*sin(x))*log(cos(x)^2) + 4*sqrt((4*cos(x) - 3*
sin(x))/cos(x))*(8*cos(x) - 3*sin(x)))/(4*cos(x) - 3*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21784, size = 42, normalized size = 1.05

$$\frac{2}{3}\sqrt{-3\tan\left(x\right)+4}+\frac{8}{3\sqrt{-3\tan\left(x\right)+4}}+\frac{1}{3}\ln\left(\left|-3\tan\left(x\right)+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(-3*tan(x) + 4) - 3*tan(x))/((-3*tan(x) + 4)^(3/2)*cos(x)^2),x, a
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*ln(abs(-3
*tan(x) + 4))
```

3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{x}{2}+\frac{\tan^{-1}\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{2}\log(\cos(x))+\frac{\tanh^{-1}\left(\frac{\tan(x)+1}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}$$

```
[Out] -x/2 + ArcTan[(1 - Tan[x])/(Sqrt[2]*Sqrt[Tan[x]])]/Sqrt[2] + ArcT
anh[(1 + Tan[x])/(Sqrt[2]*Sqrt[Tan[x]])]/Sqrt[2] + Log[Cos[x]]/2
+ Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Tan[x]])^(-1)
```

Rubi [A] time = 0.610796, antiderivative size = 133, normalized size of antiderivative = 1.58, number of steps used = 19, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923

$$-\frac{x}{2}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)$$
$$-\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}+\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}+\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] -x/2 + ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] - ArcTan[1 + Sqrt
[2]*Sqrt[Tan[x]]]/Sqrt[2] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]]
 - Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + Log[1 + S
qrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + (1 - Sqrt[Tan[x]])^(-
1)
```

Rubi in Sympy [A] time = 25.6114, size = 148, normalized size = 1.76

$$\left(\frac{1}{2}-\frac{i}{2}\right)\log\left(-\sqrt{\tan(x)}+1\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\log\left(-\sqrt{\tan(x)}+1\right)-\left(\frac{1}{4}-\frac{i}{4}\right)\log\left(-\tan(x)+i\right)$$
$$-\left(\frac{1}{4}+\frac{i}{4}\right)\log\left(\tan(x)+i\right)+\frac{\sqrt{2}\,(1-i)\,\mathrm{atan}\left(\frac{\sqrt{2}(-1+i)\sqrt{\tan(x)}}{2}\right)}{2}$$
$$-\frac{\sqrt{2}\,(1-i)\,\mathrm{atanh}\left(\frac{\sqrt{2}(-1+i)\sqrt{\tan(x)}}{2}\right)}{2}+\frac{\frac{1}{2}-\frac{i}{2}}{-\sqrt{\tan(x)}+1}+\frac{\frac{1}{2}+\frac{i}{2}}{-\sqrt{\tan(x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
```

```
[Out] (1/2 - I/2)*log(-sqrt(tan(x)) + 1) + (1/2 + I/2)*log(-sqrt(tan(x)
) + 1) - (1/4 - I/4)*log(-tan(x) + I) - (1/4 + I/4)*log(tan(x) +
I) + sqrt(2)*(1 - I)*atan(sqrt(2)*(-1 + I)*sqrt(tan(x))/2)/2 - sq
rt(2)*(1 - I)*atanh(sqrt(2)*(-1 + I)*sqrt(tan(x))/2)/2 + (1/2 - I
/2)/(-sqrt(tan(x)) + 1) + (1/2 + I/2)/(-sqrt(tan(x)) + 1)
```

Mathematica [A] time = 0.795419, size = 155, normalized size = 1.85

$$\frac{1}{2}\left(-x+\sqrt{2}\left(\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)-\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\right)+\log\left(1-\sqrt{\tan(x)}\right)-\log\left(\sqrt{\tan(x)}+1\right)+\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)-\log\left(-\tan(x)+\sqrt{2}\sqrt{\tan(x)}-1\right)}{\sqrt{2}}+\frac{2\sin(x)}{\cos(x)-\sin(x)}+\log(\cos(x)-\sin(x))+\frac{2\cos(x)\sqrt{\tan(x)}}{\cos(x)-\sin(x)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] (-x + Sqrt[2]*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]] - ArcTan[1 + Sqrt
[2]*Sqrt[Tan[x]]]) + Log[Cos[x] - Sin[x]] + Log[1 - Sqrt[Tan[x]]]
 - Log[1 + Sqrt[Tan[x]]] + (-Log[-1 + Sqrt[2]*Sqrt[Tan[x]] - Tan[
x]] + Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]])/Sqrt[2] + (2*Sin[x]
)/(Cos[x] - Sin[x]) + (2*Cos[x]*Sqrt[Tan[x]])/(Cos[x] - Sin[x]))/
2
```

Maple [A] time = 0.028, size = 97, normalized size = 1.2

$$-\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)-\frac{\sqrt{2}}{2}\arctan\left(\sqrt{2}\sqrt{\tan(x)}-1\right)-\frac{\sqrt{2}}{4}\ln\left(1\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)^{-1}\right)-\frac{\ln\left(1+(\tan(x))^{2}\right)}{4}-\left(-1+\sqrt{\tan(x)}\right)^{-1}+\ln\left(-1+\sqrt{\tan(x)}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)/(-1+tan(x)^(1/2))^2,x)
```

```
[Out] -1/2*arctan(1+2^(1/2)*tan(x)^(1/2))*2^(1/2)-1/2*arctan(2^(1/2)*ta
n(x)^(1/2)-1)*2^(1/2)-1/4*2^(1/2)*ln((1-2^(1/2)*tan(x)^(1/2)+tan(
x))/(1+2^(1/2)*tan(x)^(1/2)+tan(x)))-1/4*ln(1+tan(x)^2)-1/(-1+tan
(x)^(1/2))+ln(-1+tan(x)^(1/2))-1/2*x
```

Maxima [A] time = 1.53385, size = 158, normalized size = 1.88

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2}-2\right)\log\left(\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2}+2\right)\log\left(-\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) - \frac{1}{\sqrt{\tan\left(x\right)}-1} + \log\left(\sqrt{\tan\left(x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(sqrt(tan(x)) - 1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(ta
n(x)))) - 1/4*sqrt(2)*(sqrt(2) + 2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2*sqrt(tan(x)))) - 1/8*sqrt(2)*(sqrt(2) - 2)*log(sqrt(2)*sqrt(t
an(x)) + tan(x) + 1) - 1/8*sqrt(2)*(sqrt(2) + 2)*log(-sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1) + log(sqrt(tan(x))
 - 1)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(sqrt(tan(x)) - 1)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(x\right)}{\left(\sqrt{\tan\left(x\right)}-1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
```

```
[Out] Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)
```

GIAC/XCAS [A] time = 0.219621, size = 150, normalized size = 1.79

$$-\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)-\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)+\frac{1}{4}\sqrt{2}\ln\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)-\frac{1}{4}\sqrt{2}\ln\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)-\frac{1}{\sqrt{\tan(x)}-1}-\frac{1}{4}\ln\left(\tan(x)^2+1\right)+\ln\left(\left|\sqrt{\tan(x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(sqrt(tan(x)) - 1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x))))
 - 1/2*(sqrt(2) + 1)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)
))) + 1/4*sqrt(2)*ln(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/4*sqr
t(2)*ln(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1
) - 1/4*ln(tan(x)^2 + 1) + ln(abs(sqrt(tan(x)) - 1))
```

3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]
]/2
```

Rubi [A] time = 0.0274881, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]
]/2
```

Rubi in Sympy [A] time = 1.41027, size = 27, normalized size = 0.87

$$-\frac{\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)}{2}+\frac{\operatorname{asin}(\sin(x)-\cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/sin(2*x)**(1/2),x)
```

```
[Out] -log(sin(x) + sqrt(sin(2*x)) + cos(x))/2 + asin(sin(x) - cos(x))/
2
```

Mathematica [A] time = 0.0498089, size = 31, normalized size = 1.

$$\frac{1}{2}\left(-\sin^{-1}(\cos(x)-\sin(x))-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
)/2
```

Maple [C] time = 0.109, size = 266, normalized size = 8.6

$$-\frac{1}{2}\sqrt{-1\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(2\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}EllipticE\left(\sqrt{1+\mathrm{ta}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/sin(2*x)^(1/2),x)
```

```
[Out] -1/2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*(1+
tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ell
ipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-(1+tan(1/2*
x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((
1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+2*(1+tan(1/2*x))^(1
/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(
1/2*x))^(1/2),1/2*2^(1/2))-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)
^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(
1/2))+2*tan(1/2*x)^4-2*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1)
)^(1/2)/(tan(1/2*x)^2+1)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sqrt(sin(2*x)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.256769, size = 342, normalized size = 11.03

$$-\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+2\cos(x)\sin(x)}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+2\cos(x)^2-1}\right)+\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-2\cos(x)\sin(x)}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))-2\cos(x)^2+1}\right)+\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+1\right)}{2\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))}\right)-\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-1\right)}{2\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))}\right)-\frac{1}{8}\log\left(2\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+4\cos(x)\sin(x)+1\right)+\frac{1}{8}\log\left(-2\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+4\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sqrt(sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/8*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 2*c
os(x)*sin(x))/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + 2*
cos(x)^2 - 1)) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x)
 + sin(x)) - 2*cos(x)*sin(x))/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x
) - sin(x)) - 2*cos(x)^2 + 1)) + 1/8*arctan(-1/2*sqrt(2)*(sqrt(2)
*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1)/(sqrt(cos(x)*sin(x))*
(cos(x) - sin(x)))) - 1/8*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(cos(x
)*sin(x))*(cos(x) + sin(x)) - 1)/(sqrt(cos(x)*sin(x))*(cos(x) - s
in(x)))) - 1/8*log(2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)
) + 4*cos(x)*sin(x) + 1) + 1/8*log(-2*sqrt(2)*sqrt(cos(x)*sin(x))
*(cos(x) + sin(x)) + 4*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sqrt(sin(2*x)),x, algorithm="giac")
```

```
[Out] integrate(sin(x)/sqrt(sin(2*x)), x)
```

3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]
]/2
```

Rubi [A] time = 0.0269563, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/Sqrt[Sin[2*x]],x]

[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]
]/2
```

Rubi in Sympy [A] time = 1.3834, size = 27, normalized size = 0.87

$$\frac{\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)}{2}+\frac{\operatorname{asin}(\sin(x)-\cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/sin(2*x)**(1/2),x)

[Out] log(sin(x) + sqrt(sin(2*x)) + cos(x))/2 + asin(sin(x) - cos(x))/2
```

Mathematica [A] time = 0.0342881, size = 29, normalized size = 0.94

$$\frac{1}{2}\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
)/2
```

Maple [C] time = 0.112, size = 98, normalized size = 3.2

$$1\sqrt{-1\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}-1\right)\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}EllipticF\Bigg(\sqrt{1+\tan\Big(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(
1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*Elliptic
F((1+tan(1/2*x))^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sqrt(sin(2*x)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.253368, size = 342, normalized size = 11.03

$$\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+2\cos(x)^2-1}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+2\cos(x)\sin(x)}\right) - \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))-2\cos(x)^2+1}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-2\cos(x)\sin(x)}\right) + \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+1\right)}{2\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))}\right) - \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-1\right)}{2\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))}\right) + \frac{1}{8}\log\left(2\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+4\cos(x)\sin(x)+1\right) - \frac{1}{8}\log\left(-2\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+4\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sqrt(sin(2*x)),x, algorithm="fricas")
```

```
[Out] 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + 2*co
s(x)^2 - 1)/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 2*co
s(x)*sin(x))) - 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x)
- sin(x)) - 2*cos(x)^2 + 1)/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x)
+ sin(x)) - 2*cos(x)*sin(x))) + 1/8*arctan(-1/2*sqrt(2)*(sqrt(2)*
sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1)/(sqrt(cos(x)*sin(x))*(
cos(x) - sin(x)))) - 1/8*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(cos(x)
*sin(x))*(cos(x) + sin(x)) - 1)/(sqrt(cos(x)*sin(x))*(cos(x) - si
n(x)))) + 1/8*log(2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x))
 + 4*cos(x)*sin(x) + 1) - 1/8*log(-2*sqrt(2)*sqrt(cos(x)*sin(x))*
(cos(x) + sin(x)) + 4*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sqrt(sin(2*x)),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\sqrt{\sin(2x)}\cos(x)+\frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]
]/4 - (Cos[x]*Sqrt[Sin[2*x]])/2
```

Rubi [A] time = 0.0480131, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\sqrt{\sin(2x)}\cos(x)+\frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]
]/4 - (Cos[x]*Sqrt[Sin[2*x]])/2
```

Rubi in Sympy [A] time = 2.27453, size = 41, normalized size = 0.91

$$\frac{\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)}{4}-\frac{\sqrt{\sin(2x)}\cos(x)}{2}+\frac{\operatorname{asin}(\sin(x)-\cos(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)*sin(2*x)**(1/2),x)
```

```
[Out] log(sin(x) + sqrt(sin(2*x)) + cos(x))/4 - sqrt(sin(2*x))*cos(x)/2
 + asin(sin(x) - cos(x))/4
```

Mathematica [A] time = 0.0453947, size = 41, normalized size = 0.91

$$\frac{1}{4}\left(-\sin^{-1}(\cos(x)-\sin(x))-2\sqrt{\sin(2x)}\cos(x)+\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
 - 2*Cos[x]*Sqrt[Sin[2*x]])/4
```

Maple [C] time = 0.127, size = 171, normalized size = 3.8

$$1\sqrt{-1\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}EllipticF\left(\sqrt{1+\tan\left(\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)*sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+tan(1/2
*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF(
(1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/
2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1
/2*x))^(1/2),1/2*2^(1/2))+2*tan(1/2*x)^3-2*tan(1/2*x))/(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/
2*x)^2+1)
```

Maxima [A] time = 12.2267, size = 1, normalized size = 0.02

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(2*x))*sin(x),x, algorithm="maxima")
```

```
[Out] 0
```

Fricas [A] time = 0.26004, size = 360, normalized size = 8.

$$-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)+\frac{1}{16}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+2\cos(x)^2-1}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+2\cos(x)\sin(x)}\right)-\frac{1}{16}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))-2\cos(x)^2+1}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-2\cos(x)\sin(x)}\right)+\frac{1}{16}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+1\right)}{2\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))}\right)-\frac{1}{16}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-1\right)}{2\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))}\right)+\frac{1}{16}\log\left(2\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+4\cos(x)\sin(x)+1\right)-\frac{1}{16}\log\left(-2\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+4\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(2*x))*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/16*arctan(-(sqrt(2)*s
qrt(cos(x)*sin(x))*(cos(x) - sin(x)) + 2*cos(x)^2 - 1)/(sqrt(2)*s
qrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 2*cos(x)*sin(x))) - 1/16*a
rctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) - 2*cos(x)^
2 + 1)/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) - 2*cos(x)*
sin(x))) + 1/16*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(cos(x)*sin(x))*
(cos(x) + sin(x)) + 1)/(sqrt(cos(x)*sin(x))*(cos(x) - sin(x)))) -
 1/16*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) +
sin(x)) - 1)/(sqrt(cos(x)*sin(x))*(cos(x) - sin(x)))) + 1/16*log(
2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 4*cos(x)*sin(x)
 + 1) - 1/16*log(-2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x))
 + 4*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2x)}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(2*x))*sin(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2 + (Cos[x]*Sqrt[Sin[2*x]]
)/2 + (Sin[x]*Sqrt[Sin[2*x]])/2
```

Rubi [A] time = 0.144513, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] -Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2 + (Cos[x]*Sqrt[Sin[2*x]]
)/2 + (Sin[x]*Sqrt[Sin[2*x]])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0672367, size = 43, normalized size = 0.91

$$\frac{1}{2}\left(\sin(x)\sqrt{\sin(2x)}+\sqrt{\sin(2x)}\cos(x)-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] (-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] +
 Sin[x]*Sqrt[Sin[2*x]])/2
```

Maple [C] time = 0.207, size = 442, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))*sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(4*(tan(1/2
*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(1+tan(1/2*x))^(1/2)*(-
2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x)
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-3*(tan(1/2*x)*(-1+tan(1/2*x))*(
1+tan(1/2*x)))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)
*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*
tan(1/2*x)^2+4*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*
(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-3*(1+tan(1/2*x))^(1/2
)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/
2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x
)))^(1/2)+4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4-2*(tan(1
/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^3+4*(tan(1
/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2+2*(tan(1/2*x)*(-1+tan(1/2*
x))*(1+tan(1/2*x)))^(1/2)*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1
))^(1/2)/(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)/(tan(1
/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)^2+1)
```

Maxima [A] time = 20.9064, size = 1, normalized size = 0.02

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x) - sin(x))*sqrt(sin(2*x)),x, algorithm="maxima")
```

```
[Out] 0
```

Fricas [A] time = 0.248057, size = 181, normalized size = 3.85

$$\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x)) + \frac{1}{16}\log\left(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3 - \left(4\cos(x)^2+1\right)\sin(x) - 5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1\right) - \frac{1}{16}\log\left(-32\cos(x)^4 - 4\sqrt{2}\left(4\cos(x)^3 - \left(4\cos(x)^2+1\right)\sin(x) - 5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) - sin(x))*sqrt(sin(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1/16*log(-32*
cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*co
s(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1) -
 1/16*log(-32*cos(x)^4 - 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)
*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)
*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) - sin(x))*sqrt(sin(2*x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=61

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{\sin(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]
]]/16 + Sin[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.123812, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{\sin(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]
]]/16 + Sin[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])
```

Rubi in Sympy [A] time = 5.61283, size = 56, normalized size = 0.92

$$\frac{\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)}{16}+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{\sin(x)}{4\sqrt{\sin(2x)}}+\frac{\operatorname{asin}(\sin(x)-\cos(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] log(sin(x) + sqrt(sin(2*x)) + cos(x))/16 + sin(x)**5/(5*sin(2*x)*
*(5/2)) - sin(x)/(4*sqrt(sin(2*x))) + asin(sin(x) - cos(x))/16
```

Mathematica [A] time = 0.124732, size = 50, normalized size = 0.82

$$\frac{1}{80}\left(2\sqrt{\sin(2x)}\sec(x)\left(\sec^2(x)-6\right)+5\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x
]]]) + 2*Sec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80
```

Maple [C] time = 0.183, size = 510, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] 1/2688*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*E
llipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^14+35*(1+ta
n(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellip
ticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^12+10*tan(1/2*x
)^15+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x
))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^1
0+66*tan(1/2*x)^13+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/
2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2)
)*tan(1/2*x)^8-1014*tan(1/2*x)^11+175*(1+tan(1/2*x))^(1/2)*(-2*ta
n(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1
/2),1/2*2^(1/2))*tan(1/2*x)^6+2002*tan(1/2*x)^9+105*(1+tan(1/2*x)
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+
tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^4-2002*tan(1/2*x)^7+35*
(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+1014*tan
(1/2*x)^5+5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/
2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-66*tan(1/
2*x)^3-10*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/
2*x)^2+1)^7/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.269678, size = 409, normalized size = 6.7

$$5\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+2\cos(x)^2-1}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+2\cos(x)\sin(x)}\right)\cos(x)^3-5\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))-2\cos(x)^2+1}{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))-2\cos(x)\sin(x)}\right)\cos(x)^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(5*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) +
 2*cos(x)^2 - 1)/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) +
 2*cos(x)*sin(x)))*cos(x)^3 - 5*arctan(-(sqrt(2)*sqrt(cos(x)*sin(
x))*(cos(x) - sin(x)) - 2*cos(x)^2 + 1)/(sqrt(2)*sqrt(cos(x)*sin(
x))*(cos(x) + sin(x)) - 2*cos(x)*sin(x)))*cos(x)^3 + 5*arctan(-1/
2*sqrt(2)*(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1)/(sq
rt(cos(x)*sin(x))*(cos(x) - sin(x))))*cos(x)^3 - 5*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) - 1)/(sqrt(c
os(x)*sin(x))*(cos(x) - sin(x))))*cos(x)^3 + 5*cos(x)^3*log(2*sqr
t(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 4*cos(x)*sin(x) + 1)
 - 5*cos(x)^3*log(-2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)
) + 4*cos(x)*sin(x) + 1) - 8*sqrt(2)*(6*cos(x)^2 - 1)*sqrt(cos(x)
*sin(x)))/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=61

$$-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]
]]/16 - Cos[x]^5/(5*Sin[2*x]^(5/2)) + Cos[x]/(4*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.124381, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]
]]/16 - Cos[x]^5/(5*Sin[2*x]^(5/2)) + Cos[x]/(4*Sqrt[Sin[2*x]])
```

Rubi in Sympy [A] time = 5.74053, size = 56, normalized size = 0.92

$$-\frac{\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)}{16}+\frac{\operatorname{asin}(\sin(x)-\cos(x))}{16}+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] -log(sin(x) + sqrt(sin(2*x)) + cos(x))/16 + asin(sin(x) - cos(x))
/16 + cos(x)/(4*sqrt(sin(2*x))) - cos(x)**5/(5*sin(2*x)**(5/2))
```

Mathematica [A] time = 0.102209, size = 56, normalized size = 0.92

$$\sqrt{\sin(2x)}\left(\frac{3\csc(x)}{20}-\frac{\csc^3(x)}{40}\right)+\frac{1}{16}\left(-\sin^{-1}(\cos(x)-\sin(x))-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
)/16 + ((3*Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]
```

Maple [C]   time = 0.199, size = 1108, normalized size = 18.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] 1/160*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(192*(1+t
an(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elli
pticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-1+tan(1/2*x
))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(
1/2*x)^6-96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/
2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*
x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^
2-1))^(1/2)*tan(1/2*x)^6-(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x
)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^10-384*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*E
llipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-1+tan(1/
2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*t
an(1/2*x)^4+192*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-ta
n(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(
1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan
(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^8+
48*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*tan(1/2*x)^8+96*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^8+192*(1+tan(1/
2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE
((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-1+tan(1/2*x))*(1
+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x
)^2-96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-
1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2)*tan(1/2*x)^2+14*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x))
)^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6-144*(tan
(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)^3-tan(1
/2*x))^(1/2)*tan(1/2*x)^6-192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)
*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+14*tan(1/2*x)^4*(ta
n(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1+tan(1/2*x))*(1+t
an(1/2*x)))^(1/2)+144*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))
^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4+96*tan(1/2*x)
^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^
(1/2)+3*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/
```

```
2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)*(-1+tan(
1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan
(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1+tan
(1/2*x))*(1+tan(1/2*x)))^(1/2))/(tan(1/2*x)^2-1)/(tan(1/2*x)^3-ta
n(1/2*x))^(1/2)/(-1+tan(1/2*x))/(tan(1/2*x)*(-1+tan(1/2*x))*(1+ta
n(1/2*x)))^(1/2)/(1+tan(1/2*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.296237, size = 452, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/320*(5*(cos(x)^2 - 1)*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(co
s(x) + sin(x)) + 2*cos(x)*sin(x))/(sqrt(2)*sqrt(cos(x)*sin(x))*(c
os(x) - sin(x)) + 2*cos(x)^2 - 1))*sin(x) - 5*(cos(x)^2 - 1)*arct
an(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) - 2*cos(x)*sin
(x))/(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) - 2*cos(x)^2
+ 1))*sin(x) - 5*(cos(x)^2 - 1)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(cos(x)*sin(x))*(cos(x) + sin(x)) + 1)/(sqrt(cos(x)*sin(x))*(cos(
x) - sin(x))))*sin(x) + 5*(cos(x)^2 - 1)*arctan(-1/2*sqrt(2)*(sqr
t(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) - 1)/(sqrt(cos(x)*sin(
x))*(cos(x) - sin(x))))*sin(x) + 5*(cos(x)^2 - 1)*log(2*sqrt(2)*s
qrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 4*cos(x)*sin(x) + 1)*sin(x
) - 5*(cos(x)^2 - 1)*log(-2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) +
 sin(x)) + 4*cos(x)*sin(x) + 1)*sin(x) - 8*sqrt(2)*(6*cos(x)^2 -
5)*sqrt(cos(x)*sin(x)))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

3.408 $\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

```
[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5
```

Rubi [A] time = 0.0342945, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^5*Sin[2*x]^(3/2),x]
```

```
[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5
```

Rubi in Sympy [A] time = 1.9087, size = 15, normalized size = 0.94

$$-\frac{\sin^{\frac{5}{2}}(2x)}{5\sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)**(3/2)/sin(x)**5,x)
```

```
[Out] -sin(2*x)**(5/2)/(5*sin(x)**5)
```

Mathematica [A] time = 0.0366745, size = 16, normalized size = 1.

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]
```

```
[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5
```

Maple [C] time = 0.167, size = 508, normalized size = 31.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)^(3/2)/sin(x)^5,x)
```

```
[Out] 1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(96*(1+tan(
1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipti
cE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-1+tan(1/2*x))*
(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2
*x)^2-48*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x
))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*
(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1
))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))
^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6+tan(1/2*x
)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1+tan(1/2*x
))*(1+tan(1/2*x)))^(1/2)+28*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/
2*x)))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4+40*tan(
1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^
2-1))^(1/2)+(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(ta
n(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-28*(tan(1/2*x)*(-1+
tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)
*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1
+tan(1/2*x))*(1+tan(1/2*x)))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/
2)/(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)
```

Maxima [A] time = 2.20918, size = 2002, normalized size = 125.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 4/5*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(cos(x)^2 +
sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) +
1)^(1/4)*(((((cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*
cos(6*x) - (3*cos(2*x) + 9*sin(2*x) + 4)*cos(4*x) - 3*cos(4*x)^2
+ 6*cos(2*x)^2 - (cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) +
 1)*sin(6*x) + (9*cos(2*x) - 3*sin(2*x) + 2)*sin(4*x) - 3*sin(4*x
)^2 + 6*sin(2*x)^2 + cos(2*x) - 5*sin(2*x) - 1)*cos(1/2*arctan2(s
in(x), -cos(x) + 1)) - ((cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin
(2*x) + 1)*cos(6*x) - (3*cos(2*x) - 9*sin(2*x) + 4)*cos(4*x) - 3*
```

```
cos(4*x)^2 + 6*cos(2*x)^2 + (cos(4*x) + 2*cos(2*x) + sin(4*x) + 2
*sin(2*x) + 1)*sin(6*x) - (9*cos(2*x) + 3*sin(2*x) + 2)*sin(4*x)
- 3*sin(4*x)^2 + 6*sin(2*x)^2 + cos(2*x) + 5*sin(2*x) - 1)*sin(1/
2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) +
 1)) + (((cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*cos(
6*x) - (3*cos(2*x) - 9*sin(2*x) + 4)*cos(4*x) - 3*cos(4*x)^2 + 6*
cos(2*x)^2 + (cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*
sin(6*x) - (9*cos(2*x) + 3*sin(2*x) + 2)*sin(4*x) - 3*sin(4*x)^2
+ 6*sin(2*x)^2 + cos(2*x) + 5*sin(2*x) - 1)*cos(1/2*arctan2(sin(x
), -cos(x) + 1)) + ((cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x
) + 1)*cos(6*x) - (3*cos(2*x) + 9*sin(2*x) + 4)*cos(4*x) - 3*cos(
4*x)^2 + 6*cos(2*x)^2 - (cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin
(2*x) + 1)*sin(6*x) + (9*cos(2*x) - 3*sin(2*x) + 2)*sin(4*x) - 3*
sin(4*x)^2 + 6*sin(2*x)^2 + cos(2*x) - 5*sin(2*x) - 1)*sin(1/2*ar
ctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1))
)*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((((cos(4*x) + 2*cos
(2*x) - sin(4*x) - 2*sin(2*x) + 1)*cos(6*x) - (3*cos(2*x) - 9*sin
(2*x) + 4)*cos(4*x) - 3*cos(4*x)^2 + 6*cos(2*x)^2 + (cos(4*x) + 2
*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*sin(6*x) - (9*cos(2*x) + 3
*sin(2*x) + 2)*sin(4*x) - 3*sin(4*x)^2 + 6*sin(2*x)^2 + cos(2*x)
+ 5*sin(2*x) - 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((cos(4
*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*cos(6*x) - (3*cos(2
*x) + 9*sin(2*x) + 4)*cos(4*x) - 3*cos(4*x)^2 + 6*cos(2*x)^2 - (c
os(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*sin(6*x) + (9*c
os(2*x) - 3*sin(2*x) + 2)*sin(4*x) - 3*sin(4*x)^2 + 6*sin(2*x)^2
+ cos(2*x) - 5*sin(2*x) - 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)
))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - (((cos(4*x) + 2*cos(2*x
) + sin(4*x) + 2*sin(2*x) + 1)*cos(6*x) - (3*cos(2*x) + 9*sin(2*x
) + 4)*cos(4*x) - 3*cos(4*x)^2 + 6*cos(2*x)^2 - (cos(4*x) + 2*cos
(2*x) - sin(4*x) - 2*sin(2*x) + 1)*sin(6*x) + (9*cos(2*x) - 3*sin
(2*x) + 2)*sin(4*x) - 3*sin(4*x)^2 + 6*sin(2*x)^2 + cos(2*x) - 5*
sin(2*x) - 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((cos(4*x)
+ 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*cos(6*x) - (3*cos(2*x)
- 9*sin(2*x) + 4)*cos(4*x) - 3*cos(4*x)^2 + 6*cos(2*x)^2 + (cos(4
*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*sin(6*x) - (9*cos(2
*x) + 3*sin(2*x) + 2)*sin(4*x) - 3*sin(4*x)^2 + 6*sin(2*x)^2 + co
s(2*x) + 5*sin(2*x) - 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*s
in(1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x), co
s(2*x) + 1)))/(2*(3*cos(4*x) - 3*cos(2*x) + 1)*cos(6*x) - cos(6*x
)^2 + 6*(3*cos(2*x) - 1)*cos(4*x) - 9*cos(4*x)^2 - 9*cos(2*x)^2 +
 6*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - 9*sin(4*x)^2 + 1
8*sin(4*x)*sin(2*x) - 9*sin(2*x)^2 + 6*cos(2*x) - 1)
```

Fricas [A] time = 0.225318, size = 38, normalized size = 2.38

$$\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)^2}{5\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)**(3/2)/sin(x)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="giac")
```

```
[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

```
[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

Rubi [A] time = 0.0614223, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3/Sqrt[Sin[2*x]],x]

[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

Rubi in Sympy [A] time = 3.29005, size = 29, normalized size = 0.94

$$\frac{4\sqrt{\sin(2x)}}{5\cos(x)}+\frac{\sqrt{\sin(2x)}}{5\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cos(x)**3/sin(2*x)**(1/2),x)

[Out] 4*sqrt(sin(2*x))/(5*cos(x)) + sqrt(sin(2*x))/(5*cos(x)**3)
```

Mathematica [A] time = 0.032588, size = 20, normalized size = 0.65

$$\frac{1}{5}\sqrt{\sin(2x)}\sec(x)\left(\sec^2(x)+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

```
[Out] (Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5
```

Maple [C] time = 0.153, size = 286, normalized size = 9.2

$$\frac{1}{12}\sqrt{-1\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(5\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}\mathit{EllipticF}\left(\sqrt{1+\mathrm{ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cos(x)^3/sin(2*x)^(1/2),x)
```

```
[Out] 1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(1+
tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ell
ipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+15*(1+tan(1
/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elliptic
F((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^4-14*tan(1/2*x)^7+
15*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/
2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+2*tan
(1/2*x)^5+5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/
2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-2*tan(1/2
*x)^3+14*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2
*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)^2+1)^3
```

Maxima [A] time = 3.19371, size = 2726, normalized size = 87.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^3*sqrt(sin(2*x))),x, algorithm="maxima")
```

```
[Out] -4/5*((((((cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*cos(
6*x) + (2*cos(2*x) + 6*sin(2*x) + 1)*cos(4*x) + 2*cos(4*x)^2 - 4*
cos(2*x)^2 - (cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*
sin(6*x) - (6*cos(2*x) - 2*sin(2*x) + 3)*sin(4*x) + 2*sin(4*x)^2
- 4*sin(2*x)^2 - 4*cos(2*x) - 1)*cos(1/2*arctan2(sin(x), -cos(x)
+ 1)) - ((cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*cos(
6*x) + (2*cos(2*x) - 6*sin(2*x) + 1)*cos(4*x) + 2*cos(4*x)^2 - 4*
cos(2*x)^2 + (cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*
sin(6*x) + (6*cos(2*x) + 2*sin(2*x) + 3)*sin(4*x) + 2*sin(4*x)^2
- 4*sin(2*x)^2 - 4*cos(2*x) - 1)*sin(1/2*arctan2(sin(x), -cos(x)
+ 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) + (((cos(4*x) + 2*cos
(2*x) - sin(4*x) - 2*sin(2*x) + 1)*cos(6*x) + (2*cos(2*x) - 6*sin
(2*x) + 1)*cos(4*x) + 2*cos(4*x)^2 - 4*cos(2*x)^2 + (cos(4*x) + 2
*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*sin(6*x) + (6*cos(2*x) + 2
```

```
*sin(2*x) + 3)*sin(4*x) + 2*sin(4*x)^2 - 4*sin(2*x)^2 - 4*cos(2*x
) - 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((cos(4*x) + 2*cos
(2*x) + sin(4*x) + 2*sin(2*x) + 1)*cos(6*x) + (2*cos(2*x) + 6*sin
(2*x) + 1)*cos(4*x) + 2*cos(4*x)^2 - 4*cos(2*x)^2 - (cos(4*x) + 2
*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*sin(6*x) - (6*cos(2*x) - 2
*sin(2*x) + 3)*sin(4*x) + 2*sin(4*x)^2 - 4*sin(2*x)^2 - 4*cos(2*x
) - 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin
(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1)) + (((
(cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*cos(6*x) + (2
*cos(2*x) - 6*sin(2*x) + 1)*cos(4*x) + 2*cos(4*x)^2 - 4*cos(2*x)^
2 + (cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*sin(6*x)
 + (6*cos(2*x) + 2*sin(2*x) + 3)*sin(4*x) + 2*sin(4*x)^2 - 4*sin(2
*x)^2 - 4*cos(2*x) - 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + (
(cos(4*x) + 2*cos(2*x) + sin(4*x) + 2*sin(2*x) + 1)*cos(6*x) + (2
*cos(2*x) + 6*sin(2*x) + 1)*cos(4*x) + 2*cos(4*x)^2 - 4*cos(2*x)^
2 - (cos(4*x) + 2*cos(2*x) - sin(4*x) - 2*sin(2*x) + 1)*sin(6*x)
 - (6*cos(2*x) - 2*sin(2*x) + 3)*sin(4*x) + 2*sin(4*x)^2 - 4*sin(2
*x)^2 - 4*cos(2*x) - 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*co
s(1/2*arctan2(sin(x), cos(x) + 1)) - (((cos(4*x) + 2*cos(2*x) + s
in(4*x) + 2*sin(2*x) + 1)*cos(6*x) + (2*cos(2*x) + 6*sin(2*x) + 1
)*cos(4*x) + 2*cos(4*x)^2 - 4*cos(2*x)^2 - (cos(4*x) + 2*cos(2*x)
 - sin(4*x) - 2*sin(2*x) + 1)*sin(6*x) - (6*cos(2*x) - 2*sin(2*x)
 + 3)*sin(4*x) + 2*sin(4*x)^2 - 4*sin(2*x)^2 - 4*cos(2*x) - 1)*co
s(1/2*arctan2(sin(x), -cos(x) + 1)) - ((cos(4*x) + 2*cos(2*x) - s
in(4*x) - 2*sin(2*x) + 1)*cos(6*x) + (2*cos(2*x) - 6*sin(2*x) + 1
)*cos(4*x) + 2*cos(4*x)^2 - 4*cos(2*x)^2 + (cos(4*x) + 2*cos(2*x)
 + sin(4*x) + 2*sin(2*x) + 1)*sin(6*x) + (6*cos(2*x) + 2*sin(2*x)
 + 3)*sin(4*x) + 2*sin(4*x)^2 - 4*sin(2*x)^2 - 4*cos(2*x) - 1)*si
n(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos(
x) + 1)))*sin(1/2*arctan2(sin(2*x), cos(2*x) + 1)))/(((((2*(2*cos
(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*
sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*cos(1/2*arctan
2(sin(x), -cos(x) + 1))^2 + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*
x)^2 + 4*cos(2*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*c
os(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2*cos(2*x) + 1)*cos(
4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*
x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*cos(1/2*arctan2(sin(x), -cos(
x) + 1))^2 + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*
x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*
x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*sin(1/2*arctan2(
sin(x), cos(x) + 1))^2)*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1))^
2 + (((2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 +
sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (2*(2*cos(2*x) + 1)*co
s(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(
2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*sin(1/2*arctan2(sin(x), -co
s(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2*cos
(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*
sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*cos(1/2*arctan
2(sin(x), -cos(x) + 1))^2 + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*
x)^2 + 4*cos(2*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*s
in(1/2*arctan2(sin(x), cos(x) + 1))^2)*sin(1/2*arctan2(sin(2*x),
cos(2*x) + 1))^2)*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4
)*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2
```

```
- 2*cos(x) + 1)^(1/4))
```

Fricas [A] time = 0.225469, size = 32, normalized size = 1.03

$$\frac{\sqrt{2}\left(4\,\cos\left(x\right)^{2}+1\right)\sqrt{\cos\left(x\right)\sin\left(x\right)}}{5\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^3*sqrt(sin(2*x))),x, algorithm="fricas")

[Out] 1/5*sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x))/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cos(x)**3/sin(2*x)**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos\left(x\right)^{3}\sqrt{\sin\left(2\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)^3*sqrt(sin(2*x))),x, algorithm="giac")

[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$

Optimal. Leaf size=29

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

```
[Out] (-2*Cos[x])/(3*Sin[2*x]^(3/2)) + (4*Sin[x])/(3*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.0694859, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-2*Cos[x])/(3*Sin[2*x]^(3/2)) + (4*Sin[x])/(3*Sqrt[Sin[2*x]])
```

Rubi in Sympy [A] time = 3.44304, size = 29, normalized size = 1.

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

```
[Out] 4*sin(x)/(3*sqrt(sin(2*x))) - 2*cos(x)/(3*sin(2*x)**(3/2))
```

Mathematica [A] time = 0.0385378, size = 24, normalized size = 0.83

$$\sqrt{\sin(2x)}\left(\frac{\sec(x)}{2} - \frac{1}{6}\cot(x)\csc(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-(Cot[x]*Csc[x])/6 + Sec[x]/2)*Sqrt[Sin[2*x]]
```

Maple [C] time = 0.098, size = 121, normalized size = 4.2

$$-\frac{1}{12}\sqrt{-1\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}-1\right)\left(2\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}EllipticF\left(\sqrt{1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sin(x)/sin(2*x)^(3/2),x)
```

```
[Out] -1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1
/2*x)*(2*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x
))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)-t
an(1/2*x)^4+1)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-
tan(1/2*x))^(1/2)
```

Maxima [A] time = 2.03308, size = 1446, normalized size = 49.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(2*x)^(3/2)*sin(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(((((cos(2*x) + sin(2*x) - 1)*cos(4*x) - cos(2*x)^2 - (cos(2
*x) - sin(2*x) - 1)*sin(4*x) - sin(2*x)^2 + 2*cos(2*x) - 1)*cos(1
/2*arctan2(sin(x), -cos(x) + 1)) - ((cos(2*x) - sin(2*x) - 1)*cos
(4*x) - cos(2*x)^2 + (cos(2*x) + sin(2*x) - 1)*sin(4*x) - sin(2*x
)^2 + 2*cos(2*x) - 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(
1/2*arctan2(sin(x), cos(x) + 1)) + (((cos(2*x) - sin(2*x) - 1)*co
s(4*x) - cos(2*x)^2 + (cos(2*x) + sin(2*x) - 1)*sin(4*x) - sin(2*
x)^2 + 2*cos(2*x) - 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((
cos(2*x) + sin(2*x) - 1)*cos(4*x) - cos(2*x)^2 - (cos(2*x) - sin(
2*x) - 1)*sin(4*x) - sin(2*x)^2 + 2*cos(2*x) - 1)*sin(1/2*arctan2
(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*cos
(1/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((((cos(2*x) - sin(2*x) -
 1)*cos(4*x) - cos(2*x)^2 + (cos(2*x) + sin(2*x) - 1)*sin(4*x) -
sin(2*x)^2 + 2*cos(2*x) - 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)
) + ((cos(2*x) + sin(2*x) - 1)*cos(4*x) - cos(2*x)^2 - (cos(2*x)
- sin(2*x) - 1)*sin(4*x) - sin(2*x)^2 + 2*cos(2*x) - 1)*sin(1/2*a
rctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)
) - (((cos(2*x) + sin(2*x) - 1)*cos(4*x) - cos(2*x)^2 - (cos(2*x)
 - sin(2*x) - 1)*sin(4*x) - sin(2*x)^2 + 2*cos(2*x) - 1)*cos(1/2*
arctan2(sin(x), -cos(x) + 1)) - ((cos(2*x) - sin(2*x) - 1)*cos(4*
```

```
x) - cos(2*x)^2 + (cos(2*x) + sin(2*x) - 1)*sin(4*x) - sin(2*x)^2
 + 2*cos(2*x) - 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2
*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x), cos(2*x)
 + 1)))/(((((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*cos(1/2*ar
ctan2(sin(x), -cos(x) + 1))^2 + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(
2*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*cos(1/2*arctan
2(sin(x), cos(x) + 1))^2 + ((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x)
 + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (cos(2*x)^2 + sin
(2*x)^2 - 2*cos(2*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2
)*sin(1/2*arctan2(sin(x), cos(x) + 1))^2)*cos(1/2*arctan2(sin(2*x
), cos(2*x) + 1))^2 + (((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1
)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (cos(2*x)^2 + sin(2*x
)^2 - 2*cos(2*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*co
s(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((cos(2*x)^2 + sin(2*x)^2
- 2*cos(2*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (cos(
2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*sin(1/2*arctan2(sin(x), -co
s(x) + 1))^2)*sin(1/2*arctan2(sin(x), cos(x) + 1))^2)*sin(1/2*arc
tan2(sin(2*x), cos(2*x) + 1))^2)*(cos(2*x)^2 + sin(2*x)^2 + 2*cos
(2*x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

Fricas [A] time = 0.223991, size = 42, normalized size = 1.45

$$\frac{\sqrt{2}\left(4 \cos\left(x\right)^{2} - 3\right) \sqrt{\cos\left(x\right) \sin\left(x\right)}}{6 \left(\cos\left(x\right)^{3} - \cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(2*x)^(3/2)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*(4*cos(x)^2 - 3)*sqrt(cos(x)*sin(x))/(cos(x)^3 - cos(
x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin\left(2\,x\right)^{\frac{3}{2}}\,\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(2*x)^(3/2)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=68

$$-\frac{9\cos(x)}{16\sqrt{\sin(2x)}}+\frac{\cos(x)\cot^2(x)}{20\sqrt{\sin(2x)}}-\frac{5\cos(x)\cot(x)}{24\sqrt{\sin(2x)}}+\frac{33}{32}\tanh^{-1}\left(\frac{1}{2}\sqrt{\sin(2x)}\sec(x)\right)$$

```
[Out] (33*ArcTanh[(Sec[x]*Sqrt[Sin[2*x]])/2])/32 - (9*Cos[x])/(16*Sqrt[
Sin[2*x]]) - (5*Cos[x]*Cot[x])/(24*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[
x]^2)/(20*Sqrt[Sin[2*x]])
```

Rubi [A] time = 1.40299, antiderivative size = 95, normalized size of antiderivative = 1.4, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{33\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) - (5*Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/
2)) - (9*Cos[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (33*ArcTanh[Sqrt
[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5
```

```
[Out] Timed out
```

Mathematica [C] time = 7.45175, size = 150, normalized size = 2.21

$$\frac{\sqrt{\sin(2x)}\cos(x)(\cos(2x)-3\tan(x))\left(\frac{1}{15}\csc(x)\left(-50\cot(x)+12\csc^2(x)-147\right)-33\sqrt{\frac{\cos(x)}{2\cos(x)-2}}\sqrt{\tan\left(\frac{x}{2}\right)}\sec(x)\left(F\left(\sin^{-1}\left(\cdots\right.\right.\right.\right.}{16(-6\sin(x)+\cos(x)+\cos(3\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2
```

```
[Out] (Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))
/15 - 33*Sqrt[Cos[x]/(-2 + 2*Cos[x])]*(EllipticF[ArcSin[1/Sqrt[Ta
n[x/2]]], -1] + EllipticPi[-2/(-1 + Sqrt[5]), -ArcSin[1/Sqrt[Tan[
x/2]]], -1] + EllipticPi[(-1 + Sqrt[5])/2, -ArcSin[1/Sqrt[Tan[x/2
]]], -1])*Sec[x]*Sqrt[Tan[x/2]])*(Cos[2*x] - 3*Tan[x]))/(16*(Cos[
x] + Cos[3*x] - 6*Sin[x]))
```

Maple [C] time = 0.401, size = 761, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

```
[Out] -1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(3024*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*E
llipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(-1+tan(1/
2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*t
an(1/2*x)^2-932*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-ta
n(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(
1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*tan(1/2*x)^2-24*(tan(1/2*x)*(-1+tan(1/2*x))*(1+ta
n(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6
-3*sum((34*_alpha^3+13*_alpha^2+34*_alpha-21)*(_alpha^3+2*_alpha-
3)*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)
/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((1+tan(1/2*x))^(1
/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+
_Z^3+2*_Z^2-_Z+1))*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1
/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*2^(1/2)*(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*tan(1/2*x)^2-200*(tan(1/2*x)*(-1+tan(1/2*x))*(1+ta
n(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^5
+552*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x
)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4+24*tan(1/2*x)^4*(tan(1/2*x)*(t
an(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))
^(1/2)+1920*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)-552*(tan(1/2*x)*(-1+tan(1/2*x))*(1+ta
n(1/2*x)))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2+24*
(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(ta
n(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2+200*tan(1/2*x)*(tan(1/2*x)*(tan
(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(
1/2)-24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(-1+tan(1
/2*x))*(1+tan(1/2*x)))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(ta
n(1/2*x)*(-1+tan(1/2*x))*(1+tan(1/2*x)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2))
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.257312, size = 173, normalized size = 2.54

$$\frac{495\left(\cos(x)^2-1\right)\log\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x)+\sin(x))+\frac{3}{4}\cos(x)^2+3\cos(x)\sin(x)+\frac{1}{4}\right)\sin(x)-495\left(\cos(x)^2-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2))
```

```
[Out] 1/3840*(495*(cos(x)^2 - 1)*log(sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos
(x) + sin(x)) + 3/4*cos(x)^2 + 3*cos(x)*sin(x) + 1/4)*sin(x) - 49
5*(cos(x)^2 - 1)*log(-sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + sin
(x)) + 3/4*cos(x)^2 + 3*cos(x)*sin(x) + 1/4)*sin(x) - 8*sqrt(2)*(
147*cos(x)^2 - 50*cos(x)*sin(x) - 135)*sqrt(cos(x)*sin(x)))/((cos
(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(2x)-3\tan(x))\cos(x)^3}{\left(\sin(x)^2-\sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2))

[Out] integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*s
in(2*x)^(5/2)), x)
```

3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}\sin(x)\cos(x)\sqrt{\tan(x)\sec^4(x)}$$

```
[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

Rubi [A] time = 0.22063, antiderivative size = 29, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{2\tan^2(x)\sec^2(x)}{3\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Sec[x]^4*Tan[x]],x]

[Out] (2*Sec[x]^2*Tan[x]^2)/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])
```

Rubi in Sympy [A] time = 1.65225, size = 20, normalized size = 1.05

$$\frac{2\sqrt{\frac{\sin(x)}{\cos^5(x)}}\sin(x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((sin(x)/cos(x)**5)**(1/2),x)

[Out] 2*sqrt(sin(x)/cos(x)**5)*sin(x)*cos(x)/3
```

Mathematica [A] time = 0.0127513, size = 19, normalized size = 1.

$$\frac{2}{3}\sin(x)\cos(x)\sqrt{\tan(x)\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Sec[x]^4*Tan[x]],x]
```

```
[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

Maple [A] time = 0.201, size = 16, normalized size = 0.8

$$\frac{2\,\cos\left(x\right)\sin\left(x\right)}{3}\sqrt{\frac{\sin\left(x\right)}{\left(\cos\left(x\right)\right)^{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((sin(x)/cos(x)^5)^(1/2),x)
```

```
[Out] 2/3*(sin(x)/cos(x)^5)^(1/2)*sin(x)*cos(x)
```

Maxima [A] time = 1.5454, size = 8, normalized size = 0.42

$$\frac{2}{3}\,\tan\left(x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x)/cos(x)^5),x, algorithm="maxima")
```

```
[Out] 2/3*tan(x)^(3/2)
```

Fricas [A] time = 0.221691, size = 20, normalized size = 1.05

$$\frac{2}{3}\sqrt{\frac{\sin\left(x\right)}{\cos\left(x\right)^{5}}}\cos\left(x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x)/cos(x)^5),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)/cos(x)**5)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sin(x)}{\cos(x)^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x)/cos(x)^5),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)/cos(x)^5), x)
```

3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}+\frac{3\tan^{-1}\left(\frac{(1-\cot(x))\csc^2(x)\sqrt{\sin^4(x)\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{3\log\left(\sin(x)+\cos(x)-\sqrt{2}\cot(x)\csc(x)\sqrt{\sin^4(x)\tan(x)}\right)}{4\sqrt{2}}$$

```
[Out] (3*ArcTan[((1 - Cot[x])*Csc[x]^2*Sqrt[Sin[x]^4*Tan[x]])/Sqrt[2]])
/(4*Sqrt[2]) + (3*Log[Cos[x] + Sin[x] - Sqrt[2]*Cot[x]*Csc[x]*Sqr
t[Sin[x]^4*Tan[x]]])/(4*Sqrt[2]) - (Cot[x]*Sqrt[Sin[x]^4*Tan[x]])
/2
```

Rubi [B] time = 0.421635, antiderivative size = 204, normalized size of antiderivative = 2.22, number of steps used = 13, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818

$$-\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}+\frac{3\sec^2(x)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)}-\frac{3\sec^2(x)\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -(Cot[x]*Sqrt[Sin[x]^4*Tan[x]])/2 - (3*ArcTan[1 - Sqrt[2]*Sqrt[Ta
n[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) +
 (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x
]])/(4*Sqrt[2]*Tan[x]^(5/2)) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[x]] +
Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2))
- (3*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^
4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2))
```

Rubi in Sympy [A] time = 9.69505, size = 257, normalized size = 2.79

$$\frac{3\sqrt{2}\sqrt{\frac{\sin^5(x)}{\cos(x)}}\log\left(-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\frac{\sin(x)}{\cos(x)}+1\right)\sqrt{\cos(x)}}{16\sin^{\frac{5}{2}}(x)} - \frac{3\sqrt{2}\sqrt{\frac{\sin^5(x)}{\cos(x)}}\log\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\frac{\sin(x)}{\cos(x)}+1\right)\sqrt{\cos(x)}}{16\sin^{\frac{5}{2}}(x)} - \frac{\sqrt{\frac{\sin^5(x)}{\cos(x)}}\cos(x)}{2\sin(x)} + \frac{3\sqrt{2}\sqrt{\frac{\sin^5(x)}{\cos(x)}}\sqrt{\cos(x)}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}-1\right)}{8\sin^{\frac{5}{2}}(x)} + \frac{3\sqrt{2}\sqrt{\frac{\sin^5(x)}{\cos(x)}}\sqrt{\cos(x)}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sin^{\frac{5}{2}}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((sin(x)**5/cos(x))**(1/2),x)
```

```
[Out] 3*sqrt(2)*sqrt(sin(x)**5/cos(x))*log(-sqrt(2)*sqrt(sin(x))/sqrt(c
os(x)) + sin(x)/cos(x) + 1)*sqrt(cos(x))/(16*sin(x)**(5/2)) - 3*s
qrt(2)*sqrt(sin(x)**5/cos(x))*log(sqrt(2)*sqrt(sin(x))/sqrt(cos(x
)) + sin(x)/cos(x) + 1)*sqrt(cos(x))/(16*sin(x)**(5/2)) - sqrt(si
n(x)**5/cos(x))*cos(x)/(2*sin(x)) + 3*sqrt(2)*sqrt(sin(x)**5/cos(
x))*sqrt(cos(x))*atan(sqrt(2)*sqrt(sin(x))/sqrt(cos(x)) - 1)/(8*s
in(x)**(5/2)) + 3*sqrt(2)*sqrt(sin(x)**5/cos(x))*sqrt(cos(x))*ata
n(sqrt(2)*sqrt(sin(x))/sqrt(cos(x)) + 1)/(8*sin(x)**(5/2))
```

Mathematica [A] time = 0.108275, size = 66, normalized size = 0.72

$$-\frac{1}{8}\sqrt{\sin(2x)}\csc^3(x)\sqrt{\sin^4(x)\tan(x)}\left(2\sin(x)\sqrt{\sin(2x)} + 3\sin^{-1}(\cos(x)-\sin(x)) + 3\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + S
qrt[Sin[2*x]]] + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin
[x]^4*Tan[x]])/8
```

Maple [C] time = 0.207, size = 310, normalized size = 3.4

$$-\frac{\sqrt{32}(\cos(x)-1)(1+\cos(x))^2}{32(\sin(x))^5}\left(3i\sqrt{\frac{\cos(x)-1}{\sin(x)}}\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}}\sqrt{\frac{\sin(x)-1+\cos(x)}{\sin(x)}}\mathit{EllipticPi}\left(\sqrt{\frac{1-\cos(x)+\mathrm{si}}{\sin(x)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((sin(x)^5/cos(x))^(1/2),x)
```

```
[Out] -1/32*32^(1/2)*(cos(x)-1)*(3*I*((cos(x)-1)/sin(x))^(1/2)*((1-cos(
x)+sin(x))/sin(x))^(1/2)*((sin(x)-1+cos(x))/sin(x))^(1/2)*Ellipti
cPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*(
(cos(x)-1)/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin(x))^(1/2)*((sin(x
)-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(
1/2),1/2+1/2*I,1/2*2^(1/2))-3*((cos(x)-1)/sin(x))^(1/2)*((1-cos(x
)+sin(x))/sin(x))^(1/2)*((sin(x)-1+cos(x))/sin(x))^(1/2)*Elliptic
Pi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((co
s(x)-1)/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin(x))^(1/2)*((sin(x)-1
+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+2*cos(x)^2*2^(1/2)-2*cos(x)*2^(1/2))*(1+
cos(x))^2*(sin(x)^5/cos(x))^(1/2)/sin(x)^5
```

Maxima [A] time = 1.59812, size = 127, normalized size = 1.38

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)$$
$$-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)-\frac{\tan(x)^{\frac{3}{2}}}{2\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x)^5/cos(x)),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 3/8*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 3/16*sq
rt(2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) + 3/16*sqrt(2)*log(-
sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/2*tan(x)^(3/2)/(tan(x)^2 +
 1)
```

Fricas [A] time = 0.739585, size = 1338, normalized size = 14.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x)^5/cos(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(16*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))*cos(x)*s
in(x) + 6*(sqrt(2)*cos(x)^2 - sqrt(2))*arctan(-2*(cos(x)^2 - cos(
x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))/(sqrt(
```

```
2)*cos(x)^2 + sqrt(2)*(cos(x)^2 - 1)*sqrt((cos(x)^2 + 4*(cos(x)^3
 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*
sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 -
1)) + 2*(cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 +
1)*sin(x)/cos(x)) - sqrt(2))) + 6*(sqrt(2)*cos(x)^2 - sqrt(2))*ar
ctan(-2*(cos(x)^2 - cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 +
1)*sin(x)/cos(x))/(sqrt(2)*cos(x)^2 + sqrt(2)*(cos(x)^2 - 1)*sqrt
((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) - 2*(sqrt(2)*cos(x)^2 +
 sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/c
os(x)) - 1)/(cos(x)^2 - 1)) - 2*(cos(x)^2 + cos(x)*sin(x))*sqrt((
cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - sqrt(2))) - 6*(sqrt(2
)*cos(x)^2 - sqrt(2))*arctan(-(2*sqrt(2)*cos(x)^4 - 3*sqrt(2)*cos
(x)^2 + 2*(cos(x)^2 - cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2
+ 1)*sin(x)/cos(x)) + sqrt(2))/(sqrt(2)*(cos(x)^2 - 1)*sqrt((cos(
x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + sqrt(
2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))
 - 1)/(cos(x)^2 - 1)) + 2*(sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sin
(x) + 2*(cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 +
1)*sin(x)/cos(x)))) - 6*(sqrt(2)*cos(x)^2 - sqrt(2))*arctan((2*sq
rt(2)*cos(x)^4 - 3*sqrt(2)*cos(x)^2 - 2*(cos(x)^2 - cos(x)*sin(x)
)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + sqrt(2))/(sqr
t(2)*(cos(x)^2 - 1)*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x)
 - 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 -
2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)) + 2*(sqrt(2)*
cos(x)^3 - sqrt(2)*cos(x))*sin(x) - 2*(cos(x)^2 + cos(x)*sin(x))*
sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)))) + 3*(sqrt(2)*co
s(x)^2 - sqrt(2))*log(2*(cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x)
+ 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2
*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)) - 3*(sqrt(2)*c
os(x)^2 - sqrt(2))*log(2*(cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x)
 - 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 -
2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 -
1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sin^5(x)}{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)**5/cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)**5/cos(x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sin(x)^5/cos(x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$

Optimal. Leaf size=47

$$\frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}+\frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

```
[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[
x]^3*(Sec[x]^12*Tan[x]^2)^(1/3))/11
```

Rubi [A] time = 0.234184, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}+\frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[
x]^3*(Sec[x]^12*Tan[x]^2)^(1/3))/11
```

Rubi in Sympy [A] time = 1.75774, size = 44, normalized size = 0.94

$$\frac{3\sqrt[3]{\frac{\sin^2(x)}{\cos^{14}(x)}}\sin(x)\cos^5(x)\,{}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{17}{6}\\\frac{11}{6}\end{matrix}\middle|\sin^2(x)\right)}{5\sqrt[6]{\cos^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((sin(x)**2/cos(x)**14)**(1/3),x)
```

```
[Out] 3*(sin(x)**2/cos(x)**14)**(1/3)*sin(x)*cos(x)**5*hyper((5/6, 17/6
), (11/6,), sin(x)**2)/(5*(cos(x)**2)**(1/6))
```

Mathematica [A] time = 0.0346558, size = 29, normalized size = 0.62

$$\frac{3}{55}\sin(x)\cos(x)\left(6\cos^2(x)+5\right)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]*(5 + 6*Cos[x]^2)*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/55
```

Maple [F] time = 0.542, size = 0, normalized size = 0.

$$\int \sqrt[3]{\frac{(\sin(x))^2}{(\cos(x))^{14}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((sin(x)^2/cos(x)^14)^(1/3),x)
```

```
[Out] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

Maxima [A] time = 1.8147, size = 18, normalized size = 0.38

$$\frac{3}{11}\,\tan(x)^{\frac{11}{3}}+\frac{3}{5}\,\tan(x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)
```

Fricas [A] time = 0.21367, size = 39, normalized size = 0.83

$$\frac{3}{55}\left(6\,\cos(x)^3+5\,\cos(x)\right)\left(-\frac{\cos(x)^2-1}{\cos(x)^{14}}\right)^{\frac{1}{3}}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*si
n(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)**2/cos(x)**14)**(1/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{\sin(x)^2}{\cos(x)^{14}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((sin(x)^2/cos(x)^14)^(1/3), x)
```

3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$

Optimal. Leaf size=70

$$\frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

```
[Out] (-4*Cos[x]^5*Sin[x])/(9*(Cos[x]^11*Sin[x]^13)^(1/4)) - (8*Cos[x]^
3*Sin[x]^3)/(Cos[x]^11*Sin[x]^13)^(1/4) + (4*Cos[x]*Sin[x]^5)/(7*
(Cos[x]^11*Sin[x]^13)^(1/4))
```

Rubi [A] time = 0.317178, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-4*Cos[x]^5*Sin[x])/(9*(Cos[x]^11*Sin[x]^13)^(1/4)) - (8*Cos[x]^
3*Sin[x]^3)/(Cos[x]^11*Sin[x]^13)^(1/4) + (4*Cos[x]*Sin[x]^5)/(7*
(Cos[x]^11*Sin[x]^13)^(1/4))
```

Rubi in Sympy [A] time = 1.7978, size = 51, normalized size = 0.73

$$-\frac{4\left(\sin^{13}(x)\cos^{11}(x)\right)^{\frac{3}{4}}{}_2F_1\left(\begin{matrix}-\frac{9}{8},\frac{15}{8}\\-\frac{1}{8}\end{matrix}\middle|\sin^2(x)\right)}{9\sqrt[8]{\cos^2(x)}\sin^{12}(x)\cos^8(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

```
[Out] -4*(sin(x)**13*cos(x)**11)**(3/4)*hyper((-9/8, 15/8), (-1/8,), si
n(x)**2)/(9*(cos(x)**2)**(1/8)*sin(x)**12*cos(x)**8)
```

Mathematica [A] time = 0.0872389, size = 35, normalized size = 0.5

$$-\frac{2\sin(2x)(8\cos(2x)-16\cos(4x)+15)}{63\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-2*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[2*x])/(63*(Cos[x]^11*Sin[
x]^13)^(1/4))
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{(\cos(x))^{11}(\sin(x))^{13}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

```
[Out] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

Maxima [A] time = 1.601, size = 104, normalized size = 1.49

$$\frac{4}{23}\tan(x)^{\frac{23}{4}}+\frac{8}{15}\tan(x)^{\frac{15}{4}}+\frac{4}{7}\tan(x)^{\frac{7}{4}}-\frac{4\left(35\tan(x)^{7}+161\tan(x)^{5}+345\tan(x)^{3}-805\tan(x)\right)}{805\tan(x)^{\frac{5}{4}}}+\frac{4\left(21\tan(x)^{7}+135\tan(x)^{5}-945\tan(x)^{3}-35\tan(x)\right)}{315\tan(x)^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x)^11*sin(x)^13)^(-1/4),x, algorithm="maxima")
```

```
[Out] 4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/80
5*(35*tan(x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)
^(5/4) + 4/315*(21*tan(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*ta
n(x))/tan(x)^(13/4)
```

Fricas [A] time = 0.221892, size = 136, normalized size = 1.94

$$\frac{4\left(128\cos\left(x\right)^{4}-144\cos\left(x\right)^{2}+9\right)\left(\left(\cos\left(x\right)^{23}-6\cos\left(x\right)^{21}+15\cos\left(x\right)^{19}-20\cos\left(x\right)^{17}+15\cos\left(x\right)^{15}-6\cos\left(x\right)^{13}+\cos\left(\right.\right.\right.}{63\left(\cos\left(x\right)^{22}-6\cos\left(x\right)^{20}+15\cos\left(x\right)^{18}-20\cos\left(x\right)^{16}+15\cos\left(x\right)^{14}-6\cos\left(x\right)^{12}+\cos\left(x\right)^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x)^11*sin(x)^13)^(-1/4),x, algorithm="fricas")
```

```
[Out] 4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21
+ 15*cos(x)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(
x)^11)*sin(x))^(3/4)/(cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20
*cos(x)^16 + 15*cos(x)^14 - 6*cos(x)^12 + cos(x)^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\cos\left(x\right)^{11}\sin\left(x\right)^{13}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x)^11*sin(x)^13)^(-1/4),x, algorithm="giac")
```

```
[Out] integrate((cos(x)^11*sin(x)^13)^(-1/4), x)
```

3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{\sin(2x)}\cos(x)\sin^{-1}(\cos(x)-\sin(x))}{\sqrt{\sin(x)\cos^3(x)}}-\frac{\sin(2x)}{\sqrt{\sin(x)\cos^3(x)}}-\frac{\sqrt{\sin(2x)}\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{\sin(x)\cos^3(x)}}-\sqrt{2}\log\left(\sin(x)+\cos(x)-\sqrt{2}\sec(x)\sqrt{\sin(x)\cos^3(x)}\right)$$

```
[Out] -(Sqrt[2]*Log[Cos[x] + Sin[x] - Sqrt[2]*Sec[x]*Sqrt[Cos[x]^3*Sin[
x]]]) - (ArcSin[Cos[x] - Sin[x]]*Cos[x]*Sqrt[Sin[2*x]])/Sqrt[Cos[
x]^3*Sin[x]] - (ArcTanh[Sin[x]]*Cos[x]*Sqrt[Sin[2*x]])/Sqrt[Cos[x
]^3*Sin[x]] - Sin[2*x]/Sqrt[Cos[x]^3*Sin[x]]
```

Rubi [B] time = 2.76881, antiderivative size = 234, normalized size of antiderivative = 2.17, number of steps used = 27, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407

$$-2\sec^2(x)\sqrt{\sin(x)\cos^3(x)}-\frac{\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}}+\frac{\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sec^2(x)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}}-\frac{\sec^2(x)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{2}\sqrt{\tan(x)}}+\frac{\sec^2(x)\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{2}\sqrt{\tan(x)}}-\sqrt{2}\cot(x)\sec^2(x)^{3/2}\sqrt{\sin(x)\cos(x)}\sqrt{\sin(x)\cos^3(x)}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] -2*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]] - Sqrt[2]*ArcSinh[Tan[x]]*Cot[x
]*(Sec[x]^2)^(3/2)*Sqrt[Cos[x]*Sin[x]]*Sqrt[Cos[x]^3*Sin[x]] - (S
qrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Si
n[x]])/Sqrt[Tan[x]] + (Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*S
ec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Log[1 - Sqrt[2]*Sq
rt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(Sqrt[2]*Sqr
t[Tan[x]]) + (Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqr
t[Cos[x]^3*Sin[x]])/(Sqrt[2]*Sqrt[Tan[x]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.363698, size = 105, normalized size = 0.97

$$\frac{-4\sin(x)\cos^3(x)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\cos^2(x)\right)-3\sqrt[4]{\sin^2(x)}\cos(x)\left(2\sin(x)+\sqrt{\sin(2x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)\right)}{3\sqrt[4]{\sin^2(x)}\sqrt{\sin(x)\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] (-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2]*Sin[x] -
3*Cos[x]*(Sin[x]^2)^(1/4)*(2*Sin[x] + (-Log[Cos[x/2] - Sin[x/2]]
+ Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[x]^3*Sin
[x]]*(Sin[x]^2)^(1/4))
```

Maple [C] time = 0.488, size = 244, normalized size = 2.3

$$-2\,\frac{\cos(x)\sin(x)}{\sqrt{(\cos(x))^3\sin(x)}}+2\,\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)}{\sqrt{(\cos(x))^3\sin(x)}}Artanh\left(\frac{\cos(x)-1}{\sin(x)}\right)$$
$$+\frac{\cos(x)\sqrt{2}(\sin(x))^2}{\cos(x)-1}\left(-iEllipticPi\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+iEllipticPi\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
```

```
[Out] -2*sin(x)*cos(x)/(cos(x)^3*sin(x))^(1/2)+2*2^(1/2)*(cos(x)*sin(x)
)^(1/2)*cos(x)*arctanh((cos(x)-1)/sin(x))/(cos(x)^3*sin(x))^(1/2)
+2^(1/2)*(-I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*
I,1/2*2^(1/2))+I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+
1/2*I,1/2*2^(1/2))+2*EllipticF(((1-cos(x)+sin(x))/sin(x))^(1/2),1
/2*2^(1/2))-EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I
```

```
,1/2*2^(1/2))-EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2
*I,1/2*2^(1/2)))*cos(x)*((sin(x)-1+cos(x))/sin(x))^(1/2)*((cos(x)
-1)/sin(x))^(1/2)*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)/(cos(
x)-1)/(cos(x)^3*sin(x))^(1/2)
```

Maxima [A] time = 1.75785, size = 258, normalized size = 2.39

$$\frac{3}{5}\tan(x)^{\frac{5}{2}} + \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{\tan(x)}\right)\right) + \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{\tan(x)}\right)\right) - \frac{1}{2}\sqrt{2}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{2}\sqrt{2}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right) + \frac{1}{2}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)} + \tan(x) + 1\right) - \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)} + \tan(x) + 1\right) - \frac{\left(\tan(x)^3 + \tan(x)\right)\tan(x)^{\frac{3}{2}}}{2\left(\tan(x)^2 + 1\right)} - 4\sqrt{\tan(x)} - \frac{\left(\tan(x)^3 - 15\tan(x)\right)\tan(x)^{\frac{3}{2}} - \frac{4\left(\tan(x)^3 + 5\tan(x)\right)}{\sqrt{\tan(x)}}}{10\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)),x, algorithm="maxi
```

```
[Out] 3/5*tan(x)^(5/2) + sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(t
an(x)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))
) - 1/2*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/2*sqr
t(2)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 1/2*sqrt(2)*log(sq
rt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/2*sqrt(2)*log(-sqrt(2)*sqrt(
tan(x)) + tan(x) + 1) - 1/2*(tan(x)^3 + tan(x))*tan(x)^(3/2)/(tan
(x)^2 + 1) - 4*sqrt(tan(x)) - 1/10*((tan(x)^3 - 15*tan(x))*tan(x)
^(3/2) - 4*(tan(x)^3 + 5*tan(x))/sqrt(tan(x)))/(tan(x)^2 + 1)
```

Fricas [A] time = 0.531339, size = 761, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)),x, algorithm="fric
```

```
[Out] 1/4*sqrt(2)*(2*arctan(sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) - sin
(x))/(sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) - sqrt((4*c
os(x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)
) + cos(x))/cos(x))*cos(x) + cos(x)))*cos(x)^2 + 2*arctan(-sqrt(2
)*sqrt(cos(x)^3*sin(x))*(cos(x) - sin(x))/(sqrt(2)*sqrt(cos(x)^3*
sin(x))*(cos(x) + sin(x)) + sqrt((4*cos(x)^2*sin(x) - 2*sqrt(2)*s
qrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x))*cos(x) -
 cos(x)))*cos(x)^2 + 2*arctan((2*cos(x)^2*sin(x) + sqrt(2)*sqrt(c
```

```
os(x)^3*sin(x))*(cos(x) + sin(x)))/(2*cos(x)^3 + sqrt(2)*sqrt(cos
(x)^3*sin(x))*(cos(x) - sin(x)) + sqrt((4*cos(x)^2*sin(x) + 2*sqr
t(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x))*co
s(x) - cos(x)))*cos(x)^2 + 2*arctan(-(2*cos(x)^2*sin(x) - sqrt(2)
*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)))/(2*cos(x)^3 - sqrt(2)*s
qrt(cos(x)^3*sin(x))*(cos(x) - sin(x)) + sqrt((4*cos(x)^2*sin(x)
- 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos
(x))*cos(x) - cos(x)))*cos(x)^2 + cos(x)^2*log(2*(4*cos(x)^2*sin(
x) + 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/
cos(x)) - cos(x)^2*log(2*(4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(
x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)) + cos(x)^2*log((
cos(x)^6 - 8*cos(x)^4 + 4*sqrt(cos(x)^3*sin(x))*(cos(x)^2 - 2)*sq
rt(cos(x)*sin(x)) + 8*cos(x)^2)/cos(x)^6) - 4*sqrt(2)*sqrt(cos(x)
^3*sin(x)))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\sin(2x)} - \cos(2x)}{\sqrt{\cos(x)^3 \sin(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)),x, algorithm="giac

[Out] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)), x)
```

3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

Optimal. Leaf size=364

$$-\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)+\frac{4}{\sqrt{\tan(x)}}-\sqrt[4]{2}\coth^{-1}\left(\frac{\tan(x)+\sqrt{2}}{2^{3/4}\sqrt{\tan(x)}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\cos(x)(\cos(x)-\sin(x))}{\sqrt{2}\sqrt{\sin(x)\cos^3(x)}}\right)+\sqrt[4]{2}\tan^{-1}\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)-\sin(x)\right)}{2^{3/4}\sqrt{\sin(x)\cos^3(x)}}\right)-2\sqrt{2}\coth^{-1}\left(\frac{\cos(x)(\sin(x)+\cos(x))}{\sqrt{2}\sqrt{\sin(x)\cos^3(x)}}\right)+\sqrt[4]{2}\coth^{-1}\left(\frac{\cos(x)\left(\sin(x)+\sqrt{2}\cos(x)\right)}{2^{3/4}\sqrt{\sin(x)\cos^3(x)}}\right)$$

```
[Out] -2*Sqrt[2]*ArcCoth[(Cos[x]*(Cos[x] + Sin[x]))/(Sqrt[2]*Sqrt[Cos[x
]^3*Sin[x]])] + 2^(1/4)*ArcCoth[(Cos[x]*(Sqrt[2]*Cos[x] + Sin[x])
)/(2^(3/4)*Sqrt[Cos[x]^3*Sin[x]])] - 2^(1/4)*ArcCoth[(Sqrt[2] + T
an[x])/(2^(3/4)*Sqrt[Tan[x]])] - 2*Sqrt[2]*ArcTan[(Cos[x]*(Cos[x]
 - Sin[x]))/(Sqrt[2]*Sqrt[Cos[x]^3*Sin[x]])] + 2^(1/4)*ArcTan[(Co
s[x]*(Sqrt[2]*Cos[x] - Sin[x]))/(2^(3/4)*Sqrt[Cos[x]^3*Sin[x]])]
- 2^(1/4)*ArcTan[(Sqrt[2] - Tan[x])/(2^(3/4)*Sqrt[Tan[x]])] + 4*C
sc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] + (Csc[x]^2*Log[1 + Cos[x]^2]*
Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Csc[x]
^2*Log[Sin[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^
3])/2 + 4/Sqrt[Tan[x]] - (Csc[x]^2*Log[1 + Cos[x]^2]*Sqrt[Cos[x]*
Sin[x]^3]*Sqrt[Tan[x]])/4 + (Csc[x]^2*Log[Sin[x]]*Sqrt[Cos[x]*Sin
[x]^3]*Sqrt[Tan[x]])/2
```

Rubi [A] time = 8.69942, antiderivative size = 665, normalized size of antiderivative = 1.83, number of steps used = 66, number of rules used = 20, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.488

$$-\sqrt[4]{2}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)+\sqrt[4]{2}\tan^{-1}\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)+\frac{4}{\sqrt{\tan(x)}}+\frac{\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}}-\frac{\log\left(\tan(x)+2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}}+4\csc(x)\sec(x)\sqrt{\sin(x)\cos^3(x)}+\frac{\sqrt[4]{2}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}}-\frac{\sqrt[4]{2}\tan^{-1}\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)\sec^2(x)\sqrt{\si}}{\sqrt{\tan(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Tan[x]]
```

```
[Out] -(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 +
2^(1/4)*Sqrt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[
x]]/2^(3/4) - Log[Sqrt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4
) + 4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] - (Csc[x]^2*Log[Sec[x]^
2]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/2 + Csc[
x]^2*Log[Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Cos[x]
```

```
*Sin[x]^3] + (Csc[x]^2*Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x]^3*S
in[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Log[Tan[x]]*Sec[x]^2*Sqrt[Cos[
x]*Sin[x]^3])/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt
[Cos[x]*Sin[x]^3])/(4*Tan[x]^(3/2)) + 4/Sqrt[Tan[x]] + (2^(1/4)*A
rcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/S
qrt[Tan[x]] - (2^(1/4)*ArcTan[1 + 2^(1/4)*Sqrt[Tan[x]]]*Sec[x]^2*
Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2*Sqrt[2]*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] + (
2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3
*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[x]] +
Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Sqrt[2]*L
og[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[
x]])/Sqrt[Tan[x]] - (Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]
*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]]) + (Log[Sq
rt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin
[x]])/(2^(3/4)*Sqrt[Tan[x]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(
```

```
[Out] Timed out
```

Mathematica [C] time = 20.8127, size = 2057, normalized size = 5.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Ta
```

```
[Out] -(Cos[x]*Csc[x/2]*(4*Log[Sec[x/2]^2] - 2*Log[Tan[x/2]] - Log[1 +
Tan[x/2]^4])*Sec[x/2]*Sqrt[Cos[x]*Sin[x]^3])/(8*Sqrt[Cos[x]^3*Sin
[x]]) + ((1 + I)*((4 + 4*I)*EllipticPi[-I, -ArcSin[Sqrt[Tan[x/2]]
], -1] - (4 + 4*I)*EllipticPi[I, -ArcSin[Sqrt[Tan[x/2]]], -1] + (
-1)^(1/4)*(-EllipticPi[-(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1]
+ EllipticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1] - EllipticP
i[-(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1)^(3/
4), -ArcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^4*Sqrt[Cos[x]^3*Sin[x
]]*((2*Sqrt[2]*Sec[x]^2*Sqrt[2*Sin[2*x] + Sin[4*x]])/(3 + Cos[2*x
]) + (Sqrt[2]*Cos[2*x]*Sec[x]^2*Sqrt[2*Sin[2*x] + Sin[4*x]])/(3 +
 Cos[2*x])))/(Sqrt[Cos[x]*Sec[x/2]^2]*Sqrt[Tan[x/2]]*(-1 + Tan[x/
2]^2)*(((-1 - I)*((4 + 4*I)*EllipticPi[-I, -ArcSin[Sqrt[Tan[x/2]]]
```

```
], -1] - (4 + 4*I)*EllipticPi[I, -ArcSin[Sqrt[Tan[x/2]]], -1] + (
-1)^(1/4)*(-EllipticPi[-(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1]
+ EllipticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1] - EllipticP
i[-(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1)^(3/
4), -ArcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^6*Sqrt[Cos[x]^3*Sin[x
]]*Sqrt[Tan[x/2]])/(Sqrt[Cos[x]*Sec[x/2]^2]*(-1 + Tan[x/2]^2)^2)
- ((1/4 + I/4)*((4 + 4*I)*EllipticPi[-I, -ArcSin[Sqrt[Tan[x/2]]],
 -1] - (4 + 4*I)*EllipticPi[I, -ArcSin[Sqrt[Tan[x/2]]], -1] + (-1
)^(1/4)*(-EllipticPi[-(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1] +
EllipticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1] - EllipticPi[
-(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1)^(3/4)
, -ArcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^6*Sqrt[Cos[x]^3*Sin[x]]
)/(Sqrt[Cos[x]*Sec[x/2]^2]*Tan[x/2]^(3/2)*(-1 + Tan[x/2]^2)) + ((
1/2 + I/2)*((4 + 4*I)*EllipticPi[-I, -ArcSin[Sqrt[Tan[x/2]]], -1]
 - (4 + 4*I)*EllipticPi[I, -ArcSin[Sqrt[Tan[x/2]]], -1] + (-1)^(1
/4)*(-EllipticPi[-(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + Elli
pticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]], -1] - EllipticPi[-(-1
)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1)^(3/4), -A
rcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^4*(Cos[x]^4 - 3*Cos[x]^2*Si
n[x]^2))/(Sqrt[Cos[x]*Sec[x/2]^2]*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Tan[
x/2]]*(-1 + Tan[x/2]^2)) + ((2 + 2*I)*((4 + 4*I)*EllipticPi[-I, -
ArcSin[Sqrt[Tan[x/2]]], -1] - (4 + 4*I)*EllipticPi[I, -ArcSin[Sqr
t[Tan[x/2]]], -1] + (-1)^(1/4)*(-EllipticPi[-(-1)^(1/4), -ArcSin[
Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/
2]]], -1] - EllipticPi[-(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1]
+ EllipticPi[(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^
4*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Tan[x/2]])/(Sqrt[Cos[x]*Sec[x/2]^2]*
(-1 + Tan[x/2]^2)) - ((1/2 + I/2)*((4 + 4*I)*EllipticPi[-I, -ArcS
in[Sqrt[Tan[x/2]]], -1] - (4 + 4*I)*EllipticPi[I, -ArcSin[Sqrt[Ta
n[x/2]]], -1] + (-1)^(1/4)*(-EllipticPi[-(-1)^(1/4), -ArcSin[Sqrt
[Tan[x/2]]], -1] + EllipticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]]
, -1] - EllipticPi[-(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + El
lipticPi[(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^4*Sq
rt[Cos[x]^3*Sin[x]]*(-(Sec[x/2]^2*Sin[x]) + Cos[x]*Sec[x/2]^2*Tan
[x/2]))/((Cos[x]*Sec[x/2]^2)^(3/2)*Sqrt[Tan[x/2]]*(-1 + Tan[x/2]^
2)) + ((1 + I)*Sec[x/2]^4*Sqrt[Cos[x]^3*Sin[x]]*(((1 + I)*Sec[x/2
]^2)/(Sqrt[1 - Tan[x/2]]*(1 - I*Tan[x/2])*Sqrt[Tan[x/2]]*Sqrt[1 +
 Tan[x/2]]) - ((1 + I)*Sec[x/2]^2)/(Sqrt[1 - Tan[x/2]]*(1 + I*Tan
[x/2])*Sqrt[Tan[x/2]]*Sqrt[1 + Tan[x/2]]) + (-1)^(1/4)*(-Sec[x/2]
^2/(4*Sqrt[1 - Tan[x/2]]*Sqrt[Tan[x/2]]*Sqrt[1 + Tan[x/2]]*(1 - (
-1)^(1/4)*Tan[x/2])) + Sec[x/2]^2/(4*Sqrt[1 - Tan[x/2]]*Sqrt[Tan[
x/2]]*Sqrt[1 + Tan[x/2]]*(1 + (-1)^(1/4)*Tan[x/2])) - Sec[x/2]^2/
(4*Sqrt[1 - Tan[x/2]]*Sqrt[Tan[x/2]]*Sqrt[1 + Tan[x/2]]*(1 - (-1)
^(3/4)*Tan[x/2])) + Sec[x/2]^2/(4*Sqrt[1 - Tan[x/2]]*Sqrt[Tan[x/2
]]*Sqrt[1 + Tan[x/2]]*(1 + (-1)^(3/4)*Tan[x/2])))))/(Sqrt[Cos[x]*
Sec[x/2]^2]*Sqrt[Tan[x/2]]*(-1 + Tan[x/2]^2)))) + 4/Sqrt[Tan[x]]
- (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2/(
2*(1 - Sin[x]^2/2))]*(2 - Sin[x]^2)*Tan[x]^(3/2))/(3*(1 - Sin[x]^
2/2)^(3/4)*(-2 + Sin[x]^2)) + Sqrt[2*Sin[2*x] + Sin[4*x]]*(Sqrt[2
]*Cot[x] + Sqrt[2]*Tan[x]) + (Csc[x]^2*(4*Log[Sqrt[Tan[x]]] - Log
[2 + Tan[x]^2])*Sec[x]^2*Sqrt[2*Sin[2*x] - Sin[4*x]]*Sqrt[Tan[x]]
*(2 + Tan[x]^2))/(4*Sqrt[2]*(3 + Cos[2*x])*(1 + Tan[x]^2)^2)
```

Maple [C] time = 4.821, size = 22888, normalized size = 62.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^(1/
```

```
[Out] result too large to display
```

Maxima [A] time = 1.6281, size = 159, normalized size = 0.44

$$2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)$$
$$-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$
$$-\frac{2\left(\tan(x)^2+1\right)\sqrt{\tan(x)}}{\tan(x)^3+\tan(x)}+\frac{10}{\sqrt{\tan(x)}}-\frac{1}{2}\log\left(\tan(x)^2+1\right)+\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*sin(x)) - sqrt(ta
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - sqrt(2)*log
(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(t
an(x)) + tan(x) + 1) - 2*(tan(x)^2 + 1)*sqrt(tan(x))/(tan(x)^3 +
tan(x)) + 10/sqrt(tan(x)) - 1/2*log(tan(x)^2 + 1) + log(tan(x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*sin(x)) - sqrt(ta
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1/2)+
```

```
[Out] Timed out
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\cos(x)\sin(x)^3} - 2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)} - \sqrt{\tan(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*sin(x)) - sqrt(ta
```

```
[Out] integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*si
n(x)) - sqrt(tan(x))), x)
```

3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx$

Optimal. Leaf size=125

$$-\frac{9\sin^4(x)}{10(\sin(x)\cos^5(x))^{2/3}}-\frac{9}{4}\sec^8(x)\left(\sin(x)\cos^5(x)\right)^{4/3}+\frac{3}{14}\tan^4(x)\sqrt[3]{\sin(x)\cos^5(x)}\sqrt[3]{\tan(x)\sec^6(x)}+\frac{3}{4}\tan^2(x)\sqrt[3]{\sin(x)\cos^5(x)}\sqrt[3]{\tan(x)\sec^6(x)}+\frac{3}{2}\sqrt[3]{\sin(x)\cos^5(x)}\sqrt[3]{\tan(x)\sec^6(x)}$$

```
[Out] (-9*Sin[x]^4)/(10*(Cos[x]^5*Sin[x])^(2/3)) - (9*Sec[x]^8*(Cos[x]^
5*Sin[x])^(4/3))/4 + (3*(Cos[x]^5*Sin[x])^(1/3)*(Sec[x]^6*Tan[x])
^(1/3))/2 + (3*(Cos[x]^5*Sin[x])^(1/3)*Tan[x]^2*(Sec[x]^6*Tan[x])
^(1/3))/4 + (3*(Cos[x]^5*Sin[x])^(1/3)*Tan[x]^4*(Sec[x]^6*Tan[x])
^(1/3))/14
```

Rubi [A] time = 1.70804, antiderivative size = 141, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{9\sin^4(x)}{10(\sin(x)\cos^5(x))^{2/3}}-\frac{9\sin^2(x)\cos^2(x)}{4(\sin(x)\cos^5(x))^{2/3}}+\frac{3\sin^5(x)\cos(x)\sqrt[3]{\tan(x)\sec^6(x)}}{14(\sin(x)\cos^5(x))^{2/3}}+\frac{3\sin(x)\cos^5(x)\sqrt[3]{\tan(x)\sec^6(x)}}{2(\sin(x)\cos^5(x))^{2/3}}+\frac{3\sin^3(x)\cos^3(x)\sqrt[3]{\tan(x)\sec^6(x)}}{4(\sin(x)\cos^5(x))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-9*Cos[x]^2*Sin[x]^2)/(4*(Cos[x]^5*Sin[x])^(2/3)) - (9*Sin[x]^4)
/(10*(Cos[x]^5*Sin[x])^(2/3)) + (3*Cos[x]^5*Sin[x]*(Sec[x]^6*Tan[
x])^(1/3))/(2*(Cos[x]^5*Sin[x])^(2/3)) + (3*Cos[x]^3*Sin[x]^3*(Se
c[x]^6*Tan[x])^(1/3))/(4*(Cos[x]^5*Sin[x])^(2/3)) + (3*Cos[x]*Sin
[x]^5*(Sec[x]^6*Tan[x])^(1/3))/(14*(Cos[x]^5*Sin[x])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),
```

```
[Out] Timed out
```

Mathematica [A] time = 0.453768, size = 58, normalized size = 0.46

$$-\frac{3\sin(x)\left(924\sin(x)+252\sin(3x)-5(158\cos(x)+57\cos(3x)+9\cos(5x))\sqrt[3]{\tan(x)\sec^6(x)}\right)}{2240\left(\sin(x)\cos^5(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*
x] + 9*Cos[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x]
)^(2/3))
```

Maple [F] time = 0.808, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{\frac{\sin(x)}{(\cos(x))^7}}-3\tan(x)\right)\left((\cos(x))^5\sin(x)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

```
[Out] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

Maxima [A] time = 1.70148, size = 81, normalized size = 0.65

$$-\frac{3}{20}\tan(x)^{\frac{20}{3}}-\frac{3}{7}\tan(x)^{\frac{14}{3}}-\frac{9}{10}\tan(x)^{\frac{10}{3}}-\frac{3}{8}\tan(x)^{\frac{8}{3}}-\frac{9}{4}\tan(x)^{\frac{4}{3}}+\frac{3\left(14\tan(x)^7+60\tan(x)^5+105\tan(x)^3+140\tan(x)\right)}{280\tan(x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, algor
```

```
[Out] -3/20*tan(x)^(20/3) - 3/7*tan(x)^(14/3) - 9/10*tan(x)^(10/3) - 3/
8*tan(x)^(8/3) - 9/4*tan(x)^(4/3) + 3/280*(14*tan(x)^7 + 60*tan(x
```

```
)^5 + 105*tan(x)^3 + 140*tan(x))/tan(x)^(1/3)
```

Fricas [A] time = 0.230845, size = 76, normalized size = 0.61

$$-\frac{3\left(\cos\left(x\right)^{5}\sin\left(x\right)\right)^{\frac{1}{3}}\left(21\left(3\cos\left(x\right)^{2}+2\right)\sin\left(x\right)-5\left(9\cos\left(x\right)^{5}+3\cos\left(x\right)^{3}+2\cos\left(x\right)\right)\left(\frac{\sin(x)}{\cos(x)^{7}}\right)^{\frac{1}{3}}\right)}{140\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, algor
```

```
[Out] -3/140*(cos(x)^5*sin(x))^(1/3)*(21*(3*cos(x)^2 + 2)*sin(x) - 5*(9
*cos(x)^5 + 3*cos(x)^3 + 2*cos(x))*(sin(x)/cos(x)^7)^(1/3))/cos(x
)^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{\sin(x)}{\cos(x)^{7}}\right)^{\frac{1}{3}}-3\tan\left(x\right)}{\left(\cos\left(x\right)^{5}\sin\left(x\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, algor
```

```
[Out] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^
(2/3), x)
```

3.419 $\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx$

Optimal. Leaf size=73

$$-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}-\frac{5}{24}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}-\frac{5}{16}\cos(x)\sqrt{2\cos^2(x)+1}-\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}$$

```
[Out] (-5*ArcSinh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) - (5*Cos[x]*Sqrt[1 + 2*
Cos[x]^2])/16 - (5*Cos[x]*(1 + 2*Cos[x]^2)^(3/2))/24 - (Cos[x]*(1
 + 2*Cos[x]^2)^(5/2))/6
```

Rubi [A] time = 0.0760593, antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{1}{6}\cos(x)(\cos(2x)+2)^{5/2}-\frac{5}{24}\cos(x)(\cos(2x)+2)^{3/2}-\frac{5}{16}\cos(x)\sqrt{\cos(2x)+2}-\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

```
[Out] (-5*ArcSinh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) - (5*Cos[x]*Sqrt[2 + Co
s[2*x]])/16 - (5*Cos[x]*(2 + Cos[2*x])^(3/2))/24 - (Cos[x]*(2 + C
os[2*x])^(5/2))/6
```

Rubi in Sympy [A] time = 2.84493, size = 73, normalized size = 1.

$$-\frac{\left(2\cos^2(x)+1\right)^{\frac{5}{2}}\cos(x)}{6}-\frac{5\left(2\cos^2(x)+1\right)^{\frac{3}{2}}\cos(x)}{24}-\frac{5\sqrt{2\cos^2(x)+1}\cos(x)}{16}-\frac{5\sqrt{2}\operatorname{asinh}\left(\sqrt{2}\cos(x)\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
```

```
[Out] -(2*cos(x)**2 + 1)**(5/2)*cos(x)/6 - 5*(2*cos(x)**2 + 1)**(3/2)*c
os(x)/24 - 5*sqrt(2*cos(x)**2 + 1)*cos(x)/16 - 5*sqrt(2)*asinh(sq
rt(2)*cos(x))/32
```

Mathematica [A] time = 0.179296, size = 61, normalized size = 0.84

$$\frac{1}{96}\left(-2\sqrt{\cos(2x)+2}(92\cos(x)+23\cos(3x)+2\cos(5x))-15\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)+2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

```
[Out] (-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 1
5*Sqrt[2]*Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

Maple [A] time = 0.02, size = 56, normalized size = 0.8

$$-\frac{5\cos\left(x\right)}{24}\left(1+2\left(\cos\left(x\right)\right)^{2}\right)^{\frac{3}{2}}-\frac{\cos\left(x\right)}{6}\left(1+2\left(\cos\left(x\right)\right)^{2}\right)^{\frac{5}{2}}-\frac{5\,Arcsinh\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos\left(x\right)}{16}\sqrt{1+2\left(\cos\left(x\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*cos(x)^2)^(5/2)*sin(x),x)
```

```
[Out] -5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)
-5/32*arcsinh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^
(1/2)
```

Maxima [A] time = 1.53213, size = 74, normalized size = 1.01

$$-\frac{1}{6}\left(2\cos\left(x\right)^{2}+1\right)^{\frac{5}{2}}\cos\left(x\right)-\frac{5}{24}\left(2\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}\cos\left(x\right)-\frac{5}{32}\sqrt{2}\,\operatorname{arsinh}\left(\sqrt{2}\cos\left(x\right)\right)-\frac{5}{16}\sqrt{2\cos\left(x\right)^{2}+1}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*cos(x)^2 + 1)^(5/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*(2*cos(x)^2 + 1)^(5/2)*cos(x) - 5/24*(2*cos(x)^2 + 1)^(3/2)*
cos(x) - 5/32*sqrt(2)*arcsinh(sqrt(2)*cos(x)) - 5/16*sqrt(2*cos(x
)^2 + 1)*cos(x)
```

Fricas [A] time = 0.252754, size = 147, normalized size = 2.01

$$-\frac{1}{48}\left(32\cos(x)^5+52\cos(x)^3+33\cos(x)\right)\sqrt{2\cos(x)^2+1}+\frac{5}{256}\sqrt{2}\log\left(2048\sqrt{2}\cos(x)^8+2048\sqrt{2}\cos(x)^6+640\sqrt{2}\cos(x)^4+64\sqrt{2}\cos(x)^2-16\left(128\cos(x)^7+96\cos(x)^5+20\cos(x)^3+\cos(x)\right)\sqrt{2\cos(x)^2+1}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*cos(x)^2 + 1)^(5/2)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/48*(32*cos(x)^5 + 52*cos(x)^3 + 33*cos(x))*sqrt(2*cos(x)^2 + 1
) + 5/256*sqrt(2)*log(2048*sqrt(2)*cos(x)^8 + 2048*sqrt(2)*cos(x)
^6 + 640*sqrt(2)*cos(x)^4 + 64*sqrt(2)*cos(x)^2 - 16*(128*cos(x)^
7 + 96*cos(x)^5 + 20*cos(x)^3 + cos(x))*sqrt(2*cos(x)^2 + 1) + sq
rt(2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220378, size = 74, normalized size = 1.01

$$-\frac{1}{48}\left(4\left(8\cos(x)^2+13\right)\cos(x)^2+33\right)\sqrt{2\cos(x)^2+1}\cos(x)+\frac{5}{32}\sqrt{2}\ln\left(-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*cos(x)^2 + 1)^(5/2)*sin(x),x, algorithm="giac")
```

```
[Out] -1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*co
s(x) + 5/32*sqrt(2)*ln(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))
```

3.420 $\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}$$

```
[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]])/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[
x]^2])/16 + (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 -
4*Sin[x]^2)^(5/2))/6
```

Rubi [A] time = 0.0853401, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

```
[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]])/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[
x]^2])/16 + (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 -
4*Sin[x]^2)^(5/2))/6
```

Rubi in Sympy [A] time = 179.512, size = 70, normalized size = 1.01

$$\frac{\left(-4\sin^2(x)+5\right)^{\frac{5}{2}}\sin(x)}{6}+\frac{25\left(-4\sin^2(x)+5\right)^{\frac{3}{2}}\sin(x)}{24}+\frac{125\sqrt{-4\sin^2(x)+5}\,\sin(x)}{16}+\frac{625\,\mathrm{asin}\left(\frac{2\sqrt{5}\sin(x)}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
```

```
[Out] (-4*sin(x)**2 + 5)**(5/2)*sin(x)/6 + 25*(-4*sin(x)**2 + 5)**(3/2)
*sin(x)/24 + 125*sqrt(-4*sin(x)**2 + 5)*sin(x)/16 + 625*asin(2*sq
rt(5)*sin(x)/5)/32
```

Mathematica [A] time = 0.126779, size = 55, normalized size = 0.8

$$\frac{1}{48}(515\sin(x)+90\sin(3x)+8\sin(5x))\sqrt{2\cos(2x)+3}+\frac{625}{32}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{2\cos(2x)+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

```
[Out] (625*ArcTan[(2*Sin[x])/Sqrt[3 + 2*Cos[2*x]]])/32 + (Sqrt[3 + 2*Co
s[2*x]]*(515*Sin[x] + 90*Sin[3*x] + 8*Sin[5*x]))/48
```

Maple [A] time = 0.136, size = 103, normalized size = 1.5

$$-\frac{1}{192\,\sin(x)}\sqrt{\left(4\,(\cos(x))^2+1\right)(\sin(x))^2}\left(-512\sqrt{-4\,(\sin(x))^4+5\,(\sin(x))^2}(\sin(x))^4+2080\sqrt{-4\,(\sin(x))^4+5\,(\sin(x))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x)
```

```
[Out] -1/192*((4*cos(x)^2+1)*sin(x)^2)^(1/2)*(-512*(-4*sin(x)^4+5*sin(x
)^2)^(1/2)*sin(x)^4+2080*(-4*sin(x)^4+5*sin(x)^2)^(1/2)*sin(x)^2-
3300*(-4*sin(x)^4+5*sin(x)^2)^(1/2)-1875*arcsin(-1+8/5*sin(x)^2))
/sin(x)/(4*cos(x)^2+1)^(1/2)
```

Maxima [A] time = 1.55701, size = 72, normalized size = 1.04

$$\frac{1}{6}\left(-4\,\sin(x)^2+5\right)^{\frac{5}{2}}\sin(x)+\frac{25}{24}\left(-4\,\sin(x)^2+5\right)^{\frac{3}{2}}\sin(x)+\frac{125}{16}\sqrt{-4\,\sin(x)^2+5}\sin(x)+\frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2 + sin(x)^2)^(5/2)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*sin(x)^2 + 5)^(5/2)*sin(x) + 25/24*(-4*sin(x)^2 + 5)^(3/2
)*sin(x) + 125/16*sqrt(-4*sin(x)^2 + 5)*sin(x) + 625/32*arcsin(2/
5*sqrt(5)*sin(x))
```

Fricas [A] time = 0.282593, size = 162, normalized size = 2.35

$$\frac{1}{48}\left(128\cos\left(x\right)^{4}+264\cos\left(x\right)^{2}+433\right)\sqrt{4\cos\left(x\right)^{2}+1}\sin\left(x\right)-\frac{625}{64}\arctan\left(-\frac{\left(16\cos\left(x\right)^{2}-3\right)\sqrt{4\cos\left(x\right)^{2}+1}\sin\left(x\right)-2\left(16\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)}{32\cos\left(x\right)^{4}-18\cos\left(x\right)^{2}-\left(16\cos\left(x\right)^{3}-11\cos\left(x\right)\right)\sqrt{4\cos\left(x\right)^{2}+1}-4}\right)+\frac{625}{64}\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2 + sin(x)^2)^(5/2)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/48*(128*cos(x)^4 + 264*cos(x)^2 + 433)*sqrt(4*cos(x)^2 + 1)*sin
(x) - 625/64*arctan(-((16*cos(x)^2 - 3)*sqrt(4*cos(x)^2 + 1)*sin(
x) - 2*(16*cos(x)^3 - cos(x))*sin(x))/(32*cos(x)^4 - 18*cos(x)^2
- (16*cos(x)^3 - 11*cos(x))*sqrt(4*cos(x)^2 + 1) - 4)) + 625/64*a
rctan(sin(x)/cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.212135, size = 55, normalized size = 0.8

$$\frac{1}{48}\left(8\left(16\sin\left(x\right)^{2}-65\right)\sin\left(x\right)^{2}+825\right)\sqrt{-4\sin\left(x\right)^{2}+5}\sin\left(x\right)+\frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2 + sin(x)^2)^(5/2)*cos(x),x, algorithm="giac")
```

```
[Out] 1/48*(8*(16*sin(x)^2 - 65)*sin(x)^2 + 825)*sqrt(-4*sin(x)^2 + 5)*
sin(x) + 625/32*arcsin(2/5*sqrt(5)*sin(x))
```

3.421 $\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=58

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[
-1 - 4*Sin[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4
```

Rubi [A] time = 0.0830833, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[
-1 - 4*Sin[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4
```

Rubi in Sympy [A] time = 163.063, size = 61, normalized size = 1.05

$$\frac{\left(-4\sin^2(x)-1\right)^{\frac{3}{2}}\sin(x)}{4}-\frac{3\sqrt{-4\sin^2(x)-1}\sin(x)}{8}+\frac{3\,\mathrm{atan}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)
```

```
[Out] (-4*sin(x)**2 - 1)**(3/2)*sin(x)/4 - 3*sqrt(-4*sin(x)**2 - 1)*sin
(x)/8 + 3*atan(2*sin(x)/sqrt(-4*sin(x)**2 - 1))/16
```

Mathematica [A] time = 0.107873, size = 50, normalized size = 0.86

$$\left(\frac{1}{4}\sin(3x)-\frac{11\sin(x)}{8}\right)\sqrt{2\cos(2x)-3}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{2\cos(2x)-3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-3 + 2*Cos[2*x]]])/16 + Sqrt[-3 + 2*Cos
[2*x]]*((-11*Sin[x])/8 + Sin[3*x]/4)
```

Maple [A] time = 0.135, size = 82, normalized size = 1.4

$$-\frac{1}{32 \sin(x)}\sqrt{\left(4 (\cos(x))^2-5\right)(\sin(x))^2}\left(32\sqrt{-4(\sin(x))^4-(\sin(x))^2}(\sin(x))^2+20\sqrt{-4(\sin(x))^4-(\sin(x))^2}-3\ \mathrm{arcsin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x)
```

```
[Out] -1/32*((4*cos(x)^2-5)*sin(x)^2)^(1/2)*(32*(-4*sin(x)^4-sin(x)^2)^
(1/2)*sin(x)^2+20*(-4*sin(x)^4-sin(x)^2)^(1/2)-3*arcsin(8*sin(x)^
2+1))/sin(x)/(4*cos(x)^2-5)^(1/2)
```

Maxima [A] time = 1.5958, size = 49, normalized size = 0.84

$$\frac{1}{4}\left(-4 \sin(x)^2-1\right)^{\frac{3}{2}}\sin(x)-\frac{3}{8}\sqrt{-4 \sin(x)^2-1}\sin(x)-\frac{3}{16}i\ \mathrm{arsinh}(2 \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cos(x)^2 - 5*sin(x)^2)^(3/2)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/4*(-4*sin(x)^2 - 1)^(3/2)*sin(x) - 3/8*sqrt(-4*sin(x)^2 - 1)*si
n(x) - 3/16*I*arcsinh(2*sin(x))
```

Fricas [A] time = 0.220021, size = 528, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cos(x)^2 - 5*sin(x)^2)^(3/2)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/128*(((1536*I*e^(10*I*x) - 6912*I*e^(8*I*x) + 9408*I*e^(6*I*x)
- 3744*I*e^(4*I*x))*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - 1536*I*e^
```

```
(12*I*x) + 9216*I*e^(10*I*x) - 18816*I*e^(8*I*x) + 14976*I*e^(6*I
*x) - 3756*I*e^(4*I*x))*log(-1/2*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1
)*(4*e^(2*I*x) - 5) + 2*e^(4*I*x) - 11/2*e^(2*I*x) + 5/2) + ((-15
36*I*e^(10*I*x) + 6912*I*e^(8*I*x) - 9408*I*e^(6*I*x) + 3744*I*e^
(4*I*x))*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) + 1536*I*e^(12*I*x) -
9216*I*e^(10*I*x) + 18816*I*e^(8*I*x) - 14976*I*e^(6*I*x) + 3756*
I*e^(4*I*x))*log(sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - e^(2*I*x) -
1) + (2048*I*e^(14*I*x) - 23552*I*e^(12*I*x) + 85376*I*e^(10*I*x)
 - 144064*I*e^(8*I*x) + 151424*I*e^(6*I*x) - 117216*I*e^(4*I*x) +
 47512*I*e^(2*I*x) - 5008*I)*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) -
2048*I*e^(16*I*x) + 26624*I*e^(14*I*x) - 119424*I*e^(12*I*x) + 25
9328*I*e^(10*I*x) - 332960*I*e^(8*I*x) + 302752*I*e^(6*I*x) - 185
271*I*e^(4*I*x) + 54976*I*e^(2*I*x) - 4992*I)/(8*(16*e^(10*I*x) -
 72*e^(8*I*x) + 98*e^(6*I*x) - 39*e^(4*I*x))*sqrt(e^(4*I*x) - 3*e
^(2*I*x) + 1) - 128*e^(12*I*x) + 768*e^(10*I*x) - 1568*e^(8*I*x)
+ 1248*e^(6*I*x) - 313*e^(4*I*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.211122, size = 41, normalized size = 0.71

$$-\frac{1}{8} i \left(8 \sin\left(x\right)^{2} + 5\right) \sqrt{4 \sin\left(x\right)^{2} + 1} \sin\left(x\right) - \frac{3}{16} i \arcsin\left(2 i \sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-cos(x)^2 - 5*sin(x)^2)^(3/2)*cos(x),x, algorithm="giac")
```

```
[Out] -1/8*I*(8*sin(x)^2 + 5)*sqrt(4*sin(x)^2 + 1)*sin(x) - 3/16*I*arcs
in(2*I*sin(x))
```

3.422 $\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{3/2}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

```
[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2
)^(3/2)) + Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])
```

Rubi [A] time = 0.083873, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{3/2}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2
)^(3/2)) + Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])
```

Rubi in Sympy [A] time = 90.3384, size = 49, normalized size = 0.89

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{\frac{3}{2}}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
```

```
[Out] cos(x)/(15*sqrt(7*cos(x)**2 - 2)) - cos(x)/(15*(7*cos(x)**2 - 2)*
*(3/2)) + cos(x)/(10*(7*cos(x)**2 - 2)**(5/2))
```

Mathematica [A] time = 0.113148, size = 37, normalized size = 0.67

$$\frac{\cos(x)(56\cos(2x)+49\cos(4x)+67)}{15\sqrt{2}(7\cos(2x)+3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[
2*x])^(5/2))
```

Maple [A] time = 0.043, size = 44, normalized size = 0.8

$$\frac{\cos(x)}{10}\left(-2+7\,(\cos(x))^2\right)^{-\frac{5}{2}}-\frac{\cos(x)}{15}\left(-2+7\,(\cos(x))^2\right)^{-\frac{3}{2}}+\frac{\cos(x)}{15}\frac{1}{\sqrt{-2+7\,(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x)
```

```
[Out] 1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/
2)+1/15*cos(x)/(-2+7*cos(x)^2)^(1/2)
```

Maxima [A] time = 1.35159, size = 58, normalized size = 1.05

$$\frac{\cos(x)}{15\sqrt{7\cos(x)^2-2}}-\frac{\cos(x)}{15\left(7\cos(x)^2-2\right)^{\frac{3}{2}}}+\frac{\cos(x)}{10\left(7\cos(x)^2-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*cos(x)/sqrt(7*cos(x)^2 - 2) - 1/15*cos(x)/(7*cos(x)^2 - 2)^(
3/2) + 1/10*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

Fricas [A] time = 0.249776, size = 69, normalized size = 1.25

$$\frac{\left(98\cos(x)^5-70\cos(x)^3+15\cos(x)\right)\sqrt{7\cos(x)^2-2}}{30\left(343\cos(x)^6-294\cos(x)^4+84\cos(x)^2-8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(98*cos(x)^5 - 70*cos(x)^3 + 15*cos(x))*sqrt(7*cos(x)^2 - 2)
/(343*cos(x)^6 - 294*cos(x)^4 + 84*cos(x)^2 - 8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.212682, size = 41, normalized size = 0.75

$$\frac{\left(14\left(7\cos\left(x\right)^{2}-5\right)\cos\left(x\right)^{2}+15\right)\cos\left(x\right)}{30\left(7\cos\left(x\right)^{2}-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/30*(14*(7*cos(x)^2 - 5)*cos(x)^2 + 15)*cos(x)/(7*cos(x)^2 - 2)^
(5/2)
```

3.423 $\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

```
[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*
Sin[x]^2])
```

Rubi [A] time = 0.111612, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*
Sin[x]^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.291074, size = 54, normalized size = 1.38

$$\frac{1}{50}\left(\frac{5\sqrt{2}\sin(x)}{\sqrt{5\cos(2x)-1}}+4\sqrt{5}\tan^{-1}\left(\frac{\sqrt{10}\sin(x)}{\sqrt{5\cos(2x)-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[5]*ArcTan[(Sqrt[10]*Sin[x])/Sqrt[-1 + 5*Cos[2*x]]] + (5*S
qrt[2]*Sin[x])/Sqrt[-1 + 5*Cos[2*x]])/50
```

Maple [B] time = 0.164, size = 58, normalized size = 1.5

$$\frac{1}{250\,(\cos(x))^2-150}\left(20\,\sqrt{5}\arcsin\left(1/2\,\sin(x)\,\sqrt{10}\right)(\cos(x))^2+5\,\sin(x)\sqrt{5\,(\cos(x))^2-3}-12\,\arcsin\left(1/2\,\sin(x)\,\sqrt{10}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x)
```

```
[Out] 1/50/(5*cos(x)^2-3)*(20*5^(1/2)*arcsin(1/2*sin(x)*10^(1/2))*cos(x
)^2+5*sin(x)*(5*cos(x)^2-3)^(1/2)-12*arcsin(1/2*sin(x)*10^(1/2))*
5^(1/2))
```

Maxima [A] time = 1.78562, size = 967, normalized size = 24.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(x)/(-5*sin(x)^2 + 2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/50*(5*cos(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*cos(4*x) - 2*c
os(2*x) + 5))*sin(2*x) - 5*(cos(2*x) - 1)*sin(1/2*arctan2(5*sin(4
*x) - 2*sin(2*x), 5*cos(4*x) - 2*cos(2*x) + 5)) + 2*(-10*(2*cos(2
*x) - 5)*cos(4*x) + 25*cos(4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2
- 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*(
sqrt(5)*arctan2(1/12*sqrt(6)*(sqrt(6)*(25/36)^(1/4)*(25*cos(2*x)^
4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 - 10*cos(2*x
) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*sin(
1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25
/24*sin(2*x)^2 - 5/12*cos(2*x) + 25/24)) + 5*sin(2*x)), 5/12*sqrt
(6)*cos(2*x) + 1/2*(25/36)^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 -
 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 - 10*cos(2*x) - 23)*sin(2*x)^2
+ 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*cos(1/2*arctan2(5/12*(5
*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2*x)^2 - 5/
12*cos(2*x) + 25/24)) - 1/12*sqrt(6)) + sqrt(5)*arctan2(1/12*sqrt
(6)*(sqrt(6)*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^
3 + 2*(cos(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2
- 980*cos(2*x) + 2401)^(1/4)*sin(1/2*arctan2(1/12*(cos(2*x) - 5)*
```

```
sin(2*x), 1/24*cos(2*x)^2 - 1/24*sin(2*x)^2 - 5/12*cos(2*x) + 49/
24)) + sin(2*x)), 1/12*sqrt(6)*cos(2*x) + 1/2*(1/36)^(1/4)*(cos(2
*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos(2*x)^2 - 10*cos(2*x)
+ 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 2401)^(1/4)*cos
(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/24
*sin(2*x)^2 - 5/12*cos(2*x) + 49/24)) - 5/12*sqrt(6))))/(-10*(2*c
os(2*x) - 5)*cos(4*x) + 25*cos(4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x
)^2 - 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 - 20*cos(2*x) + 25)^(1/
4)
```

Fricas [A] time = 0.244971, size = 127, normalized size = 3.26

$$-\frac{\left(5\sqrt{5}\cos(x)^2-3\sqrt{5}\right)\arctan\left(\frac{50\sqrt{5}\cos(x)^4-80\sqrt{5}\cos(x)^2+31\sqrt{5}}{10\sqrt{5\cos(x)^2-3}\left(5\cos(x)^2-4\right)\sin(x)}\right)-5\sqrt{5\cos(x)^2-3}\sin(x)}{50\left(5\cos(x)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(x)/(-5*sin(x)^2 + 2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*c
os(x)^4 - 80*sqrt(5)*cos(x)^2 + 31*sqrt(5))/(sqrt(5*cos(x)^2 - 3)
*(5*cos(x)^2 - 4)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*co
s(x)^2 - 3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217445, size = 51, normalized size = 1.31

$$\frac{2}{25}\sqrt{5}\arcsin\left(\frac{1}{2}\sqrt{10}\sin(x)\right)-\frac{\sqrt{-5\sin(x)^2+2}\sin(x)}{10\left(5\sin(x)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*cos(x)/(-5*sin(x)^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2
+ 2)*sin(x)/(5*sin(x)^2 - 2)
```

3.424 $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}-\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)$$

```
[Out] -ArcSin[(2*Cos[x])/3]/2 - (55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2))
 + (295*Cos[x])/(243*Sqrt[9 - 4*Cos[x]^2])
```

Rubi [A] time = 0.123762, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}-\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] -ArcSin[(2*Cos[x])/3]/2 - (55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2))
 + (295*Cos[x])/(243*Sqrt[9 - 4*Cos[x]^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.412996, size = 63, normalized size = 1.31

$$\frac{2550\cos(x)-590\cos(3x)+243i(7-2\cos(2x))^{3/2}\log\left(\sqrt{7-2\cos(2x)}+2i\cos(x)\right)}{486(7-2\cos(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] (2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[
(2*I)*Cos[x] + Sqrt[7 - 2*Cos[2*x]]])/(486*(7 - 2*Cos[2*x])^(3/2)
)
```

Maple [A]   time = 0.106, size = 53, normalized size = 1.1

$$\frac{26\cos(x)}{27}\left(9-4\,(\cos(x))^2\right)^{-\frac{3}{2}}+\frac{214\cos(x)}{243}\frac{1}{\sqrt{9-4\,(\cos(x))^2}}-\frac{4\,(\cos(x))^3}{3}\left(9-4\,(\cos(x))^2\right)^{-\frac{3}{2}}-\frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x)
```

```
[Out] 26/27*cos(x)/(9-4*cos(x)^2)^(3/2)+214/243*cos(x)/(9-4*cos(x)^2)^(
1/2)-4/3*cos(x)^3/(9-4*cos(x)^2)^(3/2)-1/2*arcsin(2/3*cos(x))
```

Maxima [A]   time = 1.52975, size = 93, normalized size = 1.94

$$-2\left(\frac{2\cos(x)^2}{\left(-4\cos(x)^2+9\right)^{\frac{3}{2}}}-\frac{3}{\left(-4\cos(x)^2+9\right)^{\frac{3}{2}}}\right)\cos(x)+\frac{52\cos(x)}{243\sqrt{-4\cos(x)^2+9}}+\frac{26\cos(x)}{27\left(-4\cos(x)^2+9\right)^{\frac{3}{2}}}-\frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(2*cos(x)^2/(-4*cos(x)^2 + 9)^(3/2) - 3/(-4*cos(x)^2 + 9)^(3/2
))*cos(x) + 52/243*cos(x)/sqrt(-4*cos(x)^2 + 9) + 26/27*cos(x)/(-
4*cos(x)^2 + 9)^(3/2) - 1/2*arcsin(2/3*cos(x))
```

Fricas [A]   time = 0.251594, size = 217, normalized size = 4.52

$$-\frac{243\left(16\cos(x)^4-72\cos(x)^2+81\right)\arctan\left(-\frac{2\left(16\cos(x)^3-27\cos(x)\right)\sin(x)+\left(16\cos(x)^3-17\cos(x)\right)\sqrt{-4\cos(x)^2+9}}{32\cos(x)^4-\left(16\cos(x)^2-9\right)\sqrt{-4\cos(x)^2+9}\sin(x)-70\cos(x)^2+18}\right)+243\left(16\cos(x)^4\right.}{972\left(16\cos(x)^4-72\cos(x)^2+81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/972*(243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(-(2*(16*cos(x
)^3 - 27*cos(x))*sin(x) + (16*cos(x)^3 - 17*cos(x))*sqrt(-4*cos(x
)^2 + 9))/(32*cos(x)^4 - (16*cos(x)^2 - 9)*sqrt(-4*cos(x)^2 + 9)*
sin(x) - 70*cos(x)^2 + 18)) + 243*(16*cos(x)^4 - 72*cos(x)^2 + 81
)*arctan(sin(x)/cos(x)) + 80*(59*cos(x)^3 - 108*cos(x))*sqrt(-4*c
os(x)^2 + 9))/(16*cos(x)^4 - 72*cos(x)^2 + 81)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.212165, size = 54, normalized size = 1.12

$$-\frac{20 \left(59 \cos(x)^2 - 108\right) \sqrt{-4 \cos(x)^2 + 9} \cos(x)}{243 \left(4 \cos(x)^2 - 9\right)^2} - \frac{1}{2} \arcsin\left(\frac{2}{3} \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x
)^2 - 9)^2 - 1/2*arcsin(2/3*cos(x))
```

3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

```
[Out] -1/(4*(-5 + 4*Sin[x]^2)^(3/2)) - 5/(8*Sqrt[-5 + 4*Sin[x]^2]) + Sq
rt[-5 + 4*Sin[x]^2]/8
```

Rubi [A] time = 0.17784, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] -1/(4*(-5 + 4*Sin[x]^2)^(3/2)) - 5/(8*Sqrt[-5 + 4*Sin[x]^2]) + Sq
rt[-5 + 4*Sin[x]^2]/8
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.505549, size = 59, normalized size = 1.2

$$-\frac{2\cos(2x)\left(\sqrt{6\cos(2x)+9}-11\right)+3\left(\sqrt{6\cos(2x)+9}-8\right)-2\cos(4x)}{8(-2\cos(2x)-3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] -(2*Cos[2*x]*(-11 + Sqrt[9 + 6*Cos[2*x]]) + 3*(-8 + Sqrt[9 + 6*Co
s[2*x]]) - 2*Cos[4*x])/(8*(-3 - 2*Cos[2*x])^(3/2))
```

Maple [A] time = 0.058, size = 46, normalized size = 0.9

$$\frac{1}{2}\left(-4\,(\cos(x))^{2}-1\right)^{-\frac{3}{2}}+\frac{7\,(\cos(x))^{2}}{2}\left(-4\,(\cos(x))^{2}-1\right)^{-\frac{3}{2}}+2\,\frac{(\cos(x))^{4}}{\left(-4\,(\cos(x))^{2}-1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x)
```

```
[Out] 1/2/(-4*cos(x)^2-1)^(3/2)+7/2*cos(x)^2/(-4*cos(x)^2-1)^(3/2)+2*co
s(x)^4/(-4*cos(x)^2-1)^(3/2)
```

Maxima [A] time = 1.4443, size = 259, normalized size = 5.29

$$-\frac{(\cos(11\,x)+14\,\cos(9\,x)+58\,\cos(7\,x)+94\,\cos(5\,x)+58\,\cos(3\,x)+15\,\cos(x))\cos\left(\frac{5}{2}\,\arctan(\sin(4\,x)+3\,\sin(2\,x),-\mathrm{co}\right.}{8\left(2\,(3\,\cos(2\,x)+1)\cos(4\,x)+\mathrm{cos}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(x)*sin(3*x)/(4*sin(x)^2 - 5)^(5/2),x, algorithm="maxima"
```

```
[Out] -1/8*((cos(11*x) + 14*cos(9*x) + 58*cos(7*x) + 94*cos(5*x) + 58*c
os(3*x) + 15*cos(x))*cos(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(
4*x) - 3*cos(2*x) - 1)) - (sin(11*x) + 14*sin(9*x) + 58*sin(7*x)
+ 94*sin(5*x) + 58*sin(3*x) + 13*sin(x))*sin(5/2*arctan2(sin(4*x)
 + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) - 1)))/(2*(3*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + sin(4*x)^2 + 6*sin(4*x)*si
n(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)^(5/4)
```

Fricas [A] time = 0.238004, size = 1, normalized size = 0.02

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(x)*sin(3*x)/(4*sin(x)^2 - 5)^(5/2),x, algorithm="fricas"
```

```
[Out] 0
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.204251, size = 45, normalized size = 0.92

$$\frac{1}{8}\sqrt{4\sin(x)^2-5}-\frac{20\sin(x)^2-23}{8\left(4\sin(x)^2-5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*cos(x)*sin(3*x)/(4*sin(x)^2 - 5)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5
)^(3/2)
```

3.426 $\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$

Optimal. Leaf size=111

$$2\sqrt{\sin^2(x)-5}-\frac{2\tan^{-1}\left(\frac{\sqrt{\sin^2(x)-5}}{\sqrt{5}}\right)}{\sqrt{5}}-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+\frac{2}{5}\sqrt{\sin^2(x)-5}\csc(x)+2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{\sin^2(x)-5}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{\sin^2(x)-5}}\right)}{\sqrt{5}}$$

```
[Out] 2*ArcTan[Cos[x]/Sqrt[-5 + Sin[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sq
rt[-5 + Sin[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-5 + Sin[x]^2]/Sqrt[5
]])/Sqrt[5] - 2*ArcTanh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-5 +
 Sin[x]^2] + (2*Csc[x]*Sqrt[-5 + Sin[x]^2])/5
```

Rubi [A] time = 1.00023, antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$

$$2\sqrt{-\cos^2(x)-4}-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)}{\sqrt{5}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-\cos^2(x)-4}}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{2}{5}\sqrt{\sin^2(x)-5}\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-
5 + Sin[x]^2],x]
```

```
[Out] 2*ArcTan[Cos[x]/Sqrt[-4 - Cos[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sq
rt[-4 - Cos[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-4 - Cos[x]^2]/Sqrt[5
]])/Sqrt[5] - 2*ArcTanh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-4 -
 Cos[x]^2] + (2*Csc[x]*Sqrt[-5 + Sin[x]^2])/5
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-
5+sin(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.77235, size = 338, normalized size = 3.05

$$(16-32i)\sqrt{5}\sqrt{\frac{(1+2i)(\cos(x)-2i)}{\cos(x)+1}}\sqrt{\frac{(1-2i)(\cos(x)+2i)}{\cos(x)+1}}\cos^2\left(\frac{x}{2}\right)F\left(\sin^{-1}\left(\frac{(1+2i)\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)|-\frac{7}{25}+\frac{24i}{25}\right)-(32-64i)\sqrt{5}\sqrt{\frac{(1+2i)(\cos(x)-2i)}{\cos(x)+1}}\sqrt{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-
5 + Sin[x]^2],x]
```

```
[Out] ((16 - 32*I)*Sqrt[5]*Cos[x/2]^2*Sqrt[((1 + 2*I)*(-2*I + Cos[x]))/
(1 + Cos[x])]*Sqrt[((1 - 2*I)*(2*I + Cos[x]))/(1 + Cos[x])]*Ellip
ticF[ArcSin[((1 + 2*I)*Tan[x/2])/Sqrt[5]], -7/25 + (24*I)/25] - (
32 - 64*I)*Sqrt[5]*Cos[x/2]^2*Sqrt[((1 + 2*I)*(-2*I + Cos[x]))/(1
 + Cos[x])]*Sqrt[((1 - 2*I)*(2*I + Cos[x]))/(1 + Cos[x])]*Ellipti
cPi[3/5 + (4*I)/5, ArcSin[((1 + 2*I)*Tan[x/2])/Sqrt[5]], -7/25 +
(24*I)/25] - 5*(85 + Sqrt[10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[-9 -
Cos[2*x]]]*Sqrt[-9 - Cos[2*x]] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[
2*x]]/Sqrt[10]]*Sqrt[-9 - Cos[2*x]] + 18*Csc[x] + 2*Cos[2*x]*Csc[
x] + (10*I)*Sqrt[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sq
rt[-9 - Cos[2*x]]] + 5*Csc[x]*Sin[3*x]))/(25*Sqrt[2]*Sqrt[-9 - Co
s[2*x]])
```

Maple [A] time = 0.266, size = 130, normalized size = 1.2

$$\frac{2}{5\sin(x)}\sqrt{-5+(\sin(x))^2}+2\sqrt{-5+(\sin(x))^2}-2\ln\left(\sin(x)+\sqrt{-5+(\sin(x))^2}\right)+\frac{2\sqrt{5}}{5}\arctan\left(\sqrt{5}\frac{1}{\sqrt{-5+(\sin(x))^2}}\right)+\frac{1}{10\cos(x)}\sqrt{\left(-5+(\sin(x))^2\right)(\cos(x))^2}\left(\sqrt{5}\arctan\left(\frac{\sqrt{5}\left(3(\sin(x))^2-5\right)}{5}\frac{1}{\sqrt{-(\cos(x))^4-4(\cos(x))^2}}\right)+10\arcsin\left(1+1/\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^(1/2),x)
```

```
[Out] 2/5*(-5+sin(x)^2)^(1/2)/sin(x)+2*(-5+sin(x)^2)^(1/2)-2*ln(sin(x)+
(-5+sin(x)^2)^(1/2))+2/5*5^(1/2)*arctan(5^(1/2)/(-5+sin(x)^2)^(1/
```

```
2))+1/10*((-5+sin(x)^2)*cos(x)^2)^(1/2)*(5^(1/2)*arctan(1/5*5^(1/
2)*(3*sin(x)^2-5)/(-cos(x)^4-4*cos(x)^2)^(1/2))+10*arcsin(1+1/2*c
os(x)^2))/cos(x)/(-5+sin(x)^2)^(1/2)
```

Maxima [A] time = 1.55068, size = 155, normalized size = 1.4

$$\frac{2}{5}\sqrt{5}\arcsin\left(\frac{\sqrt{5}}{|\sin(x)|}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(\frac{\cos(x)}{2(\cos(x)+1)}-\frac{2}{\cos(x)+1}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(-\frac{\cos(x)}{2(\cos(x)-1)}-\frac{2}{\cos(x)-1}\right)+2\sqrt{\sin(x)^2-5}+\frac{2\sqrt{\sin(x)^2-5}}{5\sin(x)}-2i\operatorname{arsinh}\left(\frac{1}{2}\cos(x)\right)-2\log\left(2\sqrt{\sin(x)^2-5}+2\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)*sin(x
```

```
[Out] 2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(
1/2*cos(x)/(cos(x) + 1) - 2/(cos(x) + 1)) - 1/10*I*sqrt(5)*arcsin
h(-1/2*cos(x)/(cos(x) - 1) - 2/(cos(x) - 1)) + 2*sqrt(sin(x)^2 -
5) + 2/5*sqrt(sin(x)^2 - 5)/sin(x) - 2*I*arcsinh(1/2*cos(x)) - 2*
log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))
```

Fricas [A] time = 1.4217, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)*sin(x
```

```
[Out] 0
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-
5+sin(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.435009, size = 356, normalized size = 3.21

$$\pi\operatorname{sign}\left(-2i\sqrt{\cos(x)^2+4}-4i\right)\operatorname{sign}(\cos(x)) - \frac{1}{5}\sqrt{5}\left(\pi\operatorname{sign}\left(-2i\sqrt{\cos(x)^2+4}-4i\right)\operatorname{sign}(\cos(x)) + 2\arctan\left(\frac{\sqrt{5}\left(\frac{\left(i\sqrt{\cos(x)^2+4}+2i\right)^2}{\cos(x)^2}-1\right)\cos(x)}{5\left(-2i\sqrt{\cos(x)^2+4}-4i\right)}\right)\right) + \frac{1}{10}\sqrt{5}\left(\pi\operatorname{sign}\left(-2i\sqrt{\cos(x)^2+4}-4i\right)\operatorname{sign}(\cos(x)) + 2\arctan\left(\frac{\sqrt{5}\left(\frac{\left(-i\sqrt{\cos(x)^2+4}-2i\right)^2}{\cos(x)^2}-1\right)\cos(x)}{5\left(-2i\sqrt{\cos(x)^2+4}-4i\right)}\right)\right) - \frac{2}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\sqrt{\sin(x)^2-5}\right) + 2\sqrt{\sin(x)^2-5} + \frac{4}{\left(\sqrt{\sin(x)^2-5}-\sin(x)\right)^2+5} + 2\arctan\left(\frac{\left(\frac{\left(i\sqrt{\cos(x)^2+4}+2i\right)^2}{\cos(x)^2}-1\right)\cos(x)}{-2i\sqrt{\cos(x)^2+4}-4i}\right) + \ln\left(\left(\sqrt{\sin(x)^2-5}-\sin(x)\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)*sin(x
```

```
[Out] pi*sign(-2*I*sqrt(cos(x)^2 + 4) - 4*I)*sign(cos(x)) - 1/5*sqrt(5)
*(pi*sign(-2*I*sqrt(cos(x)^2 + 4) - 4*I)*sign(cos(x)) + 2*arctan(
1/5*sqrt(5)*((I*sqrt(cos(x)^2 + 4) + 2*I)^2/cos(x)^2 - 1)*cos(x)/
(-2*I*sqrt(cos(x)^2 + 4) - 4*I))) + 1/10*sqrt(5)*(pi*sign(-2*I*sq
rt(cos(x)^2 + 4) - 4*I)*sign(cos(x)) + 2*arctan(1/5*sqrt(5)*((-I*
sqrt(cos(x)^2 + 4) - 2*I)^2/cos(x)^2 - 1)*cos(x)/(-2*I*sqrt(cos(x
)^2 + 4) - 4*I))) - 2/5*sqrt(5)*arctan(1/5*sqrt(5)*sqrt(sin(x)^2
- 5)) + 2*sqrt(sin(x)^2 - 5) + 4/((sqrt(sin(x)^2 - 5) - sin(x))^2
 + 5) + 2*arctan(((I*sqrt(cos(x)^2 + 4) + 2*I)^2/cos(x)^2 - 1)*co
s(x)/(-2*I*sqrt(cos(x)^2 + 4) - 4*I)) + ln((sqrt(sin(x)^2 - 5) -
sin(x))^2)
```

3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

Optimal. Leaf size=112

$$\frac{5\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{1}{2}\sin(x)\sqrt{4\cos^2(x)-1}-\frac{1}{2}\sin(x)\sqrt{8\cos^2(x)-1}-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{4\cos^2(x)-1}}\right)-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{8\cos^2(x)-1}}\right)$$

```
[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])
/Sqrt[3]])/4 - (3*ArcTan[Sin[x]/Sqrt[-1 + 4*Cos[x]^2]])/4 - (3*Ar
cTan[Sin[x]/Sqrt[-1 + 8*Cos[x]^2]])/4 - (Sqrt[-1 + 4*Cos[x]^2]*Si
n[x])/2 - (Sqrt[-1 + 8*Cos[x]^2]*Sin[x])/2
```

Rubi [A] time = 0.818783, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{5\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right)-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

```
[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])
/Sqrt[3]])/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*Arc
Tan[Sin[x]/Sqrt[3 - 4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2
])/2 - (Sin[x]*Sqrt[3 - 4*Sin[x]^2])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/
```

```
[Out] Timed out
```

Mathematica [C] time = 0.350557, size = 131, normalized size = 1.17

$$\frac{1}{8}\left(-4\sin(x)\sqrt{2\cos(2x)+1}-4\sin(x)\sqrt{4\cos(2x)+3}-6i\log\left(\sqrt{2\cos(2x)+1}+2i\sin(x)\right)-5i\sqrt{2}\log\left(\sqrt{4\cos(2x)+3}+2i\sqrt{2}\sin(x)\right)-6\tan^{-1}\left(\frac{\sin(x)}{\sqrt{2\cos(2x)+1}}\right)-6\tan^{-1}\left(\frac{\sin(x)}{\sqrt{4\cos(2x)+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

```
[Out] (-6*ArcTan[Sin[x]/Sqrt[1 + 2*Cos[2*x]]] - 6*ArcTan[Sin[x]/Sqrt[3
+ 4*Cos[2*x]]] - (6*I)*Log[Sqrt[1 + 2*Cos[2*x]] + (2*I)*Sin[x]] -
 (5*I)*Sqrt[2]*Log[Sqrt[3 + 4*Cos[2*x]] + (2*I)*Sqrt[2]*Sin[x]] -
 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4*Sqrt[3 + 4*Cos[2*x]]*Sin[x])/8
```

Maple [C] time = 4.377, size = 98136, normalized size = 876.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)),x, algo
```

```
[Out] Timed out
```

Fricas [A] time = 0.277455, size = 340, normalized size = 3.04

$$-\frac{5}{32}\sqrt{2}\arctan\left(\frac{512\sqrt{2}\cos(x)^4-576\sqrt{2}\cos(x)^2+113\sqrt{2}}{16\left(16\cos(x)^2-9\right)\sqrt{8\cos(x)^2-1}\sin(x)}\right) -\frac{1}{2}\sqrt{8\cos(x)^2-1}\sin(x)-\frac{1}{2}\sqrt{4\cos(x)^2-1}\sin(x) -\frac{3}{8}\arctan\left(-\frac{\left(16\cos(x)^2-5\right)\sqrt{4\cos(x)^2-1}\sin(x)-2\left(16\cos(x)^3-7\cos(x)\right)\sin(x)}{32\cos(x)^4-30\cos(x)^2-\left(16\cos(x)^3-13\cos(x)\right)\sqrt{4\cos(x)^2-1}+4}\right) -\frac{3}{4}\arctan\left(\frac{4\cos(x)^2-2\sqrt{4\cos(x)^2-1}\cos(x)-1}{2\cos(x)\sin(x)-\sqrt{4\cos(x)^2-1}\sin(x)}\right) +\frac{3}{8}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{3}{8}\arctan\left(\frac{9\cos(x)^2-2}{2\sqrt{8\cos(x)^2-1}\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)),x, algo
```

```
[Out] -5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos
(x)^2 + 113*sqrt(2))/((16*cos(x)^2 - 9)*sqrt(8*cos(x)^2 - 1)*sin(
x))) - 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)
*sin(x) - 3/8*arctan(-((16*cos(x)^2 - 5)*sqrt(4*cos(x)^2 - 1)*sin
(x) - 2*(16*cos(x)^3 - 7*cos(x))*sin(x))/(32*cos(x)^4 - 30*cos(x)
^2 - (16*cos(x)^3 - 13*cos(x))*sqrt(4*cos(x)^2 - 1) + 4)) - 3/4*a
rctan((4*cos(x)^2 - 2*sqrt(4*cos(x)^2 - 1)*cos(x) - 1)/(2*cos(x)*
sin(x) - sqrt(4*cos(x)^2 - 1)*sin(x))) + 3/8*arctan(sin(x)/cos(x)
) + 3/8*arctan(1/2*(9*cos(x)^2 - 2)/(sqrt(8*cos(x)^2 - 1)*sin(x))
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(3x)}{\sqrt{-\sin^2(x)+3\cos^2(x)}-\sqrt{8\cos^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2)),x
```

```
[Out] Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)
**2 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\cos(3x)}{\sqrt{8\cos(x)^2-1}-\sqrt{3\cos(x)^2-\sin(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)),x, algo
```

```
[Out] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin
(x)^2)), x)
```

3.428 $\int \left(2-3 \sin^2(x)\right)^{3/5} \sin(4x)\, dx$

Optimal. Leaf size=33

$$\frac{5}{36}\left(2-3 \sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3 \sin^2(x)\right)^{13/5}$$

```
[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117
```

Rubi [A] time = 0.110782, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{5}{36}\left(2-3 \sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3 \sin^2(x)\right)^{13/5}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]

[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)

[Out] Timed out
```

Mathematica [A] time = 0.158764, size = 64, normalized size = 1.94

$$(3 \cos(2x)+1)^{3/5}\left(-\frac{5 \cos(2x)}{104\ 2^{3/5}}-\frac{5 \cos(4x)}{26\ 2^{3/5}}\right)-\frac{155\left((3 \cos(2x)+1)^{3/5}-1\right)}{936\ 2^{3/5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]

[Out] (-155*(-1 + (1 + 3*Cos[2*x])^(3/5)))/(936*2^(3/5)) + (1 + 3*Cos[2
*x])^(3/5)*((-5*Cos[2*x])/(104*2^(3/5)) - (5*Cos[4*x])/(26*2^(3/5
```

```
)))
```

Maple [A] time = 0.142, size = 38, normalized size = 1.2

$$\frac{5}{12}\left(3\left(\cos\left(x\right)\right)^{2}-1\right)^{\frac{8}{5}}-\frac{20}{117}\left(\frac{3\cos\left(2x\right)}{2}+\frac{1}{2}\right)^{\frac{13}{5}}-\frac{5}{18}\left(\frac{3\cos\left(2x\right)}{2}+\frac{1}{2}\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2-3*sin(x)^2)^(3/5)*sin(4*x),x)
```

```
[Out] 5/12*(3*cos(x)^2-1)^(8/5)-20/117*(3/2*cos(2*x)+1/2)^(13/5)-5/18*(
3/2*cos(2*x)+1/2)^(8/5)
```

Maxima [A] time = 1.48325, size = 34, normalized size = 1.03

$$-\frac{20}{117}\left(-3\sin\left(x\right)^{2}+2\right)^{\frac{13}{5}}+\frac{5}{36}\left(-3\sin\left(x\right)^{2}+2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*sin(x)^2 + 2)^(3/5)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -20/117*(-3*sin(x)^2 + 2)^(13/5) + 5/36*(-3*sin(x)^2 + 2)^(8/5)
```

Fricas [A] time = 0.23286, size = 35, normalized size = 1.06

$$-\frac{5}{468}\left(144\cos\left(x\right)^{4}-135\cos\left(x\right)^{2}+29\right)\left(3\cos\left(x\right)^{2}-1\right)^{\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*sin(x)^2 + 2)^(3/5)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -5/468*(144*cos(x)^4 - 135*cos(x)^2 + 29)*(3*cos(x)^2 - 1)^(3/5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3\,\sin\left(x\right)^{2}+2\right)^{\frac{3}{5}}\sin\left(4\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*sin(x)^2 + 2)^(3/5)*sin(4*x),x, algorithm="giac")
```

```
[Out] integrate((-3*sin(x)^2 + 2)^(3/5)*sin(4*x), x)
```

3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$

Optimal. Leaf size=33

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

```
[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Rubi [A] time = 0.0384552, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Sqrt[Cos[2*x]],x]

[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(2*x)**(1/2),x)

[Out] Timed out
```

Mathematica [A] time = 0.0705124, size = 33, normalized size = 1.

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Maple [B] time = 0.086, size = 62, normalized size = 1.9

$$-\frac{1}{8\sin(x)}\sqrt{\left(2\,(\cos(x))^2-1\right)(\sin(x))^2}\left(-\sqrt{2}\arcsin\left(4\,(\sin(x))^2-1\right)-4\sqrt{-2\,(\sin(x))^4+(\sin(x))^2}\right)\frac{1}{\sqrt{2\,(\cos(x))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(2*x)^(1/2),x)
```

```
[Out] -1/8*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(-2^(1/2)*arcsin(4*sin(x)^2-
1)-4*(-2*sin(x)^4+sin(x)^2)^(1/2))/sin(x)/(2*cos(x)^2-1)^(1/2)
```

Maxima [A] time = 1.95078, size = 659, normalized size = 19.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(2*x))*cos(x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*
(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - (cos(2*x) -
1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + arctan2(-(cos(4*x)
^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x)
, cos(4*x) + 1))*sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), co
s(4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(
cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(
1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 1) - arctan2(-(cos(4*x)^2
 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x),
cos(4*x) + 1))*sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), cos(
4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(co
s(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1))) - 1) - arctan2((cos(4*x)^2 +
sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos(
4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(
1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + arctan2((cos(4*x)^2 +
 sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos
(4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos
(1/2*arctan2(sin(4*x), cos(4*x) + 1)) - 1))
```

Fricas [A] time = 0.238081, size = 96, normalized size = 2.91

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}}{8\left(4\cos(x)^2-3\right)\sqrt{2\cos(x)^2-1}\sin(x)}\right)+\frac{1}{2}\sqrt{2\cos(x)^2-1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(2*x))*cos(x),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)
^2 + 17*sqrt(2))/((4*cos(x)^2 - 3)*sqrt(2*cos(x)^2 - 1)*sin(x)))
+ 1/2*sqrt(2*cos(x)^2 - 1)*sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos(x)\sqrt{\cos(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*cos(2*x)**(1/2),x)
```

```
[Out] Integral(cos(x)*sqrt(cos(2*x)), x)
```

GIAC/XCAS [A] time = 0.207651, size = 36, normalized size = 1.09

$$\frac{1}{4}\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)+\frac{1}{2}\sqrt{-2\sin(x)^2+1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(2*x))*cos(x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*si
n(x)
```

3.430 $\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$

Optimal. Leaf size=55

$$-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]])/(8*Sqrt[2]) + (3*Co
s[x]*Sqrt[Cos[2*x]])/8 - (Cos[x]*Cos[2*x]^(3/2))/4
```

Rubi [A] time = 0.0574155, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]])/(8*Sqrt[2]) + (3*Co
s[x]*Sqrt[Cos[2*x]])/8 - (Cos[x]*Cos[2*x]^(3/2))/4
```

Rubi in Sympy [A] time = 112.273, size = 65, normalized size = 1.18

$$-\frac{\left(2\cos^{2}(x)-1\right)^{\frac{3}{2}}\cos(x)}{4}+\frac{3\sqrt{2\cos^{2}(x)-1}\cos(x)}{8}-\frac{3\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{2\cos^{2}(x)-1}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(2*x)**(3/2)*sin(x),x)
```

```
[Out] -(2*cos(x)**2 - 1)**(3/2)*cos(x)/4 + 3*sqrt(2*cos(x)**2 - 1)*cos(
x)/8 - 3*sqrt(2)*atanh(sqrt(2)*cos(x)/sqrt(2*cos(x)**2 - 1))/16
```

Mathematica [A] time = 0.121083, size = 49, normalized size = 0.89

$$-\frac{1}{8}\sqrt{\cos(2x)}(\cos(3x)-2\cos(x))-\frac{3\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] -(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x]))/8 - (3*Log[Sqrt[2]*Cos[x
] + Sqrt[Cos[2*x]]])/(8*Sqrt[2])
```

Maple [A] time = 0.06, size = 55, normalized size = 1.

$$\frac{5\cos(x)}{8}\sqrt{2(\cos(x))^2-1}-\frac{(\cos(x))^3}{2}\sqrt{2(\cos(x))^2-1}-\frac{3\sqrt{2}}{16}\ln\left(\cos(x)\sqrt{2}+\sqrt{2(\cos(x))^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(2*x)^(3/2)*sin(x),x)
```

```
[Out] 5/8*cos(x)*(2*cos(x)^2-1)^(1/2)-1/2*cos(x)^3*(2*cos(x)^2-1)^(1/2)
-3/16*ln(cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)
```

Maxima [A] time = 2.28214, size = 1067, normalized size = 19.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4
)*(((cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x
)*sin(1/2*arctan2(sin(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*ar
ctan2(sin(4*x), cos(4*x) + 1)) - (cos(1/2*arctan2(sin(4*x), cos(4
*x)))*sin(4*x) - (cos(4*x) - 2)*sin(1/2*arctan2(sin(4*x), cos(4*x
))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 3*log
(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(s
in(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(
4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x
)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x)
, cos(4*x) + 1)) + 1) - 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*co
s(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos
(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x),
cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(
1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + 3*log(((cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), co
s(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2
*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4
*x)))^2)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x
```

```
)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 +
 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*
cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4*x),
cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1) - 3*log
(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4
*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (
cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x)
, cos(4*x)))^2)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(
cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) - 2*(cos(4*x)^2 + sin(4
*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x)
 + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1))
```

Fricas [A] time = 0.243426, size = 142, normalized size = 2.58

$$-\frac{1}{8}\left(4\cos\left(x\right)^{3}-5\cos\left(x\right)\right)\sqrt{2\cos\left(x\right)^{2}-1}+\frac{3}{128}\sqrt{2}\log\left(2048\sqrt{2}\cos\left(x\right)^{8}-2048\sqrt{2}\cos\left(x\right)^{6}+640\sqrt{2}\cos\left(x\right)^{4}-64\sqrt{2}\cos\left(x\right)^{2}-16\left(128\cos\left(x\right)^{7}-96\cos\left(x\right)^{5}+20\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sqrt{2\cos\left(x\right)^{2}-1}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/8*(4*cos(x)^3 - 5*cos(x))*sqrt(2*cos(x)^2 - 1) + 3/128*sqrt(2)
*log(2048*sqrt(2)*cos(x)^8 - 2048*sqrt(2)*cos(x)^6 + 640*sqrt(2)*
cos(x)^4 - 64*sqrt(2)*cos(x)^2 - 16*(128*cos(x)^7 - 96*cos(x)^5 +
 20*cos(x)^3 - cos(x))*sqrt(2*cos(x)^2 - 1) + sqrt(2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)**(3/2)*sin(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218005, size = 65, normalized size = 1.18

$$-\frac{1}{8}\left(4\cos\left(x\right)^{2}-5\right)\sqrt{2\cos\left(x\right)^{2}-1}\cos\left(x\right)+\frac{3}{16}\sqrt{2}\ln\left(\left|-\sqrt{2}\cos\left(x\right)+\sqrt{2\cos\left(x\right)^{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="giac")
```

```
[Out] -1/8*(4*cos(x)^2 - 5)*sqrt(2*cos(x)^2 - 1)*cos(x) + 3/16*sqrt(2)*
ln(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1)))
```

3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

```
[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Rubi [A] time = 0.0238499, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/Cos[2*x]^(5/2),x]

[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Rubi in Sympy [A] time = 1.31778, size = 15, normalized size = 0.94

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/cos(2*x)**(5/2),x)

[Out] -cos(3*x)/(3*cos(2*x)**(3/2))
```

Mathematica [A] time = 0.0370278, size = 16, normalized size = 1.

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/Cos[2*x]^(5/2),x]
```

```
[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Maple [B] time = 0.106, size = 39, normalized size = 2.4

$$\frac{\cos(x)\left(4\,(\sin(x))^2-1\right)}{12\,(\sin(x))^4-12\,(\sin(x))^2+3}\sqrt{-2\,(\sin(x))^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/cos(2*x)^(5/2),x)
```

```
[Out] 1/3/(4*sin(x)^4-4*sin(x)^2+1)*(-2*sin(x)^2+1)^(1/2)*cos(x)*(4*sin
(x)^2-1)
```

Maxima [A] time = 1.99301, size = 122, normalized size = 7.62

$$-\frac{\sqrt{2}\sin\left(\frac{3}{2}\arctan\left(\sin(4x),\cos(4x)+1\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(4x),\cos(4x)\right)\right)+\left(\sqrt{2}\cos\left(\frac{3}{2}\arctan\left(\sin(4x),\cos(4x)\right)\right)+\right.}{3\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(2)*sin(3/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(3/2*ar
ctan2(sin(4*x), cos(4*x))) + (sqrt(2)*cos(3/2*arctan2(sin(4*x), c
os(4*x))) + sqrt(2))*cos(3/2*arctan2(sin(4*x), cos(4*x) + 1)))/(c
os(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(3/4)
```

Fricas [A] time = 0.245072, size = 53, normalized size = 3.31

$$-\frac{\left(4\,\cos(x)^3-3\,\cos(x)\right)\sqrt{2\,\cos(x)^2-1}}{3\left(4\,\cos(x)^4-4\,\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*cos(x)^3 - 3*cos(x))*sqrt(2*cos(x)^2 - 1)/(4*cos(x)^4 - 4
*cos(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.262174, size = 62, normalized size = 3.88

$$\frac{\left(\left(\tan\left(\frac{1}{2}x\right)^2 - 15\right)\tan\left(\frac{1}{2}x\right)^2 + 15\right)\tan\left(\frac{1}{2}x\right)^2 - 1}{3\left(\tan\left(\frac{1}{2}x\right)^4 - 6\tan\left(\frac{1}{2}x\right)^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((tan(1/2*x)^2 - 15)*tan(1/2*x)^2 + 15)*tan(1/2*x)^2 - 1)/(t
an(1/2*x)^4 - 6*tan(1/2*x)^2 + 1)^(3/2)
```

3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$

Optimal. Leaf size=49

$$2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

```
[Out] 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - (5*ArcTan[Sin[x]/Sqrt[Cos[2*x]
]])/2 - (Sqrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.143748, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - (5*ArcTan[Sin[x]/Sqrt[Cos[2*x]
]])/2 - (Sqrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.119188, size = 49, normalized size = 1.

$$\frac{1}{2}\left(4\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-5\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\sqrt{\cos(2x)}\tan(x)\sec(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] (4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]
]] - Sqrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Maple [B] time = 0.111, size = 100, normalized size = 2.

$$-\frac{1}{4\left(\cos\left(x\right)\right)^{2}\sin\left(x\right)}\sqrt{\left(2\left(\cos\left(x\right)\right)^{2}-1\right)\left(\sin\left(x\right)\right)^{2}}\left(4\sqrt{2}\arcsin\left(4\left(\cos\left(x\right)\right)^{2}-3\right)\left(\cos\left(x\right)\right)^{2}-5\arctan\left(1/2\frac{3\left(\cos\left(x\right)\right)^{2}}{\sqrt{-2\left(\sin\left(x\right)\right)^{4}+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(2*x)^(3/2)/cos(x)^3,x)
```

```
[Out] -1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2
-3)*cos(x)^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(
1/2))*cos(x)^2+2*(-2*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2
*cos(x)^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(2x\right)^{\frac{3}{2}}}{\cos\left(x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

Fricas [A] time = 0.248311, size = 147, normalized size = 3.

$$-\frac{2\sqrt{2}\arctan\left(\frac{32\cos(x)^{4}-48\cos(x)^{2}+17}{4\left(4\sqrt{2}\cos(x)^{2}-3\sqrt{2}\right)\sqrt{2\cos(x)^{2}-1}\sin(x)}\right)\cos\left(x\right)^{2}-5\arctan\left(\frac{3\cos(x)^{2}-2}{2\sqrt{2\cos(x)^{2}-1}\sin(x)}\right)\cos\left(x\right)^{2}+2\sqrt{2\cos\left(x\right)^{2}-1}\sin\left(x\right)}{4\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/4*(32*cos(x)^4 - 48*cos(x)^2 + 17)/((4*s
qrt(2)*cos(x)^2 - 3*sqrt(2))*sqrt(2*cos(x)^2 - 1)*sin(x)))*cos(x)
^2 - 5*arctan(1/2*(3*cos(x)^2 - 2)/(sqrt(2*cos(x)^2 - 1)*sin(x)))
*cos(x)^2 + 2*sqrt(2*cos(x)^2 - 1)*sin(x))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

3.433 $\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=87

$$-\frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)}-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{63\cos(x)}{20\sqrt{\cos(2x)}}+\frac{3\sin^2(x)\cos(x)}{10\cos^{\frac{5}{2}}(2x)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]/Sqrt[2]) - (11*Cos[x])
/(20*Cos[2*x]^(3/2)) - (2*Cos[x]^3)/(3*Cos[2*x]^(3/2)) + (63*Cos[
x])/(20*Sqrt[Cos[2*x]]) + (3*Cos[x]*Sin[x]^2)/(10*Cos[2*x]^(5/2))
```

Rubi [A] time = 0.363642, antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}+\frac{3\sin^4(x)\cos(x)}{5\cos^{\frac{5}{2}}(2x)}-\frac{4\sin^2(x)\cos(x)}{5\cos^{\frac{3}{2}}(2x)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]/Sqrt[2]) - (2*Cos[x]^3
)/(3*Cos[2*x]^(3/2)) + (13*Cos[x])/(5*Sqrt[Cos[2*x]]) - (4*Cos[x]
*Sin[x]^2)/(5*Cos[2*x]^(3/2)) + (3*Cos[x]*Sin[x]^4)/(5*Cos[2*x]^(
5/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.23835, size = 62, normalized size = 0.71

$$\frac{250\cos(x)+45\cos(3x)+169\cos(5x)-120\sqrt{2}\cos^{\frac{5}{2}}(2x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{240\cos^{\frac{5}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] (250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(
5/2)*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(240*Cos[2*x]^(5/2))
```

Maple [B] time = 0.233, size = 180, normalized size = 2.1

$$-\frac{1}{240\,(\sin(x))^6-360\,(\sin(x))^4+180\,(\sin(x))^2-30}\left(120\,\ln\left(\cos(x)\sqrt{2}+\sqrt{-2\,(\sin(x))^2+1}\right)\sqrt{2}(\sin(x))^6+338\sqrt{-2\,(\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x)
```

```
[Out] -1/30/(8*sin(x)^6-12*sin(x)^4+6*sin(x)^2-1)*(120*ln(cos(x)*2^(1/2
)+(-2*sin(x)^2+1)^(1/2))*2^(1/2)*sin(x)^6+338*(-2*sin(x)^2+1)^(1/
2)*cos(x)*sin(x)^4-180*ln(cos(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*2
^(1/2)*sin(x)^4-276*(-2*sin(x)^2+1)^(1/2)*sin(x)^2*cos(x)+90*ln(c
os(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*2^(1/2)*sin(x)^2+58*(-2*sin(
x)^2+1)^(1/2)*cos(x)-15*ln(cos(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*
2^(1/2))
```

Maxima [A] time = 2.23118, size = 1832, normalized size = 21.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3 - cos(x)*sin(4*x))/(cos(2*x)^(7/2)*csc(x)^2),x, algorithm
```

```
[Out] 1/48*(((16*sqrt(2)) + 16*sqrt(2)*sin(4*x)*sin(5/2*arctan2(sin(4*x
), cos(4*x))) + ((16*sqrt(2)) + 16*sqrt(2)*cos(4*x))*cos(5/2*arct
an2(sin(4*x), cos(4*x))) + 12*sqrt(2)*cos(8*x) + 28*sqrt(2)*cos(4
*x))*cos(5/2*arctan2(sin(4*x), cos(4*x) + 1)) + 12*sqrt(2)*sqrt(c
os(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*arctan2(sin(4*x)
, cos(4*x) + 1)) - 12*(sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 +
2*sqrt(2)*cos(4*x) + sqrt(2))*cos(1/2*arctan2(sin(4*x), cos(4*x)
```

```
+ 1)) - (16*sqrt(2)*cos(5/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x)
 - ((16*sqrt(2)) + 16*sqrt(2)*cos(4*x))*sin(5/2*arctan2(sin(4*x),
 cos(4*x))) - 12*sqrt(2)*sin(8*x) - 28*sqrt(2)*sin(4*x))*sin(5/2*
arctan2(sin(4*x), cos(4*x) + 1)) - 3*(cos(4*x)^2 + sin(4*x)^2 + 2
*cos(4*x) + 1)^(1/4)*((sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 +
2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt
(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*
x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - (sqrt(2)
*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*
log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan
2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*c
os(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(
4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4
*x), cos(4*x) + 1)) + 1) + (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)
^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(((cos(1/2*arctan2(sin(4*x)
, cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*cos(1/2*
arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x), c
os(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arc
tan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2
*cos(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/
4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(
4*x), cos(4*x))) + sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1
/2*arctan2(sin(4*x), cos(4*x)))) + 1) - (sqrt(2)*cos(4*x)^2 + sqr
t(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(((cos(1/2*arc
tan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x))
)^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan
2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + s
in(4*x)^2 + 2*cos(4*x) + 1) - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(
4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2
*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4*x), cos(4*x
) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1)))/(cos(4*x)^2
+ sin(4*x)^2 + 2*cos(4*x) + 1)^(5/4) + 1/20*(((15*cos(8*x) + 70*c
os(4*x) + 43)*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 5*(3*sin(8*x
) + 14*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 12)*cos(5
/2*arctan2(sin(4*x), cos(4*x) + 1)) + 15*(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) - (5*
(3*sin(8*x) + 14*sin(4*x))*cos(5/2*arctan2(sin(4*x), cos(4*x))) -
 (15*cos(8*x) + 70*cos(4*x) + 43)*sin(5/2*arctan2(sin(4*x), cos(4
*x))))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1)) + 40*sqrt(cos(4*x
)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*arctan2(sin(4*x), cos(
4*x) + 1)))/((sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)
*cos(4*x) + sqrt(2))*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(
1/4))
```

Fricas [A] time = 5.99466, size = 223, normalized size = 2.56

$$\frac{15\left(8\sqrt{2}\cos(x)^6 - 12\sqrt{2}\cos(x)^4 + 6\sqrt{2}\cos(x)^2 - \sqrt{2}\right)\log\left(2048\sqrt{2}\cos(x)^8 - 2048\sqrt{2}\cos(x)^6 + 640\sqrt{2}\cos(x)^4 - 64\sqrt{2}\right.}{240\,(8\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3 - cos(x)*sin(4*x))/(cos(2*x)^(7/2)*csc(x)^2),x, algorithm
```

```
[Out] 1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*c
os(x)^2 - sqrt(2))*log(2048*sqrt(2)*cos(x)^8 - 2048*sqrt(2)*cos(x
)^6 + 640*sqrt(2)*cos(x)^4 - 64*sqrt(2)*cos(x)^2 - 16*(128*cos(x)
^7 - 96*cos(x)^5 + 20*cos(x)^3 - cos(x))*sqrt(2*cos(x)^2 - 1) + s
qrt(2)) + 16*(169*cos(x)^5 - 200*cos(x)^3 + 60*cos(x))*sqrt(2*cos
(x)^2 - 1))/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225052, size = 74, normalized size = 0.85

$$\frac{1}{2}\sqrt{2}\ln\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)+\frac{\left(\left(169\cos(x)^2-200\right)\cos(x)^2+60\right)\cos(x)}{15\left(2\cos(x)^2-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3 - cos(x)*sin(4*x))/(cos(2*x)^(7/2)*csc(x)^2),x, algorithm
```

```
[Out] 1/2*sqrt(2)*ln(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/1
5*((169*cos(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5
/2)
```

3.434 $\int \left(4-5 \sec^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=68

$$-\frac{5}{2} \tan(x)\sqrt{-5 \tan^2(x)-1}+8 \tan^{-1}\left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{7}{2}\sqrt{5} \tan^{-1}\left(\frac{\sqrt{5} \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)$$

```
[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(S
qrt[5]*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*
Tan[x]^2])/2
```

Rubi [A] time = 0.128738, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{5}{2} \tan(x)\sqrt{-5 \tan^2(x)-1}+8 \tan^{-1}\left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{7}{2}\sqrt{5} \tan^{-1}\left(\frac{\sqrt{5} \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(S
qrt[5]*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*
Tan[x]^2])/2
```

Rubi in Sympy [A] time = 8.19377, size = 73, normalized size = 1.07

$$-\frac{5\sqrt{-5 \tan^2(x)-1}\tan(x)}{2}+8 \operatorname{atan}\left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{7\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4-5*sec(x)**2)**(3/2),x)
```

```
[Out] -5*sqrt(-5*tan(x)**2 - 1)*tan(x)/2 + 8*atan(2*tan(x)/sqrt(-5*tan(
x)**2 - 1)) - 7*sqrt(5)*atan(sqrt(5)*tan(x)/sqrt(-5*tan(x)**2 - 1
))/2
```

Mathematica [C] time = 0.278456, size = 115, normalized size = 1.69

$$-\frac{\left(4\cos^2(x)-5\right)\sec(x)\sqrt{4-5\sec^2(x)}\left(5\sin(x)\sqrt{2\cos(2x)-3}+16i\cos^2(x)\log\left(\sqrt{2\cos(2x)-3}+2i\sin(x)\right)+7\sqrt{5}\cos^2(x)\right.}{2(2\cos(2x)-3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] -((-5 + 4*Cos[x]^2)*Sec[x]*Sqrt[4 - 5*Sec[x]^2]*(7*Sqrt[5]*ArcTan
[(Sqrt[5]*Sin[x])/Sqrt[-3 + 2*Cos[2*x]]]*Cos[x]^2 + (16*I)*Cos[x]
^2*Log[Sqrt[-3 + 2*Cos[2*x]] + (2*I)*Sin[x]] + 5*Sqrt[-3 + 2*Cos[
2*x]]*Sin[x]))/(2*(-3 + 2*Cos[2*x])^(3/2))
```

Maple [C] time = 0.572, size = 754, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4-5*sec(x)^2)^(3/2),x)
```

```
[Out] -1/2*I/(-9-4*5^(1/2))^(1/2)/(2+5^(1/2))*(-70*I*5^(1/2)*(-2*(2*5^(
1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*((2*5^(1/2)*c
os(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticPi((-9-4*5^
(1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1
/2)/(-9-4*5^(1/2))^(1/2))*sin(x)*cos(x)^2*2^(1/2)+3*I*5^(1/2)*(-2
*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*((2*5^
(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticF(I*
(2+5^(1/2))*(cos(x)-1)/sin(x),9-4*5^(1/2))*sin(x)*cos(x)^2*2^(1/2
)+64*I*5^(1/2)*(-2*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos
(x)))^(1/2)*((2*5^(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^
(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5^
(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*sin(x)*cos(x)^2
*2^(1/2)-140*I*(-2*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos
(x)))^(1/2)*((2*5^(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^
(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(9+4*5
^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*sin(x)*cos(x)^
2*2^(1/2)+6*I*(-2*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(
x)))^(1/2)*((2*5^(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(
1/2)*EllipticF(I*(2+5^(1/2))*(cos(x)-1)/sin(x),9-4*5^(1/2))*sin(x
)*cos(x)^2*2^(1/2)+128*I*(-2*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)
-5)/(1+cos(x)))^(1/2)*((2*5^(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1
+cos(x)))^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x)
,1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*sin(x
)*cos(x)^2*2^(1/2)+80*5^(1/2)*cos(x)^3-80*5^(1/2)*cos(x)^2+180*co
s(x)^3-100*5^(1/2)*cos(x)-180*cos(x)^2+100*5^(1/2)-225*cos(x)+225
)*sin(x)*cos(x)*((4*cos(x)^2-5)/cos(x)^2)^(3/2)/(cos(x)-1)/(4*cos
```

```
(x)^2-5)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*sec(x)^2 + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.240001, size = 279, normalized size = 4.1

$$\frac{7\sqrt{5}\arctan\left(\frac{2\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)^2+4\cos(x)^2-5}{\sqrt{5}\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)\sin(x)+2\sqrt{5}\cos(x)\sin(x)}\right)\cos(x)+8\arctan\left(\frac{\left(16\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+2\left(16\cos(x)^3-19\cos(x)\right)}{32\cos(x)^4-54\cos(x)^2+\left(16\cos(x)^4-17\cos(x)^2\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}+2}\right.}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*sec(x)^2 + 4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(7*sqrt(5)*arctan((2*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)^2
 + 4*cos(x)^2 - 5)/(sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x
)*sin(x) + 2*sqrt(5)*cos(x)*sin(x)))*cos(x) + 8*arctan(((16*cos(x
)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) + 2*(16*co
s(x)^3 - 19*cos(x))*sin(x))/(32*cos(x)^4 - 54*cos(x)^2 + (16*cos(
x)^4 - 17*cos(x)^2)*sqrt((4*cos(x)^2 - 5)/cos(x)^2) + 20))*cos(x)
 - 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x
)^2)*sin(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5\sec^2(x)+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4-5*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((-5*sec(x)**2 + 4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5 \sec \left(x\right)^2 + 4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*sec(x)^2 + 4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

3.435 $\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

```
[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-
1 - 5*Tan[x]^2])
```

Rubi [A] time = 0.0635397, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-
1 - 5*Tan[x]^2])
```

Rubi in Sympy [A] time = 4.07887, size = 41, normalized size = 1.02

$$\frac{\operatorname{atan}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)}{8}-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4-5*sec(x)**2)**(3/2),x)
```

```
[Out] atan(2*tan(x)/sqrt(-5*tan(x)**2 - 1))/8 - 5*tan(x)/(4*sqrt(-5*tan
(x)**2 - 1))
```

Mathematica [A] time = 0.118732, size = 64, normalized size = 1.6

$$-\frac{(2\cos(2x)-3)\sec^3(x)\left(10\sin(x)-\sqrt{2\cos(2x)-3}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{2\cos(2x)-3}}\right)\right)}{8\left(4-5\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] -((-3 + 2*Cos[2*x])*Sec[x]^3*(-(ArcTan[(2*Sin[x])/Sqrt[-3 + 2*Cos
[2*x]]]*Sqrt[-3 + 2*Cos[2*x]]) + 10*Sin[x]))/(8*(4 - 5*Sec[x]^2)^
(3/2))
```

Maple [C] time = 0.316, size = 473, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4-5*sec(x)^2)^(3/2),x)
```

```
[Out] 1/4*I/(-9-4*5^(1/2))^(1/2)/(2+5^(1/2))*(4*cos(x)^2-5)*(I*5^(1/2)*
EllipticF(I*(2+5^(1/2))*(cos(x)-1)/sin(x),9-4*5^(1/2))*sin(x)*(-2
*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*((2*5^
(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*2^(1/2)-2*I*
5^(1/2)*(-2*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(
1/2)*2^(1/2)*((2*5^(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))
^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5
^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*sin(x)+2*I*Ell
ipticF(I*(2+5^(1/2))*(cos(x)-1)/sin(x),9-4*5^(1/2))*sin(x)*(-2*(2
*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*((2*5^(1/
2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*2^(1/2)-4*I*(-2
*(2*5^(1/2)*cos(x)-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*2^(1/2
)*((2*5^(1/2)*cos(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*Elli
pticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5^(1/2)),(-9
+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*sin(x)-20*5^(1/2)*cos(x)+
20*5^(1/2)-45*cos(x)+45)*sin(x)/(cos(x)-1)/cos(x)^3/((4*cos(x)^2-
5)/cos(x)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*sec(x)^2 + 4)^(-3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.257376, size = 207, normalized size = 5.18

$$-\frac{20\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)\sin(x)-\left(4\cos(x)^2-5\right)\arctan\left(\frac{\left(16\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+2\left(16\cos(x)^3-19\cos(x)\right)\sin(x)}{32\cos(x)^4-54\cos(x)^2+\left(16\cos(x)^4-17\cos(x)^2\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}+20}\right)+\left(4\cos\right.}{16\left(4\cos(x)^2-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*sec(x)^2 + 4)^(-3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(
x)^2 - 5)*arctan(((16*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/
cos(x)^2)*sin(x) + 2*(16*cos(x)^3 - 19*cos(x))*sin(x))/(32*cos(x)
^4 - 54*cos(x)^2 + (16*cos(x)^4 - 17*cos(x)^2)*sqrt((4*cos(x)^2 -
 5)/cos(x)^2) + 20)) + (4*cos(x)^2 - 5)*arctan(sin(x)/cos(x)))/(4
*cos(x)^2 - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-5\sec^2(x)+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4-5*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((-5*sec(x)**2 + 4)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-5\sec(x)^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*sec(x)^2 + 4)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{8}\cos(x)\sqrt{5\tan^2(x)+1}-\frac{\cos(x)}{4\sqrt{5\tan^2(x)+1}}-\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)+\frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x)-\frac{5\cot(x)}{2\sqrt{5\tan^2(x)+1}}$$

```
[Out] -ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]]/4 - Cos[x]/(4*Sqrt[1 +
5*Tan[x]^2]) - (5*Cot[x])/(2*Sqrt[1 + 5*Tan[x]^2]) - (Cos[x]*Sqrt
[1 + 5*Tan[x]^2])/8 + (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2
```

Rubi [A] time = 0.352963, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}-\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)+\frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x)-\frac{5\cot(x)}{2\sqrt{5\tan^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]]/4 + Cos[x]/(4*Sqrt[-4 +
 5*Sec[x]^2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) - (5*Cot[x])
/(2*Sqrt[1 + 5*Tan[x]^2]) + (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.515383, size = 75, normalized size = 0.8

$$\frac{\csc(x)\sec(x)\left(-9\sin(x)+\sin(3x)-164\cos(2x)-4\sin(x)\sqrt{2\cos(2x)-3}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{2\cos(2x)-3}}\right)+196\right)}{16\sqrt{2\tan^2(x)+3\sec^2(x)-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (Csc[x]*Sec[x]*(196 - 164*Cos[2*x] - 9*Sin[x] - 4*ArcTan[(2*Sin[x
])/Sqrt[-3 + 2*Cos[2*x]]]*Sqrt[-3 + 2*Cos[2*x]]*Sin[x] + Sin[3*x]
))/(16*Sqrt[-2 + 3*Sec[x]^2 + 2*Tan[x]^2])
```

Maple [C] time = 0.76, size = 975, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x)
```

```
[Out] -1/8*I/(2+5^(1/2))^2/(-9+4*5^(1/2))^(1/2)/(-2+5^(1/2))^2/(4*cos(x
)^2-5)^2*(3*I*5^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*arctan
h(1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+
cos(x))^2)^(1/2))*sin(x)*cos(x)-4*I*2^(1/2)*((2*5^(1/2)*cos(x)-4*
cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*5^(1/2)*cos(x)-2*5^(
1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticF(I*(-2+5^(1/2)
)*(cos(x)-1)/sin(x),9+4*5^(1/2))+8*I*2^(1/2)*((2*5^(1/2)*cos(x)-4
*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*5^(1/2)*cos(x)-2*5^
(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticPi((-9+4*5^(1/
2))^(1/2)*(cos(x)-1)/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2
)/(-9+4*5^(1/2))^(1/2))-6*I*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*
arctanh(1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-
5)/(1+cos(x))^2)^(1/2))*sin(x)*cos(x)-4*I*2^(1/2)*((2*5^(1/2)*cos
(x)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*5^(1/2)*cos(x)
-2*5^(1/2)+4*cos(x)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticF(I*(-2+5
^(1/2))*(cos(x)-1)/sin(x),9+4*5^(1/2))*cos(x)+3*5^(1/2)*(-(4*cos(
x)^2-5)/(1+cos(x))^2)^(1/2)*arctan(2*cos(x)*(cos(x)-1)/sin(x)^2/(
-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*sin(x)*cos(x)-6*I*(-(4*cos(x
)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)*arctanh(1/2*(-16)^(1/2)*cos(x)*
(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))+3*I*5^(
1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)*arctanh(1/2*(-16
)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)
^(1/2))+3*5^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)*arc
tan(2*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(
1/2))-2*5^(1/2)*sin(x)*cos(x)^2-6*(-(4*cos(x)^2-5)/(1+cos(x))^2)^
(1/2)*sin(x)*arctan(2*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)
/(1+cos(x))^2)^(1/2))*cos(x)+8*I*2^(1/2)*((2*5^(1/2)*cos(x)-4*cos
```

```
(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*5^(1/2)*cos(x)-2*5^(1/2
)+4*cos(x)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticPi((-9+4*5^(1/2))^
(1/2)*(cos(x)-1)/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-
9+4*5^(1/2))^(1/2))*cos(x)+164*5^(1/2)*cos(x)^2-6*(-(4*cos(x)^2-5
)/(1+cos(x))^2)^(1/2)*sin(x)*arctan(2*cos(x)*(cos(x)-1)/sin(x)^2/
(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))+4*cos(x)^2*sin(x)+5*sin(x)*
5^(1/2)-328*cos(x)^2-180*5^(1/2)-10*sin(x)+360)*cos(x)^3*(-(4*cos
(x)^2-5)/cos(x)^2)^(3/2)/sin(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.296103, size = 131, normalized size = 1.39

$$\frac{2\left(4\cos(x)^2-5\right)\log\left(\sqrt{-\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)-2\sin(x)\right)\sin(x)+\left(164\cos(x)^3-\left(2\cos(x)^3-5\cos(x)\right)\sin(x)-180\cos\right.}{8\left(4\cos(x)^2-5\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(
x) - 2*sin(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*s
in(x) - 180*cos(x))*sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^
2 - 5)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,\cot(x)^2 - \sin(x)}{\left(5\,\tan(x)^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2), x)
```

3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

```
[Out] (-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/
3
```

Rubi [A] time = 0.267486, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$-\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

```
[Out] (-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/
3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.123131, size = 33, normalized size = 0.85

$$\frac{(5\cos(2x)-3)\csc(x)\left(\sec^3(x)+\sec(x)\right)}{6\sqrt{4-\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

```
[Out] ((-3 + 5*Cos[2*x])*Csc[x]*(Sec[x] + Sec[x]^3))/(6*Sqrt[4 - Cot[x]
^2])
```

Maple [A] time = 0.639, size = 61, normalized size = 1.6

$$-\frac{\left(5\left(\cos\left(x\right)\right)^{2}+2\right)\sin\left(x\right)\sqrt{4}}{12\left(\cos\left(x\right)\right)^{3}}\sqrt{-\frac{-4+5\left(\cos\left(x\right)\right)^{2}}{\left(\sin\left(x\right)\right)^{2}}}+\frac{\sin\left(x\right)}{2\cos\left(x\right)}\sqrt{-\frac{-4+5\left(\cos\left(x\right)\right)^{2}}{\left(\sin\left(x\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x)
```

```
[Out] -1/12*(5*cos(x)^2+2)*sin(x)*(-(-4+5*cos(x)^2)/sin(x)^2)^(1/2)*4^(
1/2)/cos(x)^3+1/2*(-(-4+5*cos(x)^2)/sin(x)^2)^(1/2)*sin(x)/cos(x)
```

Maxima [A] time = 2.94487, size = 85, normalized size = 2.18

$$-\frac{1}{48}\left(-\frac{1}{\tan\left(x\right)^{2}}+4\right)^{\frac{3}{2}}\tan\left(x\right)^{3}+\frac{3}{16}\sqrt{-\frac{1}{\tan\left(x\right)^{2}}+4}\tan\left(x\right)-\frac{8\tan\left(x\right)^{4}+26\tan\left(x\right)^{2}-7}{8\sqrt{2\tan\left(x\right)+1}\sqrt{2\tan\left(x\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x) - 3)/(sqrt(-cot(x)^2 + 4)*cos(x)^4),x, algorithm="maxima")
```

```
[Out] -1/48*(-1/tan(x)^2 + 4)^(3/2)*tan(x)^3 + 3/16*sqrt(-1/tan(x)^2 +
4)*tan(x) - 1/8*(8*tan(x)^4 + 26*tan(x)^2 - 7)/(sqrt(2*tan(x) + 1
)*sqrt(2*tan(x) - 1))
```

Fricas [A] time = 0.22753, size = 45, normalized size = 1.15

$$-\frac{\left(\cos\left(x\right)^{2}+1\right)\sqrt{\frac{5\cos\left(x\right)^{2}-4}{\cos\left(x\right)^{2}-1}}\sin\left(x\right)}{3\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x) - 3)/(sqrt(-cot(x)^2 + 4)*cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 + 1)*sqrt((5*cos(x)^2 - 4)/(cos(x)^2 - 1))*sin(x)/
cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2x) - 3}{\sqrt{-\cot(x)^2 + 4}\cos(x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x) - 3)/(sqrt(-cot(x)^2 + 4)*cos(x)^4),x, algorithm="giac")

[Out] integrate((cos(2*x) - 3)/(sqrt(-cot(x)^2 + 4)*cos(x)^4), x)
```

3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] -ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[Sqrt
[5 - 4*Sec[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2
])
```

Rubi [A] time = 2.21203, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

```
[Out] -ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[Sqrt
[5 - 4*Sec[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2
])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x
```

```
[Out] Timed out
```

Mathematica [B] time = 2.17114, size = 234, normalized size = 3.21

$$\frac{\sec^2(x)\left(-20\cos(2x)+\frac{\sqrt{2}(5\cos(2x)-3)^{3/2}\left(15\sqrt{3}\sin^2(x)\tanh^{-1}\left(\frac{\sqrt{5\cos(2x)-3}}{\sqrt{6}\sqrt{\cos^2(x)}}\right)-18\sqrt{5}\sin^2(x)\left(\log\left(10\sin^2(x)\right)-\log\left(5\left(\sqrt{10}\sqrt{\sin^2(x)}\sqrt{\sin^2(2x)}+\sqrt{5\cos(2x)-3}\,\mathrm{c}\right.\right.\right.\right.}{15\sqrt{\sin^2(x)}\sqrt{\sin^2(2x)}}}{60\left(5-4\sec^2(x)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

```
[Out] (Sec[x]^2*(12 - 20*Cos[2*x] + (Sqrt[2]*(-3 + 5*Cos[2*x])^(3/2)*(1
5*Sqrt[3]*ArcTanh[Sqrt[-3 + 5*Cos[2*x]]/(Sqrt[6]*Sqrt[Cos[x]^2])]
*Sin[x]^2 - 18*Sqrt[5]*(Log[10*Sin[x]^2] - Log[5*(-Sqrt[-3 + 5*Co
s[2*x]] + Cos[2*x]*Sqrt[-3 + 5*Cos[2*x]] + Sqrt[10]*Sqrt[Sin[x]^2
]*Sqrt[Sin[2*x]^2])])*Sin[x]^2 - 20*Sqrt[3]*ArcTanh[(Sqrt[6]*Cos[
x])/Sqrt[-3 + 5*Cos[2*x]]]*Sec[x]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2]
))/(15*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2])))/(60*(5 - 4*Sec[x]^2)^(3
/2))
```

Maple [B] time = 0.345, size = 1597, normalized size = 21.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(x)^2)*tan(x)^3/(cos(x)^2-2)/(5-4*sec(x)^2)^(3/2),x)
```

```
[Out] 3/5/(5+2*5^(1/2))/(-5+2*5^(1/2))/(-6+2*5^(1/2)+2^(1/2))/(-6+2*5^(
1/2)-2^(1/2))/(6+2*5^(1/2)-2^(1/2))/(-2*3^(1/2)+6^(1/2))/(6+2*5^(
1/2)+2^(1/2))/(2*3^(1/2)+6^(1/2))*(-4+5*cos(x)^2)*(-25*((-4+5*cos
(x)^2)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(-2*3^(1/2)+6^(1/2))*4^(1/
2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/(
(-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*6^(1/2)*cos(x)*2^(1/2)-50*((
-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(-2*3^(1/2)+6^(1/2
))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/si
n(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*cos(x)*2^(1/2)*3^(1/
2)-25*((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)
+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2
)+4)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*6^(1/2)*cos(x
)*2^(1/2)+50*((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*
3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4
*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*cos(x)
*2^(1/2)*3^(1/2)+72*5^(1/2)*((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)*
arctanh(1/2*5^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x)^2/((-4+5*cos
(x)^2)/(1+cos(x))^2)^(1/2))*cos(x)+50*((-4+5*cos(x)^2)/(1+cos(x))
^2)^(1/2)*arctanh(1/2/(-2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*
cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(
```

```
1+cos(x))^2)^(1/2))*6^(1/2)*cos(x)-25*arctanh(1/2/(-2*3^(1/2)+6^(
1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)
/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*6^(1/2)*2^(1/2)*(
(-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)+100*((-4+5*cos(x)^2)/(1+cos(x
))^2)^(1/2)*arctanh(1/2/(-2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(
5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)
/(1+cos(x))^2)^(1/2))*cos(x)*3^(1/2)-50*arctanh(1/2/(-2*3^(1/2)+6
^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-
4)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*2^(1/2)*((-4+5*
cos(x)^2)/(1+cos(x))^2)^(1/2)*3^(1/2)-50*((-4+5*cos(x)^2)/(1+cos(
x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(
5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)
/(1+cos(x))^2)^(1/2))*6^(1/2)*cos(x)-25*arctanh(1/2/(2*3^(1/2)+6^
(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4
)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*6^(1/2)*2^(1/2)*
((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)+100*((-4+5*cos(x)^2)/(1+cos(
x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(
5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)
/(1+cos(x))^2)^(1/2))*cos(x)*3^(1/2)+50*arctanh(1/2/(2*3^(1/2)+6^
(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4
)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*2^(1/2)*((-4+5*c
os(x)^2)/(1+cos(x))^2)^(1/2)*3^(1/2)+72*arctanh(1/2*5^(1/2)*cos(x
)*4^(1/2)*(cos(x)-1)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2
))*5^(1/2)*((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)+50*arctanh(1/2/(-
2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)
+4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*6^(1
/2)*((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2)+100*arctanh(1/2/(-2*3^(1
/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(
1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*((-4+5*cos
(x)^2)/(1+cos(x))^2)^(1/2)*3^(1/2)-50*arctanh(1/2/(2*3^(1/2)+6^(1
/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/
sin(x)^2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*6^(1/2)*((-4+5*cos
(x)^2)/(1+cos(x))^2)^(1/2)+100*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^
(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^
2/((-4+5*cos(x)^2)/(1+cos(x))^2)^(1/2))*((-4+5*cos(x)^2)/(1+cos(x
))^2)^(1/2)*3^(1/2)-240*cos(x))/cos(x)^3/((-4+5*cos(x)^2)/cos(x)^
2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),x, al
```

```
[Out] Timed out
```

Fricas [A] time = 0.314283, size = 358, normalized size = 4.9

$$-\frac{480\,\sqrt{\frac{5\,\cos(x)^2-4}{\cos(x)^2}}\cos\left(x\right)^2-18\left(5\,\sqrt{5}\cos\left(x\right)^2-4\,\sqrt{5}\right)\log\left(625\,\sqrt{5}\cos\left(x\right)^8-1000\,\sqrt{5}\cos\left(x\right)^6+500\,\sqrt{5}\cos\left(x\right)^4-80\,\sqrt{5}\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),x, al
```

```
[Out] -1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqr
t(5)*cos(x)^2 - 4*sqrt(5))*log(625*sqrt(5)*cos(x)^8 - 1000*sqrt(5
)*cos(x)^6 + 500*sqrt(5)*cos(x)^4 - 80*sqrt(5)*cos(x)^2 - 5*(125*
cos(x)^8 - 150*cos(x)^6 + 50*cos(x)^4 - 4*cos(x)^2)*sqrt((5*cos(x
)^2 - 4)/cos(x)^2) + 2*sqrt(5)) - 25*(5*sqrt(3)*cos(x)^2 - 4*sqrt
(3))*log((1921*sqrt(3)*cos(x)^8 - 3464*sqrt(3)*cos(x)^6 + 2040*sq
rt(3)*cos(x)^4 - 416*sqrt(3)*cos(x)^2 - 24*(62*cos(x)^8 - 87*cos(
x)^6 + 36*cos(x)^4 - 4*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2)
+ 16*sqrt(3))/(cos(x)^8 - 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2
+ 16)))/(5*cos(x)^2 - 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.324906, size = 231, normalized size = 3.16

$$-\frac{1}{4500}\,\sqrt{15}\sqrt{5}\left(6i\,\sqrt{15}\pi+6\,\sqrt{15}\ln\left(4\right)-25\,\ln\left(-\frac{\sqrt{15}+5}{\sqrt{15}-5}\right)\right)\operatorname{sign}\left(\cos\left(x\right)\right)-\frac{\sqrt{15}\sqrt{5}\ln\left(-\frac{2\left(\left(\sqrt{5}\cos(x)-\sqrt{5\,\cos(x)^2-4}\right)^2-4\,\sqrt{15}-16\right)}{\left|2\left(\sqrt{5}\cos(x)-\sqrt{5\,\cos(x)^2-4}\right)^2+8\,\sqrt{15}-32\right|}\right)}{180\operatorname{sign}\left(\cos\left(x\right)\right)}+\frac{\sqrt{5}\ln\left(\left(\sqrt{5}\cos\left(x\right)-\sqrt{5\,\cos\left(x\right)^2-4}\right)^2\right)}{50\operatorname{sign}\left(\cos\left(x\right)\right)}-\frac{2\,\cos\left(x\right)}{15\,\sqrt{5\,\cos\left(x\right)^2-4}\operatorname{sign}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),x, al
```

```
[Out] -1/4500*sqrt(15)*sqrt(5)*(6*I*sqrt(15)*pi + 6*sqrt(15)*ln(4) - 25
*ln(-(sqrt(15) + 5)/(sqrt(15) - 5)))*sign(cos(x)) - 1/180*sqrt(15
)*sqrt(5)*ln(-2*((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 - 4*sq
rt(15) - 16)/abs(2*(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*
sqrt(15) - 32))/sign(cos(x)) + 1/50*sqrt(5)*ln((sqrt(5)*cos(x) -
sqrt(5*cos(x)^2 - 4))^2)/sign(cos(x)) - 2/15*cos(x)/(sqrt(5*cos(x
)^2 - 4)*sign(cos(x)))
```

3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

```
[Out] (-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4
 + 9*Tan[x]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

Rubi [A] time = 1.35963, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[x]^2 +
```

```
[Out] (-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4
 + 9*Tan[x]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2
```

```
[Out] Timed out
```

Mathematica [B] time = 1.36514, size = 116, normalized size = 2.04

$$\frac{-5\tan(x)+5\cot(x)-9\csc(x)\sec(x)-6\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)\right)\sqrt{5\tan^2(x)+13\sec^2(x)-5}+6\sqrt{\frac{13-5\cos(2x)}{\cos(2x)+1}}\log\left(\tan^4\left(\frac{x}{2}\right)+7\tan\right.}{16\sqrt{\frac{13-5\cos(2x)}{\cos(2x)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec
```

```
[Out] (5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Ta
n[x/2]^2 + Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*L
og[Tan[x/2]]*Sqrt[-5 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 -
5*Cos[2*x])/(1 + Cos[2*x])])
```

Maple [B] time = 0.52, size = 117, normalized size = 2.1

$$\frac{1}{8\,(\cos(x))^3\sin(x)}\left(3\left(-\frac{5\,(\cos(x))^2-9}{(\cos(x))^2}\right)^{3/2}\ln\left(-\frac{5\,(\cos(x))^2-9}{(1+\cos(x))^2}\right)\sin(x)(\cos(x))^3-6\left(-\frac{5\,(\cos(x))^2-9}{(\cos(x))^2}\right)^{3/2}\ln\left(-\frac{\cos(}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*t
```

```
[Out] 1/8*(3*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(5*cos(x)^2-9)/(1+cos
(x))^2)*sin(x)*cos(x)^3-6*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(c
os(x)-1)/sin(x))*sin(x)*cos(x)^3-25*cos(x)^4+80*cos(x)^2-63)/(-(5
*cos(x)^2-9)/cos(x)^2)^(3/2)/sin(x)/cos(x)^3
```

Maxima [A] time = 1.52436, size = 63, normalized size = 1.11

$$-\frac{7\tan(x)}{8\sqrt{9\tan(x)^2+4}}-\frac{1}{4\sqrt{9\tan(x)^2+4}\tan(x)}+\frac{3}{8}\log\left(9\tan(x)^2+4\right)-\frac{3}{4}\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2 + 5*t
```

```
[Out] -7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(
x)) + 3/8*log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

Fricas [A] time = 0.238857, size = 113, normalized size = 1.98

$$\frac{3\left(5\cos(x)^2-9\right)\log\left(-\frac{5}{4}\cos(x)^2+\frac{9}{4}\right)\sin(x)-6\left(5\cos(x)^2-9\right)\log\left(\frac{1}{2}\sin(x)\right)\sin(x)-\left(5\cos(x)^3-7\cos(x)\right)\sqrt{-\frac{5\cos}{\cos}}}{8\left(5\cos(x)^2-9\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2 + 5*t
```

```
[Out] 1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*co
s(x)^2 - 9)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt
(-(5*cos(x)^2 - 9)/cos(x)^2))/((5*cos(x)^2 - 9)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/(4*s
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)^2 - 3\sqrt{4\sec(x)^2 + 5\tan(x)^2}\tan(x)}{\left(4\sec(x)^2 + 5\tan(x)^2\right)^{\frac{3}{2}}\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2 + 5*t
```

```
[Out] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4
*sec(x)^2 + 5*tan(x)^2)^(3/2)*sin(x)^2), x)
```

3.440 $\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2}-\frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2}+16\sqrt{5\tan^2(x)+1}-32\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

```
[Out] -32*ArcTan[Sqrt[1 + 5*Tan[x]^2]/2] + 16*Sqrt[1 + 5*Tan[x]^2] - (4
*(1 + 5*Tan[x]^2)^(3/2))/3 + (1 + 5*Tan[x]^2)^(5/2)/5
```

Rubi [A] time = 0.137757, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2}-\frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2}+16\sqrt{5\tan^2(x)+1}-32\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] -32*ArcTan[Sqrt[1 + 5*Tan[x]^2]/2] + 16*Sqrt[1 + 5*Tan[x]^2] - (4
*(1 + 5*Tan[x]^2)^(3/2))/3 + (1 + 5*Tan[x]^2)^(5/2)/5
```

Rubi in Sympy [A] time = 7.64166, size = 58, normalized size = 0.88

$$\frac{\left(5\tan^2(x)+1\right)^{\frac{5}{2}}}{5}-\frac{4\left(5\tan^2(x)+1\right)^{\frac{3}{2}}}{3}+16\sqrt{5\tan^2(x)+1}-32\,\mathrm{atan}\left(\frac{\sqrt{5\tan^2(x)+1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] (5*tan(x)**2 + 1)**(5/2)/5 - 4*(5*tan(x)**2 + 1)**(3/2)/3 + 16*sq
rt(5*tan(x)**2 + 1) - 32*atan(sqrt(5*tan(x)**2 + 1)/2)
```

Mathematica [A] time = 0.592762, size = 82, normalized size = 1.24

$$\cos^5(x)\left(5\tan^2(x)+1\right)^{5/2}\left(\frac{(74\cos(2x)+46\cos(4x)+103)\sec^5(x)}{15(3-2\cos(2x))^2}-\frac{32\log\left(2\cos(x)+\sqrt{2\cos(2x)-3}\right)}{(2\cos(2x)-3)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] Cos[x]^5*((-32*Log[2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])/(-3 + 2*Cos
[2*x])^(5/2) + ((103 + 74*Cos[2*x] + 46*Cos[4*x])*Sec[x]^5)/(15*(
3 - 2*Cos[2*x])^2))*(1 + 5*Tan[x]^2)^(5/2)
```

Maple [A] time = 0.033, size = 61, normalized size = 0.9

$$\frac{223}{15}\sqrt{1+5\,(\tan(x))^2}+5\,(\tan(x))^4\sqrt{1+5\,(\tan(x))^2}\\ -\frac{14\,(\tan(x))^2}{3}\sqrt{1+5\,(\tan(x))^2}-32\,\arctan\left(1/2\sqrt{1+5\,(\tan(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)*(1+5*tan(x)^2)^(5/2),x)
```

```
[Out] 223/15*(1+5*tan(x)^2)^(1/2)+5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*
tan(x)^2*(1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5\,\tan(x)^2+1\right)^{\frac{5}{2}}\tan(x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*tan(x)^2 + 1)^(5/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

Fricas [A] time = 0.247347, size = 68, normalized size = 1.03

$$\frac{1}{15}\left(75\,\tan(x)^4-70\,\tan(x)^2+223\right)\sqrt{5\,\tan(x)^2+1}-16\,\arctan\left(\frac{5\,\tan(x)^2-3}{4\sqrt{5\,\tan(x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*tan(x)^2 + 1)^(5/2)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*
arctan(1/4*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5\tan^{2}(x)+1\right)^{\frac{5}{2}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)
```

GIAC/XCAS [A] time = 0.205246, size = 70, normalized size = 1.06

$$\frac{1}{5}\left(5\,\tan(x)^{2}+1\right)^{\frac{5}{2}}-\frac{4}{3}\left(5\,\tan(x)^{2}+1\right)^{\frac{3}{2}}+16\,\sqrt{5\,\tan(x)^{2}+1}-32\,\arctan\left(\frac{1}{2}\,\sqrt{5\,\tan(x)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*tan(x)^2 + 1)^(5/2)*tan(x),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt
(5*tan(x)^2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))
```

3.441 $\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{3/2}}+\frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

```
[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2))
 + 1/(16*Sqrt[1 + 5*Tan[x]^2])
```

Rubi [A] time = 0.119809, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{3/2}}+\frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2))
 + 1/(16*Sqrt[1 + 5*Tan[x]^2])
```

Rubi in Sympy [A] time = 6.85549, size = 46, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{5\tan^2(x)+1}}{2}\right)}{32}+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) -
 1/(12*(5*tan(x)**2 + 1)**(3/2))
```

Mathematica [A] time = 0.536195, size = 71, normalized size = 1.31

$$\frac{(2\cos(2x)-3)\sec^5(x)\left(-6\cos(x)+8\cos(3x)-3(2\cos(2x)-3)^{3/2}\log\left(2\cos(x)+\sqrt{2\cos(2x)-3}\right)\right)}{96\left(5\tan^2(x)+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^
(3/2)*Log[2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])*Sec[x]^5)/(96*(1 + 5
*Tan[x]^2)^(5/2))
```

Maple [A] time = 0.026, size = 41, normalized size = 0.8

$$\frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{1+5\,(\tan(x))^2}\right)+\frac{1}{16}\frac{1}{\sqrt{1+5\,(\tan(x))^2}}-\frac{1}{12}\left(1+5\,(\tan(x))^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)/(1+5*tan(x)^2)^(5/2),x)
```

```
[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1
/12/(1+5*tan(x)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan(x)}{\left(5\,\tan(x)^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)
```

Fricas [A] time = 0.250353, size = 103, normalized size = 1.91

$$\frac{3\left(25\,\tan(x)^4+10\,\tan(x)^2+1\right)\arctan\left(\frac{5\,\tan(x)^2-3}{4\sqrt{5\,\tan(x)^2+1}}\right)+4\left(15\,\tan(x)^2-1\right)\sqrt{5\,\tan(x)^2+1}}{192\left(25\,\tan(x)^4+10\,\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 -
 3)/sqrt(5*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 +
 1))/(25*tan(x)^4 + 10*tan(x)^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{(5\tan^2(x)+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] Integral(tan(x)/(5*tan(x)**2 + 1)**(5/2), x)
```

GIAC/XCAS [A] time = 0.203296, size = 49, normalized size = 0.91

$$\frac{15\tan(x)^2-1}{48\left(5\tan(x)^2+1\right)^{\frac{3}{2}}}+\frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{5\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*s
qrt(5*tan(x)^2 + 1))
```

3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(
1/3))/Sqrt[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3
)^(1/3)) + (3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)]
)/(4*(a^3 - b^3)^(1/3))
```

Rubi [A] time = 0.289968, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(
1/3))/Sqrt[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3
)^(1/3)) + (3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)]
)/(4*(a^3 - b^3)^(1/3))
```

Rubi in Sympy [A] time = 14.9056, size = 167, normalized size = 1.26

$$\frac{3\log\left(-\sqrt[3]{a-b}\sqrt[3]{a^2+ab+b^2}+\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a-b}\sqrt[3]{a^2+ab+b^2}}-\frac{\log\left(\tan^2(x)+1\right)}{4\sqrt[3]{a-b}\sqrt[3]{a^2+ab+b^2}}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{1}{3}+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{3\sqrt[3]{a-b}\sqrt[3]{a^2+ab+b^2}}\right)\right)}{2\sqrt[3]{a-b}\sqrt[3]{a^2+ab+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
```

```
[Out] 3*log(-(a - b)**(1/3)*(a**2 + a*b + b**2)**(1/3) + (a**3 + b**3*t
an(x)**2)**(1/3))/(4*(a - b)**(1/3)*(a**2 + a*b + b**2)**(1/3)) -
 log(tan(x)**2 + 1)/(4*(a - b)**(1/3)*(a**2 + a*b + b**2)**(1/3))
 + sqrt(3)*atan(sqrt(3)*(1/3 + 2*(a**3 + b**3*tan(x)**2)**(1/3)/(
3*(a - b)**(1/3)*(a**2 + a*b + b**2)**(1/3))))/(2*(a - b)**(1/3)*
(a**2 + a*b + b**2)**(1/3))
```

Mathematica [C] time = 1.5005, size = 90, normalized size = 0.68

$$-\frac{3\sqrt[3]{\frac{(a^3-b^3)\cos(2x)+a^3+b^3}{b^3}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{(b^3-a^3)\cos^2(x)}{b^3}\right)}{2\sqrt[3]{\sec^2(x)\left((a^3-b^3)\cos(2x)+a^3+b^3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (-3*((a^3 + b^3 + (a^3 - b^3)*Cos[2*x])/b^3)^(1/3)*Hypergeometric
2F1[1/3, 1/3, 4/3, ((-a^3 + b^3)*Cos[x]^2)/b^3])/(2*((a^3 + b^3 +
 (a^3 - b^3)*Cos[2*x])*Sec[x]^2)^(1/3))
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int \tan(x)\frac{1}{\sqrt[3]{a^3+b^3(\tan(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

```
[Out] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

Maxima [A] time = 15.2106, size = 14, normalized size = 0.11

$$-\frac{2\cos(x)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -2*cos(x)/(a*b)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
```

```
[Out] Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.443 $\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx$

Optimal. Leaf size=69

$$\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right)+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+2\log(\cos(x))$$

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Co
s[x]] + 3*Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(
2/3))/4
```

Rubi [A] time = 0.161897, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right)+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Co
s[x]] + 3*Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(
2/3))/4
```

Rubi in Sympy [A] time = 6.69952, size = 70, normalized size = 1.01

$$\frac{3\left(-7\tan^2(x)+1\right)^{\frac{2}{3}}}{4}+3\log\left(-\sqrt[3]{-7\tan^2(x)+1}+2\right)-\log\left(\tan^2(x)+1\right)+2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{-7\tan^2(x)+1}}{3}+\frac{1}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
```

```
[Out] 3*(-7*tan(x)**2 + 1)**(2/3)/4 + 3*log(-(-7*tan(x)**2 + 1)**(1/3)
+ 2) - log(tan(x)**2 + 1) + 2*sqrt(3)*atan(sqrt(3)*((-7*tan(x)**2
 + 1)**(1/3)/3 + 1/3))
```

Mathematica [C] time = 0.101501, size = 42, normalized size = 0.61

$$-\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{8}(4\cos(2x)-3)\sec^2(x)\right)-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] (-3*(-1 + Hypergeometric2F1[2/3, 1, 5/3, ((-3 + 4*Cos[2*x])*Sec[x
]^2)/8])*(1 - 7*Tan[x]^2)^(2/3))/4
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \tan(x)\left(1-7\left(\tan(x)\right)^2\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

```
[Out] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-7\tan^{2}(x)+1\right)^{\frac{2}{3}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
```

```
[Out] Integral((-7*tan(x)**2 + 1)**(2/3)*tan(x), x)
```

GIAC/XCAS [A] time = 0.231054, size = 107, normalized size = 1.55

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\left(-7\tan(x)^{2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{4}\left(-7\tan(x)^{2}+1\right)^{\frac{2}{3}} - \ln\left(\left(-7\tan(x)^{2}+1\right)^{\frac{2}{3}}+2\left(-7\tan(x)^{2}+1\right)^{\frac{1}{3}}+4\right)+2\ln\left(\left|\left(-7\tan(x)^{2}+1\right)^{\frac{1}{3}}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*((-7*tan(x)^2 + 1)^(1/3) + 1)) + 3/4
*(-7*tan(x)^2 + 1)^(2/3) - ln((-7*tan(x)^2 + 1)^(2/3) + 2*(-7*tan
(x)^2 + 1)^(1/3) + 4) + 2*ln(abs((-7*tan(x)^2 + 1)^(1/3) - 2))
```

3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 + b^4*Cs
c[x]^2)^(1/4)/a]/a
```

Rubi [A] time = 0.137817, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 + b^4*Cs
c[x]^2)^(1/4)/a]/a
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.22919, size = 84, normalized size = 1.62

$$-\frac{\csc^2(x)\left(a^4\cos(2x)-a^4-2b^4\right)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{\left(\cos(2x)a^4-a^4-2b^4\right)\csc^2(x)}{2a^4}\right)}{3a^4\sqrt[4]{a^4+b^4\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -((-a^4 - 2*b^4 + a^4*Cos[2*x])*Csc[x]^2*Hypergeometric2F1[3/4, 1
, 7/4, -((-a^4 - 2*b^4 + a^4*Cos[2*x])*Csc[x]^2)/(2*a^4)])/(3*a^4
*(a^4 + b^4*Csc[x]^2)^(1/4))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \cot(x) \frac{1}{\sqrt[4]{a^4 + b^4 (\csc(x))^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

Maxima [A] time = 1.54554, size = 96, normalized size = 1.85

$$-\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(b^4*csc(x)^2 + a^4)^(1/4),x, algorithm="maxima")
```

```
[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 + b^4/
sin(x)^2)^(1/4))/a - 1/2*log(-a + (a^4 + b^4/sin(x)^2)^(1/4))/a
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(b^4*csc(x)^2 + a^4)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4\csc^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(b^4\csc(x)^2 + a^4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(b^4*csc(x)^2 + a^4)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/(b^4*csc(x)^2 + a^4)^(1/4), x)
```

3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 - b^4*Cs
c[x]^2)^(1/4)/a]/a
```

Rubi [A] time = 0.143375, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 - b^4*Cs
c[x]^2)^(1/4)/a]/a
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.228065, size = 85, normalized size = 1.57

$$-\frac{\csc^2(x)\left(a^4\cos(2x)-a^4+2b^4\right)\ {}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{\left(\cos(2x)a^4-a^4+2b^4\right)\csc^2(x)}{2a^4}\right)}{3a^4\sqrt[4]{a^4-b^4\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -((-a^4 + 2*b^4 + a^4*Cos[2*x])*Csc[x]^2*Hypergeometric2F1[3/4, 1
, 7/4, -((-a^4 + 2*b^4 + a^4*Cos[2*x])*Csc[x]^2)/(2*a^4)])/(3*a^4
*(a^4 - b^4*Csc[x]^2)^(1/4))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \cot(x) \frac{1}{\sqrt[4]{a^4 - b^4 (\csc(x))^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

Maxima [A] time = 1.49489, size = 100, normalized size = 1.85

$$-\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2a} - \frac{\log\left(-a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(-b^4*csc(x)^2 + a^4)^(1/4),x, algorithm="maxima")
```

```
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/
sin(x)^2)^(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(-b^4*csc(x)^2 + a^4)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt[4]{(a^2 - b^2 \csc(x))(a^2 + b^2 \csc(x))}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(-b^4 \csc(x)^2 + a^4\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/(-b^4*csc(x)^2 + a^4)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/(-b^4*csc(x)^2 + a^4)^(1/4), x)
```

3.446 $$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$$

Optimal. Leaf size=133

$$-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}+\frac{1}{2\left(1-\sqrt{1-3\sec^2(x)}\right)}+\frac{1}{4}\log\left(\sec^2(x)\right)$$
$$-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{3}\log\left(1-\sqrt{1-3\sec^2(x)}\right)+\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{1-3\sec^2(x)}+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + Log[Sec[
x]^2]/4 - (3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1 - Sqrt[1
- 3*Sec[x]^2]]/3 - (1 - 3*Sec[x]^2)^(1/6) - (1 - 3*Sec[x]^2)^(2/3
)/4 + 1/(2*(1 - Sqrt[1 - 3*Sec[x]^2]))
```

Rubi [A] time = 8.93239, antiderivative size = 174, normalized size of antiderivative = 1.31, number of steps used = 29, number of rules used = 16, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262

$$\frac{\cos^2(x)}{6}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}$$
$$-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{2}\log\left(1-\sqrt{1-3\sec^2(x)}\right)+\frac{1}{6}\cos^2(x)\sqrt{1-3\sec^2(x)}$$
$$+\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{1-3\sec^2(x)}+1}{\sqrt{3}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-3\sec^2(x)}\right)+\frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 - 3*Sec
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + ArcTanh[
Sqrt[1 - 3*Sec[x]^2]]/2 + Cos[x]^2/6 + Log[1 - Sqrt[-((3 - Cos[x]
^2)*Sec[x]^2)]]/3 - (3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1
 - Sqrt[1 - 3*Sec[x]^2]]/2 - (1 - 3*Sec[x]^2)^(1/6) + (Cos[x]^2*S
qrt[1 - 3*Sec[x]^2])/6 - (1 - 3*Sec[x]^2)^(2/3)/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2
3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 71.1464, size = 4397, normalized size = 33.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 -
```

```
[Out] (-3*(6 + ((-5 + Cos[2*x])/(1 + Cos[2*x]))^(1/3) + Cos[2*x]*((-5 +
 Cos[2*x])/(1 + Cos[2*x]))^(1/3))*(3*Sec[x]^2 + (1 - 3*Sec[x]^2)^
(1/3))*Sin[x]^2*Tan[x]*(-2 - 3*Tan[x]^2)^(5/6)*(1 + Tan[x]^2)*(2
+ 3*Tan[x]^2)*(-8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]
^2] + 4*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]
^2 + 3*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^
2)^2*((4*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2] + 3*A
ppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2])*Tan[x]^2*(30*3
^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 + 3*Tan[x]^2)^(-1)]*Sq
rt[-2 - 3*Tan[x]^2]*(1 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[x]^
2))^(1/3) + 12*3^(1/6)*Hypergeometric2F1[5/6, 5/6, 11/6, (3 + 3*T
an[x]^2)^(-1)]*(1 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[x]^2))^(
5/6) + 5*(2*Log[1 + Tan[x]^2]*(-2 - 3*Tan[x]^2)^(5/6)*(1 + Tan[x]
^2) + 9*Tan[x]^4*(4 + Sqrt[-2 - 3*Tan[x]^2]) + 3*Tan[x]^2*(20 - 2
*(-2 - 3*Tan[x]^2)^(1/3) + 5*Sqrt[-2 - 3*Tan[x]^2]) + 2*(12 - 2*(
-2 - 3*Tan[x]^2)^(1/3) + 3*Sqrt[-2 - 3*Tan[x]^2] + (-2 - 3*Tan[x]
^2)^(5/6)))) - 8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^
2]*(30*3^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 + 3*Tan[x]^2)^
(-1)]*Sqrt[-2 - 3*Tan[x]^2]*(1 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 +
 Tan[x]^2))^(1/3) + 12*3^(1/6)*Hypergeometric2F1[5/6, 5/6, 11/6,
(3 + 3*Tan[x]^2)^(-1)]*(1 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[
x]^2))^(5/6) + 5*(2*Log[1 + Tan[x]^2]*(-2 - 3*Tan[x]^2)^(5/6)*(1
+ Tan[x]^2) + 9*Tan[x]^4*(4 + Sqrt[-2 - 3*Tan[x]^2]) + Tan[x]^2*(
60 - 7*(-2 - 3*Tan[x]^2)^(1/3) + 15*Sqrt[-2 - 3*Tan[x]^2]) + 2*(1
2 - 2*(-2 - 3*Tan[x]^2)^(1/3) + 3*Sqrt[-2 - 3*Tan[x]^2] + (-2 - 3
*Tan[x]^2)^(5/6))))))/(10*2^(1/6)*(-1 + Sqrt[(-5 + Cos[2*x])/(1 +
 Cos[2*x])])*(1 - 3*Sec[x]^2)^(5/6)*(6 + (1 - 3*Sec[x]^2)^(1/3) +
 Cos[2*x]*(1 - 3*Sec[x]^2)^(1/3))*(-4 - 6*Tan[x]^2)^(5/6)*(-8*App
ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2] + (4*AppellF1[2,
1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2] + 3*AppellF1[2, 3/2, 1, 3,
 (-3*Tan[x]^2)/2, -Tan[x]^2])*Tan[x]^2)*(1152*AppellF1[1, 1/2, 1,
 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3 + 2880*AppellF1[1, 1/2
, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^5 - 1152*AppellF1[1,
 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-
3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5 - 864*AppellF1[1, 1/2, 1, 2, (
-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2
, -Tan[x]^2]*Tan[x]^5 + 1728*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)
/2, -Tan[x]^2]^2*Tan[x]^7 - 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x
```

]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x
]^2]*Tan[x]^7 + 288*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[
x]^2]^2*Tan[x]^7 - 2160*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -
Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[
x]^7 + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*App
ellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7 + 162*Ap
pellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 172
8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2,
1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 + 720*AppellF1[2,
1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9 - 1296*AppellF
1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3
, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 + 1080*AppellF1[2, 1/2, 2,
3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]
^2)/2, -Tan[x]^2]*Tan[x]^9 + 405*AppellF1[2, 3/2, 1, 3, (-3*Tan[x
]^2)/2, -Tan[x]^2]^2*Tan[x]^9 + 432*AppellF1[2, 1/2, 2, 3, (-3*Ta
n[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11 + 648*AppellF1[2, 1/2, 2, 3, (-
3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2,
-Tan[x]^2]*Tan[x]^11 + 243*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/
2, -Tan[x]^2]^2*Tan[x]^11 + 720*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]
^2)/2, -Tan[x]^2]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(1/3) - 192*Appell
F1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2,
3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^3*(-2 - 3*Tan[x]^2)^(1/3) -
144*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[
2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^3*(-2 - 3*Tan[x]
^2)^(1/3) + 1008*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^
2]^2*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) - 1032*AppellF1[1, 1/2, 1,
2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^
2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) - 774*AppellF1[
1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3,
(-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 12
8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3,
1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)
^(1/3) + 96*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*Ap
pellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3
*Tan[x]^2)^(1/3) + 108*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -T
an[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x
]^5*(-2 - 3*Tan[x]^2)^(1/3) - 1080*AppellF1[1, 1/2, 1, 2, (-3*Tan
[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan
[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 96*AppellF1[2, 1/2, 2,
3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3)
- 810*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF
1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[
x]^2)^(1/3) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]
^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(
-2 - 3*Tan[x]^2)^(1/3) + 54*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/
2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 320*AppellF1[1
, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (
-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 240
*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 3
/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^
(1/3) + 270*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*Ap
pellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3
*Tan[x]^2)^(1/3) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -T
an[x]^2]^2*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 216*AppellF1[2, 1/2
, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Ta
n[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 81*Appel

lF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*(-2 - 3*
Tan[x]^2)^(1/3) + 192*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Ta
n[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]
^9*(-2 - 3*Tan[x]^2)^(1/3) + 144*AppellF1[1, 1/2, 1, 2, (-3*Tan[x
]^2)/2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x
]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 162*AppellF1[1, 1/2, 1, 2
, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2
)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 1152*AppellF1[
1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*Sqrt[-2 - 3*
Tan[x]^2] + 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^
2]^2*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] - 1152*AppellF1[1, 1/2, 1, 2,
 (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)
/2, -Tan[x]^2]*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] - 864*AppellF1[1, 1
/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*
Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] + 1728*App
ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*Sqrt[-
2 - 3*Tan[x]^2] - 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -T
an[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x
]^7*Sqrt[-2 - 3*Tan[x]^2] + 288*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]
^2)/2, -Tan[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 2160*AppellF
1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3
, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] + 43
2*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2,
3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*Sqrt[-2 - 3*Tan[x
]^2] + 162*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*T
an[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 1728*AppellF1[1, 1/2, 1, 2, (-3*T
an[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -T
an[x]^2]*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 720*AppellF1[2, 1/2, 2,
 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2]
 - 1296*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF
1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*Sqrt[-2 - 3*
Tan[x]^2] + 1080*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^
2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*Sq
rt[-2 - 3*Tan[x]^2] + 405*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2,
 -Tan[x]^2]^2*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 432*AppellF1[2, 1/
2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11*Sqrt[-2 - 3*Tan[
x]^2] + 648*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Ap
pellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^11*Sqrt[-
2 - 3*Tan[x]^2] + 243*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Ta
n[x]^2]^2*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 384*AppellF1[1, 1/2,
1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5
/6) + 576*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Ta
n[x]^5*(-2 - 3*Tan[x]^2)^(5/6) - 384*AppellF1[1, 1/2, 1, 2, (-3*T
an[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -T
an[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(5/6) - 288*AppellF1[1, 1/2,
1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[
x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(5/6) - 576*Appell
F1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2,
3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) +
 96*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7
*(-2 - 3*Tan[x]^2)^(5/6) - 432*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^
2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^
2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) + 144*AppellF1[2, 1/2, 2, 3,
(-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/
2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) + 54*AppellF1[2, 3
/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2

```
)^(5/6) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^
2*Tan[x]^9*(-2 - 3*Tan[x]^2)^(5/6) + 216*AppellF1[2, 1/2, 2, 3, (
-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2
, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(5/6) + 81*AppellF1[2, 3/
2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*(-2 - 3*Tan[x]^2)
^(5/6)))
```

Maple [F] time = 4.865, size = 0, normalized size = 0.

$$\int \frac{\tan (x)}{(\cos (x))^2} \left(\sqrt[3]{1 - 3\ (\sec (x))^2}\,(\sin (x))^2 + 3\ (\tan (x))^2\right) \left(1 - 3\ (\sec (x))^2\right)^{-\frac{5}{6}} \left(1 - \sqrt{1 - 3\ (\sec (x))^2}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-
3*sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

```
[Out] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1
-3*sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((-3*sec(x)^2 + 1)^(1/3)*sin(x)^2 + 3*tan(x)^2)*tan(x)/((-
3*sec(x)^2 + 1)^(5/6)*(sqrt(-3*sec(x)^2 + 1) - 1)*cos(x)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((-3*sec(x)^2 + 1)^(1/3)*sin(x)^2 + 3*tan(x)^2)*tan(x)/((-
3*sec(x)^2 + 1)^(5/6)*(sqrt(-3*sec(x)^2 + 1) - 1)*cos(x)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/(1-
3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\left(-3 \sec(x)^2 + 1\right)^{\frac{1}{3}} \sin(x)^2 + 3 \tan(x)^2\right) \tan(x)}{\left(-3 \sec(x)^2 + 1\right)^{\frac{5}{6}} \left(\sqrt{-3 \sec(x)^2 + 1} - 1\right) \cos(x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((-3*sec(x)^2 + 1)^(1/3)*sin(x)^2 + 3*tan(x)^2)*tan(x)/((-
3*sec(x)^2 + 1)^(5/6)*(sqrt(-3*sec(x)^2 + 1) - 1)*cos(x)^2),x, algorithm="giac")
```

```
[Out] integrate(-((-3*sec(x)^2 + 1)^(1/3)*sin(x)^2 + 3*tan(x)^2)*tan(x)
/((-3*sec(x)^2 + 1)^(5/6)*(sqrt(-3*sec(x)^2 + 1) - 1)*cos(x)^2),
x)
```

3.447 $\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}}+\frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}+\frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}}+2\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)-\frac{11\tanh^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)}{4\sqrt{2}}$$

```
[Out] 2*ArcTanh[Tan[x]/Sqrt[Tan[x]*Tan[2*x]]] - (11*ArcTanh[(Sqrt[2]*Ta
n[x])/Sqrt[Tan[x]*Tan[2*x]]])/(4*Sqrt[2]) + Tan[x]/(2*(Tan[x]*Tan
[2*x])^(3/2)) + (2*Tan[x]^3)/(3*(Tan[x]*Tan[2*x])^(3/2)) + (3*Tan
[x])/(4*Sqrt[Tan[x]*Tan[2*x]])
```

Rubi [B] time = 1.92717, antiderivative size = 208, normalized size of antiderivative = 2.08, number of steps used = 21, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345

$$\frac{\left(1-\tan^2(x)\right)\tan(x)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}+\frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}-\frac{11\tan^{-1}\left(\sqrt{\tan^2(x)-1}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\tan^2(x)-1}}{\sqrt{2}}\right)\tan(x)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}}+\frac{\left(1-\tan^2(x)\right)\cot(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] (3*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Cot[x]*(1
 - Tan[x]^2))/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Tan[x]
*(1 - Tan[x]^2))/(3*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) - (11*
ArcTan[Sqrt[-1 + Tan[x]^2]]*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 -
 Tan[x]^2)]*Sqrt[-1 + Tan[x]^2]) + (2*ArcTan[Sqrt[-1 + Tan[x]^2]/
Sqrt[2]]*Tan[x])/(Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sqrt[-1 + Tan[x]^
2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.5532, size = 207, normalized size = 2.07

$$\frac{\tan^2(2x)\left(2\tan^2(x)-\cos(2x)\right)\left(-\frac{72\sin^2(x)\cos(2x)\tan(x)F_1\left(\frac{1}{2};-\frac{1}{2},1;\frac{3}{2};\cot^2(x),-\cot^2(x)\right)}{2F_1\left(\frac{3}{2};-\frac{1}{2},2;\frac{5}{2};\cot^2(x),-\cot^2(x)\right)+F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};\cot^2(x),-\cot^2(x)\right)-3\tan^2(x)F_1\left(\frac{1}{2};-\frac{1}{2},1;\frac{3}{2};\cot^2(x),-\cot^2(x)\right)}-4\tan^{3}\right.}{6(6\cos(2x)+\cos(4x)-3)(\tan(x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] ((-Cos[2*x] + 2*Tan[x]^2)*(-3*Cot[x] - 4*Cos[x]*Sin[x] + 18*Sin[x
]^2*Tan[x] - 4*Tan[x]^3 - 9*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Si
n[x]*Sqrt[-1 + Tan[x]^2] - (72*AppellF1[1/2, -1/2, 1, 3/2, Cot[x]
^2, -Cot[x]^2]*Cos[2*x]*Sin[x]^2*Tan[x])/(2*AppellF1[3/2, -1/2, 2
, 5/2, Cot[x]^2, -Cot[x]^2] + AppellF1[3/2, 1/2, 1, 5/2, Cot[x]^2
, -Cot[x]^2] - 3*AppellF1[1/2, -1/2, 1, 3/2, Cot[x]^2, -Cot[x]^2]
*Tan[x]^2))*Tan[2*x]^2)/(6*(-3 + 6*Cos[2*x] + Cos[4*x])*(Tan[x]*T
an[2*x])^(3/2))
```

Maple [B] time = 0.748, size = 559, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
```

```
[Out] -1/96*2^(1/2)*4^(1/2)*(cos(x)-1)^2*(-48*cos(x)^4*2^(1/2)*arctanh(
1/2*2^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x)^2/((2*cos(x)^2-1)/(1
+cos(x))^2)^(1/2))+22*cos(x)^4*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2
)+33*cos(x)^4*arctanh(1/2*4^(1/2)*(2*cos(x)^2-3*cos(x)+1)/sin(x)^
2/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))-168*cos(x)^4*ln(-4*(cos(x)
^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(
x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)+201*cos(x)^4*ln(-2*(cos(
x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x)^2+cos(x)-((2*co
s(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)+48*cos(x)^3*2^(1/2)*ar
ctanh(1/2*2^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x)^2/((2*cos(x)^2
-1)/(1+cos(x))^2)^(1/2))-33*cos(x)^3*arctanh(1/2*4^(1/2)*(2*cos(x
)^2-3*cos(x)+1)/sin(x)^2/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))+168
*cos(x)^3*ln(-4*(cos(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*c
os(x)^2+cos(x)-((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)-2
01*cos(x)^3*ln(-2*(cos(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2
```

```
*cos(x)^2+cos(x)-((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)
-36*cos(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+8*((2*cos(x)^2-1
)/(1+cos(x))^2)^(1/2))/sin(x)^3/cos(x)^3/((2*cos(x)^2-1)/(1+cos(x
))^2)^(3/2)/(sin(x)^2/(2*cos(x)^2-1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2),x, algorit
```

```
[Out] Timed out
```

Fricas [A] time = 0.336954, size = 498, normalized size = 4.98

$$-\frac{24\left(\cos\left(x\right)^{5}-\cos\left(x\right)^{3}\right)\log\left(\frac{\sqrt{2}\left(\sqrt{2}\left(32\,\cos(x)^{4}-16\,\cos(x)^{2}+1\right)\sqrt{-\frac{\cos(x)^{2}-1}{2\,\cos(x)^{2}-1}}-8\left(4\,\cos(x)^{3}-\cos(x)\right)\sin(x)\right)}{2\sqrt{-\frac{\cos(x)^{2}-1}{2\,\cos(x)^{2}-1}}}\right)\sin\left(x\right)-33\left(\sqrt{2}\cos\left(x\right)^{5}-\sqrt{2}\,\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2),x, algorit
```

```
[Out] -1/48*(24*(cos(x)^5 - cos(x)^3)*log(1/2*sqrt(2)*(sqrt(2)*(32*cos(
x)^4 - 16*cos(x)^2 + 1)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) -
8*(4*cos(x)^3 - cos(x))*sin(x))/sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2
- 1)))*sin(x) - 33*(sqrt(2)*cos(x)^5 - sqrt(2)*cos(x)^3)*log(-2*(
sqrt(2)*(32*sqrt(2)*cos(x)^6 - 16*(3*sqrt(2) - 2)*cos(x)^4 + (25*
sqrt(2) - 28)*cos(x)^2 - 3*sqrt(2) + 4)*sqrt(-(cos(x)^2 - 1)/(2*c
os(x)^2 - 1)) - 4*(8*sqrt(2)*cos(x)^5 - 2*(5*sqrt(2) - 4)*cos(x)^
3 + (4*sqrt(2) - 5)*cos(x))*sin(x))/(sqrt(2)*(32*cos(x)^6 - 48*co
s(x)^4 + 17*cos(x)^2 - 1)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1))
- 8*(4*cos(x)^5 - 5*cos(x)^3 + cos(x))*sin(x)))*sin(x) - 2*sqrt(2
)*(22*cos(x)^6 - 47*cos(x)^4 + 26*cos(x)^2 - 4)*sqrt(-(cos(x)^2 -
 1)/(2*cos(x)^2 - 1)) - 44*(cos(x)^5 - cos(x)^3)*sin(x))/((cos(x)
^5 - cos(x)^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.336899, size = 265, normalized size = 2.65

$$\frac{11\sqrt{2}\ln\left(\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sign}\left(\tan(x)^2-1\right)\mathrm{sign}\left(\tan(x)\right)}-\frac{11\sqrt{2}\ln\left(-\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sign}\left(\tan(x)^2-1\right)\mathrm{sign}\left(\tan(x)\right)}-\frac{2\sqrt{2}\left(-\tan(x)^2+1\right)^{\frac{3}{2}}+3\sqrt{2}\sqrt{-\tan(x)^2+1}}{12\,\mathrm{sign}\left(\tan(x)^2-1\right)\mathrm{sign}\left(\tan(x)\right)}+\frac{\ln\left(\frac{\sqrt{2}-\sqrt{-\tan(x)^2+1}}{\sqrt{2}+\sqrt{-\tan(x)^2+1}}\right)}{\mathrm{sign}\left(\tan(x)^2-1\right)\mathrm{sign}\left(\tan(x)\right)}-\frac{\sqrt{2}\sqrt{-\tan(x)^2+1}}{8\,\mathrm{sign}\left(\tan(x)^2-1\right)\mathrm{sign}\left(\tan(x)\right)\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2),x, algorit
```

```
[Out] 11/16*sqrt(2)*ln(sqrt(-tan(x)^2 + 1) + 1)/(sign(tan(x)^2 - 1)*sig
n(tan(x))) - 11/16*sqrt(2)*ln(-sqrt(-tan(x)^2 + 1) + 1)/(sign(tan
(x)^2 - 1)*sign(tan(x))) - 1/12*(2*sqrt(2)*(-tan(x)^2 + 1)^(3/2)
+ 3*sqrt(2)*sqrt(-tan(x)^2 + 1))/(sign(tan(x)^2 - 1)*sign(tan(x))
) + ln((sqrt(2) - sqrt(-tan(x)^2 + 1))/(sqrt(2) + sqrt(-tan(x)^2
+ 1)))/(sign(tan(x)^2 - 1)*sign(tan(x))) - 1/8*sqrt(2)*sqrt(-tan(
x)^2 + 1)/(sign(tan(x)^2 - 1)*sign(tan(x))*tan(x)^2)
```

3.448 $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$

Optimal. Leaf size=112

$$\frac{\log(\cos(x))}{2a^4}-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n}$$

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*Cos[x]^n)^(1/3))/(Sqrt[3]*a)]
)/(a^4*n)) - 3/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3)) + Log[Cos[x]]/(
2*a^4) - (3*Log[a - (a^3 - b^3*Cos[x]^n)^(1/3)])/(2*a^4*n)
```

Rubi [A] time = 0.288089, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{\log(\cos(x))}{2a^4}-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n}$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*Cos[x]^n)^(1/3))/(Sqrt[3]*a)]
)/(a^4*n)) - 3/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3)) + Log[Cos[x]]/(
2*a^4) - (3*Log[a - (a^3 - b^3*Cos[x]^n)^(1/3)])/(2*a^4*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.321408, size = 71, normalized size = 0.63

$$\frac{3\left(\sqrt[3]{1-\frac{a^3\cos^{-n}(x)}{b^3}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{a^3\cos^{-n}(x)}{b^3}\right)-1\right)}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] (3*(-1 + (1 - a^3/(b^3*Cos[x]^n))^(1/3)*Hypergeometric2F1[1/3, 1/
3, 4/3, a^3/(b^3*Cos[x]^n)]))/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3))
```

Maple [A] time = 0.042, size = 137, normalized size = 1.2

$$\frac{1}{2\,na^4}\ln\left(\left(a^3-b^3\left(\cos(x)\right)^n\right)^{\frac{2}{3}}+a\sqrt[3]{a^3-b^3\left(\cos(x)\right)^n}+a^2\right)-\frac{\sqrt{3}}{na^4}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(a+2\sqrt[3]{a^3-b^3\left(\cos(x)\right)^n}\right)\right)-3\,\frac{1}{a^3n\sqrt[3]{a^3-b^3\left(\cos(x)\right)^n}}-\frac{1}{na^4}\ln\left(-a+\sqrt[3]{a^3-b^3\left(\cos(x)\right)^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x)
```

```
[Out] 1/2/n/a^4*ln((a^3-b^3*cos(x)^n)^(2/3)+a*(a^3-b^3*cos(x)^n)^(1/3)+
a^2)-arctan(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2))*3^(1/2)
/a^4/n-3/a^3/n/(a^3-b^3*cos(x)^n)^(1/3)-1/n/a^4*ln(-a+(a^3-b^3*co
s(x)^n)^(1/3))
```

Maxima [A] time = 1.79094, size = 184, normalized size = 1.64

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^4n}+\frac{\log\left(a^2+\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}a+\left(-b^3\cos(x)^n+a^3\right)^{\frac{2}{3}}\right)}{2\,a^4n}-\frac{\log\left(-a+\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{a^4n}-\frac{3}{\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a
)/(a^4*n) + 1/2*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*c
os(x)^n + a^3)^(2/3))/(a^4*n) - log(-a + (-b^3*cos(x)^n + a^3)^(1
/3))/(a^4*n) - 3/((-b^3*cos(x)^n + a^3)^(1/3)*a^3*n)
```

Fricas [A] time = 0.26732, size = 224, normalized size = 2.

$$-\frac{2\sqrt{3}\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)-\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\log\left(a^2+\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}a+\left(-b^3\right.\right.}{2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(-b^3*cos(x)^n + a^3)^(1/3)*arctan(1/3*sqrt(3)*(a
 + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a) - (-b^3*cos(x)^n + a^3)^(1/3
)*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)
^(2/3)) + 2*(-b^3*cos(x)^n + a^3)^(1/3)*log(-a + (-b^3*cos(x)^n +
 a^3)^(1/3)) + 6*a)/((-b^3*cos(x)^n + a^3)^(1/3)*a^4*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(-b^3\cos(x)^n+a^3\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3), x)
```

3.449 $\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx$

Optimal. Leaf size=95

$$-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}+\frac{\tan^{-1}\left(\frac{1-\sqrt[3]{2\cos^9(x)+1}}{\sqrt{3}\sqrt[6]{2\cos^9(x)+1}}\right)}{3\sqrt{3}}+\frac{1}{3}\tanh^{-1}\left(\sqrt[6]{2\cos^9(x)+1}\right)-\frac{1}{9}\tanh^{-1}\left(\sqrt{2\cos^9(x)+1}\right)$$

```
[Out] ArcTan[(1 - (1 + 2*Cos[x]^9)^(1/3))/(Sqrt[3]*(1 + 2*Cos[x]^9)^(1/
6))]/(3*Sqrt[3]) + ArcTanh[(1 + 2*Cos[x]^9)^(1/6)]/3 - ArcTanh[Sq
rt[1 + 2*Cos[x]^9]]/9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15
```

Rubi [A] time = 0.472983, antiderivative size = 162, normalized size of antiderivative = 1.71, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}-\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}-\sqrt[6]{2\cos^9(x)+1}+1\right)+\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}+\sqrt[6]{2\cos^9(x)+1}+1\right)+\frac{\tan^{-1}\left(\frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[6]{2\cos^9(x)+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{9}\tanh^{-1}\left(\sqrt[6]{2\cos^9(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) - ArcT
an[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) + (2*ArcTa
nh[(1 + 2*Cos[x]^9)^(1/6)])/9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15 - L
og[1 - (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(1/3)]/18 + Log[
1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(1/3)]/18
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

```
[Out] Timed out
```

<u>Mathematica</u> [C] time = 25.4161, size = 924, normalized size = 9.73

$$\frac{}{960\ 2^{5/6}\sqrt[6]{\cos^{10}(x)\left(\tan^{10}(x)+5\tan^{8}(x)+10\tan^{6}(x)+10\tan^{4}(x)+5\tan^{2}(x)+2\sqrt{\sec^{2}(x)}+1\right)}\left(\frac{-4\left(10\sec^{2}(x)\tan^{9}(x)+40\sec^{2}(x)\tan\cdots\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] ((128 + 126*Cos[x] + 84*Cos[3*x] + 36*Cos[5*x] + 9*Cos[7*x] + Cos
[9*x])^(5/6)*Tan[x]*(5*2^(5/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -
(Sec[x]^2)^(9/2)/2]*(Sec[x]^8*(35 + 56*Cos[2*x] + 28*Cos[4*x] + 8
*Cos[6*x] + Cos[8*x] + 64*Sqrt[Sec[x]^2]))^(1/6) - 4*Cos[x]^10*(1
 + 2*Sqrt[Sec[x]^2] + 5*Tan[x]^2 + 10*Tan[x]^4 + 10*Tan[x]^6 + 5*
Tan[x]^8 + Tan[x]^10)))/(960*2^(5/6)*(Cos[x]^10*(1 + 2*Sqrt[Sec[x
]^2] + 5*Tan[x]^2 + 10*Tan[x]^4 + 10*Tan[x]^6 + 5*Tan[x]^8 + Tan[
x]^10))^(1/6)*(-((5*2^(5/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -(Se
c[x]^2)^(9/2)/2]*(Sec[x]^8*(35 + 56*Cos[2*x] + 28*Cos[4*x] + 8*Co
s[6*x] + Cos[8*x] + 64*Sqrt[Sec[x]^2]))^(1/6) - 4*Cos[x]^10*(1 +
2*Sqrt[Sec[x]^2] + 5*Tan[x]^2 + 10*Tan[x]^4 + 10*Tan[x]^6 + 5*Tan
[x]^8 + Tan[x]^10))*(Cos[x]^10*(10*Sec[x]^2*Tan[x] + 2*Sqrt[Sec[x
]^2]*Tan[x] + 40*Sec[x]^2*Tan[x]^3 + 60*Sec[x]^2*Tan[x]^5 + 40*Se
c[x]^2*Tan[x]^7 + 10*Sec[x]^2*Tan[x]^9) - 10*Cos[x]^9*Sin[x]*(1 +
 2*Sqrt[Sec[x]^2] + 5*Tan[x]^2 + 10*Tan[x]^4 + 10*Tan[x]^6 + 5*Ta
n[x]^8 + Tan[x]^10)))/(180*(Cos[x]^10*(1 + 2*Sqrt[Sec[x]^2] + 5*T
an[x]^2 + 10*Tan[x]^4 + 10*Tan[x]^6 + 5*Tan[x]^8 + Tan[x]^10))^(7
/6)) + ((15*(Sec[x]^8*(35 + 56*Cos[2*x] + 28*Cos[4*x] + 8*Cos[6*x
] + Cos[8*x] + 64*Sqrt[Sec[x]^2]))^(1/6)*(-Hypergeometric2F1[1/6,
 1/6, 7/6, -(Sec[x]^2)^(9/2)/2] + (1 + (Sec[x]^2)^(9/2)/2)^(-1/6)
)*Tan[x])/2^(1/6) - 4*Cos[x]^10*(10*Sec[x]^2*Tan[x] + 2*Sqrt[Sec[
x]^2]*Tan[x] + 40*Sec[x]^2*Tan[x]^3 + 60*Sec[x]^2*Tan[x]^5 + 40*S
ec[x]^2*Tan[x]^7 + 10*Sec[x]^2*Tan[x]^9) + 40*Cos[x]^9*Sin[x]*(1
+ 2*Sqrt[Sec[x]^2] + 5*Tan[x]^2 + 10*Tan[x]^4 + 10*Tan[x]^6 + 5*T
an[x]^8 + Tan[x]^10) + (5*Hypergeometric2F1[1/6, 1/6, 7/6, -(Sec[
x]^2)^(9/2)/2]*(8*Sec[x]^8*(35 + 56*Cos[2*x] + 28*Cos[4*x] + 8*Co
s[6*x] + Cos[8*x] + 64*Sqrt[Sec[x]^2])*Tan[x] + Sec[x]^8*(-112*Si
n[2*x] - 112*Sin[4*x] - 48*Sin[6*x] - 8*Sin[8*x] + 64*Sqrt[Sec[x]
^2]*Tan[x])))/(3*2^(1/6)*(Sec[x]^8*(35 + 56*Cos[2*x] + 28*Cos[4*x
] + 8*Cos[6*x] + Cos[8*x] + 64*Sqrt[Sec[x]^2]))^(5/6)))/(30*(Cos[
x]^10*(1 + 2*Sqrt[Sec[x]^2] + 5*Tan[x]^2 + 10*Tan[x]^4 + 10*Tan[x
```

```
]^6 + 5*Tan[x]^8 + Tan[x]^10))^(1/6))))
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \left(1+2\,(\cos(x))^9\right)^{\frac{5}{6}} \tan(x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

```
[Out] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

Maxima [A] time = 1.51956, size = 196, normalized size = 2.06

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right)$$
$$-\frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}}+\frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}+\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)$$
$$-\frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}-\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)$$
$$+\frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)-\frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*cos(x)^9 + 1)^(5/6)*tan(x),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) -
 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) -
 2/15*(2*cos(x)^9 + 1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) +
(2*cos(x)^9 + 1)^(1/6) + 1) - 1/18*log((2*cos(x)^9 + 1)^(1/3) - (
2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*cos(x)^9 + 1)^(1/6) + 1)
- 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*cos(x)^9 + 1)^(5/6)*tan(x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.241561, size = 197, normalized size = 2.07

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right)-\frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}}+\frac{1}{18}\ln\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}+\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\ln\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}-\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)+\frac{1}{9}\ln\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)-\frac{1}{9}\ln\left(\left|\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*cos(x)^9 + 1)^(5/6)*tan(x),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) -
 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) -
 2/15*(2*cos(x)^9 + 1)^(5/6) + 1/18*ln((2*cos(x)^9 + 1)^(1/3) + (
2*cos(x)^9 + 1)^(1/6) + 1) - 1/18*ln((2*cos(x)^9 + 1)^(1/3) - (2*
cos(x)^9 + 1)^(1/6) + 1) + 1/9*ln((2*cos(x)^9 + 1)^(1/6) + 1) - 1
/9*ln(abs((2*cos(x)^9 + 1)^(1/6) - 1))
```

3.450 $\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

```
[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 -
 (2 - 5*Sin[x]^3)^(5/3)/625
```

Rubi [A] time = 0.162477, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

```
[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 -
 (2 - 5*Sin[x]^3)^(5/3)/625
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 23.7909, size = 30, normalized size = 0.61

$$\frac{-25\sin^6(x)-30\sin^3(x)+36}{625\sqrt[3]{2-5\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

```
[Out] (36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))
```

Maple [F] time = 0.272, size = 0, normalized size = 0.

$$\int \cot(x)\,(\sin(x))^9 \left(2-5\,(\sin(x))^3\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

```
[Out] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

Maxima [A] time = 1.35555, size = 50, normalized size = 1.02

$$-\frac{1}{625}\left(-5\,\sin(x)^3+2\right)^{\frac{5}{3}}+\frac{2}{125}\left(-5\,\sin(x)^3+2\right)^{\frac{2}{3}}+\frac{4}{125\left(-5\,\sin(x)^3+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*sin(x)^9/(-5*sin(x)^3 + 2)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) +
4/125/(-5*sin(x)^3 + 2)^(1/3)
```

Fricas [A] time = 0.290468, size = 62, normalized size = 1.27

$$\frac{25\,\cos(x)^6-75\,\cos(x)^4+75\,\cos(x)^2+30\left(\cos(x)^2-1\right)\sin(x)+11}{625\left(5\left(\cos(x)^2-1\right)\sin(x)+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*sin(x)^9/(-5*sin(x)^3 + 2)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1
)*sin(x) + 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.204262, size = 50, normalized size = 1.02

$$-\frac{1}{625}\left(-5\sin\left(x\right)^{3}+2\right)^{\frac{5}{3}}+\frac{2}{125}\left(-5\sin\left(x\right)^{3}+2\right)^{\frac{2}{3}}+\frac{4}{125\left(-5\sin\left(x\right)^{3}+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(-5*sin(x)^3 + 2)^(4/3),x, algorithm="giac")
```

```
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) +
4/125/(-5*sin(x)^3 + 2)^(1/3)
```

3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$

Optimal. Leaf size=20

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

```
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Rubi [A] time = 0.343131, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
```

```
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Rubi in Sympy [A] time = 11.146, size = 19, normalized size = 0.95

$$-\frac{3\left(\sqrt[3]{-8\tan^2(x)+1}+1\right)^2}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2
```

```
[Out] -3*((-8*tan(x)**2 + 1)**(1/3) + 1)**2/32
```

Mathematica [B] time = 0.179395, size = 42, normalized size = 2.1

$$-\frac{3(9\cos(2x)-7)\left(\sqrt[3]{1-8\tan^2(x)}+2\right)\sec^2(x)}{64\left(1-8\tan^2(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),
```

```
[Out] (-3*(-7 + 9*Cos[2*x])*Sec[x]^2*(2 + (1 - 8*Tan[x]^2)^(1/3)))/(64*
(1 - 8*Tan[x]^2)^(2/3))
```

Maple [A] time = 0.033, size = 26, normalized size = 1.3

$$-\frac{3}{16}\sqrt[3]{1-8\,(\tan(x))^2}-\frac{3}{32}\left(1-8\,(\tan(x))^2\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

```
[Out] -3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)
```

Maxima [A] time = 1.55395, size = 116, normalized size = 5.8

$$-\frac{3\left(\frac{\left(9\,\sin(x)^2-1\right)(3\,\sin(x)-1)^{\frac{1}{3}}(\sin(x)+1)^{\frac{1}{3}}(\sin(x)-1)^{\frac{1}{3}}}{(3\,\sin(x)+1)^{\frac{1}{3}}}+\frac{2\left(9\,\sin(x)^2-1\right)(\sin(x)+1)^{\frac{2}{3}}(\sin(x)-1)^{\frac{2}{3}}}{(3\,\sin(x)+1)^{\frac{2}{3}}}\right)}{32\left(\sin(x)^2-1\right)(3\,\sin(x)-1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*tan(x)/((-8*tan(x)^2 + 1)^(2/3)*cos(x)^2
```

```
[Out] -3/32*((9*sin(x)^2 - 1)*(3*sin(x) - 1)^(1/3)*(sin(x) + 1)^(1/3)*(
sin(x) - 1)^(1/3)/(3*sin(x) + 1)^(1/3) + 2*(9*sin(x)^2 - 1)*(sin(
x) + 1)^(2/3)*(sin(x) - 1)^(2/3)/(3*sin(x) + 1)^(2/3))/((sin(x)^2
 - 1)*(3*sin(x) - 1)^(2/3))
```

Fricas [A] time = 0.228544, size = 47, normalized size = 2.35

$$-\frac{3}{32}\left(\frac{9\,\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}-\frac{3}{16}\left(\frac{9\,\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*tan(x)/((-8*tan(x)^2 + 1)^(2/3)*cos(x)^2
```

```
[Out] -3/32*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) - 3/16*((9*cos(x)^2 - 8)/
cos(x)^2)^(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3),x
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.222105, size = 34, normalized size = 1.7

$$-\frac{3}{32}\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{2}{3}}-\frac{3}{16}\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*tan(x)/((-8*tan(x)^2 + 1)^(2/3)*cos(x)^2
```

```
[Out] -3/32*(-8*tan(x)^2 + 1)^(2/3) - 3/16*(-8*tan(x)^2 + 1)^(1/3)
```

3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$

Optimal. Leaf size=27

$$\frac{3}{2}\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)-\log(\tan(x))$$

```
[Out] -Log[Tan[x]] + (3*Log[1 - (1 - 8*Tan[x]^2)^(1/3)])/2
```

Rubi [A] time = 1.63165, antiderivative size = 35, normalized size of antiderivative = 1.3, number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29

$$\frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\frac{1}{2}\log\left(1-\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
```

```
[Out] -Log[1 - Sec[x]^2]/2 + (3*Log[1 - (9 - 8*Sec[x]^2)^(1/3)])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\tan(x)}\frac{\sqrt[3]{-8x^2+1}+1}{x\left(-8x^2+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2
```

```
[Out] Integral(((-8*x**2 + 1)**(1/3) + 1)/(x*(-8*x**2 + 1)**(2/3)), (x,
 tan(x)))
```

Mathematica [C] time = 4.67344, size = 93, normalized size = 3.44

$$-\frac{3\sqrt[3]{8-\cot^2(x)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{\cot^2(x)}{8}\right)}{4\sqrt[3]{1-8\tan^2(x)}}-\frac{3\left(8-\cot^2(x)\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{\cot^2(x)}{8}\right)}{16\left(1-8\tan^2(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
```

```
[Out] (-3*(8 - Cot[x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, Cot[x]^
2/8])/(16*(1 - 8*Tan[x]^2)^(2/3)) - (3*(8 - Cot[x]^2)^(1/3)*Hyper
geometric2F1[1/3, 1/3, 4/3, Cot[x]^2/8])/(4*(1 - 8*Tan[x]^2)^(1/3
))
```

Maple [F] time = 0.471, size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{(\cos(x))^2}\left(1+\sqrt[3]{1-8\,(\tan(x))^2}\right)\left(1-8\,(\tan(x))^2\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

```
[Out] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3)
,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*cos(x)^2
```

```
[Out] Timed out
```

Fricas [A] time = 1.29129, size = 126, normalized size = 4.67

$$-\frac{1}{2}\log\left(\frac{16\left(145\cos(x)^4-200\cos(x)^2+3\left(11\cos(x)^4-8\cos(x)^2\right)\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}+3\left(19\cos(x)^4-16\cos(x)^2\right)\left(\frac{9\cos(x)}{\cos(x)}\right.\right.}{\cos(x)^4-2\cos(x)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*cos(x)^2
```

```
[Out] -1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos
(x)^2)*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*co
```

```
s(x)^2)*((9*cos(x)^2 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos
(x)^2 + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3),x
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229704, size = 55, normalized size = 2.04

$$-\frac{1}{2}\ln\left(\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{2}{3}}+\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{1}{3}}+1\right)+\ln\left(-\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*cos(x)^2
```

```
[Out] -1/2*ln((-8*tan(x)^2 + 1)^(2/3) + (-8*tan(x)^2 + 1)^(1/3) + 1) +
ln(-(-8*tan(x)^2 + 1)^(1/3) + 1)
```

3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

Optimal. Leaf size=101

$$2\sqrt[4]{5\sin^2(x)-1}-\frac{\sqrt[4]{5\sin^2(x)-1}}{2\left(\sqrt{5\sin^2(x)-1}+2\right)}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{5\sin^2(x)-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{5\sin^2(x)-1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] (-3*ArcTan[(-1 + 5*Sin[x]^2)^(1/4)/Sqrt[2]])/Sqrt[2] - ArcTanh[(-
1 + 5*Sin[x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[2]) + 2*(-1 + 5*Sin[x]^2)^
(1/4) - (-1 + 5*Sin[x]^2)^(1/4)/(2*(2 + Sqrt[-1 + 5*Sin[x]^2]))
```

Rubi [A] time = 2.30177, antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 10, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(\sqrt{4-5\cos^2(x)}+2\right)}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)*(2 + S
1 + 5*Sin[x]^2])),x]
```

```
[Out] ArcTan[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/Sqrt[2] - 2*Sqrt[2]*ArcTan
[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]] - ArcTanh[(4 - 5*Cos[x]^2)^(1/4)
/Sqrt[2]]/(2*Sqrt[2]) + 2*(4 - 5*Cos[x]^2)^(1/4) - (4 - 5*Cos[x]^
2)^(1/4)/(2*(2 + Sqrt[4 - 5*Cos[x]^2]))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**
1+5*sin(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.49761, size = 158, normalized size = 1.56

$$-\frac{-30\,5^{3/4}\sqrt{3-5\cos(2x)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{4\sec^2(x)}{5}\right)\sqrt[4]{(5\cos(2x)-3)\sec^2(x)}+28\sqrt[4]{5}\left(2-8\tan^2(x)\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{4\sec^2(x)}{5}\right)+3\cdots}{60(3-5\cos(2x))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)
1 + 5*Sin[x]^2])),x]
```

```
[Out] -(3*2^(1/4)*(-3 + 5*Cos[2*x])*(8*Sqrt[2] + Sqrt[3 - 5*Cos[2*x]] +
 10*Sqrt[2]*Cos[2*x])*Sec[x]^2 - 30*5^(3/4)*Sqrt[3 - 5*Cos[2*x]]*
Hypergeometric2F1[1/4, 1/4, 5/4, (4*Sec[x]^2)/5]*((-3 + 5*Cos[2*x
])*Sec[x]^2)^(1/4) + 28*5^(1/4)*Hypergeometric2F1[3/4, 3/4, 7/4,
(4*Sec[x]^2)/5]*(2 - 8*Tan[x]^2)^(3/4))/(60*(3 - 5*Cos[2*x])^(3/4
))
```

Maple [F] time = 0.892, size = 0, normalized size = 0.

$$\int \tan(x)\left(5\,(\cos(x))^2-\sqrt{-1+5\,(\sin(x))^2}\right)\frac{1}{\sqrt[4]{-1+5\,(\sin(x))^2}}\left(2+\sqrt{-1+5\,(\sin(x))^2}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-
1+5*sin(x)^2)^(1/2)),x)
```

```
[Out] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/
4)/(2+(-1+5*sin(x)^2)^(1/2)),x)
```

Maxima [A] time = 1.5518, size = 138, normalized size = 1.37

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\right)}{2\sqrt{2}+2\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}\right)+2\left(5\sin(x)^2-1\right)^{\frac{1}{4}}-\frac{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}{2\left(\sqrt{5\sin(x)^2-1}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/4)*(sq
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqr
t(2)*log(-2*(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/((2*sqrt(2)) + 2*(
5*sin(x)^2 - 1)^(1/4))) + 2*(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x
)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1) + 2)
```

Fricas [A] time = 62.177, size = 687, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/4)*(sq
```

```
[Out] -1/160*(70*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*arctan(2*((5
*sqrt(2)*cos(x)^2 - 4*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) + 2*sqrt(2
)*(-5*cos(x)^2 + 4)^(5/4))/(25*cos(x)^4 - 60*cos(x)^2 + 4*(5*cos(
x)^2 - 4)*sqrt(-5*cos(x)^2 + 4) + 32)) + 50*(5*sqrt(2)*cos(x)^4 -
 4*sqrt(2)*cos(x)^2)*arctan(2*(sqrt(2)*(-5*cos(x)^2 + 4)^(3/4) -
2*sqrt(2)*(-5*cos(x)^2 + 4)^(1/4))/(5*cos(x)^2 + 4*sqrt(-5*cos(x)
^2 + 4) - 8)) - 35*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(
-(125*sqrt(2)*cos(x)^6 - 1700*sqrt(2)*cos(x)^4 + 2560*sqrt(2)*cos
(x)^2 - 16*(15*cos(x)^2 - 16)*(-5*cos(x)^2 + 4)^(5/4) + 8*(25*cos
(x)^4 - 100*cos(x)^2 + 64)*(-5*cos(x)^2 + 4)^(3/4) - 16*(25*sqrt(
2)*cos(x)^4 - 60*sqrt(2)*cos(x)^2 + 32*sqrt(2))*sqrt(-5*cos(x)^2
+ 4) - 1024*sqrt(2))/(5*cos(x)^6 - 4*cos(x)^4)) - 25*(5*sqrt(2)*c
os(x)^4 - 4*sqrt(2)*cos(x)^2)*log(-(25*sqrt(2)*cos(x)^4 - 320*sqr
t(2)*cos(x)^2 - 8*(5*cos(x)^2 - 16)*(-5*cos(x)^2 + 4)^(3/4) - 16*
(5*sqrt(2)*cos(x)^2 - 8*sqrt(2))*sqrt(-5*cos(x)^2 + 4) - 16*(15*c
os(x)^2 - 16)*(-5*cos(x)^2 + 4)^(1/4) + 256*sqrt(2))/cos(x)^4) -
16*(5*cos(x)^2 - 2*(10*cos(x)^2 - 1)*sqrt(-5*cos(x)^2 + 4) - 4)*(
-5*cos(x)^2 + 4)^(3/4))/(5*cos(x)^4 - 4*cos(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(1/4)
1+5*sin(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/4)*(sq
```

```
[Out] Timed out
```

3.454 $\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx$

Optimal. Leaf size=25

$$-\frac{3}{64}\cos^{\frac{8}{3}}(2x)-\frac{3}{40}\cos^{\frac{5}{3}}(2x)$$

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*Cos[2*x]^(8/3))/64
```

Rubi [A] time = 0.0976713, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{3}{64}\cos^{\frac{8}{3}}(2x)-\frac{3}{40}\cos^{\frac{5}{3}}(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]

[Out] (-3*Cos[2*x]^(5/3))/40 - (3*Cos[2*x]^(8/3))/64
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)

[Out] Timed out
```

Mathematica [C] time = 0.519398, size = 140, normalized size = 5.6

$$-\frac{3}{40}\cos^{\frac{5}{3}}(2x)-\frac{3e^{-6ix}\sqrt[3]{1+e^{4ix}}\left(2e^{4ix}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{4ix}\right)+e^{8ix}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{4ix}\right)+\left(1+e^{4ix}\right)^{2/3}\left(1+e^{8ix}\right)\right)}{256\ 2^{2/3}\sqrt[3]{e^{-2ix}+e^{2ix}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*(1 + E^((4*I)*x))^(1/3)*((1 + E^((4*I
)*x))^(2/3)*(1 + E^((8*I)*x)) + 2*E^((4*I)*x)*Hypergeometric2F1[-
1/3, 1/3, 2/3, -E^((4*I)*x)] + E^((8*I)*x)*Hypergeometric2F1[1/3,
 2/3, 5/3, -E^((4*I)*x)]))/(256*2^(2/3)*E^((6*I)*x)*(E^((-2*I)*x)
 + E^((2*I)*x))^(1/3))
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int (\cos (x))^{4} (\cos (2 x))^{\frac{2}{3}} \tan (x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

```
[Out] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

Maxima [A] time = 1.96459, size = 1392, normalized size = 55.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x),x, algorithm="maxima")
```

```
[Out] -3/2560*2^(1/3)*(((5*cos(16/3*x)*cos(3/2*arctan2(sin(16/3*x), cos
(16/3*x))) + 16*cos(16/3*x)*cos(9/8*arctan2(sin(16/3*x), cos(16/3
*x))) + 10*cos(16/3*x)*cos(3/4*arctan2(sin(16/3*x), cos(16/3*x)))
 + 16*cos(16/3*x)*cos(3/8*arctan2(sin(16/3*x), cos(16/3*x))) + 5*
sin(16/3*x)*sin(3/2*arctan2(sin(16/3*x), cos(16/3*x))) + 16*sin(1
6/3*x)*sin(9/8*arctan2(sin(16/3*x), cos(16/3*x))) + 10*sin(16/3*x
)*sin(3/4*arctan2(sin(16/3*x), cos(16/3*x))) + 16*sin(16/3*x)*sin
(3/8*arctan2(sin(16/3*x), cos(16/3*x))) + 5*cos(16/3*x))*cos(2/3*
arctan2(sin(1/4*arctan2(sin(16/3*x), cos(16/3*x))), cos(1/4*arcta
n2(sin(16/3*x), cos(16/3*x))) + 1)) + (5*cos(3/2*arctan2(sin(16/3
*x), cos(16/3*x)))*sin(16/3*x) + 16*cos(9/8*arctan2(sin(16/3*x),
cos(16/3*x)))*sin(16/3*x) + 10*cos(3/4*arctan2(sin(16/3*x), cos(1
6/3*x)))*sin(16/3*x) + 16*cos(3/8*arctan2(sin(16/3*x), cos(16/3*x
)))*sin(16/3*x) - 5*cos(16/3*x)*sin(3/2*arctan2(sin(16/3*x), cos(
16/3*x))) - 16*cos(16/3*x)*sin(9/8*arctan2(sin(16/3*x), cos(16/3*
x))) - 10*cos(16/3*x)*sin(3/4*arctan2(sin(16/3*x), cos(16/3*x)))
- 16*cos(16/3*x)*sin(3/8*arctan2(sin(16/3*x), cos(16/3*x))) + 5*s
in(16/3*x))*sin(2/3*arctan2(sin(1/4*arctan2(sin(16/3*x), cos(16/3
*x))), cos(1/4*arctan2(sin(16/3*x), cos(16/3*x))) + 1)))*cos(2/3*
arctan2(sin(1/2*arctan2(sin(16/3*x), cos(16/3*x))) - sin(1/4*arct
an2(sin(16/3*x), cos(16/3*x))), cos(1/2*arctan2(sin(16/3*x), cos(
16/3*x))) - cos(1/4*arctan2(sin(16/3*x), cos(16/3*x))) + 1)) + ((
```

```
5*cos(3/2*arctan2(sin(16/3*x), cos(16/3*x)))*sin(16/3*x) + 16*cos
(9/8*arctan2(sin(16/3*x), cos(16/3*x)))*sin(16/3*x) + 10*cos(3/4*
arctan2(sin(16/3*x), cos(16/3*x)))*sin(16/3*x) + 16*cos(3/8*arcta
n2(sin(16/3*x), cos(16/3*x)))*sin(16/3*x) - 5*cos(16/3*x)*sin(3/2
*arctan2(sin(16/3*x), cos(16/3*x))) - 16*cos(16/3*x)*sin(9/8*arct
an2(sin(16/3*x), cos(16/3*x))) - 10*cos(16/3*x)*sin(3/4*arctan2(s
in(16/3*x), cos(16/3*x))) - 16*cos(16/3*x)*sin(3/8*arctan2(sin(16
/3*x), cos(16/3*x))) + 5*sin(16/3*x))*cos(2/3*arctan2(sin(1/4*arc
tan2(sin(16/3*x), cos(16/3*x))), cos(1/4*arctan2(sin(16/3*x), cos
(16/3*x))) + 1)) - (5*cos(16/3*x)*cos(3/2*arctan2(sin(16/3*x), co
s(16/3*x))) + 16*cos(16/3*x)*cos(9/8*arctan2(sin(16/3*x), cos(16/
3*x))) + 10*cos(16/3*x)*cos(3/4*arctan2(sin(16/3*x), cos(16/3*x))
) + 16*cos(16/3*x)*cos(3/8*arctan2(sin(16/3*x), cos(16/3*x))) + 5
*sin(16/3*x)*sin(3/2*arctan2(sin(16/3*x), cos(16/3*x))) + 16*sin(
16/3*x)*sin(9/8*arctan2(sin(16/3*x), cos(16/3*x))) + 10*sin(16/3*
x)*sin(3/4*arctan2(sin(16/3*x), cos(16/3*x))) + 16*sin(16/3*x)*si
n(3/8*arctan2(sin(16/3*x), cos(16/3*x))) + 5*cos(16/3*x))*sin(2/3
*arctan2(sin(1/4*arctan2(sin(16/3*x), cos(16/3*x))), cos(1/4*arct
an2(sin(16/3*x), cos(16/3*x))) + 1)))*sin(2/3*arctan2(sin(1/2*arc
tan2(sin(16/3*x), cos(16/3*x))) - sin(1/4*arctan2(sin(16/3*x), co
s(16/3*x))), cos(1/2*arctan2(sin(16/3*x), cos(16/3*x))) - cos(1/4
*arctan2(sin(16/3*x), cos(16/3*x))) + 1)))*(-2*(cos(1/4*arctan2(s
in(16/3*x), cos(16/3*x))) - 1)*cos(1/2*arctan2(sin(16/3*x), cos(1
6/3*x))) + cos(1/2*arctan2(sin(16/3*x), cos(16/3*x)))^2 + cos(1/4
*arctan2(sin(16/3*x), cos(16/3*x)))^2 + sin(1/2*arctan2(sin(16/3*
x), cos(16/3*x)))^2 - 2*sin(1/2*arctan2(sin(16/3*x), cos(16/3*x))
)*sin(1/4*arctan2(sin(16/3*x), cos(16/3*x))) + sin(1/4*arctan2(si
n(16/3*x), cos(16/3*x)))^2 - 2*cos(1/4*arctan2(sin(16/3*x), cos(1
6/3*x))) + 1)^(1/3)*(cos(1/4*arctan2(sin(16/3*x), cos(16/3*x)))^2
 + sin(1/4*arctan2(sin(16/3*x), cos(16/3*x)))^2 + 2*cos(1/4*arcta
n2(sin(16/3*x), cos(16/3*x))) + 1)^(1/3)
```

Fricas [A] time = 0.223395, size = 35, normalized size = 1.4

$$-\frac{3}{320}\left(20\cos\left(x\right)^{4}-4\cos\left(x\right)^{2}-3\right)\left(2\cos\left(x\right)^{2}-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x),x, algorithm="fricas")
```

```
[Out] -3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220223, size = 34, normalized size = 1.36

$$-\frac{3}{64}\left(2\cos\left(x\right)^{2}-1\right)^{\frac{8}{3}}-\frac{3}{40}\left(2\cos\left(x\right)^{2}-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x),x, algorithm="giac")
```

```
[Out] -3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)
```

3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x)-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{7}{4}\sqrt[4]{\cos(2x)}+\frac{\tan^{-1}\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\cos(2x)}+1}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(1 - Sqrt[Cos[2*x]])/(Sqrt[2]*Cos[2*x]^(1/4))]/Sqrt[2] - A
rcTanh[(1 + Sqrt[Cos[2*x]])/(Sqrt[2]*Cos[2*x]^(1/4))]/Sqrt[2] + (
7*Cos[2*x]^(1/4))/4 - Cos[2*x]^(5/4)/5 + Cos[2*x]^(9/4)/36
```

Rubi [A] time = 0.357234, antiderivative size = 154, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x)-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{7}{4}\sqrt[4]{\cos(2x)}+\frac{\log\left(\sqrt{\cos(2x)}-\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{\cos(2x)}+\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*C
os[2*x]^(1/4)]/Sqrt[2] + (7*Cos[2*x]^(1/4))/4 - Cos[2*x]^(5/4)/5
+ Cos[2*x]^(9/4)/36 + Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2
*x]]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x
]]]/(2*Sqrt[2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.73951, size = 59, normalized size = 0.58

$$\frac{2\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{\sec^2(x)}{2}\right)}{3(\cos(2x)+1)^{3/4}}+\frac{1}{360}\sqrt[4]{\cos(2x)}(-72\cos(2x)+5\cos(4x)+635)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] (Cos[2*x]^(1/4)*(635 - 72*Cos[2*x] + 5*Cos[4*x]))/360 + (2*Hyperg
eometric2F1[3/4, 3/4, 7/4, Sec[x]^2/2])/(3*(1 + Cos[2*x])^(3/4))
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int (\sin(x))^6 \tan(x) (\cos(2x))^{-\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

```
[Out] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$

Optimal. Leaf size=17

$$-\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

```
[Out] -ArcTanh[Tan[x]/Sqrt[Tan[x]*Tan[2*x]]]
```

Rubi [A] time = 0.0531911, antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\tanh^{-1}\left(\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Tan[x]*Tan[2*x]],x]

[Out] -ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((tan(x)*tan(2*x))**(1/2),x)

[Out] Timed out
```

Mathematica [B] time = 0.0789664, size = 45, normalized size = 2.65

$$-\frac{\sqrt{\cos(2x)}\sqrt{\tan(x)\tan(2x)}\csc(x)\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Tan[x]*Tan[2*x]],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Csc[x]
*Sqrt[Tan[x]*Tan[2*x]])/Sqrt[2])
```

Maple [B] time = 0.158, size = 88, normalized size = 5.2

$$-\frac{\sqrt{4}\sin(x)}{2\cos(x)-2}\sqrt{\frac{-(\cos(x))^2+1}{2(\cos(x))^2-1}}\sqrt{\frac{2(\cos(x))^2-1}{(1+\cos(x))^2}}\mathit{Artanh}\left(\frac{\cos(x)\sqrt{2}\sqrt{4}(\cos(x)-1)}{2(\sin(x))^2}\frac{1}{\sqrt{\frac{2(\cos(x))^2-1}{(1+\cos(x))^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((tan(x)*tan(2*x))^(1/2),x)
```

```
[Out] -1/2*4^(1/2)*((-cos(x)^2+1)/(2*cos(x)^2-1))^(1/2)*sin(x)*((2*cos(
x)^2-1)/(1+cos(x))^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(x)*4^(1/2)*(c
os(x)-1)/sin(x)^2/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))/(cos(x)-1)
```

Maxima [A] time = 1.67975, size = 350, normalized size = 20.59

$$\frac{1}{4}\log\left(4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2\right.$$
$$+4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2$$
$$\left.+8\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+4\right)$$
$$-\frac{1}{4}\log\left(\cos(2x)^2+\sin(2x)^2\right.$$
$$+\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2+\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)\right.$$
$$+2\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\left(\cos(2x)\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+\sin(2x)\sin\left(\frac{1}{2}\arctan(\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(tan(2*x)*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/4*log(4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*
arctan2(sin(4*x), cos(4*x) + 1))^2 + 4*sqrt(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 +
 8*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arcta
```

```
n2(sin(4*x), cos(4*x) + 1)) + 4) - 1/4*log(cos(2*x)^2 + sin(2*x)^
2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*(cos(1/2*arcta
n2(sin(4*x), cos(4*x) + 1))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x
) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(
cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(
1/2*arctan2(sin(4*x), cos(4*x) + 1))))
```

Fricas [A] time = 0.241358, size = 85, normalized size = 5.

$$\frac{1}{2}\log\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\tan\left(x\right)^{2}-3\right)\sqrt{-\frac{\tan(x)^{2}}{\tan(x)^{2}-1}}+4\tan\left(x\right)\right)}{2\left(\tan\left(x\right)^{2}+1\right)\sqrt{-\frac{\tan(x)^{2}}{\tan(x)^{2}-1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(tan(2*x)*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*sqrt(2)*(sqrt(2)*(tan(x)^2 - 3)*sqrt(-tan(x)^2/(tan(
x)^2 - 1)) + 4*tan(x))/((tan(x)^2 + 1)*sqrt(-tan(x)^2/(tan(x)^2 -
 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tan\left(x\right)\tan\left(2x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((tan(x)*tan(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)*tan(2*x)), x)
```

GIAC/XCAS [A] time = 0.25037, size = 115, normalized size = 6.76

$$\frac{1}{4}\sqrt{2}\left(\left(\sqrt{2}\ln\left(\sqrt{2}+\sqrt{-\tan\left(x\right)^{2}+1}\right)-\sqrt{2}\ln\left(\sqrt{2}-\sqrt{-\tan\left(x\right)^{2}+1}\right)\right)\mathrm{sign}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sign}\left(\tan\left(x\right)\right)+\left(\sqrt{2}\ln\left(\sqrt{2}+1\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(tan(2*x)*tan(x)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((sqrt(2)*ln(sqrt(2) + sqrt(-tan(x)^2 + 1)) - sqrt(2)
*ln(sqrt(2) - sqrt(-tan(x)^2 + 1)))*sign(tan(x)^2 - 1)*sign(tan(x
)) + (sqrt(2)*ln(sqrt(2) + 1) - sqrt(2)*ln(sqrt(2) - 1))*sign(tan
(x)))
```

3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$

Optimal. Leaf size=32

$$\tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

```
[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[
x]^2]]
```

Rubi [A] time = 0.0788192, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[
x]^2]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((cot(2*x)/cot(x))**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0749064, size = 52, normalized size = 1.62

$$\frac{\cos(x)\sqrt{\tan(x)\cot(2x)}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\right)}{\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] ((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])
*Cos[x]*Sqrt[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

Maple [C] time = 0.451, size = 240, normalized size = 7.5

$$-\frac{\sqrt{2}\left(2+\sqrt{2}\right)\cos(x)(\sin(x))^2}{2\sqrt{3+2\sqrt{2}}\left(1+\sqrt{2}\right)(\cos(x)-1)\left(2(\cos(x))^2-1\right)}\left(\mathit{EllipticF}\left(\frac{(\cos(x)-1)\left(1+\sqrt{2}\right)}{\sin(x)},3-2\sqrt{2}\right)-4\,\mathit{EllipticPi}\left(\frac{\sqrt{3+2\sqrt{}}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((cot(2*x)/cot(x))^(1/2),x)
```

```
[Out] -1/2*2^(1/2)/(3+2*2^(1/2))^(1/2)/(1+2^(1/2))*(2+2^(1/2))*(Ellipti
cF((cos(x)-1)*(1+2^(1/2))/sin(x),3-2*2^(1/2))-4*EllipticPi((3+2*2
^(1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1
/2)/(3+2*2^(1/2))^(1/2))+2*EllipticPi((3+2*2^(1/2))^(1/2)*(cos(x)
-1)/sin(x),1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2
)))*cos(x)*sin(x)^2*((2*cos(x)^2-1)/cos(x)^2)^(1/2)*(-2*(cos(x)*2
^(1/2)-2*cos(x)-2^(1/2)+1)/(1+cos(x)))^(1/2)*((cos(x)*2^(1/2)+2*c
os(x)-2^(1/2)-1)/(1+cos(x)))^(1/2)/(cos(x)-1)/(2*cos(x)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cot(2*x)/cot(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.234377, size = 107, normalized size = 3.34

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{2\cos(2x)-1}{2\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}\sin(2x)}\right)-\arctan\left(\frac{\sqrt{2}(3\cos(2x)-1)}{4\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}\sin(2x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cot(2*x)/cot(x)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*arctan(1/2*(2*cos(2*x) - 1)/(sqrt(cos(2*x)/
(cos(2*x) + 1))*sin(2*x))) - arctan(1/4*sqrt(2)*(3*cos(2*x) - 1)/
(sqrt(cos(2*x)/(cos(2*x) + 1))*sin(2*x))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\cot(2x)}{\cot(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cot(2*x)/cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(2*x)/cot(x)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cot(2*x)/cot(x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.458 $\int \frac{1}{x^5(5+x^2)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{20x^4}+\frac{1}{50x^2}-\frac{1}{250}\log\left(x^2+5\right)+\frac{\log(x)}{125}$$

```
[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Rubi [A] time = 0.033795, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{20x^4}+\frac{1}{50x^2}-\frac{1}{250}\log\left(x^2+5\right)+\frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(5 + x^2)),x]

[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Rubi in Sympy [A] time = 2.50242, size = 27, normalized size = 0.87

$$\frac{\log\left(x^2\right)}{250}-\frac{\log\left(x^2+5\right)}{250}+\frac{1}{50x^2}-\frac{1}{20x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**2+5),x)

[Out] log(x**2)/250 - log(x**2 + 5)/250 + 1/(50*x**2) - 1/(20*x**4)
```

Mathematica [A] time = 0.00568578, size = 31, normalized size = 1.

$$-\frac{1}{20x^4}+\frac{1}{50x^2}-\frac{1}{250}\log\left(x^2+5\right)+\frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(5 + x^2)),x]
```

```
[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Maple [A] time = 0.011, size = 24, normalized size = 0.8

$$-\frac{1}{20\,x^{4}}+\frac{1}{50\,x^{2}}+\frac{\ln\left(x\right)}{125}-\frac{\ln\left(x^{2}+5\right)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^2+5),x)
```

```
[Out] -1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

Maxima [A] time = 1.42888, size = 36, normalized size = 1.16

$$\frac{2\,x^{2}-5}{100\,x^{4}}-\frac{1}{250}\,\log\left(x^{2}+5\right)+\frac{1}{250}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 5)*x^5),x, algorithm="maxima")
```

```
[Out] 1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

Fricas [A] time = 0.428666, size = 41, normalized size = 1.32

$$-\frac{2\,x^{4}\log\left(x^{2}+5\right)-4\,x^{4}\log\left(x\right)-10\,x^{2}+25}{500\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 5)*x^5),x, algorithm="fricas")
```

```
[Out] -1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4
```

Sympy [A] time = 0.132341, size = 24, normalized size = 0.77

$$\frac{\log\left(x\right)}{125}-\frac{\log\left(x^{2}+5\right)}{250}+\frac{2x^{2}-5}{100x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**2+5),x)
```

```
[Out] log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)
```

GIAC/XCAS [A] time = 0.202384, size = 43, normalized size = 1.39

$$-\frac{3\,x^{4}-10\,x^{2}+25}{500\,x^{4}}-\frac{1}{250}\,\ln\left(x^{2}+5\right)+\frac{1}{250}\,\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 5)*x^5),x, algorithm="giac")
```

```
[Out] -1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*ln(x^2 + 5) + 1/250*ln(x
^2)
```

3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqr
t[5])
```

Rubi [A] time = 0.0358167, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(5 + x^2)),x]
```

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqr
t[5])
```

Rubi in Sympy [A] time = 3.24778, size = 34, normalized size = 0.87

$$-\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625}-\frac{1}{125x}+\frac{1}{75x^3}-\frac{1}{25x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**2+5),x)
```

```
[Out] -sqrt(5)*atan(sqrt(5)*x/5)/625 - 1/(125*x) + 1/(75*x**3) - 1/(25*
x**5)
```

Mathematica [A] time = 0.0205384, size = 39, normalized size = 1.

$$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(5 + x^2)),x]
```

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqr
t[5])
```

Maple [A] time = 0.008, size = 29, normalized size = 0.7

$$-\frac{1}{25\,x^5}+\frac{1}{75\,x^3}-\frac{1}{125\,x}-\frac{\sqrt{5}}{625}\arctan\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^2+5),x)
```

```
[Out] -1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.58516, size = 41, normalized size = 1.05

$$-\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 5)*x^6),x, algorithm="maxima")
```

```
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)
/x^5
```

Fricas [A] time = 0.199812, size = 50, normalized size = 1.28

$$-\frac{\sqrt{5}\left(3\,x^5\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\sqrt{5}\left(3\,x^4-5\,x^2+15\right)\right)}{1875\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 5)*x^6),x, algorithm="fricas")
```

```
[Out] -1/1875*sqrt(5)*(3*x^5*arctan(1/5*sqrt(5)*x) + sqrt(5)*(3*x^4 - 5
*x^2 + 15))/x^5
```

Sympy [A] time = 0.160203, size = 34, normalized size = 0.87

$$-\frac{\sqrt{5}\,\text{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625}-\frac{3x^4-5x^2+15}{375x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**2+5),x)
```

```
[Out] -sqrt(5)*atan(sqrt(5)*x/5)/625 - (3*x**4 - 5*x**2 + 15)/(375*x**5
)
```

GIAC/XCAS [A] time = 0.202756, size = 41, normalized size = 1.05

$$-\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 5)*x^6),x, algorithm="giac")
```

```
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)
/x^5
```

3.460 $\int \frac{1}{x(-4+x^2)^4}\,dx$

Optimal. Leaf size=58

$$\frac{1}{128(4-x^2)}+\frac{1}{64(4-x^2)^2}+\frac{1}{24(4-x^2)^3}-\frac{1}{512}\log\left(4-x^2\right)+\frac{\log(x)}{256}$$

```
[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log
[x]/256 - Log[4 - x^2]/512
```

Rubi [A] time = 0.0616172, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{128(4-x^2)}+\frac{1}{64(4-x^2)^2}+\frac{1}{24(4-x^2)^3}-\frac{1}{512}\log\left(4-x^2\right)+\frac{\log(x)}{256}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(-4 + x^2)^4),x]
```

```
[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log
[x]/256 - Log[4 - x^2]/512
```

Rubi in Sympy [A] time = 3.24212, size = 42, normalized size = 0.72

$$\frac{\log\left(x^2\right)}{512}-\frac{\log\left(-x^2+4\right)}{512}+\frac{1}{128(-x^2+4)}+\frac{1}{64(-x^2+4)^2}+\frac{1}{24(-x^2+4)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**2-4)**4,x)
```

```
[Out] log(x**2)/512 - log(-x**2 + 4)/512 + 1/(128*(-x**2 + 4)) + 1/(64*
(-x**2 + 4)**2) + 1/(24*(-x**2 + 4)**3)
```

Mathematica [A] time = 0.0264754, size = 40, normalized size = 0.69

$$\frac{-3\log\left(4-x^2\right)-\frac{4(3x^4-30x^2+88)}{(x^2-4)^3}+6\log(x)}{1536}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(-4 + x^2)^4),x]
```

```
[Out] ((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x
^2])/1536
```

Maple [A] time = 0.02, size = 60, normalized size = 1.

$$\frac{1}{1536\,(2+x)^{3}}+\frac{3}{2048\,(2+x)^{2}}+\frac{11}{8192+4096\,x}-\frac{\ln(2+x)}{512}+\frac{\ln(x)}{256}-\frac{1}{1536\,(-2+x)^{3}}+\frac{3}{2048\,(-2+x)^{2}}-\frac{11}{-8192+4096\,x}-\frac{\ln(-2+x)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^2-4)^4,x)
```

```
[Out] 1/1536/(2+x)^3+3/2048/(2+x)^2+11/4096/(2+x)-1/512*ln(2+x)+1/256*l
n(x)-1/1536/(-2+x)^3+3/2048/(-2+x)^2-11/4096/(-2+x)-1/512*ln(-2+x
)
```

Maxima [A] time = 1.33462, size = 62, normalized size = 1.07

$$-\frac{3\,x^{4}-30\,x^{2}+88}{384\,(x^{6}-12\,x^{4}+48\,x^{2}-64)}-\frac{1}{512}\,\log\left(x^{2}-4\right)+\frac{1}{512}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4)^4*x),x, algorithm="maxima")
```

```
[Out] -1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512
*log(x^2 - 4) + 1/512*log(x^2)
```

Fricas [A] time = 0.197571, size = 99, normalized size = 1.71

$$-\frac{12\,x^{4}-120\,x^{2}+3\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)\log\left(x^{2}-4\right)-6\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)\log(x)+352}{1536\,(x^{6}-12\,x^{4}+48\,x^{2}-64)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4)^4*x),x, algorithm="fricas")
```

```
[Out] -1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x^
2 - 4) - 6*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x) + 352)/(x^6 - 12*x
^4 + 48*x^2 - 64)
```

Sympy [A] time = 0.198873, size = 41, normalized size = 0.71

$$-\frac{3x^4 - 30x^2 + 88}{384x^6 - 4608x^4 + 18432x^2 - 24576} + \frac{\log(x)}{256} - \frac{\log(x^2 - 4)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**2-4)**4,x)
```

```
[Out] -(3*x**4 - 30*x**2 + 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 245
76) + log(x)/256 - log(x**2 - 4)/512
```

GIAC/XCAS [A] time = 0.20528, size = 57, normalized size = 0.98

$$\frac{11\,x^6 - 156\,x^4 + 768\,x^2 - 1408}{3072\,(x^2 - 4)^3} + \frac{1}{512}\ln\left(x^2\right) - \frac{1}{512}\ln\left(\left|x^2 - 4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4)^4*x),x, algorithm="giac")
```

```
[Out] 1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*ln
(x^2) - 1/512*ln(abs(x^2 - 4))
```

3.461 $\int \frac{1}{x(-2+x^2)^{5/2}}\,dx$

Optimal. Leaf size=52

$$\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\,(x^2-2)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/(6*(-2 + x^2)^(3/2)) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 +
 x^2]/Sqrt[2]]/(4*Sqrt[2])
```

Rubi [A] time = 0.0530273, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\,(x^2-2)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(-2 + x^2)^(5/2)),x]
```

```
[Out] -1/(6*(-2 + x^2)^(3/2)) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 +
 x^2]/Sqrt[2]]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 2.72414, size = 46, normalized size = 0.88

$$\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\sqrt{x^2-2}}{2}\right)}{8}+\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\,(x^2-2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**2-2)**(5/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*sqrt(x**2 - 2)/2)/8 + 1/(4*sqrt(x**2 - 2)) -
 1/(6*(x**2 - 2)**(3/2))
```

Mathematica [A] time = 0.07609, size = 46, normalized size = 0.88

$$\frac{1}{24}\left(\frac{2\left(3x^2-8\right)}{(x^2-2)^{3/2}}-3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(-2 + x^2)^(5/2)),x]
```

```
[Out] ((2*(-8 + 3*x^2))/(-2 + x^2)^(3/2) - 3*Sqrt[2]*ArcTan[Sqrt[2]/Sqr
t[-2 + x^2]])/24
```

Maple [A] time = 0.01, size = 37, normalized size = 0.7

$$-\frac{1}{6}\left(x^2-2\right)^{-\frac{3}{2}}+\frac{1}{4}\frac{1}{\sqrt{x^2-2}}-\frac{\sqrt{2}}{8}\arctan\left(\sqrt{2}\frac{1}{\sqrt{x^2-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^2-2)^(5/2),x)
```

```
[Out] -1/6/(x^2-2)^(3/2)+1/4/(x^2-2)^(1/2)-1/8*2^(1/2)*arctan(2^(1/2)/(
x^2-2)^(1/2))
```

Maxima [A] time = 1.53176, size = 45, normalized size = 0.87

$$-\frac{1}{8}\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right)+\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\,(x^2-2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 2)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arcsin(sqrt(2)/abs(x)) + 1/4/sqrt(x^2 - 2) - 1/6/(x^
2 - 2)^(3/2)
```

Fricas [A] time = 0.208758, size = 212, normalized size = 4.08

$$-\frac{\sqrt{2}\left(6\,x^4-19\,x^2+8\right)\sqrt{x^2-2}+6\left(2\,x^6-9\,x^4+12\,x^2-\left(2\,x^5-7\,x^3+6\,x\right)\sqrt{x^2-2}-4\right)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)}{12\left(\sqrt{2}(2\,x^5-7\,x^3+6\,x)\sqrt{x^2-2}-\sqrt{2}(2\,x^6-9\,x^4+12\,x^2-4)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 2)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2)*(6*x^4 - 19*x^2 + 8)*sqrt(x^2 - 2) + 6*(2*x^6 - 9*
x^4 + 12*x^2 - (2*x^5 - 7*x^3 + 6*x)*sqrt(x^2 - 2) - 4)*arctan(-1
/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(x^2 - 2)) - sqrt(2)*(6*x^5 - 25*x
^3 + 24*x))/(sqrt(2)*(2*x^5 - 7*x^3 + 6*x)*sqrt(x^2 - 2) - sqrt(2
)*(2*x^6 - 9*x^4 + 12*x^2 - 4))
```

Sympy [A] time = 10.6649, size = 986, normalized size = 18.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**2-2)**(5/2),x)
```

```
[Out] Piecewise((6*I*x**4*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 9
6*sqrt(2)) - 3*I*x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**
2 + 96*sqrt(2)) - 6*x**4*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sq
rt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2)*x**2*sqrt(x**2 - 2)/(24*sqrt
(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*I*x**2*log(x)/(24*s
qrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I*x**2*log(x**2)
/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*x**2*asin(
sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*
sqrt(2)*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sq
rt(2)) + 24*I*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt
(2)) - 12*I*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqr
t(2)) - 24*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 9
6*sqrt(2)), Abs(x**2)/2 > 1), (-3*I*x**4*log(x**2)/(24*sqrt(2)*x*
*4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*I*x**4*log(sqrt(-x**2/2 +
1) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 3*pi*x
**4/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 3*I*x**4*l
og(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2
)*I*x**2*sqrt(-x**2 + 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*
sqrt(2)) + 12*I*x**2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2
 + 96*sqrt(2)) - 24*I*x**2*log(sqrt(-x**2/2 + 1) + 1)/(24*sqrt(2)
*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*pi*x**2/(24*sqrt(2)*x*
*4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*x**2*log(2)/(24*sqrt(2)
*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(2)*I*sqrt(-x**2 +
 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x
**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*log(
sqrt(-x**2/2 + 1) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sq
rt(2)) - 12*pi/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) +
 12*I*log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)), Tr
ue))
```

GIAC/XCAS [A] time = 0.203643, size = 47, normalized size = 0.9

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+\frac{3x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 2)^(5/2)*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/
(x^2 - 2)^(3/2)
```

3.462 $\int \frac{(-10+x^2)^{5/2}}{x} \, dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(x^2-10\right)^{5/2}-\frac{10}{3}\left(x^2-10\right)^{3/2}+100\sqrt{x^2-10}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

```
[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2
)/5 - 100*Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]
```

Rubi [A] time = 0.0689656, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{1}{5}\left(x^2-10\right)^{5/2}-\frac{10}{3}\left(x^2-10\right)^{3/2}+100\sqrt{x^2-10}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2
)/5 - 100*Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]
```

Rubi in Sympy [A] time = 3.16264, size = 54, normalized size = 0.89

$$\frac{\left(x^2-10\right)^{\frac{5}{2}}}{5}-\frac{10\left(x^2-10\right)^{\frac{3}{2}}}{3}+100\sqrt{x^2-10}-100\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\sqrt{x^2-10}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-10)**(5/2)/x,x)
```

```
[Out] (x**2 - 10)**(5/2)/5 - 10*(x**2 - 10)**(3/2)/3 + 100*sqrt(x**2 -
10) - 100*sqrt(10)*atan(sqrt(10)*sqrt(x**2 - 10)/10)
```

Mathematica [A] time = 0.0319282, size = 47, normalized size = 0.77

$$100\sqrt{10}\tan^{-1}\left(\frac{1}{\sqrt{\frac{x^2}{10}-1}}\right)+\frac{1}{15}\sqrt{x^2-10}\left(3x^4-110x^2+2300\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 + 100*Sqrt[10]*ArcT
an[1/Sqrt[-1 + x^2/10]]
```

Maple [A] time = 0.009, size = 46, normalized size = 0.8

$$\frac{1}{5}\left(x^{2}-10\right)^{\frac{5}{2}}-\frac{10}{3}\left(x^{2}-10\right)^{\frac{3}{2}}+100\sqrt{x^{2}-10}+100\sqrt{10}\arctan\left(\frac{\sqrt{10}}{\sqrt{x^{2}-10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-10)^(5/2)/x,x)
```

```
[Out] 1/5*(x^2-10)^(5/2)-10/3*(x^2-10)^(3/2)+100*(x^2-10)^(1/2)+100*10^
(1/2)*arctan(10^(1/2)/(x^2-10)^(1/2))
```

Maxima [A] time = 1.49045, size = 57, normalized size = 0.93

$$\frac{1}{5}\left(x^{2}-10\right)^{\frac{5}{2}}-\frac{10}{3}\left(x^{2}-10\right)^{\frac{3}{2}}+100\sqrt{10}\arcsin\left(\frac{\sqrt{10}}{|x|}\right)+100\sqrt{x^{2}-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 10)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsi
n(sqrt(10)/abs(x)) + 100*sqrt(x^2 - 10)
```

Fricas [A] time = 0.210487, size = 217, normalized size = 3.56

$$-\frac{12\,x^{10}-650\,x^{8}+17875\,x^{6}-197500\,x^{4}+775000\,x^{2}-3000\left(\sqrt{10}\left(4\,x^{4}-30\,x^{2}+25\right)\sqrt{x^{2}-10}-\sqrt{10}\left(4\,x^{5}-50\,x^{3}+125\,x\right.\right.}{15\left(4\,x^{5}-50\,x^{3}-\left(4\,x^{4}-30\,x^{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 10)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/15*(12*x^10 - 650*x^8 + 17875*x^6 - 197500*x^4 + 775000*x^2 -
3000*(sqrt(10)*(4*x^4 - 30*x^2 + 25)*sqrt(x^2 - 10) - sqrt(10)*(4
*x^5 - 50*x^3 + 125*x))*arctan(-1/10*sqrt(10)*(x - sqrt(x^2 - 10)
)) - (12*x^9 - 590*x^7 + 15075*x^5 - 128750*x^3 + 287500*x)*sqrt(
x^2 - 10) - 575000)/(4*x^5 - 50*x^3 - (4*x^4 - 30*x^2 + 25)*sqrt(
x^2 - 10) + 125*x)
```

Sympy [A] time = 21.9911, size = 167, normalized size = 2.74

$$\begin{cases} \frac{x^4\sqrt{x^2-10}}{5} - \frac{22x^2\sqrt{x^2-10}}{3} + \frac{460\sqrt{x^2-10}}{3} - 100\sqrt{10}i\log(x) + 50\sqrt{10}i\log(x^2) + 100\sqrt{10}\,\mathrm{asin}\left(\frac{\sqrt{10}}{x}\right) & \text{for } \frac{|x^2|}{10} > 1 \\ \frac{ix^4\sqrt{-x^2+10}}{5} - \frac{22ix^2\sqrt{-x^2+10}}{3} + \frac{460i\sqrt{-x^2+10}}{3} + 50\sqrt{10}i\log(x^2) - 100\sqrt{10}i\log\left(\sqrt{-\frac{x^2}{10}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-10)**(5/2)/x,x)
```

```
[Out] Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 4
60*sqrt(x**2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(
x**2) + 100*sqrt(10)*asin(sqrt(10)/x), Abs(x**2)/10 > 1), (I*x**4
*sqrt(-x**2 + 10)/5 - 22*I*x**2*sqrt(-x**2 + 10)/3 + 460*I*sqrt(-
x**2 + 10)/3 + 50*sqrt(10)*I*log(x**2) - 100*sqrt(10)*I*log(sqrt(
-x**2/10 + 1) + 1), True))
```

GIAC/XCAS [A] time = 0.201529, size = 62, normalized size = 1.02

$$\frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} - 100\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\sqrt{x^2-10}\right) + 100\sqrt{x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 10)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arcta
n(1/10*sqrt(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)
```

3.463 $\int x^{1+2n} \, dx$

Optimal. Leaf size=16

$$\frac{x^{2(n+1)}}{2(n+1)}$$

```
[Out] x^(2*(1 + n))/(2*(1 + n))
```

Rubi [A] time = 0.00842035, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^{2(n+1)}}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(1 + 2*n),x]

[Out] x^(2*(1 + n))/(2*(1 + n))
```

Rubi in Sympy [A] time = 0.639888, size = 10, normalized size = 0.62

$$\frac{x^{2n+2}}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1+2*n),x)

[Out] x**(2*n + 2)/(2*(n + 1))
```

Mathematica [A] time = 0.00380172, size = 15, normalized size = 0.94

$$\frac{x^{2n+2}}{2n+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(1 + 2*n),x]
```

```
[Out] x^(2 + 2*n)/(2 + 2*n)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$\frac{x^{2+2n}}{2+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+2*n),x)
```

```
[Out] 1/2*x^(2+2*n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n + 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219531, size = 20, normalized size = 1.25

$$\frac{xx^{2n+1}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x*x^(2*n + 1)/(n + 1)
```

Sympy [A] time = 0.038167, size = 19, normalized size = 1.19

$$\begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for } 2n+1 \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1+2*n),x)
```

```
[Out] Piecewise((x**(2*n + 2)/(2*n + 2), Ne(2*n + 1, -1)), (log(x), Tru
e))
```

GIAC/XCAS [A] time = 0.198033, size = 19, normalized size = 1.19

$$\frac{x^{2n+2}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^(2*n + 2)/(n + 1)
```

3.464 $\int \frac{x^7}{(-5+x^2)^3} \, dx$

Optimal. Leaf size=46

$$\frac{x^2}{2} + \frac{75}{2\left(5-x^2\right)} - \frac{125}{4\left(5-x^2\right)^2} + \frac{15}{2}\log\left(5-x^2\right)$$

```
[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2]
)/2
```

Rubi [A] time = 0.0500866, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x^2}{2} + \frac{75}{2\left(5-x^2\right)} - \frac{125}{4\left(5-x^2\right)^2} + \frac{15}{2}\log\left(5-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(-5 + x^2)^3,x]
```

```
[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2]
)/2
```

Rubi in Sympy [A] time = 2.83898, size = 32, normalized size = 0.7

$$\frac{x^2}{2} + \frac{15\log\left(-x^2+5\right)}{2} + \frac{75}{2\left(-x^2+5\right)} - \frac{125}{4\left(-x^2+5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**2-5)**3,x)
```

```
[Out] x**2/2 + 15*log(-x**2 + 5)/2 + 75/(2*(-x**2 + 5)) - 125/(4*(-x**2
 + 5)**2)
```

Mathematica [A] time = 0.0200149, size = 36, normalized size = 0.78

$$\frac{1}{4}\left(2x^2 - \frac{150}{x^2-5} - \frac{125}{\left(x^2-5\right)^2} + 30\log\left(x^2-5\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(-5 + x^2)^3,x]
```

```
[Out] (2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4
```

Maple [A] time = 0.013, size = 33, normalized size = 0.7

$$\frac{x^2}{2} + \frac{15\ln\left(x^2-5\right)}{2} - \frac{125}{4\left(x^2-5\right)^2} - \frac{75}{2x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^2-5)^3,x)
```

```
[Out] 1/2*x^2+15/2*ln(x^2-5)-125/4/(x^2-5)^2-75/2/(x^2-5)
```

Maxima [A] time = 1.34311, size = 47, normalized size = 1.02

$$\frac{1}{2}x^2 - \frac{25\left(6x^2-25\right)}{4\left(x^4-10x^2+25\right)} + \frac{15}{2}\log\left(x^2-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^2 - 5)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 -
5)
```

Fricas [A] time = 0.198328, size = 66, normalized size = 1.43

$$\frac{2x^6 - 20x^4 - 100x^2 + 30\left(x^4-10x^2+25\right)\log\left(x^2-5\right) + 625}{4\left(x^4-10x^2+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^2 - 5)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 -
5) + 625)/(x^4 - 10*x^2 + 25)
```

Sympy [A] time = 0.137702, size = 32, normalized size = 0.7

$$\frac{x^2}{2} - \frac{150x^2 - 625}{4x^4 - 40x^2 + 100} + \frac{15\log(x^2 - 5)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**2-5)**3,x)
```

```
[Out] x**2/2 - (150*x**2 - 625)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2
- 5)/2
```

GIAC/XCAS [A] time = 0.199073, size = 49, normalized size = 1.07

$$\frac{1}{2}\,x^2 - \frac{5\left(9\,x^4 - 60\,x^2 + 100\right)}{4\,(x^2 - 5)^2} + \frac{15}{2}\ln\left(|x^2 - 5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^2 - 5)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*ln(abs(x^
2 - 5))
```

3.465 $\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx$

Optimal. Leaf size=40

$$-\frac{3}{4(1-x^2)^2}+\frac{1}{3(1-x^2)^3}+\frac{1}{8(1-x^2)^4}$$

```
[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)
```

Rubi [A] time = 0.0770468, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{3}{4(1-x^2)^2}+\frac{1}{3(1-x^2)^3}+\frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)
```

Rubi in Sympy [A] time = 5.07684, size = 29, normalized size = 0.72

$$-\frac{3}{4(-x^2+1)^2}+\frac{1}{3(-x^2+1)^3}+\frac{1}{8(-x^2+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
```

```
[Out] -3/(4*(-x**2 + 1)**2) + 1/(3*(-x**2 + 1)**3) + 1/(8*(-x**2 + 1)**
4)
```

Mathematica [A] time = 0.0129017, size = 23, normalized size = 0.57

$$\frac{-18x^4+28x^2-7}{24(x^2-1)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

```
[Out] (-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

Maple [A] time = 0.014, size = 58, normalized size = 1.5

$$\frac{1}{128\,(1+x)^4}+\frac{11}{192\,(1+x)^3}-\frac{27}{256\,(1+x)^2}-\frac{27}{256+256\,x}+\frac{1}{128\,(-1+x)^4}-\frac{11}{192\,(-1+x)^3}-\frac{27}{256\,(-1+x)^2}+\frac{27}{-256+256\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^5-4*x^3)/(x^2-1)^5,x)
```

```
[Out] 1/128/(1+x)^4+11/192/(1+x)^3-27/256/(1+x)^2-27/256/(1+x)+1/128/(-
1+x)^4-11/192/(-1+x)^3-27/256/(-1+x)^2+27/256/(-1+x)
```

Maxima [A] time = 1.3602, size = 49, normalized size = 1.22

$$-\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^5 - 4*x^3)/(x^2 - 1)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Fricas [A] time = 0.194266, size = 49, normalized size = 1.22

$$-\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^5 - 4*x^3)/(x^2 - 1)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [A] time = 0.193676, size = 34, normalized size = 0.85

$$-\frac{18x^4 - 28x^2 + 7}{24x^8 - 96x^6 + 144x^4 - 96x^2 + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
```

```
[Out] -(18*x**4 - 28*x**2 + 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2
+ 24)
```

GIAC/XCAS [A] time = 0.201339, size = 28, normalized size = 0.7

$$-\frac{18\,x^4 - 28\,x^2 + 7}{24\,(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^5 - 4*x^3)/(x^2 - 1)^5,x, algorithm="giac")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4
```

3.466 $\int x^3 \left(1+x^2\right)^{9/14} dx$

Optimal. Leaf size=27

$$\frac{7}{37}\left(x^2+1\right)^{37/14}-\frac{7}{23}\left(x^2+1\right)^{23/14}$$

```
[Out] (-7*(1 + x^2)^(23/14))/23 + (7*(1 + x^2)^(37/14))/37
```

Rubi [A] time = 0.0271874, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{7}{37}\left(x^2+1\right)^{37/14}-\frac{7}{23}\left(x^2+1\right)^{23/14}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(1 + x^2)^(9/14),x]

[Out] (-7*(1 + x^2)^(23/14))/23 + (7*(1 + x^2)^(37/14))/37
```

Rubi in Sympy [A] time = 1.79915, size = 22, normalized size = 0.81

$$\frac{7\left(x^2+1\right)^{\frac{37}{14}}}{37}-\frac{7\left(x^2+1\right)^{\frac{23}{14}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(x**2+1)**(9/14),x)

[Out] 7*(x**2 + 1)**(37/14)/37 - 7*(x**2 + 1)**(23/14)/23
```

Mathematica [A] time = 0.00846643, size = 20, normalized size = 0.74

$$\frac{7}{851}\left(x^2+1\right)^{23/14}\left(23x^2-14\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(1 + x^2)^(9/14),x]
```

```
[Out] (7*(1 + x^2)^(23/14)*(-14 + 23*x^2))/851
```

Maple [A] time = 0.006, size = 17, normalized size = 0.6

$$\frac{161 x^2 - 98}{851} \left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(9/14),x)
```

```
[Out] 7/851*(x^2+1)^(23/14)*(23*x^2-14)
```

Maxima [A] time = 1.32832, size = 26, normalized size = 0.96

$$\frac{7}{37} \left(x^2+1\right)^{\frac{37}{14}} - \frac{7}{23} \left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)^(9/14)*x^3,x, algorithm="maxima")
```

```
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

Fricas [A] time = 0.203359, size = 28, normalized size = 1.04

$$\frac{7}{851} \left(23 x^4 + 9 x^2 - 14\right) \left(x^2+1\right)^{\frac{9}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)^(9/14)*x^3,x, algorithm="fricas")
```

```
[Out] 7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)
```

Sympy [A] time = 74.1617, size = 41, normalized size = 1.52

$$\frac{7x^4 \left(x^2+1\right)^{\frac{9}{14}}}{37} + \frac{63x^2 \left(x^2+1\right)^{\frac{9}{14}}}{851} - \frac{98 \left(x^2+1\right)^{\frac{9}{14}}}{851}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(x**2+1)**(9/14),x)
```

```
[Out] 7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 9
8*(x**2 + 1)**(9/14)/851
```

GIAC/XCAS [A] time = 0.20122, size = 26, normalized size = 0.96

$$\frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}}-\frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(9/14)*x^3,x, algorithm="giac")
```

```
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

3.467 $\int \frac{x^5}{(-4+x^2)^{13/6}} \, dx$

Optimal. Leaf size=38

$$\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

```
[Out] -48/(7*(-4 + x^2)^(7/6)) - 24/(-4 + x^2)^(1/6) + (3*(-4 + x^2)^(5
/6))/5
```

Rubi [A] time = 0.0377135, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(-4 + x^2)^(13/6),x]
```

```
[Out] -48/(7*(-4 + x^2)^(7/6)) - 24/(-4 + x^2)^(1/6) + (3*(-4 + x^2)^(5
/6))/5
```

Rubi in Sympy [A] time = 2.26905, size = 32, normalized size = 0.84

$$\frac{3\left(x^2-4\right)^{\frac{5}{6}}}{5}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**2-4)**(13/6),x)
```

```
[Out] 3*(x**2 - 4)**(5/6)/5 - 24/(x**2 - 4)**(1/6) - 48/(7*(x**2 - 4)**
(7/6))
```

Mathematica [A] time = 0.0147151, size = 25, normalized size = 0.66

$$\frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(-4 + x^2)^(13/6),x]

[Out] (3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

Maple [A] time = 0.005, size = 28, normalized size = 0.7

$$\frac{(-6+3x)(2+x)\left(7x^4-336x^2+1152\right)}{35}\left(x^2-4\right)^{-\frac{13}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^2-4)^(13/6),x)

[Out] 3/35*(-2+x)*(2+x)*(7*x^4-336*x^2+1152)/(x^2-4)^(13/6)
```

Maxima [A] time = 1.34152, size = 38, normalized size = 1.

$$\frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}}-\frac{24}{(x^2-4)^{\frac{1}{6}}}-\frac{48}{7\,(x^2-4)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^2 - 4)^(13/6),x, algorithm="maxima")

[Out] 3/5*(x^2 - 4)^(5/6) - 24/(x^2 - 4)^(1/6) - 48/7/(x^2 - 4)^(7/6)
```

Fricas [A] time = 0.202315, size = 28, normalized size = 0.74

$$\frac{3\left(7x^4-336x^2+1152\right)}{35\,(x^2-4)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^2 - 4)^(13/6),x, algorithm="fricas")

[Out] 3/35*(7*x^4 - 336*x^2 + 1152)/(x^2 - 4)^(7/6)
```

Sympy [A] time = 11.7506, size = 82, normalized size = 2.16

$$\frac{21x^4}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}-\frac{1008x^2}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}+\frac{3456}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**2-4)**(13/6),x)
```

```
[Out] 21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 100
8*x**2/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456
/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6))
```

GIAC/XCAS [A] time = 0.200564, size = 35, normalized size = 0.92

$$\frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}}-\frac{24\left(7x^2-26\right)}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^2 - 4)^(13/6),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 4)^(5/6) - 24/7*(7*x^2 - 26)/(x^2 - 4)^(7/6)
```

3.468 $\int \frac{1}{(1+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{3/2}}$$

```
[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

Rubi [A] time = 0.0120243, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)^(-5/2),x]

[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

Rubi in Sympy [A] time = 0.596979, size = 27, normalized size = 0.82

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**2+1)**(5/2),x)

[Out] 2*x/(3*sqrt(2*x**2 + 1)) + x/(3*(2*x**2 + 1)**(3/2))
```

Mathematica [A] time = 0.0127785, size = 23, normalized size = 0.7

$$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)^(-5/2),x]
```

```
[Out] (x*(3 + 4*x^2))/(3*(1 + 2*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 20, normalized size = 0.6

$$\frac{x\left(4x^2+3\right)}{3}\left(2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^2+1)^(5/2),x)
```

```
[Out] 1/3*x*(4*x^2+3)/(2*x^2+1)^(3/2)
```

Maxima [A] time = 1.3432, size = 34, normalized size = 1.03

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)^(-5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)
```

Fricas [A] time = 0.202123, size = 112, normalized size = 3.39

$$-\frac{12x^5+17x^3-\left(4x^5+11x^3+6x\right)\sqrt{2x^2+1}+6x}{3\left(4x^6+12x^4+9x^2-\left(6x^4+7x^2+2\right)\sqrt{2x^2+1}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(12*x^5 + 17*x^3 - (4*x^5 + 11*x^3 + 6*x)*sqrt(2*x^2 + 1) +
6*x)/(4*x^6 + 12*x^4 + 9*x^2 - (6*x^4 + 7*x^2 + 2)*sqrt(2*x^2 + 1
) + 2)
```

Sympy [A] time = 14.5528, size = 61, normalized size = 1.85

$$\frac{4x^3}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}+\frac{3x}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**2+1)**(5/2),x)
```

```
[Out] 4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x*
*2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1))
```

GIAC/XCAS [A] time = 0.202205, size = 26, normalized size = 0.79

$$\frac{\left(4\,x^2+3\right)x}{3\,\left(2\,x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)
```

3.469 $\int \frac{1}{(-1-2x+x^2)^{5/2}} \, dx$

Optimal. Leaf size=43

$$\frac{1-x}{6\left(x^2-2x-1\right)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

```
[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x
^2])
```

Rubi [A] time = 0.0157768, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1-x}{6\left(x^2-2x-1\right)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x
^2])
```

Rubi in Sympy [A] time = 0.741166, size = 36, normalized size = 0.84

$$-\frac{-4x+4}{24\sqrt{x^2-2x-1}}+\frac{-2x+2}{12\left(x^2-2x-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x-1)**(5/2),x)
```

```
[Out] -(-4*x + 4)/(24*sqrt(x**2 - 2*x - 1)) + (-2*x + 2)/(12*(x**2 - 2*
x - 1)**(3/2))
```

Mathematica [A] time = 0.0177216, size = 26, normalized size = 0.6

$$\frac{x^3-3x^2+2}{6\left(x^2-2x-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 2*x + x^2)^(-5/2),x]

[Out] (2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))
```

Maple [A] time = 0.003, size = 23, normalized size = 0.5

$$\frac{x^3 - 3x^2 + 2}{6}\left(x^2 - 2x - 1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x-1)^(5/2),x)

[Out] 1/6*(x^3-3*x^2+2)/(x^2-2*x-1)^(3/2)
```

Maxima [A] time = 1.39652, size = 69, normalized size = 1.6

$$\frac{x}{6\sqrt{x^2 - 2x - 1}} - \frac{1}{6\sqrt{x^2 - 2x - 1}} - \frac{x}{6\left(x^2 - 2x - 1\right)^{\frac{3}{2}}} + \frac{1}{6\left(x^2 - 2x - 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 1)^(-5/2),x, algorithm="maxima")

[Out] 1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2
- 2*x - 1)^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)
```

Fricas [A] time = 0.20293, size = 123, normalized size = 2.86

$$\frac{3x^2 - 3\sqrt{x^2 - 2x - 1}(x - 1) - 6x - 1}{6\left(2x^6 - 12x^5 + 21x^4 - 4x^3 - 12x^2 - (2x^5 - 10x^4 + 13x^3 + x^2 - 5x - 1)\sqrt{x^2 - 2x - 1} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 1)^(-5/2),x, algorithm="fricas")

[Out] 1/6*(3*x^2 - 3*sqrt(x^2 - 2*x - 1)*(x - 1) - 6*x - 1)/(2*x^6 - 12
*x^5 + 21*x^4 - 4*x^3 - 12*x^2 - (2*x^5 - 10*x^4 + 13*x^3 + x^2 -
 5*x - 1)*sqrt(x^2 - 2*x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - 2x - 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x-1)**(5/2),x)
```

```
[Out] Integral((x**2 - 2*x - 1)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.209657, size = 28, normalized size = 0.65

$$\frac{(x-3)x^2 + 2}{6\,(x^2 - 2\,x - 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 1)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)
```

3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}\,x}+\frac{1}{24\sqrt{x^2-8}\,x^3}$$

```
[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt
[-8 + x^2])
```

Rubi [A] time = 0.0279512, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}\,x}+\frac{1}{24\sqrt{x^2-8}\,x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt
[-8 + x^2])
```

Rubi in Sympy [A] time = 2.01458, size = 39, normalized size = 0.83

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48x\sqrt{x^2-8}}+\frac{1}{24x^3\sqrt{x^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**2-8)**(3/2),x)
```

```
[Out] -x/(192*sqrt(x**2 - 8)) + 1/(48*x*sqrt(x**2 - 8)) + 1/(24*x**3*sq
rt(x**2 - 8))
```

Mathematica [A] time = 0.0151381, size = 28, normalized size = 0.6

$$\frac{-x^4+4x^2+8}{192x^3\sqrt{x^2-8}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]

[Out] (8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

Maple [A] time = 0.005, size = 23, normalized size = 0.5

$$-\frac{x^4-4x^2-8}{192x^3}\frac{1}{\sqrt{x^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^2-8)^(3/2),x)

[Out] -1/192*(x^4-4*x^2-8)/x^3/(x^2-8)^(1/2)
```

Maxima [A] time = 1.5198, size = 47, normalized size = 1.

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}x}+\frac{1}{24\sqrt{x^2-8}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 8)^(3/2)*x^4),x, algorithm="maxima")

[Out] -1/192*x/sqrt(x^2 - 8) + 1/48/(sqrt(x^2 - 8)*x) + 1/24/(sqrt(x^2
- 8)*x^3)
```

Fricas [A] time = 0.203978, size = 76, normalized size = 1.62

$$\frac{x^2-\sqrt{x^2-8}x-2}{6\left(x^8-12x^6+32x^4-(x^7-8x^5+8x^3)\sqrt{x^2-8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 8)^(3/2)*x^4),x, algorithm="fricas")

[Out] 1/6*(x^2 - sqrt(x^2 - 8)*x - 2)/(x^8 - 12*x^6 + 32*x^4 - (x^7 - 8
*x^5 + 8*x^3)*sqrt(x^2 - 8))
```

Sympy [A] time = 9.77293, size = 153, normalized size = 3.26

$$\begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for } 8\left|\frac{1}{x^2}\right| > 1 \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**2-8)**(3/2),x)
```

```
[Out] Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I
*x**2*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/
x**2)/(192*x**4 - 1536*x**2), 8*Abs(x**(-2)) > 1), (-x**4*sqrt(1
- 8/x**2)/(192*x**4 - 1536*x**2) + 4*x**2*sqrt(1 - 8/x**2)/(192*x
**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2), Tru
e))
```

GIAC/XCAS [A] time = 0.23372, size = 84, normalized size = 1.79

$$-\frac{x}{512\sqrt{x^2-8}} - \frac{3\left(x-\sqrt{x^2-8}\right)^4 + 96\left(x-\sqrt{x^2-8}\right)^2 + 320}{96\left(\left(x-\sqrt{x^2-8}\right)^2 + 8\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 8)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] -1/512*x/sqrt(x^2 - 8) - 1/96*(3*(x - sqrt(x^2 - 8))^4 + 96*(x -
sqrt(x^2 - 8))^2 + 320)/((x - sqrt(x^2 - 8))^2 + 8)^3
```

3.471 $\int \frac{(5+x^2)^2}{x^{13/3}}\,dx$

Optimal. Leaf size=28

$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

```
[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

Rubi [A] time = 0.0152897, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x^2)^2/x^(13/3),x]

[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

Rubi in Sympy [A] time = 1.48201, size = 24, normalized size = 0.86

$$\frac{3x^{\frac{2}{3}}}{2}-\frac{15}{2x^{\frac{4}{3}}}-\frac{15}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+5)**2/x**(13/3),x)

[Out] 3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

Mathematica [A] time = 0.00794838, size = 19, normalized size = 0.68

$$\frac{3\left(x^4-5x^2-5\right)}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x^2)^2/x^(13/3),x]
```

```
[Out] (3*(-5 - 5*x^2 + x^4))/(2*x^(10/3))
```

Maple [A] time = 0.005, size = 16, normalized size = 0.6

$$\frac{3\,x^{4}-15\,x^{2}-15}{2}x^{-\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+5)^2/x^(13/3),x)
```

```
[Out] 3/2*(x^4-5*x^2-5)/x^(10/3)
```

Maxima [A] time = 1.39666, size = 22, normalized size = 0.79

$$\frac{3}{2}\,x^{\frac{2}{3}}-\frac{15\,\left(x^{2}+1\right)}{2\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 5)^2/x^(13/3),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

Fricas [A] time = 0.199233, size = 20, normalized size = 0.71

$$\frac{3\,\left(x^{4}-5\,x^{2}-5\right)}{2\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 5)^2/x^(13/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^4 - 5*x^2 - 5)/x^(10/3)
```

Sympy [A] time = 17.4243, size = 24, normalized size = 0.86

$$\frac{3x^{\frac{2}{3}}}{2}-\frac{15}{2x^{\frac{4}{3}}}-\frac{15}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+5)**2/x**(13/3),x)
```

```
[Out] 3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

GIAC/XCAS [A] time = 0.210596, size = 22, normalized size = 0.79

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,\left(x^{2} + 1\right)}{2\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 5)^2/x^(13/3),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

3.472 $\int \frac{1}{x^7(1+x^2)^3}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4(x^2+1)^2}+5\log\left(x^2+1\right)-10\log(x)$$

```
[Out] -1/(6*x^6) + 3/(4*x^4) - 3/x^2 - 1/(4*(1 + x^2)^2) - 2/(1 + x^2)
- 10*Log[x] + 5*Log[1 + x^2]
```

Rubi [A] time = 0.0572152, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4(x^2+1)^2}+5\log\left(x^2+1\right)-10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -1/(6*x^6) + 3/(4*x^4) - 3/x^2 - 1/(4*(1 + x^2)^2) - 2/(1 + x^2)
- 10*Log[x] + 5*Log[1 + x^2]
```

Rubi in Sympy [A] time = 3.1573, size = 49, normalized size = 0.94

$$-5\log\left(x^2\right)+5\log\left(x^2+1\right)-\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}-\frac{3}{x^2}+\frac{3}{4x^4}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x**7/(x**2+1)**3,x)
```

```
[Out] -5*log(x**2) + 5*log(x**2 + 1) - 2/(x**2 + 1) - 1/(4*(x**2 + 1)**
2) - 3/x**2 + 3/(4*x**4) - 1/(6*x**6)
```

Mathematica [A] time = 0.0509151, size = 49, normalized size = 0.94

$$5\log\left(x^2+1\right)-\frac{60x^8+90x^6+20x^4-5x^2+2}{12x^6\left(x^2+1\right)^2}-10\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(12*x^6*(1 + x^2)^2) - 10
*Log[x] + 5*Log[1 + x^2]
```

Maple [A] time = 0.019, size = 47, normalized size = 0.9

$$-\frac{1}{6\,x^{6}}+\frac{3}{4\,x^{4}}-3\,x^{-2}-\frac{1}{4\,\left(x^{2}+1\right)^{2}}-2\,\left(x^{2}+1\right)^{-1}-10\,\ln\left(x\right)+5\,\ln\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^2+1)^3,x)
```

```
[Out] -1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+
1)
```

Maxima [A] time = 1.36067, size = 72, normalized size = 1.38

$$-\frac{60\,x^{8}+90\,x^{6}+20\,x^{4}-5\,x^{2}+2}{12\,\left(x^{10}+2\,x^{8}+x^{6}\right)}+5\,\log\left(x^{2}+1\right)-5\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^3*x^7),x, algorithm="maxima")
```

```
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^10 + 2*x^8 + x^6)
 + 5*log(x^2 + 1) - 5*log(x^2)
```

Fricas [A] time = 0.196667, size = 100, normalized size = 1.92

$$-\frac{60\,x^{8}+90\,x^{6}+20\,x^{4}-5\,x^{2}-60\,\left(x^{10}+2\,x^{8}+x^{6}\right)\log\left(x^{2}+1\right)+120\,\left(x^{10}+2\,x^{8}+x^{6}\right)\log\left(x\right)+2}{12\,\left(x^{10}+2\,x^{8}+x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^3*x^7),x, algorithm="fricas")
```

```
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^10 + 2*x^8 + x^6)
*log(x^2 + 1) + 120*(x^10 + 2*x^8 + x^6)*log(x) + 2)/(x^10 + 2*x^
8 + x^6)
```

Sympy [A] time = 0.243296, size = 49, normalized size = 0.94

$$-10\log(x)+5\log\left(x^2+1\right)-\frac{60x^8+90x^6+20x^4-5x^2+2}{12x^{10}+24x^8+12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**2+1)**3,x)
```

```
[Out] -10*log(x) + 5*log(x**2 + 1) - (60*x**8 + 90*x**6 + 20*x**4 - 5*x
**2 + 2)/(12*x**10 + 24*x**8 + 12*x**6)
```

GIAC/XCAS [A] time = 0.229241, size = 78, normalized size = 1.5

$$-\frac{30\,x^4+68\,x^2+39}{4\,(x^2+1)^2}+\frac{110\,x^6-36\,x^4+9\,x^2-2}{12\,x^6}+5\ln\left(x^2+1\right)-5\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^3*x^7),x, algorithm="giac")
```

```
[Out] -1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4
+ 9*x^2 - 2)/x^6 + 5*ln(x^2 + 1) - 5*ln(x^2)
```

3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

```
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Rubi [A] time = 0.0856428, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]

[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Rubi in Sympy [A] time = 4.45903, size = 24, normalized size = 0.96

$$-\frac{9x\left(1+\frac{2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)

[Out] -9*x*(1 + 2/x**2)**(7/9)/(10*sqrt(x**2 + 2))
```

Mathematica [A] time = 0.0197727, size = 25, normalized size = 1.

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]

[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Maple [A] time = 0.006, size = 22, normalized size = 0.9

$$-\frac{9x}{10}\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}\frac{1}{\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x)

[Out] -9/10*x/(x^2+2)^(1/2)*((x^2+2)/x^2)^(7/9)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{\left(x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x, algorithm="maxima")

[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

Fricas [A] time = 0.41402, size = 28, normalized size = 1.12

$$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x, algorithm="fricas")

[Out] -9/10*x*((x^2 + 2)/x^2)^(7/9)/sqrt(x^2 + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^5}{175\left(\sqrt{10}-x^2\right)^{5/2}}+\frac{x^5}{7\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}$$

```
[Out] x^5/(7*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) + x^5/(175*(Sqrt[10] - x^
2)^(5/2))
```

Rubi [A] time = 0.0616649, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

```
[Out] x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^
2)^(7/2))
```

Rubi in Sympy [A] time = 2.23194, size = 37, normalized size = 0.74

$$-\frac{x^7}{175\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{\sqrt{10}x^5}{50\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
```

```
[Out] -x**7/(175*(-x**2 + sqrt(10))**(7/2)) + sqrt(10)*x**5/(50*(-x**2
+ sqrt(10))**(7/2))
```

Mathematica [A] time = 0.0830948, size = 35, normalized size = 0.7

$$\frac{x^5\left(7\sqrt{10}-2x^2\right)}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]

[Out] (x^5*(7*Sqrt[10] - 2*x^2))/(350*(Sqrt[10] - x^2)^(7/2))
```

Maple [A] time = 0.047, size = 28, normalized size = 0.6

$$\frac{x^5\left(-2\,x^2+7\,\sqrt{10}\right)}{350}\left(-x^2+\sqrt{10}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+10^(1/2))^(9/2),x)

[Out] 1/350*x^5*(-2*x^2+7*10^(1/2))/(-x^2+10^(1/2))^(7/2)
```

Maxima [A] time = 1.50137, size = 107, normalized size = 2.14

$$\frac{x}{175\sqrt{-x^2+\sqrt{10}}}+\frac{\sqrt{10}x}{350\left(-x^2+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{x^3}{4\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\,x}{140\left(-x^2+\sqrt{10}\right)^{\frac{5}{2}}}-\frac{3\sqrt{10}x}{28\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + sqrt(10))^(9/2),x, algorithm="maxima")

[Out] 1/175*x/sqrt(-x^2 + sqrt(10)) + 1/350*sqrt(10)*x/(-x^2 + sqrt(10)
)^(3/2) + 1/4*x^3/(-x^2 + sqrt(10))^(7/2) + 3/140*x/(-x^2 + sqrt(
10))^(5/2) - 3/28*sqrt(10)*x/(-x^2 + sqrt(10))^(7/2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + sqrt(10))^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.292029, size = 132, normalized size = 2.64

$$-\frac{16\left(7\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^2+20\right)}{175\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + sqrt(10))^(9/2),x, algorithm="giac")
```

```
[Out] -16/175*(7*(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 + s
qrt(10)) - 10^(1/4))/x)^2 + 20)/(x/(sqrt(-x^2 + sqrt(10)) - 10^(1
/4)) - (sqrt(-x^2 + sqrt(10)) - 10^(1/4))/x)^7
```

3.475 $\int \frac{x^2}{(3-x^2)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{\sqrt{3-x^2}}-\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Rubi [A] time = 0.0187712, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x}{\sqrt{3-x^2}}-\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(3 - x^2)^(3/2),x]

[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Rubi in Sympy [A] time = 1.62007, size = 19, normalized size = 0.79

$$\frac{x}{\sqrt{-x^2+3}}-\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+3)**(3/2),x)

[Out] x/sqrt(-x**2 + 3) - asin(sqrt(3)*x/3)
```

Mathematica [A] time = 0.0327183, size = 24, normalized size = 1.

$$\frac{x}{\sqrt{3-x^2}}-\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(3 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Maple [A] time = 0.008, size = 22, normalized size = 0.9

$$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+x\frac{1}{\sqrt{-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+3)^(3/2),x)
```

```
[Out] -arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

Maxima [A] time = 1.51201, size = 28, normalized size = 1.17

$$\frac{x}{\sqrt{-x^2+3}}-\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2 + 3)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)
```

Fricas [A] time = 0.211874, size = 55, normalized size = 2.29

$$\frac{(x^2-3)\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)-\sqrt{-x^2+3}x}{x^2-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2 + 3)^(3/2),x, algorithm="fricas")
```

```
[Out] ((x^2 - 3)*arctan(sqrt(-x^2 + 3)/x) - sqrt(-x^2 + 3)*x)/(x^2 - 3)
```

Sympy [A] time = 0.984954, size = 49, normalized size = 2.04

$$-\frac{x^2\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}-\frac{x\sqrt{-x^2+3}}{x^2-3}+\frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+3)**(3/2),x)
```

```
[Out] -x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(-x**2 + 3)/(x**2 - 3)
 + 3*asin(sqrt(3)*x/3)/(x**2 - 3)
```

GIAC/XCAS [A] time = 0.209452, size = 39, normalized size = 1.62

$$-\frac{\sqrt{-x^2+3}x}{x^2-3} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(-x^2 + 3)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)
```

3.476 $\int \frac{(25-x^2)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=40

$$\frac{\sqrt{25-x^2}}{x}-\frac{(25-x^2)^{3/2}}{3x^3}+\sin^{-1}\left(\frac{x}{5}\right)$$

```
[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

Rubi [A] time = 0.0268027, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\sqrt{25-x^2}}{x}-\frac{(25-x^2)^{3/2}}{3x^3}+\sin^{-1}\left(\frac{x}{5}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(25 - x^2)^(3/2)/x^4,x]

[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

Rubi in Sympy [A] time = 2.41493, size = 27, normalized size = 0.68

$$\operatorname{asin}\left(\frac{x}{5}\right)+\frac{\sqrt{-x^2+25}}{x}-\frac{(-x^2+25)^{\frac{3}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+25)**(3/2)/x**4,x)

[Out] asin(x/5) + sqrt(-x**2 + 25)/x - (-x**2 + 25)**(3/2)/(3*x**3)
```

Mathematica [A] time = 0.0287412, size = 34, normalized size = 0.85

$$\sqrt{25-x^2}\left(\frac{4}{3x}-\frac{25}{3x^3}\right)+\sin^{-1}\left(\frac{x}{5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(25 - x^2)^(3/2)/x^4,x]
```

```
[Out] (-25/(3*x^3) + 4/(3*x))*Sqrt[25 - x^2] + ArcSin[x/5]
```

Maple [A] time = 0.008, size = 58, normalized size = 1.5

$$-\frac{1}{75\,x^{3}}\left(-x^{2}+25\right)^{\frac{5}{2}}+\frac{2}{1875\,x}\left(-x^{2}+25\right)^{\frac{5}{2}}+\frac{2\,x}{1875}\left(-x^{2}+25\right)^{\frac{3}{2}}+\frac{x}{25}\sqrt{-x^{2}+25}+\arcsin\left(\frac{x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+25)^(3/2)/x^4,x)
```

```
[Out] -1/75/x^3*(-x^2+25)^(5/2)+2/1875/x*(-x^2+25)^(5/2)+2/1875*x*(-x^2
+25)^(3/2)+1/25*x*(-x^2+25)^(1/2)+arcsin(1/5*x)
```

Maxima [A] time = 1.55216, size = 61, normalized size = 1.52

$$\frac{1}{25}\sqrt{-x^{2}+25}x+\frac{2\left(-x^{2}+25\right)^{\frac{3}{2}}}{75\,x}-\frac{\left(-x^{2}+25\right)^{\frac{5}{2}}}{75\,x^{3}}+\arcsin\left(\frac{1}{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 25)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 +
25)^(5/2)/x^3 + arcsin(1/5*x)
```

Fricas [A] time = 0.211659, size = 167, normalized size = 4.18

$$\frac{4\,x^{6}-525\,x^{4}+13125\,x^{2}-6\left(15\,x^{5}-500\,x^{3}-\left(x^{5}-100\,x^{3}\right)\sqrt{-x^{2}+25}\right)\arctan\left(\frac{\sqrt{-x^{2}+25}-5}{x}\right)+5\left(12\,x^{4}-475\,x^{2}+2500\right)\sqrt{-}}{3\left(15\,x^{5}-500\,x^{3}-\left(x^{5}-100\,x^{3}\right)\sqrt{-x^{2}+25}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 25)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*x^6 - 525*x^4 + 13125*x^2 - 6*(15*x^5 - 500*x^3 - (x^5 - 1
00*x^3)*sqrt(-x^2 + 25))*arctan((sqrt(-x^2 + 25) - 5)/x) + 5*(12*
x^4 - 475*x^2 + 2500)*sqrt(-x^2 + 25) - 62500)/(15*x^5 - 500*x^3
- (x^5 - 100*x^3)*sqrt(-x^2 + 25))
```

Sympy [A] time = 4.15295, size = 32, normalized size = 0.8

$$\operatorname{asin}\left(\frac{x}{5}\right)+\frac{4\sqrt{-x^2+25}}{3x}-\frac{25\sqrt{-x^2+25}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+25)**(3/2)/x**4,x)
```

```
[Out] asin(x/5) + 4*sqrt(-x**2 + 25)/(3*x) - 25*sqrt(-x**2 + 25)/(3*x**
3)
```

GIAC/XCAS [A] time = 0.239168, size = 104, normalized size = 2.6

$$-\frac{x^3\left(\frac{15\left(\sqrt{-x^2+25}-5\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+25}-5\right)^3}+\frac{5\left(\sqrt{-x^2+25}-5\right)}{8\,x}-\frac{\left(\sqrt{-x^2+25}-5\right)^3}{24\,x^3}+\arcsin\left(\frac{1}{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 25)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*x^3*(15*(sqrt(-x^2 + 25) - 5)^2/x^2 - 1)/(sqrt(-x^2 + 25) -
 5)^3 + 5/8*(sqrt(-x^2 + 25) - 5)/x - 1/24*(sqrt(-x^2 + 25) - 5)^
3/x^3 + arcsin(1/5*x)
```

3.477 $\int \frac{1}{(1-2x^2)^{7/2}} \, dx$

Optimal. Leaf size=49

$$\frac{8x}{15\sqrt{1-2x^2}} + \frac{4x}{15\,(1-2x^2)^{3/2}} + \frac{x}{5\,(1-2x^2)^{5/2}}$$

```
[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(1
5*Sqrt[1 - 2*x^2])
```

Rubi [A] time = 0.0186643, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{8x}{15\sqrt{1-2x^2}} + \frac{4x}{15\,(1-2x^2)^{3/2}} + \frac{x}{5\,(1-2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^2)^(-7/2),x]
```

```
[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(1
5*Sqrt[1 - 2*x^2])
```

Rubi in Sympy [A] time = 0.761265, size = 42, normalized size = 0.86

$$\frac{8x}{15\sqrt{-2x^2+1}} + \frac{4x}{15\,(-2x^2+1)^{\frac{3}{2}}} + \frac{x}{5\,(-2x^2+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**2+1)**(7/2),x)
```

```
[Out] 8*x/(15*sqrt(-2*x**2 + 1)) + 4*x/(15*(-2*x**2 + 1)**(3/2)) + x/(5
*(-2*x**2 + 1)**(5/2))
```

Mathematica [A] time = 0.0198556, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4-40x^2+15\right)}{15\,(1-2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^2)^(-7/2),x]

[Out] (x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))
```

Maple [A] time = 0.005, size = 25, normalized size = 0.5

$$\frac{x\left(32x^{4}-40x^{2}+15\right)\left(-2x^{2}+1\right)^{-\frac{5}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^2+1)^(7/2),x)

[Out] 1/15*x*(32*x^4-40*x^2+15)/(-2*x^2+1)^(5/2)
```

Maxima [A] time = 1.42907, size = 50, normalized size = 1.02

$$\frac{8x}{15\sqrt{-2x^{2}+1}}+\frac{4x}{15\left(-2x^{2}+1\right)^{\frac{3}{2}}}+\frac{x}{5\left(-2x^{2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 1)^(-7/2),x, algorithm="maxima")

[Out] 8/15*x/sqrt(-2*x^2 + 1) + 4/15*x/(-2*x^2 + 1)^(3/2) + 1/5*x/(-2*x
^2 + 1)^(5/2)
```

Fricas [A] time = 0.206363, size = 165, normalized size = 3.37

$$\frac{160x^{9}-520x^{7}+603x^{5}-310x^{3}-\left(32x^{9}-232x^{7}+383x^{5}-250x^{3}+60x\right)\sqrt{-2x^{2}+1}+60x}{15\left(8x^{10}-60x^{8}+110x^{6}-85x^{4}+30x^{2}+\left(20x^{8}-60x^{6}+61x^{4}-26x^{2}+4\right)\sqrt{-2x^{2}+1}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 1)^(-7/2),x, algorithm="fricas")

[Out] 1/15*(160*x^9 - 520*x^7 + 603*x^5 - 310*x^3 - (32*x^9 - 232*x^7 +
 383*x^5 - 250*x^3 + 60*x)*sqrt(-2*x^2 + 1) + 60*x)/(8*x^10 - 60*
x^8 + 110*x^6 - 85*x^4 + 30*x^2 + (20*x^8 - 60*x^6 + 61*x^4 - 26*
x^2 + 4)*sqrt(-2*x^2 + 1) - 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**2+1)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.207985, size = 47, normalized size = 0.96

$$-\frac{\left(8\left(4x^2-5\right)x^2+15\right)\sqrt{-2x^2+1}x}{15\left(2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 1)^(-7/2),x, algorithm="giac")
```

```
[Out] -1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3
```

3.478 $\int \frac{1}{(-7+6x-x^2)^{5/2}} \, dx$

Optimal. Leaf size=47

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\left(-x^2+6x-7\right)^{3/2}}$$

```
[Out] -(3 - x)/(6*(-7 + 6*x - x^2)^(3/2)) - (3 - x)/(6*Sqrt[-7 + 6*x -
x^2])
```

Rubi [A] time = 0.0188272, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\left(-x^2+6x-7\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] -(3 - x)/(6*(-7 + 6*x - x^2)^(3/2)) - (3 - x)/(6*Sqrt[-7 + 6*x -
x^2])
```

Rubi in Sympy [A] time = 0.820637, size = 37, normalized size = 0.79

$$-\frac{-4x+12}{24\sqrt{-x^2+6x-7}}-\frac{-2x+6}{12\left(-x^2+6x-7\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+6*x-7)**(5/2),x)
```

```
[Out] -(-4*x + 12)/(24*sqrt(-x**2 + 6*x - 7)) - (-2*x + 6)/(12*(-x**2 +
 6*x - 7)**(3/2))
```

Mathematica [A] time = 0.0267931, size = 29, normalized size = 0.62

$$-\frac{(x-3)\left(x^2-6x+6\right)}{6\left(-x^2+6x-7\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-7 + 6*x - x^2)^(-5/2),x]

[Out] -((-3 + x)*(6 - 6*x + x^2))/(6*(-7 + 6*x - x^2)^(3/2))
```

Maple [A] time = 0.005, size = 28, normalized size = 0.6

$$-\frac{x^3-9x^2+24x-18}{6}\left(-x^2+6x-7\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+6*x-7)^(5/2),x)

[Out] -1/6*(x^3-9*x^2+24*x-18)/(-x^2+6*x-7)^(3/2)
```

Maxima [A] time = 1.49477, size = 80, normalized size = 1.7

$$\frac{x}{6\sqrt{-x^2+6x-7}}-\frac{1}{2\sqrt{-x^2+6x-7}}+\frac{x}{6\left(-x^2+6x-7\right)^{\frac{3}{2}}}-\frac{1}{2\left(-x^2+6x-7\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 6*x - 7)^(-5/2),x, algorithm="maxima")

[Out] 1/6*x/sqrt(-x^2 + 6*x - 7) - 1/2/sqrt(-x^2 + 6*x - 7) + 1/6*x/(-x
^2 + 6*x - 7)^(3/2) - 1/2/(-x^2 + 6*x - 7)^(3/2)
```

Fricas [A] time = 0.211081, size = 63, normalized size = 1.34

$$-\frac{\left(x^3-9x^2+24x-18\right)\sqrt{-x^2+6x-7}}{6\left(x^4-12x^3+50x^2-84x+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 6*x - 7)^(-5/2),x, algorithm="fricas")

[Out] -1/6*(x^3 - 9*x^2 + 24*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^4 - 12*x^3
 + 50*x^2 - 84*x + 49)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^2+6x-7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+6*x-7)**(5/2),x)
```

```
[Out] Integral((-x**2 + 6*x - 7)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.244061, size = 47, normalized size = 1.

$$-\frac{(((x-9)x+24)x-18)\sqrt{-x^2+6\,x-7}}{6\,(x^2-6\,x+7)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 6*x - 7)^(-5/2),x, algorithm="giac")
```

```
[Out] -1/6*(((x - 9)*x + 24)*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^2 - 6*x +
7)^2
```

3.479 $\int (1-2x-2x^2)^3 \, dx$

Optimal. Leaf size=36

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Rubi [A] time = 0.0245501, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x - 2*x^2)^3,x]
```

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3+x-6\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*x**2-2*x+1)**3,x)
```

```
[Out] -8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 + x - 6*Integral
(x, x)
```

Mathematica [A] time = 0.00211189, size = 36, normalized size = 1.

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x - 2*x^2)^3,x]
```

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$x-3x^{2}+2x^{3}+4x^{4}-\frac{12x^{5}}{5}-4x^{6}-\frac{8x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2-2*x+1)^3,x)
```

```
[Out] x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

Maxima [A] time = 1.76688, size = 43, normalized size = 1.19

$$-\frac{8}{7}x^{7}-4x^{6}-\frac{12}{5}x^{5}+4x^{4}+2x^{3}-3x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 + 2*x - 1)^3,x, algorithm="maxima")
```

```
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Fricas [A] time = 0.174392, size = 1, normalized size = 0.03

$$-\frac{8}{7}x^{7}-4x^{6}-\frac{12}{5}x^{5}+4x^{4}+2x^{3}-3x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 + 2*x - 1)^3,x, algorithm="fricas")
```

```
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Sympy [A] time = 0.038343, size = 34, normalized size = 0.94

$$-\frac{8x^{7}}{7}-4x^{6}-\frac{12x^{5}}{5}+4x^{4}+2x^{3}-3x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2-2*x+1)**3,x)

[Out] -8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x
```

GIAC/XCAS [A] time = 0.204213, size = 43, normalized size = 1.19

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^2 + 2*x - 1)^3,x, algorithm="giac")

[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

3.480 $\int (-1+5x)\left(-1-x+x^2\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Rubi [A] time = 0.035186, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}-x+3\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+5*x)*(x**2-x-1)**2,x)
```

```
[Out] 5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 - x + 3*Integral(x, x
)
```

Mathematica [A] time = 0.00247187, size = 39, normalized size = 1.

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Maple [A] time = 0.003, size = 30, normalized size = 0.8

$$-x+\frac{3x^{2}}{2}+\frac{11x^{3}}{3}-\frac{3x^{4}}{4}-\frac{11x^{5}}{5}+\frac{5x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+5*x)*(x^2-x-1)^2,x)
```

```
[Out] -x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

Maxima [A] time = 1.76693, size = 39, normalized size = 1.

$$\frac{5}{6}x^{6}-\frac{11}{5}x^{5}-\frac{3}{4}x^{4}+\frac{11}{3}x^{3}+\frac{3}{2}x^{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x - 1)^2*(5*x - 1),x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Fricas [A] time = 0.176109, size = 1, normalized size = 0.03

$$\frac{5}{6}x^{6}-\frac{11}{5}x^{5}-\frac{3}{4}x^{4}+\frac{11}{3}x^{3}+\frac{3}{2}x^{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x - 1)^2*(5*x - 1),x, algorithm="fricas")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Sympy [A] time = 0.042663, size = 34, normalized size = 0.87

$$\frac{5x^{6}}{6}-\frac{11x^{5}}{5}-\frac{3x^{4}}{4}+\frac{11x^{3}}{3}+\frac{3x^{2}}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+5*x)*(x**2-x-1)**2,x)
```

```
[Out] 5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x
```

GIAC/XCAS [A] time = 0.200296, size = 39, normalized size = 1.

$$\frac{5}{6}x^6 - \frac{11}{5}x^5 - \frac{3}{4}x^4 + \frac{11}{3}x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - x - 1)^2*(5*x - 1),x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{1-2x}{6\left(2x^2-8x+1\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

```
[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 -
8*x + 2*x^2])
```

Rubi [A] time = 0.0281777, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{1-2x}{6\left(2x^2-8x+1\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 -
8*x + 2*x^2])
```

Rubi in Sympy [A] time = 2.30734, size = 39, normalized size = 0.83

$$\frac{-28x+14}{84\left(2x^2-8x+1\right)^{\frac{3}{2}}}-\frac{-8x+16}{84\sqrt{2x^2-8x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
```

```
[Out] (-28*x + 14)/(84*(2*x**2 - 8*x + 1)**(3/2)) - (-8*x + 16)/(84*sqr
t(2*x**2 - 8*x + 1))
```

Mathematica [A] time = 0.026532, size = 33, normalized size = 0.7

$$\frac{8x^3-48x^2+54x-1}{42\left(2x^2-8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]

[Out] (-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

Maple [A] time = 0.006, size = 30, normalized size = 0.6

$$\frac{8x^3 - 48x^2 + 54x - 1}{42}\left(2x^2 - 8x + 1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+3*x)/(2*x^2-8*x+1)^(5/2),x)

[Out] 1/42*(8*x^3-48*x^2+54*x-1)/(2*x^2-8*x+1)^(3/2)
```

Maxima [A] time = 1.38613, size = 80, normalized size = 1.7

$$\frac{2x}{21\sqrt{2x^2 - 8x + 1}} - \frac{4}{21\sqrt{2x^2 - 8x + 1}} - \frac{x}{3(2x^2 - 8x + 1)^{\frac{3}{2}}} + \frac{1}{6(2x^2 - 8x + 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x, algorithm="maxima")

[Out] 2/21*x/sqrt(2*x^2 - 8*x + 1) - 4/21/sqrt(2*x^2 - 8*x + 1) - 1/3*x
/(2*x^2 - 8*x + 1)^(3/2) + 1/6/(2*x^2 - 8*x + 1)^(3/2)
```

Fricas [A] time = 0.209978, size = 186, normalized size = 3.96

$$\frac{18x^6 - 132x^5 + 282x^4 - 265x^3 + 75x^2 + \left(8x^5 - 59x^4 + 103x^3 - 51x^2 + 6x\right)\sqrt{2x^2 - 8x + 1} - 6x}{3\left(100x^6 - 864x^5 + 2220x^4 - 1552x^3 + 417x^2 + (88x^5 - 502x^4 + 692x^3 - 271x^2 + 40x - 2)\sqrt{2x^2 - 8x + 1} - 48x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x, algorithm="fricas")

[Out] 1/3*(18*x^6 - 132*x^5 + 282*x^4 - 265*x^3 + 75*x^2 + (8*x^5 - 59*
x^4 + 103*x^3 - 51*x^2 + 6*x)*sqrt(2*x^2 - 8*x + 1) - 6*x)/(100*x
^6 - 864*x^5 + 2220*x^4 - 1552*x^3 + 417*x^2 + (88*x^5 - 502*x^4
+ 692*x^3 - 271*x^2 + 40*x - 2)*sqrt(2*x^2 - 8*x + 1) - 48*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+1}{(2x^2-8x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
```

```
[Out] Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)
```

GIAC/XCAS [A] time = 0.210109, size = 36, normalized size = 0.77

$$\frac{2\,(4\,(x-6)x+27)x-1}{42\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^(3/2)
```

3.482 $\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\, dx$

Optimal. Leaf size=45

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

```
[Out] (-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[
1 + 2*x - 4*x^2])
```

Rubi [A] time = 0.0415108, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] (-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[
1 + 2*x - 4*x^2])
```

Rubi in Sympy [A] time = 6.49009, size = 58, normalized size = 1.29

$$\frac{2x^2}{\left(-4x^2+2x+1\right)^{\frac{3}{2}}}+\frac{61\left(-16x+4\right)}{600\sqrt{-4x^2+2x+1}}-\frac{152x+92}{120\left(-4x^2+2x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
```

```
[Out] 2*x**2/(-4*x**2 + 2*x + 1)**(3/2) + 61*(-16*x + 4)/(600*sqrt(-4*x
**2 + 2*x + 1)) - (152*x + 92)/(120*(-4*x**2 + 2*x + 1)**(3/2))
```

Mathematica [A] time = 0.0394235, size = 33, normalized size = 0.73

$$-\frac{-488x^3+216x^2+156x+27}{75\left(-4x^2+2x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] -(27 + 156*x + 216*x^2 - 488*x^3)/(75*(1 + 2*x - 4*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 30, normalized size = 0.7

$$\frac{488\,x^{3}-216\,x^{2}-156\,x-27}{75}\left(-4\,x^{2}+2\,x+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x)
```

```
[Out] 1/75*(488*x^3-216*x^2-156*x-27)/(-4*x^2+2*x+1)^(3/2)
```

Maxima [A] time = 1.35143, size = 103, normalized size = 2.29

$$-\frac{122\,x}{75\,\sqrt{-4\,x^{2}+2\,x+1}}+\frac{2\,x^{2}}{\left(-4\,x^{2}+2\,x+1\right)^{\frac{3}{2}}}+\frac{61}{150\,\sqrt{-4\,x^{2}+2\,x+1}}-\frac{19\,x}{15\,\left(-4\,x^{2}+2\,x+1\right)^{\frac{3}{2}}}-\frac{23}{30\,\left(-4\,x^{2}+2\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3 - 8*x - 1)/(-4*x^2 + 2*x + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] -122/75*x/sqrt(-4*x^2 + 2*x + 1) + 2*x^2/(-4*x^2 + 2*x + 1)^(3/2)
 + 61/150/sqrt(-4*x^2 + 2*x + 1) - 19/15*x/(-4*x^2 + 2*x + 1)^(3/
2) - 23/30/(-4*x^2 + 2*x + 1)^(3/2)
```

Fricas [A] time = 0.205839, size = 186, normalized size = 4.13

$$-\frac{232\,x^{6}-192\,x^{5}-264\,x^{4}+103\,x^{3}+90\,x^{2}+\left(28\,x^{5}+154\,x^{4}-55\,x^{3}-78\,x^{2}-12\,x\right)\sqrt{-4\,x^{2}+2\,x+1}+12\,x}{3\left(16\,x^{6}-144\,x^{5}+60\,x^{4}+100\,x^{3}-15\,x^{2}+\left(44\,x^{5}-10\,x^{4}-65\,x^{3}+5\,x^{2}+20\,x+4\right)\sqrt{-4\,x^{2}+2\,x+1}-24\,x-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3 - 8*x - 1)/(-4*x^2 + 2*x + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(232*x^6 - 192*x^5 - 264*x^4 + 103*x^3 + 90*x^2 + (28*x^5 +
154*x^4 - 55*x^3 - 78*x^2 - 12*x)*sqrt(-4*x^2 + 2*x + 1) + 12*x)/
(16*x^6 - 144*x^5 + 60*x^4 + 100*x^3 - 15*x^2 + (44*x^5 - 10*x^4
- 65*x^3 + 5*x^2 + 20*x + 4)*sqrt(-4*x^2 + 2*x + 1) - 24*x - 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.214437, size = 55, normalized size = 1.22

$$\frac{(4\,(2\,(61\,x-27)x-39)x-27)\sqrt{-4\,x^2+2\,x+1}}{75\,(4\,x^2-2\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3 - 8*x - 1)/(-4*x^2 + 2*x + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*
x^2 - 2*x - 1)^2
```

3.483 $\int x^2 \cos^5(x)\, dx$

Optimal. Leaf size=83

$$\frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\cos^3(x)+\frac{16}{15}x\cos(x)$$

```
[Out] (16*x*Cos[x])/15 + (8*x*Cos[x]^3)/45 + (2*x*Cos[x]^5)/25 - (298*S
in[x])/225 + (8*x^2*Sin[x])/15 + (4*x^2*Cos[x]^2*Sin[x])/15 + (x^
2*Cos[x]^4*Sin[x])/5 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125
```

Rubi [A] time = 0.137297, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\cos^3(x)+\frac{16}{15}x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[x]^5,x]
```

```
[Out] (16*x*Cos[x])/15 + (8*x*Cos[x]^3)/45 + (2*x*Cos[x]^5)/25 - (298*S
in[x])/225 + (8*x^2*Sin[x])/15 + (4*x^2*Cos[x]^2*Sin[x])/15 + (x^
2*Cos[x]^4*Sin[x])/5 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125
```

Rubi in Sympy [A] time = 4.03267, size = 90, normalized size = 1.08

$$\frac{x^2\sin(x)\cos^4(x)}{5}+\frac{4x^2\sin(x)\cos^2(x)}{15}+\frac{8x^2\sin(x)}{15}+\frac{2x\cos^5(x)}{25}+\frac{8x\cos^3(x)}{45}+\frac{16x\cos(x)}{15}-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(x)**5,x)
```

```
[Out] x**2*sin(x)*cos(x)**4/5 + 4*x**2*sin(x)*cos(x)**2/15 + 8*x**2*sin
(x)/15 + 2*x*cos(x)**5/25 + 8*x*cos(x)**3/45 + 16*x*cos(x)/15 - 2
*sin(x)**5/125 + 76*sin(x)**3/675 - 298*sin(x)/225
```

Mathematica [A] time = 0.0653242, size = 67, normalized size = 0.81

$$\frac{5}{8}\left(x^2-2\right)\sin(x)+\frac{5}{432}\left(9x^2-2\right)\sin(3x)+\frac{\left(25x^2-2\right)\sin(5x)}{2000}+\frac{5}{4}x\cos(x)+\frac{5}{72}x\cos(3x)+\frac{1}{200}x\cos(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[x]^5,x]
```

```
[Out] (5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 +
x^2)*Sin[x])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*S
in[5*x])/2000
```

Maple [A] time = 0.02, size = 70, normalized size = 0.8

$$\frac{x^2\sin(x)}{5}\left(\frac{8}{3}+(\cos(x))^4+\frac{4(\cos(x))^2}{3}\right)-\frac{16\sin(x)}{15}+\frac{16x\cos(x)}{15}+\frac{2x(\cos(x))^5}{25}-\frac{2\sin(x)}{125}\left(\frac{8}{3}+(\cos(x))^4+\frac{4(\cos(x))^2}{3}\right)+\frac{8x(\cos(x))^3}{45}-\frac{\left(16+8(\cos(x))^2\right)\sin(x)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(x)^5,x)
```

```
[Out] 1/5*x^2*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)-16/15*sin(x)+16/15*x*c
os(x)+2/25*x*cos(x)^5-2/125*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)+8/
45*x*cos(x)^3-8/135*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 1.44443, size = 74, normalized size = 0.89

$$\frac{1}{200}x\cos(5x)+\frac{5}{72}x\cos(3x)+\frac{5}{4}x\cos(x)+\frac{1}{2000}\left(25x^2-2\right)\sin(5x)+\frac{5}{432}\left(9x^2-2\right)\sin(3x)+\frac{5}{8}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^
2 - 2)*sin(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(
x)
```

Fricas [A] time = 0.225764, size = 77, normalized size = 0.93

$$\frac{2}{25}\,x\cos\left(x\right)^{5}+\frac{8}{45}\,x\cos\left(x\right)^{3}+\frac{16}{15}\,x\cos\left(x\right)+\frac{1}{3375}\,\left(27\,\left(25\,x^{2}-2\right)\,\cos\left(x\right)^{4}+4\,\left(225\,x^{2}-68\right)\,\cos\left(x\right)^{2}+1800\,x^{2}-4144\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^5,x, algorithm="fricas")
```

```
[Out] 2/25*x*cos(x)^5 + 8/45*x*cos(x)^3 + 16/15*x*cos(x) + 1/3375*(27*(
25*x^2 - 2)*cos(x)^4 + 4*(225*x^2 - 68)*cos(x)^2 + 1800*x^2 - 414
4)*sin(x)
```

Sympy [A] time = 5.72153, size = 112, normalized size = 1.35

$$\frac{8x^{2}\sin^{5}\left(x\right)}{15}+\frac{4x^{2}\sin^{3}\left(x\right)\cos^{2}\left(x\right)}{3}+x^{2}\sin\left(x\right)\cos^{4}\left(x\right)+\frac{16x\sin^{4}\left(x\right)\cos\left(x\right)}{15}+\frac{104x\sin^{2}\left(x\right)\cos^{3}\left(x\right)}{45}+\frac{298x\cos^{5}\left(x\right)}{225}-\frac{4144\sin^{5}\left(x\right)}{3375}-\frac{1712\sin^{3}\left(x\right)\cos^{2}\left(x\right)}{675}-\frac{298\sin\left(x\right)\cos^{4}\left(x\right)}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(x)**5,x)
```

```
[Out] 8*x**2*sin(x)**5/15 + 4*x**2*sin(x)**3*cos(x)**2/3 + x**2*sin(x)*
cos(x)**4 + 16*x*sin(x)**4*cos(x)/15 + 104*x*sin(x)**2*cos(x)**3/
45 + 298*x*cos(x)**5/225 - 4144*sin(x)**5/3375 - 1712*sin(x)**3*c
os(x)**2/675 - 298*sin(x)*cos(x)**4/225
```

GIAC/XCAS [A] time = 0.236828, size = 74, normalized size = 0.89

$$\frac{1}{200}\,x\cos\left(5\,x\right)+\frac{5}{72}\,x\cos\left(3\,x\right)+\frac{5}{4}\,x\cos\left(x\right)+\frac{1}{2000}\,\left(25\,x^{2}-2\right)\,\sin\left(5\,x\right)+\frac{5}{432}\,\left(9\,x^{2}-2\right)\,\sin\left(3\,x\right)+\frac{5}{8}\,\left(x^{2}-2\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^
2 - 2)*sin(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(
```

x)

3.484 $\int x^3 \sin^3(x)\,dx$

Optimal. Leaf size=73

$$-\frac{2}{3}x^3\cos(x)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+2x^2\sin(x)-\frac{2\sin^3(x)}{27}-\frac{40\sin(x)}{9}+\frac{40}{9}x\cos(x)+\frac{2}{9}x\sin^2(x)\cos(x)$$

```
[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x]
 + (2*x*Cos[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^
3)/27 + (x^2*Sin[x]^3)/3
```

Rubi [A] time = 0.123912, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{2}{3}x^3\cos(x)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+2x^2\sin(x)-\frac{2\sin^3(x)}{27}-\frac{40\sin(x)}{9}+\frac{40}{9}x\cos(x)+\frac{2}{9}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x]^3,x]
```

```
[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x]
 + (2*x*Cos[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^
3)/27 + (x^2*Sin[x]^3)/3
```

Rubi in Sympy [A] time = 3.74182, size = 78, normalized size = 1.07

$$-\frac{x^3\sin^2(x)\cos(x)}{3}-\frac{2x^3\cos(x)}{3}+\frac{x^2\sin^3(x)}{3}+2x^2\sin(x)+\frac{2x\sin^2(x)\cos(x)}{9}+\frac{40x\cos(x)}{9}-\frac{2\sin^3(x)}{27}-\frac{40\sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*sin(x)**3,x)
```

```
[Out] -x**3*sin(x)**2*cos(x)/3 - 2*x**3*cos(x)/3 + x**2*sin(x)**3/3 + 2
*x**2*sin(x) + 2*x*sin(x)**2*cos(x)/9 + 40*x*cos(x)/9 - 2*sin(x)*
*3/27 - 40*sin(x)/9
```

Mathematica [A] time = 0.138985, size = 51, normalized size = 0.7

$$\frac{1}{108}\left(243\left(x^2-2\right)\sin(x)-\left(9x^2-2\right)\sin(3x)-81x\left(x^2-6\right)\cos(x)+3x\left(3x^2-2\right)\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sin[x]^3,x]
```

```
[Out] (-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 +
x^2)*Sin[x] - (-2 + 9*x^2)*Sin[3*x])/108
```

Maple [A] time = 0.037, size = 57, normalized size = 0.8

$$-\frac{x^3\left(2+(\sin(x))^2\right)\cos(x)}{3}+2x^2\sin(x)-\frac{40\sin(x)}{9}+4x\cos(x)+\frac{x^2(\sin(x))^3}{3}+\frac{2x\left(2+(\sin(x))^2\right)\cos(x)}{9}-\frac{2(\sin(x))^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*sin(x)^3,x)
```

```
[Out] -1/3*x^3*(2+sin(x)^2)*cos(x)+2*x^2*sin(x)-40/9*sin(x)+4*x*cos(x)+
1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3
```

Maxima [A] time = 1.3478, size = 66, normalized size = 0.9

$$\frac{1}{36}\left(3x^3-2x\right)\cos(3x)-\frac{3}{4}\left(x^3-6x\right)\cos(x)-\frac{1}{108}\left(9x^2-2\right)\sin(3x)+\frac{9}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x
^2 - 2)*sin(3*x) + 9/4*(x^2 - 2)*sin(x)
```

Fricas [A] time = 0.222215, size = 70, normalized size = 0.96

$$\frac{1}{9}\left(3x^3-2x\right)\cos(x)^3-\frac{1}{3}\left(3x^3-14x\right)\cos(x)-\frac{1}{27}\left(\left(9x^2-2\right)\cos(x)^2-63x^2+122\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9
*x^2 - 2)*cos(x)^2 - 63*x^2 + 122)*sin(x)
```

Sympy [A] time = 3.17871, size = 92, normalized size = 1.26

$$-x^3\sin^2(x)\cos(x)-\frac{2x^3\cos^3(x)}{3}+\frac{7x^2\sin^3(x)}{3}+2x^2\sin(x)\cos^2(x)+\frac{14x\sin^2(x)\cos(x)}{3}+\frac{40x\cos^3(x)}{9}-\frac{122\sin^3(x)}{27}-\frac{40\sin(x)\cos^2(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*sin(x)**3,x)
```

```
[Out] -x**3*sin(x)**2*cos(x) - 2*x**3*cos(x)**3/3 + 7*x**2*sin(x)**3/3
+ 2*x**2*sin(x)*cos(x)**2 + 14*x*sin(x)**2*cos(x)/3 + 40*x*cos(x)
**3/9 - 122*sin(x)**3/27 - 40*sin(x)*cos(x)**2/9
```

GIAC/XCAS [A] time = 0.223877, size = 66, normalized size = 0.9

$$\frac{1}{36}\left(3x^3-2x\right)\cos(3x)-\frac{3}{4}\left(x^3-6x\right)\cos(x)-\frac{1}{108}\left(9x^2-2\right)\sin(3x)+\frac{9}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x
^2 - 2)*sin(3*x) + 9/4*(x^2 - 2)*sin(x)
```

3.485 $\int x^2 \sin^6(x)\,dx$

Optimal. Leaf size=105

$$\frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2(x)+\frac{1}{108}\sin^5(x)\cos(x)+\frac{65\sin^3(x)\cos(x)}{1728}+\frac{245\sin(x)\cos(x)}{1152}$$

```
[Out] (-245*x)/1152 + (5*x^3)/48 + (245*Cos[x]*Sin[x])/1152 - (5*x^2*Co
s[x]*Sin[x])/16 + (5*x*Sin[x]^2)/16 + (65*Cos[x]*Sin[x]^3)/1728 -
 (5*x^2*Cos[x]*Sin[x]^3)/24 + (5*x*Sin[x]^4)/48 + (Cos[x]*Sin[x]^
5)/108 - (x^2*Cos[x]*Sin[x]^5)/6 + (x*Sin[x]^6)/18
```

Rubi [A] time = 0.164945, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2(x)+\frac{1}{108}\sin^5(x)\cos(x)+\frac{65\sin^3(x)\cos(x)}{1728}+\frac{245\sin(x)\cos(x)}{1152}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[x]^6,x]
```

```
[Out] (-245*x)/1152 + (5*x^3)/48 + (245*Cos[x]*Sin[x])/1152 - (5*x^2*Co
s[x]*Sin[x])/16 + (5*x*Sin[x]^2)/16 + (65*Cos[x]*Sin[x]^3)/1728 -
 (5*x^2*Cos[x]*Sin[x]^3)/24 + (5*x*Sin[x]^4)/48 + (Cos[x]*Sin[x]^
5)/108 - (x^2*Cos[x]*Sin[x]^5)/6 + (x*Sin[x]^6)/18
```

Rubi in Sympy [A] time = 4.21456, size = 114, normalized size = 1.09

$$\frac{5x^3}{48}-\frac{x^2\sin^5(x)\cos(x)}{6}-\frac{5x^2\sin^3(x)\cos(x)}{24}-\frac{5x^2\sin(x)\cos(x)}{16}+\frac{x\sin^6(x)}{18}+\frac{5x\sin^4(x)}{48}+\frac{5x\sin^2(x)}{16}-\frac{245x}{1152}+\frac{\sin^5(x)\cos(x)}{108}+\frac{65\sin^3(x)\cos(x)}{1728}+\frac{245\sin(x)\cos(x)}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(x)**6,x)
```

```
[Out] 5*x**3/48 - x**2*sin(x)**5*cos(x)/6 - 5*x**2*sin(x)**3*cos(x)/24
- 5*x**2*sin(x)*cos(x)/16 + x*sin(x)**6/18 + 5*x*sin(x)**4/48 + 5
```

```
*x*sin(x)**2/16 - 245*x/1152 + sin(x)**5*cos(x)/108 + 65*sin(x)**
3*cos(x)/1728 + 245*sin(x)*cos(x)/1152
```

Mathematica [A] time = 0.116413, size = 70, normalized size = 0.67

$$\frac{1440x^3 - 1620\left(2x^2-1\right)\sin(2x) + 81\left(8x^2-1\right)\sin(4x) - 4\left(18x^2-1\right)\sin(6x) - 3240x\cos(2x) + 324x\cos(4x) - 24x\cos(6x)}{13824}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[x]^6,x]
```

```
[Out] (1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 16
20*(-1 + 2*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*
x^2)*Sin[6*x])/13824
```

Maple [A] time = 0.043, size = 96, normalized size = 0.9

$$x^2\left(-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5\,(\sin(x))^3}{4}+\frac{15\,\sin(x)}{8}\right)+\frac{5\,x}{16}\right)+\frac{x\,(\sin(x))^6}{18} + \frac{\cos(x)}{108}\left((\sin(x))^5+\frac{5\,(\sin(x))^3}{4}+\frac{15\,\sin(x)}{8}\right)+\frac{115\,x}{1152}+\frac{5\,x\,(\sin(x))^4}{48} + \frac{5\,\cos(x)}{192}\left((\sin(x))^3+\frac{3\,\sin(x)}{2}\right)-\frac{5\,x\,(\cos(x))^2}{16}+\frac{5\,\cos(x)\sin(x)}{32}-\frac{5\,x^3}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(x)^6,x)
```

```
[Out] x^2*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+1/18
*x*sin(x)^6+1/108*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+115/
1152*x+5/48*x*sin(x)^4+5/192*(sin(x)^3+3/2*sin(x))*cos(x)-5/16*x*
cos(x)^2+5/32*cos(x)*sin(x)-5/24*x^3
```

Maxima [A] time = 1.37144, size = 89, normalized size = 0.85

$$\frac{5}{48}\,x^3-\frac{1}{576}\,x\cos(6\,x)+\frac{3}{128}\,x\cos(4\,x)-\frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\left(18\,x^2-1\right)\sin(6\,x)+\frac{3}{512}\left(8\,x^2-1\right)\sin(4\,x)-\frac{15}{128}\left(2\,x^2-1\right)\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x)
 - 1/3456*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15
/128*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 0.228992, size = 97, normalized size = 0.92

$$-\frac{1}{18}\,x\cos\left(x\right)^{6}+\frac{13}{48}\,x\cos\left(x\right)^{4}+\frac{5}{48}\,x^{3}-\frac{11}{16}\,x\cos\left(x\right)^{2}-\frac{1}{3456}\,\left(32\,\left(18\,x^{2}-1\right)\,\cos\left(x\right)^{5}-2\,\left(936\,x^{2}-97\right)\,\cos\left(x\right)^{3}+3\,\left(792\,x^{2}-299\right)\,\cos\left(x\right)\right)\,\sin\left(x\right)+\frac{299}{1152}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/18*x*cos(x)^6 + 13/48*x*cos(x)^4 + 5/48*x^3 - 11/16*x*cos(x)^2
 - 1/3456*(32*(18*x^2 - 1)*cos(x)^5 - 2*(936*x^2 - 97)*cos(x)^3 +
 3*(792*x^2 - 299)*cos(x))*sin(x) + 299/1152*x
```

Sympy [A] time = 9.78817, size = 192, normalized size = 1.83

$$\frac{5x^{3}\sin^{6}\left(x\right)}{48}+\frac{5x^{3}\sin^{4}\left(x\right)\cos^{2}\left(x\right)}{16}+\frac{5x^{3}\sin^{2}\left(x\right)\cos^{4}\left(x\right)}{16}+\frac{5x^{3}\cos^{6}\left(x\right)}{48}-\frac{11x^{2}\sin^{5}\left(x\right)\cos\left(x\right)}{16}-\frac{5x^{2}\sin^{3}\left(x\right)\cos^{3}\left(x\right)}{6}-\frac{5x^{2}\sin\left(x\right)\cos^{5}\left(x\right)}{16}+\frac{299x\sin^{6}\left(x\right)}{1152}+\frac{35x\sin^{4}\left(x\right)\cos^{2}\left(x\right)}{384}-\frac{125x\sin^{2}\left(x\right)\cos^{4}\left(x\right)}{384}-\frac{245x\cos^{6}\left(x\right)}{1152}+\frac{299\sin^{5}\left(x\right)\cos\left(x\right)}{1152}+\frac{25\sin^{3}\left(x\right)\cos^{3}\left(x\right)}{54}+\frac{245\sin\left(x\right)\cos^{5}\left(x\right)}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(x)**6,x)
```

```
[Out] 5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(
x)**2*cos(x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(
x)/16 - 5*x**2*sin(x)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16
 + 299*x*sin(x)**6/1152 + 35*x*sin(x)**4*cos(x)**2/384 - 125*x*si
n(x)**2*cos(x)**4/384 - 245*x*cos(x)**6/1152 + 299*sin(x)**5*cos(
x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin(x)*cos(x)**5/1152
```

GIAC/XCAS [A] time = 0.233638, size = 89, normalized size = 0.85

$$\frac{5}{48}x^3 - \frac{1}{576}x\cos(6x) + \frac{3}{128}x\cos(4x) - \frac{15}{64}x\cos(2x) - \frac{1}{3456}\left(18x^2 - 1\right)\sin(6x) + \frac{3}{512}\left(8x^2 - 1\right)\sin(4x) - \frac{15}{128}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x)
 - 1/3456*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15
/128*(2*x^2 - 1)*sin(2*x)
```

3.486 $\int x^2 \cos(x) \sin^2(x) \, dx$

Optimal. Leaf size=44

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9} + \frac{4}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

```
[Out] (4*x*Cos[x])/9 - (4*Sin[x])/9 + (2*x*Cos[x]*Sin[x]^2)/9 - (2*Sin[
x]^3)/27 + (x^2*Sin[x]^3)/3
```

Rubi [A] time = 0.0641787, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9} + \frac{4}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (4*x*Cos[x])/9 - (4*Sin[x])/9 + (2*x*Cos[x]*Sin[x]^2)/9 - (2*Sin[
x]^3)/27 + (x^2*Sin[x]^3)/3
```

Rubi in Sympy [A] time = 2.84775, size = 46, normalized size = 1.05

$$\frac{x^2 \sin^3(x)}{3} + \frac{2x \sin^2(x)\cos(x)}{9} + \frac{4x\cos(x)}{9} - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(x)*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/9 + 4*x*cos(x)/9 - 2*sin(
x)**3/27 - 4*sin(x)/9
```

Mathematica [A] time = 0.200041, size = 39, normalized size = 0.89

$$\frac{1}{54}\left(\sin(x)\left(9x^2 + \left(2 - 9x^2\right)\cos(2x) - 26\right) + 27x\cos(x) - 3x\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x]
)*Sin[x])/54
```

Maple [A] time = 0.006, size = 32, normalized size = 0.7

$$\frac{x^2 (\sin(x))^3}{3} + \frac{2x\left(2 + (\sin(x))^2\right)\cos(x)}{9} - \frac{2\,(\sin(x))^3}{27} - \frac{4\,\sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(x)*sin(x)^2,x)
```

```
[Out] 1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3-4/9*sin(
x)
```

Maxima [A] time = 1.35956, size = 47, normalized size = 1.07

$$-\frac{1}{18}x\cos(3x) + \frac{1}{2}x\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{1}{4}\left(x^2 - 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/
4*(x^2 - 2)*sin(x)
```

Fricas [A] time = 0.220398, size = 49, normalized size = 1.11

$$-\frac{2}{9}x\cos(x)^3 + \frac{2}{3}x\cos(x) - \frac{1}{27}\left(\left(9x^2 - 2\right)\cos(x)^2 - 9x^2 + 14\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -2/9*x*cos(x)^3 + 2/3*x*cos(x) - 1/27*((9*x^2 - 2)*cos(x)^2 - 9*x
^2 + 14)*sin(x)
```

Sympy [A] time = 1.68804, size = 53, normalized size = 1.2

$$\frac{x^2\sin^3(x)}{3}+\frac{2x\sin^2(x)\cos(x)}{3}+\frac{4x\cos^3(x)}{9}-\frac{14\sin^3(x)}{27}-\frac{4\sin(x)\cos^2(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(x)*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/3 + 4*x*cos(x)**3/9 - 14*
sin(x)**3/27 - 4*sin(x)*cos(x)**2/9
```

GIAC/XCAS [A] time = 0.197077, size = 47, normalized size = 1.07

$$-\frac{1}{18}\,x\cos(3\,x)+\frac{1}{2}\,x\cos(x)-\frac{1}{108}\left(9\,x^2-2\right)\sin(3\,x)+\frac{1}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/
4*(x^2 - 2)*sin(x)
```

3.487 $\int x\cos^2(x)\cot^2(x)\,dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[
x])/2
```

Rubi [A] time = 0.0753717, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[
x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos^4(x)}{\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(x)**4/sin(x)**2,x)
```

```
[Out] Integral(x*cos(x)**4/sin(x)**2, x)
```

Mathematica [A] time = 0.01407, size = 33, normalized size = 1.

$$-\frac{3x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)-x\cot(x)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

Maple [B] time = 0.16, size = 111, normalized size = 3.4

$$1\left(-\frac{1}{2}\tan\left(\frac{x}{2}\right)-\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{5}-\frac{x}{2}-\frac{3x}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{2}+\frac{3x}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{4}+\frac{x}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{6}-\frac{3x^{2}}{4}\tan\left(\frac{x}{2}\right)-\frac{3x^{2}}{2}\left(\tan\left(\frac{x}{2}\right.\right.\right.$$

$$-\ln\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x)^4/sin(x)^2,x)
```

```
[Out] (-1/2*tan(1/2*x)-1/2*tan(1/2*x)^5-1/2*x-3/2*x*tan(1/2*x)^2+3/2*x*
tan(1/2*x)^4+1/2*x*tan(1/2*x)^6-3/4*x^2*tan(1/2*x)-3/2*x^2*tan(1/
2*x)^3-3/4*x^2*tan(1/2*x)^5)/(tan(1/2*x)^2+1)^2/tan(1/2*x)-ln(tan
(1/2*x)^2+1)+ln(tan(1/2*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.23522, size = 61, normalized size = 1.85

$$\frac{4\,x\cos\left(x\right)^{3}-12\,x\cos\left(x\right)-\left(6\,x^{2}+2\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)+8\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)\sin\left(x\right)}{8\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x)
 + 8*log(1/2*sin(x))*sin(x))/sin(x)
```

Sympy [A] time = 4.86808, size = 507, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)**4/sin(x)**2,x)
```

```
[Out] -3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
- 6*x**2*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
 - 3*x**2*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) +
 2*x*tan(x/2)**6/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6
*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x
*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 2*x/(
4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(tan(x/2)**2 +
 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 8*
log(tan(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
 4*tan(x/2)) - 4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8
*tan(x/2)**3 + 4*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x
/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 8*log(tan(x/2))*tan(x/2)**
3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 4*log(tan(x/2))*
tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 4*tan(x/2
)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

GIAC/XCAS [A] time = 0.229129, size = 278, normalized size = 8.42

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\ln\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\tan\left(\frac{1}{2}\,x\right)^4 + \tan\left(\frac{1}{2}\,x\right)^5 \cdots}{8\left(\tan\left(\frac{1}{2} \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)^4/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*ln(16*tan(1/2*x)^
2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(
1/2*x)^3 - 12*x*tan(1/2*x)^4 + tan(1/2*x)^5 - 8*ln(16*tan(1/2*x)^
2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1
/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*x)^3 - 4*ln(16*tan(1/2*x)^2
/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) + 4*x + tan(1/2*
x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

3.488 $\int x \sec(x) \tan^3(x)\, dx$

Optimal. Leaf size=30

$$\frac{1}{3}x \sec^3(x) - x \sec(x) + \frac{5}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tan(x) \sec(x)$$

```
[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x
])/6
```

Rubi [A] time = 0.061406, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{3}x \sec^3(x) - x \sec(x) + \frac{5}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x
])/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin^3(x)}{\cos^4(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x)**3/cos(x)**4,x)
```

```
[Out] Integral(x*sin(x)**3/cos(x)**4, x)
```

Mathematica [B] time = 0.182542, size = 104, normalized size = 3.47

$$-\frac{1}{24} \sec^3(x) \left(4x + 2\sin(2x) + 12x\cos(2x) + 5\cos(3x)\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 15\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right) - 5\cos(3x)\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] -(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x
/2]] + 15*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x
/2]]) - 5*Cos[3*x]*Log[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))/24
```

Maple [A] time = 0.174, size = 30, normalized size = 1.

$$-\frac{x}{\cos(x)}+\frac{5\ln(\sec(x)+\tan(x))}{6}+\frac{x}{3(\cos(x))^3}-\frac{\sec(x)\tan(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)^3/cos(x)^4,x)
```

```
[Out] -x/cos(x)+5/6*ln(sec(x)+tan(x))+1/3*x/cos(x)^3-1/6*sec(x)*tan(x)
```

Maxima [A] time = 1.58442, size = 836, normalized size = 27.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(48*x*sin(3*x)*sin(2*x) + 4*(6*x*cos(5*x) + 4*x*cos(3*x) +
6*x*cos(x) + sin(5*x) - sin(x))*cos(6*x) + 12*(6*x*cos(4*x) + 6*x
*cos(2*x) + 2*x - sin(4*x) - sin(2*x))*cos(5*x) + 12*(4*x*cos(3*x
) + 6*x*cos(x) - sin(x))*cos(4*x) + 16*(3*x*cos(2*x) + x)*cos(3*x
) + 12*(6*x*cos(x) - sin(x))*cos(2*x) + 24*x*cos(x) - 5*(2*(3*cos
(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1
)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x)
)*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9
*sin(2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x)
+ 1) + 5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 +
 6*(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(s
in(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin
(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*sin(x) + 1) + 4*(6*x*sin(5*x) + 4*x*sin(3*x) + 6*x*sin
(x) - cos(5*x) + cos(x))*sin(6*x) + 4*(18*x*sin(4*x) + 18*x*sin(2
*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*sin(5*x) + 12*(4*x*sin(3*x) +
6*x*sin(x) + cos(x))*sin(4*x) + 12*(6*x*sin(x) + cos(x))*sin(2*x)
 - 4*sin(x))/(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)
^2 + 6*(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 +
```

```
6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18
*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)
```

Fricas [A] time = 0.228565, size = 63, normalized size = 2.1

$$\frac{5\cos(x)^3\log(\sin(x)+1) - 5\cos(x)^3\log(-\sin(x)+1) - 12\,x\cos(x)^2 - 2\cos(x)\sin(x) + 4\,x}{12\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) -
12*x*cos(x)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3
```

Sympy [A] time = 3.74601, size = 551, normalized size = 18.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**3/cos(x)**4,x)
```

```
[Out] 4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2
- 6) - 12*x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(
x/2)**2 - 6) - 12*x*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 +
 18*tan(x/2)**2 - 6) + 4*x/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*t
an(x/2)**2 - 6) - 5*log(tan(x/2) - 1)*tan(x/2)**6/(6*tan(x/2)**6
- 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) - 1)*tan
(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) -
15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4
+ 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*t
an(x/2)**4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**
6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(
tan(x/2) + 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*ta
n(x/2)**2 - 6) + 15*log(tan(x/2) + 1)*tan(x/2)**2/(6*tan(x/2)**6
- 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) + 1)/(6*t
an(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 2*tan(x/2)**5
/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*tan(x/
2)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)
```

GIAC/XCAS [A] time = 0.436535, size = 460, normalized size = 15.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(8*x*tan(1/2*x)^6 + 5*ln(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 5*ln(2*(tan(1/2*x)^2 - 2*tan(
1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4
- 15*ln(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*t
an(1/2*x)^4 + 15*ln(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5 - 24*x*tan(1/2*x)^2 + 15
*ln(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1
/2*x)^2 - 15*ln(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^2 + 8*x - 5*ln(2*(tan(1/2*x)^2 + 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 5*ln(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1
)/(tan(1/2*x)^2 + 1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x
)^4 + 3*tan(1/2*x)^2 - 1)
```

3.489 $\int x \sec^2(x) \tan(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2}$$

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Rubi [A] time = 0.0288477, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sec[x]^2*Tan[x],x]

[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Rubi in Sympy [A] time = 1.4015, size = 15, normalized size = 0.94

$$\frac{x}{2\cos^2(x)} - \frac{\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x)/cos(x)**3,x)

[Out] x/(2*cos(x)**2) - sin(x)/(2*cos(x))
```

Mathematica [A] time = 0.00750552, size = 16, normalized size = 1.

$$\frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sec[x]^2*Tan[x],x]
```

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Maple [A] time = 0.009, size = 13, normalized size = 0.8

$$\frac{x}{2\left(\cos\left(x\right)\right)^{2}}-\frac{\tan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)/cos(x)^3,x)
```

```
[Out] 1/2*x/cos(x)^2-1/2*tan(x)
```

Maxima [A] time = 1.33755, size = 178, normalized size = 11.12

$$\frac{4\,x\cos\left(2\,x\right)^{2}+4\,x\sin\left(2\,x\right)^{2}+\left(2\,x\cos\left(2\,x\right)+\sin\left(2\,x\right)\right)\cos\left(4\,x\right)+2\,x\cos\left(2\,x\right)+\left(2\,x\sin\left(2\,x\right)-\cos\left(2\,x\right)-1\right)\sin\left(4\,x\right)-\sin(}{2\left(2\,\cos\left(2\,x\right)+1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^{2}+4\,\cos\left(2\,x\right)^{2}+\sin\left(4\,x\right)^{2}+4\,\sin\left(4\,x\right)\sin\left(2\,x\right)+4\,\sin\left(2\,x\right)^{2}+4\,\cos\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] (4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(
4*x) + 2*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - si
n(2*x))/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2
+ sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) +
1)
```

Fricas [A] time = 0.236553, size = 20, normalized size = 1.25

$$-\frac{\cos\left(x\right)\sin\left(x\right)-x}{2\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*sin(x) - x)/cos(x)^2
```

Sympy [A] time = 2.00223, size = 128, normalized size = 8.

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{x}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{2\tan^3\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}-\frac{2\tan\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)/cos(x)**3,x)
```

```
[Out] x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)*
*2/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan
(x/2)**2 + 2) + 2*tan(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2)
 - 2*tan(x/2)/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2)
```

GIAC/XCAS [A] time = 0.204393, size = 72, normalized size = 4.5

$$\frac{x\tan\left(\frac{1}{2}x\right)^4+2x\tan\left(\frac{1}{2}x\right)^2+2\tan\left(\frac{1}{2}x\right)^3+x-2\tan\left(\frac{1}{2}x\right)}{2\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)/cos(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*t
an(1/2*x))/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

3.490 $\int x\sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=62

$$\frac{1}{2}i\mathrm{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{4}-x\log\left(1+e^{2ix}\right)-\frac{1}{2}x\sin^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((
2*I)*x)] - (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2
```

Rubi [A] time = 0.110272, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.

$$\frac{1}{2}i\mathrm{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{4}-x\log\left(1+e^{2ix}\right)-\frac{1}{2}x\sin^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[x]^2*Tan[x],x]
```

```
[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((
2*I)*x)] - (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin^3(x)}{\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x)**3/cos(x),x)
```

```
[Out] Integral(x*sin(x)**3/cos(x), x)
```

Mathematica [A] time = 0.0137042, size = 57, normalized size = 0.92

$$\frac{1}{2}i\mathrm{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{8}\sin(2x)+\frac{1}{4}x\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x]^2*Tan[x],x]
```

```
[Out] (I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyL
og[2, -E^((2*I)*x)] - Sin[2*x]/8
```

Maple [A] time = 0.077, size = 55, normalized size = 0.9

$$\frac{i}{2}x^2 + \left(\frac{i}{16} + \frac{x}{8}\right) e^{2\,ix} + \left(-\frac{i}{16} + \frac{x}{8}\right) e^{-2\,ix} - x\ln\left(1 + e^{2\,ix}\right) + \frac{i}{2} polylog\left(2, -e^{2\,ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)^3/cos(x),x)
```

```
[Out] 1/2*I*x^2+(1/16*I+1/8*x)*exp(2*I*x)+(-1/16*I+1/8*x)*exp(-2*I*x)-x
*ln(1+exp(2*I*x))+1/2*I*polylog(2,-exp(2*I*x))
```

Maxima [A] time = 2.5209, size = 89, normalized size = 1.44

$$\frac{1}{2} i\,x^2 - i\,x \arctan\left(\sin\left(2\,x\right), \cos\left(2\,x\right) + 1\right) + \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right) + \frac{1}{2} i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right) - \frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x)
- 1/2*x*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dil
og(-e^(2*I*x)) - 1/8*sin(2*x)
```

Fricas [A] time = 0.282952, size = 304, normalized size = 4.9

$$\frac{1}{2}\,x\cos\left(x\right)^{2}-\frac{1}{2}\,x\log\left(\frac{\left(i+1\right)\,\cos\left(x\right)+\left(i+1\right)\,\sin\left(x\right)+i+1}{i\,\cos\left(x\right)+\sin\left(x\right)+i}\right)-\frac{1}{2}\,x\log\left(\frac{-\left(i-1\right)\,\cos\left(x\right)-\left(i-1\right)\,\sin\left(x\right)-i+1}{-i\,\cos\left(x\right)+\sin\left(x\right)-i}\right)-\frac{1}{2}\,x\log\left(\frac{\left(i-1\right)\,\cos\left(x\right)-\left(i-1\right)\,\sin\left(x\right)+i-1}{i\,\cos\left(x\right)+\sin\left(x\right)+i}\right)-\frac{1}{2}\,x\log\left(\frac{-\left(i+1\right)\,\cos\left(x\right)+\left(i+1\right)\,\sin\left(x\right)-i-1}{-i\,\cos\left(x\right)+\sin\left(x\right)-i}\right)-\frac{1}{4}\,\cos\left(x\right)\sin\left(x\right)-\frac{1}{4}\,x+\frac{1}{2}\,i\,\mathrm{Li}_2\left(-\frac{\left(i+1\right)\,\cos\left(x\right)+\left(i+1\right)\,\sin\left(x\right)+i+1}{i\,\cos\left(x\right)+\sin\left(x\right)+i}+1\right)-\frac{1}{2}\,i\,\mathrm{Li}_2\left(-\frac{-\left(i-1\right)\,\cos\left(x\right)-\left(i-1\right)\,\sin\left(x\right)-i+1}{-i\,\cos\left(x\right)+\sin\left(x\right)-i}+1\right)+\frac{1}{2}\,i\,\mathrm{Li}_2\left(-\frac{\left(i-1\right)\,\cos\left(x\right)-\left(i-1\right)\,\sin\left(x\right)+i-1}{i\,\cos\left(x\right)+\sin\left(x\right)+i}+1\right)-\frac{1}{2}\,i\,\mathrm{Li}_2\left(-\frac{-\left(i+1\right)\,\cos\left(x\right)+\left(i+1\right)\,\sin\left(x\right)-i-1}{-i\,\cos\left(x\right)+\sin\left(x\right)-i}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)^2 - 1/2*x*log(((I + 1)*cos(x) + (I + 1)*sin(x) + I +
 1)/(I*cos(x) + sin(x) + I)) - 1/2*x*log((-(I - 1)*cos(x) - (I -
1)*sin(x) - I + 1)/(-I*cos(x) + sin(x) - I)) - 1/2*x*log(((I - 1)
*cos(x) - (I - 1)*sin(x) + I - 1)/(I*cos(x) + sin(x) + I)) - 1/2*
x*log((-(I + 1)*cos(x) + (I + 1)*sin(x) - I - 1)/(-I*cos(x) + sin
(x) - I)) - 1/4*cos(x)*sin(x) - 1/4*x + 1/2*I*dilog(-((I + 1)*cos
(x) + (I + 1)*sin(x) + I + 1)/(I*cos(x) + sin(x) + I) + 1) - 1/2*
I*dilog(-(-(I - 1)*cos(x) - (I - 1)*sin(x) - I + 1)/(-I*cos(x) +
sin(x) - I) + 1) + 1/2*I*dilog(-((I - 1)*cos(x) - (I - 1)*sin(x)
+ I - 1)/(I*cos(x) + sin(x) + I) + 1) - 1/2*I*dilog(-(-(I + 1)*co
s(x) + (I + 1)*sin(x) - I - 1)/(-I*cos(x) + sin(x) - I) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin^{3}\left(x\right)}{\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**3/cos(x),x)
```

```
[Out] Integral(x*sin(x)**3/cos(x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(x)^3}{\cos(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x),x, algorithm="giac")

[Out] integrate(x*sin(x)^3/cos(x), x)
```

3.491 $\int x \tan^3(x)\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

```
[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((
2*I)*x)] - Tan[x]/2 + (x*Tan[x]^2)/2
```

Rubi [A] time = 0.0987397, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Tan[x]^3,x]
```

```
[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((
2*I)*x)] - Tan[x]/2 + (x*Tan[x]^2)/2
```

Rubi in Sympy [A] time = 13.2233, size = 48, normalized size = 0.81

$$-\frac{ix^2}{2}+x\log\left(e^{2ix}+1\right)+\frac{x\tan^2(x)}{2}+\frac{x}{2}-\frac{\tan(x)}{2}-\frac{i\operatorname{Li}_2\left(-e^{2ix}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x)**3/cos(x)**3,x)
```

```
[Out] -I*x**2/2 + x*log(exp(2*I*x) + 1) + x*tan(x)**2/2 + x/2 - tan(x)/
2 - I*polylog(2, -exp(2*I*x))/2
```

Mathematica [A] time = 0.0125968, size = 54, normalized size = 0.92

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Tan[x]^3,x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*
x)] + (x*Sec[x]^2)/2 - Tan[x]/2
```

Maple [A] time = 0.116, size = 59, normalized size = 1.

$$-\frac{i}{2}x^2+\frac{-ie^{2\,ix}+2\,xe^{2\,ix}-i}{(1+e^{2\,ix})^2}+x\ln\left(1+e^{2\,ix}\right)-\frac{i}{2}polylog\left(2,-e^{2\,ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)^3/cos(x)^3,x)
```

```
[Out] -1/2*I*x^2+(-I*exp(2*I*x)+2*x*exp(2*I*x)-I)/(1+exp(2*I*x))^2+x*ln
(1+exp(2*I*x))-1/2*I*polylog(2,-exp(2*I*x))
```

Maxima [A] time = 1.89677, size = 288, normalized size = 4.88

$$-\frac{x^2\cos(4\,x)+i\,x^2\sin(4\,x)+x^2-(2\,x\cos(4\,x)+4\,x\cos(2\,x)+2i\,x\sin(4\,x)+4i\,x\sin(2\,x)+2\,x)\arctan(\sin(2\,x),\cos(2\,x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")
```

```
[Out] -(x^2*cos(4*x) + I*x^2*sin(4*x) + x^2 - (2*x*cos(4*x) + 4*x*cos(2
*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(2*x), co
s(2*x) + 1) + 2*(x^2 + 2*I*x + 1)*cos(2*x) + (cos(4*x) + 2*cos(2*
x) + I*sin(4*x) + 2*I*sin(2*x) + 1)*dilog(-e^(2*I*x)) - (-I*x*cos
(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*x*sin(2*x) - I*x)*log(cos
(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - (-2*I*x^2 + 4*x - 2*I)*s
in(2*x) + 2)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2
*x) - 2*I)
```

Fricas [A] time = 0.277717, size = 338, normalized size = 5.73

$$\frac{x\cos(x)^2\log\left(\frac{(i+1)\cos(x)+(i+1)\sin(x)+i+1}{i\cos(x)+\sin(x)+i}\right)+x\cos(x)^2\log\left(\frac{-(i-1)\cos(x)-(i-1)\sin(x)-i+1}{-i\cos(x)+\sin(x)-i}\right)+x\cos(x)^2\log\left(\frac{(i-1)\cos(x)-(i-1)\sin(x)+i-}{i\cos(x)+\sin(x)+i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(((I + 1)*cos(x) + (I + 1)*sin(x) + I + 1)/(I*
cos(x) + sin(x) + I)) + x*cos(x)^2*log((-(I - 1)*cos(x) - (I - 1)
*sin(x) - I + 1)/(-I*cos(x) + sin(x) - I)) + x*cos(x)^2*log(((I -
 1)*cos(x) - (I - 1)*sin(x) + I - 1)/(I*cos(x) + sin(x) + I)) + x
*cos(x)^2*log((-(I + 1)*cos(x) + (I + 1)*sin(x) - I - 1)/(-I*cos(
x) + sin(x) - I)) - I*cos(x)^2*dilog(-((I + 1)*cos(x) + (I + 1)*s
in(x) + I + 1)/(I*cos(x) + sin(x) + I) + 1) + I*cos(x)^2*dilog(-(
-(I - 1)*cos(x) - (I - 1)*sin(x) - I + 1)/(-I*cos(x) + sin(x) - I
) + 1) - I*cos(x)^2*dilog(-((I - 1)*cos(x) - (I - 1)*sin(x) + I -
 1)/(I*cos(x) + sin(x) + I) + 1) + I*cos(x)^2*dilog(-(-(I + 1)*co
s(x) + (I + 1)*sin(x) - I - 1)/(-I*cos(x) + sin(x) - I) + 1) - co
s(x)*sin(x) + x)/cos(x)^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin^3(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**3/cos(x)**3,x)
```

```
[Out] Integral(x*sin(x)**3/cos(x)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x)^3, x)
```

3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

```
[Out] 2/(1 + Cot[x]/x)
```

Rubi [A] time = 0.163048, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] 2/(1 + Cot[x]/x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+\sin(2x)}{(x\sin(x)+\cos(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

```
[Out] Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)
```

Mathematica [A] time = 0.0539933, size = 14, normalized size = 1.17

$$\frac{2x\sin(x)}{x\sin(x)+\cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] (2*x*Sin[x])/(Cos[x] + x*Sin[x])
```

Maple [C] time = 0.879, size = 44, normalized size = 3.7

$$\frac{-2\,i}{x+i}-\frac{4\,ix}{(x+i)\,(x\mathrm{e}^{2\,ix}-x+i\mathrm{e}^{2\,ix}+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x)
```

```
[Out] -2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)
```

Maxima [A] time = 3.32465, size = 105, normalized size = 8.75

$$-\frac{2\left(\cos\left(2\,x\right)^{2}+2\,x\sin\left(2\,x\right)+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)}{(x^{2}+1)\cos\left(2\,x\right)^{2}+(x^{2}+1)\sin\left(2\,x\right)^{2}+x^{2}-2\,(x^{2}-1)\cos\left(2\,x\right)+4\,x\sin\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + sin(2*x))/(x*sin(x) + cos(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^
2 + 1)*cos(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(
2*x) + 4*x*sin(2*x) + 1)
```

Fricas [A] time = 0.223665, size = 18, normalized size = 1.5

$$-\frac{2\,\cos\left(x\right)}{x\sin\left(x\right)+\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + sin(2*x))/(x*sin(x) + cos(x))^2,x, algorithm="fricas")
```

```
[Out] -2*cos(x)/(x*sin(x) + cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.231916, size = 14, normalized size = 1.17

$$-\frac{2}{x\tan\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + sin(2*x))/(x*sin(x) + cos(x))^2,x, algorithm="giac")
```

```
[Out] -2/(x*tan(x) + 1)
```

3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$

Optimal. Leaf size=20

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

```
[Out] -Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

Rubi [A] time = 0.055621, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(x*Cos[x] - Sin[x])^2,x]

[Out] -Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x*cos(x)-sin(x))**2,x)

[Out] Integral(x**2/(x*cos(x) - sin(x))**2, x)
```

Mathematica [A] time = 0.0632405, size = 19, normalized size = 0.95

$$\frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]
```

```
[Out] (Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])
```

Maple [A] time = 0.29, size = 37, normalized size = 1.9

$$1\left(-1+\left(\tan\left(\frac{x}{2}\right)\right)^{2}-2\,x\tan\left(x/2\right)\right)\left(x\left(\tan\left(\frac{x}{2}\right)\right)^{2}-x+2\,\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x*cos(x)-sin(x))^2,x)
```

```
[Out] (-1+tan(1/2*x)^2-2*x*tan(1/2*x))/(x*tan(1/2*x)^2-x+2*tan(1/2*x))
```

Maxima [A] time = 1.37374, size = 93, normalized size = 4.65

$$\frac{2\left(2\,x\cos\left(2\,x\right)+\left(x^{2}-1\right)\sin\left(2\,x\right)\right)}{\left(x^{2}+1\right)\cos\left(2\,x\right)^{2}+\left(x^{2}+1\right)\sin\left(2\,x\right)^{2}+x^{2}+2\left(x^{2}-1\right)\cos\left(2\,x\right)-4\,x\sin\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x*cos(x) - sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^
2 + 1)*sin(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1
)
```

Fricas [A] time = 0.210983, size = 26, normalized size = 1.3

$$\frac{x\sin\left(x\right)+\cos\left(x\right)}{x\cos\left(x\right)-\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x*cos(x) - sin(x))^2,x, algorithm="fricas")
```

```
[Out] (x*sin(x) + cos(x))/(x*cos(x) - sin(x))
```

Sympy [A] time = 4.85837, size = 66, normalized size = 3.3

$$-\frac{2x\tan\left(\frac{x}{2}\right)}{x\tan^{2}\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}+\frac{\tan^{2}\left(\frac{x}{2}\right)}{x\tan^{2}\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}-\frac{1}{x\tan^{2}\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x*cos(x)-sin(x))**2,x)
```

```
[Out] -2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*t
an(x/2)**2 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))
```

GIAC/XCAS [A] time = 0.201871, size = 53, normalized size = 2.65

$$-\frac{2\,x\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)^{2}+1}{x\tan\left(\frac{1}{2}\,x\right)^{2}-x+2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x*cos(x) - sin(x))^2,x, algorithm="giac")
```

```
[Out] -(2*x*tan(1/2*x) - tan(1/2*x)^2 + 1)/(x*tan(1/2*x)^2 - x + 2*tan(
1/2*x))
```

3.494 $\int a^{mx} b^{nx} \, dx$

Optimal. Leaf size=22

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Rubi [A] time = 0.0435366, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Rubi in Sympy [A] time = 2.16329, size = 22, normalized size = 1.

$$\frac{e^{x(m\log(a)+n\log(b))}}{m\log(a)+n\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(a**(m*x)*b**(n*x),x)
```

```
[Out] exp(x*(m*log(a) + n*log(b)))/(m*log(a) + n*log(b))
```

Mathematica [A] time = 0.00717626, size = 22, normalized size = 1.

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Maple [A] time = 0.009, size = 23, normalized size = 1.1

$$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(m*x)*b^(n*x),x)
```

```
[Out] a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218419, size = 30, normalized size = 1.36

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")
```

```
[Out] a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))
```

Sympy [A] time = 1.02577, size = 42, normalized size = 1.91

$$\begin{cases} \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)} & \text{for}\ m \neq -\frac{n\log(b)}{\log(a)} \\ b^{nx}xe^{-nx\log(b)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**(m*x)*b**(n*x),x)
```

```
[Out] Piecewise((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(
b)/log(a))), (b**(n*x)*x*exp(-n*x*log(b)), True))
```

GIAC/XCAS [A] time = 0.223587, size = 439, normalized size = 19.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(m*x)*b^(n*x),x, algorithm="giac")
```

```
[Out] 2*(2*(m*ln(abs(a)) + n*ln(abs(b)))*cos(-1/2*pi*m*x*sign(a) - 1/2*
pi*n*x*sign(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sign(a) + pi*n*s
ign(b) - pi*m - pi*n)^2 + 4*(m*ln(abs(a)) + n*ln(abs(b)))^2) - (p
i*m*sign(a) + pi*n*sign(b) - pi*m - pi*n)*sin(-1/2*pi*m*x*sign(a)
 - 1/2*pi*n*x*sign(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sign(a) +
 pi*n*sign(b) - pi*m - pi*n)^2 + 4*(m*ln(abs(a)) + n*ln(abs(b)))^
2))*e^((m*ln(abs(a)) + n*ln(abs(b)))*x) - 1/2*I*(-2*I*e^(1/2*I*pi
*m*x*sign(a) + 1/2*I*pi*n*x*sign(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x
)/(I*pi*m*sign(a) + I*pi*n*sign(b) - I*pi*m - I*pi*n + 2*m*ln(abs
(a)) + 2*n*ln(abs(b))) + 2*I*e^(-1/2*I*pi*m*x*sign(a) - 1/2*I*pi*
n*x*sign(b) + 1/2*I*pi*m*x + 1/2*I*pi*n*x)/(-I*pi*m*sign(a) - I*p
i*n*sign(b) + I*pi*m + I*pi*n + 2*m*ln(abs(a)) + 2*n*ln(abs(b))))
*e^((m*ln(abs(a)) + n*ln(abs(b)))*x)
```

3.495 $\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx$

Optimal. Leaf size=34

$$\frac{a^x b^{-x} - a^{-x} b^x}{\log(a) - \log(b)} - 2x$$

```
[Out] -2*x + (a^x/b^x - b^x/a^x)/(Log[a] - Log[b])
```

Rubi [A] time = 0.318668, antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{a^{-x} b^x}{\log(a) - \log(b)} + \frac{a^x b^{-x}}{\log(a) - \log(b)} - 2x$$

Antiderivative was successfully verified.

```
[In] Int[(a^x - b^x)^2/(a^x*b^x),x]

[Out] -2*x + a^x/(b^x*(Log[a] - Log[b])) - b^x/(a^x*(Log[a] - Log[b]))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a^x - b^x)^2 e^{x(-\log(a) - \log(b))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**x-b**x)**2/(a**x)/(b**x),x)

[Out] Integral((a**x - b**x)**2*exp(x*(-log(a) - log(b))), x)
```

Mathematica [A] time = 0.0543958, size = 40, normalized size = 1.18

$$\frac{a^{-x} b^x}{\log(b) - \log(a)} + \frac{a^x b^{-x}}{\log(a) - \log(b)} - 2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] -2*x + a^x/(b^x*(Log[a] - Log[b])) + b^x/(a^x*(-Log[a] + Log[b]))
```

Maple [A] time = 0.026, size = 65, normalized size = 1.9

$$\frac{1}{e^{x\ln(a)}e^{x\ln(b)}}\left(\frac{\left(e^{x\ln(b)}\right)^2}{-\ln(a)+\ln(b)}-\frac{\left(e^{x\ln(a)}\right)^2}{-\ln(a)+\ln(b)}-2\,xe^{x\ln(a)}e^{x\ln(b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^x-b^x)^2/(a^x)/(b^x),x)
```

```
[Out] (1/(-ln(a)+ln(b))*exp(x*ln(b))^2-1/(-ln(a)+ln(b))*exp(x*ln(a))^2-
2*x*exp(x*ln(a))*exp(x*ln(b)))/exp(x*ln(a))/exp(x*ln(b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^x - b^x)^2/(a^x*b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210739, size = 70, normalized size = 2.06

$$-\frac{2\,(x\log{(a)}-x\log{(b)})a^x b^x - a^{2\,x} + b^{2\,x}}{a^x b^x(\log{(a)}-\log{(b)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^x - b^x)^2/(a^x*b^x),x, algorithm="fricas")
```

```
[Out] -(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(
log(a) - log(b)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**x-b**x)**2/(a**x)/(b**x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.295168, size = 589, normalized size = 17.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^x - b^x)^2/(a^x*b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(ln(abs(a)) - ln(abs(b)))*cos(-1/2*pi*x*sign(a) + 1/2*pi*x*s
ign(b))/((pi*sign(a) - pi*sign(b))^2 + 4*(ln(abs(a)) - ln(abs(b))
)^2) - (pi*sign(a) - pi*sign(b))*sin(-1/2*pi*x*sign(a) + 1/2*pi*x
*sign(b))/((pi*sign(a) - pi*sign(b))^2 + 4*(ln(abs(a)) - ln(abs(b
)))^2))*e^(x*(ln(abs(a)) - ln(abs(b)))) - 1/2*I*(-2*I*e^(1/2*I*pi
*x*sign(a) - 1/2*I*pi*x*sign(b))/(I*pi*sign(a) - I*pi*sign(b) + 2
*ln(abs(a)) - 2*ln(abs(b))) + 2*I*e^(-1/2*I*pi*x*sign(a) + 1/2*I*
pi*x*sign(b))/(-I*pi*sign(a) + I*pi*sign(b) + 2*ln(abs(a)) - 2*ln
(abs(b))))*e^(x*(ln(abs(a)) - ln(abs(b)))) - 2*(2*(ln(abs(a)) - l
n(abs(b)))*cos(1/2*pi*x*sign(a) - 1/2*pi*x*sign(b))/((pi*sign(a)
- pi*sign(b))^2 + 4*(ln(abs(a)) - ln(abs(b)))^2) - (pi*sign(a) -
pi*sign(b))*sin(1/2*pi*x*sign(a) - 1/2*pi*x*sign(b))/((pi*sign(a)
 - pi*sign(b))^2 + 4*(ln(abs(a)) - ln(abs(b)))^2))*e^(-x*(ln(abs(
a)) - ln(abs(b)))) - 1/2*I*(2*I*e^(1/2*I*pi*x*sign(a) - 1/2*I*pi*
x*sign(b))/(I*pi*sign(a) - I*pi*sign(b) - 2*ln(abs(a)) + 2*ln(abs
(b))) - 2*I*e^(-1/2*I*pi*x*sign(a) + 1/2*I*pi*x*sign(b))/(-I*pi*s
ign(a) + I*pi*sign(b) - 2*ln(abs(a)) + 2*ln(abs(b))))*e^(-x*(ln(a
bs(a)) - ln(abs(b)))) - 2*x
```

3.496 $\int (-e^{-x} + e^x) \, dx$

Optimal. Leaf size=9

$$e^{-x} + e^x$$

```
[Out] E^(-x) + E^x
```

Rubi [A] time = 0.00747512, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$e^{-x} + e^x$$

Antiderivative was successfully verified.

```
[In]  Int[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

Rubi in Sympy [A] time = 0.637621, size = 7, normalized size = 0.78

$$e^x + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-1/exp(x)+exp(x),x)
```

```
[Out] exp(x) + exp(-x)
```

Mathematica [A] time = 0.00587105, size = 9, normalized size = 1.

$$e^{-x} + e^x$$

Antiderivative was successfully verified.

```
[In]  Integrate[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

Maple [A] time = 0.002, size = 8, normalized size = 0.9

$$(e^{x})^{-1} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-1/exp(x)+exp(x),x)

[Out] 1/exp(x)+exp(x)
```

Maxima [A] time = 1.36308, size = 9, normalized size = 1.

$$e^{(-x)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^(-x) + e^x,x, algorithm="maxima")

[Out] e^(-x) + e^x
```

Fricas [A] time = 0.204326, size = 15, normalized size = 1.67

$$\left(e^{(2x)} + 1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^(-x) + e^x,x, algorithm="fricas")

[Out] (e^(2*x) + 1)*e^(-x)
```

Sympy [A] time = 0.070997, size = 7, normalized size = 0.78

$$e^{x} + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/exp(x)+exp(x),x)

[Out] exp(x) + exp(-x)
```

GIAC/XCAS [A] time = 0.22158, size = 9, normalized size = 1.

$$e^{(-x)} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^(-x) + e^x,x, algorithm="giac")
```

```
[Out] e^(-x) + e^x
```

3.497 $\int (-e^{-x} + e^x)^2 \, dx$

Optimal. Leaf size=22

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Rubi [A] time = 0.0387579, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Rubi in Sympy [A] time = 3.53508, size = 20, normalized size = 0.91

$$\frac{e^{2x}}{2} - \log\left(e^{2x}\right) - \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1/exp(x)+exp(x))**2,x)
```

```
[Out] exp(2*x)/2 - log(exp(2*x)) - exp(-2*x)/2
```

Mathematica [A] time = 0.00924591, size = 22, normalized size = 1.

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Maple [A] time = 0.008, size = 19, normalized size = 0.9

$$\frac{(e^x)^2}{2} - \frac{1}{2\,(e^x)^2} - 2\,\ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1/exp(x)+exp(x))^2,x)
```

```
[Out] 1/2*exp(x)^2-1/2/exp(x)^2-2*ln(exp(x))
```

Maxima [A] time = 1.38412, size = 22, normalized size = 1.

$$-2\,x + \frac{1}{2}\,e^{(2\,x)} - \frac{1}{2}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(-x) - e^x)^2,x, algorithm="maxima")
```

```
[Out] -2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

Fricas [A] time = 0.205848, size = 28, normalized size = 1.27

$$-\frac{1}{2}\left(4\,xe^{(2\,x)} - e^{(4\,x)} + 1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(-x) - e^x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)
```

Sympy [A] time = 0.101697, size = 17, normalized size = 0.77

$$-2x + \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1/exp(x)+exp(x))**2,x)
```

```
[Out] -2*x + exp(2*x)/2 - exp(-2*x)/2
```

GIAC/XCAS [A] time = 0.21875, size = 32, normalized size = 1.45

$$\frac{1}{2}\left(2\,e^{(2\,x)}-1\right)e^{(-2\,x)}-2\,x+\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(-x) - e^x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(2*x) - 1)*e^(-2*x) - 2*x + 1/2*e^(2*x)
```

3.498 $\int (-e^{-x}+e^x)^3\,dx$

Optimal. Leaf size=31

$$\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

```
[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

Rubi [A] time = 0.0375142, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(-E^(-x) + E^x)^3,x]

[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

Rubi in Sympy [A] time = 3.53578, size = 24, normalized size = 0.77

$$\frac{e^{3x}}{3}-3e^x-3e^{-x}+\frac{e^{-3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1/exp(x)+exp(x))**3,x)

[Out] exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

Mathematica [A] time = 0.0124771, size = 30, normalized size = 0.97

$$\frac{1}{3}e^{-3x}\left(-9e^{2x}-9e^{4x}+e^{6x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-E^(-x) + E^x)^3,x]
```

```
[Out] (1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))
```

Maple [A] time = 0.009, size = 24, normalized size = 0.8

$$\frac{(e^x)^3}{3} - 3\,e^x + \frac{1}{3\,(e^x)^3} - 3\,(e^x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1/exp(x)+exp(x))^3,x)
```

```
[Out] 1/3*exp(x)^3-3*exp(x)+1/3/exp(x)^3-3/exp(x)
```

Maxima [A] time = 1.3546, size = 31, normalized size = 1.

$$\frac{1}{3}\,e^{(3\,x)} - 3\,e^{(-x)} + \frac{1}{3}\,e^{(-3\,x)} - 3\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^(-x) - e^x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x
```

Fricas [A] time = 0.201237, size = 32, normalized size = 1.03

$$\frac{1}{3}\left(e^{(6\,x)} - 9\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^(-x) - e^x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)
```

Sympy [A] time = 0.130581, size = 24, normalized size = 0.77

$$\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1/exp(x)+exp(x))**3,x)

[Out] exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

GIAC/XCAS [A] time = 0.200068, size = 34, normalized size = 1.1

$$-\frac{1}{3}\left(9\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{3}\,e^{(3\,x)}-3\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^(-x) - e^x)^3,x, algorithm="giac")

[Out] -1/3*(9*e^(2*x) - 1)*e^(-3*x) + 1/3*e^(3*x) - 3*e^x
```

3.499 $\int (-e^{-x}+e^x)^4\,dx$

Optimal. Leaf size=36

$$6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

```
[Out] -1/(4*E^(4*x)) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x
```

Rubi [A] time = 0.052147, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^4,x]
```

```
[Out] -1/(4*E^(4*x)) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2e^{2x}+3\log\left(e^{2x}\right)+\frac{\int^{e^{2x}} x\,dx}{2}+2e^{-2x}-\frac{e^{-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-1/exp(x)+exp(x))**4,x)
```

```
[Out] -2*exp(2*x) + 3*log(exp(2*x)) + Integral(x, (x, exp(2*x)))/2 + 2*
exp(-2*x) - exp(-4*x)/4
```

Mathematica [A] time = 0.0234141, size = 34, normalized size = 0.94

$$\frac{1}{4}\left(24x-e^{-4x}+8e^{-2x}-8e^{2x}+e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^4,x]
```

```
[Out] (-E^(-4*x) + 8/E^(2*x) - 8*E^(2*x) + E^(4*x) + 24*x)/4
```

Maple [A] time = 0.01, size = 31, normalized size = 0.9

$$\frac{(e^{x})^{4}}{4}-2\,(e^{x})^{2}-\frac{1}{4\,(e^{x})^{4}}+2\,(e^{x})^{-2}+6\,\ln(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1/exp(x)+exp(x))^4,x)
```

```
[Out] 1/4*exp(x)^4-2*exp(x)^2-1/4/exp(x)^4+2/exp(x)^2+6*ln(exp(x))
```

Maxima [A] time = 1.35233, size = 38, normalized size = 1.06

$$6\,x+\frac{1}{4}\,e^{(4\,x)}-2\,e^{(2\,x)}+2\,e^{(-2\,x)}-\frac{1}{4}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(-x) - e^x)^4,x, algorithm="maxima")
```

```
[Out] 6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)
```

Fricas [A] time = 0.203782, size = 42, normalized size = 1.17

$$\frac{1}{4}\left(24\,xe^{(4\,x)}+e^{(8\,x)}-8\,e^{(6\,x)}+8\,e^{(2\,x)}-1\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(-x) - e^x)^4,x, algorithm="fricas")
```

```
[Out] 1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)
```

Sympy [A] time = 0.153951, size = 31, normalized size = 0.86

$$6x+\frac{e^{4x}}{4}-2e^{2x}+2e^{-2x}-\frac{e^{-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1/exp(x)+exp(x))**4,x)
```

```
[Out] 6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4
```

GIAC/XCAS [A] time = 0.198424, size = 49, normalized size = 1.36

$$-\frac{1}{4}\left(18\,e^{(4\,x)} - 8\,e^{(2\,x)} + 1\right)e^{(-4\,x)} + 6\,x + \frac{1}{4}\,e^{(4\,x)} - 2\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(-x) - e^x)^4,x, algorithm="giac")
```

```
[Out] -1/4*(18*e^(4*x) - 8*e^(2*x) + 1)*e^(-4*x) + 6*x + 1/4*e^(4*x) -
2*e^(2*x)
```

3.500 $\int (-e^{-x} + e^x)^n \, dx$

Optimal. Leaf size=48

$$-\frac{(1-e^{2x})(e^x-e^{-x})^n \, {}_2F_1\left(1,\frac{n+2}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

```
[Out] -(((-E^(-x) + E^x)^n*(1 - E^(2*x))*Hypergeometric2F1[1, (2 + n)/2
, 1 - n/2, E^(2*x)])/n)
```

Rubi [A] time = 0.0863916, antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{(e^x-e^{-x})^n (1-e^{2x})^{-n} \, {}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[(-E^(-x) + E^x)^n,x]
```

```
[Out] -(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^(2*x)
])/((1 - E^(2*x))^n*n))
```

Rubi in Sympy [A] time = 5.60333, size = 48, normalized size = 1.

$$\frac{(1-e^{-2x})^{-n}(1-e^{-2x})^{n+1}(e^x-e^{-x})^n \, {}_2F_1\left(\begin{matrix}1, \frac{n}{2}+1 \\ -\frac{n}{2}+1\end{matrix}\middle| e^{-2x}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1/exp(x)+exp(x))**n,x)
```

```
[Out] (1 - exp(-2*x))**(-n)*(1 - exp(-2*x))**(n + 1)*(exp(x) - exp(-x))
**n*hyper((1, n/2 + 1), (-n/2 + 1,), exp(-2*x))/n
```

Mathematica [A] time = 0.037629, size = 52, normalized size = 1.08

$$-\frac{(e^x-e^{-x})^n (1-e^{2x})^{-n} \, {}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-E^(-x) + E^x)^n,x]

[Out] -(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^(2*x)
])/((1 - E^(2*x))^n*n))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \left(-\left(\mathrm{e}^{x}\right)^{-1}+\mathrm{e}^{x}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1/exp(x)+exp(x))^n,x)

[Out] int((-1/exp(x)+exp(x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^{(-x)}+e^{x}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e^(-x) + e^x)^n,x, algorithm="maxima")

[Out] integrate((-e^(-x) + e^x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-e^{(-x)}+e^{x}\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e^(-x) + e^x)^n,x, algorithm="fricas")

[Out] integral((-e^(-x) + e^x)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1/exp(x)+exp(x))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^{(-x)} + e^{x}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-e^(-x) + e^x)^n,x, algorithm="giac")
```

```
[Out] integrate((-e^(-x) + e^x)^n, x)
```

3.501 $\int \left(a^{-4x}-a^{2x}\right)^3\,dx$

Optimal. Leaf size=43

$$-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

```
[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*
Log[a])
```

Rubi [A] time = 0.0579412, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(-4*x) - a^(2*x))^3,x]
```

```
[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*
Log[a])
```

Rubi in Sympy [A] time = 4.75474, size = 44, normalized size = 1.02

$$-\frac{a^{6x}}{6\log(a)}+\frac{\log\left(a^{6x}\right)}{2\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{-12x}}{12\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/(a**(4*x))-a**(2*x))**3,x)
```

```
[Out] -a**(6*x)/(6*log(a)) + log(a**(6*x))/(2*log(a)) + a**(-6*x)/(2*lo
g(a)) - a**(-12*x)/(12*log(a))
```

Mathematica [A] time = 0.0456398, size = 33, normalized size = 0.77

$$-\frac{a^{-12x}-6a^{-6x}+2a^{6x}-36x\log(a)}{12\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(-4*x) - a^(2*x))^3,x]
```

```
[Out] -(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*x*Log[a])/(12*Log[a])
```

Maple [A] time = 0.03, size = 56, normalized size = 1.3

$$\frac{1}{\left(e^{2x\ln(a)}\right)^{6}}\left(-\frac{1}{12\ln(a)}+3x\left(e^{2x\ln(a)}\right)^{6}+\frac{\left(e^{2x\ln(a)}\right)^{3}}{2\ln(a)}-\frac{\left(e^{2x\ln(a)}\right)^{9}}{6\ln(a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/(a^(4*x))-a^(2*x))^3,x)
```

```
[Out] (-1/12/ln(a)+3*x*exp(2*x*ln(a))^6+1/2/ln(a)*exp(2*x*ln(a))^3-1/6/
ln(a)*exp(2*x*ln(a))^9)/exp(2*x*ln(a))^6
```

Maxima [A] time = 1.37487, size = 50, normalized size = 1.16

$$3x-\frac{a^{6x}}{6\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{-12x}}{12\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a^(2*x) - 1/a^(4*x))^3,x, algorithm="maxima")
```

```
[Out] 3*x - 1/6*a^(6*x)/log(a) + 1/2*a^(-6*x)/log(a) - 1/12*a^(-12*x)/l
og(a)
```

Fricas [A] time = 0.217355, size = 53, normalized size = 1.23

$$\frac{36\,a^{12x}x\log(a)-2\,a^{18x}+6\,a^{6x}-1}{12\,a^{12x}\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a^(2*x) - 1/a^(4*x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(36*a^(12*x)*x*log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x
)*log(a))
```

Sympy [A] time = 0.297328, size = 54, normalized size = 1.26

$$3x + \begin{cases} \frac{-24a^{6x}\log(a)^2 + 72a^{-6x}\log(a)^2 - 12a^{-12x}\log(a)^2}{144\log(a)^3} & \text{for } 144\log(a)^3 \neq 0 \\ -3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/(a**(4*x))-a**(2*x))**3,x)
```

```
[Out] 3*x + Piecewise(((-24*a**(6*x)*log(a)**2 + 72*a**(-6*x)*log(a)**2
 - 12*a**(-12*x)*log(a)**2)/(144*log(a)**3), Ne(144*log(a)**3, 0)
), (-3*x, True))
```

GIAC/XCAS [A] time = 0.203146, size = 62, normalized size = 1.44

$$-\frac{2\,a^{6\,x} + \frac{9\,a^{12\,x} - 6\,a^{6\,x} + 1}{a^{12\,x}} - 6\,\ln\left(a^{6\,x}\right)}{12\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a^(2*x) - 1/a^(4*x))^3,x, algorithm="giac")
```

```
[Out] -1/12*(2*a^(6*x) + (9*a^(12*x) - 6*a^(6*x) + 1)/a^(12*x) - 6*ln(a
^(6*x)))/ln(a)
```

3.502 $\int \left(a^{kx}+a^{lx}\right) dx$

Optimal. Leaf size=27

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Rubi [A] time = 0.0243129, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Rubi in Sympy [A] time = 1.02179, size = 19, normalized size = 0.7

$$\frac{a^{kx}}{k\log{(a)}}+\frac{a^{lx}}{l\log{(a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(a**(k*x)+a**(l*x),x)
```

```
[Out] a**(k*x)/(k*log(a)) + a**(l*x)/(l*log(a))
```

Mathematica [A] time = 0.00928911, size = 27, normalized size = 1.

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Maple [A] time = 0.004, size = 28, normalized size = 1.

$$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^(k*x)+a^(l*x),x)

[Out] a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)
```

Maxima [A] time = 1.3419, size = 36, normalized size = 1.33

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(k*x) + a^(l*x),x, algorithm="maxima")

[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

Fricas [A] time = 0.214634, size = 35, normalized size = 1.3

$$\frac{a^{lx}k+a^{kx}l}{kl\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(k*x) + a^(l*x),x, algorithm="fricas")

[Out] (a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))
```

Sympy [A] time = 0.566064, size = 29, normalized size = 1.07

$$\begin{cases}\frac{a^{kx}}{k\log(a)} & \text{for}\ k\log(a)\neq 0\\ x & \text{otherwise}\end{cases}+\begin{cases}\frac{a^{lx}}{l\log(a)} & \text{for}\ l\log(a)\neq 0\\ x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**(k*x)+a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Pi
ecewise((a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

GIAC/XCAS [A] time = 0.195233, size = 36, normalized size = 1.33

$$\frac{a^{kx}}{k\ln(a)} + \frac{a^{lx}}{l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(k*x) + a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*ln(a)) + a^(l*x)/(l*ln(a))
```

3.503 $\int \left(a^{kx}+a^{lx}\right)^2 \, dx$

Optimal. Leaf size=53

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x
))/((k + l)*Log[a])
```

Rubi [A] time = 0.135001, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x
))/((k + l)*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)+a**(l*x))**2,x)
```

```
[Out] Integral((a**(k*x) + a**(l*x))**2, x)
```

Mathematica [A] time = 0.0590813, size = 55, normalized size = 1.04

$$\frac{2a^{kx+lx}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^(k*x + l*x
))/((k + l)*Log[a])
```

Maple [A] time = 0.029, size = 59, normalized size = 1.1

$$\frac{\left(e^{kx\ln(a)}\right)^2}{2k\ln(a)} + \frac{\left(e^{lx\ln(a)}\right)^2}{2l\ln(a)} + 2\frac{e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)+a^(l*x))^2,x)
```

```
[Out] 1/2/k/ln(a)*exp(k*x*ln(a))^2+1/2/l/ln(a)*exp(l*x*ln(a))^2+2/ln(a)
/(k+l)*exp(k*x*ln(a))*exp(l*x*ln(a))
```

Maxima [A] time = 1.38941, size = 69, normalized size = 1.3

$$\frac{2a^{kx+lx}}{(k+l)\log(a)} + \frac{a^{2kx}}{2k\log(a)} + \frac{a^{2lx}}{2l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^2,x, algorithm="maxima")
```

```
[Out] 2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2
*a^(2*l*x)/(l*log(a))
```

Fricas [A] time = 0.218975, size = 84, normalized size = 1.58

$$\frac{4a^{kx}a^{lx}kl + \left(kl + l^2\right)a^{2kx} + \left(k^2 + kl\right)a^{2lx}}{2\left(k^2l + kl^2\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*
a^(2*l*x))/((k^2*l + k*l^2)*log(a))
```

Sympy [A] time = 3.38479, size = 250, normalized size = 4.72

$$\begin{cases} 4x & \text{for } a = 1 \wedge (\\ \frac{a^{2lx}}{2l\log(a)} + \frac{2a^{lx}}{l\log(a)} + x & \text{for } k = 0 \\ \frac{a^{2lx}}{2l\log(a)} + 2x - \frac{a^{-2lx}}{2l\log(a)} & \text{for } k = -l \\ \frac{a^{2kx}}{2k\log(a)} + \frac{2a^{kx}}{k\log(a)} + x & \text{for } l = 0 \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)+a**(l*x))**2,x)
```

```
[Out] Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq
(l, 0))), (a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, E
q(k, 0)), (a**(2*l*x)/(2*l*log(a)) + 2*x - a**(-2*l*x)/(2*l*log(a
)), Eq(k, -l)), (a**(2*k*x)/(2*k*log(a)) + 2*a**(k*x)/(k*log(a))
+ x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a
)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + 4*a**(
k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x
)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k*
*2*l*log(a) + 2*k*l**2*log(a)), True))
```

GIAC/XCAS [A] time = 0.241497, size = 938, normalized size = 17.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^2,x, algorithm="giac")
```

```
[Out] (2*k*cos(-pi*k*x*sign(a) + pi*k*x)*ln(abs(a))/(4*k^2*ln(abs(a))^2
 + (pi*k*sign(a) - pi*k)^2) - (pi*k*sign(a) - pi*k)*sin(-pi*k*x*s
ign(a) + pi*k*x)/(4*k^2*ln(abs(a))^2 + (pi*k*sign(a) - pi*k)^2))*
e^(2*k*x*ln(abs(a))) - 1/2*I*(-I*e^(I*pi*k*x*sign(a) - I*pi*k*x)/
(I*pi*k*sign(a) - I*pi*k + 2*k*ln(abs(a))) + I*e^(-I*pi*k*x*sign(
a) + I*pi*k*x)/(-I*pi*k*sign(a) + I*pi*k + 2*k*ln(abs(a))))*e^(2*
k*x*ln(abs(a))) + (2*l*cos(-pi*l*x*sign(a) + pi*l*x)*ln(abs(a))/(
4*l^2*ln(abs(a))^2 + (pi*l*sign(a) - pi*l)^2) - (pi*l*sign(a) - p
i*l)*sin(-pi*l*x*sign(a) + pi*l*x)/(4*l^2*ln(abs(a))^2 + (pi*l*si
gn(a) - pi*l)^2))*e^(2*l*x*ln(abs(a))) - 1/2*I*(-I*e^(I*pi*l*x*si
gn(a) - I*pi*l*x)/(I*pi*l*sign(a) - I*pi*l + 2*l*ln(abs(a))) + I*
e^(-I*pi*l*x*sign(a) + I*pi*l*x)/(-I*pi*l*sign(a) + I*pi*l + 2*l*
ln(abs(a))))*e^(2*l*x*ln(abs(a))) + 4*(2*(k*ln(abs(a)) + l*ln(abs
(a)))*cos(-1/2*pi*k*x*sign(a) - 1/2*pi*l*x*sign(a) + 1/2*pi*k*x +
 1/2*pi*l*x)/((pi*k*sign(a) + pi*l*sign(a) - pi*k - pi*l)^2 + 4*(
k*ln(abs(a)) + l*ln(abs(a)))^2) - (pi*k*sign(a) + pi*l*sign(a) -
```

```
pi*k - pi*l)*sin(-1/2*pi*k*x*sign(a) - 1/2*pi*l*x*sign(a) + 1/2*p
i*k*x + 1/2*pi*l*x)/((pi*k*sign(a) + pi*l*sign(a) - pi*k - pi*l)^
2 + 4*(k*ln(abs(a)) + l*ln(abs(a)))^2))*e^((k*ln(abs(a)) + l*ln(a
bs(a)))*x) - 1/2*I*(-4*I*e^(1/2*I*pi*k*x*sign(a) + 1/2*I*pi*l*x*s
ign(a) - 1/2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sign(a) + I*pi*l*si
gn(a) - I*pi*k - I*pi*l + 2*k*ln(abs(a)) + 2*l*ln(abs(a))) + 4*I*
e^(-1/2*I*pi*k*x*sign(a) - 1/2*I*pi*l*x*sign(a) + 1/2*I*pi*k*x +
1/2*I*pi*l*x)/(-I*pi*k*sign(a) - I*pi*l*sign(a) + I*pi*k + I*pi*l
 + 2*k*ln(abs(a)) + 2*l*ln(abs(a))))*e^((k*ln(abs(a)) + l*ln(abs(
a)))*x)
```

3.504 $\int \left(a^{kx}+a^{lx}\right)^3 \, dx$

Optimal. Leaf size=79

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

```
[Out] a^(3*k*x)/(3*k*Log[a]) + a^(3*l*x)/(3*l*Log[a]) + (3*a^((2*k + l)
*x))/((2*k + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rubi [A] time = 0.176797, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) + a^(3*l*x)/(3*l*Log[a]) + (3*a^((2*k + l)
*x))/((2*k + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)+a**(l*x))**3,x)
```

```
[Out] Integral((a**(k*x) + a**(l*x))**3, x)
```

Mathematica [A] time = 0.134699, size = 65, normalized size = 0.82

$$\frac{\frac{9a^{x(2k+l)}}{2k+l}+\frac{9a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{k}+\frac{a^{3lx}}{l}}{3\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] (a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + l)*x))/(2*k + l) + (9*a
^((k + 2*l)*x))/(k + 2*l))/(3*Log[a])
```

Maple [A] time = 0.049, size = 90, normalized size = 1.1

$$\frac{\left(e^{kx\ln(a)}\right)^3}{3k\ln(a)}+\frac{\left(e^{lx\ln(a)}\right)^3}{3l\ln(a)}+3\frac{e^{kx\ln(a)}\left(e^{lx\ln(a)}\right)^2}{\ln(a)(k+2l)}+3\frac{\left(e^{kx\ln(a)}\right)^2e^{lx\ln(a)}}{\ln(a)(2k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)+a^(l*x))^3,x)
```

```
[Out] 1/3/k/ln(a)*exp(k*x*ln(a))^3+1/3/l/ln(a)*exp(l*x*ln(a))^3+3/ln(a)
/(k+2*l)*exp(k*x*ln(a))*exp(l*x*ln(a))^2+3/ln(a)/(2*k+l)*exp(k*x*
ln(a))^2*exp(l*x*ln(a))
```

Maxima [A] time = 1.36814, size = 104, normalized size = 1.32

$$\frac{3a^{2kx+lx}}{(2k+l)\log(a)}+\frac{3a^{kx+2lx}}{(k+2l)\log(a)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] 3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*
l)*log(a)) + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

Fricas [A] time = 0.219328, size = 176, normalized size = 2.23

$$\frac{9\left(2k^2l+kl^2\right)a^{kx}a^{2lx}+9\left(k^2l+2kl^2\right)a^{2kx}a^{lx}+\left(2k^2l+5kl^2+2l^3\right)a^{3kx}+\left(2k^3+5k^2l+2kl^2\right)a^{3lx}}{3\left(2k^3l+5k^2l^2+2kl^3\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*
a^(2*k*x)*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^
```

```
3 + 5*k^2*l + 2*k*l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3
)*log(a))
```

Sympy [A] time = 56.7249, size = 665, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)+a**(l*x))**3,x)
```

```
[Out] Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq
(l, 0))), (a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) +
3*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) +
3*x - a**(-3*l*x)/(l*log(a)) - a**(-6*l*x)/(6*l*log(a)), Eq(k, -2
*l)), (2*a**(3*l*x/2)/(l*log(a)) + a**(3*l*x)/(3*l*log(a)) + 3*x
- 2*a**(-3*l*x/2)/(3*l*log(a)), Eq(k, -l/2)), (a**(3*k*x)/(3*k*lo
g(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) + x, Eq
(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 1
5*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3
*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)
*a**(l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**
3*log(a)) + 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*k**3*l*log(a) + 15*k
**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*l*x)*k**2*
l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a
**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a)
+ 6*k*l**3*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2
*l**2*log(a) + 6*k*l**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*l
og(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l
**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), Tr
ue))
```

GIAC/XCAS [A] time = 0.250829, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^3,x, algorithm="giac")
```

```
[Out] Done
```

3.505 $\int \left(a^{kx}+a^{lx}\right)^4 \, dx$

Optimal. Leaf size=98

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)
*x))/((k + l)*Log[a]) + (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) +
(4*a^((k + 3*l)*x))/((k + 3*l)*Log[a])
```

Rubi [A] time = 0.224627, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)
*x))/((k + l)*Log[a]) + (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) +
(4*a^((k + 3*l)*x))/((k + 3*l)*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx}+a^{lx}\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)+a**(l*x))**4,x)
```

```
[Out] Integral((a**(k*x) + a**(l*x))**4, x)
```

Mathematica [A] time = 0.119006, size = 80, normalized size = 0.82

$$\frac{\frac{12a^{2x(k+l)}}{k+l}+\frac{16a^{x(3k+l)}}{3k+l}+\frac{16a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}}{4\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a
^((3*k + l)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Lo
g[a])
```

Maple [A] time = 0.027, size = 109, normalized size = 1.1

$$\frac{\left(a^{kx}\right)^{4}}{4\,k\ln\left(a\right)}+4\,\frac{\left(a^{kx}\right)^{3}a^{lx}}{\ln\left(a\right)\left(3\,k+l\right)}+3\,\frac{\left(a^{kx}\right)^{2}\left(a^{lx}\right)^{2}}{\ln\left(a\right)\left(k+l\right)}+4\,\frac{a^{kx}\left(a^{lx}\right)^{3}}{\ln\left(a\right)\left(k+3\,l\right)}+\frac{\left(a^{lx}\right)^{4}}{4\,l\ln\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)+a^(l*x))^4,x)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4+4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^
(k*x))^2/ln(a)/(k+l)*(a^(l*x))^2+4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x)
)^3+1/4/ln(a)/l*(a^(l*x))^4
```

Maxima [A] time = 1.34686, size = 134, normalized size = 1.37

$$\frac{4\,a^{3\,kx+lx}}{\left(3\,k+l\right)\log\left(a\right)}+\frac{4\,a^{kx+3\,lx}}{\left(k+3\,l\right)\log\left(a\right)}+\frac{3\,a^{2\,kx+2\,lx}}{\left(k+l\right)\log\left(a\right)}+\frac{a^{4\,kx}}{4\,k\log\left(a\right)}+\frac{a^{4\,lx}}{4\,l\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] 4*a^(3*k*x + l*x)/((3*k + l)*log(a)) + 4*a^(k*x + 3*l*x)/((k + 3*
l)*log(a)) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)
/(k*log(a)) + 1/4*a^(4*l*x)/(l*log(a))
```

Fricas [A] time = 0.223424, size = 277, normalized size = 2.83

$$\frac{16\left(3\,k^{3}l+4\,k^{2}l^{2}+kl^{3}\right)a^{kx}a^{3\,lx}+12\left(3\,k^{3}l+10\,k^{2}l^{2}+3\,kl^{3}\right)a^{2\,kx}a^{2\,lx}+16\left(k^{3}l+4\,k^{2}l^{2}+3\,kl^{3}\right)a^{3\,kx}a^{lx}+\left(3\,k^{3}l+13\,k^{2}l^{2}\right.}{4\left(3\,k^{4}l+13\,k^{3}l^{2}+13\,k^{2}l^{3}+3\,kl^{4}\right)\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) + 12*(3*k
^3*l + 10*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*
k^2*l^2 + 3*k*l^3)*a^(3*k*x)*a^(l*x) + (3*k^3*l + 13*k^2*l^2 + 13
*k*l^3 + 3*l^4)*a^(4*k*x) + (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*
l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2 + 13*k^2*l^3 + 3*k*l^4)*lo
g(a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)+a**(l*x))**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.248191, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^4,x, algorithm="giac")
```

```
[Out] Done
```

3.506 $\int \left(a^{kx}+a^{lx}\right)^n\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^{x(k-l)}+1\right)\left(a^{kx}+a^{lx}\right)^n\,{}_2F_1\left(1,\frac{kn}{k-l}+1;\frac{ln}{k-l}+1;-a^{(k-l)x}\right)}{ln\log(a)}$$

```
[Out] ((1 + a^((k - l)*x))*(a^(k*x) + a^(l*x))^n*Hypergeometric2F1[1, 1
 + (k*n)/(k - l), 1 + (l*n)/(k - l), -a^((k - l)*x)])/(l*n*Log[a]
)
```

Rubi [A] time = 0.134128, antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\left(a^{x(-(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n\,{}_2F_1\left(-n,-\frac{kn}{k-l};1-\frac{kn}{k-l};-a^{-(k-l)x}\right)}{kn\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1
- (k*n)/(k - l), -a^(-((k - l)*x))])/((1 + a^(-((k - l)*x)))^n*k*
n*Log[a])
```

Rubi in Sympy [A] time = 7.44185, size = 65, normalized size = 0.9

$$\frac{\left(a^{kx}+a^{lx}\right)^n\left(a^{x(-k+l)}+1\right)^{-n}\left(a^{x(-k+l)}+1\right)^{n+1}{}_2F_1\left(\left.\begin{matrix}1,\frac{kn}{-k+l}+n+1\\\frac{kn}{-k+l}+1\end{matrix}\right|-a^{x(-k+l)}\right)}{kn\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)+a**(l*x))**n,x)
```

```
[Out] (a**(k*x) + a**(l*x))**n*(a**(x*(-k + l)) + 1)**(-n)*(a**(x*(-k +
 l)) + 1)**(n + 1)*hyper((1, k*n/(-k + l) + n + 1), (k*n/(-k + l)
 + 1,), -a**(x*(-k + l)))/(k*n*log(a))
```

Mathematica [A] time = 0.0312627, size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] Integrate[(a^(k*x) + a^(l*x))^n, x]
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)+a^(l*x))^n,x)
```

```
[Out] int((a^(k*x)+a^(l*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{kx} + a^{lx}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^n,x, algorithm="fricas")
```

```
[Out] integral((a^(k*x) + a^(l*x))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)+a**(l*x))**n,x)
```

```
[Out] Integral((a**(k*x) + a**(l*x))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) + a^(l*x))^n,x, algorithm="giac")
```

```
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

3.507 $\int \left(a^{kx}-a^{lx}\right)\,dx$

Optimal. Leaf size=28

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

```
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Rubi [A] time = 0.0209691, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[a^(k*x) - a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Rubi in Sympy [A] time = 1.13873, size = 19, normalized size = 0.68

$$\frac{a^{kx}}{k\log\left(a\right)}-\frac{a^{lx}}{l\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**(k*x)-a**(l*x),x)
```

```
[Out] a**(k*x)/(k*log(a)) - a**(l*x)/(l*log(a))
```

Mathematica [A] time = 0.0106356, size = 28, normalized size = 1.

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^(k*x) - a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$\frac{a^{kx}}{k\ln(a)} - \frac{a^{lx}}{l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^(k*x)-a^(l*x),x)
```

```
[Out] a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)
```

Maxima [A] time = 1.38007, size = 38, normalized size = 1.36

$$\frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(k*x) - a^(l*x),x, algorithm="maxima")
```

```
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

Fricas [A] time = 0.216226, size = 38, normalized size = 1.36

$$-\frac{a^{lx}k - a^{kx}l}{kl\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(k*x) - a^(l*x),x, algorithm="fricas")
```

```
[Out] -(a^(l*x)*k - a^(k*x)*l)/(k*l*log(a))
```

Sympy [A] time = 0.558248, size = 29, normalized size = 1.04

$$\begin{cases} \frac{a^{kx}}{k\log(a)} & \text{for}\: k\log(a) \neq 0 \\ x & \text{otherwise} \end{cases} - \begin{cases} \frac{a^{lx}}{l\log(a)} & \text{for}\: l\log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**(k*x)-a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Pi
ecewise((a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

GIAC/XCAS [A] time = 0.198384, size = 38, normalized size = 1.36

$$\frac{a^{kx}}{k\ln(a)} - \frac{a^{lx}}{l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(k*x) - a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*ln(a)) - a^(l*x)/(l*ln(a))
```

3.508 $\int \left(a^{kx}-a^{lx}\right)^2 \, dx$

Optimal. Leaf size=53

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x
))/((k + l)*Log[a])
```

Rubi [A] time = 0.127586, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x
))/((k + l)*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)-a**(l*x))**2,x)
```

```
[Out] Integral((a**(k*x) - a**(l*x))**2, x)
```

Mathematica [A] time = 0.0573637, size = 55, normalized size = 1.04

$$-\frac{2a^{kx+lx}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^(k*x + l*x
))/((k + l)*Log[a])
```

Maple [A] time = 0.023, size = 59, normalized size = 1.1

$$\frac{\left(e^{kx\ln(a)}\right)^2}{2k\ln(a)} + \frac{\left(e^{lx\ln(a)}\right)^2}{2l\ln(a)} - 2\frac{e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)-a^(l*x))^2,x)
```

```
[Out] 1/2/k/ln(a)*exp(k*x*ln(a))^2+1/2/l/ln(a)*exp(l*x*ln(a))^2-2/ln(a)
/(k+l)*exp(k*x*ln(a))*exp(l*x*ln(a))
```

Maxima [A] time = 1.39032, size = 69, normalized size = 1.3

$$-\frac{2a^{kx+lx}}{(k+l)\log(a)} + \frac{a^{2kx}}{2k\log(a)} + \frac{a^{2lx}}{2l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^2,x, algorithm="maxima")
```

```
[Out] -2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/
2*a^(2*l*x)/(l*log(a))
```

Fricas [A] time = 0.218455, size = 86, normalized size = 1.62

$$-\frac{4a^{kx}a^{lx}kl - (kl+l^2)a^{2kx} - (k^2+kl)a^{2lx}}{2(k^2l+kl^2)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)
*a^(2*l*x))/((k^2*l + k*l^2)*log(a))
```

Sympy [A] time = 3.47077, size = 248, normalized size = 4.68

$$\begin{cases} 0 & \text{for } a = 1 \wedge (\\ \frac{a^{2lx}}{2l\log(a)} - \frac{2a^{lx}}{l\log(a)} + x & \text{for } k = 0 \\ \frac{a^{2lx}}{2l\log(a)} - 2x - \frac{a^{-2lx}}{2l\log(a)} & \text{for } k = -l \\ \frac{a^{2kx}}{2k\log(a)} - \frac{2a^{kx}}{k\log(a)} + x & \text{for } l = 0 \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} - \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)-a**(l*x))**2,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l
, 0))), (a**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(
k, 0)), (a**(2*l*x)/(2*l*log(a)) - 2*x - a**(-2*l*x)/(2*l*log(a))
, Eq(k, -l)), (a**(2*k*x)/(2*k*log(a)) - 2*a**(k*x)/(k*log(a)) +
x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a))
 + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) - 4*a**(k*
x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*
k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2
*l*log(a) + 2*k*l**2*log(a)), True))
```

GIAC/XCAS [A] time = 0.256413, size = 938, normalized size = 17.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^2,x, algorithm="giac")
```

```
[Out] (2*k*cos(-pi*k*x*sign(a) + pi*k*x)*ln(abs(a))/(4*k^2*ln(abs(a))^2
 + (pi*k*sign(a) - pi*k)^2) - (pi*k*sign(a) - pi*k)*sin(-pi*k*x*s
ign(a) + pi*k*x)/(4*k^2*ln(abs(a))^2 + (pi*k*sign(a) - pi*k)^2))*
e^(2*k*x*ln(abs(a))) - 1/2*I*(-I*e^(I*pi*k*x*sign(a) - I*pi*k*x)/
(I*pi*k*sign(a) - I*pi*k + 2*k*ln(abs(a))) + I*e^(-I*pi*k*x*sign(
a) + I*pi*k*x)/(-I*pi*k*sign(a) + I*pi*k + 2*k*ln(abs(a))))*e^(2*
k*x*ln(abs(a))) + (2*l*cos(-pi*l*x*sign(a) + pi*l*x)*ln(abs(a))/(
4*l^2*ln(abs(a))^2 + (pi*l*sign(a) - pi*l)^2) - (pi*l*sign(a) - p
i*l)*sin(-pi*l*x*sign(a) + pi*l*x)/(4*l^2*ln(abs(a))^2 + (pi*l*si
gn(a) - pi*l)^2))*e^(2*l*x*ln(abs(a))) - 1/2*I*(-I*e^(I*pi*l*x*si
gn(a) - I*pi*l*x)/(I*pi*l*sign(a) - I*pi*l + 2*l*ln(abs(a))) + I*
e^(-I*pi*l*x*sign(a) + I*pi*l*x)/(-I*pi*l*sign(a) + I*pi*l + 2*l*
ln(abs(a))))*e^(2*l*x*ln(abs(a))) - 4*(2*(k*ln(abs(a)) + l*ln(abs
(a)))*cos(-1/2*pi*k*x*sign(a) - 1/2*pi*l*x*sign(a) + 1/2*pi*k*x +
 1/2*pi*l*x)/((pi*k*sign(a) + pi*l*sign(a) - pi*k - pi*l)^2 + 4*(
k*ln(abs(a)) + l*ln(abs(a)))^2) - (pi*k*sign(a) + pi*l*sign(a) -
```

```
pi*k - pi*l)*sin(-1/2*pi*k*x*sign(a) - 1/2*pi*l*x*sign(a) + 1/2*p
i*k*x + 1/2*pi*l*x)/((pi*k*sign(a) + pi*l*sign(a) - pi*k - pi*l)^
2 + 4*(k*ln(abs(a)) + l*ln(abs(a)))^2))*e^((k*ln(abs(a)) + l*ln(a
bs(a)))*x) - 1/2*I*(4*I*e^(1/2*I*pi*k*x*sign(a) + 1/2*I*pi*l*x*si
gn(a) - 1/2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sign(a) + I*pi*l*sig
n(a) - I*pi*k - I*pi*l + 2*k*ln(abs(a)) + 2*l*ln(abs(a))) - 4*I*e
^(-1/2*I*pi*k*x*sign(a) - 1/2*I*pi*l*x*sign(a) + 1/2*I*pi*k*x + 1
/2*I*pi*l*x)/(-I*pi*k*sign(a) - I*pi*l*sign(a) + I*pi*k + I*pi*l
+ 2*k*ln(abs(a)) + 2*l*ln(abs(a))))*e^((k*ln(abs(a)) + l*ln(abs(a
)))*x)
```

3.509 $\int \left(a^{kx}-a^{lx}\right)^3 \, dx$

Optimal. Leaf size=79

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

```
[Out] a^(3*k*x)/(3*k*Log[a]) - a^(3*l*x)/(3*l*Log[a]) - (3*a^((2*k + l)
*x))/((2*k + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rubi [A] time = 0.187399, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) - a^(3*l*x)/(3*l*Log[a]) - (3*a^((2*k + l)
*x))/((2*k + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx}-a^{lx}\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)-a**(l*x))**3,x)
```

```
[Out] Integral((a**(k*x) - a**(l*x))**3, x)
```

Mathematica [A] time = 0.133242, size = 66, normalized size = 0.84

$$\frac{-\frac{9a^{x(2k+l)}}{2k+l}+\frac{9a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{k}-\frac{a^{3lx}}{l}}{3\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] (a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + l)*x))/(2*k + l) + (9*a
^((k + 2*l)*x))/(k + 2*l))/(3*Log[a])
```

Maple [A] time = 0.04, size = 90, normalized size = 1.1

$$\frac{\left(e^{kx\ln(a)}\right)^3}{3k\ln(a)} - \frac{\left(e^{lx\ln(a)}\right)^3}{3l\ln(a)} + 3\frac{e^{kx\ln(a)}\left(e^{lx\ln(a)}\right)^2}{\ln(a)(k+2l)} - 3\frac{\left(e^{kx\ln(a)}\right)^2 e^{lx\ln(a)}}{\ln(a)(2k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)-a^(l*x))^3,x)
```

```
[Out] 1/3/k/ln(a)*exp(k*x*ln(a))^3-1/3/l/ln(a)*exp(l*x*ln(a))^3+3/ln(a)
/(k+2*l)*exp(k*x*ln(a))*exp(l*x*ln(a))^2-3/ln(a)/(2*k+l)*exp(k*x*
ln(a))^2*exp(l*x*ln(a))
```

Maxima [A] time = 1.34094, size = 104, normalized size = 1.32

$$-\frac{3a^{2kx+lx}}{(2k+l)\log(a)} + \frac{3a^{kx+2lx}}{(k+2l)\log(a)} + \frac{a^{3kx}}{3k\log(a)} - \frac{a^{3lx}}{3l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] -3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2
*l)*log(a)) + 1/3*a^(3*k*x)/(k*log(a)) - 1/3*a^(3*l*x)/(l*log(a))
```

Fricas [A] time = 0.218901, size = 177, normalized size = 2.24

$$\frac{9\left(2k^2l + kl^2\right)a^{kx}a^{2lx} - 9\left(k^2l + 2kl^2\right)a^{2kx}a^{lx} + \left(2k^2l + 5kl^2 + 2l^3\right)a^{3kx} - \left(2k^3 + 5k^2l + 2kl^2\right)a^{3lx}}{3\left(2k^3l + 5k^2l^2 + 2kl^3\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) - 9*(k^2*l + 2*k*l^2)*
a^(2*k*x)*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) - (2*k^
```

```
3 + 5*k^2*l + 2*k*l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3
)*log(a))
```

Sympy [A] time = 56.9311, size = 663, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)-a**(l*x))**3,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l
, 0))), (-a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3
*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) +
3*x + a**(-3*l*x)/(l*log(a)) - a**(-6*l*x)/(6*l*log(a)), Eq(k, -2
*l)), (2*a**(3*l*x/2)/(l*log(a)) - a**(3*l*x)/(3*l*log(a)) - 3*x
- 2*a**(-3*l*x/2)/(3*l*log(a)), Eq(k, -l/2)), (a**(3*k*x)/(3*k*lo
g(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) - x, Eq
(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 1
5*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3
*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)
*a**(l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**
3*log(a)) - 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*k**3*l*log(a) + 15*k
**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*l*x)*k**2*
l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a
**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a)
+ 6*k*l**3*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2
*l**2*log(a) + 6*k*l**3*log(a)) - 5*a**(3*l*x)*k**2*l/(6*k**3*l*l
og(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l
**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), Tr
ue))
```

GIAC/XCAS [A] time = 0.235738, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^3,x, algorithm="giac")
```

```
[Out] Done
```

3.510 $\int \left(a^{kx}-a^{lx}\right)^4\,dx$

Optimal. Leaf size=98

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)
*x))/((k + l)*Log[a]) - (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) -
(4*a^((k + 3*l)*x))/((k + 3*l)*Log[a])
```

Rubi [A] time = 0.207649, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)
*x))/((k + l)*Log[a]) - (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) -
(4*a^((k + 3*l)*x))/((k + 3*l)*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)-a**(l*x))**4,x)
```

```
[Out] Integral((a**(k*x) - a**(l*x))**4, x)
```

Mathematica [A] time = 0.110412, size = 80, normalized size = 0.82

$$\frac{\frac{12a^{2x(k+l)}}{k+l}-\frac{16a^{x(3k+l)}}{3k+l}-\frac{16a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}}{4\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a
^((3*k + l)*x))/(3*k + l) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Lo
g[a])
```

Maple [A] time = 0.026, size = 109, normalized size = 1.1

$$\frac{(a^{kx})^4}{4k\ln(a)} - 4\frac{(a^{kx})^3 a^{lx}}{\ln(a)(3k+l)} + 3\frac{(a^{kx})^2 (a^{lx})^2}{\ln(a)(k+l)} - 4\frac{a^{kx}(a^{lx})^3}{\ln(a)(k+3l)} + \frac{(a^{lx})^4}{4l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)-a^(l*x))^4,x)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^
(k*x))^2/ln(a)/(k+l)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x)
)^3+1/4/ln(a)/l*(a^(l*x))^4
```

Maxima [A] time = 1.34728, size = 134, normalized size = 1.37

$$-\frac{4a^{3kx+lx}}{(3k+l)\log(a)} - \frac{4a^{kx+3lx}}{(k+3l)\log(a)} + \frac{3a^{2kx+2lx}}{(k+l)\log(a)} + \frac{a^{4kx}}{4k\log(a)} + \frac{a^{4lx}}{4l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] -4*a^(3*k*x + l*x)/((3*k + l)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3
*l)*log(a)) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x
)/(k*log(a)) + 1/4*a^(4*l*x)/(l*log(a))
```

Fricas [A] time = 0.215607, size = 279, normalized size = 2.85

$$-\frac{16\left(3k^3l + 4k^2l^2 + kl^3\right)a^{kx}a^{3lx} - 12\left(3k^3l + 10k^2l^2 + 3kl^3\right)a^{2kx}a^{2lx} + 16\left(k^3l + 4k^2l^2 + 3kl^3\right)a^{3kx}a^{lx} - \left(3k^3l + 13k^2l\right.}{4\left(3k^4l + 13k^3l^2 + 13k^2l^3 + 3kl^4\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) - 12*(3*
k^3*l + 10*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4
*k^2*l^2 + 3*k*l^3)*a^(3*k*x)*a^(l*x) - (3*k^3*l + 13*k^2*l^2 + 1
3*k*l^3 + 3*l^4)*a^(4*k*x) - (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k
*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2 + 13*k^2*l^3 + 3*k*l^4)*l
og(a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)-a**(l*x))**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266916, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^4,x, algorithm="giac")
```

```
[Out] Done
```

3.511 $\int \left(a^{kx}-a^{lx}\right)^n\,dx$

Optimal. Leaf size=74

$$\frac{\left(1-a^{x(k-l)}\right)\left(a^{kx}-a^{lx}\right)^n\,{}_2F_1\left(1,\frac{kn}{k-l}+1;\frac{ln}{k-l}+1;a^{(k-l)x}\right)}{ln\log(a)}$$

```
[Out] ((1 - a^((k - l)*x))*(a^(k*x) - a^(l*x))^n*Hypergeometric2F1[1, 1
 + (k*n)/(k - l), 1 + (l*n)/(k - l), a^((k - l)*x)])/(l*n*Log[a])
```

Rubi [A] time = 0.125999, antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\left(1-a^{x(-(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n\,{}_2F_1\left(-n,-\frac{kn}{k-l};1-\frac{kn}{k-l};a^{-(k-l)x}\right)}{kn\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1
- (k*n)/(k - l), a^(-((k - l)*x))])/((1 - a^(-((k - l)*x)))^n*k*n
*Log[a])
```

Rubi in Sympy [A] time = 7.66933, size = 63, normalized size = 0.85

$$\frac{\left(a^{kx}-a^{lx}\right)^n\left(-a^{x(-k+l)}+1\right)^{-n}\left(-a^{x(-k+l)}+1\right)^{n+1}{}_2F_1\left(\begin{matrix}1,\frac{kn}{-k+l}+n+1\\ \frac{kn}{-k+l}+1\end{matrix}\middle|a^{x(-k+l)}\right)}{kn\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(k*x)-a**(l*x))**n,x)
```

```
[Out] (a**(k*x) - a**(l*x))**n*(-a**(x*(-k + l)) + 1)**(-n)*(-a**(x*(-k
 + l)) + 1)**(n + 1)*hyper((1, k*n/(-k + l) + n + 1), (k*n/(-k +
l) + 1,), a**(x*(-k + l)))/(k*n*log(a))
```

Mathematica [A] time = 0.0367328, size = 0, normalized size = 0.

$$\int\left(a^{kx}-a^{lx}\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] Integrate[(a^(k*x) - a^(l*x))^n, x]
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(k*x)-a^(l*x))^n,x)
```

```
[Out] int((a^(k*x)-a^(l*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{kx} - a^{lx}\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^n,x, algorithm="fricas")
```

```
[Out] integral((a^(k*x) - a^(l*x))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(k*x)-a**(l*x))**n,x)

[Out] Integral((a**(k*x) - a**(l*x))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(k*x) - a^(l*x))^n,x, algorithm="giac")

[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

3.512 $\int (1 + a^{mx}) \, dx$

Optimal. Leaf size=15

$$\frac{a^{mx}}{m \log(a)} + x$$

```
[Out] x + a^(m*x)/(m*Log[a])
```

Rubi [A] time = 0.012735, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{a^{mx}}{m \log(a)} + x$$

Antiderivative was successfully verified.

```
[In]  Int[1 + a^(m*x),x]

[Out] x + a^(m*x)/(m*Log[a])
```

Rubi in Sympy [A] time = 0.803823, size = 10, normalized size = 0.67

$$\frac{a^{mx}}{m \log(a)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1+a**(m*x),x)

[Out] a**(m*x)/(m*log(a)) + x
```

Mathematica [A] time = 0.00654237, size = 15, normalized size = 1.

$$\frac{a^{mx}}{m \log(a)} + x$$

Antiderivative was successfully verified.

```
[In]  Integrate[1 + a^(m*x),x]
```

```
[Out] x + a^(m*x)/(m*Log[a])
```

Maple [A] time = 0.003, size = 16, normalized size = 1.1

$$x+\frac{a^{mx}}{m\ln\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+a^(m*x),x)
```

```
[Out] x+a^(m*x)/m/ln(a)
```

Maxima [A] time = 1.35404, size = 20, normalized size = 1.33

$$x+\frac{a^{mx}}{m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x) + 1,x, algorithm="maxima")
```

```
[Out] x + a^(m*x)/(m*log(a))
```

Fricas [A] time = 0.211485, size = 26, normalized size = 1.73

$$\frac{mx\log\left(a\right)+a^{mx}}{m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x) + 1,x, algorithm="fricas")
```

```
[Out] (m*x*log(a) + a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.080112, size = 15, normalized size = 1.

$$x+\begin{cases}\frac{a^{mx}}{m\log\left(a\right)} & \text{for}\: m\log\left(a\right)\neq 0\\x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1+a**(m*x),x)
```

```
[Out] x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))
```

GIAC/XCAS [A] time = 0.196398, size = 20, normalized size = 1.33

$$x + \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(m*x) + 1,x, algorithm="giac")
```

```
[Out] x + a^(m*x)/(m*ln(a))
```

3.513 $\int (1+a^{mx})^2\,dx$

Optimal. Leaf size=33

$$\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rubi [A] time = 0.0281822, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a^(m*x))^2,x]
```

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a^{mx}}{m\log(a)}+\frac{\log(a^{mx})}{m\log(a)}+\frac{\int^{a^{mx}} x\,dx}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+a**(m*x))**2,x)
```

```
[Out] 2*a**(m*x)/(m*log(a)) + log(a**(m*x))/(m*log(a)) + Integral(x, (x
, a**(m*x)))/(m*log(a))
```

Mathematica [A] time = 0.0150002, size = 33, normalized size = 1.

$$\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a^(m*x))^2,x]
```

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Maple [A] time = 0.013, size = 46, normalized size = 1.4

$$\frac{(a^{mx})^2}{2\,m\ln(a)} + 2\,\frac{a^{mx}}{m\ln(a)} + \frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+a^(m*x))^2,x)
```

```
[Out] 1/2/m/ln(a)*(a^(m*x))^2+2*a^(m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 1.37321, size = 42, normalized size = 1.27

$$x + \frac{a^{2\,mx}}{2\,m\log(a)} + \frac{2\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))
```

Fricas [A] time = 0.209099, size = 39, normalized size = 1.18

$$\frac{2\,mx\log(a) + a^{2\,mx} + 4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.116857, size = 46, normalized size = 1.39

$$x + \begin{cases} \frac{a^{2mx}m\log(a)+4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } 2m^2\log(a)^2 \neq 0 \\ 3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+a**(m*x))**2,x)
```

```
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**
2*log(a)**2), Ne(2*m**2*log(a)**2, 0)), (3*x, True))
```

GIAC/XCAS [A] time = 0.197431, size = 41, normalized size = 1.24

$$\frac{2\,mx\ln\left(|a|\right) + a^{2\,mx} + 4\,a^{mx}}{2\,m\ln\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*m*x*ln(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*ln(a))
```

3.514 $\int (1 + a^{mx})^3 \, dx$

Optimal. Leaf size=50

$$\frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{a^{3mx}}{3m\log(a)} + x$$

```
[Out] x + (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) + a^(3*m*
x)/(3*m*Log[a])
```

Rubi [A] time = 0.036049, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{a^{3mx}}{3m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a^(m*x))^3,x]
```

```
[Out] x + (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) + a^(3*m*
x)/(3*m*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{3mx}}{3m\log(a)} + \frac{3a^{mx}}{m\log(a)} + \frac{\log(a^{mx})}{m\log(a)} + \frac{3\int^{a^{mx}} x\,dx}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+a**(m*x))**3,x)
```

```
[Out] a**(3*m*x)/(3*m*log(a)) + 3*a**(m*x)/(m*log(a)) + log(a**(m*x))/(
m*log(a)) + 3*Integral(x, (x, a**(m*x)))/(m*log(a))
```

Mathematica [A] time = 0.0194399, size = 41, normalized size = 0.82

$$\frac{18a^{mx} + 9a^{2mx} + 2a^{3mx} + 6mx\log(a)}{6m\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a^(m*x))^3,x]
```

```
[Out] (18*a^(m*x) + 9*a^(2*m*x) + 2*a^(3*m*x) + 6*m*x*Log[a])/(6*m*Log[
a])
```

Maple [A] time = 0.003, size = 62, normalized size = 1.2

$$\frac{(a^{mx})^3}{3\,m\ln(a)}+\frac{3\,(a^{mx})^2}{2\,m\ln(a)}+3\,\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+a^(m*x))^3,x)
```

```
[Out] 1/3/m/ln(a)*(a^(m*x))^3+3/2/m/ln(a)*(a^(m*x))^2+3*a^(m*x)/m/ln(a)
+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 1.37683, size = 62, normalized size = 1.24

$$x+\frac{a^{3\,mx}}{3\,m\log(a)}+\frac{3\,a^{2\,mx}}{2\,m\log(a)}+\frac{3\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^3,x, algorithm="maxima")
```

```
[Out] x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*
x)/(m*log(a))
```

Fricas [A] time = 0.241489, size = 53, normalized size = 1.06

$$\frac{6\,mx\log(a)+2\,a^{3\,mx}+9\,a^{2\,mx}+18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*lo
g(a))
```

Sympy [A] time = 0.14797, size = 71, normalized size = 1.42

$$x + \begin{cases} \frac{2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2+18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } 6m^3\log(a)^3 \neq 0 \\ 7x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+a**(m*x))**3,x)
```

```
[Out] x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*l
og(a)**2 + 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(6*m
**3*log(a)**3, 0)), (7*x, True))
```

GIAC/XCAS [A] time = 0.200246, size = 54, normalized size = 1.08

$$\frac{6\,mx\ln(|a|) + 2\,a^{3\,mx} + 9\,a^{2\,mx} + 18\,a^{mx}}{6\,m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*m*x*ln(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(
m*ln(a))
```

3.515 $\int (1 + a^{mx})^4 \, dx$

Optimal. Leaf size=65

$$\frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)} + x$$

```
[Out] x + (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) + (4*a^(3*m
*x))/(3*m*Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rubi [A] time = 0.0425654, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a^(m*x))^4,x]
```

```
[Out] x + (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) + (4*a^(3*m
*x))/(3*m*Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{4mx}}{4m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{4a^{mx}}{m\log(a)} + \frac{\log(a^{mx})}{m\log(a)} + \frac{6\int^{a^{mx}} x\,dx}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+a**(m*x))**4,x)
```

```
[Out] a**(4*m*x)/(4*m*log(a)) + 4*a**(3*m*x)/(3*m*log(a)) + 4*a**(m*x)/
(m*log(a)) + log(a**(m*x))/(m*log(a)) + 6*Integral(x, (x, a**(m*x
)))/(m*log(a))
```

Mathematica [A] time = 0.0231901, size = 49, normalized size = 0.75

$$\frac{48a^{mx} + 36a^{2mx} + 16a^{3mx} + 3a^{4mx} + 12mx\log(a)}{12m\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a^(m*x))^4,x]
```

```
[Out] (48*a^(m*x) + 36*a^(2*m*x) + 16*a^(3*m*x) + 3*a^(4*m*x) + 12*m*x*
Log[a])/(12*m*Log[a])
```

Maple [A] time = 0.004, size = 78, normalized size = 1.2

$$\frac{(a^{mx})^4}{4\,m\ln(a)}+\frac{4\,(a^{mx})^3}{3\,m\ln(a)}+3\,\frac{(a^{mx})^2}{m\ln(a)}+4\,\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+a^(m*x))^4,x)
```

```
[Out] 1/4/m/ln(a)*(a^(m*x))^4+4/3/m/ln(a)*(a^(m*x))^3+3/m/ln(a)*(a^(m*x
))^2+4*a^(m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 1.34543, size = 82, normalized size = 1.26

$$x+\frac{a^{4\,mx}}{4\,m\log(a)}+\frac{4\,a^{3\,mx}}{3\,m\log(a)}+\frac{3\,a^{2\,mx}}{m\log(a)}+\frac{4\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*
m*x)/(m*log(a)) + 4*a^(m*x)/(m*log(a))
```

Fricas [A] time = 0.216538, size = 63, normalized size = 0.97

$$\frac{12\,mx\log(a)+3\,a^{4\,mx}+16\,a^{3\,mx}+36\,a^{2\,mx}+48\,a^{mx}}{12\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) +
 48*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.174139, size = 88, normalized size = 1.35

$$x+\begin{cases}\frac{3a^{4mx}m^3\log(a)^3+16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3+48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } 12m^4\log(a)^4\neq 0\\ 15x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 + 16*a**(3*m*x)*m**3*
log(a)**3 + 36*a**(2*m*x)*m**3*log(a)**3 + 48*a**(m*x)*m**3*log(a
)**3)/(12*m**4*log(a)**4), Ne(12*m**4*log(a)**4, 0)), (15*x, True
))
```

GIAC/XCAS [A] time = 0.205564, size = 65, normalized size = 1.

$$\frac{12\,mx\ln(|a|)+3\,a^{4\,mx}+16\,a^{3\,mx}+36\,a^{2\,mx}+48\,a^{mx}}{12\,m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^4,x, algorithm="giac")
```

```
[Out] 1/12*(12*m*x*ln(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*
x) + 48*a^(m*x))/(m*ln(a))
```

3.516 $\int (1 + a^{mx})^n \, dx$

Optimal. Leaf size=40

$$-\frac{(a^{mx}+1)^{n+1}\ {}_2F_1(1,n+1;n+2;a^{mx}+1)}{m(n+1)\log(a)}$$

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a
^(m*x)])/(m*(1 + n)*Log[a]))
```

Rubi [A] time = 0.0421315, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{(a^{mx}+1)^{n+1}\ {}_2F_1(1,n+1;n+2;a^{mx}+1)}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a^(m*x))^n,x]
```

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a
^(m*x)])/(m*(1 + n)*Log[a]))
```

Rubi in Sympy [A] time = 2.33268, size = 31, normalized size = 0.78

$$-\frac{(a^{mx}+1)^{n+1}\ {}_2F_1\left(\begin{matrix}1, n+1 \\ n+2\end{matrix}\middle| a^{mx}+1\right)}{m\,(n+1)\log\,(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+a**(m*x))**n,x)
```

```
[Out] -(a**(m*x) + 1)**(n + 1)*hyper((1, n + 1), (n + 2,), a**(m*x) + 1
)/(m*(n + 1)*log(a))
```

Mathematica [A] time = 0.0293722, size = 52, normalized size = 1.3

$$\frac{(a^{-mx}+1)^{-n}(a^{mx}+1)^n\ {}_2F_1(-n,-n;1-n;-a^{-mx})}{mn\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a^(m*x))^n,x]
```

```
[Out] ((1 + a^(m*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -a^(-(m*x))])/(
(1 + a^(-(m*x)))^n*m*n*Log[a])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (1 + a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+a^(m*x))^n,x)
```

```
[Out] int((1+a^(m*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(m*x) + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a^{mx} + 1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^n,x, algorithm="fricas")
```

```
[Out] integral((a^(m*x) + 1)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+a**(m*x))**n,x)
```

```
[Out] Integral((a**(m*x) + 1)**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) + 1)^n,x, algorithm="giac")
```

```
[Out] integrate((a^(m*x) + 1)^n, x)
```

3.517 $\int (1 - a^{mx})\, dx$

Optimal. Leaf size=16

$$x - \frac{a^{mx}}{m \log(a)}$$

```
[Out] x - a^(m*x)/(m*Log[a])
```

Rubi [A] time = 0.0117776, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[1 - a^(m*x),x]

[Out] x - a^(m*x)/(m*Log[a])
```

Rubi in Sympy [A] time = 0.91116, size = 10, normalized size = 0.62

$$-\frac{a^{mx}}{m \log(a)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1-a**(m*x),x)

[Out] -a**(m*x)/(m*log(a)) + x
```

Mathematica [A] time = 0.00714906, size = 16, normalized size = 1.

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - a^(m*x),x]
```

```
[Out] x - a^(m*x)/(m*Log[a])
```

Maple [A] time = 0.001, size = 17, normalized size = 1.1

$$x - \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-a^(m*x),x)

[Out] x-a^(m*x)/m/ln(a)
```

Maxima [A] time = 1.34517, size = 22, normalized size = 1.38

$$x - \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-a^(m*x) + 1,x, algorithm="maxima")

[Out] x - a^(m*x)/(m*log(a))
```

Fricas [A] time = 0.223231, size = 28, normalized size = 1.75

$$\frac{mx \log(a) - a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-a^(m*x) + 1,x, algorithm="fricas")

[Out] (m*x*log(a) - a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.084571, size = 19, normalized size = 1.19

$$x + \begin{cases} -\frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1-a**(m*x),x)
```

```
[Out] x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True)
)
```

GIAC/XCAS [A] time = 0.195935, size = 22, normalized size = 1.38

$$x - \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-a^(m*x) + 1,x, algorithm="giac")
```

```
[Out] x - a^(m*x)/(m*ln(a))
```

3.518 $\int (1 - a^{mx})^2 \, dx$

Optimal. Leaf size=33

$$-\frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)} + x$$

```
[Out] x - (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rubi [A] time = 0.0281422, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - a^(m*x))^2,x]
```

```
[Out] x - (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a^{mx}}{m\log(a)} + \frac{\log(a^{mx})}{m\log(a)} + \frac{\int^{a^{mx}} x\,dx}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-a**(m*x))**2,x)
```

```
[Out] -2*a**(m*x)/(m*log(a)) + log(a**(m*x))/(m*log(a)) + Integral(x, (
x, a**(m*x)))/(m*log(a))
```

Mathematica [A] time = 0.0224551, size = 25, normalized size = 0.76

$$\frac{(a^{mx} - 4)\,a^{mx}}{2m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - a^(m*x))^2,x]
```

```
[Out] x + (a^(m*x)*(-4 + a^(m*x)))/(2*m*Log[a])
```

Maple [A] time = 0.003, size = 46, normalized size = 1.4

$$\frac{(a^{mx})^2}{2\,m\ln(a)} - 2\,\frac{a^{mx}}{m\ln(a)} + \frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-a^(m*x))^2,x)
```

```
[Out] 1/2/m/ln(a)*(a^(m*x))^2-2*a^(m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 1.34398, size = 42, normalized size = 1.27

$$x + \frac{a^{2\,mx}}{2\,m\log(a)} - \frac{2\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) - 1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))
```

Fricas [A] time = 0.215469, size = 39, normalized size = 1.18

$$\frac{2\,mx\log(a) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) - 1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.126281, size = 46, normalized size = 1.39

$$x + \begin{cases} \frac{a^{2mx}m\log(a)-4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } 2m^2\log(a)^2 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-a**(m*x))**2,x)
```

```
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**
2*log(a)**2), Ne(2*m**2*log(a)**2, 0)), (-x, True))
```

GIAC/XCAS [A] time = 0.212845, size = 41, normalized size = 1.24

$$\frac{2\,mx\ln\left(|a|\right)+a^{2\,mx}-4\,a^{mx}}{2\,m\ln\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) - 1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*m*x*ln(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*ln(a))
```

3.519 $\int (1-a^{mx})^3 \, dx$

Optimal. Leaf size=50

$$-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}+x$$

```
[Out] x - (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) - a^(3*m*
x)/(3*m*Log[a])
```

Rubi [A] time = 0.036264, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - a^(m*x))^3,x]
```

```
[Out] x - (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) - a^(3*m*
x)/(3*m*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{3mx}}{3m\log(a)}-\frac{3a^{mx}}{m\log(a)}+\frac{\log(a^{mx})}{m\log(a)}+\frac{3\int^{a^{mx}} x\,dx}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-a**(m*x))**3,x)
```

```
[Out] -a**(3*m*x)/(3*m*log(a)) - 3*a**(m*x)/(m*log(a)) + log(a**(m*x))/
(m*log(a)) + 3*Integral(x, (x, a**(m*x)))/(m*log(a))
```

Mathematica [A] time = 0.0413415, size = 35, normalized size = 0.7

$$x-\frac{a^{mx}\left(-9a^{mx}+2a^{2mx}+18\right)}{6m\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - a^(m*x))^3,x]

[Out] x - (a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/(6*m*Log[a])
```

Maple [A] time = 0.004, size = 62, normalized size = 1.2

$$-\frac{(a^{mx})^3}{3\,m\ln(a)}+\frac{3\,(a^{mx})^2}{2\,m\ln(a)}-3\,\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-a^(m*x))^3,x)

[Out] -1/3/m/ln(a)*(a^(m*x))^3+3/2/m/ln(a)*(a^(m*x))^2-3*a^(m*x)/m/ln(a
)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 1.39193, size = 62, normalized size = 1.24

$$x-\frac{a^{3\,mx}}{3\,m\log(a)}+\frac{3\,a^{2\,mx}}{2\,m\log(a)}-\frac{3\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a^(m*x) - 1)^3,x, algorithm="maxima")

[Out] x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*
x)/(m*log(a))
```

Fricas [A] time = 0.214333, size = 53, normalized size = 1.06

$$\frac{6\,mx\log(a)-2\,a^{3\,mx}+9\,a^{2\,mx}-18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a^(m*x) - 1)^3,x, algorithm="fricas")

[Out] 1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*lo
g(a))
```

Sympy [A] time = 0.15599, size = 71, normalized size = 1.42

$$x+\begin{cases} \frac{-2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2-18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } 6m^3\log(a)^3\neq 0\\ -x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-a**(m*x))**3,x)
```

```
[Out] x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*
log(a)**2 - 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(6*
m**3*log(a)**3, 0)), (-x, True))
```

GIAC/XCAS [A] time = 0.198784, size = 54, normalized size = 1.08

$$\frac{6\,mx\ln(|a|)-2\,a^{3\,mx}+9\,a^{2\,mx}-18\,a^{mx}}{6\,m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a^(m*x) - 1)^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*m*x*ln(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(
m*ln(a))
```

3.520 $\int (1 - a^{mx})^4 \, dx$

Optimal. Leaf size=65

$$-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

```
[Out] x - (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) - (4*a^(3*m
*x))/(3*m*Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rubi [A] time = 0.0437666, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - a^(m*x))^4,x]
```

```
[Out] x - (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) - (4*a^(3*m
*x))/(3*m*Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{4mx}}{4m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}-\frac{4a^{mx}}{m\log(a)}+\frac{\log(a^{mx})}{m\log(a)}+\frac{6\int^{a^{mx}} x\,dx}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-a**(m*x))**4,x)
```

```
[Out] a**(4*m*x)/(4*m*log(a)) - 4*a**(3*m*x)/(3*m*log(a)) - 4*a**(m*x)/
(m*log(a)) + log(a**(m*x))/(m*log(a)) + 6*Integral(x, (x, a**(m*x
)))/(m*log(a))
```

Mathematica [A] time = 0.0376921, size = 43, normalized size = 0.66

$$\frac{\left(36a^{mx}-16a^{2mx}+3a^{3mx}-48\right)a^{mx}}{12m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - a^(m*x))^4,x]
```

```
[Out] x + (a^(m*x)*(-48 + 36*a^(m*x) - 16*a^(2*m*x) + 3*a^(3*m*x)))/(12
*m*Log[a])
```

Maple [A] time = 0.002, size = 78, normalized size = 1.2

$$\frac{(a^{mx})^4}{4m\ln(a)} - \frac{4\,(a^{mx})^3}{3m\ln(a)} + 3\,\frac{(a^{mx})^2}{m\ln(a)} - 4\,\frac{a^{mx}}{m\ln(a)} + \frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-a^(m*x))^4,x)
```

```
[Out] 1/4/m/ln(a)*(a^(m*x))^4-4/3/m/ln(a)*(a^(m*x))^3+3/m/ln(a)*(a^(m*x
))^2-4*a^(m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 1.39896, size = 82, normalized size = 1.26

$$x + \frac{a^{4\,mx}}{4m\log(a)} - \frac{4\,a^{3\,mx}}{3m\log(a)} + \frac{3\,a^{2\,mx}}{m\log(a)} - \frac{4\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) - 1)^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*
m*x)/(m*log(a)) - 4*a^(m*x)/(m*log(a))
```

Fricas [A] time = 0.207181, size = 63, normalized size = 0.97

$$\frac{12\,mx\log(a) + 3\,a^{4\,mx} - 16\,a^{3\,mx} + 36\,a^{2\,mx} - 48\,a^{mx}}{12\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) - 1)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) -
 48*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.184514, size = 88, normalized size = 1.35

$$x+\begin{cases}\frac{3a^{4mx}m^3\log(a)^3-16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3-48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } 12m^4\log(a)^4\neq 0\\ -x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 - 16*a**(3*m*x)*m**3*
log(a)**3 + 36*a**(2*m*x)*m**3*log(a)**3 - 48*a**(m*x)*m**3*log(a
)**3)/(12*m**4*log(a)**4), Ne(12*m**4*log(a)**4, 0)), (-x, True))
```

GIAC/XCAS [A] time = 0.230891, size = 65, normalized size = 1.

$$\frac{12\,mx\ln(|a|)+3\,a^{4\,mx}-16\,a^{3\,mx}+36\,a^{2\,mx}-48\,a^{mx}}{12\,m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^(m*x) - 1)^4,x, algorithm="giac")
```

```
[Out] 1/12*(12*m*x*ln(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*
x) - 48*a^(m*x))/(m*ln(a))
```

3.521 $\int (1 - a^{mx})^n \, dx$

Optimal. Leaf size=44

$$-\frac{(1-a^{mx})^{n+1}\,{}_2F_1(1,n+1;n+2;1-a^{mx})}{m(n+1)\log(a)}$$

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a
^(m*x)])/(m*(1 + n)*Log[a]))
```

Rubi [A] time = 0.0397345, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{(1-a^{mx})^{n+1}\,{}_2F_1(1,n+1;n+2;1-a^{mx})}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - a^(m*x))^n,x]
```

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a
^(m*x)])/(m*(1 + n)*Log[a]))
```

Rubi in Sympy [A] time = 2.6184, size = 31, normalized size = 0.7

$$-\frac{(-a^{mx}+1)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|-a^{mx}+1\right)}{m\,(n+1)\log\,(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-a**(m*x))**n,x)
```

```
[Out] -(-a**(m*x) + 1)**(n + 1)*hyper((1, n + 1), (n + 2,), -a**(m*x) +
 1)/(m*(n + 1)*log(a))
```

Mathematica [A] time = 0.0322805, size = 54, normalized size = 1.23

$$\frac{(1-a^{-mx})^{-n}(1-a^{mx})^n\,{}_2F_1(-n,-n;1-n;a^{-mx})}{mn\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - a^(m*x))^n,x]
```

```
[Out] ((1 - a^(m*x))^n*Hypergeometric2F1[-n, -n, 1 - n, a^(-(m*x))])/((
1 - a^(-(m*x)))^n*m*n*Log[a])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (1 - a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-a^(m*x))^n,x)
```

```
[Out] int((1-a^(m*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a^(m*x) + 1)^n,x, algorithm="maxima")
```

```
[Out] integrate((-a^(m*x) + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-a^{mx} + 1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a^(m*x) + 1)^n,x, algorithm="fricas")
```

```
[Out] integral((-a^(m*x) + 1)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-a**(m*x))**n,x)
```

```
[Out] Integral((-a**(m*x) + 1)**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a^(m*x) + 1)^n,x, algorithm="giac")
```

```
[Out] integrate((-a^(m*x) + 1)^n, x)
```

3.522 $\int \frac{1}{b+ae^{nx}} dx$

Optimal. Leaf size=24

$$\frac{x}{b} - \frac{\log(ae^{nx} + b)}{bn}$$

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Rubi [A] time = 0.032037, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{x}{b} - \frac{\log(ae^{nx} + b)}{bn}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + a*E^(n*x))^(-1),x]

[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Rubi in Sympy [A] time = 3.13836, size = 22, normalized size = 0.92

$$-\frac{\log(ae^{nx} + b)}{bn} + \frac{\log(e^{nx})}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b+a*exp(n*x)),x)

[Out] -log(a*exp(n*x) + b)/(b*n) + log(exp(n*x))/(b*n)
```

Mathematica [A] time = 0.00754968, size = 24, normalized size = 1.

$$\frac{x}{b} - \frac{\log(ae^{nx} + b)}{bn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + a*E^(n*x))^(-1),x]
```

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Maple [A] time = 0.013, size = 31, normalized size = 1.3

$$\frac{\ln\left(e^{nx}\right)}{nb} - \frac{\ln\left(b + a e^{nx}\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b+a*exp(n*x)),x)
```

```
[Out] 1/n/b*ln(exp(n*x))-ln(b+a*exp(n*x))/b/n
```

Maxima [A] time = 1.33471, size = 31, normalized size = 1.29

$$\frac{x}{b} - \frac{\log\left(ae^{(nx)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*e^(n*x) + b),x, algorithm="maxima")
```

```
[Out] x/b - log(a*e^(n*x) + b)/(b*n)
```

Fricas [A] time = 0.233617, size = 30, normalized size = 1.25

$$\frac{nx - \log\left(ae^{(nx)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*e^(n*x) + b),x, algorithm="fricas")
```

```
[Out] (n*x - log(a*e^(n*x) + b))/(b*n)
```

Sympy [A] time = 0.131794, size = 15, normalized size = 0.62

$$\frac{x}{b} - \frac{\log\left(e^{nx} + \frac{b}{a}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b+a*exp(n*x)),x)
```

```
[Out] x/b - log(exp(n*x) + b/a)/(b*n)
```

GIAC/XCAS [A] time = 0.200428, size = 32, normalized size = 1.33

$$\frac{x}{b} - \frac{\ln\left(\left|ae^{(nx)} + b\right|\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*e^(n*x) + b),x, algorithm="giac")
```

```
[Out] x/b - ln(abs(a*e^(n*x) + b))/(b*n)
```

3.523 $\int \frac{e^x}{b+ae^{3x}} dx$

Optimal. Leaf size=100

$$\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{2\sqrt[3]{a}b^{2/3}}-\frac{\log\left(ae^{3x}+b\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*E^x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^
(1/3)*b^(2/3))) + Log[b^(1/3) + a^(1/3)*E^x]/(2*a^(1/3)*b^(2/3))
- Log[b + a*E^(3*x)]/(6*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.171921, antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(b + a*E^(3*x)),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*E^x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^
(1/3)*b^(2/3))) + Log[b^(1/3) + a^(1/3)*E^x]/(3*a^(1/3)*b^(2/3))
- Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)]/(6*a^(1/3)
*b^(2/3))
```

Rubi in Sympy [A] time = 14.4717, size = 116, normalized size = 1.16

$$\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}}-\frac{\log\left(a^{\frac{2}{3}}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{\frac{2}{3}}\right)}{6\sqrt[3]{a}b^{\frac{2}{3}}}-\frac{\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}\left(-\frac{2\sqrt[3]{a}e^x}{3}+\frac{\sqrt[3]{b}}{3}\right)}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(b+a*exp(3*x)),x)
```

```
[Out] log(a**(1/3)*exp(x) + b**(1/3))/(3*a**(1/3)*b**(2/3)) - log(a**(2
/3)*exp(2*x) - a**(1/3)*b**(1/3)*exp(x) + b**(2/3))/(6*a**(1/3)*b
**(2/3)) - sqrt(3)*atan(sqrt(3)*(-2*a**(1/3)*exp(x)/3 + b**(1/3)/
```

```
3)/b**(1/3))/(3*a**(1/3)*b**(2/3))
```

Mathematica [C] time = 0.0153797, size = 36, normalized size = 0.36

$$\frac{\text{RootSum}\left[\#1^3a+b\&,\frac{\log(e^x-\#1)-x}{\#1^2}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(b + a*E^(3*x)),x]
```

```
[Out] RootSum[b + a*#1^3 & , (-x + Log[E^x - #1])/#1^2 & ]/(3*a)
```

Maple [A] time = 0.01, size = 95, normalized size = 1.

$$\frac{1}{3\,a}\ln\left(\mathrm{e}^{x}+\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{1}{6\,a}\ln\left((\mathrm{e}^{x})^{2}-\mathrm{e}^{x}\sqrt[3]{\frac{b}{a}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,\mathrm{e}^{x}\frac{1}{\sqrt[3]{\frac{b}{a}}}-1\right)\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(b+a*exp(3*x)),x)
```

```
[Out] 1/3/a/(b/a)^(2/3)*ln(exp(x)+(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(exp
(x)^2-exp(x)*(b/a)^(1/3)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*exp(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(a*e^(3*x) + b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220812, size = 126, normalized size = 1.26

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(-\left(ab^2\right)^{\frac{1}{3}}be^x+b^2+\left(ab^2\right)^{\frac{2}{3}}e^{(2x)}\right)-2\sqrt{3}\log\left(\left(ab^2\right)^{\frac{1}{3}}e^x+b\right)-6\arctan\left(\frac{2\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}e^x-\sqrt{3}b}{3b}\right)\right)}{18\left(ab^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(a*e^(3*x) + b),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(-(a*b^2)^(1/3)*b*e^x + b^2 + (a*b^2)^(
2/3)*e^(2*x)) - 2*sqrt(3)*log((a*b^2)^(1/3)*e^x + b) - 6*arctan(1
/3*(2*sqrt(3)*(a*b^2)^(1/3)*e^x - sqrt(3)*b)/b))/(a*b^2)^(1/3)
```

Sympy [A] time = 0.198847, size = 22, normalized size = 0.22

$$\mathrm{RootSum}\left(27z^3ab^2-1,\left(i\mapsto i\log\left(3ib+e^x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(b+a*exp(3*x)),x)
```

```
[Out] RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))
```

GIAC/XCAS [A] time = 0.200282, size = 157, normalized size = 1.57

$$-\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\ln\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right|\right)}{3b}+\frac{\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2e^x\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{\left(-a^2b\right)^{\frac{1}{3}}\ln\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(-\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2x)}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(a*e^(3*x) + b),x, algorithm="giac")
```

```
[Out] -1/3*(-b/a)^(1/3)*ln(abs(-(-b/a)^(1/3) + e^x))/b + 1/3*sqrt(3)*(-
a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*e^x)/(-b/a)^(1/
3))/(a*b) + 1/6*(-a^2*b)^(1/3)*ln((-b/a)^(1/3)*e^x + (-b/a)^(2/3)
 + e^(2*x))/(a*b)
```

3.524 $\int \frac{-1+e^x}{1+e^x} \, dx$

Optimal. Leaf size=12

$$2 \log \left(e^x+1\right)-x$$

```
[Out] -x + 2*Log[1 + E^x]
```

Rubi [A] time = 0.0419974, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$2 \log \left(e^x+1\right)-x$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + E^x)/(1 + E^x),x]

[Out] -x + 2*Log[1 + E^x]
```

Rubi in Sympy [A] time = 4.74773, size = 15, normalized size = 1.25

$$x+2 \log \left(e^x+1\right)-2 \log \left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+exp(x))/(1+exp(x)),x)

[Out] x + 2*log(exp(x) + 1) - 2*log(exp(x))
```

Mathematica [A] time = 0.00524388, size = 12, normalized size = 1.

$$2 \log \left(e^x+1\right)-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + E^x)/(1 + E^x),x]

[Out] -x + 2*Log[1 + E^x]
```

Maple [A] time = 0.009, size = 14, normalized size = 1.2

$$2\ln(1+e^x)-\ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+exp(x))/(1+exp(x)),x)

[Out] 2*ln(1+exp(x))-ln(exp(x))
```

Maxima [A] time = 1.36302, size = 15, normalized size = 1.25

$$-x+2\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^x - 1)/(e^x + 1),x, algorithm="maxima")

[Out] -x + 2*log(e^x + 1)
```

Fricas [A] time = 0.218669, size = 15, normalized size = 1.25

$$-x+2\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^x - 1)/(e^x + 1),x, algorithm="fricas")

[Out] -x + 2*log(e^x + 1)
```

Sympy [A] time = 0.066218, size = 8, normalized size = 0.67

$$-x+2\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+exp(x))/(1+exp(x)),x)

[Out] -x + 2*log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.200389, size = 15, normalized size = 1.25

$$-x + 2\ln\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^x - 1)/(e^x + 1),x, algorithm="giac")
```

```
[Out] -x + 2*ln(e^x + 1)
```

3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx$

Optimal. Leaf size=47

$$\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[Out] -ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*E^(2*x)
+ 3*E^(4*x)]/12
```

Rubi [A] time = 0.0997838, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] -ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*E^(2*x)
+ 3*E^(4*x)]/12
```

Rubi in Sympy [A] time = 11.9864, size = 42, normalized size = 0.89

$$\frac{\log\left(3e^{4x}-2e^{2x}+1\right)}{12}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{3e^{2x}}{2}-\frac{1}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

```
[Out] log(3*exp(4*x) - 2*exp(2*x) + 1)/12 + sqrt(2)*atan(sqrt(2)*(3*exp
(2*x)/2 - 1/2))/12
```

Mathematica [A] time = 0.0283617, size = 44, normalized size = 0.94

$$\frac{1}{12}\left(\log\left(-2e^{2x}+3e^{4x}+1\right)+\sqrt{2}\tan^{-1}\left(\frac{3e^{2x}-1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*E^(2*x))/Sqrt[2]] + Log[1 - 2*E^(2*x) + 3
*E^(4*x)])/12
```

Maple [A] time = 0.009, size = 38, normalized size = 0.8

$$\frac{\ln\left(1-2\left(e^{x}\right)^{2}+3\left(e^{x}\right)^{4}\right)}{12}+\frac{\sqrt{2}}{12}\arctan\left(\frac{\left(6\left(e^{x}\right)^{2}-2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

```
[Out] 1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x
)^2-2)*2^(1/2))
```

Maxima [A] time = 1.53689, size = 50, normalized size = 1.06

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)}-1\right)\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(4*x)/(3*e^(4*x) - 2*e^(2*x) + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(
4*x) - 2*e^(2*x) + 1)
```

Fricas [A] time = 0.219026, size = 58, normalized size = 1.23

$$\frac{1}{24}\sqrt{2}\left(\sqrt{2}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)+2\arctan\left(\frac{3}{2}\sqrt{2}e^{(2x)}-\frac{1}{2}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(4*x)/(3*e^(4*x) - 2*e^(2*x) + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*(sqrt(2)*log(3*e^(4*x) - 2*e^(2*x) + 1) + 2*arctan(3
/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)))
```

Sympy [A] time = 0.152781, size = 22, normalized size = 0.47

$$\mathrm{RootSum}\left(96z^2 - 16z + 1, \left(i \mapsto i\log\left(8i + e^{2x} - 1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

```
[Out] RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) -
 1)))
```

GIAC/XCAS [A] time = 0.197918, size = 50, normalized size = 1.06

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)} - 1\right)\right) + \frac{1}{12}\ln\left(3e^{(4x)} - 2e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(4*x)/(3*e^(4*x) - 2*e^(2*x) + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*ln(3*e^(4
*x) - 2*e^(2*x) + 1)
```

3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}} \, dx$

Optimal. Leaf size=39

$$e^x + \frac{e^{2x}}{2} + \log(1-e^x) - \frac{1}{2}\log\left(e^{2x}+1\right) - \tan^{-1}(e^x)$$

```
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Rubi [A] time = 0.104901, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$e^x + \frac{e^{2x}}{2} + \log(1-e^x) - \frac{1}{2}\log\left(e^{2x}+1\right) - \tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

```
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{5x}+e^x}{e^{3x}-e^{2x}+e^x-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] Integral((exp(5*x) + exp(x))/(exp(3*x) - exp(2*x) + exp(x) - 1),
x)
```

Mathematica [A] time = 0.0322495, size = 39, normalized size = 1.

$$e^x + \frac{e^{2x}}{2} + \log(1-e^x) - \frac{1}{2}\log\left(e^{2x}+1\right) - \tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

```
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Maple [A] time = 0.019, size = 29, normalized size = 0.7

$$-\frac{\ln\left((\mathrm{e}^{x})^{2}+1\right)}{2}-\arctan\left(\mathrm{e}^{x}\right)+\ln\left(-1+\mathrm{e}^{x}\right)+\mathrm{e}^{x}+\frac{(\mathrm{e}^{x})^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] -1/2*ln(exp(x)^2+1)-arctan(exp(x))+ln(-1+exp(x))+exp(x)+1/2*exp(x
)^2
```

Maxima [A] time = 1.57276, size = 38, normalized size = 0.97

$$-\arctan\left(e^{x}\right)+\frac{1}{2}\,e^{(2\,x)}+e^{x}-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^(5*x) + e^x)/(e^(3*x) - e^(2*x) + e^x - 1),x, algorithm="maxima")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x
 - 1)
```

Fricas [A] time = 0.230679, size = 38, normalized size = 0.97

$$-\arctan\left(e^{x}\right)+\frac{1}{2}\,e^{(2\,x)}+e^{x}-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^(5*x) + e^x)/(e^(3*x) - e^(2*x) + e^x - 1),x, algorithm="fricas")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x
 - 1)
```

Sympy [A] time = 0.240272, size = 48, normalized size = 1.23

$$\frac{e^{2x}}{2}+e^{x}+\log\left(e^{x}-1\right)+\operatorname{RootSum}\left(2z^{2}+2z+1,\left(i\mapsto i\log\left(\frac{4i^{2}}{5}-\frac{6i}{5}+e^{x}-\frac{3}{5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] exp(2*x)/2 + exp(x) + log(exp(x) - 1) + RootSum(2*_z**2 + 2*_z +
1, Lambda(_i, _i*log(4*_i**2/5 - 6*_i/5 + exp(x) - 3/5)))
```

GIAC/XCAS [A] time = 0.199046, size = 39, normalized size = 1.

$$-\arctan\left(e^{x}\right) + \frac{1}{2}\, e^{(2\,x)} + e^{x} - \frac{1}{2}\,\ln\left(e^{(2\,x)} + 1\right) + \ln\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^(5*x) + e^x)/(e^(3*x) - e^(2*x) + e^x - 1),x, algorithm="giac")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*ln(e^(2*x) + 1) + ln(abs(e
^x - 1))
```

3.527 $\int e^{nx} (a + be^{nx})^{r/s} \, dx$

Optimal. Leaf size=30

$$\frac{s (a + be^{nx})^{\frac{r+s}{s}}}{bn(r + s)}$$

```
[Out] ((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

Rubi [A] time = 0.065609, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{s (a + be^{nx})^{\frac{r+s}{s}}}{bn(r + s)}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]

[Out] ((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

Rubi in Sympy [A] time = 5.2087, size = 20, normalized size = 0.67

$$\frac{s (a + be^{nx})^{\frac{r+s}{s}}}{bn (r + s)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)

[Out] s*(a + b*exp(n*x))**((r + s)/s)/(b*n*(r + s))
```

Mathematica [A] time = 0.0479255, size = 30, normalized size = 1.

$$\frac{s (a + be^{nx})^{\frac{r}{s}+1}}{bnr + bns}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)
```

Maple [A] time = 0.003, size = 33, normalized size = 1.1

$$\frac{1}{nb}\left(a+be^{nx}\right)^{\frac{r}{s}+1}\left(\frac{r}{s}+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(n*x)*(a+b*exp(n*x))^(r/s),x)

[Out] 1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(n*x) + a)^(r/s)*e^(n*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.22827, size = 50, normalized size = 1.67

$$\frac{\left(bse^{(nx)}+as\right)\left(be^{(nx)}+a\right)^{\frac{r}{s}}}{bnr+bns}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(n*x) + a)^(r/s)*e^(n*x),x, algorithm="fricas")

[Out] (b*s*e^(n*x) + a*s)*(b*e^(n*x) + a)^(r/s)/(b*n*r + b*n*s)
```

Sympy [A] time = 2.05022, size = 94, normalized size = 3.13

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge n = 0 \wedge r = -s \\ \frac{a^{\frac{r}{s}} e^{nx}}{n} & \text{for } b = 0 \\ x (a + b)^{\frac{r}{s}} & \text{for } n = 0 \\ \frac{\log\left(\frac{a}{b} + e^{nx}\right)}{bn} & \text{for } r = -s \\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns} + \frac{bs(a+be^{nx})^{\frac{r}{s}} e^{nx}}{bnr+bns} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n
*x)/n, Eq(b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*
x))/(b*n), Eq(r, -s)), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*
s) + b*s*(a + b*exp(n*x))**(r/s)*exp(n*x)/(b*n*r + b*n*s), True))
```

GIAC/XCAS [A] time = 0.20331, size = 43, normalized size = 1.43

$$\frac{\left(be^{(nx)} + a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(n*x) + a)^(r/s)*e^(n*x),x, algorithm="giac")
```

```
[Out] (b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

3.528 $\int \sqrt[4]{1-2e^{x/3}}\,dx$

Optimal. Leaf size=54

$$12\sqrt[4]{1-2e^{x/3}}-6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*Ar
cTanh[(1 - 2*E^(x/3))^(1/4)]
```

Rubi [A] time = 0.0507944, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$12\sqrt[4]{1-2e^{x/3}}-6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*E^(x/3))^(1/4),x]
```

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*Ar
cTanh[(1 - 2*E^(x/3))^(1/4)]
```

Rubi in Sympy [A] time = 2.53313, size = 42, normalized size = 0.78

$$12\sqrt[4]{-2e^{\frac{x}{3}}+1}-6\,\mathrm{atan}\left(\sqrt[4]{-2e^{\frac{x}{3}}+1}\right)-6\,\mathrm{atanh}\left(\sqrt[4]{-2e^{\frac{x}{3}}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-2*exp(1/3*x))**(1/4),x)
```

```
[Out] 12*(-2*exp(x/3) + 1)**(1/4) - 6*atan((-2*exp(x/3) + 1)**(1/4)) -
6*atanh((-2*exp(x/3) + 1)**(1/4))
```

Mathematica [C] time = 0.0568485, size = 70, normalized size = 1.3

$$-\frac{2\left(\sqrt[4]{2}\left(2-e^{-x/3}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{e^{-x/3}}{2}\right)+12e^{x/3}-6\right)}{\left(1-2e^{x/3}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*E^(x/3))^(1/4),x]
```

```
[Out] (-2*(-6 + 12*E^(x/3) + 2^(1/4)*(2 - E^(-x/3))^(3/4)*Hypergeometri
c2F1[3/4, 3/4, 7/4, 1/(2*E^(x/3))]))/(1 - 2*E^(x/3))^(3/4)
```

Maple [A] time = 0.01, size = 57, normalized size = 1.1

$$12\sqrt[4]{1-2\,\mathrm{e}^{x/3}}+3\,\ln\left(-1+\sqrt[4]{1-2\,\mathrm{e}^{x/3}}\right)-3\,\ln\left(1+\sqrt[4]{1-2\,\mathrm{e}^{x/3}}\right)-6\,\arctan\left(\sqrt[4]{1-2\,\mathrm{e}^{x/3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-2*exp(1/3*x))^(1/4),x)
```

```
[Out] 12*(1-2*exp(1/3*x))^(1/4)+3*ln(-1+(1-2*exp(1/3*x))^(1/4))-3*ln(1+
(1-2*exp(1/3*x))^(1/4))-6*arctan((1-2*exp(1/3*x))^(1/4))
```

Maxima [A] time = 1.56773, size = 76, normalized size = 1.41

$$12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*e^(1/3*x) + 1)^(1/4),x, algorithm="maxima")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4))
- 3*log((-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^
(1/4) - 1)
```

Fricas [A] time = 0.223122, size = 76, normalized size = 1.41

$$12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*e^(1/3*x) + 1)^(1/4),x, algorithm="fricas")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4))
- 3*log((-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^
(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{-2e^{\frac{x}{3}} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-2*exp(1/3*x))**(1/4),x)
```

```
[Out] Integral((-2*exp(x/3) + 1)**(1/4), x)
```

GIAC/XCAS [A] time = 0.209881, size = 77, normalized size = 1.43

$$12\left(-2\,e^{\left(\frac{1}{3}\,x\right)} + 1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)} + 1\right)^{\frac{1}{4}}\right) - 3\ln\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)} + 1\right)^{\frac{1}{4}} + 1\right) + 3\ln\left(\left|\left(-2\,e^{\left(\frac{1}{3}\,x\right)} + 1\right)^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*e^(1/3*x) + 1)^(1/4),x, algorithm="giac")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4))
- 3*ln((-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*ln(abs((-2*e^(1/3*x) + 1
)^(1/4) - 1))
```

3.529 $\int (a + be^{nx})^{r/s}\, dx$

Optimal. Leaf size=59

$$-\frac{s\,(a + be^{nx})^{\frac{r+s}{s}}\ {}_2F_1\left(1, \frac{r+s}{s}; \frac{r}{s} + 2; \frac{e^{nx}b}{a} + 1\right)}{an(r + s)}$$

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s,
2 + r/s, 1 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Rubi [A] time = 0.0632181, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{s\,(a + be^{nx})^{\frac{r+s}{s}}\ {}_2F_1\left(1, \frac{r+s}{s}; \frac{r}{s} + 2; \frac{e^{nx}b}{a} + 1\right)}{an(r + s)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s,
2 + r/s, 1 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Rubi in Sympy [A] time = 3.50817, size = 41, normalized size = 0.69

$$-\frac{s\,(a + be^{nx})^{\frac{r+s}{s}}\ {}_2F_1\left(\begin{matrix} 1, \frac{r+s}{s} \\ \frac{r}{s} + 2 \end{matrix}\middle| 1 + \frac{be^{nx}}{a}\right)}{an\,(r + s)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*exp(n*x))**(r/s),x)
```

```
[Out] -s*(a + b*exp(n*x))**((r + s)/s)*hyper((1, (r + s)/s), (r/s + 2,)
, 1 + b*exp(n*x)/a)/(a*n*(r + s))
```

Mathematica [A] time = 0.0511749, size = 76, normalized size = 1.29

$$\frac{s\left(\frac{ae^{-nx}}{b} + 1\right)^{-\frac{r}{s}} (a + be^{nx})^{r/s}\ {}_2F_1\left(-\frac{r}{s}, -\frac{r}{s}; 1 - \frac{r}{s}; -\frac{ae^{-nx}}{b}\right)}{nr}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^(r/s)*s*Hypergeometric2F1[-(r/s), -(r/s), 1 - r/
s, -(a/(b*E^(n*x)))])/((1 + a/(b*E^(n*x)))^(r/s)*n*r)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (a + b e^{nx})^{\frac{r}{s}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*exp(n*x))^(r/s),x)
```

```
[Out] int((a+b*exp(n*x))^(r/s),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(be^{(nx)} + a\right)^{\frac{r}{s}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(n*x) + a)^(r/s),x, algorithm="maxima")
```

```
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(be^{(nx)} + a\right)^{\frac{r}{s}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(n*x) + a)^(r/s),x, algorithm="fricas")
```

```
[Out] integral((b*e^(n*x) + a)^(r/s), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + be^{nx})^{\frac{r}{s}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*exp(n*x))**(r/s),x)
```

```
[Out] Integral((a + b*exp(n*x))**(r/s), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(be^{(nx)} + a\right)^{\frac{r}{s}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(n*x) + a)^(r/s),x, algorithm="giac")
```

```
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}} \, dx$

Optimal. Leaf size=18

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

```
[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

Rubi [A] time = 0.0457611, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/Sqrt[a^2 + E^(2*x)],x]

[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

Rubi in Sympy [A] time = 3.55023, size = 15, normalized size = 0.83

$$\operatorname{atanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)

[Out] atanh(exp(x)/sqrt(a**2 + exp(2*x)))
```

Mathematica [A] time = 0.0172602, size = 18, normalized size = 1.

$$\log\left(\sqrt{a^2+e^{2x}}+e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/Sqrt[a^2 + E^(2*x)],x]
```

```
[Out] Log[E^x + Sqrt[a^2 + E^(2*x)]]
```

Maple [A] time = 0.013, size = 15, normalized size = 0.8

$$\ln\left(e^{x}+\sqrt{a^{2}+\left(e^{x}\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(a^2+exp(2*x))^(1/2),x)
```

```
[Out] ln(exp(x)+(a^2+exp(x)^2)^(1/2))
```

Maxima [A] time = 1.4386, size = 12, normalized size = 0.67

$$\operatorname{arsinh}\left(\frac{e^{x}}{\sqrt{a^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/sqrt(a^2 + e^(2*x)),x, algorithm="maxima")
```

```
[Out] arcsinh(e^x/sqrt(a^2))
```

Fricas [A] time = 0.21664, size = 24, normalized size = 1.33

$$-\log\left(\sqrt{a^{2}+e^{(2\,x)}}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/sqrt(a^2 + e^(2*x)),x, algorithm="fricas")
```

```
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

Sympy [A] time = 0.579017, size = 31, normalized size = 1.72

$$\begin{cases} \operatorname{asinh}\left(\sqrt{\frac{1}{a^{2}}}e^{x}\right) & \text{for}\: a^{2} > 0 \\ \operatorname{acosh}\left(\sqrt{-\frac{1}{a^{2}}}e^{x}\right) & \text{for}\: a^{2} < 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((asinh(sqrt(a**(-2))*exp(x)), a**2 > 0), (acosh(sqrt(-1
/a**2)*exp(x)), a**2 < 0))
```

GIAC/XCAS [A] time = 0.207792, size = 24, normalized size = 1.33

$$-\ln\left(\sqrt{a^2 + e^{(2\,x)}} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/sqrt(a^2 + e^(2*x)),x, algorithm="giac")
```

```
[Out] -ln(sqrt(a^2 + e^(2*x)) - e^x)
```

3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}} \, dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

```
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Rubi [A] time = 0.0475626, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/Sqrt[-a^2 + E^(2*x)],x]

[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Rubi in Sympy [A] time = 3.88252, size = 15, normalized size = 0.75

$$\operatorname{atanh}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)

[Out] atanh(exp(x)/sqrt(-a**2 + exp(2*x)))
```

Mathematica [A] time = 0.0185382, size = 20, normalized size = 1.

$$\log\left(\sqrt{e^{2x}-a^2}+e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

```
[Out] Log[E^x + Sqrt[-a^2 + E^(2*x)]]
```

Maple [A] time = 0.015, size = 17, normalized size = 0.9

$$\ln\left(e^{x}+\sqrt{-a^{2}+\left(e^{x}\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-a^2+exp(2*x))^(1/2),x)
```

```
[Out] ln(exp(x)+(-a^2+exp(x)^2)^(1/2))
```

Maxima [A] time = 1.35544, size = 27, normalized size = 1.35

$$\log\left(2\sqrt{-a^{2}+e^{(2x)}}+2e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^x/sqrt(-a^2 + e^(2*x)),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)
```

Fricas [A] time = 0.213627, size = 27, normalized size = 1.35

$$-\log\left(\sqrt{-a^{2}+e^{(2x)}}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^x/sqrt(-a^2 + e^(2*x)),x, algorithm="fricas")
```

```
[Out] -log(sqrt(-a^2 + e^(2*x)) - e^x)
```

Sympy [A] time = 0.59348, size = 34, normalized size = 1.7

$$\begin{cases}\operatorname{asinh}\left(\sqrt{-\frac{1}{a^{2}}}e^{x}\right) & \text{for}\ -a^{2}>0\\ \operatorname{acosh}\left(\sqrt{\frac{1}{a^{2}}}e^{x}\right) & \text{for}\ -a^{2}<0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((asinh(sqrt(-1/a**2)*exp(x)), -a**2 > 0), (acosh(sqrt(a
**(-2))*exp(x)), -a**2 < 0))
```

GIAC/XCAS [A] time = 0.228527, size = 27, normalized size = 1.35

$$-\ln\left(-\sqrt{-a^2 + e^{(2\,x)}} + e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/sqrt(-a^2 + e^(2*x)),x, algorithm="giac")
```

```
[Out] -ln(-sqrt(-a^2 + e^(2*x)) + e^x)
```

3.532 $\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

```
[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x
)/2)])])/3
```

Rubi [A] time = 0.166763, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),x]
```

```
[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x
)/2)])])/3
```

Rubi in Sympy [A] time = 24.2406, size = 36, normalized size = 0.9

$$-\frac{2\,\text{atanh}\left(\frac{5e^{\frac{3x}{4}}-2}{4\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
```

```
[Out] -2*atanh((5*exp(3*x/4) - 2)/(4*sqrt(exp(3*x/4) + exp(3*x/2) - 2))
)/3
```

Mathematica [A] time = 0.040818, size = 55, normalized size = 1.38

$$\frac{2}{3}\log\left(2-e^{3x/4}\right)-\frac{2}{3}\log\left(-4\sqrt{e^{3x/4}+e^{3x/2}-2}-5e^{3x/4}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),
```

```
[Out] (2*Log[2 - E^((3*x)/4)])/3 - (2*Log[2 - 5*E^((3*x)/4) - 4*Sqrt[-2
 + E^((3*x)/4) + E^((3*x)/2)]])/3
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int 1\mathrm{e}^{\frac{3x}{4}}\left(-2+\mathrm{e}^{\frac{3x}{4}}\right)^{-1}\frac{1}{\sqrt{-2+\mathrm{e}^{\frac{3x}{4}}+\mathrm{e}^{\frac{3x}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

```
[Out] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x
)
```

Maxima [A] time = 1.6532, size = 53, normalized size = 1.32

$$-\frac{2}{3}\log\left(\frac{4\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+\frac{8}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(3/4*x)/(sqrt(e^(3/2*x) + e^(3/4*x) - 2)*(e^(3/4*x) - 2)),x, algorit
```

```
[Out] -2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8
/abs(e^(3/4*x) - 2) + 5)
```

Fricas [A] time = 0.22534, size = 62, normalized size = 1.55

$$-\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right)+\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(3/4*x)/(sqrt(e^(3/2*x) + e^(3/4*x) - 2)*(e^(3/4*x) - 2)),x, algorit
```

```
[Out] -2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*l
og(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
```

```
[Out] Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2
) - 2)), x)
```

GIAC/XCAS [A] time = 0.244508, size = 65, normalized size = 1.62

$$-\frac{2}{3}\ln\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right|\right)+\frac{2}{3}\ln\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(3/4*x)/(sqrt(e^(3/2*x) + e^(3/4*x) - 2)*(e^(3/4*x) - 2)),x, algorit
```

```
[Out] -2/3*ln(abs(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4)) + 2
/3*ln(abs(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x)))
```

3.533 $\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}e^{-2x}\left(e^{7x}-3\right)^{5/3}\,{}_2F_1\left(1,\frac{29}{21};\frac{5}{7};\frac{e^{7x}}{3}\right)$$

```
[Out] ((-3 + E^(7*x))^(5/3)*Hypergeometric2F1[1, 29/21, 5/7, E^(7*x)/3]
)/(6*E^(2*x))
```

Rubi [A] time = 0.0941681, antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{3^{2/3}e^{-2x}\left(e^{7x}-3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{7};\frac{5}{7};\frac{e^{7x}}{3}\right)}{2\left(3-e^{7x}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] -(3^(2/3)*(-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7,
 E^(7*x)/3])/(2*E^(2*x)*(3 - E^(7*x))^(2/3))
```

Rubi in Sympy [A] time = 6.56238, size = 46, normalized size = 1.24

$$-\frac{\left(e^{7x}-3\right)^{\frac{2}{3}}e^{-2x}{}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{7}\\ \frac{5}{7}\end{matrix}\middle|\frac{e^{7x}}{3}\right)}{2\left(-\frac{e^{7x}}{3}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)
```

```
[Out] -(exp(7*x) - 3)**(2/3)*exp(-2*x)*hyper((-2/3, -2/7), (5/7,), exp(
7*x)/3)/(2*(-exp(7*x)/3 + 1)**(2/3))
```

Mathematica [A] time = 0.0560825, size = 68, normalized size = 1.84

$$\frac{e^{-2x}\left(7\;3^{2/3}\sqrt[3]{3-e^{7x}}\,{}_2F_1\left(-\frac{2}{7},\frac{1}{3};\frac{5}{7};\frac{e^{7x}}{3}\right)+3e^{7x}-9\right)}{8\sqrt[3]{e^{7x}-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] (-9 + 3*E^(7*x) + 7*3^(2/3)*(3 - E^(7*x))^(1/3)*Hypergeometric2F1
[-2/7, 1/3, 5/7, E^(7*x)/3])/(8*E^(2*x)*(-3 + E^(7*x))^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{e^{2x}} \left(-3+e^{7x}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

```
[Out] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e^{(7x)}-3\right)^{\frac{2}{3}} e^{(-2x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(7*x) - 3)^(2/3)*e^(-2*x),x, algorithm="maxima")
```

```
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{(7x)}-3\right)^{\frac{2}{3}} e^{(-2x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(7*x) - 3)^(2/3)*e^(-2*x),x, algorithm="fricas")
```

```
[Out] integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e^{7x}-3\right)^{\frac{2}{3}} e^{-2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)

[Out] Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e^{(7x)}-3\right)^{\frac{2}{3}} e^{(-2x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^(7*x) - 3)^(2/3)*e^(-2*x),x, algorithm="giac")

[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$

Optimal. Leaf size=73

$$\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

```
[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E
^(x/2))^(9/4) + (8*(3 - E^(x/2))^(13/4))/13
```

Rubi [A] time = 0.0808552, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E
^(x/2))^(9/4) + (8*(3 - E^(x/2))^(13/4))/13
```

Rubi in Sympy [A] time = 5.63422, size = 49, normalized size = 0.67

$$\frac{8\left(-e^{\frac{x}{2}}+3\right)^{\frac{13}{4}}}{13}-8\left(-e^{\frac{x}{2}}+3\right)^{\frac{9}{4}}+\frac{216\left(-e^{\frac{x}{2}}+3\right)^{\frac{5}{4}}}{5}-216\sqrt[4]{-e^{\frac{x}{2}}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
```

```
[Out] 8*(-exp(x/2) + 3)**(13/4)/13 - 8*(-exp(x/2) + 3)**(9/4) + 216*(-e
xp(x/2) + 3)**(5/4)/5 - 216*(-exp(x/2) + 3)**(1/4)
```

Mathematica [A] time = 0.0295629, size = 44, normalized size = 0.6

$$-\frac{8}{65}\sqrt[4]{3-e^{x/2}}\left(96e^{x/2}+20e^{x}+5e^{3x/2}+1152\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] (-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/
2)))/65
```

Maple [A] time = 0.027, size = 37, normalized size = 0.5

$$\frac{8}{65}\left(5\,e^{3/2\,x}+20\,e^{x}+96\,e^{x/2}+1152\right)\left(-3+e^{\frac{x}{2}}\right)\left(3-e^{\frac{x}{2}}\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(3-exp(1/2*x))^(3/4),x)
```

```
[Out] 8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1
152)*(-3+exp(1/2*x))
```

Maxima [A] time = 1.48294, size = 66, normalized size = 0.9

$$\frac{8}{13}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{13}{4}}-8\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{9}{4}}+\frac{216}{5}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(-e^(1/2*x) + 3)^(3/4),x, algorithm="maxima")
```

```
[Out] 8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(
-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)
```

Fricas [A] time = 0.216434, size = 41, normalized size = 0.56

$$-\frac{8}{65}\left(5\,e^{\left(\frac{3}{2}\,x\right)}+96\,e^{\left(\frac{1}{2}\,x\right)}+20\,e^{x}+1152\right)\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(-e^(1/2*x) + 3)^(3/4),x, algorithm="fricas")
```

```
[Out] -8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) +
3)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2x}}{\left(-e^{\frac{x}{2}}+3\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
```

```
[Out] Integral(exp(2*x)/(-exp(x/2) + 3)**(3/4), x)
```

GIAC/XCAS [A] time = 0.219197, size = 88, normalized size = 1.21

$$-\frac{8}{13}\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{3}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}-8\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{2}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}+\frac{216}{5}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(-e^(1/2*x) + 3)^(3/4),x, algorithm="giac")
```

```
[Out] -8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3
)^2*(-e^(1/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(
-e^(1/2*x) + 3)^(1/4)
```

3.535 $\int e^{-x/2}x^3\,dx$

Optimal. Leaf size=44

$$-2e^{-x/2}x^3-12e^{-x/2}x^2-48e^{-x/2}x-96e^{-x/2}$$

```
[Out] -96/E^(x/2) - (48*x)/E^(x/2) - (12*x^2)/E^(x/2) - (2*x^3)/E^(x/2)
```

Rubi [A] time = 0.0544157, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-2e^{-x/2}x^3-12e^{-x/2}x^2-48e^{-x/2}x-96e^{-x/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(x/2),x]
```

```
[Out] -96/E^(x/2) - (48*x)/E^(x/2) - (12*x^2)/E^(x/2) - (2*x^3)/E^(x/2)
```

Rubi in Sympy [A] time = 3.04113, size = 36, normalized size = 0.82

$$-2x^3e^{-\frac{x}{2}}-12x^2e^{-\frac{x}{2}}-48xe^{-\frac{x}{2}}-96e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/exp(1/2*x),x)
```

```
[Out] -2*x**3*exp(-x/2) - 12*x**2*exp(-x/2) - 48*x*exp(-x/2) - 96*exp(-
x/2)
```

Mathematica [A] time = 0.00522756, size = 23, normalized size = 0.52

$$e^{-x/2}\left(-2x^3-12x^2-48x-96\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(x/2),x]
```

```
[Out] (-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)
```

Maple [A] time = 0.003, size = 22, normalized size = 0.5

$$-2\,\frac{x^3+6\,x^2+24\,x+48}{e^{x/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/exp(1/2*x),x)
```

```
[Out] -2*(x^3+6*x^2+24*x+48)/exp(1/2*x)
```

Maxima [A] time = 1.37629, size = 26, normalized size = 0.59

$$-2\,\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(-1/2*x),x, algorithm="maxima")
```

```
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Fricas [A] time = 0.214349, size = 26, normalized size = 0.59

$$-2\,\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(-1/2*x),x, algorithm="fricas")
```

```
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Sympy [A] time = 0.075999, size = 20, normalized size = 0.45

$$\left(-2x^3-12x^2-48x-96\right)e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/exp(1/2*x),x)
```

```
[Out] (-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)
```

GIAC/XCAS [A] time = 0.21273, size = 26, normalized size = 0.59

$$-2\left(x^{3}+6\,x^{2}+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(-1/2*x),x, algorithm="giac")
```

```
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

3.536 $\int \frac{e^{-x/2}}{x^3}\,dx$

Optimal. Leaf size=39

$$\frac{\text{Ei}\left(-\frac{x}{2}\right)}{8}-\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}$$

```
[Out] -1/(2*E^(x/2)*x^2) + 1/(4*E^(x/2)*x) + ExpIntegralEi[-x/2]/8
```

Rubi [A] time = 0.0544195, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\text{Ei}\left(-\frac{x}{2}\right)}{8}-\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(x/2)*x^3),x]
```

```
[Out] -1/(2*E^(x/2)*x^2) + 1/(4*E^(x/2)*x) + ExpIntegralEi[-x/2]/8
```

Rubi in Sympy [A] time = 3.29709, size = 29, normalized size = 0.74

$$\frac{\text{Ei}\left(-\frac{x}{2}\right)}{8}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{e^{-\frac{x}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/exp(1/2*x)/x**3,x)
```

```
[Out] Ei(-x/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

Mathematica [A] time = 0.0197055, size = 26, normalized size = 0.67

$$\frac{1}{8}\left(\text{Ei}\left(-\frac{x}{2}\right)+\frac{2e^{-x/2}(x-2)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(x/2)*x^3),x]
```

```
[Out] ((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-x/2])/8
```

Maple [A] time = 0.006, size = 31, normalized size = 0.8

$$-\frac{1}{2\,x^2}\left(\mathrm{e}^{\frac{x}{2}}\right)^{-1}+\frac{1}{4\,x}\left(\mathrm{e}^{\frac{x}{2}}\right)^{-1}-\frac{1}{8}Ei\left(1,\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/exp(1/2*x)/x^3,x)

[Out] -1/2/exp(1/2*x)/x^2+1/4/exp(1/2*x)/x-1/8*Ei(1,1/2*x)
```

Maxima [A] time = 2.15085, size = 9, normalized size = 0.23

$$-\frac{1}{4}\left(-2,\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-1/2*x)/x^3,x, algorithm="maxima")

[Out] -1/4*gamma(-2, 1/2*x)
```

Fricas [A] time = 0.20208, size = 31, normalized size = 0.79

$$\frac{x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right)+2\,(x-2)e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-1/2*x)/x^3,x, algorithm="fricas")

[Out] 1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2
```

Sympy [A] time = 2.31937, size = 32, normalized size = 0.82

$$\frac{\mathrm{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{e^{-\frac{x}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/exp(1/2*x)/x**3,x)
```

```
[Out] Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

GIAC/XCAS [A] time = 0.225322, size = 36, normalized size = 0.92

$$\frac{x^2 \mathrm{Ei}\left(-\frac{1}{2}\,x\right) + 2\,x e^{\left(-\frac{1}{2}\,x\right)} - 4\,e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-1/2*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*x*e^(-1/2*x) - 4*e^(-1/2*x))/x^2
```

3.537 $\int a^{3x}x^2\,dx$

Optimal. Leaf size=44

$$\frac{x^2a^{3x}}{3\log(a)}+\frac{2a^{3x}}{27\log^3(a)}-\frac{2xa^{3x}}{9\log^2(a)}$$

```
[Out] (2*a^(3*x))/(27*Log[a]^3) - (2*a^(3*x)*x)/(9*Log[a]^2) + (a^(3*x)
*x^2)/(3*Log[a])
```

Rubi [A] time = 0.0380386, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{x^2a^{3x}}{3\log(a)}+\frac{2a^{3x}}{27\log^3(a)}-\frac{2xa^{3x}}{9\log^2(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[a^(3*x)*x^2,x]
```

```
[Out] (2*a^(3*x))/(27*Log[a]^3) - (2*a^(3*x)*x)/(9*Log[a]^2) + (a^(3*x)
*x^2)/(3*Log[a])
```

Rubi in Sympy [A] time = 2.83781, size = 41, normalized size = 0.93

$$\frac{a^{3x}x^2}{3\log(a)}-\frac{2a^{3x}x}{9\log(a)^2}+\frac{2a^{3x}}{27\log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**(3*x)*x**2,x)
```

```
[Out] a**(3*x)*x**2/(3*log(a)) - 2*a**(3*x)*x/(9*log(a)**2) + 2*a**(3*x
)/(27*log(a)**3)
```

Mathematica [A] time = 0.0082402, size = 29, normalized size = 0.66

$$\frac{a^{3x}\left(9x^2\log^2(a)-6x\log(a)+2\right)}{27\log^3(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^(3*x)*x^2,x]

[Out] (a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)
```

Maple [A] time = 0.01, size = 28, normalized size = 0.6

$$\frac{\left(9\,x^{2}\left(\ln\left(a\right)\right)^{2}-6\,x\ln\left(a\right)+2\right)\,a^{3\,x}}{27\,\left(\ln\left(a\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^(3*x)*x^2,x)

[Out] 1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3
```

Maxima [A] time = 1.50813, size = 36, normalized size = 0.82

$$\frac{\left(9\,x^{2}\log\left(a\right)^{2}-6\,x\log\left(a\right)+2\right)a^{3\,x}}{27\,\log\left(a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(3*x)*x^2,x, algorithm="maxima")

[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

Fricas [A] time = 0.211463, size = 36, normalized size = 0.82

$$\frac{\left(9\,x^{2}\log\left(a\right)^{2}-6\,x\log\left(a\right)+2\right)a^{3\,x}}{27\,\log\left(a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(3*x)*x^2,x, algorithm="fricas")

[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

Sympy [A] time = 0.108898, size = 39, normalized size = 0.89

$$\begin{cases} \frac{a^{3x}\left(9x^2\log{(a)}^2-6x\log{(a)}+2\right)}{27\log{(a)}^3} & \text{for } 27\log{(a)}^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**(3*x)*x**2,x)
```

```
[Out] Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a
)**3), Ne(27*log(a)**3, 0)), (x**3/3, True))
```

GIAC/XCAS [A] time = 0.222489, size = 1, normalized size = 0.02

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^(3*x)*x^2,x, algorithm="giac")
```

```
[Out] Done
```

3.538 $\int e^{x^2} x \left(1 + x^2\right) dx$

Optimal. Leaf size=12

$$\frac{1}{2} e^{x^2} x^2$$

```
[Out] (E^x^2*x^2)/2
```

Rubi [A] time = 0.0718183, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{2} e^{x^2} x^2$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*x*(1 + x^2),x]

[Out] (E^x^2*x^2)/2
```

Rubi in Sympy [A] time = 3.65258, size = 8, normalized size = 0.67

$$\frac{x^2 e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*x*(x**2+1),x)

[Out] x**2*exp(x**2)/2
```

Mathematica [A] time = 0.00432201, size = 12, normalized size = 1.

$$\frac{1}{2} e^{x^2} x^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*x*(1 + x^2),x]
```

```
[Out] (E^x^2*x^2)/2
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$\frac{e^{x^2}x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x*(x^2+1),x)
```

```
[Out] 1/2*exp(x^2)*x^2
```

Maxima [A] time = 1.40663, size = 24, normalized size = 2.

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}+\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)*x*e^(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

Fricas [A] time = 0.215933, size = 12, normalized size = 1.

$$\frac{1}{2}\,x^2e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)*x*e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*e^(x^2)
```

Sympy [A] time = 0.072963, size = 8, normalized size = 0.67

$$\frac{x^2e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*x*(x**2+1),x)
```

```
[Out] x**2*exp(x**2)/2
```

GIAC/XCAS [A] time = 0.212216, size = 24, normalized size = 2.

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}+\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*x*e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

3.539 $\int \frac{x}{(e^{-x}+e^x)^2} dx$

Optimal. Leaf size=32

$$-\frac{x}{2(e^{2x}+1)}+\frac{x}{2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

```
[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4
```

Rubi [A] time = 0.0869832, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$-\frac{x}{2(e^{2x}+1)}+\frac{x}{2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(E^(-x) + E^x)^2,x]
```

```
[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4
```

Rubi in Sympy [A] time = 5.67498, size = 31, normalized size = 0.97

$$\frac{x}{2(1+e^{-2x})}-\frac{\log\left(1+e^{-2x}\right)}{4}+\frac{\log\left(e^{-2x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(exp(-x)+exp(x))**2,x)
```

```
[Out] x/(2*(1 + exp(-2*x))) - log(1 + exp(-2*x))/4 + log(exp(-2*x))/4
```

Mathematica [A] time = 0.0297949, size = 31, normalized size = 0.97

$$\frac{e^{2x}x}{2e^{2x}+2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(E^(-x) + E^x)^2,x]
```

```
[Out] (E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

Maple [A] time = 0.016, size = 26, normalized size = 0.8

$$-\frac{\ln\left((\mathrm{e}^x)^2+1\right)}{4}+\frac{(\mathrm{e}^x)^2 x}{2(\mathrm{e}^x)^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(exp(-x)+exp(x))^2,x)
```

```
[Out] -1/4*ln(exp(x)^2+1)+1/2*x*exp(x)^2/(exp(x)^2+1)
```

Maxima [A] time = 1.57644, size = 34, normalized size = 1.06

$$\frac{xe^{(2x)}}{2\left(e^{(2x)}+1\right)}-\frac{1}{4}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(e^(-x) + e^x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*x)/(e^(2*x) + 1) - 1/4*log(e^(2*x) + 1)
```

Fricas [A] time = 0.216871, size = 45, normalized size = 1.41

$$\frac{2xe^{(2x)}-\left(e^{(2x)}+1\right)\log\left(e^{(2x)}+1\right)}{4\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(e^(-x) + e^x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x*e^(2*x) - (e^(2*x) + 1)*log(e^(2*x) + 1))/(e^(2*x) + 1)
```

Sympy [A] time = 0.100881, size = 24, normalized size = 0.75

$$-\frac{x}{2}+\frac{x}{2+2e^{-2x}}-\frac{\log\left(1+e^{-2x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(exp(-x)+exp(x))**2,x)
```

```
[Out] -x/2 + x/(2 + 2*exp(-2*x)) - log(1 + exp(-2*x))/4
```

GIAC/XCAS [A] time = 0.219543, size = 54, normalized size = 1.69

$$\frac{2\,xe^{(2\,x)} - e^{(2\,x)}\ln\left(e^{(2\,x)} + 1\right) - \ln\left(e^{(2\,x)} + 1\right)}{4\,\left(e^{(2\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(e^(-x) + e^x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x*e^(2*x) - e^(2*x)*ln(e^(2*x) + 1) - ln(e^(2*x) + 1))/(e^
(2*x) + 1)
```

3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=15

$$e^x\sqrt{1-x^2}$$

```
[Out] E^x*Sqrt[1 - x^2]
```

Rubi [A] time = 0.0910112, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$e^x\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]

[Out] E^x*Sqrt[1 - x^2]
```

Rubi in Sympy [A] time = 5.63924, size = 10, normalized size = 0.67

$$\sqrt{-x^2+1}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)

[Out] sqrt(-x**2 + 1)*exp(x)
```

Mathematica [A] time = 0.173838, size = 0, normalized size = 0.

$$\int \frac{e^x\left(1-x-x^2\right)}{\sqrt{1-x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]

[Out] Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2], x]
```

Maple [A] time = 0.007, size = 20, normalized size = 1.3

$$-e^{x}(-1+x)(1+x)\frac{1}{\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x)
```

```
[Out] -exp(x)*(-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.54409, size = 28, normalized size = 1.87

$$-\frac{\left(x^{2}-1\right)e^{x}}{\sqrt{x+1}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] -(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

Fricas [A] time = 0.210252, size = 16, normalized size = 1.07

$$\sqrt{-x^{2}+1}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 1)*e^x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{e^{x}}{\sqrt{-x^{2}+1}}\right)dx-\int\frac{xe^{x}}{\sqrt{-x^{2}+1}}\,dx-\int\frac{x^{2}e^{x}}{\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(-exp(x)/sqrt(-x**2 + 1), x) - Integral(x*exp(x)/sqrt(-x
**2 + 1), x) - Integral(x**2*exp(x)/sqrt(-x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(x^2 + x - 1\right)e^x}{\sqrt{-x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1), x)
```

3.541 $\int e^{-3x}\cos(2x)\,dx$

Optimal. Leaf size=27

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

```
[Out] (-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

Rubi [A] time = 0.0201784, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.1$

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[2*x]/E^(3*x),x]

[Out] (-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

Rubi in Sympy [A] time = 1.49845, size = 26, normalized size = 0.96

$$\frac{2e^{-3x}\sin(2x)}{13}-\frac{3e^{-3x}\cos(2x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(2*x)/exp(3*x),x)

[Out] 2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

Mathematica [A] time = 0.0178167, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{-3x}(2\sin(2x)-3\cos(2x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[2*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[2*x] + 2*Sin[2*x])/(13*E^(3*x))
```

Maple [A] time = 0.01, size = 22, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{-3\,x}\cos\left(2\,x\right)}{13}+\frac{2\,\mathrm{e}^{-3\,x}\sin\left(2\,x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(2*x)/exp(3*x),x)
```

```
[Out] -3/13*exp(-3*x)*cos(2*x)+2/13*exp(-3*x)*sin(2*x)
```

Maxima [A] time = 1.37483, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\,\cos\left(2\,x\right)-2\,\sin\left(2\,x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*e^(-3*x),x, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

Fricas [A] time = 0.210814, size = 28, normalized size = 1.04

$$-\frac{3}{13}\,\cos\left(2\,x\right)e^{\left(-3\,x\right)}+\frac{2}{13}\,e^{\left(-3\,x\right)}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*e^(-3*x),x, algorithm="fricas")
```

```
[Out] -3/13*cos(2*x)*e^(-3*x) + 2/13*e^(-3*x)*sin(2*x)
```

Sympy [A] time = 0.810375, size = 26, normalized size = 0.96

$$\frac{2e^{-3x}\sin\left(2x\right)}{13}-\frac{3e^{-3x}\cos\left(2x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)/exp(3*x),x)

[Out] 2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

GIAC/XCAS [A] time = 0.198556, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\cos\left(2x\right) - 2\sin\left(2x\right)\right)e^{\left(-3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*e^(-3*x),x, algorithm="giac")

[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$

Optimal. Leaf size=35

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

```
[Out] (-30*Cos[x/2])/(13*(E^x)^(1/3)) + (6*Sin[x/2])/(13*(E^x)^(1/3))
```

Rubi [A] time = 0.171011, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

```
[Out] (-30*Cos[x/2])/(13*(E^x)^(1/3)) + (6*Sin[x/2])/(13*(E^x)^(1/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{\frac{x}{3}}\int\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)e^{-\frac{x}{3}}\,dx}{\sqrt[3]{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
```

```
[Out] exp(x/3)*Integral((sin(x/2) + cos(x/2))*exp(-x/3), x)/exp(x)**(1/
3)
```

Mathematica [A] time = 0.0222583, size = 26, normalized size = 0.74

$$\frac{6\left(\sin\left(\frac{x}{2}\right)-5\cos\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

```
[Out] (6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))
```

Maple [C] time = 0.079, size = 32, normalized size = 0.9

$$\left(-\frac{15}{169}-\frac{3\,i}{169}\right)\left(13\,\mathrm{e}^{i/2x}+12\,\mathrm{e}^{-i/2x}-5\,i\mathrm{e}^{-\frac{i}{2}x}\right)\frac{1}{\sqrt[3]{\mathrm{e}^{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x)
```

```
[Out] (-15/169-3/169*I)/exp(x)^(1/3)*(13*exp(1/2*I*x)+12*exp(-1/2*I*x)-
5*I*exp(-1/2*I*x))
```

Maxima [A] time = 1.44572, size = 53, normalized size = 1.51

$$-\frac{6}{13}\left(3\,\cos\left(\frac{1}{2}\,x\right)+2\,\sin\left(\frac{1}{2}\,x\right)\right)e^{\left(-\frac{1}{3}\,x\right)}-\frac{6}{13}\left(2\,\cos\left(\frac{1}{2}\,x\right)-3\,\sin\left(\frac{1}{2}\,x\right)\right)e^{\left(-\frac{1}{3}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x) + sin(1/2*x))*e^(-1/3*x),x, algorithm="maxima")
```

```
[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*
x) - 3*sin(1/2*x))*e^(-1/3*x)
```

Fricas [A] time = 0.230245, size = 28, normalized size = 0.8

$$-\frac{30}{13}\,\cos\left(\frac{1}{2}\,x\right)e^{\left(-\frac{1}{3}\,x\right)}+\frac{6}{13}\,e^{\left(-\frac{1}{3}\,x\right)}\sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x) + sin(1/2*x))*e^(-1/3*x),x, algorithm="fricas")
```

```
[Out] -30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)
```

Sympy [A] time = 1.4763, size = 29, normalized size = 0.83

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^{x}}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
```

```
[Out] 6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))
```

GIAC/XCAS [A] time = 0.231833, size = 53, normalized size = 1.51

$$-\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}-\frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(1/2*x) + sin(1/2*x))*e^(-1/3*x),x, algorithm="giac")
```

```
[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*
x) - 3*sin(1/2*x))*e^(-1/3*x)
```

3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$

Optimal. Leaf size=57

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

```
[Out] (-4*Cos[(3*x)/2]*Log[3])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2)) + (8*
Sin[(3*x)/2])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2))
```

Rubi [A] time = 0.0495452, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*Cos[(3*x)/2]*Log[3])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2)) + (8*
Sin[(3*x)/2])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2))
```

Rubi in Sympy [A] time = 2.67675, size = 63, normalized size = 1.11

$$\frac{3\sin\left(\frac{3x}{2}\right)}{2\left(\frac{9\log(3)^2}{16}+\frac{9}{4}\right)\sqrt[4]{3^{3x}}}-\frac{3\log(3)\cos\left(\frac{3x}{2}\right)}{4\left(\frac{9\log(3)^2}{16}+\frac{9}{4}\right)\sqrt[4]{3^{3x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
```

```
[Out] 3*sin(3*x/2)/(2*(9*log(3)**2/16 + 9/4)*(3**(3*x))**(1/4)) - 3*log
(3)*cos(3*x/2)/(4*(9*log(3)**2/16 + 9/4)*(3**(3*x))**(1/4))
```

Mathematica [A] time = 0.0643294, size = 37, normalized size = 0.65

$$-\frac{4\left(\log(3)\cos\left(\frac{3x}{2}\right)-2\sin\left(\frac{3x}{2}\right)\right)}{3\sqrt[4]{27^x}\left(4+\log^2(3)\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 +
Log[3]^2))
```

Maple [C] time = 0.072, size = 55, normalized size = 1.

$$-\frac{-4\,i\mathrm{e}^{-\frac{3\,i}{2}x}+2\,\mathrm{e}^{-3/2\,ix}\ln(3)+4\,i\mathrm{e}^{\frac{3\,i}{2}x}+2\,\ln(3)\,\mathrm{e}^{3/2\,ix}}{(6\,i+3\,\ln(3))\,(-2\,i+\ln(3))}\frac{1}{\sqrt[4]{27^{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3/2*x)/(3^(3*x))^(1/4),x)
```

```
[Out] -2/3/(2*I+ln(3))/(-2*I+ln(3))/(27^x)^(1/4)*(-2*I*exp(-3/2*I*x)+ex
p(-3/2*I*x)*ln(3)+2*I*exp(3/2*I*x)+ln(3)*exp(3/2*I*x))
```

Maxima [A] time = 1.57372, size = 39, normalized size = 0.68

$$-\frac{4\left(\cos\left(\frac{3}{2}\,x\right)\,\log(3)-2\,\sin\left(\frac{3}{2}\,x\right)\right)3^{-\frac{3}{4}\,x}}{3\left(\log(3)^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")
```

```
[Out] -4/3*(cos(3/2*x)*log(3) - 2*sin(3/2*x))*3^(-3/4*x)/(log(3)^2 + 4)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [A] time = 4.74624, size = 70, normalized size = 1.23

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2+12\sqrt[4]{3^{3x}}}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2+12\sqrt[4]{3^{3x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
```

```
[Out] 8*sin(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4
)) - 4*log(3)*cos(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**
(3*x))**(1/4))
```

GIAC/XCAS [A] time = 0.215425, size = 51, normalized size = 0.89

$$-\frac{4}{3}\left(\frac{\cos\left(\frac{3}{2}x\right)\ln(3)}{\ln(3)^2+4}-\frac{2\sin\left(\frac{3}{2}x\right)}{\ln(3)^2+4}\right)e^{\left(-\frac{3}{4}x\ln(3)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="giac")
```

```
[Out] -4/3*(cos(3/2*x)*ln(3)/(ln(3)^2 + 4) - 2*sin(3/2*x)/(ln(3)^2 + 4)
)*e^(-3/4*x*ln(3))
```

3.544 $\int e^{mx}\cos^2(x)\,dx$

Optimal. Leaf size=54

$$\frac{2e^{mx}}{m\left(m^2+4\right)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

```
[Out] (2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E
^(m*x)*Cos[x]*Sin[x])/(4 + m^2)
```

Rubi [A] time = 0.0453896, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2e^{mx}}{m\left(m^2+4\right)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(m*x)*Cos[x]^2,x]
```

```
[Out] (2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E
^(m*x)*Cos[x]*Sin[x])/(4 + m^2)
```

Rubi in Sympy [A] time = 3.27636, size = 48, normalized size = 0.89

$$\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}+\frac{2e^{mx}}{m\left(m^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(m*x)*cos(x)**2,x)
```

```
[Out] m*exp(m*x)*cos(x)**2/(m**2 + 4) + 2*exp(m*x)*sin(x)*cos(x)/(m**2
+ 4) + 2*exp(m*x)/(m*(m**2 + 4))
```

Mathematica [A] time = 0.0367763, size = 39, normalized size = 0.72

$$\frac{e^{mx}\left(m^2\cos(2x)+m^2+2m\sin(2x)+4\right)}{2m\left(m^2+4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(m*x)*Cos[x]^2,x]
```

```
[Out] (E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))
```

Maple [A] time = 0.019, size = 45, normalized size = 0.8

$$\frac{me^{mx}\cos(2x)}{2m^2+8}+\frac{e^{mx}\sin(2x)}{m^2+4}+\frac{e^{mx}}{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(m*x)*cos(x)^2,x)
```

```
[Out] 1/2*m/(m^2+4)*exp(m*x)*cos(2*x)+1/(m^2+4)*exp(m*x)*sin(2*x)+1/2*e
xp(m*x)/m
```

Maxima [A] time = 1.37879, size = 61, normalized size = 1.13

$$\frac{m^2\cos(2x)e^{(mx)}+2\,me^{(mx)}\sin(2x)+\left(m^2+4\right)e^{(mx)}}{2\left(m^3+4m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*e^(m*x),x, algorithm="maxima")
```

```
[Out] 1/2*(m^2*cos(2*x)*e^(m*x) + 2*m*e^(m*x)*sin(2*x) + (m^2 + 4)*e^(m
*x))/(m^3 + 4*m)
```

Fricas [A] time = 0.216397, size = 50, normalized size = 0.93

$$\frac{2\,m\cos(x)e^{(mx)}\sin(x)+\left(m^2\cos(x)^2+2\right)e^{(mx)}}{m^3+4m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*e^(m*x),x, algorithm="fricas")
```

```
[Out] (2*m*cos(x)*e^(m*x)*sin(x) + (m^2*cos(x)^2 + 2)*e^(m*x))/(m^3 + 4
*m)
```

Sympy [A] time = 5.79584, size = 269, normalized size = 4.98

$$\begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = 0 \\ -\frac{xe^{-2ix}\sin^2(x)}{4} + \frac{ixe^{-2ix}\sin(x)\cos(x)}{2} + \frac{xe^{-2ix}\cos^2(x)}{4} + \frac{ie^{-2ix}\sin^2(x)}{2} + \frac{3e^{-2ix}\sin(x)\cos(x)}{4} & \text{for } m = -2i \\ -\frac{xe^{2ix}\sin^2(x)}{4} - \frac{ixe^{2ix}\sin(x)\cos(x)}{2} + \frac{xe^{2ix}\cos^2(x)}{4} - \frac{ie^{2ix}\sin^2(x)}{8} - \frac{3ie^{2ix}\cos^2(x)}{8} & \text{for } m = 2i \\ \frac{m^2e^{mx}\cos^2(x)}{m^3+4m} + \frac{2me^{mx}\sin(x)\cos(x)}{m^3+4m} + \frac{2e^{mx}\sin^2(x)}{m^3+4m} + \frac{2e^{mx}\cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(m*x)*cos(x)**2,x)
```

```
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m,
 0)), (-x*exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)
/2 + x*exp(-2*I*x)*cos(x)**2/4 + I*exp(-2*I*x)*sin(x)**2/2 + 3*ex
p(-2*I*x)*sin(x)*cos(x)/4, Eq(m, -2*I)), (-x*exp(2*I*x)*sin(x)**2
/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 + x*exp(2*I*x)*cos(x)**2/4 -
I*exp(2*I*x)*sin(x)**2/8 - 3*I*exp(2*I*x)*cos(x)**2/8, Eq(m, 2*I)
), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*sin(x)*co
s(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x
)*cos(x)**2/(m**3 + 4*m), True))
```

GIAC/XCAS [A] time = 0.229083, size = 58, normalized size = 1.07

$$\frac{1}{2}\left(\frac{m\cos(2x)}{m^2+4} + \frac{2\sin(2x)}{m^2+4}\right)e^{(mx)} + \frac{e^{(mx)}}{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*e^(m*x),x, algorithm="giac")
```

```
[Out] 1/2*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4))*e^(m*x) + 1/2*e
^(m*x)/m
```

3.545 $\int e^{mx}\sin^3(x)\,dx$

Optimal. Leaf size=82

$$\frac{me^{mx}\sin^3(x)}{m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}$$

```
[Out] (-6*E^(m*x)*Cos[x])/(9 + 10*m^2 + m^4) + (6*E^(m*x)*m*Sin[x])/(9
+ 10*m^2 + m^4) - (3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x
)*m*Sin[x]^3)/(9 + m^2)
```

Rubi [A] time = 0.0688139, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{me^{mx}\sin^3(x)}{m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(m*x)*Sin[x]^3,x]
```

```
[Out] (-6*E^(m*x)*Cos[x])/(9 + 10*m^2 + m^4) + (6*E^(m*x)*m*Sin[x])/(9
+ 10*m^2 + m^4) - (3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x
)*m*Sin[x]^3)/(9 + m^2)
```

Rubi in Sympy [A] time = 5.11385, size = 78, normalized size = 0.95

$$\frac{me^{mx}\sin^3(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{(m^2+1)(m^2+9)}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}-\frac{6e^{mx}\cos(x)}{(m^2+1)(m^2+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(m*x)*sin(x)**3,x)
```

```
[Out] m*exp(m*x)*sin(x)**3/(m**2 + 9) + 6*m*exp(m*x)*sin(x)/((m**2 + 1)
*(m**2 + 9)) - 3*exp(m*x)*sin(x)**2*cos(x)/(m**2 + 9) - 6*exp(m*x
)*cos(x)/((m**2 + 1)*(m**2 + 9))
```

Mathematica [A] time = 0.291401, size = 64, normalized size = 0.78

$$\frac{e^{mx}\left(-3\left(m^2+9\right)\cos(x)+3\left(m^2+1\right)\cos(3x)-2m\sin(x)\left(\left(m^2+1\right)\cos(2x)-m^2-13\right)\right)}{4\left(m^4+10m^2+9\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(m*x)*Sin[x]^3,x]
```

```
[Out] (E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 -
 m^2 + (1 + m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

Maple [A] time = 0.016, size = 68, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{mx}\cos(x)}{4\,m^2+4}+\frac{3\,m\mathrm{e}^{mx}\sin(x)}{4\,m^2+4}+\frac{3\,\mathrm{e}^{mx}\cos(3\,x)}{4\,m^2+36}-\frac{m\mathrm{e}^{mx}\sin(3\,x)}{4\,m^2+36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(m*x)*sin(x)^3,x)
```

```
[Out] -3/4/(m^2+1)*exp(m*x)*cos(x)+3/4*m/(m^2+1)*exp(m*x)*sin(x)+3/4/(m
^2+9)*exp(m*x)*cos(3*x)-1/4*m/(m^2+9)*exp(m*x)*sin(3*x)
```

Maxima [A] time = 1.41198, size = 99, normalized size = 1.21

$$\frac{3\left(m^2+1\right)\cos(3\,x)\,e^{(mx)}-3\left(m^2+9\right)\cos(x)\,e^{(mx)}-\left(m^3+m\right)e^{(mx)}\sin(3\,x)+3\left(m^3+9\,m\right)e^{(mx)}\sin(x)}{4\,(m^4+10\,m^2+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(m^2 + 1)*cos(3*x)*e^(m*x) - 3*(m^2 + 9)*cos(x)*e^(m*x) -
(m^3 + m)*e^(m*x)*sin(3*x) + 3*(m^3 + 9*m)*e^(m*x)*sin(x))/(m^4 +
 10*m^2 + 9)
```

Fricas [A] time = 0.215859, size = 88, normalized size = 1.07

$$\frac{\left(m^3-\left(m^3+m\right)\cos(x)^2+7\,m\right)e^{(mx)}\sin(x)+3\left(\left(m^2+1\right)\cos(x)^3-\left(m^2+3\right)\cos(x)\right)e^{(mx)}}{m^4+10\,m^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)*sin(x)^3,x, algorithm="fricas")
```

```
[Out] ((m^3 - (m^3 + m)*cos(x)^2 + 7*m)*e^(m*x)*sin(x) + 3*((m^2 + 1)*c
os(x)^3 - (m^2 + 3)*cos(x))*e^(m*x))/(m^4 + 10*m^2 + 9)
```

Sympy [A] time = 25.7219, size = 651, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(m*x)*sin(x)**3,x)
```

```
[Out] Piecewise((x*exp(-3*I*x)*sin(x)**3/8 - 3*I*x*exp(-3*I*x)*sin(x)**
2*cos(x)/8 - 3*x*exp(-3*I*x)*sin(x)*cos(x)**2/8 + I*x*exp(-3*I*x)
*cos(x)**3/8 + 5*I*exp(-3*I*x)*sin(x)**3/12 + 3*exp(-3*I*x)*sin(x
)**2*cos(x)/8 - I*exp(-3*I*x)*sin(x)*cos(x)**2/8, Eq(m, -3*I)), (
3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I*x)*sin(x)**2*cos(x)/8 +
3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)*cos(x)**3/8 +
I*exp(-I*x)*sin(x)**3/4 - 3*exp(-I*x)*sin(x)**2*cos(x)/8 + 3*I*ex
p(-I*x)*sin(x)*cos(x)**2/8, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8
 + 3*I*x*exp(I*x)*sin(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)
**2/8 + 3*I*x*exp(I*x)*cos(x)**3/8 - I*exp(I*x)*sin(x)**3/4 - 3*e
xp(I*x)*sin(x)**2*cos(x)/8 - 3*I*exp(I*x)*sin(x)*cos(x)**2/8, Eq(
I, m)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*exp(3*I*x)*sin(x)**2*co
s(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(3*I*x)*cos(x
)**3/8 - 5*I*exp(3*I*x)*sin(x)**3/12 + 3*exp(3*I*x)*sin(x)**2*cos
(x)/8 + I*exp(3*I*x)*sin(x)*cos(x)**2/8, Eq(m, 3*I)), (m**3*exp(m
*x)*sin(x)**3/(m**4 + 10*m**2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*co
s(x)/(m**4 + 10*m**2 + 9) + 7*m*exp(m*x)*sin(x)**3/(m**4 + 10*m**
2 + 9) + 6*m*exp(m*x)*sin(x)*cos(x)**2/(m**4 + 10*m**2 + 9) - 9*e
xp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) - 6*exp(m*x)*cos(x)
**3/(m**4 + 10*m**2 + 9), True))
```

GIAC/XCAS [A] time = 0.216811, size = 85, normalized size = 1.04

$$-\frac{1}{4}\left(\frac{m\sin\left(3\,x\right)}{m^2+9}-\frac{3\,\cos\left(3\,x\right)}{m^2+9}\right)e^{(mx)}+\frac{3}{4}\left(\frac{m\sin\left(x\right)}{m^2+1}-\frac{\cos\left(x\right)}{m^2+1}\right)e^{(mx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9))*e^(m*x) + 3/4*
(m*sin(x)/(m^2 + 1) - cos(x)/(m^2 + 1))*e^(m*x)
```

3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$

Optimal. Leaf size=79

$$\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

```
[Out] (-48*Cos[x/3])/(65*Sqrt[E^x]) - (2*Cos[x/3]^3)/(5*Sqrt[E^x]) + (3
2*Sin[x/3])/(65*Sqrt[E^x]) + (4*Cos[x/3]^2*Sin[x/3])/(5*Sqrt[E^x]
)
```

Rubi [A] time = 0.0754341, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x/3]^3/Sqrt[E^x],x]
```

```
[Out] (-48*Cos[x/3])/(65*Sqrt[E^x]) - (2*Cos[x/3]^3)/(5*Sqrt[E^x]) + (3
2*Sin[x/3])/(65*Sqrt[E^x]) + (4*Cos[x/3]^2*Sin[x/3])/(5*Sqrt[E^x]
)
```

Rubi in Sympy [A] time = 4.5828, size = 68, normalized size = 0.86

$$\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
```

```
[Out] 4*sin(x/3)*cos(x/3)**2/(5*sqrt(exp(x))) + 32*sin(x/3)/(65*sqrt(ex
p(x))) - 2*cos(x/3)**3/(5*sqrt(exp(x))) - 48*cos(x/3)/(65*sqrt(ex
p(x)))
```

Mathematica [A] time = 0.0288074, size = 36, normalized size = 0.46

$$\frac{90\sin\left(\frac{x}{3}\right)+26\sin(x)-135\cos\left(\frac{x}{3}\right)-13\cos(x)}{130\sqrt{e^x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x/3]^3/Sqrt[E^x],x]
```

```
[Out] (-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E
^x])
```

Maple [C] time = 0.147, size = 62, normalized size = 0.8

$$\left(-\frac{1}{1300}-\frac{i}{650}\right)\left(65\,\mathrm{e}^{ix}-180\,i\mathrm{e}^{\frac{i}{3}x}+315\,\mathrm{e}^{i/3x}-360\,i\mathrm{e}^{-\frac{i}{3}x}-45\,\mathrm{e}^{-i/3x}-39\,\mathrm{e}^{-ix}-52\,i\mathrm{e}^{-ix}\right)\frac{1}{\sqrt{\mathrm{e}^{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(1/3*x)^3/exp(x)^(1/2),x)
```

```
[Out] (-1/1300-1/650*I)/exp(x)^(1/2)*(65*exp(I*x)-180*I*exp(1/3*I*x)+31
5*exp(1/3*I*x)-360*I*exp(-1/3*I*x)-45*exp(-1/3*I*x)-39*exp(-I*x)-
52*I*exp(-I*x))
```

Maxima [A] time = 1.38784, size = 36, normalized size = 0.46

$$-\frac{1}{130}\left(135\,\cos\left(\frac{1}{3}\,x\right)+13\,\cos\left(x\right)-90\,\sin\left(\frac{1}{3}\,x\right)-26\,\sin\left(x\right)\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/3*x)^3*e^(-1/2*x),x, algorithm="maxima")
```

```
[Out] -1/130*(135*cos(1/3*x) + 13*cos(x) - 90*sin(1/3*x) - 26*sin(x))*e
^(-1/2*x)
```

Fricas [A] time = 0.249542, size = 57, normalized size = 0.72

$$\frac{4}{65}\left(13\,\cos\left(\frac{1}{3}\,x\right)^{2}+8\right)e^{\left(-\frac{1}{2}\,x\right)}\sin\left(\frac{1}{3}\,x\right)-\frac{2}{65}\left(13\,\cos\left(\frac{1}{3}\,x\right)^{3}+24\,\cos\left(\frac{1}{3}\,x\right)\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/3*x)^3*e^(-1/2*x),x, algorithm="fricas")
```

```
[Out] 4/65*(13*cos(1/3*x)^2 + 8)*e^(-1/2*x)*sin(1/3*x) - 2/65*(13*cos(1
/3*x)^3 + 24*cos(1/3*x))*e^(-1/2*x)
```

Sympy [A] time = 4.23361, size = 76, normalized size = 0.96

$$\frac{32\sin^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{48\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{84\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{74\cos^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
```

```
[Out] 32*sin(x/3)**3/(65*sqrt(exp(x))) - 48*sin(x/3)**2*cos(x/3)/(65*sq
rt(exp(x))) + 84*sin(x/3)*cos(x/3)**2/(65*sqrt(exp(x))) - 74*cos(
x/3)**3/(65*sqrt(exp(x)))
```

GIAC/XCAS [A] time = 0.233189, size = 45, normalized size = 0.57

$$-\frac{9}{26}\left(3\cos\left(\frac{1}{3}x\right)-2\sin\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}-\frac{1}{10}\left(\cos\left(x\right)-2\sin\left(x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(1/3*x)^3*e^(-1/2*x),x, algorithm="giac")
```

```
[Out] -9/26*(3*cos(1/3*x) - 2*sin(1/3*x))*e^(-1/2*x) - 1/10*(cos(x) - 2
*sin(x))*e^(-1/2*x)
```

3.547 $\int e^{2x} \cos^2(x) \sin^2(x) \, dx$

Optimal. Leaf size=36

$$\frac{e^{2x}}{16} - \frac{1}{40} e^{2x} \sin(4x) - \frac{1}{80} e^{2x} \cos(4x)$$

```
[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

Rubi [A] time = 0.0705463, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{e^{2x}}{16} - \frac{1}{40} e^{2x} \sin(4x) - \frac{1}{80} e^{2x} \cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

Rubi in Sympy [A] time = 5.31334, size = 29, normalized size = 0.81

$$-\frac{e^{2x} \sin(4x)}{40} - \frac{e^{2x} \cos(4x)}{80} + \frac{e^{2x}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
```

```
[Out] -exp(2*x)*sin(4*x)/40 - exp(2*x)*cos(4*x)/80 + exp(2*x)/16
```

Mathematica [A] time = 0.0298775, size = 21, normalized size = 0.58

$$-\frac{1}{80} e^{2x} (2 \sin(4x) + \cos(4x) - 5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] -(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))/80
```

Maple [A] time = 0.008, size = 28, normalized size = 0.8

$$-\frac{e^{2x}\cos(4x)}{80}-\frac{e^{2x}\sin(4x)}{40}+\frac{(e^{x})^{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)*cos(x)^2*sin(x)^2,x)
```

```
[Out] -1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)+1/16*exp(x)^2
```

Maxima [A] time = 1.34544, size = 36, normalized size = 1.

$$-\frac{1}{80}\cos(4x)e^{(2x)}-\frac{1}{40}e^{(2x)}\sin(4x)+\frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*e^(2*x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/80*cos(4*x)*e^(2*x) - 1/40*e^(2*x)*sin(4*x) + 1/16*e^(2*x)
```

Fricas [A] time = 0.218727, size = 54, normalized size = 1.5

$$-\frac{1}{10}\left(2\cos(x)^{3}-\cos(x)\right)e^{(2x)}\sin(x)-\frac{1}{20}\left(2\cos(x)^{4}-2\cos(x)^{2}-1\right)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*e^(2*x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(2*cos(x)^3 - cos(x))*e^(2*x)*sin(x) - 1/20*(2*cos(x)^4 - 2
*cos(x)^2 - 1)*e^(2*x)
```

Sympy [A] time = 8.3612, size = 70, normalized size = 1.94

$$\frac{e^{2x}\sin^{4}(x)}{20}+\frac{e^{2x}\sin^{3}(x)\cos(x)}{10}+\frac{e^{2x}\sin^{2}(x)\cos^{2}(x)}{5}-\frac{e^{2x}\sin(x)\cos^{3}(x)}{10}+\frac{e^{2x}\cos^{4}(x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
```

```
[Out] exp(2*x)*sin(x)**4/20 + exp(2*x)*sin(x)**3*cos(x)/10 + exp(2*x)*s
in(x)**2*cos(x)**2/5 - exp(2*x)*sin(x)*cos(x)**3/10 + exp(2*x)*co
s(x)**4/20
```

GIAC/XCAS [A] time = 0.209015, size = 32, normalized size = 0.89

$$-\frac{1}{80}\left(\cos\left(4x\right) + 2\sin\left(4x\right)\right)e^{(2x)} + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*e^(2*x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/80*(cos(4*x) + 2*sin(4*x))*e^(2*x) + 1/16*e^(2*x)
```

3.548 $\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx$

Optimal. Leaf size=36

$$\frac{e^{3x}}{24} - \frac{1}{60}e^{3x}\sin(6x) - \frac{1}{120}e^{3x}\cos(6x)$$

```
[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

Rubi [A] time = 0.0706062, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{e^{3x}}{24} - \frac{1}{60}e^{3x}\sin(6x) - \frac{1}{120}e^{3x}\cos(6x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

Rubi in Sympy [A] time = 5.71341, size = 29, normalized size = 0.81

$$-\frac{e^{3x}\sin(6x)}{60} - \frac{e^{3x}\cos(6x)}{120} + \frac{e^{3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

```
[Out] -exp(3*x)*sin(6*x)/60 - exp(3*x)*cos(6*x)/120 + exp(3*x)/24
```

Mathematica [A] time = 0.0247324, size = 21, normalized size = 0.58

$$-\frac{1}{120}e^{3x}(2\sin(6x) + \cos(6x) - 5)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] -(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))/120
```

Maple [A] time = 0.019, size = 28, normalized size = 0.8

$$\frac{e^{3x}}{24}-\frac{e^{3x}\cos(6x)}{120}-\frac{e^{3x}\sin(6x)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x)
```

```
[Out] 1/24*exp(3*x)-1/120*exp(3*x)*cos(6*x)-1/60*exp(3*x)*sin(6*x)
```

Maxima [A] time = 1.37877, size = 36, normalized size = 1.

$$-\frac{1}{120}\cos(6x)e^{(3x)}-\frac{1}{60}e^{(3x)}\sin(6x)+\frac{1}{24}e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)^2*e^(3*x)*sin(3/2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)
```

Fricas [A] time = 0.25263, size = 68, normalized size = 1.89

$$-\frac{1}{15}\left(2\cos\left(\frac{3}{2}x\right)^{3}-\cos\left(\frac{3}{2}x\right)\right)e^{(3x)}\sin\left(\frac{3}{2}x\right)-\frac{1}{30}\left(2\cos\left(\frac{3}{2}x\right)^{4}-2\cos\left(\frac{3}{2}x\right)^{2}-1\right)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)^2*e^(3*x)*sin(3/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*
cos(3/2*x)^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)
```

Sympy [A] time = 8.30839, size = 99, normalized size = 2.75

$$\frac{e^{3x}\sin^{4}\left(\frac{3x}{2}\right)}{30}+\frac{e^{3x}\sin^{3}\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{15}+\frac{2e^{3x}\sin^{2}\left(\frac{3x}{2}\right)\cos^{2}\left(\frac{3x}{2}\right)}{15}-\frac{e^{3x}\sin\left(\frac{3x}{2}\right)\cos^{3}\left(\frac{3x}{2}\right)}{15}+\frac{e^{3x}\cos^{4}\left(\frac{3x}{2}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

```
[Out] exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15
+ 2*exp(3*x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)
*cos(3*x/2)**3/15 + exp(3*x)*cos(3*x/2)**4/30
```

GIAC/XCAS [A] time = 0.203925, size = 32, normalized size = 0.89

$$-\frac{1}{120}\left(\cos\left(6\,x\right) + 2\,\sin\left(6\,x\right)\right)e^{\left(3\,x\right)} + \frac{1}{24}\,e^{\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3/2*x)^2*e^(3*x)*sin(3/2*x)^2,x, algorithm="giac")
```

```
[Out] -1/120*(cos(6*x) + 2*sin(6*x))*e^(3*x) + 1/24*e^(3*x)
```

3.549 $\int e^{mx}\tan^2(x)\,dx$

Optimal. Leaf size=58

$$-\frac{e^{mx}}{m}+\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};-e^{2ix}\right)}{m+2i}$$

```
[Out] -(E^(m*x)/m) + (4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*
m, 2 - (I/2)*m, -E^((2*I)*x)])/(2*I + m)
```

Rubi [A] time = 0.130186, antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{4e^{mx}\,{}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)}{m}-\frac{4e^{mx}\,{}_2F_1\left(2,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)}{m}-\frac{e^{mx}}{m}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(m*x)*Tan[x]^2,x]
```

```
[Out] -(E^(m*x)/m) + (4*E^(m*x)*Hypergeometric2F1[1, (-I/2)*m, 1 - (I/2
)*m, -E^((2*I)*x)])/m - (4*E^(m*x)*Hypergeometric2F1[2, (-I/2)*m,
 1 - (I/2)*m, -E^((2*I)*x)])/m
```

Rubi in Sympy [A] time = 10.2565, size = 63, normalized size = 1.09

$$\frac{4e^{mx}{}_2F_1\left(\begin{matrix}1,-\frac{im}{2}\\-\frac{im}{2}+1\end{matrix}\middle|-e^{2ix}\right)}{m}-\frac{4e^{mx}{}_2F_1\left(\begin{matrix}2,-\frac{im}{2}\\-\frac{im}{2}+1\end{matrix}\middle|-e^{2ix}\right)}{m}-\frac{e^{mx}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(m*x)*tan(x)**2,x)
```

```
[Out] 4*exp(m*x)*hyper((1, -I*m/2), (-I*m/2 + 1,), -exp(2*I*x))/m - 4*e
xp(m*x)*hyper((2, -I*m/2), (-I*m/2 + 1,), -exp(2*I*x))/m - exp(m*
x)/m
```

Mathematica [A] time = 0.354975, size = 97, normalized size = 1.67

$$\frac{e^{mx}\left(\frac{im^2e^{2ix}\,{}_2F_1\left(1,1-\frac{im}{2};2-\frac{im}{2};-e^{2ix}\right)}{m+2i}-im\,{}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)+m\tan(x)-1\right)}{m}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(m*x)*Tan[x]^2,x]
```

```
[Out] (E^(m*x)*(-1 + (I*E^((2*I)*x)*m^2*Hypergeometric2F1[1, 1 - (I/2)*
m, 2 - (I/2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[
1, (-I/2)*m, 1 - (I/2)*m, -E^((2*I)*x)] + m*Tan[x]))/m
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int e^{mx} (\tan (x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(m*x)*tan(x)^2,x)
```

```
[Out] int(exp(m*x)*tan(x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(mx)} \tan (x)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(m*x)*tan(x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{mx}\tan^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(m*x)*tan(x)**2,x)

[Out] Integral(exp(m*x)*tan(x)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(mx)}\tan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)*tan(x)^2,x, algorithm="giac")

[Out] integrate(e^(m*x)*tan(x)^2, x)
```

3.550 $\int e^{mx} \csc^2(x)\, dx$

Optimal. Leaf size=45

$$-\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)}{m+2i}$$

```
[Out] (-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m
, E^((2*I)*x)])/(2*I + m)
```

Rubi [A] time = 0.0337854, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)}{m+2i}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(m*x)*Csc[x]^2,x]
```

```
[Out] (-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m
, E^((2*I)*x)])/(2*I + m)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{\sin^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(m*x)/sin(x)**2,x)
```

```
[Out] Integral(exp(m*x)/sin(x)**2, x)
```

Mathematica [A] time = 0.260766, size = 90, normalized size = 2.

$$\frac{e^{mx}\left(me^{2ix}\,{}_2F_1\left(1,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)+(m+2i)\left({}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};e^{2ix}\right)-i\cot(x)\right)\right)}{-2+im}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(m*x)*Csc[x]^2,x]
```

```
[Out] (E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/
2)*m, E^((2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1
, (-I/2)*m, 1 - (I/2)*m, E^((2*I)*x)])))/(-2 + I*m)
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{(\sin(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(m*x)/sin(x)^2,x)
```

```
[Out] int(exp(m*x)/sin(x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/sin(x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{e^{(mx)}}{\cos(x)^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/sin(x)^2,x, algorithm="fricas")
```

```
[Out] integral(-e^(m*x)/(cos(x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{\sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(m*x)/sin(x)**2,x)

[Out] Integral(exp(m*x)/sin(x)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/sin(x)^2,x, algorithm="giac")

[Out] integrate(e^(m*x)/sin(x)^2, x)
```

3.551 $\int e^{mx}\sec^3(x)\,dx$

Optimal. Leaf size=51

$$\frac{8e^{(m+3i)x}\,{}_2F_1\left(3,\frac{1}{2}(3-im);\frac{1}{2}(5-im);-e^{2ix}\right)}{m+3i}$$

```
[Out] (8*E^((3*I + m)*x)*Hypergeometric2F1[3, (3 - I*m)/2, (5 - I*m)/2,
 -E^((2*I)*x)])/(3*I + m)
```

Rubi [A] time = 0.0661715, antiderivative size = 77, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$(-m+i)\left(-e^{(m+i)x}\right)\,{}_2F_1\left(1,\frac{1}{2}(1-im);\frac{1}{2}(3-im);-e^{2ix}\right)-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(m*x)*Sec[x]^3,x]
```

```
[Out] -(E^((I + m)*x)*(I - m)*Hypergeometric2F1[1, (1 - I*m)/2, (3 - I*
m)/2, -E^((2*I)*x)]) - (E^(m*x)*m*Sec[x])/2 + (E^(m*x)*Sec[x]*Tan
[x])/2
```

Rubi in Sympy [A] time = 5.70676, size = 68, normalized size = 1.33

$$-\frac{me^{mx}}{2\cos(x)}+\frac{e^{mx}\sin(x)}{2\cos^2(x)}+\frac{\left(m^2+1\right)e^{ix}e^{mx}\,{}_2F_1\left(\begin{matrix}1,-\frac{im}{2}+\frac{1}{2}\\-\frac{im}{2}+\frac{3}{2}\end{matrix}\middle|-e^{2ix}\right)}{m+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(m*x)/cos(x)**3,x)
```

```
[Out] -m*exp(m*x)/(2*cos(x)) + exp(m*x)*sin(x)/(2*cos(x)**2) + (m**2 +
1)*exp(I*x)*exp(m*x)*hyper((1, -I*m/2 + 1/2), (-I*m/2 + 3/2,), -e
xp(2*I*x))/(m + I)
```

Mathematica [A] time = 0.435017, size = 66, normalized size = 1.29

$$\frac{1}{2}e^{mx}\left(\sec(x)(\tan(x)-m)+2(m-i)e^{ix}\,{}_2F_1\left(1,\frac{1}{2}-\frac{im}{2};\frac{3}{2}-\frac{im}{2};-e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(m*x)*Sec[x]^3,x]
```

```
[Out] (E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m,
3/2 - (I/2)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x])))/2
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{\left(\cos \left(x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(m*x)/cos(x)^3,x)
```

```
[Out] int(exp(m*x)/cos(x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{\left(mx\right)}}{\cos\left(x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(m*x)/cos(x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(m*x)/cos(x)**3,x)
```

```
[Out] Integral(exp(m*x)/cos(x)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(m*x)/cos(x)^3, x)
```

3.552 $\int \frac{e^x}{1+\cos(x)} \, dx$

Optimal. Leaf size=28

$$(1-i)e^{(1+i)x} \, {}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)
]
```

Rubi [A] time = 0.0524635, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$(1-i)e^{(1+i)x} \, {}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 + Cos[x]),x]
```

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^x}{\cos^2\left(\frac{x}{2}\right)} \, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1+cos(x)),x)
```

```
[Out] Integral(exp(x)/cos(x/2)**2, x)/2
```

Mathematica [B] time = 0.0709591, size = 89, normalized size = 3.18

$$-\frac{(1+i)e^x \cos\left(\frac{x}{2}\right)\left((1+i)\, {}_2F_1\left(-i,1;1-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-e^{ix}\, {}_2F_1\left(1,1-i;2-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-(1-i)\sin\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 + Cos[x]),x]
```

```
[Out] ((-1 - I)*E^x*Cos[x/2]*((1 + I)*Cos[x/2]*Hypergeometric2F1[-I, 1,
 1 - I, -E^(I*x)] - E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I,
2 - I, -E^(I*x)] - (1 - I)*Sin[x/2]))/(1 + Cos[x])
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{e^x}{1+\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1+cos(x)),x)
```

```
[Out] int(exp(x)/(1+cos(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)\int\frac{e^x\sin(x)}{\cos(x)^2+\sin(x)^2+2\,\cos(x)+1}\,dx-e^x\sin(x)\right)}{\cos(x)^2+\sin(x)^2+2\,\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] -2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(co
s(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^x}{\cos(x)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] integral(e^x/(cos(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\cos(x) + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1+cos(x)),x)
```

```
[Out] Integral(exp(x)/(cos(x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\cos(x) + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(e^x/(cos(x) + 1), x)
```

3.553 $\int \frac{e^x}{1-\cos(x)} dx$

Optimal. Leaf size=26

$$(-1+i)e^{(1+i)x} {}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

```
[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)
]
```

Rubi [A] time = 0.0517265, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$(-1+i)e^{(1+i)x} {}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 - Cos[x]),x]
```

```
[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^x}{\sin^2\left(\frac{x}{2}\right)} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1-cos(x)),x)
```

```
[Out] Integral(exp(x)/sin(x/2)**2, x)/2
```

Mathematica [B] time = 0.0761144, size = 84, normalized size = 3.23

$$\frac{(1+i)e^x \sin\left(\frac{x}{2}\right)\left((1+i)\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+e^{ix}\,{}_2F_1\left(1,1-i;2-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1-i)\cos\left(\frac{x}{2}\right)\right)}{\cos(x)-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 - Cos[x]),x]
```

```
[Out] ((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2
F1[-I, 1, 1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1,
 1 - I, 2 - I, E^(I*x)]*Sin[x/2]))/(-1 + Cos[x])
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{e^{x}}{1-\cos (x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1-cos(x)),x)
```

```
[Out] int(exp(x)/(1-cos(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(\cos (x)^{2}+\sin (x)^{2}-2 \cos (x)+1\right) \int \frac{e^{x} \sin(x)}{\cos(x)^{2}+\sin(x)^{2}-2 \cos(x)+1}\, dx-e^{x} \sin (x)\right)}{\cos (x)^{2}+\sin (x)^{2}-2 \cos (x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] 2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{e^{x}}{\cos (x)-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] integral(-e^x/(cos(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^x}{\cos(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1-cos(x)),x)
```

```
[Out] -Integral(exp(x)/(cos(x) - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{e^x}{\cos(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(cos(x) - 1),x, algorithm="giac")
```

```
[Out] integrate(-e^x/(cos(x) - 1), x)
```

3.554 $\int \frac{e^x}{1+\sin(x)} \, dx$

Optimal. Leaf size=30

$$(-1+i)e^{(1-i)x} \, {}_2F_1\left(1+i,2;2+i;-ie^{-ix}\right)$$

```
[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^
(I*x)]
```

Rubi [A] time = 0.0562652, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$(-1+i)e^{(1-i)x} \, {}_2F_1\left(1+i,2;2+i;-ie^{-ix}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 + Sin[x]),x]
```

```
[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^
(I*x)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^x}{\sin^2\left(\frac{x}{2}+\frac{\pi}{4}\right)} \, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1+sin(x)),x)
```

```
[Out] Integral(exp(x)/sin(x/2 + pi/4)**2, x)/2
```

Mathematica [B] time = 0.513181, size = 61, normalized size = 2.03

$$\frac{2e^x \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)} - (1-i)(\sinh(x)+\cosh(x))(1-(1-i)\,{}_2F_1(-i,1;1-i;i\cos(x)-\sin(x)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 + Sin[x]),x]
```

```
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hyp
ergeometric2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[
x])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{e^{x}}{1+\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1+sin(x)),x)
```

```
[Out] int(exp(x)/(1+sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\cos(x)\,e^{x}-\left(\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1\right)\int \frac{\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}\,dx\right)}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(c
os(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + s
in(x)^2 + 2*sin(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{x}}{\sin(x)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] integral(e^x/(sin(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1+sin(x)),x)
```

```
[Out] Integral(exp(x)/(sin(x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(sin(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(e^x/(sin(x) + 1), x)
```

3.555 $\int \frac{e^x}{1-\sin(x)} \, dx$

Optimal. Leaf size=30

$$(1+i)e^{(1+i)x} \, {}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

```
[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(
I*x)]
```

Rubi [A] time = 0.0571934, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$(1+i)e^{(1+i)x} \, {}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 - Sin[x]),x]
```

```
[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(
I*x)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^x}{\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)} \, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1-sin(x)),x)
```

```
[Out] Integral(exp(x)/cos(x/2 + pi/4)**2, x)/2
```

Mathematica [B] time = 0.552607, size = 61, normalized size = 2.03

$$\frac{2e^x \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)} + (1+i)(\sinh(x)+\cosh(x))(1-(1+i)\, {}_2F_1(-i,1;1-i;\sin(x)-i\cos(x)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 - Sin[x]),x]
```

```
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hyp
ergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Si
nh[x])
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{e^{x}}{1-\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1-sin(x)),x)
```

```
[Out] int(exp(x)/(1-sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\cos(x)e^{x}-\left(\cos(x)^{2}+\sin(x)^{2}-2\sin(x)+1\right)\int\frac{\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}-2\sin(x)+1}\,dx\right)}{\cos(x)^{2}+\sin(x)^{2}-2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(sin(x) - 1),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(co
s(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + si
n(x)^2 - 2*sin(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{e^{x}}{\sin(x)-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(sin(x) - 1),x, algorithm="fricas")
```

```
[Out] integral(-e^x/(sin(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^{x}}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1-sin(x)),x)
```

```
[Out] -Integral(exp(x)/(sin(x) - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{e^{x}}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(sin(x) - 1),x, algorithm="giac")
```

```
[Out] integrate(-e^x/(sin(x) - 1), x)
```

3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=15

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

```
[Out] -((E^x*Sin[x])/(1 - Cos[x]))
```

Rubi [A] time = 0.0492604, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]

[Out] -((E^x*Sin[x])/(1 - Cos[x]))
```

Rubi in Sympy [A] time = 3.31384, size = 12, normalized size = 0.8

$$-\frac{e^x\sin(x)}{-\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)

[Out] -exp(x)*sin(x)/(-cos(x) + 1)
```

Mathematica [A] time = 0.0514769, size = 11, normalized size = 0.73

$$-e^x\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] -(E^x*Cot[x/2])
```

Maple [B] time = 0.079, size = 33, normalized size = 2.2

$$1\left(-\mathrm{e}^{x}\left(\tan\left(\frac{x}{2}\right)\right)^{2}-\mathrm{e}^{x}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-1}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1-sin(x))/(1-cos(x)),x)

[Out] (-exp(x)*tan(1/2*x)^2-exp(x))/(tan(1/2*x)^2+1)/tan(1/2*x)
```

Maxima [A] time = 1.93374, size = 30, normalized size = 2.

$$-\frac{2\,e^{x}\sin\left(x\right)}{\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x) - 1)*e^x/(cos(x) - 1),x, algorithm="maxima")

[Out] -2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 0.214006, size = 16, normalized size = 1.07

$$-\frac{(\cos\left(x\right)+1)e^{x}}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x) - 1)*e^x/(cos(x) - 1),x, algorithm="fricas")

[Out] -(cos(x) + 1)*e^x/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(\sin\left(x\right)-1)\,e^{x}}{\cos\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)
```

```
[Out] Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)
```

GIAC/XCAS [A] time = 0.214244, size = 14, normalized size = 0.93

$$-\frac{e^{x}}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x) - 1)*e^x/(cos(x) - 1),x, algorithm="giac")
```

```
[Out] -e^x/tan(1/2*x)
```

3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=41

$$\frac{e^x \sin(x)}{1-\cos(x)} - (2-2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

```
[Out] (-2 + 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*
x)] + (E^x*Sin[x])/(1 - Cos[x])
```

Rubi [A] time = 0.174846, antiderivative size = 45, normalized size of antiderivative = 1.1, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-4ie^x\,{}_2F_1\left(-i,1;1-i;e^{ix}\right) + 2ie^x - \frac{e^x \sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)] -
(E^x*Sin[x])/(1 - Cos[x])
```

Rubi in Sympy [A] time = 14.7949, size = 34, normalized size = 0.83

$$-4ie^x\,{}_2F_1\left(\left.\begin{matrix}1,-i\\1-i\end{matrix}\right| e^{ix}\right) + 2ie^x - \frac{e^x \sin(x)}{-\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] -4*I*exp(x)*hyper((1, -I), (1 - I,), exp(I*x)) + 2*I*exp(x) - exp
(x)*sin(x)/(-cos(x) + 1)
```

Mathematica [B] time = 0.111739, size = 100, normalized size = 2.44

$$\frac{2e^x \sin\left(\frac{x}{2}\right)(\sin(x)+1)\left(2i\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1+i)e^{ix}\,{}_2F_1\left(1,1-i;2-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{(\cos(x)-1)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I,
 E^(I*x)]*Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I,
2 - I, E^(I*x)]*Sin[x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2]
+ Sin[x/2])^2)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{x} \left(1+\sin \left(x \right)\right)}{1-\cos \left(x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \left(2 \left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2 \cos \left(x\right)+1\right) \int \frac{e^{x} \sin(x)}{\cos(x)^{2}+\sin(x)^{2}-2 \cos(x)+1}\, dx-e^{x} \sin \left(x\right)\right)}{\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2 \cos \left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) + 1)*e^x/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] 2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(c
os(x)^2 + sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{e^{x} \sin \left(x\right)+e^{x}}{\cos \left(x\right)-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) + 1)*e^x/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^x}{\cos(x)-1}\,dx - \int \frac{e^x \sin(x)}{\cos(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] -Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x
) - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) + 1)*e^x/(cos(x) - 1),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) + 1)*e^x/(cos(x) - 1), x)
```

3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{e^x\sin(x)}{\cos(x)+1}$$

```
[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Rubi [A] time = 0.0452424, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{e^x\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]

[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Rubi in Sympy [A] time = 2.67281, size = 10, normalized size = 0.83

$$\frac{e^x\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)

[Out] exp(x)*sin(x)/(cos(x) + 1)
```

Mathematica [A] time = 0.0452962, size = 10, normalized size = 0.83

$$e^x\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] E^x*Tan[x/2]
```

Maple [A] time = 0.069, size = 8, normalized size = 0.7

$$e^{x} \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1+sin(x))/(1+cos(x)),x)
```

```
[Out] exp(x)*tan(1/2*x)
```

Maxima [A] time = 1.9357, size = 30, normalized size = 2.5

$$\frac{2 e^{x} \sin(x)}{\cos(x)^{2} + \sin(x)^{2} + 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x) + 1)*e^x/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] 2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

Fricas [A] time = 0.215085, size = 15, normalized size = 1.25

$$\frac{e^{x} \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x) + 1)*e^x/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] e^x*sin(x)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sin(x) + 1) e^{x}}{\cos(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)
```

```
[Out] Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)
```

GIAC/XCAS [A] time = 0.222806, size = 9, normalized size = 0.75

$$e^x \tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x) + 1)*e^x/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] e^x*tan(1/2*x)
```

3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{e^x\sin(x)}{\cos(x)+1}+(2-2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

```
[Out] (2 - 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*
x)] - (E^x*Sin[x])/(1 + Cos[x])
```

Rubi [A] time = 0.173968, antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-4ie^x\,{}_2F_1\left(-i,1;1-i;-e^{ix}\right)+2ie^x+\frac{e^x\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)] +
 (E^x*Sin[x])/(1 + Cos[x])
```

Rubi in Sympy [A] time = 13.0771, size = 36, normalized size = 0.86

$$-4ie^x\,{}_2F_1\left(\begin{matrix}1,-i\\1-i\end{matrix}\middle|-e^{ix}\right)+2ie^x+\frac{e^x\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] -4*I*exp(x)*hyper((1, -I), (1 - I,), -exp(I*x)) + 2*I*exp(x) + ex
p(x)*sin(x)/(cos(x) + 1)
```

Mathematica [B] time = 0.117179, size = 87, normalized size = 2.07

$$-\frac{2e^x\cos\left(\frac{x}{2}\right)\left(2i\,{}_2F_1\left(-i,1;1-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-(1+i)e^{ix}\,{}_2F_1\left(1,1-i;2-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (-2*E^x*Cos[x/2]*((2*I)*Cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I,
-E^(I*x)] - (1 + I)*E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I,
2 - I, -E^(I*x)] - Sin[x/2]))/(1 + Cos[x])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{e^{x} (1-\sin (x))}{1+\cos (x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2 \left(2 \left(\cos (x)^{2}+\sin (x)^{2}+2 \cos (x)+1\right) \int \frac{e^{x} \sin (x)}{\cos (x)^{2}+\sin (x)^{2}+2 \cos (x)+1} dx-e^{x} \sin (x)\right)}{\cos (x)^{2}+\sin (x)^{2}+2 \cos (x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) - 1)*e^x/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] -2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(
cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 +
 sin(x)^2 + 2*cos(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{e^{x} \sin (x)-e^{x}}{\cos (x)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) - 1)*e^x/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{e^{x}}{\cos(x)+1}\right)dx-\int\frac{e^{x}\sin(x)}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] -Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(
x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{(\sin(x)-1)e^{x}}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) - 1)*e^x/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) - 1)*e^x/(cos(x) + 1), x)
```

3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{e^x\cos(x)}{1-\sin(x)}+(2+2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

```
[Out] (2 + 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E
^(I*x)] - (E^x*Cos[x])/(1 - Sin[x])
```

Rubi [A] time = 0.199746, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-4ie^x\,{}_2F_1\left(-i,1;1-i;-ie^{ix}\right)+2ie^x+\frac{e^x\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x
)] + (E^x*Cos[x])/(1 - Sin[x])
```

Rubi in Sympy [A] time = 19.6037, size = 39, normalized size = 0.85

$$-4ie^x\,{}_2F_1\left(\begin{matrix}1,-i\\1-i\end{matrix}\middle|-e^{i\left(x+\frac{\pi}{2}\right)}\right)+2ie^x+\frac{e^x\cos(x)}{-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] -4*I*exp(x)*hyper((1, -I), (1 - I,), -exp(I*(x + pi/2))) + 2*I*ex
p(x) + exp(x)*cos(x)/(-sin(x) + 1)
```

Mathematica [A] time = 0.664691, size = 72, normalized size = 1.57

$$\frac{1}{2}(\cos(x)-1)\csc^2\left(\frac{x}{2}\right)\left(4i(\sinh(x)+\cosh(x))\,{}_2F_1(-i,1;1-i;\sin(x)-i\cos(x))-\frac{e^x\left((1+2i)\cot\left(\frac{x}{2}\right)+(1-2i)\right)}{\cot\left(\frac{x}{2}\right)-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] ((-1 + Cos[x])*Csc[x/2]^2*(-((E^x*((1 - 2*I) + (1 + 2*I)*Cot[x/2]
))/(-1 + Cot[x/2])) + (4*I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*
Cos[x] + Sin[x]]*(Cosh[x] + Sinh[x])))/2
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \frac{e^{x} \left(1-\cos \left(x\right)\right)}{1-\sin \left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \left(\cos \left(x\right) e^{x} - 2 \left(\cos \left(x\right)^{2} + \sin \left(x\right)^{2} - 2 \, \sin \left(x\right) + 1\right) \int \frac{\cos\left(x\right) e^{x}}{\cos\left(x\right)^{2} + \sin\left(x\right)^{2} - 2 \, \sin\left(x\right) + 1}\, dx\right)}{\cos \left(x\right)^{2} + \sin \left(x\right)^{2} - 2 \, \sin \left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) - 1)*e^x/(sin(x) - 1),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(
cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 +
sin(x)^2 - 2*sin(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(\cos \left(x\right) - 1)e^{x}}{\sin \left(x\right) - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) - 1)*e^x/(sin(x) - 1),x, algorithm="fricas")
```

```
[Out] integral((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(x) - 1)\, e^x}{\sin(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(x) - 1)e^x}{\sin(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) - 1)*e^x/(sin(x) - 1),x, algorithm="giac")
```

```
[Out] integrate((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)} dx$

Optimal. Leaf size=14

$$\frac{e^x \cos(x)}{1-\sin(x)}$$

```
[Out] (E^x*Cos[x])/(1 - Sin[x])
```

Rubi [A] time = 0.0375474, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{e^x \cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]

[Out] (E^x*Cos[x])/(1 - Sin[x])
```

Rubi in Sympy [A] time = 3.03878, size = 10, normalized size = 0.71

$$\frac{e^x \cos(x)}{-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)

[Out] exp(x)*cos(x)/(-sin(x) + 1)
```

Mathematica [A] time = 0.0635755, size = 23, normalized size = 1.64

$$-\frac{e^x \left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan\left(\frac{x}{2}\right)-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] -((E^x*(1 + Tan[x/2]))/(-1 + Tan[x/2]))
```

Maple [B] time = 0.106, size = 53, normalized size = 3.8

$$1\left(-e^{x}\tan\left(\frac{x}{2}\right)-e^{x}\left(\tan\left(\frac{x}{2}\right)\right)^{2}-e^{x}\left(\tan\left(\frac{x}{2}\right)\right)^{3}-e^{x}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-1}\left(-1+\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1+cos(x))/(1-sin(x)),x)
```

```
[Out] (-exp(x)*tan(1/2*x)-exp(x)*tan(1/2*x)^2-exp(x)*tan(1/2*x)^3-exp(x
))/(tan(1/2*x)^2+1)/(-1+tan(1/2*x))
```

Maxima [A] time = 1.92892, size = 30, normalized size = 2.14

$$\frac{2\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}-2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x) + 1)*e^x/(sin(x) - 1),x, algorithm="maxima")
```

```
[Out] 2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [A] time = 0.215362, size = 32, normalized size = 2.29

$$\frac{(\cos(x)+1)e^{x}+e^{x}\sin(x)}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x) + 1)*e^x/(sin(x) - 1),x, algorithm="fricas")
```

```
[Out] ((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{e^{x}}{\sin(x)-1}\,dx-\int\frac{e^{x}\cos(x)}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)
```

```
[Out] -Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x
) - 1), x)
```

GIAC/XCAS [A] time = 0.221522, size = 27, normalized size = 1.93

$$-\frac{e^{x} \tan\left(\frac{1}{2} \, x\right) + e^{x}}{\tan\left(\frac{1}{2} \, x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x) + 1)*e^x/(sin(x) - 1),x, algorithm="giac")
```

```
[Out] -(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)
```

3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{e^x\cos(x)}{\sin(x)+1}-(2+2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;ie^{ix}\right)$$

```
[Out] (-2 - 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, I*E^(
I*x)] + (E^x*Cos[x])/(1 + Sin[x])
```

Rubi [A] time = 0.197733, antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$4ie^x\,{}_2F_1\left(i,1;1+i;-ie^{-ix}\right)-2ie^x-\frac{e^x\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] (-2*I)*E^x + (4*I)*E^x*Hypergeometric2F1[I, 1, 1 + I, (-I)/E^(I*x
)] - (E^x*Cos[x])/(1 + Sin[x])
```

Rubi in Sympy [A] time = 20.0212, size = 37, normalized size = 0.86

$$-4ie^x\,{}_2F_1\left(\begin{matrix}1,-i\\1-i\end{matrix}\middle|e^{i\left(x+\frac{\pi}{2}\right)}\right)+2ie^x-\frac{e^x\cos{(x)}}{\sin{(x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

```
[Out] -4*I*exp(x)*hyper((1, -I), (1 - I,), exp(I*(x + pi/2))) + 2*I*exp
(x) - exp(x)*cos(x)/(sin(x) + 1)
```

Mathematica [A] time = 0.243974, size = 73, normalized size = 1.7

$$\frac{1}{2}(\cos(x)+1)\sec^2\left(\frac{x}{2}\right)\left(\frac{e^x\left((1+2i)\tan\left(\frac{x}{2}\right)-(1-2i)\right)}{\tan\left(\frac{x}{2}\right)+1}\right.$$
$$\left.-4i(\sinh(x)+\cosh(x))\,{}_2F_1(-i,1;1-i;i\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] ((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I,
I*Cos[x] - Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 +
2*I)*Tan[x/2]))/(1 + Tan[x/2])))/2
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{x} (1+\cos (x))}{1+\sin (x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

```
[Out] int(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2 \left(\cos (x) e^{x}-2 \left(\cos (x)^{2}+\sin (x)^{2}+2 \sin (x)+1\right) \int \frac{\cos (x) e^{x}}{\cos (x)^{2}+\sin (x)^{2}+2 \sin (x)+1}\, dx\right)}{\cos (x)^{2}+\sin (x)^{2}+2 \sin (x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 1)*e^x/(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate
(cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 +
 sin(x)^2 + 2*sin(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(\cos (x)+1) e^{x}}{\sin (x)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 1)*e^x/(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(x)+1)\,e^x}{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

```
[Out] Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 1)*e^x/(sin(x) + 1),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx$

Optimal. Leaf size=13

$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

```
[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Rubi [A] time = 0.0364646, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]

[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Rubi in Sympy [A] time = 3.05096, size = 12, normalized size = 0.92

$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)

[Out] -exp(x)*cos(x)/(sin(x) + 1)
```

Mathematica [A] time = 0.0596314, size = 23, normalized size = 1.77

$$-\frac{e^x\left(\cot\left(\frac{x}{2}\right)-1\right)}{\cot\left(\frac{x}{2}\right)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] -((E^x*(-1 + Cot[x/2]))/(1 + Cot[x/2]))
```

Maple [B] time = 0.091, size = 51, normalized size = 3.9

$$1\left(e^{x}\tan\left(\frac{x}{2}\right)+e^{x}\left(\tan\left(\frac{x}{2}\right)\right)^{3}-e^{x}\left(\tan\left(\frac{x}{2}\right)\right)^{2}-e^{x}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-1}\left(1+\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

```
[Out] (exp(x)*tan(1/2*x)+exp(x)*tan(1/2*x)^3-exp(x)*tan(1/2*x)^2-exp(x)
)/(tan(1/2*x)^2+1)/(1+tan(1/2*x))
```

Maxima [A] time = 1.92338, size = 30, normalized size = 2.31

$$-\frac{2\,\cos\left(x\right)e^{x}}{\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x) - 1)*e^x/(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

Fricas [A] time = 0.214799, size = 32, normalized size = 2.46

$$-\frac{(\cos\left(x\right)+1)e^{x}-e^{x}\sin\left(x\right)}{\cos\left(x\right)+\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x) - 1)*e^x/(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] -((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{e^{x}}{\sin\left(x\right)+1}\right)dx-\int\frac{e^{x}\cos\left(x\right)}{\sin\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

```
[Out] -Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(
x) + 1), x)
```

GIAC/XCAS [A] time = 0.218446, size = 28, normalized size = 2.15

$$\frac{e^{x}\tan\left(\frac{1}{2}\,x\right) - e^{x}}{\tan\left(\frac{1}{2}\,x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x) - 1)*e^x/(sin(x) + 1),x, algorithm="giac")
```

```
[Out] (e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)
```

3.564 $\int e^x x \cos(x) \, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi [A] time = 0.0599082, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi in Sympy [A] time = 2.98909, size = 27, normalized size = 0.9

$$\frac{xe^x \sin(x)}{2} + \frac{xe^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Mathematica [A] time = 0.0288897, size = 18, normalized size = 0.6

$$\frac{1}{2}e^x((x-1)\sin(x) + x\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x*Cos[x],x]
```

```
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] time = 0., size = 20, normalized size = 0.7

$$\frac{e^{x} x \cos (x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right) e^{x} \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x*cos(x),x)

[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A] time = 1.36129, size = 23, normalized size = 0.77

$$\frac{1}{2} x \cos (x) e^{x}+\frac{1}{2} (x-1) e^{x} \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*e^x,x, algorithm="maxima")

[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A] time = 0.22617, size = 23, normalized size = 0.77

$$\frac{1}{2} x \cos (x) e^{x}+\frac{1}{2} (x-1) e^{x} \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*e^x,x, algorithm="fricas")

[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] time = 1.10913, size = 27, normalized size = 0.9

$$\frac{x e^{x} \sin (x)}{2}+\frac{x e^{x} \cos (x)}{2}-\frac{e^{x} \sin (x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x*cos(x),x)

[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

GIAC/XCAS [A] time = 0.239756, size = 20, normalized size = 0.67

$$\frac{1}{2}\,(x\cos(x) + (x-1)\sin(x))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*e^x,x, algorithm="giac")

[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.565 $\int e^x x^2 \sin(x)\, dx$

Optimal. Leaf size=50

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x]
)/2 + (E^x*x^2*Sin[x])/2
```

Rubi [A] time = 0.167055, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x^2*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x]
)/2 + (E^x*x^2*Sin[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2 e^x \sin(x)}{2} - \frac{x^2 e^x \cos(x)}{2} - 2\int x \left(\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 - 2*Integral(x*(exp(x
)*sin(x)/2 - exp(x)*cos(x)/2), x)
```

Mathematica [A] time = 0.0431513, size = 25, normalized size = 0.5

$$\frac{1}{2}e^x \left(\left(x^2-1\right)\sin(x) - (x-1)^2\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x^2*Sin[x],x]
```

```
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Maple [A] time = 0.008, size = 27, normalized size = 0.5

$$\left(-\frac{x^2}{2}+x-\frac{1}{2}\right)\mathrm{e}^{x}\cos(x)+\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x^2*sin(x),x)
```

```
[Out] (-1/2*x^2+x-1/2)*exp(x)*cos(x)+(1/2*x^2-1/2)*exp(x)*sin(x)
```

Maxima [A] time = 1.3933, size = 35, normalized size = 0.7

$$-\frac{1}{2}\left(x^2-2x+1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Fricas [A] time = 6.30064, size = 35, normalized size = 0.7

$$-\frac{1}{2}\left(x^2-2x+1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Sympy [A] time = 3.21248, size = 48, normalized size = 0.96

$$\frac{x^2e^x\sin(x)}{2}-\frac{x^2e^x\cos(x)}{2}+xe^x\cos(x)-\frac{e^x\sin(x)}{2}-\frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - e
xp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.198486, size = 34, normalized size = 0.68

$$-\frac{1}{2}\left(\left(x^{2}-2\,x+1\right)\,\cos\left(x\right)-\left(x^{2}-1\right)\,\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

3.566 $\int e^{-3x}x^2\sin(x)\,dx$

Optimal. Leaf size=75

$$-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{9}{250}e^{-3x}\sin(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{13}{250}e^{-3x}\cos(x)$$

```
[Out] (-13*Cos[x])/(250*E^(3*x)) - (3*x*Cos[x])/(25*E^(3*x)) - (x^2*Cos
[x])/(10*E^(3*x)) - (9*Sin[x])/(250*E^(3*x)) - (4*x*Sin[x])/(25*E
^(3*x)) - (3*x^2*Sin[x])/(10*E^(3*x))
```

Rubi [A] time = 0.2228, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{9}{250}e^{-3x}\sin(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{13}{250}e^{-3x}\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sin[x])/E^(3*x),x]
```

```
[Out] (-13*Cos[x])/(250*E^(3*x)) - (3*x*Cos[x])/(25*E^(3*x)) - (x^2*Cos
[x])/(10*E^(3*x)) - (9*Sin[x])/(250*E^(3*x)) - (4*x*Sin[x])/(25*E
^(3*x)) - (3*x^2*Sin[x])/(10*E^(3*x))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3x^2e^{-3x}\sin(x)}{10}-\frac{x^2e^{-3x}\cos(x)}{10}-2\int x\left(-\frac{3e^{-3x}\sin(x)}{10}-\frac{e^{-3x}\cos(x)}{10}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(x)/exp(3*x),x)
```

```
[Out] -3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 2*Integr
al(x*(-3*exp(-3*x)*sin(x)/10 - exp(-3*x)*cos(x)/10), x)
```

Mathematica [A] time = 0.0406455, size = 38, normalized size = 0.51

$$\frac{1}{250}e^{-3x}\left(-\left(75x^2+40x+9\right)\sin(x)-\left(25x^2+30x+13\right)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sin[x])/E^(3*x),x]
```

```
[Out] (-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(25
0*E^(3*x))
```

Maple [A] time = 0.007, size = 36, normalized size = 0.5

$$\left(-\frac{x^2}{10}-\frac{3x}{25}-\frac{13}{250}\right)\mathrm{e}^{-3x}\cos(x)+\left(-\frac{3x^2}{10}-\frac{4x}{25}-\frac{9}{250}\right)\mathrm{e}^{-3x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(x)/exp(3*x),x)
```

```
[Out] (-1/10*x^2-3/25*x-13/250)*exp(-3*x)*cos(x)+(-3/10*x^2-4/25*x-9/25
0)*exp(-3*x)*sin(x)
```

Maxima [A] time = 1.38843, size = 45, normalized size = 0.6

$$-\frac{1}{250}\left(\left(25x^2+30x+13\right)\cos(x)+\left(75x^2+40x+9\right)\sin(x)\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))
*e^(-3*x)
```

Fricas [A] time = 0.214124, size = 50, normalized size = 0.67

$$-\frac{1}{250}\left(25x^2+30x+13\right)\cos(x)e^{(-3x)}-\frac{1}{250}\left(75x^2+40x+9\right)e^{(-3x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/250*(25*x^2 + 30*x + 13)*cos(x)*e^(-3*x) - 1/250*(75*x^2 + 40*
x + 9)*e^(-3*x)*sin(x)
```

Sympy [A] time = 3.66555, size = 80, normalized size = 1.07

$$-\frac{3x^2e^{-3x}\sin(x)}{10}-\frac{x^2e^{-3x}\cos(x)}{10}-\frac{4xe^{-3x}\sin(x)}{25}-\frac{3xe^{-3x}\cos(x)}{25}-\frac{9e^{-3x}\sin(x)}{250}-\frac{13e^{-3x}\cos(x)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(x)/exp(3*x),x)
```

```
[Out] -3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(
-3*x)*sin(x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/25
0 - 13*exp(-3*x)*cos(x)/250
```

GIAC/XCAS [A] time = 0.204716, size = 45, normalized size = 0.6

$$-\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos(x)+\left(75\,x^2+40\,x+9\right)\sin(x)\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))
*e^(-3*x)
```

3.567 $\int e^{x/2}x^2\cos^3(x)\,dx$

Optimal. Leaf size=187

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{24}{125}e^{x/2}\sin(x)-\frac{24}{25}e^{x/2}x\sin(x)-\frac{792e^{x/2}\sin(3x)}{50653}-\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{132}{125}e^{x/2}$$

```
[Out] (-132*E^(x/2)*Cos[x])/125 + (18*E^(x/2)*x*Cos[x])/25 + (48*E^(x/2
)*x^2*Cos[x])/185 + (2*E^(x/2)*x^2*Cos[x]^3)/37 - (428*E^(x/2)*Co
s[3*x])/50653 + (70*E^(x/2)*x*Cos[3*x])/1369 - (24*E^(x/2)*Sin[x]
)/125 - (24*E^(x/2)*x*Sin[x])/25 + (96*E^(x/2)*x^2*Sin[x])/185 +
(12*E^(x/2)*x^2*Cos[x]^2*Sin[x])/37 - (792*E^(x/2)*Sin[3*x])/5065
3 - (24*E^(x/2)*x*Sin[3*x])/1369
```

Rubi [A] time = 0.753444, antiderivative size = 253, normalized size of antiderivative = 1.35, number of steps used = 31, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{1218672e^{x/2}\sin(x)}{6331625}-\frac{32556e^{x/2}x\sin(x)}{34225}-\frac{816e^{x/2}\sin(3x)}{50653}-\frac{12e^{x/2}x\sin(3x)}{1369}+$$

Antiderivative was successfully verified.

```
[In]  Int[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (-6687696*E^(x/2)*Cos[x])/6331625 + (24792*E^(x/2)*x*Cos[x])/3422
5 + (48*E^(x/2)*x^2*Cos[x])/185 + (16*E^(x/2)*Cos[x]^3)/50653 - (
8*E^(x/2)*x*Cos[x]^3)/1369 + (2*E^(x/2)*x^2*Cos[x]^3)/37 - (432*E
^(x/2)*Cos[3*x])/50653 + (72*E^(x/2)*x*Cos[3*x])/1369 - (1218672*
E^(x/2)*Sin[x])/6331625 - (32556*E^(x/2)*x*Sin[x])/34225 + (96*E^
(x/2)*x^2*Sin[x])/185 + (96*E^(x/2)*Cos[x]^2*Sin[x])/50653 - (48*
E^(x/2)*x*Cos[x]^2*Sin[x])/1369 + (12*E^(x/2)*x^2*Cos[x]^2*Sin[x]
)/37 - (816*E^(x/2)*Sin[3*x])/50653 - (12*E^(x/2)*x*Sin[3*x])/136
9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{12x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{37}+\frac{96x^2e^{\frac{x}{2}}\sin(x)}{185}+\frac{2x^2e^{\frac{x}{2}}\cos^3(x)}{37}+\frac{48x^2e^{\frac{x}{2}}\cos(x)}{185}-2\int x\left(\frac{12e^{\frac{x}{2}}\sin(x)\cos^2(x)}{37}+\frac{96e^{\frac{x}{2}}\sin(x)}{185}+\frac{2e^{\frac{x}{2}}\cos^3(x)}{37}+\frac{48e^{\frac{x}{2}}\cos(x)}{185}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

```
[Out] 12*x**2*exp(x/2)*sin(x)*cos(x)**2/37 + 96*x**2*exp(x/2)*sin(x)/18
5 + 2*x**2*exp(x/2)*cos(x)**3/37 + 48*x**2*exp(x/2)*cos(x)/185 -
2*Integral(x*(12*exp(x/2)*sin(x)*cos(x)**2/37 + 96*exp(x/2)*sin(x
)/185 + 2*exp(x/2)*cos(x)**3/37 + 48*exp(x/2)*cos(x)/185), x)
```

Mathematica [A] time = 0.209466, size = 72, normalized size = 0.39

$$\frac{e^{x/2}\left(303918\left(25x^2-40x-8\right)\sin(x)+750\left(1369x^2-296x-264\right)\sin(3x)+151959\left(25x^2+60x-88\right)\cos(x)+125\left(1369x\right.}{12663250}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*
x + 1369*x^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750
*(-264 - 296*x + 1369*x^2)*Sin[3*x]))/12663250
```

Maple [A] time = 0.023, size = 78, normalized size = 0.4

$$\frac{3\cos(x)}{4}\left(\frac{2x^2}{5}+\frac{24x}{25}-\frac{176}{125}\right)e^{\frac{x}{2}}-\frac{3\sin(x)}{4}\left(-\frac{4x^2}{5}+\frac{32x}{25}+\frac{32}{125}\right)e^{\frac{x}{2}}+\frac{\cos(3x)}{4}\left(\frac{2x^2}{37}+\frac{280x}{1369}-\frac{1712}{50653}\right)e^{\frac{x}{2}}-\frac{\sin(3x)}{4}\left(-\frac{12x^2}{37}+\frac{96x}{1369}+\frac{3168}{50653}\right)e^{\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1/2*x)*x^2*cos(x)^3,x)
```

```
[Out] 3/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-3/4*(-4/5*x^2+32/
25*x+32/125)*exp(1/2*x)*sin(x)+1/4*(2/37*x^2+280/1369*x-1712/5065
3)*exp(1/2*x)*cos(3*x)-1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(
1/2*x)*sin(3*x)
```

Maxima [A] time = 1.37078, size = 104, normalized size = 0.56

$$\frac{1}{101306}\left(1369x^2+5180x-856\right)\cos(3x)e^{\left(\frac{1}{2}x\right)}+\frac{3}{250}\left(25x^2+60x-88\right)\cos(x)e^{\left(\frac{1}{2}x\right)}+\frac{3}{50653}\left(1369x^2-296x-264\right)e^{\left(\frac{1}{2}x\right)}\sin(3x)+\frac{3}{125}\left(25x^2-40x-8\right)e^{\left(\frac{1}{2}x\right)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^3*e^(1/2*x),x, algorithm="maxima")
```

```
[Out] 1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25
*x^2 + 60*x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x -
264)*e^(1/2*x)*sin(3*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin
(x)
```

Fricas [A] time = 0.25207, size = 97, normalized size = 0.52

$$\frac{12}{6331625}\left(125\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2+273800\,x^2-497280\,x-93056\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)+\frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3+24\left(34225\,x^2+74740\,x-135952\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^3*e^(1/2*x),x, algorithm="fricas")
```

```
[Out] 12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 -
497280*x - 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5
180*x - 856)*cos(x)^3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))
*e^(1/2*x)
```

Sympy [A] time = 21.9322, size = 202, normalized size = 1.08

$$\frac{96x^2e^{\frac{x}{2}}\sin^3(x)}{185}+\frac{48x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185}+\frac{156x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185}+\frac{58x^2e^{\frac{x}{2}}\cos^3(x)}{185}-\frac{32256xe^{\frac{x}{2}}\sin^3(x)}{34225}+\frac{19392xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225}-\frac{34656xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225}+\frac{26392xe^{\frac{x}{2}}\cos^3(x)}{34225}-\frac{1116672e^{\frac{x}{2}}\sin^3(x)}{6331625}-\frac{6525696e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625}-\frac{1512672e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625}-\frac{6739696e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

```
[Out] 96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x
)/185 + 156*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)
*cos(x)**3/185 - 32256*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x
/2)*sin(x)**2*cos(x)/34225 - 34656*x*exp(x/2)*sin(x)*cos(x)**2/34
225 + 26392*x*exp(x/2)*cos(x)**3/34225 - 1116672*exp(x/2)*sin(x)*
```

```
*3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x)/6331625 - 1512672*
exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*cos(x)**3/63
31625
```

GIAC/XCAS [A] time = 0.200491, size = 99, normalized size = 0.53

$$\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{3}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)^3*e^(1/2*x),x, algorithm="giac")
```

```
[Out] 1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*
x - 264)*sin(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x)
 + 2*(25*x^2 - 40*x - 8)*sin(x))*e^(1/2*x)
```

3.568 $\int e^{2x}x^2\sin(4x)\,dx$

Optimal. Leaf size=87

$$\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)-\frac{11}{500}e^{2x}\sin(4x)+\frac{2}{25}e^{2x}x\cos(4x)+\frac{1}{250}e^{2x}\cos(4x)$$

```
[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2
*Cos[4*x])/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])
/50 + (E^(2*x)*x^2*Sin[4*x])/10
```

Rubi [A] time = 0.247479, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)-\frac{11}{500}e^{2x}\sin(4x)+\frac{2}{25}e^{2x}x\cos(4x)+\frac{1}{250}e^{2x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2
*Cos[4*x])/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])
/50 + (E^(2*x)*x^2*Sin[4*x])/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2e^{2x}\sin(4x)}{10}-\frac{x^2e^{2x}\cos(4x)}{5}-2\int x\left(\frac{e^{2x}\sin(4x)}{10}-\frac{e^{2x}\cos(4x)}{5}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)*x**2*sin(4*x),x)
```

```
[Out] x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 - 2*Integral
(x*(exp(2*x)*sin(4*x)/10 - exp(2*x)*cos(4*x)/5), x)
```

Mathematica [A] time = 0.112473, size = 40, normalized size = 0.46

$$\frac{1}{500}e^{2x}\left(\left(50x^2+30x-11\right)\sin(4x)+\left(-100x^2+40x+2\right)\cos(4x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*S
in[4*x]))/500
```

Maple [A] time = 0.007, size = 40, normalized size = 0.5

$$\left(-\frac{x^2}{5}+\frac{2x}{25}+\frac{1}{250}\right)\mathrm{e}^{2x}\cos(4x)+\left(\frac{x^2}{10}+\frac{3x}{50}-\frac{11}{500}\right)\mathrm{e}^{2x}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)*x^2*sin(4*x),x)
```

```
[Out] (-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500
)*exp(2*x)*sin(4*x)
```

Maxima [A] time = 1.37048, size = 55, normalized size = 0.63

$$-\frac{1}{250}\left(50x^2-20x-1\right)\cos(4x)e^{(2x)}+\frac{1}{500}\left(50x^2+30x-11\right)e^{(2x)}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(2*x)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*
x - 11)*e^(2*x)*sin(4*x)
```

Fricas [A] time = 0.212232, size = 55, normalized size = 0.63

$$-\frac{1}{250}\left(50x^2-20x-1\right)\cos(4x)e^{(2x)}+\frac{1}{500}\left(50x^2+30x-11\right)e^{(2x)}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(2*x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*
x - 11)*e^(2*x)*sin(4*x)
```

Sympy [A] time = 3.20309, size = 85, normalized size = 0.98

$$\frac{x^2e^{2x}\sin(4x)}{10}-\frac{x^2e^{2x}\cos(4x)}{5}+\frac{3xe^{2x}\sin(4x)}{50}+\frac{2xe^{2x}\cos(4x)}{25}-\frac{11e^{2x}\sin(4x)}{500}+\frac{e^{2x}\cos(4x)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)*x**2*sin(4*x),x)
```

```
[Out] x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*
x)*sin(4*x)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/
500 + exp(2*x)*cos(4*x)/250
```

GIAC/XCAS [A] time = 0.237365, size = 53, normalized size = 0.61

$$-\frac{1}{500}\left(2\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)-\left(50\,x^2+30\,x-11\right)\sin\left(4\,x\right)\right)e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(2*x)*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin
(4*x))*e^(2*x)
```

3.569 $\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=185

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)$$
$$+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin(x)+\frac{792e^{x/2}\sin(3x)}{50653}+\frac{6}{25}e^{x/2}x\cos(x)-$$

```
[Out] (-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2
*Cos[x])/10 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*
x])/1369 - (E^(x/2)*x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (
8*E^(x/2)*x*Sin[x])/25 + (E^(x/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Si
n[3*x])/50653 + (24*E^(x/2)*x*Sin[3*x])/1369 - (3*E^(x/2)*x^2*Sin
[3*x])/37
```

Rubi [A] time = 0.55239, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)$$
$$+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin(x)+\frac{792e^{x/2}\sin(3x)}{50653}+\frac{6}{25}e^{x/2}x\cos(x)-$$

Antiderivative was successfully verified.

```
[In]  Int[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2
*Cos[x])/10 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*
x])/1369 - (E^(x/2)*x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (
8*E^(x/2)*x*Sin[x])/25 + (E^(x/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Si
n[3*x])/50653 + (24*E^(x/2)*x*Sin[3*x])/1369 - (3*E^(x/2)*x^2*Sin
[3*x])/37
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2e^{\frac{x}{2}}\sin(x)}{5}-\frac{3x^2e^{\frac{x}{2}}\sin(3x)}{37}+\frac{x^2e^{\frac{x}{2}}\cos(x)}{10}-\frac{x^2e^{\frac{x}{2}}\cos(3x)}{74}-\frac{\int x\left(\frac{4e^{\frac{x}{2}}\sin(x)}{5}+\frac{2e^{\frac{x}{2}}\cos(x)}{5}\right)dx}{2}+\frac{\int x\left(\frac{12e^{\frac{x}{2}}\sin(3x)}{37}+\frac{2e^{\frac{x}{2}}\cos(3x)}{37}\right)dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

```
[Out] x**2*exp(x/2)*sin(x)/5 - 3*x**2*exp(x/2)*sin(3*x)/37 + x**2*exp(x
/2)*cos(x)/10 - x**2*exp(x/2)*cos(3*x)/74 - Integral(x*(4*exp(x/2
)*sin(x)/5 + 2*exp(x/2)*cos(x)/5), x)/2 + Integral(x*(12*exp(x/2)
*sin(3*x)/37 + 2*exp(x/2)*cos(3*x)/37), x)/2
```

Mathematica [A] time = 0.304697, size = 76, normalized size = 0.41

$$\frac{e^{x/2}\left(50653\left(2\left(25x^2-40x-8\right)\sin(x)+\left(25x^2+60x-88\right)\cos(x)\right)-125\left(6\left(1369x^2-296x-264\right)\sin(3x)+\left(1369x^2+51\right.\right.\right.}{12663250}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25
*x^2)*Sin[x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-26
4 - 296*x + 1369*x^2)*Sin[3*x])))/12663250
```

Maple [A] time = 0.009, size = 78, normalized size = 0.4

$$\frac{\cos(x)}{4}\left(\frac{2x^2}{5}+\frac{24x}{25}-\frac{176}{125}\right)e^{\frac{x}{2}}-\frac{\sin(x)}{4}\left(-\frac{4x^2}{5}+\frac{32x}{25}+\frac{32}{125}\right)e^{\frac{x}{2}}-\frac{\cos(3x)}{4}\left(\frac{2x^2}{37}+\frac{280x}{1369}-\frac{1712}{50653}\right)e^{\frac{x}{2}}+\frac{\sin(3x)}{4}\left(-\frac{12x^2}{37}+\frac{96x}{1369}+\frac{3168}{50653}\right)e^{\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x)
```

```
[Out] 1/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-1/4*(-4/5*x^2+32/
25*x+32/125)*exp(1/2*x)*sin(x)-1/4*(2/37*x^2+280/1369*x-1712/5065
3)*exp(1/2*x)*cos(3*x)+1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(
1/2*x)*sin(3*x)
```

Maxima [A] time = 1.38076, size = 104, normalized size = 0.56

$$-\frac{1}{101306}\left(1369x^2+5180x-856\right)\cos(3x)e^{\left(\frac{1}{2}x\right)}+\frac{1}{250}\left(25x^2+60x-88\right)\cos(x)e^{\left(\frac{1}{2}x\right)}-\frac{3}{50653}\left(1369x^2-296x-264\right)e^{\left(\frac{1}{2}x\right)}\sin(3x)+\frac{1}{125}\left(25x^2-40x-8\right)e^{\left(\frac{1}{2}x\right)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)*e^(1/2*x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(2
5*x^2 + 60*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x -
 264)*e^(1/2*x)*sin(3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*si
n(x)
```

Fricas [A] time = 0.221643, size = 97, normalized size = 0.52

$$-\frac{4}{6331625}\left(375\left(1369\,x^{2}-296\,x-264\right)\cos\left(x\right)^{2}-444925\,x^{2}+534280\,x+126056\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)-\frac{2}{6331625}\left(125\left(1369\,x^{2}+5180\,x-856\right)\cos\left(x\right)^{3}-\left(444925\,x^{2}+1245420\,x-1194616\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)*e^(1/2*x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 +
534280*x + 126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 +
5180*x - 856)*cos(x)^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x
))*e^(1/2*x)
```

Sympy [A] time = 21.8472, size = 202, normalized size = 1.09

$$\frac{52x^{2}e^{\frac{x}{2}}\sin^{3}\left(x\right)}{185}+\frac{26x^{2}e^{\frac{x}{2}}\sin^{2}\left(x\right)\cos\left(x\right)}{185}-\frac{8x^{2}e^{\frac{x}{2}}\sin\left(x\right)\cos^{2}\left(x\right)}{185}+\frac{16x^{2}e^{\frac{x}{2}}\cos^{3}\left(x\right)}{185}-\frac{11552xe^{\frac{x}{2}}\sin^{3}\left(x\right)}{34225}+\frac{13464xe^{\frac{x}{2}}\sin^{2}\left(x\right)\cos\left(x\right)}{34225}-\frac{9152xe^{\frac{x}{2}}\sin\left(x\right)\cos^{2}\left(x\right)}{34225}+\frac{6464xe^{\frac{x}{2}}\cos^{3}\left(x\right)}{34225}-\frac{504224e^{\frac{x}{2}}\sin^{3}\left(x\right)}{6331625}-\frac{2389232e^{\frac{x}{2}}\sin^{2}\left(x\right)\cos\left(x\right)}{6331625}-\frac{108224e^{\frac{x}{2}}\sin\left(x\right)\cos^{2}\left(x\right)}{6331625}-\frac{2175232e^{\frac{x}{2}}\cos^{3}\left(x\right)}{6331625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

```
[Out] 52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x
)/185 - 8*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*c
os(x)**3/185 - 11552*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2
)*sin(x)**2*cos(x)/34225 - 9152*x*exp(x/2)*sin(x)*cos(x)**2/34225
 + 6464*x*exp(x/2)*cos(x)**3/34225 - 504224*exp(x/2)*sin(x)**3/63
31625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/6331625 - 108224*exp(x/
2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)**3/6331625
```

GIAC/XCAS [A] time = 0.233029, size = 99, normalized size = 0.54

$$-\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{1}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(x)*e^(1/2*x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296
*x - 264)*sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x
) + 2*(25*x^2 - 40*x - 8)*sin(x))*e^(1/2*x)
```

3.570 $\int \cosh(x)\,dx$

Optimal. Leaf size=2

$$\sinh(x)$$

```
[Out] Sinh[x]
```

Rubi [A] time = 0.0059424, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cosh(x),x)
```

```
[Out] Integral(cosh(x), x)
```

Mathematica [A] time = 0.00343822, size = 2, normalized size = 1.

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cosh(x),x)

[Out] sinh(x)
```

Maxima [A] time = 1.3895, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x, algorithm="maxima")

[Out] sinh(x)
```

Fricas [A] time = 0.19907, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x, algorithm="fricas")

[Out] sinh(x)
```

Sympy [A] time = 0.103228, size = 2, normalized size = 1.

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x)

[Out] sinh(x)
```

GIAC/XCAS [A] time = 0.205672, size = 15, normalized size = 7.5

$$-\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

3.571 $\int \sinh(x)\,dx$

Optimal. Leaf size=2

$$\cosh(x)$$

```
[Out] Cosh[x]
```

Rubi [A] time = 0.00627967, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x],x]

[Out] Cosh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sinh(x),x)

[Out] Integral(sinh(x), x)
```

Mathematica [A] time = 0.0033963, size = 2, normalized size = 1.

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x],x]

[Out] Cosh[x]
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sinh(x),x)
```

```
[Out] cosh(x)
```

Maxima [A] time = 1.37059, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)
```

Fricas [A] time = 0.199181, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x, algorithm="fricas")
```

```
[Out] cosh(x)
```

Sympy [A] time = 0.102313, size = 2, normalized size = 1.

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x)
```

```
[Out] cosh(x)
```

GIAC/XCAS [A] time = 0.199236, size = 15, normalized size = 7.5

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x
```

3.572 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

```
[Out] Log[Cosh[x]]
```

Rubi [A] time = 0.00655869, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tanh[x],x]

[Out] Log[Cosh[x]]
```

Rubi in Sympy [A] time = 22.7456, size = 10, normalized size = 3.33

$$-\frac{\log\left(-\tanh^2(x)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tanh(x),x)

[Out] -log(-tanh(x)**2 + 1)/2
```

Mathematica [A] time = 0.00558818, size = 3, normalized size = 1.

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tanh[x],x]

[Out] Log[Cosh[x]]
```

Maple [A] time = 0.002, size = 4, normalized size = 1.3

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tanh(x),x)
```

```
[Out] ln(cosh(x))
```

Maxima [A] time = 1.37232, size = 4, normalized size = 1.33

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x, algorithm="maxima")
```

```
[Out] log(cosh(x))
```

Fricas [A] time = 0.209271, size = 24, normalized size = 8.

$$-x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [A] time = 0.120688, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x)
```

```
[Out] x - log(tanh(x) + 1)
```

GIAC/XCAS [A] time = 0.201614, size = 15, normalized size = 5.

$$-x + \ln\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + ln(e^(2*x) + 1)
```

3.573 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

```
[Out] Log[Sinh[x]]
```

Rubi [A] time = 0.00726553, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Coth[x],x]

[Out] Log[Sinh[x]]
```

Rubi in Sympy [A] time = 36.4378, size = 17, normalized size = 5.67

$$-\frac{\log\left(-\tanh^2(x)+1\right)}{2}+\frac{\log\left(\tanh^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(coth(x),x)

[Out] -log(-tanh(x)**2 + 1)/2 + log(tanh(x)**2)/2
```

Mathematica [A] time = 0.00569506, size = 3, normalized size = 1.

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Coth[x],x]

[Out] Log[Sinh[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(coth(x),x)
```

```
[Out] ln(sinh(x))
```

Maxima [A] time = 1.40444, size = 4, normalized size = 1.33

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x, algorithm="maxima")
```

```
[Out] log(sinh(x))
```

Fricas [A] time = 0.212569, size = 24, normalized size = 8.

$$-x + \log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [A] time = 0.423998, size = 12, normalized size = 4.

$$x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x)
```

```
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

GIAC/XCAS [A] time = 0.20795, size = 16, normalized size = 5.33

$$-x + \ln\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(coth(x),x, algorithm="giac")
```

```
[Out] -x + ln(abs(e^(2*x) - 1))
```

3.574 $\int \operatorname{sech}(x)\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sinh(x))$$

```
[Out] ArcTan[Sinh[x]]
```

Rubi [A] time = 0.0063651, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sech[x],x]

[Out] ArcTan[Sinh[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sech(x),x)

[Out] Integral(1/cosh(x), x)
```

Mathematica [B] time = 0.0102759, size = 9, normalized size = 3.

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sech[x],x]

[Out] 2*ArcTan[Tanh[x/2]]
```

Maple [A] time = 0.003, size = 4, normalized size = 1.3

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sech(x),x)

[Out] arctan(sinh(x))
```

Maxima [A] time = 1.32907, size = 4, normalized size = 1.33

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x),x, algorithm="maxima")

[Out] arctan(sinh(x))
```

Fricas [A] time = 0.227617, size = 11, normalized size = 3.67

$$2\arctan(\cosh(x)+\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x),x, algorithm="fricas")

[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x),x)
```

```
[Out] Integral(sech(x), x)
```

GIAC/XCAS [A] time = 0.197831, size = 7, normalized size = 2.33

$$2 \arctan \left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

3.575 $\int \operatorname{csch}(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]
```

Rubi [A] time = 0.00719738, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]]
```

Rubi in Sympy [A] time = 1.65586, size = 5, normalized size = 1.

$$-\operatorname{atanh}(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(csch(x),x)
```

```
[Out] -atanh(cosh(x))
```

Mathematica [B] time = 0.00729465, size = 17, normalized size = 3.4

$$\log\left(\sinh\left(\frac{x}{2}\right)\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x],x]
```

```
[Out] -Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

Maple [A] time = 0.013, size = 6, normalized size = 1.2

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csch(x),x)

[Out] ln(tanh(1/2*x))
```

Maxima [A] time = 1.33546, size = 7, normalized size = 1.4

$$\log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x),x, algorithm="maxima")

[Out] log(tanh(1/2*x))
```

Fricas [A] time = 0.20445, size = 23, normalized size = 4.6

$$-\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x),x, algorithm="fricas")

[Out] -log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x),x)
```

```
[Out] Integral(csch(x), x)
```

GIAC/XCAS [A] time = 0.19852, size = 19, normalized size = 3.8

$$-\ln\left(e^{x} + 1\right) + \ln\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x),x, algorithm="giac")
```

```
[Out] -ln(e^x + 1) + ln(abs(e^x - 1))
```

3.576 $\int \cosh^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] x/2 + (Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.0126256, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cosh[x]^2,x]

[Out] x/2 + (Cosh[x]*Sinh[x])/2
```

Rubi in Sympy [A] time = 4.14878, size = 17, normalized size = 1.21

$$\frac{\operatorname{atanh}(\tanh(x))}{2}+\frac{\tanh(x)}{2\left(-\tanh^2(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cosh(x)**2,x)

[Out] atanh(tanh(x))/2 + tanh(x)/(2*(-tanh(x)**2 + 1))
```

Mathematica [A] time = 0.00299472, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cosh[x]^2,x]
```

```
[Out] x/2 + Sinh[2*x]/4
```

Maple [A] time = 0.009, size = 11, normalized size = 0.8

$$\frac{x}{2} + \frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2,x)
```

```
[Out] 1/2*x+1/2*cosh(x)*sinh(x)
```

Maxima [A] time = 1.37792, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 0.202795, size = 14, normalized size = 1.

$$\frac{1}{2}\cosh(x)\sinh(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cosh(x)*sinh(x) + 1/2*x
```

Sympy [A] time = 0.194999, size = 24, normalized size = 1.71

$$-\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)**2,x)
```

```
[Out] -x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
```

GIAC/XCAS [A] time = 0.207565, size = 32, normalized size = 2.29

$$-\frac{1}{8}\left(2\,e^{(2\,x)} + 1\right)e^{(-2\,x)} + \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*e^(2*x) + 1)*e^(-2*x) + 1/2*x + 1/8*e^(2*x)
```

3.577 $\int \sinh^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

```
[Out] Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

Rubi [A] time = 0.0190271, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sinh[x]^5,x]
```

```
[Out] Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

Rubi in Sympy [A] time = 4.79018, size = 17, normalized size = 0.89

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sinh(x)**5,x)
```

```
[Out] cosh(x)**5/5 - 2*cosh(x)**3/3 + cosh(x)
```

Mathematica [A] time = 0.00322191, size = 23, normalized size = 1.21

$$\frac{5\cosh(x)}{8}-\frac{5}{48}\cosh(3x)+\frac{1}{80}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sinh[x]^5,x]
```

```
[Out] (5*Cosh[x])/8 - (5*Cosh[3*x])/48 + Cosh[5*x]/80
```

Maple [A] time = 0.053, size = 18, normalized size = 1.

$$\left(\frac{8}{15}+\frac{(\sinh(x))^4}{5}-\frac{4\,(\sinh(x))^2}{15}\right)\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sinh(x)^5,x)
```

```
[Out] (8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)
```

Maxima [A] time = 1.32751, size = 47, normalized size = 2.47

$$\frac{1}{160}\,e^{(5\,x)}-\frac{5}{96}\,e^{(3\,x)}+\frac{5}{16}\,e^{(-x)}-\frac{5}{96}\,e^{(-3\,x)}+\frac{1}{160}\,e^{(-5\,x)}+\frac{5}{16}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/16
0*e^(-5*x) + 5/16*e^x
```

Fricas [A] time = 0.198142, size = 57, normalized size = 3.

$$\frac{1}{80}\,\cosh(x)^5+\frac{1}{16}\,\cosh(x)\sinh(x)^4-\frac{5}{48}\,\cosh(x)^3+\frac{1}{16}\left(2\,\cosh(x)^3-5\,\cosh(x)\right)\sinh(x)^2+\frac{5}{8}\,\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 5/48*cosh(x)^3 + 1/16*(
2*cosh(x)^3 - 5*cosh(x))*sinh(x)^2 + 5/8*cosh(x)
```

Sympy [A] time = 1.64429, size = 29, normalized size = 1.53

$$\sinh^4(x)\cosh(x)-\frac{4\sinh^2(x)\cosh^3(x)}{3}+\frac{8\cosh^5(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)**5,x)
```

```
[Out] sinh(x)**4*cosh(x) - 4*sinh(x)**2*cosh(x)**3/3 + 8*cosh(x)**5/15
```

GIAC/XCAS [A] time = 0.202153, size = 50, normalized size = 2.63

$$\frac{1}{480}\left(150\, e^{(4\, x)} - 25\, e^{(2\, x)} + 3\right) e^{(-5\, x)} + \frac{1}{160}\, e^{(5\, x)} - \frac{5}{96}\, e^{(3\, x)} + \frac{5}{16}\, e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)^5,x, algorithm="giac")
```

```
[Out] 1/480*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 1/160*e^(5*x) - 5
/96*e^(3*x) + 5/16*e^x
```

3.578 $\int \tanh^4(x)\,dx$

Optimal. Leaf size=14

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

```
[Out] x - Tanh[x] - Tanh[x]^3/3
```

Rubi [A] time = 0.0179731, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4,x]

[Out] x - Tanh[x] - Tanh[x]^3/3
```

Rubi in Sympy [A] time = 40.0089, size = 14, normalized size = 1.

$$-\frac{\tanh^3(x)}{3}-\tanh(x)+\operatorname{atanh}(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(tanh(x)**4,x)

[Out] -tanh(x)**3/3 - tanh(x) + atanh(tanh(x))
```

Mathematica [A] time = 0.00551939, size = 18, normalized size = 1.29

$$x-\frac{4\tanh(x)}{3}+\frac{1}{3}\tanh(x)\operatorname{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4,x]
```

```
[Out] x - (4*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3
```

Maple [B] time = 0.017, size = 26, normalized size = 1.9

$$-\frac{(\tanh(x))^3}{3}-\tanh(x)-\frac{\ln(-1+\tanh(x))}{2}+\frac{\ln(1+\tanh(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tanh(x)^4,x)
```

```
[Out] -1/3*tanh(x)^3-tanh(x)-1/2*ln(-1+tanh(x))+1/2*ln(1+tanh(x))
```

Maxima [A] time = 1.3713, size = 51, normalized size = 3.64

$$x-\frac{4\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+2\right)}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)^4,x, algorithm="maxima")
```

```
[Out] x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*x) +
e^(-6*x) + 1)
```

Fricas [A] time = 0.2035, size = 92, normalized size = 6.57

$$\frac{(3\,x+4)\cosh(x)^3+3\,(3\,x+4)\cosh(x)\sinh(x)^2-12\,\cosh(x)^2\sinh(x)-4\,\sinh(x)^3+3\,(3\,x+4)\cosh(x)}{3\left(\cosh(x)^3+3\,\cosh(x)\sinh(x)^2+3\,\cosh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cos
h(x)^2*sinh(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 +
3*cosh(x)*sinh(x)^2 + 3*cosh(x))
```

Sympy [A] time = 0.271181, size = 10, normalized size = 0.71

$$x - \frac{\tanh^3(x)}{3} - \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)**4,x)
```

```
[Out] x - tanh(x)**3/3 - tanh(x)
```

GIAC/XCAS [A] time = 0.20209, size = 35, normalized size = 2.5

$$x + \frac{4\left(3\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)}{3\,\left(e^{(2\,x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)^4,x, algorithm="giac")
```

```
[Out] x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3
```

3.579 $\int \mathrm{csch}^3(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\mathrm{csch}(x)$$

```
[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2
```

Rubi [A] time = 0.02023, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\mathrm{csch}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2
```

Rubi in Sympy [A] time = 4.06, size = 17, normalized size = 1.06

$$\frac{\operatorname{atanh}(\cosh(x))}{2}+\frac{\cosh(x)}{2\left(-\cosh^2(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(csch(x)**3,x)
```

```
[Out] atanh(cosh(x))/2 + cosh(x)/(2*(-cosh(x)**2 + 1))
```

Mathematica [B] time = 0.00619391, size = 47, normalized size = 2.94

$$-\frac{1}{8}\mathrm{csch}^2\left(\frac{x}{2}\right)-\frac{1}{8}\mathrm{sech}^2\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csch[x]^3,x]
```

```
[Out] -Csch[x/2]^2/8 + Log[Cosh[x/2]]/2 - Log[Sinh[x/2]]/2 - Sech[x/2]^
2/8
```

Maple [A] time = 0.059, size = 11, normalized size = 0.7

$$-\frac{\coth(x)\operatorname{csch}(x)}{2} + Artanh\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csch(x)^3,x)
```

```
[Out] -1/2*coth(x)*csch(x)+arctanh(exp(x))
```

Maxima [A] time = 1.32647, size = 61, normalized size = 3.81

$$\frac{e^{(-x)} + e^{(-3x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{2}\log\left(e^{(-x)} + 1\right) - \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x)^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*log(e^(-x)
+ 1) - 1/2*log(e^(-x) - 1)
```

Fricas [A] time = 0.212668, size = 285, normalized size = 17.81

$$-\frac{2\cosh(x)^{3} + 6\cosh(x)\sinh(x)^{2} + 2\sinh(x)^{3} - \left(\cosh(x)^{4} + 4\cosh(x)\sinh(x)^{3} + \sinh(x)^{4} + 2\left(3\cosh(x)^{2} - 1\right)\sinh(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^
4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)
^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(
x) + sinh(x) + 1) + (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
+ 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - co
sh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(3*cosh(x)^2 +
 1)*sinh(x) + 2*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
```

```
x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3
 - cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x)**3,x)
```

```
[Out] Integral(csch(x)**3, x)
```

GIAC/XCAS [A] time = 0.207146, size = 61, normalized size = 3.81

$$-\frac{e^{(-x)} + e^{x}}{\left(e^{(-x)} + e^{x}\right)^{2} - 4} + \frac{1}{4} \ln\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{4} \ln\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csch(x)^3,x, algorithm="giac")
```

```
[Out] -(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + 1/4*ln(e^(-x) + e^x + 2)
 - 1/4*ln(e^(-x) + e^x - 2)
```

3.580 $\int \operatorname{sech}^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{8}\tanh(x)\operatorname{sech}(x)$$

```
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x
])/4
```

Rubi [A] time = 0.0288209, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{8}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x
])/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/cosh(x)**5,x)
```

```
[Out] Integral(cosh(x)**(-5), x)
```

Mathematica [A] time = 0.0110161, size = 30, normalized size = 1.15

$$\frac{3}{4}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{8}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh
[x])/4
```

Maple [A] time = 0.05, size = 21, normalized size = 0.8

$$\left(\frac{(\operatorname{sech}(x))^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)+\frac{3\arctan(e^x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/cosh(x)^5,x)
```

```
[Out] (1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))
```

Maxima [A] time = 1.49697, size = 82, normalized size = 3.15

$$\frac{3\,e^{(-x)}+11\,e^{(-3\,x)}-11\,e^{(-5\,x)}-3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}-\frac{3}{4}\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)^(-5),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*
x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))
```

Fricas [A] time = 0.213171, size = 622, normalized size = 23.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)^(-5),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(
x)^2 + 11)*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x
))*sinh(x)^4 + (105*cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 1
1*cosh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*cosh(x))*sinh(x)
^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x
)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*si
nh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(
x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(
```

```
7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(
x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x
) + 1)*arctan(cosh(x) + sinh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 -
 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh(x))/(cosh(x)^8 + 8*cosh(x)*si
nh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6
 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*c
osh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(
x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*c
osh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^
5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [A] time = 8.57482, size = 422, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/cosh(x)**5,x)
```

```
[Out] 3*tanh(x/2)**8*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6
+ 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 5*tanh(x/2)**7/(4*tanh
(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 +
4) + 12*tanh(x/2)**6*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/
2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*tanh(x/2)**5/(
4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)
**2 + 4) + 18*tanh(x/2)**4*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*t
anh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 3*tanh(x/2
)**3/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tan
h(x/2)**2 + 4) + 12*tanh(x/2)**2*atan(tanh(x/2))/(4*tanh(x/2)**8
+ 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 5*ta
nh(x/2)/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*
tanh(x/2)**2 + 4) + 3*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
```

GIAC/XCAS [A] time = 0.229546, size = 81, normalized size = 3.12

$$\frac{3}{16}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^{3} + 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{2}} + \frac{3}{8}\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)^(-5),x, algorithm="giac")
```

```
[Out] 3/16*pi - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x)
- e^x)^2 + 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.581 $\int \sinh^4(x) \tanh(x) \, dx$

Optimal. Leaf size=18

$$\frac{\cosh^4(x)}{4} - \cosh^2(x) + \log(\cosh(x))$$

```
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Rubi [A] time = 0.0424013, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{\cosh^4(x)}{4} - \cosh^2(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sinh[x]^4*Tanh[x],x]

[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tanh(x)**5/sech(x)**4,x)

[Out] Timed out
```

Mathematica [A] time = 0.00704923, size = 20, normalized size = 1.11

$$-\frac{3}{8}\cosh(2x) + \frac{1}{32}\cosh(4x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sinh[x]^4*Tanh[x],x]

[Out] (-3*Cosh[2*x])/8 + Cosh[4*x]/32 + Log[Cosh[x]]
```

Maple [A] time = 0.018, size = 17, normalized size = 0.9

$$\frac{(\sinh(x))^4}{4} - \frac{(\sinh(x))^2}{2} + \ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tanh(x)^5/sech(x)^4,x)
```

```
[Out] 1/4*sinh(x)^4-1/2*sinh(x)^2+ln(cosh(x))
```

Maxima [A] time = 1.5143, size = 47, normalized size = 2.61

$$-\frac{1}{64}\left(12\,e^{(-2\,x)} - 1\right)e^{(4\,x)} + x - \frac{3}{16}\,e^{(-2\,x)} + \frac{1}{64}\,e^{(-4\,x)} + \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*x) - 1)*e^(4*x) + x - 3/16*e^(-2*x) + 1/64*e^(-4*
x) + log(e^(-2*x) + 1)
```

Fricas [A] time = 0.2254, size = 347, normalized size = 19.28

$$\frac{\cosh(x)^8 + 8\,\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\,\cosh(x)^2 - 3\right)\sinh(x)^6 - 12\,\cosh(x)^6 + 8\left(7\,\cosh(x)^3 - 9\,\cosh(x)\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^
2 - 3)*sinh(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sin
h(x)^5 - 64*x*cosh(x)^4 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 - 32*x)*
sinh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(x)^3 - 32*x*cosh(x))*sinh(x)
^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 - 96*x*cosh(x)^2 - 3)*sinh(x)^
2 - 12*cosh(x)^2 + 64*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)*log(2*cosh(x)/(c
osh(x) - sinh(x))) + 8*(cosh(x)^7 - 9*cosh(x)^5 - 32*x*cosh(x)^3
- 3*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*co
sh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\operatorname{sech}^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)**5/sech(x)**4,x)
```

```
[Out] Integral(tanh(x)**5/sech(x)**4, x)
```

GIAC/XCAS [A] time = 0.207452, size = 58, normalized size = 3.22

$$\frac{1}{64}\left(48\,e^{(4\,x)} - 12\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - x + \frac{1}{64}\,e^{(4\,x)} - \frac{3}{16}\,e^{(2\,x)} + \ln\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tanh(x)^5/sech(x)^4,x, algorithm="giac")
```

```
[Out] 1/64*(48*e^(4*x) - 12*e^(2*x) + 1)*e^(-4*x) - x + 1/64*e^(4*x) -
3/16*e^(2*x) + ln(e^(2*x) + 1)
```

3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$

Optimal. Leaf size=31

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

```
[Out] (-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))
/19
```

Rubi [A] time = 0.0525972, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] (-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))
/19
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0659674, size = 27, normalized size = 0.87

$$\operatorname{sech}^{\frac{3}{4}}(x)\left(-\frac{4}{19}\operatorname{sech}^4(x)+\frac{8\operatorname{sech}^2(x)}{11}-\frac{4}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^{\frac{3}{4}} (\tanh(x))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sech(x)^(3/4)*tanh(x)^5,x)
```

```
[Out] int(sech(x)^(3/4)*tanh(x)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \tanh(x)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

Fricas [A] time = 0.219261, size = 485, normalized size = 15.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -4/627*2^(3/4)*(209*cosh(x)^8 + 1672*cosh(x)*sinh(x)^7 + 209*sinh
(x)^8 + 76*(77*cosh(x)^2 + 5)*sinh(x)^6 + 380*cosh(x)^6 + 152*(77
*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(1463*cosh(x)^4 + 570*cos
h(x)^2 + 87)*sinh(x)^4 + 870*cosh(x)^4 + 8*(1463*cosh(x)^5 + 950*
cosh(x)^3 + 435*cosh(x))*sinh(x)^3 + 4*(1463*cosh(x)^6 + 1425*cos
h(x)^4 + 1305*cosh(x)^2 + 95)*sinh(x)^2 + 380*cosh(x)^2 + 8*(209*
cosh(x)^7 + 285*cosh(x)^5 + 435*cosh(x)^3 + 95*cosh(x))*sinh(x) +
 209)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 + 1))^(3/4)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4
*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*c
```

```
osh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4
 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(
x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2
 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(
x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \tanh(x)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTanh[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0843094, antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Cosh[x])^(-1),x]

[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*cosh(x)),x)

[Out] Integral(1/(a + b*cosh(x)), x)
```

Mathematica [A] time = 0.0379954, size = 41, normalized size = 1.

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Cosh[x])^(-1),x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
]
```

Maple [A] time = 0.017, size = 36, normalized size = 0.9

$$2\,{\frac {1}{\sqrt {(a+b)(a-b)}}{\it Artanh} \left({\frac {(a-b)\tanh(x/2)}{\sqrt {(a+b)(a-b)}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cosh(x) + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226393, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\,a^{3}-2\,ab^{2}+2\,\left(a^{2}b-b^{3}\right)\,\cosh(x)+2\,\left(a^{2}b-b^{3}\right)\,\sinh(x)-\left(b^{2}\,\cosh(x)^{2}+b^{2}\,\sinh(x)^{2}+2\,ab\,\cosh(x)+2\,a^{2}-b^{2}+2\,\left(b^{2}\,\cosh(x)+ab\right)\,\sinh(x)\right)\sqrt{a^{2}-b^{2}}}{b\,\cosh(x)^{2}+b\,\sinh(x)^{2}+2\,a\,\cosh(x)+2\,(b\,\cosh(x)+a)\,\sinh(x)+b}\right)}{\sqrt{a^{2}-b^{2}}},\ \frac{2\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cosh(x) + a),x, algorithm="fricas")
```

```
[Out] [log(-(2*a^3 - 2*a*b^2 + 2*(a^2*b - b^3)*cosh(x) + 2*(a^2*b - b^3
)*sinh(x) - (b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^
```

```
2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))*sqrt(a^2 - b^2))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b
))/sqrt(a^2 - b^2), 2*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a)/(a^2 - b^2))/sqrt(-a^2 + b^2)]
```

Sympy [A] time = 20.1161, size = 126, normalized size = 3.07

$$\begin{cases} \tilde{\infty} \operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for}\ a = 0 \wedge b = 0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for}\ a = b \\ -\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for}\ a = -b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2)/
b, Eq(a, b)), (-1/(b*tanh(x/2)), Eq(a, -b)), (-log(-sqrt(a/(a - b
) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sq
rt(a/(a - b) + b/(a - b))) + log(sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a
 - b))), True))
```

GIAC/XCAS [A] time = 0.215767, size = 43, normalized size = 1.05

$$\frac{2\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cosh(x) + a),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.584 $\int \frac{1}{(1+\cosh(x))^2} \, dx$

Optimal. Leaf size=25

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

```
[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rubi [A] time = 0.0275973, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cosh[x])^(-2),x]

[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\cosh(x)+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+cosh(x))**2,x)

[Out] Integral((cosh(x) + 1)**(-2), x)
```

Mathematica [A] time = 0.0189139, size = 16, normalized size = 0.64

$$\frac{\sinh(x)(\cosh(x)+2)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cosh[x])^(-2),x]
```

```
[Out] ((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

Maple [A] time = 0.01, size = 16, normalized size = 0.6

$$-\frac{1}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{1}{2}\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+cosh(x))^2,x)
```

```
[Out] -1/6*tanh(1/2*x)^3+1/2*tanh(1/2*x)
```

Maxima [A] time = 1.35703, size = 66, normalized size = 2.64

$$\frac{2\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\,\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x) + 1)^(-2),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) +
 3*e^(-2*x) + e^(-3*x) + 1)
```

Fricas [A] time = 0.204283, size = 78, normalized size = 3.12

$$-\frac{2\,(3\,\cosh(x)+3\,\sinh(x)+1)}{3\,\left(\cosh(x)^3+3\,(\cosh(x)+1)\sinh(x)^2+\sinh(x)^3+3\,\cosh(x)^2+3\,\left(\cosh(x)^2+2\,\cosh(x)+1\right)\sinh(x)+3\,\cosh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x) + 1)^(-2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x) + 3*sinh(x) + 1)/(cosh(x)^3 + 3*(cosh(x) + 1)*sin
h(x)^2 + sinh(x)^3 + 3*cosh(x)^2 + 3*(cosh(x)^2 + 2*cosh(x) + 1)*
sinh(x) + 3*cosh(x) + 1)
```

Sympy [A] time = 0.623633, size = 14, normalized size = 0.56

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+cosh(x))**2,x)
```

```
[Out] -tanh(x/2)**3/6 + tanh(x/2)/2
```

GIAC/XCAS [A] time = 0.22186, size = 19, normalized size = 0.76

$$-\frac{2\left(3\,e^{x}+1\right)}{3\left(e^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x) + 1)^(-2),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^x + 1)/(e^x + 1)^3
```

3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0790553, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi in Sympy [A] time = 59.3487, size = 42, normalized size = 1.08

$$-\frac{b\log\left(a+b\tanh\left(x\right)\right)}{a^2-b^2}-\frac{\log\left(-\tanh\left(x\right)+1\right)}{2\left(a+b\right)}+\frac{\log\left(\tanh\left(x\right)+1\right)}{2\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*tanh(x)),x)
```

```
[Out] -b*log(a + b*tanh(x))/(a**2 - b**2) - log(-tanh(x) + 1)/(2*(a + b
)) + log(tanh(x) + 1)/(2*(a - b))
```

Mathematica [A] time = 0.0620722, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.024, size = 55, normalized size = 1.4

$$-\frac{b\ln\left(a+b\tanh\left(x\right)\right)}{\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(1+\tanh\left(x\right)\right)}{2\,a-2\,b}-\frac{\ln\left(-1+\tanh\left(x\right)\right)}{2\,a+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)),x)
```

```
[Out] -b/(a+b)/(a-b)*ln(a+b*tanh(x))+1/(2*a-2*b)*ln(1+tanh(x))-1/(2*a+2
*b)*ln(-1+tanh(x))
```

Maxima [A] time = 1.36427, size = 55, normalized size = 1.41

$$-\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(x) + a),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 0.223995, size = 57, normalized size = 1.46

$$\frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(x) + a),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x)))
)/(a^2 - b^2)
```

Sympy [A] time = 1.14998, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}\left(x - \log\left(\tanh\left(x\right)+1\right) + \log\left(\tanh\left(x\right)\right)\right) & \text{for } a = 0 \wedge b = 0 \\ -\frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)-2b} + \frac{x}{2b\tanh\left(x\right)-2b} + \frac{1}{2b\tanh\left(x\right)-2b} & \text{for } a = -b \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)+2b} + \frac{x}{2b\tanh\left(x\right)+2b} - \frac{1}{2b\tanh\left(x\right)+2b} & \text{for } a = b \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} - \frac{b\log\left(\frac{a}{b}+\tanh\left(x\right)\right)}{a^2-b^2} + \frac{b\log\left(\tanh\left(x\right)+1\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) &
Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b
) + 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) +
2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)),
(x/a, Eq(b, 0)), (a*x/(a**2 - b**2) - b*x/(a**2 - b**2) - b*log(a
/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2), T
rue))
```

GIAC/XCAS [A] time = 0.211538, size = 58, normalized size = 1.49

$$-\frac{b\ln\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*tanh(x) + a),x, algorithm="giac")
```

```
[Out] -b*ln(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.586 $\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.061871, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{b\cosh(x)-\sqrt{-a^2}}\,dx}{2\sqrt{-a^2}} - \frac{\int \frac{1}{b\cosh(x)+\sqrt{-a^2}}\,dx}{2\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+b**2*cosh(x)**2),x)
```

```
[Out] Integral(1/(b*cosh(x) - sqrt(-a**2)), x)/(2*sqrt(-a**2)) - Integr
al(1/(b*cosh(x) + sqrt(-a**2)), x)/(2*sqrt(-a**2))
```

Mathematica [A] time = 0.0417095, size = 31, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Maple [B] time = 0.062, size = 98, normalized size = 3.2

$$\frac{1}{2a}\ln\left(\sqrt{a^2+b^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2+2a\tanh(x/2)+\sqrt{a^2+b^2}\right)\frac{1}{\sqrt{a^2+b^2}}-\frac{1}{2a}\ln\left(\sqrt{a^2+b^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2-2a\tanh(x/2)+\sqrt{a^2+b^2}\right)\frac{1}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+b^2*cosh(x)^2),x)
```

```
[Out] 1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1
/2*x)+(a^2+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*t
anh(1/2*x)^2-2*a*tanh(1/2*x)+(a^2+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cosh(x)^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221579, size = 431, normalized size = 13.9

$$\frac{\log\left(-\frac{8a^5+12a^3b^2+4ab^4+4\left(a^3b^2+ab^4\right)\cosh(x)^2+8\left(a^3b^2+ab^4\right)\cosh(x)\sinh(x)+4\left(a^3b^2+ab^4\right)\sinh(x)^2-\left(b^4\cosh(x)^4+4b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8a^4\cdots\right.}{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2a^2+b^2\right)\cosh(x)^2+2\left(3b^2\cosh(x)^2+\cdots\right.}\cdots\right.}{2\sqrt{a^2+b^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cosh(x)^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(8*a^5 + 12*a^3*b^2 + 4*a*b^4 + 4*(a^3*b^2 + a*b^4)*cosh
(x)^2 + 8*(a^3*b^2 + a*b^4)*cosh(x)*sinh(x) + 4*(a^3*b^2 + a*b^4)
```

```
*sinh(x)^2 - (b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(
x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 +
2*(3*b^4*cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^
3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2))/(b^2*cos
h(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2
)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a^2 + b^2)*sinh(x)^2 + b^2 +
 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*cosh(x))*sinh(x)))/(sqrt(a^2 +
b^2)*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+b**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.237243, size = 107, normalized size = 3.45

$$\frac{\ln\left(\frac{b^2e^{(2x)}+2a^2+b^2-2\sqrt{a^2+b^2}|a|}{b^2e^{(2x)}+2a^2+b^2+2\sqrt{a^2+b^2}|a|}\right)}{2\sqrt{a^2+b^2}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cosh(x)^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*ln((b^2*e^(2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*abs(a))/(b^
2*e^(2*x) + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 +
b^2)*abs(a))
```

3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.0654445, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a-b\cosh(x)}\,dx}{2a}+\frac{\int \frac{1}{a+b\cosh(x)}\,dx}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2-b**2*cosh(x)**2),x)
```

```
[Out] Integral(1/(a - b*cosh(x)), x)/(2*a) + Integral(1/(a + b*cosh(x))
, x)/(2*a)
```

Mathematica [A] time = 0.0423853, size = 35, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Maple [B] time = 0.034, size = 74, normalized size = 2.1

$$\frac{1}{a}Artanh\left((a-b)\tanh\left(\frac{x}{2}\right)\frac{1}{\sqrt{(a+b)(a-b)}}\right)\frac{1}{\sqrt{(a+b)(a-b)}} + \frac{1}{a}Artanh\left((a+b)\tanh\left(\frac{x}{2}\right)\frac{1}{\sqrt{(a+b)(a-b)}}\right)\frac{1}{\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2-b^2*cosh(x)^2),x)
```

```
[Out] 1/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(
1/2))+1/a/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a
-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b^2*cosh(x)^2 - a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234597, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{8\,a^5-12\,a^3b^2+4\,ab^4-4\left(a^3b^2-ab^4\right)\cosh(x)^2-8\left(a^3b^2-ab^4\right)\cosh(x)\sinh(x)-4\left(a^3b^2-ab^4\right)\sinh(x)^2-\left(b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\cdots\right.}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4-2\left(2\,a^2-b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)\cdots\right.}\right.}{2\sqrt{a^2-b^2}a}\cdots, -\frac{\arctan\left(-\frac{\left(b^2\cosh(x)^2+2\,b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2-2\,a^2+b^2\right)\sqrt{-a^2+b^2}}{2\left(a^3-ab^2\right)}\right)}{\sqrt{-a^2+b^2}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b^2*cosh(x)^2 - a^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a^5 - 12*a^3*b^2 + 4*a*b^4 - 4*(a^3*b^2 - a*b^4)*cos
h(x)^2 - 8*(a^3*b^2 - a*b^4)*cosh(x)*sinh(x) - 4*(a^3*b^2 - a*b^4
)*sinh(x)^2 - (b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh
(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 +
 2*(3*b^4*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)
^3 - (2*a^2*b^2 - b^4)*cosh(x))*sinh(x))*sqrt(a^2 - b^2))/(b^2*co
sh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^
2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 - 2*a^2 + b^2)*sinh(x)^2 + b^2
+ 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*cosh(x))*sinh(x)))/(sqrt(a^2 -
 b^2)*a), -arctan(-1/2*(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b
^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)/(a^3 - a*b^2))/(sqrt
(-a^2 + b^2)*a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2-b**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21404, size = 68, normalized size = 1.94

$$-\frac{\arctan\left(\frac{b^2e^{(2x)}-2a^2+b^2}{2\sqrt{-a^2+b^2}a}\right)}{\sqrt{-a^2+b^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b^2*cosh(x)^2 - a^2),x, algorithm="giac")
```

```
[Out] -arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sq
rt(-a^2 + b^2)*a)
```

3.588 $\int \frac{1}{1-\sinh^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rubi [A] time = 0.0353424, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{-\sinh(x)-1}\,dx}{4}+\frac{i\int \frac{1}{-\sinh(x)+i}\,dx}{4}-\frac{\int \frac{1}{\sinh(x)-1}\,dx}{4}+\frac{i\int \frac{1}{\sinh(x)+i}\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-sinh(x)**4),x)
```

```
[Out] -Integral(1/(-sinh(x) - 1), x)/4 + I*Integral(1/(-sinh(x) + I), x
)/4 - Integral(1/(sinh(x) - 1), x)/4 + I*Integral(1/(sinh(x) + I)
, x)/4
```

Mathematica [A] time = 0.0555097, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

Maple [B] time = 0.032, size = 55, normalized size = 2.2

$$1\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-1}+\frac{\sqrt{2}}{4}Artanh\left(\frac{\sqrt{2}}{4}\left(2\tanh\left(x/2\right)+2\right)\right)+\frac{\sqrt{2}}{4}Artanh\left(\frac{\sqrt{2}}{4}\left(2\tanh\left(x/2\right)-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-sinh(x)^4),x)
```

```
[Out] tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2
*x)+2)*2^(1/2))+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2)
)
```

Maxima [A] time = 1.505, size = 99, normalized size = 3.96

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-e^{(-x)}+1\right)}{2\sqrt{2}+2e^{(-x)}-2}\right)-\frac{1}{8}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-e^{(-x)}-1\right)}{2\sqrt{2}+2e^{(-x)}+2}\right)+\frac{1}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sinh(x)^4 - 1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-2*(sqrt(2) - e^(-x) + 1)/((2*sqrt(2)) + 2*e^(-x)
 - 2)) - 1/8*sqrt(2)*log(-2*(sqrt(2) - e^(-x) - 1)/((2*sqrt(2)) +
 2*e^(-x) + 2)) + 1/(e^(-2*x) + 1)
```

Fricas [A] time = 0.220252, size = 157, normalized size = 6.28

$$\frac{\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\log\left(\frac{3\left(3\sqrt{2}-4\right)\cosh(x)^{2}-8\left(2\sqrt{2}-3\right)\cosh(x)\sinh(x)+3\left(3\sqrt{2}-4\right)\sinh(x)^{2}-3\sqrt{2}+4}{\cosh(x)^{2}+\sinh(x)^{2}-3}\right)-4\sqrt{2}}{4\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sinh(x)^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((3*(3*sq
rt(2) - 4)*cosh(x)^2 - 8*(2*sqrt(2) - 3)*cosh(x)*sinh(x) + 3*(3*s
qrt(2) - 4)*sinh(x)^2 - 3*sqrt(2) + 4)/(cosh(x)^2 + sinh(x)^2 - 3
)) - 4*sqrt(2))/(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) +
sqrt(2)*sinh(x)^2 + sqrt(2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-sinh(x)**4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.201693, size = 65, normalized size = 2.6

$$-\frac{1}{8}\sqrt{2}\ln\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{1}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sinh(x)^4 - 1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*ln(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2
*e^(2*x) - 6)) - 1/(e^(2*x) + 1)
```

3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3(\tanh(x)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh
[x]))
```

Rubi [A] time = 0.225939, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{1}{3(\tanh(x)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh
[x]))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0667827, size = 37, normalized size = 1.12

$$\frac{1}{18}\left(3\sinh(2x)-3\cosh(2x)+8\sqrt{3}\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sin
h[2*x])/18
```

Maple [C] time = 0.116, size = 78, normalized size = 2.4

$$-\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{2i}{9}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2i}{9}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\left(-1+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x)
```

```
[Out] -2/3/(tanh(1/2*x)+1)^2+2/3/(tanh(1/2*x)+1)+2/9*I*3^(1/2)*ln(tanh(
1/2*x)^2+(-1-I*3^(1/2))*tanh(1/2*x)+1)-2/9*I*3^(1/2)*ln(tanh(1/2*
x)^2+(-1+I*3^(1/2))*tanh(1/2*x)+1)
```

Maxima [A] time = 1.52357, size = 95, normalized size = 2.88

$$\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{6}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4
)*sqrt(2))) - 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e
^(-x) - 3^(1/4)*sqrt(2))) - 1/6*e^(-2*x)
```

Fricas [A] time = 0.229257, size = 103, normalized size = 3.12

$$-\frac{8\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}\right)\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)+\sqrt{3}}{6\left(\sqrt{3}\cosh(x)^{2}+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*arctan(-1/3*(
sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + sqrt(3)
)/(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x
)^2)
```

Sympy [A] time = 4.66666, size = 102, normalized size = 3.09

$$\frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{9\sinh(x)+9\cosh(x)}+\frac{3\sinh(x)}{9\sinh(x)+9\cosh(x)}+\frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{9\sinh(x)+9\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))
/(9*sinh(x) + 9*cosh(x)) + 3*sinh(x)/(9*sinh(x) + 9*cosh(x)) + 4*
sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(
9*sinh(x) + 9*cosh(x))
```

GIAC/XCAS [A] time = 0.205021, size = 30, normalized size = 0.91

$$\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)-\frac{1}{6}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x, algorithm="giac")
```

```
[Out] 4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6*e^(-2*x)
```

3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Rubi [A] time = 0.0563583, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\operatorname{atanh}(\tanh(3x))}{12}+\int\cosh(2x)\,dx+\frac{\int\cosh(4x)\,dx}{2}+\frac{\tanh(3x)}{12\left(-\tanh^2(3x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] atanh(tanh(3*x))/12 + Integral(cosh(2*x), x) + Integral(cosh(4*x)
, x)/2 + tanh(3*x)/(12*(-tanh(3*x)**2 + 1))
```

Mathematica [A] time = 0.0153374, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Maple [A] time = 0.058, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sinh(2x)}{8}+\frac{\sinh(4x)}{16}+\frac{\sinh(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

Maxima [A] time = 1.3573, size = 57, normalized size = 1.9

$$\frac{1}{96}\left(3\,e^{(-2\,x)}+6\,e^{(-4\,x)}+2\right)e^{(6\,x)}+\frac{1}{4}\,x-\frac{1}{16}\,e^{(-2\,x)}-\frac{1}{32}\,e^{(-4\,x)}-\frac{1}{48}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*cosh(2*x)*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x
) - 1/32*e^(-4*x) - 1/48*e^(-6*x)
```

Fricas [A] time = 0.209451, size = 59, normalized size = 1.97

$$\frac{1}{4}\cosh(x)\sinh(x)^5+\frac{1}{12}\left(10\cosh(x)^3+3\cosh(x)\right)\sinh(x)^3+\frac{1}{4}\left(\cosh(x)^5+\cosh(x)^3+\cosh(x)\right)\sinh(x)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*cosh(2*x)*cosh(x),x, algorithm="fricas")
```

```
[Out] 1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
 + 1/4*(cosh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x
```

Sympy [A] time = 21.7791, size = 114, normalized size = 3.8

$$\frac{x\sinh(x)\sinh(2x)\cosh(3x)}{4} - \frac{x\sinh(x)\sinh(3x)\cosh(2x)}{4} - \frac{x\sinh(2x)\sinh(3x)\cosh(x)}{4} + \frac{x\cosh(x)\cosh(2x)\cosh(3x)}{4} - \frac{\sinh(x)\cosh(2x)\cosh(3x)}{24} - \frac{\sinh(2x)\cosh(x)\cosh(3x)}{6} + \frac{3\sinh(3x)\cosh(x)\cosh(2x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4
 - x*sinh(2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x
)/4 - sinh(x)*cosh(2*x)*cosh(3*x)/24 - sinh(2*x)*cosh(x)*cosh(3*x
)/6 + 3*sinh(3*x)*cosh(x)*cosh(2*x)/8
```

GIAC/XCAS [A] time = 0.199502, size = 65, normalized size = 2.17

$$-\frac{1}{96}\left(22\,e^{(6\,x)} + 6\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{4}\,x + \frac{1}{48}\,e^{(6\,x)} + \frac{1}{32}\,e^{(4\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(3*x)*cosh(2*x)*cosh(x),x, algorithm="giac")
```

```
[Out] -1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)*e^(-6*x) + 1/4*x +
 1/48*e^(6*x) + 1/32*e^(4*x) + 1/16*e^(2*x)
```

3.591 $\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$

Optimal. Leaf size=30

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Rubi [A] time = 0.0551376, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\operatorname{atanh}\left(\tanh\left(\frac{3x}{2}\right)\right)}{6}-\int\cosh(2x)\,dx-\frac{\int\cosh(5x)\,dx}{2}+\frac{\tanh\left(\frac{3x}{2}\right)}{6\left(-\tanh^2\left(\frac{3x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] atanh(tanh(3*x/2))/6 - Integral(cosh(2*x), x) - Integral(cosh(5*x
), x)/2 + tanh(3*x/2)/(6*(-tanh(3*x/2)**2 + 1))
```

Mathematica [A] time = 0.0158769, size = 30, normalized size = 1.

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Maple [A] time = 0.138, size = 23, normalized size = 0.8

$$-\frac{x}{4}+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] -1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

Maxima [A] time = 1.37259, size = 57, normalized size = 1.9

$$-\frac{1}{240}\left(10\,e^{(-2\,x)}-15\,e^{(-3\,x)}-6\right)e^{(5\,x)}-\frac{1}{4}\,x-\frac{1}{16}\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}-\frac{1}{40}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(3/2*x)*sinh(5/2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(
-2*x) + 1/24*e^(-3*x) - 1/40*e^(-5*x)
```

Fricas [A] time = 0.212649, size = 150, normalized size = 5.

$$\begin{aligned}&6\,\cosh\left(\frac{1}{2}\,x\right)^{3}\,\sinh\left(\frac{1}{2}\,x\right)^{7}+\frac{1}{2}\,\cosh\left(\frac{1}{2}\,x\right)\,\sinh\left(\frac{1}{2}\,x\right)^{9}\\&+\frac{1}{10}\left(126\,\cosh\left(\frac{1}{2}\,x\right)^{5}-5\,\cosh\left(\frac{1}{2}\,x\right)\right)\sinh\left(\frac{1}{2}\,x\right)^{5}\\&+\frac{1}{6}\left(36\,\cosh\left(\frac{1}{2}\,x\right)^{7}-10\,\cosh\left(\frac{1}{2}\,x\right)^{3}+3\,\cosh\left(\frac{1}{2}\,x\right)\right)\sinh\left(\frac{1}{2}\,x\right)^{3}\\&+\frac{1}{2}\left(\cosh\left(\frac{1}{2}\,x\right)^{9}-\cosh\left(\frac{1}{2}\,x\right)^{5}+\cosh\left(\frac{1}{2}\,x\right)^{3}\right)\sinh\left(\frac{1}{2}\,x\right)-\frac{1}{4}\,x\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(3/2*x)*sinh(5/2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 6*cosh(1/2*x)^3*sinh(1/2*x)^7 + 1/2*cosh(1/2*x)*sinh(1/2*x)^9 + 1
/10*(126*cosh(1/2*x)^5 - 5*cosh(1/2*x))*sinh(1/2*x)^5 + 1/6*(36*c
osh(1/2*x)^7 - 10*cosh(1/2*x)^3 + 3*cosh(1/2*x))*sinh(1/2*x)^3 +
1/2*(cosh(1/2*x)^9 - cosh(1/2*x)^5 + cosh(1/2*x)^3)*sinh(1/2*x) -
 1/4*x
```

Sympy [A] time = 20.4118, size = 139, normalized size = 4.63

$$-\frac{x\sinh(x)\sinh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4}+\frac{x\sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)}{4}+\frac{x\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\cosh(x)}{4}-\frac{x\cosh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4}+\frac{4\sinh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{15}-\frac{3\sinh\left(\frac{3x}{2}\right)\cosh(x)\cosh\left(\frac{5x}{2}\right)}{20}+\frac{\sinh\left(\frac{5x}{2}\right)\cosh(x)\cosh\left(\frac{3x}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] -x*sinh(x)*sinh(3*x/2)*cosh(5*x/2)/4 + x*sinh(x)*sinh(5*x/2)*cosh
(3*x/2)/4 + x*sinh(3*x/2)*sinh(5*x/2)*cosh(x)/4 - x*cosh(x)*cosh(
3*x/2)*cosh(5*x/2)/4 + 4*sinh(x)*cosh(3*x/2)*cosh(5*x/2)/15 - 3*s
inh(3*x/2)*cosh(x)*cosh(5*x/2)/20 + sinh(5*x/2)*cosh(x)*cosh(3*x/
2)/12
```

GIAC/XCAS [A] time = 0.202078, size = 65, normalized size = 2.17

$$\frac{1}{240}\left(137\,e^{(5\,x)}-15\,e^{(3\,x)}+10\,e^{(2\,x)}-6\right)e^{(-5\,x)}-\frac{1}{4}\,x+\frac{1}{40}\,e^{(5\,x)}-\frac{1}{24}\,e^{(3\,x)}+\frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(3/2*x)*sinh(5/2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/240*(137*e^(5*x) - 15*e^(3*x) + 10*e^(2*x) - 6)*e^(-5*x) - 1/4*
x + 1/40*e^(5*x) - 1/24*e^(3*x) + 1/16*e^(2*x)
```

3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right)+\frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\sqrt{2}\tan^{-1}\left(\operatorname{sech}(x)\sqrt{\sinh(x)\cosh(x)}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\operatorname{sech}(x)\sqrt{\sinh(x)\cosh(x)}\right)$$

```
[Out] Sqrt[2]*ArcTan[Sech[x]*Sqrt[Cosh[x]*Sinh[x]]] + ArcTan[Sinh[x]/Sq
rt[Sinh[2*x]]]/6 - (Sqrt[2]*ArcTanh[Sech[x]*Sqrt[Cosh[x]*Sinh[x]]
])/3 + Cosh[x]/Sqrt[Sinh[2*x]]
```

Rubi [A] time = 1.58, antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$-\frac{2\sinh(x)\tanh^{-1}\left(\sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}+\frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\frac{2\sinh(x)\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}+\frac{\sinh(x)\tan^{-1}\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*x])),x]
```

```
[Out] Cosh[x]/Sqrt[Sinh[2*x]] + (2*ArcTan[Sqrt[Tanh[x]]]*Sinh[x])/(Sqrt
[Sinh[2*x]]*Sqrt[Tanh[x]]) + (ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]*Sinh[
x])/(3*Sqrt[2]*Sqrt[Sinh[2*x]]*Sqrt[Tanh[x]]) - (2*ArcTanh[Sqrt[T
anh[x]]]*Sinh[x])/(3*Sqrt[Sinh[2*x]]*Sqrt[Tanh[x]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**
```

```
[Out] Timed out
```

Mathematica [C] time = 31.186, size = 487, normalized size = 7.06

$$-\frac{\sqrt{\sinh(2x)}\coth(x)(\tanh(x)-\cosh(2x))}{-2\sinh(x)+\cosh(x)+\cosh(3x)} + \frac{\cosh(x)(\tanh(x)-\cosh(2x))\left(\frac{16(-1)^{5/12}\sinh^{\frac{3}{2}}(2x)\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}\left(2\left(\sqrt[3]{-1}-1\right)\left(i;\sin^{-1}\left((-1)^{3/4}\sqrt{\tanh\left(\frac{x}{2}\right)}\right)\middle|-1\right)+\left(3-3i\sqrt{3}\right)\left(-i;i\sinh^{-1}\left(\sqrt[4]{-\cdots}\right.\right.\right.}{3\left(\sqrt{3}-i\right)(\cosh(x)+1)^3\sqrt{\tanh\cdots}}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2
```

```
[Out] -((Coth[x]*Sqrt[Sinh[2*x]]*(-Cosh[2*x] + Tanh[x]))/(Cosh[x] + Cos
h[3*x] - 2*Sinh[x])) + (Cosh[x]*((-6*(-1)^(1/4)*Sqrt[1 + Coth[x/2
]^2]*(EllipticF[I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1] - Elli
pticPi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1] -
EllipticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1
])*Sqrt[Sinh[2*x]]*Sqrt[Tanh[x/2]]*Sqrt[Tanh[x/2] + Tanh[x/2]^3])
/((1 + Cosh[x])*Sqrt[Sinh[2*x]/(1 + Cosh[x])^2]*(1 + Tanh[x/2]^2)
) + (16*(-1)^(5/12)*((3 - (3*I)*Sqrt[3])*EllipticPi[-I, I*ArcSinh
[(-1)^(1/4)*Sqrt[Tanh[x/2]]], -1] + 2*(-1 + (-1)^(1/3))*EllipticP
i[I, ArcSin[(-1)^(3/4)*Sqrt[Tanh[x/2]]], -1] + I*(I + Sqrt[3])*El
lipticPi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)*Sqrt[Tanh[x/2]]], -1]
+ 2*(-1 + (-1)^(1/3))*EllipticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(1/4
)*Sqrt[Tanh[x/2]]], -1])*Sinh[2*x]^(3/2)*Sqrt[Tanh[x/2] + Tanh[x/
2]^3])/(3*(-I + Sqrt[3])*(1 + Cosh[x])^3*(Sinh[2*x]/(1 + Cosh[x])
^2)^(3/2)*Sqrt[Tanh[x/2]]*Sqrt[1 + Tanh[x/2]^2]))*(-Cosh[2*x] + T
anh[x]))/(2*(Cosh[x] + Cosh[3*x] - 2*Sinh[x]))
```

Maple [C] time = 0.299, size = 609, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x)
```

```
[Out] -1/12*((tanh(1/2*x)^2+1)*tanh(1/2*x)/(tanh(1/2*x)^2-1)^2)^(1/2)*(
tanh(1/2*x)^2-1)*(12*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-t
anh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1
/2*x)+I))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/
2*x))^(1/2)-9*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2
*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticF((-I*(tanh(1/2*x)+I)
)^(1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-4*I*(-
I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*t
```

```
anh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-12*(-I*(tanh(
1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*
x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-4*(-I*(tanh(1/2*x)+I)
)^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)
*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((t
anh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-I*2^(1/2)*sum(_alpha*(I*_alpha
+1+I)*(-I*(tanh(1/2*x)+I))^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*t
anh(1/2*x))^(1/2)/((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*EllipticP
i((-I*(tanh(1/2*x)+I))^(1/2),_alpha+1-I,1/2*2^(1/2)),_alpha=RootO
f(_Z^2+_Z+1))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*(tanh(1/2*x)^
3+tanh(1/2*x))^(1/2)+6*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)*tanh(1/2
*x)^2+6*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2))/(tanh(1/2*x)^2+1)/tanh
(1/2*x)/(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(\cosh(2x) - \tanh(x))\cosh(x)}{\left(\sinh(x)^2 + \sinh(2x)\right)\sqrt{\sinh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh(2*x
```

```
[Out] -integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))
*sqrt(sinh(2*x))), x)
```

Fricas [A] time = 0.24017, size = 405, normalized size = 5.87

$$-\frac{\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\arctan\left(\frac{\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + 3\sqrt{2}}{8\sqrt{\frac{\cosh(x)\sinh(x)}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}}}\right) + 6\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh(2*x
```

```
[Out] -1/12*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(1/8
*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)
^2 + 3*sqrt(2))/sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))) + 6*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh
(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*arctan(1/2/sqrt(cosh(x)*sinh(x
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) - (sqrt(2)*cosh(x
)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*lo
```

```
g(2*cosh(x)^4 + 8*cosh(x)^3*sinh(x) + 12*cosh(x)^2*sinh(x)^2 + 8*
cosh(x)*sinh(x)^3 + 2*sinh(x)^4 - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) - 1) - 12*sqrt(2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(1/2)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(\cosh(2x) - \tanh(x))\cosh(x)}{(\sinh(x)^2 + \sinh(2x))\sqrt{\sinh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh(2*x
```

```
[Out] integrate(-(cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))
*sqrt(sinh(2*x))), x)
```

3.593 $\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

```
[Out] -Cosh[x]/(27*(-9 + 4*Cosh[x]^2)^(3/2)) + (2*Cosh[x])/(243*Sqrt[-9
 + 4*Cosh[x]^2])
```

Rubi [A] time = 0.0691723, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] -Cosh[x]/(27*(-9 + 4*Cosh[x]^2)^(3/2)) + (2*Cosh[x])/(243*Sqrt[-9
 + 4*Cosh[x]^2])
```

Rubi in Sympy [A] time = 3.11215, size = 34, normalized size = 0.92

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
```

```
[Out] 2*cosh(x)/(243*sqrt(4*cosh(x)**2 - 9)) - cosh(x)/(27*(4*cosh(x)**
2 - 9)**(3/2))
```

Mathematica [A] time = 0.0611392, size = 26, normalized size = 0.7

$$\frac{\cosh(x)(4\cosh(2x)-23)}{243(2\cosh(2x)-7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))
```

Maple [A] time = 0.019, size = 30, normalized size = 0.8

$$-\frac{\cosh(x)}{27}\left(-9+4\left(\cosh(x)\right)^{2}\right)^{-\frac{3}{2}}+\frac{2\cosh(x)}{243}\frac{1}{\sqrt{-9+4\left(\cosh(x)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x)
```

```
[Out] -1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^
2)^(1/2)
```

Maxima [A] time = 1.43846, size = 169, normalized size = 4.57

$$-\frac{1855\,e^{(-2\,x)}-8485\,e^{(-4\,x)}+5285\,e^{(-6\,x)}-980\,e^{(-8\,x)}+56\,e^{(-10\,x)}-106}{12150\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}+\frac{980\,e^{(-2\,x)}-5285\,e^{(-4\,x)}+8485\,e^{(-6\,x)}-1855\,e^{(-8\,x)}+106\,e^{(-10\,x)}-56}{12150\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12150*(1855*e^(-2*x) - 8485*e^(-4*x) + 5285*e^(-6*x) - 980*e^(
-8*x) + 56*e^(-10*x) - 106)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*
e^(-x) + e^(-2*x) + 1)^(5/2)) + 1/12150*(980*e^(-2*x) - 5285*e^(-
4*x) + 8485*e^(-6*x) - 1855*e^(-8*x) + 106*e^(-10*x) - 56)/((3*e^
(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) + e^(-2*x) + 1)^(5/2))
```

Fricas [A] time = 0.274774, size = 3929, normalized size = 106.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x, algorithm="fricas")
```

[Out] 1/2*(256*cosh(x)^18 + 3584*cosh(x)*sinh(x)^17 + 256*sinh(x)^18 + 256*(89*cosh(x)^2 - 24)*sinh(x)^16 - 6144*cosh(x)^16 + 12288*(7*cosh(x)^3 - 6*cosh(x))*sinh(x)^15 + 16*(13120*cosh(x)^4 - 24576*cosh(x)^2 + 3617)*sinh(x)^14 + 57872*cosh(x)^14 + 32*(10304*cosh(x)^5 - 37632*cosh(x)^3 + 18117*cosh(x))*sinh(x)^13 + 16*(17472*cosh(x)^6 - 139776*cosh(x)^4 + 156299*cosh(x)^2 - 17013)*sinh(x)^12 - 272208*cosh(x)^12 - 64*(832*cosh(x)^7 + 34944*cosh(x)^5 - 91481*cosh(x)^3 + 34410*cosh(x))*sinh(x)^11 - 2*(256256*cosh(x)^8 - 3613896*cosh(x)^4 + 3661584*cosh(x)^2 - 342187)*sinh(x)^10 + 684374*cosh(x)^10 - 4*(183040*cosh(x)^9 - 878592*cosh(x)^7 - 445016*cosh(x)^5 + 2998560*cosh(x)^3 - 1083665*cosh(x))*sinh(x)^9 - 2*(256256*cosh(x)^10 - 2635776*cosh(x)^8 + 4368936*cosh(x)^6 + 3516120*cosh(x)^4 - 5362095*cosh(x)^2 + 484868)*sinh(x)^8 - 969736*cosh(x)^8 - 16*(3328*cosh(x)^11 - 219648*cosh(x)^9 + 895752*cosh(x)^7 - 494064*cosh(x)^5 - 741915*cosh(x)^3 + 306774*cosh(x))*sinh(x)^7 + 4*(69888*cosh(x)^12 - 2184468*cosh(x)^8 + 4168080*cosh(x)^6 + 802515*cosh(x)^4 - 2513896*cosh(x)^2 + 228401)*sinh(x)^6 + 913604*cosh(x)^6 + 8*(41216*cosh(x)^13 - 279552*cosh(x)^11 + 222508*cosh(x)^9 + 988128*cosh(x)^7 - 396669*cosh(x)^5 - 1445332*cosh(x)^3 + 515771*cosh(x))*sinh(x)^5 + 2*(104960*cosh(x)^14 - 1118208*cosh(x)^12 + 3613896*cosh(x)^10 - 3516120*cosh(x)^8 + 1605030*cosh(x)^6 - 5445720*cosh(x)^4 + 4141118*cosh(x)^2 - 325331)*sinh(x)^4 - 650662*cosh(x)^4 + 16*(5376*cosh(x)^15 - 75264*cosh(x)^13 + 365924*cosh(x)^11 - 749640*cosh(x)^9 + 741915*cosh(x)^7 - 722666*cosh(x)^5 + 633709*cosh(x)^3 - 145279*cosh(x))*sinh(x)^3 + 2*(11392*cosh(x)^16 - 196608*cosh(x)^14 + 1250392*cosh(x)^12 - 3661584*cosh(x)^10 + 5362095*cosh(x)^8 - 5027792*cosh(x)^6 + 4141118*cosh(x)^4 - 1673802*cosh(x)^2 + 112164)*sinh(x)^2 + 224328*cosh(x)^2 + 4*(896*cosh(x)^17 - 18432*cosh(x)^15 + 144936*cosh(x)^13 - 550560*cosh(x)^11 + 1083665*cosh(x)^9 - 1227096*cosh(x)^7 + 1031542*cosh(x)^5 - 581116*cosh(x)^3 + 107790*cosh(x))*sinh(x) - (512*cosh(x)^16 + 6144*cosh(x)*sinh(x)^15 + 512*sinh(x)^16 + 256*(128*cosh(x)^2 - 41)*sinh(x)^14 - 10496*cosh(x)^14 + 512*(196*cosh(x)^3 - 205*cosh(x))*sinh(x)^13 + 32*(5824*cosh(x)^4 - 14104*cosh(x)^2 + 2527)*sinh(x)^12 + 80864*cosh(x)^12 + 256*(728*cosh(x)^5 - 4100*cosh(x)^3 + 2527*cosh(x))*sinh(x)^11 - 16*(79376*cosh(x)^4 - 131404*cosh(x)^2 + 18085)*sinh(x)^10 - 289360*cosh(x)^10 - 32*(9152*cosh(x)^7 + 7216*cosh(x)^5 - 101080*cosh(x)^3 + 54255*cosh(x))*sinh(x)^9 - 2*(219648*cosh(x)^8 - 865920*cosh(x)^6 - 606480*cosh(x)^4 + 1880840*cosh(x)^2 - 240425)*sinh(x)^8 + 480850*cosh(x)^8 - 8*(36608*cosh(x)^9 - 346368*cosh(x)^7 + 485184*cosh(x)^5 + 289360*cosh(x)^3 - 240425*cosh(x))*sinh(x)^7 + (1731840*cosh(x)^8 - 6792576*cosh(x)^6 + 4051040*cosh(x)^4 + 1923400*cosh(x)^2 - 326917)*sinh(x)^6 - 326917*cosh(x)^6 + 2*(93184*cosh(x)^11 - 115456*cosh(x)^9 - 1940736*cosh(x)^7 + 4051040*cosh(x)^5 - 961700*cosh(x)^3 - 326917*cosh(x))*sinh(x)^5 + (186368*cosh(x)^12 - 1270016*cosh(x)^10 + 1212960*cosh(x)^8 + 4051040*cosh(x)^6 - 4808500*cosh(x)^4 + 326917*cosh(x)^2 + 72171)*sinh(x)^4 + 72171*cosh(x)^4 + 4*(25088*cosh(x)^13 - 262400*cosh(x)^11 + 808640*cosh(x)^9 - 578720*cosh(x)^7 - 480850*cosh(x)^5 + 326917*cosh(x)^3)*sinh(x)^3 + (32768*cosh(x)^14 - 451328*cosh(x)^12 + 2102464*cosh(x)^10 - 3761680*cosh(x)^8 + 1923400*cosh(x)^6 + 326917*cosh(x)^4 - 144342*cosh(x)^2 - 4374)*sinh(x)^2 - 4374*cosh(x)^2 + 2*(3072*cosh(x)^15 - 52480*cosh(x)^13 + 323456*cosh(x)^11 - 868080*cosh(x)^9 + 961700*cosh(x)^7 - 326917*cosh(x)^5 + 4374*cosh(x))*sinh(x))*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 17934)/(1024

*cosh(x)^22 + 18432*cosh(x)*sinh(x)^21 + 1024*sinh(x)^22 + 1024*(
151*cosh(x)^2 - 35)*sinh(x)^20 - 35840*cosh(x)^20 + 20480*(39*cos
h(x)^3 - 28*cosh(x))*sinh(x)^19 + 128*(22040*cosh(x)^4 - 33040*co
sh(x)^2 + 4111)*sinh(x)^18 + 526208*cosh(x)^18 + 256*(27816*cosh(
x)^5 - 73920*cosh(x)^3 + 28793*cosh(x))*sinh(x)^17 + 128*(100776*
cosh(x)^6 - 442680*cosh(x)^4 + 366391*cosh(x)^2 - 32844)*sinh(x)^
16 - 4204032*cosh(x)^16 + 2048*(7752*cosh(x)^7 - 57120*cosh(x)^5
+ 86571*cosh(x)^3 - 24703*cosh(x))*sinh(x)^15 + 4*(2480640*cosh(x
)^8 - 40212480*cosh(x)^6 + 108446080*cosh(x)^4 - 67766272*cosh(x)
^2 + 4969137)*sinh(x)^14 + 19876548*cosh(x)^14 - 8*(826880*cosh(x
)^9 + 14622720*cosh(x)^7 - 85651328*cosh(x)^5 + 104629504*cosh(x)
^3 - 25107253*cosh(x))*sinh(x)^13 - 4*(6449664*cosh(x)^10 - 11880
960*cosh(x)^8 - 148127616*cosh(x)^6 + 395612672*cosh(x)^4 - 21995
0523*cosh(x)^2 + 14276325)*sinh(x)^12 - 57105300*cosh(x)^12 - 16*
(2149888*cosh(x)^11 - 15841280*cosh(x)^9 + 4790656*cosh(x)^7 + 10
4610688*cosh(x)^5 - 132860169*cosh(x)^3 + 29580362*cosh(x))*sinh(
x)^11 - 4*(6449664*cosh(x)^12 - 87127040*cosh(x)^10 + 251652544*c
osh(x)^8 + 71751680*cosh(x)^6 - 716355497*cosh(x)^4 + 412292930*c
osh(x)^2 - 25234392)*sinh(x)^10 + 100937568*cosh(x)^10 - 8*(82688
0*cosh(x)^13 - 31682560*cosh(x)^11 + 181529920*cosh(x)^9 - 246774
528*cosh(x)^7 - 184136667*cosh(x)^5 + 378020580*cosh(x)^3 - 85057
450*cosh(x))*sinh(x)^9 + 4*(2480640*cosh(x)^14 + 11880960*cosh(x)
^12 - 251652544*cosh(x)^10 + 778761984*cosh(x)^8 - 405583893*cosh
(x)^6 - 707446635*cosh(x)^4 + 477715480*cosh(x)^2 - 28833252)*sin
h(x)^8 - 115333008*cosh(x)^8 + 16*(992256*cosh(x)^15 - 7311360*co
sh(x)^13 - 4790656*cosh(x)^11 + 123387264*cosh(x)^9 - 207118626*c
osh(x)^7 - 44503620*cosh(x)^5 + 181428620*cosh(x)^3 - 41154953*co
sh(x))*sinh(x)^7 + 2*(6449664*cosh(x)^16 - 80424960*cosh(x)^14 +
296255232*cosh(x)^12 - 143503360*cosh(x)^10 - 811167786*cosh(x)^8
 + 326555208*cosh(x)^6 + 1388794400*cosh(x)^4 - 822107664*cosh(x)
^2 + 47077041)*sinh(x)^6 + 94154082*cosh(x)^6 + 8*(890112*cosh(x)
^17 - 14622720*cosh(x)^15 + 85651328*cosh(x)^13 - 209221376*cosh(
x)^11 + 184136667*cosh(x)^9 - 89007240*cosh(x)^7 + 301517692*cosh
(x)^5 - 311984526*cosh(x)^3 + 58966813*cosh(x))*sinh(x)^5 + (2821
120*cosh(x)^18 - 56663040*cosh(x)^16 + 433784320*cosh(x)^14 - 158
2450688*cosh(x)^12 + 2865421988*cosh(x)^10 - 2829786540*cosh(x)^8
 + 2777588800*cosh(x)^6 - 2789614240*cosh(x)^4 + 1039551278*cosh(
x)^2 - 53093873)*sinh(x)^4 - 53093873*cosh(x)^4 + 8*(99840*cosh(x
)^19 - 2365440*cosh(x)^17 + 22162176*cosh(x)^15 - 104629504*cosh(
x)^13 + 265720338*cosh(x)^11 - 378020580*cosh(x)^9 + 362857240*co
sh(x)^7 - 311984526*cosh(x)^5 + 165492714*cosh(x)^3 - 24496633*co
sh(x))*sinh(x)^3 + (154624*cosh(x)^20 - 4229120*cosh(x)^18 + 4689
8048*cosh(x)^16 - 271065088*cosh(x)^14 + 879802092*cosh(x)^12 - 1
649171720*cosh(x)^10 + 1910861920*cosh(x)^8 - 1644215328*cosh(x)^
6 + 1039551278*cosh(x)^4 - 285758382*cosh(x)^2 + 14806051)*sinh(x
)^2 + 14806051*cosh(x)^2 + 2*(9216*cosh(x)^21 - 286720*cosh(x)^19
 + 3685504*cosh(x)^17 - 25295872*cosh(x)^15 + 100429012*cosh(x)^1
3 - 236642896*cosh(x)^11 + 340229800*cosh(x)^9 - 329239624*cosh(x
)^7 + 235867252*cosh(x)^5 - 97986532*cosh(x)^3 + 14258985*cosh(x)
)*sinh(x) - 4*(512*cosh(x)^20 + 8192*cosh(x)*sinh(x)^19 + 512*sin
h(x)^20 + 256*(236*cosh(x)^2 - 63)*sinh(x)^18 - 16128*cosh(x)^18
+ 1536*(176*cosh(x)^3 - 147*cosh(x))*sinh(x)^17 + 384*(2108*cosh(
x)^4 - 3738*cosh(x)^2 + 543)*sinh(x)^16 + 208512*cosh(x)^16 + 153
6*(1088*cosh(x)^5 - 3528*cosh(x)^3 + 1629*cosh(x))*sinh(x)^15 + 1
92*(11968*cosh(x)^6 - 68880*cosh(x)^4 + 69504*cosh(x)^2 - 7399)*s
inh(x)^14 - 1420608*cosh(x)^14 + 384*(4352*cosh(x)^7 - 54096*cosh

```
(x)^5 + 106428*cosh(x)^3 - 36995*cosh(x))*sinh(x)^13 - 6*(113152*
cosh(x)^8 + 2935296*cosh(x)^6 - 12649728*cosh(x)^4 + 10181024*cos
h(x)^2 - 907017)*sinh(x)^12 + 5442102*cosh(x)^12 - 16*(226304*cos
h(x)^9 - 209664*cosh(x)^7 - 4743648*cosh(x)^5 + 8878800*cosh(x)^3
 - 2721051*cosh(x))*sinh(x)^11 - (4978688*cosh(x)^10 - 32288256*c
osh(x)^8 + 171893568*cosh(x)^4 - 141494652*cosh(x)^2 + 11566737)*
sinh(x)^10 - 11566737*cosh(x)^10 - 2*(1810432*cosh(x)^11 - 230630
40*cosh(x)^9 + 59634432*cosh(x)^7 + 15626688*cosh(x)^5 - 10884204
0*cosh(x)^3 + 34700211*cosh(x))*sinh(x)^9 - 3*(226304*cosh(x)^12
- 10762752*cosh(x)^10 + 59634432*cosh(x)^8 - 78133440*cosh(x)^6 -
 27210510*cosh(x)^4 + 50122527*cosh(x)^2 - 4200776)*sinh(x)^8 + 1
2602328*cosh(x)^8 + 24*(69632*cosh(x)^13 + 139776*cosh(x)^11 - 49
69536*cosh(x)^9 + 15626688*cosh(x)^7 - 10884204*cosh(x)^5 - 38555
79*cosh(x)^3 + 2100388*cosh(x))*sinh(x)^7 + 3*(765952*cosh(x)^14
- 5870592*cosh(x)^12 + 78133440*cosh(x)^8 - 152378856*cosh(x)^6 +
 53978106*cosh(x)^4 + 16803104*cosh(x)^2 - 2010729)*sinh(x)^6 - 6
032187*cosh(x)^6 + 6*(278528*cosh(x)^15 - 3462144*cosh(x)^13 + 12
649728*cosh(x)^11 - 5208896*cosh(x)^9 - 43536816*cosh(x)^7 + 5397
8106*cosh(x)^5 - 8401552*cosh(x)^3 - 2010729*cosh(x))*sinh(x)^5 +
 3*(269824*cosh(x)^16 - 4408320*cosh(x)^14 + 25299456*cosh(x)^12
- 57297856*cosh(x)^10 + 27210510*cosh(x)^8 + 53978106*cosh(x)^6 -
 42007760*cosh(x)^4 + 2010729*cosh(x)^2 + 362436)*sinh(x)^4 + 108
7308*cosh(x)^4 + 12*(22528*cosh(x)^17 - 451584*cosh(x)^15 + 34056
96*cosh(x)^13 - 11838400*cosh(x)^11 + 18140340*cosh(x)^9 - 771115
8*cosh(x)^7 - 4200776*cosh(x)^5 + 2010729*cosh(x)^3)*sinh(x)^3 +
(60416*cosh(x)^18 - 1435392*cosh(x)^16 + 13344768*cosh(x)^14 - 61
086144*cosh(x)^12 + 141494652*cosh(x)^10 - 150367581*cosh(x)^8 +
50409312*cosh(x)^6 + 6032187*cosh(x)^4 - 2174616*cosh(x)^2 - 6447
7)*sinh(x)^2 - 64477*cosh(x)^2 + 2*(4096*cosh(x)^19 - 112896*cosh
(x)^17 + 1251072*cosh(x)^15 - 7103040*cosh(x)^13 + 21768408*cosh(
x)^11 - 34700211*cosh(x)^9 + 25204656*cosh(x)^7 - 6032187*cosh(x)
^5 + 64477*cosh(x))*sinh(x))*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1276303)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224391, size = 54, normalized size = 1.46

$$\frac{\left(\left(2\,e^{(2\,x)} - 21\right)e^{(2\,x)} - 21\right)e^{(2\,x)} + 2}{486\,\left(e^{(4\,x)} - 7\,e^{(2\,x)} + 1\right)^{\frac{3}{2}}} - \frac{1}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x, algorithm="giac")
```

```
[Out] 1/486*(((2*e^(2*x) - 21)*e^(2*x) - 21)*e^(2*x) + 2)/(e^(4*x) - 7*
e^(2*x) + 1)^(3/2) - 1/243
```

3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

Rubi [A] time = 0.15536, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

Rubi in Sympy [A] time = 14.7041, size = 22, normalized size = 0.76

$$2\sqrt{-\sinh^2(x)+1}+\frac{2}{\sqrt{-\sinh^2(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
```

```
[Out] 2*sqrt(-sinh(x)**2 + 1) + 2/sqrt(-sinh(x)**2 + 1)
```

Mathematica [A] time = 0.342549, size = 36, normalized size = 1.24

$$\frac{2\left(-\cosh(2x)-\sqrt{9-3\cosh(2x)}+5\right)}{\sqrt{6-2\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (2*(5 - Sqrt[9 - 3*Cosh[2*x]] - Cosh[2*x]))/Sqrt[6 - 2*Cosh[2*x]]
```

Maple [C] time = 0.062, size = 28, normalized size = 1.

$$\text{` int/indef0`}\left(-2\,\frac{(\sinh(x))^3}{\left((\sinh(x))^2-1\right)\sqrt{1-(\sinh(x))^2}},\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-2*sinh(x)^3/(sinh(x)^2-1)/(1-sinh(x)^2)^(1/2),sinh(
x))
```

Maxima [A] time = 1.73691, size = 239, normalized size = 8.24

$$-\frac{16\,e^{(-x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}+\frac{62\,e^{(-3\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}-\frac{16\,e^{(-5\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}+\frac{e^{(-7\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}+\frac{e^{x}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] -16*e^(-x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x)
+ 1)^(3/2)) + 62*e^(-3*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(
-x) - e^(-2*x) + 1)^(3/2)) - 16*e^(-5*x)/((2*e^(-x) + e^(-2*x) -
1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2)) + e^(-7*x)/((2*e^(-x) +
 e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2)) + e^x/((2*e
^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2))
```

Fricas [A] time = 0.218261, size = 217, normalized size = 7.48

$$\frac{\sqrt{2}\left(\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2-5\right)\sinh(x)^2-10\,\cosh(x)^2+4\left(\cosh(x)^3-5\,\cosh\right.\right.}{\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+\sinh(x)^5+2\left(5\,\cosh(x)^2-3\right)\sinh(x)^3-6\,\cosh(x)^3+2\left(5\,\cosh(x)^3-9\,\cosh(x)\right)\sinh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 - 5)*sinh(x)^2 - 10*cosh(x)^2 + 4*(cosh(x)^3 - 5*cosh(x))*si
nh(x) + 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(
x)^5 + 2*(5*cosh(x)^2 - 3)*sinh(x)^3 - 6*cosh(x)^3 + 2*(5*cosh(x)
^3 - 9*cosh(x))*sinh(x)^2 + (5*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh
(x) + cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(-(\sinh(x)-1)(\sinh(x)+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
```

```
[Out] Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/
2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(2x)\sinh(x)^2}{\left(-\sinh(x)^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2), x)
```

3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rubi [A] time = 0.0326159, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Cosh[x]/Sqrt[Cosh[2*x]],x]

[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh{(x)}}{\sqrt{\cosh{(2x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cosh(x)/cosh(2*x)**(1/2),x)

[Out] Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

Mathematica [A] time = 0.0674553, size = 15, normalized size = 1.

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

```
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Maple [B] time = 0.07, size = 63, normalized size = 4.2

$$\frac{\sqrt{2}}{4\sinh(x)}\sqrt{\left(2\left(\cosh(x)\right)^2-1\right)\left(\sinh(x)\right)^2}\ln\left(\sqrt{2}\left(\sinh(x)\right)^2+\frac{\sqrt{2}}{4}+\sqrt{2\left(\sinh(x)\right)^4+\left(\sinh(x)\right)^2}\right)\frac{1}{\sqrt{2\left(\cosh(x)\right)^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cosh(x)/cosh(2*x)^(1/2),x)
```

```
[Out] 1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+1/4*2^
(1/2)+(2*sinh(x)^4+sinh(x)^2)^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2
-1)^(1/2)
```

Maxima [A] time = 1.56167, size = 138, normalized size = 9.2

$$\frac{1}{8}\sqrt{2}\left(\frac{2e^{(-2x)}}{\sqrt{e^{(-4x)}+1}}-\log\left(\sqrt{e^{(-4x)}+1}e^{(2x)}+1\right)+\log\left(\sqrt{e^{(-4x)}+1}e^{(2x)}-1\right)\right)+\frac{1}{8}\sqrt{2}\left(\log\left(\sqrt{e^{(-4x)}+1}+1\right)-\log\left(\sqrt{e^{(-4x)}+1}-1\right)\right)-\frac{\sqrt{2}e^{(-2x)}}{4\sqrt{e^{(-4x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)/sqrt(cosh(2*x)),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(2*e^(-2*x)/sqrt(e^(-4*x) + 1) - log(sqrt(e^(-4*x) +
1)*e^(2*x) + 1) + log(sqrt(e^(-4*x) + 1)*e^(2*x) - 1)) + 1/8*sqrt
(2)*(log(sqrt(e^(-4*x) + 1) + 1) - log(sqrt(e^(-4*x) + 1) - 1)) -
 1/4*sqrt(2)*e^(-2*x)/sqrt(e^(-4*x) + 1)
```

Fricas [A] time = 0.220735, size = 651, normalized size = 43.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)/sqrt(cosh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (
28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*c
osh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4
+ 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(
x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2
 + sqrt(2)*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*co
sh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^3 + (15*cosh(x)^4 - 18*cosh(x)^2 + 4)*sinh(x)^2 + 4*cos
h(x)^2 + 2*(3*cosh(x)^5 - 6*cosh(x)^3 + 4*cosh(x))*sinh(x) - 4)*s
qrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)
^3 - 4*cosh(x))*sinh(x) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sin
h(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log((cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*s
qrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + 1)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)/cosh(2*x)**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x)/sqrt(cosh(2*x)),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

3.596 $\int x \tanh^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Rubi [A] time = 0.0307238, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[x]^2,x]

[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-x\tanh(x) - \frac{\log\left(-\tanh^2(x) + 1\right)}{2} + \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*tanh(x)**2,x)

[Out] -x*tanh(x) - log(-tanh(x)**2 + 1)/2 + Integral(x, x)
```

Mathematica [A] time = 0.00775351, size = 16, normalized size = 1.

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Maple [A] time = 0.025, size = 28, normalized size = 1.8

$$\frac{x^2}{2} - 2x + 2\frac{x}{1 + e^{2x}} + \ln\left(1 + e^{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*tanh(x)^2,x)
```

```
[Out] 1/2*x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))
```

Maxima [A] time = 1.63203, size = 66, normalized size = 4.12

$$-\frac{xe^{(2x)}}{e^{(2x)} + 1} + \frac{x^2 + (x^2 - 2x)e^{(2x)}}{2\left(e^{(2x)} + 1\right)} + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*
x) + 1) + log(e^(2*x) + 1)
```

Fricas [A] time = 0.214682, size = 126, normalized size = 7.88

$$\frac{(x^2 - 4x)\cosh(x)^2 + 2(x^2 - 4x)\cosh(x)\sinh(x) + (x^2 - 4x)\sinh(x)^2 + x^2 + 2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x\right.}{2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2
 - 4*x)*sinh(x)^2 + x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [A] time = 0.207648, size = 17, normalized size = 1.06

$$\frac{x^2}{2} - x\tanh(x) + x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tanh(x)**2,x)
```

```
[Out] x**2/2 - x*tanh(x) + x - log(tanh(x) + 1)
```

GIAC/XCAS [A] time = 0.228031, size = 69, normalized size = 4.31

$$\frac{x^2e^{(2x)} + x^2 - 4\,xe^{(2x)} + 2\,e^{(2x)}\ln\left(e^{(2x)} + 1\right) + 2\ln\left(e^{(2x)} + 1\right)}{2\left(e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) + x^2 - 4*x*e^(2*x) + 2*e^(2*x)*ln(e^(2*x) + 1)
+ 2*ln(e^(2*x) + 1))/(e^(2*x) + 1)
```

3.597 $\int x \coth^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x\coth(x) + \log(\sinh(x))$$

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Rubi [A] time = 0.0328149, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x^2}{2} - x\coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[x*Coth[x]^2,x]
```

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{\tanh(x)} - \frac{\log\left(-\tanh^2(x) + 1\right)}{2} + \frac{\log\left(\tanh^2(x)\right)}{2} + \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*coth(x)**2,x)
```

```
[Out] -x/tanh(x) - log(-tanh(x)**2 + 1)/2 + log(tanh(x)**2)/2 + Integra
l(x, x)
```

Mathematica [A] time = 0.00773175, size = 16, normalized size = 1.

$$\frac{x^2}{2} - x\coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Coth[x]^2,x]
```

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Maple [A] time = 0.022, size = 28, normalized size = 1.8

$$\frac{x^{2}}{2}-2x-2\,\frac{x}{-1+\mathrm{e}^{2x}}+\ln\left(-1+\mathrm{e}^{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*coth(x)^2,x)
```

```
[Out] 1/2*x^2-2*x-2*x/(-1+exp(2*x))+ln(-1+exp(2*x))
```

Maxima [A] time = 1.47587, size = 72, normalized size = 4.5

$$-\frac{xe^{(2x)}}{e^{(2x)}-1}-\frac{x^{2}-\left(x^{2}-2x\right)e^{(2x)}}{2\left(e^{(2x)}-1\right)}+\log\left(e^{x}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*coth(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*
x) - 1) + log(e^x + 1) + log(e^x - 1)
```

Fricas [A] time = 0.209913, size = 128, normalized size = 8.

$$\frac{\left(x^{2}-4x\right)\cosh\left(x\right)^{2}+2\left(x^{2}-4x\right)\cosh\left(x\right)\sinh\left(x\right)+\left(x^{2}-4x\right)\sinh\left(x\right)^{2}-x^{2}+2\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right.\right.}{2\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2
 - 4*x)*sinh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A] time = 1.67047, size = 22, normalized size = 1.38

$$\frac{x^2}{2} + x - \frac{x}{\tanh(x)} - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*coth(x)**2,x)
```

```
[Out] x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))
```

GIAC/XCAS [A] time = 0.219455, size = 72, normalized size = 4.5

$$\frac{x^2 e^{(2x)} - x^2 - 4xe^{(2x)} + 2e^{(2x)}\ln\left(e^{(2x)} - 1\right) - 2\ln\left(e^{(2x)} - 1\right)}{2\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*coth(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) - x^2 - 4*x*e^(2*x) + 2*e^(2*x)*ln(e^(2*x) - 1)
- 2*ln(e^(2*x) - 1))/(e^(2*x) - 1)
```

3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=20

$$e^x x - e^x + \frac{e^{2x}}{2}$$

```
[Out] -E^x + E^(2*x)/2 + E^x*x
```

Rubi [A] time = 0.1087, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$e^x x - e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] -E^x + E^(2*x)/2 + E^x*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(x+e^x)^2}{2} - e^x - \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] (x + exp(x))**2/2 - exp(x) - Integral(x, x)
```

Mathematica [A] time = 0.0641025, size = 23, normalized size = 1.15

$$(x-1)\sinh(x) + \frac{1}{2}\cosh(2x) + (x+\sinh(x)-1)\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] Cosh[2*x]/2 + (-1 + x)*Sinh[x] + Cosh[x]*(-1 + x + Sinh[x])
```

Maple [A] time = 0.112, size = 14, normalized size = 0.7

$$(-1+x)\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2\,x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] (-1+x)*exp(x)+1/2*exp(2*x)
```

Maxima [A] time = 1.3682, size = 18, normalized size = 0.9

$$(x-1)e^{x}+\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x + 1/2*e^(2*x)
```

Fricas [A] time = 0.215303, size = 27, normalized size = 1.35

$$\frac{2\,x+\cosh\left(x\right)+\sinh\left(x\right)-2}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + cosh(x) + sinh(x) - 2)/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.662627, size = 26, normalized size = 1.3

$$\frac{x}{-\sinh\left(x\right)+\cosh\left(x\right)}+\frac{\sinh\left(x\right)}{-\sinh\left(x\right)+\cosh\left(x\right)}-\frac{1}{-\sinh\left(x\right)+\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] x/(-sinh(x) + cosh(x)) + sinh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(
x) + cosh(x))
```

GIAC/XCAS [A] time = 0.218583, size = 15, normalized size = 0.75

$$\frac{1}{2}\,(2\,x + e^{x} - 2)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(2*x + e^x - 2)*e^x
```

3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=15

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

```
[Out] x - (1 - x)*Tanh[x/2]
```

Rubi [A] time = 0.20023, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]

[Out] x - (1 - x)*Tanh[x/2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+e^x}{\cosh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)

[Out] Integral((x + exp(x))/(cosh(x) + 1), x)
```

Mathematica [A] time = 0.0392386, size = 20, normalized size = 1.33

$$\frac{\sinh(x)\left(x+x\coth\left(\frac{x}{2}\right)-1\right)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] ((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

Maple [A] time = 0.032, size = 16, normalized size = 1.1

$$2x - 2\,\frac{-1+x}{1+\mathrm{e}^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

```
[Out] 2*x-2*(-1+x)/(1+exp(x))
```

Maxima [A] time = 1.3595, size = 47, normalized size = 3.13

$$x + \frac{2\,xe^{x}}{e^{x}+1} - \frac{2}{e^{(-x)}+1} + \log\left(\cosh\left(x\right)+1\right) - 2\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + cosh(x) + sinh(x))/(cosh(x) + 1),x, algorithm="maxima")
```

```
[Out] x + 2*x*e^x/(e^x + 1) - 2/(e^(-x) + 1) + log(cosh(x) + 1) - 2*log
(e^x + 1)
```

Fricas [A] time = 0.203108, size = 27, normalized size = 1.8

$$\frac{2\left(x\cosh\left(x\right)+x\sinh\left(x\right)+1\right)}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + cosh(x) + sinh(x))/(cosh(x) + 1),x, algorithm="fricas")
```

```
[Out] 2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] time = 0.651869, size = 12, normalized size = 0.8

$$x\tanh\left(\frac{x}{2}\right) + x - \tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

```
[Out] x*tanh(x/2) + x - tanh(x/2)
```

GIAC/XCAS [A] time = 0.21597, size = 19, normalized size = 1.27

$$\frac{2\,(xe^{x}+1)}{e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + cosh(x) + sinh(x))/(cosh(x) + 1),x, algorithm="giac")
```

```
[Out] 2*(x*e^x + 1)/(e^x + 1)
```

3.600 $\int e^{2x}\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=20

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Rubi [A] time = 0.0381935, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Rubi in Sympy [A] time = 2.14405, size = 27, normalized size = 1.35

$$-\frac{e^{2x}}{3\sinh^2(x)}-\frac{e^{2x}\cosh(x)}{3\sinh^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/sinh(x)**4,x)
```

```
[Out] -exp(2*x)/(3*sinh(x)**2) - exp(2*x)*cosh(x)/(3*sinh(x)**3)
```

Mathematica [A] time = 0.0120733, size = 29, normalized size = 1.45

$$-\frac{8\left(-3e^{2x}+3e^{4x}+1\right)}{3\left(e^{2x}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (-8*(1 - 3*E^(2*x) + 3*E^(4*x)))/(3*(-1 + E^(2*x))^3)
```

Maple [A] time = 0.11, size = 20, normalized size = 1.

$$-(\tanh(x))^{-2} - \frac{1}{3(\tanh(x))^3} - (\tanh(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/sinh(x)^4,x)
```

```
[Out] -1/tanh(x)^2-1/3/tanh(x)^3-1/tanh(x)
```

Maxima [A] time = 1.3664, size = 30, normalized size = 1.5

$$\frac{8}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(2*x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] 8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)
```

Fricas [A] time = 0.203387, size = 101, normalized size = 5.05

$$-\frac{8\left(4\cosh(x)^2 + 4\cosh(x)\sinh(x) + 4\sinh(x)^2 - 3\right)}{3\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 2\right)\sinh(x)^2 - 4\cosh(x)^2 + 4\left(\cosh(x)^3 - \cosh(x)\right)\sinh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(2*x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x
)^2 - 4*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2x}}{\sinh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/sinh(x)**4,x)
```

```
[Out] Integral(exp(2*x)/sinh(x)**4, x)
```

GIAC/XCAS [A] time = 0.23012, size = 32, normalized size = 1.6

$$-\frac{8\left(3\,e^{(4\,x)}-3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] -8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

3.601 $\int e^{-2x}\mathbf{sech}^4(x)\,dx$

Optimal. Leaf size=13

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

```
[Out] -8/(3*(1 + E^(2*x))^3)
```

Rubi [A] time = 0.0282667, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sech[x]^4/E^(2*x),x]

[Out] -8/(3*(1 + E^(2*x))^3)
```

Rubi in Sympy [A] time = 2.18981, size = 27, normalized size = 2.08

$$\frac{e^{-2x}\sinh(x)}{3\cosh^3(x)}-\frac{e^{-2x}}{3\cosh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/exp(2*x)/cosh(x)**4,x)

[Out] exp(-2*x)*sinh(x)/(3*cosh(x)**3) - exp(-2*x)/(3*cosh(x)**2)
```

Mathematica [B] time = 0.0123171, size = 32, normalized size = 2.46

$$\frac{8e^{2x}\left(3e^{2x}+e^{4x}+3\right)}{3\left(e^{2x}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sech[x]^4/E^(2*x),x]
```

```
[Out] (8*E^(2*x)*(3 + 3*E^(2*x) + E^(4*x)))/(3*(1 + E^(2*x))^3)
```

Maple [B] time = 0.058, size = 52, normalized size = 4.

$$-2\,\frac{-\left(\tanh\left(x/2\right)\right)^{5}+2\,\left(\tanh\left(x/2\right)\right)^{4}-10/3\,\left(\tanh\left(x/2\right)\right)^{3}+2\,\left(\tanh\left(x/2\right)\right)^{2}-\tanh\left(x/2\right)}{\left(\left(\tanh\left(x/2\right)\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/exp(2*x)/cosh(x)^4,x)
```

```
[Out] -2*(-tanh(1/2*x)^5+2*tanh(1/2*x)^4-10/3*tanh(1/2*x)^3+2*tanh(1/2*
x)^2-tanh(1/2*x))/(tanh(1/2*x)^2+1)^3
```

Maxima [A] time = 1.35261, size = 101, normalized size = 7.77

$$\frac{8\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{8\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{8}{3\,\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-2*x)/cosh(x)^4,x, algorithm="maxima")
```

```
[Out] 8*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*e^(-4*x)/
(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8/3/(3*e^(-2*x) + 3*e^
(-4*x) + e^(-6*x) + 1)
```

Fricas [A] time = 0.203236, size = 138, normalized size = 10.62

$$-\frac{8}{3\,\left(\cosh\left(x\right)^{6}+6\,\cosh\left(x\right)\sinh\left(x\right)^{5}+\sinh\left(x\right)^{6}+3\,\left(5\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{4}+3\,\cosh\left(x\right)^{4}+4\,\left(5\,\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\right)\mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-2*x)/cosh(x)^4,x, algorithm="fricas")
```

```
[Out] -8/3/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^
2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh
(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2
 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-2x}}{\cosh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/exp(2*x)/cosh(x)**4,x)
```

```
[Out] Integral(exp(-2*x)/cosh(x)**4, x)
```

GIAC/XCAS [A] time = 0.224659, size = 14, normalized size = 1.08

$$-\frac{8}{3\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-2*x)/cosh(x)^4,x, algorithm="giac")
```

```
[Out] -8/3/(e^(2*x) + 1)^3
```

3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)} \, dx$

Optimal. Leaf size=9

$$\frac{e^{2x}}{2}$$

```
[Out] E^(2*x)/2
```

Rubi [A] time = 0.027043, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

Rubi in Sympy [A] time = 2.5804, size = 5, normalized size = 0.56

$$\frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] exp(2*x)/2
```

Mathematica [A] time = 0.00199029, size = 9, normalized size = 1.

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

Maple [B] time = 0.001, size = 14, normalized size = 1.6

$$\frac{e^{x}}{2\cosh\left(x\right)-2\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] 1/2*exp(x)/(cosh(x)-sinh(x))
```

Maxima [A] time = 1.35773, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cosh(x) - sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*x)
```

Fricas [A] time = 0.204769, size = 22, normalized size = 2.44

$$\frac{\cosh\left(x\right)+\sinh\left(x\right)}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cosh(x) - sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.719767, size = 12, normalized size = 1.33

$$\frac{e^{x}}{-2\sinh\left(x\right)+2\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] exp(x)/(-2*sinh(x) + 2*cosh(x))
```

GIAC/XCAS [A] time = 0.195779, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cosh(x) - sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*e^(2*x)
```

3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx$

Optimal. Leaf size=13

$$\frac{e^{(m-1)x}}{m-1}$$

```
[Out] E^((-1 + m)*x)/(-1 + m)
```

Rubi [A] time = 0.047424, antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{e^{-(1-m)x}}{1-m}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -(1/(E^((1 - m)*x)*(1 - m)))
```

Rubi in Sympy [A] time = 4.33957, size = 10, normalized size = 0.77

$$-\frac{e^{x(m-1)}}{-m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
```

```
[Out] -exp(x*(m - 1))/(-m + 1)
```

Mathematica [A] time = 0.0184985, size = 18, normalized size = 1.38

$$\frac{e^{mx}(\cosh(x)-\sinh(x))}{m-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] (E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)
```

Maple [A] time = 0.004, size = 18, normalized size = 1.4

$$\frac{e^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/(cosh(x)+sinh(x)),x)
```

```
[Out] 1/(-1+m)*exp(m*x)/(cosh(x)+sinh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(m*x)/(cosh(x) + sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.203085, size = 34, normalized size = 2.62

$$\frac{\cosh(mx)+\sinh(mx)}{(m-1)\cosh(x)+(m-1)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^(m*x)/(cosh(x) + sinh(x)),x, algorithm="fricas")
```

```
[Out] (cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))
```

Sympy [A] time = 0.964104, size = 32, normalized size = 2.46

$$\begin{cases} \frac{e^{mx}}{m\sinh(x)+m\cosh(x)-\sinh(x)-\cosh(x)} & \text{for } m \neq 1 \\ \frac{xe^{x}}{\sinh(x)+\cosh(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
```

```
[Out] Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)),
Ne(m, 1)), (x*exp(x)/(sinh(x) + cosh(x)), True))
```

GIAC/XCAS [A]   time = 0.203376, size = 22, normalized size = 1.69

$$\frac{e^{(mx)}}{me^x - e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(m*x)/(cosh(x) + sinh(x)),x, algorithm="giac")
```

```
[Out] e^(m*x)/(m*e^x - e^x)
```

3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.0238061, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(Cosh[x] + Sinh[x]),x]

[Out] x
```

Rubi in Sympy [A] time = 2.31914, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(cosh(x)+sinh(x)),x)

[Out] x
```

Mathematica [A] time = 0.00075932, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(Cosh[x] + Sinh[x]),x]

[Out] x
```

Maple [A] time = 0.026, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(cosh(x)+sinh(x)),x)

[Out] x
```

Maxima [A] time = 1.34282, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cosh(x) + sinh(x)),x, algorithm="maxima")

[Out] x
```

Fricas [A] time = 0.187559, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cosh(x) + sinh(x)),x, algorithm="fricas")

[Out] x
```

Sympy [A] time = 0.670552, size = 10, normalized size = 10.

$$\frac{xe^x}{\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(cosh(x)+sinh(x)),x)

[Out] x*exp(x)/(sinh(x) + cosh(x))
```

<u>GIAC/XCAS</u> [A] time = 0.200462, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(cosh(x) + sinh(x)),x, algorithm="giac")

[Out] x
```

3.605 $\int \frac{e^x}{1-\cosh(x)} \, dx$

Optimal. Leaf size=22

$$-\frac{2}{1-e^x} - 2\log\left(1-e^x\right)$$

```
[Out] -2/(1 - E^x) - 2*Log[1 - E^x]
```

Rubi [A] time = 0.0426349, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{2}{1-e^x} - 2\log\left(1-e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 - Cosh[x]),x]

[Out] -2/(1 - E^x) - 2*Log[1 - E^x]
```

Rubi in Sympy [A] time = 6.70457, size = 15, normalized size = 0.68

$$-2\log\left(-e^x+1\right) - \frac{2}{-e^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)/(1-cosh(x)),x)

[Out] -2*log(-exp(x) + 1) - 2/(-exp(x) + 1)
```

Mathematica [A] time = 0.0360618, size = 20, normalized size = 0.91

$$\frac{2}{e^x-1} - 2\log\left(1-e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Cosh[x]),x]

[Out] 2/(-1 + E^x) - 2*Log[1 - E^x]
```

Maple [A] time = 0.022, size = 24, normalized size = 1.1

$$\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - 2\ \ln\left(\tanh\left(x/2\right)\right) + 2\ \ln\left(\tanh\left(x/2\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1-cosh(x)),x)
```

```
[Out] 1/tanh(1/2*x)-2*ln(tanh(1/2*x))+2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.34534, size = 22, normalized size = 1.

$$\frac{2}{e^x - 1} - 2\ \log\left(e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(cosh(x) - 1),x, algorithm="maxima")
```

```
[Out] 2/(e^x - 1) - 2*log(e^x - 1)
```

Fricas [A] time = 0.20476, size = 35, normalized size = 1.59

$$-\frac{2\left(\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right) - 1\right)}{\cosh\left(x\right) + \sinh\left(x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(cosh(x) - 1),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh
(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^x}{\cosh\left(x\right) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1-cosh(x)),x)

[Out] -Integral(exp(x)/(cosh(x) - 1), x)
```

GIAC/XCAS **[A]** time = 0.197069, size = 23, normalized size = 1.05

$$\frac{2}{e^{x} - 1} - 2\ln\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^x/(cosh(x) - 1),x, algorithm="giac")

[Out] 2/(e^x - 1) - 2*ln(abs(e^x - 1))
```

3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx$

Optimal. Leaf size=13

$$e^x+\frac{2}{e^x+1}$$

```
[Out] E^x + 2/(1 + E^x)
```

Rubi [A] time = 0.0530461, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$e^x+\frac{2}{e^x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

```
[Out] E^x + 2/(1 + E^x)
```

Rubi in Sympy [A] time = 2.53638, size = 10, normalized size = 0.77

$$\frac{e^x\sinh(x)}{\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

```
[Out] exp(x)*sinh(x)/(cosh(x) + 1)
```

Mathematica [A] time = 0.0240954, size = 18, normalized size = 1.38

$$\frac{e^x+e^{2x}+2}{e^x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

```
[Out] (2 + E^x + E^(2*x))/(1 + E^x)
```

Maple [A] time = 0.02, size = 18, normalized size = 1.4

$$-\tanh\left(\frac{x}{2}\right)-2\left(\tanh\left(x/2\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

```
[Out] -tanh(1/2*x)-2/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.34819, size = 15, normalized size = 1.15

$$\frac{2}{e^{x}+1}+e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sinh(x) + 1)*e^x/(cosh(x) + 1),x, algorithm="maxima")
```

```
[Out] 2/(e^x + 1) + e^x
```

Fricas [A] time = 0.204252, size = 28, normalized size = 2.15

$$\frac{3\cosh\left(x\right)-\sinh\left(x\right)+1}{\cosh\left(x\right)-\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sinh(x) + 1)*e^x/(cosh(x) + 1),x, algorithm="fricas")
```

```
[Out] (3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\sinh\left(x\right)+1\right)e^{x}}{\cosh\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

```
[Out] Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)
```

GIAC/XCAS [A] time = 0.205245, size = 15, normalized size = 1.15

$$\frac{2}{e^x + 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sinh(x) + 1)*e^x/(cosh(x) + 1),x, algorithm="giac")
```

```
[Out] 2/(e^x + 1) + e^x
```

3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx$

Optimal. Leaf size=15

$$e^x - \frac{2}{1-e^x}$$

```
[Out] E^x - 2/(1 - E^x)
```

Rubi [A] time = 0.0588138, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$e^x - \frac{2}{1-e^x}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

```
[Out] E^x - 2/(1 - E^x)
```

Rubi in Sympy [A] time = 3.21279, size = 12, normalized size = 0.8

$$-\frac{e^x \sinh(x)}{-\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

```
[Out] -exp(x)*sinh(x)/(-cosh(x) + 1)
```

Mathematica [A] time = 0.0247699, size = 20, normalized size = 1.33

$$\frac{-e^x + e^{2x} + 2}{e^x - 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

```
[Out] (2 - E^x + E^(2*x))/(-1 + E^x)
```

Maple [A] time = 0.029, size = 18, normalized size = 1.2

$$\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - 2\,(\tanh(x/2) - 1)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

```
[Out] 1/tanh(1/2*x)-2/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.38186, size = 15, normalized size = 1.

$$\frac{2}{e^x - 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x) - 1)*e^x/(cosh(x) - 1),x, algorithm="maxima")
```

```
[Out] 2/(e^x - 1) + e^x
```

Fricas [A] time = 0.2077, size = 30, normalized size = 2.

$$-\frac{3\,\cosh(x) - \sinh(x) - 1}{\cosh(x) - \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x) - 1)*e^x/(cosh(x) - 1),x, algorithm="fricas")
```

```
[Out] -(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sinh(x) - 1)\,e^x}{\cosh(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

```
[Out] Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)
```

GIAC/XCAS [A] time = 0.198143, size = 15, normalized size = 1.

$$\frac{2}{e^x - 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sinh(x) - 1)*e^x/(cosh(x) - 1),x, algorithm="giac")
```

```
[Out] 2/(e^x - 1) + e^x
```

3.608 $\int x^m \log(x)\,dx$

Optimal. Leaf size=26

$$\frac{x^{m+1}\log(x)}{m+1}-\frac{x^{m+1}}{(m+1)^2}$$

```
[Out] -(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

Rubi [A] time = 0.0210334, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{x^{m+1}\log(x)}{m+1}-\frac{x^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[x],x]

[Out] -(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

Rubi in Sympy [A] time = 1.60283, size = 20, normalized size = 0.77

$$\frac{x^{m+1}\log(x)}{m+1}-\frac{x^{m+1}}{(m+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*ln(x),x)

[Out] x**(m + 1)*log(x)/(m + 1) - x**(m + 1)/(m + 1)**2
```

Mathematica [A] time = 0.0138108, size = 19, normalized size = 0.73

$$\frac{x^{m+1}((m+1)\log(x)-1)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[x],x]
```

```
[Out] (x^(1 + m)*(-1 + (1 + m)*Log[x]))/(1 + m)^2
```

Maple [A] time = 0.013, size = 34, normalized size = 1.3

$$\frac{x\ln(x)\,\mathrm{e}^{m\ln(x)}}{1+m}-\frac{x\mathrm{e}^{m\ln(x)}}{m^2+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*ln(x),x)
```

```
[Out] 1/(1+m)*x*ln(x)*exp(m*ln(x))-1/(m^2+2*m+1)*x*exp(m*ln(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m*log(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.214468, size = 34, normalized size = 1.31

$$\frac{((m+1)x\log(x)-x)x^m}{m^2+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m*log(x),x, algorithm="fricas")
```

```
[Out] ((m + 1)*x*log(x) - x)*x^m/(m^2 + 2*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*ln(x),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \log(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^m*log(x), x)
```

3.609 $\int x^m \log^2(x)\,dx$

Optimal. Leaf size=42

$$\frac{2x^{m+1}}{(m+1)^3}+\frac{x^{m+1}\log^2(x)}{m+1}-\frac{2x^{m+1}\log(x)}{(m+1)^2}$$

```
[Out] (2*x^(1 + m))/(1 + m)^3 - (2*x^(1 + m)*Log[x])/(1 + m)^2 + (x^(1
+ m)*Log[x]^2)/(1 + m)
```

Rubi [A] time = 0.0406657, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2x^{m+1}}{(m+1)^3}+\frac{x^{m+1}\log^2(x)}{m+1}-\frac{2x^{m+1}\log(x)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*Log[x]^2,x]
```

```
[Out] (2*x^(1 + m))/(1 + m)^3 - (2*x^(1 + m)*Log[x])/(1 + m)^2 + (x^(1
+ m)*Log[x]^2)/(1 + m)
```

Rubi in Sympy [A] time = 3.2409, size = 39, normalized size = 0.93

$$\frac{x^{m+1}\log(x)^2}{m+1}-\frac{2x^{m+1}\log(x)}{(m+1)^2}+\frac{2x^{m+1}}{(m+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*ln(x)**2,x)
```

```
[Out] x**(m + 1)*log(x)**2/(m + 1) - 2*x**(m + 1)*log(x)/(m + 1)**2 + 2
*x**(m + 1)/(m + 1)**3
```

Mathematica [A] time = 0.0188515, size = 30, normalized size = 0.71

$$\frac{x^{m+1}\left((m+1)^2\log^2(x)-2(m+1)\log(x)+2\right)}{(m+1)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*Log[x]^2,x]
```

```
[Out] (x^(1 + m)*(2 - 2*(1 + m)*Log[x] + (1 + m)^2*Log[x]^2))/(1 + m)^3
```

Maple [A] time = 0.014, size = 61, normalized size = 1.5

$$\frac{x\left(\ln\left(x\right)\right)^{2}\mathrm{e}^{m\ln(x)}}{1+m}+2\,\frac{x\mathrm{e}^{m\ln(x)}}{m^{3}+3\,m^{2}+3\,m+1}-2\,\frac{x\ln\left(x\right)\mathrm{e}^{m\ln(x)}}{m^{2}+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*ln(x)^2,x)
```

```
[Out] 1/(1+m)*x*ln(x)^2*exp(m*ln(x))+2/(m^3+3*m^2+3*m+1)*x*exp(m*ln(x))
-2/(m^2+2*m+1)*x*ln(x)*exp(m*ln(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m*log(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.22421, size = 61, normalized size = 1.45

$$\frac{\left(\left(m^{2}+2\,m+1\right)x\log\left(x\right)^{2}-2\left(m+1\right)x\log\left(x\right)+2\,x\right)x^{m}}{m^{3}+3\,m^{2}+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m*log(x)^2,x, algorithm="fricas")
```

```
[Out] ((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3
+ 3*m^2 + 3*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*ln(x)**2,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.207145, size = 122, normalized size = 2.9

$$-\frac{2\,mxe^{(m\ln(x))}\ln(x)}{(m^2+2\,m+1)(m+1)}+\frac{x^{m+1}\ln(x)^2}{m+1}-\frac{2\,xe^{(m\ln(x))}\ln(x)}{(m^2+2\,m+1)(m+1)}+\frac{2\,xe^{(m\ln(x))}}{(m^2+2\,m+1)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m*log(x)^2,x, algorithm="giac")
```

```
[Out] -2*m*x*e^(m*ln(x))*ln(x)/((m^2 + 2*m + 1)*(m + 1)) + x^(m + 1)*ln
(x)^2/(m + 1) - 2*x*e^(m*ln(x))*ln(x)/((m^2 + 2*m + 1)*(m + 1)) +
 2*x*e^(m*ln(x))/((m^2 + 2*m + 1)*(m + 1))
```

3.610 $\int \frac{\log^2(x)}{x^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{16}{27x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}$$

```
[Out] -16/(27*x^(3/2)) - (8*Log[x])/(9*x^(3/2)) - (2*Log[x]^2)/(3*x^(3/
2))
```

Rubi [A] time = 0.031638, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{16}{27x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^2/x^(5/2),x]
```

```
[Out] -16/(27*x^(3/2)) - (8*Log[x])/(9*x^(3/2)) - (2*Log[x]^2)/(3*x^(3/
2))
```

Rubi in Sympy [A] time = 2.03656, size = 34, normalized size = 1.

$$-\frac{2\log{(x)}^2}{3x^{\frac{3}{2}}}-\frac{8\log{(x)}}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**2/x**(5/2),x)
```

```
[Out] -2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/
2))
```

Mathematica [A] time = 0.00791862, size = 21, normalized size = 0.62

$$-\frac{2\left(9\log^2(x)+12\log(x)+8\right)}{27x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^2/x^(5/2),x]
```

```
[Out] (-2*(8 + 12*Log[x] + 9*Log[x]^2))/(27*x^(3/2))
```

Maple [A] time = 0.026, size = 23, normalized size = 0.7

$$-\frac{16}{27}x^{-\frac{3}{2}}-\frac{8\ln(x)}{9}x^{-\frac{3}{2}}-\frac{2(\ln(x))^2}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^2/x^(5/2),x)
```

```
[Out] -16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

Maxima [A] time = 1.35767, size = 30, normalized size = 0.88

$$-\frac{2\log(x)^2}{3x^{\frac{3}{2}}}-\frac{8\log(x)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

Fricas [A] time = 0.208768, size = 23, normalized size = 0.68

$$-\frac{2\left(9\log(x)^2+12\log(x)+8\right)}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)
```

Sympy [A] time = 10.0105, size = 34, normalized size = 1.

$$-\frac{2\log(x)^2}{3x^{\frac{3}{2}}}-\frac{8\log(x)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**2/x**(5/2),x)
```

```
[Out] -2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/
2))
```

GIAC/XCAS [A] time = 0.23798, size = 30, normalized size = 0.88

$$-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*ln(x)^2/x^(3/2) - 8/9*ln(x)/x^(3/2) - 16/27/x^(3/2)
```

3.611 $\int (a+bx)\log(x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log(x)\left(2ax+bx^2\right)-ax-\frac{bx^2}{4}$$

```
[Out] -(a*x) - (b*x^2)/4 + ((2*a*x + b*x^2)*Log[x])/2
```

Rubi [A] time = 0.022857, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{1}{2}\log(x)\left(2ax+bx^2\right)-ax-\frac{bx^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)*Log[x],x]

[Out] -(a*x) - (b*x^2)/4 + ((2*a*x + b*x^2)*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\log(x)}{2b}-ax-\frac{b\int x\,dx}{2}+\frac{(a+bx)^2\log(x)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)*ln(x),x)

[Out] -a**2*log(x)/(2*b) - a*x - b*Integral(x, x)/2 + (a + b*x)**2*log(
x)/(2*b)
```

Mathematica [A] time = 0.00240627, size = 28, normalized size = 0.97

$$-ax+ax\log(x)-\frac{bx^2}{4}+\frac{1}{2}bx^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)*Log[x],x]
```

```
[Out] -(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

Maple [A] time = 0.003, size = 25, normalized size = 0.9

$$\frac{bx^2 \ln(x)}{2} - \frac{bx^2}{4} + \ln(x)\, xa - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)*ln(x),x)
```

```
[Out] 1/2*b*x^2*ln(x)-1/4*b*x^2+ln(x)*x*a-a*x
```

Maxima [A] time = 1.3364, size = 34, normalized size = 1.17

$$-\frac{1}{4}\, bx^2 - ax + \frac{1}{2}\left(bx^2 + 2\, ax\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

Fricas [A] time = 0.226597, size = 34, normalized size = 1.17

$$-\frac{1}{4}\, bx^2 - ax + \frac{1}{2}\left(bx^2 + 2\, ax\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*log(x),x, algorithm="fricas")
```

```
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

Sympy [A] time = 0.108082, size = 22, normalized size = 0.76

$$-ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)*ln(x),x)

[Out] -a*x - b*x**2/4 + (a*x + b*x**2/2)*log(x)
```

GIAC/XCAS [A] time = 0.213729, size = 32, normalized size = 1.1

$$\frac{1}{2}\,bx^2\ln\left(x\right) - \frac{1}{4}\,bx^2 + ax\ln\left(x\right) - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*log(x),x, algorithm="giac")

[Out] 1/2*b*x^2*ln(x) - 1/4*b*x^2 + a*x*ln(x) - a*x
```

3.612 $\int (a+bx)^3 \log(x)\,dx$

Optimal. Leaf size=67

$$-\frac{a^4\log(x)}{4b}-a^3x-\frac{3}{4}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*
Log[x])/(4*b) + ((a + b*x)^4*Log[x])/(4*b)
```

Rubi [A] time = 0.0664176, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{a^4\log(x)}{4b}-a^3x-\frac{3}{4}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^3*Log[x],x]
```

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*
Log[x])/(4*b) + ((a + b*x)^4*Log[x])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4\log(x)}{4b}-a^3x-\frac{3a^2b\int x\,dx}{2}-\frac{ab^2x^3}{3}-\frac{b^3x^4}{16}+\frac{(a+bx)^4\log(x)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**3*ln(x),x)
```

```
[Out] -a**4*log(x)/(4*b) - a**3*x - 3*a**2*b*Integral(x, x)/2 - a*b**2*
x**3/3 - b**3*x**4/16 + (a + b*x)**4*log(x)/(4*b)
```

Mathematica [A] time = 0.0205071, size = 68, normalized size = 1.01

$$\frac{1}{48}x\left(-48a^3-36a^2bx+12\log(x)\left(4a^3+6a^2bx+4ab^2x^2+b^3x^3\right)-16ab^2x^2-3b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^3*Log[x],x]
```

```
[Out] (x*(-48*a^3 - 36*a^2*b*x - 16*a*b^2*x^2 - 3*b^3*x^3 + 12*(4*a^3 +
 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*Log[x]))/48
```

Maple [A] time = 0.003, size = 72, normalized size = 1.1

$$\frac{b^3x^4\ln(x)}{4}-\frac{b^3x^4}{16}+ab^2x^3\ln(x)-\frac{ab^2x^3}{3}+\frac{3a^2bx^2\ln(x)}{2}-\frac{3a^2bx^2}{4}+\ln(x)\,xa^3-a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^3*ln(x),x)
```

```
[Out] 1/4*b^3*x^4*ln(x)-1/16*b^3*x^4+a*b^2*x^3*ln(x)-1/3*a*b^2*x^3+3/2*
a^2*b*x^2*ln(x)-3/4*a^2*b*x^2+ln(x)*x*a^3-a^3*x
```

Maxima [A] time = 1.35796, size = 93, normalized size = 1.39

$$-\frac{1}{16}b^3x^4-\frac{1}{3}ab^2x^3-\frac{3}{4}a^2bx^2-a^3x+\frac{1}{4}\left(b^3x^4+4ab^2x^3+6a^2bx^2+4a^3x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*
x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

Fricas [A] time = 0.207007, size = 93, normalized size = 1.39

$$-\frac{1}{16}b^3x^4-\frac{1}{3}ab^2x^3-\frac{3}{4}a^2bx^2-a^3x+\frac{1}{4}\left(b^3x^4+4ab^2x^3+6a^2bx^2+4a^3x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^3*log(x),x, algorithm="fricas")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*
x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

Sympy [A] time = 0.168731, size = 71, normalized size = 1.06

$$-a^3x - \frac{3a^2bx^2}{4} - \frac{ab^2x^3}{3} - \frac{b^3x^4}{16} + \left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**3*ln(x),x)
```

```
[Out] -a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*
x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*log(x)
```

GIAC/XCAS [A] time = 0.231184, size = 96, normalized size = 1.43

$$\frac{1}{4}b^3x^4\ln(x) - \frac{1}{16}b^3x^4 + ab^2x^3\ln(x) - \frac{1}{3}ab^2x^3 + \frac{3}{2}a^2bx^2\ln(x) - \frac{3}{4}a^2bx^2 + a^3x\ln(x) - a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*ln(x) - 1/16*b^3*x^4 + a*b^2*x^3*ln(x) - 1/3*a*b^2*x^
3 + 3/2*a^2*b*x^2*ln(x) - 3/4*a^2*b*x^2 + a^3*x*ln(x) - a^3*x
```

3.613 $\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx$

Optimal. Leaf size=23

$$-35x+3x\log^3(x)-17x\log^2(x)+34x\log(x)$$

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Rubi [A] time = 0.0266347, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-35x+3x\log^3(x)-17x\log^2(x)+34x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]

[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Rubi in Sympy [A] time = 0.996133, size = 26, normalized size = 1.13

$$3x\log\left(x\right)^3-17x\log\left(x\right)^2+34x\log\left(x\right)-35x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-1-8*ln(x)**2+3*ln(x)**3,x)

[Out] 3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

Mathematica [A] time = 0.00392011, size = 23, normalized size = 1.

$$-35x+3x\log^3(x)-17x\log^2(x)+34x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]

[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$-35\,x+34\,x\ln\left(x\right)-17\,x\left(\ln\left(x\right)\right)^{2}+3\,x\left(\ln\left(x\right)\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-1-8*ln(x)^2+3*ln(x)^3,x)
```

```
[Out] -35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

Maxima [A] time = 1.34375, size = 49, normalized size = 2.13

$$3\left(\log\left(x\right)^{3}-3\,\log\left(x\right)^{2}+6\,\log\left(x\right)-6\right)x-8\left(\log\left(x\right)^{2}-2\,\log\left(x\right)+2\right)x-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3*log(x)^3 - 8*log(x)^2 - 1,x, algorithm="maxima")
```

```
[Out] 3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(
x) + 2)*x - x
```

Fricas [A] time = 0.205039, size = 31, normalized size = 1.35

$$3\,x\log\left(x\right)^{3}-17\,x\log\left(x\right)^{2}+34\,x\log\left(x\right)-35\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3*log(x)^3 - 8*log(x)^2 - 1,x, algorithm="fricas")
```

```
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

Sympy [A] time = 0.115071, size = 26, normalized size = 1.13

$$3x\log\left(x\right)^{3}-17x\log\left(x\right)^{2}+34x\log\left(x\right)-35x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1-8*ln(x)**2+3*ln(x)**3,x)
```

```
[Out] 3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

GIAC/XCAS [A] time = 0.211696, size = 31, normalized size = 1.35

$$3\,x\ln\left(x\right)^{3} - 17\,x\ln\left(x\right)^{2} + 34\,x\ln\left(x\right) - 35\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(3*log(x)^3 - 8*log(x)^2 - 1,x, algorithm="giac")
```

```
[Out] 3*x*ln(x)^3 - 17*x*ln(x)^2 + 34*x*ln(x) - 35*x
```

3.614 $\int (1 + x^4) (1 - 2\log(x) + \log^3(x)) \, dx$

Optimal. Leaf size=60

$$\frac{169x^5}{625} + \frac{1}{5}x^5\log^3(x) - \frac{3}{25}x^5\log^2(x) - \frac{44}{125}x^5\log(x) - 3x + x\log^3(x) - 3x\log^2(x) + 4x\log(x)$$

```
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log
[x]^2 - (3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Rubi [A] time = 0.142764, antiderivative size = 73, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{169x^5}{625} + \frac{1}{5}x^5\log^3(x) - \frac{3}{25}x^5\log^2(x) + \frac{6}{125}x^5\log(x) - \frac{2}{5}(x^5 + 5x)\log(x) - 3x + x\log^3(x) - 3x\log^2(x) + 6x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 6*x*Log[x] + (6*x^5*Log[x])/125 - (2*(5*x
+ x^5)*Log[x])/5 - 3*x*Log[x]^2 - (3*x^5*Log[x]^2)/25 + x*Log[x]^
3 + (x^5*Log[x]^3)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 1) \left(\log(x)^3 - 2\log(x) + 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

```
[Out] Integral((x**4 + 1)*(log(x)**3 - 2*log(x) + 1), x)
```

Mathematica [A] time = 0.00741785, size = 60, normalized size = 1.

$$\frac{169x^5}{625} + \frac{1}{5}x^5\log^3(x) - \frac{3}{25}x^5\log^2(x) - \frac{44}{125}x^5\log(x) - 3x + x\log^3(x) - 3x\log^2(x) + 4x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log
[x]^2 - (3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Maple [A] time = 0.004, size = 53, normalized size = 0.9

$$-3\,x+\frac{169\,x^5}{625}+4\,x\ln\left(x\right)-\frac{44\,x^5\ln\left(x\right)}{125}-3\,x\left(\ln\left(x\right)\right)^2-\frac{3\,x^5\left(\ln\left(x\right)\right)^2}{25}+x\left(\ln\left(x\right)\right)^3+\frac{x^5\left(\ln\left(x\right)\right)^3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)*(1-2*ln(x)+ln(x)^3),x)
```

```
[Out] -3*x+169/625*x^5+4*x*ln(x)-44/125*x^5*ln(x)-3*x*ln(x)^2-3/25*x^5*
ln(x)^2+x*ln(x)^3+1/5*x^5*ln(x)^3
```

Maxima [A] time = 1.35298, size = 89, normalized size = 1.48

$$\frac{1}{625}\left(125\,\log\left(x\right)^3-75\,\log\left(x\right)^2+30\,\log\left(x\right)-6\right)x^5-\frac{2}{25}\,x^5(5\,\log\left(x\right)-1)+\frac{1}{5}\,x^5+\left(\log\left(x\right)^3-3\,\log\left(x\right)^2+6\,\log\left(x\right)-6\right)x-2\,x(\log\left(x\right)-1)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x, algorithm="maxima")
```

```
[Out] 1/625*(125*log(x)^3 - 75*log(x)^2 + 30*log(x) - 6)*x^5 - 2/25*x^5
*(5*log(x) - 1) + 1/5*x^5 + (log(x)^3 - 3*log(x)^2 + 6*log(x) - 6
)*x - 2*x*(log(x) - 1) + x
```

Fricas [A] time = 0.20669, size = 65, normalized size = 1.08

$$\frac{169}{625}\,x^5+\frac{1}{5}\left(x^5+5\,x\right)\log\left(x\right)^3-\frac{3}{25}\left(x^5+25\,x\right)\log\left(x\right)^2-\frac{4}{125}\left(11\,x^5-125\,x\right)\log\left(x\right)-3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x, algorithm="fricas")
```

```
[Out] 169/625*x^5 + 1/5*(x^5 + 5*x)*log(x)^3 - 3/25*(x^5 + 25*x)*log(x)
^2 - 4/125*(11*x^5 - 125*x)*log(x) - 3*x
```

Sympy [A] time = 0.159022, size = 51, normalized size = 0.85

$$\frac{169x^5}{625} - 3x + \left(-\frac{44x^5}{125} + 4x\right)\log\left(x\right) + \left(-\frac{3x^5}{25} - 3x\right)\log\left(x\right)^2 + \left(\frac{x^5}{5} + x\right)\log\left(x\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

```
[Out] 169*x**5/625 - 3*x + (-44*x**5/125 + 4*x)*log(x) + (-3*x**5/25 -
3*x)*log(x)**2 + (x**5/5 + x)*log(x)**3
```

GIAC/XCAS [A] time = 0.204316, size = 70, normalized size = 1.17

$$\frac{1}{5}\,x^5\ln\left(x\right)^3 - \frac{3}{25}\,x^5\ln\left(x\right)^2 - \frac{44}{125}\,x^5\ln\left(x\right) + \frac{169}{625}\,x^5 + x\ln\left(x\right)^3 - 3\,x\ln\left(x\right)^2 + 4\,x\ln\left(x\right) - 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x, algorithm="giac")
```

```
[Out] 1/5*x^5*ln(x)^3 - 3/25*x^5*ln(x)^2 - 44/125*x^5*ln(x) + 169/625*x
^5 + x*ln(x)^3 - 3*x*ln(x)^2 + 4*x*ln(x) - 3*x
```

3.615 $\int \frac{1}{x^3 \log^4(x)}\, dx$

Optimal. Leaf size=43

$$-\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}$$

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*L
og[x]^2) - 2/(3*x^2*Log[x])
```

Rubi [A] time = 0.0905949, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Log[x]^4),x]
```

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*L
og[x]^2) - 2/(3*x^2*Log[x])
```

Rubi in Sympy [A] time = 4.31623, size = 44, normalized size = 1.02

$$-\frac{4\,\text{Ei}\left(-2\log\left(x\right)\right)}{3} - \frac{2}{3x^2\log\left(x\right)} + \frac{1}{3x^2\log\left(x\right)^2} - \frac{1}{3x^2\log\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/ln(x)**4,x)
```

```
[Out] -4*Ei(-2*log(x))/3 - 2/(3*x**2*log(x)) + 1/(3*x**2*log(x)**2) - 1
/(3*x**2*log(x)**3)
```

Mathematica [A] time = 0.00837652, size = 43, normalized size = 1.

$$-\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Log[x]^4),x]
```

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*L
og[x]^2) - 2/(3*x^2*Log[x])
```

Maple [A] time = 0.01, size = 37, normalized size = 0.9

$$-\frac{1}{3\,x^{2}\left(\ln\left(x\right)\right)^{3}}+\frac{1}{3\,x^{2}\left(\ln\left(x\right)\right)^{2}}-\frac{2}{3\,x^{2}\ln\left(x\right)}+\frac{4\,Ei\left(1,2\,\ln\left(x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/ln(x)^4,x)
```

```
[Out] -1/3/x^2/ln(x)^3+1/3/x^2/ln(x)^2-2/3/x^2/ln(x)+4/3*Ei(1,2*ln(x))
```

Maxima [A] time = 1.79426, size = 11, normalized size = 0.26

$$-8\,\left(-3,2\,\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3*log(x)^4),x, algorithm="maxima")
```

```
[Out] -8*gamma(-3, 2*log(x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,x^{2}\log\left(x\right)^{3}\,log_{i}ntegral\left(\frac{1}{x^{2}}\right)+2\,\log\left(x\right)^{2}-\log\left(x\right)+1}{3\,x^{2}\log\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3*log(x)^4),x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^2*log(x)^3*log_integral(x^(-2)) + 2*log(x)^2 - log(x) +
 1)/(x^2*log(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\int \frac{1}{x^3\log{(x)}}\,dx}{3}+\frac{-2\log{(x)}^2+\log{(x)}-1}{3x^2\log{(x)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/ln(x)**4,x)
```

```
[Out] -4*Integral(1/(x**3*log(x)), x)/3 + (-2*log(x)**2 + log(x) - 1)/(
3*x**2*log(x)**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\log{(x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3*log(x)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log(x)^4), x)
```

3.616 $\int \frac{\log(x)}{a+bx}\,dx$

Optimal. Leaf size=29

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

```
[Out] (Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Rubi [A] time = 0.036958, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]/(a + b*x),x]
```

```
[Out] (Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Rubi in Sympy [A] time = 2.80544, size = 22, normalized size = 0.76

$$\frac{\log(x)\log\left(\frac{a+bx}{a}\right)}{b}+\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)/(b*x+a),x)
```

```
[Out] log(x)*log((a + b*x)/a)/b + polylog(2, -b*x/a)/b
```

Mathematica [A] time = 0.0109402, size = 26, normalized size = 0.9

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)+\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]/(a + b*x),x]

[Out] (Log[x]*Log[1 + (b*x)/a] + PolyLog[2, -((b*x)/a)])/b
```

Maple [C] time = 0.016, size = 32, normalized size = 1.1

$$\frac{1}{b}dilog\left(\frac{bx+a}{a}\right)+\frac{\ln\left(x\right)}{b}\ln\left(\frac{bx+a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)/(b*x+a),x)

[Out] dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

Maxima [A] time = 1.32739, size = 34, normalized size = 1.17

$$\frac{\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(b*x + a),x, algorithm="maxima")

[Out] (log(b*x/a + 1)*log(x) + dilog(-b*x/a))/b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(x\right)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(b*x + a),x, algorithm="fricas")

[Out] integral(log(x)/(b*x + a), x)
```

Sympy [A] time = 4.68795, size = 151, normalized size = 5.21

$$\begin{cases} \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b} + \frac{i\pi\log\left(\frac{a}{b}+x\right)}{b} - \frac{\operatorname{Li}_{2}\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for} \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{\operatorname{Li}_{2}\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for} \\ -\frac{G_{2,2}^{2,0}\left(\begin{matrix} & 1, 1 \\ 0, 0 & \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} - \frac{i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1, 1 \\ 0, 0 & \end{matrix} \middle| \frac{a}{b}+x\right)}{b} + \frac{G_{2,2}^{0,2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} + \frac{i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{b} - \frac{\operatorname{Li}_{2}\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/(b*x+a),x)
```

```
[Out] Piecewise((log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylo
g(2, b*(a/b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b +
 x))/b - I*pi*log(1/(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, A
bs(1/(a/b + x)) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), a/b +
 x)*log(a/b)/b - I*pi*meijerg(((), (1, 1)), ((0, 0), ()), a/b + x
)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)*log(a/b)/b + I
*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - polylog(2, b
*(a/b + x)/a)/b, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(b*x + a),x, algorithm="giac")
```

```
[Out] integrate(log(x)/(b*x + a), x)
```

3.617 $\int \frac{\log(x)}{(a+bx)^2} \, dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

```
[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

Rubi [A] time = 0.0264434, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(a + b*x)^2,x]
```

```
[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

Rubi in Sympy [A] time = 3.3948, size = 26, normalized size = 0.9

$$-\frac{\log(x)}{b(a+bx)} + \frac{\log(x)}{ab} - \frac{\log(a+bx)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(ln(x)/(b*x+a)**2,x)
```

```
[Out] -log(x)/(b*(a + b*x)) + log(x)/(a*b) - log(a + b*x)/(a*b)
```

Mathematica [A] time = 0.0192764, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(a + b*x)^2,x]
```

```
[Out] ((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a
```

Maple [A] time = 0.01, size = 30, normalized size = 1.

$$\frac{x\ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(b*x+a)^2,x)
```

```
[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

Maxima [A] time = 1.329, size = 51, normalized size = 1.76

$$-\frac{\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}}{b} - \frac{\log(x)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x + a)^2,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)
```

Fricas [A] time = 0.247128, size = 46, normalized size = 1.59

$$\frac{bx\log(x) - (bx+a)\log(bx+a)}{ab^2x + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x + a)^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)
```

Sympy [A] time = 0.701272, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{ab + b^2x} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/(b*x+a)**2,x)
```

```
[Out] -log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)
```

GIAC/XCAS [A] time = 0.200366, size = 49, normalized size = 1.69

$$-\frac{\ln(x)}{(bx+a)b}+\frac{\ln\left(\left|-\frac{a}{bx+a}+1\right|\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(b*x + a)^2,x, algorithm="giac")
```

```
[Out] -ln(x)/((b*x + a)*b) + ln(abs(-a/(b*x + a) + 1))/(a*b)
```

3.618 $\int \frac{\log^n(x)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{\log^{n+1}(x)}{n+1}$$

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Rubi [A] time = 0.027003, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 1.67372, size = 8, normalized size = 0.67

$$\frac{\log(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**n/x,x)
```

```
[Out] log(x)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.00357997, size = 12, normalized size = 1.

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 13, normalized size = 1.1

$$\frac{(\ln (x))^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^n/x,x)
```

```
[Out] ln(x)^(1+n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^n/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218214, size = 16, normalized size = 1.33

$$\frac{\log (x)^n \log (x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^n/x,x, algorithm="fricas")
```

```
[Out] log(x)^n*log(x)/(n + 1)
```

Sympy [A] time = 1.2244, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log (x)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log (\log (x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**n/x,x)
```

```
[Out] Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), Tru
e))
```

GIAC/XCAS [A] time = 0.198872, size = 16, normalized size = 1.33

$$\frac{\ln(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^n/x,x, algorithm="giac")
```

```
[Out] ln(x)^(n + 1)/(n + 1)
```

3.619 $\int \frac{(a+b\log(x))^n}{x} \, dx$

Optimal. Leaf size=19

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0467822, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Log[x])^n/x,x]

[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Rubi in Sympy [A] time = 2.35429, size = 14, normalized size = 0.74

$$\frac{(a+b\log{(x)})^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*ln(x))**n/x,x)

[Out] (a + b*log(x))**(n + 1)/(b*(n + 1))
```

Mathematica [A] time = 0.0152306, size = 18, normalized size = 0.95

$$\frac{(a+b\log(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Log[x])^n/x,x]
```

```
[Out] (a + b*Log[x])^(1 + n)/(b + b*n)
```

Maple [A] time = 0.003, size = 20, normalized size = 1.1

$$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*ln(x))^n/x,x)

[Out] (a+b*ln(x))^(1+n)/b/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*log(x) + a)^n/x,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220611, size = 30, normalized size = 1.58

$$\frac{(b\log(x)+a)(b\log(x)+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*log(x) + a)^n/x,x, algorithm="fricas")

[Out] (b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)
```

Sympy [A] time = 1.75653, size = 31, normalized size = 1.63

$$\begin{cases} \frac{\begin{cases} \frac{(a+b\log(x))^{n+1}}{n+1} & \text{for}\ n\neq -1 \\ \log(a+b\log(x)) & \text{otherwise} \end{cases}}{b} & \text{for}\ b\neq 0 \\ a^n\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*ln(x))**n/x,x)
```

```
[Out] Piecewise((Piecewise(((a + b*log(x))**(n + 1)/(n + 1), Ne(n, -1))
, (log(a + b*log(x)), True))/b, Ne(b, 0)), (a**n*log(x), True))
```

GIAC/XCAS [A] time = 0.208722, size = 26, normalized size = 1.37

$$\frac{(b\ln(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*log(x) + a)^n/x,x, algorithm="giac")
```

```
[Out] (b*ln(x) + a)^(n + 1)/(b*(n + 1))
```

3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\log(x))}{b}$$

```
[Out] Log[a + b*Log[x]]/b
```

Rubi [A] time = 0.038478, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\log(a+b\log(x))}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*Log[x])),x]

[Out] Log[a + b*Log[x]]/b
```

Rubi in Sympy [A] time = 2.28236, size = 8, normalized size = 0.73

$$\frac{\log\left(a+b\log\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*ln(x)),x)

[Out] log(a + b*log(x))/b
```

Mathematica [A] time = 0.00375372, size = 11, normalized size = 1.

$$\frac{\log(a+b\log(x))}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*Log[x])),x]
```

```
[Out] Log[a + b*Log[x]]/b
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$\frac{\ln \left(a+b\ln \left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*ln(x)),x)
```

```
[Out] ln(a+b*ln(x))/b
```

Maxima [A] time = 1.36142, size = 15, normalized size = 1.36

$$\frac{\log \left(b\log \left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*log(x) + a)*x),x, algorithm="maxima")
```

```
[Out] log(b*log(x) + a)/b
```

Fricas [A] time = 0.209172, size = 15, normalized size = 1.36

$$\frac{\log \left(b\log \left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*log(x) + a)*x),x, algorithm="fricas")
```

```
[Out] log(b*log(x) + a)/b
```

Sympy [A] time = 0.132304, size = 8, normalized size = 0.73

$$\frac{\log \left(\frac{a}{b}+\log \left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*ln(x)),x)
```

```
[Out] log(a/b + log(x))/b
```

GIAC/XCAS [A] time = 0.259998, size = 41, normalized size = 3.73

$$\frac{\ln\left(\frac{1}{4}\,\pi^2 b^2 (\mathrm{sign}\,(x) - 1)^2 + (b\ln\left(|x|\right) + a)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*log(x) + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(1/4*pi^2*b^2*(sign(x) - 1)^2 + (b*ln(abs(x)) + a)^2)/b
```

3.621 $\int \frac{(a+b\log(x))^{-n}}{x}\,dx$

Optimal. Leaf size=23

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

```
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Rubi [A] time = 0.0554726, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*Log[x])^n),x]

[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Rubi in Sympy [A] time = 2.58341, size = 14, normalized size = 0.61

$$\frac{(a+b\log(x))^{-n+1}}{b(-n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((a+b*ln(x))**n),x)

[Out] (a + b*log(x))**(-n + 1)/(b*(-n + 1))
```

Mathematica [A] time = 0.0175808, size = 21, normalized size = 0.91

$$\frac{(a+b\log(x))^{1-n}}{b-bn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*Log[x])^n),x]
```

```
[Out] (a + b*Log[x])^(1 - n)/(b - b*n)
```

Maple [A] time = 0.026, size = 24, normalized size = 1.

$$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((a+b*ln(x))^n),x)
```

```
[Out] (a+b*ln(x))^(1-n)/b/(1-n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*log(x) + a)^n*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219061, size = 36, normalized size = 1.57

$$-\frac{b\log(x)+a}{(bn-b)(b\log(x)+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*log(x) + a)^n*x),x, algorithm="fricas")
```

```
[Out] -(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)
```

Sympy [A] time = 82.0779, size = 71, normalized size = 3.09

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=1 \\ a^{-n}\log(x) & \text{for } b=0 \\ \frac{\log\left(\frac{a}{b}+\log(x)\right)}{b} & \text{for } n=1 \\ -\frac{a}{bn(a+b\log(x))^n-b(a+b\log(x))^n}-\frac{b\log(x)}{bn(a+b\log(x))^n-b(a+b\log(x))^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/((a+b*ln(x))**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (a**(-n)*log(x), Eq(b,
 0)), (log(a/b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n
 - b*(a + b*log(x))**n) - b*log(x)/(b*n*(a + b*log(x))**n - b*(a
+ b*log(x))**n), True))
```

GIAC/XCAS [A] time = 0.237205, size = 30, normalized size = 1.3

$$-\frac{(b\ln(x)+a)^{-n+1}}{b(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*log(x) + a)^n*x),x, algorithm="giac")
```

```
[Out] -(b*ln(x) + a)^(-n + 1)/(b*(n - 1))
```

3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

```
[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

Rubi [A] time = 0.0599383, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]

[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

Rubi in Sympy [A] time = 3.52305, size = 15, normalized size = 0.94

$$\operatorname{atanh}\left(\frac{\log(x)}{\sqrt{a^2+\log(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2+ln(x)**2)**(1/2),x)

[Out] atanh(log(x)/sqrt(a**2 + log(x)**2))
```

Mathematica [B] time = 0.00720858, size = 46, normalized size = 2.88

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] -Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]]/2 + Log[1 + Log[x]/Sqrt[a^2
 + Log[x]^2]]/2
```

Maple [A] time = 0.008, size = 15, normalized size = 0.9

$$\ln\left(\ln(x) + \sqrt{a^2 + (\ln(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2+ln(x)^2)^(1/2),x)
```

```
[Out] ln(ln(x)+(a^2+ln(x)^2)^(1/2))
```

Maxima [A] time = 1.38121, size = 12, normalized size = 0.75

$$\operatorname{arsinh}\left(\frac{\log(x)}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + log(x)^2)*x),x, algorithm="maxima")
```

```
[Out] arcsinh(log(x)/sqrt(a^2))
```

Fricas [A] time = 0.203896, size = 24, normalized size = 1.5

$$-\log\left(\sqrt{a^2 + \log(x)^2} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + log(x)^2)*x),x, algorithm="fricas")
```

```
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a^2 + \log(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)), x)
```

GIAC/XCAS [A] time = 0.207753, size = 24, normalized size = 1.5

$$-\ln\left(\sqrt{a^2 + \ln(x)^2} - \ln(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + log(x)^2)*x),x, algorithm="giac")
```

```
[Out] -ln(sqrt(a^2 + ln(x)^2) - ln(x))
```

3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=18

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

```
[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

Rubi [A] time = 0.0692984, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]

[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

Rubi in Sympy [A] time = 3.98792, size = 15, normalized size = 0.83

$$\operatorname{atanh}\left(\frac{\log(x)}{\sqrt{-a^2+\log(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)

[Out] atanh(log(x)/sqrt(-a**2 + log(x)**2))
```

Mathematica [B] time = 0.00717018, size = 50, normalized size = 2.78

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] -Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]]/2 + Log[1 + Log[x]/Sqrt[-a
^2 + Log[x]^2]]/2
```

Maple [A] time = 0.008, size = 17, normalized size = 0.9

$$\ln\left(\ln(x) + \sqrt{-a^2 + (\ln(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-a^2+ln(x)^2)^(1/2),x)
```

```
[Out] ln(ln(x)+(-a^2+ln(x)^2)^(1/2))
```

Maxima [A] time = 1.36991, size = 27, normalized size = 1.5

$$\log\left(2\sqrt{-a^2 + \log(x)^2} + 2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + log(x)^2)*x),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))
```

Fricas [A] time = 0.207752, size = 27, normalized size = 1.5

$$-\log\left(\sqrt{-a^2 + \log(x)^2} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + log(x)^2)*x),x, algorithm="fricas")
```

```
[Out] -log(sqrt(-a^2 + log(x)^2) - log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + log(x)^2)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$

Optimal. Leaf size=18

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Rubi [A] time = 0.0663763, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]

[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Rubi in Sympy [A] time = 3.99253, size = 15, normalized size = 0.83

$$\operatorname{atan}\left(\frac{\log(x)}{\sqrt{a^2-\log(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2-ln(x)**2)**(1/2),x)

[Out] atan(log(x)/sqrt(a**2 - log(x)**2))
```

Mathematica [A] time = 0.00765847, size = 18, normalized size = 1.

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$\arctan\left(\ln(x)\frac{1}{\sqrt{a^2-(\ln(x))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2-ln(x)^2)^(1/2),x)
```

```
[Out] arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

Maxima [A] time = 1.52835, size = 12, normalized size = 0.67

$$\arcsin\left(\frac{\log(x)}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - log(x)^2)*x),x, algorithm="maxima")
```

```
[Out] arcsin(log(x)/sqrt(a^2))
```

Fricas [A] time = 0.208032, size = 34, normalized size = 1.89

$$-2\arctan\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - log(x)^2)*x),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**2-ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))), x)
```

GIAC/XCAS [A] time = 0.224048, size = 14, normalized size = 0.78

$$\arcsin\left(\frac{\ln(x)}{a}\right)\operatorname{sign}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - log(x)^2)*x),x, algorithm="giac")
```

```
[Out] arcsin(ln(x)/a)*sign(a)
```

3.625 $\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Rubi [A] time = 0.148976, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Rubi in Sympy [A] time = 9.88801, size = 17, normalized size = 0.77

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2+\log(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)

[Out] -atanh(sqrt(a**2 + log(x)**2)/a)/a
```

Mathematica [A] time = 0.014428, size = 32, normalized size = 1.45

$$\frac{\log(\log(x))}{a} - \frac{\log\left(a\sqrt{a^2+\log^2(x)}+a^2\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] Log[Log[x]]/a - Log[a^2 + a*Sqrt[a^2 + Log[x]^2]]/a
```

Maple [A] time = 0.006, size = 37, normalized size = 1.7

$$-1\ln\left(\frac{1}{\ln(x)}\left(2\,a^2+2\,\sqrt{a^2}\sqrt{a^2+(\ln(x))^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+ln(x)^2)^(1/2))/ln(x)
)
```

Maple [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + log(x)^2)*x*log(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238844, size = 59, normalized size = 2.68

$$-\frac{\log\left(a+\sqrt{a^2+\log(x)^2}-\log(x)\right)-\log\left(-a+\sqrt{a^2+\log(x)^2}-\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + log(x)^2)*x*log(x)),x, algorithm="fricas")
```

```
[Out] -(log(a + sqrt(a^2 + log(x)^2) - log(x)) - log(-a + sqrt(a^2 + lo
g(x)^2) - log(x)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a^2 + \log(x)^2}\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)
```

GIAC/XCAS [A] time = 19.4756, size = 4, normalized size = 0.18

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 + log(x)^2)*x*log(x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.626 $\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Rubi [A] time = 0.174372, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Rubi in Sympy [A] time = 10.9284, size = 17, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-\log(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)

[Out] -atanh(sqrt(a**2 - log(x)**2)/a)/a
```

Mathematica [A] time = 0.0174592, size = 34, normalized size = 1.42

$$\frac{\log(\log(x))}{a}-\frac{\log\left(a\sqrt{a^2-\log^2(x)}+a^2\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] Log[Log[x]]/a - Log[a^2 + a*Sqrt[a^2 - Log[x]^2]]/a
```

Maple [A] time = 0.007, size = 39, normalized size = 1.6

$$-1\ln\left(\frac{1}{\ln(x)}\left(2a^2+2\sqrt{a^2}\sqrt{a^2-(\ln(x))^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-ln(x)^2)^(1/2))/ln(x)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - log(x)^2)*x*log(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.199954, size = 36, normalized size = 1.5

$$\frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - log(x)^2)*x*log(x)),x, algorithm="fricas")
```

```
[Out] log(-(a - sqrt(a^2 - log(x)^2))/log(x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{(a-\log(x))\,(a+\log(x))}\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))*log(x)), x)
```

GIAC/XCAS [A] time = 20.3169, size = 4, normalized size = 0.17

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a^2 - log(x)^2)*x*log(x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.627 $\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}} \, dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Rubi [A] time = 0.175648, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Rubi in Sympy [A] time = 10.7661, size = 15, normalized size = 0.65

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-a^2+\log(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
```

```
[Out] atan(sqrt(-a**2 + log(x)**2)/a)/a
```

Mathematica [C] time = 0.0211195, size = 38, normalized size = 1.65

$$-\frac{i \log\left(\frac{2\sqrt{\log^2(x)-a^2}}{\log(x)}-\frac{2ia}{\log(x)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ((-I)*Log[((-2*I)*a)/Log[x] + (2*Sqrt[-a^2 + Log[x]^2])/Log[x]])/
a
```

Maple [A] time = 0.006, size = 43, normalized size = 1.9

$$-1\ln\left(\frac{1}{\ln(x)}\left(-2a^2+2\sqrt{-a^2}\sqrt{-a^2+(\ln(x))^2}\right)\right)\frac{1}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x)
```

```
[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+ln(x)^2)^(1/2))/l
n(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + log(x)^2)*x*log(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.21002, size = 36, normalized size = 1.57

$$\frac{2\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}-\log(x)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + log(x)^2)*x*log(x)),x, algorithm="fricas")
```

```
[Out] 2*arctan((sqrt(-a^2 + log(x)^2) - log(x))/a)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}\log\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))*log(x)), x)
```

GIAC/XCAS [A] time = 0.208225, size = 28, normalized size = 1.22

$$\frac{\arctan\left(\frac{\sqrt{-a^2+\ln(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-a^2 + log(x)^2)*x*log(x)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a^2 + ln(x)^2)/a)/a
```

3.628 $\int \frac{\log(\log(x))}{x} \, dx$

Optimal. Leaf size=11

$$\log(x)\log(\log(x)) - \log(x)$$

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rubi [A] time = 0.01359, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\log(x)\log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Log[x]]/x,x]

[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rubi in Sympy [A] time = 1.30619, size = 10, normalized size = 0.91

$$-(-\log(\log(x)) + 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(ln(x))/x,x)

[Out] -(-log(log(x)) + 1)*log(x)
```

Mathematica [A] time = 0.00159256, size = 11, normalized size = 1.

$$\log(x)\log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Log[x]]/x,x]

[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$-\ln(x) + \ln(x)\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(ln(x))/x,x)
```

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Maxima [A] time = 1.35973, size = 15, normalized size = 1.36

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Fricas [A] time = 0.209839, size = 15, normalized size = 1.36

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Sympy [A] time = 0.550549, size = 10, normalized size = 0.91

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(ln(x))/x,x)
```

```
[Out] log(x)*log(log(x)) - log(x)
```

GIAC/XCAS [A] time = 0.205675, size = 15, normalized size = 1.36

$$\ln(x)\ln(\ln(x)) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))/x,x, algorithm="giac")

[Out] ln(x)*ln(ln(x)) - ln(x)
```

3.629 $\int \frac{\log^2(\log(x))}{x} \, dx$

Optimal. Leaf size=20

$$\log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Rubi [A] time = 0.0295178, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Log[x]]^2/x,x]

[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x))^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(ln(x))**2/x,x)

[Out] Integral(log(log(x))**2/x, x)
```

Mathematica [A] time = 0.0022322, size = 20, normalized size = 1.

$$\log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Log[x]]^2/x,x]

[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Maple [A] time = 0.003, size = 21, normalized size = 1.1

$$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)(\ln(\ln(x)))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(ln(x))^2/x,x)

[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

Maxima [A] time = 1.35044, size = 20, normalized size = 1.

$$\left(\log(\log(x))^2 - 2\log(\log(x)) + 2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^2/x,x, algorithm="maxima")

[Out] (log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

Fricas [A] time = 0.225012, size = 27, normalized size = 1.35

$$\log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^2/x,x, algorithm="fricas")

[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

Sympy [A] time = 0.614676, size = 24, normalized size = 1.2

$$\log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(ln(x))**2/x,x)
```

```
[Out] log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)
```

GIAC/XCAS [A] time = 0.214266, size = 27, normalized size = 1.35

$$\ln(x)\ln(\ln(x))^2 - 2\ln(x)\ln(\ln(x)) + 2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^2/x,x, algorithm="giac")
```

```
[Out] ln(x)*ln(ln(x))^2 - 2*ln(x)*ln(ln(x)) + 2*ln(x)
```

3.630 $\int \frac{\log^3(\log(x))}{x} dx$

Optimal. Leaf size=29

$$\log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x
]*Log[Log[x]]^3
```

Rubi [A] time = 0.0344065, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x
]*Log[Log[x]]^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(ln(x))**3/x,x)
```

```
[Out] Integral(log(log(x))**3/x, x)
```

Mathematica [A] time = 0.00241203, size = 29, normalized size = 1.

$$\log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x
]*Log[Log[x]]^3
```

Maple [A] time = 0.003, size = 30, normalized size = 1.

$$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)(\ln(\ln(x)))^2+\ln(x)(\ln(\ln(x)))^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(ln(x))^3/x,x)
```

```
[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

Maxima [A] time = 1.33847, size = 30, normalized size = 1.03

$$\left(\log(\log(x))^3-3\log(\log(x))^2+6\log(\log(x))-6\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^3/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

Fricas [A] time = 0.213367, size = 39, normalized size = 1.34

$$\log(x)\log(\log(x))^3-3\log(x)\log(\log(x))^2+6\log(x)\log(\log(x))-6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^3/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(
x)) - 6*log(x)
```

Sympy [A] time = 0.688063, size = 36, normalized size = 1.24

$$\log(x)\log(\log(x))^3-3\log(x)\log(\log(x))^2+6\log(x)\log(\log(x))-6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(ln(x))**3/x,x)
```

```
[Out] log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(lo
g(x)) - 6*log(x)
```

GIAC/XCAS [A] time = 0.223345, size = 39, normalized size = 1.34

$$\ln(x)\ln(\ln(x))^3 - 3\ln(x)\ln(\ln(x))^2 + 6\ln(x)\ln(\ln(x)) - 6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^3/x,x, algorithm="giac")
```

```
[Out] ln(x)*ln(ln(x))^3 - 3*ln(x)*ln(ln(x))^2 + 6*ln(x)*ln(ln(x)) - 6*l
n(x)
```

3.631 $\int \frac{\log^4(\log(x))}{x} \, dx$

Optimal. Leaf size=38

$$\log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*L
og[x]*Log[Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Rubi [A] time = 0.0403575, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*L
og[x]*Log[Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x))^4}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(ln(x))**4/x,x)
```

```
[Out] Integral(log(log(x))**4/x, x)
```

Mathematica [A] time = 0.00248211, size = 38, normalized size = 1.

$$\log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*L
og[x]*Log[Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Maple [A] time = 0.003, size = 39, normalized size = 1.

$$24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)(\ln(\ln(x)))^2 - 4\ln(x)(\ln(\ln(x)))^3 + \ln(x)(\ln(\ln(x)))^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(ln(x))^4/x,x)
```

```
[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x)
)^3+ln(x)*ln(ln(x))^4
```

Maxima [A] time = 1.37078, size = 39, normalized size = 1.03

$$\left(\log(\log(x))^4 - 4\log(\log(x))^3 + 12\log(\log(x))^2 - 24\log(\log(x)) + 24\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^4/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^4 - 4*log(log(x))^3 + 12*log(log(x))^2 - 24*log(log(
x)) + 24)*log(x)
```

Fricas [A] time = 0.236263, size = 51, normalized size = 1.34

$$\log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^4/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log
(x))^2 - 24*log(x)*log(log(x)) + 24*log(x)
```

Sympy [A] time = 0.74834, size = 48, normalized size = 1.26

$$\log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(ln(x))**4/x,x)
```

```
[Out] log(x)*log(log(x))**4 - 4*log(x)*log(log(x))**3 + 12*log(x)*log(l
og(x))**2 - 24*log(x)*log(log(x)) + 24*log(x)
```

GIAC/XCAS [A] time = 0.210627, size = 51, normalized size = 1.34

$$\ln(x)\ln(\ln(x))^4 - 4\ln(x)\ln(\ln(x))^3 + 12\ln(x)\ln(\ln(x))^2 - 24\ln(x)\ln(\ln(x)) + 24\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^4/x,x, algorithm="giac")
```

```
[Out] ln(x)*ln(ln(x))^4 - 4*ln(x)*ln(ln(x))^3 + 12*ln(x)*ln(ln(x))^2 -
24*ln(x)*ln(ln(x)) + 24*ln(x)
```

3.632 $\int \frac{\log^n(\log(x))}{x} \, dx$

Optimal. Leaf size=24

$$(-\log(\log(x)))^{-n} \log^n(\log(x)) \text{Gamma}(n+1, -\log(\log(x)))$$

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Rubi [A] time = 0.0497788, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$(-\log(\log(x)))^{-n} \log^n(\log(x)) \text{Gamma}(n+1, -\log(\log(x)))$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Log[x]]^n/x,x]
```

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x))^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(ln(x))**n/x,x)
```

```
[Out] Integral(log(log(x))**n/x, x)
```

Mathematica [A] time = 0.0177344, size = 24, normalized size = 1.

$$(-\log(\log(x)))^{-n} \log^n(\log(x)) \text{Gamma}(n+1, -\log(\log(x)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Log[x]]^n/x,x]
```

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(\ln (\ln (x)))^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(ln(x))^n/x,x)
```

```
[Out] int(ln(ln(x))^n/x,x)
```

Maxima [A] time = 1.45075, size = 39, normalized size = 1.62

$$-(-\log (\log (x)))^{-n-1} \log (\log (x))^{n+1}\ (n+1, -\log (\log (x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^n/x,x, algorithm="maxima")
```

```
[Out] -(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(lo
g(x)))
```

Fricas [A] time = 0.276443, size = 19, normalized size = 0.79

$$\cos (\pi n)\ (n+1, -\log (\log (x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^n/x,x, algorithm="fricas")
```

```
[Out] cos(pi*n)*gamma(n + 1, -log(log(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log (\log (x))^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(ln(x))**n/x,x)
```

```
[Out] Integral(log(log(x))**n/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(log(x))^n/x,x, algorithm="giac")
```

```
[Out] integrate(log(log(x))^n/x, x)
```

3.633 $\int \frac{\cot(x)}{\log(\sin(x))} dx$

Optimal. Leaf size=4

$$\log(\log(\sin(x)))$$

```
[Out] Log[Log[Sin[x]]]
```

Rubi [A] time = 0.0304205, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]/Log[Sin[x]],x]

[Out] Log[Log[Sin[x]]]
```

Rubi in Sympy [A] time = 2.18413, size = 5, normalized size = 1.25

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)/ln(sin(x)),x)

[Out] log(log(sin(x)))
```

Mathematica [A] time = 0.0042337, size = 4, normalized size = 1.

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]/Log[Sin[x]],x]

[Out] Log[Log[Sin[x]]]
```

Maple [A] time = 0.012, size = 5, normalized size = 1.3

$$\ln(\ln(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)/ln(sin(x)),x)
```

```
[Out] ln(ln(sin(x)))
```

Maxima [A] time = 1.44547, size = 5, normalized size = 1.25

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/log(sin(x)),x, algorithm="maxima")
```

```
[Out] log(log(sin(x)))
```

Fricas [A] time = 0.215226, size = 5, normalized size = 1.25

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/log(sin(x)),x, algorithm="fricas")
```

```
[Out] log(log(sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/ln(sin(x)),x)
```

```
[Out] Integral(cot(x)/log(sin(x)), x)
```

GIAC/XCAS [A] time = 0.221748, size = 7, normalized size = 1.75

$$\ln(|\ln(\sin(x))|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)/log(sin(x)),x, algorithm="giac")

[Out] ln(abs(ln(sin(x))))
```

3.634 $\int (\cos(x) + \sec(x)) \tan(x) \, dx$

Optimal. Leaf size=7

$$\sec(x) - \cos(x)$$

```
[Out] -Cos[x] + Sec[x]
```

Rubi [A] time = 0.0698325, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\sec(x) - \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x] + Sec[x])*Tan[x],x]

[Out] -Cos[x] + Sec[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos^2(x) + 1)\tan(x)}{\cos(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/cos(x)+cos(x))*tan(x),x)

[Out] Integral((cos(x)**2 + 1)*tan(x)/cos(x), x)
```

Mathematica [A] time = 0.00657245, size = 7, normalized size = 1.

$$\sec(x) - \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x] + Sec[x])*Tan[x],x]

[Out] -Cos[x] + Sec[x]
```

Maple [A] time = 0.016, size = 10, normalized size = 1.4

$$(\cos(x))^{-1} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/cos(x)+cos(x))*tan(x),x)

[Out] 1/cos(x)-cos(x)
```

Maxima [A] time = 1.35082, size = 12, normalized size = 1.71

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/cos(x) + cos(x))*tan(x),x, algorithm="maxima")

[Out] 1/cos(x) - cos(x)
```

Fricas [A] time = 0.24735, size = 16, normalized size = 2.29

$$-\frac{\cos(x)^2 - 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/cos(x) + cos(x))*tan(x),x, algorithm="fricas")

[Out] -(cos(x)^2 - 1)/cos(x)
```

Sympy [A] time = 4.06795, size = 7, normalized size = 1.

$$-\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/cos(x)+cos(x))*tan(x),x)
```

```
[Out] -cos(x) + 1/cos(x)
```

GIAC/XCAS [A] time = 0.224792, size = 12, normalized size = 1.71

$$\frac{1}{\cos\left(x\right)} - \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/cos(x) + cos(x))*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x) - cos(x)
```

3.635 $\int \log(\cosh(x)) \sinh(x) \, dx$

Optimal. Leaf size=11

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

```
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Rubi [A] time = 0.0204005, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]]*Sinh[x],x]

[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\log(\cosh(x)) \int \sinh(x) \, dx - \int \tanh(x) \int \sinh(x) \, dx \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(ln(cosh(x))*sinh(x),x)

[Out] log(cosh(x))*Integral(sinh(x), x) - Integral(tanh(x)*Integral(sin
h(x), x), x)
```

Mathematica [A] time = 0.00202133, size = 11, normalized size = 1.

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]]*Sinh[x],x]

[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Maple [A] time = 0.005, size = 12, normalized size = 1.1

$$-\cosh(x) + \cosh(x)\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(cosh(x))*sinh(x),x)
```

```
[Out] -cosh(x)+cosh(x)*ln(cosh(x))
```

Maxima [A] time = 1.41434, size = 15, normalized size = 1.36

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)*log(cosh(x)) - cosh(x)
```

Fricas [A] time = 0.207714, size = 62, normalized size = 5.64

$$-\frac{\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log(\cosh(x)) + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
*log(cosh(x)) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sin
h(x))
```

Sympy [A] time = 1.39592, size = 10, normalized size = 0.91

$$\log(\cosh(x))\cosh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(cosh(x))*sinh(x),x)

[Out] log(cosh(x))*cosh(x) - cosh(x)
```

GIAC/XCAS [A] time = 0.227228, size = 43, normalized size = 3.91

$$\frac{1}{2}\left(e^{(-x)} + e^{x}\right)\ln\left(\frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}\right) - \frac{1}{2}e^{(-x)} - \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cosh(x))*sinh(x),x, algorithm="giac")

[Out] 1/2*(e^(-x) + e^x)*ln(1/2*e^(-x) + 1/2*e^x) - 1/2*e^(-x) - 1/2*e^
x
```

3.636 $\int \log(\cosh(x)) \tanh(x) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\cosh(x))$$

```
[Out] Log[Cosh[x]]^2/2
```

Rubi [A] time = 0.0248128, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{2}\log^2(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Cosh[x]]*Tanh[x],x]
```

```
[Out] Log[Cosh[x]]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(-\tanh^2(x)+1\right)\log\left(\frac{1}{\cosh^2(x)}\right)}{4}-\frac{\log\left(-\tanh^2(x)+1\right)\log\left(\cosh(x)\right)}{2}-\frac{\int \log\left(\frac{1}{\cosh^2(x)}\right)\tanh(x)\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(cosh(x))*tanh(x),x)
```

```
[Out] -log(-tanh(x)**2 + 1)*log(cosh(x)**(-2))/4 - log(-tanh(x)**2 + 1)
*log(cosh(x))/2 - Integral(log(cosh(x)**(-2))*tanh(x), x)/2
```

Mathematica [A] time = 0.00377164, size = 9, normalized size = 1.

$$\frac{1}{2}\log^2(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Cosh[x]]*Tanh[x],x]
```

```
[Out] Log[Cosh[x]]^2/2
```

Maple [A] time = 0.011, size = 8, normalized size = 0.9

$$\frac{(\ln(\cosh(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x))*tanh(x),x)
```

```
[Out] 1/2*ln(cosh(x))^2
```

Maxima [A] time = 1.35679, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\cosh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(x))^2
```

Fricas [A] time = 0.213216, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\cosh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(cosh(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log(\cosh(x))\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(cosh(x))*tanh(x),x)
```

```
[Out] Integral(log(cosh(x))*tanh(x), x)
```

GIAC/XCAS [A] time = 0.223165, size = 22, normalized size = 2.44

$$\frac{1}{2}\ln\left(\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cosh(x))*tanh(x),x, algorithm="giac")
```

```
[Out] 1/2*ln(1/2*e^(-x) + 1/2*e^x)^2
```

3.637 $\int \log\left(x-\sqrt{1+x^2}\right)\,dx$

Optimal. Leaf size=26

$$\sqrt{x^2+1}+x\log\left(x-\sqrt{x^2+1}\right)$$

```
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0115053, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\sqrt{x^2+1}+x\log\left(x-\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x - Sqrt[1 + x^2]],x]

[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Rubi in Sympy [A] time = 0.867183, size = 20, normalized size = 0.77

$$x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x-(x**2+1)**(1/2)),x)

[Out] x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

Mathematica [A] time = 0.021139, size = 26, normalized size = 1.

$$\sqrt{x^2+1}+x\log\left(x-\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x - Sqrt[1 + x^2]],x]

[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Maple [A] time = 0.003, size = 23, normalized size = 0.9

$$x\ln\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x-(x^2+1)^(1/2)),x)
```

```
[Out] x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(x-\sqrt{x^2+1}\right)-x+\arctan\left(x\right)+\int-\frac{x}{x^3-\left(x^2+1\right)^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x - sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x
^2 + 1)^(3/2) + x), x)
```

Fricas [A] time = 0.217292, size = 30, normalized size = 1.15

$$x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x - sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

Sympy [A] time = 18.7891, size = 20, normalized size = 0.77

$$x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x-(x**2+1)**(1/2)),x)
```

```
[Out] x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

GIAC/XCAS [A] time = 0.222063, size = 30, normalized size = 1.15

$$x\ln\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x - sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] x*ln(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

3.638 $\int \frac{\log(-1+x)}{x^3} \, dx$

Optimal. Leaf size=35

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

```
[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Rubi [A] time = 0.0294976, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[-1 + x]/x^3,x]

[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Rubi in Sympy [A] time = 1.96245, size = 26, normalized size = 0.74

$$-\frac{\log(x)}{2}+\frac{\log(-x+1)}{2}+\frac{1}{2x}-\frac{\log(x-1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(-1+x)/x**3,x)

[Out] -log(x)/2 + log(-x + 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

Mathematica [A] time = 0.003851, size = 35, normalized size = 1.

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[-1 + x]/x^3,x]
```

```
[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Maple [A] time = 0.01, size = 26, normalized size = 0.7

$$-\frac{\ln(x)}{2}+\frac{1}{2x}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+x)/x^3,x)
```

```
[Out] -1/2*ln(x)+1/2/x+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2
```

Maxima [A] time = 1.34424, size = 34, normalized size = 0.97

$$\frac{1}{2x}-\frac{\log(x-1)}{2x^2}+\frac{1}{2}\log(x-1)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x - 1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(x - 1) - 1/2*log(x)
```

Fricas [A] time = 0.216945, size = 35, normalized size = 1.

$$-\frac{x^2\log(x)-(x^2-1)\log(x-1)-x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x - 1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*log(x) - (x^2 - 1)*log(x - 1) - x)/x^2
```

Sympy [A] time = 0.14776, size = 26, normalized size = 0.74

$$-\frac{\log(x)}{2}+\frac{\log(x-1)}{2}+\frac{1}{2x}-\frac{\log(x-1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(-1+x)/x**3,x)
```

```
[Out] -log(x)/2 + log(x - 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

GIAC/XCAS [A] time = 0.222051, size = 36, normalized size = 1.03

$$\frac{1}{2x} - \frac{\ln(x-1)}{2x^2} + \frac{1}{2}\ln(|x-1|) - \frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x - 1)/x^3,x, algorithm="giac")
```

```
[Out] 1/2/x - 1/2*ln(x - 1)/x^2 + 1/2*ln(abs(x - 1)) - 1/2*ln(abs(x))
```

3.639 $\int \left(-e^{-x}+e^x\right) \log \left(1+e^{2x}\right) \, dx$

Optimal. Leaf size=32

$$-2e^x+e^{-x}\log \left(e^{2x}+1\right)+e^x\log \left(e^{2x}+1\right)$$

```
[Out] -2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

Rubi [A] time = 0.0898013, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-2e^x+e^{-x}\log \left(e^{2x}+1\right)+e^x\log \left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^x\log \left(e^{2x}+1\right)-4\int^{e^x}\frac{1}{2}\,dx+e^{-x}\log \left(e^{2x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

```
[Out] exp(x)*log(exp(2*x) + 1) - 4*Integral(1/2, (x, exp(x))) + exp(-x)
*log(exp(2*x) + 1)
```

Mathematica [A] time = 0.0326213, size = 24, normalized size = 0.75

$$\left(e^{-x}+e^x\right)\log \left(e^{2x}+1\right)-2e^x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

Maple [A] time = 0.026, size = 32, normalized size = 1.

$$\frac{(e^x)^2 \ln\left((e^x)^2+1\right) - 2\,(e^x)^2 + \ln\left((e^x)^2+1\right)}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

```
[Out] (exp(x)^2*ln(exp(x)^2+1)-2*exp(x)^2+ln(exp(x)^2+1))/exp(x)
```

Maxima [A] time = 1.36099, size = 27, normalized size = 0.84

$$\left(e^{(-x)} + e^x\right) \log\left(e^{(2\,x)} + 1\right) - 2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^(-x) - e^x)*log(e^(2*x) + 1),x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

Fricas [A] time = 0.212635, size = 35, normalized size = 1.09

$$\left(\left(e^{(2\,x)} + 1\right) \log\left(e^{(2\,x)} + 1\right) - 2\,e^{(2\,x)}\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^(-x) - e^x)*log(e^(2*x) + 1),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ShapeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

```
[Out] Exception raised: ShapeError
```

GIAC/XCAS [A] time = 0.214888, size = 27, normalized size = 0.84

$$\left(e^{(-x)}+e^{x}\right)\ln\left(e^{(2x)}+1\right)-2e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^(-x) - e^x)*log(e^(2*x) + 1),x, algorithm="giac")
```

```
[Out] (e^(-x) + e^x)*ln(e^(2*x) + 1) - 2*e^x
```

3.640 $\int e^{3x/2}\log(-1+e^x)\,dx$

Optimal. Leaf size=52

$$-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log(e^x-1)+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)$$

```
[Out] (-4*E^(x/2))/3 - (4*E^((3*x)/2))/9 + (4*ArcTanh[E^(x/2)])/3 + (2*
E^((3*x)/2)*Log[-1 + E^x])/3
```

Rubi [A] time = 0.0565285, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log(e^x-1)+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^((3*x)/2)*Log[-1 + E^x],x]
```

```
[Out] (-4*E^(x/2))/3 - (4*E^((3*x)/2))/9 + (4*ArcTanh[E^(x/2)])/3 + (2*
E^((3*x)/2)*Log[-1 + E^x])/3
```

Rubi in Sympy [A] time = 5.49044, size = 44, normalized size = 0.85

$$\frac{2e^{\frac{3x}{2}}\log(e^x-1)}{3}-\frac{4e^{\frac{3x}{2}}}{9}-\frac{4e^{\frac{x}{2}}}{3}+\frac{4\operatorname{atanh}\left(e^{\frac{x}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(3/2*x)*ln(-1+exp(x)),x)
```

```
[Out] 2*exp(3*x/2)*log(exp(x) - 1)/3 - 4*exp(3*x/2)/9 - 4*exp(x/2)/3 +
4*atanh(exp(x/2))/3
```

Mathematica [A] time = 0.0541965, size = 57, normalized size = 1.1

$$\frac{2}{9}\left(-3\log\left(1-e^{x/2}\right)+3\log\left(e^{x/2}+1\right)+e^{x/2}\left(-2e^x+3e^x\log(e^x-1)-6\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^((3*x)/2)*Log[-1 + E^x],x]
```

```
[Out] (2*(-3*Log[1 - E^(x/2)] + 3*Log[1 + E^(x/2)] + E^(x/2)*(-6 - 2*E^
x + 3*E^x*Log[-1 + E^x])))/9
```

Maple [A] time = 0.037, size = 43, normalized size = 0.8

$$\frac{2\ln\left(-1+\mathrm{e}^{x}\right)}{3}\mathrm{e}^{\frac{3x}{2}}-\frac{4}{9}\mathrm{e}^{\frac{3x}{2}}-\frac{4}{3}\mathrm{e}^{\frac{x}{2}}-\frac{2}{3}\ln\left(-1+\mathrm{e}^{\frac{x}{2}}\right)+\frac{2}{3}\ln\left(\mathrm{e}^{\frac{x}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(3/2*x)*ln(-1+exp(x)),x)
```

```
[Out] 2/3*exp(3/2*x)*ln(-1+exp(x))-4/9*exp(3/2*x)-4/3*exp(1/2*x)-2/3*ln
(-1+exp(1/2*x))+2/3*ln(exp(1/2*x)+1)
```

Maxima [A] time = 1.35095, size = 57, normalized size = 1.1

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(3/2*x)*log(e^x - 1),x, algorithm="maxima")
```

```
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*
log(e^(1/2*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

Fricas [A] time = 0.226361, size = 57, normalized size = 1.1

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(3/2*x)*log(e^x - 1),x, algorithm="fricas")
```

```
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*
log(e^(1/2*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(3/2*x)*ln(-1+exp(x)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.207844, size = 58, normalized size = 1.12

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\ln\left(e^{x}-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\ln\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\ln\left(\left|e^{\left(\frac{1}{2}x\right)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(3/2*x)*log(e^x - 1),x, algorithm="giac")
```

```
[Out] 2/3*e^(3/2*x)*ln(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*l
n(e^(1/2*x) + 1) - 2/3*ln(abs(e^(1/2*x) - 1))
```

3.641 $\int \cos^3(x)\log(\sin(x))\,dx$

Optimal. Leaf size=30

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3
)/3
```

Rubi [A] time = 0.05559, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3
)/3
```

Rubi in Sympy [A] time = 2.88135, size = 29, normalized size = 0.97

$$-\frac{\log(\sin(x))\sin^3(x)}{3}+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3*ln(sin(x)),x)
```

```
[Out] -log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x) + sin(x)**3/9 - sin
(x)
```

Mathematica [A] time = 0.0351261, size = 25, normalized size = 0.83

$$\frac{1}{18}\sin(x)(15\log(\sin(x))+\cos(2x)(3\log(\sin(x))-1)-17)$$

Antiderivative was successfully verified.

```
[In]   Integrate[Cos[x]^3*Log[Sin[x]],x]
```

```
[Out] ((-17 + 15*Log[Sin[x]] + Cos[2*x]*(-1 + 3*Log[Sin[x]]))*Sin[x])/1
8
```

Maple [C] time = 0.053, size = 126, normalized size = 4.2

$$\frac{i}{24}e^{-3\,i x}\ln(2\,\sin(x)) - \frac{i}{72}e^{-3\,i x} - \frac{11\,i}{24}e^{-i x} - \frac{i}{24}e^{3\,i x}\ln(2\,\sin(x)) + \frac{i}{72}e^{3\,i x} + \frac{11\,i}{24}e^{i x} - \frac{i}{24}\ln(2)\,e^{-3\,i x} - \frac{3\,i}{8}\ln(2)\,e^{-i x} + \frac{3\,i}{8}e^{-i x}\ln(2\,\sin(x)) + \frac{i}{24}\ln(2)\,e^{3\,i x} + \frac{3\,i}{8}\ln(2)\,e^{i x} - \frac{3\,i}{8}e^{i x}\ln(2\,\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(cos(x)^3*ln(sin(x)),x)
```

```
[Out] 1/24*I*exp(-3*I*x)*ln(2*sin(x))-1/72*I*exp(-3*I*x)-11/24*I*exp(-I
*x)-1/24*I*exp(3*I*x)*ln(2*sin(x))+1/72*I*exp(3*I*x)+11/24*I*exp(
I*x)-1/24*I*ln(2)*exp(-3*I*x)-3/8*I*ln(2)*exp(-I*x)+3/8*I*exp(-I*
x)*ln(2*sin(x))+1/24*I*ln(2)*exp(3*I*x)+3/8*I*ln(2)*exp(I*x)-3/8*
I*exp(I*x)*ln(2*sin(x))
```

Maxima [A] time = 1.34985, size = 34, normalized size = 1.13

$$\frac{1}{9}\,\sin(x)^3 - \frac{1}{3}\left(\sin(x)^3 - 3\,\sin(x)\right)\log(\sin(x)) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")
```

```
[Out] 1/9*sin(x)^3 - 1/3*(sin(x)^3 - 3*sin(x))*log(sin(x)) - sin(x)
```

Fricas [A] time = 0.221986, size = 32, normalized size = 1.07

$$\frac{1}{3}\left(\cos(x)^2 + 2\right)\log(\sin(x))\sin(x) - \frac{1}{9}\left(\cos(x)^2 + 8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*log(sin(x))*sin(x) - 1/9*(cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 10.1912, size = 42, normalized size = 1.4

$$\frac{2\log(\sin(x))\sin^3(x)}{3} + \log(\sin(x))\sin(x)\cos^2(x) - \frac{8\sin^3(x)}{9} - \sin(x)\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3*ln(sin(x)),x)
```

```
[Out] 2*log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x)*cos(x)**2 - 8*sin(
x)**3/9 - sin(x)*cos(x)**2
```

GIAC/XCAS [A] time = 0.210022, size = 35, normalized size = 1.17

$$-\frac{1}{3}\ln(\sin(x))\sin(x)^3 + \frac{1}{9}\sin(x)^3 + \ln(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*log(sin(x)),x, algorithm="giac")
```

```
[Out] -1/3*ln(sin(x))*sin(x)^3 + 1/9*sin(x)^3 + ln(sin(x))*sin(x) - sin
(x)
```

3.642 $\int \log(\tan(x)) \sec^4(x)\,dx$

Optimal. Leaf size=30

$$-\frac{\tan^3(x)}{9}-\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

```
[Out] -Tan[x] + Log[Tan[x]]*Tan[x] - Tan[x]^3/9 + (Log[Tan[x]]*Tan[x]^3
)/3
```

Rubi [A] time = 0.0878993, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{\tan^3(x)}{9}-\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] -Tan[x] + Log[Tan[x]]*Tan[x] - Tan[x]^3/9 + (Log[Tan[x]]*Tan[x]^3
)/3
```

Rubi in Sympy [A] time = 4.73309, size = 49, normalized size = 1.63

$$\frac{2\log(\tan(x))\sin(x)}{3\cos(x)}+\frac{\log(\tan(x))\sin(x)}{3\cos^3(x)}-\frac{8\sin(x)}{9\cos(x)}-\frac{\sin(x)}{9\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(ln(tan(x))/cos(x)**4,x)
```

```
[Out] 2*log(tan(x))*sin(x)/(3*cos(x)) + log(tan(x))*sin(x)/(3*cos(x)**3
) - 8*sin(x)/(9*cos(x)) - sin(x)/(9*cos(x)**3)
```

Mathematica [A] time = 0.0895325, size = 29, normalized size = 0.97

$$\frac{1}{9}\tan(x)\left(\sec^2(x)(6\log(\tan(x))+3\cos(2x)\log(\tan(x))-1)-8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] ((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Ta
n[x])/9
```

Maple [C] time = 0.488, size = 782, normalized size = 26.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(tan(x))/cos(x)^4,x)
```

```
[Out] -4/3*I*(3*exp(2*I*x)+1)/(1+exp(2*I*x))^3*ln(1+exp(2*I*x))+2/9*(3*
Pi+3*I*ln(1+exp(2*I*x))+3*Pi*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x))
)^3-3*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1)
/(1+exp(2*I*x)))*Pi+9*Pi*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*
exp(2*I*x)+9*Pi*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3*exp(2*I*x)+
3*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I/(1+exp(2*I*x)))*cs
gn(I*(exp(2*I*x)-1))*Pi+9*Pi*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x))
)*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*exp(2*I*x)-9*Pi*csgn(I*(exp
(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*exp(2*I*x)-9*
Pi*csgn(I/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2
*exp(2*I*x)-9*Pi*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(
2*I*x)-1)/(1+exp(2*I*x)))^2*exp(2*I*x)+9*I*exp(2*I*x)*ln(exp(2*I*
x)-1)+3*I*ln(1+exp(2*I*x))*exp(6*I*x)-3*I*ln(exp(2*I*x)-1)*exp(6*
I*x)+9*I*ln(1+exp(2*I*x))*exp(4*I*x)-9*I*ln(exp(2*I*x)-1)*exp(4*I
*x)+9*I*ln(1+exp(2*I*x))*exp(2*I*x)+3*csgn((exp(2*I*x)-1)/(1+exp(
2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*Pi-3*csgn(I*(exp(2
*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-3*csgn(I*(ex
p(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))*Pi-9*Pi*csgn
((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*exp(2*I*x)-18*I*exp(2*I*x)+3*I*
ln(exp(2*I*x)-1)-6*I*exp(4*I*x)+9*Pi*exp(2*I*x)+3*Pi*csgn((exp(2*
I*x)-1)/(1+exp(2*I*x)))^3+9*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+e
xp(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*exp(2*I*x)-8*I-
3*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi)/(1+exp(2*I*x))^3
```

Maxima [A] time = 1.34377, size = 34, normalized size = 1.13

$$-\frac{1}{9}\tan(x)^3 + \frac{1}{3}\left(\tan(x)^3 + 3\tan(x)\right)\log(\tan(x)) - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/9*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*log(tan(x)) - tan(x)
```

Fricas [A] time = 0.228799, size = 53, normalized size = 1.77

$$\frac{3\left(2\cos\left(x\right)^{2}+1\right)\log\left(\frac{\sin(x)}{\cos(x)}\right)\sin\left(x\right)-\left(8\cos\left(x\right)^{2}+1\right)\sin\left(x\right)}{9\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(2*cos(x)^2 + 1)*log(sin(x)/cos(x))*sin(x) - (8*cos(x)^2 +
 1)*sin(x))/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(tan(x))/cos(x)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.207278, size = 35, normalized size = 1.17

$$\frac{1}{3}\ln\left(\tan\left(x\right)\right)\tan\left(x\right)^{3}-\frac{1}{9}\tan\left(x\right)^{3}+\ln\left(\tan\left(x\right)\right)\tan\left(x\right)-\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(tan(x))/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*ln(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + ln(tan(x))*tan(x) - tan(
x)
```

3.643 $\int \frac{\log(\cos(\frac{x}{2}))}{1+\cos(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

```
[Out] -x/2 + (Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + Tan[x/2]
```

Rubi [A] time = 0.0565151, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

```
[Out] -x/2 + (Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + Tan[x/2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)\tan\left(\frac{x}{2}\right)}{2\cos(x)+2}\,dx+\frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

```
[Out] Integral(sin(x)*tan(x/2)/(2*cos(x) + 2), x) + log(cos(x/2))*sin(x
)/(cos(x) + 1)
```

Mathematica [A] time = 0.0842314, size = 32, normalized size = 1.14

$$-\frac{\sin(x)\left(x\cot\left(\frac{x}{2}\right)-2\left(\log\left(\cos\left(\frac{x}{2}\right)\right)+1\right)\right)}{2(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

```
[Out] -((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(2*(1 + Cos[x]))
```

Maple [C] time = 0.217, size = 164, normalized size = 5.9

$$\frac{-2\,i\ln\left(e^{\frac{i}{2}x}\right)}{e^{ix}+1}+\frac{1}{e^{ix}+1}\left(-i\ln\left(e^{ix}+1\right)e^{ix}+\pi\,csgn\left(i\left(e^{ix}+1\right)\right)\,csgn\left(ie^{-\frac{i}{2}x}\right)\,csgn\left(i\cos\left(\frac{x}{2}\right)\right)-\pi\,csgn\left(i\left(e^{ix}+1\right)\right)\left(csgn\left(i\cos\left(\frac{x}{2}\right)\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(cos(1/2*x))/(1+cos(x)),x)
```

```
[Out] -2*I/(exp(I*x)+1)*ln(exp(1/2*I*x))+(-I*ln(exp(I*x)+1)*exp(I*x)+Pi
*csgn(I*(exp(I*x)+1))*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))-Pi
*csgn(I*(exp(I*x)+1))*csgn(I*cos(1/2*x))^2-Pi*csgn(I*exp(-1/2*I*x
))*csgn(I*cos(1/2*x))^2+Pi*csgn(I*cos(1/2*x))^3-x*exp(I*x)+I*ln(e
xp(I*x)+1)-2*I*ln(2)+2*I-x)/(exp(I*x)+1)
```

Maxima [A] time = 1.37029, size = 76, normalized size = 2.71

$$\frac{\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{x\cos\left(x\right)^{2}+x\sin\left(x\right)^{2}+2\,x\cos\left(x\right)+x-4\,\sin\left(x\right)}{2\,\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cos(1/2*x))/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] log(cos(1/2*x))*sin(x)/(cos(x) + 1) - 1/2*(x*cos(x)^2 + x*sin(x)^
2 + 2*x*cos(x) + x - 4*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) +
1)
```

Fricas [A] time = 0.252857, size = 43, normalized size = 1.54

$$-\frac{x\cos\left(\frac{1}{2}\,x\right)-2\,\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)-2\,\sin\left(\frac{1}{2}\,x\right)}{2\,\cos\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cos(1/2*x))/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(1/2*x) - 2*log(cos(1/2*x))*sin(1/2*x) - 2*sin(1/2*x))
/cos(1/2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

```
[Out] Integral(log(cos(x/2))/(cos(x) + 1), x)
```

GIAC/XCAS [A] time = 0.218164, size = 58, normalized size = 2.07

$$-\frac{1}{2}\,x - \frac{2\,\ln\left(\cos\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)} + \tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(cos(1/2*x))/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] -1/2*x - 2*ln(cos(1/2*x))*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 +
 1) - 1)) + tan(1/2*x)
```

3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{2x}{3}+\frac{8\sin(x)}{9(\cos(x)+1)}-\frac{\sin(x)}{9(\cos(x)+1)^2}+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

```
[Out] (-2*x)/3 - Sin[x]/(9*(1 + Cos[x])^2) + (8*Sin[x])/(9*(1 + Cos[x])
) - (Log[Sin[x]]*Sin[x])/(3*(1 + Cos[x])^2) + (2*Log[Sin[x]]*Sin[
x])/(3*(1 + Cos[x]))
```

Rubi [A] time = 0.192911, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583

$$-\frac{2x}{3}+\frac{8\sin(x)}{9(\cos(x)+1)}-\frac{\sin(x)}{9(\cos(x)+1)^2}+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] (-2*x)/3 - Sin[x]/(9*(1 + Cos[x])^2) + (8*Sin[x])/(9*(1 + Cos[x])
) - (Log[Sin[x]]*Sin[x])/(3*(1 + Cos[x])^2) + (2*Log[Sin[x]]*Sin[
x])/(3*(1 + Cos[x]))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\cos(x)+\cos(2x)+1}{(\cos(x)+1)^2}\,dx}{3}+\frac{2\log(\sin(x))\sin(x)}{3(\cos(x)+1)}-\frac{\log(\sin(x))\sin(x)}{3(\cos(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
```

```
[Out] -Integral((cos(x) + cos(2*x) + 1)/(cos(x) + 1)**2, x)/3 + 2*log(s
in(x))*sin(x)/(3*(cos(x) + 1)) - log(sin(x))*sin(x)/(3*(cos(x) +
1)**2)
```

Mathematica [A] time = 0.195948, size = 56, normalized size = 0.93

$$-\frac{1}{18}\sec^3\left(\frac{x}{2}\right)\left(9x\cos\left(\frac{x}{2}\right)+3x\cos\left(\frac{3x}{2}\right)-\sin\left(\frac{x}{2}\right)(3\log(\sin(x))+\cos(x)(6\log(\sin(x))+8)+7)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] -(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]
] + Cos[x]*(8 + 6*Log[Sin[x]]))*Sin[x/2]))/18
```

Maple [B] time = 0.141, size = 106, normalized size = 1.8

$$-\frac{1}{9\,(\sin(x))^3}\left(12\,\sin(x)\,(\cos(x))^2\arctan\left(\frac{\cos(x)-1}{\sin(x)}\right)-6\,(\cos(x))^3\ln(2)-6\,(\cos(x))^3\ln(1/2\,\sin(x))-8\,(\cos(x))^3+9\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*ln(sin(x))/(1+cos(x))^2,x)
```

```
[Out] -1/9*(12*sin(x)*cos(x)^2*arctan((cos(x)-1)/sin(x))-6*cos(x)^3*ln(
2)-6*cos(x)^3*ln(1/2*sin(x))-8*cos(x)^3+9*cos(x)^2*ln(2)+9*cos(x)
^2*ln(1/2*sin(x))-12*arctan((cos(x)-1)/sin(x))*sin(x)+9*cos(x)^2+
6*cos(x)-3*ln(2)-3*ln(1/2*sin(x))-7)/sin(x)^3
```

Maxima [A] time = 1.58134, size = 116, normalized size = 1.93

$$\frac{1}{6}\left(\frac{3\,\sin(x)}{\cos(x)+1}-\frac{\sin(x)^3}{(\cos(x)+1)^3}\right)\log\left(\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)+\frac{5\,\sin(x)}{6\,(\cos(x)+1)}-\frac{\sin(x)^3}{18\,(\cos(x)+1)^3}-\frac{4}{3}\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x))/(cos(x) + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)*log(2*sin(x
)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))) + 5/6*sin(x)/(cos
(x) + 1) - 1/18*sin(x)^3/(cos(x) + 1)^3 - 4/3*arctan(sin(x)/(cos(
x) + 1))
```

Fricas [A] time = 0.218074, size = 72, normalized size = 1.2

$$-\frac{6\,x\cos(x)^2-3\,(2\,\cos(x)+1)\log(\sin(x))\sin(x)+12\,x\cos(x)-(8\,\cos(x)+7)\sin(x)+6\,x}{9\,\left(\cos(x)^2+2\,\cos(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x))/(cos(x) + 1)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(6*x*cos(x)^2 - 3*(2*cos(x) + 1)*log(sin(x))*sin(x) + 12*x*c
os(x) - (8*cos(x) + 7)*sin(x) + 6*x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A] time = 26.6043, size = 107, normalized size = 1.78

$$-\frac{2x}{3}+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^3\left(\frac{x}{2}\right)}{6}-\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{2}-\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^3\left(\frac{x}{2}\right)}{6}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}{2}-\frac{\log(2)\tan^3\left(\frac{x}{2}\right)}{6}-\frac{\tan^3\left(\frac{x}{2}\right)}{18}+\frac{\log(2)\tan\left(\frac{x}{2}\right)}{2}+\frac{5\tan\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
```

```
[Out] -2*x/3 + log(tan(x/2)**2 + 1)*tan(x/2)**3/6 - log(tan(x/2)**2 + 1
)*tan(x/2)/2 - log(tan(x/2))*tan(x/2)**3/6 + log(tan(x/2))*tan(x/
2)/2 - log(2)*tan(x/2)**3/6 - tan(x/2)**3/18 + log(2)*tan(x/2)/2
+ 5*tan(x/2)/6
```

GIAC/XCAS [A] time = 0.268546, size = 49, normalized size = 0.82

$$-\frac{1}{18}\tan\left(\frac{1}{2}x\right)^3-\frac{1}{6}\left(\tan\left(\frac{1}{2}x\right)^3-3\tan\left(\frac{1}{2}x\right)\right)\ln(\sin(x))-\frac{2}{3}x+\frac{5}{6}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x))/(cos(x) + 1)^2,x, algorithm="giac")
```

```
[Out] -1/18*tan(1/2*x)^3 - 1/6*(tan(1/2*x)^3 - 3*tan(1/2*x))*ln(sin(x))
 - 2/3*x + 5/6*tan(1/2*x)
```

3.645 $\int \frac{\cos^{-1}(x)^2}{x^5} dx$

Optimal. Leaf size=65

$$-\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{12x^2} + \frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x} + \frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3} + \frac{\log(x)}{3}$$

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*ArcCos[x])/(6*x^3) + (Sqrt[1 - x^2]*
ArcCos[x])/(3*x) - ArcCos[x]^2/(4*x^4) + Log[x]/3
```

Rubi [A] time = 0.173195, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{12x^2} + \frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x} + \frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3} + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*ArcCos[x])/(6*x^3) + (Sqrt[1 - x^2]*
ArcCos[x])/(3*x) - ArcCos[x]^2/(4*x^4) + Log[x]/3
```

Rubi in Sympy [A] time = 7.98202, size = 53, normalized size = 0.82

$$\frac{\log(x)}{3} + \frac{\sqrt{-x^2+1}\,\mathrm{acos}(x)}{3x} - \frac{1}{12x^2} + \frac{\sqrt{-x^2+1}\,\mathrm{acos}(x)}{6x^3} - \frac{\mathrm{acos}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acos(x)**2/x**5,x)
```

```
[Out] log(x)/3 + sqrt(-x**2 + 1)*acos(x)/(3*x) - 1/(12*x**2) + sqrt(-x*
*2 + 1)*acos(x)/(6*x**3) - acos(x)**2/(4*x**4)
```

Mathematica [A] time = 0.0347489, size = 52, normalized size = 0.8

$$-\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{12x^2} + \frac{\sqrt{1-x^2}\,(2x^2+1)\cos^{-1}(x)}{6x^3} + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - Arc
Cos[x]^2/(4*x^4) + Log[x]/3
```

Maple [A] time = 0.066, size = 52, normalized size = 0.8

$$-\frac{1}{12\,x^{2}}-\frac{(\arccos(x))^{2}}{4\,x^{4}}+\frac{\ln(x)}{3}+\frac{\arccos(x)}{6\,x^{3}}\sqrt{-x^{2}+1}+\frac{\arccos(x)}{3\,x}\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccos(x)^2/x^5,x)
```

```
[Out] -1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1
/2)/x^3+1/3*arccos(x)*(-x^2+1)^(1/2)/x
```

Maxima [A] time = 1.84669, size = 69, normalized size = 1.06

$$\frac{1}{6}\left(\frac{2\sqrt{-x^{2}+1}}{x}+\frac{\sqrt{-x^{2}+1}}{x^{3}}\right)\arccos(x)-\frac{1}{12\,x^{2}}-\frac{\arccos(x)^{2}}{4\,x^{4}}+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) - 1/12/x^
2 - 1/4*arccos(x)^2/x^4 + 1/3*log(x)
```

Fricas [A] time = 0.24294, size = 59, normalized size = 0.91

$$\frac{4\,x^{4}\log(x)+2\left(2\,x^{3}+x\right)\sqrt{-x^{2}+1}\arccos(x)-x^{2}-3\,\arccos(x)^{2}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x) + 2*(2*x^3 + x)*sqrt(-x^2 + 1)*arccos(x) - x^2
 - 3*arccos(x)^2)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acos(x)**2/x**5,x)
```

```
[Out] Integral(acos(x)**2/x**5, x)
```

GIAC/XCAS [A] time = 0.229496, size = 140, normalized size = 2.15

$$-\frac{1}{48}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3}-\frac{9\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x)-\frac{2\,x^2+1}{12\,x^2}-\frac{\arccos(x)^2}{4\,x^4}+\frac{1}{6}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1
)^3 - 9*(sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arcc
os(x) - 1/12*(2*x^2 + 1)/x^2 - 1/4*arccos(x)^2/x^4 + 1/6*ln(x^2)
```

3.646 $\int x^2 \sin^{-1}(x)^2\,dx$

Optimal. Leaf size=61

$$-\frac{2x^3}{27}+\frac{1}{3}x^3\sin^{-1}(x)^2+\frac{2}{9}\sqrt{1-x^2}x^2\sin^{-1}(x)+\frac{4}{9}\sqrt{1-x^2}\sin^{-1}(x)-\frac{4x}{9}$$

```
[Out] (-4*x)/9 - (2*x^3)/27 + (4*Sqrt[1 - x^2]*ArcSin[x])/9 + (2*x^2*Sq
rt[1 - x^2]*ArcSin[x])/9 + (x^3*ArcSin[x]^2)/3
```

Rubi [A] time = 0.147264, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$-\frac{2x^3}{27}+\frac{1}{3}x^3\sin^{-1}(x)^2+\frac{2}{9}\sqrt{1-x^2}x^2\sin^{-1}(x)+\frac{4}{9}\sqrt{1-x^2}\sin^{-1}(x)-\frac{4x}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*ArcSin[x]^2,x]
```

```
[Out] (-4*x)/9 - (2*x^3)/27 + (4*Sqrt[1 - x^2]*ArcSin[x])/9 + (2*x^2*Sq
rt[1 - x^2]*ArcSin[x])/9 + (x^3*ArcSin[x]^2)/3
```

Rubi in Sympy [A] time = 7.07051, size = 54, normalized size = 0.89

$$\frac{x^3\operatorname{asin}^2(x)}{3}-\frac{2x^3}{27}+\frac{2x^2\sqrt{-x^2+1}\operatorname{asin}(x)}{9}-\frac{4x}{9}+\frac{4\sqrt{-x^2+1}\operatorname{asin}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*asin(x)**2,x)
```

```
[Out] x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(-x**2 + 1)*asin(x)/9
- 4*x/9 + 4*sqrt(-x**2 + 1)*asin(x)/9
```

Mathematica [A] time = 0.0370118, size = 42, normalized size = 0.69

$$\frac{1}{27}\left(9x^3\sin^{-1}(x)^2-2\left(x^2+6\right)x+6\sqrt{1-x^2}\left(x^2+2\right)\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*ArcSin[x]^2,x]
```

```
[Out] (-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*Arc
Sin[x]^2)/27
```

Maple [A] time = 0.036, size = 37, normalized size = 0.6

$$\frac{x^3 (\arcsin(x))^2}{3} + \frac{2\arcsin(x)(x^2+2)}{9}\sqrt{-x^2+1} - \frac{2x^3}{27} - \frac{4x}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arcsin(x)^2,x)
```

```
[Out] 1/3*x^3*arcsin(x)^2+2/9*arcsin(x)*(x^2+2)*(-x^2+1)^(1/2)-2/27*x^3
-4/9*x
```

Maxima [A] time = 1.57421, size = 63, normalized size = 1.03

$$\frac{1}{3}x^3\arcsin(x)^2 - \frac{2}{27}x^3 + \frac{2}{9}\left(\sqrt{-x^2+1}x^2 + 2\sqrt{-x^2+1}\right)\arcsin(x) - \frac{4}{9}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(sqrt(-x^2 + 1)*x^2 + 2*sqrt
(-x^2 + 1))*arcsin(x) - 4/9*x
```

Fricas [A] time = 0.231204, size = 49, normalized size = 0.8

$$\frac{1}{3}x^3\arcsin(x)^2 - \frac{2}{27}x^3 + \frac{2}{9}(x^2+2)\sqrt{-x^2+1}\arcsin(x) - \frac{4}{9}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*sqrt(-x^2 + 1)*arc
sin(x) - 4/9*x
```

Sympy [A] time = 0.943585, size = 54, normalized size = 0.89

$$\frac{x^3\operatorname{asin}^2(x)}{3}-\frac{2x^3}{27}+\frac{2x^2\sqrt{-x^2+1}\operatorname{asin}(x)}{9}-\frac{4x}{9}+\frac{4\sqrt{-x^2+1}\operatorname{asin}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*asin(x)**2,x)
```

```
[Out] x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(-x**2 + 1)*asin(x)/9
- 4*x/9 + 4*sqrt(-x**2 + 1)*asin(x)/9
```

GIAC/XCAS [A] time = 0.213155, size = 77, normalized size = 1.26

$$\frac{1}{3}\left(x^2-1\right)x\arcsin(x)^2+\frac{1}{3}x\arcsin(x)^2-\frac{2}{9}\left(-x^2+1\right)^{\frac{3}{2}}\arcsin(x)-\frac{2}{27}\left(x^2-1\right)x+\frac{2}{3}\sqrt{-x^2+1}\arcsin(x)-\frac{14}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x)^2 + 1/3*x*arcsin(x)^2 - 2/9*(-x^2 + 1)^
(3/2)*arcsin(x) - 2/27*(x^2 - 1)*x + 2/3*sqrt(-x^2 + 1)*arcsin(x)
 - 14/27*x
```

3.647 $\int x^3 \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=53

$$\frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{6}x^3 \tan^{-1}(x) + \frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right) + \frac{1}{2}x \tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)^2$$

```
[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x
^4*ArcTan[x]^2)/4 - Log[1 + x^2]/3
```

Rubi [A] time = 0.179567, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$

$$\frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{6}x^3 \tan^{-1}(x) + \frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right) + \frac{1}{2}x \tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*ArcTan[x]^2,x]
```

```
[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x
^4*ArcTan[x]^2)/4 - Log[1 + x^2]/3
```

Rubi in Sympy [A] time = 11.0979, size = 44, normalized size = 0.83

$$\frac{x^4 \operatorname{atan}^2(x)}{4} - \frac{x^3 \operatorname{atan}(x)}{6} + \frac{x^2}{12} + \frac{x \operatorname{atan}(x)}{2} - \frac{\log\left(x^2+1\right)}{3} - \frac{\operatorname{atan}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*atan(x)**2,x)
```

```
[Out] x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(
x**2 + 1)/3 - atan(x)**2/4
```

Mathematica [A] time = 0.0110817, size = 37, normalized size = 0.7

$$\frac{1}{12}\left(3\left(x^4-1\right)\tan^{-1}(x)^2 + x^2 - 4\log\left(x^2+1\right) - 2\left(x^2-3\right)x \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*ArcTan[x]^2,x]
```

```
[Out] (x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Lo
g[1 + x^2])/12
```

Maple [A] time = 0.016, size = 42, normalized size = 0.8

$$\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{(\arctan(x))^2}{4}+\frac{x^4(\arctan(x))^2}{4}-\frac{\ln(x^2+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arctan(x)^2,x)
```

```
[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^
4*arctan(x)^2-1/3*ln(x^2+1)
```

Maxima [A] time = 1.53726, size = 59, normalized size = 1.11

$$\frac{1}{4}x^4\arctan(x)^2+\frac{1}{12}x^2-\frac{1}{6}\left(x^3-3x+3\arctan(x)\right)\arctan(x)+\frac{1}{4}\arctan(x)^2-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*ar
ctan(x) + 1/4*arctan(x)^2 - 1/3*log(x^2 + 1)
```

Fricas [A] time = 0.227866, size = 49, normalized size = 0.92

$$\frac{1}{4}\left(x^4-1\right)\arctan(x)^2+\frac{1}{12}x^2-\frac{1}{6}\left(x^3-3x\right)\arctan(x)-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x)*arctan(x)
- 1/3*log(x^2 + 1)
```

Sympy [A] time = 0.994043, size = 44, normalized size = 0.83

$$\frac{x^4 \operatorname{atan}^2(x)}{4} - \frac{x^3 \operatorname{atan}(x)}{6} + \frac{x^2}{12} + \frac{x \operatorname{atan}(x)}{2} - \frac{\log(x^2+1)}{3} - \frac{\operatorname{atan}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*atan(x)**2,x)
```

```
[Out] x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(
x**2 + 1)/3 - atan(x)**2/4
```

GIAC/XCAS [A] time = 0.212072, size = 58, normalized size = 1.09

$$\frac{1}{4}x^4 \arctan(x)^2 - \frac{1}{6}x^3 \arctan(x) + \frac{1}{12}x^2 + \frac{1}{2}x \arctan(x) - \frac{1}{4}\arctan(x)^2 - \frac{1}{3}\ln(i x^2 + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(x)^2 - 1/6*x^3*arctan(x) + 1/12*x^2 + 1/2*x*arctan
(x) - 1/4*arctan(x)^2 - 1/3*ln(I*x^2 + I)
```

3.648 $\int \frac{\tan^{-1}(x)^2}{x^5} \, dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}(x)^2}{4x^4} - \frac{\tan^{-1}(x)}{6x^3} - \frac{1}{12x^2} + \frac{1}{3}\log\left(x^2+1\right) - \frac{2\log(x)}{3} + \frac{1}{4}\tan^{-1}(x)^2 + \frac{\tan^{-1}(x)}{2x}$$

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4
 - ArcTan[x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Rubi [A] time = 0.201209, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.

$$-\frac{\tan^{-1}(x)^2}{4x^4} - \frac{\tan^{-1}(x)}{6x^3} - \frac{1}{12x^2} + \frac{1}{3}\log\left(x^2+1\right) - \frac{2\log(x)}{3} + \frac{1}{4}\tan^{-1}(x)^2 + \frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4
 - ArcTan[x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Rubi in Sympy [A] time = 12.6269, size = 53, normalized size = 0.87

$$-\frac{\log\left(x^2\right)}{3} + \frac{\log\left(x^2+1\right)}{3} + \frac{\operatorname{atan}^2(x)}{4} + \frac{\operatorname{atan}(x)}{2x} - \frac{1}{12x^2} - \frac{\operatorname{atan}(x)}{6x^3} - \frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x)**2/x**5,x)
```

```
[Out] -log(x**2)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1
/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

Mathematica [A] time = 0.0133414, size = 56, normalized size = 0.92

$$\frac{\left(x^4-1\right)\tan^{-1}(x)^2}{4x^4} - \frac{1}{12x^2} + \frac{1}{3}\log\left(x^2+1\right) + \frac{\left(3x^2-1\right)\tan^{-1}(x)}{6x^3} - \frac{2\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcT
an[x]^2)/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Maple [A] time = 0.019, size = 48, normalized size = 0.8

$$-\frac{1}{12\,x^{2}}-\frac{\arctan\left(x\right)}{6\,x^{3}}+\frac{\arctan\left(x\right)}{2\,x}+\frac{\left(\arctan\left(x\right)\right)^{2}}{4}-\frac{\left(\arctan\left(x\right)\right)^{2}}{4\,x^{4}}-\frac{2\,\ln\left(x\right)}{3}+\frac{\ln\left(x^{2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x)^2/x^5,x)
```

```
[Out] -1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*a
rctan(x)^2/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

Maxima [A] time = 1.50342, size = 86, normalized size = 1.41

$$\frac{1}{6}\left(\frac{3\,x^{2}-1}{x^{3}}+3\,\arctan\left(x\right)\right)\arctan\left(x\right)-\frac{3\,x^{2}\arctan\left(x\right)^{2}-4\,x^{2}\log\left(x^{2}+1\right)+8\,x^{2}\log\left(x\right)+1}{12\,x^{2}}-\frac{\arctan\left(x\right)^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*x^2 - 1)/x^3 + 3*arctan(x))*arctan(x) - 1/12*(3*x^2*arcta
n(x)^2 - 4*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2 - 1/4*arctan(
x)^2/x^4
```

Fricas [A] time = 0.232671, size = 72, normalized size = 1.18

$$\frac{4\,x^{4}\log\left(x^{2}+1\right)-8\,x^{4}\log\left(x\right)+3\,\left(x^{4}-1\right)\arctan\left(x\right)^{2}-x^{2}+2\,\left(3\,x^{3}-x\right)\arctan\left(x\right)}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x^2 + 1) - 8*x^4*log(x) + 3*(x^4 - 1)*arctan(x)^2
 - x^2 + 2*(3*x^3 - x)*arctan(x))/x^4
```

Sympy [A] time = 1.70025, size = 53, normalized size = 0.87

$$-\frac{2\log(x)}{3}+\frac{\log(x^2+1)}{3}+\frac{\operatorname{atan}^2(x)}{4}+\frac{\operatorname{atan}(x)}{2x}-\frac{1}{12x^2}-\frac{\operatorname{atan}(x)}{6x^3}-\frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x)**2/x**5,x)
```

```
[Out] -2*log(x)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1/
(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^5, x)
```

3.649 $\int x^3 \csc^{-1}(x)^2 \, dx$

Optimal. Leaf size=63

$$\frac{1}{4}x^4\csc^{-1}(x)^2+\frac{x^2}{12}+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)+\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)+\frac{\log(x)}{3}$$

```
[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3
*ArcCsc[x])/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3
```

Rubi [A] time = 0.104295, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$\frac{1}{4}x^4\csc^{-1}(x)^2+\frac{x^2}{12}+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)+\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*ArcCsc[x]^2,x]
```

```
[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3
*ArcCsc[x])/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int^{\operatorname{acsc}(x)}\frac{x^2}{\sin^4(x)\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*acsc(x)**2,x)
```

```
[Out] -Integral(x**2/(sin(x)**4*tan(x)), (x, acsc(x)))
```

Mathematica [A] time = 0.0458619, size = 42, normalized size = 0.67

$$\frac{1}{12}\left(3x^4\csc^{-1}(x)^2+x^2+2\sqrt{1-\frac{1}{x^2}}\left(x^2+2\right)x\csc^{-1}(x)+4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*ArcCsc[x]^2,x]
```

```
[Out] (x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]
^2 + 4*Log[x])/12
```

Maple [A] time = 0.073, size = 56, normalized size = 0.9

$$\frac{x^4 (\operatorname{arccsc}(x))^2}{4} + \frac{\operatorname{arccsc}(x)\, x^3}{6}\sqrt{\frac{x^2-1}{x^2}} + \frac{x^2}{12} + \frac{\operatorname{arccsc}(x)\, x}{3}\sqrt{\frac{x^2-1}{x^2}} - \frac{\ln\left(x^{-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arccsc(x)^2,x)
```

```
[Out] 1/4*x^4*arccsc(x)^2+1/6*arccsc(x)*((x^2-1)/x^2)^(1/2)*x^3+1/12*x^
2+1/3*arccsc(x)*((x^2-1)/x^2)^(1/2)*x-1/3*ln(1/x)
```

Maxima [A] time = 1.41225, size = 74, normalized size = 1.17

$$\frac{1}{4}\, x^4 \operatorname{arccsc}(x)^2 + \frac{1}{12}\, x^2 + \frac{1}{6}\left(x^3\left(-\frac{1}{x^2}+1\right)^{\frac{3}{2}} + 3\, x\sqrt{-\frac{1}{x^2}+1}\right)\operatorname{arccsc}(x) - \frac{1}{12}\log\left(x^2\right) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arccsc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/12*x^2 + 1/6*(x^3*(-1/x^2 + 1)^(3/2) + 3*
x*sqrt(-1/x^2 + 1))*arccsc(x) - 1/12*log(x^2) + 1/2*log(x)
```

Fricas [A] time = 0.236001, size = 47, normalized size = 0.75

$$\frac{1}{4}\, x^4 \operatorname{arccsc}(x)^2 + \frac{1}{6}\left(x^2+2\right)\sqrt{x^2-1}\operatorname{arccsc}(x) + \frac{1}{12}\, x^2 + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arccsc(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/6*(x^2 + 2)*sqrt(x^2 - 1)*arccsc(x) + 1/1
2*x^2 + 1/3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{acsc}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*acsc(x)**2,x)
```

```
[Out] Integral(x**3*acsc(x)**2, x)
```

GIAC/XCAS [A] time = 0.248377, size = 1, normalized size = 0.02

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arccsc(x)^2,x, algorithm="giac")
```

```
[Out] sage2
```

3.650 $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$

Optimal. Leaf size=148

$$-\frac{3}{128x^4}-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}-\frac{45}{128x^2}+\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{9\sec^{-1}(x)^2}{16x^2}-\frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3}+\frac{3}{32}\sec^{-1}(x)^4-\frac{45}{128}\sec^{-1}(x)^2$$

```
[Out] -3/(128*x^4) - 45/(128*x^2) - (3*Sqrt[1 - x^(-2)]*ArcSec[x])/(32*
x^3) - (45*Sqrt[1 - x^(-2)]*ArcSec[x])/(64*x) - (45*ArcSec[x]^2)/
128 + (3*ArcSec[x]^2)/(16*x^4) + (9*ArcSec[x]^2)/(16*x^2) + (Sqrt
[1 - x^(-2)]*ArcSec[x]^3)/(4*x^3) + (3*Sqrt[1 - x^(-2)]*ArcSec[x]
^3)/(8*x) + (3*ArcSec[x]^4)/32 - ArcSec[x]^4/(4*x^4)
```

Rubi [A] time = 0.21998, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{3}{128x^4}-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}-\frac{45}{128x^2}+\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{9\sec^{-1}(x)^2}{16x^2}-\frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3}+\frac{3}{32}\sec^{-1}(x)^4-\frac{45}{128}\sec^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSec[x]^4/x^5,x]
```

```
[Out] -3/(128*x^4) - 45/(128*x^2) - (3*Sqrt[1 - x^(-2)]*ArcSec[x])/(32*
x^3) - (45*Sqrt[1 - x^(-2)]*ArcSec[x])/(64*x) - (45*ArcSec[x]^2)/
128 + (3*ArcSec[x]^2)/(16*x^4) + (9*ArcSec[x]^2)/(16*x^2) + (Sqrt
[1 - x^(-2)]*ArcSec[x]^3)/(4*x^3) + (3*Sqrt[1 - x^(-2)]*ArcSec[x]
^3)/(8*x) + (3*ArcSec[x]^4)/32 - ArcSec[x]^4/(4*x^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\operatorname{asec}(x)} x^4\cos^4(x)\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x)**4/x**5,x)
```

```
[Out] Integral(x**4*cos(x)**4*tan(x), (x, asec(x)))
```

Mathematica [A] time = 0.0842493, size = 92, normalized size = 0.62

$$\frac{4\left(3x^4-8\right)\sec^{-1}(x)^4-45x^2+16\sqrt{1-\frac{1}{x^2}}x\left(3x^2+2\right)\sec^{-1}(x)^3-6\sqrt{1-\frac{1}{x^2}}x\left(15x^2+2\right)\sec^{-1}(x)+\left(-45x^4+72x^2+24\right)\text{se}}{128x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSec[x]^4/x^5,x]
```

```
[Out] (-3 - 45*x^2 - 6*Sqrt[1 - x^(-2)]*x*(2 + 15*x^2)*ArcSec[x] + (24
+ 72*x^2 - 45*x^4)*ArcSec[x]^2 + 16*Sqrt[1 - x^(-2)]*x*(2 + 3*x^2
)*ArcSec[x]^3 + 4*(-8 + 3*x^4)*ArcSec[x]^4)/(128*x^4)
```

Maple [A] time = 0.099, size = 165, normalized size = 1.1

$$-\frac{(\operatorname{arcsec}(x))^4}{4x^4}+\frac{(\operatorname{arcsec}(x))^3}{8x^3}\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-1}{x^2}}\right)+\frac{3(\operatorname{arcsec}(x))^2}{16x^4}$$
$$-\frac{3\operatorname{arcsec}(x)}{64x^3}\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-1}{x^2}}\right)+\frac{45(\operatorname{arcsec}(x))^2}{128}-\frac{3}{128x^4}$$
$$-\frac{45}{128x^2}+\frac{9(\operatorname{arcsec}(x))^2}{16x^2}-\frac{9\operatorname{arcsec}(x)}{16x}\left(x\operatorname{arcsec}(x)+\sqrt{\frac{x^2-1}{x^2}}\right)+\frac{9}{32}-\frac{9(\operatorname{arcsec}(x))^4}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x)^4/x^5,x)
```

```
[Out] -1/4*arcsec(x)^4/x^4+1/8*arcsec(x)^3*(3*arcsec(x)*x^3+3*x^2*((x^2
-1)/x^2)^(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+3/16*arcsec(x)^2/x^4-3/
64*arcsec(x)*(3*arcsec(x)*x^3+3*x^2*((x^2-1)/x^2)^(1/2)+2*((x^2-1
)/x^2)^(1/2))/x^3+45/128*arcsec(x)^2-3/128/x^4-45/128/x^2+9/16*ar
csec(x)^2/x^2-9/16*arcsec(x)*(x*arcsec(x)+((x^2-1)/x^2)^(1/2))/x+
9/32-9/32*arcsec(x)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)^4/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.238045, size = 104, normalized size = 0.7

$$\frac{4\left(3x^4-8\right)\operatorname{arcsec}(x)^4-3\left(15x^4-24x^2-8\right)\operatorname{arcsec}(x)^2-45x^2+2\left(8\left(3x^2+2\right)\operatorname{arcsec}(x)^3-3\left(15x^2+2\right)\operatorname{arcsec}(x)\right)\sqrt{x}}{128x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)^4/x^5,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(3*x^4 - 8)*arcsec(x)^4 - 3*(15*x^4 - 24*x^2 - 8)*arcsec
(x)^2 - 45*x^2 + 2*(8*(3*x^2 + 2)*arcsec(x)^3 - 3*(15*x^2 + 2)*ar
csec(x))*sqrt(x^2 - 1) - 3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asec}^4(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x)**4/x**5,x)
```

```
[Out] Integral(asec(x)**4/x**5, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcsec}(x)^4}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)^4/x^5,x, algorithm="giac")
```

```
[Out] integrate(arcsec(x)^4/x^5, x)
```

3.651 $\int \sqrt{1-x^2} \sin^{-1}(x) \, dx$

Optimal. Leaf size=34

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2$$

```
[Out] -x^2/4 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rubi [A] time = 0.0500607, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]*ArcSin[x],x]

[Out] -x^2/4 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x\sqrt{-x^2+1}\operatorname{asin}(x)}{2}+\frac{\operatorname{asin}^2(x)}{4}-\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)*(-x**2+1)**(1/2),x)

[Out] x*sqrt(-x**2 + 1)*asin(x)/2 + asin(x)**2/4 - Integral(x, x)/2
```

Mathematica [A] time = 0.0138799, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(-x^2+2\sqrt{1-x^2}x\sin^{-1}(x)+\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]*ArcSin[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

Maple [A] time = 0.065, size = 31, normalized size = 0.9

$$\frac{\arcsin\left(x\right)}{2}\left(x\sqrt{-x^{2}+1}+\arcsin\left(x\right)\right)-\frac{\left(\arcsin\left(x\right)\right)^{2}}{4}-\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)*(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2-1/4*x^
2
```

Maxima [A] time = 1.49736, size = 41, normalized size = 1.21

$$-\frac{1}{4}\,x^{2}+\frac{1}{2}\left(\sqrt{-x^{2}+1}x+\arcsin\left(x\right)\right)\arcsin\left(x\right)-\frac{1}{4}\,\arcsin\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*arcsin(x),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arc
sin(x)^2
```

Fricas [A] time = 0.230856, size = 35, normalized size = 1.03

$$\frac{1}{2}\,\sqrt{-x^{2}+1}x\arcsin\left(x\right)-\frac{1}{4}\,x^{2}+\frac{1}{4}\,\arcsin\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2
```

Sympy [A] time = 31.7608, size = 48, normalized size = 1.41

$$\left(\begin{cases} \frac{x\sqrt{-x^2+1}}{2} + \frac{\operatorname{asin}(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}\right) \operatorname{asin}(x) - \begin{cases} \text{NaN} & \text{for } x < -1 \\ \frac{x^2}{4} + \frac{\operatorname{asin}^2(x)}{4} - \frac{\pi^2}{16} - \frac{1}{4} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(-x**2 + 1)/2 + asin(x)/2, (x > -1) & (x < 1)))*
asin(x) - Piecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2
/16 - 1/4, x < 1), (nan, True))
```

GIAC/XCAS [A] time = 0.213836, size = 36, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x) - \frac{1}{4}x^2 + \frac{1}{4}\arcsin(x)^2 + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8
```

3.652 $\int \sqrt{1-x^2} \cos^{-1}(x) \, dx$

Optimal. Leaf size=34

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rubi [A] time = 0.050542, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x\sqrt{-x^2+1}\,\mathrm{acos}\,(x)}{2}-\frac{\mathrm{acos}^2\,(x)}{4}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 1)*acos(x)/2 - acos(x)**2/4 + Integral(x, x)/2
```

Mathematica [A] time = 0.0118167, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(x^2+2\sqrt{1-x^2}x\cos^{-1}(x)-\cos^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Maple [A] time = 0.065, size = 33, normalized size = 1.

$$-\frac{\arccos(x)}{2}\left(-x\sqrt{-x^2+1}+\arccos(x)\right)+\frac{(\arccos(x))^2}{4}+\frac{x^2}{4}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccos(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*
x^2-1/4
```

Maxima [A] time = 1.49377, size = 41, normalized size = 1.21

$$\frac{1}{4}x^2+\frac{1}{2}\left(\sqrt{-x^2+1}x+\arcsin(x)\right)\arccos(x)+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*arccos(x),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcs
in(x)^2
```

Fricas [A] time = 0.230259, size = 35, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*arccos(x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A] time = 31.937, size = 48, normalized size = 1.41

$$\left(\begin{cases} \frac{x\sqrt{-x^2+1}}{2} + \frac{\operatorname{asin}(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}\right) \operatorname{acos}(x) + \begin{cases} \text{NaN} & \text{for } x < -1 \\ \frac{x^2}{4} + \frac{\operatorname{asin}^2(x)}{4} - \frac{\pi^2}{16} - \frac{1}{4} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(-x**2 + 1)/2 + asin(x)/2, (x > -1) & (x < 1)))*
acos(x) + Piecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2
/16 - 1/4, x < 1), (nan, True))
```

GIAC/XCAS [A] time = 0.211954, size = 36, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+1}x \arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2 - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*arccos(x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

3.653 $\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{x}{3}$$

```
[Out] -x/3 + x^3/9 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

Rubi [A] time = 0.0505711, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{x}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x],x]

[Out] -x/3 + x^3/9 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

Rubi in Sympy [A] time = 3.02564, size = 20, normalized size = 0.67

$$\frac{x^3}{9}-\frac{x}{3}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}\operatorname{acos}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*acos(x)*(-x**2+1)**(1/2),x)

[Out] x**3/9 - x/3 - (-x**2 + 1)**(3/2)*acos(x)/3
```

Mathematica [A] time = 0.0248096, size = 26, normalized size = 0.87

$$\frac{1}{9}\left(x^3-3\left(1-x^2\right)^{3/2}\cos^{-1}(x)-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] (-3*x + x^3 - 3*(1 - x^2)^(3/2)*ArcCos[x])/9
```

Maple [C] time = 0.208, size = 134, normalized size = 4.5

$$-\frac{i+3\arccos(x)}{72}\left(4ix^3-4x^2\sqrt{-x^2+1}-3ix+\sqrt{-x^2+1}\right)+\frac{\arccos(x)+i}{8}\left(ix-\sqrt{-x^2+1}\right)-\frac{\arccos(x)-i}{8}\left(ix+\sqrt{-x^2+1}\right)+\frac{-i+3\arccos(x)}{72}\left(4ix^3+4x^2\sqrt{-x^2+1}-3ix-\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arccos(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/72*(I+3*arccos(x))*(4*I*x^3-4*x^2*(-x^2+1)^(1/2)-3*I*x+(-x^2+1
)^(1/2))+1/8*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))-1/8*(arccos(x)-I)
*(I*x+(-x^2+1)^(1/2))+1/72*(-I+3*arccos(x))*(4*I*x^3+4*x^2*(-x^2+
1)^(1/2)-3*I*x-(-x^2+1)^(1/2))
```

Maxima [A] time = 1.49901, size = 30, normalized size = 1.

$$\frac{1}{9}x^3-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)-\frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x*arccos(x),x, algorithm="maxima")
```

```
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

Fricas [A] time = 0.234238, size = 36, normalized size = 1.2

$$\frac{1}{9}x^3+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}\arccos(x)-\frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x*arccos(x),x, algorithm="fricas")
```

```
[Out] 1/9*x^3 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x) - 1/3*x
```

Sympy [A] time = 1.42644, size = 37, normalized size = 1.23

$$\frac{x^3}{9} + \frac{x^2\sqrt{-x^2+1}\,\mathrm{acos}\,(x)}{3} - \frac{x}{3} - \frac{\sqrt{-x^2+1}\,\mathrm{acos}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] x**3/9 + x**2*sqrt(-x**2 + 1)*acos(x)/3 - x/3 - sqrt(-x**2 + 1)*a
cos(x)/3
```

GIAC/XCAS [A] time = 0.211673, size = 30, normalized size = 1.

$$\frac{1}{9}\,x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x*arccos(x),x, algorithm="giac")
```

```
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

3.654 $\int \left(1-x^2\right)^{3/2} \sin^{-1}(x)\,dx$

Optimal. Leaf size=59

$$\frac{x^4}{16}-\frac{5x^2}{16}+\frac{1}{4}\left(1-x^2\right)^{3/2}x\sin^{-1}(x)+\frac{3}{8}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

```
[Out] (-5*x^2)/16 + x^4/16 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 + (x*(1 -
x^2)^(3/2)*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rubi [A] time = 0.0810063, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{x^4}{16}-\frac{5x^2}{16}+\frac{1}{4}\left(1-x^2\right)^{3/2}x\sin^{-1}(x)+\frac{3}{8}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2)/16 + x^4/16 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 + (x*(1 -
x^2)^(3/2)*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{16}+\frac{x\left(-x^2+1\right)^{\frac{3}{2}}\operatorname{asin}(x)}{4}+\frac{3x\sqrt{-x^2+1}\operatorname{asin}(x)}{8}+\frac{3\operatorname{asin}^2(x)}{16}-\frac{5\int x\,dx}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**4/16 + x*(-x**2 + 1)**(3/2)*asin(x)/4 + 3*x*sqrt(-x**2 + 1)*as
in(x)/8 + 3*asin(x)**2/16 - 5*Integral(x, x)/8
```

Mathematica [A] time = 0.0293751, size = 42, normalized size = 0.71

$$\frac{1}{16}\left(x^4-5x^2-2\sqrt{1-x^2}\left(2x^2-5\right)x\sin^{-1}(x)+3\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcS
in[x]^2)/16
```

Maple [A] time = 0.063, size = 58, normalized size = 1.

$$\frac{\arcsin(x)}{8}\left(-2\sqrt{-x^2+1}x^3+5x\sqrt{-x^2+1}+3\arcsin(x)\right)-\frac{3(\arcsin(x))^2}{16}+\frac{(x^2-1)^2}{16}-\frac{3x^2}{16}+\frac{3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(3/2)*arcsin(x),x)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*x*(-x^2+1)^(1/2)+3*arcsin(
x))-3/16*arcsin(x)^2+1/16*(x^2-1)^2-3/16*x^2+3/16
```

Maxima [A] time = 1.51325, size = 68, normalized size = 1.15

$$\frac{1}{16}x^4-\frac{5}{16}x^2+\frac{1}{8}\left(2\left(-x^2+1\right)^{\frac{3}{2}}x+3\sqrt{-x^2+1}x+3\arcsin(x)\right)\arcsin(x)-\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2 + 1)^(3/2)*x + 3*sqrt(-x^2 + 1
)*x + 3*arcsin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

Fricas [A] time = 0.235883, size = 53, normalized size = 0.9

$$\frac{1}{16}x^4-\frac{1}{8}\left(2x^3-5x\right)\sqrt{-x^2+1}\arcsin(x)-\frac{5}{16}x^2+\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/16*x^4 - 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 1)*arcsin(x) - 5/16*x^2
+ 3/16*arcsin(x)^2
```

Sympy [A] time = 3.22033, size = 53, normalized size = 0.9

$$\frac{x^4}{16} - \frac{x^3\sqrt{-x^2+1}\,\mathrm{asin}\,(x)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{-x^2+1}\,\mathrm{asin}\,(x)}{8} + \frac{3\,\mathrm{asin}^2\,(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**4/16 - x**3*sqrt(-x**2 + 1)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(-
x**2 + 1)*asin(x)/8 + 3*asin(x)**2/16
```

GIAC/XCAS [A] time = 0.215583, size = 68, normalized size = 1.15

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin\left(x\right) + \frac{3}{8}\sqrt{-x^2+1}x\arcsin\left(x\right) + \frac{1}{16}\left(x^2-1\right)^2 - \frac{3}{16}x^2 + \frac{3}{16}\arcsin\left(x\right)^2 + \frac{9}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x)
 + 1/16*(x^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128
```

3.655 $\int x\left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx$

Optimal. Leaf size=37

$$\frac{x^5}{25}-\frac{2x^3}{15}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{x}{5}$$

```
[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

Rubi [A] time = 0.0554275, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x^5}{25}-\frac{2x^3}{15}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]

[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

Rubi in Sympy [A] time = 3.05189, size = 27, normalized size = 0.73

$$\frac{x^5}{25}-\frac{2x^3}{15}+\frac{x}{5}-\frac{\left(-x^2+1\right)^{\frac{5}{2}}\operatorname{asin}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(-x**2+1)**(3/2)*asin(x),x)

[Out] x**5/25 - 2*x**3/15 + x/5 - (-x**2 + 1)**(5/2)*asin(x)/5
```

Mathematica [A] time = 0.0352308, size = 33, normalized size = 0.89

$$\frac{1}{75}\left(3x^5-10x^3-15\left(1-x^2\right)^{5/2}\sin^{-1}(x)+15x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (15*x - 10*x^3 + 3*x^5 - 15*(1 - x^2)^(5/2)*ArcSin[x])/75
```

Maple [A] time = 0.068, size = 37, normalized size = 1.

$$-\frac{\arcsin(x)\left(x^2-1\right)^2}{5}\sqrt{-x^2+1}+\frac{\left(3x^4-10x^2+15\right)x}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^2+1)^(3/2)*arcsin(x),x)
```

```
[Out] -1/5*arcsin(x)*(x^2-1)^2*(-x^2+1)^(1/2)+1/75*(3*x^4-10*x^2+15)*x
```

Maxima [A] time = 1.4915, size = 36, normalized size = 0.97

$$\frac{1}{25}x^5-\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}\arcsin(x)-\frac{2}{15}x^3+\frac{1}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*x*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x
```

Fricas [A] time = 0.228442, size = 50, normalized size = 1.35

$$\frac{1}{25}x^5-\frac{2}{15}x^3-\frac{1}{5}\left(x^4-2x^2+1\right)\sqrt{-x^2+1}\arcsin(x)+\frac{1}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*x*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin
(x) + 1/5*x
```

Sympy [A] time = 5.54244, size = 63, normalized size = 1.7

$$\frac{x^5}{25}-\frac{x^4\sqrt{-x^2+1}\operatorname{asin}(x)}{5}-\frac{2x^3}{15}+\frac{2x^2\sqrt{-x^2+1}\operatorname{asin}(x)}{5}+\frac{x}{5}-\frac{\sqrt{-x^2+1}\operatorname{asin}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**5/25 - x**4*sqrt(-x**2 + 1)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqr
t(-x**2 + 1)*asin(x)/5 + x/5 - sqrt(-x**2 + 1)*asin(x)/5
```

GIAC/XCAS [A] time = 0.227314, size = 46, normalized size = 1.24

$$\frac{1}{25}\,x^{5} - \frac{1}{5}\,{\left(x^{2} - 1\right)}^{2} \sqrt{-x^{2} + 1} \arcsin\left(x\right) - \frac{2}{15}\,x^{3} + \frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*x*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 +
1/5*x
```

3.656 $\int x^3 \left(1-x^2\right)^{3/2} \cos^{-1}(x)\, dx$

Optimal. Leaf size=61

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{2x}{35}$$

```
[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*Arc
Cos[x])/5 + ((1 - x^2)^(7/2)*ArcCos[x])/7
```

Rubi [A] time = 0.125726, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{2x}{35}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*Arc
Cos[x])/5 + ((1 - x^2)^(7/2)*ArcCos[x])/7
```

Rubi in Sympy [A] time = 7.70813, size = 48, normalized size = 0.79

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}-\frac{2x}{35}+\frac{\left(-x^2+1\right)^{\frac{7}{2}}\operatorname{acos}(x)}{7}-\frac{\left(-x^2+1\right)^{\frac{5}{2}}\operatorname{acos}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

```
[Out] -x**7/49 + 8*x**5/175 - x**3/105 - 2*x/35 + (-x**2 + 1)**(7/2)*ac
os(x)/7 - (-x**2 + 1)**(5/2)*acos(x)/5
```

Mathematica [A] time = 0.052059, size = 47, normalized size = 0.77

$$-\frac{1}{35}\left(5x^2+2\right)\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{x\left(75x^6-168x^4+35x^2+210\right)}{3675}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] -(x*(210 + 35*x^2 - 168*x^4 + 75*x^6))/3675 - ((1 - x^2)^(5/2)*(2
 + 5*x^2)*ArcCos[x])/35
```

Maple [C] time = 0.378, size = 430, normalized size = 7.1

$$\frac{i+7\arccos(x)}{6272}\left(64ix^7-64\sqrt{-x^2+1}x^6-112ix^5+80\sqrt{-x^2+1}x^4+56ix^3-24x^2\sqrt{-x^2+1}-7ix+\sqrt{-x^2+1}\right)$$
$$-\frac{i+5\arccos(x)}{3200}\left(16ix^5-16\sqrt{-x^2+1}x^4-20ix^3+12x^2\sqrt{-x^2+1}+5ix-\sqrt{-x^2+1}\right)$$
$$-\frac{i+3\arccos(x)}{384}\left(4ix^3-4x^2\sqrt{-x^2+1}-3ix+\sqrt{-x^2+1}\right)+\frac{3\arccos(x)+3i}{128}\left(ix-\sqrt{-x^2+1}\right)$$
$$-\frac{3\arccos(x)-3i}{128}\left(ix+\sqrt{-x^2+1}\right)+\frac{-i+3\arccos(x)}{384}\left(4ix^3+4x^2\sqrt{-x^2+1}-3ix-\sqrt{-x^2+1}\right)$$
$$+\frac{-i+5\arccos(x)}{3200}\left(16ix^5+16\sqrt{-x^2+1}x^4-20ix^3-12x^2\sqrt{-x^2+1}+5ix+\sqrt{-x^2+1}\right)$$
$$-\frac{-i+7\arccos(x)}{6272}\left(64ix^7+64\sqrt{-x^2+1}x^6-112ix^5-80\sqrt{-x^2+1}x^4+56ix^3+24x^2\sqrt{-x^2+1}-7ix-\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-x^2+1)^(3/2)*arccos(x),x)
```

```
[Out] 1/6272*(I+7*arccos(x))*(64*I*x^7-64*(-x^2+1)^(1/2)*x^6-112*I*x^5+
80*(-x^2+1)^(1/2)*x^4+56*I*x^3-24*x^2*(-x^2+1)^(1/2)-7*I*x+(-x^2+
1)^(1/2))-1/3200*(I+5*arccos(x))*(16*I*x^5-16*(-x^2+1)^(1/2)*x^4-
20*I*x^3+12*x^2*(-x^2+1)^(1/2)+5*I*x-(-x^2+1)^(1/2))-1/384*(I+3*a
rccos(x))*(4*I*x^3-4*x^2*(-x^2+1)^(1/2)-3*I*x+(-x^2+1)^(1/2))+3/1
28*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))-3/128*(arccos(x)-I)*(I*x+(-
x^2+1)^(1/2))+1/384*(-I+3*arccos(x))*(4*I*x^3+4*x^2*(-x^2+1)^(1/2
)-3*I*x-(-x^2+1)^(1/2))+1/3200*(-I+5*arccos(x))*(16*I*x^5+16*(-x^
2+1)^(1/2)*x^4-20*I*x^3-12*x^2*(-x^2+1)^(1/2)+5*I*x+(-x^2+1)^(1/2
))-1/6272*(-I+7*arccos(x))*(64*I*x^7+64*(-x^2+1)^(1/2)*x^6-112*I*
x^5-80*(-x^2+1)^(1/2)*x^4+56*I*x^3+24*x^2*(-x^2+1)^(1/2)-7*I*x-(-
x^2+1)^(1/2))
```

Maxima [A] time = 1.54128, size = 66, normalized size = 1.08

$$-\frac{1}{49}x^7+\frac{8}{175}x^5-\frac{1}{105}x^3-\frac{1}{35}\left(5\left(-x^2+1\right)^{\frac{5}{2}}x^2+2\left(-x^2+1\right)^{\frac{5}{2}}\right)\arccos(x)-\frac{2}{35}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*x^3*arccos(x),x, algorithm="maxima")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(-x^2 + 1)^(5/2)*x^2
+ 2*(-x^2 + 1)^(5/2))*arccos(x) - 2/35*x
```

Fricas [A] time = 0.229817, size = 63, normalized size = 1.03

$$-\frac{1}{49}x^7+\frac{8}{175}x^5-\frac{1}{105}x^3-\frac{1}{35}\left(5x^6-8x^4+x^2+2\right)\sqrt{-x^2+1}\arccos\left(x\right)-\frac{2}{35}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*x^3*arccos(x),x, algorithm="fricas")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*x^6 - 8*x^4 + x^2 + 2
)*sqrt(-x^2 + 1)*arccos(x) - 2/35*x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.230798, size = 81, normalized size = 1.33

$$-\frac{1}{49}x^7+\frac{8}{175}x^5-\frac{1}{105}x^3-\frac{1}{35}\left(5\left(x^2-1\right)^3\sqrt{-x^2+1}+7\left(x^2-1\right)^2\sqrt{-x^2+1}\right)\arccos\left(x\right)-\frac{2}{35}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*x^3*arccos(x),x, algorithm="giac")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2
 + 1) + 7*(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x
```

3.657 $\int \frac{(1-x^2)^{3/2}\cos^{-1}(x)}{x}\,dx$

Optimal. Leaf size=95

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)-\frac{x^3}{9}+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{4x}{3}+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right)$$

```
[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcC
os[x])/3 + (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2,
 (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Rubi [A] time = 0.242765, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)-\frac{x^3}{9}+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{4x}{3}+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcC
os[x])/3 + (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2,
 (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^3}{9}+\frac{4x}{3}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}\operatorname{acos}(x)}{3}+\sqrt{-x^2+1}\operatorname{acos}(x)-\int^{\operatorname{acos}(x)}\frac{x}{\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(3/2)*acos(x)/x,x)
```

```
[Out] -x**3/9 + 4*x/3 + (-x**2 + 1)**(3/2)*acos(x)/3 + sqrt(-x**2 + 1)*
acos(x) - Integral(x/cos(x), (x, acos(x)))
```

Mathematica [A] time = 0.316394, size = 119, normalized size = 1.25

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{36}\left(12\left(1-x^2\right)^{3/2}\cos^{-1}(x)+9x-\cos\left(3\cos^{-1}(x)\right)\right)+x-\cos^{-1}(x)\log\left(1-ie^{i\cos^{-1}(x)}\right)+\cos^{-1}(x)\log\left(1+ie^{i\cos^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] x + Sqrt[1 - x^2]*ArcCos[x] + (9*x + 12*(1 - x^2)^(3/2)*ArcCos[x]
 - Cos[3*ArcCos[x]])/36 - ArcCos[x]*Log[1 - I*E^(I*ArcCos[x])] +
ArcCos[x]*Log[1 + I*E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*Arc
Cos[x])] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Maple [C] time = 0.258, size = 227, normalized size = 2.4

$$\frac{i+3\arccos(x)}{72}\left(4ix^3-4x^2\sqrt{-x^2+1}-3ix+\sqrt{-x^2+1}\right)-\frac{5\arccos(x)+5i}{8}\left(ix-\sqrt{-x^2+1}\right)+\frac{5\arccos(x)-5i}{8}\left(ix+\sqrt{-x^2+1}\right)-\frac{-i+3\arccos(x)}{72}\left(4ix^3+4x^2\sqrt{-x^2+1}-3ix-\sqrt{-x^2+1}\right)+\arccos(x)\ln\left(1+i\left(x+i\sqrt{-x^2+1}\right)\right)-\arccos(x)\ln\left(1-i\left(x+i\sqrt{-x^2+1}\right)\right)-i\,dilog\left(1+i\left(x+i\sqrt{-x^2+1}\right)\right)+i\,dilog\left(1-i\left(x+i\sqrt{-x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(3/2)*arccos(x)/x,x)
```

```
[Out] 1/72*(I+3*arccos(x))*(4*I*x^3-4*x^2*(-x^2+1)^(1/2)-3*I*x+(-x^2+1)
^(1/2))-5/8*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))+5/8*(arccos(x)-I)*
(I*x+(-x^2+1)^(1/2))-1/72*(-I+3*arccos(x))*(4*I*x^3+4*x^2*(-x^2+1
)^(1/2)-3*I*x-(-x^2+1)^(1/2))+arccos(x)*ln(1+I*(x+I*(-x^2+1)^(1/2
)))-arccos(x)*ln(1-I*(x+I*(-x^2+1)^(1/2)))-I*dilog(1+I*(x+I*(-x^2
+1)^(1/2)))+I*dilog(1-I*(x+I*(-x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arccos(x)/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(x^2-1\right)\sqrt{-x^2+1}\arccos\left(x\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arccos(x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}\operatorname{acos}\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(3/2)*acos(x)/x,x)
```

```
[Out] Integral((-(x - 1)*(x + 1))**(3/2)*acos(x)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arccos(x)/x,x, algorithm="giac")
```

```
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

3.658 $\int \frac{(1-x^2)^{3/2}\sin^{-1}(x)}{x^6}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}$$

```
[Out] -1/(20*x^4) + 1/(5*x^2) - ((1 - x^2)^(5/2)*ArcSin[x])/(5*x^5) + L
og[x]/5
```

Rubi [A] time = 0.0909712, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

```
[Out] -1/(20*x^4) + 1/(5*x^2) - ((1 - x^2)^(5/2)*ArcSin[x])/(5*x^5) + L
og[x]/5
```

Rubi in Sympy [A] time = 5.21241, size = 36, normalized size = 0.88

$$\frac{\log(x^2)}{10}+\frac{1}{5x^2}-\frac{1}{20x^4}-\frac{(-x^2+1)^{\frac{5}{2}}\operatorname{asin}(x)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

```
[Out] log(x**2)/10 + 1/(5*x**2) - 1/(20*x**4) - (-x**2 + 1)**(5/2)*asin
(x)/(5*x**5)
```

Mathematica [A] time = 0.0511058, size = 36, normalized size = 0.88

$$-\frac{-4x^5\log(x)-4x^3+4(1-x^2)^{5/2}\sin^{-1}(x)+x}{20x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

```
[Out] -(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/(20*x^5
)
```

Maple [C] time = 0.664, size = 201, normalized size = 4.9

$$-\frac{2i}{5}\arcsin(x) + \frac{1}{(100x^8 - 200x^6 + 200x^4 - 100x^2 + 20)x^5}\left(-\sqrt{-x^2+1}x^4 + ix^5 + 2x^2\sqrt{-x^2+1} - \sqrt{-x^2+1}\right)\left(20\arcsin(x)x^8 - 4ix^8 - 4\right. + \frac{1}{5}\ln\left(\left(ix + \sqrt{-x^2+1}\right)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(3/2)*arcsin(x)/x^6,x)
```

```
[Out] -2/5*I*arcsin(x)+1/20*(-(-x^2+1)^(1/2)*x^4+I*x^5+2*x^2*(-x^2+1)^(
1/2)-(-x^2+1)^(1/2))*(20*arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^(1/2)*x
^7-40*arcsin(x)*x^6+I*x^6+9*(-x^2+1)^(1/2)*x^5+40*arcsin(x)*x^4-6
*(-x^2+1)^(1/2)*x^3-20*x^2*arcsin(x)+x*(-x^2+1)^(1/2)+4*arcsin(x)
)/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+1/5*ln((I*x+(-x^2+1)^(1/2))^2
-1)
```

Maxima [A] time = 1.49161, size = 47, normalized size = 1.15

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}\arcsin(x)}{5x^5} + \frac{4x^2-1}{20x^4} + \frac{1}{10}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)*arcsin(x)/x^5 + 1/20*(4*x^2 - 1)/x^4 + 1/10
*log(x^2)
```

Fricas [A] time = 0.2627, size = 59, normalized size = 1.44

$$\frac{4x^5\log(x) + 4x^3 - 4\left(x^4 - 2x^2 + 1\right)\sqrt{-x^2+1}\arcsin(x) - x}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(4*x^5*log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*a
rcsin(x) - x)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}\operatorname{asin}\left(x\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

```
[Out] Integral((-(x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)
```

GIAC/XCAS [A] time = 0.22152, size = 182, normalized size = 4.44

$$-\frac{1}{160}\left(\frac{x^{5}\left(\frac{5\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}}-\frac{10\left(\sqrt{-x^{2}+1}-1\right)^{4}}{x^{4}}-1\right)}{\left(\sqrt{-x^{2}+1}-1\right)^{5}}+\frac{10\left(\sqrt{-x^{2}+1}-1\right)}{x}-\frac{5\left(\sqrt{-x^{2}+1}-1\right)^{3}}{x^{3}}+\frac{\left(\sqrt{-x^{2}+1}-1\right)^{5}}{x^{5}}\right)\arcsin\left(x\right)-\frac{3\,x^{4}-4\,x^{2}+1}{20\,x^{4}}+\frac{1}{10}\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 1)^(3/2)*arcsin(x)/x^6,x, algorithm="giac")
```

```
[Out] -1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) -
1)^4/x^4 - 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x
- 5*(sqrt(-x^2 + 1) - 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcs
in(x) - 1/20*(3*x^4 - 4*x^2 + 1)/x^4 + 1/10*ln(x^2)
```

3.659 $\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=34

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2$$

```
[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rubi [A] time = 0.0981733, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x\sqrt{-x^2+1}\operatorname{asin}(x)}{2} + \frac{\operatorname{asin}^2(x)}{4} + \frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(-x**2 + 1)*asin(x)/2 + asin(x)**2/4 + Integral(x, x)/2
```

Mathematica [A] time = 0.0178506, size = 28, normalized size = 0.82

$$\frac{1}{4}\left(x^2 - 2\sqrt{1-x^2}x\sin^{-1}(x) + \sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

Maple [A] time = 0.059, size = 32, normalized size = 0.9

$$\frac{\arcsin(x)}{2}\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)-\frac{(\arcsin(x))^2}{4}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arcsin(x)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)*(-x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2+1/4*x
^2
```

Maxima [A] time = 1.4913, size = 43, normalized size = 1.26

$$\frac{1}{4}x^2-\frac{1}{2}\left(\sqrt{-x^2+1}x-\arcsin(x)\right)\arcsin(x)-\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcs
in(x)^2
```

Fricas [A] time = 0.217913, size = 35, normalized size = 1.03

$$-\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2
```

Sympy [A] time = 0.488454, size = 26, normalized size = 0.76

$$\frac{x^2}{4}-\frac{x\sqrt{-x^2+1}\operatorname{asin}(x)}{2}+\frac{\operatorname{asin}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**2/4 - x*sqrt(-x**2 + 1)*asin(x)/2 + asin(x)**2/4
```

GIAC/XCAS [A] time = 0.220856, size = 36, normalized size = 1.06

$$-\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8
```

3.660 $\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=61

$$\frac{x^4}{16} + \frac{3x^2}{16} - \frac{3}{8}\sqrt{1-x^2}x\sin^{-1}(x) - \frac{1}{4}\sqrt{1-x^2}x^3\sin^{-1}(x) + \frac{3}{16}\sin^{-1}(x)^2$$

```
[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt
[1 - x^2]*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rubi [A] time = 0.170617, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{x^4}{16} + \frac{3x^2}{16} - \frac{3}{8}\sqrt{1-x^2}x\sin^{-1}(x) - \frac{1}{4}\sqrt{1-x^2}x^3\sin^{-1}(x) + \frac{3}{16}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt
[1 - x^2]*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{16} - \frac{x^3\sqrt{-x^2+1}\,\mathrm{asin}\,(x)}{4} - \frac{3x\sqrt{-x^2+1}\,\mathrm{asin}\,(x)}{8} + \frac{3\,\mathrm{asin}^2\,(x)}{16} + \frac{3\int x\,dx}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**4/16 - x**3*sqrt(-x**2 + 1)*asin(x)/4 - 3*x*sqrt(-x**2 + 1)*as
in(x)/8 + 3*asin(x)**2/16 + 3*Integral(x, x)/8
```

Mathematica [A] time = 0.063141, size = 42, normalized size = 0.69

$$\frac{1}{16}\left(x^4 + 3x^2 - 2\sqrt{1-x^2}\left(2x^2+3\right)x\sin^{-1}(x) + 3\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin
[x]^2)/16
```

Maple [A] time = 0.069, size = 53, normalized size = 0.9

$$\frac{\arcsin(x)}{8}\left(-2\sqrt{-x^2+1}x^3-3x\sqrt{-x^2+1}+3\arcsin(x)\right)-\frac{3(\arcsin(x))^2}{16}+\frac{x^4}{16}+\frac{3x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*arcsin(x)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3-3*x*(-x^2+1)^(1/2)+3*arcsin(
x))-3/16*arcsin(x)^2+1/16*x^4+3/16*x^2
```

Maxima [A] time = 1.49297, size = 70, normalized size = 1.15

$$\frac{1}{16}x^4+\frac{3}{16}x^2-\frac{1}{8}\left(2\sqrt{-x^2+1}x^3+3\sqrt{-x^2+1}x-3\arcsin(x)\right)\arcsin(x)-\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*arcsin(x)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 + 3/16*x^2 - 1/8*(2*sqrt(-x^2 + 1)*x^3 + 3*sqrt(-x^2 + 1
)*x - 3*arcsin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

Fricas [A] time = 0.222698, size = 53, normalized size = 0.87

$$\frac{1}{16}x^4-\frac{1}{8}\left(2x^3+3x\right)\sqrt{-x^2+1}\arcsin(x)+\frac{3}{16}x^2+\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*arcsin(x)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/16*x^4 - 1/8*(2*x^3 + 3*x)*sqrt(-x^2 + 1)*arcsin(x) + 3/16*x^2
+ 3/16*arcsin(x)^2
```

Sympy [A] time = 1.85282, size = 53, normalized size = 0.87

$$\frac{x^4}{16} - \frac{x^3\sqrt{-x^2+1}\operatorname{asin}(x)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{-x^2+1}\operatorname{asin}(x)}{8} + \frac{3\operatorname{asin}^2(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**4/16 - x**3*sqrt(-x**2 + 1)*asin(x)/4 + 3*x**2/16 - 3*x*sqrt(-
x**2 + 1)*asin(x)/8 + 3*asin(x)**2/16
```

GIAC/XCAS [A] time = 0.219396, size = 68, normalized size = 1.11

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin(x) - \frac{5}{8}\sqrt{-x^2+1}x\arcsin(x) + \frac{1}{16}\left(x^2-1\right)^2 + \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2 - \frac{23}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*arcsin(x)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) - 5/8*sqrt(-x^2 + 1)*x*arcsin(x)
 + 1/16*(x^2 - 1)^2 + 5/16*x^2 + 3/16*arcsin(x)^2 - 23/128
```

3.661 $\int \frac{x \sin^{-1}(x)}{(1-x^2)^{3/2}} \, dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Rubi [A] time = 0.0551174, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Rubi in Sympy [A] time = 2.84078, size = 14, normalized size = 0.74

$$-\operatorname{atanh}(x) + \frac{\operatorname{asin}(x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -atanh(x) + asin(x)/sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.0512091, size = 32, normalized size = 1.68

$$\frac{1}{2}\left(\frac{2\sin^{-1}(x)}{\sqrt{1-x^2}} + \log(1-x) - \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ((2*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/2
```

Maple [B] time = 0.067, size = 46, normalized size = 2.4

$$-\frac{\arcsin(x)}{x^2-1}\sqrt{-x^2+1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+x\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arcsin(x)/(-x^2+1)^(3/2),x)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+x/(-x^2+1)^
(1/2))
```

Maxima [A] time = 1.54947, size = 34, normalized size = 1.79

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}}-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x)/(-x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 0.223584, size = 59, normalized size = 3.11

$$-\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)+2\sqrt{-x^2+1}\arcsin(x)}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x)/(-x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 +
 1)*arcsin(x))/(x^2 - 1)
```

Sympy [A] time = 5.40632, size = 20, normalized size = 1.05

$$-\begin{cases}\operatorname{acoth}(x) & \text{for } x^2>1\\ \operatorname{atanh}(x) & \text{for } x^2<1\end{cases}+\frac{\operatorname{asin}(x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + asin(x)/
sqrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.213926, size = 36, normalized size = 1.89

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2}\ln(|x+1|) + \frac{1}{2}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x)/(-x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*ln(abs(x + 1)) + 1/2*ln(abs(x - 1)
)
```

3.662 $\int \frac{x \cos^{-1}(x)}{(1-x^2)^{3/2}}\, dx$

Optimal. Leaf size=17

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} + \tanh^{-1}(x)$$

```
[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

Rubi [A] time = 0.051085, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*ArcCos[x])/(1 - x^2)^(3/2),x]

[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

Rubi in Sympy [A] time = 2.95559, size = 14, normalized size = 0.82

$$\operatorname{atanh}(x) + \frac{\operatorname{acos}(x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*acos(x)/(-x**2+1)**(3/2),x)

[Out] atanh(x) + acos(x)/sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.0503308, size = 32, normalized size = 1.88

$$\frac{1}{2}\left(\frac{2\cos^{-1}(x)}{\sqrt{1-x^2}} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2
```

Maple [B] time = 0.075, size = 47, normalized size = 2.8

$$-\frac{\arccos(x)}{x^2-1}\sqrt{-x^2+1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}-x\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)/(-x^2+1)^(3/2),x)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(1/(-x^2+1)^(1/2)-x/(-x^2+1)^
(1/2))
```

Maxima [A] time = 1.49289, size = 34, normalized size = 2.

$$\frac{\arccos(x)}{\sqrt{-x^2+1}}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A] time = 0.226343, size = 59, normalized size = 3.47

$$\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\sqrt{-x^2+1}\arccos(x)}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 +
1)*arccos(x))/(x^2 - 1)
```

Sympy [A] time = 7.73623, size = 20, normalized size = 1.18

$$\begin{cases}\operatorname{acoth}(x) & \text{for } x^2>1\\ \operatorname{atanh}(x) & \text{for } x^2<1\end{cases}+\frac{\operatorname{acos}(x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*acos(x)/(-x**2+1)**(3/2),x)
```

```
[Out] Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + acos(x)/s
qrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.214696, size = 36, normalized size = 2.12

$$\frac{\arccos\left(x\right)}{\sqrt{-x^{2}+1}}+\frac{1}{2}\ln\left(|x+1|\right)-\frac{1}{2}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arccos(x)/(-x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*ln(abs(x + 1)) - 1/2*ln(abs(x - 1)
)
```

3.663 $\int \frac{\sin^{-1}(x)}{(1-x^2)^{5/2}} \, dx$

Optimal. Leaf size=62

$$-\frac{1}{6(1-x^2)}+\frac{1}{3}\log\left(1-x^2\right)+\frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}}+\frac{x\sin^{-1}(x)}{3(1-x^2)^{3/2}}$$

```
[Out] -1/(6*(1 - x^2)) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*x*ArcSi
n[x])/(3*Sqrt[1 - x^2]) + Log[1 - x^2]/3
```

Rubi [A] time = 0.0614553, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{1}{6(1-x^2)}+\frac{1}{3}\log\left(1-x^2\right)+\frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}}+\frac{x\sin^{-1}(x)}{3(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] -1/(6*(1 - x^2)) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*x*ArcSi
n[x])/(3*Sqrt[1 - x^2]) + Log[1 - x^2]/3
```

Rubi in Sympy [A] time = 4.91892, size = 49, normalized size = 0.79

$$\frac{2x\operatorname{asin}(x)}{3\sqrt{-x^2+1}}+\frac{x\operatorname{asin}(x)}{3(-x^2+1)^{\frac{3}{2}}}+\frac{\log((x-1)(x+1))}{3}-\frac{1}{6(-x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)/(-x**2+1)**(5/2),x)
```

```
[Out] 2*x*asin(x)/(3*sqrt(-x**2 + 1)) + x*asin(x)/(3*(-x**2 + 1)**(3/2)
) + log((x - 1)*(x + 1))/3 - 1/(6*(-x**2 + 1))
```

Mathematica [A] time = 0.0931762, size = 45, normalized size = 0.73

$$\frac{1}{6}\left(\frac{1}{x^2-1}+2\log\left(1-x^2\right)-\frac{2x\left(2x^2-3\right)\sin^{-1}(x)}{(1-x^2)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] ((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) +
 2*Log[1 - x^2])/6
```

Maple [A] time = 0.059, size = 63, normalized size = 1.

$$\frac{\arcsin(x)\,x}{3\,(x^2-1)^2}\sqrt{-x^2+1}+\frac{1}{6\,x^2-6}-\frac{2\,\arcsin(x)\,x}{3\,x^2-3}\sqrt{-x^2+1}+\frac{\ln\left(-x^2+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x*arcsin(x)*(-x^2+1)^(1/2)/(x^2-1)^2+1/6/(x^2-1)-2/3*(-x^2+1)
^(1/2)/(x^2-1)*arcsin(x)*x+1/3*ln(-x^2+1)
```

Maxima [A] time = 1.51415, size = 65, normalized size = 1.05

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{-x^2+1}}+\frac{x}{(-x^2+1)^{\frac{3}{2}}}\right)\arcsin(x)+\frac{1}{6\,(x^2-1)}+\frac{1}{3}\,\log\left(-3\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/(-x^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(-x^2 + 1) + x/(-x^2 + 1)^(3/2))*arcsin(x) + 1/6/(x^
2 - 1) + 1/3*log(-3*x^2 + 3)
```

Fricas [A] time = 0.225259, size = 82, normalized size = 1.32

$$-\frac{2\left(2\,x^3-3\,x\right)\sqrt{-x^2+1}\,\arcsin(x)-x^2-2\left(x^4-2\,x^2+1\right)\log\left(x^2-1\right)+1}{6\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/(-x^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*x^3 - 3*x)*sqrt(-x^2 + 1)*arcsin(x) - x^2 - 2*(x^4 - 2
*x^2 + 1)*log(x^2 - 1) + 1)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)/(-x**2+1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22125, size = 73, normalized size = 1.18

$$-\frac{\left(2\,x^2-3\right)\sqrt{-x^2+1}x\arcsin\left(x\right)}{3\left(x^2-1\right)^2}-\frac{2\,x^2-3}{6\left(x^2-1\right)}+\frac{1}{3}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/(-x^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*
x^2 - 3)/(x^2 - 1) + 1/3*ln(abs(x^2 - 1))
```

3.664 $\int \frac{x^3 \sin^{-1}(x)}{(1-x^2)^{3/2}} \, dx$

Optimal. Leaf size=36

$$\sqrt{1-x^2}\sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

```
[Out] -x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[
x]
```

Rubi [A] time = 0.117813, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\sqrt{1-x^2}\sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] -x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[
x]
```

Rubi in Sympy [A] time = 7.31756, size = 27, normalized size = 0.75

$$-x + \sqrt{-x^2+1}\operatorname{asin}(x) - \operatorname{atanh}(x) + \frac{\operatorname{asin}(x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -x + sqrt(-x**2 + 1)*asin(x) - atanh(x) + asin(x)/sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.0755278, size = 40, normalized size = 1.11

$$\frac{1}{2}\left(-\frac{2\left(x^2-2\right)\sin^{-1}(x)}{\sqrt{1-x^2}} - 2x + \log(1-x) - \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] (-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log
[1 + x])/2
```

Maple [C] time = 0.214, size = 102, normalized size = 2.8

$$\frac{\arcsin(x)+i}{2}\left(ix+\sqrt{-x^2+1}\right)-\frac{\arcsin(x)-i}{2}\left(ix-\sqrt{-x^2+1}\right)-\frac{\arcsin(x)}{x^2-1}\sqrt{-x^2+1}+\ln\left(ix+\sqrt{-x^2+1}-i\right)-\ln\left(ix+\sqrt{-x^2+1}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arcsin(x)/(-x^2+1)^(3/2),x)
```

```
[Out] 1/2*(arcsin(x)+I)*(I*x+(-x^2+1)^(1/2))-1/2*(I*x-(-x^2+1)^(1/2))*(
arcsin(x)-I)-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+ln(I*x+(-x^2+1)^(1/
2)-I)-ln(I*x+(-x^2+1)^(1/2)+I)
```

Maxima [A] time = 1.51074, size = 61, normalized size = 1.69

$$-\left(\frac{x^2}{\sqrt{-x^2+1}}-\frac{2}{\sqrt{-x^2+1}}\right)\arcsin(x)-x-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsin(x)/(-x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(x^2/sqrt(-x^2 + 1) - 2/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(
x + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 0.231147, size = 77, normalized size = 2.14

$$-\frac{2x^3-2(x^2-2)\sqrt{-x^2+1}\arcsin(x)+(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2x}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsin(x)/(-x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^3 - 2*(x^2 - 2)*sqrt(-x^2 + 1)*arcsin(x) + (x^2 - 1)*lo
g(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21538, size = 54, normalized size = 1.5

$$\left(\sqrt{-x^2+1}+\frac{1}{\sqrt{-x^2+1}}\right)\arcsin\left(x\right)-x-\frac{1}{2}\ln\left(|x+1|\right)+\frac{1}{2}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsin(x)/(-x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(-x^2 + 1) + 1/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*ln(abs(x
+ 1)) + 1/2*ln(abs(x - 1))
```

3.665 $\int \frac{\sin^{-1}(x)}{x(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)$$

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] -
ArcTanh[x] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*A
rcSin[x])]
```

Rubi [A] time = 0.174725, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] -
ArcTanh[x] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*A
rcSin[x])]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\operatorname{atanh}(x)+\int^{\operatorname{asin}(x)}\frac{x}{\sin(x)}\,dx+\frac{\operatorname{asin}(x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

```
[Out] -atanh(x) + Integral(x/sin(x), (x, asin(x))) + asin(x)/sqrt(-x**2
 + 1)
```

Mathematica [A] time = 0.254424, size = 112, normalized size = 1.81

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}+\sin^{-1}(x)\log\left(1-e^{i\sin^{-1}(x)}\right)-\sin^{-1}(x)\log\left(1+e^{i\sin^{-1}(x)}\right)+\log\left(\cos\left(\frac{1}{2}\sin^{-1}(x)\right)-\sin\left(\frac{1}{2}\sin^{-1}(x)\right)\right)-\log\left(\sin\left(\frac{1}{2}\sin^{-1}(x)\right)+\cos\left(\frac{1}{2}\sin^{-1}(x)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - Ar
cSin[x]*Log[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[Arc
Sin[x]/2]] - Log[Cos[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog
[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

Maple [C] time = 0.222, size = 97, normalized size = 1.6

$$-\frac{\arcsin(x)}{x^2-1}\sqrt{-x^2+1}+2\,i\arctan\left(ix+\sqrt{-x^2+1}\right)+i\,dilog\left(1+ix+\sqrt{-x^2+1}\right)-\arcsin(x)\ln\left(1+ix+\sqrt{-x^2+1}\right)+i\,dilog\left(ix+\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)/x/(-x^2+1)^(3/2),x)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+2*I*arctan(I*x+(-x^2+1)^(1/2))+
I*dilog(1+I*x+(-x^2+1)^(1/2))-arcsin(x)*ln(1+I*x+(-x^2+1)^(1/2))+
I*dilog(I*x+(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\arcsin\left(x\right)}{\left(x^{3}-x\right)\sqrt{-x^{2}+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] integral(-arcsin(x)/((x^3 - x)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asin}\left(x\right)}{x\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral(asin(x)/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin\left(x\right)}{\left(-x^{2}+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

3.666 $\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{6x^2}-\frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3}-\frac{2\log(x)}{3}$$

```
[Out] 1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1 - x^2]*
ArcCos[x])/(3*x) - (2*Log[x])/3
```

Rubi [A] time = 0.154835, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{1}{6x^2}-\frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3}-\frac{2\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] 1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1 - x^2]*
ArcCos[x])/(3*x) - (2*Log[x])/3
```

Rubi in Sympy [A] time = 6.68549, size = 46, normalized size = 0.85

$$-\frac{2\log(x)}{3}-\frac{2\sqrt{-x^2+1}\,\mathrm{acos}\,(x)}{3x}+\frac{1}{6x^2}-\frac{\sqrt{-x^2+1}\,\mathrm{acos}\,(x)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] -2*log(x)/3 - 2*sqrt(-x**2 + 1)*acos(x)/(3*x) + 1/(6*x**2) - sqrt
(-x**2 + 1)*acos(x)/(3*x**3)
```

Mathematica [A] time = 0.0371372, size = 38, normalized size = 0.7

$$\frac{-4x^3\log(x)-2\sqrt{1-x^2}\left(2x^2+1\right)\cos^{-1}(x)+x}{6x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] (x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3
)
```

Maple [A] time = 0.075, size = 43, normalized size = 0.8

$$\frac{1}{6\,x^{2}}-\frac{2\,\ln\left(x\right)}{3}-\frac{\arccos\left(x\right)}{3\,x^{3}}\sqrt{-x^{2}+1}-\frac{2\,\arccos\left(x\right)}{3\,x}\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccos(x)/x^4/(-x^2+1)^(1/2),x)
```

```
[Out] 1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*
(-x^2+1)^(1/2)/x
```

Maxima [A] time = 1.49825, size = 57, normalized size = 1.06

$$-\frac{1}{3}\left(\frac{2\,\sqrt{-x^{2}+1}}{x}+\frac{\sqrt{-x^{2}+1}}{x^{3}}\right)\arccos\left(x\right)+\frac{1}{6\,x^{2}}-\frac{2}{3}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x)/(sqrt(-x^2 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) + 1/6/x^
2 - 2/3*log(x)
```

Fricas [A] time = 0.227472, size = 49, normalized size = 0.91

$$-\frac{4\,x^{3}\log\left(x\right)+2\,\left(2\,x^{2}+1\right)\sqrt{-x^{2}+1}\arccos\left(x\right)-x}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x)/(sqrt(-x^2 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^3*log(x) + 2*(2*x^2 + 1)*sqrt(-x^2 + 1)*arccos(x) - x)/
x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217484, size = 128, normalized size = 2.37

$$\frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3}-\frac{9\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos\left(x\right)+\frac{2\,x^2+1}{6\,x^2}-\frac{1}{3}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x)/(sqrt(-x^2 + 1)*x^4),x, algorithm="giac")
```

```
[Out] 1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)
^3 - 9*(sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arcco
s(x) + 1/6*(2*x^2 + 1)/x^2 - 1/3*ln(x^2)
```

3.667 $\int x\sqrt{1-x^2}\cos^{-1}(x)^2\,dx$

Optimal. Leaf size=66

$$\frac{2}{9}x^3\cos^{-1}(x)+\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2-\frac{2}{3}x\cos^{-1}(x)$$

```
[Out] (4*Sqrt[1 - x^2])/9 + (2*(1 - x^2)^(3/2))/27 - (2*x*ArcCos[x])/3
+ (2*x^3*ArcCos[x])/9 - ((1 - x^2)^(3/2)*ArcCos[x]^2)/3
```

Rubi [A] time = 0.142883, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{2}{9}x^3\cos^{-1}(x)+\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2-\frac{2}{3}x\cos^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (4*Sqrt[1 - x^2])/9 + (2*(1 - x^2)^(3/2))/27 - (2*x*ArcCos[x])/3
+ (2*x^3*ArcCos[x])/9 - ((1 - x^2)^(3/2)*ArcCos[x]^2)/3
```

Rubi in Sympy [A] time = 8.30509, size = 56, normalized size = 0.85

$$\frac{2x^3\operatorname{acos}(x)}{9}-\frac{2x\operatorname{acos}(x)}{3}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}\operatorname{acos}^2(x)}{3}+\frac{2\left(-x^2+1\right)^{\frac{3}{2}}}{27}+\frac{4\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
```

```
[Out] 2*x**3*acos(x)/9 - 2*x*acos(x)/3 - (-x**2 + 1)**(3/2)*acos(x)**2/
3 + 2*(-x**2 + 1)**(3/2)/27 + 4*sqrt(-x**2 + 1)/9
```

Mathematica [A] time = 0.0518136, size = 50, normalized size = 0.76

$$\frac{1}{27}\left(-2\sqrt{1-x^2}\left(x^2-7\right)-9\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2+6x\left(x^2-3\right)\cos^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 -
x^2)^(3/2)*ArcCos[x]^2)/27
```

Maple [C] time = 0.188, size = 158, normalized size = 2.4

$$-\frac{6\,i\arccos(x)+9\,(\arccos(x))^2-2}{216}\left(4\,ix^3-4\,x^2\sqrt{-x^2+1}-3\,ix+\sqrt{-x^2+1}\right)+\frac{(\arccos(x))^2-2+2\,i\arccos(x)}{8}\left(ix-\sqrt{-x^2+1}\right)-\frac{(\arccos(x))^2-2-2\,i\arccos(x)}{8}\left(ix+\sqrt{-x^2+1}\right)+\frac{-6\,i\arccos(x)+9\,(\arccos(x))^2-2}{216}\left(4\,ix^3+4\,x^2\sqrt{-x^2+1}-3\,ix-\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arccos(x)^2*(-x^2+1)^(1/2),x)
```

```
[Out] -1/216*(6*I*arccos(x)+9*arccos(x)^2-2)*(4*I*x^3-4*x^2*(-x^2+1)^(1
/2)-3*I*x+(-x^2+1)^(1/2))+1/8*(arccos(x)^2-2+2*I*arccos(x))*(I*x-
(-x^2+1)^(1/2))-1/8*(arccos(x)^2-2-2*I*arccos(x))*(I*x+(-x^2+1)^(
1/2))+1/216*(-6*I*arccos(x)+9*arccos(x)^2-2)*(4*I*x^3+4*x^2*(-x^2
+1)^(1/2)-3*I*x-(-x^2+1)^(1/2))
```

Maxima [A] time = 1.48028, size = 70, normalized size = 1.06

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2-\frac{2}{27}\sqrt{-x^2+1}x^2+\frac{2}{9}\left(x^3-3\,x\right)\arccos(x)+\frac{14}{27}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x*arccos(x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 + 2/9
*(x^3 - 3*x)*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

Fricas [A] time = 0.224732, size = 55, normalized size = 0.83

$$\frac{2}{9}\left(x^3-3\,x\right)\arccos(x)+\frac{1}{27}\left(9\left(x^2-1\right)\arccos(x)^2-2\,x^2+14\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x*arccos(x)^2,x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 - 3*x)*arccos(x) + 1/27*(9*(x^2 - 1)*arccos(x)^2 - 2*x^2
 + 14)*sqrt(-x^2 + 1)
```

Sympy [A] time = 4.28532, size = 78, normalized size = 1.18

$$\frac{2x^3\operatorname{acos}(x)}{9}+\frac{x^2\sqrt{-x^2+1}\operatorname{acos}^2(x)}{3}-\frac{2x^2\sqrt{-x^2+1}}{27}-\frac{2x\operatorname{acos}(x)}{3}-\frac{\sqrt{-x^2+1}\operatorname{acos}^2(x)}{3}+\frac{14\sqrt{-x^2+1}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
```

```
[Out] 2*x**3*acos(x)/9 + x**2*sqrt(-x**2 + 1)*acos(x)**2/3 - 2*x**2*sqr
t(-x**2 + 1)/27 - 2*x*acos(x)/3 - sqrt(-x**2 + 1)*acos(x)**2/3 +
14*sqrt(-x**2 + 1)/27
```

GIAC/XCAS [A] time = 0.224758, size = 72, normalized size = 1.09

$$\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2-\frac{2}{27}\sqrt{-x^2+1}x^2-\frac{2}{3}x\arccos(x)+\frac{14}{27}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x*arccos(x)^2,x, algorithm="giac")
```

```
[Out] 2/9*x^3*arccos(x) - 1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(
-x^2 + 1)*x^2 - 2/3*x*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

3.668 $\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=73

$$-\frac{3x^2}{8}-\frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3+\frac{3}{4}x^2\sin^{-1}(x)^2+\frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x)+\frac{1}{8}\sin^{-1}(x)^4-\frac{3}{8}\sin^{-1}(x)^2$$

```
[Out] (-3*x^2)/8 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/4 - (3*ArcSin[x]^2)/8
+ (3*x^2*ArcSin[x]^2)/4 - (x*Sqrt[1 - x^2]*ArcSin[x]^3)/2 + ArcSi
n[x]^4/8
```

Rubi [A] time = 0.272341, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{3x^2}{8}-\frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3+\frac{3}{4}x^2\sin^{-1}(x)^2+\frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x)+\frac{1}{8}\sin^{-1}(x)^4-\frac{3}{8}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2)/8 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/4 - (3*ArcSin[x]^2)/8
+ (3*x^2*ArcSin[x]^2)/4 - (x*Sqrt[1 - x^2]*ArcSin[x]^3)/2 + ArcSi
n[x]^4/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3x^2\operatorname{asin}^2(x)}{4}-\frac{x\sqrt{-x^2+1}\operatorname{asin}^3(x)}{2}+\frac{3x\sqrt{-x^2+1}\operatorname{asin}(x)}{4}+\frac{\operatorname{asin}^4(x)}{8}-\frac{3\operatorname{asin}^2(x)}{8}-\frac{3\int x\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 3*x**2*asin(x)**2/4 - x*sqrt(-x**2 + 1)*asin(x)**3/2 + 3*x*sqrt(-
x**2 + 1)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8 - 3*Integral(
x, x)/4
```

Mathematica [A] time = 0.0377257, size = 60, normalized size = 0.82

$$\frac{1}{8}\left(-3x^2-4x\sqrt{1-x^2}\sin^{-1}(x)^3+\left(6x^2-3\right)\sin^{-1}(x)^2+6x\sqrt{1-x^2}\sin^{-1}(x)+\sin^{-1}(x)^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2
- 4*x*Sqrt[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8
```

Maple [A] time = 0.089, size = 69, normalized size = 1.

$$\frac{(\arcsin(x))^3}{2}\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)+\frac{3\,(\arcsin(x))^2\,(x^2-1)}{4}+\frac{3\,\arcsin(x)}{4}\left(x\sqrt{-x^2+1}+\arcsin(x)\right)-\frac{3\,(\arcsin(x))^2}{8}-\frac{3\,x^2}{8}-\frac{3\,(\arcsin(x))^4}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)^3*(-x*(-x^2+1)^(1/2)+arcsin(x))+3/4*arcsin(x)^2*(x^
2-1)+3/4*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-3/8*arcsin(x)^2-3
/8*x^2-3/8*arcsin(x)^4
```

Maxima [A] time = 10.2824, size = 194, normalized size = 2.66

$$-\frac{1}{2}\left(\sqrt{-x^2+1}x-\arcsin(x)\right)\arcsin(x)^3-\frac{1}{8}\arctan\left(x,\sqrt{x+1}\sqrt{-x+1}\right)^4+\frac{3}{4}\left(x^2-\arctan\left(x,\sqrt{x+1}\sqrt{-x+1}\right)^2\right)\arcsin(x)^2-\frac{3}{8}x^2+\frac{1}{4}\left(2\arctan\left(x,\sqrt{x+1}\sqrt{-x+1}\right)^3+3\sqrt{-x^2+1}x-3\arctan\left(x,\sqrt{-x^2+1}\right)\right)\arcsin(x)+\frac{3}{8}\arctan\left(x,\sqrt{x+1}\sqrt{-x+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x)^3 - 1/8*arctan2(x,
sqrt(x + 1)*sqrt(-x + 1))^4 + 3/4*(x^2 - arctan2(x, sqrt(x + 1)*s
qrt(-x + 1))^2)*arcsin(x)^2 - 3/8*x^2 + 1/4*(2*arctan2(x, sqrt(x
+ 1)*sqrt(-x + 1))^3 + 3*sqrt(-x^2 + 1)*x - 3*arctan2(x, sqrt(-x^
2 + 1)))*arcsin(x) + 3/8*arctan2(x, sqrt(x + 1)*sqrt(-x + 1))^2
```

Fricas [A] time = 0.254375, size = 66, normalized size = 0.9

$$\frac{1}{8}\arcsin(x)^4 + \frac{3}{8}\left(2x^2 - 1\right)\arcsin(x)^2 - \frac{3}{8}x^2 - \frac{1}{4}\left(2x\arcsin(x)^3 - 3x\arcsin(x)\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*
x*arcsin(x)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

Sympy [A] time = 1.83688, size = 66, normalized size = 0.9

$$\frac{3x^2\operatorname{asin}^2(x)}{4} - \frac{3x^2}{8} - \frac{x\sqrt{-x^2+1}\operatorname{asin}^3(x)}{2} + \frac{3x\sqrt{-x^2+1}\operatorname{asin}(x)}{4} + \frac{\operatorname{asin}^4(x)}{8} - \frac{3\operatorname{asin}^2(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(-x**2 + 1)*asin(x)**3/2 +
 3*x*sqrt(-x**2 + 1)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

GIAC/XCAS [A] time = 0.230822, size = 81, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)^3 + \frac{1}{8}\arcsin(x)^4 + \frac{3}{4}\left(x^2 - 1\right)\arcsin(x)^2 + \frac{3}{4}\sqrt{-x^2+1}x\arcsin(x) - \frac{3}{8}x^2 + \frac{3}{8}\arcsin(x)^2 + \frac{3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 -
1)*arcsin(x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*a
rcsin(x)^2 + 3/16
```

3.669 $\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=32

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)$$

```
[Out] x/(4*(1 + x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1 + x^2))
```

Rubi [A] time = 0.0622412, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] x/(4*(1 + x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1 + x^2))
```

Rubi in Sympy [A] time = 2.69631, size = 22, normalized size = 0.69

$$\frac{x}{4(x^2+1)} + \frac{\operatorname{atan}(x)}{4} - \frac{\operatorname{atan}(x)}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*atan(x)/(x**2+1)**2,x)
```

```
[Out] x/(4*(x**2 + 1)) + atan(x)/4 - atan(x)/(2*(x**2 + 1))
```

Mathematica [A] time = 0.0123289, size = 21, normalized size = 0.66

$$\frac{(x^2-1)\tan^{-1}(x) + x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{x}{4\,x^2+4}+\frac{\arctan\left(x\right)}{4}-\frac{\arctan\left(x\right)}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(x^2+1)^2,x)

[Out] 1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)
```

Maxima [A] time = 1.52968, size = 35, normalized size = 1.09

$$\frac{x}{4\left(x^2+1\right)}-\frac{\arctan\left(x\right)}{2\left(x^2+1\right)}+\frac{1}{4}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2 + 1)^2,x, algorithm="maxima")

[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

Fricas [A] time = 0.215204, size = 26, normalized size = 0.81

$$\frac{\left(x^2-1\right)\arctan\left(x\right)+x}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2 + 1)^2,x, algorithm="fricas")

[Out] 1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 1.2587, size = 31, normalized size = 0.97

$$\frac{x^2\operatorname{atan}\left(x\right)}{4x^2+4}+\frac{x}{4x^2+4}-\frac{\operatorname{atan}\left(x\right)}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan(x)/(x**2+1)**2,x)
```

```
[Out] x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)
```

GIAC/XCAS [A] time = 0.207636, size = 35, normalized size = 1.09

$$\frac{x}{4\,(x^2+1)} - \frac{\arctan(x)}{2\,(x^2+1)} + \frac{1}{4}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

3.670 $\int \frac{x \tan^{-1}(x)}{(1+x^2)^3} \, dx$

Optimal. Leaf size=44

$$\frac{3x}{32(x^2+1)} + \frac{x}{16(x^2+1)^2} - \frac{\tan^{-1}(x)}{4(x^2+1)^2} + \frac{3}{32}\tan^{-1}(x)$$

```
[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - Ar
cTan[x]/(4*(1 + x^2)^2)
```

Rubi [A] time = 0.0548796, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3x}{32(x^2+1)} + \frac{x}{16(x^2+1)^2} - \frac{\tan^{-1}(x)}{4(x^2+1)^2} + \frac{3}{32}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^3,x]
```

```
[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - Ar
cTan[x]/(4*(1 + x^2)^2)
```

Rubi in Sympy [A] time = 2.77009, size = 37, normalized size = 0.84

$$\frac{3x}{32(x^2+1)} + \frac{x}{16(x^2+1)^2} + \frac{3\,\mathrm{atan}(x)}{32} - \frac{\mathrm{atan}(x)}{4(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*atan(x)/(x**2+1)**3,x)
```

```
[Out] 3*x/(32*(x**2 + 1)) + x/(16*(x**2 + 1)**2) + 3*atan(x)/32 - atan(
x)/(4*(x**2 + 1)**2)
```

Mathematica [A] time = 0.0158686, size = 36, normalized size = 0.82

$$\frac{x(3x^2+5) + (3x^4+6x^2-5)\tan^{-1}(x)}{32(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*ArcTan[x])/(1 + x^2)^3,x]

[Out] (x*(5 + 3*x^2) + (-5 + 6*x^2 + 3*x^4)*ArcTan[x])/(32*(1 + x^2)^2)
```

Maple [A] time = 0.003, size = 37, normalized size = 0.8

$$\frac{x}{16\,(x^2+1)^2}+\frac{3\,x}{32\,x^2+32}+\frac{3\,\arctan(x)}{32}-\frac{\arctan(x)}{4\,(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arctan(x)/(x^2+1)^3,x)

[Out] 1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2
+1)^2
```

Maxima [A] time = 1.50602, size = 53, normalized size = 1.2

$$\frac{3\,x^3+5\,x}{32\,(x^4+2\,x^2+1)}-\frac{\arctan(x)}{4\,(x^2+1)^2}+\frac{3}{32}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)/(x^2 + 1)^3,x, algorithm="maxima")

[Out] 1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2
+ 3/32*arctan(x)
```

Fricas [A] time = 0.212534, size = 51, normalized size = 1.16

$$\frac{3\,x^3+\left(3\,x^4+6\,x^2-5\right)\arctan(x)+5\,x}{32\,(x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)/(x^2 + 1)^3,x, algorithm="fricas")

[Out] 1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 +
 1)
```

Sympy [A] time = 2.15552, size = 88, normalized size = 2.

$$\frac{3x^4\operatorname{atan}(x)}{32x^4+64x^2+32}+\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\operatorname{atan}(x)}{32x^4+64x^2+32}+\frac{5x}{32x^4+64x^2+32}-\frac{5\operatorname{atan}(x)}{32x^4+64x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan(x)/(x**2+1)**3,x)
```

```
[Out] 3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x*
*2 + 32) + 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4
 + 64*x**2 + 32) - 5*atan(x)/(32*x**4 + 64*x**2 + 32)
```

GIAC/XCAS [A] time = 0.208432, size = 46, normalized size = 1.05

$$\frac{3\,x^3+5\,x}{32\,(x^2+1)^2}-\frac{\arctan(x)}{4\,(x^2+1)^2}+\frac{3}{32}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)/(x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32
*arctan(x)
```

3.671 $\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0772449, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*ArcTan[x])/(1 + x^2),x]

[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 5.42486, size = 19, normalized size = 0.83

$$x\,\text{atan}\,(x)-\frac{\log\left(x^2+1\right)}{2}-\frac{\text{atan}^2\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*atan(x)/(x**2+1),x)

[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

Mathematica [A] time = 0.00577889, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Maple [A] time = 0., size = 20, normalized size = 0.9

$$x \arctan(x) - \frac{(\arctan(x))^2}{2} - \frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1),x)
```

```
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Maxima [A] time = 1.56222, size = 32, normalized size = 1.39

$$(x - \arctan(x))\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.215553, size = 26, normalized size = 1.13

$$x \arctan(x) - \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.591431, size = 19, normalized size = 0.83

$$x \operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*atan(x)/(x**2+1),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

GIAC/XCAS [A] time = 0.210767, size = 28, normalized size = 1.22

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \ln\left(-i x^2 - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*ln(-I*x^2 - I)
```

3.672 $\int \frac{x^3 \tan^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=67

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}i\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)+\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcT
an[x]*Log[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi [A] time = 0.155049, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}i\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)+\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcT
an[x]*Log[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi in Sympy [A] time = 9.80396, size = 49, normalized size = 0.73

$$\frac{x^2\,\text{atan}\,(x)}{2}-\frac{x}{2}+\log\left(\frac{2i}{-x+i}\right)\text{atan}\,(x)+\frac{i\,\text{atan}^2\,(x)}{2}+\frac{\text{atan}\,(x)}{2}+\frac{i\,\text{Li}_2\left(\frac{-x-i}{-x+i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*atan(x)/(x**2+1),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + log(2*I/(-x + I))*atan(x) + I*atan(x)**2/2
 + atan(x)/2 + I*polylog(2, (-x - I)/(-x + I))/2
```

Mathematica [A] time = 0.0537018, size = 53, normalized size = 0.79

$$\frac{1}{2}\left(-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(x)}\right)+\tan^{-1}(x)\left(x^2+2\log\left(1+e^{2i\tan^{-1}(x)}\right)+1\right)-x-i\tan^{-1}(x)^2\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] (-x - I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[1 + E^((2*I)*Arc
Tan[x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[x])])/2
```

Maple [C] time = 0.052, size = 128, normalized size = 1.9

$$\frac{x^2\arctan(x)}{2}-\frac{\arctan(x)\ln\left(x^2+1\right)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}-\frac{i}{4}\ln\left(x^2+1\right)\ln(x-i)+\frac{i}{8}(\ln(x-i))^2+\frac{i}{4}\ln(x-i)\ln\left(-\frac{i}{2}(x+i)\right)+\frac{i}{4}dilog\left(-\frac{i}{2}(x+i)\right)+\frac{i}{4}\ln\left(x^2+1\right)\ln(x+i)-\frac{i}{8}(\ln(x+i))^2-\frac{i}{4}\ln(x+i)\ln\left(\frac{i}{2}(x-i)\right)-\frac{i}{4}dilog\left(\frac{i}{2}(x-i)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arctan(x)/(x^2+1),x)
```

```
[Out] 1/2*x^2*arctan(x)-1/2*arctan(x)*ln(x^2+1)-1/2*x+1/2*arctan(x)-1/4
*I*ln(x^2+1)*ln(x-I)+1/8*I*ln(x-I)^2+1/4*I*ln(x-I)*ln(-1/2*I*(x+I
))+1/4*I*dilog(-1/2*I*(x+I))+1/4*I*ln(x^2+1)*ln(x+I)-1/8*I*ln(x+I
)^2-1/4*I*ln(x+I)*ln(1/2*I*(x-I))-1/4*I*dilog(1/2*I*(x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\arctan(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\arctan(x)}{x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}(x)}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*atan(x)/(x**2+1),x)
```

```
[Out] Integral(x**3*atan(x)/(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(x)}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

3.673 $\int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=34

$$-\frac{1}{4(x^2+1)} - \frac{x\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)^2$$

```
[Out] -1/(4*(1 + x^2)) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

Rubi [A] time = 0.0753278, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{1}{4(x^2+1)} - \frac{x\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*ArcTan[x])/(1 + x^2)^2,x]

[Out] -1/(4*(1 + x^2)) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

Rubi in Sympy [A] time = 5.12067, size = 26, normalized size = 0.76

$$-\frac{x\,\mathrm{atan}(x)}{2(x^2+1)} + \frac{\mathrm{atan}^2(x)}{4} - \frac{1}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*atan(x)/(x**2+1)**2,x)

[Out] -x*atan(x)/(2*(x**2 + 1)) + atan(x)**2/4 - 1/(4*(x**2 + 1))
```

Mathematica [A] time = 0.0116563, size = 28, normalized size = 0.82

$$\frac{(x^2+1)\tan^{-1}(x)^2 - 2x\tan^{-1}(x) - 1}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))
```

Maple [A] time = 0.01, size = 29, normalized size = 0.9

$$-\frac{1}{4\,x^2+4}-\frac{x\arctan\left(x\right)}{2\,x^2+2}+\frac{\left(\arctan\left(x\right)\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1)^2,x)
```

```
[Out] -1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

Maxima [A] time = 1.51529, size = 54, normalized size = 1.59

$$-\frac{1}{2}\left(\frac{x}{x^2+1}-\arctan\left(x\right)\right)\arctan\left(x\right)-\frac{\left(x^2+1\right)\arctan\left(x\right)^2+1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(
x)^2 + 1)/(x^2 + 1)
```

Fricas [A] time = 0.225648, size = 35, normalized size = 1.03

$$\frac{\left(x^2+1\right)\arctan\left(x\right)^2-2\,x\arctan\left(x\right)-1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(x)}{(x^2+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(x)/(x^2 + 1)^2, x)
```

3.674 $\int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=79

$$-\frac{1}{2} i \text{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right) - \frac{x}{4(x^2+1)} + \frac{\tan^{-1}(x)}{2(x^2+1)} - \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{4} \tan^{-1}(x) - \log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

```
[Out] -x/(4*(1 + x^2)) - ArcTan[x]/4 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*
ArcTan[x]^2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2
/(1 + I*x)]
```

Rubi [A] time = 0.177409, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{1}{2} i \text{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right) - \frac{x}{4(x^2+1)} + \frac{\tan^{-1}(x)}{2(x^2+1)} - \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{4} \tan^{-1}(x) - \log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -x/(4*(1 + x^2)) - ArcTan[x]/4 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*
ArcTan[x]^2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2
/(1 + I*x)]
```

Rubi in Sympy [A] time = 10.5117, size = 56, normalized size = 0.71

$$-\frac{x}{4(x^2+1)} - \log\left(\frac{2i}{-x+i}\right) \operatorname{atan}(x) - \frac{i \operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{4} - \frac{i \operatorname{Li}_2\left(\frac{-x-i}{-x+i}\right)}{2} + \frac{\operatorname{atan}(x)}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*atan(x)/(x**2+1)**2,x)
```

```
[Out] -x/(4*(x**2 + 1)) - log(2*I/(-x + I))*atan(x) - I*atan(x)**2/2 -
atan(x)/4 - I*polylog(2, (-x - I)/(-x + I))/2 + atan(x)/(2*(x**2
+ 1))
```

Mathematica [A] time = 0.0389717, size = 64, normalized size = 0.81

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(x)}\right)+\frac{1}{2}i\tan^{-1}(x)^2-\tan^{-1}(x)\log\left(1+e^{2i\tan^{-1}(x)}\right)-\frac{1}{8}\sin\left(2\tan^{-1}(x)\right)+\frac{1}{4}\tan^{-1}(x)\cos\left(2\tan^{-1}(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Lo
g[1 + E^((2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x]
)] - Sin[2*ArcTan[x]]/8
```

Maple [C] time = 0.045, size = 139, normalized size = 1.8

$$\frac{\arctan(x)\ln(x^2+1)}{2}+\frac{\arctan(x)}{2x^2+2}+\frac{i}{4}\ln(x^2+1)\ln(x-i)-\frac{i}{8}(\ln(x-i))^2-\frac{i}{4}\ln(x-i)\ln\left(-\frac{i}{2}(x+i)\right)-\frac{i}{4}\mathit{dilog}\left(-\frac{i}{2}(x+i)\right)-\frac{i}{4}\ln(x^2+1)\ln(x+i)+\frac{i}{8}(\ln(x+i))^2+\frac{i}{4}\ln(x+i)\ln\left(\frac{i}{2}(x-i)\right)+\frac{i}{4}\mathit{dilog}\left(\frac{i}{2}(x-i)\right)-\frac{x}{4x^2+4}-\frac{\arctan(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arctan(x)/(x^2+1)^2,x)
```

```
[Out] 1/2*arctan(x)*ln(x^2+1)+1/2*arctan(x)/(x^2+1)+1/4*I*ln(x^2+1)*ln(
x-I)-1/8*I*ln(x-I)^2-1/4*I*ln(x-I)*ln(-1/2*I*(x+I))-1/4*I*dilog(-
1/2*I*(x+I))-1/4*I*ln(x^2+1)*ln(x+I)+1/8*I*ln(x+I)^2+1/4*I*ln(x+I
)*ln(1/2*I*(x-I))+1/4*I*dilog(1/2*I*(x-I))-1/4*x/(x^2+1)-1/4*arct
an(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\arctan(x)}{(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\arctan\left(x\right)}{x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\arctan\left(x\right)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

3.675 $\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=89

$$i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{x}{4(x^2+1)}+\frac{1}{2}x^2\tan^{-1}(x)-\frac{\tan^{-1}(x)}{2(x^2+1)}-\frac{x}{2}+i\tan^{-1}(x)^2+\frac{3}{4}\tan^{-1}(x)+2\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

```
[Out] -x/2 + x/(4*(1 + x^2)) + (3*ArcTan[x])/4 + (x^2*ArcTan[x])/2 - Ar
cTan[x]/(2*(1 + x^2)) + I*ArcTan[x]^2 + 2*ArcTan[x]*Log[2/(1 + I*
x)] + I*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi [A] time = 0.371818, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$

$$i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{x}{4(x^2+1)}+\frac{1}{2}x^2\tan^{-1}(x)-\frac{\tan^{-1}(x)}{2(x^2+1)}-\frac{x}{2}+i\tan^{-1}(x)^2+\frac{3}{4}\tan^{-1}(x)+2\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -x/2 + x/(4*(1 + x^2)) + (3*ArcTan[x])/4 + (x^2*ArcTan[x])/2 - Ar
cTan[x]/(2*(1 + x^2)) + I*ArcTan[x]^2 + 2*ArcTan[x]*Log[2/(1 + I*
x)] + I*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi in Sympy [A] time = 16.9948, size = 68, normalized size = 0.76

$$\frac{x^2\operatorname{atan}(x)}{2}-\frac{x}{2}+\frac{x}{4(x^2+1)}+2\log\left(\frac{2i}{-x+i}\right)\operatorname{atan}(x)+i\operatorname{atan}^2(x)+\frac{3\operatorname{atan}(x)}{4}+i\operatorname{Li}_2\left(\frac{-x-i}{-x+i}\right)-\frac{\operatorname{atan}(x)}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*atan(x)/(x**2+1)**2,x)
```

```
[Out] x**2*atan(x)/2 - x/2 + x/(4*(x**2 + 1)) + 2*log(2*I/(-x + I))*ata
n(x) + I*atan(x)**2 + 3*atan(x)/4 + I*polylog(2, (-x - I)/(-x + I
)) - atan(x)/(2*(x**2 + 1))
```

Mathematica [A] time = 0.210256, size = 70, normalized size = 0.79

$$\frac{1}{8}\left(-8i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(x)}\right)+4\left(x^2+1\right)\tan^{-1}(x)-4x-8i\tan^{-1}(x)^2+16\tan^{-1}(x)\log\left(1+e^{2i\tan^{-1}(x)}\right)+\sin\left(2\tan^{-1}(x)\right)-2\tan^{-1}(x)\cos\left(2\tan^{-1}(x)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*C
os[2*ArcTan[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] - (8*
I)*PolyLog[2, -E^((2*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8
```

Maple [C] time = 0.048, size = 149, normalized size = 1.7

$$\frac{x^2\arctan(x)}{2}-\arctan(x)\ln\left(x^2+1\right)-\frac{\arctan(x)}{2x^2+2}+\frac{i}{4}\left(\ln(x-i)\right)^2+\frac{i}{2}\ln(x-i)\ln\left(-\frac{i}{2}(x+i)\right)-\frac{i}{2}\ln(x-i)\ln\left(x^2+1\right)+\frac{i}{2}dilog\left(-\frac{i}{2}(x+i)\right)-\frac{i}{4}\left(\ln(x+i)\right)^2-\frac{i}{2}\ln(x+i)\ln\left(\frac{i}{2}(x-i)\right)+\frac{i}{2}\ln(x+i)\ln\left(x^2+1\right)-\frac{i}{2}dilog\left(\frac{i}{2}(x-i)\right)-\frac{x}{2}+\frac{3\arctan(x)}{4}+\frac{x}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*arctan(x)/(x^2+1)^2,x)
```

```
[Out] 1/2*x^2*arctan(x)-arctan(x)*ln(x^2+1)-1/2*arctan(x)/(x^2+1)+1/4*I
*ln(x-I)^2+1/2*I*ln(x-I)*ln(-1/2*I*(x+I))-1/2*I*ln(x-I)*ln(x^2+1)
+1/2*I*dilog(-1/2*I*(x+I))-1/4*I*ln(x+I)^2-1/2*I*ln(x+I)*ln(1/2*I
*(x-I))+1/2*I*ln(x+I)*ln(x^2+1)-1/2*I*dilog(1/2*I*(x-I))-1/2*x+3/
4*arctan(x)+1/4*x/(x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5\arctan(x)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*arctan(x)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5 \arctan(x)}{x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*arctan(x)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(x)}{(x^2 + 1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*arctan(x)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

3.676 $\int \frac{(1+x^2)\tan^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\log\left(x^2+1\right)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

```
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Rubi [A] time = 0.0600282, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727

$$-\log\left(x^2+1\right)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)*ArcTan[x])/x^2,x]

[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Rubi in Sympy [A] time = 4.00374, size = 22, normalized size = 1.

$$x\,\mathrm{atan}\,(x)+\frac{\log\left(x^2\right)}{2}-\log\left(x^2+1\right)-\frac{\mathrm{atan}\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*atan(x)/x**2,x)

[Out] x*atan(x) + log(x**2)/2 - log(x**2 + 1) - atan(x)/x
```

Mathematica [A] time = 0.00665661, size = 22, normalized size = 1.

$$-\log\left(x^2+1\right)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)*ArcTan[x])/x^2,x]
```

```
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Maple [A] time = 0.007, size = 23, normalized size = 1.1

$$-\frac{\arctan(x)}{x}+x\arctan(x)+\ln(x)-\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)/x^2,x)
```

```
[Out] -arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)
```

Maxima [A] time = 1.53725, size = 28, normalized size = 1.27

$$\left(x-\frac{1}{x}\right)\arctan(x)-\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)*arctan(x)/x^2,x, algorithm="maxima")
```

```
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)
```

Fricas [A] time = 0.231375, size = 35, normalized size = 1.59

$$\frac{\left(x^2-1\right)\arctan(x)-x\log\left(x^2+1\right)+x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 1)*arctan(x)/x^2,x, algorithm="fricas")
```

```
[Out] ((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x
```

Sympy [A] time = 0.629659, size = 19, normalized size = 0.86

$$x\operatorname{atan}(x)+\log(x)-\log\left(x^2+1\right)-\frac{\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*atan(x)/x**2,x)

[Out] x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x
```

GIAC/XCAS [A] time = 0.206921, size = 34, normalized size = 1.55

$$\left(x - \frac{1}{x}\right) \arctan\left(x\right) - \ln\left(x^{2} + 1\right) + \frac{1}{2} \ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)/x^2,x, algorithm="giac")

[Out] (x - 1/x)*arctan(x) - ln(x^2 + 1) + 1/2*ln(x^2)
```

3.677 $\int \frac{(1+x^2)\tan^{-1}(x)}{x^5}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{12x^3}-\frac{\left(x^2+1\right)^2\tan^{-1}(x)}{4x^4}-\frac{1}{4x}$$

```
[Out] -1/(12*x^3) - 1/(4*x) - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

Rubi [A] time = 0.0394017, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{12x^3}-\frac{\left(x^2+1\right)^2\tan^{-1}(x)}{4x^4}-\frac{1}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x])/x^5,x]

[Out] -1/(12*x^3) - 1/(4*x) - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

Rubi in Sympy [A] time = 2.49253, size = 27, normalized size = 0.87

$$-\frac{1}{4x}-\frac{1}{12x^3}-\frac{\left(x^2+1\right)^2\operatorname{atan}(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)*atan(x)/x**5,x)

[Out] -1/(4*x) - 1/(12*x**3) - (x**2 + 1)**2*atan(x)/(4*x**4)
```

Mathematica [A] time = 0.00913647, size = 39, normalized size = 1.26

$$-\frac{\tan^{-1}(x)}{4x^4}-\frac{1}{12x^3}-\frac{\tan^{-1}(x)}{2x^2}-\frac{1}{4x}-\frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^5,x]
```

```
[Out] -1/(12*x^3) - 1/(4*x) - ArcTan[x]/4 - ArcTan[x]/(4*x^4) - ArcTan[
x]/(2*x^2)
```

Maple [A] time = 0.014, size = 30, normalized size = 1.

$$-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4}-\frac{1}{12x^3}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*arctan(x)/x^5,x)
```

```
[Out] -1/4*arctan(x)/x^4-1/2*arctan(x)/x^2-1/4*arctan(x)-1/12/x^3-1/4/x
```

Maxima [A] time = 1.4903, size = 42, normalized size = 1.35

$$-\frac{3x^2+1}{12x^3}-\frac{(2x^2+1)\arctan(x)}{4x^4}-\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arcta
n(x)
```

Fricas [A] time = 0.215338, size = 35, normalized size = 1.13

$$-\frac{3x^3+3(x^4+2x^2+1)\arctan(x)+x}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + x)/x^4
```

Sympy [A] time = 1.63614, size = 34, normalized size = 1.1

$$-\frac{\operatorname{atan}(x)}{4}-\frac{1}{4x}-\frac{\operatorname{atan}(x)}{2x^2}-\frac{1}{12x^3}-\frac{\operatorname{atan}(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*atan(x)/x**5,x)
```

```
[Out] -atan(x)/4 - 1/(4*x) - atan(x)/(2*x**2) - 1/(12*x**3) - atan(x)/(
4*x**4)
```

GIAC/XCAS [A] time = 0.209827, size = 42, normalized size = 1.35

$$-\frac{3\,x^2+1}{12\,x^3}-\frac{\left(2\,x^2+1\right)\,\arctan\left(x\right)}{4\,x^4}-\frac{1}{4}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arcta
n(x)
```

3.678 $\int \frac{(1+x^2)^2 \tan^{-1}(x)}{x^5} \, dx$

Optimal. Leaf size=63

$$\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{\tan^{-1}(x)}{4x^4}-\frac{1}{12x^3}-\frac{\tan^{-1}(x)}{x^2}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)$$

```
[Out] -1/(12*x^3) - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - Arc
Tan[x]/x^2 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rubi [A] time = 0.156104, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{\tan^{-1}(x)}{4x^4}-\frac{1}{12x^3}-\frac{\tan^{-1}(x)}{x^2}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -1/(12*x^3) - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - Arc
Tan[x]/x^2 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rubi in Sympy [A] time = 8.63027, size = 51, normalized size = 0.81

$$-\frac{3\,\text{atan}\,(x)}{4}+\frac{i\,\text{Li}_2\,(-ix)}{2}-\frac{i\,\text{Li}_2\,(ix)}{2}-\frac{3}{4x}-\frac{\text{atan}\,(x)}{x^2}-\frac{1}{12x^3}-\frac{\text{atan}\,(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**2*atan(x)/x**5,x)
```

```
[Out] -3*atan(x)/4 + I*polylog(2, -I*x)/2 - I*polylog(2, I*x)/2 - 3/(4*
x) - atan(x)/x**2 - 1/(12*x**3) - atan(x)/(4*x**4)
```

Mathematica [A] time = 0.0294378, size = 60, normalized size = 0.95

$$\frac{1}{2}i(\text{PolyLog}(2,-ix)-\text{PolyLog}(2,ix))-\frac{\tan^{-1}(x)}{4x^4}-\frac{1}{12x^3}-\frac{\tan^{-1}(x)}{x^2}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -1/(12*x^3) - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - Arc
Tan[x]/x^2 + (I/2)*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

Maple [C] time = 0.029, size = 79, normalized size = 1.3

$$-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{x^2}+\arctan(x)\ln(x)-\frac{3\arctan(x)}{4}-\frac{1}{12x^3}-\frac{3}{4x}+\frac{i}{2}\ln(x)\ln(1+ix)-\frac{i}{2}\ln(x)\ln(1-ix)+\frac{i}{2}\mathit{dilog}(1+ix)-\frac{i}{2}\mathit{dilog}(1-ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^2*arctan(x)/x^5,x)
```

```
[Out] -1/4*arctan(x)/x^4-arctan(x)/x^2+arctan(x)*ln(x)-3/4*arctan(x)-1/
12/x^3-3/4/x+1/2*I*ln(x)*ln(1+I*x)-1/2*I*ln(x)*ln(1-I*x)+1/2*I*di
log(1+I*x)-1/2*I*dilog(1-I*x)
```

Maxima [A] time = 1.87779, size = 96, normalized size = 1.52

$$-\frac{3\pi x^4\log\left(x^2+1\right)-12x^4\arctan(x)\log(x)+6ix^4\mathrm{Li}_2(ix+1)-6ix^4\mathrm{Li}_2(-ix+1)+9x^3+3\left(3x^4+4x^2+1\right)\arctan(x)+}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2*arctan(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*pi*x^4*log(x^2 + 1) - 12*x^4*arctan(x)*log(x) + 6*I*x^4*
dilog(I*x + 1) - 6*I*x^4*dilog(-I*x + 1) + 9*x^3 + 3*(3*x^4 + 4*x
^2 + 1)*arctan(x) + x)/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+2x^2+1\right)\arctan(x)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2*arctan(x)/x^5,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^2 \operatorname{atan}(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**2*atan(x)/x**5,x)
```

```
[Out] Integral((x**2 + 1)**2*atan(x)/x**5, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^2 \arctan(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2*arctan(x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2*arctan(x)/x^5, x)
```

3.679 $\int \frac{\tan^{-1}(x)}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.110146, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[x]/(x^2*(1 + x^2)),x]

[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 6.72582, size = 26, normalized size = 0.93

$$\frac{\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}^2(x)}{2}-\frac{\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x)/x**2/(x**2+1),x)

[Out] log(x**2)/2 - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

Mathematica [A] time = 0.00826388, size = 28, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.012, size = 25, normalized size = 0.9

$$-\frac{\arctan(x)}{x}-\frac{(\arctan(x))^2}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x)/x^2/(x^2+1),x)
```

```
[Out] -arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.50925, size = 36, normalized size = 1.29

$$-\left(\frac{1}{x}+\arctan(x)\right)\arctan(x)+\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log(x^2+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)/((x^2 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
 + log(x)
```

Fricas [A] time = 0.223709, size = 39, normalized size = 1.39

$$-\frac{x\arctan(x)^2+x\log(x^2+1)-2x\log(x)+2\arctan(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)/((x^2 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/
x
```

Sympy [A] time = 0.993274, size = 22, normalized size = 0.79

$$\log(x)-\frac{\log(x^2+1)}{2}-\frac{\operatorname{atan}^2(x)}{2}-\frac{\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x)/x**2/(x**2+1),x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)}{(x^2+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)/((x^2 + 1)*x^2),x, algorithm="giac")
```

```
[Out] integrate(arctan(x)/((x^2 + 1)*x^2), x)
```

3.680 $\int \frac{\tan^{-1}(x)^2}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - L
og[1 + x^2]/2
```

Rubi [A] time = 0.1145, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[x]^2/x^3,x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - L
og[1 + x^2]/2
```

Rubi in Sympy [A] time = 7.81244, size = 36, normalized size = 0.92

$$\frac{\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}^2(x)}{2}-\frac{\operatorname{atan}(x)}{x}-\frac{\operatorname{atan}^2(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x)**2/x**3,x)
```

```
[Out] log(x**2)/2 - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x
)**2/(2*x**2)
```

Mathematica [A] time = 0.0178759, size = 38, normalized size = 0.97

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{\left(-x^2-1\right)\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[x]^2/x^3,x]
```

```
[Out] -(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[
1 + x^2]/2
```

Maple [A] time = 0.003, size = 34, normalized size = 0.9

$$-\frac{\arctan(x)}{x}-\frac{(\arctan(x))^2}{2}-\frac{(\arctan(x))^2}{2\,x^2}+\ln(x)-\frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x)^2/x^3,x)
```

```
[Out] -arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2
+1)
```

Maxima [A] time = 1.52359, size = 49, normalized size = 1.26

$$-\left(\frac{1}{x}+\arctan(x)\right)\arctan(x)+\frac{1}{2}\arctan(x)^2-\frac{\arctan(x)^2}{2\,x^2}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/
x^2 - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 0.218751, size = 51, normalized size = 1.31

$$-\frac{(x^2+1)\arctan(x)^2+x^2\log(x^2+1)-2\,x^2\log(x)+2\,x\arctan(x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x)^2 + x^2*log(x^2 + 1) - 2*x^2*log(x) + 2
*x*arctan(x))/x^2
```

Sympy [A] time = 1.02925, size = 32, normalized size = 0.82

$$\log{(x)} - \frac{\log{(x^2+1)}}{2} - \frac{\operatorname{atan}^2{(x)}}{2} - \frac{\operatorname{atan}{(x)}}{x} - \frac{\operatorname{atan}^2{(x)}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x)**2/x**3,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x)**2/
(2*x**2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan{(x)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^3, x)
```

3.681 $\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5}\,dx$

Optimal. Leaf size=60

$$-\frac{\tan^{-1}(x)}{6x^3}-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)-\frac{\left(x^2+1\right)^2\tan^{-1}(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{\tan^{-1}(x)}{2x}$$

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*
ArcTan[x]^2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6
```

Rubi [A] time = 0.135943, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$-\frac{\tan^{-1}(x)}{6x^3}-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)-\frac{\left(x^2+1\right)^2\tan^{-1}(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*
ArcTan[x]^2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6
```

Rubi in Sympy [A] time = 7.71448, size = 53, normalized size = 0.88

$$\frac{\log\left(x^2\right)}{6}-\frac{\log\left(x^2+1\right)}{6}-\frac{\operatorname{atan}(x)}{2x}-\frac{1}{12x^2}-\frac{\operatorname{atan}(x)}{6x^3}-\frac{\left(x^2+1\right)^2\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*atan(x)**2/x**5,x)
```

```
[Out] log(x**2)/6 - log(x**2 + 1)/6 - atan(x)/(2*x) - 1/(12*x**2) - ata
n(x)/(6*x**3) - (x**2 + 1)**2*atan(x)**2/(4*x**4)
```

Mathematica [A] time = 0.0232026, size = 56, normalized size = 0.93

$$\frac{-2\left(3x^3+x\right)\tan^{-1}(x)+x^2\left(4x^2\log(x)-2x^2\log\left(x^2+1\right)-1\right)-3\left(x^2+1\right)^2\tan^{-1}(x)^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] (-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 +
 4*x^2*Log[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)
```

Maple [A] time = 0.026, size = 57, normalized size = 1.

$$-\frac{(\arctan(x))^2}{4x^4}-\frac{(\arctan(x))^2}{2x^2}-\frac{(\arctan(x))^2}{4}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{\ln(x^2+1)}{6}-\frac{1}{12x^2}+\frac{\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*arctan(x)^2/x^5,x)
```

```
[Out] -1/4*arctan(x)^2/x^4-1/2*arctan(x)^2/x^2-1/4*arctan(x)^2-1/6*arct
an(x)/x^3-1/2*arctan(x)/x-1/6*ln(x^2+1)-1/12/x^2+1/3*ln(x)
```

Maxima [A] time = 1.52303, size = 96, normalized size = 1.6

$$-\frac{1}{6}\left(\frac{3x^2+1}{x^3}+3\arctan(x)\right)\arctan(x)+\frac{3x^2\arctan(x)^2-2x^2\log(x^2+1)+4x^2\log(x)-1}{12x^2}-\frac{(2x^2+1)\arctan(x)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((3*x^2 + 1)/x^3 + 3*arctan(x))*arctan(x) + 1/12*(3*x^2*arct
an(x)^2 - 2*x^2*log(x^2 + 1) + 4*x^2*log(x) - 1)/x^2 - 1/4*(2*x^2
 + 1)*arctan(x)^2/x^4
```

Fricas [A] time = 0.229341, size = 73, normalized size = 1.22

$$-\frac{2x^4\log(x^2+1)-4x^4\log(x)+3(x^4+2x^2+1)\arctan(x)^2+x^2+2(3x^3+x)\arctan(x)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(2*x^4*log(x^2 + 1) - 4*x^4*log(x) + 3*(x^4 + 2*x^2 + 1)*ar
ctan(x)^2 + x^2 + 2*(3*x^3 + x)*arctan(x))/x^4
```

Sympy [A] time = 1.75213, size = 61, normalized size = 1.02

$$\frac{\log(x)}{3}-\frac{\log(x^2+1)}{6}-\frac{\operatorname{atan}^2(x)}{4}-\frac{\operatorname{atan}(x)}{2x}-\frac{\operatorname{atan}^2(x)}{2x^2}-\frac{1}{12x^2}-\frac{\operatorname{atan}(x)}{6x^3}-\frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*atan(x)**2/x**5,x)
```

```
[Out] log(x)/3 - log(x**2 + 1)/6 - atan(x)**2/4 - atan(x)/(2*x) - atan(
x)**2/(2*x**2) - 1/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x
**4)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)\arctan(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)*arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)*arctan(x)^2/x^5, x)
```

3.682 $\int \frac{x^3 \tan^{-1}(x)^2}{(1+x^2)^3} \, dx$

Optimal. Leaf size=79

$$\frac{5}{32(x^2+1)} - \frac{1}{32(x^2+1)^2} + \frac{3x\tan^{-1}(x)}{16(x^2+1)} + \frac{x^4\tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3\tan^{-1}(x)}{8(x^2+1)^2} - \frac{3}{32}\tan^{-1}(x)^2$$

```
[Out] -1/(32*(1 + x^2)^2) + 5/(32*(1 + x^2)) + (x^3*ArcTan[x])/(8*(1 +
x^2)^2) + (3*x*ArcTan[x])/(16*(1 + x^2)) - (3*ArcTan[x]^2)/32 + (
x^4*ArcTan[x]^2)/(4*(1 + x^2)^2)
```

Rubi [A] time = 0.21289, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3}{32(x^2+1)} + \frac{3x\tan^{-1}(x)}{16(x^2+1)} - \frac{x^4}{32(x^2+1)^2} + \frac{x^4\tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3\tan^{-1}(x)}{8(x^2+1)^2} - \frac{3}{32}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] -x^4/(32*(1 + x^2)^2) + 3/(32*(1 + x^2)) + (x^3*ArcTan[x])/(8*(1
+ x^2)^2) + (3*x*ArcTan[x])/(16*(1 + x^2)) - (3*ArcTan[x]^2)/32 +
 (x^4*ArcTan[x]^2)/(4*(1 + x^2)^2)
```

Rubi in Sympy [A] time = 11.9563, size = 73, normalized size = 0.92

$$\frac{x^4\operatorname{atan}^2(x)}{4(x^2+1)^2} - \frac{x^4}{32(x^2+1)^2} + \frac{x^3\operatorname{atan}(x)}{8(x^2+1)^2} + \frac{3x\operatorname{atan}(x)}{16(x^2+1)} - \frac{3\operatorname{atan}^2(x)}{32} + \frac{3}{32(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*atan(x)**2/(x**2+1)**3,x)
```

```
[Out] x**4*atan(x)**2/(4*(x**2 + 1)**2) - x**4/(32*(x**2 + 1)**2) + x**
3*atan(x)/(8*(x**2 + 1)**2) + 3*x*atan(x)/(16*(x**2 + 1)) - 3*ata
n(x)**2/32 + 3/(32*(x**2 + 1))
```

Mathematica [A] time = 0.0228365, size = 47, normalized size = 0.59

$$\frac{5x^2 + 2\left(5x^2+3\right)x\tan^{-1}(x) + \left(5x^4-6x^2-3\right)\tan^{-1}(x)^2 + 4}{32(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] (4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*Arc
Tan[x]^2)/(32*(1 + x^2)^2)
```

Maple [A] time = 0.037, size = 78, normalized size = 1.

$$\frac{(\arctan(x))^2}{4(x^2+1)^2} - \frac{(\arctan(x))^2}{2x^2+2} + \frac{5x^3\arctan(x)}{16(x^2+1)^2} + \frac{3x\arctan(x)}{16(x^2+1)^2} + \frac{5(\arctan(x))^2}{32} - \frac{1}{32(x^2+1)^2} + \frac{5}{32x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arctan(x)^2/(x^2+1)^3,x)
```

```
[Out] 1/4*arctan(x)^2/(x^2+1)^2-1/2*arctan(x)^2/(x^2+1)+5/16*x^3*arctan
(x)/(x^2+1)^2+3/16*x*arctan(x)/(x^2+1)^2+5/32*arctan(x)^2-1/32/(x
^2+1)^2+5/32/(x^2+1)
```

Maxima [A] time = 1.51415, size = 127, normalized size = 1.61

$$\frac{1}{16}\left(\frac{5x^3+3x}{x^4+2x^2+1} + 5\arctan(x)\right)\arctan(x) - \frac{(2x^2+1)\arctan(x)^2}{4(x^4+2x^2+1)} - \frac{5(x^4+2x^2+1)\arctan(x)^2 - 5x^2 - 4}{32(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)^2/(x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*x^3 + 3*x)/(x^4 + 2*x^2 + 1) + 5*arctan(x))*arctan(x) -
1/4*(2*x^2 + 1)*arctan(x)^2/(x^4 + 2*x^2 + 1) - 1/32*(5*(x^4 + 2*
x^2 + 1)*arctan(x)^2 - 5*x^2 - 4)/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 0.219073, size = 69, normalized size = 0.87

$$\frac{(5x^4 - 6x^2 - 3)\arctan(x)^2 + 5x^2 + 2(5x^3 + 3x)\arctan(x) + 4}{32(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)^2/(x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((5*x^4 - 6*x^2 - 3)*arctan(x)^2 + 5*x^2 + 2*(5*x^3 + 3*x)*a
rctan(x) + 4)/(x^4 + 2*x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}^2(x)}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*atan(x)**2/(x**2+1)**3,x)
```

```
[Out] Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(x)^2}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arctan(x)^2/(x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)^2/(x^2 + 1)^3, x)
```

3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=107

$$\frac{i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x}$$

```
[Out] -(1/Sqrt[x^2]) - (Sqrt[-1 + x^2]*ArcSec[x])/x - ((2*I)*Sqrt[x^2]*
ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (I*Sqrt[x^2]*PolyLog[2, (-
I)*E^(I*ArcSec[x])])/x - (I*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x]
)])/x
```

Rubi [A] time = 0.291478, antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -(1/Sqrt[x^2]) - (Sqrt[1 - x^(-2)]*Sqrt[x^2]*ArcSec[x])/x - ((2*I
)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (I*Sqrt[x^2]*P
olyLog[2, (-I)*E^(I*ArcSec[x])])/x - (I*Sqrt[x^2]*PolyLog[2, I*E^
(I*ArcSec[x])])/x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{x}+\frac{\sqrt{x^2}\int^{\operatorname{acos}\left(\frac{1}{x}\right)}\frac{x}{\cos(x)}\,dx}{x}-\frac{\sqrt{x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
```

```
[Out] -sqrt(1 - 1/x**2)*sqrt(x**2)*acos(1/x)/x + sqrt(x**2)*Integral(x/
cos(x), (x, acos(1/x)))/x - sqrt(x**2)/x**2
```

Mathematica [A] time = 0.212723, size = 116, normalized size = 1.08

$$-\frac{\sqrt{1-\frac{1}{x^2}}\left(-ix\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)+ix\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)+\sqrt{1-\frac{1}{x^2}}x\sec^{-1}(x)-x\sec^{-1}(x)\log\left(1-ie^{i\sec^{-1}(x)}\right)\right.}{\sqrt{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[
x]*Log[1 - I*E^(I*ArcSec[x])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec
[x])] - I*x*PolyLog[2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E
^(I*ArcSec[x])]))/Sqrt[-1 + x^2])
```

Maple [C] time = 0.263, size = 199, normalized size = 1.9

$$-\frac{\text{arcsec}(x)+i}{2x}\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}-\frac{\text{arcsec}(x)-i}{2x}\left(i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}$$
$$-x\sqrt{\frac{x^2-1}{x^2}}\left(\text{arcsec}(x)\ln\left(1+i\left(x^{-1}+i\sqrt{1-x^{-2}}\right)\right)-\text{arcsec}(x)\ln\left(1-i\left(x^{-1}+i\sqrt{1-x^{-2}}\right)\right)-i\,dilog\left(1+i\left(x^{-1}+i\sqrt{1-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x)*(x^2-1)^(1/2)/x^2,x)
```

```
[Out] -1/2/(x^2-1)^(1/2)/x*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)+
I)-1/2/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)-I
)/x-((x^2-1)/x^2)^(1/2)*x*(arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2
)))-arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))-I*dilog(1+I*(1/x+I*
(1-1/x^2)^(1/2)))+I*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2))))/(x^2-1)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-1}\,\text{arcsec}\,(x)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}\,\text{arcsec}\,(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

3.684 $\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx$

Optimal. Leaf size=106

$$-\frac{7x\log(x)}{3\sqrt{x^2}}+\frac{(x^2-1)^{5/2}\csc^{-1}(x)}{3x^2}-\frac{5(x^2-1)^{3/2}\csc^{-1}(x)}{3x^2}-\frac{5\sqrt{x^2-1}\csc^{-1}(x)}{2x^2}-\frac{5x\csc^{-1}(x)^2}{4\sqrt{x^2}}+\frac{2x^4+3}{12x\sqrt{x^2}}$$

```
[Out] (3 + 2*x^4)/(12*x*Sqrt[x^2]) - (5*Sqrt[-1 + x^2]*ArcCsc[x])/(2*x^
2) - (5*(-1 + x^2)^(3/2)*ArcCsc[x])/(3*x^2) + ((-1 + x^2)^(5/2)*A
rcCsc[x])/(3*x^2) - (5*x*ArcCsc[x]^2)/(4*Sqrt[x^2]) - (7*x*Log[x]
)/(3*Sqrt[x^2])
```

Rubi [A] time = 0.300395, antiderivative size = 133, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533

$$\frac{x\sqrt{x^2}}{6}-\frac{7\sqrt{x^2}\log(x)}{3x}+\frac{1}{3}(x^2)^{3/2}\left(1-\frac{1}{x^2}\right)^{5/2}\csc^{-1}(x)-\frac{5}{3}\sqrt{x^2}\left(1-\frac{1}{x^2}\right)^{3/2}\csc^{-1}(x)-\frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}}-\frac{5\sqrt{x^2}\csc^{-1}(x)^2}{4x}+\frac{\sqrt{x^2}}{4x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] Sqrt[x^2]/(4*x^3) + (x*Sqrt[x^2])/6 - (5*Sqrt[1 - x^(-2)]*ArcCsc[
x])/(2*Sqrt[x^2]) - (5*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcCsc[x])/3
+ ((1 - x^(-2))^(5/2)*(x^2)^(3/2)*ArcCsc[x])/3 - (5*Sqrt[x^2]*Arc
Csc[x]^2)/(4*x) - (7*Sqrt[x^2]*Log[x])/(3*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\left(1-\frac{1}{x^2}\right)^{\frac{5}{2}}\sqrt{x^2}\,\text{asin}\left(\frac{1}{x}\right)}{3}+\frac{x\sqrt{x^2}}{6}-\frac{5\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}\sqrt{x^2}\,\text{asin}\left(\frac{1}{x}\right)}{3}+\frac{\sqrt{x^2}\log\left(\frac{1}{x^2}\right)}{3x}+\frac{5\sqrt{x^2}\log\left(\frac{1}{x}\right)}{3x}-\frac{5\sqrt{x^2}\,\text{asin}^2\left(\frac{1}{x}\right)}{4x}+\frac{5\sqrt{x^2}\int^{\frac{1}{x}}x\,dx}{6x}-\frac{5\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\,\text{asin}\left(\frac{1}{x}\right)}{2x^2}-\frac{\sqrt{x^2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
```

```
[Out] x**2*(1 - 1/x**2)**(5/2)*sqrt(x**2)*asin(1/x)/3 + x*sqrt(x**2)/6
- 5*(1 - 1/x**2)**(3/2)*sqrt(x**2)*asin(1/x)/3 + sqrt(x**2)*log(x
**(-2))/(3*x) + 5*sqrt(x**2)*log(1/x)/(3*x) - 5*sqrt(x**2)*asin(1
/x)**2/(4*x) + 5*sqrt(x**2)*Integral(x, (x, 1/x))/(6*x) - 5*sqrt(
1 - 1/x**2)*sqrt(x**2)*asin(1/x)/(2*x**2) - sqrt(x**2)/(6*x**3)
```

Mathematica [A] time = 0.372801, size = 86, normalized size = 0.81

$$\frac{\sqrt{x^2-1}\left(4x^2+\csc^{-1}(x)\left(8\sqrt{1-\frac{1}{x^2}}x\left(x^2-7\right)-6\sin\left(2\csc^{-1}(x)\right)\right)+48\log\left(\frac{1}{x}\right)-8\log(x)-30\csc^{-1}(x)^2-3\cos\left(2\csc^{-1}(\right.\right.}{24\sqrt{1-\frac{1}{x^2}}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] (Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48
*Log[x^(-1)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x
^2) - 6*Sin[2*ArcCsc[x]])))/(24*Sqrt[1 - x^(-2)]*x)
```

Maple [C] time = 0.438, size = 305, normalized size = 2.9

$$-\frac{5x(\operatorname{arccsc}(x))^2}{4}\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}}+\frac{2\operatorname{arccsc}(x)+i}{16x^2}\left(i\sqrt{\frac{x^2-1}{x^2}}x^3-2i\sqrt{\frac{x^2-1}{x^2}}x-2x^2+2\right)\frac{1}{\sqrt{x^2-1}}$$
$$-\frac{-i+2\operatorname{arccsc}(x)}{16x^2}\left(i\sqrt{\frac{x^2-1}{x^2}}x^3-2i\sqrt{\frac{x^2-1}{x^2}}x+2x^2-2\right)\frac{1}{\sqrt{x^2-1}}-\frac{14i}{3}x\operatorname{arccsc}(x)\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}}$$
$$+\frac{1}{6x^4-90x^2+378}\left(x^4+7i\sqrt{\frac{x^2-1}{x^2}}x-8x^2+7\right)\left(2\operatorname{arccsc}(x)x^4+\sqrt{\frac{x^2-1}{x^2}}x^3-30\operatorname{arccsc}(x)x^2-7\sqrt{\frac{x^2-1}{x^2}}x+126\operatorname{ar}\right.$$
$$+\frac{7x}{3}\sqrt{\frac{x^2-1}{x^2}}\ln\left(\left(\frac{i}{x}+\sqrt{1-x^{-2}}\right)^2-1\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)^(5/2)*arccsc(x)/x^3,x)
```

```
[Out] -5/4/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arccsc(x)^2+1/16/(x^2-1)
^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2
*x^2+2)*(2*arccsc(x)+I)-1/16/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(
1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*(-I+2*arccsc(x))-14/3
*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arccsc(x)+1/6/(x^2-1)^(1/2
)*(x^4+7*I*((x^2-1)/x^2)^(1/2)*x-8*x^2+7)*(2*arccsc(x)*x^4+((x^2-
1)/x^2)^(1/2)*x^3-30*arccsc(x)*x^2-7*((x^2-1)/x^2)^(1/2)*x+126*ar
ccsc(x)-7*I)/(x^4-15*x^2+63)+7/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2
```

```
)*x*ln((I/x+(1-1/x^2)^(1/2))^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

Fricas [A] time = 0.230014, size = 69, normalized size = 0.65

$$\frac{2\,x^4-15\,x^2\operatorname{arccsc}(x)^2-28\,x^2\log(x)+2\left(2\,x^4-14\,x^2-3\right)\sqrt{x^2-1}\operatorname{arccsc}(x)+3}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^4 - 15*x^2*arccsc(x)^2 - 28*x^2*log(x) + 2*(2*x^4 - 14*
x^2 - 3)*sqrt(x^2 - 1)*arccsc(x) + 3)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

3.685 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3\sqrt{x^2}}-\frac{1}{9\left(x^2\right)^{3/2}}+\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

```
[Out] -1/(9*(x^2)^(3/2)) + 1/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x
])/(3*x^3)
```

Rubi [A] time = 0.0894826, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{1}{3\sqrt{x^2}}-\frac{1}{9\left(x^2\right)^{3/2}}+\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

```
[Out] -1/(9*(x^2)^(3/2)) + 1/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x
])/(3*x^3)
```

Rubi in Sympy [A] time = 4.7384, size = 39, normalized size = 0.95

$$\frac{1}{3\sqrt{x^2}}-\frac{1}{9x^2\sqrt{x^2}}+\frac{\left(x^2-1\right)^{\frac{3}{2}}\operatorname{asec}(x)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] 1/(3*sqrt(x**2)) - 1/(9*x**2*sqrt(x**2)) + (x**2 - 1)**(3/2)*asec
(x)/(3*x**3)
```

Mathematica [A] time = 0.0502664, size = 48, normalized size = 1.17

$$\frac{\sqrt{1-\frac{1}{x^2}}x\left(3x^2-1\right)+3\left(x^2-1\right)^2\sec^{-1}(x)}{9x^3\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x
^3*Sqrt[-1 + x^2])
```

Maple [C] time = 0.405, size = 198, normalized size = 4.8

$$\frac{i+3\,\mathrm{arcsec}\,(x)}{72\,x^3}\left(-3\,i\sqrt{\frac{x^2-1}{x^2}}x^3+x^4+4\,i\sqrt{\frac{x^2-1}{x^2}}x-5\,x^2+4\right)\frac{1}{\sqrt{x^2-1}}+\frac{\mathrm{arcsec}\,(x)+i}{8\,x}\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{\mathrm{arcsec}\,(x)-i}{8\,x}\left(i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{-i+3\,\mathrm{arcsec}\,(x)}{72\,x^3}\left(-5\,x^2+4+3\,i\sqrt{\frac{x^2-1}{x^2}}x^3+x^4-4\,i\sqrt{\frac{x^2-1}{x^2}}x\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x)*(x^2-1)^(1/2)/x^4,x)
```

```
[Out] 1/72/(x^2-1)^(1/2)/x^3*(-3*I*((x^2-1)/x^2)^(1/2)*x^3+x^4+4*I*((x^
2-1)/x^2)^(1/2)*x-5*x^2+4)*(I+3*arcsec(x))+1/8/(x^2-1)^(1/2)/x*(-
I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)+I)+1/8/(x^2-1)^(1/2)*(I
*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)-I)/x+1/72/(x^2-1)^(1/2)/
x^3*(-5*x^2+4+3*I*((x^2-1)/x^2)^(1/2)*x^3+x^4-4*I*((x^2-1)/x^2)^(
1/2)*x)*(-I+3*arcsec(x))
```

Maxima [A] time = 1.57957, size = 36, normalized size = 0.88

$$\frac{\left(x^2-1\right)^{\frac{3}{2}}\mathrm{arcsec}\,(x)}{3\,x^3}+\frac{3\,x^2-1}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)/x^3 + 1/9*(3*x^2 - 1)/x^3
```

Fricas [A] time = 0.221462, size = 31, normalized size = 0.76

$$\frac{3 \left(x^2-1\right)^{\frac{3}{2}} \operatorname{arcsec}(x)+3 x^2-1}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(x^2 - 1)^(3/2)*arcsec(x) + 3*x^2 - 1)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.223023, size = 101, normalized size = 2.46

$$-\frac{2 \arctan\left(-x+\sqrt{x^2-1}\right)}{3 \operatorname{sign}(x)}+\frac{2 \left(3 \left(x-\sqrt{x^2-1}\right)^4+1\right) \arccos\left(\frac{1}{x}\right)}{3 \left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3}+\frac{3 x^2-1}{9 x^3 \operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*arctan(-x + sqrt(x^2 - 1))/sign(x) + 2/3*(3*(x - sqrt(x^2 -
1))^4 + 1)*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2
 - 1)/(x^3*sign(x))
```

3.686 $\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{5}{6}\coth^{-1}\left(\sqrt{x^2}\right)$$

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) + (5*ArcCoth[Sqrt[x^2]])/6 - (x*ArcSec[x]
)/(3*(-1 + x^2)^(3/2)) + (2*x*ArcSec[x])/(3*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0576145, antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{5x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x*ArcSec[x])/(3*(-1 + x^2)^(3/2)) + (2
*x*ArcSec[x])/(3*Sqrt[-1 + x^2]) + (5*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rubi in Sympy [A] time = 4.48212, size = 63, normalized size = 0.97

$$\frac{x^2}{6(-x^2+1)\sqrt{x^2}}+\frac{5x\,\text{atanh}\,(x)}{6\sqrt{x^2}}+\frac{2x\,\text{asec}\,(x)}{3\sqrt{x^2-1}}-\frac{x\,\text{asec}\,(x)}{3(x^2-1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] x**2/(6*(-x**2 + 1)*sqrt(x**2)) + 5*x*atanh(x)/(6*sqrt(x**2)) + 2
*x*asec(x)/(3*sqrt(x**2 - 1)) - x*asec(x)/(3*(x**2 - 1)**(3/2))
```

Mathematica [A] time = 0.154264, size = 67, normalized size = 1.03

$$\frac{\sqrt{1-\frac{1}{x^2}}x\left(-5\left(x^2-1\right)\log(1-x)+5\left(x^2-1\right)\log(x+1)-2x\right)+4x\left(2x^2-3\right)\sec^{-1}(x)}{12\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] (4*x*(-3 + 2*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x - 5*(-1 +
x^2)*Log[1 - x] + 5*(-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

Maple [C] time = 0.414, size = 128, normalized size = 2.

$$\frac{x}{6\,x^4-12\,x^2+6}\sqrt{x^2-1}\left(4\,\mathrm{arcsec}\,(x)\,x^2-\sqrt{\frac{x^2-1}{x^2}}x-6\,\mathrm{arcsec}\,(x)\right)\\ -\frac{5\,x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(x^{-1}+i\sqrt{1-x^{-2}}-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{5\,x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(1+x^{-1}+i\sqrt{1-x^{-2}}\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(4*arcsec(x)*x^2-((x^2-1)/x^2)^
(1/2)*x-6*arcsec(x))-5/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1
/x+I*(1-1/x^2)^(1/2)-1)+5/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*l
n(1+1/x+I*(1-1/x^2)^(1/2))
```

Maxima [A] time = 1.50952, size = 65, normalized size = 1.

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{x^2-1}}-\frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\mathrm{arcsec}\,(x)-\frac{x}{6\,(x^2-1)}+\frac{5}{12}\,\log\,(x+1)-\frac{5}{12}\,\log\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(x^2 - 1) - x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^
2 - 1) + 5/12*log(x + 1) - 5/12*log(x - 1)
```

Fricas [A] time = 0.228902, size = 101, normalized size = 1.55

$$-\frac{2\,x^3-4\left(2\,x^3-3\,x\right)\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)-5\left(x^4-2\,x^2+1\right)\,\log\,(x+1)+5\left(x^4-2\,x^2+1\right)\,\log\,(x-1)-2\,x}{12\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*x^3 - 4*(2*x^3 - 3*x)*sqrt(x^2 - 1)*arcsec(x) - 5*(x^4 -
 2*x^2 + 1)*log(x + 1) + 5*(x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x
^4 - 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.259447, size = 78, normalized size = 1.2

$$\frac{(2x^2-3)x\arccos\left(\frac{1}{x}\right)}{3(x^2-1)^{\frac{3}{2}}}+\frac{5\ln(|x+1|)}{12\,\mathrm{sign}(x)}-\frac{5\ln(|x-1|)}{12\,\mathrm{sign}(x)}-\frac{x}{6(x^2-1)\mathrm{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*x^2 - 3)*x*arccos(1/x)/(x^2 - 1)^(3/2) + 5/12*ln(abs(x + 1
))/sign(x) - 5/12*ln(abs(x - 1))/sign(x) - 1/6*x/((x^2 - 1)*sign(
x))
```

3.687 $\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

Optimal. Leaf size=51

$$\frac{\sqrt{x^2}}{6(1-x^2)} - \frac{1}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{x^3 \sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - ArcCoth[Sqrt[x^2]]/6 - (x^3*ArcSec[x])/
(3*(-1 + x^2)^(3/2))
```

Rubi [A] time = 0.101729, antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x \tanh^{-1}(x)}{6\sqrt{x^2}} - \frac{x^3 \sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x^3*ArcSec[x])/(3*(-1 + x^2)^(3/2)) -
(x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rubi in Sympy [A] time = 5.38584, size = 46, normalized size = 0.9

$$-\frac{x^3 \operatorname{asec}(x)}{3(x^2-1)^{\frac{3}{2}}} + \frac{x^2}{6(-x^2+1)\sqrt{x^2}} - \frac{x \operatorname{atanh}(x)}{6\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] -x**3*asec(x)/(3*(x**2 - 1)**(3/2)) + x**2/(6*(-x**2 + 1)*sqrt(x*
*2)) - x*atanh(x)/(6*sqrt(x**2))
```

Mathematica [A] time = 0.139068, size = 61, normalized size = 1.2

$$\frac{\sqrt{1-\frac{1}{x^2}}x\left(\left(x^2-1\right)\log(1-x)-\left(x^2-1\right)\log(x+1)-2x\right)-4x^3\sec^{-1}(x)}{12\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 -
 x] - (-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

Maple [C] time = 0.442, size = 121, normalized size = 2.4

$$-\frac{x^2}{6x^4-12x^2+6}\sqrt{x^2-1}\left(2x\operatorname{arcsec}(x)+\sqrt{\frac{x^2-1}{x^2}}\right)+\frac{x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(x^{-1}+i\sqrt{1-x^{-2}}-1\right)\frac{1}{\sqrt{x^2-1}}-\frac{x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(1+x^{-1}+i\sqrt{1-x^{-2}}\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] -1/6*(x^2-1)^(1/2)*x^2/(x^4-2*x^2+1)*(2*x*arcsec(x)+((x^2-1)/x^2)
^(1/2))+1/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2
)^(1/2)-1)-1/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1+1/x+I*(1-
1/x^2)^(1/2))
```

Maxima [A] time = 1.46852, size = 62, normalized size = 1.22

$$-\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}}+\frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\operatorname{arcsec}(x)-\frac{x}{6(x^2-1)}-\frac{1}{12}\log(x+1)+\frac{1}{12}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x/sqrt(x^2 - 1) + x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2
 - 1) - 1/12*log(x + 1) + 1/12*log(x - 1)
```

Fricas [A] time = 0.226634, size = 92, normalized size = 1.8

$$-\frac{4\sqrt{x^2-1}x^3\operatorname{arcsec}(x)+2x^3+\left(x^4-2x^2+1\right)\log(x+1)-\left(x^4-2x^2+1\right)\log(x-1)-2x}{12\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(x^2 - 1)*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*
log(x + 1) - (x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.256302, size = 72, normalized size = 1.41

$$-\frac{x^3 \arccos\left(\frac{1}{x}\right)}{3\left(x^2-1\right)^{\frac{3}{2}}} - \frac{\ln\left(|x+1|\right)}{12\,\mathrm{sign}\left(x\right)} + \frac{\ln\left(|x-1|\right)}{12\,\mathrm{sign}\left(x\right)} - \frac{x}{6\left(x^2-1\right)\mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arccos(1/x)/(x^2 - 1)^(3/2) - 1/12*ln(abs(x + 1))/sign(x
) + 1/12*ln(abs(x - 1))/sign(x) - 1/6*x/((x^2 - 1)*sign(x))
```

3.688 $\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{x}{6\sqrt{x^2}\,(1-x^2)}-\frac{2x\log(x)}{3\sqrt{x^2}}+\frac{x\log\left(x^2-1\right)}{3\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3\,(x^2-1)^{3/2}}$$

```
[Out] x/(6*Sqrt[x^2]*(1 - x^2)) - ArcSec[x]/(3*(-1 + x^2)^(3/2)) - ArcS
ec[x]/Sqrt[-1 + x^2] - (2*x*Log[x])/(3*Sqrt[x^2]) + (x*Log[-1 + x
^2])/(3*Sqrt[x^2])
```

Rubi [A] time = 0.158123, antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{x}{6\sqrt{x^2}\,(1-x^2)}-\frac{2x\log(x)}{3\sqrt{x^2}}+\frac{x\log\left(1-x^2\right)}{3\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3\,(x^2-1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] x/(6*Sqrt[x^2]*(1 - x^2)) - ArcSec[x]/(3*(-1 + x^2)^(3/2)) - ArcS
ec[x]/Sqrt[-1 + x^2] - (2*x*Log[x])/(3*Sqrt[x^2]) + (x*Log[1 - x^
2])/(3*Sqrt[x^2])
```

Rubi in Sympy [A] time = 8.65708, size = 71, normalized size = 0.87

$$-\frac{x\log\left(x^2\right)}{3\sqrt{x^2}}+\frac{x\log\left(-x^2+1\right)}{3\sqrt{x^2}}+\frac{x}{6\,(-x^2+1)\,\sqrt{x^2}}-\frac{\operatorname{asec}(x)}{\sqrt{x^2-1}}-\frac{\operatorname{asec}(x)}{3\,(x^2-1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] -x*log(x**2)/(3*sqrt(x**2)) + x*log(-x**2 + 1)/(3*sqrt(x**2)) + x
/(6*(-x**2 + 1)*sqrt(x**2)) - asec(x)/sqrt(x**2 - 1) - asec(x)/(3
*(x**2 - 1)**(3/2))
```

Mathematica [A] time = 0.228939, size = 72, normalized size = 0.88

$$\frac{-\frac{(x^2-1)\left(4(x^2-1)\log(x)-2(x^2-1)\log(1-x^2)+1\right)}{\sqrt{1-\frac{1}{x^2}}\,x}-2\left(3x^2-2\right)\sec^{-1}(x)}{6\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-2*(-2 + 3*x^2)*ArcSec[x] - ((-1 + x^2)*(1 + 4*(-1 + x^2)*Log[x]
 - 2*(-1 + x^2)*Log[1 - x^2]))/(Sqrt[1 - x^(-2)]*x))/(6*(-1 + x^2
)^(3/2))
```

Maple [C] time = 0.615, size = 197, normalized size = 2.4

$$-\frac{4\,i}{3}x\mathrm{arcsec}\left(x\right)\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}}$$
$$+\frac{1}{6\,x^2\left(4\,x^6-11\,x^4+10\,x^2-3\right)}\sqrt{x^2-1}\left(2\,i\sqrt{\frac{x^2-1}{x^2}}x^3-2\,i\sqrt{\frac{x^2-1}{x^2}}x-3\,x^2+2\right)\left(8\,\mathrm{arcsec}\left(x\right)x^4+2\,ix^4+3\,\sqrt{\frac{x^2-1}{x^2}}x^3-\right.$$
$$+\frac{2\,x}{3}\sqrt{\frac{x^2-1}{x^2}}\ln\left(\left(x^{-1}+i\sqrt{1-x^{-2}}\right)^2-1\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] -4/3*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)+1/6*(x^2-1)^
(1/2)/x^2*(2*I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-
3*x^2+2)*(8*arcsec(x)*x^4+2*I*x^4+3*((x^2-1)/x^2)^(1/2)*x^3-6*arc
sec(x)*x^2-4*I*x^2-2*((x^2-1)/x^2)^(1/2)*x+2*I)/(4*x^6-11*x^4+10*
x^2-3)+2/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln((1/x+I*(1-1/x^2
)^(1/2))^2-1)
```

Maxima [A] time = 1.433, size = 86, normalized size = 1.05

$$-\frac{1}{3}\left(\frac{3\,x^2}{\left(x^2-1\right)^{\frac{3}{2}}}-\frac{2}{\left(x^2-1\right)^{\frac{3}{2}}}\right)\mathrm{arcsec}\left(x\right)+\frac{1}{3\left(x^2-1\right)}$$
$$-\frac{1}{2\left(x+1\right)\left(x-1\right)}+\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)-\frac{2}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2/(x^2 - 1)^(3/2) - 2/(x^2 - 1)^(3/2))*arcsec(x) + 1/3/
(x^2 - 1) - 1/2/((x + 1)*(x - 1)) + 1/3*log(x + 1) + 1/3*log(x -
1) - 2/3*log(x)
```

Fricas [A] time = 0.225422, size = 93, normalized size = 1.13

$$-\frac{2\left(3x^2-2\right)\sqrt{x^2-1}\,\mathrm{arcsec}\left(x\right)+x^2-2\left(x^4-2x^2+1\right)\log\left(x^2-1\right)+4\left(x^4-2x^2+1\right)\log\left(x\right)-1}{6\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^
2 + 1)*log(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^
2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.231543, size = 86, normalized size = 1.05

$$-\frac{\left(3x^2-2\right)\arccos\left(\frac{1}{x}\right)}{3\left(x^2-1\right)^{\frac{3}{2}}}-\frac{\ln\left(x^2\right)}{3\,\mathrm{sign}\left(x\right)}+\frac{\ln\left(\left|x^2-1\right|\right)}{3\,\mathrm{sign}\left(x\right)}-\frac{2x^2-1}{6\left(x^2-1\right)\mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*ln(x^2)/sign(x
) + 1/3*ln(abs(x^2 - 1))/sign(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sig
n(x))
```

3.689 $\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=175

$$\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x}-\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}+\frac{\sqrt{x^2}\,(2-3x^2)}{6\,(x^2-1)}-\frac{5x\sec^{-1}(x)}{2\sqrt{x^2-1}}-\frac{13}{6}\coth^{-1}\left(\sqrt{x^2}\right)-\frac{5i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x}+\frac{x^5\sec^{-1}(x)}{2\,(x^2-1)^{3/2}}-\frac{5x^3\sec^{-1}(x)}{6\,(x^2-1)^{3/2}}$$

```
[Out] (Sqrt[x^2]*(2 - 3*x^2))/(6*(-1 + x^2)) - (13*ArcCoth[Sqrt[x^2]])/
6 - (5*x^3*ArcSec[x])/(6*(-1 + x^2)^(3/2)) + (x^5*ArcSec[x])/(2*(
-1 + x^2)^(3/2)) - (5*x*ArcSec[x])/(2*Sqrt[-1 + x^2]) - ((5*I)*Sq
rt[x^2]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (((5*I)/2)*Sqrt[x^
2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x - (((5*I)/2)*Sqrt[x^2]*Pol
yLog[2, I*E^(I*ArcSec[x])])/x
```

Rubi [A] time = 0.46795, antiderivative size = 232, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733

$$\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x}-\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}+\frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)}-\frac{3\sqrt{x^2}}{4}-\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}}+\frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x}-\frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x}-\frac{13\sqrt{x^2}\coth^{-1}(x)}{6x}-\frac{5i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -5/(12*(1 - x^(-2))*Sqrt[x^2]) - (3*Sqrt[x^2])/4 + Sqrt[x^2]/(4*(
1 - x^(-2))) - (13*Sqrt[x^2]*ArcCoth[x])/(6*x) - (5*Sqrt[x^2]*Arc
Sec[x])/(6*(1 - x^(-2))^(3/2)*x) - (5*Sqrt[x^2]*ArcSec[x])/(2*Sqr
t[1 - x^(-2)]*x) + (x*Sqrt[x^2]*ArcSec[x])/(2*(1 - x^(-2))^(3/2))
 - ((5*I)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (((5*I
)/2)*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x - (((5*I)/2)*S
qrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}}-\frac{3\sqrt{x^2}}{4}+\frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)}-\frac{13\sqrt{x^2}\operatorname{atanh}\left(\frac{1}{x}\right)}{6x}+\frac{5\sqrt{x^2}\int^{\operatorname{acos}\left(\frac{1}{x}\right)}\frac{x}{\cos(x)}\,dx}{2x}-\frac{5\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{2x\sqrt{1-\frac{1}{x^2}}}-\frac{5\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{6x\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}}-\frac{5\sqrt{x^2}}{12x^2\left(1-\frac{1}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] x*sqrt(x**2)*acos(1/x)/(2*(1 - 1/x**2)**(3/2)) - 3*sqrt(x**2)/4 +
 sqrt(x**2)/(4*(1 - 1/x**2)) - 13*sqrt(x**2)*atanh(1/x)/(6*x) + 5
*sqrt(x**2)*Integral(x/cos(x), (x, acos(1/x)))/(2*x) - 5*sqrt(x**
2)*acos(1/x)/(2*x*sqrt(1 - 1/x**2)) - 5*sqrt(x**2)*acos(1/x)/(6*x
*(1 - 1/x**2)**(3/2)) - 5*sqrt(x**2)/(12*x**2*(1 - 1/x**2))
```

Mathematica [B] time = 2.77508, size = 383, normalized size = 2.19

$$-\frac{x^5\left(-60i\sqrt{1-\frac{1}{x^2}}\sin^2\left(2\sec^{-1}(x)\right)\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)+60i\sqrt{1-\frac{1}{x^2}}\sin^2\left(2\sec^{-1}(x)\right)\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)-30\sqrt{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -(x^5*(22*ArcSec[x] + 40*ArcSec[x]*Cos[2*ArcSec[x]] - 30*ArcSec[x
]*Cos[4*ArcSec[x]] - 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 - I*E^(I
*ArcSec[x])] + 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSe
c[x])] + 26*Sqrt[1 - x^(-2)]*Log[Cos[ArcSec[x]/2]] - 26*Sqrt[1 -
x^(-2)]*Log[Sin[ArcSec[x]/2]] + 16*Sin[2*ArcSec[x]] - (60*I)*Sqrt
[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec[x])]*Sin[2*ArcSec[x]]^2
+ (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSec[x])]*Sin[2*Arc
Sec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*ArcSec[
x]] + 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] +
13*Log[Cos[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/
2]]*Sin[3*ArcSec[x]] - 4*Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 -
I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]] - 15*ArcSec[x]*Log[1 + I*E^(I
*ArcSec[x])]*Sin[5*ArcSec[x]] - 13*Log[Cos[ArcSec[x]/2]]*Sin[5*Ar
cSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Sin[5*ArcSec[x]]))/(96*(-1 +
x^2)^(3/2))
```

Maple [C] time = 0.675, size = 240, normalized size = 1.4

$$\frac{x}{6\,x^4-12\,x^2+6}\sqrt{x^2-1}\left(3\,\mathrm{arcsec}\,(x)\,x^4-3\,\sqrt{\frac{x^2-1}{x^2}}x^3-20\,\mathrm{arcsec}\,(x)\,x^2+2\,\sqrt{\frac{x^2-1}{x^2}}x+15\,\mathrm{arcsec}\,(x)\right)$$
$$+\frac{i}{6}x\sqrt{\frac{x^2-1}{x^2}}\left(15\,i\mathrm{arcsec}\,(x)\ln\left(1+i\left(x^{-1}+i\sqrt{1-x^{-2}}\right)\right)-15\,i\mathrm{arcsec}\,(x)\ln\left(1-i\left(x^{-1}+i\sqrt{1-x^{-2}}\right)\right)-13\,i\ln\left(x^{-1}+i\sqrt{1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(3*arcsec(x)*x^4-3*((x^2-1)/x^2
)^(1/2)*x^3-20*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x+15*arcsec(x)
)+1/6*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*(15*I*arcsec(x)*ln(1+
I*(1/x+I*(1-1/x^2)^(1/2)))-15*I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)
^(1/2)))-13*I*ln(1/x+I*(1-1/x^2)^(1/2)-1)+13*I*ln(1+1/x+I*(1-1/x^
2)^(1/2))+15*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))-15*dilog(1-I*(1/x
+I*(1-1/x^2)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^6\,\mathrm{arcsec}\,(x)}{(x^4-2\,x^2+1)\sqrt{x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^6*arcsec(x)/((x^4 - 2*x^2 + 1)*sqrt(x^2 - 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

```
[Out] 1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

Rubi [A] time = 0.0818331, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]

[Out] 1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

Rubi in Sympy [A] time = 3.97558, size = 20, normalized size = 0.87

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\,\mathrm{asec}\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x)/x**2/(x**2-1)**(1/2),x)

[Out] 1/sqrt(x**2) + sqrt(x**2 - 1)*asec(x)/x
```

Mathematica [A] time = 0.0328063, size = 35, normalized size = 1.52

$$\frac{\sqrt{1-\frac{1}{x^2}}x+\left(x^2-1\right)\sec^{-1}(x)}{x\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])
```

Maple [C] time = 0.223, size = 76, normalized size = 3.3

$$\frac{\operatorname{arcsec}(x)+i}{2x}\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{\operatorname{arcsec}(x)-i}{2x}\left(i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x)/x^2/(x^2-1)^(1/2),x)
```

```
[Out] 1/2/(x^2-1)^(1/2)/x*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)+I
)+1/2/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)-I)
/x
```

Maxima [A] time = 1.52973, size = 23, normalized size = 1.

$$\frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)/(sqrt(x^2 - 1)*x^2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arcsec(x)/x + 1/x
```

Fricas [A] time = 0.221303, size = 22, normalized size = 0.96

$$\frac{\sqrt{x^2-1}\operatorname{arcsec}(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)/(sqrt(x^2 - 1)*x^2),x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 - 1)*arcsec(x) + 1)/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x)/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220471, size = 68, normalized size = 2.96

$$\frac{2\arccos\left(\frac{1}{x}\right)}{\left(x-\sqrt{x^2-1}\right)^2+1}-\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\mathrm{sign}\left(x\right)}+\frac{1}{x\mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x)/(sqrt(x^2 - 1)*x^2),x, algorithm="giac")
```

```
[Out] 2*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1) - 2*arctan(-x + sqrt(x^
2 - 1))/sign(x) + 1/(x*sign(x))
```

3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2}}{6\left(x^2-1\right)}-\frac{11}{6}\coth^{-1}\left(\sqrt{x^2}\right)+\frac{\left(8x^4-12x^2+3\right)\csc^{-1}(x)}{3x\left(x^2-1\right)^{3/2}}$$

```
[Out] -(1/Sqrt[x^2]) + Sqrt[x^2]/(6*(-1 + x^2)) - (11*ArcCoth[Sqrt[x^2]
])/6 + ((3 - 12*x^2 + 8*x^4)*ArcCsc[x])/(3*x*(-1 + x^2)^(3/2))
```

Rubi [A] time = 0.165217, antiderivative size = 91, normalized size of antiderivative = 1.3, number of steps used = 5, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533

$$-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}-\frac{11x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] -(1/Sqrt[x^2]) - Sqrt[x^2]/(6*(1 - x^2)) + ArcCsc[x]/(x*(-1 + x^2
)^(3/2)) - (4*x*ArcCsc[x])/(3*(-1 + x^2)^(3/2)) + (8*x*ArcCsc[x])
/(3*Sqrt[-1 + x^2]) - (11*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rubi in Sympy [A] time = 10.8913, size = 87, normalized size = 1.24

$$-\frac{x^2}{6\left(-x^2+1\right)\sqrt{x^2}}-\frac{11x\,\mathrm{atanh}\,(x)}{6\sqrt{x^2}}+\frac{8x\,\mathrm{acsc}\,(x)}{3\sqrt{x^2-1}}-\frac{4x\,\mathrm{acsc}\,(x)}{3\left(x^2-1\right)^{\frac{3}{2}}}-\frac{1}{\sqrt{x^2}}+\frac{\mathrm{acsc}\,(x)}{x\left(x^2-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
```

```
[Out] -x**2/(6*(-x**2 + 1)*sqrt(x**2)) - 11*x*atanh(x)/(6*sqrt(x**2)) +
 8*x*acsc(x)/(3*sqrt(x**2 - 1)) - 4*x*acsc(x)/(3*(x**2 - 1)**(3/2
)) - 1/sqrt(x**2) + acsc(x)/(x*(x**2 - 1)**(3/2))
```

Mathematica [A] time = 0.177279, size = 79, normalized size = 1.13

$$\frac{\sqrt{1-\frac{1}{x^2}}x\left(-10x^2+11\left(x^2-1\right)x\log(1-x)-11\left(x^2-1\right)x\log(x+1)+12\right)+4\left(8x^4-12x^2+3\right)\csc^{-1}(x)}{12x\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] (4*(3 - 12*x^2 + 8*x^4)*ArcCsc[x] + Sqrt[1 - x^(-2)]*x*(12 - 10*x
^2 + 11*x*(-1 + x^2)*Log[1 - x] - 11*x*(-1 + x^2)*Log[1 + x]))/(1
2*x*(-1 + x^2)^(3/2))
```

Maple [C] time = 0.48, size = 203, normalized size = 2.9

$$\frac{\operatorname{arccsc}(x)+i}{2x}\left(i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{\operatorname{arccsc}(x)-i}{2x}\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{x}{6x^4-12x^2+6}\sqrt{x^2-1}\left(10\operatorname{arccsc}(x)x^2+\sqrt{\frac{x^2-1}{x^2}}x-12\operatorname{arccsc}(x)\right)+\frac{11x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(\frac{i}{x}+\sqrt{1-x^{-2}}-i\right)\frac{1}{\sqrt{x^2-1}}-\frac{11x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(\frac{i}{x}+\sqrt{1-x^{-2}}+i\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccsc(x)/x^2/(x^2-1)^(5/2),x)
```

```
[Out] 1/2/(x^2-1)^(1/2)/x*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arccsc(x)+I)
+1/2/(x^2-1)^(1/2)*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arccsc(x)-I)
/x+1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(10*arccsc(x)*x^2+((x^2-1)/x
^2)^(1/2)*x-12*arccsc(x))+11/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*
x*ln(I/x+(1-1/x^2)^(1/2)-I)-11/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2
)*x*ln(I/x+(1-1/x^2)^(1/2)+I)
```

Maxima [A] time = 1.62524, size = 105, normalized size = 1.5

$$\frac{1}{3}\left(\frac{8x}{\sqrt{x^2-1}}-\frac{4x}{(x^2-1)^{\frac{3}{2}}}+\frac{3}{(x^2-1)^{\frac{3}{2}}x}\right)\operatorname{arccsc}(x)-\frac{3x^2-2}{2(x^3-x)}+\frac{2x}{3(x^2-1)}-\frac{11}{12}\log(x+1)+\frac{11}{12}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x)/((x^2 - 1)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*(8*x/sqrt(x^2 - 1) - 4*x/(x^2 - 1)^(3/2) + 3/((x^2 - 1)^(3/2)
*x))*arccsc(x) - 1/2*(3*x^2 - 2)/(x^3 - x) + 2/3*x/(x^2 - 1) - 11
/12*log(x + 1) + 11/12*log(x - 1)
```

Fricas [A] time = 0.255627, size = 109, normalized size = 1.56

$$-\frac{10\,x^4-4\,\left(8\,x^4-12\,x^2+3\right)\sqrt{x^2-1}\,\mathrm{arccsc}\left(x\right)-22\,x^2+11\,\left(x^5-2\,x^3+x\right)\log\left(x+1\right)-11\,\left(x^5-2\,x^3+x\right)\log\left(x-1\right)+12}{12\,\left(x^5-2\,x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x)/((x^2 - 1)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] -1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*sqrt(x^2 - 1)*arccsc(x) -
22*x^2 + 11*(x^5 - 2*x^3 + x)*log(x + 1) - 11*(x^5 - 2*x^3 + x)*l
og(x - 1) + 12)/(x^5 - 2*x^3 + x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.261763, size = 142, normalized size = 2.03

$$\frac{1}{3}\left(\frac{\left(5\,x^2-6\right)x}{\left(x^2-1\right)^{\frac{3}{2}}}+\frac{6}{\left(x-\sqrt{x^2-1}\right)^2+1}\right)\arcsin\left(\frac{1}{x}\right)+\frac{2\,\arctan\left(-x+\sqrt{x^2-1}\right)}{\mathrm{sign}\left(x\right)}-\frac{11\ln\left(|x+1|\right)}{12\,\mathrm{sign}\left(x\right)}+\frac{11\ln\left(|x-1|\right)}{12\,\mathrm{sign}\left(x\right)}-\frac{5\,x^2-6}{6\left(x^3-x\right)\mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x)/((x^2 - 1)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] 1/3*((5*x^2 - 6)*x/(x^2 - 1)^(3/2) + 6/((x - sqrt(x^2 - 1))^2 + 1
))*arcsin(1/x) + 2*arctan(-x + sqrt(x^2 - 1))/sign(x) - 11/12*ln(
abs(x + 1))/sign(x) + 11/12*ln(abs(x - 1))/sign(x) - 1/6*(5*x^2 -
 6)/((x^3 - x)*sign(x))
```

3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=74

$$\frac{24\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2-1}\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{x^2-1}\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

```
[Out] (24*Sqrt[-1 + x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (12*Sqrt[-1 +
x^2]*ArcCsc[x]^2)/x - (4*ArcCsc[x]^3)/Sqrt[x^2] + (Sqrt[-1 + x^2]
*ArcCsc[x]^4)/x
```

Rubi [A] time = 0.288679, antiderivative size = 101, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*Sqrt[1 - x^(-2)]*Sqrt[x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (1
2*Sqrt[1 - x^(-2)]*Sqrt[x^2]*ArcCsc[x]^2)/x - (4*ArcCsc[x]^3)/Sqr
t[x^2] + (Sqrt[1 - x^(-2)]*Sqrt[x^2]*ArcCsc[x]^4)/x
```

Rubi in Sympy [A] time = 11.2252, size = 107, normalized size = 1.45

$$\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\operatorname{asin}^4\left(\frac{1}{x}\right)}{x}-\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\operatorname{asin}^2\left(\frac{1}{x}\right)}{x}+\frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x}-\frac{4\sqrt{x^2}\operatorname{asin}^3\left(\frac{1}{x}\right)}{x^2}+\frac{24\sqrt{x^2}\operatorname{asin}\left(\frac{1}{x}\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(1 - 1/x**2)*sqrt(x**2)*asin(1/x)**4/x - 12*sqrt(1 - 1/x**2)*
sqrt(x**2)*asin(1/x)**2/x + 24*sqrt(1 - 1/x**2)*sqrt(x**2)/x - 4*
sqrt(x**2)*asin(1/x)**3/x**2 + 24*sqrt(x**2)*asin(1/x)/x**2
```

Mathematica [A] time = 0.0619605, size = 76, normalized size = 1.03

$$\frac{24\left(x^2-1\right)+\left(x^2-1\right)\csc^{-1}(x)^4-4\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)^3-12\left(x^2-1\right)\csc^{-1}(x)^2+24\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)}{x\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*
ArcCsc[x]^2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCs
c[x]^4)/(x*Sqrt[-1 + x^2])
```

Maple [C] time = 0.286, size = 114, normalized size = 1.5

$$\frac{(\mathrm{arccsc}\,(x))^4-12\,(\mathrm{arccsc}\,(x))^2+4\,i\,(\mathrm{arccsc}\,(x))^3+24-24\,i\mathrm{arccsc}\,(x)}{2\,x}\left(i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{(\mathrm{arccsc}\,(x))^4-12\,(\mathrm{arccsc}\,(x))^2-4\,i\,(\mathrm{arccsc}\,(x))^3+24+24\,i\mathrm{arccsc}\,(x)}{2\,x}\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x)
```

```
[Out] 1/2/(x^2-1)^(1/2)/x*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arccsc(x)^4-
12*arccsc(x)^2+4*I*arccsc(x)^3+24-24*I*arccsc(x))+1/2/(x^2-1)^(1/
2)*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arccsc(x)^4-12*arccsc(x)^2-4
*I*arccsc(x)^3+24+24*I*arccsc(x))/x
```

Maxima [A] time = 1.53464, size = 78, normalized size = 1.05

$$\frac{\sqrt{x^2-1}\,\mathrm{arccsc}\,(x)^4}{x}-12\sqrt{-\frac{1}{x^2}+1}\,\mathrm{arccsc}\,(x)^2-\frac{4\,\mathrm{arccsc}\,(x)^3}{x}+24\sqrt{-\frac{1}{x^2}+1}+\frac{24\,\mathrm{arccsc}\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4
*arccsc(x)^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

Fricas [A] time = 0.224569, size = 50, normalized size = 0.68

$$-\frac{4 \operatorname{arccsc}(x)^3 - \left(\operatorname{arccsc}(x)^4 - 12 \operatorname{arccsc}(x)^2 + 24\right)\sqrt{x^2-1} - 24 \operatorname{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2),x, algorithm="fricas")
```

```
[Out] -(4*arccsc(x)^3 - (arccsc(x)^4 - 12*arccsc(x)^2 + 24)*sqrt(x^2 -
1) - 24*arccsc(x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccsc}(x)^4}{\sqrt{x^2-1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2),x, algorithm="giac")
```

```
[Out] integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2), x)
```

3.693 $\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$

Optimal. Leaf size=133

$$\frac{x\sec^{-1}(x)^3}{8\sqrt{x^2}}-\frac{3\sqrt{x^2-1}\sec^{-1}(x)^2}{8x^2}+\frac{9x\sec^{-1}(x)}{64\sqrt{x^2}}-\frac{3\sec^{-1}(x)}{8x\sqrt{x^2}}+\frac{\sqrt{x^2-1}\,(17x^2-2)}{64x^4}-\frac{(x^2-1)^{3/2}\sec^{-1}(x)^2}{4x^4}+\frac{(x^2-1)^2\sec^{-1}(x)}{8x^3\sqrt{x^2}}$$

```
[Out] (Sqrt[-1 + x^2]*(-2 + 17*x^2))/(64*x^4) - (3*ArcSec[x])/(8*x*Sqrt
[x^2]) + (9*x*ArcSec[x])/(64*Sqrt[x^2]) + ((-1 + x^2)^2*ArcSec[x]
)/(8*x^3*Sqrt[x^2]) - (3*Sqrt[-1 + x^2]*ArcSec[x]^2)/(8*x^2) - ((
-1 + x^2)^(3/2)*ArcSec[x]^2)/(4*x^4) + (x*ArcSec[x]^3)/(8*Sqrt[x^
2])
```

Rubi [A] time = 0.308679, antiderivative size = 172, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}}+\frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}}-\frac{9\sqrt{x^2}\csc^{-1}(x)}{64x}+\frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x}-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x}-\frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (15*Sqrt[1 - x^(-2)])/(64*Sqrt[x^2]) + (1 - x^(-2))^(3/2)/(32*Sqr
t[x^2]) - (9*Sqrt[x^2]*ArcCsc[x])/(64*x) - (3*Sqrt[x^2]*ArcSec[x]
)/(8*x^3) + ((1 - x^(-2))^2*Sqrt[x^2]*ArcSec[x])/(8*x) - (3*Sqrt[
1 - x^(-2)]*ArcSec[x]^2)/(8*Sqrt[x^2]) - ((1 - x^(-2))^(3/2)*ArcS
ec[x]^2)/(4*Sqrt[x^2]) + (Sqrt[x^2]*ArcSec[x]^3)/(8*x)
```

Rubi in Sympy [A] time = 14.8492, size = 180, normalized size = 1.35

$$\frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{8x}+\frac{\sqrt{x^2}\operatorname{acos}^3\left(\frac{1}{x}\right)}{8x}-\frac{9\sqrt{x^2}\operatorname{asin}\left(\frac{1}{x}\right)}{64x}-\frac{\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}\sqrt{x^2}\operatorname{acos}^2\left(\frac{1}{x}\right)}{4x^2}+\frac{\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}\sqrt{x^2}}{32x^2}-\frac{3\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\operatorname{acos}^2\left(\frac{1}{x}\right)}{8x^2}+\frac{15\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{64x^2}-\frac{3\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

```
[Out] (1 - 1/x**2)**2*sqrt(x**2)*acos(1/x)/(8*x) + sqrt(x**2)*acos(1/x)
**3/(8*x) - 9*sqrt(x**2)*asin(1/x)/(64*x) - (1 - 1/x**2)**(3/2)*s
qrt(x**2)*acos(1/x)**2/(4*x**2) + (1 - 1/x**2)**(3/2)*sqrt(x**2)/
(32*x**2) - 3*sqrt(1 - 1/x**2)*sqrt(x**2)*acos(1/x)**2/(8*x**2) +
 15*sqrt(1 - 1/x**2)*sqrt(x**2)/(64*x**2) - 3*sqrt(x**2)*acos(1/x
)/(8*x**3)
```

Mathematica [A] time = 0.324673, size = 84, normalized size = 0.63

$$\frac{\sqrt{x^2-1}\left(32\sec^{-1}(x)^3+4\sec^{-1}(x)\left(\cos\left(4\sec^{-1}(x)\right)-16\cos\left(2\sec^{-1}(x)\right)\right)+8\sec^{-1}(x)^2\left(\sin\left(4\sec^{-1}(x)\right)-8\sin\left(2\sec^{-1}(x\right.\right.\right.}{256\sqrt{1-\frac{1}{x^2}}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x
]] + Cos[4*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] +
 8*ArcSec[x]^2*(-8*Sin[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sq
rt[1 - x^(-2)]*x)
```

Maple [C] time = 0.455, size = 327, normalized size = 2.5

$$\begin{aligned}
&\frac{x\left(\mathrm{arcsec}\left(x\right)\right)^{3}}{8}\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}}\\
&-\frac{4\,i\mathrm{arcsec}\left(x\right)+8\,\left(\mathrm{arcsec}\left(x\right)\right)^{2}-1}{512\,x^4}\left(i\sqrt{\frac{x^2-1}{x^2}}x^5-8\,i\sqrt{\frac{x^2-1}{x^2}}x^3+4\,x^4+8\,i\sqrt{\frac{x^2-1}{x^2}}x-12\,x^2+8\right)\frac{1}{\sqrt{x^2-1}}\\
&-\frac{2\,\left(\mathrm{arcsec}\left(x\right)\right)^{2}-1+2\,i\mathrm{arcsec}\left(x\right)}{16\,x^2}\left(i\sqrt{\frac{x^2-1}{x^2}}x^3-2\,i\sqrt{\frac{x^2-1}{x^2}}x+2\,x^2-2\right)\frac{1}{\sqrt{x^2-1}}\\
&+\frac{2\,\left(\mathrm{arcsec}\left(x\right)\right)^{2}-1-2\,i\mathrm{arcsec}\left(x\right)}{16\,x^2}\left(i\sqrt{\frac{x^2-1}{x^2}}x^3-2\,i\sqrt{\frac{x^2-1}{x^2}}x-2\,x^2+2\right)\frac{1}{\sqrt{x^2-1}}\\
&+\frac{-4\,i\mathrm{arcsec}\left(x\right)+8\,\left(\mathrm{arcsec}\left(x\right)\right)^{2}-1}{512\,x^4}\left(i\sqrt{\frac{x^2-1}{x^2}}x^5-8\,i\sqrt{\frac{x^2-1}{x^2}}x^3-4\,x^4+8\,i\sqrt{\frac{x^2-1}{x^2}}x+12\,x^2-8\right)\frac{1}{\sqrt{x^2-1}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x)
```

```
[Out] 1/8/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)^3-1/512/(x^2-1)
^(1/2)/x^4*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3
+4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*(4*I*arcsec(x)+8*arcse
c(x)^2-1)-1/16/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*(
(x^2-1)/x^2)^(1/2)*x+2*x^2-2)*(2*arcsec(x)^2-1+2*I*arcsec(x))+1/1
6/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^
(1/2)*x-2*x^2+2)*(2*arcsec(x)^2-1-2*I*arcsec(x))+1/512/(x^2-1)^(1
/2)/x^4*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3-4*
x^4+8*I*((x^2-1)/x^2)^(1/2)*x+12*x^2-8)*(-4*I*arcsec(x)+8*arcsec(
x)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.226313, size = 80, normalized size = 0.6

$$\frac{8\,x^4\,\mathrm{arcsec}\,(x)^3+\left(17\,x^4-40\,x^2+8\right)\,\mathrm{arcsec}\,(x)-\left(8\,\left(5\,x^2-2\right)\,\mathrm{arcsec}\,(x)^2-17\,x^2+2\right)\sqrt{x^2-1}}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(8*x^4*arcsec(x)^3 + (17*x^4 - 40*x^2 + 8)*arcsec(x) - (8*(5
*x^2 - 2)*arcsec(x)^2 - 17*x^2 + 2)*sqrt(x^2 - 1))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-1\right)^{\frac{3}{2}} \operatorname{arcsec}\left(x\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="giac")

[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$

Optimal. Leaf size=110

$$\frac{2(1-21x^2)}{27(x^2)^{3/2}}+\frac{(x^2-1)\sec^{-1}(x)^2}{3(x^2)^{3/2}}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}-\frac{4\sqrt{x^2-1}\sec^{-1}(x)}{3x}+\frac{(x^2-1)^{3/2}\sec^{-1}(x)^3}{3x^3}-\frac{2(x^2-1)^{3/2}\sec^{-1}(x)}{9x^3}$$

```
[Out] (2*(1 - 21*x^2))/(27*(x^2)^(3/2)) - (4*Sqrt[-1 + x^2]*ArcSec[x])/
(3*x) - (2*(-1 + x^2)^(3/2)*ArcSec[x])/(9*x^3) + (2*ArcSec[x]^2)/
(3*Sqrt[x^2]) + ((-1 + x^2)*ArcSec[x]^2)/(3*(x^2)^(3/2)) + ((-1 +
 x^2)^(3/2)*ArcSec[x]^3)/(3*x^3)
```

Rubi [A] time = 0.312982, antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{14}{9\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x}+\frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}-\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x}-\frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x}+\frac{2\sqrt{x^2}}{27x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] -14/(9*Sqrt[x^2]) + (2*Sqrt[x^2])/(27*x^4) - (4*Sqrt[1 - x^(-2)]*
Sqrt[x^2]*ArcSec[x])/(3*x) - (2*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcS
ec[x])/(9*x) + (2*ArcSec[x]^2)/(3*Sqrt[x^2]) + ((1 - x^(-2))*ArcS
ec[x]^2)/(3*Sqrt[x^2]) + ((1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x]^
3)/(3*x)
```

Rubi in Sympy [A] time = 14.2463, size = 153, normalized size = 1.39

$$\frac{\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}\sqrt{x^2}\operatorname{acos}^3\left(\frac{1}{x}\right)}{3x}-\frac{2\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{9x}-\frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\operatorname{acos}\left(\frac{1}{x}\right)}{3x}+\frac{\left(1-\frac{1}{x^2}\right)\sqrt{x^2}\operatorname{acos}^2\left(\frac{1}{x}\right)}{3x^2}+\frac{2\sqrt{x^2}\operatorname{acos}^2\left(\frac{1}{x}\right)}{3x^2}-\frac{14\sqrt{x^2}}{9x^2}+\frac{2\sqrt{x^2}}{27x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] (1 - 1/x**2)**(3/2)*sqrt(x**2)*acos(1/x)**3/(3*x) - 2*(1 - 1/x**2
)**(3/2)*sqrt(x**2)*acos(1/x)/(9*x) - 4*sqrt(1 - 1/x**2)*sqrt(x**
2)*acos(1/x)/(3*x) + (1 - 1/x**2)*sqrt(x**2)*acos(1/x)**2/(3*x**2
) + 2*sqrt(x**2)*acos(1/x)**2/(3*x**2) - 14*sqrt(x**2)/(9*x**2) +
 2*sqrt(x**2)/(27*x**4)
```

Mathematica [A] time = 0.0892186, size = 92, normalized size = 0.84

$$\frac{2\sqrt{1-\frac{1}{x^2}}x\left(1-21x^2\right)+9\left(x^2-1\right)^2\sec^{-1}(x)^3+9\sqrt{1-\frac{1}{x^2}}x\left(3x^2-1\right)\sec^{-1}(x)^2-6\left(7x^4-8x^2+1\right)\sec^{-1}(x)}{27x^3\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec
[x] + 9*Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2
)^2*ArcSec[x]^3)/(27*x^3*Sqrt[-1 + x^2])
```

Maple [C] time = 0.48, size = 250, normalized size = 2.3

$$\frac{9\,i\,(\mathrm{arcsec}\,(x))^2+9\,(\mathrm{arcsec}\,(x))^3-2\,i-6\,\mathrm{arcsec}\,(x)}{216\,x^3}\left(-3\,i\sqrt{\frac{x^2-1}{x^2}}x^3+x^4+4\,i\sqrt{\frac{x^2-1}{x^2}}x-5\,x^2+4\right)\frac{1}{\sqrt{x^2-1}}$$
$$+\frac{(\mathrm{arcsec}\,(x))^3-6\,\mathrm{arcsec}\,(x)+3\,i\,(\mathrm{arcsec}\,(x))^2-6\,i}{8\,x}\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}$$
$$+\frac{(\mathrm{arcsec}\,(x))^3-6\,\mathrm{arcsec}\,(x)-3\,i\,(\mathrm{arcsec}\,(x))^2+6\,i}{8\,x}\left(i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)\frac{1}{\sqrt{x^2-1}}$$
$$+\frac{-9\,i\,(\mathrm{arcsec}\,(x))^2+9\,(\mathrm{arcsec}\,(x))^3+2\,i-6\,\mathrm{arcsec}\,(x)}{216\,x^3}\left(-5\,x^2+4+3\,i\sqrt{\frac{x^2-1}{x^2}}x^3+x^4-4\,i\sqrt{\frac{x^2-1}{x^2}}x\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x)
```

```
[Out] 1/216/(x^2-1)^(1/2)/x^3*(-3*I*((x^2-1)/x^2)^(1/2)*x^3+x^4+4*I*((x
^2-1)/x^2)^(1/2)*x-5*x^2+4)*(9*I*arcsec(x)^2+9*arcsec(x)^3-2*I-6*
arcsec(x))+1/8/(x^2-1)^(1/2)/x*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(
arcsec(x)^3-6*arcsec(x)+3*I*arcsec(x)^2-6*I)+1/8/(x^2-1)^(1/2)*(I
*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(arcsec(x)^3-6*arcsec(x)-3*I*arcsec
(x)^2+6*I)/x+1/216/(x^2-1)^(1/2)/x^3*(-5*x^2+4+3*I*((x^2-1)/x^2)^
```

```
(1/2)*x^3+x^4-4*I*((x^2-1)/x^2)^(1/2)*x)*(-9*I*arcsec(x)^2+9*arcs
ec(x)^3+2*I-6*arcsec(x))
```

Maxima [A] time = 1.67185, size = 100, normalized size = 0.91

$$\frac{\left(x^2-1\right)^{\frac{3}{2}}\operatorname{arcsec}(x)^3}{3x^3}+\frac{\left(3x^2-1\right)\operatorname{arcsec}(x)^2}{3x^3}-\frac{2\left(21x^2-1\right)}{27x^3}-\frac{2\left(7x^4-8x^2+1\right)\operatorname{arcsec}(x)}{9\sqrt{x+1}\sqrt{x-1}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2
/x^3 - 2/27*(21*x^2 - 1)/x^3 - 2/9*(7*x^4 - 8*x^2 + 1)*arcsec(x)/
(sqrt(x + 1)*sqrt(x - 1)*x^3)
```

Fricas [A] time = 0.245998, size = 77, normalized size = 0.7

$$\frac{9\left(3x^2-1\right)\operatorname{arcsec}(x)^2-42x^2+3\left(3\left(x^2-1\right)\operatorname{arcsec}(x)^3-2\left(7x^2-1\right)\operatorname{arcsec}(x)\right)\sqrt{x^2-1}+2}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(
x)^3 - 2*(7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4, x)
```

3.695 $\int \sin^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$

Optimal. Leaf size=55

$$(a+x)\sin^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)-\frac{\sqrt{2}a\sqrt{\frac{x-a}{a+x}}}{\sqrt{\frac{a}{a+x}}}$$

```
[Out] -((Sqrt[2]*a*Sqrt[(-a + x)/(a + x)])/Sqrt[a/(a + x)]) + (a + x)*A
rcSin[Sqrt[(-a + x)/(a + x)]]
```

Rubi [B] time = 1.34713, antiderivative size = 118, normalized size of antiderivative = 2.15, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}(a+x)+x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{a\sqrt{\frac{a}{a+x}}\tanh^{-1}\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] -(Sqrt[2]*Sqrt[a/(a + x)]*Sqrt[-((a - x)/(a + x))]*(a + x)) + x*A
rcSin[Sqrt[-((a - x)/(a + x))]] - (a*Sqrt[a/(a + x)]*ArcTanh[Sqrt
[-((a - x)/(a + x))]/(Sqrt[2]*Sqrt[-(a/(a + x))])])/Sqrt[-(a/(a +
 x))]
```

Rubi in Sympy [A] time = 78.8106, size = 80, normalized size = 1.45

$$-\sqrt{a}\sqrt{\frac{a}{a+x}}\sqrt{a+x}\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{a+x}}\right)+x\,\mathrm{asin}\left(\sqrt{\frac{-a+x}{a+x}}\right)-\sqrt{2}\sqrt{\frac{a}{a+x}}(a+x)\sqrt{-\frac{2a}{a+x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] -sqrt(a)*sqrt(a/(a + x))*sqrt(a + x)*asin(sqrt(2)*sqrt(a)/sqrt(a
+ x)) + x*asin(sqrt((-a + x)/(a + x))) - sqrt(2)*sqrt(a/(a + x))*
(a + x)*sqrt(-2*a/(a + x) + 1)
```

Mathematica [A] time = 0.222995, size = 99, normalized size = 1.8

$$x\sin^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)+\frac{\sqrt{\frac{a}{a+x}}\left(\sqrt{2}\sqrt{a}\sqrt{x-a}\tan^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right)+2a-2x\right)}{\sqrt{2}\sqrt{\frac{x-a}{a+x}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x +
Sqrt[2]*Sqrt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a]
)]))/(Sqrt[2]*Sqrt[(-a + x)/(a + x)])
```

Maple [A] time = 0.045, size = 85, normalized size = 1.6

$$x\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{2}}{2}\sqrt{-a+x}\sqrt{\frac{a}{a+x}}\left(\sqrt{a}\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-a+x}\frac{1}{\sqrt{a}}\right)-2\sqrt{-a+x}\right)\frac{1}{\sqrt{\frac{-a+x}{a+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(((-a+x)/(a+x))^(1/2)),x)
```

```
[Out] x*arcsin(((-a+x)/(a+x))^(1/2))+1/2/((-a+x)/(a+x))^(1/2)*(-a+x)^(1
/2)*2^(1/2)*(a/(a+x))^(1/2)*(a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1
/2)*2^(1/2)/a^(1/2))-2*(-a+x)^(1/2))
```

Maxima [A] time = 1.52682, size = 139, normalized size = 2.53

$$a\left(\frac{2\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1}+\frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}+1}+\frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sqrt(-(a - x)/(a + x))),x, algorithm="maxima")
```

```
[Out] a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt(
(a - x)/(a + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/
(a + x) + 1)/(sqrt(-(a - x)/(a + x)) - 1))
```

Fricas [A] time = 0.230716, size = 69, normalized size = 1.25

$$-\sqrt{2}(a+x)\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}}+(a+x)\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sqrt(-(a - x)/(a + x))),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)
*arcsin(sqrt(-(a - x)/(a + x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sqrt(-(a - x)/(a + x))),x, algorithm="giac")
```

```
[Out] integrate(arcsin(sqrt(-(a - x)/(a + x))), x)
```

3.696 $\int \tan^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

Optimal. Leaf size=40

$$x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a
+ x))]]
```

Rubi [A] time = 0.0749986, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a
+ x))]]
```

Rubi in Sympy [A] time = 2.7696, size = 26, normalized size = 0.65

$$-a\,\text{atanh}\left(\sqrt{\frac{-a+x}{a+x}}\right)+x\,\text{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] -a*atanh(sqrt((-a + x)/(a + x))) + x*atan(sqrt((-a + x)/(a + x)))
```

Mathematica [A] time = 0.0950292, size = 75, normalized size = 1.88

$$x\tan^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)-\frac{a\sqrt{x-a}\log\left(\sqrt{x-a}\sqrt{a+x}+x\right)}{2\sqrt{\frac{x-a}{a+x}}\sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*Log[x + Sqrt[-
a + x]*Sqrt[a + x]])/(2*Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

Maple [A] time = 0.023, size = 64, normalized size = 1.6

$$x\arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)-\frac{(-a+x)\,a}{2}\ln\left(x+\sqrt{-a^2+x^2}\right)\frac{1}{\sqrt{\frac{-a+x}{a+x}}}\frac{1}{\sqrt{(a+x)\,(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(((-a+x)/(a+x))^(1/2)),x)
```

```
[Out] x*arctan(((-a+x)/(a+x))^(1/2))-1/2/((-a+x)/(a+x))^(1/2)*(-a+x)/((
a+x)*(-a+x))^(1/2)*a*ln(x+(-a^2+x^2)^(1/2))
```

Maxima [A] time = 1.50061, size = 120, normalized size = 3.

$$\frac{1}{2}\,a\left(\frac{4\,\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1}-2\,\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(-(a - x)/(a + x))),x, algorithm="maxima")
```

```
[Out] 1/2*a*(4*arctan(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) - 2
*arctan(sqrt(-(a - x)/(a + x))) - log(sqrt(-(a - x)/(a + x)) + 1)
 + log(sqrt(-(a - x)/(a + x)) - 1))
```

Fricas [A] time = 0.232585, size = 78, normalized size = 1.95

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{1}{2}\,a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\frac{1}{2}\,a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(-(a - x)/(a + x))),x, algorithm="fricas")
```

```
[Out] x*arctan(sqrt(-(a - x)/(a + x))) - 1/2*a*log(sqrt(-(a - x)/(a + x
)) + 1) + 1/2*a*log(sqrt(-(a - x)/(a + x)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.249637, size = 66, normalized size = 1.65

$$\frac{1}{2}\,a\ln\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sign}\left(a+x\right)+x\arctan\left(\frac{\sqrt{-a^2+x^2}\operatorname{sign}\left(a+x\right)}{a+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(-(a - x)/(a + x))),x, algorithm="giac")
```

```
[Out] 1/2*a*ln(abs(-x + sqrt(-a^2 + x^2)))*sign(a + x) + x*arctan(sqrt(
-a^2 + x^2)*sign(a + x)/(a + x))
```

3.697 $\int \frac{\tan^{-1}(x)}{(1+x)^3} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4(x+1)}+\frac{1}{4}\log(x+1)-\frac{\tan^{-1}(x)}{2(x+1)^2}$$

```
[Out] -1/(4*(1 + x)) - ArcTan[x]/(2*(1 + x)^2) + Log[1 + x]/4 - Log[1 +
 x^2]/8
```

Rubi [A] time = 0.0671878, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4(x+1)}+\frac{1}{4}\log(x+1)-\frac{\tan^{-1}(x)}{2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] -1/(4*(1 + x)) - ArcTan[x]/(2*(1 + x)^2) + Log[1 + x]/4 - Log[1 +
 x^2]/8
```

Rubi in Sympy [A] time = 4.49992, size = 31, normalized size = 0.79

$$\frac{\log\left(x+1\right)}{4}-\frac{\log\left(x^2+1\right)}{8}-\frac{1}{4\left(x+1\right)}-\frac{\operatorname{atan}\left(x\right)}{2\left(x+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x)/(1+x)**3,x)
```

```
[Out] log(x + 1)/4 - log(x**2 + 1)/8 - 1/(4*(x + 1)) - atan(x)/(2*(x +
1)**2)
```

Mathematica [A] time = 0.0260444, size = 35, normalized size = 0.9

$$\frac{1}{8}\left(-\log\left(x^2+1\right)-\frac{2}{x+1}+2\log(x+1)-\frac{4\tan^{-1}(x)}{(x+1)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] (-2/(1 + x) - (4*ArcTan[x])/(1 + x)^2 + 2*Log[1 + x] - Log[1 + x^
2])/8
```

Maple [A] time = 0.006, size = 32, normalized size = 0.8

$$-\frac{1}{4+4x}-\frac{\arctan(x)}{2\,(1+x)^{2}}+\frac{\ln(1+x)}{4}-\frac{\ln\left(x^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x)/(1+x)^3,x)
```

```
[Out] -1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

Maxima [A] time = 1.51731, size = 42, normalized size = 1.08

$$-\frac{1}{4\,(x+1)}-\frac{\arctan(x)}{2\,(x+1)^{2}}-\frac{1}{8}\,\log\left(x^{2}+1\right)+\frac{1}{4}\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)/(x + 1)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*l
og(x + 1)
```

Fricas [A] time = 0.225717, size = 68, normalized size = 1.74

$$-\frac{\left(x^{2}+2\,x+1\right)\log\left(x^{2}+1\right)-2\left(x^{2}+2\,x+1\right)\log(x+1)+2\,x+4\,\arctan(x)+2}{8\,(x^{2}+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)/(x + 1)^3,x, algorithm="fricas")
```

```
[Out] -1/8*((x^2 + 2*x + 1)*log(x^2 + 1) - 2*(x^2 + 2*x + 1)*log(x + 1)
 + 2*x + 4*arctan(x) + 2)/(x^2 + 2*x + 1)
```

Sympy [A] time = 1.25266, size = 153, normalized size = 3.92

$$\frac{2x^2\log(x+1)}{8x^2+16x+8}-\frac{x^2\log(x^2+1)}{8x^2+16x+8}+\frac{x^2}{8x^2+16x+8}+\frac{4x\log(x+1)}{8x^2+16x+8}-\frac{2x\log(x^2+1)}{8x^2+16x+8}+\frac{2\log(x+1)}{8x^2+16x+8}-\frac{\log(x^2+1)}{8x^2+16x+8}-\frac{4\operatorname{atan}(x)}{8x^2+16x+8}-\frac{1}{8x^2+16x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x)/(1+x)**3,x)
```

```
[Out] 2*x**2*log(x + 1)/(8*x**2 + 16*x + 8) - x**2*log(x**2 + 1)/(8*x**
2 + 16*x + 8) + x**2/(8*x**2 + 16*x + 8) + 4*x*log(x + 1)/(8*x**2
 + 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 + 16*x + 8) + 2*log(x +
1)/(8*x**2 + 16*x + 8) - log(x**2 + 1)/(8*x**2 + 16*x + 8) - 4*at
an(x)/(8*x**2 + 16*x + 8) - 1/(8*x**2 + 16*x + 8)
```

GIAC/XCAS [A] time = 0.201316, size = 43, normalized size = 1.1

$$-\frac{1}{4(x+1)}-\frac{\arctan(x)}{2(x+1)^2}-\frac{1}{8}\ln\left(x^2+1\right)+\frac{1}{4}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x)/(x + 1)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*ln(x^2 + 1) + 1/4*ln
(abs(x + 1))
```

3.698 $\int -\frac{\tan^{-1}(a-x)}{a+x}\,dx$

Optimal. Leaf size=122

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1+\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)+\log\left(\frac{2}{1-i(a-x)}\right)\tan^{-1}(a-x)-\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)\tan^{-1}(a-x)$$

```
[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a +
 x))/((I - 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I
*(a - x))] + (I/2)*PolyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(
a - x)))]
```

Rubi [A] time = 0.161205, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1+\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)+\log\left(\frac{2}{1-i(a-x)}\right)\tan^{-1}(a-x)-\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)\tan^{-1}(a-x)$$

Antiderivative was successfully verified.

```
[In]  Int[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a +
 x))/((I - 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I
*(a - x))] + (I/2)*PolyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(
a - x)))]
```

Rubi in Sympy [A] time = 10.8327, size = 80, normalized size = 0.66

$$\frac{i\log\left(\frac{a+x}{2a-i}\right)\log\left(i\left(a-x\right)+1\right)}{2}-\frac{i\log\left(\frac{a+x}{2a+i}\right)\log\left(-i\left(a-x\right)+1\right)}{2}-\frac{i\operatorname{Li}_{2}\left(\frac{i(a-x)-1}{2ia-1}\right)}{2}+\frac{i\operatorname{Li}_{2}\left(\frac{i(a-x)+1}{2ia+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-atan(a-x)/(a+x),x)
```

```
[Out] I*log((a + x)/(2*a - I))*log(I*(a - x) + 1)/2 - I*log((a + x)/(2*
a + I))*log(-I*(a - x) + 1)/2 - I*polylog(2, (I*(a - x) - 1)/(2*I
```

```
*a - 1))/2 + I*polylog(2, (I*(a - x) + 1)/(2*I*a + 1))/2
```

Mathematica [B] time = 0.339064, size = 256, normalized size = 2.1

$$-\tan^{-1}(a-x)\left(\frac{1}{2}\log\left(a^2-2ax+x^2+1\right)+\log\left(-\sin\left(\tan^{-1}(2a)-\tan^{-1}(a-x)\right)\right)\right)$$
$$+\frac{1}{2}\left(i\text{PolyLog}\left(2,-e^{-2i\tan^{-1}(a-x)}\right)+i\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(a-x)-\tan^{-1}(2a)\right)}\right)\right.$$
$$+\frac{1}{4}i\left(\pi-2\tan^{-1}(a-x)\right)^2+i\left(\tan^{-1}(2a)-\tan^{-1}(a-x)\right)^2$$
$$-\left(\pi-2\tan^{-1}(a-x)\right)\log\left(1+e^{-2i\tan^{-1}(a-x)}\right)+\log\left(\frac{2}{\sqrt{(a-x)^2+1}}\right)\left(\pi-2\tan^{-1}(a-x)\right)$$
$$-2\left(\tan^{-1}(a-x)-\tan^{-1}(2a)\right)\log\left(1-e^{2i\left(\tan^{-1}(a-x)-\tan^{-1}(2a)\right)}\right)$$
$$\left.-2\left(\tan^{-1}(2a)-\tan^{-1}(a-x)\right)\log\left(-2\sin\left(\tan^{-1}(2a)-\tan^{-1}(a-x)\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] -(ArcTan[a - x]*(Log[1 + a^2 - 2*a*x + x^2]/2 + Log[-Sin[ArcTan[2
*a] - ArcTan[a - x]]])) + ((I/4)*(Pi - 2*ArcTan[a - x])^2 + I*(Ar
cTan[2*a] - ArcTan[a - x])^2 - (Pi - 2*ArcTan[a - x])*Log[1 + E^(
(-2*I)*ArcTan[a - x])] - 2*(-ArcTan[2*a] + ArcTan[a - x])*Log[1 -
 E^((2*I)*(-ArcTan[2*a] + ArcTan[a - x]))] + (Pi - 2*ArcTan[a - x
])*Log[2/Sqrt[1 + (a - x)^2]] - 2*(ArcTan[2*a] - ArcTan[a - x])*L
og[-2*Sin[ArcTan[2*a] - ArcTan[a - x]]] + I*PolyLog[2, -E^((-2*I)
*ArcTan[a - x])] + I*PolyLog[2, E^((2*I)*(-ArcTan[2*a] + ArcTan[a
 - x]))])/2
```

Maple [C] time = 0.026, size = 102, normalized size = 0.8

$$-\ln(a+x)\arctan(a-x)+\frac{i}{2}\ln(a+x)\ln\left(\frac{a-x+i}{2a+i}\right)$$
$$-\frac{i}{2}\ln(a+x)\ln\left(\frac{-a+x+i}{i-2a}\right)+\frac{i}{2}\mathit{dilog}\left(\frac{a-x+i}{2a+i}\right)-\frac{i}{2}\mathit{dilog}\left(\frac{-a+x+i}{i-2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-arctan(a-x)/(a+x),x)
```

```
[Out] -ln(a+x)*arctan(a-x)+1/2*I*ln(a+x)*ln((a-x+I)/(2*a+I))-1/2*I*ln(a
+x)*ln((-a+x+I)/(I-2*a))+1/2*I*dilog((a-x+I)/(2*a+I))-1/2*I*dilog
```

```
((-a+x+I)/(I-2*a))
```

Maxima [A] time = 1.63018, size = 159, normalized size = 1.3

$$-\frac{1}{2}\arctan\left(\frac{a+x}{4a^2+1}, \frac{2(a^2+ax)}{4a^2+1}\right)\log\left(a^2-2ax+x^2+1\right) + \frac{1}{2}\arctan\left(-a+x\right)\log\left(\frac{a^2+2ax+x^2}{4a^2+1}\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-ia+ix+1}{2ia-1}\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-ia+ix-1}{2ia+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-arctan(a - x)/(a + x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2((a + x)/(4*a^2 + 1), 2*(a^2 + a*x)/(4*a^2 + 1))*log(
a^2 - 2*a*x + x^2 + 1) + 1/2*arctan(-a + x)*log((a^2 + 2*a*x + x^
2)/(4*a^2 + 1)) - 1/2*I*dilog(-(-I*a + I*x + 1)/(2*I*a - 1)) + 1/
2*I*dilog(-(-I*a + I*x - 1)/(2*I*a + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(-a+x\right)}{a+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-arctan(a - x)/(a + x),x, algorithm="fricas")
```

```
[Out] integral(arctan(-a + x)/(a + x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-atan(a-x)/(a+x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\arctan(a-x)}{a+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-arctan(a - x)/(a + x),x, algorithm="giac")
```

```
[Out] integrate(-arctan(a - x)/(a + x), x)
```

3.699 $\int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

```
[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Rubi [A] time = 0.0541507, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]

[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)

[Out] Exception raised: ValueError
```

Mathematica [A] time = 0.0118858, size = 28, normalized size = 1.

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]

[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int 1 \arcsin\left(\sqrt{-x^2+1}\right) \frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)

[Out] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

Maxima [A] time = 1.54754, size = 27, normalized size = 0.96

$$\arcsin\left(\sqrt{-x^2+1}\right)\arcsin\left(x\right)+\frac{1}{2}\arcsin\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1),x, algorithm="maxima")

[Out] arcsin(sqrt(-x^2 + 1))*arcsin(x) + 1/2*arcsin(x)^2
```

Fricas [A] time = 0.222461, size = 19, normalized size = 0.68

$$-\frac{1}{2}\arcsin\left(\sqrt{-x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1),x, algorithm="fricas")

[Out] -1/2*arcsin(sqrt(-x^2 + 1))^2
```

Sympy [A] time = 4.28677, size = 22, normalized size = 0.79

$$-\frac{\sqrt{x^2}\operatorname{asin}^2\left(\sqrt{-x^2+1}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(x**2)*asin(sqrt(-x**2 + 1))**2/(2*x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

3.700 $\int \frac{x \tan^{-1}\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=31

$$\sqrt{x^2+1}\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

```
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Rubi [A] time = 0.0592013, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\sqrt{x^2+1}\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]

[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Rubi in Sympy [A] time = 3.13254, size = 26, normalized size = 0.84

$$\sqrt{x^2+1}\,\text{atan}\left(\sqrt{x^2+1}\right)-\frac{\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)

[Out] sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

Mathematica [A] time = 0.0126611, size = 31, normalized size = 1.

$$\sqrt{x^2+1}\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Maple [A] time = 0.012, size = 26, normalized size = 0.8

$$-\frac{\ln\left(x^2+2\right)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

```
[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

Maxima [A] time = 1.3351, size = 34, normalized size = 1.1

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(sqrt(x^2 + 1))/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Fricas [A] time = 0.254788, size = 34, normalized size = 1.1

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(sqrt(x^2 + 1))/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Sympy [A] time = 6.25808, size = 26, normalized size = 0.84

$$\sqrt{x^2+1}\,\mathrm{atan}\left(\sqrt{x^2+1}\right)-\frac{\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

GIAC/XCAS [A] time = 0.198816, size = 34, normalized size = 1.1

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(sqrt(x^2 + 1))/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*ln(x^2 + 2)
```

3.701 $\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{x+1}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] -Sqrt[1 + x]/(3*(1 - x)) + (2*ArcSin[x])/(3*(1 - x)^(3/2)) - ArcT
anh[Sqrt[1 + x]/Sqrt[2]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0701697, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{\sqrt{x+1}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] -Sqrt[1 + x]/(3*(1 - x)) + (2*ArcSin[x])/(3*(1 - x)^(3/2)) - ArcT
anh[Sqrt[1 + x]/Sqrt[2]]/(3*Sqrt[2])
```

Rubi in Sympy [A] time = 5.25042, size = 46, normalized size = 0.81

$$-\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{6}-\frac{\sqrt{x+1}}{3\,(-x+1)}+\frac{2\,\text{asin}\,(x)}{3\,(-x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)/(1-x)**(5/2),x)
```

```
[Out] -sqrt(2)*atanh(sqrt(2)*sqrt(x + 1)/2)/6 - sqrt(x + 1)/(3*(-x + 1)
) + 2*asin(x)/(3*(-x + 1)**(3/2))
```

Mathematica [A] time = 0.136826, size = 61, normalized size = 1.07

$$\frac{1}{6}\left(-\frac{2\left(\sqrt{1-x^2}-2\sin^{-1}(x)\right)}{(1-x)^{3/2}}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] ((-2*(Sqrt[1 - x^2] - 2*ArcSin[x]))/(1 - x)^(3/2) - Sqrt[2]*ArcTa
nh[Sqrt[1 - x^2]/Sqrt[2 - 2*x]])/6
```

Maple [A] time = 0.01, size = 70, normalized size = 1.2

$$\frac{2\arcsin(x)}{3}(1-x)^{-\frac{3}{2}}-\frac{1}{6}\sqrt{1+x}\left(\sqrt{2}Artanh\left(\sqrt{2}\frac{1}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{-(1-x)^{2}+2-2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)/(1-x)^(5/2),x)
```

```
[Out] 2/3*arcsin(x)/(1-x)^(3/2)-1/6/(1-x)^(1/2)*(1+x)^(1/2)*(2^(1/2)*ar
ctanh(2^(1/2)/(1+x)^(1/2))*(1-x)+2*(1+x)^(1/2))/(-(1-x)^2+2-2*x)^
(1/2)
```

Maxima [A] time = 1.52608, size = 77, normalized size = 1.35

$$-\frac{1}{6}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{x+1}}{\sqrt{-x+1}}+\frac{4}{\sqrt{-x+1}}\right)+\frac{\sqrt{x+1}}{3(x-1)}+\frac{2\arcsin(x)}{3(-x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/(-x + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*log(2*sqrt(2)*sqrt(x + 1)/sqrt(-x + 1) + 4/sqrt(-x +
 1)) + 1/3*sqrt(x + 1)/(x - 1) + 2/3*arcsin(x)/(-x + 1)^(3/2)
```

Fricas [A] time = 0.227102, size = 135, normalized size = 2.37

$$\frac{\sqrt{2}\left((x^{2}-2x+1)\log\left(-\frac{\sqrt{2}(x^{2}+2x-3)+4\sqrt{-x^{2}+1}\sqrt{-x+1}}{x^{2}-2x+1}\right)+2\left(2\sqrt{2}\arcsin(x)-\sqrt{2}\sqrt{-x^{2}+1}\right)\sqrt{-x+1}\right)}{12(x^{2}-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/(-x + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*((x^2 - 2*x + 1)*log(-(sqrt(2)*(x^2 + 2*x - 3) + 4*s
qrt(-x^2 + 1)*sqrt(-x + 1))/(x^2 - 2*x + 1)) + 2*(2*sqrt(2)*arcsi
n(x) - sqrt(2)*sqrt(-x^2 + 1))*sqrt(-x + 1))/(x^2 - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asin}(x)}{(-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)/(1-x)**(5/2),x)
```

```
[Out] Integral(asin(x)/(-x + 1)**(5/2), x)
```

GIAC/XCAS [A] time = 0.216891, size = 78, normalized size = 1.37

$$\frac{1}{12}\sqrt{2}\ln\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+\frac{\sqrt{x+1}}{3(x-1)}-\frac{2\arcsin(x)}{3(x-1)\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/(-x + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*ln((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1)))
+ 1/3*sqrt(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))
```

3.702 $\int (-1+x)^{5/2} \csc^{-1}(x)\,dx$

Optimal. Leaf size=82

$$\frac{4x\sqrt{x^2-1}\left(3x^2-19x+83\right)}{105\sqrt{x^2}\sqrt{x-1}}+\frac{4x\tanh^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{x-1}}\right)}{7\sqrt{x^2}}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

```
[Out] (4*x*Sqrt[-1 + x^2]*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]*Sqrt[x
^2]) + (2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*x*ArcTanh[Sqrt[-1 + x^
2]/Sqrt[-1 + x]])/(7*Sqrt[x^2])
```

Rubi [A] time = 0.21222, antiderivative size = 140, normalized size of antiderivative = 1.71, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{4(x+1)^3\sqrt{x-1}}{35\sqrt{1-\frac{1}{x^2}}x}-\frac{20(x+1)^2\sqrt{x-1}}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4(x+1)\sqrt{x-1}}{\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{x+1}\sqrt{x-1}\tanh^{-1}\left(\sqrt{x+1}\right)}{7\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[-1 + x]*(1 + x))/(Sqrt[1 - x^(-2)]*x) - (20*Sqrt[-1 + x]*
(1 + x)^2)/(21*Sqrt[1 - x^(-2)]*x) + (4*Sqrt[-1 + x]*(1 + x)^3)/(
35*Sqrt[1 - x^(-2)]*x) + (2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*Sqrt
[-1 + x]*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]])/(7*Sqrt[1 - x^(-2)]*x)
```

Rubi in Sympy [A] time = 5.36338, size = 143, normalized size = 1.74

$$\frac{4x(x+1)^2\sqrt{x^2-1}}{35\sqrt{x-1}\sqrt{x^2}}-\frac{20x(x+1)\sqrt{x^2-1}}{21\sqrt{x-1}\sqrt{x^2}}+\frac{4x\sqrt{x^2-1}}{\sqrt{x-1}\sqrt{x^2}}+\frac{4x\sqrt{x^2-1}\,\mathrm{atanh}\left(\sqrt{x+1}\right)}{7\sqrt{x-1}\sqrt{x+1}\sqrt{x^2}}+\frac{2(x-1)^{\frac{7}{2}}\,\mathrm{acsc}(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)**(5/2)*acsc(x),x)
```

```
[Out] 4*x*(x + 1)**2*sqrt(x**2 - 1)/(35*sqrt(x - 1)*sqrt(x**2)) - 20*x*
(x + 1)*sqrt(x**2 - 1)/(21*sqrt(x - 1)*sqrt(x**2)) + 4*x*sqrt(x**
2 - 1)/(sqrt(x - 1)*sqrt(x**2)) + 4*x*sqrt(x**2 - 1)*atanh(sqrt(x
```

```
 + 1))/(7*sqrt(x - 1)*sqrt(x + 1)*sqrt(x**2)) + 2*(x - 1)**(7/2)*
acsc(x)/7
```

Mathematica [A] time = 0.105234, size = 72, normalized size = 0.88

$$\frac{4\sqrt{1-\frac{1}{x^2}}x\left(3x^2-19x+83\right)}{105\sqrt{x-1}}+\frac{4}{7}\tanh^{-1}\left(\frac{\sqrt{1-\frac{1}{x^2}}x}{\sqrt{x-1}}\right)+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (
2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/S
qrt[-1 + x]])/7
```

Maple [A] time = 0.024, size = 76, normalized size = 0.9

$$\frac{2\,\mathrm{arccsc}\,(x)}{7}(-1+x)^{\frac{7}{2}}+\frac{4}{105\,x}\sqrt{-1+x}\sqrt{1+x}\left(3\,(-1+x)^2\sqrt{1+x}-13\,(-1+x)\sqrt{1+x}+15\,\mathit{Artanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)\frac{1}{\sqrt{\frac{(-1+x)(1+x)}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)^(5/2)*arccsc(x),x)
```

```
[Out] 2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+
x)^2*(1+x)^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67
*(1+x)^(1/2))/((-1+x)*(1+x)/x^2)^(1/2)/x
```

Maxima [A] time = 1.59334, size = 69, normalized size = 0.84

$$\frac{2}{7}(x-1)^{\frac{7}{2}}\,\mathrm{arccsc}\,(x)+\frac{4}{35}(x+1)^{\frac{5}{2}}-\frac{20}{21}(x+1)^{\frac{3}{2}}+4\sqrt{x+1}+\frac{2}{7}\log\left(\sqrt{x+1}+1\right)-\frac{2}{7}\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^(5/2)*arccsc(x),x, algorithm="maxima")
```

```
[Out] 2/7*(x - 1)^(7/2)*arccsc(x) + 4/35*(x + 1)^(5/2) - 20/21*(x + 1)^
(3/2) + 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x
 + 1) - 1)
```

Fricas [A] time = 0.234169, size = 198, normalized size = 2.41

$$\frac{2\left(6\,x^{4}-38\,x^{3}+15\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)\sqrt{x^{2}-1}\operatorname{arccsc}\left(x\right)+160\,x^{2}+15\,\sqrt{x^{2}-1}\sqrt{x-1}\log\left(\frac{x^{2}+\sqrt{x^{2}-1}\sqrt{x-1}-1}{x^{2}-1}\right)-15}{105\,\sqrt{x^{2}-1}\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^(5/2)*arccsc(x),x, algorithm="fricas")
```

```
[Out] 2/105*(6*x^4 - 38*x^3 + 15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x
^2 - 1)*arccsc(x) + 160*x^2 + 15*sqrt(x^2 - 1)*sqrt(x - 1)*log((x
^2 + sqrt(x^2 - 1)*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*sqrt(x^2 - 1)
*sqrt(x - 1)*log(-(x^2 - sqrt(x^2 - 1)*sqrt(x - 1) - 1)/(x^2 - 1)
) + 38*x - 166)/(sqrt(x^2 - 1)*sqrt(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)**(5/2)*acsc(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.308175, size = 266, normalized size = 3.24

$$\frac{2}{3}\left(x-1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{x}\right)+\frac{2}{105}\left(15\left(x-1\right)^{\frac{7}{2}}+42\left(x-1\right)^{\frac{5}{2}}+35\left(x-1\right)^{\frac{3}{2}}\right)\arcsin\left(\frac{1}{x}\right)$$
$$-\frac{4}{15}\left(3\left(x-1\right)^{\frac{5}{2}}+5\left(x-1\right)^{\frac{3}{2}}\right)\arcsin\left(\frac{1}{x}\right)+\frac{4\left(3\left(x+1\right)^{\frac{5}{2}}-11\left(x+1\right)^{\frac{3}{2}}+14\sqrt{x+1}\right)}{105\operatorname{sign}\left(\left(x-1\right)^{\frac{3}{2}}+\sqrt{x-1}\right)}$$
$$-\frac{8\left(\left(x+1\right)^{\frac{3}{2}}-4\sqrt{x+1}\right)}{15\operatorname{sign}\left(\left(x-1\right)^{\frac{3}{2}}+\sqrt{x-1}\right)}+\frac{2\ln\left(\sqrt{x+1}+1\right)}{7\operatorname{sign}\left(\left(x-1\right)^{\frac{3}{2}}+\sqrt{x-1}\right)}$$
$$-\frac{2\ln\left(\sqrt{x+1}-1\right)}{7\operatorname{sign}\left(\left(x-1\right)^{\frac{3}{2}}+\sqrt{x-1}\right)}+\frac{4\sqrt{x+1}}{3\operatorname{sign}\left(\left(x-1\right)^{\frac{3}{2}}+\sqrt{x-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^(5/2)*arccsc(x),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2)*arcsin(1/x) + 2/105*(15*(x - 1)^(7/2) + 42*(x -
 1)^(5/2) + 35*(x - 1)^(3/2))*arcsin(1/x) - 4/15*(3*(x - 1)^(5/2)
 + 5*(x - 1)^(3/2))*arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 11*(x
+ 1)^(3/2) + 14*sqrt(x + 1))/sign((x - 1)^(3/2) + sqrt(x - 1)) -
8/15*((x + 1)^(3/2) - 4*sqrt(x + 1))/sign((x - 1)^(3/2) + sqrt(x
- 1)) + 2/7*ln(sqrt(x + 1) + 1)/sign((x - 1)^(3/2) + sqrt(x - 1))
 - 2/7*ln(sqrt(x + 1) - 1)/sign((x - 1)^(3/2) + sqrt(x - 1)) + 4/
3*sqrt(x + 1)/sign((x - 1)^(3/2) + sqrt(x - 1))
```

3.703 $\int \sin^{-1}(\sinh(x))\mathbf{sech}^4(x)\,dx$

Optimal. Leaf size=49

$$-\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right)+\frac{1}{6}\sqrt{1-\sinh^2(x)}\,\text{sech}(x)-\frac{1}{3}\tanh^3(x)\sin^{-1}(\sinh(x))+\tanh(x)\sin^{-1}(\sinh(x))$$

```
[Out] (-2*ArcSin[Cosh[x]/Sqrt[2]])/3 + (Sech[x]*Sqrt[1 - Sinh[x]^2])/6
+ ArcSin[Sinh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

Rubi [A] time = 0.234634, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$

$$\frac{1}{6}\sqrt{2-\cosh^2(x)}\,\text{sech}(x)-\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right)-\frac{1}{3}\tanh^3(x)\sin^{-1}(\sinh(x))+\tanh(x)\sin^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] (-2*ArcSin[Cosh[x]/Sqrt[2]])/3 + (Sqrt[2 - Cosh[x]^2]*Sech[x])/6
+ ArcSin[Sinh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(sinh(x))*sech(x)**4,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.32502, size = 66, normalized size = 1.35

$$\frac{1}{12}\Big(8i\log\left(\sqrt{3-\cosh(2x)}+i\sqrt{2}\cosh(x)\right)+\sqrt{6-2\cosh(2x)}\,\text{sech}(x)$$
$$+4(\cosh(2x)+2)\tanh(x)\text{sech}^2(x)\sin^{-1}(\sinh(x))\Big)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] ((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]] + Sqrt[6 - 2*
Cosh[2*x]]*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*
Tanh[x])/12
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \arcsin\left(\sinh\left(x\right)\right)\left(\operatorname{sech}\left(x\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(sinh(x))*sech(x)^4,x)
```

```
[Out] int(arcsin(sinh(x))*sech(x)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(3\,e^{(2\,x)}+1\right)\arctan\left(e^{(2\,x)}-1,\sqrt{e^{(2\,x)}+2\,e^{x}-1}\sqrt{-e^{(2\,x)}+2\,e^{x}+1}\right)+16\left(e^{(6\,x)}+3\,e^{(4\,x)}+3\,e^{(2\,x)}+1\right)\int-\frac{}{\left(e^{(8\,x)}-4\,e^{(6\,x)}-10\right.}}{3\left(e^{(6\,x)}+3\,e^{(4\,x)}+3\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x
 - 1)*sqrt(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^
(2*x) + 1)*integrate(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x
) + 2*e^x - 1) + 1/2*log(-e^(2*x) + 2*e^x + 1))/((e^(8*x) - 4*e^(
6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e^(log(e^(2*x) + 2*e^x - 1) +
log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(10*x) - e^(8*x) + 12
*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*e^(4*x) + 3
*e^(2*x) + 1)
```

Fricas [A] time = 0.248265, size = 566, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*
cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*
sinh(x) + 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)) + 4*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 +
 sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*c
osh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 +
1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x)
)*sinh(x) + 1)*arctan(1/2*sqrt(2)*(3*cosh(x)^2 + 6*cosh(x)*sinh(x
) + 3*sinh(x)^2 - 1)/sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))) - 8*(3*cosh(x)^2 + 6*cosh(x)*
sinh(x) + 3*sinh(x)^2 + 1)*arctan(1/2*sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))))/(cosh(x)^6 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 +
6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(
x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(sinh(x))*sech(x)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \arcsin(\sinh(x))\,\mathrm{sech}(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="giac")
```

```
[Out] integrate(arcsin(sinh(x))*sech(x)^4, x)
```

3.704 $\int \cot^{-1}(\cosh(x)) \coth(x) \mathbf{csch}^3(x)\, dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x
]]*Csch[x]^3)/3
```

Rubi [A] time = 0.195529, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x
]]*Csch[x]^3)/3
```

Rubi in Sympy [A] time = 25.8673, size = 36, normalized size = 1.

$$\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\tanh(x)}{2}\right)}{12}+\frac{1}{6\tanh(x)}-\frac{\text{acot}(\cosh(x))}{3\sinh^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

```
[Out] sqrt(2)*atanh(sqrt(2)*tanh(x)/2)/12 + 1/(6*tanh(x)) - acot(cosh(x
))/(3*sinh(x)**3)
```

Mathematica [C] time = 0.238485, size = 144, normalized size = 4.

$$\frac{1}{48}\operatorname{csch}^3(x)\left(-2\cosh(x)+2\cosh(3x)-16\cot^{-1}(\cosh(x)) - 3i\sqrt{2}\sinh(x)\tan^{-1}\left(1-i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)+3i\sqrt{2}\sinh(x)\tan^{-1}\left(1+i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right) + i\sqrt{2}\sinh(3x)\tan^{-1}\left(1-i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)-i\sqrt{2}\sinh(3x)\tan^{-1}\left(1+i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] (Csch[x]^3*(-16*ArcCot[Cosh[x]] - 2*Cosh[x] + 2*Cosh[3*x] - (3*I)
*Sqrt[2]*ArcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[x] + (3*I)*Sqrt[2]*
ArcTan[1 + I*Sqrt[2]*Tanh[x/2]]*Sinh[x] + I*Sqrt[2]*ArcTan[1 - I*
Sqrt[2]*Tanh[x/2]]*Sinh[3*x] - I*Sqrt[2]*ArcTan[1 + I*Sqrt[2]*Tan
h[x/2]]*Sinh[3*x]))/48
```

Maple [C] time = 0.556, size = 830, normalized size = 23.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccot(cosh(x))*cosh(x)/sinh(x)^4,x)
```

```
[Out] 4/3*I*exp(3*x)/(-1+exp(2*x))^3*ln(exp(2*x)+1+2*I*exp(x))-1/24*(-8
-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1-2*I*exp(x)))*csgn(exp(-x)*(exp(
2*x)+1-2*I*exp(x)))^2*exp(3*x)-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2
*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))*exp(3*x)+16*Pi*
csgn(I*exp(-x)*(exp(2*x)+1-2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1-
2*I*exp(x)))*exp(3*x)-16*Pi*csgn(I*(exp(2*x)+1-2*I*exp(x)))*csgn(
I*exp(-x)*(exp(2*x)+1-2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*(exp(2
*x)+1+2*I*exp(x)))*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(
3*x)+16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(
exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x))*csgn(I*e
xp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)-16*Pi*csgn(I*exp(-x))*
csgn(I*exp(-x)*(exp(2*x)+1-2*I*exp(x)))^2*exp(3*x)-16*Pi*csgn(I*e
xp(-x))*csgn(I*(exp(2*x)+1+2*I*exp(x)))*csgn(I*exp(-x)*(exp(2*x)+
1+2*I*exp(x)))*exp(3*x)+16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)
))^2*exp(3*x)-16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3
*x)-16*Pi*csgn(exp(-x)*(exp(2*x)+1-2*I*exp(x)))^3*exp(3*x)+16*Pi*
csgn(exp(-x)*(exp(2*x)+1-2*I*exp(x)))^2*exp(3*x)+32*I*exp(3*x)*ln
(exp(2*x)+1-2*I*exp(x))-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(
x)))^3*exp(3*x)-8*exp(4*x)-2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)+2^(
1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)+16*Pi*csgn(I*exp(-x))*csgn(I*(exp
(2*x)+1-2*I*exp(x)))*csgn(I*exp(-x)*(exp(2*x)+1-2*I*exp(x)))*exp(
3*x)+16*exp(2*x)-ln(exp(2*x)+(2^(1/2)-1)^2)*2^(1/2)*exp(6*x)+ln(e
xp(2*x)+(1+2^(1/2))^2)*2^(1/2)*exp(6*x)+3*ln(exp(2*x)+(2^(1/2)-1)
```

```
^2)*2^(1/2)*exp(4*x)-3*ln(exp(2*x)+(1+2^(1/2))^2)*2^(1/2)*exp(4*x
)-3*ln(exp(2*x)+(2^(1/2)-1)^2)*2^(1/2)*exp(2*x)+3*ln(exp(2*x)+(1+
2^(1/2))^2)*2^(1/2)*exp(2*x)+16*Pi*csgn(I*exp(-x)*(exp(2*x)+1-2*I
*exp(x)))^3*exp(3*x))/(-1+exp(2*x))^3
```

Maxima [A] time = 1.52274, size = 73, normalized size = 2.03

$$-\frac{1}{24}\sqrt{2}\log\left(-\frac{2\left(2\sqrt{2}-e^{(-2x)}-3\right)}{4\sqrt{2}+2e^{(-2x)}+6}\right)-\frac{1}{3\left(e^{(-2x)}-1\right)}-\frac{\operatorname{arccot}(\cosh(x))}{3\sinh(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(-2*(2*sqrt(2) - e^(-2*x) - 3)/((4*sqrt(2)) + 2*
e^(-2*x) + 6)) - 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)
^3
```

Fricas [A] time = 0.230645, size = 629, normalized size = 17.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(2)*cosh(x)^4 + 16*sqrt(2)*cosh(x)*sinh(x)^3 + 4*sqrt
(2)*sinh(x)^4 + 8*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 8*s
qrt(2)*cosh(x)^2 - 32*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*si
nh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*arctan(2
*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)) + (cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x
)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*si
nh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)
^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)*log((3*(3
*sqrt(2) - 4)*cosh(x)^2 - 8*(2*sqrt(2) - 3)*cosh(x)*sinh(x) + 3*(
3*sqrt(2) - 4)*sinh(x)^2 + 3*sqrt(2) - 4)/(cosh(x)^2 + sinh(x)^2
+ 3)) + 16*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + 4*sqrt
(2))/(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*s
inh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2
)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)
^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh
(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) - sqrt(2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.259781, size = 95, normalized size = 2.64

$$\frac{1}{24}\sqrt{2}\ln\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+\frac{1}{3\left(e^{(2x)}-1\right)}+\frac{8\arctan\left(\frac{2}{e^{(-x)}+e^{x}}\right)}{3\left(e^{(-x)}-e^{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*ln(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) +
 3)) + 1/3/(e^(2*x) - 1) + 8/3*arctan(2/(e^(-x) + e^x))/(e^(-x) -
 e^x)^3
```

3.705 $\int e^x \sin^{-1}(\tanh(x))\,dx$

Optimal. Leaf size=28

$$e^x \sin^{-1}(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

```
[Out] E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

Rubi [B] time = 0.0870904, antiderivative size = 67, normalized size of antiderivative = 2.39, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$

$$-e^{-x}\sqrt{\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\left(e^{2x}+1\right)\log\left(e^{2x}+1\right) - e^x \sin^{-1}\left(\frac{1-e^{2x}}{e^{2x}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Int[E^x*ArcSin[Tanh[x]],x]
```

```
[Out] -(E^x*ArcSin[(1 - E^(2*x))/(1 + E^(2*x))]) - (Sqrt[E^(2*x)/(1 + E
^(2*x))^2]*(1 + E^(2*x))*Log[1 + E^(2*x)])/E^x
```

Rubi in Sympy [A] time = 9.42831, size = 42, normalized size = 1.5

$$-\sqrt{\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\left(e^{2x}+1\right)e^{-x}\log\left(e^{2x}+1\right) + e^x \operatorname{asin}(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*asin(tanh(x)),x)
```

```
[Out] -sqrt(exp(2*x)/(exp(2*x) + 1)**2)*(exp(2*x) + 1)*exp(-x)*log(exp(
2*x) + 1) + exp(x)*asin(tanh(x))
```

Mathematica [B] time = 0.851244, size = 64, normalized size = 2.29

$$e^x \sin^{-1}\left(\frac{e^{2x}-1}{e^{2x}+1}\right) - e^{-x}\sqrt{\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\left(e^{2x}+1\right)\log\left(e^{2x}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[E^x*ArcSin[Tanh[x]],x]
```

```
[Out] E^x*ArcSin[(-1 + E^(2*x))/(1 + E^(2*x))] - (Sqrt[E^(2*x)/(1 + E^(
2*x))^2]*(1 + E^(2*x))*Log[1 + E^(2*x)])/E^x
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int e^x \arcsin(\tanh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*arcsin(tanh(x)),x)
```

```
[Out] int(exp(x)*arcsin(tanh(x)),x)
```

Maxima [A] time = 1.63831, size = 22, normalized size = 0.79

$$\arcsin(\tanh(x))e^x - \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(tanh(x))*e^x,x, algorithm="maxima")
```

```
[Out] arcsin(tanh(x))*e^x - log(e^(2*x) + 1)
```

Fricas [A] time = 0.227128, size = 35, normalized size = 1.25

$$(\cosh(x) + \sinh(x))\arctan(\sinh(x)) - \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(tanh(x))*e^x,x, algorithm="fricas")
```

```
[Out] (cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - si
nh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{asin}\left(\tanh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*asin(tanh(x)),x)
```

```
[Out] Integral(exp(x)*asin(tanh(x)), x)
```

GIAC/XCAS [A] time = 0.203641, size = 39, normalized size = 1.39

$$\arcsin\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)e^{x}-\ln\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(tanh(x))*e^x,x, algorithm="giac")
```

```
[Out] arcsin((e^(2*x) - 1)/(e^(2*x) + 1))*e^x - ln(e^(2*x) + 1)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```